# THE WORLD TRADE NETWORK

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#### Abstract

This paper uses the tools of network analysis to graphically and analytically represent the characteristics of world trade. The structure of the World Trade Network is compared over time, detecting and interpreting patterns of trade ties among countries, and network indices are also used in a gravity model regression. The results show that trade integration at the world level has been increasing but it is still far from being complete, with the exception of some areas, that there is a strong heterogeneity in countries' choice of trade partners, and that WTO members are more closely connected than the rest of the world. The structural difference between the extensive and the intensive margin of trade is also highlighted.

KEYWORDS: International Trade, Network Analysis, Gravity, WTO, Extensive and Intensive Margins of Trade. JEL CLASSIFICATION: C02, F10, F14.

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# 1 Introduction

A natural way of representing the trade flow between two countries is by means of a straight line segment connecting two points representing the trading countries. The segment can be directed, like an arrow, if we knew that the flow originates from one of the two countries and is bound to the second one. We could also attach a value to it indicating the strength of the flow, or we can make the drawing even more complex, including additional information about the countries or the links. If we do the same for all countries in the world, our drawing of international trade flows becomes a graph and, including in the picture all supplementary information about vertices and links, the result would be a network: the World Trade Network.

Independently from the emergence of topology and graph theory in mathematics and of social network analysis in anthropology and sociology,<sup>1</sup> international economists have conceived international trade as a network since long ago. The picture reproduced in figure 1 is taken from Hilgerdt (1943) and is a modified version of a chart included in the the volume *The Network of World Trade* by the League of Nations published in 1942 (League of Nations, 1942). The purpose of that study was to describe the pattern of international trade before World War II, so to guide welfare promoting national trade policies not based on "... the nature of the trade of the country formulating its policy only, but on the nature of the essential oneness of the trade of the world." Such emphasis on the interconnectedness of national trade policies is based on a view of world trade clearly described in the introduction of the volume:

International trade is much more than the exchange of goods between one country and another; it is an intricate network that cannot be rent without loss. (League of Nations, 1942, p.7)

In order to provide a perception of such an intricate network Folke Hilgerdt and the other researchers of the Economic Intelligence Service of the League

<sup>&</sup>lt;sup>1</sup> Graph theory, born in the 18th century, has rapidly developed in the 1950s with the inclusion of probability and the development of random graphs and is now a well recognized branch of mathematics (see Bollobás (2002) for a comprehensive modern treatment). Building on this approach, Social Network Analysis developed at the turn of the twentieth century, through the intellectual effort of sociologists, psychologists and anthropologists. The interest was mainly on the characteristics of small networks and on community relations and individual interactions. The discipline was fully established in the 1970s. In the same years the interest expanded from small to large networks and on the study of their characteristics, such the number of degrees of separation in social networks (the "Small World" problem). On the origin of social network analysis see Scott (2000, ch.2) and for a general overview see Wasserman and Faust (1994).

of Nations did use a graph or, what was called by sociologists in the tradition of Jacob L. Moreno, a sociogram.<sup>2</sup>



Figure 1: A natural way of representing international trade is through a network. The figure is from Folke Hilgerdt (1943), "The Case for Multilateral Trade", p. 394.

The conventions followed in drawing the graph in figure 1 are evocative rather than mathematical or associated to any political or economic relations, and the same has been the case for other examples of the same sort in the past (Saul, 1954, p. 61) or in present times (Feenstra and Taylor, 2008, p.6). Only recently economists and social network scholars have started to go beyond graphical visualisation and dig into the structural characteristics of the World Trade Network and into its properties.

The benefit of representing a network of trade flows is to give emphasis to the relationship between the countries in the network and the structure,

<sup>&</sup>lt;sup>2</sup> The countries considered in the League of Nations volume represented the ninetenths of the world's trade in 1928. Only the three largest trading countries - the United Kingdom, the United States, and Germany – are shown separately; the other countries were grouped in three categories: the 'Tropics' (including Central Africa, the tropical agricultural and the mineral producing countries of Latin America and tropical Asia), the 'Regions of recent settlement in the temperate belts' (including the British dominions of South Africa, Canada, Oceania, and Argentina, Uruguay, and Paraguay), and 'Europe' with the exception of the United Kingdom and Germany. See League of Nations (1942), Table 20-23, Table 44 and Annex 3 for details on the classification and country data. As an example, imports of the United States from the 'Tropics' were 1,820 and exports of the United States to the 'Tropics' were 870: the trade balance was (-)950; imports of the 'Tropics' from the United States were 1,010 and exports of the 'Tropics' to the United States were 1,650: the trade balance was 640. The difference between imports (exports) of the United States and exports (imports) of the 'Tropics' are due to transport cost and insurance freight.

or the systemic feature, of the network itself. Not surprisingly, this is exactly the purpose of network analysis (NA). In fact, both graph theory and NA place more emphasis on the relationship between vertices in the graph and on the structure of the system itself, rather than on vertices' attributes, that are generally left in the background. The application of NA to international trade can, therefore, nicely complement other empirical analyses of trade, in particular the gravity model of international trade (Harrigan, 2003; Anderson and van Wincoop, 2003; Helpman, Melitz and Rubinstein, 2008), which instead put countries' characteristics or dyadic relationships at the fore front of the analysis, and that even if recognizing the importance of the structure of the system - generally represented by a Multilateral Resistance term - it leaves its analysis on the background. NA can be therefore fruitfully used to address some of the recently discussed issues in the empirics of international trade where systemic effects can be relevant, such as the role of the extensive and the intensive margins in trade dynamics (Hummels and Kleanow, 2005; Felbermayr and Kohler, 2005), or the 'triangular' relations in trade and the presence of trade creation and trade diversion in Regional Trade Agreements (Magee, 2008; Egger and Larch, 2008), or the role of international institutions such the WTO (Rose, 2004; Subramanian and Wei, 2007) and of new emerging countries in the network, and how the system changes because of these.

In this paper, after presenting the main tools of NA and some of the results obtained in previous applications of this approach to trade (section 2), we use NA to explore the World Trade Network and its changes over time (section 3), and address some issues debated in the recent trade literature: the role of the WTO in international trade, the existence of regional blocks, the dimensions of the extensive and intensive margin of trade (section 4). The results obtained through this analysis provide a measure of trade integration at the world level, showing that the world is still far from being fully connected, but that full-connection (or network completeness) is already evident in some sub-regional components of the World Trade Network. This evidence also indicates a strong heterogeneity in the countries' choice of partners, and that the WTO membership characterizes trade integration at the extensive margin and not only at the intensive margin.

# 2 International trade as a network

The World Trade Network,<sup>3</sup> defined as  $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{W}, \mathcal{P})$ , is composed of two distinct parts. The first one is the graph  $\mathcal{G} = (\mathcal{V}, \mathcal{L})$ , where  $\mathcal{V} = \{2, 3, \ldots, n\}$ is a set of vertices (countries) and  $\mathcal{L} = \{0, 1, \ldots, m\}$  is a set of links (trade flows) between pairs of vertices. The links are directed, going from the exporting country, *i*, to the importing country, *j*, where  $\mathcal{L}_{ij} \in [0, 1]$ , and  $\mathcal{G}$  is a simple directed graph. The second part includes all additional information on relevant characteristics of the links, included in the line value function  $\mathcal{W}$ , and the vertices, included in the vertex value function  $\mathcal{P}$ . The  $w_{ij}$  positive elements in  $\mathcal{W}$  act as dyadic weights on  $\mathcal{G}$ , modifying its original binary structure and transforming the simple directed graph in a weighted network, where  $w_{ij}$  indicates the strength of the link between country *i* and country *j* (e.g., export volumes). The elements in  $\mathcal{P}$  include instead country-specific values (e.g., income, population, geographical location). We will analyze the World Trade Network as a simple directed graph in most of the paper.<sup>4</sup>

In describing the World Trade Network we will make use of the summary statistics generally used in NA. All formal derivation is relegated in the Appendix. The first basic notion of connectivity of a vertex i to the network is the concept of *degree*. In the case of a simple directed graph the degree of a vertex is just the total number of other vertices  $j \neq i$  to which i is connected. In our specific case the *indegree* of vertex i is the number of countries from which the country is importing, while the *outdegree* would be the number of countries to which country i is exporting, i.e. the extensive margin. The  $d \in \mathcal{V}$  countries (directly or indirectly) linked to country i constitute its (first-order or higher-order) neighborhood  $\mathcal{V}_i^d$ , and vertex i would have a high *clustering coefficient* if its neighborhood is highly connected (the proportion of the vertex's neighbors which are neighbors of each others is high).

In general, the *density* of a network is higher the higher the number of its vertices pertaining to the same direct neighborhood. If all n vertices are linked together, the network is complete and its density is  $\gamma = 1$ . Moreover, we can focus on a specific neighborhood, calculating in a similar way

 $<sup>^{3}</sup>$  We include all technical analysis in the Appendix. The interested reader can find updated and beautifully organized surveys of the application of NA to economics in the volumes by Vega Redondo (2007); Goyal (2007) and Jackson (2008).

<sup>&</sup>lt;sup>4</sup> We use a simple directed graph, where  $\mathcal{L}_{ij} \in \{0, 1\}$ , in all the analysis (sections 3.1, 3.2, 3.4, 4.1 and 4.2). Also in section 4.3 we transformed the weighted network with a line value function  $\mathcal{W}$  were the links' weights  $w_{ij}$  are deflated import volumes into a simple directed graphs indicating the structure of extensive and intensive margins of trade. For an analysis of the weighted trade network see Bhattacharya et al. (2008) and Fagiolo et al. (2008).

its ego-density. The position of each vertex i, with respect to the whole network or its neighborhood, can be measured in term of its centrality. This can be evaluated looking at its relative degree (degree centrality,  $C_i^d$ ) or in term of geodesic distance (closeness centrality,  $C_i^c$ ), calculating the shortest path between i and all other vertices, or in terms of its mediating position (betweeness centrality,  $C_i^b$ ), calculating the number of paths between vertices that goes through i.<sup>5</sup>

Until the 1990s, most applications of NA to international trade flows mainly used these network statistics to study the structural equivalence of countries' position in the the network, or the existence of asymmetries in trade flows. Relevant methodological problems addressed in that context are concerned with which flows should be considered, and which distance or centrality measure can capture correctly the position of a country in the system. For example, in their seminal contribution, Smith and White (1992) analyze the trade flow of a limited number of commodities, and they characterize the structure of the trade network with a relational distance algorithm,<sup>6</sup> finding evidence of a tripartition of countries in a core, a semi-periphery and a periphery, that evolves slowly over time. This partition is obtained only from data on trade relationships, without considering attributes of individual countries. Not surprisingly the countries in the core resulted to be characterized by higher average GDP per capita than countries in the semi-periphery, which were in turn better off than countries in the periphery.

The stream of research that started in the 2000s was instead related to the concept of complex networks. This wave of works focused on the topological properties of the World Trade Network, and was more interested in finding the inner characteristics of the whole system than in defining its partitions. Serrano and Boguna (2003) show that the World Trade Network in the year 2000 was displaying the typical properties of a complex network. In particular: (i) a *scale-free* degree distribution, implying a high level of degree heterogeneity; (ii) a *small-world* property, stating that the average path length between any pair of vertices grows logarithmically with the system size; (iii) a high *clustering coefficient*, meaning that the neighbors of a given vertex are interconnected with high probability; (iv) *degree-degree correlation*, measuring the probability that a vertex of degree-*d* is connected

<sup>&</sup>lt;sup>5</sup> The measures of centrality are numerous and can be based on very different relational concepts. See Bonacich (1987) for an early and influential analysis, and Jackson (2008, ch.2) for a modern treatment.

<sup>&</sup>lt;sup>6</sup> The REGE algorithm used by Smith and White is based on the similarity of sectoral trade volumes between countries, measured recursively. See Smith and White (1992) for more details on the methodology used and for comparison with previous analysis using different techniques.

to a vertex of degree-d, an important property in defining the hierarchical organization of the network.

Complex or scale-free networks (Barabási, 2002) - juxtaposed to random networks - can easily arise in a social context because of the effects of cooperative and/or competitive forces at work between units of the network, influencing the network structure (Vega Redondo, 2007). The finding that the World Trade Network is a complex network was an important result. International trade occurs because of economic competition between firms and countries, and it is a mutually beneficial (cooperative) activity: a random distribution of linkages between countries is therefore very unlikely. If the world trade system can be defined as a self-organized complex network, it can be studied as a whole, whose changes are also driven by collective phenomena.

From these results, some more recent works moved to discuss the topological properties of the world trade network considering different specifications of the countries' links. Garlaschelli and Loffredo (2005) and Kali and Reyes (2007) consider the World Trade Network as a directed network, confirming the strongly hierarchical structure and the scale-free property of the trade network, underlying once more that speaking of a *representative* country in international trade does not make much sense. Fagiolo et al. (2008) study a symmetric weighted trade network, where links between countries are not only counted in terms of number of flows, but the links are weighted by the average trade flow  $\left(\frac{imports+exports}{2}\right)$  between countries. This approach confirms the large differences existing between countries in term of their role in international trade, showing that countries that are less and more weakly connected tend to have trade relations with intensively connected countries, that play the role of 'hubs'. This *disassortative* nature of the trade network is evident both studying the unweighted network and the weighted one.<sup>7</sup> Serrano et al. (2007), also using a weighted trade network, find high global and local heterogeneity not only among countries, but also in trade flow characteristics.

Overall, the existing evidence suggests that using NA to study international trade flows might yield interesting insights and new results. For example, one of the main elements emerging from the works discussed above - and not so evident in other contexts - is that trade flows, partners and links, are strongly heterogeneous among countries, and specific countries play very

<sup>&</sup>lt;sup>7</sup> An *assortative* network is defined as a network where better connected nodes tend to link with other well-connected nodes, while in a *disassortative* network, nodes with many links are connected to poorly connected nodes. This characteristic is studied through the degree-degree correlation (Newman, 2002). See also Jackson (2008, ch.3) on the related notion of *homophily*.

different roles in the network structure, an evidence challenging the traditional assumption of "old", "new", and "new new" trade models. Moreover, the distribution of degrees, the disassortative nature of trade links, the high clustering coefficients, offer a structure that must be matched by aggregated trade models, pretty much the same way firm-level evidence on the heterogeneous characteristics of international firms (Bernard and Jensen, 1995; Bernard, Jensen, Redding and Schott, 2007) has induced a change in trade models (Eaton and Kortum, 2002; Melitz, 2003, and Tybout, 2003, for a survey).<sup>8</sup>

Therefore, when analyzing a country's trade patterns, not only its individual characteristics should be taken into account, but also its interactions with its actual or potential trade partners, and its position in the network of trade flows. This is what we will start exploring in the following sections.

# 3 Characteristics of the World Trade Network

A strong perception concerning the current wave of globalization is that the characteristics of international trade have changed over time, with an acceleration of modifications occurring in the last decades: before the global financial crisis the amount of trade kept increasing substantially more than world production, on average by more than 6 per cent per year. Even after the dramatic drop of 2009, trade shows an impressive resilience. Over the years, the composition of trade flows changed, with a higher share of trade in inputs, intermediate goods and services, making countries even more deeply interconnected; and the geographical composition of trade also changed, with an increasing role of the emerging countries, especially in Asia (WTO, 2010). NA can contribute to the analysis of such changes: as international trade links shift and re-arrange, this would become evident through the change of the network structure. The extent of these changes over time is the first thing we want to verify, using the tools of NA to represent the structure of the world trading system and to assess the changes in its topological properties.

<sup>&</sup>lt;sup>8</sup> There are also important dynamic implications of the Scale-Free topology of the World Trade Network that we will not discuss here: Scale-Free Networks are more robust to structural failures, yet are more vulnerable to targeted shocks, and they have a vanishing epidemic threshold in diffusion processes (Barabási, 2002).

## 3.1 The trade dataset

In our analysis of the World Trade Network, we use the same dataset used by Subramanian and Wei (2007),<sup>9</sup> to make possible the direct comparison of our results with the results obtained by others scholars using the same dataset but different empirical approaches.<sup>10</sup> Our trade data are aggregate bilateral imports, as reported by the importing country and measured in US dollars, reported in the *IMF Direction of Trade Statistics*. We use data for six decades, from 1950 to 2000, deflated by US CPI (at 1982-83 prices).<sup>11</sup> Given that these flows are reported by importers, we can directly calculate the indegree of countries, but of course we can also compute the outdegree for each vertex, as we know the origin of each import flow.

The description of the characteristics of the dataset is presented in Table 1. World trade tends to be concentrated among a sub-group of countries and a small percentage of the total number of flows accounts for a disproportionally large share of world trade. In 1950, 340 trade flows making up to 90 per cent of the total reported trade were 20.6 per cent of the the 1649 total number of flows, and the top 1 per cent of flows accounted for 29.25 per cent of world trade. Of the 60 reporting countries, 57 were contributing in 1950 to the 90 per cent of total trade. In 2000 the first percentage shrinks to 7.2 per cent, pointing to a large increase in the number of very small flows, while the second expanded to 58.17 per cent, indicating an increasing relevance of the largest flows; and only 82 countries out of the 157 reporting countries make the same 90 per cent.

It is also interesting to see that the number of trade partners is quite different if we consider import sources rather than export destinations. While the typical number of partners tends to increase over time, exports markets are relatively more limited in number, suggesting the existence of difficulties in penetrating new foreign markets, while import sources are more highly diversified, in line with the idea of promoting competition from import sources.

<sup>&</sup>lt;sup>9</sup> The dataset used by Subramanian and Wei (2007) is downloadable from the website http://www.nber.org/~wei/data.html. In what follows we use S-W to indicate the source of these data.

<sup>&</sup>lt;sup>10</sup> In particular, our results in section 4.1 can be compared with Rose (2004) and Subramanian and Wei (2007), among others.

<sup>&</sup>lt;sup>11</sup> As mentioned, the choice of the trade data to use is not neutral in describing the network. Even if generally before the 1990s, import data were more reliable in terms of coverage and completeness, the use of import data can give rise to a network structure that is different than the one found with exports - as shown by Kali and Reyes (2007) and by De Benedictis and Tajoli (2008) - or with trade flows (the average of exports and imports). The same is true in a gravity context (see Subramanian and Wei (2007) on the use of trade flows in Rose (2004)).

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Table

	1950	1960	1970	1980	1990	2000
Countries reporting trade flows	60	113	130	143	145	157
Total number of flows	1649	3655	6593	8180	10289	11938
Value of total imports (million US dollars at constant prices)	1585.04	3205.92	6459.40	19529.49	22217.38	34100.35
Countries making up 50 per cent of trade	23	26	28	31	23	24
Flows making up 50 per cent of trade	46	20	72	89	68	78
Countries making up 90 per cent of trade	57	88	66	95	82	82
Flows making up 90 per cent of trade	340	634	794	894	748	855
share of world imports accounted for by the top 10 per cent flows	77.52	81.50	87.73	88.87	93.16	93.45
(number of flows)	(165)	(365)	(659)	(818)	(1029)	(1194)
share of world imports accounted for by the top 1 per cent flows	29.25	37.34	48.68	48.39	58.64	58.17
(number of flows)	(17)	(36)	(99)	(82)	(103)	(119)
Average of export markets	27.95	32.65	50.72	57.20	70.96	76.04
Median of export markets	24	25.5	45	52	60	67
Average of import markets	29.45	38.88	56.35	68.17	74.02	78.54
Median of import markets	27	33	57	64	66	71.50
Correlation between tot.import value and flows by country	0.68	0.69	0.58	0.59	0.54	0.53
Correlation between tot export value and flows by country	0.58	0.60	0.56	0.53	0.56	0.55
<i>Source</i> : our elaboration on S-W dataset. This dataset is based on I	MF's Dire	ction of T	rade Statis	stics. Bilate	eral imports	are reported
by the importing country and measured in US dollars and deflated	l by US C	PI (1982-	1983 price	s). The de	finitions of	the summary
statistics can be found in the Appendix.						

Unsurprisingly, the larger countries account for a generally larger share of world trade and have more partners. But the relationship between economic size and number of partners is far from perfect, as indicated by the (relatively stable) correlation between the total value of trade flows and the number of partners for each country.

In assessing changes over time, a relevant problem is that the dataset is not a balanced panel and the number of countries (i.e. of vertices in our network) changes over time (and so does the value of total trade). This occurs for a number of reasons: in the past, a large number of countries (especially the smallest and poorest ones) were not reporting trade data, either because of the lack of officially recorded data, or because they belonged to an isolated political bloc. Additional problems in assessing our dataset come from the fact that over time new countries were born (e.g. the Czech Republic and Slovakia), and a few disappeared (e.g. Yugoslavia). Therefore in our dataset missing observations are considered as zero reported trade flow between two countries.<sup>12</sup> To reduce the number of 'meaningless zeros', until 1990 we keep in the sample 157 countries and we have 176 countries in 2000, as many new countries came into existence (and some disappeared) after the disintegration of the former Soviet Union and the Comecon bloc. Of course, the change in the number of vertices is per se a relevant change in the network structure, but on the other end to stick only to the countries that are present over the entire period limits artificially the network introducing other biases. Furthermore, in computing some indices, we included only the countries for which we had at least one trade flow recorded, and we dropped the countries for which data were completely missing.<sup>13</sup>

### **3.2** Properties of the trade network

In Tables 2 through 4 we compare some of the trade network characteristics over time, considering different groups of countries. In Table 2, all officially existing countries appearing in the dataset are included. Therefore we have a high number of vertices, which increases in 2000 because of the birth of

<sup>&</sup>lt;sup>12</sup> On some of the problems of the IMF DoTs dataset in describing world trade see Felbermayr and Kohler (2006) and references therein, and on some possible ways to fix the zeros/missing values in the dataset for the years 1995-2004 see Gaulier and Zignago (2008) and the CEPII webpage.

<sup>&</sup>lt;sup>13</sup>Working at the aggregate level, we are confident that some missing trade links in our dataset (for example for well-linked countries such as Malta or United Arab Emirates, showing zero links in some years) are due to unreported data and do not indicate that the country does not trade at all. Therefore, removing vertices without any reported data will eliminate both some meaningful (but unobserved) links and some meaningless zeros, but it should not introduce a systematic bias, even if it changes the size of the network.

new countries after the disintegration of the former Soviet Union. In Table 3, we included in the network in each year only the countries for which at least one trade flow was recorded, i.e. excluding unlinked countries. At the same time, it is more difficult to compare the trade network over time because of the inherent change in its structure given the changing number of vertices. Therefore, we computed the network indices also over the balanced panel composed of the constant subset of 113 countries for which observations are available, and these are reported in Table 4.

Looking at the number of trade links among countries measured as the number of arcs, this has increased sensibly over time. We observe an increasing trend in the density of the network in all the samples presented in Tables 2 through 4. Density declines slightly in 2000 compared to ten years earlier, but this is explained by the increase in the size of the trade network in terms of vertices,<sup>14</sup> and it is in any case higher than in 1980. The stronger fall in density in 2000 in Table 4 (where new countries are not considered) than in Table 3 shows the relevance of the trade links with the new group of transition countries.

The rising trend in the network density confirms what other measures of economic integration indicate, that linkages between countries have been increasing in the second half of the twentieth century. Here we consider the number of linkages, and we are not weighting for the value of trade carried by each flow, therefore this indicator is showing something different than the standard openness measures that consider openness at the individual country level. An increase in density means that on average each country has a larger number of trade partners, and that the entire system is more intensely connected. Still in 2000, though, the density index is below 0.50 if we include all countries in the sample, meaning that the network is not regular and is far from being complete, or in other words that most countries do not trade with all other countries, but they rather select their partners.

The change in density was not uniform across the network, as the change in the centralization indices suggest. The decline in the betweenness centralization index,  $C^b$ , in all the tables from 1960 to 2000 implies that the increase in trade linkages has been fairly widespread, reducing the role of hubs in the network. The reduction in total betweenness until 1980 in Table 3 indicates a reduction in the average network geodesic distance between vertices,  $\bar{\delta}_{ij}$ , making the world 'smaller'. But distance seems to increase again in the last

<sup>&</sup>lt;sup>14</sup> Larger networks are expected to have a lower density, because an increase in the number of vertices requires a much more than proportional increase in the number of links to keep the density constant. The quotient  $\gamma = \frac{m}{m_{\text{max}}}$ , defining density, is  $\frac{1649}{157 \times 156} = 0.0673$  in 1950, and  $\frac{11938}{176 \times 175} = 0.3876$  in 2000.

decades: this effect is related to the increase in the size of the network. In Table 4, where the network size is constant, the fall in total betweenness (and the reduction in the geodesic distance) is monotonic over time. In line with this evidence is the trend in closeness centralization,  $C^c$ , (which is also influenced by the size of the network). Considering inward flows (imports), until the 1980s trade was increasingly concentrated around a core group of markets, while in more recent years closeness centralization declines, especially with respect to in-degree centralization, and it might signal of the rise of a new group of emerging countries, whose involvement in international trade is increasing the size of the world. Once again, if the network size is kept constant, both closeness centralization indices monotonically decline.

 Table 2: Trade network indices over time with all countries included

	1950	1960	1970	1980	1990	2000
No. Countries	157	157	157	157	157	176
No. Arcs	1649	3655	6593	8180	10289	11938
Density	0.067	0.149	0.269	0.334	0.420	0.388
In-Degree Closeness Centralization	0.306	0.489	0.523	0.561	0.506	0.507
Out-Degree Closeness Centralization	0.287	0.450	0.477	0.432	0.468	0.478
Betweenness Centralization	0.007	0.033	0.025	0.027	0.014	0.013

Source: Our elaboration on S-W data.

Table 3: Trade network indices over time with only reporting countries

	1950	1960	1970	1980	1990	2000
No. Countries	60	113	130	143	145	157
No. Arcs	1649	3655	6593	8180	10289	11938
Density	0.466	0.289	0.393	0.403	0.493	0.487
In-Degree Closeness Centralization	0.526	0.601	0.565	0.580	0.511	0.519
Out-Degree Closeness Centralization	0.474	0.546	0.510	0.438	0.469	0.484
In-Degree St.Dev.	14.132	24.024	30.790	37.052	37.49	39.073
Out-Degree St.Dev.	15.550	26.307	31.983	32.869	35.864	41.416
Betweenness Centralization	0.042	0.063	0.036	0.032	0.016	0.016
Total Betweenness	0.468	0.552	0.518	0.443	0.472	0.487

*Note*: Reporting countries included in the computations are the ones for which at least one trade flow is recorded. *Source*: Our elaboration on S-W data.

From Tables 2, 3 and 4 we can also see that in-degree centralization is always higher that out-degree centralization, confirming a systematic difference in the structure of imports and export flows. These differences can be better appreciated looking at the distribution of indegrees and outdegrees in Figure 2.

	1960	1970	1980	1990	2000
No. Countries	113	113	113	113	113
No. Arcs	3655	5807	6522	7355	6964
Density	0.289 [*]	0.459 [*]	0.515 [*]	0.581 [*]	0.550 [*]
In-Degree Closeness Centralization	0.6005	0.5190	0.4800	0.3866	0.3547
Out-Degree Closeness Centralization	0.5464	0.4920	0.3809	0.3776	0.3547
In-Degree St.Dev.	24.02	26.16	30.01	28.04	28.54
Out-Degree St.Dev.	26.31	28.78	25.91	27.84	30.72
Betweenness Centralization	0.0627	0.0308	0.0155	0.0097	0.0065
Total Betweenness	0.5516	0.4991	0.3853	0.3466	0.2685

Table 4: Trade network indices over time - balanced panel

*Note*: Here the network and its indices are computed including only the group of countries for which data are available over the entire time span 1960-2000.

[\*] indicates that the density is significantly different from the null hypothesis of  $\gamma=1$  with p=0.0002. Source: Our elaboration on S-W data.



50

100

150

Variations 2000 - 1980

50

29

35

Figure 2: The empirical distribution of indegrees and outdegrees

The empirical distribution of indegrees is plotted in the left upper quadrant, while the one of outdegrees is in the right upper quadrant (1960-dashed line, 1980-pointed line, 2000-continuous line). The distributions for 1950, 1970 and 1990 are not drown to facilitate visualization. Lower quadrants include the histograms of difference in degrees between 1980 and 2000 for indegrees (left quadrant) and outdegrees (right quadrant).

Over time, the distribution of indegrees and outdegrees shifted to the right, and changed remarkably its shape, indicating the change in the characteristics of the trade network. From a 1960 network with many countries with very few trade linkages, in 1980 there is a strong increase in the number of countries with an average number of linkages. This change is even stronger in the last decades, as shown also by the variations occurring between 1980 and 2000: there are a few countries that decrease the number of linkages, a few countries increasing a lot their linkages, while most of the change occurs in the intermediate range. In the year 2000, the result of these changes is a indegree distribution where many countries have an 'average' number of trade links, but it exists also a significant group of countries that is importing from a very large number of partners. This bi-modality shows up also looking at exports, even if the distribution here is 'flatter', and slightly more shifted to the left. Overall, in 2000 the average number of trade links has increased remarkably, and countries have more import sources than export markets. The heterogeneity shown in the distributions makes it impossible to talk of a 'representative' country in terms of geographical trade patterns: both distributions show very 'fat tails' and a high variance. Indeed, over time the heterogeneity in the network has increased, creating two main groups of countries, one with an average (or slightly below average) number of partners and another group with many more links, and with a *continuum* of countries in intermediate situations in between. It seems that now the core-periphery partition studied in the past has become obsolete, giving rise to a more complex structure.

A further relevant question is to what extent our results showing a selection of partners and the world trade network being different from a complete network are statistically meaningful. To do that we have to consider the information on network indices in a probabilistic light. Focusing on Table 4, the density of the World Trade Network in 1960,  $\gamma_{1960}$ , is 0.289 and can also be interpreted as the average value of the links in the network,  $\frac{3655}{113\times112}$ . Since the link  $\mathcal{L}_{ij}$  between any two countries  $\mathcal{V}_i$  and  $\mathcal{V}_j$  has been coded as a binary variable,  $\gamma$  is also the proportion of possible links that assume a value of 1, or, in other terms, the probability that any given link between two random countries is present (28.9 per cent chance).

We can test if the difference between the observed value of  $\gamma_{1960}$  from a null hypothesis of  $\gamma_{1960} = 1$  (as in a complete network) is just do to random variation. We do it by bootstrapping the adjacency matrix corresponding to  $\mathcal{N}_{1960}$ , and computing the estimated sampling variance of  $\gamma_{1960}$  by drawing 5000 random sub-samples from our network, and constructing a sampling distribution of density measures. The estimated standard error for  $\gamma_{1960}$  is 0.040 with a z-score of -17.801 and an average bootstrap density of 0.287 which is significantly different from the null with a p=0.0002.

Doing the same for any time slice of the World Trade Network  $\mathcal{N}_T$  - as it is reported in Table 4 - we came out with the same answer: the null hypothesis that the World Trade Network is a complete network is rejected.

We can also test if the observed increase in the World Trade Network density between 1960 and 1990 (and the further drop in 2000) is just due to randomness. To do that we make a pairwise comparison between subsequent time slices of  $\mathcal{N}_T$  finding that the observed difference in density arises very rarely by chance (the *p* is alway below 0.003) until 1990, while the observed change between 1990 and 2000 is statistically significant with a two-tailed probability of *p*=0.173, casting doubts on the trend of the reported data in the 2000s.

## 3.3 Countries' positions in the trade network

Moving to consider the countries' position within the network, we also see some relevant changes over time. In 1960, the country with the highest indegree was the United Kingdom, an heritage of the past colonial empire. The US show instead the highest out-degree in 1960, followed by the UK and by other European countries. In 1980 the UK is still first in terms of in-degree, but also in terms of out-degree, and the first places in terms of the number of links are all taken by European countries, confirming also with this index the high level of international integration of European countries. The effect of the European integration is further enhanced in terms of vertices' degrees in 1990, but the ranking changes in 2000, when the US display the highest degree both as a sender and as a receiver. Over time we see also an clear increase of degree for many less developed countries, with a rapid increase in the number of trading partners and the position in the ranking especially of South-East Asian nations.

These changes in position are confirmed by the vertex centrality indices,  $C_i^c$ . In 1960, the highest centrality indices are found for European countries, followed by the US It is worth noticing that the position in terms of indegree or outdegree closeness centrality is often different for a country. As  $C_i^c$  is an inverse measure of distance of vertex  $\mathcal{V}_i$  from all the others in the network, and is related to the number of direct linkages that a country holds,<sup>15</sup> a country with higher centrality in terms of outdegree than in terms of indegree is *closer* to its trading partners as an exporter than as an importer. This seems to be the case of Hong Kong, which can be seen as an export platform, but also of the US before the year 2000, as both countries are ranked higher

<sup>&</sup>lt;sup>15</sup> The formula for Closeness centrality  $C_i^c$  is the Appendix (see equation 6).

in terms of outdegree closeness centrality until the last observation period. The US become the more central vertex of the network in terms of indegree and outdegree only in the year 2000, sharing the position with Germany, with exactly the same centrality index. Unsurprisingly, the rank correlation between indegree and outdegree rankings is high and positive, ranging from 0.77 in 1980 to 0.95 in 2000. The same is true for the correlation between indegree closeness centrality indices, which goes from 0.71 in 1980 to 0.93 in 2000, meaning that countries with many inward linkages tend to have also many outward linkages, and their position in the network as importers is correlated to their position as exporters. But it is interesting to notice that this correlation increases over time: while until the 1980s the world was to some extent divided in 'importers' and 'exporters', this is certainly not the case now.

The betweenness centrality index,  $C_i^b$ , captures instead the role of a country as a 'hub' in the trade network.<sup>16</sup> Generally we expect a positive correlation with closeness centrality, as the position in the network may enhance the role of a hub, but some factors other than position and distance may give rise to hubs. In the trade network, the correlation between indegree closeness centrality and betweenness centrality indices is positive, but not very high, going from 0.54 in 1980 to 0.62 in 2000.

In Figure 3, the World Trade Network is visualized showing for each vertex its betweenness centrality (the size of the vertex) and its position in the network in terms of structural distance from the other vertices. In 1960 there is a clear center formed by a group of European countries and the US In terms of betweenness centrality index, the US were ranked third in 1960 (see Figure 3), but then moved down to the seventh-eighth position until 2000, when they reached the first position again together with Germany. But in 2000 the center of the network appears more crowded and less well-defined. Looking at the countries with the highest scores in terms of betweenness centrality, we observe some 'regional hubs', and their change in position over time: France, India and Morocco high in rank in the 1960, Hong Kong's centrality increasing over time between the 1960s and the 1990s, and the slightly lower rank of Switzerland with the increase of the integration within the EU.

## 3.4 Interpreting the World Trade Network properties

In order to assess the results presented in the previous sections, we should know which are the predictions of international trade models in terms of the

<sup>&</sup>lt;sup>16</sup> The formula for Betweenness centrality  $\mathcal{C}_i^b$  is the Appendix (see equation 8).

Indeg	ree closen	less centrality	Outde	gree closer	ness centrality	Bet	Betweenness centrality	
Rank	Index	Country	Rank	Index	Country	Rank	Index	Country
				1960	)			
1	0.6438	UK	1	0.5987	USA	1	0.0344	France
2	0.5954	Netherlands	2	0.5861	UK	2	0.0327	UK
3	0.5866	France	3	0.5740	France	3	0.0283	USA
4	0.5822	Japan	3	0.5740	Germany	4	0.0182	Netherlands
5	0.5656	USA	3	0.5740	Netherlands	5	0.0179	Japan
6	0.5616	Germany	6	0.5624	Italy	6	0.0140	Germany
6	0.5616	Italy	7	0.5568	Sweden	7	0.0126	Italy
8	0.5387	Sweden	7	0.5568	Japan	8	0.0121	Switzerland
8	0.5387	Switzerland	9	0.5406	Switzerland	9	0.0108	Canada
10	0.5350	Canada	10	0.5354	Denmark	10	0.0097	Sweden
11	0.5244	Norway	11	0.5303	India	11	0.0091	India
12	0.5142	Austria	12	0.5156	Canada	12	0.0072	Denmark
13	0.5012	Denmark	13	0.5016	Norway	13	0.0070	Austria
13	0.5012	Greece	13	0.5016	Spain	14	0.0068	Norway
15	0.4858	Finland	15	0.4928	Austria	15	0.0053	Morocco
				1980	)			
1	0.8920	UK	1	0.7643	UK	1	0.0287	UK
2	0.8453	France	1	0.7643	Germany	2	0.0175	Germany
2	0.8453	Germany	3	0.7580	USA	3	0.0167	France
4	0.8344	Italy	3	0.7580	Netherlands	4	0.0160	Italy
5	0.8291	Spain	3	0.7580	Canada	5	0.0155	Netherlands
6	0.8186	Netherlands	3	0.7580	Japan	6	0.0151	Japan
6	0.8186	Japan	7	0.7517	France	7	0.0149	USA
8	0.8134	USA	8	0.7455	Italy	8	0.0144	Spain
9	0.7984	Denmark	9	0.7395	Switzerland	9	0.0129	Denmark
10	0.7839	Switzerland	10	0.7335	Denmark	10	0.0120	Switzerland
11	0.7745	Ireland	10	0.7335	Sweden		0.0105	Sweden
12	0.7653	Portugal	12	0.7162	Spain	12	0.0096	Australia
13	0.7608	Saudi Arabia	13	0.7051	Hong Kong	13	0.0085	Canada
14	0.7433	Sweden	14	0.6997	China	14	0.0085	Portugal
15	0.7391	Greece	15	0.6839	Brazil	15	0.0085	Ireland
15	0.7391	Australia	15	0.6839	India	16	0.0083	Hong Kong
				2000	)			
1	0.8920	USA	1	0.8636	USA	1	0.0149	USA
1	0.8920	Germany	1	0.8636	UK	1	0.0149	Germany
3	0.8808	UK	1	0.8636	France	3	0.0141	UK
3	0.8808	France	1	0.8636	Germany	4	0.0141	France
5	0.8752	Italy	5	0.8580	Italy	5	0.0134	Italy
5	0.8752	Netherlands	5	0.8580	Japan	6	0.0132	Japan
7	0.8590	Japan	7	0.8523	Netherlands	7	0.0130	Netherlands
7	0.8590	Spain	7	0.8523	Spain	8	0.0121	Spain
9	0.8537	Canada	9	0.8413	India	9	0.0115	Canada
10	0.8434	Belgium	10	0.8360	Denmark	10	0.0106	Korea
11	0.8186	Korea	11	0.8306	Switzerland	11	0.0104	Belgium
12	0.8138	Thailand	11	0.8306	Canada	12	0.0096	Malaysia
13	0.8091	Portugal	11	0.8306	Korea	13	0.0093	Australia
14	0.8044	Malaysia	14	0.8254	Malaysia	14	0.0092	Denmark
15	0.7998	Switzerland	15	0.8202	Sweden	15	0.0091	Thailand

Table 5:	Countries'	centrality	in	the	World	Trade	Networl	K
Table 5:	Countries'	centrality	in	the	World	Trade	Networl	

Source: our elaboration on S-W data.



#### Figure 3: The World Trade Network 1950-2000

The networks have been drown using the software Pajer Using the force-directed Kamada-Kawai algorithm (see de

Nooy et al. (2005) for details). Colors of nodes indicate continents and were chosen using ColorBrewer, a web tool for selecting color schemes for thematic maps: dark blue is North America, light blue is Europe, dark red is Oceania, light red is Africa, dark green is Asia and the Middle East, light green is Latin America.

structure of the trade network. Unfortunately, most trade models deal with the pattern of trade of individual countries, and do not have much to say about the structure of the whole system, and about the number of trade flows that we should observe between countries.

But this issue needs to be tackled in empirical work, and to compare our results we can consider the most commonly used and successful empirical specification, the gravity model of trade, that can be derived from different theoretical models. This specification yields a stark prediction in terms of the network structure. In its basic form, the gravity equation is written as<sup>17</sup>

$$\mathcal{L}_{ij} = A \cdot \frac{GDP_i \cdot GDP_j}{D_{ij}}.$$
(1)

Therefore, according to these specifications, as long as two countries,  $\mathcal{V}_i$ and  $\mathcal{V}_j$ , have positive GDP in the vertex value function  $\mathcal{P}$ , and the physical distance between them  $D_{ij}$  included in the line value function  $\mathcal{W}$ , is less than infinite, and the goods produced in the two countries are not perfect substitutes, we should see a positive trade link between them (i.e.  $\mathcal{L}_{ij}=1$ ). In other words, according to the basic gravity model we should expect to observe a complete trade network with density  $\gamma$  equal to 1. If this is our benchmark, we can say that the density we found of about 0.50 is still quite low, and even if density has generally increased over time, we are still very far from a fully integrated world.

Of course, the basic gravity specification can be improved and modified to produce some of the zero flows that we observe in the real world. First of all, in the empirical applications the variable  $D_{ij}$  is not meant to capture only geographical distance, which is of course never infinite, but it can represent other types of barriers to trade and frictions, that might indeed stop trade completely.

A way to find in the model a number of trade links below the maximum and not identical for all countries is by introducing heterogeneity in countries' characteristics (differences in countries' production costs, and eventually in preferences) and in firms' export propensity. Deardorff (1998) proposes an equation derived by a frictionless Heckscher-Ohlin model with many goods and factors, where no trade between a pair of countries  $\mathcal{V}_i$  and  $\mathcal{V}_j$  can be observed if the production specialization of country i is perfectly negatively correlated with the preferences of country j, or in other words if country ihappens to be specialized in goods that country j does not demand at all:

<sup>&</sup>lt;sup>17</sup>In a model with identical countries producing differentiated goods under monopolistic competition and Dixit-Stiglitz consumers' preference for variety, the equation obtained will be only slightly modified:  $\mathcal{L}_{ij} = A \cdot \frac{GDP_i \cdot GDP_j}{D_{ij}^{\sigma}}$  where  $\sigma$  is the elasticity of substitution between varieties.

$$\mathcal{L}_{ij} = \frac{GDP_i \cdot GDP_j}{GDP_W} \left( 1 + \sum_k \lambda_k \widetilde{\alpha}_{ik} \widetilde{\beta}_{jk} \right)$$
(2)

Here the sign of the summation in equation 2 is given by the weighted covariance between  $\tilde{\alpha}_{ik}$  and  $\tilde{\beta}_{jk}$ , which represent the deviations of the exporter production shares and importers consumption shares from world averages. With a covariance of -1 the term in parenthesis becomes zero and no trade is observed between country  $\mathcal{V}_i$  and  $\mathcal{V}_j$ . In this context, where the role of distance is disregarded, and therefore trade costs do not play a role, the increase in the network density that we observe in Section 3.2 can imply that the similarity in production patterns and preferences in the world is slowly increasing over time, but that countries' heterogeneity is still quite strong. Furthermore, this equation also allows some countries to be more 'central' than others in terms of the number of trade links that they have, and this centrality is not related to geographical distance. In fact, a country is more likely to have more trade links if its production and consumption share are closer to the world average.<sup>18</sup>

A sharp reduction in the number of trade links between countries is also observed if there are fixed costs of exporting. If these costs are specific to the exporter-importer pair, the distribution of trade links can be very heterogeneous across countries. Helpman *et al.* (2008) show that the combination of fixed export costs and firm level heterogeneity in productivity, combined with cross-country variation in efficiency, implies that any given country need not serve all foreign markets. A higher productivity (or a lower production cost) for a country in this model implies a larger number of bilateral trade flows. The evidence provided in the previous sections of many countries trading with a limited number of partners and of the number of linkages increasing gradually over time is in line with this model. The asymmetries in trade flows observed in the data are explained by the systematic variation in trade opportunities according to the characteristics of trade partners, that influence the fixed and variable costs of serving a foreign market. The observed increase in the number of trading partners over time in our data is in line with the reduction of the costs to reach a foreign market, even if the cost is still high enough to give rise to a selection of partners.

Both the model suggested by Deardorff (1998) and by Helpman *et al.* 

<sup>&</sup>lt;sup>18</sup> Similar reasoning applies to the concept of country's *remoteness* and *multilateral* resistance à la Anderson and van Wincoop (2003). Anderson and van Wincoop assume however that firms are homogeneous within each country and that consumers love of variety, this ensures that all goods are traded everywhere. In this model there is no extensive margin and all change in trade volumes occurs in the intensive margin.

(2008) predict an heterogeneous effect of the reduction of trade costs on different countries. In Deardorff (1998), especially trade between distant countries should expand when transport cost decline, and in Helpman *et al.* (2008), less developed countries should have a stronger response at the extensive margin. A differentiated response to the reduction of trade barriers is also found by Chaney (2008), assuming a different substitutability between goods coming from countries with different characteristics. This means that lowering the trade barriers should affect not only the amount or the number of trade flows, but also the structure of the network, changing countries' relative positions. The results we find are in line with these predictions. The decline of the centralization indices over time shows that many of the changes occurring in the trade network are taking place at the periphery of the system.

# 4 Applications of network analysis to trade issues

Given that the World Trade Network is not a random network, but it presents well-defined characteristics, a number of issues can be analyzed considering the structural characteristics of such network and its changes over time. In what follows, we propose some applications of this type of analysis, by testing if network indices add explanatory power to the gravity specification of bilateral trade flows, and we address the question of whether the WTO has promoted international trade by comparing the entire World Trade Network with the network composed by WTO members. We also compare regional trade networks, where barriers to trade are reduced by geographical proximity and sometimes by trade agreements, to the world trade system to observe if there are systematic differences across regions.

# 4.1 Gravity models and the trade network

In their modern applications, also gravity models of trade assume that bilateral trade between two countries can affect trade between a different pair of trading partners. Such effect is introduced through the multilateral resistance term (Anderson and van Wincoop, 2003), but in practice those effects are frequently treated as unobserved heterogeneity and controlled for with country-fixed effects estimators. This procedure is, however, correct only in a cross-country framework but not in a panel (Baldwin and Taglioni, 2006), since the multilateral resistance term is time-varying. NA allows to address more properly the issue of multilateral effects of bilateral flow introducing time-varying network indices in a gravity equation to estimate bilateral trade flows. The use of indices that capture the position of a country relative to all the others in the trade network and with respect to the entire trading system allows to consider the assumption of interdependence between bilateral trade flows appropriately, and to see how a bilateral trade link between country i and country j can be affected by the links to all partners of the two countries.<sup>19</sup>

Specifically, considering the expenditure function of an importing country *i*, given its economic size (measured by GDP), we can expect that there is a optimal overall amount of imports that its domestic demand can absorb. Therefore, controlling for its economic size, we expect that a large number of sources of imports (the indegree of a country in our trade network) implies on average a lower amount of imports per source, or in other words, that the coefficient of the indegree of a country in a standard gravity equation where the dependent variable is the value of bilateral imports should have a negative sign. Instead, the sign of the outdegree variable of the importing country would crucially depends on the technological interplay between the import of intermediate goods and the export of finite products (and vice versa), the sign would be positive for countries heavily involved in international fragmentation of production and in case of complex goods (De Benedictis and Tajoli, 2010). It is more difficult to formulate an expectation at the aggregate level, where input-output linkages are unaccounted for: the number of export market could simply be unrelated in this specification to the amount of bilateral imports of a country.

Centrality measures in NA indicate the relative importance of a country in the network. Country *i*, with a high  $C_i^c$ , being a relevant trader, attracts a large amount of imports, everything else equal. Therefore, we expect a positive sign of the coefficient of the indegree and outdegree centrality measures of the importing country, especially if the country plays a relevant role in the international fragmentation of production.

A note of caution is however necessary in interpreting these results. In this first attempt to include some summary statistics of the World Trade Network in a standard gravity equation we could not be guided by a consensual structural model that jointly considers the characteristics of the trading partners and the structure of the network. This exposes the estimation to three potential shortcomings. The first one is due to omitted variable bias, that we tried to limit using conventional controls; the second one is due to the violation of independence of observations, that we dealt with cluster-

<sup>&</sup>lt;sup>19</sup>Also in Baier and Bergstrand (2009, p.78) some notion of 'economic density' is derived and included in the analysis of bilateral trade volumes based on a gravity model.

ing the standard errors on country-pairs and bootstrapping; the third is the endogeneity bias due to the fact that the network's summary statistics are endogenous by definition. This last problem is not, however, a peculiarity of these estimates, but it occurs whenever peers' effects are introduced in similar regressions. Here we do not have a definitive solution to Manski (1994) reflection problem,<sup>20</sup> and we leave this issue open for future research.

Having said so, in Table 6, we present the results of a gravity model regression obtained using the S-W database, and the estimates obtained adding the log of network indices as explanatory variables. Both specifications are presented with time fixed effects and with (columns 2, 4, and 5) and without (columns 1, and 3) country fixed effects. All the coefficients of the standard gravity specification present the expected sign and significance. The network indices used in the regressions refer to the position in the network of the importing country *i*. They are very significant (column 3) and the signs displayed can be interpreted as suggested above: there is a decrease in the marginal advantage of increasing the indegree, the outdegree is not significant, and the country's central position in the network is enhancing the magnitude of its bilateral imports.

It is also interesting to observe how the standard gravity coefficients are affected by the introduction of such indices. The coefficient of the geographical distance between countries is very moderately affected, and this is not surprising, given that the countries' position in the network can be independent from their geographical positions. Instead the coefficient of the importer's GDP is reduced sensibly when country's fixed effects are not included (column 3). This result seems to indicate that the GDP of the importing market itself becomes less important when we consider the links that the country has with other markets. The major effect is on the WTO membership parameter: controlling for network effects makes the coefficient not different from zero, statistically. Therefore, this result indicates that what seems to matter is not the WTO membership *per sè* but the degree of connectiveness of a country. As we will see in next session the WTO can contribute to generate that connectivity, but it is not the only possible mechanism that can guarantee it.

When we include country-fixed effects (column 4), only the country-time varying network characteristics remain significant. Since the level of centrality of a country does not display high volatility, all its effect is captured

<sup>&</sup>lt;sup>20</sup>We also use an IV-estimator, instrumenting on the lagged values of the network variables, but the instruments did not pass the tests for week-instruments and we preferred to make a step back to a fixed-effect estimation rather than incurring in severe bias do to the poor performance of the instruments. We also could not address the issue of selection since the data did not include absent trade flows.

by the country-fixed effect. Interestingly, the WTO's coefficient is now significant, confirming our hypothesis that WTO's membership and country's network position are highly correlated, also when we deal with residual heteroskedasticity bootstrapping the standard errors (column 5).

Overall, the country's position in the network provides some additional explanations about its capacity to attract trade flows. These preliminary results are quite promising for the use of these network indicators, complementing the more traditional gravity model variables.

### 4.2 The role of the WTO in the trade network

The role of the WTO in fostering economic integration has been central for a long time in the discussions on trade policy. A recent new wave of empirical investigations on this issue was started by Rose (2004), that in a series of works questions whether there is any evidence that the WTO has increased world trade, giving a negative answer. A different interpretation of Rose's findings is given by Subramanian and Wei (2007), who find that "the WTO promotes trade, strongly but unevenly". They reach this conclusion by carefully examining countries' different positions in the WTO system. The GATT/WTO agreements provide an asymmetric treatment to different trade flows, according to their origins and destinations (developed or less developed countries, members or non-members, new or old members) and according to the sector. Therefore, the impact of the WTO is not expected to be the same for all countries. Controlling for these differences, Subramanian and Wei (2007) indeed find a positive 'WTO effect', albeit differentiated among countries. In their work, they explicitly take into account countries' heterogeneity within the system, and this seems an important aspect to consider. But both this work and the one by Rose measure the WTO effect on trade at the country level. What we try to do with NA is to see the impact of the WTO agreements on the entire system.

In Table 7 we present network indicators for WTO members. Here too the number of vertices in our network changes over time, as GATT/WTO membership increases, increasing sensibly the size of the network over time. The density of the network therefore is affected by this change in size, and it appears to decline between 1950 and 1970, then to increase until 1990, to decline slightly again in 2000, with the large increase in the number of vertices. In any case, if we compare the density of the WTO network with the one of the World Trade Network in Table 4, this is significantly higher in every year.<sup>21</sup> Of course, the direction of causality cannot immediately be

<sup>&</sup>lt;sup>21</sup> To run a formal test of this evidence we bootstrapped the adjacency matrix of the

	(1)	(2)	(3)	(4)	(5)
$Log Distance_{ij}$	-1.158***	-1.500***	-1.122***	-1.518***	-1.518***
<u> </u>	(-46.68)	(-60.09)	(-39.61)	(-52.09)	(-53.82)
Log real $GDP_i$	0.920***	0.900***	0.727***	0.820***	0.820**
	(111.3)	(9.90)	(40.48)	(4.25)	(3.18)
Log real $GDP_i$	1.092***	0.915***	1.134***	0.765***	0.765**
	(129.71)	(9.88)	(118.35)	(4.11)	(3.07)
Common Language Dummy $_{ij}$	0.560***	0.386***	0.598***	0.377***	0.377***
	(11.58)	(8.42)	(11.74)	(7.51)	(8.06)
Land Border $\text{Dummy}_{ij}$	0.281**	0.403***	$0.273^{*}$	0.352**	0.352***
-	(2.71)	(3.96)	(2.40)	(3.11)	(3.61)
Currency Union $_{ij}$	$0.392^{*}$	$1.032^{***}$	$0.551^{**}$	1.030***	1.030***
-	(2.15)	(5.94)	(2.58)	(4.96)	(5.73)
FTA $\operatorname{dummy}_{ij}$	$1.791^{***}$	$0.400^{***}$	$1.447^{***}$	0.324***	0.324***
-	(25.89)	(5.36)	(19.84)	(4.29)	(4.77)
$GSP Dummy_{ij}$	$0.989^{***}$	$0.518^{***}$	$0.597^{***}$	0.387***	0.387***
-	(27.64)	(14.17)	(15.31)	(9.86)	(11.95)
Importer WTO member <sub><math>i</math></sub>	$0.142^{***}$	$0.125^{*}$	0.067	0.291**	0.291*
	(3.36)	(2.39)	(1.08)	(2.71)	(2.22)
$\operatorname{Log} \operatorname{Indegree}_i$			-0.0179***	-0.0137*	-0.0137
			(-8.63)	(-2.28)	(-1.58)
$\operatorname{Log} \operatorname{Outdegree}_i$			-0.00004	0.00836	0.00836
			(-0.01)	(0.67)	(0.51)
$Log InClose Centr_i$			$3.421^{***}$	4.246	4.246
			(6.82)	(1.94)	(1.30)
$Log OutClose Centr_i$			6.377***	-1.840	-1.840
			(5.52)	(-0.52)	(-0.41)
Constant	-16.97***	-8.68***	-19.51***	-7.408*	-8.424
	(-55.29)	(-5.55)	(-34.33)	(-2.17)	(-1.27)
Root MSE	2.169	1.798	2.014	1.657	1.657
export dummy	no	yes	no	yes	yes
import dummy	no	yes	no	yes	yes
time dummy	yes	yes	yes	yes	yes
other controlls (SW)	yes	yes	yes	yes	yes

Table 6: Expanded gravity model of world trade

Note: Dependent variable: import flows from country j to country i. t-statistics in parentheses, clustered at the country-pair level, or bootstrapped (in column 5). The Subramanian and Wei (SW) extra controls are common currency, colonial linkages, being part of the same country or empire.

\* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

Source: Our elaboration on S-W data.

determined, but we can certainly say that GATT/WTO members have many more trade linkages than non-members and the WTO system is much more closely interconnected than the whole world trade system.

The higher density indicators emerging from NA show that WTO members have a higher number of trade linkages, and not only trade more in volumes. If we assume that there is a fixed cost for firms to enter in a new foreign market, it is possible that WTO membership opens up new markets by lowering the entry cost (for example by increasing transparency, as the institution aims to do), an effect that shows up in the increased number of linkages. This possible explanation is consistent to what we found in the previous section: what matters is not WTO membership *per sè*, but the connection the membership can convey.

	1950	1960	1970	1980	1990	2000
Countries	24	35	75	85	98	124
Arcs	345	764	2966	3979	6021	8699
Share of total recorded arcs	20.92	20.9	44.99	48.64	58.52	72.87
Density	0.6250	0.6420	0.5344	0.5573	0.6334	0.5704
In-Degree Centralization	0.3006	0.308	0.4308	0.4239	0.3496	0.4168
Out-Degree Centralization	0.2552	0.2474	0.4034	0.3275	0.3183	0.384
In-Degree St.Dev.	6.6946	9.5961	19.1034	23.2229	24.9187	30.6184

| 5.9499 | 8.4936 | 19.3716 | 20.2412 |

22.4931 31.2289

 Table 7: WTO network indices over time

Figures and indices refer to the countries member of the WTO in each given year. Source: our elaboration on S-W data.

Out-Degree St.Dev.

The issue of whether the effects of the WTO are evenly distributed can be addressed looking at the other network indices presented in Table 7. Considering the centralization indices, we see that they are lower that the indices found for the entire network. This tells that the WTO system is less centralized than the world trade system as a whole. This could be the result of the fact that WTO membership allows an easier access to the markets of other members, spreading out linkages and reducing the separation between countries (which is inversely related to centralization). Over time, centralization

trade links between WTO members, drawing 5000 sub-samples for every time-slice from 1960 to 2000, and for any time-slice we tested the null hypothesis of equality in density with the correspondent complete adjacency matrix  $\mathcal{N}_t$  including non-WTO members (we considered as expected densities the values included in Table 4). The test rejected the null with a p < 0.0005 for t=1960, 1990; with a p < 0.007 for t=1970, 2000; and with a p = 0.0172 for t=1980. Only in this time slice the probability that the higher density among TWO members can be due to random variation is above 1 per cent.

does not show an uniform trend, and it is possible that with the increase in membership, the WTO system has become more hierarchical.

The observation of the standard deviation of degrees in the network brings to similar conclusions. The dispersion in terms of number of trade linkages with other countries is always lower for WTO members than for all trading countries. This can be interpreted as an indicator that the WTO system is more 'even' than the whole world trading system, as the number of trading opportunities taken by WTO members is more uniformly spread than for the other countries. But we see that the standard deviation of degrees for WTO members increases over time, and more rapidly than for the entire network. This is another result pointing to the increase in heterogeneity in the WTO network.

Figure 4: GATT/WTO membership in 1950 and 2000.



GATT/WTO members in light blue. The size of the circle is proportional to the betweenness of the vertex.

Figure 4 shows the World Trade Network in 1950 and 2000, divided between GATT/WTO members and non-members. In 1950, countries appear divided between a central group, a more peripheral group close to the center, and an outer circle. The center appears composed mainly by GATT/WTO member countries, that also display some of the highest betweenness centrality indices. This visual analysis confirms the important role in the trade network of a multilateral agreement, even if this in 1950 was covering only a small number of countries. The central role of the WTO is confirmed in 2000, when the center of the network is all taken by WTO members. The only sizable country close to the center that is not a WTO member appears to be China, at the time negotiating its membership.

## 4.3 Is international trade regionalized?

Another debated point that can be addressed using NA is whether international trade is regionalized, or in other words organized around trading blocs, possibly formed through regional agreements (see Pomfret, 2007; Baier et al., 2008). Such trading blocs can be formed in different ways, and NA is a useful additional tool to study their formation and existence within the network. But here we address a more specific question: we want to verify if there are more trade flows between (relatively) geographically close countries that belong to the same continent and even more between countries belonging to a trade agreement. To do so, we analyze some of the characteristics of continental subnetworks of trade, reported in Table 8.

	World	Europe (EU)	America	Asia $(ASEAN)$	Africa	Oceania
Countries	$\begin{array}{c c c} 1980 & 130 \\ 2000 & 157 \end{array}$	$\begin{array}{c c} 23 & (9) \\ 32 & (15) \end{array}$	33 33	$\begin{vmatrix} 28\\ 38 (10) \end{vmatrix}$	49 45	9 9
Arcs	19808180200011938	463 826	651 757	517 849	530 618	$\begin{array}{c} 45 \\ 49 \end{array}$
Regional share of arcs	1980     1.000       2000     1.000	$0.057 \\ 0.069$	0.080	0.063	$0.065 \\ 0.052$	$0.006 \\ 0.004$
Density	$\begin{array}{c c} 1980 & 0.403 \\ 2000 & 0.487 \end{array}$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	0.617	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	0.225 0.312	$0.625 \\ 0.681$

 Table 8: Regional trade networks

Source: our elaboration on S-W data.

If we consider density as an indicator of trade intensity within each continental subnetwork, we see that both in 1980 and in 2000, the density of trade flows in each continent - with the exception of Africa - is sensibly higher than the world density, implying that among countries belonging to the same continent there are proportionally more trade flows than with a random country elsewhere in the world. In this respect world trade is indeed regionalized.<sup>22</sup> It is also important to notice that the total number of intra-regional trade flows in 1980 amounted to 27 per cent of the total number of world trade flows, and it declined to 26 per cent in 2000, limiting the relevance that can be assigned to regionalization.<sup>23</sup>

<sup>&</sup>lt;sup>22</sup> This finding is in line with the evidence gathered through gravity models, showing that geographical distance is important in trade relations, as well as sharing a border and other proximity indicators.

 $<sup>^{23}</sup>$  A view of the World Trade Network complementary to the one of looking separately at each continental subnetwork is to consider continents as vertices, and building a very simplified network with only five or six (if America is split in North and Latin America) vertices. The main characteristic of such a simplified network is to have density equal to 1, or to be complete, i.e. all continents trade with all the other continents. Even if

But we can also see that over time, the density index within some continents declines, while world density tends to increase. This is true for Europe, that in 1980 is close to being a complete network, while in 2000 its density is much lower. This is also due to the increase in the number of trading countries in Europe after the Soviet era, and especially to the increase in the heterogeneity of countries in the region. A further important source of heterogeneity in Europe is the affiliation to the European Union (EU). The EU sub-continental network is a complete network with density equal to 1, showing the strength of the economic links between EU members. European countries not belonging to the EU have a quite different position in the European network, as shown also in Figure 5.

Figure 5 presents the continental sub-networks, and it shows that in 2000 also Europe itself (panel (a) in the figure) is divided in different groups of countries. The graphical representation of the network that places countries taking into account not their geographical distance but distance within the network structure only in terms of trade linkages, places Germany at the center, surrounded by the large European Union members, and then by the smaller countries of Western Europe, while the Central-Eastern European countries in 2000 were in more peripheral positions.

The other continents present slightly different network shapes, but it is generally easy to identify a country or a small group of countries taking the central position in the network. For example, in America, there is central role for the NAFTA countries (US, Canada and Mexico), and in Asia for Japan and Korea. Regional trade agreements seem to strengthen the proximity effect also for the group of Asian countries belonging to ASEAN. The network formed by this sub-group is much higher that the density of the whole continent. On the other hand, Africa not only displays a low density, but also a number of very peripheral countries, that appear distant even from the local trade network.

## 4.4 The extensive and intensive margins of world trade

In recent years much of the discussions on the evolution of world trade was carried out using the concepts of intensive and extensive margins. A change through time of a bilateral trading relationship that already existed at the beginning of the period is called the *intensive margin* of trade. Trade also increases if a trading bilateral relationship between countries that have not traded with each other in the past is newly established, this is called the

the amount of inter-continental trade flows is very different, this shows that no continent isolated from another, and in this respect we can talk about a global trade network.



Figure 5: The continental trade sub-networks in 2000.

extensive margin of trade.

These concepts that have been quantified by Felbermayr and Kohler (2005). They show that about 60 per cent of world trade growth from 1950 to 1997 comes from the intensive margin. Helpman, Melitz and Rubinstein (2008) also confirm and reinforce this fact: "the rapid growth of world trade from 1970 to 1997 was predominantly due to the growth of the volume of trade among countries that traded with each other in 1970 rather than due to the expansion of trade among new trade partners". Moreover, Lawless (2008) finds that in a traditional gravity setup, such as the one expressed in equation 1, distance  $D_{ij}$  has a negative effect on both margins, but the magnitude of the coefficient is considerably larger and more significant for the extensive margin, and that most of the variables capturing language, geography, infrastructure and import cost barriers work solely through the extensive margin. This important facts give new light to the link between trade costs and the evolution of the volume of world trade.

If trade evolves along two margins also the World Trade Network can be decomposed in its extensive and intensive simple subnetworks, studying the two effects at a systemic level rather than at a county level. The example for trade changes between 1980 and 1990, reported in Table 9, is constructed starting from the two time slices  $\mathcal{N}_{1980}$  and  $\mathcal{N}_{1990}$  of the weighted network of world trade, with a line value function  $\mathcal{W}$  where the links' weights  $w_{ij}$  are the deflated import volumes. We then calculated the weighted adjacency matrix of the differences in trade volumes between 1980 and 1990 and deconstructed these flows in three components: the extensive margin, due to the expansion of trade among new trade partners (having  $w_{ij} = 0$  in  $\mathcal{N}_{1980}$ ); the positive component of the intensive margin, including non negative changes through time of bilateral trading relationships already established in 1980 ( $w_{ij} > 0$ in  $\mathcal{N}_{1980}$ ; and the negative component of the intensive margin, including reductions through time of bilateral trading relationships already established in 1980.<sup>24</sup> The resulting weighted networks were then reduces to simple directed networks transforming all non zero values in  $a_{ij} = 1$ .

The characteristics of these three components of the evolution of the World Trade Network are summarized in Table 9. The number of active nodes in the three networks is 109 (Iraq, Liberia, Réunion, and Somalia did not report any flow in 1990), resulting in 7355 links. Only 23.7 per cent of these links are due to newly established trade partnerships, confirming that the intensive margin plays a major role on the whole trading system, shaping the change in the network. What is also remarkable is that the

 $<sup>^{24}</sup>$ We excluded 910 flows characterized by missing observations in 1990. The resulting total number of flows is 7355 as reported in Table 4.

	Extensive margin	Intensive margin (positive)	Intensive margin (negative)
Countries (active)	113 (109*)	113 (109*)	113 (109*)
Arcs	1743	2813	2799
Share of total recorded arcs	23.70	38.25	38.05
Density	0.138	0.222	0.221
In-Degree average (St.Dev.)	15.99(15.98)	25.81(18.01)	24.76(16.30)
Out-Degree average (St.Dev.)	15.99(5.52)	25.81(17.93)	24.76(13.35)
In-Degree Closeness Centralization	0.457	0.430	#
Out-Degree Closeness Centralization	0.460	0.430	#
Betweenness Centralization	0.470	0.320	0.047

Table 9: The extensive margin and the intensive margins of trade: 1980-1990

Note: Figures and indices refer to the 113 countries included in Table 4. (\*) Iraq, Liberia, Réunion, and Somalia were inactive in the extensive and the intensive margin (both positive and negative); (\$) Closeness Centralization could not be

computed since the network is not strongly connected. Source: our elaboration on S-W data.

number of trade flows decreasing the intensive margin is very large, showing a redirection of trade links. The two components of the intensive margin are in facts about equal in terms of links and density.

In comparison with the extensive margin network, both the intensive margin networks appear more dense. The average in and outdegree is higher and also the degree dispersion is higher, while the betweenness centralization,  $C^b$ , is lower (much lower in the case of the negative intensive margin).

Finally, the fact that the extensive margin network is less dense and more centralized indicates that the evolution of the World Trade Network along the extensive margin is primarily due to the active role of a limited number of countries, in particular Mexico, Nigeria, Tunisia, and China.

# 5 Conclusion

Using the tools of NA, in this paper we examined a number of issues related to the international trading system. Through the indices describing the network's properties, such as density, closeness, betweenness and degree distribution, we show graphically and analytically that the world trade network has indeed changed in the past decades. In particular, the trading system has become more intensely interconnected, while the heterogeneity among countries increased; the average structural network distance has decreased and then increased again, and the position of many countries in the network changed. Furthermore, the analysis shows that trade policies do play a role in shaping the trade network.

An important feature of these results is that they pertain to the trading system as a whole, which is the object of analysis in this context, and are not due to a specific country or group of countries. The use of such 'system indices' in a gravity regression shows that they can provide additional explanatory power to the traditional country's variables. This is probably the main contribution of NA to empirical investigations on trade: giving a unified view of the system characteristics, while underlying the heterogeneity of its components and its complexity. This approach can have relevant implications both for trade policy and for the modeling of trade relations.

# 6 APPENDIX

## 6.1 Definition of a Network

A *network* consists of a graph plus some additional information on the vertices or the lines of the graph.<sup>25</sup>

In its general form, a network

$$\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{W}, \mathcal{P}) \tag{3}$$

consists of a graph  $\mathcal{G} = (\mathcal{V}, \mathcal{L})$ , where  $\mathcal{V} = \{1, 2, \ldots, n\}$  is a set of vertices and  $\mathcal{L}$  is a set of lines between pairs of vertices.<sup>26</sup> A simple undirected graph contains neither multiple edges nor loops. A simple directed graph contains no multiple arcs, so that  $\mathcal{L} \subseteq \mathcal{V} \times \mathcal{V}$ . A directed network can be symmetrized replacing unilateral and bidirectional arcs by edges.

In simple graphs,  $\mathcal{L}$  is a binary variable, and  $\mathcal{L}_{ij} \in \{0, 1\}$  denotes the link between two vertices *i* and *j*, taking on a value of 1 if there exists a link between *i* and *j* and 0 otherwise.<sup>27</sup> Weighted networks add to simple graph some additional information on the lines of the graph. The additional information is contained in the line value function  $\mathcal{W}$ , where line values are positive weights associated to each line, usually indicating the strength of the relation. In the *ij* case,  $w_{ij}$  is the link's weight.

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{L} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, two vertices are said to be adjacent if they are connected by a line.

 $<sup>^{25}</sup>$ The additional information can be exogenous or can be endogenously computed.

<sup>&</sup>lt;sup>26</sup> In the literature, vertices can also be called *nodes* connected by *links* instead of lines (Goyal, 2007; Vega-Redondo, 2007). We will exclusively use the letter  $\mathcal{N}$  for network, while we will use the terms line and link interchangeably.

<sup>&</sup>lt;sup>27</sup> Another convenient way (Vega-Redondo, 2007) of representing simple graphs is through its adjacency matrix, a  $\mathcal{V} \times \mathcal{V}$ -dimensional matrix denoted by *a* such that

The additional information on the vertices is contained in the *vertex value* function  $\mathcal{P}$ , assembling different properties or characteristics of the vertices.  $\mathcal{P}$  can be innocuous (containing vertices' labels) or can be relevant in clustering vertices and containing possible related covariates.

## 6.2 Dimensions of a Network

The *size* of a network is expressed by the number of vertices  $n = |\mathcal{V}|$  and the number of lines  $m = |\mathcal{L}|$ . In a simple undirected graph  $m \leq \frac{1}{2}n(n-1)$ .<sup>28</sup> A small network includes some tens vertices, middle size networks includes some hundred vertices, large networks contain thousands or millions of vertices.

The set of vertices that are connected to any given  $\mathcal{V}_i \in \mathcal{V}$  defines its neighborhood  $\mathcal{V}_i^d \equiv \{j \in \mathcal{V} : ij \in \mathcal{L}\}, ^{29}$  where  $d \geq 0$  denotes the number of neighbors of  $\mathcal{V}_i$ .  $\mathcal{V}_i^d$  is the *d*-neighborhood of  $\{\mathcal{V}^i\}_{i\in\mathcal{V}}$ , and the neighborhood of  $\mathcal{V}_i$  is of the *d*-degree.<sup>30</sup> Since, in simple directed graphs, a vertex can be both a sender and a receiver, the *indegree* of a vertex is the number of arcs it receives, and the *outdegree* is the number of arcs it sends. The degree distribution of a network is the frequency distribution of vertices with degree  $d = 0, 1, \ldots, n -$ 1. The average degree of a network is generally used to measure the cohesion of a network, and, in the context of random networks, networks are defined in terms of a given degree distribution's statistical properties.<sup>31</sup> A complete network,  $\mathcal{N}^c$ , is a network in which every vertex is linked to every other vertex, and d = n - 1. In an empty network, d = 0.

The notion of neighborhood is associated to the one of clustering. The *clustering coefficient* of a vertex  $\mathcal{V}_i$  is the proportion of a vertex's neighbors which are neighbors of each other. The clustering coefficient for the network as a whole can be derived taking a weighted or an unweighted average across vertices in the network.

<sup>&</sup>lt;sup>28</sup> In a simple directed graph (no parallel arcs)  $m \le n^2$ .

<sup>&</sup>lt;sup>29</sup> Therefore, any network  $\mathcal{N}$  is the set of neighborhoods for all vertices,  $\{\mathcal{V}^i\}_{i\in\mathcal{V}}$ .

<sup>&</sup>lt;sup>30</sup>The analysis on neighborhoods can be further extended. If in a simple undirected network  $\mathcal{V}_i^d$  is the neighborhood of  $\mathcal{V}_i$  including only the vertices immediately connected to it: the *first-order* neighborhood. The *second-order* network is the set of vertices which are at a *geodesic distance* equal to 2 from  $\mathcal{V}_i$ , where the geodesic distance is the shortest path joining two vertices. Analogously, the *r*th-degree neighborhood of  $\mathcal{V}_i$  included the vertices at a geodesic distance of *r*.

<sup>&</sup>lt;sup>31</sup>Specific examples of degree distributions used in random graph analysis are the binomial, the Poisson, the geometric, and the power-law distributions (Vega-Redondo, 2007).

### 6.3 Structural properties of a Network

The density of a network is the number of lines in a simple network, expressed as a proportion of the maximum possible number of lines. It is defined by the quotient  $\gamma = \frac{m}{m_{\text{max}}}$ , where  $m_{\text{max}}$  is the number of lines in a complete network with the same number of vertices.<sup>32</sup> Accordingly, a complete network is a network with maximum density.

The position of every vertex in a network is measured in terms of *central-ity*. The simplest measure of centrality of  $\mathcal{V}_i$  is the number of its neighbors, i.e. its degree. The standardized *degree centrality* of a vertex is its degree divided by the maximum possible degree:

$$C_i^d = \frac{d}{n-1} \tag{4}$$

The degree centralization of a network is defined relatively to the maximum attainable centralization. The minimum degree for any component of the network is 0 and the maximum possible degree is n - 1. If  $C_i^d *$  is the centrality of the vertex that attains the maximum centrality score, the variation in the degree of vertices is the summed absolute differences between the centrality scores of the vertices and the maximum centrality score among them. So, as the maximum attainable centrality is (n-2)(n-1), the degree centralization of a network is

$$C^{d} = \frac{\sum_{i=1}^{n} |C_{i}^{d} - C_{i}^{d} *|}{(n-2)(n-1)}.$$
(5)

and the higher the variation in the degree of vertices the higher the centralization of a network. The degree centralization of any regular network is 0, while a star has a degree centralization of  $1.3^{33}$ 

If degree centralization is associated to direct links, when connections in a network acquire some relevance one should give prominence also to indirect links. This brings to the concept of *distance* in networks, namely the number of steps needed to connect two vertices  $\mathcal{V}_i$  and  $\mathcal{V}_j$ . The shortest the distance

 $<sup>^{32}</sup>$  In this definition of density, multiple lines and weights eventually contained in the *line value function* W - the line values – are disregarded.

<sup>&</sup>lt;sup>33</sup> The variation in the degree of vertices in a star grows with n. In a pure star network with one core and n-1 vertices in the periphery, the core has a maximum degree of n-1 and the peripheries have a minimum degree of 1. Hence, the variation in the degree of vertices amounts to (n-1)(n-2):(vertices in the periphery contribute)  $(n-1)\times((n-1)-1)$  and (the core contributes)  $1 \times ((n-1) - (n-1))$ ). This expression grows in n, and divided by the maximum degree variation (n-2)(n-1), yields a degree centralization of 1. With standardized measure the maximum degree variation is (n-2) and the variation in the degree of vertices amounts to (n-2) as well.

between two vertices the closest is the connection between them. A *path* is a sequence of lines in which no vertex in between the two vertices at the extremes occurs more than once, and a *geodesic distance*,  $\delta_{ij}$  is the shortest path between two vertices.

The notion of geodesic distance is at the bulk of a second definition of centrality: *Closeness centrality*. The closeness centrality of a vertex  $\mathcal{V}_i$  is the number of other vertices divided by the sum of all distances between  $\mathcal{V}_i$  and all others  $\mathcal{V}_{j\neq i}$ .

$$\mathcal{C}_i^c = \frac{n-1}{\sum_{\substack{i\neq i \\ i\neq i}}^{n-1} \delta_{ij}}.$$
(6)

At the aggregate network level, if, as in the case of degree centralization,  $C_i^c *$  is the centrality of the vertex that attains the maximum closeness centrality score, the degree of *closeness centralization* of a network is (Freeman, 1979; Goyal, 2007)

$$C^{c} = \frac{\sum_{i=1}^{n} |\mathcal{C}_{i}^{c} - \mathcal{C}_{i}^{c} *|}{(n-2)(n-1)/(2n-3)}.$$
(7)

The closeness centralization is, therefore, the variation in the closeness centrality of vertices divided by the maximum variation in closeness centrality scores possible in a network of the same size. The closeness centrality of a pure star is  $1.^{34}$ 

A different notion of centrality is based on the intuition that a vertex  $\mathcal{V}_i$  is central if it is essential in the indirect link between  $\mathcal{V}_k$  and  $\mathcal{V}_j$ . A vertex that is located on the geodesic distance between many pairs of vertices plays a central role in the network, and in a pure star, the core is central because it is necessary for all periphery vertices in order to be mutually reachable. This concept of centrality is based on betweenness, so it is called *betweenness centrality*.

The betweenness centrality of vertex  $\mathcal{V}_i$  is the proportion of all geodesic distances between pairs of other vertices that include this vertex (Vega-Redondo, 2007):

$$C_i^b = \sum_{j \neq k} \frac{\delta_{jk}^i}{\delta_{jk}} \tag{8}$$

<sup>&</sup>lt;sup>34</sup> Closeness centrality and degree centrality are equal for some networks, such as the star. However, this is not always the case in general. Furthermore, if an undirected network is not connected or a directed network is not strongly connected, there are no path between all vertices. In this case, one can take into account only the vertices that are reachable and weight the summed distance by the percentage of vertices that are reachable (de Nooy, Mrvar and Batagelj, 2005).

where  $\delta_{jk}$  is the total number of shortest paths joining any two vertices  $\mathcal{V}_k$ and  $\mathcal{V}_j$ , and  $\delta_{jk}^i$  is the number of those paths that not only connect  $\mathcal{V}_k$  and  $\mathcal{V}_j$ , but also go through  $\mathcal{V}_i$ . The core of a star network has maximum betweenness centrality, because all geodesic distances between pairs of other vertices include the core. In contrast, all other vertices have minimum betweenness centrality, because they are not located between other vertices.

The *betweenness centralization* is the variation in the betweenness centrality of vertices divided by the maximum variation in betweenness centrality scores possible in a network of the same size.

$$\mathcal{C}^b = \sum_{i=1}^n |\mathcal{C}^b_i - \mathcal{C}^b_i *|.$$
(9)

The total betweenness  $\sum_{i=1}^{n} C_i^b$  is proportional to the average network distance, with the factor of proportionality being the number of possible vertex pairs (Vega-Redondo, 2007).

The notion of betweenness centrality has important strategic implications. The central vertex could, in fact, exploit its position to its advantage.

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