Incomplete Pass-Through in a Model of Retailers - Wholesalers Relationships

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Abstract

Recent empirical work documents that the pass-through of cost shocks to prices is very low, and delayed. Moreover, delayed pass-through mostly occur at the wholesale rather than at the retail level. To explain these facts, this paper develops a model of wholesalers-retailers relationships where incomplete pass-through arises endogenously. The model is based on two key assumptions with strong empirical support. First, both retailers and wholesalers invest resources to form new, long-term, business relationships. Second, once a business relationship is formed, the wholesale prices and the quantities of the intermediate good exchanged are set in a bilateral bargaining between wholesalers and retailers.

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1 Introduction

A number of empirical studies documents that marginal cost shocks are not fully passed through to prices at the firm level and that prices are substantially less volatile than costs. This is in stark contrast with the standard framework of monopolistic competition used in macro models, the one developed by Dixit and Stiglitz (1977), which implies a complete pass-through of cost to prices. There are many theoretical reasons proposed as to why prices are more stable than marginal costs. The most recent literature, trying to explain the low pass-through to prices of exchange rate shocks, has mainly focused on three factors: the existence of local distribution costs, markup adjustments (due, for instance, to a variable elasticity of demand), and pure nominal rigidities (menu costs).

In this paper we propose a novel explanation based on the presence of product market imperfections in the relationship between wholesalers and retailers. In our model incomplete pass-through arises endogenously as a consequence of two key assumptions. First, both retailers and wholesalers spend resources to form new long term business relationships. Second, once a business relationship is formed, the wholesale prices and the quantities of the intermediate good exchanged are determined in a bilateral bargaining between wholesalers and retailers. The model can explain both the low and delayed pass-through of cost shocks to wholesale prices, and the almost complete pass-through of wholesale prices to retail prices.

There is a vast empirical evidence on the importance of business to business (B2B) long term relationships and product market imperfections. For example, Blinder et al. (1998) find that, in the US, 85% of firms surveyed engage mainly in long term relationships with their customers, and that 77% of their customers are other firms. These long-term relationships are mainly covered by contracts, and these contracts typically last one year. Surveys for other industrialized economies usually corroborate these findings (See e.g. Fabiani et al. (2006) for the Euro Area or Apel et al. (2005) for Sweden). As noted by Pierrard and Matha (2009) firms allocate a non-negligible amount of resources in the search of customers or suppliers. The need for advertising, marketing, promotions etc. provided almost 600,000 jobs in 2006. This represents almost 0.5% of total US employment. A similar amount of people were engaged in purchasing and buying occupations. Moreover, total annual expenditure in all media advertising represents on average 2.5% of GDP over the last decade.

Negotiations among retailers and wholesalers seem to be the rule rather than the excep-

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2 Examples include implicit contracts, social customs, customer markets and theories of countercyclical markups (See Ball and Romer (1990) and the references therein). More recent examples are the modern DSGE models with non-constant elasticities of consumer demand (e.g. Dotsey and King (2005)), costly information (Wiederholt and Mackowiak (2006)) or nominal rigidities (e.g Christiano, Eichenbaum and Evans (2005)).
tion. Zbaracky et al. (2004) find that customer communications and price negotiation costs account for almost 75% of the total price adjustment cost and are 20 times bigger than the size of the menu costs. Fabiani et al. (2006) find, on the basis of surveys conducted by nine Eurosystem national central banks, that the existence of implicit and explicit contracts with customers is considered as the most important explanation for rigid prices. Friberg and Wilander (2008) report that the invoicing currency for export is predominantly set through a negotiation between the exporter and the importer.3

Very recent findings of Nakamura (2008), Gopinath and Itskhoki (2010) and Nakamura and Zerom (2010) suggest that delayed pass-through mostly occur at the wholesale rather than at the retail level. Nakamura (2008) studies a large panel data set of retailers in the US to analyse the pass-through of costs to wholesale and retail prices. Her results suggest that most of the observed price variation arises from retail-level4 rather than manufacturer-level demand and supply shocks: wholesale prices seem to be more sticky than retail prices. Gopinath and Itskhoki (2010) review the closed and open economy empirical literature on real rigidities and reveal a consistent finding across studies: the variable markup channel for real rigidities plays little role for retail prices but appears to be quite important for wholesale prices.5 Nakamura and Zerom (2010) study the pass-through of commodity price shocks in the coffee industry. They find that both for wholesale and retail prices, a 1% increase in coffee commodity costs lead to an increase in prices of approximately 0.3% over the subsequent 6 quarters. This implies that the majority of incomplete pass-through arises at the level of wholesale prices. Based on these findings, they argue that "it is wholesale price rigidity that matters" and that "studies that focus exclusively on retail prices may be incomplete in an important way".6

In this paper we investigate the implications of B2B long term relationships and bargaining for the response of prices and quantities to cost shocks. The central element of the model is the introduction of search and matching frictions a’ la Diamond-Mortensen-Pissarides in the relationship between wholesalers and retailers. Both retailers and wholesalers face search and matching costs to form new business relationships. Wholesalers need search effort (marketing, advertising and sale managers) to find new customers; retailers produce effort (e.g. by employing purchase agents) to find wholesalers to buy their products. The total amount of trade of intermediate goods depends on two margins: an extensive margin (the number of customers) and an intensive margin (the quantity exchanged in each match). The presence of search costs governs the response of the extensive margin

3Goldberg and Tille (2009) find that larger transactions are more likely to be invoiced in the importer's currency, and show that this is consistent with a model where currency invoicing is chosen in a bargaining between exporters and importers.

4Temporary sales are the main determinant of these variations.

5After reviewing the literature, Gopinath and Itskhoki (2010) use unpublished international price data and exchange rate shocks to evaluate the importance of real rigidities in price setting. They show that the pass-through of import prices to exchange rate shocks, even conditionally on changing, is very low and delayed. This suggests the presence of important real rigidities in the wholesale sector.

and creates a surplus related to each business relationship. Retailers and wholesalers bargain over this surplus and set wholesale prices and quantities according to their relative bargaining power.

The analysis proceed in two steps. We first restrict the analysis to the case in which intermediated trade only occurs along the extensive margin. Following a purely transitory cost shock, we get zero pass-through to retail prices and only partial (and proportional to the retailer’s bargaining power) pass-through to wholesale prices. Hence, in an environment where firms are hit by idiosyncratic cost shocks, our model yields price rigidity and time-varying markups. If the cost shock is persistent, the pass-through to retail prices increases, but remains quite low. Moreover, we show that the distinction between wholesalers and retailers is potentially important as cost shocks to retailers and/or wholesalers may have different implications for the dynamics of wholesale and retail prices.

We then open up the intensive margin and allow firms to bargain also over the quantity exchanged in each match. The introduction of an intensive margin of adjustment strongly affects the results. The degree of pass-through to wholesale and retail prices is found to be strictly related to three factors: (1) the relative bargaining power of retailers in the negotiations, (2) the persistence of the cost shock and (3) the elasticity of the demand of retailers for wholesale goods along the intensive margin. Interestingly, while retailers’ bargaining power has a strong and monotonic effect on the pass-through to wholesale prices, its influence on retail prices and consumption is non-linear, and rather limited. This happens because in the model bargaining power mainly affects the distribution of the rents related to a business relationship, while the reaction of retail prices ultimately depends on the costs of rapidly adjusting the marketing and distribution infrastructure needed to sell the final goods.

The model can be easily reconciled with the findings of Nakamura and Zerom (2010) if three conditions are satisfied: (1) the cost shock is sufficiently persistent, (2) the demand of retailers for the goods produced by wholesalers is not too elastic and (3) wholesalers have most of the bargaining power.

The repeated nature of the interactions between firms points towards an intriguing issue: observed wholesale prices may not be allocative, in the sense that they may not affect the retail prices faced by consumers nor their consumption decisions. This issue is very relevant, especially if one takes into account the recent empirical evidence, which suggests that nominal price stickyness arises mainly at the wholesale rather than at the retail level. As recognized at least since Barro (1977), the stabilizing role of monetary policy when prices are sticky crucially depends on prices being allocative. The business to business model provides a natural laboratory to address this issue.

We show that wholesale prices in our setup have no direct influence on the intensive margin of trade, but affect the value of business relationships and thus the incentive to engage in search activities. For this reason, the allocative power of wholesale prices depends
on the perceived persistence of the price change, and on the efficiency of the matching process. If wholesale price changes are long-lasting and search externalities are substantial, then wholesale prices still retain a large, and very persistent, allocative role. In all the other circumstances, the allocative power of wholesale prices is likely to be small, much smaller than in the standard monopolistic competition model.

The rest of the paper is structured as follows. Section 2 discusses the related literature. In Section 3 we derive the benchmark model. In Section 4 we analyse the role of search frictions by assuming that intermediate trade takes place only along the extensive margin. In Section 5 we open up the intensive margin and analyse how the results change when firms bargain over both prices and quantities. Section 6 addresses the issue of the allocative power of wholesale prices. Section 7 concludes.

2 Related literature

Very recent research has started to investigate the role of long term relationships in the interaction between firms and customers. Notable examples are Hall (2008), Arseneau and Chugh (2007), Kleshchelski and Vincent (2009) and Ravn et al. (2010). Hall (2008) develops a model of consumers’ search and seller recruiting where firms invest heavily in advertising in order to attract final consumers because they receive a large share of the surplus. He focuses on the magnitude and distribution of the rents associated with customer relationships and on the tightness of the retail markets under alternative distributions of the rents. Arseneau and Chugh (2007) derive a similar model of retailer-consumer relationships and explore the effects of different bargaining assumptions. They show that in the presence of search frictions prices play a distributive as well as an allocative role, and explore how concerns for fairness influence price dynamics. Notice that both Hall (2008) and Arseneau and Chugh (2007) focus on the relationship between final consumers and firms, and not on business to business relationships between firms, as we do here. This is conceptually an important difference, since a bilateral bargaining between firms is arguably more realistic than between retailers and consumers.

Of these papers, the ones more closely related to our are Kleshchelski and Vincent (2009) and Ravn et al. (2010), as they both focus on industry dynamics and provide theoretical explanations of the low pass-through of cost shocks to prices. Kleshchelski and Vincent (2009) construct a model in which firms care about the size of their consumer base, because consumers incur costs to switch sellers. Consequently, firms face an intertemporal trade-off between increasing current profits and building market shares for the future. Ravn et al. (2010) provide a theoretical explanation of the incomplete pass-through of marginal costs disturbances to prices based on a relative deep-habit demand for retail goods. When habits are formed at the level of individual goods, following a cost increase firms find it optimal to narrow profits margins in the current period to limit the decline in future
habitual demand triggered by the price increase. Kleshchelski and Vincent (2009) and Ravn et al. (2010) share with our approach the idea that firms form long-term relationships, but differ in two key respects: they focus on retail firms-consumer relationships, and do not allow for bilateral negotiations between buyers and sellers.

From a modeling perspective, our paper builds on the work of Drozd and Nosal (2010a) and Matha and Pierrard (2009). Drozd and Nosal (2010a) propose an international business cycle model where international trade takes place only through matches between retailers and intermediate goods producers. The wholesale price is set in a bilateral bargaining between producers and retailers. The model is found to perform well in replicating the movements of international prices and quantities. Matha and Pierrard (2009) extend a standard closed economy business cycle model allowing for search and matching frictions between wholesalers and retailers and bilateral bargaining. They investigate the cyclical properties of such a model, and find that the search and matching model is able to produce hump-shaped dynamics for all variables, a highly persistent output and a realistic representation of the product market variables such as search and prices. Three main aspects differentiate the present paper from Drozd and Nosal (2010a) and Matha and Pierrard (2009). First, we have a different focus, as we investigate the implications of business to business relationships and bargaining for the degree of pass-through of cost shocks to prices. Second, our analysis follows a partial equilibrium approach, as we focus only on industry dynamics. This approach has the advantage of analytical tractability and allows a closer match with the empirical work on cost pass-through. Third, and perhaps more importantly, we allow wholesalers and retailers to bargain not only over wholesale prices, but also over wholesale quantities. In Drozd and Nosal (2010a) and Matha and Pierrard (2009), in fact, firms can increase trade only by matching with new firms while the quantity of the intermediate good exchanged per match is exogenous. We endogenize the intensive margin and allow firms to increase trade also by adjusting the goods exchanged in a match. We will show that the intensive margin plays an important role in the degree of pass-through of cost shocks to prices. 

3 The model

In this section we develop a tractable model of the relationship between wholesalers and retailers. The model builds on the Diamond-Mortensen-Pissarides search and matching model and adapts its basic concepts to business to business relationships.

The economy is comprised of a continuum of sectors in the unit interval. In each

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7 More recently, in independent research, Gopinath and Itskhoki (2010) develop a static bargaining model between one final good producers and a number of intermediate good suppliers to provide a microfoundation for a quantitative model with variable markups at the wholesale level but constant markups at the retail level. Our paper shares with Gopinath and Itskhoki (2010) the idea that introducing negotiations between firms is important to understand pricing dynamics, but differs in many important aspects. Most importantly, their model is static, and abstracts from the need to build long term business relationships.
sector, there is a continuum of wholesalers and retailers. Goods are produced by wholesalers, transformed by retailers and consumed by households. Retailers sell the final good to consumers in a perfectly competitive environment. Trade frictions are present in the relationship between wholesalers and retailers. More precisely, as in Matha and Pierrard (2009) the product market consists of a two-sided search market between retailers and wholesalers. Wholesalers provide search effort (marketing or advertising expenditure) to find new buyers; retail firms provide search effort (e.g. by purchasing agents) to find new suppliers. Once buyers and sellers are matched, they set the wholesale price and the quantity exchanged per match in a bilateral Nash bargaining.

Following Ravn et al. (2010) and Kleshchelski and Vincent (2009), since we are interested in analysing firms dynamics in an industry, we study the model in partial equilibrium and analyse the effect of search frictions and bargaining on the dynamics of a single sector, sector $i$. This permits a closer link with the empirical literature on cost pass-through and may provide a potential guide for future work on pass through. Moreover, this approach has the advantage of analytical tractability and allows us to dig deeper into the main mechanism of the model.

### 3.1 Demand for retail goods

Following Kleshchelski and Vincent (2009), we assume the economy is composed of a continuum of sectors, each producing a good indexed by $i$. In each sector, there is an infinite number of firms, each selling a different brand $j$. While goods $i$ are imperfect substitutes, brands are homogeneous and perfectly substitutable.$^8$

The demand for the good produced in industry $i$ is given by

$$c^i_t = \left( \frac{p^i_t}{P_t} \right)^{-\phi} C_t$$

where $\phi$ is the elasticity of substitution between the goods produced in different industries. Following Ravn et al. (2010), since we focus only on industry dynamics taking as given the aggregate price and consumption levels $P_t$ and $C_t$, we simplify the demand function for the retail good $i$ to:

$$c_t = A (p_t)^{-\phi}$$

where $A$ is a positive constant. Notice that in the rest of the chapter, to simplify the notation, we drop the industry superscript $i$.

### 3.2 Wholesale firms

The industry $i$ is composed of a continuum of wholesale firms. In order to sell their products, wholesale producers need to establish customer relationships with retailers. We

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$^8$In this paper we use the term sector and industry interchangeably.
assume the aggregate number of business to business (B2B) relationships in industry \(i\), \(T_t\), follows the law of motion:

\[
T_{t+1} = (1 - \delta) \left( T_t + M_t \right)
\]

where \(\delta\) is the rate at which business relationships are destroyed, which we take as exogenous. \(M_t\), the number of new B2B relationships, is a constant return to scale function of the search effort of retailers \(d_t\) (e.g., from purchase managers) and the search effort \(a_t\) (advertising and marketing) by wholesalers:

\[
M_t = \tilde{m} \alpha_t \xi d_t^{1-\xi}
\]

Total B2B volumes depend on the number of relationships \(T_t\) (extensive margin) and the units bought for each relationship \(q_t\) (intensive margin). Wholesalers take as given \(k_a = \frac{M_t}{a_t} = \tilde{m} \left( \theta_t \right)^{-1-\xi}\), the number of new matches per unit of effort. \(\theta_t = \frac{a_t}{d_t}\) is the product market tightness of industry \(i\), defined as the ratio of advertisement effort per purchasing effort.

The law of motion of the customer base for wholesaler \(j\) in sector \(i\) is:

\[
T_t (j) = (1 - \delta) \left( T_{t-1} (j) + a_{t-1} (j) k_{t-1}^a \right)
\]

Notice that \(T_t (j)\) is a state variable, as it takes time (one month, under our calibration) to establish a business relationship.

The marginal cost of producing one intermediate variety, \(mc_t (j)\), is assumed to be exogenous and independent of scale. Moreover, wholesale firms face a search cost to establish new business relationships that is convex in the search intensity of wholesalers \(x_{wt} (j) = \frac{a(j)}{T_t (j)}\):

\[
\frac{\gamma}{2} (x_{wt} (j))^2 T_t (j)
\]

Firms are assumed to discount future profits at the constant rate \(\beta \in (0, 1)^{10}\).

Wholesalers maximize the expected present value of future profits

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ p_{Wt} (j) - mc_t (j) \right] q_t (j) T_t (j) - \frac{\gamma}{2} (x_{wt} (j))^2 T_t (j) \right\}
\]

subject to the law of motion of the customer base (1). At the beginning of the period the firm chooses the advertising effort \(x_{wt} (j)\); wholesale prices \(p_{Wt} (j)\) and quantities \(q_t (j)\)

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9 This specification of search costs, which has been used in the labor search literature by Gertler and Trigari (2009) and Thomas (2008), greatly simplifies the bargaining problem because it implies that the bargained price does not depend on the number of B2B relationships that each firm has in place. It thus permits to avoid the problem that in labor economics is known as intrafirm bargaining.

10 See also Ravn et al. (2010). In a full-fledged general equilibrium model, the discount factor of the firm would be an endogenous variable given by the representative household’s intertemporal marginal rate of substitution.
are decided after the successful match in a bilateral bargaining between wholesalers and retailers.

The solution to the maximization problem gives the following first order conditions:

\[
\gamma \frac{x_{wt}(j)}{k^R_t} = \beta (1 - \delta) E_t W_{t+1}(j) \quad (2)
\]

\[
W_t(j) = [p_{Wt}(j) - mc_t(j)] q_t(j) + \frac{\gamma}{2} (x_{wt}(j))^2 + \beta (1 - \delta) E_t W_{t+1}(j) \quad (3)
\]

The first condition equates the expected search cost of an additional match (the left hand side), to its expected benefit, which is given by the expected value of a business relationship. The second condition determines the value of an existing business relationship for a wholesale firm, \( W_t(j) \), which consists of the total profit from an established relationship \( (p_{Wt}(j) - mc_t(j)) q_t(j) \), plus the saving in the costs of establishing a new match, \( \frac{\gamma}{2} (x_{wt}(j))^2 \), plus the expected continuation value. Notice that the introduction of search frictions transforms the wholesale problem into an intertemporal problem, as both the search intensity and the value of an existing relationship depend on the expected future value of a business relationship.

### 3.3 Retail firms

In sector \( i \), there is a continuum of retailers buying tradable goods from wholesalers and selling them to households. As wholesalers, retailers choose at the beginning of the period the amount to invest in forming business relationships, captured by the search rate \( x_{Rt}(r) = \frac{d_t(r)}{d_t(r)} \). Retailers take as given the rate at which search effort leads to a new match, defined as:

\[
k_t^R = \frac{M_t}{d_t} = \tilde{m} \theta_t^c
\]

and the search cost to establish new matches, that is convex in the search intensity \( x_{Rt}(r) \):

\[
\frac{\gamma}{2} (x_{Rt}(r))^2 T_t(r)
\]

Once matched with wholesalers, each retailer \( r \) has a technology which transforms wholesale goods into retail goods. It is important to specify at this point that in order to introduce a meaningful intensive margin of adjustment, we need to introduce a cost of changing the quantity sold per match. If changing \( q_t(r) \) were costless, firms would find it optimal to have few matches (since it is costly to establish long-term relationships) and satisfy changes in demand with changes in \( q_t(r) \).

This would go against the spirit of our model, which is meant to be one in which firms must engage in search and matching in order to expand their production, and would make the problem not well-defined.

\[11\]

As we show in Section 3.8, the number of matches converges to zero if the production adjustment cost \( \psi \rightarrow 0 \).
To address this aspect, we introduce costs in changing the quantity sold per match through the production function of wholesalers. Specifically, we assume that for each match $k$, retailers have a technology that transforms $q_t(k)$ units of the wholesale good into $(q_t(k) - \omega_t(k))$ units of retail goods, where $\omega_t(k) = \frac{\psi(q_t(k) - \bar{q})^2}{2}$ is an adjustment cost in the units bought per match. Intuitively, $\bar{q}$ is the quantity per match that maximizes the efficiency of the production process of retailers. Deviations from this optimal amount per match decrease the marginal productivity of the wholesale variety. The aggregate production of retailer $r$ is thus given by:

$$y_t(r) = \frac{1}{z_t(r)} \int_0^{T_t(r)} (q_t(k) - \omega_t(k)) \, dk$$

$$= \frac{(q_t(r) - \omega_t(r))}{z_t(r)} T_t(r)$$

(4)

where we have imposed symmetry among matches and $z_t(r)$ is intended to capture a cost shock of transforming wholesale goods into retail goods. Notice that this production function has the following attractive features:

1) It displays diminishing returns to $q_t$ for deviations from the technically optimal level $\bar{q}$ both upwards and downwards.

2) It introduces an incentive for retailers to buy from different wholesalers (similar to a love for varieties).

3) It includes both the linear case and the extensive-margin-only case as special cases. More precisely, for $\psi \to 0$, the production function is linear in $q_t(k)$ and retailers can adjust their production on the intensive margin very easily. For $\psi \to \infty$, $q_t(k) = \bar{q}$ for all $t$, the intensive margin is closed, and firms can adjust production only by establishing new business relationships.

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12 One may think of many stories or microfoundations behind our production function. For example, one may assume that retail firms have a preference for having multiple suppliers in order to reduce the risks related to having only one supplier (e.g., the risk of a delay in delivery or the risk of bankruptcy of a supplier). Alternatively, one may think of retailers having several outlets being dispersed in the territory. In this case, they may prefer to establish relationships with local suppliers in order to reduce transportation or logistic costs.

13 A natural alternative would be to endogenize the intensive margin by assuming that each retailer buys differentiated goods from a range of wholesalers and has a “love of variety” motive (common in the trade literature) that leads him to value buying from many wholesalers in itself. The production function of retailers would be:

$$y_t = \frac{1}{z_t} \left[ \int_0^{T_t} q_{ti}^{\rho} \, di \right]^{1/\rho}$$

with $\rho < 1$. The main reason why we chose a different specification is that the production function (4) is more flexible, since it nests both the linear case ($\psi \to 0$) and the extensive-margin-only case ($\psi \to \infty$) as special cases. This allows us to analyse more neatly the role of the intensive margin of trade adjustment for the cost pass-through.
Retail firms maximize the expected present value of future profits

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ p_t y_t (r) - p_{W_t} (r) q_t (r) T_t (r) - \frac{\gamma}{2} (x_{R_t} (r))^2 T_t (r) \right\}$$

subject to the law of motion of the customer base

$$T_t (r) = (1 - \delta) (T_{t-1} (r) + d_{t-1} (r) k_{t-1}^R) \quad (5)$$

and the production function (4). Notice that since retail firms sell the final good in a perfectly competitive market, they take the final price of the retail good, $p_t$, as given in the maximization problem. The solution to the problem gives:

$$\frac{x_{R_t} (r)}{k_t^R} = \beta (1 - \delta) E_t J_{t+1} (r)$$

$$J_t (r) = \frac{p_t}{\omega_t (r)} (q_t (r) - \omega_t (r)) - p_{W_t} (r) q_t (r) + \frac{\gamma}{2} (x_{R_t} (r))^2 + \beta (1 - \delta) E_t J_{t+1} (r)$$

The first condition equates the expected search costs of an additional match, to its expected benefit, which is given by the expected value to a retailer of a B2B relationship. The second equation determines the value of a business relationship for a retailer, $J_t (r)$, which consists of the gross profits from an established relationship $\left( \frac{p_t (q_t (r) - \omega_t (r))}{\omega_t (r)} - p_{W_t} (r) q_t (r) \right)$, plus the saving in the costs of establishing a B2B relationship, plus the expected continuation value.

### 3.4 The bargaining problem

As emphasized by Hall (2005) and (2008), the presence of a surplus associated with existing long-term relationships implies that many wholesale prices (and quantities) are consistent with equilibrium. Existing B2B relationships are privately efficient as long as they generate a positive surplus for both the parties involved in the bargaining. Therefore, any price path such that $W_t (j) \geq 0$ and $J_t (r) \geq 0$ for all $t$ is consistent with equilibrium. This is an interesting insight because it admits the possibility, using Hall’s (2007) language, of equilibrium sticky prices in customer markets.14

In this paper, we follow the labor market literature and assume that the surplus sharing is a solution to a Nash (1950) bargaining problem. In Nash bargaining, each wholesaler $j$ and retailer $r$ jointly choose wholesale prices and quantities to maximize the Nash product $S_t (j, r)$ according to their relative bargaining power:

$$\arg \max_{p_{W_t}, q_t} S_t (j, r) = \left[ (W_t (j))^{1-\eta} (J_t (r))^\eta \right]$$

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14See also Blanchard and Gali (2010) for a similar argument in the context of a labor search model. Arseneau and Chugh (2007) exploit this insight and analyse the implications of different pricing schemes on the price dynamics in a model with consumer search.
where $\eta$ is the bargaining power of retailers. The solution to the maximization problem with respect to the wholesale price gives the optimal sharing rule

$$\eta W_t (j) = (1 - \eta) J_t (r) \quad (6)$$

which implies:

$$p_{Wt} (j, r) = \eta \left\{ mc_t (j) - \frac{\gamma (x_{wt} (j))^2}{2q_t (j, r)} \right\} + (1 - \eta) \left\{ \frac{p_t}{z_t (r)} \left( 1 - \frac{\omega_t (r)}{q_t (j, r)} \right) + \frac{\gamma (x_{Rt} (r))^2}{2q_t (j, r)} \right\} \quad (7)$$

The wholesale price depends not only on the costs of producers, but also on the valuation of retailers. The bargained price is a weighted average between two terms. The first, $mc_t (j) - \frac{\gamma (x_{wt} (j))^2}{2q_t (j, r)}$, represents the minimum amount that wholesalers are willing to accept, which depends on marginal costs and on the saving in the cost of forming another business relationship. The second term, $\frac{p_t}{z_t (r)} \left( 1 - \frac{\omega_t (r)}{q_t (j, r)} \right) + \frac{\gamma (x_{Rt} (r))^2}{2q_t (j, r)}$, represents the maximum price that retailers can accept, which is the sum of the marginal revenue obtained in the retail market and the saving in the costs of establishing another B2B relationship for retailers.

The weights on the two terms depend on the bargaining power of the two parties. If wholesalers have no bargaining power ($\eta = 1$), retailers get the entire surplus from a business relationship and $p_{Wt} (j, r)$ is strictly related to marginal costs. Vice versa if wholesalers have all bargaining power ($\eta = 0$), wholesalers get all the surplus from a relationship and $p_{Wt} (j, r)$ follows closely the evolution of retail prices. Wholesale prices thus play a distributive role, on top of the standard allocative role.

The optimal sharing rule (6) also implies:

$$\eta \gamma_{k_t} x_{wt} (j) = (1 - \eta) \frac{\gamma}{k_t} x_{Rt} (r)$$

Aggregating across all firms, this gives, in terms of log deviations:

$$\delta_t = 0$$

$$\alpha_t = \delta_t$$

The assumption of complete symmetry between the search problem of wholesalers and retailers implies a one to one relationship between changes in search effort by retailers ($\delta_t$) and wholesalers ($\alpha_t$). As a consequence, the product market tightness $\delta_t$ is invariant to shocks.$^{15}$

While the bargained price is set in a way to split the surplus between the two parties in proportion to their bargaining power, wholesalers and retailers choose $q_t (j, r)$ in a way to maximize the total surplus from a long term relationship. Specifically, the solution of

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$^{15}$The bargaining power shock that we study in Section 6 will break this tight link.
the maximization problem with respect to quantities gives:

\[
\frac{p_t}{z_t(r)} \psi (q_t (j, r) - \bar{q}) = \frac{p_t}{z_t(r)} - mc_t (j)
\]  

which states that the marginal benefit of an additional unit sold in the retail market, which is given by the total profit margin \( \frac{p_t}{z_t(r)} - mc_t (j) \), needs to be equal to the marginal cost of increasing the quantity per match \( q_t (j, r) \) above \( \bar{q} \), which is an increasing function of the adjustment cost parameter \( \psi \).

To get further intuition, we can rewrite (8) as:

\[
q_t (j, r) = \bar{q} + \frac{1}{\psi} \left( \frac{\mu_t^{\text{tot}} (j, r)}{\mu_t^{\text{tot}} (j, r)} - 1 \right)
\]  

where \( \mu_t^{\text{tot}} (j, r) = \frac{p_t}{z_t(r)mc_t(j)} \) is the total gross mark-up of retail prices over marginal costs. The volume of trade per match is an increasing function of the total profit margin of retailers and wholesalers. As long as \( \mu_t^{\text{tot}} (j, r) > 1 \), the bargained \( q_t (j, r) \) tends to be above \( \bar{q} \) because retailers and wholesalers agree on a production strategy that tries to exploit the market power related to the presence of search frictions. More importantly, notice that, since wholesalers and retailers decide together \( q_t (j, r) \) in order to maximize the total surplus of a match, the units traded in each match depend directly on the final retail price but are set independently from the wholesale price. This rises the important question of whether wholesale prices play a role in the allocation of resources in the economy; a question to which we will return later.

3.5  Aggregations

Industry level relations are found by aggregating across all retailers \( r \) and all wholesalers \( j \) under the assumption of complete symmetry across firms. For instance, the aggregate business to business dynamics are

\[
T_t = (1 - \delta) \left( T_{t-1} + a_{t-1}k_{t-1}^a \right)
\]

\[
= (1 - \delta) \left( T_{t-1} + d_{t-1}k_{t-1}^R \right)
\]

and the aggregate demand for the final good of industry \( i \) become:

\[
c_t = \left[ \int_0^1 c_t (r) \, dr \right] = A (p_t)^{-\phi} = y_t = \left[ \int_0^1 y_t (r) \, dr \right]
\]

All other equations are identical to the individual firm’s case and are therefore not repeated here.
3.6 Search externalities and the constrained efficient allocation

In a decentralized equilibrium, wholesalers and retailers decide their search intensity taking as given \( k_a^t \) and \( k_R^t \), the rates at which additional effort leads to a new match. Each firm thus sets its optimal amount of search without internalising the effects on other firms, with the result that the sum of all individual decisions is conducive to an aggregate suboptimal outcome.

The constrained efficient allocation can be found by solving the problem of a benevolent social planner who faces the same technological constraints and search frictions that are present in the decentralized economy. The solution of the social planner’s problem leads to the following result, which is further explained in the appendix\(^{16}\).

**Proposition 1** The decentralized equilibrium is constrained efficient only if the Hosios condition \( \eta = 1 - \xi \) holds.

**Proof.** In Appendix A. ■

Proposition 1 requires that each firm’s social and private gain from participating in the matching process be equal. When the retailers’ bargaining power, \( \eta \), is larger than the elasticity of the matching function with respect to retailers’ search activities, \( 1 - \xi \), retailers’ private gains from participating into the matching process are too large, and retailers overinvest in forming new business relationships, while wholesalers underinvest in it. The opposite happens for \( \eta < 1 - \xi \). Only when \( \eta = 1 - \xi \) firms internalize the congestions that they create in the product market in a way that leads to an efficient matching process.

3.7 The mark-up in the long run

The presence of search frictions makes the mark-up endogenous and time varying. If we define the total mark-up of the retail price over marginal production costs as \( \mu_{tot}^t = \frac{p_i^t}{z mc} \), its long run level is an increasing function of the search costs of retailers and wholesalers

\[
\mu_{tot}^t = \frac{q}{(q - \omega)} + \frac{\gamma}{z mc (q - \omega)} \left\{ x_w \left( \frac{a_1 k_a}{2} - \frac{x_w}{2} \right) + x_R \left( \frac{a_1}{k_R} - \frac{x_R}{2} \right) \right\}
\]

where \( a_1 = \frac{1 - (1 - \delta) \beta}{\beta (1 - \delta)} \) and, for reasonable calibrations, \( \left\{ a_1 \left( \frac{x_w}{k_a} + \frac{x_R}{k_R} \right) - \left( \frac{x_w^2}{2} + \frac{x_R^2}{2} \right) \right\} > 0 \). Notice that, because of product market imperfections, wholesalers and retailers enjoy a mark-up even though goods are perfect substitutes. As we show in the following section, this mark-up is decreasing in the steady state value of \( q \) while it is non-linear in the bargaining power of retailers \( \eta \).

\(^{16}\)To derive the constrained efficient allocation in a partial equilibrium setup, we follow Hosios (1990). See also Matha and Pierrard (2009) for a similar analysis in a general equilibrium setting.
The gross surplus from an existing relationship is split between retailers and wholesalers according to their relative bargaining power. Wholesalers get:

\[ \mu^W = \frac{p_W}{mc} = \eta \left\{ 1 - \frac{\gamma x^2_w}{2mcq} \right\} + (1 - \eta) \left\{ \frac{p}{zm} \frac{(q - \omega)}{q} + \frac{\gamma x^2_R}{2mcq} \right\} \]

which is increasing in the bargaining power of wholesalers \((1 - \eta)\), while retailers get

\[ \mu^R = \frac{p_t}{zp_W} = \frac{\mu^{tot}}{\mu^W} \]

which is increasing in \(\eta\).

### 3.8 Calibration and steady state

The model is calibrated at the monthly frequency. The discount rate \(\beta\) is set to 0.996. The elasticity of substitution across industries is set to \(\phi = 3.5\), consistent with the results of Nakamura and Zerom (2010), who find a median price elasticity of 3.46 in the coffee industry.\(^{17}\) The elasticity of the matching function to the marketing effort by wholesalers, \(\xi\), and the bargaining power of retailers, \(\eta\), are set to 0.5, as in Matha and Pierrard (2009).

We set the efficiency of the matching technology \(\bar{m} = 0.4\), which implies that the monthly rate at which search effort leads to new business relationships is \(k^R = 0.4\). The separation rate \(\delta\) is set to 0.10, which roughly corresponds to a quarterly rate of \(\delta = 0.25\), the value used by Matha and Pierrard (2009). The search effort parameter \(\gamma\) is chosen such that, when the intensive margin is closed (that is, for \(\psi \to \infty\)) the total mark-up on the final good is 1.10. This gives a value \(\gamma = 1.1996\).

Two crucial parameters in the determination of the steady are \(\psi\), which captures the curvature of the demand of retailers for the variety produced by each wholesaler, and \(\eta\), which represents the bargaining power of retailers. Table 1 shows how the steady state changes for different values of these parameters.\(^{18}\)

Consider first the impact of the adjustment costs along the intensive margin. For \(\psi = 100000\), the intensive margin is closed, \(q = \bar{q} = 1\) and the total markup of retail prices over marginal costs is 10 percent. Lowering \(\psi\) to 1 the model displays both an intensive and an extensive margin of adjustment. Firms optimally trade-off the costs of increasing production along the extensive margin (paying the search and matching cost) with the costs of increasing production along the intensive margin. The steady state stock of business relationships decreases while the quantity sold per match increases to \(q = 1.087 > \bar{q}\). The

---

\(^{17}\)Ravn et al. (2010) set \(\phi = 6\) and Kleshchelski and Vincent (2009) set \(\phi = 5\). As it will be clear later, in our setting a higher \(\phi\) reduces the pass-through to retail prices. Our main results are thus robust to changes in \(\phi\).

\(^{18}\)To perform the steady state analysis, we set \(\gamma = 1.1996\) as in the baseline calibration, and let the number of B2B relationships, the units sold per match and the wholesale and retail prices adjust endogenously to changes in \(\psi\) and \(\eta\). We set \(\eta = 0.5\) when we study the impact of adjustment costs on the steady state, while we set \(\psi = 1\) when we analyse the steady state effect of the bargaining power parameter \(\eta\).
higher \( q \) tends to depress prices and markups, which are now (slightly) smaller. If we set the adjustment costs \( \psi \) close to zero (\( \psi = 0.00001 \)) firms lose any incentive to engage in B2B relationships, as they find it optimal to have very few matches and satisfy changes in demand with changes in \( q_t \). The steady-state stock of B2B relationships goes down to \( T = 0.007 \) while the quantity per match goes to \( q = 141.32 \). The increase in \( q \) depresses prices and reduces markups, which are now close to zero. This shows the need to have some frictions along the intensive margin in order to explain why firms spend resources in building business relationships.

<table>
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<th>Adjustment costs</th>
<th>( T )</th>
<th>( q )</th>
<th>( \theta )</th>
<th>( p )</th>
<th>( p_W )</th>
<th>( \mu^R )</th>
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<td>1.046</td>
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<td>1.096</td>
</tr>
<tr>
<td>( \psi = 0.00001 )</td>
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<td>141.32</td>
<td>1.000</td>
<td>1.001</td>
<td>1.000</td>
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<th>1.132</th>
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<td>1.015</td>
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Table 1: Adjustment costs, bargaining power and the steady state

Consider now the role of the bargaining power of retailers, \( \eta \). In the baseline calibration, wholesalers and retailers have the same bargaining power (\( \eta = 0.5 \)), the number of B2B relationships is relatively high, and the total mark-up on a final product is around 10 percent. Intuitively, since in the market there are many buyers and many sellers searching for new customers (\( \theta = \frac{d}{a} = 1 \)), the product market is fluid and this facilitates the formation of new matches. Technically, the fact that we impose the retailers’ bargaining power to be equal to the elasticity of retailers’ search intensity in the matching function, i.e. \( \eta = 1 - \xi \), implies that the search externalities are internalized and that the matching process is Pareto efficient. As soon as we move \( \eta \) away from \( 1 - \xi = 0.5 \), the stock of business relationships decreases while the quantity exchanged per match and the total mark-up \( \mu^{tot} \) increase. This higher mark-up reflects the inefficiencies in the matching process. When wholesalers have most of the bargaining power (\( \eta = 0.1 \)), they get most of the surplus from a business relationship and have a strong incentive to invest in advertising and marketing activities. At the same time, the incentive of retailers to spend resources in searching new suppliers is very low. As a consequence, the product market is very ‘tight’ (\( \theta = 3 \)), the process of matching becomes sclerotic and the steady state number of B2B relationships decreases. Something similar - even though on the opposite side of the market - happens when retailers have most of the bargaining power (\( \eta = 0.9 \)). Interestingly, the assumption
of complete symmetry between the search problem of retailers and wholesalers implies that symmetric deviations from $\eta = 0.5$ upwards and downwards have identical effects on the stock of relationships and on the final retail price. The main difference lies in the evolution of the wholesale price: when $\eta$ is high, wholesale prices are low and most of the profits go to retailers; when $\eta$ is low, wholesale prices are high and wholesalers get most of the rents.

4 The role of the extensive margin

The pass-through of marginal cost shocks is incomplete if a one percent increase in marginal costs leads to a less than 1 percent increase in prices. To determine whether in our model pass-through is incomplete, we characterize the impulse responses of wholesale and retail prices to innovations in marginal costs. We assume that the marginal cost shock is industry-specific and follows an AR process of order 1

$$\tilde{mc}_t = \lambda \tilde{mc}_{t-1} + \varepsilon_t$$

where $\lambda \in [0, 1)$ denotes the serial correlation of marginal costs and $\varepsilon_t$ is an i.i.d. shock.

We start by analyzing the response along the extensive margin. To do so, we close the intensive margin of adjustment by letting $\psi \to \infty$. In this case, the quantity traded per match is fixed ($q_t = \bar{q} = 1$ for all $t$) and the bargained wholesale price simplifies to (in terms of log-deviations from steady state):

$$\hat{p}_W t = \eta \left\{ \frac{mc}{pW} \hat{mc}_t - \frac{\gamma x_w^2}{pW} \hat{x}_W t \right\} + (1 - \eta) \left\{ \frac{p}{zpW} (\hat{p}_t - \hat{z}_t) + \frac{\gamma x_R^2}{pW} \hat{x}_R t \right\}$$

where variables with hat denote log-deviations from steady state. Notice that, ceteris paribus, a rise in marginal costs $\hat{mc}_t$ or retail prices $\hat{p}_t$ tend to increase the bargained price $\hat{p}_W t$, while cost shocks to retailers ($\hat{z}_t$) tend to lower $\hat{p}_W t$.

We proceed in two steps. We initially restrict attention to purely transitory cost shocks ($\lambda = 0$), for which it is possible to find simple analytical solutions. We then study the response to persistent cost shocks.

4.1 Transitory cost shocks

When the marginal cost shock is purely transitory ($\lambda = 0$), the model becomes static because firms do not have incentives to change their search effort and the following lemma holds.

**Lemma 2** If $\psi \to \infty$, following a purely transitory marginal cost shock ($\lambda = 0$), the pass-through to retail prices is zero and the pass-through to wholesale prices is proportional to
the bargaining power of retailers:

\[
\begin{align*}
\hat{p}_t &= 0 \\
\hat{p}_{Wt} &= \eta \frac{mc}{pw} \tilde{mc}_t
\end{align*}
\]

**Proof.** In Appendix C. ■

The zero pass-through result in this model stems directly from the presence of search frictions. Notice in fact that when firms can only increase production by forming new business relationships, output becomes a state variable, that can change only with one month delay. Since through the demand function there is a one to one relationship between consumption and prices, the presence of matching frictions prevents consumption and retail prices to move on impact. At the same time, if the marginal cost shock is completely transitory, firms have no incentive to create/destroy B2B relationships by changing the search effort level, and they absorb the shock completely through mark-up movements. The wholesale price shares the burden of the markup adjustment between wholesalers and retailers according to their relative bargaining power.

At first sight, the idea that the reaction of wholesale prices to marginal cost shocks increases with the bargaining power of retailers may seem counterintuitive. One may have expected in fact that retailers would force wholesalers to absorb the shock without changing the bargained price \( p_W t \). To understand better this result, consider again eq. (11). The wholesale price depends on the reservation price of wholesalers and the reservation price of retailers. When wholesalers have most of the bargaining power, i.e. for \( \eta \to 0 \), they get most of the surplus from a business relationship and the wholesale price becomes strictly related to the retailers’ valuation of the wholesale good, \( \frac{p}{z_{pw}} (\hat{p}_t - \hat{z}_t) \). At the limit, marginal cost shocks do not affect wholesale prices. When retailers have most of the bargaining power, i.e. for large values of \( \eta \), the wholesale price becomes strictly related to the marginal cost of production of the wholesale good, \( \frac{mc}{pw} \tilde{mc}_t \). The reaction of wholesale prices to marginal cost shocks is in this case much stronger. At the limit, for \( \eta \to 1 \) we have \( \frac{mc}{pw} = 1 \) and the pass-through to wholesale prices is complete.

Table 2 displays the response of marginal costs, prices and mark-ups to a purely temporary one-percent increase in marginal costs (\( \lambda = 0 \)) under our baseline calibration. To help the comparison with existing models, we also include in Table 2 the results obtained in a standard Dixit-Stiglitz model and in the "pricing to habit" model proposed by Ravn, Schmitt-Grohe and Uribe (2010, denoted as R-SG-U in the Table).\(^19\)

\(^{19}\) All variables are measured in percent deviations from their respective steady-state values. The values for the "pricing to habit" model are taken by Ravn et al. (2010).
In the Dixit-Stiglitz model, prices move one for one with marginal costs and markups are unaffected by the disturbance: cost pass-through is complete.

In the "pricing to habit" model, firms increase retail prices but proportionally less than the increase in marginal costs. 81 percent of the increase in costs is passed to prices while 19 percent is absorbed by a (desired) markup adjustment. Incomplete pass-through in Ravn et al. (2010) is the consequence of an intertemporal tradeoff: increasing current prices prevents a strong decline of current profit margins but, at the same time, it leads to a decline in current sales and to a reduction in the stock of habitual demand, which weakens the strength of future demand.

The firm’s dynamics in the B2B model are quite different. In the period of impact, wholesale prices increase only by around 0.5 (the bargaining power of retailers) while the retail price remains fixed. The shock is fully absorbed through mark-up movements. In the following period, the marginal cost shock disappears and all variables return to their steady state level. Hence, in an environment where firms are hit by idiosyncratic cost shocks, our model yields price rigidity and time-varying markups.

It is interesting at this point to compare the results of a marginal cost shock to the ones obtained when the cost shock affects retailers rather than wholesalers, or when it affects both simultaneously. The following two lemmas summarize these results.

**Lemma 3** If \( \psi \to \infty \), following a purely transitory retail cost shock \( z_t \), the pass-through to retail and wholesale prices is:

\[
\hat{p}_t = \frac{1}{\phi} \hat{z}_t \\
\hat{p}_{WT} = -(1 - \eta) \left\{ \eta \frac{p}{z_{PW}} \frac{\phi - 1}{\phi} \hat{z}_t \right\}
\]

**Proof.** In Appendix C. \( \blacksquare \)

---

20More exactly, the wholesale price goes up by \( \eta \frac{mc}{PW} \approx 0.48 \) (See Lemma 1).
Lemma 4 If $\psi \to \infty$, following a purely transitory common cost shock $v_t$, the pass-through to retail and wholesale prices is:

$$
\hat{p}_t = \frac{1}{\phi} \hat{v}_t \\
\hat{p}_{Wt} = \left[ \eta \frac{mc}{pw} - (1 - \eta) \frac{p}{zpW} \frac{\phi - 1}{\phi} \right] \hat{v}_t
$$

Proof. In Appendix C. ■

Consider first a retail cost shock $z_t$ (one may think for example at a tax or regulation change that only affects retailers). An increase in $z_t$ affects directly the production of retailers, which decreases proportionally. This leads to an increase in retail prices, which is larger the lower is the elasticity of demand $\phi$. This is because when goods are more substitutable, an increase in the sectoral price relative to the aggregate price level results in a larger drop in consumption. Interestingly, the shock also affects the wholesale price that decreases with $\hat{z}_t$. Intuitively, an increase in retail costs reduces the surplus from a B2B relationship for retail firms which pass, through bargaining, part of this negative shock to wholesalers. The responsiveness of wholesale prices to a retail cost shock is higher the lower is the bargaining power of retailers, $\eta$.

Finally, consider the effect of a cost shock $\hat{v}_t$ that is common to wholesalers and retailers. Similarly to before, consumption decreases and the retail price increases on impact. The main difference is that now the pass-through to wholesale prices is very weak. To understand the reason, consider again the bargained wholesale price (eq. 11). When the cost is common to wholesalers and retailers, that is $\hat{mc}_t \equiv \hat{z}_t$, two offsetting effects appear. On the one side, the increase in the costs for wholesalers tends to increase the bargained price by $\eta \frac{mc}{pw}$. On the other side, the increase of retailers’ costs worsens retailers’ reservation price by $(1 - \eta) \frac{p}{zpW} \frac{\phi - 1}{\phi}$, and thus tends to reduce, ceteris paribus, wholesale prices. These two effects tend to offset each other and mitigate the response of wholesale prices to industry shocks. For this reason, when the shock is common to retailers and wholesalers inside an industry, our model predicts a much lower degree of pass-through of cost shocks to wholesale prices. More specifically, wholesale prices are positively correlated with common cost shocks when $\eta \frac{mc}{pw} > (1 - \eta) \frac{p}{zpW} \frac{\phi - 1}{\phi}$ and negatively correlated in the opposite case.

This analysis suggests that it may be important to distinguish in empirical work between shocks that affect wholesalers, retailers or both, as this may lead to different implications for retail and wholesale prices. For example, Nakamura (2008) finds that wholesale prices are less volatile than retail prices. The same is true in our model (without the need to introduce price rigidities) under two conditions: (1) the cost shocks to retailers and wholesalers are highly correlated and (2) retail cost shocks are predominant, and retailers have most of the bargaining power, i.e. $\eta \to 1$. 

20
4.2 Persistent cost shocks

In the previous section we saw that following a transitory shock, firms are reluctant to engage in costly search activity and prefer to absorb the shock through mark-up adjustments. When the cost shock is expected to persist over time, the results change considerably. In what follows, we restrict the analysis to marginal cost shocks since they are arguably the most relevant case and the most studied empirically.

Table 3 shows the impact of a mildly persistent marginal cost shock. Specifically, to make our results comparable to the ones obtained by Ravn et al. (2010), we set $\lambda = 0.5^3$. The important difference from the previous section is that when the shock is expected to last in the future, firms have incentives to react to the shock by reducing their search effort. On impact, advertising and marketing activities by wholesalers and retailers (captured by $a_t$) decrease by 1.23 percent. This, starting from the second period, reduces the stock of B2B relationships and the total production of the industry, and induces an increase of the retail price. The pass-through to retail prices is still zero on impact, because it takes one period to make the new business relationship operational, and remains quite low afterwards, because it is costly to adjust the marketing and distribution infrastructure needed to sell the final products. The pass-through to wholesale prices is almost proportional to the bargaining power of retailers on impact, and persists now longer over time.

Notice that the low degree of pass-through to retail prices stands in stark contrast with both the Dixit-Stiglitz model, where the pass-through is complete, and the "pricing to habit" model by Ravn et al. (2010), where the pass-through is almost complete. Such a low pass-through is not far from empirical estimates. For example Hellerstein (2008) find that, in the beer industry, firms pass-through an average of 11 percent of a foreign-cost shock to their retail prices. Nakamura and Zerom (2010) find that, in the coffee industry, the pass-through of a persistent cost shock to retail prices is around 10 percent in the first quarter and around 25 percent after six quarters.

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</table>

Table 3: Persistent marginal cost shock and pass-through

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There are two main reasons why the pass-through to retail prices is so low. First, the presence of search frictions make it quite costly and resource consuming to adjust along the extensive margin. Second, the marginal cost is not very persistent and fades away quite rapidly, reducing the incentive to engage in costly advertising and marketing activities.
5 The role of the intensive margin

In the B2B model, when trade can only occur at the extensive margin, the pass-through of cost disturbances to wholesale and retail prices is very low. This result stems from two features. On the one side, the presence of search frictions implies that firms find it difficult and costly to adjust the production process to shocks, as they can only increase production by establishing new business relationships, which is a costly and time-consuming process. As a consequence retail prices and quantities do not change as easily as in a frictionless world. On the other side, the introduction of bilateral bargaining between firms implies that the responsiveness of wholesale prices to shocks depend crucially on the negotiation capabilities of the parties involved. Wholesale prices play mainly a distributive role, as they determine which party of the negotiation gets a higher proportion of the rents, while changes in production mainly depend on the expected future profitability of the business, as captured by the value of each long-term relationship.

The previous results build on a quite strong assumption: trade among firms takes place only along the extensive margin. We now relax this assumption and study the effect of introducing the intensive margin on the degree of pass-through to prices. We start again analyzing the response to a purely transitory marginal cost shock.

5.1 Transitory cost shocks

If the marginal cost shock is purely transitory, it is again possible to find recursively a simple solution to the model. The following lemma summarizes the results.

Lemma 5 If marginal cost shocks $\hat{mc}_t$ are purely transitory, i.e. $\lambda = 0$, the solution of the model is:

$$
\hat{T}_t = 0 \\
\hat{q}_t = -B_q\hat{mc}_t \\
\hat{p}_t = -\frac{1}{\phi} \left( \hat{T}_t + \kappa \hat{q}_t \right) = \frac{\kappa}{\phi} B_q \hat{mc}_t \\
\hat{p}_{WT} = \eta \frac{mc}{PW} \hat{mc}_t + \left( 1 - \eta \right) \frac{p(q - \omega)}{z_{PW}q} \hat{p}_t - A_q \hat{q}_t
$$

where $B_q = \frac{1}{\psi \mu + \phi + \psi}$ captures the elasticity of $\hat{q}_t$ to changes in the total profit margin, $\kappa = \frac{q}{q - \omega} \left( 1 - \psi (q - \bar{q}) \right)$ captures the increase in retailers’ production related to an increase in $\hat{q}_t$, and $A_q \simeq \left( 1 - \eta \right) \frac{p(q - \omega)}{z_{PW}q} \kappa \phi$ represents the elasticity of the wholesale price to changes in $\hat{q}_t$.

---

22 $B_q$ is decreasing in $\psi$ and increasing in $\phi$. $B_q \to 0$ if $\psi \to \infty$. $A_q$ is decreasing in $\psi$ and $\eta$, and converges to 0 for $\psi \to \infty$ and for $\eta \to 1$. $\kappa$ is increasing in $\psi$ and $\kappa \to 1$ for $\psi \to \infty$. 

---
Proof. In Appendix D. ■

The key to understand the previous lemma is to notice that when the shock is expected to disappear in the future, firms do not have incentives to adjust along the extensive margin and the problem becomes static (i.e. $\hat{x}_{wt} = \hat{x}_{Rt} = \hat{T}_t = 0$). In this case, the response of retail quantities and prices depends on the adjustment costs along the intensive margin. The lower is the adjustment cost $\psi$, the easier it is for retailers to adjust their production and distribution structure, the larger is the elasticity of $\hat{q}_t$ to changes in $\mathfrak{m}C_t$. In turn, a strong reduction in the production of retail goods increases retail prices with an elasticity that depends on $\phi$, the elasticity of the demand for the good produced in the industry. Ceteris paribus, the lower is $\phi$, the higher the pass-through to retail prices. The pass-through to retail prices is complete only if the adjustment along the intensive margin is completely frictionless, i.e. if $\psi \to 0$.

Wholesale prices are affected by three channels. First, there is the direct 'marginal cost channel', captured by $\eta_{\text{mc}}$ in Lemma 5. This term captures the direct influence of the marginal costs of wholesalers on the bargained price and is higher, the higher is the bargaining power of retailers. The second channel is related to the retailers’ reservation price and is captured by $(1 - \eta) \frac{p(q-\omega)}{z_{pWq}} B_q$ in Lemma 5. This term is larger, the more retail prices react to cost shocks or the higher is the bargaining power of wholesalers. The final term captures the 'bargained quantity effect' and is represented by $A_q B_q$. This term captures the fact that wholesalers are willing to offer to retailers a lower price - a sort of discount - if retailers accept to buy more units of the intermediate good. An increase in marginal costs provokes a reduction in $\hat{q}_t$, which leads, through the 'bargained quantity effect', to an increase in the wholesale price $\hat{p}_{Wt}$. This effect is stronger the lower are $\psi$ and $\eta$. The combined effect of these three channels implies that the pass-through to wholesale prices is complete when two conditions are met: (1) when the adjustment costs go to zero, $\psi \to 0$ or (2) when retailers have all the bargaining power, $\eta \to 1$.

Importantly, wholesale prices in this case play only a distributive role, not an allocative one. Notice in fact that the dynamics of the prices and quantities of the retail goods ($\hat{p}_t = -\frac{\delta}{\kappa} \hat{q}_t$ and $\hat{y}_t = \kappa \hat{q}_t$) do not depend on the evolution of wholesale prices, which only play the role of distributing the rents among wholesalers and retailers. As suggested by Barro (1977), this may have important policy implications.
this case strictly related to three factors: (1) the relative bargaining power of retailers in the negotiations, (2) the elasticity of the demand of retailers for wholesale goods along the intensive margin and (3) the persistence of the cost shock itself. We now turn to analyze the effect of each of these factors.

5.2 Persistent cost shocks

Figure 1 shows the impact of marginal cost shocks for different values of the persistence parameter \( \lambda \). The impulse responses are drawn for \( \psi = 1 \).

The degree of pass-through and - especially - the persistence of the price increase are strongly increasing in the persistence of the cost shock. The more persistent the shock is, the larger the incentive for firms to react by reducing the advertising and marketing effort. If the shock is transitory, firms are reluctant to reduce their advertising and marketing effort, as they expect costs to go back quickly to their normal level. If the shock is persistent, firms do not mind losing business relationships, because cost conditions are expected not to be favorable for many periods. For the same reason, the persistence of the shock determines crucially whether firms are willing to absorb the disturbance through the intensive or the extensive margin. When the shock is temporary, most of the adjustment goes through the intensive margin. The higher the persistence of the shock, the more the adjustment goes through the extensive margin.\(^{23}\)

\(^{23}\) This is consistent with the empirical evidence of Ruhl (2008), who finds that the extensive margin of trade responds to permanent but not to transitory shocks.
5.3 The role of adjustment costs on the intensive margin

Figure 2 shows the effect of production adjustment costs on the dynamics of the model following a persistent marginal cost shock. Consistent with the findings of Nakamura and Zerom (2010) that cost shocks in the coffee industry are highly persistent, we set $\lambda = 0.95$.\(^{24}\)

We present three cases. For $\psi = 100000$, firms are allowed to adjust production only at the extensive margin. For $\psi = 0.1$, retailers can adjust production easily along the intensive margin. $\psi = 1$ presents an intermediate case.

The degree of pass-through to wholesale and retail prices is profoundly affected by the curvature of retailers' demand on the intensive margin, as captured by $\psi$. Pass-through to wholesale and retail prices is low - and delayed - for medium to high level of adjustment costs ($\psi = 1$ or $\psi = 100000$) while it increases considerably when adjusting the quantity traded per match is relatively cheap. The introduction of an intensive margin allows firms to adjust production much faster to marginal cost shocks and thus increases the responsiveness of retail prices (and consequently of wholesale prices) to cost disturbances. Notice however that, for reasonable calibrations, introducing an intensive margin is not enough to generate complete pass-through: pass-through to retail prices remains below 0.6 even when $\psi = 0.1$.\(^{25}\)

5.4 The role of bargaining power

To understand the effects of bargaining power on the dynamics of the model, Figure 3 draws the cost pass-through to wholesale and retail prices for different values of $\eta$. In the first column of Fig. 3 the pass-through is computed as the *impact* response of prices to a one percent change in marginal costs. In the second column, the pass-through is computed as the response of prices to a marginal cost shock *after one year.*

The bargaining power of retailers affects differently the pass-through to wholesale and retail prices. The pass-through to wholesale prices is increasing in $\eta$, both on impact and after one year. The pass-through to retail prices, instead, is non-monotonic in $\eta$: it is maximum when the Hosios condition is met, and decreases symmetrically as we move away from $\eta = 1 - \xi$. Interestingly, for $\eta < 1 - \xi$ a higher pass-through to wholesale prices translates into a higher pass-through to retail prices, while for $\eta > 1 - \xi$ a higher pass-through to wholesale prices goes together with a reduction of the pass-through to retail prices.

These results are explained almost entirely by the presence of search externalities (see Fig. 6 in the Appendix). When $\eta = 1 - \xi = 0.5$, the search externalities are internalized

\(^{24}\)Nakamura and Zerom (2010) find that, in the coffee industry, a Dickey-Fuller test for the hypothesis of a unit root cannot be rejected at the 5% level. For simplicity, we focus here on very persistent, but stationary, cost processes.

\(^{25}\)In our model there are two ways to achieve complete pass-through to both retail and wholesale prices. The first way is to eliminate the curvature on $q$, i.e. let $\psi \to 0$. The second way is to eliminate search frictions, i.e. let $\gamma \to 0$. 

---

---
Figure 2: Intensive margin and pass-through

Figure 3: Bargaining power and pass-through
and the matching process is Pareto efficient. The large variation along the extensive margin is what leads to a larger pass-through to retail prices. When retailers have most of the bargaining power ($\eta = 0.9$), instead, the product market is tight, the process of matching is sclerotic and the variation along the extensive margin more expensive. Similarly, when wholesalers have most of the bargaining power ($\eta = 0.1$), too many sellers chase too few buyers. Overall, however, the effect of $\eta$ on the pass-through to retail goods is small compared with its effect on the pass-through to wholesale prices. This raises again questions about the allocative role of wholesale prices in our model.

5.5 Reconciling the model with Nakamura and Zerom (2010)

Nakamura and Zerom (2010), studying the coffee industry, find that (1) the pass-through of cost shocks to wholesale and retail prices is quite low (around 0.25 percent for each); (2) it is delayed, in the sense that most of the adjustment takes place in the second quarter and (3) most of the delayed pass-through occurs at the wholesale level, in the sense that wholesale and retail prices move very closely together.

The coffee market presents features that are not captured perfectly by our model. In particular, in the coffee market there are a few large wholesalers with some market power, a feature from which we abstract here. Nevertheless, if we assume that the findings of Nakamura and Zerom (2010) are common features among many markets, as the findings by Gopinath and Itskhoki (2010) seem to suggest, it is interesting to determine whether, and under which conditions, our model can address the above facts even without assuming explicitly price stickyness.

We find three conditions to be important to account for these three facts. First, we need persistent marginal cost shocks, as the ones studied by Nakamura and Zerom (2010), in order to account for delayed pass-through. We set $\lambda = 0.95$. Second, we need wholesalers to have most of the bargaining power. When wholesalers have high bargaining power, in fact, wholesale prices are closely related to retail prices as wholesalers internalize most of the surplus from a match. We set $\eta = 0.1$. Third, we need a relatively strong curvature along the intensive margin ($\psi = 10$), that prevents quantities to change strongly on impact. Table 4, which displays the evolution of the key variable of the model under the three conditions, shows that our model can, under the above conditions, account reasonably well for the three facts mentioned above.

6 Are wholesale prices allocative?

The repeated nature of the interactions between firms points towards a very interesting issue: observed wholesale prices may not be allocative, in the sense that they may not affect
Table 4: Reconciling the model with Nakamura and Zerom (2010)

<table>
<thead>
<tr>
<th>Month</th>
<th>( p_{Wt} )</th>
<th>( p_t )</th>
<th>( q_t )</th>
<th>( T_t )</th>
<th>( a_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.116</td>
<td>0.020</td>
<td>-0.083</td>
<td>0</td>
<td>-1.892</td>
</tr>
<tr>
<td>3</td>
<td>0.223</td>
<td>0.154</td>
<td>-0.060</td>
<td>-0.488</td>
<td>-1.640</td>
</tr>
<tr>
<td>6</td>
<td>0.283</td>
<td>0.233</td>
<td>-0.043</td>
<td>-0.778</td>
<td>-1.419</td>
</tr>
<tr>
<td>9</td>
<td>0.309</td>
<td>0.274</td>
<td>-0.030</td>
<td>-0.932</td>
<td>-1.226</td>
</tr>
<tr>
<td>12</td>
<td>0.314</td>
<td>0.290</td>
<td>-0.021</td>
<td>-0.995</td>
<td>-1.058</td>
</tr>
</tbody>
</table>

the retail prices faced by consumers nor their consumption decisions. This issue is likely to have important policy implications given that the recent empirical evidence suggests that nominal price stickyness arises mainly at the wholesale rather than at the retail level. Indeed, as recognized at least since Barro (1977), the stabilizing role of monetary policy when prices are sticky crucially depends on prices being allocative.

In our model, the allocative power of wholesale prices depends on the persistence of the price change. When the price change is purely transitory, wholesale prices play only a distributive but not an allocative role\(^{26}\). When the price change is expected to last into the future, wholesale prices potentially play an allocative role on top of the distributive role. This happens because the incentives for firms to engage in costly search activities depend on the expected benefits of a B2B relationship, which are in turn influenced by the future expected wholesale price. The questions that remain to be addressed are: how does it work, and how big is this allocative role of wholesale prices?

To answer these questions, we analyse what happens into the model when the relative bargaining power of the firms changes stochastically. Specifically, we assume that retailers’ bargaining power varies according to the law of motion:

\[
\hat{\eta}_t = \lambda_\eta \hat{\eta}_{t-1} + u_t
\]

where \( \lambda_\eta \in [0, 1) \) denotes the serial correlation of bargaining power shocks and \( u_t \) is an i.i.d. shock. The introduction of the bargaining shock only affects the evolution of the wholesale price, which is now determined as

\[
\hat{p}_{Wt} = (1 - \eta) \frac{p(q - \omega)}{zpqW} (\hat{p}_t - \hat{z}_t) + \eta \frac{mc}{pqW} \hat{mc}_t + (1 - \eta) \frac{\gamma x_{Rt}^2}{pqW} \hat{x}_{Rt} - \eta \frac{\gamma x_{Wt}^2}{pqW} \hat{x}_{Wt} - A_\eta \hat{q}_t - A_\eta \hat{a}_t
\]

where \( A_\eta = \frac{1}{p_{Wt}} \{ 1 - \beta (1 - \delta) \lambda_\eta \} \).\(^{27}\) Notice that a negative shock to the bargaining power

\(^{26}\)See Corollary 8 in Section 5.1 and Figure 7 in the Appendix.

\(^{27}\)Notice that the persistence of the bargaining power shock \( \lambda_\eta \) reduces, ceteris paribus, the response of wholesale prices to the bargaining power shock. This is a consequence of the repeated nature of the interactions between firms which leads firms to take into account, in the negotiations, the expected continuation value of a match. Retailers, for instance, are willing to accept a higher wholesale price today if they expect to get a high share of the surplus in the future.
of retailers raises \( \hat{p}_{W,t} \) and is thus equivalent to an exogenous shock to wholesale prices.

Figure 4 shows the model’s dynamics in response to a negative bargaining power shock. The persistence of the shock is set to \( \lambda_\eta = 0.95 \). In order to facilitate the comparison of the results, the bargaining shock is scaled such that, independently of the calibration, wholesale prices increase by one percent on impact.

A persistent increase in wholesale prices raises the expected value of business relationships to wholesalers while reduces the one to retailers. For this reason, wholesalers increase their search intensity while retailers reduce it; but the strength of these responses changes with the initial bargaining power of the parties. When wholesalers have most of the bargaining power (\( \eta = 0.1 < 1 - \xi \)), the product market is very tight on the side of wholesalers, and the bargaining power shock only worsens the situation, leading to a drop-out of a significant fraction of searching retailers. The formation of new matches is strongly reduced, and is only partially offset by the increase in the units sold per match. Total consumption decreases and the pass-through to retail prices is positive, but delayed. On the contrary, when wholesalers are the weak party in the negotiations (\( \eta = 0.9 > 1 - \xi \)), the bargaining power shock reduces the tightness of the market, and improves the efficiency of the matching process. The number of business relationships increases, leading to an increase in consumption and to a reduction of retail prices. When the Hosios condition is verified (\( \eta = 0.5 = 1 - \xi \)), the additional search effort by wholesalers exactly offset the reduction of retailers’ search effort, and the stock of business relationships, final consumption and retail prices are unaffected. Wholesale prices do not have any allocative power in this limiting
To assess how big is the allocative role of wholesale prices, Figure 5 compares the effects of wholesale price increases in the B-2-B model with the ones obtained in the Dixit-Stiglitz (1977) monopolistic competition model. In the B-2-B model the allocative power of wholesale prices, as measured by the reaction of final consumption to a one percent wholesale price shock, can still be quite large - even larger than in the Dixit-Stiglitz model - if search externalities are substantial. However, while in the standard monopolistic competition model consumption reacts strongly on impact, and then decays very fast, in the B-2-B model the reaction is delayed, and much more persistent, with a maximum which is reached, under our calibration, only after 15 months.

These results suggest two conclusions regarding the allocative power of wholesale prices. First, persistent wholesale price changes still retain some signalling power also in the presence of long-term contracts and efficient bargaining, but this effect works entirely through the incentives for firms to engage in costly advertising and purchasing activities. For this reason, the effect is considerably delayed and much more persistent than in the standard monopolistic competition model. Second, the effect of wholesale price changes depends on the presence and evolution of search externalities: when $\eta < 1 - \xi$ wholesale price changes...
lead to an increase in retail prices and a reduction in final consumption, as in the standard monopolistic competition model; when $\eta > 1 - \xi$, instead, an increase in wholesale prices reduces retail prices and increases final consumption.

7 Conclusion

In this paper, we have derived a simple model of wholesalers-retailers relationships and we have demonstrated that dynamic frictions of building business relationships have the potential to explain the low and delayed pass-through to wholesale prices that we observe in many empirical studies. This result stems from two main features. On the one side, the presence of search frictions implies that firms find it difficult to rapidly adjust the production and distribution process to shocks, as to increase production they need to establish new business relationships, which is a costly and time-consuming process. As a consequence retail prices and quantities do not change as easily as in a frictionless world. On the other side, the introduction of bilateral bargaining between firms implies that the responsiveness of wholesale prices to shocks depends crucially on the negotiation capabilities of the parties involved. Specifically, we show that the pass-through to wholesale prices is strongly increasing in the bargaining power of retailers, a somehow counterintuitive result.

Our analysis can be extended along several dimensions. From the modelling side, it would be interesting to incorporate negotiation costs into the bargaining problem, or allow for infrequent negotiations. This would be coherent with the evidence that most contracts among firms have a duration of 1 year, and would naturally lead to real effects of nominal shocks. Moreover, the model can be easily incorporated in full-fledged general equilibrium models. This would allow us to study how long-term contracts and bargaining between firms affect the dynamics of modern economies. From the empirical side, this work provides a number of testable implications. Our model predicts that the pass-through to wholesale prices should be higher in sectors where retailers have high bargaining power, while this does not need to be true for retail prices. Moreover, our analysis suggests that the pass-through to both retail and wholesale prices should be higher in sectors where shocks tend to be more persistent, or where it is easier for firms to adjust production along the intensive margin. We plan to test empirically these hypotheses in future research.
References


Appendix

A. Constrained Efficient Allocation

To derive the constrained efficient allocation in a partial equilibrium setup, we follow Hosios (1990). We define the constrained efficient allocation as the optimal allocation a social planner may achieve as a market equilibrium\(^29\). This allocation can be found by solving the problem of a benevolent social planner who faces the same technological constraints and search frictions that are present in the decentralized economy. The implicit assumption is thus that the social planner is not able to circumvent the search frictions required to form a match; he can however internalize the effect of variations in product market tightness on search costs and on the resource constraint.

Proposition 9 The decentralized equilibrium is constrained efficient only if \(\eta = 1 - \xi\) (Hosios condition).

Proof. The social planner chooses \(\{y_t, q_t, T_t, a_t, d_t\}\) to maximize

\[
\max_{\{y_t, q_t, T_t, a_t, d_t\}} \beta t \left\{ p_t y_t - mc_t q_t T_t - \frac{\gamma}{2} \left( \frac{d_t}{T_t} \right)^2 T_t - \frac{\gamma}{2} \left( \frac{a_t}{T_t} \right)^2 T_t \right\}
\] (13)

subject to the technological constraints on the extensive (matching frictions) and intensive margin (adjutment costs):

\[
T_t = (1 - \delta) \left( T_{t-1} + \bar{m}_a^{\xi} d_{t-1}^{1-\xi} \right)
\]

\[
y_t = \left( q_t - \psi (q_t - \bar{q})^2 \right) T_t \frac{z_t}{z_t}
\]

Notice that in (13) we have used the fact that, given symmetry in preferences and technology, efficiency requires that identical quantities of each good be produced by each wholesaler and retailer. The social planner problem gives the following first order conditions:

\[
\frac{p_t}{z_t} \psi (q_t - \bar{q}) = \frac{p_t}{z_t} - mc_t
\] (14)

\[
\tau_t = \frac{p_t}{z_t} \left( q_t - \psi (q_t - \bar{q})^2 \right) - mc_t q_t + \frac{\gamma}{2} x_{Rt} + \frac{\gamma}{2} x_{wt} + \beta (1 - \delta) \tau_{t+1}
\] (15)

\[
\frac{\gamma x_{Wt}}{\bar{m}_a^{(1-\xi)}} = \xi \beta (1 - \delta) \tau_{t+1}
\] (16)

\[
\frac{\gamma x_{Rt}}{\bar{m}_a^{\xi}} = (1 - \xi) \beta (1 - \delta) \tau_{t+1}
\] (17)

\(^{29}\)See also Mas-Colell, Whinston and Green (1995).
where $\tau_t$ captures the social value of a match. We can now compare it with the first order conditions of the decentralized solution, which can be rewritten as:

$$\frac{p_t}{z_t} \psi (q_t - \bar{q}) = \frac{p_t}{z_t} - mc_t$$  \hspace{1cm} (18)$$

$$\tau_t = \frac{p_t}{z_t} \left( q_t - \psi (q_t - \bar{q})^2 \right) - mc_t q_t + \frac{\gamma}{2} x_{wt}^2 + \frac{\gamma}{2} x_{Rt}^2 + (1 - \delta) \beta\tau_{t+1}$$ \hspace{1cm} (19)$$

$$\frac{\gamma x_{wt}}{\bar{m} \theta_t^{0.99(1-\xi)}} = \beta (1 - \delta) W_{t+1} = (1 - \eta) \beta (1 - \delta) \tau_{t+1}$$ \hspace{1cm} (20)$$

$$\frac{\gamma x_{Rt}}{\bar{m} \theta_t^{0.99}} = \beta (1 - \delta) J_{t+1} = \eta \beta (1 - \delta) \tau_{t+1}$$ \hspace{1cm} (21)$$

Comparing (14) – (17) with (18) – (21), it is easy to show that the condition $1 - \eta = \xi$ is necessary and sufficient for the equivalence of the constrained efficient and the decentralized solution.

**B. The benchmark model in log deviations**

The model is solved log-linearizing around the steady state. The resulting system of equation can be reduced to the following:

- **Wholesale prices**
  $$\hat{p}_{Wt} = \eta \left\{ \frac{mc}{pw} m_{ct} - \frac{\gamma x_{wt}}{pw}\hat{x}_{Wt} \right\} + (1 - \eta) \left\{ \frac{p (q - \omega)}{pwzq} (\hat{p}_t - \hat{z}_t) + \frac{\gamma x_{Rt}}{pwq}\hat{x}_{Rt} \right\} - A_q \hat{q}_t$$ \hspace{1cm} (22)$$

- **Bargained quantities**
  $$\hat{q}_t = \frac{1}{\psi} \frac{zmc}{pq} (\hat{p}_t - \hat{z}_t - \bar{mc}_t) = \frac{1}{\psi} \frac{zmc}{pq} (\hat{p}_t^{tot})$$ \hspace{1cm} (23)$$

- **Law of motion business to business relationships:**
  $$\hat{a}_t - (1 - \xi) \hat{a}_t = \frac{1}{\delta} \hat{T}_{t+1} - \frac{1 - \delta}{\delta} \hat{T}_t$$

- **Market clearing condition**
  $$-\phi \hat{p}_t = \hat{T}_t - \hat{z}_t + \kappa \hat{q}_t = \hat{y}_t$$ \hspace{1cm} (24)$$
• **Product market tightness and search intensities**

\[
\begin{align*}
\hat{\theta}_t &= \hat{\alpha}_t - \hat{\delta}_t = \hat{\theta}_t^R - \hat{\theta}_t^a = \hat{x}_{ut} - \hat{x}_{Rt}^i \\
\hat{x}_{ut} &= \hat{\alpha}_t - \hat{T}_t \\
\hat{x}_{Rt} &= \hat{\delta}_t - \hat{T}_t
\end{align*}
\]

• **Wholesalers: search condition**

\[
\hat{x}_{ut} - \hat{k}_t^a = \hat{x}_{ut} + (1 - \xi) \hat{\theta}_t = E_t \hat{W}_{t+1}
\]

• **Retailers: search condition**

\[
\hat{x}_{Rt} - \hat{k}_t^R = \hat{x}_{Rt} - \xi \hat{\theta}_t = E_t \hat{J}_{t+1}
\]

• **Wholesalers: value of a match**

\[
\hat{W}_t = p_W q_W \hat{p}_{Wt} - mcq \hat{m}_c - (p_W - mc) q_W \hat{q}_t + \gamma x_w^2 \hat{x}_{Wt} + (1 - \delta) \beta W E_t \hat{W}_{t+1}
\]

• **Retailers: value of a match**

\[
\begin{align*}
\hat{J}_t &= -p_W q_W \hat{p}_{Wt} + \frac{p(q - \omega)}{z} (\hat{p}_t - \hat{z}_t) + \gamma x_R^2 \hat{x}_{Rt} + (1 - \delta) \beta J E_t \hat{J}_{t+1} \\
&\quad - \left( p_W q - \frac{p(q - \omega)}{z} \right) \hat{q}_t
\end{align*}
\]

**C. Proof of Lemma 1, Lemma 2 and Lemma 3**

Consider the case in which the intensive margin is closed, i.e. \( \psi \rightarrow \infty \). Using \( \hat{q}_t = 0 \) and \( \hat{\theta}_t = 0 \), the model in log-deviations simplifies to:

\[
\begin{align*}
\hat{q}_t &= 0 \\
-\phi \hat{p}_t &= \hat{T}_t - \hat{z}_t \\
p_W q_W \hat{p}_{Wt} &= \eta mcq \hat{m}_c - (1 - \eta) \left\{ \frac{p(q - \omega)}{z} \phi - 1 \hat{z}_t \right\} \\
\hat{x}_{ut} &= E_t \hat{W}_{t+1} = \hat{x}_{Rt} = E_t \hat{J}_{t+1} \\
W \hat{W}_t &= p_W q_W \hat{p}_{Wt} - mcq \hat{m}_c + \gamma x_w^2 \hat{x}_{Wt} + (1 - \delta) \beta W E_t \hat{W}_{t+1} \\
\hat{J}_t &= -p_W q_W \hat{p}_{Wt} + \frac{p(q - \omega)}{z} (\hat{p}_t - \hat{z}_t) + \gamma x_R^2 \hat{x}_{Rt} + (1 - \delta) \beta J E_t \hat{J}_{t+1}
\end{align*}
\]

If the marginal cost shocks and the retail cost shocks are purely transitory, i.e. \( \lambda_{mc} = \lambda_z = 0 \), the model has a simple solution. When the shock is purely transitory, in fact, it does not affect the expected future value of a business relationship and thus wholesalers
and retailers do not have incentives to vary their search intensity ($\tilde{x}_{wt} = E_t \tilde{W}_{t+1} = \tilde{x}_{Rt} = E_t \tilde{J}_{t+1} = 0$). This in turn implies that the number of B2B relationships is not affected by the shock ($\tilde{J}_t = 0$). In other words, a purely transitory shock does not lead to intertemporal substitution and the model becomes static. The solution of the model is:

$$\begin{align*}
\hat{q}_t &= 0 \\
\hat{x}_{wt} &= \hat{x}_{Rt} = \hat{T}_t = 0 \\
\hat{p}_t &= \frac{1}{\phi} \hat{z}_t \\
\hat{p}_{Wt} &= \eta \frac{mc}{p_W} \hat{mc}_t - (1 - \eta) \left\{ \frac{p}{zp_W} \frac{\phi - 1}{\phi} \hat{z}_t \right\}
\end{align*}$$

Lemma 1, Lemma 2 and Lemma 3 follow by focusing the attention on (1) a marginal cost shock $\hat{mc}_t$, (2) a retail shock $\hat{z}_t$ or (3) a common shock $\hat{v}_t$, defined as $\hat{v}_t = \hat{mc}_t = \hat{z}_t$.

**D. Proof of Lemma 4**

While in the main text we focus only on the marginal cost shock $\hat{mc}_t$, in this appendix we provide the complete solution to the model when both marginal cost shocks $\hat{mc}_t$ and retail shocks $\hat{z}_t$ are purely transitory.

Consider the complete log-linearized model in Appendix A. If the marginal cost shocks and the retail cost shocks are purely transitory, i.e. $\lambda_{mc} = \lambda_z = 0$, it is possible to find recursively a relatively simple solution to the model. The key is again to notice that when the shock is expected to disappear in the future, firms do not have incentives to adjust along the extensive margin and the problem becomes static (i.e. $\tilde{x}_{wt} = \tilde{x}_{Rt} = \tilde{T}_t = 0$).

From (24) we can write:

$$\hat{p}_t = -\frac{1}{\phi} \left( \hat{T}_t - \hat{z}_t + \kappa \hat{q}_t \right) = \frac{1}{\phi} \left( \hat{z}_t - \kappa \hat{q}_t \right)$$

(25)

where $\kappa$ captures the curvature of the production function of retailers with respect to $\hat{q}_t$. Introduce (25) into (23) to get:

$$\begin{align*}
\hat{q}_t &= \frac{1}{\psi} \frac{zmc}{pq} \hat{p}_t - \frac{1}{\psi \mu^{tot} q} \hat{z}_t - \hat{mc}_t \\
\hat{q}_t &= \frac{\Phi_q}{1 + \frac{\phi}{q^{tot} \Phi_q}} \left( \frac{\phi - 1}{\phi} \hat{z}_t + \hat{mc}_t \right) = -B_q \left( \frac{\phi - 1}{\phi} \hat{z}_t + \hat{mc}_t \right)
\end{align*}$$

(26)

where $\Phi_q = \frac{1}{\psi \mu^{tot} q}$ is a decreasing function of $\psi$ and $B_q = \frac{\Phi_q}{1 + \frac{\phi}{q^{tot} \Phi_q}}$ captures the elasticity of $\hat{q}_t$ to changes in the total profit margin.
Using this, we get

\[ \hat{p}_t = \frac{1}{\phi} \left( \dot{z}_t + \kappa B_q \left( \frac{\phi - 1}{\phi} \dot{z}_t + \tilde{m}c_t \right) \right) \]

\[ \hat{p}_t = \frac{1}{\phi} \left( 1 + \kappa B_q \frac{\phi - 1}{\phi} \right) \dot{z}_t + \frac{\kappa}{\phi} B_q \tilde{m}c_t \]

\[ = B_z \dot{z}_t + \frac{\kappa}{\phi} B_q \tilde{m}c_t \]  

(27)

where \( B_z = \frac{1}{\phi} \left( 1 + \kappa B_q \frac{\phi - 1}{\phi} \right) \) and \( \frac{\kappa}{\phi} B_q \) are decreasing in \( \psi \) and \( \phi \).

Finally use (26) and (27) into (22) to get:

\[ \hat{p}_{Wt} = \left\{ \kappa \frac{mc}{p_W} + (1 - \kappa) \frac{p(q - \omega)}{z_{pWq}} B_{mc} + A_q B_q \right\} \tilde{m}c_t - \left\{ (1 - \kappa) \left\{ \frac{p(q - \omega)}{z_{pWq}} (1 - B_z) \right\} - A_q B_q \frac{\phi - 1}{\phi} \right\} \dot{z}_t \]

where \( A_q \) captures the elasticity of the wholesale price to changes in \( \dot{q}_t \), and is a decreasing function of \( \psi \).
E. Other Figures

Figure 6: Bargaining power and the response to a persistent marginal cost shock
Figure 7: Response to a transitory wholesale price shock: monopolistic competition model vs. B-2-B model