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# Monetary Policy and Herd Behavior in New-Tech Investment

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| An old               | question          |                    |                      |            |                                |
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- Should monetary policy react to perceived asset-price bubbles?
- This question has been hotly debated since the 1990s-2000s boom and bust in new-tech equity prices.
- This paper contributes to the debate, focusing on bubbles in new-tech equity prices.
- Its original contribution stems from modeling these bubbles as the result of (rational) herd behavior.



- To that question, a majority of central bankers (e.g. Greenspan, 2002; Bernanke, 2002; Trichet, 2005) answered "no" - prior to the current crisis.
- They view a monetary policy reaction to a perceived asset-price bubble as an 'insurance-against-bubbles policy':
  - raising the interest rate entails a cost, whether there is a bubble or not;
  - it brings an uncertain benefit, which depends on whether or not there is a bubble, and, if there is one, how effective the interest-rate hike is in reducing its size or duration.

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| Central              | bankers'          | answer             | 11 |                       |                                |

- Therefore, they stress the following conditions for the desirability of such a monetary policy reaction:
  - the central bank should be sufficiently certain that there is a bubble;
  - It the bubble should be sufficiently sensitive to interest-rate hikes.

- They view these conditions as unlikely to be met in practice.
- They conclude that, in most if not all cases, such a monetary policy reaction is not desirable.

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- We build a simple general-equilibrium model in which, because bubbles are modeled as the result of herd behavior, these two conditions can be met.
- We assume that a new technology becomes available, whose productivity will be known with certainty only in the medium term.
- Entrepreneurs sequentially choose (in an exogenous ordering) whether to invest in the old or the new technology, each of them on the basis of both:
  - the previous entrepreneurs' investment decisions that she observes;
  - a private signal that she receives about the productivity of the new technology.

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 Challenging this view II

- Herd behavior may arise as the result of an informational cascade (Banerjee, 1992; Bikhchandani *et al.*, 1992).
- This is a situation in which, because the first entrepreneurs choose to invest in the new technology as they receive encouraging private signals about its productivity, the following entrepreneurs rationally choose to invest in the new technology too whatever their own private signal.
- In this context, monetary policy tightening, by making borrowing dearer for the entrepreneurs, can make them invest in the new technology if and only if they receive an encouraging private signal about its productivity.
- In doing so, it makes their investment decision reveal their private signal. Therefore, it prevents herd behavior and the bubble in new-tech equity prices.

- With this explanation of bubbles in new-tech equity prices, the two conditions mentioned above can be met in the model:
  - the central bank should be sufficiently certain that there is actually a bubble: the central bank can identify herd behavior with certainty, even though it then knows less about the productivity of the new technology than each entrepreneur;
  - the bubble should be sufficiently sensitive to modest interest-rate hikes: given the fragility of informational cascades, a modest monetary policy intervention can be enough to interrupt herd behavior (even though it may not interrupt investment in the new technology).
- We show that the 'insurance-against-bubbles policy' can be *ex ante* preferable, in terms of social welfare, to the laisser-faire policy.

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- Our paper is related to a literature pioneered by Bernanke and Gertler (1999, 2001).
- This literature addresses the following question: should the monetary policy rule make the interest rate react to asset prices, in addition to standard variables, during an asset-price boom that may correspond to a bubble?
- One important difference between our paper and this literature concerns the way in which bubbles are modeled.

| Another              | related           | iteratu            | re                 |                       |                            |  |
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- Our paper is also related to the literature on the role of informational cascades in the business cycle.
- Within this literature, the paper closest to ours is that of Chamley and Gale (1994), which models investment collapses as the result of herd behavior.

• One important difference between the two papers is that, unlike them, we conduct a general-equilibrium analysis.

Motivation Literatures of the model

### General features of the model

- We consider an economy populated with:
  - infinitely lived households;
  - overlapping generations of finitely lived entrepreneurs;
  - a central bank.
- We limit our analysis to outcomes symmetric across entrepreneurs and across households (*i.e.* there is one representative household and, in each generation, one representative entrepreneur).
- Time is discrete.
- There is a single good that is non-storable and can be consumed or invested.

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| Techno     | logies      |           |             |            |                      |

- A production project requires  $\kappa_t$  units of good at date t and delivers  $Y_{t+N} = A_{t+N} L_{t+N}^{\alpha}$  units of good at date t + N, where  $A_{t+N}$  is a productivity parameter,  $L_{t+N}$  is labor services and  $0 < \alpha < 1$ .
- At a given date t, different technologies may be available. A given technology z ∈ ℝ is defined by an investment cost κ<sub>t</sub> = κ(z) and by a productivity parameter A<sub>t+N</sub> = A(z).
- A production project needs a newborn entrepreneur to be undertaken, and a newborn entrepreneur can undertake only one production project.



- The representative household supplies inelastically one unit of labor *per* period.
- Her utility function is:

$$U_t = extsf{E}_{\Omega(h,t)} \sum_{j=0}^\infty eta^j \ln( extsf{c}_{t+j})$$

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where  $\Omega(h, t)$  is her information set at date t,  $c_{t+j}$  her consumption at date t + j, and  $0 < \beta < 1$ .



- The representative entrepreneur born at date t lives until date t + N and consumes only at that date.
- Her utility function is:

$$V_t = \beta^N E_{\Omega(e,t)} c^e_{t+N}$$

where  $\Omega(e, t)$  is her information set and  $c_{t+N}^{e}$  her consumption at date t + N.

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### Timing of entrepreneurs' actions

- At date t, the representative entrepreneur born at date t:
  - chooses a technology z;
  - borrows  $\kappa_t = \kappa(z)$  from the representative household (at the *N*-period gross real interest rate  $\frac{1}{q_t}$ );
  - invests in the technology z.

• At date t + N, the representative entrepreneur born at date t:

- employs the repres. household for  $L_{t+N}$  and produces  $Y_{t+N}$ ;
- uses  $Y_{t+N}$  to pay wage  $w_{t+N}$  and reimburse debt  $\frac{\kappa_t}{q_t}$  to the representative household and to consume  $c_{t+N}^e$ .

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Timing of households' actions

- At each date t, the representative household:
  - works  $L_t$  for the representative aged N + 1 entrepreneur and receives wage  $w_t$  from her;
  - receives debt reimbursement  $\frac{\kappa_{t-N}}{q_{t-N}}$  from her;
  - lends  $\kappa_t$  to the representative newborn entrepreneur (at the N-period gross real interest rate  $\frac{1}{a_t}$ );

• consumes  $c_t$ .

| Moneta     | ary policy  | /           |             |            |            |                      |
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- We focus on the real-interest-rate transmission channel of monetary policy, *i.e.* monetary policy has an effect on the economy only through its effect on the real interest rate.
- This is done by modeling monetary policy as a tax (or subsidy)  $\tau_t$  on lending together with a positive (or negative) lump-sum transfer  $T_t$  to the representative household.
- We show that this is the reduced form of a model with money.



• The intertemporal budget constraint of the representative household at date *t* is therefore

$$c_t + \tau_t q_t B_{t+N} \leq B_t + w_t L_t + T_t,$$

where  $B_t$  denotes the quantity of bonds bought by the repres. household at date t - N and paying interest at date t.

• The Euler equation is therefore

$$\tau_t q_t = \beta^N E_{\Omega(h,t)} \left[ \frac{c_t}{c_{t+N}} \right]$$

In this economy, a competitive equilibrium is a sequence of investment decisions  $\{z_t\}$ , prices  $\{q_t, w_t\}$  and quantities  $\{B_t, c_t, c_t^e, L_t\}$  for  $t \ge 1$  such that, given initial conditions and for an exogenous sequence of technological possibilities and monetary policy interventions  $\{\tau_t\}$ ,

- household's consumption and bonds holding solve her maximization problem given prices;
- investment decision z<sub>t</sub> maximizes expected intertemporal utility of newborn entrepreneurs given prices;
- labor demand L<sub>t</sub> maximizes aged N + 1 entrepreneurs' profits given prices;

• labor, bonds and good markets clear.



- Until date 0 included, there is only one (non-trivial) technology available, noted  $\overline{z}$ , and the economy is at the corresponding steady state.
- A new technology, noted z, becomes unexpectedly available at date 1 and remains available thereafter:
  - this new technology requires more investment than the old one:  $\kappa(z) > \kappa(\overline{z})$ ;
  - it may be "good" or "bad", *i.e.* it may lead N periods later to a productivity parameter A (z) > A (z̄) or to the same productivity parameter A (z̄) as the old technology z̄;
  - from date *N* + 1 onwards, whether the new technology is good or bad is common knowledge (even when there is no investment in the new technology at date 1).

At each date  $t \in \{1, ..., N\}$ , the representative household and the central bank

- start period t with the prior  $\mu_{t-1}$  on the probability that the new technology is good ( $\mu_0$  being exogenous);
- observe the representative newborn entrepreneur's investment decision, noted  $I_t$  ( $I_t = 1$  if new-tech investment and  $I_t = 0$  if old-tech investment);
- end period t with the posterior  $\mu_t$ .

Therefore,  $\mu_t \neq \mu_{t-1}$  if and only if the observation of  $I_t$  provides some new information about the productivity of the new technology.

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At each date  $t \in \{1, ..., N\}$ , the representative entrepreneur born at date t

- starts period t with the prior  $\boldsymbol{\mu}_{t-1}$  on the probability that the new technology is good;
- receives a private signal  $S_t$ , either good news  $(S_t = 1)$  or bad news  $(S_t = 0)$  about the productivity of the new technology;
- ends therefore period t with the posterior

$$\begin{split} \widetilde{\mu}_{t} &= S_{t} \frac{\mu_{t-1}\lambda}{\mu_{t-1}\lambda + (1 - \mu_{t-1})(1 - \lambda)} \\ &+ (1 - S_{t}) \frac{\mu_{t-1}(1 - \lambda)}{\mu_{t-1}(1 - \lambda) + (1 - \mu_{t-1})\lambda} \end{split}$$

where  $\lambda$  denotes the probability that a private signal (whether good or bad) is right.



- At each date  $t \in \{1, ..., N\}$ , the representative household and the central bank observe  $I_t$  but not  $S_t$ . If there is a unique equilibrium and, at this equilibrium,
  - $I_t = S_t$  whatever  $S_t \in \{0, 1\}$ , then they can infer  $S_t$  from  $I_t$ and therefore  $\mu_t = \tilde{\mu}_t$  (no cascade);
  - $I_t = 0$  whatever  $S_t \in \{0, 1\}$ , then they cannot infer  $S_t$  from  $I_t$ and therefore  $\mu_t = \mu_{t-1} \neq \tilde{\mu}_t$  (low cascade);
  - $I_t = 1$  whatever  $S_t \in \{0, 1\}$ , then they cannot infer  $S_t$  from  $I_t$ and therefore  $\mu_t = \mu_{t-1} \neq \tilde{\mu}_t$  (high cascade).

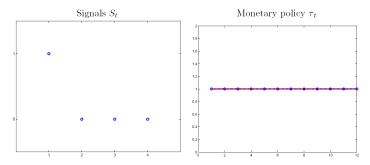
• We impose a necessary and sufficient condition (NSC) on the structural parameters for the absence of multiple equilibria.

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- Whether or not there is a cascade at date t depends on the public prior  $\mu_{t-1}$ , not on the private posterior  $\tilde{\mu}_t$ . Therefore,
  - under laisser-faire, if there is a cascade at date t, then there is also a cascade at date t+1 (as  $\mu_t=\mu_{t-1}$ ) and at all following dates until date N included (hence the terms "cascade" and "herd behavior");
  - the central bank can infer from µ<sub>t-1</sub> whether there is a cascade at date t: central bankers' first condition is met.

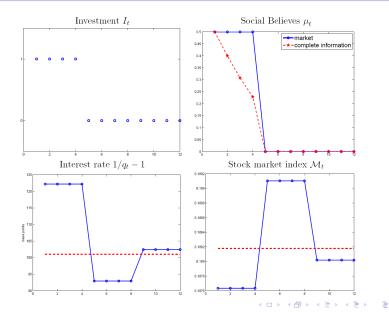


• Parameter values (the period being one year): N = 4,  $\beta = 0.99$ ,  $\alpha = 0.7$ , p = 0.4,  $\lambda = 0.6$ ,  $\mu_0 = 0.4$ ,  $\kappa(\overline{z}) = 0.1$ ,  $\kappa(z) = 0.11$ ,  $A(\overline{z}) = 1$ , A(z) = 1.1.



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### A simulated path with no policy and a cascade II



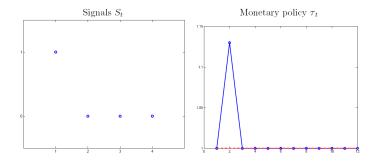
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- A monetary policy intervention raising the interest rate can interrupt a high cascade by increasing the cost of investing in the new technology relatively to the cost of investing in the old technology (as  $\kappa(z) > \kappa(\overline{z})$ ).
- A monetary policy intervention interrupting a cascade at a given date t may increase the welfare of households and entrepreneurs born between dates t + 1 and N included by revealing  $S_t$  to them.

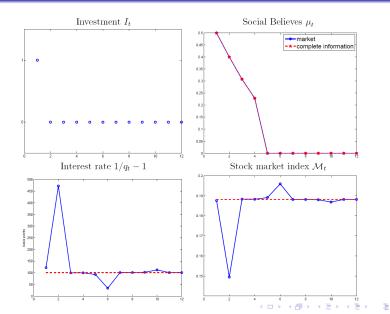
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# A simulated path with a policy preventing a cascade I

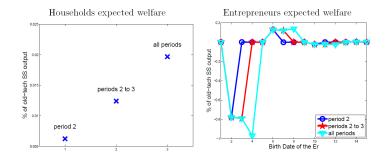


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### A simulated path with a policy preventing a cascade II







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- We assume that N = 3 and  $z \simeq \overline{z}$ .
- We impose a NSC on the parameters for:
  - under laisser-faire, the absence of cascade at t = 1 and the existence of a high cascade at t = 2 when  $S_1 = 1$ ;
  - the existence of an arbitrarily small monetary policy intervention at t = 2 able to avoid the high cascade. Hence, central bankers' second condition is met.
- We show that, for some calibrations, this monetary policy intervention increases social welfare (with weights  $\overline{c}$  for households and 1,  $\beta$ ,  $\beta^2$ ... for entrepreneurs, so that social welfare corresponds to GDP in this linearized case).
- We also show that, whatever the calibration, this intervention does not lead to a Pareto-superior outcome as it lowers the current entrepreneur's welfare.

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Analytical results in a simple case II

- Whether these analytical results obtained in a simple case understate (+) or overstate (-) the case for a monetary policy intervention at t = 2, compared to the numerical results obtained in more general cases, depends on the size of the following effects:
  - (+) the relaxation of the assumption N = 3 spreads the gains of the monetary policy intervention over more periods;
  - (+) the relaxation of the assumption that z is arbitrarily close to  $\overline{z}$  makes households' risk aversion matter in welfare computations;
  - (-) the relaxation of the assumption that parameters are such that the monetary policy intervention can be arbitrarily small increases the distortion caused by this intervention.

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| Conclus              | sion I            |                    |                       |                                |

- This paper develops a dynamic general equilibrium model in which informational cascades can occur in equilibrium.
- In this model, entrepreneurs receive private information about the productivity of a new technology, and invest or not in that new technology, borrowing from households. While entrepreneurs' information is private, their investment decisions are public.
- When entrepreneurs' private information cannot be inferred (by households and subsequent entrepreneurs) from their public investment decisions, there is an informational cascade, and investment decisions are characterized by herd behavior. We call such a situation a stock price bubble.

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| Conclu     | sion II     |           |             |           |                      |

- Monetary policy can be used to eliminate those bubbles, even though the central bank has less information than entrepreneurs about the productivity of the new technology.
- In some particular circumstances, even a modest monetary policy intervention can be enough for that matter, and may improve social welfare from an *ex ante* point of view.

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| Conclu               | sion III          |                    |                      |                                |

- These results suggest that, insofar as booms in new-tech equity prices can be modeled as the result of herd behavior, the two conditions most commonly stressed by central bankers for the desirability of a monetary policy reaction to these booms may prove less demanding than they seem at first sight.
- Of course, these results are only suggestive as they are obtained in particular cases and as our simplistic model fails to capture many important dimensions of the debate.

## Overview of the resolution

We impose a SC on the parameters for:  $\forall t > N$ ,  $I_t = 1$  if the new technology turns out to be good and  $I_t = 0$  otherwise. For  $t \in \{1, ..., N\}$ , we proceed in four steps:

- we get  $q_t = q(\tau_t, \mu_t, I_t)$  from the Euler equation;
- (2) we get  $I_t = I(\tau_t, \mu_t, S_t)$  from step 1 and from the consideration of entrepreneurs' investment decision problem;

### **(3)** we deduce from step 2 that:

- a low cascade is supported by an equilibrium if and only if  $\forall S_t \in \{0, 1\}$ ,  $I(\tau_t, \mu_{t-1}, S_t) = 0$ ;
- a **high cascade** is supported by an equilibrium if and only if  $\forall S_t \in \{0, 1\}$ ,  $I(\tau_t, \mu_{t-1}, S_t) = 1$ ;
- the **absence of cascade** is supported by an equilibrium if and only if  $\forall S_t \in \{0; 1\}$ ,  $I(\tau_t, \tilde{\mu}_t, S_t) = S_t$ ;
- we show that the three conditions obtained in step 3 are mutually exclusive (*i.e.* there are no multiple equilibria).

• For  $t \in \{1, ..., N\}$ , when  $I_t = 0$ , the Euler equation becomes:

$$\tau_{t}q_{t} = \beta^{N} \left[ \alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^{N}} \right] \times \left[ \frac{\mu_{t}}{\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{q_{t}}} + \frac{1 - \mu_{t}}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_{t}}} \right]$$

 We impose a NSC on the parameters for the existence of a strictly positive real number qt solution of this equation for all µt ∈ [0; 1].

• Then  $q_t$ , which we note  $q(\tau_t, \mu_t, \mathbf{0})$ , is unique.

• For  $t \in \{1, ..., N\}$ , when  $I_t = 1$ , the Euler equation becomes:

$$\tau_{t}q_{t} = \beta^{N} \left[ \alpha A(\overline{z}) - \kappa(\mathbf{z}) + \frac{\kappa(\overline{z})}{\beta^{N}} \right] \times \left[ \frac{\mu_{t}}{\alpha A(\mathbf{z}) - \kappa(z) + \frac{\kappa(\mathbf{z})}{q_{t}}} + \frac{1 - \mu_{t}}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\mathbf{z})}{q_{t}}} \right]$$

• We impose a NSC on the parameters for the existence of a strictly positive real number  $q_t$  solution of this equation for all  $\mu_t \in [0, 1]$ .

• Then  $q_t$ , which we note  $q(\tau_t, \mu_t, \mathbf{1})$ , is unique.

 $\forall t \in \{1, ..., N\}$ , noting  $\mu_t^{\mathbf{0}}$  the value taken by  $\mu_t$  when  $I_t = \mathbf{0}$ , we get:

 $I_t = \mathbf{0}$  is supported by an equilibrium

$$\Leftrightarrow \quad V_t \left( I_t = 0 \right) > V_t \left( I_t = 1 \right) \text{ when } q_t = q \left( \tau_t, \mu_t^{\mathbf{0}}, \mathbf{0} \right)$$

$$\Leftrightarrow \quad (1 - \alpha) A \left( \overline{z} \right) - \frac{\kappa \left( \overline{z} \right)}{q \left( \tau_t, \mu_t^{\mathbf{0}}, \mathbf{0} \right)} > \widetilde{\mu}_t \left[ \left( 1 - \alpha \right) A \left( z \right) - \frac{\kappa \left( z \right)}{q \left( \tau_t, \mu_t^{\mathbf{0}}, \mathbf{0} \right)} \right]$$

$$+ \left( 1 - \widetilde{\mu}_t \right) \left[ \left( 1 - \alpha \right) A \left( \overline{z} \right) - \frac{\kappa \left( z \right)}{q \left( \tau_t, \mu_t^{\mathbf{0}}, \mathbf{0} \right)} \right]$$

$$\Leftrightarrow \quad \widetilde{\mu}_t q \left( \tau_t, \mu_t^{\mathbf{0}}, \mathbf{0} \right) < B \equiv \frac{\kappa \left( z \right) - \kappa \left( \overline{z} \right)}{\left( 1 - \alpha \right) \left[ A \left( z \right) - A \left( \overline{z} \right) \right]}$$

(*i.e.* the interest rate must be high enough).

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Step 2: NSC for  $I_t = 1$  to be supported by an equilibrium

 $\forall t \in \{1, ..., N\}$ , noting  $\mu_t^1$  the value taken by  $\mu_t$  when  $I_t = 1$ , we get:

 $I_t = 1$  is supported by an equilibrium

$$\Leftrightarrow V_t (I_t = 0) < V_t (I_t = 1) \text{ when } q_t = q \left(\tau_t, \mu_t^1, \mathbf{1}\right)$$

$$\Leftrightarrow (1 - \alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{q(\tau_t, \mu_t^1, \mathbf{1})} < \widetilde{\mu}_t \left[ (1 - \alpha) A(z) - \frac{\kappa(z)}{q(\tau_t, \mu_t^1, \mathbf{1})} \right]$$

$$+ (1 - \widetilde{\mu}_t) \left[ (1 - \alpha) A(\overline{z}) - \frac{\kappa(z)}{q(\tau_t, \mu_t^1, \mathbf{1})} \right]$$

$$\Leftrightarrow \widetilde{\mu}_t q \left(\tau_t, \mu_t^1, \mathbf{1}\right) > B \equiv \frac{\kappa(z) - \kappa(\overline{z})}{(1 - \alpha) [A(z) - A(\overline{z})]}$$

(*i.e.* the interest rate must be **low** enough).

- We impose a NSC on the parameters for,  $\forall (\mu_t^0, \mu_t^1) \in [0; 1]^2$ , at most one of the two previous conditions to be met. This enables us to get  $I_t$  as a function, noted I, of  $\tau_t$ ,  $\mu_t$  and  $S_t$ .
- Noting µ<sub>t</sub><sup>0</sup> (µ<sub>t</sub><sup>1</sup>) the value taken by µ<sub>t</sub> when S<sub>t</sub> = 0 (S<sub>t</sub> = 1), using function I and the NSC imposed above, we show that, in equilibrium, ∀t ∈ {1, ..., N}, there are only three possibilities and these possibilities are mutually exclusive:
  - either  $\tilde{\mu}_{t}^{1}q\left(\tau_{t}, \mu_{t-1}, 0\right) < B$ , then there is a low cascade;
  - or  $\tilde{\mu}_t^0 q\left(\tau_t, \mu_{t-1}, 1\right) > B$ , then there is a **high cascade**;
  - or  $\tilde{\mu}_t^0 q\left(\tau_t, \tilde{\mu}_t^0, 0\right) < B$  and  $\tilde{\mu}_t^1 q\left(\tau_t, \tilde{\mu}_t^1, 1\right) > B$ , then there is **no cascade**.

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• In particular, monetary policy must be tightened to interrupt a high cascade.