Booms and Busts: Understanding Housing Market Dynamics

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Booms and busts

- There are many episodes in which real estate prices rise dramatically.

- Sometimes protracted booms are followed by protracted busts.
  - Japan, U.S., U.K., Finland, Belgium, Denmark, Finland, New Zealand, Switzerland, Norway.

- Other times protracted booms lead to seemingly permanently higher house prices.
  - Spain (late 1980s), Canada (late 1980s), Australia (late 1980s), New Zealand (1990s).
Real Home Prices in the United States
Price Index (1970=100)

Finland

Sweden

Norway

Denmark
Booms and busts

- Standard models can generate large price differences across steady states with different fundamentals.

- Examples of fundamentals that can generate large price differences include: borrowing constraints, income growth, demographics, transactions costs, zoning.
It is difficult to generate protracted price movements in standard rational expectations models because expected changes in future fundamentals are quickly capitalized into prices.

Protracted booms can be generated by assuming that agents receive increasingly positive signals about future fundamentals.

Booms and busts can be generated by assuming that agents first receive increasingly positive signals about future fundamentals and then increasingly negative signals.

Problem: in many episodes is difficult to find observable fundamentals that are closely correlated with observed movements in home prices.
This paper

- We develop a model that generates protracted booms as well as booms-bust episodes.

- Key features of the model:
  - Heterogeneity in beliefs about long-run fundamentals.
    - Agents can Bayesian update but the data don’t convey useful information about long-run fundamentals.
  - Social interactions that change the fraction of agents with different beliefs.
This paper

- To isolate the importance of these features we assume that information about long-run fundamentals is constant over time.

- Our emphasis on uncertainty about long-run fundamentals is related to the literature on long-run risk.

- Our emphasis on belief heterogeneity is related to work by Harrison and Kreps (1976) and Scheinkman and Xiong (2003).
A matching model

- Our starting point is an extended version of Piazzesi and Schneider’s (2009) model.

- Key insight from their paper: in a matching model a small number of optimistic agents can have a large impact on housing prices because these agents are the marginal traders.
We consider a scenario in which agents become aware of a possible change in long-run fundamentals.

Initially only a small set of agents are confident that the change in fundamentals is likely to be large.

The vast majority of agents have diffuse priors over how large the change in long-run fundamentals is likely to be.
Epidemic model of social dynamics

- We endogenize the fraction of agents in each group using an epidemic model of social dynamics.

- Agents meet randomly. At any given meeting the probability that agent $i$ adopts the priors of agent $j$ is proportional to the relative entropy of the two priors.

- Agents with tighter priors are more likely to convert other agents to their beliefs.

- Our model generates dynamics that are similar to those of the infectious diseases models proposed by Bernoulli (1766) and Kermack and McKendrick (1927).
Key results

- The model generates two types of boom-bust episodes.

- Type-one episode: the entire boom-bust episode occurs before uncertainty about long-run fundamentals is realized.

- Type-two episode: the boom occurs before uncertainty is realized. The bust occurs in response to a realization of long-run fundamentals than is lower than the expectation of a subset of the agents in the economy.

- The model can easily produce boom-bust episodes that have the general magnitude and pattern exhibited by U.S. housing prices.
Matching model with homogeneous expectations.

- An expected improvement in fundamentals.
- Transition dynamics: understanding the importance of the extensive margin.

Matching model with heterogeneous expectations.

- An epidemic model of social dynamics.
- Two types of boom-bust episodes.
A matching model

- There is a continuum of agents with measure one.
- Agents are either homeowners or renters.
- All agents have quasi-linear utility and discount utility at rate $\beta$.
- There is a fixed stock of homes, $k < 1$, in the economy.
  - In practice booms and busts occur in areas in which the elasticity of home supply is limited by zoning laws, scarcity of land, and infrastructure constraints.
- There is a rental market with $1 - k$ homes.
In each period home owners derive utility $\varepsilon$ from their house.

The value function of a home owner, $H_t$, is given by:

$$H_t = \varepsilon + \beta [(1 - \eta) H_{t+1} + \eta U_{t+1}].$$

With probability $\eta$ the match goes sour and the home owner is forced to sell his home.

We denote the value function of this home seller by $U_t$. 
Home sellers

- The probability that a sale occurs is $p_t$.

- Once a home is sold the home seller becomes a renter.

- The value of $U_t$ is given by:
  \[
  U_t = p_t \left[ P_t (1 - \phi) + \beta R_{t+1} \right] + (1 - p_t) U_{t+1}.
  \]

- $P_t =$ expected price received by home seller.

- $R_t =$ value function renter at time $t$.

- $\phi =$ sale transactions costs.
Renters

- There are two types of renters: natural home buyers and natural renters.

- Natural buyers derive more utility from owning a home than natural renters.
Natural home buyers

- These agents have a value function $B_t$ and derive a flow utility of $\varepsilon^b$ from renting a home.

- They choose to rent or buy.

$$B_{t}^{rent} = \varepsilon^b + \beta B_{t+1},$$

$$B_{t}^{buy} = q_t \left\{ \varepsilon^b - P^b_t + \beta [(1 - \eta)H_{t+1} + \eta U_{t+1}] \right\} + (1 - q_t)B_{t}^{rent},$$

$$B_t = \max \left( B_{t}^{rent}, B_{t}^{buy} \right).$$

- $q_t$ = probability of buying a home.
- $P^b_t$ = expected price paid by a natural home buyer.
Natural renters

Their value function is $R_t$.

In present-value terms their expected utility of owning a home is lower than that of a natural buyer by an amount $\kappa \varepsilon$.

In each period a fraction $\lambda$ of natural renters have a preference shock and become natural home buyers.

\[
R_{t}^{rent} = \varepsilon^r + \beta \left( (1 - \lambda)R_{t+1} + \lambda B_{t+1} \right),
\]
\[
R_{t}^{buy} = q_t \left\{ \varepsilon^r - P_t^r + \beta \left[ (1 - \eta)H_{t+1} + \eta U_{t+1} - \kappa \varepsilon \right] \right\} + (1 - q_t)R_{t}^{rent},
\]
\[
R_t = \max \left( R_t^{rent}, R_t^{buy} \right).
\]

$P_t^r$ = expected purchase price for a natural renter.
Composition of the population

- $h_t = \text{fraction of home owners.}$
- $u_t = \text{fraction of home sellers.}$
- $b_t = \text{fraction of natural buyers.}$
- $r_t = \text{fraction of natural renters.}$

\[ h_t + u_t = k. \]

\[ b_t + r_t = 1 - k. \]

- The state of the system is represented by two of these four variables.
Buyers and sellers

Indicator functions

- $J^b_t = 1$ if it is optimal for natural buyers to buy a house and zero otherwise.
- $J^r_t = 1$ if it is optimal for natural renters to buy a house and zero otherwise.

Agents can only own one home, so the only potential buyers are natural renters and natural buyers:

$$\text{Buyers}_t = b_t J^b_t + r_t J^r_t.$$ 

There is no short selling and homeowners only sell when the match with their house goes sour:

$$\text{Sellers}_t = u_t.$$
When a match occurs the transactions price is determined by generalized Nash bargaining.

The bargaining power of buyers and sellers is $\theta$ and $1 - \theta$, respectively.

There are two types of matches:
- A natural buyer and a seller;
- A natural renter and a seller.

To determine transactions prices we need to compute the reservation prices of buyers and sellers.
The reservation price, $\bar{P}_t^b$, makes these agents indifferent between buying and selling.

\[
B_t^{\text{rent}} = B_t^{\text{buy}},
\]

\[
\beta B_{t+1} = q_t \left\{ -P_t^b + \beta \left[ (1 - \eta)H_{t+1} + \eta U_{t+1} \right] \right\} + (1 - q_t)\beta B_{t+1}.
\]

Solving for $\bar{P}_t^b$:

\[
\bar{P}_t^b = \beta \left[ (1 - \eta)H_{t+1} + \eta U_{t+1} - B_{t+1} \right].
\]
The reservation price, $\bar{P}_t^r$, makes these agents indifferent between buying and selling.

\[ R_t^{rent} = R_t^{buy}, \]
\[ \beta \left[ (1 - \lambda)R_{t+1} + \lambda B_{t+1} \right] = q_t \left\{ -\bar{P}_t^r + \beta \left[ (1 - \eta)H_{t+1} + \eta U_{t+1} - \kappa \varepsilon \right] \right\} + (1 - q_t) \beta \left[ (1 - \lambda)R_{t+1} + \lambda B_{t+1} \right]. \]

Solving for $\bar{P}_t^r$:

\[ \bar{P}_t^r = \beta \left[ (1 - \eta)H_{t+1} + \eta U_{t+1} \right] - \beta \left[ (1 - \lambda)R_{t+1} + \lambda B_{t+1} \right] - \kappa \varepsilon. \]
Reservation price of home sellers

- To simplify the analysis we assume that the home sellers reservation price, $\bar{P}^u$, is an exogenous constant.

- We endogeneize this reservation price in an extended version of the model.

- Computing the equilibrium when $\bar{P}^u$ is endogenous and there are social dynamics is very difficult.
Price paid by a natural buyer ($P^b_t$):

$$P^b_t = \theta \bar{P}^b_t + (1 - \theta) \bar{P}^u.$$

Price paid by a natural renter ($P^r_t$):

$$P^r_t = \theta \bar{P}^r_t + (1 - \theta) \bar{P}^u.$$

Average price received by seller ($P_t$):

$$P_t = \frac{b_t J^b_t P^b_t + r_t J^r_t P^r_t}{b_t J^b_t + r_t J^r_t}.$$

$\bar{P}^u = \text{reservation price of the seller.}$
Timing

- Preference shocks occur in the beginning of the period.
  - With probability $\eta$ home owners become home sellers.
  - With probability $\lambda$ natural renters become natural buyers.

- Transactions occur at the end of the period.
  - Home sellers attempt to sell their house.
  - Home buyers attempt to buy a house.
The number of homes sold, $m_t$, is determined by the matching function:

$$m_t = \mu \left( \text{Sellers}_t \right)^\alpha \left( \text{Buyers}_t \right)^{1-\alpha} .$$

Probability of selling a house:

$$p_t = m_t / \text{Sellers}_t .$$

Probability of buying a house:

$$q_t = m_t / \text{Buyers}_t .$$
Population dynamics

- **Home owners:**
  \[ h_{t+1} = (1 - \eta) h_t + q_t \left( J_t^b (b_t + \lambda r_t) + J_t^r (1 - \lambda) r_t \right). \]

- **Home sellers:**
  \[ u_{t+1} = (u_t + \eta h_t) (1 - p_t). \]

- **Natural home buyers:**
  \[ b_{t+1} = (b_t + \lambda r_t) \left( 1 - q_t J_t^b \right). \]

- **Natural renters:**
  \[ r_{t+1} = (1 - \lambda) r_t \left( 1 - q_t J_t^r \right) + p_t (u_t + \eta h_t). \]
Steady state

- We choose $\kappa$ so that only natural buyers purchase homes in steady state.
  - $\kappa$ is the parameter that controls the difference between the utility of owning a home for a natural buyer and a natural renter.

- We choose the parameter $\eta$ so that the probability of buying and selling are the same in the steady state.

  $$p = q.$$
Time period = one month.

$\mu = 1/6$

- Average time to sell a house in steady state = 6 months.

$\alpha = 0.5$;

$\lambda = 0.02$;

- Chosen so that in a steady state in which $p = q$ the value is $\eta$ is 0.008.
- This value of $\eta$ implies that home owners sell their house on average every 10 years.

$k = 0.7$;

- 70 percent of the population owns homes.

$\beta = 0.995$;

- Implies 6 percent annual mortgage rate.
Parameters for numerical example

- \( \phi = 0.05; \)
  - Transactions costs of selling a home (percentage of sale price).
- \( \varepsilon = 5. \)
  - Controls level of steady state price but does not affect dynamics.
- \( \varepsilon^b = \varepsilon^r = 1. \)
  - Values chosen so that in steady state only natural buyers buy homes.
- \( \kappa = 40. \)
  - Value chosen so that it is not optimal for natural renters to buy homes in the steady state.
  - The steady state utility of a natural renter who buys a home is 28 percent lower than that of a natural home buyer.
Suppose that at time zero agents suddenly anticipate that, with probability $1 - a$, the utility of owning a home rises from $\varepsilon$ to $\varepsilon^* > \varepsilon$.

The result is a large instantaneous jump in $P_t$.

There are no transition dynamics. The economy converges immediately to a new steady state with a higher price.

So, even with matching frictions, when beliefs are homogeneous, anticipated future changes in fundamentals are immediately reflected in today’s price.
We now study an experiment that highlights the importance of the extensive margin, i.e. the number of buyers. The resulting intuition is useful for understanding the dynamics in the more complicated model.

Suppose that the initial number of natural buyers is higher than in the steady state, \( b_0 > b \).

Since \( b_0 + r_0 = 1 - k \), the initial number of natural renters is below the steady state.

Since there are more buyers, the probability of buying is below its steady state value.

The probability of selling is above its steady state value.
Transitional dynamics

- There is a persistent rise in the value of $U_t$ for two reasons.
- More buyers implies that the probability of selling is higher.
- The sale price, $P_t$, is higher.

$$U_t = p_t \left[ P_t (1 - \phi) + \beta R_{t+1} \right] + (1 - p_t) U_{t+1}. $$
Transitional dynamics

- Home owners must sell with probability $\eta$, in which case their value function is $U_t$.

$$H_t = \varepsilon + \beta \left[ (1 - \eta) H_{t+1} + \eta U_{t+1} \right].$$

- So a rise in $U_{t+1}$ induces a rise in $H_t$. 
Natural buyers want to buy a home but cannot buy with probability one due to the matching friction.

The persistent decline in the probability of buying produces a persistent fall in $B_t$.

So the reservation price (which equates the value of buying and renting) rises:

$$\bar{P}_t^b = \beta [(1 - \eta)H_{t+1} + \eta U_{t+1} - B_{t+1}] .$$
Along the transition path only natural buyers want to buy homes.

The transactions price, $P_t$ is given by:

$$P_t = \theta \bar{P}_t^b + (1 - \theta) \bar{P}_t^u.$$

So $P_t$ rises.

These effects become smaller as the number of natural buyers converges to the steady state.

So $P_t$ converges to the steady state value from above.
Transitional dynamics

Price

Buyers

Probability of buying

B's Reservation Price

Sellers

Probability of selling
Transitional dynamics

![Transitional dynamics graphs](image-url)
Transitional dynamics

- **h**: Probability of holding stocks over years.
- **u**: Probability of updating expectations over years.
- **Sales**: Trend in sales over years.
- **b**: Probability of buying stocks over years.
- **r**: Probability of selling stocks over years.
- **Probabilities of Selling and Buying**: Graphs show the probabilities of selling and buying stocks over years.
Effects of an increase in the number of buyers

Key results

- Reduces the probability of buying.
- Lowers the value function of being a buyer.
- Raises the value function of being a seller.
- Generates a persistent increase in home prices.
Generating boom-busts

The previous results suggest that we can generate a boom-bust episode if, for some reason, there is a persistent increase in the number of buyers followed by a persistent decrease.

One strategy is to look for observable fundamentals that can generate this pattern.

Problem: in many boom-bust episodes it is hard to find observable fundamentals that are correlated with housing prices.
Introducing heterogeneous beliefs

- We now consider a model in which uncertain news about future fundamentals triggers heterogeneity in beliefs.

- Social dynamics change the fraction of agents with different beliefs.

- These changes induce a rise and fall in the number of house buyers.

- The net result is a type-one boom-bust episode.
An epidemic model of social dynamics

- Before time zero the economy is in a steady state with homogenous priors.

- At time zero agents learn that, with probability $(1 - a)$, long-run fundamentals will change.

- Agents fall into three categories depending on their priors about these fundamentals.

- Borrowing from the terminology used in the epidemiology literature we call these agents “infected,” “cured,” and “vulnerable.”

- We denote by $i_t$, $c_t$, and $v_t$ the time $t$ fraction of infected, cured and vulnerable agents, respectively.
Agent types are publicly observable.

Priors are common knowledge, so higher-order beliefs do not play a role.

Agents can Bayesian update but there is no useful information to update their priors about long-run risk.

In today’s talk we consider only the case in which agents do not take into account that they might change their views as a result of social interactions.
A simple experiment

- At time zero:
  - Almost everybody in the population is vulnerable, i.e. they have diffuse priors about future fundamentals.
  - There is a very small fraction of cured and infected agents.

- Infected agents expect an improvement in fundamentals.
  \[ E^i(\varepsilon^*) > \varepsilon. \]

- Cured and vulnerable do not expect an improvement in fundamentals.
  \[ E^c(\varepsilon^*) = E^v(\varepsilon^*) = \varepsilon. \]
An epidemic model of social dynamics

- We use the entropy of the probability distribution $f^j(\varepsilon^*)$ to measure the uncertainty of an agents’ views,

$$e^j = - \sum_{i=1}^{n} f^j(\phi_i) \ln \left[ f^j(\phi_i) \right].$$

- When $f^j(\phi)$ is a uniform probability distribution agents have maximal entropy and $e^i = \ln(n)$. 
An epidemic model of social dynamics

- Agents meet randomly at the beginning of the period.

- When two different agents meet, the high-entropy agent adopts the priors of the low-entropy agent with probability $\gamma$.

- The value of $\gamma$ depends on the ratios of the two entropies:

$$\gamma_{lj} = \max(1 - e^l / e^j, 0).$$

- We adopt this assumption for two reasons.
  - It strikes us as plausible;
  - It is a reduced form way of capturing environments in which some agents have private signals or different data processing capabilities.
Our assumption about $\gamma$ is consistent with evidence from the psychology literature that people are more persuaded by those who are confident.

Financial advisors who express a high level of confidence are seen as more credible and trustworthy than advisors who express modest confidence, even when their performance is the same (Price and Stone, 2004).

Advisors who express more confidence earn greater trust, are more likely to have their advice followed, and engender more confidence in those receiving their advice (Sniezek and Van Swol (2001)).
An epidemic model of social dynamics

- To simplify we assume that the pdfs of “infected” and “cured” agents are different but have the same entropy, \( e^i = e^c \).

- So, when infected and cured agents meet no one changes their views about long-run fundamentals.

- The pdf of the vulnerable agents is diffuse, so it has high entropy.

\[
e^v > e^c = e^i.
\]

- When a vulnerable agent meets an infected or cured agent he is converted to their views with probability:

\[
\gamma = 1 - e^i / e^v = 1 - e^c / e^v.
\]
An epidemic model of social dynamics

- There are $i_t \nu_t$ encounters between infected and vulnerables at time $t$.
  - So, $\gamma i_t \nu_t$ vulnerable agents become infected.

- There are $c_t \nu_t$ encounters between cured and vulnerables at time $t$.
  - So, $\gamma c_t \nu_t$ vulnerable agents become cured.

- We assume that with a very small probability $\delta_i$, infected agents become cured.

- Priors and the laws of social dynamics are public information.

- Agents take into account future changes in the fraction of the population that holds different views.
Preference shocks occur in the beginning of the period.
- With probability $\eta$ home owners become home sellers.
- With probability $\lambda$ natural renters become natural buyers.

Social interactions occur in the middle of the period.
- Agents meet other agents and potentially change their views.

Transactions occur at the end of the period.
- Home sellers can potentially sell.
- Natural buyers and natural renters can potentially buy.
An epidemic model of social dynamics

The model generates dynamics that are similar to those of the epidemic models of Bernoulli (1766) and Kermack and McKendrick (1927).

\[ i_{t+1} = i_t + \gamma i_t \nu_t - \delta_i i_t, \]

\[ c_{t+1} = c_t + \gamma c_t \nu_t + \delta_i i_t, \]

\[ \nu_{t+1} = \nu_t - \gamma \nu_t (c_t + i_t). \]
\[ \begin{align*}
\text{Vulnerable} & \quad \gamma v_t c_t \\
\text{Cured} & \quad \gamma v_t i_t \\
\text{Infected} & \quad \delta
\end{align*} \]
The value function after uncertainty is realized depends on the realized value of $\varepsilon^*$ and on the state variables, $h_t$ and $b_t$:

$$H(\varepsilon^*, h_t, b_t) = \varepsilon^* + \beta [(1 - \eta)H(\varepsilon^*, h_{t+1}, b_{t+1}) + \eta U(\varepsilon^*, h_{t+1}, b_{t+1})].$$

Before uncertainty is realized the value function is given by:

$$H^j_t = \varepsilon + \beta \left[(1 - \eta)E_t^j(H^j_{t+1}) + \eta E_t^j(U^j_{t+1})\right].$$

$E^j(.)$ is the expectation based on the pdf of agent $j$:

$$E_t^j(H^j_{t+1}) = aH^j_{t+1} + (1 - a)E_t \bar{H}^j_{t+1},$$

$$E_t \bar{H}^j_{t+1} = \sum_{d=1}^{n} f^j(\varepsilon^*_d)H(\varepsilon^*_d, h_{t+1}, b_{t+1}).$$
Price determination

- Prices are determined as in the model with homogenous agents.

- There are six different possible prices each corresponding to a match between a seller and the following types of buyers:
  - $b^i, b^c, b^v, r^i, r^c$, and $r^v$.

- The price received by the seller is a weighted average of the prices paid by renters for whom it is optimal to sell.
An epidemic model of social dynamics

Parameters for numerical example

\[ E^c(\epsilon^*) = E^v(\epsilon^*) = \epsilon \]

\[ E^i(\epsilon^*) \approx 2\epsilon \]

- We assume that the vulnerable have a uniform pdf, so they have maximal entropy.

- We want the fraction of the population that changes its views is relatively small, say less than 10 percent.
  - This property requires that the entropy of the infected and the cured not be too low relative to the entropy of a uniform pdf.

- We assume that the support of the distribution for \( \epsilon^* \) has six elements, which makes it easy to construct a parameterization with the features just discussed.
An epidemic model of social dynamics
Parameters for numerical example

- Support of the distribution of new fundamentals:
  \[ \mathbf{\mathbf{\varepsilon}^*} \subseteq \{0.5, 1, 1.5, 2, 5, 20\} . \]

- Pdfs, entropy and expected value of \( \mathbf{\mathbf{\varepsilon}^*} \):
  \[
  f^i = [0.12 \ 0.12 \ 0.12 \ 0.12 \ 0.12 \ 0.40]; \quad e^i = 1.6; \quad E^i(\mathbf{\mathbf{\varepsilon}^*}) = 9.2, \\
  f^c = [0.12 \ 0.12 \ 0.12 \ 0.12 \ 0.40 \ 0.12]; \quad e^c = 1.6; \quad E^c(\mathbf{\mathbf{\varepsilon}^*}) = 5.0, \\
  f^v = [1/6 \ 1/6 \ 1/6 \ 1/6 \ 1/6 \ 1/6]; \quad e^v = 1.8; \quad E^v(\mathbf{\mathbf{\varepsilon}^*}) = 5.0.
  \]

\[ \gamma = 1 - e^i / e^v = 0.0854. \]
\( \delta_i = 0.009. \)

- We assume that at time zero there is a very small number of infected and cured natural renters:

\[
\begin{align*}
    r_i^0 &= 10e^{-5}; \\
    r_c^0 &= 10e^{-5}.
\end{align*}
\]

- Everyone else is vulnerable.
An epidemic model of social dynamics

- Average home prices rise and then fall.

- Even though agents have perfect foresight up to the resolution of long-run uncertainty, the initial rise in price is very small (less than one percent).

- Prices start falling when the number of potential buyers falls.

- The number of transactions is positively correlated with the average home price.

- The boom features a “sellers market,” the probability of selling is high and the probability of buying is low.
The rise and fall in prices is highly correlated with movements in the number of potential buyers.

The initial rise in the number of potential buyers relative to the steady state is $10^{-5}$.

Over time the number of potential buyers rises from 2.7 percent in the initial steady state to 7 percent at the peak of housing boom. Thereafter the number of potential buyers declines.

A subset of agents who have high expectations about long-run fundamentals exhibit speculative behavior.

- Natural renters who would not normally buy enter the housing market because they become infected.
Potential buyers (percent of population) vs. Average house price

- Potential buyers: The number of potential buyers increases with population percentage, reaching a peak and then decreasing.
- Average house price: The average house price also increases with population percentage, reaching a peak and then decreasing.
Analyzing the movements in potential buyers

- The number of potential buyers at time $t$ has two components: natural buyers, $b_t + \lambda r_t$, and infected natural renters, $(1 - \lambda)r_t^i$.

- The stock of infected natural renters at time $t + 1$ consists of two groups.
  1. Infected natural renters at time $t$ who tried to buy but were not successful.
  2. Infected home sellers who became natural renters after selling their homes.

\[
\begin{align*}
\frac{r_t^i}{t+1} &= (1 - q_t)[r_t^i(1 - \lambda) + \gamma r_t^v(1 - \lambda)i_t - \delta r_t^i(1 - \lambda)] + \\
&\quad p_t [(u_t^i + \eta h_t^i) + \gamma (u_t^v + \eta h_t^v)i_t - \delta(u_t^i + \eta h_t^i)]
\end{align*}
\]
The change in the number of natural buyers after the preference shock $\lambda$ materializes has two components:

- Inflow of natural buyers, $\lambda r_{t+1}$;
- Outflow of natural buyers who purchased homes: $-q_t(b_t + \lambda r_t)$.

Both inflows and outflows fall but outflows fall more than inflows.

As infected natural renters enter the housing market the probability of buying falls. This fall leads to a reduction in the outflow of natural buyers, creating a rise in the number of potential buyers.
Analyzing the movements in potential buyers

Potential buyers

Infected natural renters

Natural buyers

Natural buyers: inflows and outflows
A naive econometrician trying to explain the boom and bust episode using conventional fundamentals would fail.

There is a change in the environment that leads agents to think that with some probability events can improve in the long run.

There is no observable shock or event that is associated with the fall in home prices.

This fall reflects entirely the non-linearities associated with the social dynamics in our model.
What happens when uncertainty is resolved?

- There is a discontinuous jump up or down in housing prices.
  - The price rises if the expectations of the infected agents are correct;
  - It falls if the expectations of the cured agents are correct.

- We don’t observe these types of jumps in the data.

- The discontinuity reflects the stark nature of information revelation.

- This feature can be eliminated if more information about long-run fundamentals is gradually revealed.
Two types of boom-busts

- We can get a boom-bust episode under two scenarios.

  - Scenario 1: the boom-bust happens without resolution of uncertainty about long-run fundamentals.

  - Scenario 2: the boom-bust happens with resolution of uncertainty about long-run fundamentals.
    - Here $\delta_i$ can be zero.
To disentangle the impact of search frictions from social dynamics we consider a model in which only the latter is operative.

Consider an economy in which there is a fixed supply, $k$, of an asset.

In addition, suppose that each agent can only own one unit of the asset and there is no short-selling.

The asset pays a dividend $\varepsilon$ per period. With probability $1 - a$ the dividend increases permanently to a level $\varepsilon^*$. 

Agents expectations about $\varepsilon^*$ are:

$$E^c(\varepsilon^*) = E^v(\varepsilon^*) = \varepsilon, \quad E^i(\varepsilon^*) > \varepsilon.$$
The price is determined by the marginal trader.

The marginal trader is an infected agent when \( i_t \geq k \) and a cured/vulnerable agent otherwise.

Prices before the realization of uncertainty are given by:

\[
P_t = \varepsilon + \beta \left[ aE^i(\varepsilon^*) + (1 - a)P_{t+1} \right], \quad \text{if } i_t \geq k,
\]
\[
P_t = \varepsilon + \beta \left[ aE^c(\varepsilon^*) + (1 - a)P_{t+1} \right], \quad \text{if } i_t < k,
\]

Consider the case in which for \( t \geq T, i_t < k \). In this case:

\[
P_T = \varepsilon / (1 - \beta).
\]

Using this terminal condition we can compute recursively the prices that obtain if uncertainty is not realized.
Social Dynamics

- If $i_t < k$ for all $t$, the cured/vulnerable agents are always the marginal trader and the price is constant at $\varepsilon/(1 - \beta)$.

- The interesting case is when there are time periods in which $i_t > k$. 
Social Dynamics

- Infected
- Cured
- Vulnerable
- Price
Social Dynamics

- We need a large infection so that $i_t > k$.

- The boom is not very large. The price is always lower than the price the infected agents would be willing to pay.

- There is a large price jump at time zero.
Conclusion

- It is generally difficult to generate boom-bust episodes.

- We are exploring models in which agents have different views about long-run fundamentals.

- These views can spread in a way that resembles epidemics.

- These epidemics can generate boom-bust cycles.
Agent are atomistic so they have no incentive to misrepresent their type to influence social dynamics.

Example: a cured home seller poses as infected to raise the fraction of infected in the population and increase the price at which he can sell his home.

However, agents might want to misrepresent their type to bargain over price. We abstract from this possibility by assuming that agent types are publicly observable.
To keep $h_t$ constant the number of home owners who have to sell ($\eta h$) must be equal to the number of renters who become home owners ($q(b + \lambda r)$).

$$\eta h = q(b + \lambda r),$$

Once $h_t$ is constant, $u_t$ is also constant since:

$$h_t + u_t = k.$$

To keep $b_t$ constant the number of natural buyers who buy a home ($q(b + \lambda r)$) must be equal to the number of natural renters who become natural buyers ($\lambda r$).

$$\lambda r = q(b + \lambda r).$$

Once $b_t$ is constant, $r_t$ is also constant since:

$$b_t + r_t = 1 - k.$$
Timing of social dynamics model

Example

- Fraction of vulnerable happy renters in the beginning of the period is $r_t^v$.

- Preference shock occurs:
  \[
  (r_t^v)' = r_t^v (1 - \lambda).
  \]

- Social interactions occur:
  \[
  (r_t^v)'' = (r_t^v)' - \gamma (r_t^v)' c_t - \gamma (r_t^v)' i_t.
  \]

- Purchases and sales occur.
  - Natural renters might buy a home
  - Home sellers might sell their home
  \[
  r_{t+1}^v = (r_t^v)'' - q_t J_t^r (r_t^v)'' + p_t (u_t^v)''.
  \]
Solution algorithm

- We need to compute a fixed point for the sequence of indicator functions \( \{ J^b_t^j \}, \{ J^r_t^j \} \) which characterize buying decisions for \( j = i, c, v \).

- Guess initial paths for \( \{ J^b_t^j \} \) and \( \{ J^r_t^j \} \).

- Given initial conditions solve for the path that occurs before uncertainty is realized for the fractions of home owners, home sellers, natural buyers, and natural renters of different types.
  - Example:

\[
r_{t+1}^v = r_t^v (1 - \lambda) (1 - \gamma c_t - \gamma i_t) (1 - q_t J^r_t) + p_t u_{t+1}.
\]
Solution algorithm

- At every time $t$ we compute the path that can occur if uncertainty is resolved at time $t + 1$.

- When uncertainty is resolved at time $t + 1$ the relevant state variables are $h_{t+1}$ and $b_{t+1}$.

- Solve the deterministic path that can occur for every possible realization of $\varepsilon^*$. 

- Compute the values of $H_{t+1}$, $U_{t+1}$, $B_{t+1}$, and $R_{t+1}$, for each of the six values of $\varepsilon^*$ that can materialize.
Computes the expected value of the value functions at time $t+1$ when uncertainty is realized ($E(j)(\bar{H}_{t+1}), E(j)(\bar{U}_{t+1}), E(j)(\bar{B}_{t+1}), E(j)(\bar{R}_{t+1})$) using the pdf of the different agents, $j = i, c, v$.

Computes the value function using backward induction from limiting steady state.

$$H^j_t = \varepsilon + \beta(1 - \eta)[aH^j_{t+1} + (1 - a)E_t \bar{H}^j_{t+1}] + \beta \eta[aU^j_{t+1} + (1 - a)E_t \bar{U}^j_{t+1}].$$
There is substantial dispersion in prices.

The highest price is paid by $R^i$ agents.

The second highest price is paid by $B^i$.

$B^c$ and $B^v$ agents pay the lowest price.
Why do infected natural renters pay the highest price?

- This result reflects our assumption that the fixed cost of buying for this agent is proportional to $\varepsilon$.
- We made this assumption to ensure that the natural renter rents in steady state regardless of the value of $\varepsilon$.
- This agent expects the utility of being a home owner will be high in the future.
- He is better off buying before uncertainty is realized because the current $\varepsilon$ is relatively low.
- Transactions costs are lower before the realization of the shock.
- The price paid by $B^i$ agents is lower than that paid by $R^i$ agents.
  - These agents expect the same high utility of being a home owner as the $R^i$ agents.
  - Unlike the $R^i$ agents his transactions costs are independent of $\varepsilon^*$. 