New Monetarist Economics: Models[*]

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Abstract

The purpose of this paper is to discuss some of the models used in New Monetarist Economics, which is our label for a body of recent work on money, banking, payments systems, asset markets, and related topics. A key principle in New Monetarism is that solid microfoundations are critical for understanding monetary issues. We survey recent papers on monetary theory, showing how they build on common foundations. We then lay out a tractable benchmark version of the model that allows us to address a variety of issues. We use it to analyze some classic economic topics, like the welfare effects of inflation, the relationship between money and capital accumulation, and the Phillips curve. We also extend the benchmark model in new ways, and show how it can be used to generate new insights in the study of payments, banking, and asset markets.

1 Introduction

Our goal is to present some models in current use, plus work in progress, in a distinct school of thought in monetary economics. Any school needs a name, and we call ours *New Monetarist Economics*. A key principle in New Monetarism is that we need solid microfoundations for institutions that facilitate the process of exchange – institutions like money, banks, financial intermediaries more generally, and so on – if we are to make progress in monetary economics. That this view is not universally accepted is clear from the fact that many currently popular models used for monetary policy analysis either have no money (or banks or related institutions), or if they do, they slip it in with ad hoc approaches by assuming a cash-in-advance constraint or by putting money in utility or production functions (some even resort to putting government bonds and commercial bank reserves in utility or production functions). We do not go far into methodology or history of thought here, but we will say this by way of explaining our name. New Monetarists find much that is appealing in Old Monetarism, epitomized by the writings of Friedman and his followers, although we also disagree with them in several important ways. And New Monetarists have little in common with Old or New Keynesians, although this may have as much to do with the way they approach monetary economics and microfoundations generally as with sticky prices. An extended discussion of these issues has been relegated to a companion paper.[1]

New Monetarism encompasses a body of research on monetary theory and policy, and on banking, financial intermediation, payments, and asset markets, that has occurred over the last few decades. In monetary economics, this includes the seminal work using overlapping generations models by Lucas (1972) and some of the contributors to the

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[1] In "New Monetarist Economics: Methods" (Williamson and Wright 2010) we lay out what we think are the unifying principles of New Monetarism, and indicate where and why it differs from Old Monetarism, and New or Old Keynesianism. We also argue that the New Keynesian consensus that some people think characterizes the current state of affairs, at least among more policy-oriented monetary and macro economists, is not healthy. The Old Keynesians had Old Monetarists continuously engaging them in debate over issues and models. We think the current situation would be healthier if there was more discussion and appreciation of alternatives to textbook New Keynesianism. This is one of the reasons we were interested in writing this essay. A discussion along these lines was originally meant to be included here, but to keep the Handbook chapter focused, on the advise of the editors, we moved that material to the companion paper.

Models of Monetary Economies volume edited by Kareken and Wallace (1980), although antecedents exist, including Samuelson (1958), of course. More recently, much monetary theory has adopted the search and matching approach, early examples of which are Kiyotaki and Wright (1989, 1993), although there are also antecedents for this, including Jones (1976) and Diamond (1982, 1984). In the economics of banking, intermediation, and payments, which builds on advances in information theory that occurred mainly in the 1970s, examples of what we have in mind include Diamond and Dybvig (1983), Diamond (1984), Williamson (1986, 1987), Bernanke and Gertler (1989), and Freeman (1996). Much of this research is abstract and theoretical in nature, but the literature has turned more recently to empirical and policy issues.

A key principle, laid out first in the introduction to Kareken and Wallace (1980), and elaborated in Wallace (1998), is that progress can be made in monetary theory and policy analysis only by modeling monetary arrangements explicitly. In line with the arguments of Lucas (1976), to conduct a policy experiment in an economic model, the model must be structurally invariant to the experiment under consideration. One interpretation is the following: if we are considering experiments involving the operating characteristics of the economy under different monetary policy rules, we need a model in which economic agents hold money not because it enters utility or production functions, in a reduced-form fashion, but because money ameliorates some fundamental frictions. Of course the view that monetary theory should "look frictions in the face" goes back to Hicks (1935). Notice that here we are talking about explicit descriptions of frictions *in the exchange process*, as opposed to frictions *in the price setting process*, like the nominal rigidities in Keynesian theory, where money does not help (it is really the cause of the problem).

We now know that there are various ways to explicitly model frictions. There are many important frictions to consider in monetary and financial economics, including private information, limited commitment, and spatial separation, and this potentially makes the modeling difficult. There is an element of art and skill in capturing key

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frictions while allowing for tractability. Overlapping generations models can be simple, although one can also complicate them as one likes. Much research in monetary theory in the last 20 years, as mentioned above, has been conducted using matching models, building on ideas in search and game theory.[2] Matching models are very tractable for many questions in monetary economics, though a key insight that eventually arose from this literature is that spatial separation per se is not the critical friction making money essential. As emphasized by Kocherlakota (1998), with credit due to earlier work by Ostroy (see Ostroy and Starr 1990) and Townsend (1987, 1989), money is essential because it overcomes a double coincidence of wants problem in the context of limited commitment and imperfect record keeping. Perfect record keeping would imply that efficient allocations can be supported through insurance and credit markets, or various other institutions, without money. Random bilateral matching among a large number of agents is a convenient way to generate a double coincidence problem, and to motivate incomplete record keeping, but it is not the only way, as we discuss.

While it is important to understand the above issues, New Monetarism is not just about the role of currency in the exchange process. It also attempts to study a host of related institutions. An important departure from Old Monetarism is to take seriously the role of financial intermediaries and their interactions with the central bank. Developments in intermediation and payment theories over the last 25 years are critical to our understanding of credit and banking arrangements. By way of example, a difference between Old and New Monetarists regarding the role of intermediation is reflected in their respective evaluations of Friedman's (1960) proposal for 100% reserve requirements on transactions deposits. His argument was based on the premise that tight control of the money supply by the central bank was key to controlling the price level. Since transactions deposits at banks are part of what he means by money, and the money multiplier is subject to randomness, even if we could perfectly control the stock of outside money,

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[2] Individual contributions to the search and matching literature will be discussed in detail below. The previous Handbook or Monetary Economics has a survey by Ostroy and Starr (1990) of earlier attempts at building microfoundations for money using mainly general equilibrium theory, as well as a survey of overlapping generations models by Brock (1990).

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inside money would move around unless we impose 100% reserves. Old Monetarists therefore viewed 100% reserves as desirable. What this ignores is that banks perform a socially beneficial function in transforming illiquid assets into liquid liabilities, and 100% reserve requirements inefficiently preclude this activity.

The 1980s saw important developments in the theory of banking and financial intermediation. One influential contribution was the model of Diamond and Dybvig (1983), which we now understand to be a useful approach to studying banking as liquidity transformation and insurance (it does however require some auxiliary assumptions to produce anything resembling a banking panic or run; see Ennis and Keister 2008). Other work involved well-diversified intermediaries economizing on monitoring costs, including Diamond (1984) and Williamson (1986). In these models, financial intermediation is an endogenous phenomenon. The resulting intermediaries are well-diversified, process information in some manner, and transform assets in terms of liquidity, maturity, or other characteristics. The theory of financial intermediation has also been useful in helping us understand the potential for instability in banking and the financial system (Ennis and Keister 2008), and how the structure of intermediation and financial contracting can affect aggregate shocks (Williamson 1987, Bernanke and Gertler 1989).

A relatively new sub-branch of this theory studies the economics of payments. This involves the study of payments systems, particularly among financial institutions, such as Fedwire in the US, where central banks can play an important role. Freeman (1996) provides an early contribution, and Nosal and Rocheteau (2009) provide a recent survey. The key insights from this literature are related to the role played by outside money and central bank credit in the clearing and settlement of debt, and the potential for systemic risk as a result of intraday credit. Even while payment systems are working well, this area is important, since the cost of failure is potentially big, given the volume of payments processed through such systems each day. New Monetarist economics not only has something to say about these issues, it is almost by definition the only approach that does. How can one hope to understand payments and settlement without explicitly

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modeling the exchange process?

Our objective is to explain the kinds of models people are using to study these issues. As an overview, what we do is this. First we survey the papers on monetary theory with microfoundations building on matching theory, showing how several models that are apparently different actually build on common foundations. Indeed, they can all be considered special cases of a general specification. We then lay out a benchmark version of the model that is very tractable, but still allows us to address a variety of important issues. We show how it can be used to analyze classic economic topics, like the welfare effects of inflation, the relationship between money and capital accumulation, and the short- and long-run Phillips curve. We then extend the benchmark model in some new ways, and show through a series of applications how it can be used to generate new insights in the study of payments, banking, and asset markets.

To go into more detail, in Section 2 we start with models of monetary economies that are very simple because of the assumption that money, and sometimes also goods, are indivisible. We try to say why the models are interesting, and why they were constructed as they were – what lies behind the abstractions and simplifications. In Section 3 we move to more recent models, with divisible money. These models are better suited to address many empirical and policy issues, but are still tractable enough to deliver sharp analytic results. We lay out a benchmark New Monetarist model, based on Lagos and Wright (2005), and show how it can be used to address various issues. Again, we explain what lies behind the assumptions, and we discuss some of its basic properties (e.g. money is neutral but not superneutral, the Friedman rule is typically optimal, but may not yield the first best, etc.). We also show how this benchmark can be extended to incorporate capital accumulation, unemployment, and other phenomena. As one example, we generate a traditional Phillips curve – a negative relation between inflation and unemployment – that is structurally stable in the long run. In this example, anticipated policy *can* exploit this trade-off, but it *ought not*: the Friedman rule is still optimal. This illustrates the value of being explicit about micro details.

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While much of the material in Sections 2 and 3 is already in the literature, Section 4 presents novel applications. First, we show how the benchmark model can be used to formalize Friedman's (1968) view about the short-run Phillips curve, using a signal extraction problem as in Lucas (1972). This yields some conclusions that are similar to those of Friedman and Lucas, but also some that are different. We then use the model to illustrate New Keynesian ideas by introducing sticky prices. This generates policy conclusions similar to those in Clarida et al. (1999) or Woodford (2003), but there are also differences, again illustrating how details matter. In addition, we present a New Monetarist model of *endogenously* sticky prices, with some very different policy implications. Although some of the applications in this Section re-derive known results, in a different context, they also serve to make it clear that other approaches are not inconsistent with our model. One should not shy away from New Monetarism even if one believes sticky prices, imperfect information, and related ingredients are critical, since these are relatively easily incorporated into micro-based theories of the exchange process.[3]

In Section 5, we discuss applications related to banking and payments. These extensions contain more novel modeling choices and results, although the substantive issues have been raised in earlier work. One example incorporates ideas from payments economics similar in spirit to Freeman (1996), but the analysis looks different through the lens of the New Monetarist approach. Another example incorporates existing ideas in the theory of banking emulating from Diamond and Dybvig (1983), but again the details look different. In particular, we have genuinely monetary versions of these models, which seems relevant, or at least realistic, since money plays a big role in actual banking and

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[3] Since part of our mandate from the editors was to illustrate how standard results in other literatures can be recast in the context of modern monetary theory, we thought is would be good to discuss topics such as the relationship between money and capital, the long- and short-run Phillips curve, signal extraction, and sticky prices. But our New Keynsian application should not be read as condonation of the practice of assuming nominal rigidities in an ad hoc fashion. It is rather meant to show that even if one can't live without such assumptions, this does not mean one cannot think seriously about money, banking etc. Also, our examples are meant to be simple, but one can elaborate as one wishes. Craig and Rocheteau (2007) e.g. have a version of our benchmark model with sticky prices as in Benabou (1988) and Diamond (1993), while Aruoba and Schorfheide (2009) have a version on par with a typical New Keynesian model that they estimate. Similarly, Faig and Li (2009) have a more involved version with signal extraction that they take to data. The goal here is mainly to illustrate basic qualitative effects, although in various places we discuss aspects of calibration and report some quantitative results.

payments systems (previous attempts to build monetary versions of Diamond-Dybvig include Freeman 1988 and Champ et al. 1996). In Section 6, we present another application, exploring a New Monetarist approach to asset markets. This approach emphasizes liquidity, and studies markets where asset trade can be complicated by various frictions.

We think these applications illustrate the power and flexibility of the New Monetarist approach. As we hope readers will appreciate, the various models may differ with respect to details, but they share many features and build upon common principles. This is true for the simplest models of monetary exchange, as well as the extensions that integrate banking, credit arrangements, payments mechanisms, and asset markets. We think that this is not only interesting in terms of economic theory, but that there are also lessons to be learned for understanding the current economic situation and shaping future policy. To the extent that the recent crisis has at its roots problems related to banking, mortgage markets, and other credit arrangements, or information problems in asset markets, one cannot address the issues without models that take seriously the exchange process. We do not claim New Monetarist economics provides all of the answers for all of the recent economic problems; we do believe it has a great deal to contribute to the discussion.

2 Basic Monetary Theory

An elementary model in the spirit of New Monetarist Economics is a version of the first-generation monetary search theory, long the lines of Kiyotaki and Wright (1993), which is a stripped-down version of Kiyotaki and Wright (1989, 1991), and uses methods from equilibrium search theory (e.g Diamond 1982). This model makes some strong assumptions, which will be relaxed later, but even with these assumptions in place it captures something of the essence of money as an institution that facilitates exchange. What makes exchange difficult in the first place is a double-coincidence problem, generated by specialization and random matching, combined with limited commitment and imperfect memory. Frictions like this, or at least informal descriptions thereof, have been informally discussed in economics for a long time, and certainly versions of the

double-coincidence problem can be found in Adam Smith, and much further back, if one looks. The goal of recent theory is to formalize these ideas, to see which are valid under what assumptions, and hopefully to develop new insights along the way.

Before proceeding, since we start with search-based models, it is perhaps worth saying why. Clearly, random matching is an extreme assumption, but it captures well the notion that people trade with each other and not only against budget constraints. Of course, it is all too easy to criticize. As Howitt (2005) puts it: "In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters." Based in part on such criticism, much of the theory, including the models in this section, has been redone using directed rather than random search (Corbae et al. 2003; Julien et al. 2008). While some results change, the basic theory remains intact. Hence we start with random matching, hoping readers understand that the theory also works with directed search. Later, search is replaced by preference and technology shocks.

2.1 The Simplest Model

Time is discrete and continues forever. There is a $[0, 1]$ continuum of infinite-lived agents. To make exchange interesting, these agents specialize in production and consumption of differentiated commodities, and trade bilaterally. It is an old idea that specialization is intimately related to monetary exchange, so we want this in the environment. Although there are many ways to set it up, here we assume the following: There is a set of goods, that for now are indivisible and nonstorable. Each agent produces, at cost $C \geq 0$, goods in some subset, and derives utility $U > C$ from consuming goods in a different subset. It is formally equivalent, but for some applications it helps the discussion, to consider a pure exchange scenario. Thus, if each agent is endowed with a good each period that he can consume to yield utility C, but he may meet someone with another good that

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gives him utility U, the analysis is basically the same, except C is interpreted as an opportunity cost rather than a production cost.

Let α be the probability of meeting someone each period. There are different types of potential trade meetings. Let σ be the probability that you like what your partner can produce but not vice versa – a single coincidence meeting – and δ the probability that you like what he can produce and vice versa – a double coincidence meeting.[4] The environment is symmetric, and for the representative agent, the efficient allocation clearly involves producing whenever someone in a meeting likes what his partner can produce. Let V^C be the payoff from this cooperative allocation, described recursively by

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\begin{aligned}
V^C &= \alpha\sigma(U + \beta V^C) + \alpha\sigma(-C + \beta V^C) + \alpha\delta(U - C + \beta V^C) \\
&\quad + (1 - 2\alpha\sigma - \alpha\delta)\beta V^C \\
&= \beta V^C + \alpha(\sigma + \delta)(U - C).
\end{aligned}
$$

If agents could commit, ex ante, they would all agree to execute the efficient allocation. If they cannot commit, we have to worry about ex post incentive conditions.

The binding condition is this: to get agents to produce in single-coincidence meetings we require $-C + \beta V^C \geq \beta V^D$, where V^D is the deviation payoff, depending on what punishments we have at our disposal. Suppose we can punish a deviator by allowing him in the future to only trade in double-coincidence meetings. It is interesting to consider other punishments, but this one has a nice interpretation in terms of what a mechanism designer can see and do. We might like to trigger to autarky – no trade at all – after a deviation, but it is not so obvious we can enforce this in double-coincidence meetings. Having trade only in double-coincidence meetings – a pure barter system – is self enforcing, and implies payoff $V^B = \alpha\delta(U - C)/(1 - \beta)$. If we take the deviation payoff to be continuing with pure barter, $V^D = V^B$, the relevant incentive condition can

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[4]Many extensions and variations are possible. In Kiyotaki and Wright (1991) e.g. agents derive utility from all goods, but prefer some over others, and the set of goods they accept is determined endogenously. In Kiyotaki and Wright (1989) or Ayagari and Wallace (1991, 1992) there are N goods and N types of agents, where type n consumes good n and produces good $n+1$ (mod N). In this case, $N = 2$ implies $\sigma = 0$ and $\delta = 1/2$, while $N \geq 3$ implies $\sigma = 1/N$ and $\delta = 0$. The case $N = 3$ has been used to good effect by Wicksell (1967) and Jevons (1875).

be reduced to

$$[1 - \beta(1 - \alpha\sigma)] \, C \leq \beta\alpha\sigma U. \tag{1}$$

If every potential trade meeting involves a double-coincidence, i.e. if $\sigma = 0$, then pure barter suffices to achieve efficiency and there is no incentive problem. But with $\sigma > 0$, given imperfect commitment, (1) tells us that we can achieve efficiency iff production is not too expensive (C is small), search and specialization frictions are not too severe (α and σ are big), etc.[5] If (1) holds, one can interpret exchange as a credit system, as in Sanches and Williamson (2009), but there is no role for money. A fundamental result in Kocherlakota (1998) is that money is not essential – i.e. it does nothing to expand the set of incentive-feasible allocations – when we can use trigger strategies as described above. Obviously this requires that deviations can be observed and recalled. Lack of perfect monitoring or record keeping, often referred to as incomplete memory, is necessary for money to be essential.

There are several way to formalize this. Given a large number of agents that match randomly, suppose that they observe what happens in their own but not in other meetings. Then, if an agent deviates, the probability someone he meets later will know it is 0. This is often described by saying agents are anonymous. In addition to Kocherlakota (1988), see Kocherlakota and Wallace (1988), Wallace (2001), Araujo (2004), Aliprantis et al. (2006, 2007), and Araujo et al. (2010) for more discussion. Also note that we only need *some* meetings to be anonymous; in applications below we assume that with a given probability meetings are monitored, and credit may be used in those meetings. But for now, we assume all meetings are anonymous, so there is no credit, and hence no one ever produces in single-coincidence meetings. In this case, absent money, we are left with only direct barter.

Therefore we want to introduce money. Although we soon generalize this, for now, there are $M \in (0, 1)$ units of some object that agents can store in units $m \in \{0, 1\}$. This object is worthless in consumption and does not aid in production, and so if it is used

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[5] Don't get confused by the fact that $\sigma = 0$ implies (1) fails. It is true that if there were no single-coincidence meetings then we could not sustain cooperative trade in single-coincidence meetings, but it does not matter.

as a medium of exchange it is, by definition, fiat money (Wallace 1980). One could also assume the object gives off a flow utility $y > 0$ – say a dividend yield – and interpret it as commodity money. Alternatively, if $y < 0$, we can interpret it as a storage cost. To ease the presentation we set $y = 0$ for now (but see Section 6). While m may not have all the properties that undergraduate textbooks say money tends to or ought to have, and in particular it lacks divisibility, it does have other desirable properties, like storability, portability, and recognizability. We assume it is initially distributed randomly across agents, and from then on the matching process is such that, conditional on a meeting, your partner has $m = 1$ with probability M and $m = 0$ with probability $1 - M$.

Let V_m be the payoff to an agent with money holdings $m \in \{0, 1\}$. Then the value function of an agent with $m = 0$ is given by

$$V_0 = \beta V_0 + \alpha\delta(U - C) + \alpha\sigma M \max_\xi \xi \left[-C + \beta(V_1 - V_0)\right], \tag{2}$$

since he can still barter in double-coincidence meetings, and now has another option: if he meets someone with money who likes his good but cannot produce anything he likes, he could trade for cash, and ξ is the probability he agrees to do so. Similarly, the value function of an agent with $m = 1$ is

$$V_1 = \beta V_1 + \alpha\delta(U - C) + \alpha\sigma(1 - M)\Xi \left[U + \beta (V_0 - V_1)\right], \tag{3}$$

because he can still barter, and now he also can make a cash offer in single-coincidence meetings, which is accepted with a some probability Ξ that he takes as given.[6]

The best response condition gives the maximizing choice of ξ taking Ξ as given: $\xi = 1$ or 0 or [0, 1] as $-C + \beta(V_1 - V_0)$ is positive or negative or 0, where V_1 and V_0 are functions of Ξ obtained by solving (2)-(3). An equilibrium is a list $\{\xi, V_0, V_1\}$ satisfying (2)-(3) and the best response condition. Obviously $\xi = 0$ always constitutes an equilibrium, and $\xi = 1$ does as well iff

$$[1 - \beta + \beta\alpha\sigma(1 - M)] C \leq \beta\alpha\sigma(1 - M)U$$

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[6] The presentation here is slightly different from the original search models, which usually assumed agents with money could not produce. The version here is arguably more natural, and for some issues simpler. See Rupert et al. (2001) for an extended discussion and references.

(there are also mixed strategy equilibria, but one can argue they are not robust, as in Shevchenko and Wright 2004). Hence, there is a monetary equilibrium $\xi = 1$ iff C is below an upper bound. This bound is less than the one we had for credit equilibrium when triggers were available. Moreover, even if we can support $\xi = 1$, payoffs are lower with money than with triggers. So when monitoring or memory is bad, money may allow us to do better than barter, but not as well as perfect credit. In other words, money may be a substitute, but it is not a perfect substitute, for credit.

This model is crude, with its indivisibilities, but without doubt it captures the notion that money is a beneficial institution that facilitates exchange. This contrasts with cash-in-advance models, where money is a hindrance, or sticky-price models, where money plays a purely detrimental role when it is assumed agents must quote prices in dollars and not allowed to change them easily. Also note that, contrary to standard asset-pricing theory, in monetary equilibria an intrinsically worthless object has positive value. Naturally, it is valued as a medium of exchange, or for its liquidity. Monetary equilibria have good welfare properties relative to barter, even if they do not achieve first best. The fact that $\xi = 0$ is always an equilibrium points to the tenuousness of fiat money. Yet it is also robust, in the sense that the equilibrium with $\xi = 1$ survives even if we endow the fiat object with some bad characteristics, like a transaction or storage cost, or if we tax it, as long as the costs or taxes are not too big. So, while it may be crude, the model makes many predictions that ring true.[7]

2.2 Prices

Up to now prices were fixed, since every trade involves a one-for-one swap. Beginning the second generation of papers in this literature, Shi (1995) and Trejos-Wright (1995)

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[7]Other applications of these first-generation models include the following: Kiyotaki and Wright (1989), Aiyagari and Wallace (1991), Kehoe et al. (1993), and Wright (1995), among others, allow goods to be storable and discuss commodity money. Kiyotaki and Wright (1991,1993), Shi (1997), and Camera et al. (2003) endogenize specialization in production and consumption. Kiyotaki et al. (1994) and Zhu (1997) pursue issues in international monetary economics. Williamson and Wright (1994), Li (1995), and Kim (1996) introduce private information to show how money can ameliorate lemons problems. Li (1994,1995) discusses the optimal taxation of money in the presence of search externalities. Ritter (1995) asks how fiat currency might first get introduced. Green and Weber (1996) discuss counterfeiting. Cavalcanti et al. (1999), He et al. (2005) and Lester (2009) study banking and payments issues.

endogenize prices by keeping $m \in \{0, 1\}$ but allowing divisible goods. Although we relax $m \in \{0, 1\}$ soon enough, the advantage of this approach is that one can talk about prices while maintaining a simple fixed distribution of money holdings across agents: it is still the case that at any point in time M agents each hold $m = 1$ and $1 - M$ agents each hold 0. When a producer gives output x to a consumer, their instantaneous utilities are $U = u(x)$ and $C = c(x)$, where $u' > 0$, $c' > 0$, $u'' < 0$, $c'' \geq 0$, and $u(0) = c(0) = 0$. Letting x^* solve $u'(x^*) = c'(x^*)$, it is easy to show that the efficient outcome is for agents to produce x^* in every meeting where their partner likes their output. A credit system with perfect memory could support this if β is big enough. We instead want to talk about monetary equilibria, so we assume imperfect memory, as discussed above.

We focus on the case where money is accepted with probability $\xi = 1$, and to ease the presentation, we start with $\delta = 0$ so there is no direct barter. Now, to determine x in a monetary exchange, we use the generalized Nash bargaining solution.[8] One virtue of this is simplicity; another is the well-known result that Nash bargaining can be interpreted as a natural limit of a simple non-cooperative bargaining game (see e.g. Binmore et al. 1992). Letting the bargaining power of the consumer be θ and letting threat points be given by continuation values, x then solves

$$\max \left[u(x) + \beta V_0 - \beta V_1\right]^\theta \left[-c(x) + \beta V_1 - \beta V_0\right]^{1-\theta}. \tag{4}$$

For now we consider the notion of a stationary equilibrium, or steady state, which is a list $\{x, V_0, V_1\}$ such that: given V_0 and V_1, x solves (4); and given x, V_0 and V_1 solve (2) and (3).

For the sake of illustration, consider the case $\theta = 1$, which means that buyers get to make take-it-or-leave-it offers, so that $c(x) = \beta(V_1 - V_0)$. Solving for $V_1 - V_0$ from (2)-(3), this reduces to

$$c(x) = \frac{\beta \alpha \sigma (1 - M) u(x)}{1 - \beta + \beta \alpha \sigma (1 - M)}. \tag{5}$$

This condition holds at $x = 0$, which is a nonmonetary equilibrium, and at a unique

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[8]Other solution concepts can also be used: Curtis and Wright (2004) use price posting; Julien et al. (2008) use auctions in a version with some multilateral meetings; and Wallace and Zhou (2007a,2007b) use mechanism design.

monetary equilibrium $x > 0$, where it is easy to check $\partial x / \partial M < 0$, so the price level $p = 1/x$ increases with the number of buyers. When we relax $\delta = 0$, there are generically either multiple monetary equilibria or no monetary equilibria. This generalization is straightforward, although note that $\delta > 0$ means one has to also solve for x in a barter exchange, which generally differs from the x in a monetary trade (for general results with $\delta > 0$, as well as any bargaining power θ and alternative specifications for the threat points, see Rupert et al. 2001).

In the symmetric case $\theta = 1/2$ and $M = 1/2$, which is the one used in Shi (1995) and Trejos-Wright (1995), it can be shown that $x < x^*$ in any equilibrium. Hence, monetary exchange does not achieve the efficient allocation. However, it is easy to verify that $x \to x^*$ as $\beta \to 1$. To understand this, consider an Arrow-Debreu version of this environment, which means the same preferences and technology but no frictions. In such an economy, since given agents can turn their production into instantaneous consumption through the market, they choose $x = x^*$. But in our economy, they must first turn production into cash, which can then only be used in the future. Therefore, as long as $\beta < 1$, agents are willing to produce less than they would in a frictionless model. Now, one can get x to increase, say by raising θ, and for big enough θ we may have $x > x^*$, but the model still illustrates a basic tendency for $x < x^*$, other things being symmetric.[9]

Before moving on, we briefly mention nonstationary equilibria in this simple setup. For illustration, assume $\delta = 0$, and add a flow utility y of holding $m = 1$; as discussed above, if $y > 0$ then m is commodity money, and if $y < 0$ then m has a storage cost. Also, purely for convenience, we move to continuous time by letting the length of a period (in both the search and bargaining processes) vanish, implying

$$rV_0 = \alpha \sigma M \left[-c(x) + V_1 - V_0 \right] + \dot{V}_0$$

$$rV_1 = y + \alpha \sigma (1 - M) \left[u(x) + V_0 - V_1 \right] + \dot{V}_1.$$

[9] One can argue that $x > x^*$ is an artifact of indivisible money here as follows: if we allow lotteries, which are useful with $m \in \{0, 1\}$, and in a sense approximate divisible m, it can be shown show that x can never exceed x^* (see Berentsen et al. 2002 and Berentsen and Rocheteau 2004). Soon enough we can check this in models that have divisible money.

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Subtracting yields a differential equation in the difference

$$\dot{V}_1 - \dot{V}_0 = -y - \alpha\sigma(1-M)u(x) - \alpha\sigma Mc(x) + (r + \alpha\sigma)(V_1 - V_0). \qquad (6)$$

To reduce notation, without loss in generality, set $\alpha\sigma = 1$, and let $c(x) = x$. Also, assume for simplicity $\theta = 1$. Then we get $V_1 - V_0 = x$, $\dot{V}_1 - \dot{V}_0 = \dot{x}$, and

$$\dot{x} = -y + (r + 1 - M)\,x - (1 - M)u(x),$$

Define $F(x)$ by the RHS of (6). Then equilibrium can be defined as a nonnegative time path for x satisfying $\dot{q} = F(x)$, plus a side condition that says buyers want to trade, $u(x) + V_0 - V_1 \geq 0$ (the seller wants to trade by construction when $\theta = 1$). This side condition holds if and only if $x \leq \bar{x}$, where $u(\bar{x}) = \bar{x}$, and tells us that an equilibrium path for x cannot leave $[0, \bar{x}]$. By plotting $F(x)$ versus x it is now easy to see the following:

1. When $y = 0$, which means fiat money, there are two steady states, $x = 0$ and $x = x^0 \in (0, \bar{x})$, plus a continuum of dynamic equilibria starting from any $x \in (0, x^0)$ and converging to 0.

2. When $y > 0$, which means commodity money, the $F(x)$ curve shifts down. As long as y is not too big the unique equilibrium is a steady state with $x = x^y \in (x^0, \bar{x})$, since no other path satisfying $\dot{x} = F(x)$ remains in $[0, \bar{x}]$. This illustrates the venerable idea that commodity money can eliminate indeterminacies associated with fiat money. If y gets too big, however, then $x^y > \bar{x}$, which means an agent with $m = 1$ prefers to hoard rather than spend it, and is reminiscent of Gresham's Law (or at least it would be if we introduce a second money, which is easy enough to do).

3. When $y < 0$, there is always a steady state equilibrium with $x = 0$, where agents freely dispose of money, and if $|y|$ is big then this is the only equilibrium. If $|y|$ is not too big then there are two steady states in $(0, x^0)$, say x^1 and x^2, plus a continuum of dynamic equilibria starting at any $x \in (0, x^2)$ and converging to x^1.

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These results illustrate some interesting properties of fiat and commodity money systems, and show how different types of interesting dynamic equilibria may emerge (as is true in most monetary theories, of course). There are many other applications of this simple model, but without further ado, we now move to relax the inventory restriction $m \in \{0, 1\}$.[10]

2.3 Distributions

Although there are various alternatives, consider the approach in Molico (2006), who allows $m \in [0, \infty)$.[11] This means that we have to deal with the endogenous distribution of money across agents, $F(m)$, while previously this was trivial. Now, in a single-coincidence meeting where the consumer has m and the producer has $\tilde{m}$, let $x(m, \tilde{m})$ be the amount of output and $d(m, \tilde{m})$ the amount of money traded. Again setting $\delta = 0$, for expositional purposes, the generalization of (2)-(3) is

$$
\begin{aligned}
V(m) \quad = \quad & \beta V(m) + \alpha\sigma \int \{u[x(m, \tilde{m})] + \beta V[m - d(m, \tilde{m})] - \beta V(m)\} \, dF(\tilde{m}) \\
& + \alpha\sigma \int \{-c[x(\tilde{m}, m)] + \beta V[m + d(\tilde{m}, m)] - \beta V(m)\} \, dF(\tilde{m}). \quad (7)
\end{aligned}
$$

The first term is the expected value of buying from a producer with $\tilde{m}$ dollars, and the second the expected value of selling to a consumer with $\tilde{m}$ dollars (notice how the roles of m and $\tilde{m}$ are reversed in the two integrals).

In this model, we can easily add injections of new currency, say by lump sum or proportional transfers, which was not so easy with $m \in \{0, 1\}$. With lump sum transfers, we simply change m on the RHS to $m + \mu M$, where M is the aggregate money supply, governed by $M_{t+1} = (1 + \mu)M_t$. This greatly extends the class of policies that can

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[10] A few applications include the following: Shi (1996) introduces bilateral borrowing and lending to study the relation between money and credit. Aiyagair et al. (1996) study the interaction between money and bonds. Shi (1995), Coles and Wright (1998) and Ennis (2001) further study nonstationary equilibria. Wallace (1997) and Katzman et al. (2003) study the inflation-output relation. Wallace and Zhou (1997) study currency shortages. Velde et al. (1999), Burdett et a. (2001), Ales et al. (2008), and Redish and Weber (2010) use the model to analyze various issues in monetary history. Lee et al. (2005) study denomination structures. Williamson (1999) considers private money. Cavalcanti and Wallace (1999a,1999b) introduce banks. Trejos (1999) studies private information. Li (1999), Johri and Leach (2002), and Shevchenko (2004) study middlemen. Nosal and Wallace (2007) analyze counterfeiting.

[11] Other approaches to relaxing $m \in \{0, 1\}$ include Camera and Corbae (1999), Deviatov and Wallace (1998), Berentsen (2002), and Zhu (2003,2004). There is also a series of papers following up on Green and Zhou (1997); rather than list them all here, see the references in Jean et al. (2009). Some of these models assume $m \in \{0, 1...\overline{m}\}$, where the upper bound $\overline{m}$ may or may not be finite. The value function in (7) below is still valid in such cases, including the case $\overline{m} = 1$ studied above.

be analyzed. However, to illustrate the basic idea, for now we keep $M = \int m dF(m)$ fixed. Then a stationary equilibrium is a list of functions $\{V(\cdot), x(\cdot), d(\cdot), F(\cdot)\}$ such that: given $x(m, \tilde{m})$, $d(m, \tilde{m})$ and $F(m)$, $V(m)$ solves (7); given $V(m)$, $x(m, \tilde{m})$ and $d(m, \tilde{m})$ are determined by some bargaining solution, such as

$$\max \left[u(x) + \beta V(m - d) - \beta V(m) \right]^{\theta} \left[-c(x) + \beta V(\tilde{m} + d) - \beta V(\tilde{m}) \right]^{1-\theta} \qquad (8)$$

where the maximization is s.t. $d \leq m$; and given $x(m, \tilde{m})$ and $d(m, \tilde{m})$, $F(m)$ solves a stationary condition omitted in the interest of space. From this we can calculate other interesting objects, such as the distribution of $p(m, \tilde{m}) = d(m, \tilde{m})/x(m, \tilde{m})$.

This model is complicated, even using numerical methods. Heterogeneous-agent, incomplete-market, macro models of the sort analyzed by Huggett (1993) or Krusell and Smith (1998) also have an endogenous distribution as a state variable, but the agents in those models do not care about this distribution per se – they only care about prices. Of course prices depend on the distribution, but one can typically characterize accurately prices as functions of a small number of moments. In a search model, agents care about $F(m)$ directly, since they are trading with each other and not merely against their budget equations. Still, Molico (2006) computes equilibria, and the model is used to discuss issues such as the effects of inflation. See also Chiu and Molico (2006, 2008). An alternative approach used by Dressler (2009, 2010) is to assume competitive pricing, rather than bargaining (see below). This makes computation easier, on a par with Huggett-Krusell-Smith models. But while it is easier, this approach also loses some of the interesting elements from bargaining models, including the endogenous distribution of prices.

3   A Benchmark Model

Some search models with divisible money use devices that allow one to avoid having to track $F(m)$. There are two main approaches.[12] The first, originating with Shi (1997), uses the assumption of large households to render the distribution degenerate. Thus,

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[12] Recently, Menzio et al. (2009) proposed a new method for dealing with distributions, based on directed search,.

each decision making unit consists of many members who search randomly, as in the above models, but at the end of each trading round they return to the homestead, where they share the money they bring back with their siblings. Loosely speaking, by the large of large numbers, each household starts the next trading round with the same m. The large household is a natural extension for random-matching models of the "worker-shopper pair" discussed in the cash-in-advance literature (Lucas 1980). A number of interesting papers use this environment; rather than cite them all here, we refer the reader to Shi (2006). We focus instead on a different approach, following Lagos and Wright (2005), and use markets instead of families.

We use the Lagos-Wright model because it allows us to address a variety of other issues, in addition to rendering the distribution of money tractable (although some of the applications could in principle also use Shi's model). In particular, it serves to reduce the gap between monetary theory with some claim to microfoundations and standard macroeconomics. Whatever one thinks of the models discussed above, they are pretty far from mainstream macro. As Azariadis (1993) put it, "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1982) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." And as Kiyotaki and Moore (2002) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets."

To pursue the analogy, the setup in Lagos-Wright allows one to bring competitive markets back on board, in a way that can make monetary theory much closer to standard macro, as we show below. And rather than complicating matters, integrating competitive markets and search markets makes the analysis easier. We also believe this is a realistic way to think about economic activity. In reality, there is some activity in our economic

lives that is relatively centralized – it is fairly easy to trade, credit is available, we take prices as given, etc. – which can be well captured by the notion of a competitive market. But there is also much activity that is relatively decentralized – it is not easy to find trading partners, it can be hard to get credit, etc. – as captured by search theory. One might imagine that there are various alternative ways to integrate search and competitive markets. Here we present one that we think is useful.

3.1 The Environment

We now divide each period into two subperiods. In one, agents interact in a decentralized market, or DM, with frictions as in the search models discussed above. In the other, they interact in a frictionless centralized market, or CM, as in standard general equilibrium theory. Sometimes the setup is described by saying the DM convenes during the day and the CM at night; this story about day and night is not important for the theory, but we sometimes use it when it helps keep the timing straight.[13] There is one consumption good x in the DM and another X in the CM, although it is easy to have x come in many varieties, or to interpret X as a vector, as in standard GE theory (Rocheteau et al. 2008). For now x and X are produced one-for-one using labor h and H, but this is relaxed later. The implication is that for now the real wage in the CM is $w = 1$.

Preferences in any period, encompassing one DM and CM, are described by a standard utility function $\mathcal{U}(x, h, X, H)$. What is important for tractability, although not for the theory, in general, is quasi-linearity: $\mathcal{U}$ should be linear in either X or H. To be clear, with general preferences, the model requires numerical methods (see Chiu and Molico 2008); with quasi-linearity, we can derive many results analytically. Actually, as discussed below, we can use general utility and still get analytic tractability if we assume indivisible labor. For now, we assume divisible labor and take quasi-linearity as the benchmark.

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[13]One can also proceed differently without changing basic results. Williamson (2007) e.g. assumes both markets are always open and agents randomly transit between them. For some issues, it is also interesting to have more than one round of trade in the DM between meetings of the CM, as in Camera et al. (2005) and Ennis (2008), or more than one period of CM trade between meetings of the DM, as in Telyukova and Wright (2008). Chiu and Molico (2006) allow agents to transit between markets whenever they like, at a cost, embedding something like the model of Baumol (1952) and Tobin (1956) into general equilibrium where money is essential, but that requires numerical methods.

Here we assume $\mathcal{U}$ is linear in H, and in fact for now we assume

$$\mathcal{U} = u(x) - c(h) + U(X) - H;$$

later we consider cases where $\mathcal{U}$ is not necessarily separable in $(x, h, X,)$.

If we shut down the CM, these are the same preferences used in Molico, and the models become equivalent. Since the Molico model collapses to Shi-Trejos-Wright when we impose $m \in \{0, 1\}$, and to Kiyotaki-Wright when we further make x indivisible, these ostensibly different environments can be interpreted as special cases of one framework. Faig (2006,2008) further argues that the alternating-market model and the large household model in Shi (1997) can be encompassed in a more general setup. We think this is good, but not because we want one all-purpose vehicle for every issue in monetary economics. Rather, we do not want people to get the impression that New Monetarist economics consists of a huge set of mutually inconsistent models. The models reviewed so far, as well as the extensions below to incorporate banking, a payment system, and asset markets, all use similar fundamental building blocks, even if some applications make certain special assumptions.[14]

In the DM, the value function $V(\cdot)$ would be described exactly by (7) in the last section, except for one thing: wherever $\beta V(\cdot)$ appears on the RHS, replace it with $W(\cdot)$, since before going to the next DM agents now get to visit the CM, and $W(\cdot)$ denotes the CM payoff. In particular,

$$
\begin{aligned}
W(m) &= \max_{X, H, \hat{m}} \{U(X) - H + \beta V(\hat{m})\} \\
\text{st } X &= \phi(m - \hat{m}) + H - T,
\end{aligned}
$$

where ϕ is the value of money, or the inverse of the nominal price level, in the CM, and T is a lump sum tax. Assuming an interior solution (see Lagos-Wright 2005 for details),

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[14]An assumption not made explicit in early presentations of the model, but clarified by the work of Aliprantis et al. (2006,2007) is that in the CM agents observe only prices, and not other agents' actions. If they did observe others' actions there is a potential to use triggers, rendering money inessential. Aliprantis et al. (2007) also describe variations on the environment where triggers cannot be used, and hence money is essential, even if agents' actions can be observed in the CM. This was perhaps less of an issue in models with no CM – or perhaps not, since multilateral trade is neither necessary nor sufficient for public observability or communication. Some of these issues are not yet completely settled. For a recent discussion, see Araujo et al. (2010).

we can eliminate H and write

$$W(m) = \phi m - T + \max_X \{U(X) - X\} + \max_{\hat{m}} \{-\phi\hat{m} + \beta V(\hat{m})\}.$$

>From this several results are immediate: $W(m)$ is linear with slope ϕ; $X = X^*$ where $U'(X^*) = 1$; and $\hat{m}$ is independent of wealth $\phi m - T$.

Based on this last result, we should expect (and we would be right) a degenerate $F(\hat{m})$, where everyone takes the same $\hat{m} = M$ out of the CM, regardless of the m they brought in.[15] Using the fact that $F(\cdot)$ is degenerate and $W'(m) = \phi$, and replacing $\beta V(\cdot)$ with $W(\cdot)$, (7) simplifies rather dramatically to

$$V(m) = W(m) + \alpha\sigma \{u[x(m,M)] - \phi d(m,M)\} + \alpha\sigma \{-c[x(M,m)] + \phi d(M,m)\}. \quad (9)$$

Effectively, the CM here is a settlement subperiod where agents reset their liquidity positions. Without this feature the analysis is more difficult, and we think it is nice to have a benchmark model that is tractable. By analogy, while models with heterogeneous agents and incomplete markets are obviously interesting, it is nice to have the basic neoclassical growth theory with complete markets and homogeneous agents as a benchmark. Since serious monetary theory with complete markets and homogeneous agents is a non-starter, we need to find another benchmark, and this is our suggestion.

A degenerate distribution is not all we get in terms of tractability. Replacing $\beta V(\cdot)$ with $W(\cdot)$ and using $W'(m) = \phi$, the bargaining solution (8) reduces to

$$\max [u(x) - \phi d]^\theta [-c(x) + \phi d]^{1-\theta}$$

st $d \leq m$. In any equilibrium the constraint binds (see Lagos and Wright 2005). Inserting $d = m$, taking the FOC for x, and rearranging, we get $\phi m = g(x)$, where

$$g(x) \equiv \frac{\theta c(x)u'(x) + (1-\theta)u(x)c'(x)}{\theta u'(x) + (1-\theta)c'(x)}. \quad (10)$$

This expression may look complicated but it is easy to use, and simplifies a lot in some special cases – e.g. $\theta = 1$ implies $g(x) = c(x)$, and real balances paid to the producer

---

[15]The fact that $\hat{m}$ is independent of m does not quite imply that all agents choose the same $\hat{m}$. In a version of the model with some multilateral meetings, and auctions instead of bargaining, Galenianos and Kircher (2008) show that agents are indifferent over $\hat{m}$ in some set, and equilibrium entails a nondegenerate distribution $F(\hat{m})$. This cannot happen in our baseline model.

ϕm exactly compensate him for his cost. More generally, it says ϕm is determined by the sharing rule:

$$\phi m = \frac{\theta u'(x)}{\theta u'(x) + (1 - \theta)c'(x)}c(x) + \frac{(1 - \theta)c'(x)}{\theta u'(x) + (1 - \theta)c'(x)}u(x).$$

Notice $\partial x/\partial m = \phi/g'(x) > 0$, so bringing more money increases DM consumption, but in a nonlinear way, unless $\theta = 1$ and $c(x) = x$.

We have established $d(m, \tilde{m}) = m$ and $x(m, \tilde{m})$ depends on m but not $\tilde{m}$. Differentiating (9), we get

$$V'(m) = (1 - \alpha\sigma)\phi + \alpha\sigma\phi u'(x)/g'(x). \tag{11}$$

The marginal benefit of DM money is the value of carrying it into the next CM with probability $1 - \alpha\sigma$, plus the value of spending it on x with probability $\alpha\sigma$. Updating this one period and combining it with the FOC for $\hat{m}$ from the CM, we arrive at

$$\phi_t = \beta\phi_{t+1}\left[1 + \ell(x_{t+1})\right], \tag{12}$$

where

$$\ell(x) \equiv \alpha\sigma\left[\frac{u'(x)}{g'(x)} - 1\right]. \tag{13}$$

The function defined in (13) is the *liquidity premium*, giving the marginal value of spending a dollar, as opposed to carrying it forward, times the probability $\alpha\sigma$ of spending it. Using the bargaining solution $\phi m = g(x)$ plus market clearing $m = M$, (12) becomes

$$\frac{g(x_t)}{M_t} = \beta\frac{g(x_{t+1})}{M_{t+1}}\left[1 + \ell(x_{t+1})\right]. \tag{14}$$

Equilibrium can be defined as a list including $V(\cdot)$, $W(\cdot)$, $x(\cdot)$, and so on, satisfying the obvious conditions, but (14) reduces all this to a simple difference equation determining a path for x, given a path for M. Here we focus on stationary equilibria, where x and ϕM are constant (nonstationary equilibria, including sunspot, cyclic and chaotic equilibria, are studied in Lagos and Wright 2003). For this to make sense, we impose $M_{t+1} = (1 + \mu)M_t$ with μ constant. Of course, one has to also consider the consolidated monetary-fiscal budget constraint $G = T + \mu\phi M$, where G is government consumption in the CM. But notice that it does not matter for (14) whether changes in M are offset

by changing T or G. Individuals would of course prefer lower taxes, other things being equal, but this does not affect their decisions about real balances or consumption in the quasi-linear model. Hence we actually do not have to specify how money transfers are accomplished for the purpose of describing equilibrium x and ϕ. In a stationary equilibrium, or steady state, (14) simplifies to $1 + \mu = \beta[1 + \ell(x)]$.

Before moving to results, we illustrate one aspect of the framework's flexibility by showing what happens if we replace Nash bargaining with Walrasian pricing in the DM.[16] This can be motivated by interpreting agents as meeting in large groups in the DM, rather than bilaterally, and assuming that whether one is a buyer or seller is determined by preference and technology shocks, rather than by whom one meets. It might help to think about labor search models, like Mortensen-Pissarides (1994), which uses bargaining, and Lucas-Prescott (1974), which uses price taking. A standard interpretation of the latter is that workers and firms meet on islands representing "local labor markets," but on each island there are enough workers and firms that it makes sense to take wages parametrically. The same is true in monetary models. Specialization and anonymity can lead to an essential role for money despite agents meeting in large groups.

Assume for now that the shocks determining if an agent is to be a producer or a consumer in the DM are realized after the CM closes. Then we have

$$V(m) = \gamma V^b(m) + \gamma V^s(m) + (1 - 2\gamma)W(m),$$

where γ is the probability of being a buyer and the probability of being a seller (so that we have the same number of each, but this is easy to relax), while $V^b(m)$ and $V^s(m)$ are the payoffs. These payoffs solve

$$V^b(m) = \max\{u(x) + W(m - \hat{p}x)\} \text{ s.t. } \hat{p}x \leq m$$

$$V^s(m) = \max\{-c(x) + W(m + \hat{p}x)\}$$

[16] The use of price taking instead of bargaining in this model follows Rocheteau and Wright (2005). They also consider price posting with directed search, as do Faig and Huangfu (2007). Other mechanisms people consider include the following: Aruoba et al. (2007) use several alternative (to Nash) bargaining solutions. Galeanois and Kircher (2008) and Duttu et al. (2009) use auctions. Faig and Jerez (2006), Ennis (2008), Dong and Jiang (2009) and Sanches and Williamson (2010) study pricing with private information. Hu et al. (2009) use pure mechanism design. And as we show explicitly in Section 4.3 below, one can also use price posting with random search.

where $\hat{p}$ is the DM price of x in terms of dollars, which obviously is different from the CM price $p = 1/\phi$ in general. One can show the constraint for buyers binds, $\hat{p}x = m$, just like in the bargaining model. The seller optimizes by choosing x so that price (in units of CM consumption goods) equals marginal cost, or $\hat{p}\phi_{t+1} = c'(x)$. From an agents CM first-order condition, in (13) we replace $\frac{u'(x)}{g'(x)}$ with $\frac{u'(x)}{c'(x)}$, and then market clearing implies that we replace $g(x_t)$ with $x_t c'(x_t)$ and $\alpha\sigma$ with γ in (14). Everything else in the model is exactly the same.

3.2   Results

We have defined monetary equilibrium in the benchmark model, where money has a desirable role, similar to the role it had in the more primitive search-based models in the previous section. We now discuss some of its properties. To facilitate comparison to the literature, we proceed as follows. Suppose one uses standard methods to price real and nominal bonds between any two meetings of the CM, assuming these bonds cannot be traded in the DM (say, maybe because they are merely book entries that cannot be transferred between agents, although we are well aware that this deserves much more discussion). Then the real and nominal interest rates r and i satisfy $1 + r = 1/\beta$ and $1 + i = (1 + \mu)/\beta$, where the latter is a version of the standard Fisher equation. Then we can rewrite the steady state condition $1 + \mu = \beta\left[1 + \ell(x)\right]$ derived above as

$$\ell(x) = i. \tag{15}$$

In the Walrasian version of the model, the same condition holds, except in the formula for $\ell(x) = \alpha\sigma\left[u'(x)/g'(x) - 1\right]$ we replace $\alpha\sigma$ with γ and $g'(x)$ with $c'(x)$.

Notice (15) equates the marginal benefit of liquidity to its cost, given by the nominal interest rate, as is standard. In what follows we assume $i > 0$, although we do consider the limit $i \to 0$ (it is not possible to have $i < 0$ in equilibrium). A stationary monetary equilibrium, or steady state, is *almost* any solution $x > 0$ to (15). We say *almost* because this condition is really just the FOC for the CM choice of $\hat{m}$, and in principle one needs to check the SOC to be sure we have a maximum, and when there are multiple

solutions we have to be sure we pick the global maximum. The existence of a solution to $\ell(x) = i$ is immediate given standard assumptions like $u'(0) = \infty$, and if $\ell(x)$ is monotone then $\ell'(x) < 0$ at the solution, which means it is unique and satisfies the SOC. In this case, there exists a unique stationary monetary equilibrium. Unfortunately, however, $\ell(x)$ is not generally monotone.[17] Still, one can establish, as in Wright (2010), that there is generically a unique stationary monetary equilibrium even if $\ell(x)$ is not monotone. Basically this is because, even if there are multiple local maximizers solving (15), generically only one of them constitutes a global maximizer for the underlying CM problem.

This establishes the existence and uniqueness of stationary monetary equilibrium. In terms of welfare and policy implications, the first simple observation is that it is equivalent here for policy makers to target either the money growth rate or the inflation rate, since both are equal to μ; or they can target the nominal interest rate i, which is tied to μ through the Fisher equation. Second, it is clear that the initial stock of money M_0 is irrelevant for the real allocation (money is neutral), but the growth rate μ is not (money is not super neutral). These are properties shared by many monetary models, including typical overlapping-generations, cash-in-advance, and money-in-the-utility-function constructs. Next, since $\ell'(x) < 0$ in equilibrium, (??) implies $\partial x / \partial i < 0$. Hence DM output is unambiguously decreasing in i, because i represents the cost of participating in monetary exchange – or, in other words, because inflation is a tax on DM activity. Since CM output $X = X^*$ is independent of i in this basic setup, total output is decreasing in i. However, X is not generally independent of i if we allow nonseparable utility (see Section 3.5).

One can also show that x is increasing in bargaining power θ. And one can show $x < x^*$ for all $i > 0$, and in fact, $x = x^*$ if and only if $i = 0$ and $\theta = 1$.[18] The condition

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[17] Under some additional assumptions one can show $\ell(x)$ is monotone. One such assumption is $\theta \approx 1$. Another is that $c(x)$ is linear and $u(x)$ displays decreasing absolute risk aversion. In the version with Walrasian pricing, concave $u(\cdot)$ and convex $c(\cdot)$ guarantee the monotonicity of $\ell(x)$.

[18] The argument is straightforward, if slightly messy. First compute $g'(x)$ and check $u'(x^*) < g'(x^*)$, which by (13) means $\ell(x^*) < 0$. Hence, $x < x^*$. We can actually say more. One can show $x < \bar{x}$ where $\bar{x}$ solves $u'(x) = g'(x)$, and $\bar{x} < x^*$ unless $\theta = 1$. In fact, $\bar{x}$ is the x that maximizes a buyer's surplus, $u(x) - \phi\hat{m} = u(x) - g(x)$, which we use below.

$i = 0$ is the Friedman rule, and is standard, while $\theta = 1$ is a version of the Hosios (1990) condition describing how to efficiently split the surplus. This latter condition is specific to monetary theory with bargaining. To understand it, note that in general there is a holdup problem in money demand analogous to the usual problem with ex ante investments and ex post negotiations. Thus, agents make an investment when they acquire cash in the CM, which pays off in single-coincidence meetings in the DM since it allows them to trade. But if $\theta < 1$ producers capture some of the gains from trade, leading agents to initially underinvest in $\hat{m}$. The Hosios condition tells us that investment is efficient when the payoff to the investor is commensurate with his contribution to the total surplus, which in this case means $\theta = 1$, since it is the money of the buyer (not the seller) that allows the pair to trade.

There is reason to think that this is important in terms of quantitative and policy analysis, and not merely a technical detail. To make the case, first consider the typical quantitative exercise using something like a cash-in-advance model, without other explicit frictions, where one asks about the welfare cost of fully anticipated inflation. If as is standard we measure this cost by asking agents what fraction of consumption they would be willing give up to go from, say, 10% inflation to the Friedman rule, the answer is generally very low. There are many such studies, but we can summarize them accurately by saying that consumers would be willing to give up around 1/2 of 1%, or perhaps slightly more, but rarely above 1%, of their consumption. See Cooley and Hansen (1989) for a representative paper, Lucas (2000) for a somewhat different analysis, or Craig and Rocheteau (2008) for a survey. This has lead many economists to conclude that the distortion introduced by inflation is not large.

Why is the distortion implied by those models so small? It seems hard to reconcile with the aversion many politicians and regular people seem to have to inflation. The intuition is actually straightforward. In the standard cash-in-advance or other reduced-form model, at the Friedman rule we get the first best. Hence, by the envelope theorem, the derivative of welfare with respect to i is 0 at the Friedman rule, and a small inflation

matters little. This is indeed consistent with what one finds in our benchmark model when we set $\theta = 1$ and calibrate other parameters using standard methods. But if $\theta < 1$ then the envelope theorem does not apply, since while $i = 0$ is still optimal it is a corner solution (remember that $i < 0$ is not feasible). Hence, the derivative of welfare is not 0 at $i = 0$, and a small deviation from $i = 0$ has a first-order effect. The exact magnitude of the effect of course depends on parameter values, but in calibrated versions of the model it can be an order of magnitude bigger than the cost found in reduced-form models. These results lead New Monetarists to rethink the previously conventional wisdom that anticipated inflation does not matter much.

One should look at the literature for all of the details, but we can sketch the basic method here. Assume $U(X) = \log(X)$, $u(x) = Ax^{1-a}/(1-a)$, and $c(x) = x$. Then calibrate the parameters as follows. First set $\beta = 1/(1+r)$ where r is the average real rate in the data (which data and which real rate are interesting issues). In terms of arrival rates, we can at best identify $\alpha\sigma$, so normalize $\alpha = 1$. In fact, it is not that easy to identify $\alpha\sigma$, so for simplicity set σ to its maximum value of $\sigma = 1/2$, although this is not very important for the results. We need to set bargaining power θ, as discussed below. Then, as in Cooley and Hansen (1989), Lucas (2000), and virtually all other quantitative monetary models, we set the remaining parameters A and a to match the so-called money demand observations, which means the empirical relationship between i and the inverse of velocity, M/PY.

The relationship between M/PY and i is interpreted as money demand by imagining agents setting real balances M/P proportional to income Y, with a factor of proportionality that depends on the opportunity cost i. Here, with $U(X) = \log(X)$, real CM output is $X^* = 1$ (a normalization), and so nominal CM output is $PX = 1/\phi$. Nominal DM output is $\alpha\sigma M$, since in every single-coincidence meeting M dollars change hands. Hence, total nominal output is $PY = 1/\phi + \alpha\sigma M$. Using $\phi M = g(x)$, we get

$$\frac{M}{PY} = \frac{g(x)}{1 + \alpha\sigma g(x)}, \tag{16}$$

and since x is decreasing in i, so is M/PY. This is the money demand curve implied by

theory.[19] Given θ, $g(x)$ depends on preferences, and we can pick the parameters a and A of $u(x)$, by various methods, to fit (16) to the data (assuming, for simplicity, say, that each observation corresponds to a stationary equilibrium of the model, although one can also do something more sophisticated). Roughly speaking, average M/PY identifies A, and the elasticity wrt i identifies a.

To do this one has to choose an empirical measure of M, which is typically $M1$. People have tried other measures, and it does make a difference (as it would in any model of money, with or without microfoundations). One might think a more natural measure would be $M0$ based on a narrow interpretation of the theory, but this may be taking the model too literally. In any case, this empirical research program is ongoing, and some of the modeling approaches used to incorporate financial intermediation and alternative assets into the benchmark model (see Sections 5 and 6) are potentially useful in matching the theory with measurement.

This describes how one can quantify the benchmark model. The only nonstandard parameter is bargaining power θ, which does not show up in theories with price taking, and so we spend some time on it. A natural target for calibrating θ is the markup, price over marginal cost, since it seems intuitive that this should convey information about buyers' bargaining power. One can compute the average markup implied by the model using standard formulae as in Aruoba et al. (2009) and set θ so that this number matches the data. In terms of data, evidence discussed by Faig and Jerez (2005) from the Annual Retail Trade Survey describes markups across retailers as follows: At the low end, in Warehouse Clubs, Superstores, Automotive Dealers, and Gas Stations, markups range between 1.17 and 1.21; and at the high end, in Specialty Foods, Clothing, Footware, and Furniture, they range between 1.42 and 1.44. Aruoba et al. (2009) target 1.3, right in the middle of these data. Lagos and Wright (2005), used 1.1, as one might see in other macro applications (e.g. Basu and Fernald 1997). However, in this range, the exact value of θ turns out to not matter too much.

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[19]In another guise, holding M and P constant and plotting the same relationship in (Y, i) space, it becomes the LM curve from undergraduate Keynesian economics.

It is now routine to compute the cost of inflation. What is the final answer? It is hard to summarize all the results with one number, since the exact results depend on many factors, such as the sample period, frequency (monthly, quarterly, or annual), whether one includes complications like capital or fiscal policy, and so on. However, it is safe to say that Lagos and Wright (2005) can get agents to willingly give up 5% of consumption to eliminate a 10% inflation, which is an order of magnitude larger than previous findings. In the model with capital presented in Section 3.4 below, Aruoba et al. (2009) report findings closer to 3%, which is still quite large. There are many recent studies using variants of the benchmark model that come up with similar numbers (again see Craig and Rocheteau 2008). Two points to take away from this are the following: First, inflation may well be more costly than most economists used to think. Second, getting into the details of monetary theory, which in this application means thinking about search and bargaining, can make a big difference for quantitative as well as qualitative work.

3.3 Unanticipated Inflation

So far we have been concerned only about fully anticipated inflation; we now describe one way to introduce aggregate shocks.[20] Suppose the money supply is given by $M_t = \zeta_t M_{t-1}$, where we now include time subscripts explicitly, and $\zeta_t = 1 + \mu_t$ in the earlier notation. Assume ζ_t is i.i.d., drawn from some distribution G. Also, suppose that at the start of the DM at each date t, agents receive a perfect signal about the value of ζ_t to be implemented in the CM later that period, which in general affects ϕ_t. However, when they chose $\hat{m}_t$ in the CM at t they do not know ζ_{t+1}. Then the CM problem is as before, except we replace $\beta V(\hat{m}_t)$ with $\beta E_t V_{t+1}(\hat{m}_t)$. Thus, the relevant FOC becomes

$$\phi_t = \beta E_t V_{t+1}^{'}(\hat{m}_t). \tag{17}$$

In the DM, at $t+1$, upon observing ζ_{t+1}, buyers are holding $\hat{m}_t$ and cannot increase it, as they might like to do when inflation is higher than expected. Here we must get into a technicality that comes with Nash bargaining. It turns out that the surplus of the

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[20]Although there are many ways one could apply this extension, we do not do much here other than present it, in the spirit of using the Handbook as a teaching tool. As with many of the subsections to follow, one could skip this and move on to more substantive material without much loss in continuity.

buyer $u(x) - \phi\hat{m} = u(x) - g(x)$ is not globally increasing in x; typically there is some $\bar{x}$ satisfying $u'(\bar{x}) = g'(\bar{x})$ where the surplus is maximized, and $\bar{x} < x^*$ unless $\theta = 1$ (see Aruoba et al. 2007 for more discussion). Hence, if a buyer has more than required to buy $\bar{x}$ he would rather not bring it all to the bargaining table. This is not a problem in the deterministic case, since agents never choose $\hat{m}$ to purchase more than $\bar{x}$; now, however, it could be that the realized ζ_{t+1} and ϕ_{t+1} are sufficiently low that buyers can afford more than $\bar{x}$. In this case we assume that they leave some of their cash "at home" before going shopping in the DM.[21] In any case, we assume buyers after seeing ζ_{t+1} decide how much money to take shopping, which is in real terms denoted z. Letting $\bar{z} = g(\bar{x})$, nominal expenditure in the DM is

$$d_{t+1} = \begin{cases} \hat{m}_t & \text{if } \phi_{t+1}\hat{m}_t < \bar{z} \\ \bar{z}/\phi_{t+1} & \text{if } \phi_{t+1}\hat{m}_t \geq \bar{z} \end{cases}$$

Given i.i.d. shocks, it makes sense to look for a stationary equilibrium where real balances are constant: $\phi_t M_t = z \forall t$. This implies $\phi_t/\phi_{t+1} = \zeta_{t+1}$ and

$$d_{t+1} = \begin{cases} \bar{z}\zeta_{t+1}/\phi_t & \text{if } \zeta_{t+1} < \phi_t\hat{m}_t/\bar{z} \\ \hat{m}_t & \text{if } \zeta_{t+1} \geq \phi_t\hat{m}_t/\bar{z} \end{cases}$$

Therefore we can write

$$\begin{aligned} E_t V_{t+1}(\hat{m}_t) &= \alpha\sigma \int_0^{\phi_t\hat{m}_t/\bar{z}} \left[u(\bar{x}) + W_{t+1}\left(\hat{m}_t - \bar{z}\zeta_{t+1}/\phi_t\right) \right] dG(\zeta_{t+1}) \\ &+ \alpha\sigma \int_{\phi_t\hat{m}_t/\bar{z}}^{\infty} \left[u(x_{t+1}) + W_{t+1}(0) \right] dG(\zeta_{t+1}) \\ &+ \alpha\sigma E_t \left[-c\left(x_{t+1}^s\right) + W_{t+1}(\hat{m}_t + d_{t+1}^s) \right] + (1 - 2\alpha\sigma) E_t W_{t+1}(\hat{m}_t), \end{aligned} \tag{18}$$

where x_{t+1}^s and d_{t+1}^s are the terms of trade when selling, which as above do not depend on the seller's money. Indeed, the bargaining solution is still given by

$$g(x_{t+1}) = \phi_{t+1}\hat{m}_t = z/\zeta_{t+1}.$$

Using this, we can differentiate (18) and insert $E_t V_{t+1}'(\hat{m}_t)$ into (17) to get

$$1 + r = \alpha\sigma \int_{z/\bar{z}}^{\infty} \left[\frac{u'(x_{t+1})}{g'(x_{t+1})} - 1 \right] \frac{dG(\zeta_{t+1})}{\zeta_{t+1}} + E_t\left(\frac{1}{\zeta_{t+1}}\right). \tag{19}$$

---

[21] This is not meant to be a big deal, and we could proceed differently, but here we are following earlier models where agents sometimes leave something behind when they go to the DM. See Geromichalos et al. (2007), Lagos and Rocheteau (2008), and Lester et al. (2009). The issue could be avoided if we set $\theta = 1$, or we use an alternative pricing mechanism, like proportional instead of Nash bargaining, or Walrasian price taking, since in these cases buyers' surplus is globally inreasing in m.

To find the equilibrium , simply solve (19) for z. In fact, note that no-arbitrage implies the following version of the Fisher equation for our stochastic economy,

$$1 + i_t = \frac{1+r}{E_t(1/\zeta_{t+1})}, \tag{20}$$

where $1 + r = 1/\beta$. Given this, (19) can be rewritten

$$i_{t+1}\zeta_t E_t \left(\frac{1}{\zeta_{t+1}} \right) = \int_{z/\bar{z}}^{\infty} \ell(x_{t+1}) dG(\zeta_{t+1}) \tag{21}$$

where $\ell(x)$ is the marginal benefit of liquidity defined in (13). In the stochastic version, agents still equate the cost and benefit of liquidity at the margin, but since they need to take expectations (21) replaces (15).

Also, in the stochastic economy we need to be a more careful with central bank policy, since setting the nominal rate i is not the same as pinning down a path for M. That is, a given i is consistent with many different stochastic processes for money growth, as long as the average return on cash $E_t(1/\zeta_{t+1})$ satisfies (20). Nevertheless, it is not hard to verify that the Friedman rule, $i_t = 0$ for all t, is optimal, and that it still achieves the first best iff $\theta = 1$. But there can be many paths for M_t that are consistent with $i_t = 0$ for all t. See Lagos (2009) for an in-depth analysis of these issues. We return now to the effects of fully-anticipated inflation.

3.4   Money and Capital

Because of worries about the theory being "removed" from mainstream macro, we sketch the extension that includes investment and fiscal policy in Aruoba et al. (2009). For simplicity, we ignore long-run technical change (see Waller 2010). Also, in this version, capital K is a factor of production, but it does not compete with M as media of exchange. To motivate this, one can assume K is not portable, making it hard to trade directly in the DM, but of course this does not explain why claims to capital cannot circulate. On the one hand, this is no different from the result that agents in the DM cannot trade claims to future income: this is precluded by imperfect commitment and monitoring. On the other hand, if capital trades in the CM, one can imagine certified claims on K that

might also circulate in the DM. We think monetary theorists do not yet have a definitive stance on this issue, but one approach is to introduce additional informational frictions. It would suffice e.g. to assume counterfeit claims to K can be costlessly produced, and are not recognizable in the DM, even if they are in the CM. Then agents will not accept claims to K in the DM, and M must serve as the medium of exchange.[22]

Assume the CM technology produces output $f(K, H)$ that can be allocated to consumption or investment, while the DM technology is represented by a cost function $c(x, k)$ that gives an agent's disutility of producing x when he has k, where lower (upper) case denotes individual (aggregate) capital. The CM problem is

$$
\begin{aligned}
W(m, k) &= \max_{X, H, \hat{m}, \hat{k}} \left\{ U(X) - H + \beta V(\hat{m}, \hat{k}) \right\} \qquad (22) \\
\text{st } x &= \phi(m - \hat{m}) + w(1 - t_h) H + [1 + (\rho - \Delta)(1 - t_k)] k - \hat{k} - T,
\end{aligned}
$$

where ρ is the rental rate, Δ the depreciation rate, and we incorporate income taxes in the CM. The FOC for $(X, \hat{m}, \hat{k})$ are

$$
\begin{aligned}
U'(X) &= \frac{1}{(1 - t_h) w} \\
\frac{\phi}{w(1 - t_h)} &= \beta V_1(\hat{m}, \hat{k}) \qquad (23) \\
\frac{1}{w(1 - t_h)} &= \beta V_2(\hat{m}, \hat{k}).
\end{aligned}
$$

Generalizing what we found in the baseline model, $(\hat{m}, \hat{k})$ is independent of (m, k), and W is linear with $W_1(m, k) = \phi/w(1 - t_h)$ and $W_2(m, k) = [1 + (\rho - \Delta)(1 - t_k)]/w(1 - t_h)$.

In the DM, instead of assuming that agents may be consumers or producers depending on who they meet, we now proceed as follows. After the CM closes, as discussed earlier, we assume agents draw preference and technology shocks determining whether they can consume or produce, with γ denoting the probability of being a consumer and of being a producer. Then the DM opens and consumers and producers are matched bilaterally.

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[22] This line not especially elegant, but seems logically consistent. Lester et al. (2009, 2010) attempt to take the idea more seriously, following models of money and private information like Williamson-Wright (1994) or Berentsen-Rocheteau (2004), and earlier suggestions by Freenman (1989), but it raises technical challenges. A promising route has been proposed by Rocheteau (2009) (see also Li and Rocheteau 2009, 2010). Alternatively, Lagos and Rocheteau (2008) allow K and M to both be used as media of exchange, and show M can still be essential if K is not sufficiently productive or the need for liquidity is great, although in that model K and M must pay the same return in equilibrium.

This story helps motivate why capital cannot be used for DM payments: one can say that it is fixed in place physically, and consumers have to travel without their capital to producers' locations to trade. Thus, producers can use their capital as an input in the DM but consumers cannot use their capital as payment. With preference and technology shocks, the equations again look exactly the same as when we had random matching and specialization except γ replaces $\alpha\sigma$. Also, it is possible under this interpretation to easily replace Nash bargaining with Walrasian pricing, which allows us to quantify the holdup problems.

Using bargaining for now, one can again show $d = m$, and that the Nash outcome depends on the consumer's m but not the producer's M, and on the producer's K but not the consumer's k. Abusing notation slightly, $x = x(m, K)$ solves $g(x, K) = \phi m / w (1 - t_h)$, where

$$g(x, K) \equiv \frac{\theta c(x, K) u'(x) + (1 - \theta) u(x) c_1(x, K)}{\theta u'(x) + (1 - \theta) c_1(x, K)}$$

generalizes (10). Then we have the following version of (9)

$$
\begin{aligned}
V(m, k) &= W(m, k) + \gamma \left\{ u\left[x(m, K)\right] - \frac{\phi m}{w (1 - t_h)} \right\} \\
&\quad + \gamma \left\{ \frac{\phi M}{w (1 - t_h)} - c\left[x(M, k), k\right] \right\}.
\end{aligned}
$$

Differentiating this, then inserting V_1 and V_2, market clearing $k = K$ and $m = M$, and equilibrium prices $\phi = w (1 - t_h) g(x, K)/M$, $\rho = f_1(K, H)$, and $w = f_2(K, H)$, into (23), we have

$$U'(X_t) = \frac{1}{(1 - t_h) f_2(K_t, H_t)} \tag{24}$$

$$\frac{g(x_t, K_t)}{M_t} = \frac{\beta g(x_{t+1}, K_{t+1})}{M_{t+1}} \left[1 - \gamma + \gamma \frac{u'(x_{t+1})}{g_1(x_{t+1}, K_{t+1})} \right] \tag{25}$$

$$U'(X) = \beta U'(X_{t+1}) \left\{ 1 + [f_1(K_{t+1}, H_{t+1}) - \Delta] (1 - t_k) \right\} \tag{26}$$

$$\quad - \beta \gamma \left[c_2(x, K) - c_1(x, K) \frac{g_2(x, K)}{g_1(x, K)} \right].$$

And we have the resource constraint

$$X_t + G = f(K_t, H_t) + (1 - \Delta) K_t - K_{+1}. \tag{27}$$

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Equilibrium is defined as (positive, bounded) paths for $\{x, X, K, H\}$ satisfying (24)-(27), given monetary and fiscal policy, plus an initial condition K_0. As a special case, in nonmonetary equilibrium we have $x = 0$ while $\{X, H, K\}$ solves the system ignoring (25) and setting the last term in (26) to 0. Those conditions are exactly the equilibrium conditions for $\{X, H, K\}$ in the standard nonmonetary growth model described e.g. in Hansen (1985).[23] So we nest standard real business cycle theory as a special case. In monetary equilibria, we get something even more interesting. The last term in (26) generally captures the idea that if a producer buys an extra unit of capital in the CM, his marginal cost is lower in the DM for a given x, but x increases as an outcome of bargaining. This is a holdup problem on investment, parallel to the one on money demand discussed earlier. With a double holdup problem there is no value of θ that delivers efficiency, which has implications for the model's empirical performance and welfare predictions.

Aruoba et al. (2009) calibrate the model with bargaining and with price taking and compare the quantitative predictions. Interestingly, although the bargaining version generates a somewhat bigger welfare cost of inflation, the price-taking version generates much bigger effects of monetary policy on investment. Intuitively this is because K in the bargaining version is relatively low and unresponsive to what happens in the DM due to the holdup problem. That is, the returns to investing accrue mostly from CM trade, since the seller has to split with the buyer whatever surplus arises from having more K in the DM. This makes K unresponsive to taxing DM trade via inflation. In the price-taking version he effects of inflation on K are big compared to what has been found in earlier work, because with no holdup problem, the returns to investing are affected by taxing DM trade. One can put this model to many other uses, such as quantifying the impact of these holdup problems.

We do not have space to go into all the numerical results, but we do want to emphasize the methodological point that it is not hard to integrate modern monetary theory and

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[23] At least, in the deterministic version of Hansen (1985), but at this stage it is not hard to add technoloy and other shocks, as in Aruoba (2009), Aruoba and Shorfeide (2008), or Telyukova and Visschers (2009).

mainstream macro. The only quantitative result we mention is this. In case one wonders what fraction of output is produced in the DM, it is easy to see the answer is less than 10%. To verify this, note the following: Since there are γ buyers in the DM each period, and they each spend M, the share of total output produced in the DM is $\gamma M/PY = \gamma/v$, where $v = PY/M$ is velocity. If M is measured by $M1$ then v is around 5 in annual data, and since $\gamma \leq 1/2$, we are done. For actual calibrated values of γ, the share is slightly less than this upper bound. Of course if we change the frequency (from annual to quarterly e.g.) PY changes, but so does the calibrated value of γ, keeping the DM share about the same. This would not work in standard cash-in-advance models, where agents always spend all their money each period. This is important because it shows that details, like stochastic trading opportunities, as well as the two-sector structure, matter, even though 90% of output here is produced in a CM that looks exactly like standard neoclassical growth theory.

3.5 The Long-Run Phillips Curve

In the baseline model, without capital, we saw that DM output is decreasing in anticipated inflation, while CM output is independent of anticipated inflation. It is not true that CM output is independent of anticipated inflation in the model with capital in the previous section, because we assumed K enters $c(x, K)$. If this is not the case, and $c_K(x, K) = 0$, then the last term in (26) vanishes, K drops out of (25), and the system dichotomizes: we can independently solve (25) for the DM allocation x and the other three equations for the CM allocation (X, K, H). In this dichotomous case, monetary policy affects x but not (X, K, H). This is why we assumed K enters $c(x, K)$. In this section, without capital, we break the dichotomy using nonseparable utility. In fact, here we take the Phillips curve literally, and model the relation between inflation and unemployment. To make this precise, first, we introduce another friction to generate unemployment in the CM, and second, we re-cast the DM as a pure exchange market, so that unemployment is determined exclusively in the CM.

To give some background, a principle explicated in Friedman (1968) is that, while

there may exist a Phillips curve trade-off between inflation and unemployment in the short run, there is no trade-off in the long run. The *natural rate* of unemployment is defined as "the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labor and product markets" (although, as Lucas 1980 notes, Friedman was "not able to put such a system down on paper"). Friedman (1968) said monetary policy cannot engineer deviations from the natural rate in the long run. However, he tempered this view in Friedman (1977) where he said "There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, provided allowance is made for the effect of price change on the real cost of holding money balances." Here we take this real balance effect seriously.

Of the various ways to model unemployment, in this presentation we adopt the indivisible labor model of Rogerson (1988).[24] This has a nice bonus feature: we do not need quasi-linearity, because in indivisible-labor models agents act *as if* utility were quasi-linear. To make the point, we revert to the case where X is produced one-for-one with H, but now $H \in \{0, 1\}$ for each individual. Also, as we said, to derive cleaner results we use a version where there is no production in the DM. Instead, agents have an endowment $\bar{x}$, and gains from trade arise due to preference shocks. Thus, DM utility is $v^j(x, X, H)$ where j is a shock realized after (X, H) is chosen in the CM. Suppose $j = b$ or s with equal probability, where $\partial v^b(\cdot)/\partial x > \partial v^s(\cdot)/\partial x$, and then in the DM everyone that draws b is matched with someone that draws s. The indices b and s indicate which agents will be buyers and sellers in matches, for obvious reasons. We also assume here that there is discounting between one DM and the next CM, but not between the CM and DM, but this is not important. What is interesting is nonseparability in $v^j(x, X, H)$.

As in any indivisible labor model, agents choose a *lottery* $(\ell, X_1, X_0, \hat{m}_1, \hat{m}_0)$ in the

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[24]The approach follows Rocheteau et al. (2007) and Dong (2009). Alternatvely, Berentsen et al. (2009) and Liu (2009) use the unemployment theory in Mortensen and Pissarides (1994).

CM where ℓ is the probability of working $H = 1$, while X_H and $\hat{m}_H$ are CM purchases of goods and cash conditional on H (if one does not like lotteries, the equilibrium can also be supported using pure Arrow-Debreu contingent commodity markets, as in Shell and Wright 1993). There is no direct utility generated in the CM; utility is generated by combining (X, H) with x in the DM. Hence,

$$W(m) = \max_{\ell, X_1, X_0, \hat{m}_1, \hat{m}_0} \{\ell V(\hat{m}_1, X_1, 1) + (1 - \ell)V(\hat{m}_0, X_0, 0)\} \tag{28}$$
$$\text{st } 0 \leq \phi m - \ell\phi\hat{m}_1 - (1 - \ell)\phi\hat{m}_0 + w\ell - T - \ell X_1 - (1 - \ell)X_0.$$

As is well known, X and $\hat{m}$ depend on H, in general, but if V is separable between X and H then $X_0 = X_1$, and if V is separable between $\hat{m}$ and H then $\hat{m}_1 = \hat{m}_0$. But the function V is endogenous. This is another argument for making the role of money explicit, instead of, say, simply sticking it in the utility function: one cannot simply *assume* V is separable (or homothetic or whatever), one has to *derive* its properties, and this imposes discipline on both theory and quantitative work.[25]

Letting λ be the Lagrangian multiplier for the budget constraint, FOC for an interior solution are

$$0 = V_2(\hat{m}_H, X_H, H) - \lambda, \text{ for } H = 0, 1 \tag{29}$$

$$0 = V_1(\hat{m}_H, X_H, H) - \lambda\phi, \text{ for } H = 0, 1 \tag{30}$$

$$0 = V(\hat{m}_0, X_0, 0) - V(\hat{m}_1, X_1, 1) + \lambda(X_1 - X_0 - 1 + \phi\hat{m}_1 - \phi\hat{m}_0) \tag{31}$$

$$0 = \ell - \ell X_1 - (1 - \ell)X_0 + \phi[m + \gamma M - \ell\hat{m}_1 - (1 - \ell)\hat{m}_0]. \tag{32}$$

One can guarantee $\ell \in (0, 1)$, and show the FOC characterize the unique solution, even though the objective function is not generally quasi-concave (Rocheteau et al. 2007). Given $V(\cdot)$, (29)-(31) constitute 5 equations that can be solved under weak regularity conditions for $(X_1, X_0, \hat{m}_1, \hat{m}_0, \lambda)$, independent of ℓ and m. Then (32) can be solved for individual labor supply as a function of money holdings at the start of the period, $\ell = \ell(m)$. Notice $\hat{m}_H$ may depend on H, but not m, and hence we get at most a two-point distribution in the DM. Also, $W(m)$ is again linear, with $W'(m) = \lambda\phi$. This is

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[25] This point is played up in Aruoba and Chugh (2008), in the context of optimal tax theory, where proprties of $V(\cdot)$ can matter a lot for the results.

what we meant above when we said that agents act as if they had quasi-linear preferences in the model with indivisible labor and lotteries.

In DM meetings, for simplicity we assume take-it-or-leave-it offers by the buyer ($\theta = 1$). Also, although it is important to allow buyers' preferences to be nonseparable, we do not need this for sellers, so we make their preferences separable. Then as in the baseline model, the DM terms of trade do not depend on anything in a meeting except the buyer's m: in equilibrium, he pays $d = m$, and chooses the x that makes the seller just willing to accept, independent of the seller's (X, H). In general, buyers in the DM who were employed or unemployed in the CM get a different x since they have different m. In any case, we can use the methods discussed above to describe $V(\cdot)$, differentiate it, and insert the results into (29)-(31) to get conditions determining $(x_1, x_0, X_1, X_0, \lambda)$. From this we can compute aggregate employment $\bar{\ell} = \ell(M)$.

It is now routine to see how endogenous variables depend on policy. First, it is easy to check $\partial x / \partial i < 0$, since as in any such model the first-order effect of inflation is to reduce DM trade. A calculation then implies that the effect on unemployment depends on the cross derivatives of buyers' utility function as follows:

1. if $v^b(x, X, H)$ is separable between (X, H) and x, then $\partial \bar{\ell} / \partial i = 0$

2. if $v^b(x, X, H)$ is separable between (x, X) and H, then $\partial \bar{\ell} / \partial i > 0$ iff $v^b_{Xx} < 0$

3. if $v^b(x, X, H)$ is separable between (x, H) and X, then $\partial \bar{\ell} / \partial i > 0$ iff $v^b_{xH} < 0$

The economic intuition is simple. Consider case 2. Since inflation reduces x, if x and X are complements then it also reduces X, and hence reduces the $\bar{\ell}$ used to produce X; but if x and X are substitutes then inflation increases X and $\bar{\ell}$. In other words, when x and X are substitutes, inflation causes agents to move from DM to CM goods, increasing CM production and reducing unemployment. A similar intuition applies in Case 3, depending on whether x is a complement or substitute for leisure. In either case, we can get a downward-sloping Phillips curve under simple and natural conditions, without any complications like imperfect information or nominal rigidities. This relation is exploitable

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by policy makers in the long run: given the right cross derivatives, it is indeed feasible to achieve permanently lower unemployment by running a higher anticipated inflation, as Keynesians used to (still?) think. But it is not optimal: it is easy to check that the efficient policy is still Friedman's prescription, $i = 0$.

3.6   Benchmark Summary

We believe this benchmark delivers a lot of insight. A model with only CM trade could not capture the fundamental role of money, which is why one has to resort to short cuts like cash-in-advance or money-in-the-utility-function specifications. The earlier work on microfoundations with only DM trade gets at the salient role of money, but requires harsh restrictions or it becomes analytically intractable. There are other devices, including Shi (1997) and Menzio et al. (2009), which achieve some similar results, but one reason to like this benchmark model is that, in addition to imparting tractability, it integrates search and competitive markets, and this reduces the gap between the microfoundations literature and mainstream macro. Alternating markets themselves do not yield tractability; we also need something like quasi-linearity, or indivisibilities. This does not seem a huge price to pay, especially for anyone who uses the indivisible labor model anyway, but we could also dispense with these assumptions if we were willing to rely on numerical methods.[26]

Before we move to new results, however, we mention a variation by Rocheteau and Wright (2005), since this is something we use in several applications below. This extension considers an environment with two permanently distinct types, called buyers and sellers, where the former are always consumers in the DM and the latter are always

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[26]There are many applications of this model. A sample includes: Aruoba and Chugh (2007), Gomis–Porqueras and Peralta-Alva (2009), Martin (2007) and Waller (2009) study optimal monetary and fiscal policy. Banks are introduced by Berentsen et al. (2008), Chiu and Meh (2009), Li (2007), He et al. (2007), and Becivenga and Camera (2008). Boel and Camera (2006) and Berentsen and Waller (2009) study the interaction between money and bonds. Hoerova et al. (2007), Berentsen and Monnet (2008), Kahn (2009) and Andolfatto (2010a, 2010b) discuss details of monetary policy implementation. Guerrieri and Lorenzoni (2009) analyze the effects of liquidity on business cycles. Lagos and Rocheteau (2005), Lui et a. (2010), and Nosal (2010) study how velocity (or the time it takes to spend one's money) depend on inflation. These last applications are also relevant for the following reason. One sometimes hears that anything one can do with a search-based theory could be replicated with a cash-in-advance or money-in-the-utility-function specification. That is definitely *not* the case in these papers, which are concerned mainly about the effect of inflation on search behavior (as is true of some papers in the first-generation, including Li 1994, 1995).

producers in the DM. One could not have permanent buyers or permanent sellers in the DM if there were no CM, since no one would produce in one DM if they cannot spend the proceeds in a subsequent DM. Here sellers may want to produce in every DM, since they can spend the money in the CM, and buyers may want to work in every CM, since they need the money for the DM. Monetary equilibrium no longer entails a degenerate distribution, but all sellers choose $m = 0$, while all buyers choose the same $m > 0$. Notice that with two types the distribution of money holdings is degenerate only conditional on type, as we encountered earlier in Section 3.5, but this is still tractable. Indeed, the key property of the model in terms of tractability is that the choice of $\hat{m}$ is history independent, not that it is the same for all agents.

Having two types is interesting for several reasons, including the fact that one can introduce a generalized matching technology, and one can incorporate a participation decision for either sellers or buyers. By way of analogy, Pissarides (2000) has two types (workers and firms), while Diamond (1982) has only one (traders), which allows the former to consider more general matching and entry. Note also that, in a sense, having two types makes the model similar to the models presented in Section 2 with $m \in \{0, 1\}$. And there are many applications where two types just seems more natural. Actually, for all of this, we do not really need permanently distinct types: it would be equivalent to have types determined each period, as long as the realization occurs before the CM closes – the important distinction concerns whether agents can choose $\hat{m}$ conditional on type. This would be the case e.g. if we took the model at the end of Section 3.1, with preference and technology shocks in the DM replacing random matching, but alternatively assumed the realizations of the these shocks were known before agents chose $\hat{m}$.

4 New Models of Old Ideas

Although one of our goals is to survey existing models, we also want to present new material. In this section we lay out some new models of ideas in earlier Monetarist or Keynesian traditions. This shows how similar results can be derived in our framework,

although sometimes with interesting differences. We first introduce additional informational frictions to show how signal extraction problems can lead to a short-run Phillips curve, as in Old Monetarist economics. Then we analyze what happens when prices are sticky, for some unspecified reason, as in Keynesian models. Then we give a New Monetarist spin on sticky prices with some very different implications. As discussed in the introduction, there are some papers in New Monetarist economics that already explore some of these issues, with embellishments that allow one to take the theories to the data. The goal here is to come up with simple models to illustrate basic qualitative properties, although we also discuss a few empirical implications.

4.1 The Old Monetarist Phillips Curve

Here we discuss some ideas about the correlations defining the short-run Phillips curve, and the justification for predictable monetary policy, in Old Monetarist economics. Given that we already discussed a model where unemployment appears explicitly in Section 3.5, we now for simplicity take the Phillips curve to mean a positive relation between money growth or inflation, on the one hand, and output, on the other hand. Also, we use the setup where there are two distinct types called buyers and sellers. In particular, there is a unit mass of agents, half buyers and half sellers. Further, during a period CM trade occurs first, followed by DM trade, and we sometimes describe the CM and DM subperiods as the day and night markets to keep track of the timing. Finally, to yield clean results we sometimes use $u(x) = \log x$.[27]

We already studied a certain type of unanticipated inflation in Section 3.3, but in order to build a model in the spirit of Lucas (1972), we now include both real and monetary shocks. First, some fraction of the population is inactive each period: a fraction ω_t of buyers participates in both markets in period t, while the fraction $1 - \omega_t$ rests. As well, a fraction ω_t of sellers will not participate in the DM of period t and in the CM of period $t + 1$. Assume that ω_t is a random variable, and realizations are not publicly observable. Second, the money growth rate μ_t is random, and realizations are

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[27]Many applilcations of the general framework assume $u(0) = 0$, for technical reasons; we do not need this because we assume $\theta = 1$ in the bargaining solution below.

not publicly observable. So that agents have no direct information on the current money injection, only indirect information coming from prices, we add some new actors to the story. We call them government agents, and assume that in the CM in each period t, a new set of such agents appears. They have linear utility $X - H$, and can produce X one-for-one with H. If $\mu_t > 0$, the central bank prints money and gives it to these agents, and they collectively consume $\phi_t M_{t-1} \mu_t$, and if $\mu_t < 0$ they retire money by collectively producing $-\phi_t M_{t-1} \mu_t$. Their role is purely a technical one, designed to make signal extraction interesting.

In the CM, agents learn last period's money stock M_{t-1} and observe the price ϕ_t, but not the current aggregate shocks ω_t and μ_t. For an individual buyer acquiring money in the CM, the current value of money may be high (low), either because the demand for money is high (low) or because money growth is low (high). To ease the presentation, assume take-it-or-leave-it offers by buyers in the DM, $\theta = 1$, and assume that a seller's cost function is $c(h) = h$. This implies

$$x_t = \beta m_t E[\phi_{t+1} \mid \phi_t]. \tag{33}$$

An active buyer's FOC from the CM reduces by the usual manipulations to

$$-\phi_t + \beta E[\phi_{t+1} \mid \phi_t] u'(x_t) = 0. \tag{34}$$

Given that the mass of buyers is $1/2$, market clearing implies

$$\omega_t m_t / 2 = (1 + \mu_t) M_{t-1}. \tag{35}$$

If μ_t were a continuous random variable, in principle we could solve for an equilibrium as in Lucas (1972). For illustrative purposes, however, we adopt the approach in Wallace (1992), using a finite state space (see also Wallace 1980). To make the point, it suffices to assume μ_t and ω_t are independent i.i.d. processes, where μ_t is μ_1 or $\mu_2 < \mu_1$ each with probability $1/2$, and ω_t is ω_1 or $\omega_2 < \omega_1$ each with probability $1/2$. We then assume that

$$\frac{\omega_1}{1 + \mu_1} = \frac{\omega_2}{1 + \mu_2}, \tag{36}$$

so that agents cannot distinguish between high money demand and high money growth, on the one hand, or low money demand and low money growth, on the other.

Using (33)-(35) we obtain closed-form solutions for prices and quantities. Let $\phi(i,j)$ and $q(i,j)$ denote the CM price and the DM quantity when $(\mu_t, \omega_t) = (\mu_i, \omega_j)$. Then

$$\phi(i,j) = \frac{\omega_j}{2(1+\mu_i)M_{t-1}}, \text{ for } i = 1, 2 \tag{37}$$

$$q(i,j) = \frac{\beta(\omega_1 + \omega_2)(2 + \mu_1 + \mu_2)}{4(1+\mu_1)(1+\mu_2)\omega_j}, \text{ for } (i,j) = (1,2), (2,1) \tag{38}$$

$$q(1,1) = q(2,2) = \frac{\beta(\omega_1 + \omega_2)^2(2 + \mu_1 + \mu_2)}{8(1+\mu_1)(1+\mu_2)\omega_1\omega_2}. \tag{39}$$

Let total output in the day and night be $Q^d(i,j)$ and $Q^n(i,j)$ in state (μ_i, ω_j). Given $\mu_1 > \mu_2 \geq 0$, we have

$$Q^d(i,j) = \phi_t M_t = \omega_j/2, \tag{40}$$

for $i, j = 1, 2$ from (37). Further, from (38), (39), and 36,

$$Q^n(1,2) = Q^n(2,1) = \frac{\beta(\omega_1 + \omega_2)(2 + \mu_1 + \mu_2)}{8(1+\mu_1)(1+\mu_2)} \tag{41}$$

$$Q^n(1,1) = \frac{\beta(\omega_1 + \omega_2)^2(\mu_1 + \mu_2)}{16\mu_1\mu_2\omega_2} \tag{42}$$

$$Q^n(2,2) = \frac{\beta(\omega_1 + \omega_2)^2(2 + \mu_1 + \mu_2)}{16(1+\mu_1)(1+\mu_2)\omega_1}. \tag{43}$$

Total real output is $Q(i,j) = Q^d(i,j) + Q^n(i,j)$. From (40), Q^d depends only on the real shock. That is, when the number of active buyers is high (low), money demand is high (low), and the price of money is high (low). Thus, active buyers collectively produce more (less) in the day to acquire money when the number of active buyers is high (low). And at night, one can show that $Q^n(2,2) < Q^n(1,2) = Q^n(2,1) < Q^n(1,1)$.

Figure 1 displays the scatter plot of aggregate output Q against money growth μ, using time series observations generated by the model. The four dots represent money and output in each of the four states, depicting a clear positive correlation between μ and Q. This results from agents' confusion, since if there were full information about the shocks we would have

$$Q^n(i,j) = \frac{\beta(\omega_1 + \omega_2)(2 + \mu_1 + \mu_2)}{8(1+\mu_1)(1+\mu_2)} \text{ for all } (i,j)$$

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as in Figure 2. Confusion results from the fact that, if money growth and money demand are both high (low), then agents' subjective expectation of ϕ_{t+1} is greater (less) than the objective expectation, so more (less) output is produced in the DM than under full information. Except for technical details, this nonneutrality of money is essentially that in Lucas (1972) and Wallace (1980,1992).

A standard narrative associated with ideas in Friedman (1968) and Lucas (1972,1976) is that 1960s and 1970s macroeconomic policy erred because policy makers treated the dots in (their empirical version of) Figure 1 as capturing a structural relationship between money growth and output. Policy makers took for granted that more output is good and more inflation is bad, and they took the observed correlation as evidence that if the central bank permanently increased money growth this would achieve permanently higher output. Although we saw in Section 3.5 that permanent trade-offs are a theoretical possibility, the point to be emphasized is that observed empirical relations by no means constitute evidence that there is an actual trade-off. What happens in this example if we permanently set money growth to μ_1? The data points we would generate would be the two squares in Figure 1, with high (low) output when money demand is high (low). Rather than increasing output, higher inflation lowers output in all states of the world.

What is optimal policy? If we can find a monetary policy rule that achieves $x = x^*$ in all states, it is optimal. From (34), we require $\phi_t = \beta E\left[\phi_{t+1}\right]$, from which we can obtain

$$1 + \mu_{t+1} = \frac{\beta \omega_{t+1}}{\omega_t}. \tag{44}$$

This is the Friedman rule, dictating that the money supply decrease on average at the rate of time preference, with higher (lower) money growth when money demand is high (low) relative to the previous period. It might appear hard for the monetary authority to implement such a rule, because it seems to require that they know the shock ω_t. However, all we need is $\phi_{t+1} = \phi_t/\beta$, so they need not observe the shock, and can attain efficiency simply by engineering a constant rate of deflation. In equilibrium, the price level is predictable, and carries no information about the aggregate state. It is not

necessary for the price level to reveal aggregate information, since efficiency requires that buyers acquire the same real balances in the CM and receive the same quantity in the DM, independent of the shocks.

In a sense, these results are consistent with the thrust of Friedman (1968) and Lucas (1972). Monetary policy can confuse price signals, and this can result in a nonneutrality that generates a Phillips curve. However, the policy prescription derived from the model is in line with Friedman (1969) rather than Friedman (1968): the optimal money growth rate is not constant, and should respond to aggregate real disturbances to correct intertemporal distortions. This feature of the model appears consistent with some of the reasons that money growth targeting by central banks failed in practice in the 1970s and 1980s. Of course we do not intend the model in this section to be taken literally. It is meant mainly as an example to illustrate once again, but here in the context of our benchmark framework, the pitfalls of naive policy making based on empirical correlations that are incorrectly assumed to be structural.[28]

4.2 New Keynesian Sticky Prices

We now modify our benchmark model to incorporate sticky prices, capturing ideas in New Keynesian economics along the lines of Woodford (2003) and Clarida et al. (1999). We will first construct a cashless version, as does Woodford (2003), where all transactions are carried out using credit, then modify it to include currency transactions. New Keynesian models typically use monopolistic competition, where individual firms set prices, usually according to a Calvo (1983) mechanism. Here, to fit into our benchmark model, we assume that some prices are sticky in the DM. Again we use the version with permanently distinct buyer and seller types, with the mass of each set to $1/2$, and set $c(h) = h$.

In the cashless model, in spite of the fact that money is not held or exchanged, prices are denominated in dollars. Sticky price modelers do not usually attempt any justification for this, other than stating that they observe this. We follow in that tradition in this

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[28] Faif and Li (2009) have a more general quantitative analysis of signal extraction and the cost of unanticipated inflation. They find that the welfare cost of signal extraction is very low. They also find the cost of anticipated inflation is fairly low, but note that they use Walrasian pricing and not Nash bargaining in their DM.

section. As in the benchmark model, the price of money in the CM, ϕ_t, is flexible. In the DM, each buyer-seller pair conducts a credit transaction where goods are received by the buyer in exchange for a promise to pay in the next CM. To support these credit transactions we assume that there is perfect memory or record keeping in every meeting. That is, if a buyer defaults on an obligation, it is observed and an exogenous legal system imposes a severe punishment. Thus, in equilibrium, all borrowers pay their debts.

In the DM, suppose that in an individual match the terms of trade between a buyer and seller is either flexible with probability 1/2, or fixed with probability 1/2. In a flexible match, the buyer makes a take-it-or-leave-it offer. Let $1/\psi_t$ be the number of dollars a buyer offers to pay in the following CM for each unit produced by a flexible-price seller in the DM, and s_t^1 be the quantity of goods produced by the seller. Then the bargaining outcome satisfies $s_t^1 = \beta s_t^1 \phi_{t+1}/\psi_t$, so that $\psi_t = \beta\phi_{t+1}$. Now, assume that in each fixed-price exchange in the DM, the seller is constrained to offering a contract that permits buyers to purchase as much as they like in exchange for $1/\psi_{t-1}$ dollars in the next CM per unit purchased. In a flexible price contract, the buyer chooses $s_t^1 = x^*$. However, in a fixed-price contract, the buyer chooses the quantity s_t^2 to maximize $u(s_t^2) - s_t^2 \phi_{t+1}/\phi_t$, which gives

$$u'(s_t^2) = \phi_{t+1}/\phi_t. \tag{45}$$

So far there is nothing to determine the sequence $\{\phi_t\}_{t=0}^\infty$. In Woodford (2003), one solution approach involves first determining the price of a nominal bond. In our model, in the CM of period t the price z_t in units of money of a promise to pay one unit of money in the CM during period $t+1$ is given by

$$z_t = \beta\phi_{t+1}/\phi_t. \tag{46}$$

Following Woodford one could then argue that z_t can somehow be set by the central bank, perhaps in accordance with a Taylor rule. Then, given determinacy of z_t, we can solve for $\{\phi_t\}_{t=0}^\infty$ from (46). It seems consistent with New Keynesian logic to consider $\{\phi_t\}_{t=0}^\infty$ as an exogenous sequence of prices that can be set by policy. In terms of what

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matters, it is equivalent to say that government sets the path for the inflation rate, $\pi_t = \phi_{t-1}/\phi_t$.

From (??) the path for inflation is irrelevant for s_t^1, but from (45) s_t^2 is increasing in π_{t+1}. In fixed-price transactions, buyers write a credit contract under which the nominal payment in the CM is determined by the flexible-price contract from the previous period. When inflation increases, the implicit real interest rate on credit in fixed-price contracts falls, and the buyer purchases more. Note that, when the buyer in a fixed-price meeting at t repays the loan in period $t+1$, he produces $s_t^2/\beta\pi_{t+1}$. Generally, the effect of inflation depends on preferences, but If we set $u(x) = \log x$, then CM production is invariant to the path of π_t, and the only component of aggregate output affected by inflation is production in fixed-price DM meetings. From (45), $s_t^2 = \pi_{t+1}$, so there is a short-run and long-run Phillips curve: a temporarily higher rate of inflation increases output temporarily, and a permanently higher rate increases it permanently. The model predicts that the Phillips curve exists in the data and can be exploited by policy. Should policy exploit this? No. Equilibrium is generally inefficient due to sticky prices, and this shows up in a suboptimal quantity of output in fixed-price contracts. For efficiency, we require that $s_t^2 = x^*$ which implies from (45) $\phi_t = \phi$, which means 0 inflation. Further, from (46), the optimal nominal bond price consistent with price stability is $z_t = \beta$, the "Wicksellian natural rate."

To get money to play a role, assume a fraction α of meetings are non-monitored in the DM, so the seller does not have access to the buyer's history, and anything that happens in the meeting is private information to the pair.[29] Further, assume the same set of sellers engage in non-monitored meetings for all t. The remaining fraction $1 - \alpha$ of DM meetings are monitored, as in the cashless economy: the seller observes the buyer's history and their interaction is public information. The buyer and seller continue to be matched into the beginning of the next day, before the CM opens, so default is

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[29]This setup has a superficial resemblance to reduced-form models with cash goods and credit goods (Stokey and Lucas 1987), just like the baseline model has a resemblance to simple cash-in-advance models. This is as it should be, since reduced-form models were designed to be descriptive of reality, but it should be clear that there are ingredients in the models presented here that are not in those models.

publicly observable, and we continue to impose punishments that preclude default. The CM, where money and goods are traded, opens in the latter part of the day, and here only prices (not individual actions) are observable. As with credit transactions, half of the money transactions have flexible and half have fixed prices. The type of meeting (monitored or nonmonitored, flexible-price or fixed-price) is determined at random, but a buyer knows in the CM what type of meeting he will have in the following DM.

As in the cashless model, the quantities of goods traded in flexible-price and fixed-price credit transactions are s_t^1 and s_t^2, with $s_t^1 = x^*$ and s_t^2 determined by (45). For flexible-price transactions where there is no monitoring and money is needed, the buyer carries m_t^1 from the CM to the DM and makes a take-it-or-leave-it offer, which involves giving up all the money for

$$x_t^1 = \beta \phi_{t+1} m_t^1, \tag{47}$$

so the implicit flexible price of goods in terms of money is $1/\beta\phi_{t+1}$. In a fixed-price money transaction, the seller must charge a price equal to the flexible money price in the previous period. Therefore, a buyer in a fixed-price money transaction carries m_t^2 into the meeting and spends it all to get x_t^2, where

$$x_t^2 = \beta \phi_t m_t^2. \tag{48}$$

As buyers choose money balances optimally in the daytime, we obtain the following FOC for buyers in monetary flexible-price and fixed-price transactions, respectively:

$$-\phi_t + \beta \phi_{t+1} u'(x_t^1) = 0 \tag{49}$$

$$-\phi_t + \beta \phi_t u'(x_t^2) = 0. \tag{50}$$

Assume that money is injected by the government by lump-sum transfers to sellers during the day, and that M grows at rate μ. In equilibrium, the entire money stock must be held by buyers at the end of the day who will be engaged in monetary transactions at night. Thus, we have the equilibrium condition

$$\frac{\alpha}{2}\left(m_t^1 + m_t^2\right) = M_t \tag{51}$$

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Now, consider the equilibrium where $1/\phi_t$ grows at the rate μ and all real quantities are constant for all t. From (45) and (47)-(51), equilibrium quantities are

$$s_t^1 = x^*$$

$$u'(s_t^2) = 1/(1+\mu)$$

$$u'(x_t^1) = (1+\mu)/\beta$$

$$u'(x_t^2) = 1/\beta.$$

In equilibrium the money growth rate is equal to the inflation rate, and higher money growth increases output in fixed-price relative to flexible-price transactions.

From a policy perspective, we cannot support the efficient allocation $s_t^i = x_t^i = x^*$ for $i = 1, 2$. However, we can maximize the weighted average welfare criterion

$$W(\mu) = \frac{\alpha}{2} \left[u(x_t^1) - x_t^1 + u(x_t^2) - x_t^2 \right] + \frac{(1-\alpha)}{2} \left[u(s_t^1) - s_t^1 + u(s_t^2) - s_t^2 \right].$$

Then we have

$$W'(\mu) = \frac{\alpha}{2\beta u''(x_t^1)} \left(\frac{1+\mu}{\beta} - 1 \right) - \frac{(1-\alpha)}{2(1+\mu)^2 u''(s_t^2)} \left(\frac{1}{1+\mu} - 1 \right). \tag{52}$$

From (52) one can check that the optimal money growth rate is between the Friedman rule and a constant price level. This reflects a trade-off between two distortions: inflation distorts the relative price of flexible- and fixed-price goods, which is corrected by price stability; and inflation results in the standard intertemporal distortion, in that too little of the flexible-price good is purchased with cash, which is corrected by the Friedman rule. We are not the first to point this out (Aruoba and Schorfhiede 2009 provide references to the literature); we simply recast this tradeoff in terms of our New Monetarist model.

What do we learn from this? A central principle of New Monetarism is that it is important to be explicit about the frictions underlying the role for money and related institutions. What do models with explicit frictions tell us that New Keynesian models do not? One line of argument in Woodford (2003) is that it is sufficient to use a cashless model to analyze monetary policy, and the intertemporal monetary distortions corrected by the Friedman rule are secondary to sticky price considerations. Further, he argues

that one can construct monetary economies that behave essentially identically to the cashless economy, so that it is sufficient to analyze the cashless limit. This cashless limit is achieved here if we let $\alpha \to 0$. In the model, quantities traded in different types of transactions are independent of α, and the only effects of changing α are on the price level and the fraction of credit trades. As well, the optimal money growth rate tends to rise as α decreases, with $\mu^* \to 0$ as $\alpha \to 0$.

So while we can construct explicitly a cashless limit in our model, it is apparent to us that confining policy analysis to the cashless economy is not innocuous. A key feature of equilibrium in our model is that the behavior of prices is tied to the aggregate money stock, in line with the quantity theory of money. Thus the model with both cash and credit gives the central bank control over a monetary quantity, not direct control over market interest rates, prices, or inflation. In reality, central banks intervene mainly through exchanges of their liabilities for other assets and lending to financial institutions. Though central banks may conduct such interventions to target an interest rate, it seems important to model accurately the means by which this is done. How else could one evaluate e.g. whether it is preferable in the short run for the central bank to target a short-term nominal interest rate or the growth rate in the money stock?

Moreover, we have to emphasize that it is important to be agnostic, ex ante, concerning which frictions are relevant for policy, and recall from Section 3.2 that New Monetarist models predict that quantitatively the cost of inflation can be quite high. Aruoba and Schorfheide (2010) build a full-fledged model incorporating both New Keynesian rigidities and elements of our New Monetarist framework, and estimate it using Bayesian methods to explicitly compare the two channels identified above, what they call the Friedman channel, and the New Keynesian channel (inefficiency generated by sticky prices and monopolistic competition). They estimate their model under 4 different scenarios, having to do with whether there is Nash bargaining or Walrasian pricing in the DM, and whether they try to fit the short- or long-run elasticity of money demand. In the version with bargaining designed to fit the short-run elasticity, despite a

reasonably-sized New Keynesian friction, the Friedman rule turns out to be optimal after all. The other 3 versions yield optimal inflation rates of -1.5%, -1% and -0.75%. Even considering parameter uncertainty, they never find an optimal inflation rate very close to 0, and conclude that the two channels are about equally important. Moreover, micro-foundations matter for this: in a similar model, except that money demand is generated in by putting M in the utility function, 0 inflation is close to optimal.

So while one can build nominal rigidities into our model and examine cashless limits, we are not at all convinced that it is harmless to ignore monetary matters or to sweep all of the frictions other than sticky prices under the carpet. Further, we are generally uncomfortable with sticky-price models even when there are explicit costs to changing prices. The source of these menu costs is typically unexplained, and once one opens the door to such costs of adjustment it seems that one should consider many other similar types of costs in the model if we are to take them seriously. Again, our motivation for presenting a New Keynesian sticky-price model is mainly to show that if one thinks it is desirable to have nominal rigidities in a model, this is not inconsistent with being relatively explicit about the exchange process or the role of money and related institutions.

4.3   New Monetarist Sticky Prices

Temporarily leaving aside qualms about exactly how one introduces stickiness into the model, we have to admit that it is desirable to do so, for the simple reason that stickiness seems to be a feature of reality. How can New Monetarists – or Old Monetarists or New Classicists or anyone else – ignore this? Indeed, it is apparent to us that this is one of the main driving forces, if not the main force that makes Keynesians Keynesian. Consider Ball and Mankiw (1994), who we think are fairly representative. As they put it, "We believe that sticky prices provide the most natural explanation of monetary non-neutrality since so many prices are, in fact, sticky." Moreover, "based on microeconomic evidence, we believe that sluggish price adjustment is the best explanation for monetary nonneutrality." And "As a matter of logic, nominal stickiness requires a cost of nominal adjustment." *Fait accompli.*

But healthy science has to be willing to challenge and confront all aspects of theory, even fundamental canons like those passed down by Ball and Mankiw. To show one way to potentially confront the sticky-price issue, here we sketch the recent analysis by Head et al. (2010). What they show is that some natural models generate nominal price stickiness endogenously, as a result, and not an assumption. These models seem consistent not just with the broad observation that prices are, in fact, sticky, but also with some of the more detailed micro evidence discussed below. Yet, as we will soon see, such models have policy implications that are very different from those of Keynesian economics. That is, these models predict that sticky prices can emerge without Calvo (1983) pricing, Mankiw (1985) costs, or other such devices, and yet these models are consistent with monetary neutrality. And they certainly do not imply that Keynesian monetary policy prescriptions are either feasible or desirable.[30]

Consider the benchmark New Monetarist model with one change: we swap out the Nash bargaining module for price setting by sellers as in Burdett and Judd (1983). The Burdett-Judd model has every seller posting a price p taking as given the distribution of other prices, say $F(p)$, and then buyers search for prices in the sense of sampling from $F(p)$. What prevents the distribution from collapsing to a single price, as in Diamond (1971), is that buyers generally get to sample more than one draw from $F(p)$. Although there are many ways to set this up, let us assume here that the representative buyer gets to see n prices with probability α_n. Also, assume for simplicity that they each want to buy 1 unit of an indivisible good, and that each seller can satisfy any demand at cost c per unit. What drives Burdett-Judd pricing is this: Suppose all sellers charge $\bar{p}$; then any buyer that samples more than one seller will pick one at random; this gives any individual seller an incentive to shade down to $\bar{p} - \varepsilon$. In the end, equilibrium must have a nondegenerate $F(p)$. Quite naturally, sellers posting high p make more per unit, while sellers posting low p earn less per unit but make it up on the volume, so that in

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[30]This model presented in this section, while based on Head et al. (2101), has antecedents in Head and Kumar (2005) and Head et al. (2008). The idea is obviously also related to earlier work by Caplin and Spulber (1987), although their model is really very different, as are some of the implications.

equilibrium their profits are the same.[31]

Taking as given for now the price distribution, the DM value function for a buyer can be written

$$V(m) = W(m) + \sum_n \alpha_n \int_{\underline{p}}^m (u - \phi p) \, dJ_n(p) \tag{53}$$

where J_n is the distribution of the lowest p sampled from $F(\cdot)$ given $n \geq 1$ draws. When a buyer samples $n > 1$ prices, he obviously buys at the lowest one, generating a distribution of transactions prices (those actually paid, as opposed to posted) denoted by $J(p)$, which generally differs from $F(p)$. For ease of presentation, from now on we assume $\alpha_n = 0$ for $n \geq 3$. The distribution of transactions prices in this case is simply

$$J(p) = \frac{\alpha_1 F(p) + \alpha_2 \left\{ 1 - [1 - F(p)]^2 \right\}}{\alpha_1 + \alpha_2}.$$

One can also define the distribution of prices posted in real terms $H(z)$, where $z = \phi p$, as well as the distribution of real transactions prices.

For the same reason trade is monetary in all the models presented above, sellers in this model post prices in nominal terms – in dollars, since it is dollars that buyers must trade for goods. So posting nominal prices is natural, although of course they could post in other units, like the number of dollars needed to buy X in the next CM. In any case, profit from posting p is

$$\Pi(p) = (\phi p - c) \, b \left\{ \alpha_1 + 2\alpha_2 \left[1 - F(p) \right] \right\}, \tag{54}$$

where b is the buyer-seller ratio. Notice the number of units sold is the measure of buyers who show up with no other option, $b\alpha_1$, plus the measure who show up with a second option that is not as good, $2b\alpha_2 \left[1 - F(p) \right]$. This multiplied by $\phi p - c$ is profit in real

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[31] One reason to work with the Burdett-Judd model is that it can generate price dispersion even without inflation – obviously, since the original version is a nonmonetary model. This is consitent with the observation that we see price dispersion in the data even during periods when inflation was very low (see e.g. Campbell and Eden 2007). That observation is a problem for Calvo pricing models, since the only reason for dispersion in the baseline version of that model is inflation: all firms set p in nominal terms and are only allowed to adjust it at random times, so that at any point during an inflation some (who got to adust recently) will have a price above others (who did not). Without inflation all sellers charge the same price. Of course there are other ways to generate price dispersion. But Burdett-Judd seems reasonable, is certainly tractable, and can be generalized along many interesting dimensions. Additionally, we like that similar search-type frictions are at the heart of what makes money essential and what drives price dispersion.

terms. Let $\mathcal{F}$ be the support of the price distribution. Then profit maximization means:

$$\Pi(p) = \bar{\Pi} \ \forall p \in \mathcal{F} \text{ and } \Pi(p) \leq \bar{\Pi} \ \forall p \notin \mathcal{F}. \tag{55}$$

It is standard to show in Burdett-Judd models that the distribution can have no mass points, and $\mathcal{F} = [\underline{p}, \overline{p}]$ is an interval. At the upper bound, profit is

$$\Pi(\overline{p}) = \overline{\overline{\Pi}} = (\phi\overline{p} - c)b\alpha_1 \tag{56}$$

since the highest price seller only serves customers with no other option. Combining (56) and (54), we can immediately solve for the closed form of the price distribution,

$$F(p) = 1 - \frac{\alpha_1}{2\alpha_2}\left(\frac{\phi\overline{p} - \phi p}{\phi p - c}\right). \tag{57}$$

To get the bounds, simply note that $\overline{p} = M$, assuming all buyers choose the same $\hat{m} = M$ in the CM, as in the benchmark model, and solve $F(\underline{p}) = 0$ for

$$\underline{p} = \frac{\alpha_1 \phi\overline{p} + 2\alpha_2 c}{(\alpha_1 + 2\alpha_2)\phi}.$$

>From this one easily gets the real distribution $H(z)$, given the CM price level $1/\phi$.

Consider a stationary equilibrium where all real variables, including distributions, are constant while all nominal variables grow at the same rate as M. We need to satisfy two conditions: given $\phi = z/M$, the distributions are as constructed above; and given the distributions, z solves a version of our benchmark CM problem (see below). One can also generalize the model to allow entry by buyers into the DM, at some participation cost. This determines the buyer-seller ratio b and therefore we can determine the arrival rates α_n endogenously through a standard matching technology, which is of interest for reasons discussed below.

This is textbook Burdett-Judd, except that we are in a monetary economy, which raises a slight complication. There are typically many equilibria in models with fiat money, price posting and indivisible goods, for reasons related to coordination, and one needs some sort of refinement to make things determinate.[32] Since any possible

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[32] See Jean et al. (2010). We can of course relax the assumption of indivisible goods, and the results go through, but this increases the algebra and raises other issues, like whether sellers post a price, a price-quantity pair, a price-quantity schedule, etc. So here we keep goods indivisible.

equilibrium is qualitatively the same, for our purposes, and we do not want to get into refinement issues here, we simply select the equilibrium that satisfies

$$i = \alpha_1 H'(\overline{z})(u - \overline{z}). \tag{58}$$

This seems the natural analog to the unique stationary monetary equilibrium in our benchmark model, as (58) equates the marginal cost of carrying a dollar to the benefit, which is the probability of sampling a price which in real terms is $\overline{z}$, times the surplus $u - \overline{z}$. One can show that an equilibrium of this form exists for any nominal rate below some threshold nominal interest rate.

What happens in equilibrium? Although the distribution of real prices $H(z)$ is pinned down, individual sellers do not care where they are in the support of that distribution, since all $p \in \mathcal{F}$ earn equal profit. As we said, it is natural to imagine sellers posting prices in nominal terms, not because a dollar is some abstract unit of account, but because it is a medium of exchange. What happens when M increases? In a stationary equilibrium ϕ decreases, and since the real distribution $H(z)$ is invariant, the nominal distribution $F(p)$ shifts to the right. But for any seller that was at t charging $p_t \in \mathcal{F}_t$, when M_t increases to M_{t+1} and $\mathcal{F}_t$ shifts to $\mathcal{F}_{t+1}$, as long as p_t is still in $\mathcal{F}_{t+1}$ there is no incentive to raise the price. Sure, profit per sale goes down, but he makes it up on the volume! Of course, he could change to some other $p_{t+1} \in \mathcal{F}_{t+1}$, and some sellers typically must change, because we need the right number of sellers at each p to keep the same real distribution (see Head et al. 2010 for details). But many sellers with prices posted in nominal terms may not bother to adjust in any period. Thus sticky prices emerge as an equilibrium outcome, even though we let sellers adjust whenever they want, at no cost.

Many sellers not adjusting nominal prices even as the aggregate price level rises is exactly what Ball and Mankiw correctly claim to observe in the real world (although they were evidently wrong to think this implies we need menu costs in models as a matter of logic). The model is consistent with this, but also with many other observations. Consider this list of facts that people think are noteworthy:[33]

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[33] Klenow and Malin (2010) in their chapter of this Handbook emphasize facts 1, 2, 4, 6, 7 and 8. Nakamura and Steinsson (2008) cover facts 3 and 5. Both also provide many other references.

1. Prices change slowly, with a median frequency of adjustment between 4 and 7 months, or 8 and 10 months, depending on details.

2. The frequency of price changes varies a lot across goods

3. The size of prices changes varies a lot across goods.

4. All sellers that change prices at a point in time do not all change to the same price.

5. About 1/3 of price changes are reductions even during general inflation.

6. Hazard rates for price changes are flat or declining, with an eventual spike.

7. Many price changes are quite small.

8. Frequency of price changes is positively related to inflation.

The New Monetarist sticky price model can in principle match all of these observations, although only time will tell just how well. But it is already known that other more popular models do not do so well, including the basic Calvo-pricing and menu-cost models. Some parts of this claim are obvious, like the fact that standard (s, S) models predict all sellers should jump to same price when they do change, in contradiction of item 3 (although we are aware there are "fixes" one can tack on). Other parts of our claim are perhaps less obvious. Consider item 7, the fact that many p changes are small. As Klenow and Krystov (2008) say, this is "hard to reconcile with the large menu costs needed to rationalize large average price changes." The model presented here has no problem with this.[34] There is clearly more work to be done on taking this kind of model to the data and, again, time will tell. But given the success at matching the data with the labor market version of Burdett-Judd, the well known Burdett-Mortensen (1998) model, there is reason to think it is worth pursuing.

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[34]Admittedly, at least in part the reason the model has no problem with some of these observations is that it has a lot of indeterminacy. Still, our main point is that other models do not do very well. For instance, to be precise, define a small price change as less than 5%. Klenow and Krystov (2008) report around 39% of changes are small in the data, and cannot match this in their model. In the Golosov and Lucas (2005) model, which was designed to generate approximate monetary neutrality, less than 10% of prices changes are small. Midrigan (2007) can match the observation in question with some effort, but then he loses approximate neutrality. The model here can match the facts easily and is consistent with exact neutrality.

To return to the issue of monetary neutrality and implications for policy, here, we use the extended version of the model where the measure of buyers in the DM b is determined endogenously by an entry condition. First, as we said above, the distribution of real prices is invariant to the price level along the equilibrium path, although of course there are real effects to changing the inflation rate, as in any New Monetarist model. Moreover, a one-time surprise increase in M will be exactly neutral: $F(p)$ shifts up with the aggregate price level, while all real variables, including $H(z)$, b, etc. stay the same. This is *very different* from what happens in a Keynesian version of the model, where prices are sticky for Calvo reasons. In such a model, when the surprise increase in M hits, it is not possible for all nominal prices to adjust (in a menu-cost version, it may be possible, but it is not generally going to happen, and the story is similar). The distribution of nominal prices will not shift the way our model predicts, and the shape of the real price distribution changes.

Generally, in a Keynesian version of the model, after a surprise M increase, buyers will expect lower real prices – there are some real bargains out there with many sellers stuck at low prices. This increases b, and hence output, since there are more buyer-seller matches. Indeed, in the very short run, when Calvo has not yet allowed any seller to adjust, the increase in M lowers all real prices. This sets off a shopping frenzy, which means a production boom, as sellers are obliged to meet demand at the posted prices. We do not here go into whether a central bank would want to engineer such a boom, or whether they could do so systematically over time. Instead we emphasize the following. Suppose we concede the observation that some prices are sticky. We have demonstrated that this does not imply monetary injections are nonneutral, let alone that particular Keynesian policy prescriptions are feasible or desirable. To be clear, the New Monetarist position is not that nonneutralities do not exist, and this chapter contains many examples where obviously money matters (e.g. Section 4.1). Our position is that the observation that prices appear to be sticky in the data does not logically imply that Keynesian models or policy implications are correct.

5 Money, Payments, and Banking

In this section we analyze extensions of the benchmark model that incorporate payments arrangements, along the lines of Freeman (1996), and banks, along the lines of Diamond and Dybvig (1983). The goal is to construct environments where outside money is important not only for accomplishing the exchange of goods but for supporting credit arrangements.

5.1 A Payments Model

For this application we include two types of buyers and two types of sellers. It is convenient to refer to CM meetings as occurring in the day and DM transactions at night. A fraction α of buyers and a fraction α of sellers are type 1 buyers and sellers, respectively, and they meet in the night in non-monitored matches. When a type 1 buyer meets a type 1 seller, they can trade only if the former has money. As well, there are $1 - \alpha$ type 2 buyers and $1 - \alpha$ type 2 sellers, who are monitored at night and hence can trade on credit, which again is perfectly enforced. During the day, we will have a more elaborate set of meetings among agents, with limited participation in the CM. This is slightly complicated, but we think that it is an improvement over some of the models used in the payments literature, including Freeman (1996).

Thus, in the morning of the day, type 1 sellers and type 2 buyers meet in a Walrasian market where money trades for goods at the price ϕ_t^1, and type 2 buyers can produce. Type 1 buyers and type 2 sellers do not participate in this market. Then, at mid-day, bilateral meetings occur between type 2 buyers and type 2 sellers who were matched the previous night. This is essentially another DM, but neither buyers or sellers produce, and this market is only an opportunity for the type 2 buyers to settle their debts. Finally, in the afternoon, type 1 buyers meet in a second Walrasian market with type 2 sellers, with the price of money denoted by ϕ_t^2. Here, type 1 buyers can produce. Neither type 2 buyers or type 1 sellers participate in this second CM. The government can make lump-sum money transfers in the Walrasian markets during the day, so that there are

two opportunities to intervene each period. We assume these interventions are lump-sum transfers in equal quantities to sellers. As in the benchmark model, we must have $\phi_t^i \geq \beta\phi_{t+1}^i$, for $i = 1, 2$.

We are interested in an equilibrium where trade occurs as follows. First, in order to purchase goods during the night, type 1 buyers need money, which they acquire in the afternoon Walrasian market. They trade all this money at night for goods, so that type 1 sellers go into the next day with all the money. In the Walrasian market in the next morning, type 2 buyers produce in exchange for the money held by type 1 sellers. Then, at midday, type 2 buyers meet type 2 sellers and use money to settle their debts acquired in the previous night. Then, in the second Walrasian market, during afternoon, type 2 sellers exchange money for the goods produced by type 1 buyers. Finally, at night, meetings between type 1 buyers and sellers involve the exchange of money for goods, while meetings between type 2 buyers and sellers are exchanges of IOU's for goods. For clarity, we show agents' itineraries and patterns of trade in Figure 3.

In bilateral meetings at night, buyers make take-it-or-leave-it offers. Letting x_t denote the quantity of goods received by a type 1 buyer at night, his optimal choice of money balances yields the FOC

$$-\phi_t^2 + \beta\phi_{t+1}^1 u'(x_t) = 0. \tag{59}$$

To repay the debt that supported the purchase of s_t units of goods, the type 2 buyer must acquire money in Walrasian market 1 at price ϕ_{t+1}^1, and give it to a type 2 seller, who then exchanges the money for goods in Walrasian market 2 at the price ϕ_{t+1}^2. Therefore, s_t satisfies the FOC

$$-\phi_{t+1}^1 + \phi_{t+1}^2 u'(s_t) = 0. \tag{60}$$

Let M_t^i denote the quantity of money (post transfer) supplied in the ith Walrasian market during the day, for $i = 1, 2$. Then market clearing in Walrasian markets 1 and 2 implies

$$(1 - \alpha)s_{t-1} = \beta\phi_t^2 M_t^1, \tag{61}$$

$$\alpha x_t = \beta\phi_{t+1}^1 M_t^2. \tag{62}$$

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To solve for equilibrium, substitute for prices in (59) and (60) using (61) and (62) to obtain

$$-\frac{\alpha x_t}{M_t^2} + \frac{(1-\alpha)s_t u'(s_t)}{M_{t+1}^1} = 0 \tag{63}$$

$$-\frac{(1-\alpha)s_{t-1}}{\beta M_t^1} + \frac{\alpha q_t u'(x_t)}{M_t^2} = 0. \tag{64}$$

Given $\{M_t^1, M_t^2\}_{t=0}^{\infty}$, we can determine $\{x_t, s_t\}_{t=0}^{\infty}$ from (63) and (64), and then $\{\phi_t^1, \phi_t^2\}_{t=0}^{\infty}$ can be determined from (61) and (62). Note that, in general, intervention in both Walrasian markets matters. For example, suppose that $M_t^1/M_t^2 = 1 + \gamma$ for all t, $M_{t+1}^i/M_t^i = 1 + \mu$, where $\gamma > -1$ and $\mu \geq \beta$ so that the ratio of money in the two markets is constant for all t and in individual Walrasian markets money grows at a constant rate over time. Further, suppose $u(x) = \ln x$. Then, in an equilibrium where $s_t = s$ for all t and $x_t = x$ for all t, (63) and (64) yield

$$x = \frac{(1-\alpha)}{\alpha(1+\gamma)(1+\mu)}$$
$$s = \frac{\alpha\beta(1+\gamma)}{(1-\alpha)}$$

A higher money growth rate μ decreases the quantity of goods traded in cash transactions during the night, as is standard. However, a higher γ (relatively more cash in the first Walrasian market) increases the quantity of goods bought on credit and reduces goods bought with cash at night. What is efficient in general? To maximize the total surplus in the two types of trades, we need $x_t = s_t = x^*$. From (63) and (64), this gives $\mu = \beta - 1$ and $\gamma = [1 - \alpha(1 + \beta)]/\alpha\beta$. At the optimum, in line with the Friedman rule, money should shrink over time at the rate of time preference, but we also need a monetary injection in the first market that increases with the fraction of credit relative to cash transactions so as to support the optimal clearing and settlement of credit.

Outside money plays two roles here: it is used as currency in some transactions, and it is used to accommodate credit in other transactions where it is needed to settle debts. This second role is similar to the one played by central bank balances in interbank payments systems, such as Fedwire. In the model, central bank intervention in the morning Walrasian market relative to the afternoon Walrasian market stands in for real-world in-

tervention via daylight overdrafts, while intervention in the afternoon Walrasian market relative to the next Walrasian market is similar to real-world central bank intervention in overnight financial markets. There are two dimensions to monetary policy, and both are important. The optimal policy sets both the intraday nominal interest rate (the nominal interest rate on bonds issued in the morning and paying off in the afternoon Walrasian market) and the overnight rate (the nominal interest rate on bonds issued in the afternoon Walrasian market paying off the next morning) to zero.

It seems clear that it would not be easy to come up with such insights without modeling the details of the exchange process carefully. Although the example is obviously special, it is not contrived. It is meant to capture some of what goes on in actual economies, albeit in an abstract and stylized way. This is a nascent research area, and we think there are many possible applications and extensions of these types of models. Nosal and Rocheteau (2010), Chapman et al. (2009), and the references contained therein provide additional examples and references to other work on payments.

5.2 Banking

We now extend the benchmark model by including banking, in the spirit of Diamond and Dybvig (1983). The original Diamond-Dybvig model appears to have been intended mainly as a model of bank runs and deposit insurance. Subsequent research (see Ennis and Keister 2008) has shown that auxiliary assumptions are required to obtain runs, and it is not so clear that there is a role for government deposit insurance in this modeling framework. However, what survives is a nice model of financial intermediaries that act to provide insurance against liquidity needs, and they do so by diversifying across liquid and illiquid assets. The model does not capture all important features of banks, such as the fact that they issue liabilities that compete with government currency in transactions. And since it ignores monetary factors, the basic framework cannot be used to address some key features of historical banking panics, like currency shortages and high nominal interest rates (Friedman and Schwartz 1963).

Champ et al. (1996) provide an attempt to capture these features by integrating

Diamond-Dybvig banks into an overlapping generations model of money. But that model is incomplete, and has the unfortunate implication that, at the optimum, the central bank should intermediate all liquid assets. In this subsection, we build on Champ et al. (1996) in the context of our benchmark model. This is an example of how recent advances in monetary theory allow us to do more than we could in the earlier overlapping generations framework. In the model constructed here, currency and bank liabilities are both used in transactions, and a diversified bank provides risk sharing services that avoid waste. Thus, there is a Diamond-Dybvig risk-sharing role for banks, but banking provides other efficiency gains as well.

We begin with a version of the model with no aggregate uncertainty. Again we refer to the first subperiod with CM exchange as day, and the second with DM exchange as night, and there are α type 1 sellers who engage in non-monitored exchange at night using currency and $1-\alpha$ type 2 sellers who engage in monitored exchange at night using credit. At night there are α type 1 buyers each matched with a type 1 seller, and $1-\alpha$ type 2 buyers each matched with a type 2 seller, but a buyer's type is random, revealed at the end of the previous day after production and portfolio decisions are made. There is an intertemporal storage technology which takes goods produced by buyers during the afternoon of the day, and yields R goods per unit invested during the morning of the next day, ,with $R > 1/\beta$. All buyers and type 1 sellers are together in the Walrasian market that opens during the afternoon of the day, while only type 2 sellers are present during the morning of the day.

First suppose banking is prohibited. To trade with a type 2 seller at night, a buyer needs to store goods during the day before meeting the seller. Since the trade is monitored, the seller is able to verify that a claim to storage offered for goods is valid. To trade with a type 1 seller at night, a buyer needs cash, as in a non-monitored trade sellers do not accept claims to storage. Claims to storage are useless for type 1 sellers, as they do not participate in the morning CM where the storage pays off. Thus, during the afternoon of the day, the buyer acquires nominal money balances m_t and stores k_t

units of output and, again assuming given take-it-or-leave-it offers at night, solves

$$\max_{m_t, x_t} -\phi_t m_t - k_t + \alpha u(\beta \phi_{t+1} m_t) + (1 - \alpha) \left[u(\beta R k_t) + \beta \phi_{t+1} m_t \right].$$

The FOC are

$$-\phi_t + \beta \phi_{t+1} \left[\alpha u'(x_t) + 1 - \alpha \right] = 0, \tag{65}$$

$$-1 + (1 - \alpha)\beta R u'(s_t) = 0, \tag{66}$$

where x_t is the quantity traded at night in non-monitored exchange, and s_t the quantity traded in monitored exchange.

Assume that the monetary authority makes lump sum transfers during the afternoon of the day to buyers. Then the Friedman rule is optimal: the money supply grows at the rate $\beta - 1$ and $\phi_{t+1}/\phi_t = 1/\beta$. This implies from (65) that $x_t = x^*$ in monetary exchange. However, claims to storage are of no use to buyers, so if a buyer does not meet a type 2 seller, his storage is wasted, even if we run the Friedman rule.

Now consider what happens if banks can accept deposits from buyers, in the form of goods, and use them to acquire money or storage. The bank maximizes the expected utility of its depositors. Since all buyers are identical, consider an equilibrium where all depositors make the same deposit,

$$d_t = \phi_t m_t + k_t, \tag{67}$$

Here, k_t and m_t denote, respectively, storage and money acquired by the bank. If the bank is perfectly diversified, as it will be in equilibrium, it offers agents who wish to withdraw $\hat{m}_t = m_t/\alpha$ dollars, and permits those who do not withdraw to trade claims to $\hat{k}_t = k_t/(1 - \alpha)$ units of storage. Since the bank maximizes the expected utility of the representative depositor, in equilibrium, k_t and m_t solve

$$\max_{m_t, x_t} \left\{ -\phi_t m_t - k_t + \alpha u \left(\frac{\beta \phi_{t+1} m_t}{\alpha} \right) + (1 - \alpha) u \left(\frac{\beta k_t R}{1 - \alpha} \right) \right\}.$$

As above, let x_t denote the quantity of output exchanged during the night in a non-monitored transaction, and s_t the quantity of output exchanged in a monitored

transaction. Then, the FOC for an optimum are

$$u'(x_t) = \frac{\phi_t}{\beta\phi_{t+1}}, \tag{68}$$

$$u'(s_t) = \frac{1}{\beta R}, \tag{69}$$

determining x_t and s_t, respectively.Compare (65) and (66) with (68) and (69). Letting μ denote the money growth rate, we will have $\phi_t/\phi_{t+1} = 1 + \mu$. Therefore, if $\mu > \beta - 1$ then x_t is smaller in the equilibrium without banks than with banks. This is because, without banks, money is held by all buyers but cannot be used in exchange in monitored transactions, as type 2 sellers will not accept it. When $\mu = \beta - 1$, $x_t = x^*$ whether or not there are banks, as there is no opportunity cost to holding money from one day to the next. Note that s_t is always larger with banks than without. This is because, if there are no banks, storage might be wasted if a buyer has a non-monitored meeting at night. Anticipating this, buyers invest less in storage than if the bank implicitly provides insurance.

Thus, banking acts to increase consumption in the night and to eliminate wasted storage, increasing welfare. As in Diamond-Dybvig, there is an insurance role for banks, in that banks allow agents to economize on currency and promote investment in higher-yielding assets. However, there is also an efficiency gain, in that storage is not wasted. With banking, the quantity of goods x_t exchanged for money during the night is efficient under the Friedman rule, which by (68) gives $x_t = x^*$. A policy that we can analyze in this model is Friedman's recommendation for 100% reserve requirements. This effectively shuts down financial intermediation and constrains buyers to holding outside money and investing independently, rather than holding deposits backed by money and storage. We then revert to the outcome without banks, which we know is inferior.

One can also consider the case of aggregate uncertainty, where α_t is a random variable, capturing fluctuations in the demand for liquidity. Assume that α_t is publicly observable, but is not realized until the end of the day, after consumption and production decisions have been made. For convenience, assume α_t is i.i.d. Now, analogous to (??), the bank

solves

$$\max_{m_t, x_t, \hat{m}_t} \left\{ -\phi_t m_t - k_t + E_t \left[\alpha_t u \left(\frac{\beta \phi_{t+1}(m_t - \hat{m}_t)}{\alpha_t} \right) + \beta \phi_{t+1} \hat{m}_t + (1 - \alpha_t) u \left(\frac{\beta k_t R}{1 - \alpha_t} \right) \right] \right\},$$

where $\hat{m}_t$ is the quantity of money per depositor which is not spent at night.[35] The FOC for m_t and k_t are

$$-\phi_t + \beta E_t \left\{ \phi_{t+1} \max \left[1, u' \left(\frac{\beta \phi_{t+1} m_t}{\alpha_t} \right) \right] \right\} = 0, \tag{70}$$

$$-1 + \beta R E_t u' \left(\frac{\beta R k_t}{1 - \alpha_t} \right) = 0. \tag{71}$$

Now, letting M_t denote the money stock per buyer, from (70) the stochastic process for prices $\{\phi_t\}_{t=0}^{\infty}$ solves

$$-\phi_t + \beta E_t \left\{ \phi_{t+1} \max \left[1, u' \left(\frac{\beta \phi_{t+1} M_t}{\alpha_t} \right) \right] \right\} = 0, \tag{72}$$

given $\{M_t\}_{t=0}^{\infty}$.

First, suppose that $M_t = M_0(1 + \mu)^t$ with money growth accomplished through lump sum transfers to buyers in the day. Then, in a stationary equilibrium, we have $\phi_t = \phi_0(1 + \mu)^{-t}$ and from (72) we get

$$\frac{1 + \mu}{\beta} = E \left\{ \max \left[1, u' \left(\frac{\beta \phi_0 M_0}{(1 + \mu)\alpha} \right) \right] \right\}, \tag{73}$$

which solves for ϕ_0 (dropping t subscripts for convenience). Let $G(\phi_0)$ denote the right-hand side of (73). We have $G(0) = \infty$, and $G(\phi_0) = 1$ for $\phi_0 \geq x^*(1 + \mu)\bar{\alpha}/\beta M_0$ where $\bar{\alpha}$ is the largest value in the support of the α distribution. Further, $G(\cdot)$ is strictly decreasing and continuous for $0 < \phi_0 < x^*(1 + \mu)\bar{\alpha}/\beta M_0$. Therefore, if $\mu > \beta - 1$ then from (73) there is a unique solution for ϕ_0, and there will be realizations of α_t such that the quantity of goods traded in non-monitored meetings at night is less than x^*.

Further, consider a nominal bond that pays off one unit of outside money in the following day, and is exchanged at the end of the current day after α_t becomes known. The nominal interest rate on this bond is given by

$$r_t = \max \left[1, u' \left(\frac{\beta \phi_0 M_0}{(1 + \mu)\alpha_t} \right) \right] - 1.$$

[35] It is irrelevant whether this money is withdrawn by the depositor at the end of the day, or left in the bank until the next day and then withdrawn.

The nominal interest rate fluctuates with α_t. In general, when α_t is large, currency is scarce and the nominal rate is high. States of the world where there are currency shortages and the withdrawal demand at banks is high are associated with high nominal interest rates, as was the case historically during banking panics.

>From (73), note that one optimal monetary policy is $\mu = \beta$ – the standard Friedman rule. Then, for any $\phi_0 \geq \phi^*$, ϕ_0 is an equilibrium price of money at the first date, where

$$\phi^* = \frac{x^*(1+\mu)\bar{\alpha}}{\beta M_0}.$$

In any of these equilibria, the nominal interest rate is zero for all t and each buyer consumes x^* in non-monitored meetings during the night. Thus, there exists a continuum of equilibria given $\mu = \beta$ and in any of these equilibria there are states of the world where some portion of the money stock is not spent in monetary transactions at night.

There exist other money supply rules that support a zero nominal interest rate. Suppose that we look for a monetary policy rule such that the nominal rate is always zero and all cash is spent at night each period. From (72), we first require that

$$\frac{\beta \phi_{t+1} M_t}{\alpha_t} = x^*,$$

so that there is efficient trade in all non-monitored meetings in the night and all cash is spent. From this we obtain

$$\frac{M_t}{M_{t-1}} = \frac{\beta E_t(\alpha_t)}{\alpha_{t-1}},$$

which is an optimal policy rule with the characteristics we are looking for. Under this rule, agents in non-monitored meetings at night anticipate there will be a monetary contraction the next day if the demand for liquidity is high. This will tend to increase the value of money in non-monitored transactions, so that efficient trades can be made. This monetary rule is active, acting to accommodate fluctuations in the demand for liquidity, as opposed to the passive (constant money growth) rule that achieves the same result.

As discussed above, there exist optimal policy rules here that look nothing like the prescription in Friedman (1968). Typically, achieving a zero nominal interest rate in all

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states of the world can be implemented through various monetary policies that do not entail constant growth of the money stock.[36] More broadly, we think there is a lot to be learned by carefully modeling banking and the interaction with monetary policy as part of larger general equilibrium models of the exchange process. It is inevitable that these models will be somewhat complicated, at least compared to the simplest examples of money being used as a medium of exchange in Section 2. But the payoff to getting the models right is a better understanding of banking and financial intermediation, which seems very hard to dismiss as unimportant or uninteresting in this day and age.

6   Finance

The class of models presented here has recently been used to study asset markets. This work is potentially very productive, as it allows one to examine how frictions and policy affect the liquidity of assets, their prices, and the trading volume in these markets. Moreover, although this may come as a surprise to some people, who seem to think that financial markets are as close to a frictionless ideal as there is, it is also one of the most natural applications of the search-and-bargaining approach. As Duffie et al. (2008) put it, "Many assets, such as mortgage-backed securities, corporate bonds, government bonds, US federal funds, emerging-market debt, bank loans, swaps and many other derivatives, private equity, and real estate, are traded in over-the-counter (OTC) markets. Traders in these markets search for counterparties, incurring opportunity or other costs. When counterparties meet, their bilateral relationship is strategic; prices are set through a bargaining process that reflects each investor's alternatives to immediate trade." Since the models people use to formalize these ideas are closely related to those used in monetary theory, we provide a taste of these applications using the New Monetarist model.[37]

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[36] As Lagos (2009) shows, a path for the money stock that implements the Friedman rule needs only satisfy two week properties. Roughly, the money stock must go to zero in the limit, and it must grow on average at a rate higher than minus the rate of time preference.

[37] Papers that we have in mind in monetary economics include Ravikumar and Shao (2006), Lagos (2008), Lester et al. (2009), Rocheteau (2009), Jacquet and Tan (2009), Ferraris and Watanabe (2010), and Geromichalos et al. (2010). Contributions more in finance include Duffie et al. (2005, 2008), Weill (2007, 2008), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Lagos et al. (2009), and Silveira and Wright (2010).

6.1 Asset Trading and Pricing

One of the first papers in finance to use the search-and-bargaining approach is Duffie et al. (2005). They worked with a version of the second-generation monetary models presented in Section 2.2, which means in particular that agents can hold only $a \in \{0, 1\}$ units of an asset. Even under this restriction many interesting results emerge, and it would be worth discussing how they adapt the model for their purposes. However, instead, we present a model capturing similar ideas using our benchmark model where agents can hold any amount $a \in \mathbb{R}_+$ of an asset.[38]

One should think now of assets as (shares in or claims on) "trees" paying dividends each period in "fruit" as in the standard Lucas (1978) asset-pricing model. In this application, agents will value assets for their yield or dividend, which we denote by y. Thus, an agent holding a units of the asset has a claim on ay units of 'fruit," where here dividends accrue and are consumed in the DM. Let A be the fixed supply of the asset, and denote its CM price by ϕ, which is constant because we focus on steady states. Then the CM problem is

$$W(a) = \max\{U(X) - H + \beta V(\hat{a})\}$$

$$X = H + \phi a - \phi\hat{a}.$$

As usual, this implies $U'(X) = 1$, $\phi = \beta V'(\hat{a})$, and $W'(a) = \phi$.

In the DM, agents get utility from consuming dividends, subject to preference shocks realized after the CM closes but before the DM opens. Let π_H and $\pi_L = 1 - \pi_H$ be the probability of a high and a low shock, implying utility for agents with a units of the asset $u_H(ay)$ and $u_L(ay)$, respectively, with $u'_H(x) > u'_L(x)$ for all x. There is generally gain from trade between an agent who draws the L shock and one who draws H. In the literature, L is often referred to as a liquidity shock, because it stands in for agents needing to sell assets to meet liquidity needs – i.e. while the model literally has agents trading claims to "trees" because of changes in their utility from "fruit" it is meant to

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[38]Lagos and Rocheteau (2009) provide a different extension of Duffie et al. (2005), which also allows $a \in \mathbb{R}_+$, but here we stay closer to our benchmark model.

capture more generally the idea that sometimes one has to sell assets for any number of reasons, including a need for ready cash. Of course, one could say the papers ought to model the need for liquidity more explicitly; we would concur, and people are working on this. In any case, agents in the DM meet bilaterally and at random. Let σ_H be the probability an agent with shock H meets one with shock L, and σ_L the probability that an agent with L meets one with H. In a meeting where one agent has L and the other H, the former transfers q units of the asset to the latter in exchange for a payment p, interpreted as an IOU for p units of X to be delivered in the next CM, assumed again to be perfectly enforced.[39]

To reduce notation, define the trade surplus for H and for L as

$$
\begin{aligned}
S_H(a) &= u_H\left[(a+q)y\right] - u_H(ay) + \phi q - p \\
S_L(a) &= u_L\left[(a-q)y\right] - u_L(ay) - \phi q + p,
\end{aligned}
$$

using $W'(a) = \phi$. This allows us to write the DM payoff as

$$
V(a) = \pi_H \sigma_H S_H(a) + \pi_L \sigma_L S_L(a) + \pi_L u_L(ay) + \pi_H u_H(ay) + W(a).
$$

In terms of bargaining, when type H with a meets type L with A, the solution is

$$
\max S_H(a)^\theta S_L(A)^{1-\theta}.
$$

It is easy to see that this is solved by (q, p) solving

$$
u'_H\left[(a+q)y\right] = u'_L\left[(A-q)y\right] \tag{74}
$$

$$
p = \phi q + (1-\theta)\left\{u_H\left[(a+q)y\right] - u_H(ay)\right\} + \theta\left\{u_L(Ay) - u_L\left[(A-q)y\right]\right\}. \tag{75}
$$

Inserting this p into $S_H(a)$ and $S_L(a)$, and inserting these into $V(a)$, we get

$$
\begin{aligned}
V(a) &= \pi_H \sigma_H \theta \left\{u_H\left[(a+q)y\right] - u_H(ay) + u_L\left[(A-q)y\right] - u_L(Ay)\right\} \\
&\quad + \pi_L \sigma_L (1-\theta)\left\{u_H\left[(A+q)y\right] - u_H(Ay) + u_L\left[(a-q)y\right] - u_L(ay)\right\} \\
&\quad + \pi_H u_H(ay) + \pi_L u_L(ay) + W(a),
\end{aligned}
$$

---

[39] One may recognize this specification as similar to Section 3.5, in the sense that there is no production, but simply exchange between agents with different preference shocks. However, in this application we assume perfect credit.

where we are careful to note that a is the asset position of the individual whose value function we are considering and A is the position of someone he meets.

Differentiating, we get

$$
\begin{aligned}
V'(a) &= \pi_H \sigma_H \theta u'_H \left[(a+q)y \right] y + \pi_L \sigma_L (1-\theta) u'_L \left[(a-q)y \right] y \\
&\quad + \pi_H (1 - \sigma_H \theta) u'_H (ay) y + \pi_L \left[1 - \sigma_L (1-\theta) \right] u'_L (ay) y + \phi.
\end{aligned}
$$

For concreteness, consider a matching technology with $\sigma_H = \sigma \pi_L$ and $\sigma_L = \sigma \pi_H$, which is basically the matching technology introduced back in Section 2.1. Then, since $u'_H \left[(a+q)y \right] = u'_L \left[(a-q)y \right]$, we can write

$$
V'(a) = \sigma \pi_H \pi_L u'_H \left[(a+q)y \right] y + \pi_H (1 - \sigma \pi_L \theta) u'_H (ay) y + \pi_L \left[1 - \sigma \pi_H (1-\theta) \right] u'_L (ay) y + \phi.
$$

Substituting this into the FOC from the CM, $\phi = \beta V'(\hat{a})$, then setting $a = A$, $\pi_L = 1 - \pi_H$, and $r = (1-\beta)/\beta$, we get

$$
\begin{aligned}
r\phi &= \sigma \pi_H (1 - \pi_H) u'_H \left[(A+q)y \right] y + \pi_H (1 - \sigma\theta + \sigma \pi_H \theta) u'_H (Ay) y \quad (76) \\
&\quad + (1 - \pi_H)(1 - \sigma \pi_H + \sigma \pi_H \theta) u'_L (Ay) y.
\end{aligned}
$$

We can now describe equilibrium recursively: first find the q that solves (74); then the DM asset price p solves (75) and the CM asset price ϕ solves (76), where it turns out that p and ϕ are independent conditional on q.

Notice from (76) that the CM asset price per period, $r\phi$, is a weighted average of three terms: the marginal value of the asset when you trade, which is independent of your shock, $u'_H \left[(A+q)y \right] y = u'_L \left[(A-q)y \right] y$; the marginal value when you do not trade but have a high shock, $u'_H (Ay) y$; and the marginal value when you do not trade but have a low shock, $u'_L (Ay) y$. If agents are risk neutral in the DM, $u(x) = x$, then $r\phi = y$, which means the asset is priced at its fundamental value (the capitalized value of the dividend stream). If agents are risk averse the asset price will adjust for the fact that its value is random. Even if $\sigma = 1$, so there are no fundamental search frictions, in the sense that you always meet someone, if matching is random you could meet the wrong type and so there is risk.

Suppose we set $\pi_L = \pi_H = 1/2$ and change the matching technology, so that every agent with H meets one with L, which means agents always have the opportunity to rebalance their asset holdings in the DM. It is not hard to rework the analysis to get

$$r\phi = \frac{1}{2} u'_H \left[(A + q)y \right] y + \frac{1 - \theta}{2} u'_H (Ay) y + \frac{\theta}{2} u'_L (Ay) y. \tag{77}$$

In this case there is no risk per se, since everyone gets to rebalance their asset position, and $u'_H \left[(A + q)y \right] y = u'_L \left[(A - q)y \right] y$. But due to bargaining power the asset price can be priced differently from its fundamental value. In this special case without search or matching risk we have

$$\frac{\partial r\phi}{\partial \theta} = -\frac{y}{2} \left[u'_H (Ay) - u'_L (Ay) \right] < 0,$$

so increasing the bargaining power of the agent buying the asset in the DM reduce the asset's price in the CM.

Returning to the more general case, with search and matching frictions, (74) implies a similar result,

$$\frac{\partial r\phi}{\partial \theta} = \sigma \pi_H (1 - \pi_H) \left[u'_L (A) - u'_H (A) \right] y < 0.$$

And in terms of the baseline arrival rate, we have

$$\frac{\partial r\phi}{\partial \sigma} = \pi_H (1 - \pi_H) \left\{ \theta \left[u'_H (A + q) - u'_H (A) \right] + (1 - \theta) \left[u'_L (A - q) - u'_L (A) \right] \right\} y.$$

Since $u'_H (A + q) < u'_H (A)$ and $u'_L (A - q) > u'_L (A)$, this will be negative for big θ and positive for small θ. The important point is that search and bargaining frictions in the DM affect the asset price in the CM. And in terms of probabilities,

$$\frac{\partial r\phi}{\partial \pi_H} = \sigma (1 - 2\pi_H) \left[u'_L (A - q) - u'_L (A) \right] y + (1 - \sigma\theta + 2\sigma\theta\pi_H) \left[u'_H (A) - u'_L (A) \right] y,$$

which is ambiguous, in general, but is definitely positive for $\pi_H \leq 1/2$. So the distribution of liquidity shocks in the DM naturally matter for asset CM prices, too.

This setup is similar in spirit to Duffie et al. (2005), even if the details differ. Models like this have been used to study a variety of issues. One application is to introduce middlemen – dealers, or brokers, say – that can buy assets from L types and sell to H

types. Weill (2007) studies the behavior of such intermediaries, not only in steady state, but along dynamic transition paths after a crisis. A crisis is modeled as a reduction in π_H, which stands in for the idea that many people want to sell assets while few want to buy. He actually uses a second-generation version of the model, with $a \in \{0, 1\}$. Weill et al. (2009) use a generalized model with $a \in \mathbb{R}_+$. An interesting question is whether intermediaries provide enough liquidity, in the sense of buying and holding assets while the economy recovers from a crisis. One can use the model to study the effects of various central bank interventions, including the recent Fed policy of buying up certain assets. The analysis and results are too involved to go into detail, but at least we get to illustrate the types of issues people have been studying with these models.

6.2 Capital Markets

Here is an another way to model assets markets, which is not far from the benchmark monetary model, except for two twists. First, we have assets other than money acting as a medium of exchange; second, the gains from trade come not from producing goods for consumers, but from reallocating capital across producers. Suppose again that there are two types, buyers and sellers, with a mass 1/2 of each. A buyer's CM utility is $U(X) - H$, while a seller's CM utility is $X - H$. In the CM, in addition to agents being able to produce X one-for-one with H, agents can also produce it using capital: anyone with k units of capital can produce $f(k)$ units of X, where $f' > 0$, $f'' < 0$, $f'(0) = \infty$, $f'(\infty) = 0$ and $f(0) = 0$. Sellers, but not buyers, also have a technology to convert X into k one-for-one, after the CM closes. Capital produced at t becomes productive at the beginning of the CM at $t+1$, after which (for simplicity) it depreciates 100 per cent. No one produces or consumes in the DM in this application – it is only a market for asset exchange.

In addition to k, there is a second asset, a, which as in the previous section one can think of as a share in a Lucas "tree." Now we normalize the supply of "trees" to $A = 1/2$, and assume the dividend y is realized in units of X in the CM. Shares are now used in the DM as a means of payment. Of course, money can be considered an

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asset with dividend $y = 0$ (and, moreover, the quantity of the asset can be augmented by the government through transfers and taxes). In the DM, each buyer is matched with a seller with probability σ, and similarly for sellers. Again, buyers and sellers do not produce or consume in the DM, and matches represent only opportunities for trading assets. To generate potential gains from trade, we assume that the technology prohibits buyers from holding capital when the CM closes (say because the buyers have left the CM before capital production takes place). Thus, a match in the DM is an opportunity for a buyer to exchange shares for capital.

First, consider the case where $\sigma = 0$, which means we shut down the DM. Then a will be priced according to fundamentals, $\phi_t = \hat{\phi}$ for all t where $\hat{\phi} = \beta y / (1 - \beta)$. In other words, the share price is the present value of future dividends, and the rate of return on shares is equal to the rate of time preference r.[40] Sellers acquire capital in each CM that they cannot trade, since we are here assuming $\sigma = 0$, so they accumulate only for production. Letting k_{t+1} denote the capital produced by a seller in in period t, and k_{t+1}^s, k_{t+1}^b, respectively, the quantities of capital held by each seller and buyer at the beginning of the CM in period $t + 1$, we have $k_{t+1} = k_{t+1}^s = \hat{k}$ and $k_t^b = 0$, where $f'(\hat{k}) = 1/\beta$. As with shares, the return on capital also equals the rate of time preference.

Now consider the case where $\sigma > 0$. If the buyer has a shares and the seller has k units of capital, the buyer can transfer d shares to the seller for k^b units of capital. The generalized Nash bargaining solution is

$$\max_{d, k^b} \left[f(k^b) - d(\phi_{t+1} + y) \right]^\theta \left[f(k - k^b) + d(\phi_{t+1} + y) - f(k) \right]^{1-\theta},$$

subject to $d \leq a$ and $k^b \leq k$. The second constraint does not bind since $f'(0) = \infty$. Without loss of generality, we will consider equilibria where buyers always exchange all of their shares for capital in the DM if they are matched with a seller and either: (i) Sellers hold part of the stock of shares; or (ii) Buyers hold the entire stock of shares at the end of the CM. That is, we consider cases where the first constraint binds for buyers,

---

[40]Define the return on the share r_s by $1 + r_s = (\phi + y)/\phi$. Then $r_s = y/\phi = \beta/(1 - \beta)$ when shares are priced fundamentally.

so $d = a$, and k^b solves $a = z(k^b, k)$, where

$$z(k^b, k) = \left(\frac{1}{\phi_{t+1} + y}\right) \left\{ \frac{\theta f'(k^b)[f(k) - f(k - k^b)] + (1 - \theta)f'(k - k^b)f(k^b)}{\theta f'(k^b) + (1 - \theta)f'(k - k^b)} \right\}. \quad (78)$$

Then a buyer's problem in the CM is

$$\max_{k^b_{t+1}} \left\{ -\phi_t z(k^b_{t+1}, k_{t+1}) + \beta \left[\sigma f(k^b_{t+1}) + (1 - \sigma)z(k^b_{t+1}, k_{t+1}) \left(\phi_{t+1} + y\right) \right] \right\}, \quad (79)$$

while a seller's problem is

$$\max_{k_{t+1}} \left\{ -k_{t+1} + \beta\sigma \left[f(k_{t+1} - k^b_{t+1}) + \beta z(k^b_{t+1}, k_{t+1}) \left(\phi_{t+1} + y\right) \right] + \beta(1 - \sigma)f(k_{t+1}) \right\}$$

$$(80)$$

The FOC's from these problems yield

$$\left[\frac{\phi_{t+1} + y}{\phi_t} \right] \left[\frac{\sigma f'(k^b_{t+1})}{z_1(k^b_{t+1}, k_{t+1}) \left(\phi_{t+1} + y\right)} + 1 - \sigma \right] = \frac{1}{\beta}, \quad (81)$$

$$\sigma \left[f'(k_{t+1} - k^b_{t+1}) + z_2(k^b_{t+1}, k_{t+1}) \left(\phi_{t+1} + y\right) \right] + (1 - \sigma)f'(k_{t+1}) = \frac{1}{\beta}. \quad (82)$$

Note that in (81),

$$\ell(k^b_{t+1}, k_{t+1}) = \frac{f'(k^b_{t+1})}{z_1(k^b_{t+1}, k_{t+1}) \left(\phi_{t+1} + y\right)} \quad (83)$$

represents a liquidity premium on shares, analogous to the one in the baseline model. The larger is $\ell(k^b_{t+1}, k_{t+1})$ the greater is the departure of the share price from its fundamental value, and the lower is the return on the asset $\left(\phi_{t+1} + y\right)/\phi_t$. When $\ell(k^b_{t+1}, k_{t+1}) = 1$ there is no liquidity premium.

We first look for an equilibrium where some shares are held in equilibrium by sellers at the end of the CM. This implies that $\phi_t = \hat{\phi}$ and shares are priced according to fundamentals. Thus there is no liquidity premium on shares, since they are priced according to how they are valued at the margin by sellers, who do not trade shares in the DM. In this equilibrium where liquidity is not scarce, let $\bar{k}$ denote the quantity of capital produced at the end of the CM by each seller, and $\bar{k}^b$ the quantity of capital carried into the CM by each buyer. Then (83) implies

$$f'(\bar{k}^b) = \frac{yz_1(\bar{k}^b, \bar{k})}{1 - \beta}. \quad (84)$$

Also, substituting for the price of shares in (82) gives

$$\sigma \left[f'(\bar{k} - \bar{k}^b) + \frac{yz_2(\bar{k}^b, \bar{k})}{1 - \beta} \right] + (1 - \sigma) f'(\bar{k}) = \frac{1}{\beta}, \tag{85}$$

and we require that the quantity of shares brought by each seller to the DM is $a \leq 1$, or,

$$z(\bar{k}^b, \bar{k}) \leq 1. \tag{86}$$

Thus, an equilibrium where liquidity is not scarce and shares trade at their fundamental value, with no liquidity premium, consists of quantities $\bar{k}$ and $\bar{k}^b$ solving (84) and (85), and satisfying the inequality (86).

Now, consider an equilibrium where liquidity is scarce, in the sense that buyers hold the entire stock of shares at the end of the CM for transactions purposes, and sellers hold zero. Then, in a steady state

$$z(k^b, k) = 1, \tag{87}$$

and we can use (81), (82), and (87) to solve for ϕ, k, and k^b.

To consider an extreme case, let $\theta = 1$. Then, from (78) we have

$$z(k^b, k) = \left(\frac{1}{\phi_{t+1} + y} \right) [f(k) - f(k - k^b)], \tag{88}$$

and we can write (81) and (82) as

$$\left[\frac{\phi_{t+1} + y}{\phi_t} \right] \left[\frac{\sigma f'(k^b_{t+1})}{f'(k_{t+1} - k^b_{t+1})} + 1 - \sigma \right] = \frac{1}{\beta}, \tag{89}$$

$$f'(k_{t+1}) = \frac{1}{\beta}. \tag{90}$$

Thus from (90), in any equilibrium, we will have $k_{t+1} = \hat{k}$, the same total quantity of capital as in an equilibrium with $\sigma = 0$ where capital cannot be traded. We get this result as sellers receive no surplus from trading capital when $\theta = 1$.

Now, in an equilibrium where liquidity is not scarce, we will have $\phi = \hat{\phi}$, and so from (89) we have $k^b = \frac{\hat{k}}{2}$ in a steady state, so capital is efficiently allocated between buyers and sellers in DM trade. Further, from (86), an equilibrium where liquidity is not scarce exists iff

$$f(\hat{k}) - f\left(\frac{\hat{k}}{2} \right) < \frac{y}{1 - \beta}, \tag{91}$$

Thus, if y is sufficiently large, the share price is high enough when shares are at their fundamental value that there is efficient trade in the DM and capital is efficiently allocated in DM trade. This efficient allocation occurs because, given $\theta = 1$, there is no holdup problem for buyers. However, a holdup problem for sellers exists, and they tend to underaccumulate capital relative to what is efficient.

Now, consider a steady state equilibrium where liquidity is scarce. Here, from (88), (89), and (87), we obtain

$$\frac{\phi}{\phi + y} = \beta \left[\frac{\sigma f'(k^b)}{f'(\hat{k} - k^b)} + 1 - \sigma \right], \tag{92}$$

$$\phi = f(\hat{k}) - f(\hat{k} - k^b) - y, \tag{93}$$

which solve for ϕ and k^b. It is straightforward to show that the solution is unique, and the equilibrium exists iff the solution satisfies $\phi \leq \hat{\phi}$ or

$$f(\hat{k}) - f\left(\frac{\hat{k}}{2}\right) \geq \frac{y}{1 - \beta}.$$

In this equilibrium we have $k^b \leq \hat{k}/2$, so that the allocation of capital between buyers and sellers is inefficient in equilibrium with insufficient liquidity. Further, from (83), our measure of the liquidity premium is

$$\ell(k^b, k) = \frac{f'(k^b)}{f'(\hat{k} - k^b)},$$

which increases as k^b falls given $\hat{k}$ – i.e. as capital allocation becomes more inefficient. From (92) and (93), it is straightforward to show that k^b increases with σ and with y. First, an increase in the frequency of trade, which increases the frequency with which buyers and sellers can more efficiently allocate capital, also increases the efficiency of capital allocation in each trade. Second, an increase in the dividend, which increases the price of shares, results in a more efficient allocation of capital by enhancing the supply of liquidity.

Now, consider the other extreme case where $\theta = 0$ and sellers have all the bargaining power in the DM. Here, from (78), we have

$$z(k^b, k) = \frac{f(k^b)}{\phi_{t+1} + y}.$$

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Then, from (81), $\phi = \hat{\phi}$. In this case, as the buyer receives no DM surplus, and shares always trade at their fundamental value. From (82), optimization by sellers implies that k and k^b must satisfy

$$\sigma f'(k - k^b) + (1 - \sigma)f'(k) = \frac{1}{\beta}. \tag{94}$$

However, the quantity of capital that a buyer trading in the DM carries into the next CM, k^b, is indeterminate. We obtain this result since buyers receive no surplus in the DM and therefore are indifferent in equilibrium concerning the quantity of shares they take to the DM. Given that $\phi = \hat{\phi}$, sellers are also indifferent concerning the quantity of shares they carry from one CM into the next CM. We only require that k^b be small enough that the fundamental value of the stock of shares is sufficient to buy k^b in the DM, i.e. $k^b \in [0, \tilde{k}]$, where $\tilde{k}$ solves

$$f(\tilde{k}) = \frac{y}{1 - \beta}$$

With $\theta = 0$, the holdup problem for buyers in the DM is as severe as possible and so the quantity of capital k is in general inefficiently allocated between buyers and sellers in the DM. However, since $\theta = 0$ implies no holdup problem for sellers, then given k^b, (94) tells us that sellers accumulate capital efficiently. An increase in σ, since it increases the frequency of trade, will in general raise the quantity of capital from (94), given k^b.

This simple model captures the idea that assets are potentially valued for more than their simple returns, and in particular the asset price can include a liquidity premium.[41] This seems important in practice, since money is not the only asset whose value depends at least in part on its use in facilitating transactions. For example, T-bills play an important role in overnight lending in financial markets, where they are commonly used as collateral. Potentially, models like this, which allow us to examine the determinants of the liquidity premium, can help to explain the apparently anomalous behavior of relative asset returns and asset prices. See Lagos (2008) for one such application.

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[41] In the special case where m is money, $y = 0$, we can let the stock be augmented by government through lump-sum transfers. The fundamental equilibrium is then the non-monetary equilibrium where $\phi = 0$. There is also a steady state monetary equilibrium where $\phi > 0$ and $\phi_t/\phi_{t+1} = 1 + \mu$, with μ the money growth rate. In this acse there always exists an equilibrium with insufficient liquidity for $\mu > \beta - 1$. In general, the is optimal.policy is again the Friedman rule $\mu = \beta - 1$.

A clear message of this application is that asset markets are important for allocation and efficiency. If the yields on liquid assets are low or these assets are hard to trade, this tends to reduce investment in productive capital, and also to result in an inefficient allocation of capital across productive units. Further, bargaining power in asset exchange matters for efficiency as well as prices. Just as in our benchmark monetary model, the greater the bargaining power of buyers the more likely that trades will be efficient in decentralized exchange. Here, greater bargaining power for buyers increases the efficiency with which capital is allocated. However, in contrast to the benchmark model, greater bargaining power for buyers also increases inefficiency in that it tends reduce investment.

A model like this is potentially useful for analyzing phenomena related to the recent financial crisis, since it captures a mechanism by which asset exchange and asset prices are important for investment and allocative efficiency. It may seem that to directly address the reasons for credit market problems during a crisis would require models with lending and collateral. However, it is a very short step from a model like the one presented here, where liquid assets are used in exchange, to one where assets serve as collateral in credit contracts. A key feature of our model in this respect is that, if the future payoffs on liquid assets are expected to be low, and one might think now about mortgage-backed securities, then this can reduce investment and cause allocative inefficiency, both of which reduce aggregate output.

7   Conclusion

New Monetarists are committed to modeling approaches that are explicit about the frictions that make monetary exchange and related arrangements socially useful, and that capture the relationship among credit, banking, and currency transactions. Ideally, economic models that are designed for analyzing and evaluating monetary policy should be able to answer basic questions concerning the necessity and role of central banking, the superiority of one type of central bank operating procedure over another, and the differences in the effects of central bank lending and open market operations. New

Monetarist economists have made progress in understanding the basic frictions that make monetary exchange an equilibrium or an efficient arrangement, and in understanding the mechanisms by which policy can affect allocations and welfare. However, much remains to be learned about many issues, including the sources of short-run nonneutralities and their quantitative significance, as well as the role of central banking.

This chapter takes stock of how the New Monetarist approach builds on advances in the theory of money and theories of financial intermediation and payments, constructing a basis for progress in the science and practice of monetary economics. We conclude by borrowing from Hahn (1973), who went on to become an editor of the previous Handbook. He begins his analysis by suggesting "The natural place to start is by taking the claim that money has something to do with the activity of exchange, seriously." He concludes as follows: "I should like to end on a defensive note. To many who would call themselves monetary economists the problems which I have been discussing must seem excessively abstract and unnecessary. ... Will this preoccupation with foundations, they may argue, help one iota in formulating monetary policy or in predicting the consequences of parameter changes? Are not IS and LM sufficient unto the day? ... It may well be that the approaches here utilized will not in the event improve our advise to the Bank of England; I am rather convinced that it will make a fundamental difference to the way in which we view a decentralized economy."

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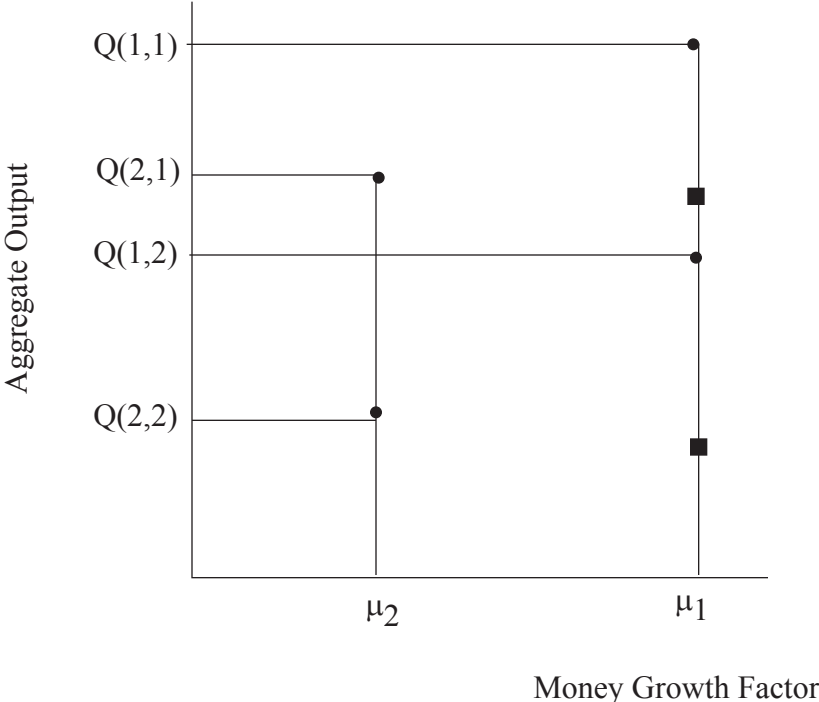
Figure 1: Imperfect Information
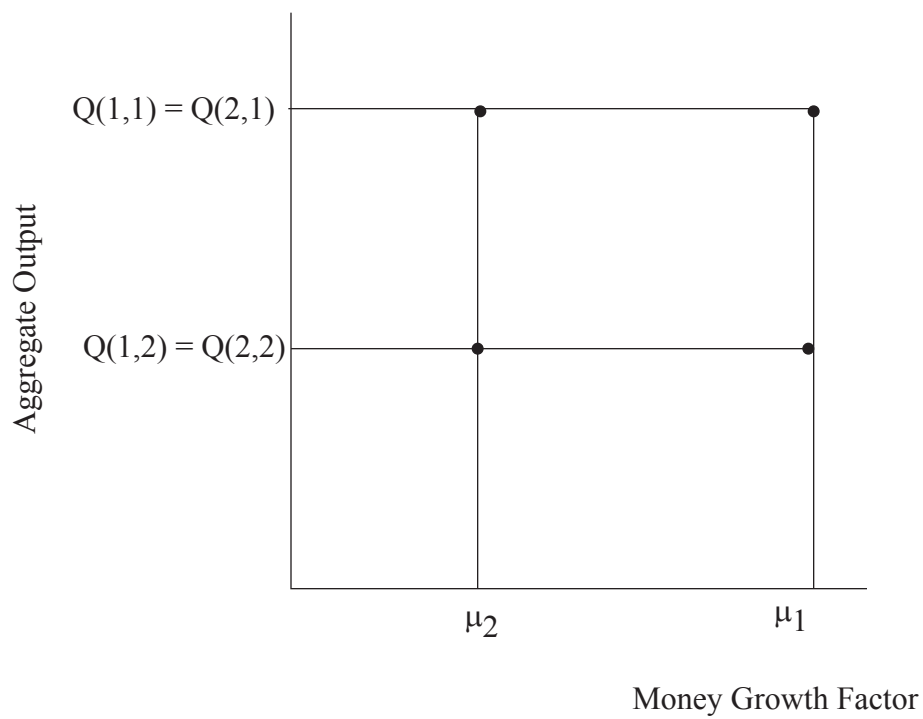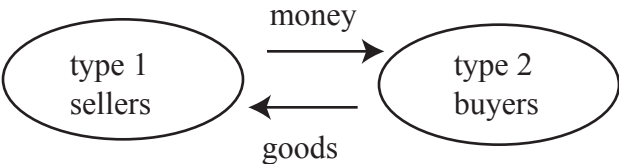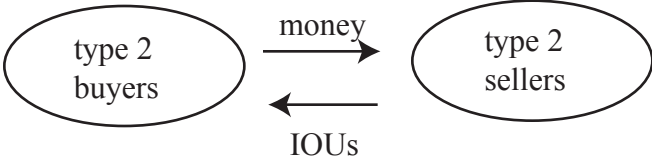
Figure 2: Perfect Information

Aggregate Output (y-axis)

$Q(1,1) = Q(2,1)$

$Q(1,2) = Q(2,2)$

μ_2 μ_1

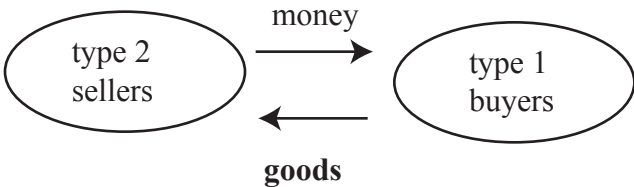Money Growth Factor

Figure 3: Interaction in the Payments System Model

DAY

Walrasian Market 1

type 1 sellers → money → type 2 buyers
type 1 sellers ← goods ← type 2 buyers

Credit Settlement

type 2 buyers → money → type 2 sellers
type 2 buyers ← IOUs ← type 2 sellers

Walrasian Market 2

type 2 sellers → money → type 1 buyers
type 2 sellers ← goods ← type 1 buyers

NIGHT

Random Matches - Cash Transactions

type 1 buyers → money → type 1 sellers
type 1 buyers ← goods ← type 1 sellers
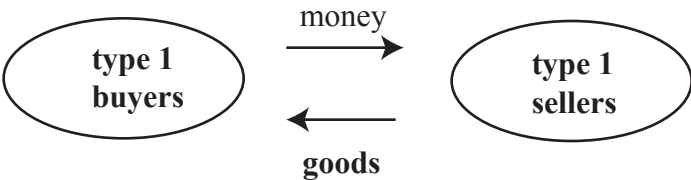
Random Matches - Credit Transactions

type 2 buyers → IOUs → type 2 sellers
type 2 buyers ← goods ← type 2 sellers