Financial Constraints and Firm Dynamics

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Abstract

The ability of firms to access external financial resources represents a key factor influencing several dimensions that relate to firm dynamics. However, while recent qualitative evidence suggests the existence of heterogeneous and asymmetric reactions of firms to financing constraints (FCs), the literature on the empirics of size-growth dynamics focuses on the effects of FCs on the average growth rate and long-term evolution in terms of firm size distribution. In this paper we extend the analysis to a wider range of possible FC effects on firm growth, including its autoregressive and heteroskedastic structure and the degree of asymmetry in the distribution of growth shocks. We measure FCs with an official credit rating index which, by directly capturing the borrowers' opinion of a firm's financial soundness, affects, in turn, the availability and cost of external resources. Our investigations reveal that FCs operate through several channels. In the short term, FCs reduce the average firm growth rate, induce an anti-correlation in growth shocks and a milder dependence of the volatility of growth rates on size. Financing constraints also operate through asymmetric "threshold effects", both preventing potentially fast growing firms from enjoying attractive growth opportunities, and further deteriorating the growth prospects of already slow growing firms. The sub-diffusive nature of the growth process of constrained firms is compatible with the distinctive properties of their size distribution.

JEL codes: C14, D21, G30, L11

Keywords: Financial constraints, Firm size distribution, Firm growth, Credit ratings, Asymmetric exponential power distribution

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1 Introduction

Firms' ability to access external financial resources represents a factor influencing several dimensions of firm dynamics, as the links between financial and operational activities of firms involve decisions pertaining to investment strategies, the ability to enter or survive in a market, job creation and destruction, innovative activity, and internationalization patterns.

Within the broad research area on the relationship between financial and operating aspects of firm's activity, a relatively well developed tradition of empirical studies have sought to identify the effect of financing problems on size-growth trajectories of firms (for reviews, see Whited, 2006; Fagiolo and Luzzi, 2006; Oliveira and Fortunato, 2006). A first major problem in this identification rests in the intrinsically difficult task of measuring financial constraints. In fact, FCs are not directly observable, as it is not possible to know whether banks or other financial institutions have refused a loan or if particularly high interest rates are imposed on a given firm. To overcome this difficulty, researchers have typically followed two strategies, resorting either to "hard" data – i.e. from annual reports – or to survey data. When hard data are used, the practice has typically been to build a binary classification of financially constrained versus non-constrained firms on the basis of a firm's relative ranking in the distribution of some variable, which is presumed to related to the need, availability and cost of external finance. Following the literature on financing constraints to investment (Fazzari et al., 1988; Kaplan and Zingales, 1997, 2000), most studies have used cash flow or cash flow sensitivity of cash (Almeida et al., 2004) as a measure of FCs. However, other variables, such as leverage, availability of collateral and interest coverage, have also been used. In parallel, attention has been devoted to measures of FCs based on multivariate indexes, designed to summarise several aspects of firm financial structure in a single measure, at the same time allowing to capture different degrees of FC, avoiding a simple binary categorization (Cleary, 1999; Lamont et al., 2001; Whited and Wu, 2006; Musso and Schiavo, 2008). The alternative approach to measure FCs consists of classifications based on survey data. These surveys typically involve managers or entrepreneurs, who are asked to make a self-assessment of whether firms have been rationed or not, whether the cost and the amount of granted loans were in line with their expectations and needs, and more generally, about the difficulties they have faced in accessing financing from banks or other institutions (Kaplan and Zingales, 1997; Winker, 1999; Angelini and Generale, 2008; Campello et al., 2009).

None of the proposed approaches are without their pitfalls, and there is no clear consensus on how different ways of measuring FCs can impact the results obtained. On the one hand, both univariate or multivariate proxies derived from business registry data inevitably give an indirect measure of FCs, as they implicitly assume that poor records of firms with respect to the chosen variables get translated into a bank's unwillingness to grant credit. Survey based measures of FCs, on the other hand, are seemingly closer to answering the question as to whether a firm has actually been constrained or not. Yet, survey data are well known to suffer from misreporting and sample selection bias, whose effect is difficult to quantify. Moreover, by collecting the opinion of the credit seeker about their own financing conditions, survey data look at the demand side of credit relations. However, given the strong asymmetries characterizing capital markets, one might argue that the opinion of the credit supplier on the credit seeker plays the crucial role in determining credit conditions.

Once a measure of FCs has been selected for the analysis, the standard approach followed in the literature has been to include such an FC proxy among the regressors of a standard growth model. Alternatively, FCs are modeled as dummy variables, indicating that a firm belongs to some specific FC class, typically constrained versus non-constrained. The conclusion is usually that FCs negatively affect firm growth, and that this effect is stronger for younger and smaller firms (see Angelini and Generale, 2008).

This kind of specifications, however, can only identify location-shift effects in the conditional distribution of growth rates, accounted for by the influence of the FC proxy on conditional average growth, or by FC class dependent deviations. Although there is a general agreement that FCs downplay growth prospects of firms, there is no clear reason why this reduction should translate *exclusively* into a negative shift of average growth rates. In fact, there are various pieces of evidence that make the shift assumption seem simplistic.

Firstly, the evidence that FCs problems affect several dimensions of firms' behaviour and decisions, such investment/divestment strategies (Fazzari et al., 1988; Devereux and Schiantarelli, 1990; Bond et al., 2003), cash management policies (Campello et al., 2009), or R&D and innovation strategies (Hall, 2002; Brown et al., 2009), clearly suggests that the role played by FC is likely to be complex and structured. Secondly, recent qualitative evidence on firms' reactions to the current crisis (see Campello et al., 2009) suggests that firms undertake heterogeneous responses to FC problems: there are firms that tend to abandon some investment projects, despite their potential, while other firms, especially those which are already experiencing poor growth dynamics, tend to display a much higher propensity to sell off productive assets as a way to generate funds. Heterogeneous responses can induce different effects in different quantiles of the (conditional) growth rates distribution.

To account for these differences, in this paper we develop an analytical framework which is flexible enough to allow for a consistent analysis of the many possible channels through which FCs can affect firm growth. We extend the usual autoregressive growth model by introducing a parametric specification of the heteroskedasticity of growth rates and by allowing for different degree of asymmetry in the growth shocks of firms subject to different strength of FC. The first extension is motivated by the robust empirical observation that smaller firms experience more volatile growth patterns (among others, see Hymer and Pashigian, 1962; Amaral et al., 1997; Bottazzi and Secchi, 2005). Heteroskedasticity is typically viewed as a factor to wash away in obtaining consistent estimates (Hall, 1987; Evans, 1987; Dunne et al., 1988). Conversely, we consider it part of the phenomenon under study, and we want to understand if FCs have a role in explaining this relationship. The second extension, the assessment of possible asymmetries, entails investigating if and how FCs affect the overall shape of growth rates distribution, a topic so far largely neglected (see Fagiolo and Luzzi, 2006, for the only exception we are aware of). Our specification enables us to reconcile the effects of FCs on firm growth dynamics with the observed differences in the firm size distribution (FSD) of constrained and non-constrained firms. Exploring such differences is of recent interest and the evidence is both scant and controversial. Cabral and Mata (2003) found that the evolution of the FSD is determined by firms ceasing to be financially constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) concluded that FCs are not the main determinant of FSD evolution. A partial explanation for such seemingly contrasting evidence may come from the different proxies of FC employed. Indeed, Cabral and Mata (2003) measured FCs with age, assuming that younger firms are more constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) adopted reported cash flow and a measure of FC based on survey data, respectively.

We perform our analysis using a measure of FCs based on credit ratings. In our view this alleviates many of the problems intrinsically related to the identification of FCs. Indeed, credit ratings, by their very definition, are based on hard data, and therefore do not suffer from the biases inherently affecting survey measures. At the same time, they share the advantages of multivariate indicators of FCs, especially in terms of the opportunity they offer to account for different degrees of exposure to FCs. What is however unique to credit ratings is that they do not only provide a picture of a wide range of potential sources of financial problems. They also represent the expectations of credit suppliers on the ability of firms to meet obligations, thus getting closer to measuring whether or not credit is granted to a particular firm. The reliability and widespread use of the specific ratings adopted in our study strongly suggest that they serve as an actual benchmark for the lending decisions of banks.

Using a large panel of manufacturing Italian firms, we provide new evidence on the interplay of FCs and age in determining the variability of growth shocks across firms of different sizes, and on the way FCs can affect the shape of both the firm size and firm growth rates distributions. Our findings suggest that FCs indeed create an asymmetric effect on growth rates distribution. They prevent attractive growth opportunities from being seized by constrained but yet potentially fast growing firms. This effect is particularly strong for younger firms. At the same, and especially among older firms, FCs tend to be associated with a further depression in the growth prospects of already slow-growing firms. These effects results, over the longer run, in a diverse evolution in the size distribution of more severely constrained firms.

The paper is organized as follows. Section 2 describes the data, introduces our FC measure and provides a first descriptive account of the relevance of the FC phenomenon. Section 3 analyzes the time evolution of the FSD. In Section 4 we develop our baseline framework and derive the hypotheses that offers a solid guidance in targeting our empirical investigations and in interpreting our results. Section 5 presents the main results of our analysis of FC effects on the patterns of firm growth, also investigating the effects of FCs on the firm growth rates distribution. Section 6 tests the robustness of the findings with respect to a set of potentially relevant determinants of size-growth dynamics and firms' financing decisions. In Section 7 we summarise our findings and draw conclusions.

2 Financing constraints: definition and basic facts

We employ a large database of Italian firms maintained by the Italian Company Account Data Service (Centrale dei Bilanci, CeBi). CeBi was originally founded as a joint agency of the Bank of Italy and the Italian Banking Association in the early 1980s to assist in supervising risk exposure in the Italian banking system. Today CeBi is a private company owned by major Italian banks, which continue to exploit its services in gathering and sharing information about firms. The long term institutional role of CeBi ensures high levels of data reliability, substantially limiting measurement errors. The dataset is of a business register type, collecting annual reports for virtually all *limited liability*. This group cover a significant proportion of Italian firms. With regard to manufacturing, which is the macro-sector we are focusing on, the dataset accounts for about 45% of total employment and about 65% of aggregate value added.[1] The data available fort the present study follow approximately $200,000$ firms over the period 1999-2003.[2] For each firm, we were able to access a subset of the original list of variables included in the annual reports. We derive Age of the firms from the foundation year of each firm, and we proxy firm size through real Total Sales.[3] The decision to prefer Total Sales over Number of Employees as a measure of size is because in our data, as well as in the Italian accounting system, employment figures are only reported in the notes of financial statements, and are therefore likely to be affected by less reliable annual updates.

The key variable for the purposes of this paper is the rating index that CeBi produces for all the firms included in the dataset, which we use to build a a proxy of financial constraints. In fact, credit ratings not only yield a summary ranking of the financial conditions of firms, but also account

[1] These shares are computed with respect to National Accounts data, as reported by Eurostat. Pistaferri et al. (2010) found similar figures, and also report that the CeBi database covers about 50% of total employment, and around 7% in terms of the number of firms.

[2] Reported results thus refer to 2000-2003 as one year is obviously lost in the computation of growth rates (see below).

[3] Nominal sales are deflated via 3-digit sectoral production price indexes made available by the Italian Statistical office, base year 2000. A basic cleaning procedure to remove a few outlying observations is applied. See the appendix for details.

for the “opinion [of credit suppliers] on the future obligor’s *overall* capacity to meet its financial obligations”(Crouhy et al., 2001). Credit ratings thus represent a crucial variable affecting both the availability and cost of external financial resources. In addition, credit ratings meet all the features which a growing consensus within the literature as identified as crucial for a meaningful measure of financial constraints (Cleary, 1999; Lamont et al., 2001). Firstly, they result from a multivariate analysis, typically stressing the financial side of a firm’s activities, thus summarising a wide range of dimensions of firm performance. Secondly, they are recomputed in each year, and thus allowed to change over time. Thirdly, they do not force the researcher to work with a binary categorization of constrained versus non-constrained firms. In contrast, the graduation of scores attributed by credit ratings to the different firms allow for different degrees of difficulty in accessing external funds to be assessed.

These features are shared by CeBi ratings and ratings issued by international agencies like Moody’s or Standard & Poor’s. However, CeBi ratings have two particular advantages. Firstly, they are available for *all* the firms included in the dataset, while credit files from international rating institutions bias the scope of analysis towards a smaller and much less representative sub-sample of firms. Secondly, the CeBi index is perceived as an ’official credit rating’, due to the tight link that have been established between CeBi and major Italian banks. This justifies the heavy reliance of banks on CeBi ratings: it is generally true that a firm with very poor rating is not likely to receive credit.

The CeBi index ranks firms with a score ranging from 1 to 9, in decreasing order of credit-worthiness: 1- high reliability, 2-reliability, 3-ample solvency, 4-solvency, 5-vulnerability, 6-high vulnerability, 7-risk, 8-high risk, and 9-extremely high risk. The ranking is purely ordinal: for example, a firm rated as 9 does not implicitly have 9 times probability of default compared to a firm rated with a 1. We define three classes of firms subject to different degrees of financial constraints: Non Financially Constrained (NFC), Mildly Financially Constrained (MFC) and Highly Financially Constrained (HFC), corresponding respectively to firms rated from 1 to 4, 5 to 7, and 8 to 9. The assignment to the three classes is based on one-period lagged values of ratings. This is because the CeBi index is updated at the end of each year: it is therefore the rating in t-1 that is relevant for credit suppliers when they have to decide whether to provide credit in year t.[4]

Table 1 shows descriptive statistics. According to our definition financing problems appear to represent a significant phenomenon: 10% of the whole sample is affected by severe difficulties in raising external resources (cfr. the HFC class), while almost half of our sample (48%) faces less severe, but still significant problems (cfr. the MFC class). This is in partial conflict with a result reported in Angelini and Generale (2008) on a smaller Italian dataset. Secondly, FCs are a pervasive phenomenon, affecting firms of different sizes and ages: more than 5% of old firms are in the HFC class, and the mean size of HFC firms is comparable with the corresponding mean size in the other two classes.[5] However, confirming a robust finding in the literature, FCs seem more relevant among young and small firms: 20% of young firms are HFC, against 5% found in the group of older firms and, moreover, the median size of HFC firms is, in all age classes, almost one third smaller as compared to the other FC classes.

[4]In order to check the sensitivity of our results, we also considered 2 alternative assignment procedures. In the first one, firms were assigned to our three classes on the basis of the worst rating displayed over the sample period. According to the second assignment rule, we restricted the analysis to firms that did never change their financial status over the whole time window (i.e., based on their ratings in the different years, they always fell in the same FC class). Our main conclusions were not influenced by the choice of assignment procedure. All the results are available upon request.

[5]The very high mean found within HFC old firms, 47.760, is explained by the presence of a one very big firm (the biggest in the dataset) which is old and HFC over the sample period. The mean size falls to 18,415 if we exclude this firm from the sample.

Table 1: FINANCIAL CONSTRAINTS BY AGE CLASSES

Firm's age (years)	Whole Sample		Non Financially Constrained		Mildly Financially Constrained		Highly Financially constrained	
	Number of firms	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)
0-4	38,020	1.795	10,356	1.804	20,408	1.970	7,256	1.293
		(0.606)	(27.2)	(0.525)	(53.7)	(0.719)	(19.1)	(0.449)
5-10	52,150	3.369	18,269	4.115	27,862	3.248	6,019	1.666
		(0.860)	(35.0)	(0.844)	(53.4)	(0.995)	(11.5)	(0.439)
11-20	62,977	7.093	29,130	8.210	29,408	6.400	4,439	4.354
		(1.522)	(55.9)	(1.606)	(46.7)	(1.663)	(7.0)	(0.525)
21-30	35,579	10.139	18,966	11.147	15,080	9.544	1,533	3.520
		(2.674)	(53.3)	(2.719)	(42.4)	(2.921)	(4.3)	(0.696)
31-∞	20,645	25.917	11,374	26.600	8,213	22.157	1,058	47.760
		(4.516)	(55.1)	(4.919)	(39.8)	(4.764)	(5.1)	(1.345)
Total	209,371	7.577	88,095	9.614	100,971	6.386	20,305	4.662
		(1.301)	(42.1)	(1.548)	(48.2)	(1.371)	(9.7)	(0.494)

Size as real sales, millions of euro.

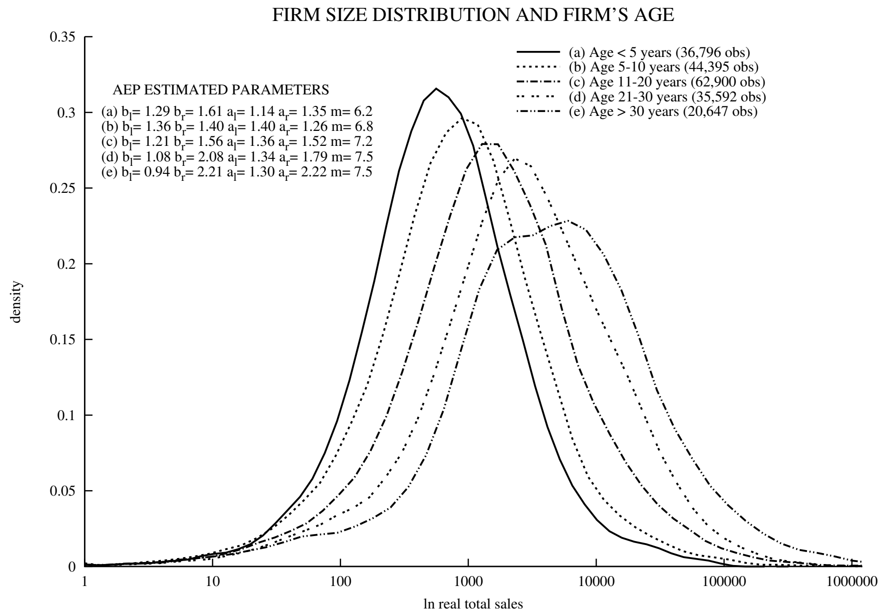

Figure 1: FSD and firm's Age. Densities estimates are obtained using the Epanenchnikov kernel with the bandwidth set using the simple heuristic described in Silverman (1986). Pooled data from 2000, 2001, 2002 and 2003.

3 Financing constraints and the evolution of FSD

Figure 1 reports kernel estimates of the empirical density of real sales by age.[6] Results broadly confirm the basic stylized facts observed in previous studies where size is proxied with employment: the FSD is right-skewed and both the mode and the width of the distribution increase with age. This visual impression is confirmed by a Fligner and Policello test for stochastic dominance. The FSD of older firms dominates those of younger firms, meaning that a firm randomly drawn from the group of older firms is, with a probability significantly higher than 50%, bigger than a firm randomly extracted from the group of younger firms.[7]

However, from the graphical analysis alone it is difficult to provide a precise statement on the validity of a second common piece of evidence reported in the literature, namely that the degree of FSD skewness diminishes with age. Most studies agree on this point, although Angelini and Generale (2008) report that the FSD appears to be more symmetric when using Total Sales, instead of Number of Employees. To provide a meaningful quantitative assessment of this issue, we consider the Asymmetric Exponential Power (AEP) distribution. This is a family of distributions introduced in Bottazzi and Secchi (2006b) to cope with asymmetries and leptokurtosis and at the same time allowing for a continuous variation from non-normality to normality. The AEP density

$$f_{\mathrm{AEP}}(x;\mathbf{p}) = \frac{1}{C}\ e^{-\left(\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}\theta(m-x)+\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}\theta(x-m)\right)} \tag{1}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m)$, $\theta(x)$ is the Heaviside theta function and where the normalization con-

[6]Since we cannot follow cohorts of firms in our data, a comparison across firms of different age provide the most effective way we have to have a clue on the intertemporal evolution of FSD within each FC class.

[7]This test is presented in Fligner and Policello (1981), while Bottazzi et al. (2008) provide details on the interpretation of the test in the case of asymmetric samples. A pair-wise comparison of the distribution in Fig 1 confirms significant differences, with negligible p-scores (less than 16^{-6} in all cases).

stant reads $C = a_l A_0(b_l) + a_r A_0(b_r)$ with

$$A_k(x) = x^{\frac{k+1}{x}-1}\,\Gamma\left(\frac{k+1}{x}\right)\ , \tag{2}$$

is characterised by 5 parameters. Two positive shape parameters, b_r and b_l, describe the tail behaviour in the upper and lower tail, respectively. Two positive scale parameters, a_r and a_l, are associated with the width of the distribution above and below the modal value, in turn captured through the location parameter m, representing the mode. Maximum Likelihood (ML) estimates of the AEP parameters are reported in Figure 1.[8] They reveal two different patterns in the evolution of FSD skewness, for small and big firms respectively. The left tail becomes fatter as age increases (b_l decreases while a_l is approximately stable), so that in the lower part of the distribution size differences are bigger among older firms. In the right-hand side of the distribution there is a shift in probability mass from the tail to the central part of the distribution (b_r increases with age), together with an overall increase in the width of support (larger a_r).[9]

The asymmetric effect that age seems to exert on the evolution of the FSD can be better understood by looking at the sub populations relative to the various FC classes. Figure 2 reports kernel estimates of the FSD obtained in the different FC classes, directly comparing young (less than 5 years) and old (more than 30 years) firms in each class.[10] The results (top left and right, and bottom left panels) suggest that the evolution of the FSD is rather similar for NFC and MFC firms, while HFC firms display a different dynamics. This difference essentially concerns two characteristics: the intensity of location-variance shift effects and the distinctive evolution of the shape in the right tail. As apparent from the plots, both location and variance effects are much milder among HFC firms than in the other two classes. This is confirmed in the bottom-right panel, where we proxy location and width of the FSD, respectively, with the median size and the estimates of the right width AEP parameter, a_r, and then explore how these two measures vary by age and FC class. Both measures are very similar across the FC classes when firms are young. Then, as age increases, it is possible to identify two diverging trends, one common to the NFC and MFC classes, and a second specific to HFC firms. The median size of NFC and MFC firms increases more than tenfold from young to old firms, while the median size of HFC firms increases only by a factor of 5. Similarly, the estimates of a_r reveal that FSD dispersion increases significantly with age for NFC and HFC firms, while the increase is much more modest for HFC firms.

A similar diverging pattern also emerges concerning the estimates of b_r, the parameter that describes the right tail behaviour. As age increases there is a progressive Gaussianization across NFC and MFC firms, while HFC firms constantly display a fat right tail. Indeed from very similar values for young firms ($\sim$1.4, $\sim$1.7 and $\sim$1.6 for the NFC, MFC and HFC classes respectively), the estimated coefficients polarize into two groups when old firms are considered: old NFC and MFC firms display values of b_r close to 2 and hence approximately consistent with a Gaussian distribution, while for old HFC firms the estimated b_r drops from 1.6 to 0.9.

In summary, while the distribution of young firms is similar across different FC classes, clear-cut differences appear when older firms are considered. This shows that the aggregate FSD of older

[8] Bottazzi and Secchi (2006b) prove that the Maximum Likelihood (ML) estimates of the AEP parameters are consistent on the whole parameter space, and when sufficiently large values of the shape parameters are considered, they are also asymptotically efficient and normal. Moreover it is shown that with a sample size of at least 100 observations, the bias associated with ML estimates, although present, becomes negligible.

[9] Notice that the Extended Generalized Gamma distribution applied in Cabral and Mata (2003), which possesses only one shape parameter, would not have allowed to independently account for the observed different behaviour in the left and right tail of the FSD.

[10] Other age classes are not reported for the sake of clarity.

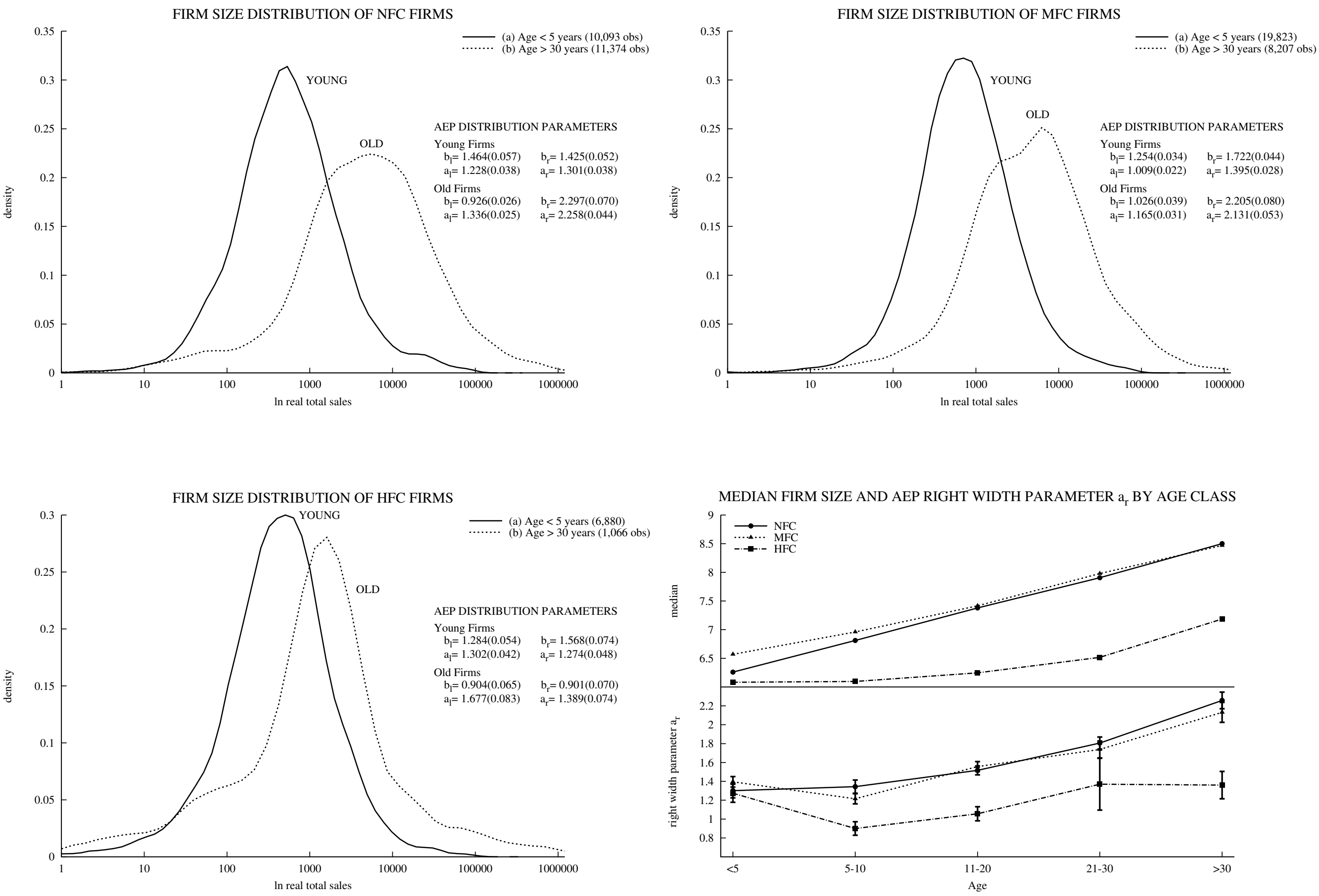

Figure 2: Kernel density of the FSD for young (less than 5 years) and old (more than 30 years) firms in different FC classes. Estimates are obtained using the Epanenchnikov kernel with the bandwidth set using the simple heuristic described in Silverman (1986). Pooled data over the period 2000-2003. Bottom right panel reports the median firm size and the AEP right width parameter, a_r, by age class.

firms reported in Figure 1 results in fact from a mixture of the FSD of financially constrained firms, which are responsible for the fat left tail behaviour observed in the aggregate, and of the FSD of non-constrained firms, which account for the Gaussian behaviour in the right tail.

4 Analytical Framework

We start from the phenomenological model of industrial dynamics based on the classical work by Gibrat (1931). Let s_t be the logarithm of firm size at time t. The simple integrated process $s_t = s_{t-1} + \epsilon_t$ with *iid* distributed shocks ϵ_t, referred to as the "Law of Proportionate Effect", has been shown to yield a good first order description of the observed dynamics of firm size (see among others Mansfield, 1962; Kumar, 1985; Hall, 1987; Bottazzi and Secchi, 2003). In order to account for the various effects of FC on firms growth dynamics we consider a generalized version, at the same allowing for FC-class specific patterns in the relevant coefficient

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\ s_{t-1} + \sigma_{FC}(s_{t-1})\epsilon_{FC,t}\ , \tag{3}$$

where λ captures an autoregressive components in the (log) levels of firm size, σ is a function describing the heteroskedastic structure of the process and ϵ are assumed to be independent of size.

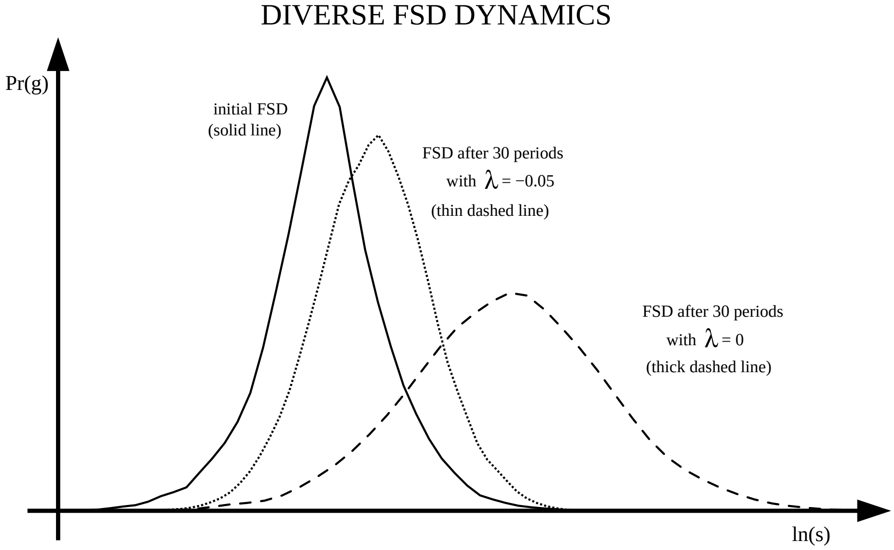

Figure 3: Evolution of the FSD for two different values of the autoregressive coefficient λ in equation (3). In the simulations we consider $15,000$ firms and we set $M_\epsilon = 0.25$ and $V_\epsilon = 0.5$.

The inclusion of an AR(1) structure accounts for the fact that smaller (surviving) firms are often reported to grow faster (see Lotti et al., 2003, for an in-depth review of the empirical literature).[11] The function σ introduces a dependence of the standard deviation of growth shocks on size. The presence of such a dependence has been extensively reported in a number of empirical studies. The common finding is that volatility is bigger for smaller firms, and that the relationship displays an exponential decrease (see the discussion and references in Bottazzi and Secchi, 2005).

According to the model in Equation (3) FCs are allowed to produce an effect on size-growth dynamics through four different channels: the drift term c, the autoregressive term λ, the heteroskedastic term $\sigma(s_{t-1})$ and distribution of growth shocks, ϵ. Let us outline what is the economic interpretation of these channels, and the predictions that can be made.

The coefficient λ is related to the long-term dynamics of the evolution of size. Too see how let us neglect, for the sake of simplicity, the FC subscript and the heteroskedasticity correction, and let the mean and variance of the size distribution at time t be M_{s_t} and V_{s_t}, respectively. Under the hypothesis of a constant λ, their evolution from $t = 0$ to $t = T$ is given by

$$M_{s_T} = (1+\lambda)^T M_{s_0} + \frac{(1+\lambda)^T - 1}{\lambda} M_\epsilon \ , \quad V_{s_T} = (1+\lambda)^{2T} V_{s_0} + \frac{(1+\lambda)^{2T} - 1}{(1+\lambda)^2 - 1} V_\epsilon$$

where M_ϵ and V_ϵ are the mean and variance of the shocks ϵ.[12] When $\lambda = 0$, as in the benchmark Gibrat's model, we have a diffusion process: the time evolution of s_T follows a unit root process (discrete Brownian motion) converging to a log-normal FSD with indefinitely increasing variance and zero mean. Conversely, when $\lambda < 0$ the process is sub-diffusive and the FSD converges in probability to a stationary distribution with finite variance $V_\epsilon/(1-(1+\lambda)^2)$. The analysis in Section 3 suggests that $\lambda < 0$ may be the case for more severely constrained firms. Figure 3 shows that even small differences in the value of λ, can quickly produce significantly different FSD shapes.

[11] In general the AR(1) specification can be replaced with a more general linear model. For the present discussion the one-lag structure is sufficient and the inclusion of further lags does not generate significant modifications in the value of λ.

[12] See the Appendix for a formal derivation.

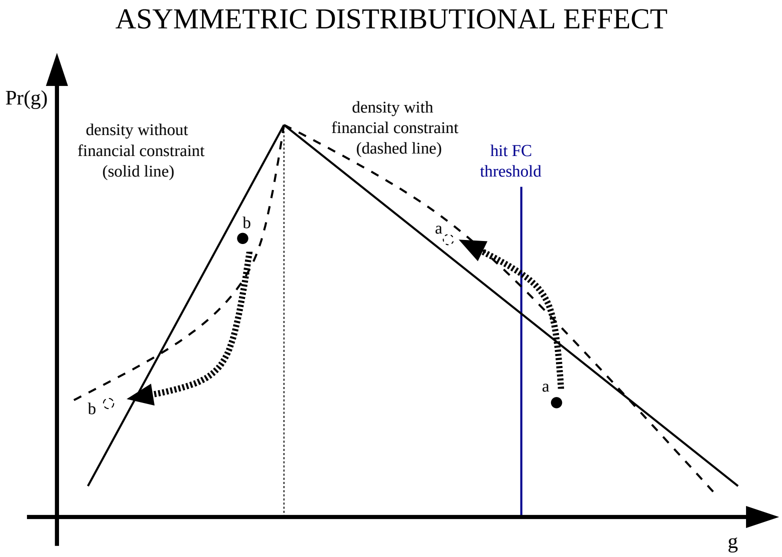

Figure 4: Possible effects of financing constraints on the growth rates distribution.

Differences in c across FC classes provide information on the effect of FCs on the central tendency of the distributions, i.e. on the aforementioned location-shift effects across constrained or non-constrained firms. This is the kind of effect typically captured by the regression models proposed in the literature. If it is assumed that FCs reduce the set or the amount of growth opportunities attainable by constrained firms, then the expectation would be that most severely constrained firms have the lowest c.

Furthermore, differences in σ across FC classes would reveal a "heteroskedasticity effect" due to FCs, suggesting that FCs also produce changes in the way the variance of growth depends on size. The aforementioned result that the variance of growth rates decreases with size has been interpreted as a portfolio effect (Bottazzi and Secchi, 2005): since larger firms are typically more diversified than small firms (in terms of products, lines of business, plants,...) they can balance negative and positive shocks hitting their single branches (at least if the various activities are weakly or on mildly correlated). According to this interpretation, we can conjecture that FCs, by reducing the range of attainable new growth opportunities, also make the actual degree of diversification more similar across smaller and larger firms. The model therefore predicts weaker heteroskedasticity effects within the group of the most severely constrained firms.

Finally, concerning the possible effects of FC on the shape of the growth shocks distribution, we can sketch some predictions based on the qualitative findings in Campello et al. (2009). In Figure 4 the solid line corresponds to a Laplace distribution of shocks (a "tent" on a log-scale) and represents the non-constrained benchmark, chosen because invariably observed in empirical data across different countries and at different levels of sectoral aggregation (cfr. Stanley et al., 1996; Bottazzi and Secchi, 2006a).[13] The dashed line describes the possible distributional effect that could plausibly emerge under the influence of binding FCs. One possible effect is a "pinioning the wings" effect: FCs prevent firms that face potentially good growth opportunities from actually seizing some of them (beyond a certain 'hit FC' threshold), thus forcing these firms to abandon or postpone investment projects. Although positive growth is still attainable in presence of FCs, these firms would have enjoyed much higher growth records if they had not hit by FCs. Such an effect

[13] A first attempt to explain the emergence of this stylized fact, based on the idea of dynamic increasing returns, is in Bottazzi and Secchi (2006a).

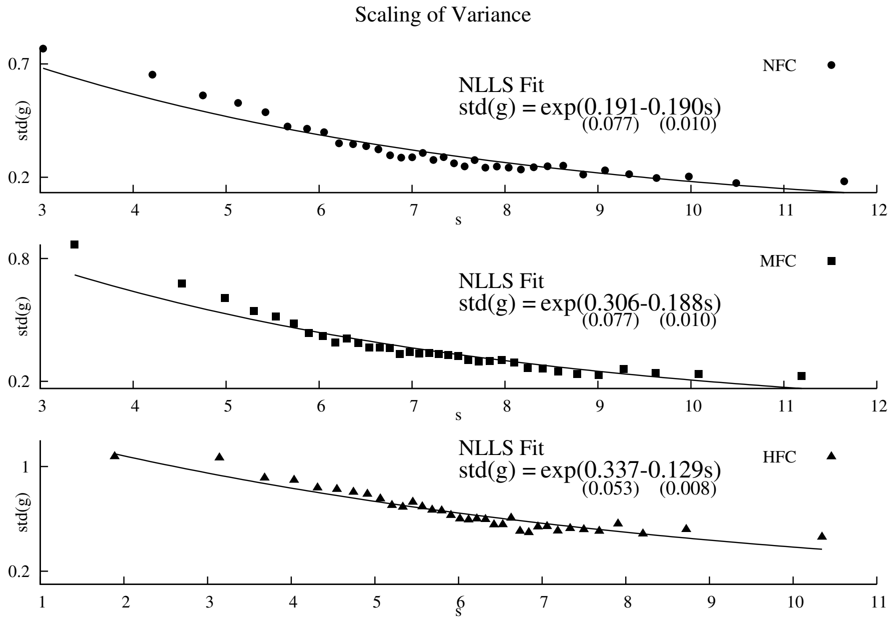

Figure 5: Empirical relation between the standard deviation of growth and firm size, by FC classes.

would imply a slimming down of the right tail of the growth shocks distribution (cfr. 'case a' in Figure 4). Another possibility is a "loss reinforcing" effect of FCs. This predicts that firms who are already facing losses in market shares will experience a further deterioration in their negative growth rates in the presence of credit constraints problems, for example because they are forced to sell productive assets and divest activities. This would then be reflected in a shift of mass towards the bottom extreme, generating a fatter left tail (cfr. 'case b' in Figure 4).

5 Main results

A preliminary step in estimating equation (3) involves modeling heteroskedasticity. We model $\sigma_{FC}(s_{t-1})$ starting from the data. Consider the standard definition of growth rates in terms of log-differences of size

$$g_{i,t} = s_{i,t} - s_{i,t-1} \quad . \tag{4}$$

For each FC class, we plot the standard deviation of g computed within bins (quantiles) of the log-size distribution vs. the average log-size of the bin. Figure 5 report results obtained with 35 size bins, however the whole procedure is very robust in terms of choice of the number of bins. Scatter plots of the data tend to agree with previous investigations finding that the relationship displays an exponential decrease. This is confirmed, for all FC classes, by the Non-Linear Least Squares estimates reported in the graphs. Notice that the relationship does not seem to depend on age. In fact, within each FC class, we did not observe any statistically significant difference in the estimated relation when considering young versus old firms.[14]

Taking this evidence into account, we insert an explicit exponential heteroskedasticity term $\sigma_{FC}(s_{t-1}) = \exp(b_{FC}\ s_{t-1})$ in our baseline model to obtain

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\ s_{t-1} + \exp(b_{FC}\ s_{t-1})\epsilon_{FC,t} \quad . \tag{5}$$

[14]Results available upon request.

A further important modeling issue concerns an appropriate treatment of the distribution of residuals. Since previous studies have documented that the distribution of growth shocks, once heteroskedasticity has been properly modeled, are well approximated by a Laplace distribution, a first choice would be to allow for Laplacian residuals via Least Absolute Deviation (LAD) estimates. However, consistently with the discussion in Section 4, we are also interested into possible asymmetries in the distribution of growth shocks, and therefore we estimate equation 5 via Maximum Likelihood assuming an Asymmetric Laplace distribution in the residuals (ALAD).[15]

Table 2 presents the results (cfr. Model 1) obtained in each FC class. A first notable finding concerns the cross-class pattern in the estimated autoregressive component. The estimated λ is not significant for NFC firms, while it is significant but practically zero in the MFC class. This suggests that an integrated process can represent a good approximation for the evolution of size in these two classes. Conversely, the estimated λ is significantly negative for HFC firms (about -0.02, roughly three times bigger), revealing that strong FCs give rise to sizeable deviations from the Gibrat's benchmark. The result is compatible with the lack of Gaussianization in the right tail of the FSD noted in Figure 2 for the HFC firms.

The patterns in the constant terms are in line with expectations: average growth rate is positive for non constrained firms, while statistically equal to zero in the other classes. Confirming intuition and standard results in the literature, FC problems reduce the average growth rate.

The estimates of the coefficients b, confirming the graphical investigation reported in Figure 5, reveal the clear-cut role of FC in explaining the heteroskedasticity of growth shocks. For NFC and MFC firms the estimated value is very close to -0.20 (which is strikingly similar to those reported in other studies on different data). This means that, in these two classes, the standard deviation of growth rates among large firms (say with s_{t-1} equal to 10), is approximately three times smaller than the standard deviation about among small firms (say with $s_{t-1} = 4$). Instead, among HFC firms the estimated b is about -0.16, implying a smaller reduction in growth dispersion when moving from smaller to bigger firms, as compared to the other two classes (growth dispersion of bigger firms is about twice that of smaller firms). This is clearly consistent with the intuition that FC create a threshold effect, thus reducing the span of growth opportunities that firms can access. According to the aforementioned "portfolio theory" interpretation, the result suggests that the diversification advantage of bigger firms is considerably reduced by the effect of FCs.

Finally, the estimated values for a_l and a_r reported in Table 2 suggest a relatively symmetric distribution of residuals. Consider however that the ALAD estimation assumes an exact Laplace shape (i.e., assumes that b_l=b_r=1). For a more general assessment of the possible presence of asymmetry it is worthwhile investigating the structure of the residuals with respect to different age classes. This is done in Figure 6 where we show kernel estimates of the empirical distributions of the residuals for young-NFC firms (top-left), young-HFC firms (top-right), old-NFC firms (bottom-left), and old-HFC firms (bottom-right).[16] The ML estimates of the AEP coefficients b_l, b_r, a_l, a_r are reported in each panel, and differences in tail behaviour are quantified by an AEP fit (solid line). A comparison across the estimates confirms the tent-shape approximation. However, the age-class disaggregation shows that FCs produce an apparent effect. The very presence of such a sizeable effect is an interesting finding *per se*. Consider that in fact location- and variance-shift effects of FC are already captured through c and σ, respectively. Thus, what remains in the residuals is only the result of asymmetric tail effects induced by FCs. Let us first focus on young firms (compare the two top pan-

[15] This corresponds to assume the error term follows an AEP distribution with $b_l = b_r = 1$ and with a_l and a_r estimated from data.

[16] The distributions of MFC firms are not presented here to keep figures more readable. The results (available upon request), substantially mimicking the findings observed for NFC firms, do not affect the main conclusions of our reasoning.

Table 2: REGRESSION ANALYSIS[a]

	FC CLASS	Main Estimates Model 1	Robustness checks Model 2A	Model 2B
	NFC			
b		-0.200*(0.001)	-0.194*(0.001)	-0.193*(0.0010)
constant		0.019*(0.001)	0.022*(0.001)	0.024*(0.0024)
$\ln(S_{i,t-1})$		-0.0001(0.0003)	-0.007*(0.001)	-0.008*(0.0007)
$\ln(Age_{i,t})$			-0.025*(0.001)	-0.026*(0.0008)
$\ln(Assets^{b}_{i,t-1})$			0.011*(0.001)	0.011*(0.0005)
$\ln(GOM^{b}_{i,t-1})$			0.0001(0.0005)	0.0004(0.0005)
a_l, a_r		0.201, 0.176	0.197, 0.171	0.198, 0.170
Number of observations		89344	85382	85382
	MFC			
b		-0.204*(0.001)	-0.195*(0.001)	-0.195*(0.001)
constant		-0.002(0.001)	0.0004(0.0003)	-0.002(0.001)
$\ln(S_{i,t-1})$		-0.0063*(0.0004)	-0.017*(0.001)	-0.017*(0.001)
$\ln(Age_{i,t})$			-0.041*(0.001)	-0.041*(0.001)
$\ln(Assets^{b}_{i,t-1})$			0.015*(0.001)	0.014*(0.001)
$\ln(GOM^{b}_{i,t-1})$			0.005*(0.0004)	0.005*(0.0004)
a_l, a_r		0.231, 0.224	0.224, 0.216	0.223, 0.216
Number of observations		102321	97437	97437
	HFC			
b		-0.164*(0.002)	-0.152*(0.0026)	-0.151*(0.003)
constant		0.006(0.003)	0.024*(0.003)	0.016*(0.004)
$\ln(S_{i,t-1})$		-0.019*(0.002)	-0.046*(0.002)	-0.046*(0.002)
$\ln(Age_{i,t})$			-0.106*(0.003)	-0.108*(0.003)
$\ln(Assets^{b}_{i,t-1})$			0.037*(0.002)	0.036(0.002)
$\ln(GOM^{b}_{i,t-1})$			0.006*(0.001)	0.007*(0.001)
a_l, a_r		0.448, 0.425	0.431, 0.395	0.430, 0.395
Number of observations		20911	18834	18834

[a] ALAD estimates, standard errors in parenthesis.
[b] Assets is proxied with Net Tangible Assets. Gross Operating Margin(GOM) has been transformed to avoid negative numbers.
[*] Significantly different from zero at 1% level.

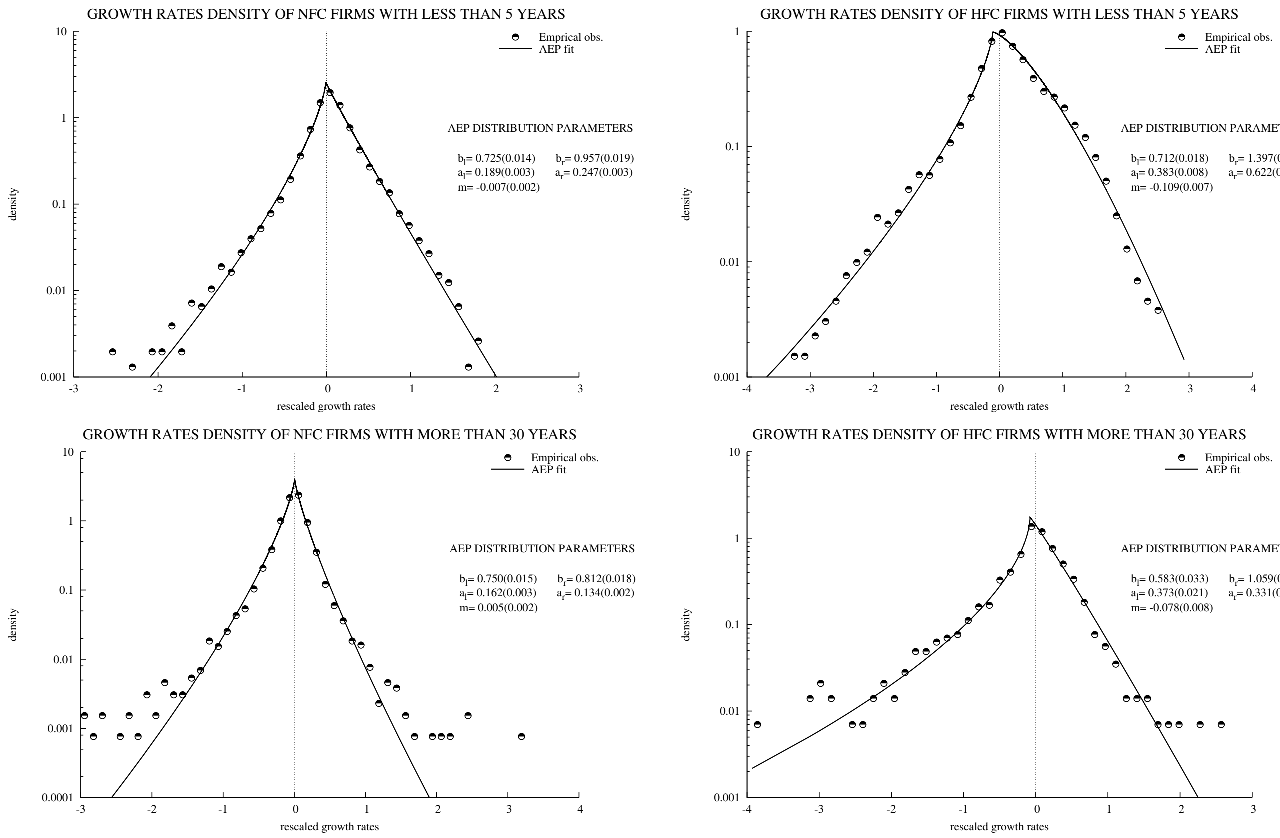

Figure 6: Growth rates distributions and financial constraints

els in Figure 6). If we move from NFC to HFC firms, what we observe is a clear-cut slimming down of the right tail: there is a leftward shift in probability mass from the right tail to the central part of the distribution (b_r increases from about $\sim$0.96 for HFC firms, and to almost 1.37 for the NFC). Correspondingly, the right width parameter also shows a slight increases: a_r goes from about 0.83 to about 0.99. In contrast, the left tails of the two distributions do not display any significant difference: for young firms, both a_l and b_l are very similar in the two groups. The picture changes completely when we consider old firms (bottom panels in Figure 6). In this case the differences between NCF and FC firms are stronger on the left-hand side of the supports. HFC firms have a fatter left tail, suggesting that FCs produce a shift in probability mass towards the left tail: b_l decreases from 0.75 to almost 0.58.[17] The findings are in line with the existence of two types of FC effects described in Section 4 (recall Figure 4), and also suggest that such effects operate differently on different age classes. The "pinioning the wings" effect of FCs mainly affects young firms, while older firms are those mostly affected by the "loss reinforcing" effect of FCs.

6 Robustness checks

Our baseline framework (5) clearly leaves out important factors that are likely to play a role in size-growth dynamics. In this respect, we have seen that age can be a major candidate, exerting interesting effects on the distributional properties of residuals. Of course, there could also be others. In this section the robustness of the FC effects collected so far is tested by enlarging the set of

[17]There is also an effect on the right side of the supports, qualitatively similar to that across young firms, and resulting in a fatter right tail for NFC firms. For old firms, however, the effect is almost negligible.

explanatory variables considered.

The relatively short time dimension of the data does not allow to perform reliable panel estimates, which would help to control for unobserved firm-specific heterogeneity. However, we can extend the set of regressors to control for the potentially relevant factors which we can observe. Firstly, the inclusion of firm age is mandatory, given the high correlation of age with size, and the significant effects that age has been shown to have on the distributional properties of both size and growth. Secondly, there are two dimensions that need to be controlled for, availability of internally generated resources and availability of collateral. These are crucial since they interacts with external FCs in determining overall financial resources available, and thus the investment strategies of firms. The rationale behind the inclusion of a proxy for collateral is that, as predicted by theory and confirmed by evidence (Angelini and Generale, 2008), the availability of hard capital can ease the access to external financing. We measure the availability of collateral using the stock of Net Tangible Assets (labeled ASSETS). Further, we proxy internal resources with the logarithm of Gross Operating Margin (GOM, equivalent to the EBIDTA), thus yielding a measure of the profit margin generated by the operational activities of a firm.[18] Given the relatively high frequency of negative GOM in the sample (about 30%, in line with other studies on Italy, see Bottazzi et al., 2008), negative GOM values were transformed to 1 before taking logs. In fact, for the purposes of our analysis, negative and null internal resources can be considered equivalent, as in both cases there is a need for the firm to completely rely on external resources in financing the operations.

Taking these variables into account, we first perform Maximum Likelihood ALAD estimates of the following model

$$\begin{aligned} s_t - s_{t-1} = c_{_{FC}} + \lambda_{_{FC}}\ s_{t-1} + \beta_{1_{FC}}\ \ln(age_t) + \beta_{2_{FC}}\ \ln(GOM_{t-1}) + \\ \beta_{3_{FC}}\ \ln(ASSETS_{t-1}) + \exp(b_{_{FC}}\ s_{t-1})\epsilon_{t_{FC}} \end{aligned} \tag{6}$$

where both GOM and ASSETS enter with 1-period lag, at least partially accounting for simultaneity issues concerning these variables, and we again model heteroskedasticity via an exponential correction.[19] Results are reported in Table 2 under the heading “Model 2A”. The most notable change induced by the inclusion of controls is that deviations from the Gibrat’s benchmark of $\lambda = 0$ can now be observed in all the classes. That is, as it is frequently reported in studies exploring augmented Gibrat’s regression, additional regressors absorb part of the size coefficient. However, the estimates of λ across the FC classes reproduce the pattern previously obtained from our baseline model: the autoregressive coefficient has a much lower value for the HFC class, thus confirming that the negative impact of size on growth rates is stronger for financially constrained firms. Estimates of the heteroskedasticity parameter b are basically unaffected by the addition of further regressors and confirm the patterns emerging from the simplest specification.

In general, the effects of added covariates present interesting cross-class differences. Age displays a negative and significant coefficient in all classes, in agreement with the expectation that on average older firms grow less than younger firms. The magnitude increases with the strength of FCs, however, thus revealing that the detrimental effect of age is stronger in HFC firms. It should also be noted that age is the regressor with the strongest effect (highest coefficient in absolute value). Next,

[18]Bottazzi et al. (2009) show that GOM is not completely absorbed in the information included in CeBi ratings, thus confirming that ratings tend to summarise financial activities better than operating activities. The use of GOM implies, by definition, that we do not consider the cash flow generated by non operating earnings and losses. These should not be very relevant, however, since we are working with manufacturing firms. Moreover, due to the limited data availability, we cannot consider the cash flows absorbed by taxes. Assuming, as a first approximation, a constant tax rate, this would amount to a constant shift in the value of our regressor.

[19]As done for size, both GOM and ASSETS were deflated with appropriate sectoral price indexes, at the 3-digit level of industry disaggregation.

Table 3: Growth Rates Distributions – Robustness checks

	AEP Parameters			
	b_l	b_r	a_l	a_r
YOUNG (age < 5)				
NFC	0.729(0.0143)	0.975(0.0199)	0.188(0.0028)	0.244(0.0034)
HFC	0.713(0.0185)	1.436(0.0405)	0.374(0.0076)	0.602(0.0104)
OLD (age > 30)				
NFC	0.751(0.0155)	0.823(0.0189)	0.159(0.0025)	0.134(0.0022)
HFC	0.717(0.0465)	0.988(0.0813)	0.384(0.0197)	0.314(0.0177)

[a] AEP fit of residuals from Equation (6), Pavitt class dummies also included. Standard errors in parenthesis.

concerning the role of ASSETS, we find a positive and significant effect, stronger for HFC firms: the availability of collateral becomes more beneficial for growth when FC are stronger. Similarly, the availability of internal resources has some beneficial effects on growth only when FCs are more severe, while internal resources do not seem to be crucial for NFC firms (GOM is not significant for NFC, positive and significant for MFC and HFC). However, even when significant, the magnitudes of GOM coefficients are negligible in practical terms, suggesting that internal resources have (if any) a second order effect compared to other regressors.

A further important check that we perform concerns the possible role of sector-specific dynamics. It is well known that a firm's dependence on external financing varies across industrial sectors (Rajan and Zingales, 1998), so that it is likely that firms operating in different industries would display, on average, a different degree of exposure to FC problems. There is also evidence (Hall, 2002) that such sectoral differences in modes of financing, and thus differential exposure to FCs, are very likely to vary depending on the sources and procedures of innovation activity of firms. In order to control for these industry-wide factors, we re-estimate Equation (6) adding dummies based on the classical Pavitt taxonomy of sectoral patterns of innovation (Pavitt, 1984). The results (cfr. Model 2B in Table 2) are clearly in line with previous estimates: all the coefficients remain unchanged in practical terms.[20]

Finally, we also investigate whether the distributional properties of growth shocks are affected by the inclusion of the new regressors. To this purpose we perform AEP estimates of the empirical distribution of residuals of Model 2B, separately for young and old firms. Note that within this specification the possible location-shift effects due to age are captured by the coefficient in the regression. Also recall that, as shown in Section 5, once controlling for size, age does not have any residual effect on the variance of growth rates. The analysis of the distributional shape of the residuals by

[20] We also explored a further specification considering 2-period lags of size, ASSETS and GOM. This allows for a check of varying effects over time, and provides a further control for possible endogeneity of covariates at $t-1$. The estimates of λ retain their signs and magnitudes, again displaying negligible values for NFC firms and then increasingly negative as FCs become stronger. Second lag coefficients of GOM and ASSETS absorb part of the first lag effects of these variables. The most noticeable difference compared to the estimates presented in Table 2 is a significant reduction in the age coefficient, whose magnitude becomes comparable with that of the other regressors, and also comparable across FC classes. The results are available upon request.

age class, therefore, tells us whether there are additional effects of age in the tails. The estimated AEP parameters, reported in Table 3, are not significantly different from the estimates found with the simplest model specification (apart from a small increase in the b_l parameter for HFC firms).

Overall, our main conclusions remain the same even with the inclusion of other relevant determinants of size-growth dynamics, such firm age and the availability of internal financial resources or collateral, and remain unchanged when also we control for differences in sectoral patterns of innovation.

7 Conclusion

Credit ratings represent a good measure of a firm's access to external resources. They are heavily relied on by banks and investors in granting and pricing credit lines, thus representing an important benchmark or a key ingredient in lending decisions. They summarise several dimensions of a firm's financial conditions, and offer an alternative to the rather strict binary distinction between constrained versus non-constrained firms traditionally employed in the literature, thus yielding a proxy of different degrees of credit problems. Using credit ratings to build a proxy for financial constraints, we investigated the effects of these constraints on the size-growth dynamics of firms. Previous literature suggests that these effects are likely to be many and varied. We thus extended the typical autoregressive linear analysis by including a parametric description of heteroskedasticity and assuming a more flexible (and robust) characterization of growth shocks. Our model confirms the qualitative predictions. The effects of FC on growth dynamics are sizeable and operates through several channels. Firstly, they magnify the negative effect of size on expected growth rates: the lower average growth rate that typically characterizes large versus small firms becomes even lower when FCs are presents. This is consistent with the observed evolution in time of the firm size distribution (FSD) of financially constrained and non-constrained firms. As the age of the firms increases, the FSD of non constrained firms evolves toward a Gaussian shape, while the FSD of financially constrained firms remains more peaked. This is the typical signature of the sub-diffusive nature of the growth process associated with a negative autoregressive coefficient. Since our measure of FCs is varies over time (firms can in principle belong to different FC classes in different years), the fact that we identify significant differences in the size distribution of different FC classes suggests a relatively high degree of persistence across the different groups. This is an interesting aspect of the FC phenomenon, which we cannot however test directly, given the relatively short temporal span of our database.

The second effect of FCs is on the relationship between firm size and variance of growth rates. Larger firms generally display a lower variability in their growth rates. This observation has been related to a portfolio effect: larger firms tend to be more diversified, and thus to the extent that the different activities are weakly related, diversification produce a lower volatility in aggregate growth rates. FCs seem to reduce the ability of larger firms to exploit their diversified structure. Indeed for more severely constrained firms, the reduction in the growth rates variability with size is weaker than for unconstrained firms.

Furthermore, once the autoregressive structure and the heteroskedasticity effects are controlled for, our model reveals that FCs have a further asymmetric effect on the tails of the growth rates distribution. We are able to identify a loss reinforcing effect: firms who are already witnessing a reduction in sales, see their performance worsened in the presence of FCs. This is plausibly the quantitative effect of activity dismissal and divestment. At the same time, however, firms experiencing positive growth rates are likely to see their growth potentials depressed if hit by FCs. In fact, credit problems generate a ”pinioning the wings” effect which prevents constrained firms from fully seizing the available growth opportunities. The economic consequences of these two effects are different. While the loss reinforcing effect can be seen as a natural market selection mechanism, generating, at least in the long run, a more efficient reallocation of productive resources, the pinioning effect plausibly translates into a net loss of growth opportunities. The fact that the pinioning mechanism is more common across younger firms makes this analysis more convincing and is compatible with the presence of frictions and inefficiencies in the capital market.

It is worth asking if our measure of FCs can also be considered as a proxy for the overall availability of financial resources, capturing at the same time difficulties in accessing external finance as well as shortage of internal financial resources. We tend to believe it can, as indeed internal resources constitute the best guarantee to potential lenders that firms will be able to pay back the interests due. As a result, firms with sound and healthy financial conditions and reasonable levels of profits are almost automatically assigned high ratings. Instead, the scarcity of internal resources, whether generated by poor operating performances or by unsound financial conditions, is very likely to be punished with bad ratings. In any case, our conclusions are still valid even when we explicitly add a control for the availability of internal resources. Indeed, while the presence of positive profit margins produces a shift in the average growth rate, both the pinioning and loss reinforcing effects of FCs on the distributional properties of growth rates remain unchanged, as does the reduced ability of larger and financially constrained firms to exploit diversification economies.

In summary, we have shown that FC problems do have relevant effects on the operating activities of firms. In order to identify these effects, however, one has to do more work than just relying upon standard linear regression framework. FC effects are indeed manifold and impact on several aspects of firm growth dynamics, ranging well beyond a shift in the expected growth rates.

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8 APPENDIX

8.1 Cleaning anomalous observations

We removed a few anomalous data from our sample. Cleaning was performed using Total Sales as a reference variable. For each firm, a missing value was inserted, in the place of the original value of Total Sales, when the latter lay outside the interval

$$[Median(TS_i)/10; Median(TS_i) * 10] \quad , \tag{7}$$

where the median is computed over the years for which data are available for firm i. Table 4 shows yearly descriptive statistics computed before and after this cleaning procedure was applied. It is apparent that the cleaning procedure did not introduce any relevant change to the data.

Table 4: TOTAL SALES[a] DESCRIPTIVE STATISTICS

BEFORE CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5700.82	1014.00	48730.09	57.89	4894.16	1.00	5634948.00	109689.00
2001	5972.90	1011.00	73679.67	141.82	29897.12	1.00	17547260.00	113405.00
2002	5804.92	973.00	67304.35	146.66	32359.62	1.00	16484840.00	116084.00
2003	5639.77	953.00	64724.22	147.42	32317.38	1.00	15803760.00	115777.00
AFTER CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5754.55	1046.00	47700.57	58.99	5192.76	1.00	5634948.00	107250.00
2001	5878.64	1025.00	69435.93	159.48	37224.24	1.00	17547260.00	112036.00
2002	5806.96	992.00	67093.95	150.02	33371.72	1.00	16484840.00	113849.00
2003	5688.46	981.00	65417.79	147.67	32063.94	1.00	15803760.00	111810.00

[a] Nominal Total Sales in thousands of Euro.

8.2 Asymptotic behaviour of the autoregressive process

Start from the model of firm size evolution as described in (3), where the shocks ϵ are independent and identically distributed according to a probability density f with mean c. Let s_0 be the initial size of the firm. By dropping the heteroskedastic term (i.e. setting $\sigma(s_t) = 1$) for simplicity, and by recursive application of (3), the size after T time steps, s_T, can be written as the weighted sum of T independent random variables

$$s_T = (1+\lambda)^T\, s_0 + \sum_{\tau=0}^{T-1} (1+\lambda)^\tau\, \epsilon_{t-\tau} \ .$$

Consider the cumulant generating function of the size at time T, $\tilde{g}_{s_T}$, defined as the logarithm of the Fourier transform of the unconditional distribution

$$\tilde{g}_{s_T}(k) = \log \mathrm{E}[e^{iks_T}] \ .$$

Due to the i.i.d. nature of the shocks it is immediate to see that

$$\tilde{g}_{s_T}(k) = \tilde{g}_{s_0}((1+\lambda)^T k) + \sum_{\tau=0}^{T-1} \tilde{f}((1+\lambda)^\tau k)$$

where $\tilde{g}_{s_0}$ and $\tilde{f}$ are the cumulants of the initial size distribution and of the shocks distribution, respectively. As a consequence, if the initial size distribution and the shocks distribution possess the cumulant of order n, C^n, then the size distribution at time T also possesses it, and thus, with obvious notation

$$C^n_{s_T} = \frac{d^m}{dk^m}\, \tilde{g}_{s_T}(k) \bigg|_{k=0} = (1+\lambda)^{mT}\, C^n_{s_0} + \frac{(1+\lambda)^{mT} - 1}{(1+\lambda)^m - 1}\, C^n_\epsilon \ .$$

Equation (4) in Section 4 directly follows by noting that the mean and the variance are the first and second cumulants, respectively: $M = C^1$ and $V = C^2$.