PROBABILISTIC DOMINANCE AND STATUS QUO BIAS

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Abstract. Decision makers have a strong tendency to retain the status quo unless an alternative which may be a significant improvement comes along. This well-documented phenomenon is termed status quo bias. In many complicated choice problems governed by a status quo, decision makers consider alternatives that yield inferior outcomes in some criteria if they simultaneously yield improvements in a set of other criteria that is of relative importance. In an uncertain environment, we present the *probabilistic dominance* approach to status quo bias – an alternative is considered acceptable to replace the status quo only if it yields a better outcome than the status quo with sufficiently high probability. Probabilistic dominance is applied and behaviorally characterized in a choice model that allows for a scope of biases towards the status quo, general enough to accommodate unanimity but also standard expected utility maximization. In addition, we show that the probabilistic dominance choice model predicts the well-known endowment effect and provide with the means of calculating the buying and selling prices.

Keywords: Unanimity, probabilistic dominance, status quo bias, endowment effect, regret.

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1. Introduction

Standard choice models describe a decision maker who ought to choose one or a few elements from a collection of alternatives. These models typically describe methods that decision makers might use based on their utility, beliefs, ambiguity, information and alike. Payoff-irrelevant factors such as an initial endowment or a default reference point are considered irrelevant to the rational assessment of the alternatives and are frequently ignored. However, a growing amount of empirical data suggests that such factors affect behavior and that standard models are insufficient to describe real-life decision-making processes. It is well-established by now that individuals exhibit a strong tendency to retain the current state of affairs, unless an alternative comes along, which may be a significant improvement. This phenomenon is traditionally referred to as *status quo bias* (Samuelson and Zeckhauser (1988)).

Accumulated experimental findings relate status quo bias to anticipated regret. In particular, the sensation of regret is stronger when bad outcomes are a result of choosing a new alternative than when they result from retaining the status quo (e.g., Kahneman and Tversky (1982), Inman and Zeelenberg (2002) and Zeelenberg, van den Bos, van Dijk, and Pieters (2002)). On the other hand, Shafir, Simonson, and Tversky (1993) and Inman and Zeelenberg (2002) argue that with sufficient justification in favor of the alternative to the status quo, this distinction decreases.

In the decision theoretic literature, a prominent approach involving status quo considerations is that of *unanimity*. According to this approach, an option is considered a candidate to replace the status quo if and only if it is considered better than the status quo with respect to every possible criterion. Examples include Bossert and Sprumont (2003), Masatlioglu and Ok (2005) and Ortoleva (2010). Criteria are determined subjectively by the decision maker according to the nature of the decision problem in study. For example, Bossert and Sprumont as well as Masatlioglu and Ok consider attributes of outcomes in a general framework, and Ortoleva (similar to Bewley (2002)) considers probability distributions over the state space.

The unanimity approach describes a decision maker who is unwilling to perform any sort of trade-off considerations relative to the status quo. In particular, she is willing to endure no regret upon the replacement of the status quo with another alternative. However, behavior in complicated problems, such as accepting a new job offer or relocating to a different country, often does not conform with this approach. Decision makers may often allow for compromises in some criteria, trading-off simultaneous improvements in other criteria that are subjectively important enough. For instance, an employee might consider enduring additional working hours if she draws more satisfaction out of her new position or earns more money, and an assistant professor might take an associate professor position in a different state despite the uncertainty whether such a move will be successful or not, as long as she evaluates the chances of success to be sufficiently high.

Taking a choice-theoretic approach, the purpose of this paper is to present, what we refer to as, the *Probabilistic Dominance Model* of decision making under uncertainty—the decision maker considers an alternative choosable only if it yields a better outcome than the status quo with sufficiently high probability.

1.1. **Main results.** We adopt a setup similar to that in Masatlioglu and Ok (2005) but focus on inherent uncertainty as in Anscombe and Aumann (1963). In this set up, a choice problem is either a collection of acts, or a collection of acts and a status quo (which is an act) in this collection. In particular, it is assumed that the status quo is always a feasible alternative.[1] The decision maker is characterized by a choice correspondence that assigns a non-empty sub-collection of alternatives for each choice problem, with or without a status quo.

We present a set of axioms implying that the decision maker is associated with a subjective prior over the state space and two binary relations. The first admits an expected utility representation with respect to the subjective prior. The second admits a *probabilistic dominance* representation: one act is preferred to another if the probability (with respect to the same subjective prior) that the first act will return a better alternative than the second exceeds a certain threshold. The decision maker uses the following procedure to make her choices: whenever there is no status quo, the decision maker simply maximizes the first relation among all feasible alternatives. That is, she acts as a standard subjective expected utility maximizer. However, if the choice problem is governed by a status quo, she first applies probabilistic dominance considerations to

[1]See the discussion section for a simple extension of the model presented here to a more general one in which the status quo may not be feasible or exogenous.

eliminate all alternatives that do not return an alternative at least as good as the one returned by the status quo with high enough probability. She then maximizes the first relation over the collection of feasible acts that survived the elimination stage described above (note that this collection is never empty, as the status quo always survives the elimination stage).

To illustrate the proposed model, consider for instance the assistant professor example mentioned above. When contemplating the different offers, the assistant professor compares each offer to the one from the university she is currently employed at, trying to assess the quality of life she and her family would experience both from professional and personal aspects. Out of all offers, she is willing to consider only those that guarantee, with (subjectively) sufficiently high probability, a quality of life at least as good as the one she would experience in case she accepted the offer from her current working place. Then, out off all the offers she considers, she chooses the one that maximizes the expected well-being.

In the suggested decision model, the first step describes how the decision maker resolves her bias towards the status quo. She groups states to construct *decisive events*—events which are sufficiently significant for the decision maker to consider the alternatives that, within such events, perform at least as well as the status quo.[2] This procedure conforms with the findings relating status quo bias to anticipated regret. Probabilistic dominance considerations guarantee that the probability the decision maker will regret having moved away from the status quo, once uncertainty is resolved, is not too high. Alternatively, probabilistic dominance relations could be considered as ensuring some level of confidence (that is, providing sufficient justification) when replacing the status quo.[3] For example, the assistant professor would like to convince her spouse (or alternatively, herself[4]) that

[2]Some experimental evidence for such comparisons can be found in Ritov (1996). In that study, when choosing between two alternatives, a major factor subjects exhibited was a concern to minimize the probability of regret.

[3]This interpretation of the first step of the suggested choice procedure raises a notion of regret different than those that had previously appeared in the literature, such as in Savage (1954), Bell (1982), Loomes and Sugden (1982) and Sarver (2008). All that matters for the decision maker is the probability of feeling regret.

[4]See the discussion in Shafir et al., (1993) page 33.

a particular offer is 'good enough' in the sense that the chance that it will improve their quality of life is significant.

The scope of the probabilistic threshold in the representation allows for a range of biases towards the status quo. The higher the threshold, the more difficult it is for an act to survive the first stage of the decision making process. So, the higher the threshold parameter, the more biased towards the status quo the decision maker is. Note that the model is general enough to allow for no bias at all, so that choices follow a 'standard' maximization of expected utility (characterized by a threshold of 0), but also for a strong bias towards the status quo as expressed in the unanimity approach (which is characterized by a threshold of 1).

The chosen framework allows us, when comparing between the choices of two individuals, to present a definition of *revealing more bias towards the status quo*. The intuition behind the definition is the following. Assume that two decision makers, Ilsa and Rick, both choose the same alternative given a status quo free choice problem. Assume in addition that at the same choice problem Rick no longer chooses this alternative once a status quo is present. If Ilsa reveals more bias towards the status quo than Rick, then in the presence of the same status quo, one would expect her to alter her choice as well. We study the implications of this definition to the suggested model. As a simple illustration, assume that both Ilsa and Rick follow the probabilistic dominance choice model and that they share beliefs and tastes. Intuitively, Ilsa reveals more bias towards the status quo than Rick if and only if Ilsa's probabilistic threshold parameter is at least as high as that of Rick.

As an application of our choice model, we show that it predicts that the decision maker will demand more in order to give up a commodity than she would be willing to pay to acquire it. Thaler (1980) termed this well-known phenomenon the *endowment effect*.[5] We provide a simple way of calculating the buying and selling prices in terms of utility, and characterize by means of the representation those acts for which the decision maker exhibits the endowment effect.

1.2. **Related literature.** Status quo bias was first captured and termed by Samuelson and Zeckhauser (1988). Through laboratory and field experiments, they indicated the

[5]Additional examples are Knetsch and Sinden (1984) and Kahneman, Knetsch, and Thaler (1990).

strong affinity of individuals to retain the alternative which is the status quo. This observation has lead to a significant number of empirical studies on the presence of status quo bias (and related effects such as the endowment effect and reference-dependence) in important choices. Examples are 401(k) pension plans (Madrian and Shea (2001), Agnew, Balduzzi, and Sundén (2003) and Choi, Laibson, Madrian, and Metrick (2004)), electrical services (Hartman, Doane, and Woo (1991)) and car insurance (Johnson, Hershey, Meszaros, and Kunreuther (1993)).

These findings promoted the development of decision making models attempting to capture status quo effects. Tversky and Kahneman (1991) presented a reference-dependent choice model based on loss aversion. The intuition behind this model is that the status quo affects the utility of the decision maker in such a manner that relative losses loom larger than corresponding gains. Sugden (2003) and Munro and Sugden (2003) axiomatize reference-dependent preferences in the spirit of Tversky and Kahneman (1991)'s 'loss-aversion'. Köszegi and Rabin (2006) develop a reference-dependent preferences model, which adhere to loss-aversion as well, extracting the reference point out of equilibrium.

Masatlioglu and Ok (2005) and (2009) present a new approach to modeling status quo bias. Instead of affecting the underlying utility as in loss aversion, the presence of the status quo imposes constraints on what is choosable and what is not.[6] These constraints make some of the alternatives appear as inferior to the status quo and therefore unchoosable.[7] In Masatlioglu and Ok (2005), the strong axioms describing the decision maker's bias towards the status quo, yield (as discussed above) unanimity type of constraints.[8]

Aware of the restrictive nature of their status quo bias axioms, Masatlioglu and Ok (2009) consider a much broader model. In particular it is broader than the model presented here. For example, it is possible in the second stage of the process for the decision maker to consider some, *but not all*, alternatives that with high probability yield a good

[6]Rubinstein and Zhou (1999) consider a similar approach where the choosable alternatives are those closest to the status quo.

[7]See Masatlioglu and Ok (2009) for elaborated discussions on the differences between the two approaches.

[8]Sagi (2006) studies the implications of an axiom bearing the essence of the Masatlioglu and Ok (2005) status quo bias axioms. The example given by Sagi, when considered in the present framework, describes an attitude towards the status quo which is identical to unanimity.

outcome relative to the status quo, as well as alternatives that yield a good outcome relative to the status quo with much lower probabilities.

1.3. **Organization.** In the following section we discuss the main model in detail, describing the framework in Section 2.1, introducing the representation in Section 2.2 and the axioms in Section 2.3. Section 3 provides the main results. The representation theorem and uniqueness result appear in Section 3.1. In Section 3.2 we present a natural comparative notion of revealed bias towards the status quo and provide its implications to the probabilistic dominance model. An application of the model is given in Section 3.3 in which we show how it predicts the well-known endowment effect and provide with a simple way of calculating the buying and selling prices. In Section 4 we axiomatize probabilistic dominance relations in an Anscombe–Aumann framework and discuss some properties of these relations. Section 5 suggests additional models where the probabilistic dominance approach could be applied, such as choice with endogenous reference points. Probabilistic dominance in a Savagean framework is axiomatized in Appendix A. Lastly, all the proofs appear in Appendix B.

2. The main model

2.1. **The framework.** We follow the setup and notation in Masatlioglu and Ok (2005) with the difference that the objects of choice are assumed to be Anscombe and Aumann acts. Formally, let X be a non–empty finite set of *outcomes*, and let $\Delta(X)$ be the set of all *lotteries* (probability distributions) over X.[9] Given $p \in \Delta(X)$ and $x \in X$, we denote by $p(x)$ the probability p assigns to the outcome x. Let S be a finite non–empty set of *states of nature*. Now, consider the collection $\mathcal{F} = \Delta(X)^S$ of all functions from states of nature to lotteries. Such functions are referred to as *acts*. We denote by $\mathcal{F}_c$ the collection of all constant acts. Following the standard abuse of notation, we denote by p the constant act that assigns the lottery p to every state of nature. Similarly, given an act f and a state s, we write $f(s)$ to represent the constant act that returns the lottery $f(s)$ in every state of nature.

Mixtures (convex combinations) of acts are performed state-wise. For $f, g \in \mathcal{F}$ and $\alpha \in [0, 1]$, we denote by $f \oplus_\alpha g$ the act $\alpha f + (1-\alpha)g$ that returns $\alpha f(s) + (1-\alpha)g(s)$, for

[9]Given a finite set A, $\Delta(A)$ denotes the collection of all probability distributions over A.

each state $s \in S$. And, if A is a non-empty collection of acts, we write $A \oplus_\alpha g$ to denote the collection $\{f \oplus_\alpha g \ : \ f \in A\}$. In addition, for any act $f \in \mathcal{F}$, state $s^* \in S$, lottery $p \in \Delta(X)$ and $\lambda \in [0,1]$, we define $f \oplus_\lambda^{s^*} p \in \mathcal{F}$ by $(f \oplus_\lambda^{s^*} p)(s^*) := \lambda f(s^*) + (1-\lambda)p$ and $(f \oplus_\lambda^{s^*} p)(s) := f(s)$ if $s \neq s^*$. Given a collection of acts A, $A \oplus_\lambda^{s^*} p$ denotes the collection $\{f \oplus_\lambda^{s^*} p \ : \ f \in A\}$.

Let $\mathfrak{F}$ denote the set of all nonempty closed subsets of $\mathcal{F}$.[10] The symbol $\diamond$ will be used to denote an object that does not belong to $\mathcal{F}$. By a *choice problem* we mean a list (A, f) where $A \in \mathfrak{F}$ and either $f \in A$ or $f = \diamond$. The set of all choice problems is denoted by $\mathcal{C}(\mathcal{F})$. If $f \in A$, then the choice problem (A, f) is referred to as a *choice problem with a status quo*. The interpretation is that the agent has to make a choice from the set A while the alternative f is her default option. We denote by $\mathcal{C}_{sq}(\mathcal{F})$ the set of all choice problems with a status quo. Finally, the notation $(A, \diamond)$, with $A \in \mathfrak{F}$ is used to represent a choice problem without a status quo.

The decision maker (henceforth, DM) is associated with a *choice correspondence*, that is, a map $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ such that

$$c(A, f) \subseteq A \text{ for all } (A, f) \in \mathcal{C}(\mathcal{F}).$$ [11]

2.2. **Representation.** We define a probabilistic dominance choice model as follows:

Definition 1. *A correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ is a* probabilistic dominance choice correspondence *if there exist an affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a $\theta \in [0,1]$ such that, for all $A \in \mathcal{F}$,*

$$c(A, \diamond) = \arg\max_{f \in A} \int_S u(f(s)) d\pi$$

[10] All results remain true, with no modifications, if we assume $\mathfrak{F}$ to be the set of all non-empty finite subsets of $\mathcal{F}$.

[11] As in some of the papers discussing choice in the presence of status quo, it is also possible to consider a system of preference relations as the primitive. This may lead to unresolved issues though. For example, if a status quo f is preferred to both g and h, then in the presence of f as a status quo, the ranking between g and h is never revealed. Thus, it seems more natural for the primitive to be a choice correspondence. Apesteguia and Ballester (2009b) study the relation between the two possible primitives.

and, for all $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A,g) = \underset{f \in \mathcal{D}(A,g,\pi,\theta)}{\arg\max} \int_S u(f(s))d\pi,$$

where, for each $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A,g,\pi,\theta) := \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$.

The interpretation of this choice procedure is the following. In the absence of a status quo, the agent acts as a standard subjective expected utility maximizer. As elaborated in the Introduction, when faced with a decision problem governed by a status quo the agent wishes to be confident that, regardless of her choice, she will obtain an outcome at least as good as the one she would have obtained had she retained the status quo, with sufficiently high probability. She first eliminates all those alternatives that do not ensure such a sufficiently high level of confidence. Following the elimination, she acts as a standard expected utility maximizer as in choice problems without a status quo.[12]

Two points should be noted regarding the threshold parameter θ. One is that θ captures the degree of confidence the DM wishes to ensure when moving away from the status quo. Fixing the beliefs and tastes, it is intuitive that the larger the threshold, the more bias towards the status quo the DM is going to exhibit (this intuition is made formal, even for the case where beliefs are not fixed, in Section 3.2). When $\theta = 0$ the DM exhibits no bias at all towards the status quo and acts as a standard expected utility maximizer. At the other end of the threshold range, a DM who is characterized by $\theta = 1$ is not willing to take any chances of loosing by moving away from the status quo and displays the extreme approach of unanimity. Second, the bias towards the status quo is not continuously shifting as θ changes. This point is closely related to the non-uniqueness of θ in the representation of c and it is discussed in detail in Section 3.1.

[12]In a different framework and in a status quo free context, sequentially rationalizable choices were also studied by Manzini and Mariotti (2007), Cherepanov, Feddersen, and Sandroni (2009) and Apesteguia and Ballester (2009a). Two-staged decision processes are also of importance in the marketing literature (see Sheridan, Richards, and Slocum (1975) and Gensch (1987) and references therein). Experimental and empirical evidence point to the fact that, when facing a choice problem, individuals tend to eliminate some alternatives according to a crude, non-compensatory initial rule, followed by a more thoughtful and compensatory process for deciding on the final choice.

2.3. **Axioms.** We impose the following properties on a choice correspondence c.

A1 *WARP.* If $(A, h), (B, h) \in \mathcal{C}(\mathcal{F})$ are such that $B \subseteq A$ and $c(A, h) \cap B \neq \emptyset$, then $c(B, h) = c(A, h) \cap B$.

The first postulate is an adaptation of the standard Weak Axiom of Revealed Preference to the environment here. It says that if we keep the status quo (or the absence of status quo) fixed, then c satisfies that postulate. This axiom implies that, for each $f \in \mathcal{F} \cup \{\diamond\}$, there exists a complete preorder $\succeq_f$ that represents the DM's behavior when the default option is f.

A2 *Independence.* For any $f \in \mathcal{F}$, $A \in \mathfrak{F}$ and $\lambda \in [0,1]$, $c(A \oplus_\lambda f, \diamond) = c(A, \diamond) \oplus_\lambda f$.

Our Independence axiom is also standard. It says that the agent's choices in problems without a status quo satisfy the well-known Independence postulate.

A3 *Continuity.* For any $f, g, h \in \mathcal{F}$, the following sets are closed: $\{\alpha : f \oplus_\alpha g \in c(\{f \oplus_\alpha g, h\}, \diamond)\}$, $\{\alpha : h \in c(\{f \oplus_\alpha g, h\}, \diamond)\}$, $\{\alpha : f \oplus_\alpha g \in c(\{f \oplus_\alpha g, h\}, h)\}$ and $\{\alpha : h \in c(\{f \oplus_\alpha g, h\}, f \oplus_\alpha g)\}$.

The first part of our continuity condition, which talks about choices in problems without a status quo, is entirely standard. The second part imposes a similar continuity condition in problems with a status quo.

A4 *Unambiguous Transitivity.* If $f(s) \in c(\{f(s), g(s)\}, \diamond)$ for all $s \in S$, then, for any $h \in \mathcal{F}$, $g \in c(\{g, h\}, h)$ implies $f \in c(\{f, h\}, h)$ and $h \in c(\{f, h\}, f)$ implies $h \in c(\{g, h\}, g)$.

As the name says, the Unambiguous Transitivity postulate imposes some transitivity in the agent's choices once it is revealed that an act dominates the other in a unanimous sense. The main idea is that we consider deviations from standard rationality that ultimately stem from the uncertainty in the world and the possibility of bias towards the status quo. But, assuming that g is chosen in the presence of h, despite h being the status quo, it is natural to assume that the bias towards h will reduce whenever presented with an act f that is unambiguously better than g, in the sense that f performs better than g in all states. So the axiom imposes that such an act f is also chosen in the presence of h when h is the status quo. Similarly, if h is such a good option that it is chosen in the presence of f even when f is the status quo, then it is natural to think that h will also be chosen when confronted with a status quo g that is unambiguously worse than f.

A5 *Status quo Irrelevance (SQI).* For any $(A, f) \in \mathcal{C}_{sq}$, if there does not exist a non-singleton $B \subseteq A$ with $(B, f) \in \mathcal{C}_{sq}$ and $\{f\} = c(B, f)$, then $c(A, f) = c(A, \diamond)$.

The postulate above was first introduced by Masatlioglu and Ok (2009). It begins with a set A such that in no subset of A the bias towards the status quo f is strong enough to make it the only choice. In other words, the bias towards f is not significant, in the sense that the DM always considers moving away from it as something acceptable. The axiom then requires that such a default option does not affect the DM's behavior. That is, she resolves her problems as if there existed no status quo. This is the essence behind **A5**.

A6 *Single-state Mixing Irrelevance (SSMI).* For any $s^* \in S$, $\lambda \in (0, 1)$, $p \in \Delta(X)$ and $(A, f) \in \mathcal{C}_{sq}$, if $c(B \oplus_\lambda^{s^*} p, \diamond) = c(B, \diamond) \oplus_\lambda^{s^*} p$ for all $B \subseteq A$, then $c(A \oplus_\lambda^{s^*} p, f \oplus_\lambda^{s^*} p) = c(A, f) \oplus_\lambda^{s^*} p$.

The SSMI axiom is the most novel property studied in this section. To the best of our knowledge the implications of this type of postulate have never been investigated in the literature. Choosing from a set of alternatives, in a given state the agent may or may not obtain a better outcome than the status quo. Mixing all the alternatives (including the status quo) in this state with some other outcome will not change the relative result in this state (or any other). Such a mixing yields a decision problem which is a very simple variation of the original one: no matter what the DM chooses, if s^* is the realized state, then with probability λ she obtains the outcome p, and with probability $1-\lambda$ her outcome is no different than in the problem when there is no such mixing.

It seems that if such a mixture does not affect the DM's choices when she objectively assesses the different alternatives (that is, in the status quo free problem), then in the presence of a status quo the insignificance of the mixture should be enhanced. Having a bias towards the status quo, insignificant changes might be perceived even more insignificant than when there is no status quo and thus will not change the choice.[13] This is the idea behind Axiom **A6**.

A7 *Binary Consistency.* Let $p, q, r \in \Delta(X)$. If f and g are acts such that $f(S) \cup g(S) \subseteq \{p, r\}$, then $\{f\} = c(\{f, g, q\}, q)$ implies that $\{f\} = c(\{f, g, q\}, \diamond)$.

To describe **A7** in detail, define the relation $\succcurlyeq \subseteq \mathcal{F} \times \mathcal{F}$ by $f' \succcurlyeq g'$ if and only if $f' \in c(\{f', g'\}, \diamond)$ and suppose, without loss of generality, that $p \succcurlyeq r$. It is possible to show that unless $p \succ q \succ r$, Binary Consistency is implied by the axiomatic structure above. However, if indeed $p \succ q \succ r$, f and g can be interpreted as acts that either return a lottery that is better than the status quo lottery q or a lottery that is worse. When we learn that f is the unique choice out of $\{f, g, q\}$ when q is the status quo, we must conclude that the event in which f returns the good lottery is more salient, under some subjective criterion, than the event in which g returns the good lottery. The axiom is then a consistency property that imposes that this saliency is sustained in the status quo free problem.

Remark 1. *The axioms SQI and SSMI are written in a stronger format than what is needed. It is easy to see that in the presence of WARP it is enough to require that those axioms be valid for sets with three or less elements. We note that if we replace A5 and A6 by the simpler versions (only for sets with less than four elements) discussed above, then, except for continuity, our axiomatic system is entirely testable.*

[13] If there exists no bias towards the status quo (meaning the status quo plays no role in the choice problem), the resulting problem is similar to the status quo free problem and no changes in choice should be observed, either.

3. Main results

3.1. The representation theorem.

We now provide the main representation result of the paper.

Theorem 1. *Given a choice correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ the following are equivalent:*
(1) c satisfies ***A1-A7****;*
(2) c is a probabilistic dominance choice correspondence.

Remark 2. *If c is not trivial,*[14] *then* ***A1-A7*** *imply that the prior π is unique and the function u is non-constant and unique up to positive linear transformations. The same is not true for θ – following the proof of Theorem 1, it is clear that θ can be chosen from some interval. For example, consider a state space with two states and a probabilistic dominance choice correspondence that is represented by a non-constant utility and prior distribution assigning probability* 0.5 *for each state. Holding the utility and prior fixed, one would obtain identical choices for every $0 \leq \theta \leq 0.5$. For each such θ, in every choice problem the corresponding procedure would simply pick those alternatives with highest expected utility.*

Next we present an auxiliary result, which is interesting on its own, partially hints to the proof of Theorem 1, and resolves the uniqueness issue of the probabilistic threshold parameter θ representing c.

Proposition 1. *Given a choice correspondence $c : \mathcal{C}(\mathcal{F}) \to \mathfrak{F}$ the following are equivalent:*
(1) c satisfies ***A1-A6****;*
(2) there exists an affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a non-empty collection of events $\mathcal{T} \subseteq 2^S$ such that, for all $A \in \mathcal{F}$,

$$c(A, \diamond) = \operatorname*{arg\,max}_{f \in A} \int_S u(f(s))d\pi$$

and, for all $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A, g) = \operatorname*{arg\,max}_{f \in \mathcal{D}(A,g,\mathcal{T})} \int_S u(f(s))d\pi,$$

[14] A choice correspondence c is trivial if, for all $c(A, g) \in \mathcal{C}(\mathcal{F})$, $c(A, g) = A$.

where, for each $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A, g, \mathcal{T}) := \{f \in A : u(f(s)) \geq u(g(s))$ *for all* $s \in T$ *for some* $T \in \mathcal{T}\}$.

Moreover, if c *is non-trivial then* π *is unique,* u *is non-constant and unique up to positive linear transformations, and there exists a unique collection of events* $\mathcal{T}$ *that is closed under containment,*[15] *satisfies that* $\pi(T) > 0$ *for all* $T \in \mathcal{T}$ *and represents* c *in the sense above.*

The interpretation of the proposition is similar to that of Theorem 1. The events in the collection $\mathcal{T}$ are called *decisive*. In the presence of a status quo, during the first stage of the decision process, an act has to perform at least as well as the status quo for all states inside at least one of the *decisive events*. Differently from Theorem 1, Proposition 1 imposes no restrictions on the collection of events $\mathcal{T}$. In particular, $\mathcal{T}$ need not be consistent with the agent's subjective belief π and there might exist some decisive event $T \in \mathcal{T}$ and an event $T' \notin \mathcal{T}$ such that $\pi(T) < \pi(T')$. Such behavior can be explained by the intuitive–deliberate choice theory described by Kahneman (2003), but this raises two remarks. First, it is hard to believe that a DM would base his choices on a low probability event but not on an event with higher probability. And second, going through all the difficulties entailed in computing the subjective prior, it seems unnatural that the DM would not revise the first stage of the decision process and reconstruct $\mathcal{T}$ so that it contained all high probability events. In short, the probabilistic dominance choice correspondence characterized in Theorem 1 is a special case of the representation above where $\mathcal{T} := \{T \subseteq S : \pi(T) \geq \theta\}$.

Remark 3. *In Proposition 1, the uniqueness properties of the collection* $\mathcal{T}$ *of decisive events which represents* c *are quite natural. First, given any* $T \in \mathcal{T}$, *if an act* f *dominates the status quo* g *over an event* T' *that contains* T, *then it dominates* g *over* T *itself, thus* f *passes to the second stage of the decision process. Second, consider a* π*-null event* T *and assume that* f *dominates the status quo* g *only over* T. *This implies that the expected utility of* f *is strictly lower than that of* g, *thus* f *will never be chosen when* g *is the status quo. This shows that zero probability events are completely inconsequential*

[15] A collection of events $\mathcal{T}$ is closed under containment if $T \in \mathcal{T}$ and $T \subseteq T'$ implies that $T' \in \mathcal{T}$.

for the choice procedure described in Proposition 1, therefore, including or not including them in the collection $\mathcal{T}$ does not affect the DM's choices.[16]

3.2. **Comparative status quo bias.** Consider two DMs, I and II, associated with choice correspondences c_1 and c_2, respectively. Assume that there exists a status quo free choice problem for which both DMs choose f. Now, assume that there exists a status quo where II's bias towards it is salient in the sense that its presence alters her choice (relative to the status quo free problem) and f is no longer chosen. If I exhibits more bias towards the status quo than II, then, in the presence of the same status quo, she will not choose f as well. This is the idea behind the following comparative notion.

Definition 2. c_1 reveals more bias towards the status quo *than* c_2 *if, for every* $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$ *such that* $f \in c_1(A, \diamond)$ *and* $f \in c_2(A, \diamond)$*,*

$$f \notin c_2(A, g) \implies f \notin c_1(A, g).$$

The following proposition reinforces the intuition behind our interpretation of probabilistic dominance choice and in particular, the formation of decisive events.

Proposition 2. *Suppose* c_1 *and* c_2 *have representations as in Proposition 1, with the same utility function* u*, priors* π_1 *and* π_2*, and collections of decisive events* $\mathcal{T}_1$ *and* $\mathcal{T}_2$*, respectively. Then, the following are equivalent:*
(1) c_1 *reveals more bias towards the status quo than* c_2*;*
(2) the tuple $(u, \pi_2, \mathcal{T}_1 \cup \mathcal{T}_2)$ *also represents* c_2 *in the sense of Proposition 1.*

What the proposition suggests is simply that, revealing less status quo bias is equivalent to imposing less restrictions on the considered alternatives by having a richer collection of decisive events (up to null events). In the context of Theorem 1, if the DMs share utilities and beliefs then Proposition 2 implies that no matter what the threshold θ_2 is, there exists a threshold $\theta_1 \geq \theta_2$ which represents c_1. This is formally presented in the next proposition.

[16]Although correct in the current formulation, Proposition 1 leads to a revised definition of $\mathcal{D}(A, g, \pi, \theta)$ in Theorem 1: $\mathcal{D}(A, g, \pi, \theta)$ could be defined by $\{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\} \cap \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} > 0\}$.

Proposition 3. *Suppose c_1 and c_2 have representations as in Theorem 1, with the same utility function u and prior π. Then, the following are equivalent:*
(1) c_1 reveals more bias towards the status quo than c_2;
(2) There exists θ_1 that represents c_1 such that $\theta_1 \geq \theta_2$ for every θ_2 representing c_2.

3.3. **The endowment effect.** People often demand more in order to give up a commodity than they would be willing to pay to acquire it (see Kahneman, Knetsch, and Thaler (1991) for a survey). This phenomenon, intrinsically related to status quo bias is the well-documented *endowment effect* (Thaler (1980)). We show that our model predicts this phenomenon in terms of unambiguous gains of utility. Also, we provide a simple way to compute, in terms of utility, the willingness to accept (WTA) and willingness to pay (WTP).

Consider a probabilistic dominance choice correspondence c that has a representation with a non-constant and affine function $u : \Delta(X) \to \mathbb{R}$, a prior π over S and a threshold parameter $\theta \in [0, 1]$. We define the function $S_c : \mathcal{F} \to u(\Delta(X))$ by[17]

$$S_c(f) := \inf\{u(p) : p \in \Delta(X) \text{ and } p \in c(\{p, f\}, f)\}$$

and the function $B_c : \mathcal{F} \to u(\Delta(X))$ by

$$B_c(f) := \sup\{u(p) : p \in \Delta(X) \text{ and } f \in c(\{p, f\}, p)\}.$$

Intuitively, $S_c(f)$ is the minimum unambiguous gain the individual would require in order to give away the act f (WTA in terms of utility) and $B_c(f)$ is the maximum unambiguous gain the individual would be willing to give away in order to have f (WTP in terms of utility).

For a constant act p to be chosen from that pair $\{f, p\}$ whenever f is the status quo, two conditions must be fulfilled. It must dominate f with probability at least θ, and its utility must be as high as the expected utility induced by f. Thus, given the representation of c, we have that

$$S_c(f) = \max\left\{\int_S u(f(s))d\pi, \min\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \leq v\} \geq \theta\}\right\}. \tag{1}$$

[17]Similar definitions, in an environment with no uncertainty and explicit potential prices, can be found in Masatlioglu and Ok (2005).

Applying similar arguments as above (for the case where the constant act p is the status quo when choosing between $\{f,p\}$), we obtain

$$(2)\qquad B_c(f) = \min\left\{\int_S u(f(s))d\pi, \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}\right\}.$$

Loosely speaking, the two terms above, $\min\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \leq v\} \geq \theta\}$ and $\max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$, are expressions for the left θ quantile and the right $1-\theta$ quantile of the distribution of $u(f)$ with respect to π. We denote them $\widetilde{Q}_{\pi,\theta}(f)$ and $\widehat{Q}_{\pi,1-\theta}(f)$ respectively. Thus, the WTA and WTP of any act can be calculated using only its expected value and these two quantiles.

When c can be represented with $\theta = 0$ and the DM is not biased to status quo, choosing as a standard expected utility maximizer, for every act f we have that $\widehat{Q}_{\pi,1}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,0}(f)$, meaning that $B_c(f) = S_c(f)$. The question is how $B_c(f)$ and $S_c(f)$ relate whenever c reveals a bias towards the status quo and cannot be represented with $\theta = 0$.

From Eq. (1) and (2) it is easy to verify that, for a particular act f, the WTA is at least as high as the WTP. Also, a necessary and sufficient condition for both to be equal is $\widehat{Q}_{\pi,1-\theta}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,\theta}(f)$. Furthermore, if c cannot be represented with $\theta = 0$, there always exists an act f for which the latter inequalities do not hold. This implies that the WTA for this specific act is strictly hight than WTP. We formalize these results in the next proposition:

Proposition 4. *Suppose that u, π and θ represent a non-trivial probabilistic dominance choice correspondence c as in Theorem 1. Then,*
(a) $S_c(f) \geq B_c(f)$ for every $f \in \mathcal{F}$.
(b) For every $f \in \mathcal{F}$, $S_c(f) = B_c(f)$ if and only if $\widehat{Q}_{\pi,1-\theta}(f) \geq \int_S u(f(s))d\pi \geq \widetilde{Q}_{\pi,\theta}(f)$.
(c) The following statements are equivalent:
  (1) There exists $f \in \mathcal{F}$ with $S_c(f) > B_c(f)$;
  (2) There exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$; and
  (3) $\theta = 0$ does not represent c.

4. Probabilistic dominance

Given a representation as in Theorem 1, define the relation $\succeq$ by

$$f \succeq g \iff \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta.$$

We say that $\succeq$ has a *probabilistic dominance* representation and refer to $\succeq$ as a probabilistic dominance relation. As mentioned in the Introduction, $\succeq$ can be interpreted as representing the fact that the probability that the agent would regret choosing f instead of g is below the threshold $1 - \theta$.

Being the first of the two steps constructing the choice procedure described in Theorem 1, we discuss in detail this family of relations in the current section.

4.1. **Framework and axioms.** Again, let S be a finite set of states of the world, X be a finite set of alternatives, $\Delta(X)$ be the set of lotteries on X and $\mathcal{F} := \Delta(X)^S$ be the space of Anscombe and Aumman acts. The primitive of the model now is a binary relation $\succeq \subseteq \mathcal{F} \times \mathcal{F}$. For any $f, g \in \mathcal{F}$ and event $T \subseteq S$ we write $f \trianglerighteq_T g$ to represent the fact that $f(s) \succeq g(s)$ for all $s \in T$.

The following is a list of basic assumptions (axioms) about $\succeq$:

B1 *Relation.* $\succeq$ is complete over $\mathcal{F}_c$, reflexive and non-trivial.[18]

B2 *Unambiguous Transitivity.* (i) $f \succeq g$ and $g \trianglerighteq_S h$ imply $f \succeq h$; and (ii) $f \succeq g$ and $h \trianglerighteq_S f$ imply $h \succeq g$.

B3 *Continuity.* For any $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succeq h\}$ and $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \preceq h\}$ are closed.

We concentrate on deviations from standard rationality that are ultimately due to uncertainty. Therefore, we assume that the restriction of $\succeq$ to constant acts satisfies all the standard postulates of rationality. Specifically, **B1** imposes that the restriction of $\succeq$ to constant acts is complete. This restriction is transitive thanks to **B2** and satisfies continuity due to **B3**. Lastly, the restriction of $\succeq$ to constant acts satisfies the standard independence axiom thanks to **B4** that will be presented below.

[18] By non-trivial we mean that $\succeq \neq \mathcal{F} \times \mathcal{F}$.

In the presence of uncertainty, we allow for the possibility that the DM is undecided and we do not require her decisions to be transitive. That is, we do not ask $\succeq$ to be complete nor transitive in general. Despite that, assume that the DM is able to conclude that $f \succeq g$. On top of that, assume that the act h is dominated by g (or dominates f) in the sense that it assigns to every state $s \in S$ a lottery that is weakly worse (better) than the lottery returned by g (f). So, in some sense, h is worse (better) than g (f) independent of any uncertainty related considerations. **B2** requires that in such a situation the DM considers $f \succeq h$ ($h \succeq g$). Note that **B2**, along with the reflexivity of $\succeq$, implies that $\succeq$ satisfies the standard monotonicity axiom. That is, for any two acts f and g, if $f \trianglerighteq_S g$, then $f \succeq g$. Also, as we have discussed above, **B2** implies that $\succeq$ is transitive over $\mathcal{F}_c$.

B3 is a standard technical continuity condition of preferences under uncertainty.

As (one of the interpretations) we have discussed in the Introduction, we have in mind a DM that considers the anticipated (ex-post) regret at the time she makes her decisions. For example, consider a CEO subordinate to a board of directors. The CEO has to take an important strategic decision for the company at the present time, but its ramifications depend on the state of nature yet to be realized. Once the state is realized, the board of directors can evaluate the CEO's decision, as they know what outcome each optional alternative would have yielded. In such a situation, it makes sense to think that the CEO's main concern will be if her decision will (ex post) prove to have been the correct one or not.

Consider any two acts f and g. Fix a state $s^* \in S$, a lottery $p \in \Delta(X)$ and $\lambda \in (0,1)$. If the DM's choices over constant acts satisfy the standard independence axiom, then $\lambda f(s^*) + (1-\lambda)p \succeq \lambda g(s^*) + (1-\lambda)p$ if and only if $f(s^*) \succeq g(s^*)$. So, mixing f and g in a single state with the same lottery does not change the states where f performs better than g or g performs better than f. Given the discussion above, we must conclude that the CEO's decision about any two acts f and g has to agree with her decision about $f \oplus_\lambda^{s^*} p$ and $g \oplus_\lambda^{s^*} p$, for any $s^* \in S$, $p \in \Delta(X)$ and $\lambda \in (0,1)$. This is exactly what the postulate below imposes.[19]

[19]Behind this discussion there is the assumption that no intensity considerations about how better or how worse a given act performs in each state are relevant for the DM's decisions. There is empirical evidence that the beliefs of obtaining a worse outcome than the alternative is the dominant aspect (out

B4 *State-wise Independence.* For any two acts f and g, lottery $p \in \Delta(X)$, state $s^* \in S$ and $\lambda \in (0,1)$, $f \succeq g$ if and only if $f \oplus_\lambda^{s^*} p \succeq g \oplus_\lambda^{s^*} p$.

We note that a simple inductive argument shows that **B4** is stronger than the standard Independence axiom, **B4',** below.[20]

B4' *Independence.* For any acts $f, g, h \in \mathcal{F}$ and $\lambda \in (0,1)$, $f \succeq g$ if and only if $\lambda f + (1-\lambda)h \succeq \lambda g + (1-\lambda)h$.

Similarly to the result in the previous section, **B1–B4** yield the following useful preliminary result:

Proposition 5. *Given a binary relation $\succeq$ over $\mathcal{F}$, the following are equivalent:*
*(1) $\succeq$ satisfies **B1–B4**;*
(2) there exist a non–constant and affine function $u : \Delta(X) \to \mathbb{R}$ and a non–empty collection of events $\mathcal{T} \subseteq \mathbf{2}^S \setminus \{\emptyset\}$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ if and only if } u(f(s)) \geq u(g(s)) \text{ for all } s \in T, \text{ for some } T \in \mathcal{T}. \tag{3}$$

Moreover, u is unique up to positive linear transformations and there exists a unique collection of events $\mathcal{T}$ that is closed under containment and represents $\succeq$ in the sense above.

Call an event $T \subseteq S$ *decisive* if $f(s) \succeq g(s)$ for all $s \in T$ implies that $f \succeq g$. Proposition 5 states, informally, that the axioms discussed above are equivalent to the existence of a collection of decisive events that completely characterizes $\succeq$.

4.2. **Complementarities.** Consider the following example:

Example 1. *Suppose $S = \{s_1, s_2, s_3, s_4\}$, $p \succ r$, and the DM's preferences satisfy axioms **B1–B4** with $\{s_1, s_3\}, \{s_2, s_4\}$ as decisive events. That is, for every two acts f and g,*

of the two mentioned above) that affects the agent's choice when anticipated regret is in play (e.g., Ritov (1996) and references within). Of course, completely disregarding the intensity of the anticipated regret is a bit extreme. We consider it as a useful simplification that delivers a tractable model.

[20] This is also clear from claim 1 in the proof of Proposition 5, in Appendix B.

$f \succeq g$ if, and only if, $\{s_1, s_3\} \subseteq \{s : f(s) \succeq g(s)\}$ or $\{s_2, s_4\} \subseteq \{s : f(s) \succeq g(s)\}$. Now, let $f := (p, r, p, r)$, $f' := (r, p, r, p)$, $g := (p, p, r, r)$ and $g' := (r, r, p, p)$. We have that $\frac{1}{2}g + \frac{1}{2}g' \trianglerighteq_S \frac{1}{2}f + \frac{1}{2}f'$, $f \succeq p$ and $f' \succeq p$, but neither $g \succeq p$ nor $g' \succeq p$ are true.

In Example 1, we see that the decision rule represented by $\succeq$ exhibits a form of perfect complementarity between the states s_1 and s_3, and the states s_2 and s_4. That is, for the DM the fact that an act f returns a better outcome than another act g in state s_1 is only relevant if it is also true that f returns a better outcome than g in state s_3. The same thing occurs with states s_2 and s_4. The example shows that such complementarities yield the following result: even though both f and f' are preferred to p, and even though both g and g' are not preferred to p, the mixture of g's unambiguously dominates the (similar) mixture of f's. The following axiom states that such complementarities between states should not exist.

B5 *No State–Complementarities.* For any $p, q, r \in \Delta(X)$ and finite sequences of acts, $f_1, ..., f_m$ and $g_1, ...g_m$, such that, for all $i = 1, ..., m$, $f_i(S) \cup g_i(S) \subseteq \{p, r\}$, if $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ for some $\lambda \in \Delta(m)$ and $f_i \succeq q$ for all i, then $g_i \succeq q$ for some i.

We can now prove the following result:

Theorem 2. *Given a binary relation $\succeq$ over $\mathcal{F}$, the following are equivalent:*
*(1) $\succeq$ satisfies **B1–B5**;*
(2) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$, a probability distribution π on S and a $\theta \in (0, 1]$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \quad \text{if and only if} \quad \pi\big(\{s \in S : \ u(f(s)) \geq u(g(s))\}\big) \geq \theta. \tag{4}$$

Of course, the representation in Theorem 2 is a particular case of the representation obtained in Proposition 5. In the representation above, the collection of decisive events is given by

$$\mathcal{T} := \{T \subseteq S : \pi(T) \geq \theta\}. \tag{5}$$

Example 1 shows that given a collection $\mathcal{T}$ of decisive events, **B5** is necessary for the existence of a probability measure and a threshold parameter that yield $\mathcal{T}$ as described in Eq. 5. Note that in Example 1 the events $\{s_1, s_3\}$ and $\{s_2, s_4\}$ are decisive. So, if $\succeq$

could be represented as in the statement of Theorem 2 we would have $\pi(\{s_1, s_3\}) \geq \theta$ and $\pi(\{s_2, s_4\}) \geq \theta$. It is not hard to see that this would imply that at least one of the events in the collection $\{\{s_1, s_2\}, \{s_1, s_4\}, \{s_3, s_2\}, \{s_3, s_4\}\}$ would also have probability greater than or equal to θ and, therefore, would also be decisive. This contradicts the assumptions within the example.

4.3. **Additional properties and particular cases.** In this section we very briefly discuss some properties and special cases of the representation introduced in Theorem 2.

Uniqueness. Following the proof of Theorem 2, it is clear that there is no unique couple (π, θ) that represents $\succeq$. Consider the following example.

Example 2. *Let $S = \{s_1, s_2, s_3\}$, $\pi(s) = \frac{1}{3}$ for every $s \in S$, and $\theta = \frac{2}{3}$. According to Theorem 2, an act f is (weakly) preferred to an act g if and only if f (weakly) dominates g in at least two out of the three states. However, this is also true for every $\theta \in (\frac{1}{3}, \frac{2}{3}]$ (given that the distribution stated above is fixed). It is possible to construct a different distribution that along with an appropriate θ would represent the same preferences. For example, define π' by $\pi'(s_1) = \frac{1}{6}$, $\pi'(s_2) = \frac{1}{3} + \varepsilon$ and $\pi'(s_3) = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is small enough. Now, given such $\varepsilon > 0$, for every $\theta' \in (\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, the couple (π', θ') represents the same relation as (π, θ).*

For a relation $\succeq$ satisfying the axioms discussed above, denote by $\mathcal{H}_{\succeq}$ the collection of elements $(\pi, \theta) \in \Delta(S) \times (0, 1]$ such that (π, θ) represents $\succeq$. A property of the collection $\mathcal{H}_{\succeq}$ is convexity. That is, whenever both (π, θ) and (π', θ') are elements of $\mathcal{H}_{\succeq}$, then $(\alpha\pi + (1 - \alpha)\pi', \alpha\theta + (1 - \alpha)\theta')$ is also an element of $\mathcal{H}_{\succeq}$, for every $\alpha \in (0, 1)$. It can be easily shown that any pair $(\pi, \theta), (\pi', \theta') \in \mathcal{H}_{\succeq}$ has to induce exactly the same collection of decisive events. This is formalized in the following result:

Proposition 6. *(π, θ) and (π', θ') represent the same relation $\succeq$ if, and only if, $\{T \subseteq S : \pi(T) \geq \theta\} = \{T \subseteq S : \pi'(T) \geq \theta'\}$.*

Completeness. The relations we have been working with so far are both incomplete and intransitive, in general. We now characterize a complete binary relation that satisfies **B1–B5**.

It is clear that whenever $\theta \leq 1/2$, the pair $(\pi, \theta) \in \Delta(S) \times (0, 1]$ induces a relation $\succeq$ that is complete. It is not hard to see that the converse is true in the sense of the proposition below.

Proposition 7. *Suppose that $\succeq$ is complete and satisfies **B1–B5**. Then, $(\pi, \theta) \in \mathcal{H}_{\succeq}$ implies that $(\pi, \min\{1/2, \theta\}) \in \mathcal{H}_{\succeq}$.*

Thus, whenever $\succeq$ is complete we can represent it with some $\theta \leq 1/2$.

Transitivity and Bewley's Knightian preferences. Above we studied the characterization of complete probabilistic dominance relations. In this subsection we focus on transitivity. We show that in this case one obtains a particular form of Bewley's Knightian preferences (see Bewley (2002)). Informally, transitivity is equivalent to the existence of an event $T \subseteq S$ such that, f is (weakly) preferred to g if and only if f (weakly) dominates g over the event T. Also, every $\pi \in \Delta(S)$ such that *support*$(\pi) = T$ and $\theta = 1$ would represent such relations, in the sense of Eq. 4. It turns out that given transitivity, **B5** is redundant. We formalize this observation in the next proposition.

Proposition 8. *Consider a binary relation $\succeq$. The following are equivalent:*
*(1) $\succeq$ satisfies transitivity, **B1-B4** ;*
(2) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$ and a probability distribution π over S such that, for every $f, g \in \mathcal{F}$

$$f \succeq g \quad \textit{if and only if} \quad \pi\big(\{s \in S : \; u(f(s)) \geq u(g(s))\}\big) = 1.$$

(3) there exist a non–constant and affine utility function $u : \Delta(X) \to \mathbb{R}$ and an event $\emptyset \neq T \subseteq S$ such that for every $f, g \in \mathcal{F}$

$$f \succeq g \quad \textit{if and only if} \quad u(f(s)) \geq u(g(s)) \textit{ for every } \; s \in T.$$

5. Additional models of probabilistic dominance

As discussed in the Introduction, probabilistic dominance can be applied to other choice models. We discuss some possible directions in this section.

5.1. **A general model of probabilistic dominance.** The representation in Theorem 2 is a particular case of a general class of representations. We say that a relation $\succeq$ has a *second–order probabilistic dominance representation* if there exist a non–constant and affine function $u : \Delta(X) \to \mathbb{R}$, a probability distribution μ over $\Delta(S)$ and a $\theta \in (0, 1]$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ iff } \mu\left(\pi \in \Delta(S) : \int u(f(s))\, d\pi \geq \int u(g(s))\, d\pi\right) \geq \theta. \tag{6}$$

The representation above reduces to the one in Theorem 2 whenever the support of the measure μ includes only degenerate priors.

A second-order probabilistic dominance representation is a particular case of Lehrer and Teper (2010)'s justifiable preferences. They introduce the notion of justifiability to axiomatic decision theory and axiomatize Knightian preferences that adhere to justifications and incorporate multiple–multiple priors. They characterize a binary relation $\succeq$ over acts, such that there exist a vN–M utility function u and a collection of closed and convex sets of probability distributions $\mathcal{P}$ over the state space, where $f \succeq g$ if and only if there exists $P \in \mathcal{P}$ such that, with respect to every $p \in P$, the expected value of $u(f)$ is at least as high as that of $u(g)$. Formally, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ if and only if } \max_{P \in \mathcal{P}} \min_{\pi \in P} \{\pi \cdot (u(f) - u(g))\} \geq 0.$$

Such preferences are characterized by **B1–B3** and **B4'**,

Given a relation as in Theorem 2 and denoting by $\mathcal{T}$ the collection of decisive events, the collection $\mathcal{P}$ of sets of probability distributions over S given by $\{conv\{\mathbb{1}_s\}_{s \in T} : \; T \in \mathcal{T}\}$, is a justifiable preferences representation of the same relation. In particular, the characterization of justifiable preferences is the second-order probabilities version of Proposition 5.

Second-order probabilistic dominance can be incorporated into a complete decision process to obtain a more general result than Theorem 1. For example, a DM can be associated with a utility function u, a prior $\pi \in \Delta(S)$, a prior μ over $\Delta(S)$ and a $\theta \in [0, 1]$ such that, for all $A \in \mathcal{F}$,

$$c(A, \diamond) = \arg\max_{f \in A} \int_S u(f(s)) d\pi$$

and, for all $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$,

$$c(A,g) = \underset{f \in \mathcal{D}(A,g,\mu,\theta)}{\arg\max} \int_S u(f(s))d\pi$$

where, for each $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$, $\mathcal{D}(A,g,\mu,\theta) := \{f \in A : \mu\{\pi' : \int_S u(f(s))d\pi' \geq \int_S u(g(s))d\pi'\} \geq \theta\}$.

The interpretation is similar to that of Theorem 1. In the first stage of the decision process the DM employs second–order probabilistic dominance considerations and, out of the alternatives that are not eliminated, in the second stage she chooses an act that maximizes expected utility.[21]

We were not able (neither for the binary relation model nor the choice model) to find a condition that guarantees that the sets of multiple-multiple priors $\mathcal{P}$ in the justifiable preferences representation could be generated by a probability measure μ over $\Delta(S)$ and a threshold parameter $\theta \in (0,1]$, as in Eq. 6. This is left for future study.

5.2. **Unfeasible status quo.** In some situations, the status quo is not a feasible alternative. The probabilistic dominance approach could still be applied for such problems. Consider the following choice procedure:

$$c(A,\diamond) = \underset{f \in A}{\arg\max} \int_S u(f(s))d\pi,$$

and for all (A,g) where g need not be an element of A,

$$c(A,g) = \underset{f \in \mathcal{D}(A,g,\pi,\theta)}{\arg\max} \int_S u(f(s))d\pi,$$

where $\mathcal{D}(A,g,\pi,\theta) = \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$ if $\{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$ is not empty, and $\mathcal{D}(A,g,\pi,\theta) = A$ otherwise.

Similar to the representation in Theorem 1, the DM considers only alternatives that dominate the status quo with high enough probability. Since the status quo need not be feasible, there may not be such an alternative. In this case the DM chooses according to expected utility maximization.

[21]The second stage of the decision process can be any general preferences. For example, smooth ambiguity preferences (Klibanoff, Marinacci, and Mukerji (2005)) with respect to μ and some increasing real function φ.

5.3. **Endogenous references.** Probabilistic dominance considerations can also be applied for choice problems where the reference is not observable, but can rather be extracted from behavior. In such a framework, choice problems would be described only by a collection of feasible alternatives A, and the choice c can be described by the existence of an endogenous reference g such that

$$c(A) = \underset{f \in \mathcal{D}(A,g,\pi,\theta)}{\arg\max} \int_S u(f(s)) d\pi,$$

where $\mathcal{D}(A, g, \pi, \theta) = \{f \in A : \pi\{s : u(f(s)) \geq u(g(s))\} \geq \theta\}$.

The interpretation is similar to the one in the previous subsection, with the difference that the reference is extracted endogenously from the DM's behavior.

Appendix A. Results in a Savagean framework

Suppose now that X is a separable metric space, S is a finite non–empty set of states of nature and call a function $f : S \to X$ an act. As we did in the main text, we use the symbol $\mathcal{F}$ to represent the set of all acts. The set of all constant acts is referred to as $\mathcal{F}_c$. The primitive of the model is a relation $\succeq$ on $\mathcal{F}$. We consider the following properties of $\succeq$:

C1 *Relation.* $\succeq$ is complete, transitive and continuous over $\mathcal{F}_c$, reflexive and non–trivial, in general.[22]

C2 *Negative Neutrality.* For any acts f and g in $\mathcal{F}$ and state $s^* \in S$, if $f \succeq g$, but $g(s^*) \succ f(s^*)$, then $x\{s^*\}f \succeq y\{s^*\}g$, for any $x, y \in X$.

C3 *Positive Neutrality.* For any acts f and g in $\mathcal{F}$ and consequences x and y in X, if $f \succeq g$ and $x \succeq y$, then $x\{s\}f \succeq y\{s\}g$ for all $s \in S$.

C4 *No State Complementarities.* For any three consequences $x, y, z \in X$ and finite sequences of acts, $f_1, ..., f_m$ and $g_1, ...g_m$ such that, for all $i = 1, ..., m$, $f_i(S) \cup g_i(S) \subseteq \{x, z\}$, if $f_i \succeq y$ for all i and, for all $s \in S$, $\#\{i : f_i(s) = x\} = \#\{i : g_i(s) = x\}$, then $g_i \succeq y$ for

[22] When we say that $\succeq$ is continous over $\mathcal{F}_c$ we mean that, for any two convergent sequences in X, $x^m \to x$ and $y^m \to y$, if $x^m \succeq y^m$ for all m, then $x \succeq y$.

some i.

In the axiomatization in the Anscombe and Aumann framework, the idea that the DM was concerned only about which act would give the best alternative, state by state, was incorporated into the State-wise Independence axiom. Such a property obviously cannot be written in the current framework. Here we replace **B4** by **C2** and **C3**.

We are interested on a DM that, state by state, only cares about the ordinal comparison between the consequences returned in these states. Condition **C2** begins with two acts f and g such that $g(s^*) \succ f(s^*)$. So, in s^* the situation is the least favorable for the act f. Despite that, **C2** assumes that $f \succeq g$. Now, if we replace $f(s^*)$ and $g(s^*)$ by any other two consequences this can only help f and, therefore, the DM should not reverse her decision.

Axiom **C3** follows a similar motivation as **C2**. Now we start with two consequences x and y such that $x \succeq y$ and two acts f and g such that $f \succeq g$. The axiom then says that if, in any state s, we replace the consequences $f(s)$ and $g(s)$ by x and y, respectively, this should not reverse the DM's decision. The idea is that since $x \succeq y$ is already the most favorable situation for f, such a change cannot make the DM change her mind.

Axiom **C4** has a similar interpretation as the correspondent property in the axiomatization in the Anscombe and Aumann framework. It rules out complementarities among the states and is also incompatible with Example 1. This postulate is reminiscent of the social choice literature. More specifically, it is very similar to the Strong Neutrality axiom (see Fishburn (1970)).

We can now state the main result of this section:

Theorem 3. *Given a binary relation $\succeq \subseteq \mathcal{F} \times \mathcal{F}$, the following are equivalent:*
*(1) $\succeq$ satisfies **C1–C4**;*
(2) there exist a non-constant, continuous function $u : X \to \mathbb{R}$, a probability distribution π on S and a threshold parameter $\theta \in (0, 1]$ such that, for every $f, g \in \mathcal{F}$,

$$f \succeq g \text{ if and only if } \pi\left(\{s \in S : u(f(s)) \succeq u(g(s))\}\right) \geq \theta.$$

The representation above is basically the same as in Theorem 2, but in a Savagean world.

Appendix B. Proofs

B.1. **Proof of Proposition 1.** It is routine to show that the representation implies the axioms, so we only show that the axioms are sufficient for the representation. Define the relation $\succcurlyeq \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq g$ iff $f \in c(\{f,g\},\diamond)$. By WARP, $\succcurlyeq$ is a complete preorder and, for any $A \in \mathfrak{F}$, $c(A,\diamond) = \arg\max(A,\succcurlyeq)$.[23] By Independence, $\succcurlyeq$ satisfies the standard Independence axiom. Continuity implies that $\succcurlyeq$ is continuous.[24] Now suppose that $f(s) \succcurlyeq g(s)$ for all $s \in S$. Unambiguous Transitivity implies that $f \in c(\{f,g\},g)$. By SQI, $f \in c(\{f,g\},\diamond)$, which is equivalent to saying that $f \succcurlyeq g$. That is, $\succcurlyeq$ satisfies Monotonicity. We have just shown that $\succcurlyeq$ satisfies all conditions for an Anscombe and Aumann representation. So, there exists an affine function $u : \Delta(X) \to \mathbb{R}$ and a prior π over S such that, for any $A \in \mathfrak{F}$,

$$c(A,\diamond) = \underset{f\in A}{\arg\max} \int_S u(f(s))d\pi. \tag{7}$$

Now define the relation $\succcurlyeq^* \subseteq \mathcal{F} \times \mathcal{F}$ by $f \succcurlyeq^* g$ iff $f \in c(\{f,g\},g)$. We note that $\succcurlyeq^*$ is a reflexive binary relation. For each $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$, define $\mathcal{D}((A,g),\succcurlyeq^*) := \{f \in A : f \succcurlyeq^* g\}$. We need the following claim:

Claim 1. *For every $(A,g) \in \mathcal{C}_{sq}(\mathcal{F})$,*

$$c(A,g) = \underset{f\in\mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} \int_S u(f(s))d\pi.$$

Proof of Claim. By WARP, $f \in c(A,g)$ implies that $f \in c(\{f,g\},g)$. So, $c(A,g) \subseteq \mathcal{D}((A,g),\succcurlyeq^*)$. Now pick any $(f,h) \in c(A,g) \times \mathcal{D}((A,g),\succcurlyeq^*)$. By WARP, $f \in c(\{f,g,h\},g)$, and, by SQI, $c(\{f,g,h\},g) = c(\{f,g,h\},\diamond)$. Now (7) implies that $\int_S u(f(s))d\pi \geq \int_S u(h(s))d\pi$. We conclude that $c(A,g) \subseteq \underset{f\in\mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} \int_S u(f(s))d\pi$. In particular, this shows that $\emptyset \neq \underset{f\in\mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} \int_S u(f(s))d\pi$. Now pick $h \in \underset{f\in\mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} \int_S u(f(s))d\pi$ and $f \in c(A,g)$. By SQI we know that $c(\{f,g,h\},g) = c(\{f,g,h\},\diamond)$, which, by (7), implies that $h \in c(\{f,g,h\},g)$. But, by WARP, $c(\{f,g,h\},g) = c(A,g) \cap \{f,g,h\}$. We conclude that $\underset{f\in\mathcal{D}((A,g),\succcurlyeq^*)}{\arg\max} \int_S u(f(s))d\pi \subseteq c(A)$. $\|$

[23] **Notation:** By $\arg\max(A,\succcurlyeq)$ we mean the set $\{f \in A : f \succcurlyeq g \text{ for all } g \in A\}$.

[24] To be precise, $\succcurlyeq$ satisfies the following property: for every $f,g,h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succcurlyeq h\}$ and $\{\alpha \in [0,1] : h \succcurlyeq \alpha f + (1-\alpha)g\}$ are closed. This property is sometimes called Archimedean Continuity.

Fix any $\lambda : S \to [0,1]$. For any acts f and g define $f \oplus_{\lambda(.)} g$ to be the act such that $\left(f \oplus_{\lambda(.)} g\right)(s) = \lambda(s)f(s) + (1-\lambda(s))g(s)$ for each $s \in S$. We can prove the following claim:

Claim 2. *Suppose $f \succ g$ and $f \oplus_{\lambda(.)} h \succ g \oplus_{\lambda(.)} h$ for some λ such that $\lambda(s) \in (0,1]$ for all $s \in S$. Then, $f \succcurlyeq^* g$ if and only if $f \oplus_{\lambda(.)} h \succcurlyeq^* g \oplus_{\lambda(.)} h$.*

Proof of Claim. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Order the states in S in a way that the states not in T come first. That is, write S as $S := \{s_1, s_2, ..., s_{|S|}\}$ with $s_i \notin T$ for $i = 1, ..., (|S| - |T|)$. For each $i = 1, ..., |S|$, let $\lambda^i : S \to (0,1]$ be defined by $\lambda^i(s) := \lambda(s)$ if $s \leq i$ and $\lambda^i(s) := 1$ for $s > i$. The representation of $c(\cdot, \diamond)$ implies that $\{f \oplus_{\lambda^i(.)} h\} = c(\{f \oplus_{\lambda^i(.)} h, g \oplus_{\lambda^i(.)} h\}, \diamond)$ for all i. But then, Statewise Independence implies that $f \succcurlyeq^* g$ iff $f \oplus_{\lambda^1(.)} h \succcurlyeq^* g \oplus_{\lambda^1(.)} h$ iff ... iff $f \oplus_{\lambda(.)} h \succcurlyeq^* g \oplus_{\lambda(.)} h$. ‖

Without loss of generality, we may assume that $u(\Delta(X)) = [0,1]$. Let $\overline{p}$ and $\underline{p}$ be such that $u(\overline{p}) = 1$ and $u(\underline{p}) = 0$. Call an event T a candidate for decisiveness if,

$$\overline{p}T\underline{p} \succcurlyeq^* \lambda\overline{p} + (1-\lambda)\underline{p} \text{ for some } \lambda \in (0,1). \tag{8}$$

Let $\mathcal{T}$ be the collection of all such events. We can now prove the following claim:

Claim 3. *If $f \succcurlyeq g$, then $f \succcurlyeq^* g$ if and only if $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Suppose first that f and g are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$ and $f \succ g$. Since $f \succ g$, it must be the case that $\pi(T) > 0$. By the representation of $\succcurlyeq$, this implies that if $\lambda^* \in (0,1)$ is small enough then $\overline{f} := \overline{p}T\underline{p} \succ \lambda^*\overline{p} + (1-\lambda^*)\underline{p} =: \underline{g}$. Let $h \in \mathcal{F}$ be any act such that $\lambda^* < u(h(s)) < 1$ for all $s \in T$ and $0 < u(h(s)) < \lambda^*$ for all $s \in S \setminus T$. Now define $f^\alpha := \alpha f + (1-\alpha)h$ and $g^\alpha := \alpha g + (1-\alpha)h$ for some $\alpha \in (0,1)$ small enough so that, for every $s \in T$, $\lambda^* < u(g^\alpha(s)) < u(f^\alpha(s)) < 1$ and, for every $s \in S \setminus T$, $0 < u(f^\alpha(s)) < u(g^\alpha(s)) < \lambda^*$. By the representation of $\succcurlyeq$, it is clear that $f^\alpha \succ g^\alpha$, so, by the previous claim, $f \succcurlyeq^* g \iff f^\alpha \succcurlyeq^* g^\alpha$. Finally, let $\lambda : S \to (0,1)$ and $j \in \mathcal{F}$ be such that, for every $s \in T$, $u(f^\alpha(s)) = \lambda(s) + (1-\lambda(s))u(j(s))$ and $u(g^\alpha(s)) = \lambda(s)\lambda^* + (1-\lambda(s))u(j(s))$ and, for every $s \in S \setminus T$, $u(f^\alpha(s)) = (1-\lambda(s))u(j(s))$ and

$u(g^\alpha(s)) = \lambda(s)\lambda^* + (1-\lambda(s))u(j(s))$.[25] Two applications of unambiguous transitivity give us that $f^\alpha \succcurlyeq^* g^\alpha \iff (\overline{p}T\underline{p}) \oplus_{\lambda(.)} j \succcurlyeq^* (\lambda^*\overline{p} + (1-\lambda^*)\underline{p}) \oplus_{\lambda(.)} j$, but, by the previous claim, $(\overline{p}T\underline{p}) \oplus_{\lambda(.)} j \succcurlyeq^* (\lambda^*\overline{p} + (1-\lambda^*)\underline{p}) \oplus_{\lambda(.)} j \iff \overline{p}T\underline{p} \succcurlyeq^* \lambda^*\overline{p} + (1-\lambda^*)\underline{p}$. Suppose now that it is not true that $u(f(s)) \neq u(g(s))$ for all $s \in S$ and $f \succ g$. Define, for each $\gamma \in (0,1)$, $f^\gamma := \gamma\overline{p} + (1-\gamma)f$ and $g^\gamma := \gamma\underline{p} + (1-\gamma)g$. Let $\gamma^* \in (0,1)$ be small enough so that $\{s \in S : u(f^{\gamma^*}(s)) \geq u(g^{\gamma^*}(s))\} = T$. It is clear that $f^\gamma \succ g^{\hat{\gamma}}$, $u(f^\gamma(s)) \neq u(g^{\hat{\gamma}}(s))$ and $\{s \in S : u(f^\gamma(s)) \geq u(g^{\hat{\gamma}}(s))\} = T$ for all $\hat{\gamma} \in (0, \gamma^*]$ and $\gamma \in (0, \hat{\gamma}]$. If $f \succcurlyeq^* g$, two applications of unambiguous transitivity give us that $f^{\gamma^*} \succcurlyeq g^{\gamma^*}$ and, by what we have just proved, $T \in \mathcal{T}$. Conversely, if $T \in \mathcal{T}$, again by what we have just proved, $f^\gamma \succcurlyeq^* g^{\hat{\gamma}}$ for all $\hat{\gamma} \in (0, \gamma^*]$ and $\gamma \in (0, \hat{\gamma}]$. But then continuity implies that, for all $\hat{\gamma} \in (0, \gamma^*]$, we have $f \succcurlyeq^* g^{\hat{\gamma}}$. But now another application of continuity gives that $f \succcurlyeq^* g$. $\|$

Combining Claims 1 and 3 above we obtain the desired representation.

Now, assuming non-triviality, if there exists $A \in \mathfrak{F}$ such that $c(A, \diamond) \subset A$, then the uniqueness of a non-constant u and π representing c is immediate. If there exists $(A, f) \in \mathcal{C}_{sq}(\mathcal{F})$ such that $c(A, f) \subset A$, then u is non-constant which in turn implies its uniqueness and π's uniqueness. As for $\mathcal{T}$, Eq. 8 implies that it is closed under containment and does not contain π-null events. Moreover, Eq. 8 implies that non-null events cannot be omitted or included to $\mathcal{T}$, thus it is the unique collection, which is closed under containment and contains no π-null events, that represents c. $\square$

B.2. **Proof of Theorem 1.** Again, the arguments that show that the representation implies the axioms are routine, so we only show that the axioms are sufficient for the representation. By Proposition 1, we know that c can be represented in that fashion for some affine $u : \Delta(X) \to \mathbb{R}$, some prior π over S and some class of events $\mathcal{T}$. It is easy to see that we can assume, without loss of generality, that $\mathcal{T}$ is such that $\mathcal{T} \ni T \subseteq \hat{T} \implies \hat{T} \in \mathcal{T}$ and $\pi(T) > 0$ for all $T \in \mathcal{T}$. The result will be proved if we can show that there exists a $\theta \in [0,1]$ such that, for any $T \subseteq S$, $T \in \mathcal{T}$ if and only if $\pi(T) \geq \theta$. Fix some event $T \in \mathcal{T}$ and some event $\hat{T} \notin \mathcal{T}$. Now pick $p, q, r \in \Delta(X)$ such that $u(r) < u(q) < u(p)$ and $\pi(T)u(p) + (1-\pi(T))u(r) > u(q)$. Let $f := pTr$ and

[25] To be precise, choose j and λ so that, for $s \in T$, $\lambda(s) := \frac{u(f^\alpha(s)) - u(g^\alpha(s))}{1-\lambda^*}$ and $u(j(s)) = \frac{(1-\lambda^*)u(f^\alpha(s)) - (u(f^\alpha(s)) - u(g^\alpha(s)))}{(1-\lambda^*) - (u(f^\alpha(s)) - u(g^\alpha(s)))}$, and, for $s \in S \setminus T$, $\lambda(s) := \frac{u(g^\alpha(s)) - u(f^\alpha(s))}{\lambda^*}$ and $u(j(s)) = \frac{\lambda^* u(f^\alpha(s))}{\lambda^* - (u(g^\alpha(s)) - u(f^\alpha(s)))}$.

$g := p\hat{T}r$. By the representation in Proposition 1, we must have $\{f\} = c(\{f, g, q\}, q)$. But, by Binary Consistency, this implies that $\{f\} = c(\{f, g, q\}, \diamond)$, which, again by the representation in Proposition 1, implies that $\pi(T) > \pi(\hat{T})$. Notice that T and $\hat{T}$ were entirely generic in the analysis above, so if we define $\theta := \min\{\pi(T) : T \in \mathcal{T}\}$ we have the desired characterization of $\mathcal{T}$. $\square$

B.3. **Proof of Proposition 2.** (2) $\implies$ (1). Suppose $(A, g) \in \mathcal{C}_{sq}(\mathcal{F})$ and $f \in A$ are such that $f \in c_1(A, g)$ and $f \in c_2(A, \diamond)$. Since $f \in c_1(A, g)$, we know that $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}_1 \subseteq \mathcal{T}_2$. From $f \in c_2(A, \diamond)$ we know that $f \in \arg\max_{f \in A} \int_S u(f(s))d\pi_2$. But then it is clear from the representation of c_2 that $f \in c_2(A, g)$.

(1) $\implies$ (2). Define the collection of events $\mathcal{T}_2^o$ by $\mathcal{T}_2^o := \mathcal{T}_2 \cup \{T \in \mathcal{T}_1 : \pi_2(T) = 0\}$. Notice that $\int_S u(g(s))d\pi_2 > \int_S u(f(s))d\pi_2$.for any two acts such that $\pi_2(\{s \in S : u(f(s)) \geq u(g(s))\}) = 0$. This implies that $(u, \pi_2, \mathcal{T}_2^o)$ is also a representation of c_2. Now fix any $T \in \mathcal{T}_1$ with $\pi_2(T) > 0$. It is possible to find two acts f and g such that $\int_S u(f(s))d\pi_1 \geq \int_S u(g(s))d\pi_1$, $\int_S u(f(s))d\pi_2 \geq \int_S u(g(s))d\pi_2$ and $\{s \in S : u(f(s)) \geq u(g(s))\} = T$. By the representations of c_1 and c_2, this implies that $f \in c_1(\{f, g\}, g)$ and $f \in c_2(\{f, g\}, \diamond)$. But then (1) implies that $f \in c_2(\{f, g\}, g)$ which can happen only if $T \in \mathcal{T}_2$. But then $\mathcal{T}_1 \cup \mathcal{T}_2 = \mathcal{T}_2^o$, which completes the proof of the proposition. $\square$

B.4. **Proof of Proposition 4.** (a) and (b) are straight forward and the proof is omitted. To see that (c) holds, if u, π and $\theta = 0$ represent c, then c simply maximizes expected utility and $S_c(f) = B_c(f)$ for all $f \in \mathcal{F}$. This shows that (1) implies (3). Also, it is clear that if, for all $s \in S$, $\pi(s) > 0$ implies $\pi(s) \geq \theta$, then u, π and $\theta = 0$ also represent c. That is, (3) implies to (2). So we only have to show that (2) implies (1). Suppose that there exists $s^* \in S$ such that $\theta > \pi(s^*) > 0$. Pick $p, q \in \Delta(X)$ such that $u(p) > u(q)$. Consider the act f such that $f(s^*) = p$ and $f(s) = q$ for all $s \neq s^*$. Notice that $\int_S u(f(s))d\pi > u(q) = \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$. Since $S_c(f) \geq \int_S u(f(s))d\pi$ and $B_c(f) \leq \max\{v \in u(\Delta(X)) : \pi\{s : u(f(s)) \geq v\} \geq \theta\}$, we conclude that $S_c(f) > B_c(f)$. $\square$

B.5. **Proof of Proposition 5.** The arguments that show that the representation implies the axioms are routine, so we only show the sufficiency part of the proof. Similar to what we did in the proof of Proposition 1, for any $\lambda : S \to [0, 1]$ and acts f and g, define $f \oplus_{\lambda(.)} g$

to be the act such that $\left(f \oplus_{\lambda(.)} g\right)(s) = \lambda(s)f(s) + (1-\lambda(s))g(s)$ for each $s \in S$. We need the following claim:

Claim 1. *For any acts f, g and h, and $\lambda : S \to (0,1]$, $f \succeq g$ if and only if $f \oplus_{\lambda(.)} h \succeq g \oplus_{\lambda(.)} h$.*

Proof of Claim. Order the states in S in any way. For each $i = 1, ..., |S|$, let $\lambda^i : S \to (0,1]$ be defined by $\lambda^i(s) := \lambda(s)$ if $s \leq i$ and $\lambda^i(s) := 1$ for $s > i$. Now notice that Statewise Independence implies that $f \succeq g$ iff $f \oplus_{\lambda^1(.)} h \succeq g \oplus_{\lambda^1(.)} h$ iff ... iff $f \oplus_{\lambda(.)} h \succeq g \oplus_{\lambda(.)} h$. $\|$

In particular, the claim above implies that the restriction of $\succeq$ to constant acts satisfies the standard Independence axiom. By **B1**, **B2** and **B3**, it is also complete, transitive, continuous and non-trivial. By the expected utility theorem, we know that there exists a non-constant and affine function $u : \Delta(X) \to \mathbb{R}$ such that, for any $p, q \in \Delta(X)$, $p \succeq q$ if and only if $u(p) \geq u(q)$. Without loss of generality we may assume that $u(\Delta(X)) = [0,1]$. Let $\overline{p}$ and $\underline{p}$ be two lotteries such that $u(\overline{p}) = 1$ and $u(\underline{p}) = 0$. Call an event T a candidate for decisiveness if $\overline{p}T\underline{p} \succeq \underline{p}T\overline{p}$. Let $\mathcal{T}$ be the class of all such events. The next two claims show that, for any two acts f and g, $f \succeq g$ if and only if $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.

Claim 2. *If $f, g \in \mathcal{F}$ are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$, then $f \succeq g$ if, and only if, $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Suppose the acts f and g are such that $u(f(s)) \neq u(g(s))$ for all $s \in S$. Let $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. Define $\lambda : S \to (0,1]$ by $\lambda(s) := u(f(s)) - u(g(s))$ if $s \in T$ and $\lambda(s) := u(g(s)) - u(f(s))$ if $s \notin T$. Let h be any act such that, for each $s \in T$, $u(h(s)) = \frac{u(g(s))}{1-(u(f(s))-u(g(s)))}$, and, for each $s \notin T$, $u(h(s)) = \frac{u(f(s))}{1-(u(g(s))-u(f(s)))}$. Define $f^\lambda := (\overline{p}T\underline{p}) \oplus_{\lambda(.)} h$ and $g^\lambda := (\underline{p}T\overline{p}) \oplus_{\lambda(.)} h$. By construction, $u(f(s)) = u(f^\lambda(s))$ and $u(g(s)) = u(g^\lambda(s))$ for all $s \in S$. Two applications of Unambiguous Transitivity imply that $f \succeq g \iff f^\lambda \succeq g^\lambda$. But, by the previous claim, $f^\lambda \succeq g^\lambda \iff T \in \mathcal{T}$. $\|$

Claim 3. *For any two acts f and g, $f \succeq g$ if, and only if, $\{s \in S : u(f(s)) \geq u(g(s))\} \in \mathcal{T}$.*

Proof of Claim. Fix any two acts f and g and define $T := \{s \in S : u(f(s)) \geq u(g(s))\}$. By Claim 1, $f \succeq g \iff \frac{1}{2}f + \frac{1}{2}\underline{p} \succeq \frac{1}{2}g + \frac{1}{2}\underline{p}$, so, we can assume, without loss of generality, that $u(f(s)) < 1$ for all $s \in S$. For any $\alpha \in (0,1)$, define $f^\alpha := \alpha f + (1-\alpha)\overline{p}$. It is clear that, for α close enough to one, $\{s \in S : u(f^\alpha(s)) \geq u(g(s))\} = T$. Moreover, for such α it is clear that $u(f^\alpha(s)) \neq u(g(s))$ for all $s \in S$. By the previous claim, we learn that, for all α close enough to one, $f^\alpha \succeq g \iff T \in \mathcal{T}$. If $f \succeq g$, then Unambiguous Transitivity implies that $f^\alpha \succeq g$ for all $\alpha \in (0,1)$ and, consequently, $T \in \mathcal{T}$. Conversely, if $T \in \mathcal{T}$, then, for all α close enough to one we have $f^\alpha \succeq g$. But then Continuity implies that $f \succeq g$. ‖

The claim above completes the proof of the proposition. □

B.6. **Proof of Theorem 2.** Since $\succeq$ satisfies **B1–B4**, it has a representation as in Proposition 5 for some collection of events $\mathcal{T}$. Suppose $T_1, ..., T_m$, and $\hat{T}_1, ..., \hat{T}_m$ are two finite sequences of events such that $T_i \in \mathcal{T}$ and $\hat{T}_i \notin \mathcal{T}$ for all i. Pick any two lotteries$p, q \in \Delta(X)$ such that $u(p) > u(q)$. The representation in Proposition 5 implies that, for all i, $pT_iq \succeq p \succeq p\hat{T}_iq$. But then **B5** implies that for no $\lambda \in \Delta(m)$ it can be true that $\sum_{i=1}^m \lambda_i(p\hat{T}_iq) \trianglerighteq_S \sum_{i=1}^m \lambda_i(pT_iq)$. By the representation in Proposition 5, this is equivalent to saying that for no $\lambda \in \Delta(m)$ can it be true that $\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}$. That is, the collection $\mathcal{T}$ satisfies the following property:

Property $*$ For any sequences of events $\{T_1, ..., T_m\} \subseteq \mathcal{T}$, $\{\hat{T}_1, ..., \hat{T}_m\} \subseteq 2^S \setminus \mathcal{T}$ and $\lambda \in \Delta(m)$, it cannot be true that $\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}$.

Now let $\mathcal{E} := conv\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T} \text{ and } \hat{T} \notin \mathcal{T}\}$.[26] $\mathcal{E}$ is a closed and convex subset of $\mathbb{R}^S$, and, by Property $*$, it is disjoint from $\mathbb{R}^S_-$. Therefore, by the separating hyperplane theorem, there exists a non-null vector $\pi \in \mathbb{R}^S$ such that $\pi \cdot x > 0$ and $\pi \cdot y \leq 0$, for every $x \in \mathcal{E}$ and $y \in \mathbb{R}^S_-$. [27] The vector π must be non–negative, since if there exists $s^* \in S$ for which $\pi(s^*) < 0$, then $\pi \cdot (-\mathbb{1}_{\{s^*\}}) > 0$, in contradiction to the separation by π. Without lost of generality, π can be normalized to be a probability distribution on S. Now note that $\pi \cdot x > 0$ for every $x \in \mathcal{E}$. In particular, $\pi \cdot (\mathbb{1}_T - \mathbb{1}_{\hat{T}}) > 0$ for every $T \in \mathcal{T}$ and $\hat{T} \notin \mathcal{T}$. That is, $\pi(T) > \pi(\hat{T})$ for every $T \in \mathcal{T}$ and $\hat{T} \notin \mathcal{T}$. Thus,

[26]By $conv\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T} \text{ and } \hat{T} \notin \mathcal{T}\}$ we mean the convex hull of the set $\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T} \text{ and } \hat{T} \notin \mathcal{T}\}$.

[27]For $p, x \in \mathbb{R}^S$ we denote the inner product of p and x by $p \cdot x = \sum_{s \in S} x_s \cdot p_s$.

letting $\theta := \min_{T\in\mathcal{T}} \pi(T)$, we have that $\pi(T) \geq \theta$ if, and only if, $T \in \mathcal{T}$. Finally, as θ is a probability of some event in $\mathcal{T}$, we must have $\theta \in [0,1]$. Since $\emptyset \notin \mathcal{T}$, it must be the case that $\theta > 0$.

Conversely, suppose that $\succeq$ has a representation as in the statement of the theorem. By Proposition 5, we know that $\succeq$ satisfies **B1-B4**. To see that it also satisfies **B5**, pick any three lotteries $p, q, r \in \Delta(X)$ and finite sequences of acts $f_1, ..., f_m$ and $g_1, ..., g_m$ such that, for all $i = 1, ..., m,\ f_i(S) \cup g_i(S) \subseteq \{p, r\}$. Suppose that $\lambda \in \Delta(m)$ is such that $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ and $f_i \succeq q$ for all $i = 1, ..., m$. Without loss of generality suppose that $u(p) \geq u(r)$. If $u(r) \geq u(q)$, then the representation implies that $g_i \succeq q$ for all i. If $u(q) > u(p)$, then the representation would imply that it cannot be true that $f_i \succeq q$ for any i. So, the only interesting case remaining is when $u(p) \geq u(q) > u(r)$. For each $i = 1, ..., m$ let $T_i := \{s \in S : u(f_i(s)) = p\}$ and $\hat{T}_i := \{s \in S : u(g_i(s)) = p\}$. By the representation of $\succeq$, it must be the case that $\pi(T_i) \geq \theta$ for $i = 1, ..., m$. Now note that saying that $\sum_{i=1}^m \lambda_i g_i \trianglerighteq_S \sum_{i=1}^m \lambda_i f_i$ is equivalent to say that $\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i} \geq \sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}$. But this implies that $\sum_{i=1}^m \lambda_i \pi(\hat{T}_i) = \pi \cdot (\sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i}) \geq \pi \cdot (\sum_{i=1}^m \lambda_i \mathbb{1}_{T_i}) = \sum_{i=1}^m \lambda_i \pi(T_i)$. This can be true only if $\pi(\hat{T}_i) \geq \pi(T_i) \geq \theta$, for some $i \in \{1, ..., m\}$, but for such i the representation of $\succeq$ implies that $g_i \succeq q$. $\square$

B.7. **Proof of Proposition 7.** Suppose $\succeq$ is complete and $(\pi, \theta) \in \mathcal{H}_{\succeq}$ is such that $\theta > 1/2$. Suppose that there exists $T \subseteq S$ with $\frac{1}{2} \leq \pi(T) < \theta$ and fix $p, q \in \Delta(X)$ with $u(p) > u(q)$. But observe that the representation of $\succeq$ would imply that pTq and qTp are not comparable. We conclude that $\pi(T) < \frac{1}{2}$ for all $T \subseteq S$ with $\pi(T) < \theta$. But now it is clear that $(\pi, 1/2) \in \mathcal{H}_{\succeq}$. $\square$

B.8. **Proof of Proposition 8.** It is obvious that (2) and (3) are equivalent and, by Proposition 5, it is clear that they imply (1). So, we only need to show that (1) implies (3). By Proposition 5, $\succeq$ has a representation as in Eq. 3. Now take any minimal set $T \in \mathcal{T}$.[28] We now show that $T \subseteq \hat{T}$ for any $\hat{T} \in \mathcal{T}$. To see that, suppose that there exists $\hat{T} \in \mathcal{T}$ such that $T \setminus \hat{T} \neq \emptyset$ and pick two lotteries p and q such that $p \succ q$. Now note that $q \succeq qTp$, $qTp \succeq q(T \cap \hat{T})p$, but, since T is minimal, it cannot be true that $q \succeq q(T \cap \hat{T})p$. We conclude that $T \subseteq \hat{T}$ for any $\hat{T} \in \mathcal{T}$. $\square$

[28]By minimal we mean that there is no $T' \in \mathcal{T}$ such that $T' \subsetneq T$.

B.9. **Proof of Theorem 3.** The proof that (2) implies **C1**, **C2** and **C3** is straightforward. We now show that the representation implies **C4**. Pick any three consequences $x, y, z \in X$ and any and two finite sequences of acts $f_1, ..., f_m$ and $g_1, ..., g_m$ such that, for all $i = 1, ..., m$, $f_i(S) \cup g_i(S) \subseteq \{x, z\}$. Suppose that, for all $s \in S$, $\#\{i : f_i(s) = x\} = \#\{i : g_i(s) = x\}$ and $f_i \succeq y$ for all $i = 1, ..., m$. Without loss of generality suppose that $u(x) \geq u(z)$. If $u(z) \geq u(y)$, then the representation implies that $g_i \succeq y$ for all i. If $u(y) > u(x)$, then the representation would imply that it cannot be true that $f_i \succeq y$ for any i. So, the only interesting case remaining is when $u(x) \geq u(y) > u(z)$. For each $i = 1, ..., m$ let $T_i := \{s \in S : u(f_i(s)) = x\}$ and $\hat{T}_i := \{s \in S : u(g_i(s)) = x\}$. By the representation of $\succeq$, it must be the case that $\pi(T_i) \geq \theta$ for $i = 1, ..., m$. Now note that saying that $\#\{i : f_i(s) = x\} = \#\{i : g_i(s) = x\}$ for all $s \in S$ is equivalent to saying that $\sum_{i=1}^{m} \mathbb{1}_{T_i} = \sum_{i=1}^{m} \mathbb{1}_{\hat{T}_i}$. But this implies that $\sum_{i=1}^{m} \pi(T_i) = \pi \cdot (\sum_{i=1}^{m} \mathbb{1}_{T_i}) = \pi \cdot (\sum_{i=1}^{m} \mathbb{1}_{\hat{T}_i}) = \sum_{i=1}^{m} \pi(\hat{T}_i)$. This can be true only if $\pi(\hat{T}_i) \geq \pi(T_i) \geq \theta$, for some $i \in \{1, ..., m\}$. For such i the representation of $\succeq$ implies that $g_i \succeq q$. It remains to show that (1) implies (2).

By **C1**, the restriction of $\succeq$ to $\mathcal{F}_c$ satisfies all the conditions in Debreu's representation theorem, so there exists a continuous function $u : X \to \mathbb{R}$ such that, for any $x, y \in X$, $x \succeq y$ if, and only if, $u(x) \geq u(y)$. Now, define an event $T \subseteq S$ to be *decisive* if, for any acts f and g, $u(f(s)) \succeq u(g(s))$ for all $s \in T$ implies that $f \succeq g$. Now let $\mathcal{T} := \{T \subseteq S : T \text{ is decisive}\}$. Notice that, since $\succeq$ is non-trivial, $\emptyset \notin \mathcal{T}$. Now observe that **C2** and **C3** immediately imply that, for any two acts f and g, if $f \succeq g$ then $\{s : u(f(s)) \succeq u(g(s))\} \in \mathcal{T}$. Pick $x, y, z \in X$ such that $u(x) \geq u(y) > u(z)$. Take any two finite sequences of events, $T_1, ..., T_m$ and $\hat{T}_1, ..., \hat{T}_m$ such that $T_i \in \mathcal{T}$ for all i and $\sum_{i=1}^{m} \mathbb{1}_{T_i} = \sum_{i=1}^{m} \mathbb{1}_{\hat{T}_i}$. For each i, define $f_i := xT_iz$ and $g_i := x\hat{T}_iz$. By our previous observation, it must be the case that $f_i \succeq y$ for all i and, for all $s \in S$, $\#\{i : f_i(s) = x\} = \#\{i : g_i(s) = x\}$. But then **C4** implies that $g_i \succeq y$ for some i. That is, $\hat{T}_i \in \mathcal{T}$ for some i. This discussion shows that the collection of events $\mathcal{T}$ satisfies the following property:

C4' For any finite sequences of events $T_1, ..., T_m$ and $\hat{T}_1, ..., \hat{T}_m$, if T_i is decisive for all i and $\sum_{i=1_i}^{m} \mathbb{1}_{T_i} = \sum_{i=1}^{m} \mathbb{1}_{\hat{T}_i}$, then T_i is decisive for some i.

Let $\mathcal{E} = conv\left\{\mathbb{1}_T - \mathbb{1}_{\hat{T}} : T \in \mathcal{T}, \hat{T} \notin \mathcal{T}\right\}$. We first need the following claim:

Claim 1. *If $\mathcal{E} \cap \mathbb{R}^S_- \neq \emptyset$, then $0 \in \mathcal{E}$.*

Proof of Claim. Suppose $\xi \in \mathcal{E} \cap \mathbb{R}^S_-$. This means that there exist $T_1, ..., T_m \in \mathcal{T}$, $\hat{T}_1, ..., \hat{T}_m \notin \mathcal{T}$ and $(\lambda_1, ..., \lambda_m) \in \Delta^m$ such that $\sum_{i=1}^m \lambda_i \mathbb{1}_{T_i} - \sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i} = \xi$. Call a state s irrelevant if, for any event $T \subseteq S$, T is decisive if and only if $T \setminus \{s\}$ is decisive. We can assume, without loss of generality, that for all i, T_i and $\hat{T}_i$ contain only relevant states. Let $K = \{s \in S : \xi(s) < 0\}$. Since all $s \in K$ are relevant, for each $s \in K$, there exists a decisive set T_s such that $\hat{T}_s := T_s \setminus \{s\} \notin \mathcal{T}$. But then, if we define, for each $s^* \in K$, $\hat{\lambda}_{s^*} = \frac{-\xi(s^*)}{1+\sum_{s\in K} -\xi(s)}$, and define, for $i = 1, ..., m$, $\hat{\lambda}_i = \frac{\lambda_i}{1+\sum_{s\in K} -\xi(s)}$, we have $\sum_{i\in K\cup\{1,...,m\}} \hat{\lambda}_i \left(\mathbb{1}_{A_i} - \mathbb{1}_{B_i}\right) = 0$. ‖

We also need the following claim:

Claim 2. *If $0 \in \mathcal{E}$, then there exist $T_1, ..., T_m \in \mathcal{T}$, $\hat{T}_1, ..., \hat{T}_m \notin \mathcal{T}$ such that $\sum_{i=1}^m \mathbb{1}_{T_i} = \sum_{i=1}^m \mathbb{1}_{\hat{T}_i}$.*

Proof of Claim. If $0 \in \mathcal{E}$ then there exist $T_1, ..., T_m \in \mathcal{T}$, $\hat{T}_1, ..., \hat{T}_m \notin \mathcal{T}$ and $(\lambda_1, ..., \lambda_m) \in \Delta^m$ such that $\sum_{i=1}^m \lambda_i \mathbb{1}_{T_i} = \sum_{i=1}^m \lambda_i \mathbb{1}_{\hat{T}_i}$. Of course, we can assume that $\lambda_i > 0$ for all i. That is, the homogeneous system of linear equations $\sum_{i=1}^m \mu_i \left(\mathbb{1}_{A_i} - \mathbb{1}_{B_i}\right) = 0$ has a solution with all μ_i's being positive real numbers. But since all the coefficients in the system above are rational numbers, this implies that the system also has a solution with all μ_i's being natural numbers.[29] Since the sequences of sets in the statement of the claim can have repetitions this gives the desired result. ‖

The two claims above show that **C4'** implies that $\mathcal{E} \cap \mathbb{R}^S_- = \emptyset$. Now we can repeat the argument in the end of the proof of Theorem 2 to finish the proof of this theorem. □

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[29]For a proof of this fact see Grillet (2001, page 49), for example.

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