On Risk Sharing in Village Economies: Structural Estimation and Testing*

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December 7, 2010

Abstract

This paper studies the role of limited commitment and heterogeneity in explaining the consumption allocation in village economies. I estimate the dynamic contract determining self-enforcing insurance transfers in a structural manner, and allow the coefficient of relative risk aversion to differ across households and groups of households to face different exogenous income processes. I then statistically compare models in terms of how well they can predict consumption shares in Thai villages. I find that the heterogenous model explains the consumption allocation significantly better than the homogenous version and the benchmarks of perfect risk sharing and autarky. Enforcement constraints bind more often with heterogeneous households, implying less risk sharing. The paper then examines how social policies would interact with existing informal insurance arrangements. I simulate the effects of counterfactual transfers targeting the poor on consumption by both eligible and ineligible households. I also study the crowing-out effect of aggregate insurance.

Keywords: risk sharing, limited commitment, dynamic contracts, heterogeneity, Thailand

JEL codes: C52, D10, D52

*PRELIMINARY AND INCOMPLETE. This paper is based on a chapter of my doctoral thesis at the Toulouse School of Economics. I also benefitted from the hospitality of the European University Institute while advancing this project. Discussions with Árpád Ábrahám, Orazio Attanasio, Russell Cooper, Pierre Dubois, Johannes Gierlinger, Jinyong Hahn, Ethan Ligon, Thierry Magnac, Ramon Marimon, Maurizio Mazzocco, and Alessandro Mennuni have greatly improved this paper. I also thank participants of the Panel Data Conference in Bonn, the Summer Workshop in Budapest, ESEM in Barcelona, SED in Montreal, NorMac in Mustio, ESWC in Shanghai, and seminar participants at the EUI, TSE, Queen Mary, University of Alicante, and UCLA for useful comments and suggestions. All errors are mine. Generous funding from Agence Nationale de la Recherche (ANR-06-BLAN-0104) is gratefully acknowledged.

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1 Introduction

Households living in rural areas of low-income countries face a great amount of risk. Revenue from agricultural production is usually low and volatile, as a result of extreme weather conditions, such as erratic monsoon rains in South Asia. Further, outside job opportunities are often lacking. In addition, access to financial instruments to insure against consumption fluctuations is limited. In such an environment, households in a community rely on one another for insurance.

There exists ample empirical evidence that households in poor villages do not fully share the risks they face, although they do achieve a remarkable amount of insurance (Townsend, 1994; Grimard, 1997; Dubois, 2000; Dercon and Krishnan, 2003a,b; and others).[1] There is also direct evidence that households make state-contingent transfers to one another (e.g. Udry, 1994). The literature has focused on two imperfections to explain the observed partial insurance, namely private information (Wang, 1995; Ligon, 1998; Ales and Maziero, 2009) and lack of commitment. This paper focuses on the case where limited commitment is the friction that may cause a deviation from the first best. Households in small communities can often observe shocks faced by their neighbors (bad harvest, or illness), but no authority exists to enforce informal risk-sharing contracts. Considering imperfect information as an alternative or additional friction is left for future research.

The model of risk sharing with limited commitment has been developed by Thomas and Worrall (1988), Coate and Ravallion (1993), Kocherlakota (1996), and Ligon, Thomas, and Worrall (2002) (LTW hereafter),[2] and its implications are supported by mounting empirical evidence (Fafchamps, 1999; Attanasio and Ríos-Rull, 2000; Foster and Rosenzweig, 2001; LTW; Dubois, Jullien, and Magnac (2008), and others). LTW is the only paper, to my knowledge, that estimates dynamic risk sharing contracts in a structural manner in general, and the model of risk sharing with limited commitment in particular. I extend their work in several dimensions. First, I show that the discount factor is identified by binding enforcement constraints. I also take measurement error into account consistently when estimating the model.

The existing evidence on comparing different models of risk sharing is of reduced form. LTW do not perform any statistical tests on parameters or model selection. This paper provides a test of perfect risk sharing, where the alternative is a well-specified model of partial

[1] See also the seminal papers by Cochrane (1991) and Mace (1991) for tests of perfect risk sharing in the United States.

[2] See also Alvarez and Jermann (2000) for a decentralization of the constrained-effient allocation, trading Arrow-Debreu securities with endogenous solvency constraints.

insurance. In particular, I compare the model of perfect risk sharing, autarky, and risk sharing with limited commitment in terms of how well they can predict the consumption allocation in Thai villages, given income shares. I apply likelihood ratio-based tests introduced by Vuong (1989).[3] The approach of this paper could be used to compare models of partial risk sharing with different or additional frictions.

I compare models not just in terms of constraints to risk sharing, but also whether households are heterogenous with respect to their risk preferences and income processes. Allowing for preference heterogeneity across households is an important extension, because efficient risk sharing has two main implications. First, incomes should be pooled. Second, less risk-averse households should bear more uninsurable risk (Borch, 1962; Wilson, 1968). Assuming that risk preferences are homogeneous, we exclude an additional motive for risk sharing. The first paper that considers heterogeneity in risk preferences when testing perfect risk sharing is Altug and Miller (1990). In that paper the authors test efficiency by allowing preferences to depend on household demographic variables. Dubois (2000) specifies an isoelastic utility function, and allows the coefficient of relative risk aversion to depend on observables. Mazzocco and Saini (2010) construct nonparametric tests of perfect risk sharing allowing for preference heterogeneity. The present paper tests perfect risk sharing against a well-specified alternative, but considers only parametric models. In particular, households' relative risk aversion coefficients may depend on observable household characteristics. I also investigate whether differences in income risk faced by households matter. Differences in income processes are rarely taken into account in the literature on risk sharing.[4] I allow groups of households to face different income processes that are exogenous, in addition to heterogeneity coming from the presence of idiosyncratic risk.

Afterwards, this paper provides examples of using the estimated model to predict how social policies would affect the consumption allocation, taking into account that these policies interact with existing informal insurance arrangements. Since the estimation is done in a structural manner, the effects of counterfactual social policies on the consumption allocation can be simulated.

Attanasio and Ríos-Rull (2000) argue that, under limited commitment, formal insurance provided by the state may crowd out informal insurance transfers to the extent that welfare

[3] These tests are appropriate under misspecification in general, and in the case of simulated estimators and in the presence of approximation errors in particular, that may be important here because of the value function iteration when solving the model.

[4] An exception is Schulhofer-Wohl (2010), who uses an experimental measure of risk aversion, and finds evidence that occupational choice is affected by risk preferences in the United States. He argues that this should be taken into account when evaluating how well people are able to mitigate the adverse effects of risk they face.

decreases. They then provide reduced form evidence on the crowding-out of informal transfers as a result of the Progresa program in Mexico, but do not use the model to predict the transfers. Further, recent evidence on the same program suggests that conditional cash transfers targeting the poor also increase consumption by ineligible households (Angelucci and De Giorgi, 2009). The authors argue that risk sharing is the explanation for the consumption pattern in the data. The model of risk sharing with limited commitment implies this partial sharing of the transfer. In addition, using the structural estimation results of this paper, the policy effects on both eligible and ineligible households can be predicted ex ante.[5]

I simulate the effects of counterfactual transfers targeting the poor, as well as the effects of introducing aggregate insurance. I compare the policy effects when the transfer is assumed to increase consumption directly, and when it raises income. In this way, I quantify the mistake made when predicting policy effects if informal insurance arrangements are ignored. I also study the effects of the introduction of formal aggregate insurance by simulation. This research may provide guidance for the evaluation and design of redistributive policies or micro-insurance programs as well, taking into account existing informal arrangements to share risk.

This paper is also related to the literature on explaining consumption inequality given income inequality. Krueger and Perri (2006) study whether limited risk sharing due to enforcement constraints can account for the fact that within-group cross-sectional consumption inequality increased less than income inequality in the United States over the period 1980-2003. Their model provides the desirable qualitative predictions. However, it implies too much risk sharing when calibrated to US data.[6] On the other hand, Blundell, Pistaferri, and Preston (2008) document that income shocks have become less persistent, and thereby easier to insure against. The present paper estimates a structural model of how consumption is allocated, given income, and could predict the effects of changes in the variance of transitory shocks and in the persistence of households' income process on the consumption allocation, thus on cross-sectional consumption inequality.

The rest of the paper is structured as follows. Section 2 details the theoretical models of risk sharing, building on Kocherlakota (1996), LTW, Kehoe and Perri (2002), and others. In section 3, the empirical models are set up, and (simulated) maximum likelihood estimators are derived allowing for measurement error in consumption. Section 4 presents the household survey data from Thailand. Section 5 contains the estimation results for the structural

[5]See Todd and Wolpin (2006, 2008) on ex-ante program evaluation.

[6]Note that, in their quantitative analysis, Krueger and Perri (2006) consider an economy with production, while I consider an endowment economy, and focus on the sharing rule.

models, both with and without preference and income risk heterogeneity, as well as the statistical tests to compare the models. Policy simulations are presented in section 6. In section 7, some extensions to the empirical models are discussed. In particular, I consider a utility function that depends on time-varying household observables, unobservable individual effects, and preference shocks. In addition, I allow for measurement error in income, as well as in consumption. Concluding remarks are presented in section 8.

2 Models of risk sharing

Suppose that there are N infinitely-lived, risk-averse households in a community. They consume a private and perishable consumption good c. Each household i maximizes its expected lifetime utility,

$$E_0 \sum_{t=1}^{\infty} \delta^t u_i\left(c_{it}\right),$$

where E_0 is the expected value at time 0 calculated with respect to the probability measure describing the common beliefs, $\delta \in (0,1)$ is the (common) discount factor, and c_{it} is the consumption of household i at time t. The instantaneous preferences of household i are described by the isoelastic (CRRA) utility function

$$u_i\left(c_{it}\right) = \frac{c_{it}^{1-\sigma_i} - 1}{1-\sigma_i}, \tag{1}$$

where $\sigma_i > 0$ is the coefficient of relative risk aversion of household i.

Suppose that random income, denoted Y_i for household i, follows a Markov process and is independent across households. Let s_t denote the state of the world that describes the income realizations of all households in the community at time t, and s^t denote the history of states, that is, $s^t = (s_t, ..., s_1)$. The distribution of Y_i, $\forall i$, is common knowledge ex ante, and so are income realizations ex post at each time t. That is, there are no informational problems. Note also that income is exogenous. In other words, the effect of risk on choices among different income generating processes is ignored. In addition, individual savings are assumed to be absent.[7] I interpret the model as predicting consumption shares, given income shares and aggregate consumption in the community. In other words, any difference between household consumption and income is thought of as a transfer to or from the rest of the community, and not as saving or dissaving explicitly.

[7] Ligon, Thomas, and Worrall (2000) allow for individual savings in the model of risk sharing with limited commitment. In this case, randomization is needed to make the problem convex.

This section considers three models in turn. First, it considers the model of perfect risk sharing. Second, subsection 2.2 mentions the benchmark of autarky. Third, the model of risk sharing with limited commitment is detailed in subsection 2.3.

2.1 Perfect risk sharing

To find the Pareto-optimal allocations, we solve the social planner's problem. The (utilitarian) social planner maximizes a weighted sum of households' excepted lifetime utilities,

$$\max_{\{c_{it}(s^t)\}} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi\left(s^t\right) u_i\left(c_{it}\left(s^t\right)\right),$$

where λ_i is the (initial) Pareto-weight of household i in the social planner's objective, and $\pi\left(s^t\right)$ is the probability of state s^t occurring, subject to the resource constraint

$$\sum_i c_{it}\left(s^t\right) \leq \sum_i y_{it}\left(s^t\right), \forall s^t, \forall t,$$

where $y_{it}\left(s^t\right)$ is the income, or, endowment, of household i at time t and state s^t.

The well-known result that

$$\frac{u_k'\left(c_{kt}\left(s^t\right)\right)}{u_i'\left(c_{it}\left(s^t\right)\right)} = \frac{\lambda_i}{\lambda_k}, \forall s^t, \forall t, \tag{2}$$

that is, the ratio of marginal utilities for any two households i and k is constant over time and across states of the world, follows from the first order conditions of the social planner's problem (Borch, 1962; Wilson, 1968). Equation (2) implies that all idiosyncratic risks are insured away, and households share aggregate risk efficiently. In particular, less risk-averse households bear more uninsurable risk. With the utility function (1), condition (2), for any s^t and t, is

$$\frac{c_{kt}^{-\sigma_k}}{c_{it}^{-\sigma_i}} = \frac{\lambda_i}{\lambda_k}. \tag{3}$$

2.2 Autarky

When households are in autarky, the problem is trivial, since individual savings have been assumed absent. The model predicts that

$$c_{it}\left(s^t\right) = y_{it}\left(s^t\right), \forall s^t, \forall t, \forall i. \tag{4}$$

Let $U_i^{aut}\left(s^t\right)$ denote the expected lifetime utility, or, the value function, of household i in autarky at state s^t and time t. Under the assumption that income is Markovian, the value of autarky can be computed by iterating the Bellman equation

$$U_i^{aut}\left(s_t\right) = u_i\left(y_{it}\left(s_t\right)\right) + \delta \sum_{s_{t+1}} \pi\left(s_{t+1} \mid s_t\right) U_i^{aut}\left(s_{t+1}\right). \tag{5}$$

2.3 Risk sharing with limited commitment

To find the constrained-efficient consumption allocations, I follow Kehoe and Perri (2002) (for the case of an endowment economy), and solve the following problem: The social planner maximizes a weighted sum of households' expected lifetime utilities,

$$\max_{\{c_{it}(s^t)\}} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi\left(s^t\right) u_i\left(c_{it}\left(s^t\right)\right),$$

where $\pi\left(s^t\right)$ is the probability of history $s^t = \left(s_1, s_2, ..., s_t\right)$ occurring, and $c_{it}\left(s^t\right)$ denotes the consumption of household i when history s^t has occurred; subject to the resource constraints

$$\sum_i c_{it}\left(s^t\right) \leq \sum_i y_{it}\left(s_t\right), \forall s^t, \forall t, \tag{6}$$

and the enforcement constraints,

$$\sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi\left(s^r \mid s^t\right) u_i\left(c_{ir}\left(s^r\right)\right) \geq U_i^{aut}\left(s_t\right), \forall s^t, \forall t, \forall i, \tag{7}$$

where $\pi\left(s^r \mid s^t\right)$ is the probability of history s^r occurring given that history s^t has occurred up to time t. The right hand side has been defined in equation (5). The problem is not recursive, because future decision variables enter into today's enforcement constraints. Therefore, even if income is i.i.d. or follows a Markov process, consumption may depend on the whole history of income realizations.

The enforcement constraints (7) assume that, if a household deviates, other households in the community do not enter into any risk sharing arrangement with it in the future. Note that reversion to autarky is the most severe subgame perfect punishment in this environment. In other words, it is an optimal penal code in the sense of Abreu (1988). We might also call reversion to autarky a trigger strategy, or the breakdown of trust. Future research should examine whether alternative specifications of the outside option would improve the model's fit to data. Alternatives include allowing for storage, community punishment for reneging, and limiting the time length of exclusion from insurance arrangements.

Denoting the multiplier on the enforcement constraint of household i (7) by $\delta^t \pi\left(s^t\right) \mu_i\left(s^t\right)$, and the multiplier on the resource constraint (6) by $\delta^t \pi\left(s^t\right) \rho\left(s^t\right)$, when history s^t has occurred, the Lagrangian is

$$\sum_{t=1}^{\infty}\sum_{s^t}\delta^t\pi\left(s^t\right)\Bigg[\sum_i\lambda_i u_i\left(c_{it}\left(s^t\right)\right)$$
$$+\mu_i\left(s^t\right)\left(\sum_{r=t}^{\infty}\sum_{s^r}\delta^{r-t}\pi\left(s^r\mid s^t\right)u_i\left(c_{ir}\left(s^r\right)\right)-U_i^{aut}\left(s_t\right)\right)$$
$$+\rho\left(s^t\right)\left(\sum_i y_{it}\left(s_t\right)-c_{it}\left(s^t\right)\right)\Bigg].$$

Using the ideas of Marcet and Marimon (2009), the Lagrangian can also be written in the form

$$\sum_{t=1}^{\infty}\sum_{s^t}\delta^t\pi\left(s^t\right)\Bigg[\sum_i M_i\left(s^{t-1}\right)u_i\left(c_{it}\left(s^t\right)\right)$$
$$+\mu_i\left(s^t\right)\left(u_i\left(c_{it}\left(s^t\right)\right)-U_i^{aut}\left(s_t\right)\right)+\rho\left(s^t\right)\left(\sum_i y_{it}\left(s_t\right)-c_{it}\left(s^t\right)\right)\Bigg],$$

where $M_i\left(s^t\right)=M_i\left(s^{t-1}\right)+\mu_i\left(s^t\right)$ with $M_i\left(s^0\right)=\lambda_i$ (see also Kehoe and Perri, 2002). In words, $M_i\left(s^t\right)$ is the initial weight of household i plus the sum of the Lagrange multipliers on its enforcement constraints along the history s^t.

The first order condition with respect to $c_{it}\left(s^t\right)$ can be written as

$$M_i\left(s^t\right)u_i'\left(c_{it}\left(s^t\right)\right)-\rho\left(s^t\right)=0. \tag{8}$$

There are also standard first order conditions relating to the resource and enforcement constraints, with complementarity slackness conditions. To illustrate this, let us consider two households sharing risk, households i and k. Combining the first order conditions (8) for these two households for history s^t at time t, we have

$$\frac{u_k'\left(c_{kt}\left(s^t\right)\right)}{u_i'\left(c_{it}\left(s^t\right)\right)}=\frac{M_i\left(s^t\right)}{M_k\left(s^t\right)}=\frac{\lambda_i+\mu_i\left(s^1\right)+\mu_i\left(s^2\right)+...+\mu_i\left(s^t\right)}{\lambda_k+\mu_k\left(s^1\right)+\mu_k\left(s^2\right)+...+\mu_k\left(s^t\right)}\equiv x_i\left(s^t\right), \tag{9}$$

where $x_i\left(s^t\right)$ can be thought of as the relative Pareto-weight assigned to household i when history s^t has occurred, normalizing the weight of household k to 1 at each time t.

The vector of relative weights $x\left(s^t\right)$, with elements $x_i\left(s^t\right)$ defined in (9), can be used as a state variable in order to rewrite the problem in a recursive form (Marcet and Marimon, 2009). The current income state s_t does not tell us everything we need to know about the past. Only (s_t, x_{t-1}) does this, where x_{t-1} is the vector of relative weights, equal to the ratio of marginal utilities, inherited from the previous period. In other words, x_{t-1} is a sufficient statistic for everything that happened in the past. The solution consists of policy functions for the consumption allocation and the new relative weight, with support over the extended state space (s_t, x_{t-1}). That is, $c_{it}(s_t, x_{t-1})$, $\forall i$, and $x_t(s_t, x_{t-1})$ are to be determined. At last, the value functions can be written recursively as

$$V_i(s_t, x_{t-1}) = u_i(c_{it}(s_t, x_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1}) V_i(s_{t+1}, x_t(s_t, x_{t-1})). \tag{10}$$

The solution is fully characterized by a set of state-dependent intervals on the relative weight x_i, that give the possible relative weights in each income state for household i (LTW). Denote the interval for household i for state s by $[\underline{x}_i^s, \overline{x}_i^s]$. Denote by x_{it} the new relative weight of household i to be found at time t. Suppose that last period the ratio of marginal utilities was $x_{i,t-1}$, and today the income state is s. The relative weight of household i today is determined by the following updating rule (LTW):

$$x_{it} = \begin{cases} \overline{x}_i^s & \text{if} \quad x_{i,t-1} > \overline{x}_i^s \\ x_{i,t-1} & \text{if} \quad x_{i,t-1} \in [\underline{x}_i^s, \overline{x}_i^s] \\ \underline{x}_i^s & \text{if} \quad x_{i,t-1} < \underline{x}_i^s \end{cases} \tag{11}$$

Numerical dynamic programming allows us to solve for the optimal intervals, and thereby the consumption allocation, given the income processes, utility functions, and discount rates of the households. As the number of periods tends to infinity, the initial weights in the social planner's objective only matters if perfect risk sharing is self-enforcing (Kocherlakota, 1996).

Let F_{Y_i} (F_{Y_k}) summarize the income process of household i's (k's). Remember that σ_i (σ_k) parametrizes the utility function of household i (k). Then, we can solve numerically for the optimal intervals that depend on σ_i, σ_k, δ, F_{Y_i}, and F_{Y_k}, and then we can find x_{it} given s_t (or, y_{it} and y_{kt}) and $x_{i,t-1}$. Details are in the Appendix. Once we know x_{it}, $\forall i$, the first order conditions (9) and the resource constraint (6) give the consumption allocation predicted by the model.

3 Empirical models

Let us first specify the utility function (1). Assume that σ_i is a linear function of observables. In particular,

$$\sigma_i = 1 + z_i'\beta,$$

where β is a parameter vector to be estimated, and z_i represents a vector of time-invariant observable covariates of household i. Note that z_i does not contain an (additional) constant, as in Dubois (2000). A normalization is needed, because the consumption risk borne by each household is determined by its risk tolerance relative to the average risk tolerance in the community. Further, if the coefficient on the constant were a free parameter, then, taking all households as risk neutral, any consumption allocation would be Pareto optimal.[8] Remember that the above theoretical models assume perfect information, thus the preferences of each household are known to everybody in the community, but the econometrician only observes z_i, $\forall i$.

Assume that consumption is measured with a multiplicative measurement error that is log-normally distributed. Let c_{it}^* denote consumption observed by the econometrician, and let $\exp(\varepsilon_{it})$ be the multiplicative measurement error in household i's consumption at time t. Then, we may write

$$c_{it}^* = \exp(\varepsilon_{it})\, c_{it},$$

where ε_{it} is independently and identically distributed (i.i.d.) across households and time, and $\varepsilon_{it} \sim N(0, \gamma^2)$, where γ^2 is to be estimated.[9] Note that true consumption c_{it} is observed by all households in the community. Measurement error in income is ignored for now, and is introduced in an extension in section 7.

I model the allocation of observed consumption $c_t^* \equiv (c_{1t}^*, ..., c_{it}^*, ..., c_{Nt}^*)$, for $t = 2, ..., T$, determined by the history of income realizations, time-constant household characteristics, observed consumption at time 1, c_1^*, and parameters. In mathematical terms, we would like to know how the following conditional density could be specified based on the above models of risk sharing:

$$f\left(c_T^*, ..., c_2^* \mid c_1^*, y_T, ..., y_1, Z; \beta, \delta, \gamma^2, F_Y, \lambda\right), \tag{12}$$

where y_t, for $t = 1, ..., T$, is the vector of income realizations for households at time t, $Z = [z_1, ..., z_i, ..., z_N]'$ is the matrix of household observables for all households, $\theta = (\beta, \delta, \gamma^2, F_Y)$ are the structural parameters to be estimated[10] where F_Y summarizes households' income processes, and the vector λ is a nuisance parameter. Each of the above theoretical models allows us to factorize the density (12). In particular, we may write

$$\prod_{t=2,...,T} f\left(c_t^* \mid c_{t-1}^*, y_t, Z; \beta, \delta, \gamma^2, F_Y, x_{t-1}\right), \tag{13}$$

[8]This is because marginal utility is constant for risk-neutral households. Thus any consumption allocation would keep the ratio of marginal utilities constant.

[9]I have allowed for measurement error in consumption to account for the error term in our estimating equations. In the consumption insurance literature, preference shocks are often used to introduce randomness, or, as in Cochrane (1991), consumption growth is measured with error. These alternative assumptions are not suitable in the case of risk sharing with limited commitment, as will be explained below.

[10]Below θ often denotes a subset of these parameters, and is used as a short form for 'structural parameters to be estimated.'

where x_{t-1} is the state variable, which has elements $x_{i,t-1}$ and is not observed. I deal with this issue below.

For the limited commitment case, LTW have shown that the updating rule (11) holds for both the 2- and the N-household case. In the empirical part of their paper, they approximate the N-household economy by looking at each household i sharing risk with the 'rest of the community.' I follow their approach in this paper. This results in important gains in computation time, since the N-household case would require solving the model with $N + (N-1)$ state variables, since the state variables would be each household's income and the relative Pareto-weights. I often call the rest of the community household k. Household k can also be thought of as the chief of the community, coordinating transfers. I evaluate the models in terms of how well they explain the allocation of consumption in each community at each time t, but take changes in aggregate consumption as given. Equivalently, I study how well the models explain each household's consumption relative to mean consumption in the community.

I normalize the coefficient of relative risk aversion of household k to 1, that is, $u_k(c_{kt}) = \log c_{kt}$, and think of explanatory variables in the utility function as deviations from their community mean hereafter, abusing notation.[11] This means assuming that the village chief's risk aversion coefficient is the equal to average risk aversion. Normalize also the Pareto-weight of household k to 1, that is, $\lambda_k = 1$. This is without loss of generality, since only relative Pareto-weights matter. Further, I assume that c_{kt} is well measured, since the variance of the measurement error in mean consumption in the community is only a fraction of the variance of the measurement error in each household's consumption. This assumption is only for notational simplicity. When preferences are heterogeneous, I use the logarithm of household size at the first month to capture heterogeneity in risk preferences, that is, I include it as z_i. I do not aim to find the best way to capture differences in the curvature of the current utility function across households. I only want to see whether some heterogeneity would improve the model's fit to data, and if there is a statistical difference between a heterogenous and a homogenous model, whether predicted policy effects differ in an economic sense.

The next three subsections detail in turn how the model of perfect risk sharing (subsection 3.1), autarky (3.2), and risk sharing with limited commitment (3.3) are estimated. The estimations are done using (simulated) pseudo maximum likelihood estimators, and Vuong's (1989) tests are applied to statistically compare the models. Subsection 3.4 expands on model selection.

[11] Note also that, when preferences are homogeneous, meaning $\beta = 0$ here, the coefficient of relative risk aversion is normalized to 1, that is, $u_i(c_{it}) = \log c_{it}$, $\forall i$.

3.1 Perfect risk sharing

In the case of perfect risk sharing, the current consumption allocation should only depend on current and not past exogenous variables. It depends neither on the discount factor, nor on income processes. However, it depends on the time-constant unobservables, $x_{t-1} = \lambda$, $\forall t$. That is, the state variable is constant and equal to the initial relative Pareto-weights in the social planner's objective. Thus (13) can be written as

$$\prod_{t=2,\ldots,T} f\left(c_t^* \mid y_t, Z; \beta, \gamma^2, \lambda\right). \tag{14}$$

Further, c_t^* only depends on today's income realizations through aggregate income.

Let us consider household i and the 'average' household, k. Taking the logarithm of the first order condition with respect to (true) consumption for these two households, equation (3), noting that $\sigma_k = 1$ and $\lambda_k = 1$, we obtain

$$\sigma_i \log c_{it} - \log c_{kt} = \log \lambda_i.$$

Replacing for σ_i and rearranging give

$$\log\left(\frac{c_{it}}{c_{kt}}\right) = -z_i'\beta \log c_{it} + \log \lambda_i. \tag{15}$$

In terms of measured consumption c_{it}^*, (15) reads

$$\log\left(\frac{c_{it}^*}{c_{kt}}\right) = -z_i'\beta \log c_{it}^* + \log \lambda_i + (1 + z_i'\beta)\,\varepsilon_{it}.$$

Now, let us take first differences to eliminate $\log \lambda_i$. Doing so and rearranging yields

$$\log\left(\frac{c_{it}^*}{c_{kt}}\right) = \log\left(\frac{c_{i,t-1}^*}{c_{k,t-1}}\right) - z_i'\beta \log\left(\frac{c_{it}^*}{c_{i,t-1}^*}\right) + (1 + z_i'\beta)\left(\varepsilon_{it} - \varepsilon_{i,t-1}\right). \tag{16}$$

Estimating (16), I implicitly assume that the ratio of marginal utilities observed at time $t-1$ contains all the information available on λ_i.

Let $\psi^2(\theta) \equiv 2\left(1 + z_i'\beta\right)^2 \gamma^2$, and

$$d_{it}^{prs}(\theta) \equiv \left[\log\left(\frac{c_{it}^*}{c_{kt}}\right) - \log\left(\frac{c_{i,t-1}^*}{c_{k,t-1}}\right) + z_i'\beta \log\left(\frac{c_{it}^*}{c_{i,t-1}^*}\right)\right] / \psi(\theta).$$

Then, we may write the likelihood of observation it as

$$L_{it}^{prs}(\theta) = \phi\left(d_{it}^{prs}(\theta)\right),$$

where ϕ is the density of the standard normal distribution. Finally, the pseudo maximum likelihood estimator (MLE) maximizes

$$\ell^{prs}(\theta) = \sum_{i=1}^{N} \sum_{t=2}^{T} \log \phi\left(d_{it}^{prs}(\theta)\right), \tag{17}$$

with respect to θ, that is, β and the variance γ^2. The model is also estimated without preference heterogeneity for comparison. This means setting $\beta = 0$. Thus the only parameter that remains to be estimated is γ^2.

I do not assume that the model is correctly specified, therefore I compute the variance-covariance matrix of the estimated parameters without assuming that the information matrix equality holds. I also take into account serial correlation. In particular, the variance-covariance matrix is estimated by $\hat{A}^{-1}\hat{B}\hat{A}^{-1}$, where $\hat{A}$ is the estimated Hessian, that is,

$$\hat{A} = \textstyle\sum_{i=1}^{N} \sum_{t=2}^{T} -\nabla_\theta^2 \ell_{it}(\hat{\theta}), \quad \text{and} \quad \hat{B} = \sum_{i=1}^{N} \sum_{t=2}^{T} \hat{s}_{it}\hat{s}_{it}' + \sum_{i=1}^{N} \sum_{t=2}^{T} \sum_{r\neq t} \hat{s}_{ir}\hat{s}_{it}',$$

where $\hat{s}_{it} = \nabla_\theta \ell_{it}(\hat{\theta})'$ is the score evaluated at the estimated parameters, and where the second term in the expression for $\hat{B}$ accounts for serial correlation (Wooldridge, 2002). Both the first and second derivatives of the log-likelihood function can be computed analytically here.

3.2 Autarky

In the autarky case, there is no state variable. Further, preferences do not matter. Therefore, we may simply write the likelihood of the consumption allocation for $t = 2, ..., T$ as

$$\prod_{t=2,\ldots,T} f\left(c_t^* \mid y_t; \gamma^2\right). \tag{18}$$

Taking the logarithm of (4) and introducing measured consumption give

$$\log c_{it}^* = \log y_{it} + \varepsilon_{it}.$$

The consumption of household i relative to mean consumption is

$$\log\left(\frac{c_{it}^*}{c_{kt}}\right) = \log\left(\frac{y_{it}}{y_{kt}}\right) + \varepsilon_{it}, \tag{19}$$

where I have just added and subtracted $\log c_{kt} = \log y_{kt}$ to have the same dependent variable in the equation to be estimated as above. In terms of the allocation of consumption within a community, the autarky model says that the consumption share of household i should be the same as its income share.

Let

$$d_{it}^{aut}(\theta) \equiv (\log c_{it}^{*} - \log y_{it})/\gamma.$$

The likelihood of observation it is

$$L_{it}^{aut}(\theta) = \phi\left(d_{it}^{aut}(\theta)\right),$$

and the log-likehood function to be maximized is

$$\ell^{aut}(\theta) = \sum_{i=1}^{N}\sum_{t=2}^{T} \log\,\phi\left(d_{it}^{aut}(\theta)\right). \tag{20}$$

The only parameter to be estimated is γ^2. I allow for misspecification and serial correlation when computing the variance of the estimated parameter.

3.3 Risk sharing with limited commitment

Remember that in the limited commitment case, the (true) ratio of marginal utilities from the last period x_{t-1} is the state variable in the recursive version of the model. It is a sufficient statistic for everything that happened in the past, including the initial condition. In other words, instead of conditioning on the whole history of income state realizations s^t, and the initial Pareto-weights in the social planner's objective λ, it is sufficient to condition on the current income state s_t and x_{t-1}. However, unlike in the perfect risk sharing case, the consumption allocation may also depend on the discount factor δ and the income processes F_Y. Remember also that x_{t-1} is not observed in (13). Therefore, we have to condition on x_{t-1}^{*} with elements $x_{i,t-1}^{*} = \left(c_{i,t-1}^{*}\right)^{1+z_i'\beta}/c_{k,t-1}$, the observable ratio of marginal utilities at time $t-1$, instead of x_{t-1} with elements $x_{i,t-1} = \left(c_{i,t-1}\right)^{1+z_i'\beta}/c_{k,t-1}$. With measurement error, all past values of consumption could be informative of $x_{i,t-1}$. For tractability, I only deal with the density (13).

Let us consider once again household i sharing risk with household k. According to the theoretical model of section 2.3, the first order condition can be written as

$$\log\left(\frac{c_{it}}{c_{kt}}\right) = -z_i'\beta\,\log c_{it} + \log x_{it}, \tag{21}$$

replacing x_{it} for λ_i in (15). Given x_{t-1}, preferences, current income realizations, and income processes, we can solve numerically for the relative Pareto-weight of household i at time t. In mathematical terms, we can compute $x_{it}(s_t, x_{t-1})$, $\forall i$ (see equations (10) and (11)).

Let $g()$ denote the function relating x_{it} to observables, parameters, and $x_{i,t-1}$. That is, $x_{it} = g(y_t, x_{i,t-1}, z_i; \theta)$, with $\theta = (\beta, \delta, F_Y)$. Note that, in general, $g()$ cannot be expressed analytically, but its value can be computed given any set of argument values. Replacing for x_{it} in (21) gives

$$\log\left(\frac{c_{it}}{c_{kt}}\right) = -z_i'\beta \log c_{it} + \log g\left(y_t, x_{i,t-1}, z_i; \theta\right). \tag{22}$$

Remember that, instead of c_{it}, the econometrician observes $c_{it}^* = \exp\left(\varepsilon_{it}\right) c_{it}$. Then, in terms of observable consumption (22) is

$$\log\left(\frac{c_{it}^*}{c_{kt}}\right) = -z_i'\beta \log c_{it}^* + \log g\left(y_t, x_{i,t-1}, z_i; \theta\right) + \left(1 + z_i'\beta\right)\varepsilon_{it}. \tag{23}$$

The first econometric issue is the identification of the discount factor δ. The second issue is that measurement error influences the updating of the state variable. That is, among the arguments of $g()$ in equation (23), instead of $x_{i,t-1}$ only

$$x_{i,t-1}^* = \left(\exp\left(\varepsilon_{i,t-1}\right)\right)^{1+z_i'\beta} x_{i,t-1} \tag{24}$$

is observed. I deal with these issues in the next two subsections.

3.3.1 Identifying the discount factor

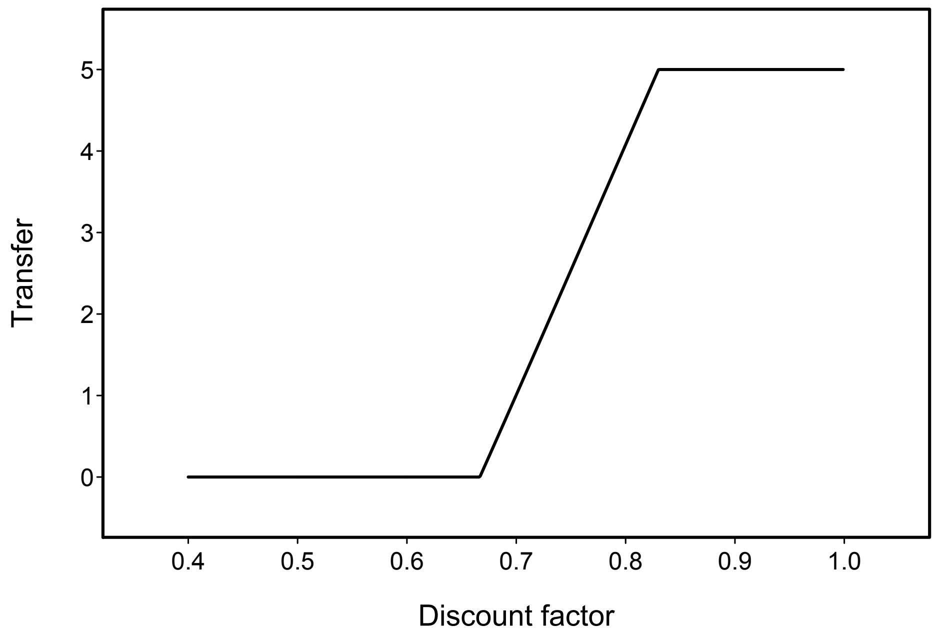
In the perfect risk sharing case, the predicted consumption allocation is independent of δ, the discount factor. The question is whether we can identify this parameter in the case of risk sharing with limited commitment. Proposition 1 states that the answer is yes, if some but not perfect risk sharing occurs.

Before giving a formal proof, I illustrate the role of δ in determining the consumption allocation with an example. Let us consider two ex-ante identical households, whose preferences are described by the utility function $u() = log()$. Both households face the random prospect $\tilde{Y} = (20, 1/2; 10, 1/2)$, and I assume that their incomes are perfectly negatively correlated. Thus, there are only two income states, $\{(y_{1t} = 20, y_{2t} = 10), (y_{1t} = 10, y_{2t} = 20)\}$, $\forall t$. Finally, assume that $\lambda = 1$. If agents stay in autarky, nothing is transferred in both states. If perfect risk sharing occurs, a transfer of 5 is made in both states, and both households consume 15. The model of risk sharing with limited commitment can predict any transfer between 0 and 5, and the exact amount depends on the households' patience, that is, on δ (see Figure 1). For example, if $\delta = 0.7$, the transfer is 1.0074; if $\delta = 0.75$, it is 2.5382; if $\delta = 0.8$, 4.0772 is transferred.

Denote by $\overline{\delta}$ the discount factor such that, $\forall \delta \geq \overline{\delta}$, perfect risk sharing is self-enforcing, and by $\underline{\delta}$ the discount factor such that, $\forall \delta \leq \underline{\delta}$, all households stay in autarky.[12]

[12]LTW have shown that $\overline{\delta}$ and $\underline{\delta}$ exist.

Figure 1: The transfer from the household earning 20 today, to its risk sharing partner receiving 10, as a function of δ



Proposition 1. *The parameter δ is identified if $\delta \in (\underline{\delta}, \overline{\delta})$, that is, if some informal insurance is achieved and at least one enforcement constraint binds.*

Proof. Let us prove this for the case with homogeneous risk preferences. The argument for the heterogeneous case is similar. Compared to the perfect risk sharing case, additional information can only come from binding enforcement constraints. Suppose that at time t households i's enforcement constraint is binding. That is, observed consumption is such that a positive transfer is made, but the ratio of marginal utilities is not the same as at time $t-1$.[13] Let us rewrite (26) with equality and with $z_i'\beta = 1 - \sigma = 0$. Simple algebra then gives

$$\log y_{it}(s_t) - \log c_{it}(s_t, x_{t-1}) = \delta \sum_{s_{t+1}} \pi(s_{t+1}) \left[V_i(s_{t+1}, x_t(s_t, x_{t-1})) - V_i^{aut}(s_{t+1}) \right],$$

where the left hand side is the utility cost of the transfer household i makes today, and the right hand side is the welfare gain of sharing risk according to the informal insurance contract, rather than staying in autarky in the future. If the right hand side is strictly monotonic and continuous in δ, and only this constraint ever binds, we could perfectly match household i's consumption at time t from the data, with an appropriately chosen unique δ.

[13]Considering the example above, a transfer strictly greater than 0, but strictly less than 5 is observed.

The expected future gain of insurance is strictly increasing in δ, for $\delta \in \left(\underline{\delta}, \overline{\delta}\right)$, since a higher δ relaxes all enforcement constraints. Note that as δ approaches 1, perfect risk sharing (the first best) is self-enforcing by the well-known folk theorem. On the other extreme, when it is close to 0, no voluntary transfers are made. In between, the higher δ is, the closer transfers get to their first-best level. That is, when δ is higher, more informal insurance is achieved, and consumption is smoother across income states. In other words, a higher δ means a better enforcement technology.

It is easy to see that $V_i^{aut}(s_{t+1})$ is continuous in δ. As for $V_i(s_{t+1}, x_t)$, LTW have shown that the limits of the optimal state-dependent intervals, which fully characterize the solution of the model, are continuous in δ. Since $V_i(s_{t+1}, x_t)$ is a continuous function of these limits, it is itself continuous in δ. It follows that one binding enforcement constraint identifies δ. $\square$

3.3.2 Measurement error

Let $\varepsilon_{i,t-1}^j$ denote a realization of measurement error in household i's consumption at time $t-1$, drawn from the distribution $N(0, \gamma^2)$. Knowing $x_{i,t-1}^*$ and $\varepsilon_{i,t-1}^j$, we can easily compute $x_{i,t-1}$ (see equation (24)). Then, $g(y_t, x_{i,t-1}, z_i; \theta) = g\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)$. To deal with the fact that measurement error enters the updating of the state variable, I first write the likelihood of each it observation conditional on $\varepsilon_{i,t-1}^j$. Then, averaging the conditional likelihood over J draws, I integrate $\varepsilon_{i,t-1}$ out. That is, in the case of risk sharing with limited commitment with measurement error, I use a simulated pseudo maximum likelihood estimator (SMLE).

Conditional on $\varepsilon_{i,t-1}^j$, (23) becomes

$$\log\left(\frac{c_{it}^*}{c_{kt}}\right) = \log\left(\frac{\hat{c}_{it}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)}{\hat{c}_{kt}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)}\right) - z_i'\beta \log\left(\frac{c_{it}^*}{\hat{c}_{it}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)}\right) + (1 + z_i'\beta)\,\varepsilon_{it}. \tag{25}$$

Note that, when perfect risk sharing is self-enforcing, (25) is equivalent to (16). This is because

$$\begin{aligned}&\log\left(\frac{\hat{c}_{it}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)}{\hat{c}_{kt}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)}\right) + z_i'\beta \log \hat{c}_{it}\left(y_t, x_{i,t-1}^*, \varepsilon_{i,t-1}^j, z_i; \theta\right)\\ &= \log\left(\frac{c_{i,t-1}^*}{c_{k,t-1}}\right) + z_i'\beta \log c_{i,t-1}^* - (1 + z_i'\beta)\,\varepsilon_{it-1}\end{aligned}$$

in that case. Similarly, we get back the estimating equation of autarky, equation (19), if $\delta \leq \underline{\delta}$.

Using (5) and (10), and replacing for the utility function using (1) with $\sigma_i = 1 + z_i'\beta$, the enforcement constraint of household i at time t, that the predicted consumption allocation has to satisfy, can be written in a recursive form as

$$\frac{c_{it}\left(s_t, x_{t-1}\right)^{-z_i'\beta} - 1}{-z_i'\beta} + \delta \sum_{s_{t+1}} \pi\left(s_{t+1}\right) V_i\left(s_{t+1}, x_t\left(s_t, x_{t-1}\right)\right) \geq$$

$$\geq \frac{y_i\left(s_t\right)^{-z_i'\beta} - 1}{-z_i'\beta} + \delta \sum_{s_{t+1}} \pi\left(s_{t+1}\right) V_i^{aut}\left(s_{t+1}\right). \tag{26}$$

This inequality is to be used in the numerical solution of the model, with

$$c_{it}\left(s_t, x_{t-1}\right) = \hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right) \quad \text{and} \quad x_{it}\left(s_t, x_{t-1}\right) = g\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right).$$

Let us now consider alternative assumptions to generate residuals in the estimating equation (25). Preference shocks are not suitable, because today's shock would drop out of the equation. Assuming that consumption growth is measured with error, as in Cochrane (1991) for example, we could not take measurement error properly into account in the limited commitment case, since we would have to draw $\varepsilon^j_{i,t-1}$ from a random walk.

3.3.3 The simulated pseudo maximum likelihood function

Let $\psi^2(\theta) \equiv (1 + z_i'\beta)^2 \gamma^2$, and

$$d^{lc}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right) \equiv \left[\log\left(\frac{c^*_{it}}{c_{kt}}\right) - \log\left(\frac{\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)}{\hat{c}_{kt}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)}\right)\right. \tag{27}$$

$$\left. + z_i'\beta \log\left(\frac{c^*_{it}}{\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)}\right) - \left(1 + z_i'\beta\right) \varepsilon^j_{i,t-1}\right] / \psi(\theta).$$

Then, the likelihood of observation it given $\varepsilon^j_{i,t-1}$ is

$$L^{lc}_{it}(\theta) = \phi\left(d^{lc}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)\right).$$

Making J draws for $\varepsilon^j_{i,t-1}$, the simulated pseudo likelihood of observation it is

$$L^{lc}_{it}(\theta) = \frac{1}{J} \sum_{j=1}^{J} \phi\left(d^{lc}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)\right).$$

Finally, the simulated pseudo log-likelihood function to be maximized is

$$\ell^{lc}(\theta) = \sum_{i=1}^{N} \sum_{t=2}^{T} \log\left(\frac{1}{J} \sum_{j=1}^{J} \phi\left(d^{lc}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)\right)\right). \tag{28}$$

Allowing for misspecification, the SMLE consistently estimates the pseudo-true values of the parameters and is asymptotically normal, if both the number of it observations, that

I denote by M, and the number of simulations J tend to infinity, and $\sqrt{M}/J \to 0$ (see Gouriéroux and Monfort, 1997, for example). When computing the variance-covariance matrix, the information matrix equality is not assumed to hold, and possible serial correlation is taken into account, as in the perfect risk sharing case. The score and the Hessian are computed numerically.

3.3.4 Estimation

The estimation is done in three steps. A preliminary step (i) involves estimating an autoregressive process for households' income and fitting a Markov chain to that process. Then, (ii) the inner optimization computes the consumption allocation predicted by the model, given the observable covariates and parameters. Finally, (iii) the log-likelihood (28) is maximized over the remaining structural parameters, $\theta = (\beta, \delta, \gamma^2)$. Now I turn to the details of each of these steps.

(i) Estimation of the income processes. First of all, the fact that income may be negative has to be dealt with. I follow Rosenzweig and Wolpin (1993) and assume that households have a form of disaster insurance, meaning that they can at least consume at the subsistence level. The subsistence level is chosen to be the mean of the lowest 5% of measured consumption in the village. Those who earn more than this level are thought of as having a claim on current village consumption proportional to their additional income.

I assume that the logarithm of each household's income follows an AR(1) process, common to all households in a village or a group of households. I estimate

$$\log(y_{it}) = (1-\rho)\mu + \rho \log(y_{i,t-1}) + \xi_{it}.$$

The parameters are pinned down by the following moments:

$$\begin{aligned} \mu &= E(\log(y_{it})) \\ \rho &= Corr(\log(y_{it}), \log(y_{i,t-1})) \\ \sigma_\xi^2 &= (1-\rho^2) Var(\log(y_{it})) \end{aligned}$$

I choose the support points for the Markov chain following Kennan (2006). In particular, the points are quantiles of the income distribution. I then apply Tauchen (1986)'s method to the logarithm of these points to compute the transition matrix. To find a Markov chain approximation for the village mean income, I simulate the household processes assuming that they are independent, compute mean income, and then perform the same steps as for household incomes.

(ii) Inner optimization. We have to solve the model of risk sharing with limited commitment to find the predicted consumption allocation. The Bellman equation (10) is solved by iteration. A grid is defined over the continuous state variable x_i. At iteration h, we solve for the new consumption values in states where an enforcement constraint is binding using (26) with equality, while the ratio of marginal utilities stays constant in other states. The values from iteration $h-1$ are kept for $V_i(s_{t+1}, x_t)$ in (26). At the first iteration, the values of perfect risk sharing are used. The algorithm to solve for the constrained-efficient risk sharing contract, given observables and structural parameters, does not impose much additional difficulty relative to the case without preference heterogeneity and heterogeneity in Y_i, except for computation time. Computation time is proportional to the number of households. The appendix gives more details on the algorithm. This step leads to the predicted consumption values, $\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)$ and $\hat{c}_{kt}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)$, $\forall i$, to be replaced in equation (25).

(iii) Outer optimization. The likelihood maximization is done, after a grid search, using a standard optimization algorithm available in R.[14] The preliminary grid search is necessary, since we can only identify δ on the interval $\left(\underline{\delta}, \overline{\delta}\right)$, see Proposition 1. It also provides good initial guesses for the parameter values.

The maximum-likelihood estimates of the structural parameters (β, δ, and γ^2) are obtained by iterating between the dynamic program that solves for the predicted consumption allocation and the likelihood maximization routine. For comparison, the model is also estimated without preference heterogeneity. In that case only δ and γ^2 are to be estimated.

3.3.5 Simulation and approximation

The estimation of the model of risk sharing with limited commitment involves both simulation and approximation. I take the number of simulations $J = 50$, because I have maximum 240 it observations per village, thus $J > \sqrt{M}$. Computation time is only moderately increased when adding additional draws, because the optimal intervals, which fully characterize the solution of the model, do not have to be recomputed.[15]

The continuous state variable x_i has to be discretized, and I use a 30-point grid.[16] Computation time is approximatively proportional to the number of income states, which is $8 \times 5 = 40$, and the number of grid points on x_i. Increasing the number of grid points would

[14]See www.r-project.org.

[15]Below I check that the results are robust to changing J. In particular, I repeat the estimation of the model of risk sharing with limited commitment with heterogeneous households setting $J = 100$.

[16]A robustness check with 100 grid points for a couple of villages verifies that the parameter estimates are not sensitive to changing the number of grid points.

be beneficial in better approximating the true solution of the model. We are limited by the cost in terms of computation time. As Ackerberg, Geweke, and Hahn (2009) point out, in terms of the asymptotic properties of the maximum likelihood estimator, approximation error in computed dynamic models has similar effects as a limited number of simulations, and the results from the literature on simulated maximum likelihood estimation apply (Hajivassiliou and Ruud, 1994, Gouriéroux and Monfort, 1997).

3.4 Model selection

To statistically compare the above models, I use model selection tests introduced by Vuong (1989). Vuong proposes likelihood ratio-based statistics to compare nested and non-nested models. These statistics allow us to test the null hypothesis that two competing models are equally close to the true data generating process, against the alternative that one model is closer. Neither model has to be correctly specified.

The tests are based on the difference between the log likelihood values of the two models being compared. Suppose that we want to compare model 1 and model 2 using M observations. Denote the log likelihood of observation m for model 1 (2) at the estimated parameter vector by ℓ_m^1 (ℓ_m^2). The likelihood ratio is defined as

$$LR = \sum_{m=1}^{M} \left(\ell_m^1 - \ell_m^2\right).$$

Denote the number of parameters to be estimated by q_1 (q_2) for model 1 (2).

If the two models are non-nested, then, under the null hypothesis that the two models are equally close to the true data generating process,

$$\frac{LR}{\sqrt{M}\hat{\omega}} \Rightarrow N(0,1),$$

where $\hat{\omega}$ is the estimated standard deviation of the likelihood ratio, that is,

$$\hat{\omega}^2 = \frac{1}{M}\sum_{m=1}^{M}\left(\ell_m^1 - \ell_m^2\right)^2 - \left(\frac{1}{M}\sum_{m=1}^{M}\left(\ell_m^1 - \ell_m^2\right)\right)^2,$$

and where $\Rightarrow$ means convergence in distribution. If the two models are nested, and we want to allow for the possibility that the unconstrained model is not correctly specified, then, under the null,

$$2LR \Rightarrow M_{q_1+q_2}(\cdot;\hat{\kappa}),$$

where $M_{q_1+q_2}(\cdot;\hat{\kappa})$ is the cumulative distribution function of a weighted sum of (q_1+q_2) χ^2 distributions with degrees of freedom equal to 1 (Vuong, 1989).[17] The p-values of the weighted χ^2 distribution are simulated. I do 100,000 replications.

I compare seven models: risk sharing with limited commitment with heterogeneous preferences and heterogeneous income processes (LC^{u_i,r_i}),[18] with heterogeneous preferences but homogeneous income processes ($\text{LC}^{u_i,r}$), with homogeneous preferences but heterogeneity in income processes (LC^{u,r_i}), the homogenous case ($\text{LC}^{u,r}$), and the benchmark models of perfect risk sharing with heterogeneous preferences (PRS^{u_i}), perfect risk sharing with homogeneous preferences (PRS^{u}), and autarky (AUT). LC^{u_i,r_i} nests LC^{u,r_i} and the benchmark models. $\text{LC}^{u_i,r}$ nests $\text{LC}^{u,r}$ as well as the benchmark models. LC^{u,r_i} and $\text{LC}^{u,r}$ nest PRS^{u} and AUT. PRS^{u_i} nests PRS^{u}. The remaining combinations are non-nested.

4 Data

I use the publicly available part of the Townsend Thai Monthly Surveys. Data from the first 24 monthly interviews are currently available (120 months of data exist). The household roster has not been released yet, thus some basic information about households is missing, such as age and education of household members.

The Monthly Baseline Survey was conducted in August 1998 on 720 households in 16 villages in 4 tambon,[19] one in each of 4 provinces[20] in Thailand. The provinces are Buriram and Srisaket in the poor Northeast region, and Lopburi and Chachoengsao in the richer Central region. 45 households per village were randomly selected to be included in the monthly survey, and reinterviewed monthly from September 1998. Information about some frequently purchased items, such as food, is collected weekly. This data set is thus likely to provide better measures of income and consumption. Data collection and release are ongoing, therefore the survey will provide a long panel on households. This advantage is, however,

[17] The weights $\hat{\kappa}$ can be computed by finding the real, nonzero eigenvalues of the matrix

$$\begin{bmatrix} -\hat{B}^1(\hat{A}^1)^{-1} & -\hat{B}^{1,2}(\hat{A}^2)^{-1} \\ \hat{B}^{2,1}(\hat{A}^1)^{-1} & \hat{B}^2(\hat{A}^2)^{-1} \end{bmatrix},$$

where $\hat{A}^1 = \sum_{i=1}^N \sum_{t=2}^T -\nabla_\theta^2 \ell_{it}^1$, $\hat{B}^1 = \sum_{i=1}^N \sum_{t=2}^T \hat{s}_{it}^1 \hat{s}_{it}^{1\prime}$, similarly for model 2, and $\hat{B}^{1,2} = \hat{B}^{2,1\prime} = \sum_{i=1}^N \sum_{t=2}^T \hat{s}_{it}^1 \hat{s}_{it}^{2\prime}$.

[18] The upper index u_i refers to heterogeneous preferences, and the upper index r_i refers to heterogenous income processes, emphasizing the heterogeneity in income risk faced. Upper index u will refer to homogenous preferences and r to homogenous income processes.

[19] Tambon is a local government unit in Thailand. As of the 2009 there are 7255 tambon, not including Bangkok.

[20] There are 75 chagwat (provinces) in Thailand.

Table 1: Descriptive statistics

Variable	Mean	Sd	Min	Max	Observations
Non-durable consumption[a]	8580.3	6176.0	117.5	67366.0	3262
Non-dur. cons. per capita[a]	2228.9	1474.3	29.4	11321.5	3262
Income[a]	14547.2	28497.1	−148638.8	362722.9	3262
Income per capita[a]	3675.5	6746.6	−37159.7	55313.9	3262
Household size	4.153	1.762	1	13	3262
Log household size	1.327	0.463	0	2.565	3262

[a]Measured in current Thai baht per quarter. Prices hardly changed in the period. In particular, according to the Thai Ministry of Trade, the Rural Consumer Price Index varied between 87.8 and 87.9 for the years 1998-2000 with 2002 serving as the base. In 2002, approximately 42 Thai baht were worth 1 US dollar.

cannot be exploited in this paper.

I construct a measure of non-durable consumption and income using both the weekly and monthly interviews. Some big expenses and revenues are assigned to a month in the raw data. For the rest, I first assign consumption and income to days taking into account the exact dates of interviews, and then aggregate to quarters. I drop the top 1% per capita consumption and income observations. I end up with a 7-period panel starting with the 4$^{\text{th}}$ quarter of 1998 and ending with the 2$^{\text{nd}}$ quarter of 2000, and a balanced panel of 491 households. While I do not have sufficient data available to consider annual consumption and income, aggregation to quarters is likely to reduce measurement error if recall errors are uncorrelated across months. To parametrize preference heterogeneity, I use the logarithm of household size in October 1998 (the first month used).

On average, daily nondurable consumption per capita is 83.26 Thai baht, which is about 2.25 2002 US dollars and about 2.75 2010 US dollars. The difference between non-durable consumption reflects durable consumption and investment.

Before turning to the structural results, I present reduced form evidence on how nundurable consumption reacts to income. I regress (i) the logarithm of consumption on logarithm of income and (ii) the logarithm of per capita consumption on the logarithm of per capita income, controlling for village×quarter dummies. I then add household size or the logarithm of household size as a control. Finally, I add household fixed effects. Table 2 presents results for total household consumption and Table 3 for per capita consumption. Unfortunately, adult-equivalent consumption cannot be constructed with the data currently available, because the age of household members in not available.

Tables 2 shows that a 1 percent increase in total household income leads to an increase of 0.17 to 0.21 percent (p-value: 0.000) in total household nondurable consumption. Table 3

Table 2: Dependent variable: logarithm of nondurable consumption

Logarithm of income	0.212***	0.176***	0.170***	0.070***	0.069***	0.067***
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Household size		0.112***			0.105***	
		(0.005)			(0.013)	
Log household size			0.443***			0.418***
			(0.019)			(0.050)
Village×time dummies	yes	yes	yes	yes	yes	yes
Household fixed effects	no	no	no	yes	yes	yes
Observations	2854	2854	2854	2854	2854	2854

Robust standard errors are in parentheses. *** indicates significance at the 1% level.

Table 3: Dependent variable: logarithm of per capita nondurable consumption

Logarithm of per	0.182***	0.171***	0.172***	0.078***	0.068***	0.068***
capita income	(0.011)	(0.011)	(0.011)	(0.010)	(0.010)	(0.010)
Household size		−0.091***			−0.126***	
		(0.005)			(0.013)	
Log household size			−0.357***			−0.500***
			(0.017)			(0.054)
Village×time dummies	yes	yes	yes	yes	yes	yes
Household fixed effects	no	no	no	yes	yes	yes
Observations	2852	2852	2852	2852	2852	2852

Robust standard errors are in parentheses. *** indicates significance at the 1% level.

shows that for per capita consumption, the coefficient is between 0.17 and 0.18. Controlling for household fixed effects in addition, the coefficient is about 0.07 with $p = 0.000$ (both for total and per capita nondurable consumption), thus this reduced-form test strongly rejects perfect risk sharing.

5 Structural Estimation and model selection results

I consider seven models: risk sharing with limited commitment with and without preference heterogeneity and with heterogeneous and homogeneous income processes (LC^{u_i,r_i}, $\mathrm{LC}^{u_i,r}$, LC^{u,r_i}, and $\mathrm{LC}^{u,r}$), perfect risk sharing with and without heterogeneous preferences (PRS^{u_i} and PRS^{u} respectively), and autarky (AUT). I measure consumption and income in per capita terms. Aggregate consumption in the village is assumed to be exogenous, and I look at each household's consumption relative to the village mean. Vuong's (1989) tests are performed to statistically compare the seven models pairwise. The computations have been done using the software R (see www.r-project.org).

I drop the village with only 17 households in the final sample. For the remaining 15 villages, data for between 24 and 40 households are available.

Before turning to the main results, some preliminary steps are necessary (see subsection 3.3.4). I estimate an AR(1) process for the logarithm of households' endowments. Remember that endowments differ from incomes because of the assumed availability of disaster insurance. I denote the endowments by y too, abusing notation. The persistence of the logarithm of the endowment (ρ) is between 0.14 and 0.76. The variance of the shocks (σ_ξ^2) is between 0.22 and 0.88 across the 15 villages.

The support points of the Markov chain on the level of income are chosen as suggested by Kennan (2006). Households' income may take 7 different values. Then Tauchen (1986)'s method is used to find the transition matrix. When income processes are heterogenous, I estimate processes for four groups separately. The four groups are created based on whether the household has mean income below or above the village median, and whether the standard deviation of its income is below or above the median standard deviation.

The average correlation coefficient of any two households' incomes in the 15 villages is on average 0.016, therefore the assumption that households incomes are independent is consistent with the data. I estimate an AR(1) process for the village mean income on simulated data from the household income processes. I then proceed to find the approximating Markov chain as for households.

Note: I present some results for one village only, the 7^{th} village in the data set. In my final sample, there are 34 households in this village. Computations for the whole sample are in progress. For Village 7, the estimated AR(1) process for household' endowment is

$$\log(y_{it}) = (1 - 0.289) \times 1.660 + 0.289 \log(y_{i,t-1})$$

and $\sigma_\xi^2 = 0.319$. *For the village mean income, the estimated AR(1) process is*

$$\log(y_{kt}) = (1 - 0.181) \times 1.846 + 0.181 \log(y_{k,t-1})$$

and $\sigma_\xi^2 = 0.008$.

5.1 Main results

Table 4 shows the structural estimation results for nondurable consumption for all the models for Village 7. The second panel shows the model selection test statistics, conducting Vuong's (1989) tests for nested and non-nested models as appropriate.

(...)

Table 4: Risk sharing in Village 7, nondurable consumption

	LC^{u_i,r_i}	$\text{LC}^{u_i,r}$	LC^{u,r_i}	$\text{LC}^{u,r}$	PRS^{u_i}	PRS^{u}	AUT
β (preference heterogeneity)					0.860^{***}		
					(0.168)		
δ (annual discount factor)			0.925	0.901^{***}			
			()	(0.001)			
γ^2 (var of measurement error)			0.082	0.062^{***}	0.049^{***}	0.049^{***}	0.319^{***}
			()	(0.013)	(0.006)	(0.007)	(0.028)
Log likelihood			-29.92	-31.47	-46.80	-51.59	-172.9
Observations			204	204	204	204	204
	Vuong's tests						

Notes: LC^{u_i,r_i} ($\text{LC}^{u_i,r}$): equation (25) is estimated by maximizing the log-likelihood function (28) assuming heterogeneous (homogeneous) income risk when estimating F_{Y_i}, $\forall i$. LC^{u,r_i} ($\text{LC}^{u,r}$): equation (25) with $\beta = 0$ is estimated assuming that income risk is heterogeneous (homogeneous). PRS^{u_i}: equation (16) is estimated by maximizing the log-likelihood function (17). PRS^{u}: equation (16) with $\beta = 0$ is estimated. AUT: equation (19) is estimated by maximizing the log-likelihood function (20). In the first panel, standard errors are in parentheses. They have been calculated taken into account misspecification and serial correlation. In the second panel, p-values of Vuong's tests are in parentheses, indicating whether the model of the line can be rejected to be as close to the true generating process as the model of the column. In the case of nested models, the p-values are simulated. *** indicates significance at the 1% level.

5.2 Robustness checks

I first look at whether the results are robust to changing the number of simulations J in the limited commitment case. I increase J to 100. Second, I check for the effects of approximation error when computing the solution of the risk sharing with limited commitment model. In particular, I increase the number of grid points to 50 when discretizing the state variable x_i. Third, I increase the number of income states for each household to 8. I only do the estimation with each of these changes for the $\text{LC}^{u,r}$ model. The results are presented in Table ??.

6 Policy simulations

In this section, I examine the effects of two types of counterfactual policies. First, I consider counterfactual transfers targeting the poor. The poor are defined as having below median mean consumption, that is 109 rupees per week per adult equivalent. I look at the effects of (i) a one-time transfer and (ii) a permanent transfer to the poor on consumption by eligible and ineligible households.

Second, I consider the introduction of formal aggregate insurance. As a result, village mean consumption can be perfectly smoothed. Attanasio and Ríos-Rull (2000) show that in the context of the model, welfare may decrease as a result of aggregate insurance. I study how much self-enforcing insurance would be crowded out by this type of insurance according to the estimated model.

6.1 Transfers to the poor

I simulate the effects of the following two social policies: (i) the poor receive a one-time unconditional cash transfer of 1000 baht at time t, and (ii) the poor receive an unconditional cash transfer of 1000 baht in all periods starting from time t. Mean income of the poor is about 4000 baht per quarter, thus the transfer increases income by 25 percent on average. This is similar in magnitude to the transfer received by households in the Progresa program in Mexico, see Gertler (2004), for example.[21] I consider the introduction of the policy at each time $t = 2, ..., 7$, and average over the predicted consumption changes at the time of the introduction of the program. I assume that the one-time transfer is seen as a shock, and that it increases permanent income so little that the decision power of recipients does not increase. In the case of a permanent transfer, there is a change in income processes, which are important in determining insurance transfers when commitment is limited. Since this means that decision powers change, the informal risk sharing contract is renegotiated.

Were households in autarky, or, under the policymaker's assumption the transfer increases consumption directly, eligible households' consumption should increase by 1000 baht, while consumption by ineligible households should be unaffected. On the other hand, if households share risk perfectly, the transfers to the poor become part of the common pool, and consumption by each household should increase by about 500 baht. Any variation in consumption changes should come from differences in risk preferences.

(...)

6.2 Aggregate insurance

(...)

7 Extensions

In this section, I develop a more general empirical model. In particular, I allow preferences to depend on (i) unobservable individual effects and (ii) time-varying household characteristics;

[21] Progresa is a conditional cash transfer program. In this exercise, I cannot take such conditionality into account. In other words, I do not look at changes in household behavior other than with respect to insurance transfers. Neither do I deal with how poor households could be identified in practice.

further, (iii) preference shocks are introduced, and (iv) income is measured with error as well as consumption. I show that the predicted consumption allocation is not affected by the individual effects, and argue that the other extensions are possible in theory. They are, however, infeasible due to prohibitively long computation time.

Instead of (1), let us now specify the utility function as

$$u_{it}\left(c_{it}\right)=\exp\left(\xi_{it}\right)\frac{c_{it}^{1-\sigma_{it}}-1}{1-\sigma_{it}}, \tag{29}$$

where

$$\xi_{it}=\eta_i+w_{it}'\alpha+\varepsilon_{it}^{\eta},$$

and the coefficient of relative risk aversion is

$$\sigma_{it}=1+z_{it}'\beta,$$

where η_i is a time-constant unobservable individual effect, w_{it} and z_{it} are vectors of observable characteristics of household i at time t, α and β are parameter vectors to be estimated, and ε_{it}^{η} is a normally distributed preference shock with mean 0 and variance γ_{η}^2. Note that σ_{it} is only allowed to depend on observable covariates. Finally, let y_{it}^* be income observed by the econometrician. Measurement error in income is assumed to be multiplicative and log-normally distributed, that is,

$$y_{it}^*=\exp\left(\varepsilon_{it}^{y}\right)y_{it},$$

and $\varepsilon_{it}^{y}\sim N\left(0,\gamma_y^2\right)$.

7.1 Perfect risk sharing

The first order condition for household i, sharing risk with the rest of the community, can now be written as

$$\sigma_{it}\log c_{it}-\xi_{it}-\log c_{kt}=\log\lambda_i.$$

Replacing for ξ_{it} and σ_{it}, in terms of measured consumption we have

$$\left(1+z_{it}'\beta\right)\log c_{it}^*-\left(1+z_{it}'\beta\right)\varepsilon_{it}-\eta_i-w_{it}'\alpha-\varepsilon_{it}^{\eta}-\log c_{kt}=\log\lambda_i.$$

First differencing and rearranging give

$$\begin{aligned}\log\left(\frac{c_{it}^*}{c_{kt}}\right)=&\log\left(\frac{c_{i,t-1}^*}{c_{k,t-1}}\right)-z_{it}'\beta\log c_{it}^*+z_{i,t-1}'\beta\log c_{i,t-1}^*+w_{it}'\alpha-w_{i,t-1}'\alpha\\&+\varepsilon_{it}^{\eta}-\varepsilon_{i,t-1}^{\eta}+\left(1+z_{it}'\beta\right)\varepsilon_{it}-\left(1+z_{i,t-1}'\beta\right)\varepsilon_{i,t-1}.\end{aligned}$$

Thus η_i drops out along with $\log\lambda_i$. The error term is now distributed as

$$N\left(0,2\gamma_{\eta}^2+\left(1+z_{it}'\beta\right)^2\gamma^2+\left(1+z_{i,t-1}'\beta\right)^2\gamma^2\right).$$

7.2 Autarky

In the autarky case, preferences do not play a role. Thus only the additional measurement error in income has to be taken into account. Instead of (19), the equation to be estimated is

$$\log\left(\frac{c^*_{it}}{c_{kt}}\right) = \log\left(\frac{y^*_{it}}{y_{kt}}\right) + \varepsilon_{it} - \varepsilon^y_{it},$$

and γ^2 and γ^2_y are not jointly identified.

7.3 Risk sharing with limited commitment

Instead of (23), the first order condition now is

$$\log\left(\frac{c^*_{it}}{c_{kt}}\right) = -z_i'\beta \log c^*_{it} + \eta_i + w_{it}'\alpha + \varepsilon^\eta_{it} + \log g\left(y_t, x_{i,t-1}, Z_i, W_i; \theta, \eta_i\right) + \left(1 + z_i'\beta\right)\varepsilon_{it}, \quad (30)$$

where Z_i and W_i are matrices of observable covariates of household i at different times, and θ now includes the vector α as well.

Let us first deal with the individual effect η_i, assuming $z_{it} = z_i$, $\forall t$, and ignoring the other time-varying components of the utility function (w_{it} and ε^η_{it}) for the moment. Suppose that we know the realization of measurement error in household i's consumption at time $t-1$, denoted $\varepsilon^j_{i,t-1}$, drawn from the distribution of $\varepsilon_{i,t-1}$, $N\left(0, \gamma^2\right)$. I show that

$$\begin{aligned} g\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta, \eta_i\right) &= \exp\left(-\eta_i\right) g\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right) \\ &= \exp\left(-\eta_i\right) \frac{\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)^{1+z_i'\beta}}{\hat{c}_{kt}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)}, \end{aligned}$$

where $\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)$ and $\hat{c}_{kt}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right)$ are consumption by households i and k respectively, predicted by the model, normalizing $\eta_i = 0$. Replacing this in equation (30), η_i drops out. The fact that the function $g()$ is homogeneous of order one in η_i is the direct consequence of the following claim.

Claim 1. $\hat{c}_{it}\left(y_t, x^*_{i,t-1}, \varepsilon^j_{i,t-1}, z_i; \theta\right) = \hat{c}_{it}\left(y_t, c^*_{i,t-1}, z_i, \varepsilon^j_{i,t-1}; \theta, \eta_i\right)$. *That is, the consumption allocation predicted by the model of risk sharing with limited commitment does not depend on the individual effects.*

Proof. To see this, let us take a closer look at the enforcement constraint of some household

i, that can be written as (7) in general. Replacing the utility function (1) in (7) gives

$$\exp(\eta_i)\frac{c_{it}(s^t)^{1-\sigma_i}-1}{1-\sigma_i}+\sum_{r=t+1}^{\infty}\sum_{s^r}\delta^{r-t}\pi(s^r)\exp(\eta_i)\frac{c_{ir}(s^r)^{1-\sigma_i}-1}{1-\sigma_i}\geq$$
$$\geq\exp(\eta_i)\frac{y_{it}(s_t)^{1-\sigma_i}-1}{1-\sigma_i}+\sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi(s_r)\exp(\eta_i)\frac{y_{ir}(s_r)^{1-\sigma_i}-1}{1-\sigma_i}. \quad (31)$$

Both sides can be divided by $\exp(\eta_i)$, thereby eliminating the individual effects. When no enforcement constraint is binding, we are back to perfect risk sharing, where $\exp(\eta_i)$ appears multiplicatively on both sides of $x_{it}=x_{i,t-1}$. □

Then, with the utility function (29), a typical enforcement constraint can be written as

$$\exp(w_{it}'\alpha+\varepsilon_{it}^{\eta})\frac{c_{it}(s^t)^{-z_{it}'\beta}-1}{-z_{it}'\beta}+\sum_{r=t+1}^{\infty}\sum_{s^r}\delta^{r-t}\pi(s^r)\exp(w_{ir}'\alpha+\varepsilon_{ir}^{\eta})\frac{c_{ir}(s^r)^{-z_{ir}'\beta}-1}{-z_{ir}'\beta}\geq$$
$$\geq\exp(w_{it}'\alpha+\varepsilon_{it}^{\eta})\frac{y_{it}(s_t)^{-z_{it}'\beta}-1}{-z_{it}'\beta}+\sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi(s_r)\exp(w_{ir}'\alpha+\varepsilon_{ir}^{\eta})\frac{y_{ir}(s_r)^{-z_{ir}'\beta}-1}{-z_{ir}'\beta}.$$

Note that future household characteristics and preference shocks enter into today's enforcement constraint. Therefore, some assumptions have to be made on households' expectations about their future characteristics and preference shocks.

If we assume that households are myopic, in the sense that they do not expect their characteristics to change relative to the rest of the village, and further that they do not expect preference shocks to differ from today's, then we may divide the above equation by $\exp(w_{it}'\alpha+\varepsilon_{it}^{\eta})=\exp(w_{ir}'\alpha+\varepsilon_{ir}^{\eta}), \forall r>t$. In this case computation time is only multiplied by the number of time periods. However, the assumption that households are always surprised by a change in any of their characteristics is very strong.

Alternatively, we may assume that households form rational expectations, and their expectations about their characteristics next period are the observed values. Preference shocks can be integrated out using simulation, if we make some assumption about their distribution. To keep things tractable, I assume that preference shocks are i.i.d. over time and across households. Given household characteristics and a realization of the preference shock today, the enforcement constraint can be written in a recursive form as

$$\exp(w_{it}'\alpha+\varepsilon_{it}^{\eta})\left[\frac{\hat{c}_i(s_t,x_{t-1})^{-z_{it}'\beta}-1}{-z_{it}'\beta}-\frac{y_i(s_t)^{-z_{it}'\beta}-1}{-z_{it}'\beta}\right]\geq\delta\sum_{s_{t+1}}\pi(s_{t+1})\times$$
$$\int\left[V_i^{aut}\left(s_{t+1},w_{i,t+1},z_{i,t+1},\varepsilon_{i,t+1}^{\eta}\right)-V_i\left(s_{t+1},x_t(s_t,x_{t-1}),w_{i,t+1},z_{i,t+1},\varepsilon_{i,t+1}^{\eta}\right)\right]f\left(\varepsilon_{i,t+1}^{\eta}\right)d\varepsilon_{i,t+1}^{\eta}.$$

Thus the model should be solved on an extended state space that includes household characteristics. In practice, a grid has to be defined over each characteristic. Today's preference shock also has to be integrated out by simulation. Computation time thus becomes prohibitively long.

Note that, once we have computed the consumption allocation predicted by the model, $\hat{c}_{it}\left(y_t, c^*_{i,t-1}, \varepsilon^j_{i,t-1}, Z_i, W_i; \theta\right)$ and $\hat{c}_{kt}\left(y_t, c^*_{i,t-1}, \varepsilon^j_{i,t-1}, Z_i, W_i; \theta\right)$, then

$$\log g\left(y_t, x_{i,t-1}, Z_i, W_i; \theta, \eta_i\right) = \exp\left(-\eta_i - w'_{it}\alpha - \varepsilon^\eta_{it}\right) \frac{\hat{c}_{it}\left(y_t, c^*_{i,t-1}, \varepsilon^j_{i,t-1}, Z_i, W_i; \theta\right)^{1+z'_i\beta}}{\hat{c}_{kt}\left(y_t, c^*_{i,t-1}, \varepsilon^j_{i,t-1}, Z_i, W_i; \theta\right)}.$$

Thus, along with η_i, $w'_{it}\alpha$ and ε^η_{it} drop out from equation (30). Therefore, adding measurement error in consumption is necessary to account for the residual in the estimating equation.

Finally, let us look at what changes if income is measured with error, as well as consumption. First, given the income processes, measurement error does not affect the optimal intervals that characterize the constrained efficient risk sharing contract, since the model is solved using a grid on income. Second, the introduction of measurement error in income may plague the estimation of the income process. We could perturbate observed income, and then recompute the model given a matrix of draws of measurement error. Computation time is then proportional to the number of matrices of draws. Third, today's income observation directly affects consumption predicted by the model. Once again, simulation is a simple way to deal with this problem, and the solution of the model does not have to be recomputed, thus computation time increases only moderately due to this third point.

8 Concluding remarks

This paper first performed statistical tests to compare seven models of risk sharing. Preliminary structural estimation results suggest that limitations in the enforcement of informal insurance contracts and heterogeneity in preferences and in income risk are important in explaining the consumption allocation in sixteen villages in rural Thailand.

Using structural estimation results, this paper then simulated the effects of a simple social policy and the introduction of aggregate insurance. The model predicts that consumption by both eligible and ineligible households should increase, consistently with the empirical findings of Angelucci and De Giorgi (2009). Research on the structural modeling of how consumption is allocated between households in poor communities can serve as an input for policy evaluation and design. Policy makers and members of non-governmental organizations could have a better understanding of the effects of their programs, such as redistributive policies or micro-insurance programs, by taking into account existing informal arrangements to share risk.

Several interesting extensions are possible. First, whether heterogeneity in the discount factor across households is important should also be addressed. Second, other models of risk sharing could be incorporated into the analysis, like the model of risk sharing with private information (Wang, 1995). In a recent paper, Kinnan (2010) finds that asymmetric information about income realizations is important in accounting for partial insurance in these Thai villages. Whether such a model is useful for predicting the consumption allocation and quantitative policy effects is to be studied. Another important task for future work is to allow for individual savings, as in Ligon, Thomas, and Worrall (2000). Fourth, it is to be examined whether introducing more heterogeneity across households would mean that the model of risk sharing sharing with limited enforcement could better capture the amount of risk sharing in other contexts, in particular, in the United States, since the homogenous model predicts too much insurance (Krueger and Perri, 2006). Finally, when complete markets do not exist to insure against income fluctuations, households are expected to choose safer jobs, or safer production technologies in agriculture. In other words, they smooth income, not just consumption (Morduch, 1995). These ideas could be formalized in the context of this paper, endogenizing income by allowing households to choose between several income generating processes. Then, the cost of imperfect consumption insurance in terms of lower expected incomes could be quantified.

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Appendix

This appendix details how to compute the consumption allocation predicted by the model. That is, we want to find $\hat{c}_{it}$ relative to mean consumption in the community $\hat{c}_{kt}$, for each household i at each time t, taking preference parameters and the income processes as given.[22]

Consider some household i sharing risk with the rest of the community, 'household' k. Take preference parameters β, the discount factor δ, and the income processes F_{Y_i} and F_{Y_k} as given. The aim is to solve for the state-dependent optimal intervals on the relative Pareto weight of household i, that fully characterize the solution (LTW).

First, define a grid over the continuous variable x_i (I define the same points for all s_t). The support of the grid is the range of ratios of marginal utilities of household k and i given the income and consumption observations. I define an equidistant grid on log x_i of 30 points. Second, guess a solution for the value functions, that is, guess $V_i^0\left(s_t, x_{i,t-1}\right)$, for each grid point. The algorithm does not converge from any initial guess for the value functions, but the value of perfect risk sharing will do.[23]

Then, proceed to update the guess. Suppose we are at iteration h. Let us look at grid point $(\tilde{s}_t, \tilde{x}_{i,t-1})$. Three cases have to be distinguished: (a) neither enforcement constraint binds, (b) the enforcement constraint for household i binds, and (c) the enforcement constraint for household k binds. Note that the two enforcement constraints cannot bind at the same time, because only one of the two households may be called upon to make a positive net transfer.

We first suppose that the enforcement constraints do not bind, that is, we try to keep x_i constant. This means setting $\hat{x}_{it}^h = \tilde{x}_{i,t-1}$ at state $\tilde{s}_t$, where the upper index h refers to iteration h. Then, using the first order condition and the resource constraint, we get the consumption allocation $\left(\hat{c}_{it}^h, \hat{c}_{kt}^h\right)$. Now the enforcement constraints have to be checked. This means verifying whether

$$u_i\left(\hat{c}_{it}^h\right) + \delta \sum_{s_{t+1}} \pi\left(s_{t+1} \mid \tilde{s}_t\right) V_i^{h-1}\left(s_{t+1}, \hat{x}_{it}^h\right) \geq U_i^{aut}\left(\tilde{s}_t\right) \tag{32}$$

[22]The first step in estimation involves determining these distributions (see main text), while the last step is the maximization over the remaining structural parameters, which is done using a standard optimization algorithm available in R (function optim() with method BFGS with bounds (L-BFGS-B), which is a quasi-Newton method). See www.r-project.org. Here, I am talking about the computation between these steps.

[23]Characterizing the convergence properties of the algorithm is left for future research. However, we know that the algorithm does not converge to the constrained-efficient solution from any initial guess for the value functions. For example, if we set the guesses for $V_i^0\left(s_t, x_{i,t-1}\right)$ equal to the autarkic values, every iteration yields these same autarkic values. This is natural, since autarky is also a subgame perfect Nash equilibrium (SPNE).

and

$$\log\, \hat{c}_{kt}^{h} + \delta \sum_{s_{t+1}} \pi\left(s_{t+1} \mid \tilde{s}_t\right) V_k^{h-1}\left(s_{t+1}, \hat{x}_{kt}^{h}\right) \geq U_k^{aut}\left(\tilde{s}_t\right). \tag{33}$$

Note the upper index $h-1$ for V_i and V_k, that is, we use the value function from the previous iteration.

(a) *The enforcement constraints* (32) *and* (33) *do not bind.* This is the easy case, since we have already computed $\hat{x}_{it}^{h}$ and the consumption allocation assuming that the enforcement constraints do not bind. What remains to be done is to set

$$V_i^h\left(\tilde{s}_t, \tilde{x}_{i,t-1}\right) = u_i\left(\hat{c}_{it}^{h}\right) + \delta \sum_{s_{t+1}} \pi\left(s_{t+1} \mid \tilde{s}_t\right) V_i^{h-1}\left(s_{t+1}, \hat{x}_{it}^{h}\right)$$

and

$$V_k^h\left(\tilde{s}_t, \tilde{x}_{i,t-1}\right) = \log\, \hat{c}_{kt}^{h} + \delta \sum_{s_{t+1}} \pi\left(s_{t+1} \mid \tilde{s}_t\right) V_k^{h-1}\left(s_{t+1}, \hat{x}_{it}^{h}\right).$$

(b) *The enforcement constraint* (32) *is binding.* We look for $\hat{c}_{it}^{h}$ and $\hat{x}_{it}^{h}$ to satisfy (32) with equality and the first order condition. Since we do not know $V_i^{h-1}\left(s_{t+1}, \hat{x}_{it}^{h}\right)$ for any value of $\hat{x}_{it}^{h}$, only for the points on the grid, I do linear interpolation. Finally, we update the value function as in case (a).

(c) *The enforcement constraint* (33) *is binding.* We proceed similarly as in case (b).

Now we are done with grid point $(\tilde{s}_t, \tilde{x}_{t-1})$. We have to do the above steps at all other grid points as well. Then the h^{th} iteration is complete. We continue iterating until the policy functions converge. In the end, we find the range of x_i's that can be optimal in some state s to find the solution in the form $[\underline{x}_i(s), \overline{x}_i(s)]$.

Computing the consumption of household i at time t, relative to mean consumption in the community, as predicted by the model is then done as follows. Remember that $c_{i,t-1}^{*}$ is the observed consumption by household i at time $t-1$, and $\varepsilon_{i,t-1}^{j}$ is a realization of measurement error drawn from $N\left(0, \gamma^2\right)$, and γ^2 is given. I compute $x_{i,t-1} = \left(\exp\left(-\varepsilon_{i,t-1}^{j}\right) c_{i,t-1}^{*}\right)^{1+z_i'\beta} / c_{k,t-1}$, and check whether it is in the optimal interval for today's state s_t. Since only a discrete number of income states have been considered, I map observed incomes into the income states of the model by picking the closest point for each household. We have to consider the above three cases.

(a) If $x_{i,t-1} \in [\underline{x}_i(s), \overline{x}_i(s)]$, then we set $x_{it} = x_{i,t-1}$.

(b) If $x_{i,t-1} < \underline{x}_i(s)$, then we determine it from (32) with equality, using a linear interpolation of the value functions from the last iteration.

(c) If $x_{i,t-1} < \overline{x}_i(s_t, x_i)$, then we use (33) with equality, and proceed similarly as in case (b).

Finally, we use the first order condition and the resource constraint to determine the predicted consumption allocation. We may then write the likelihood of observation it, given $\varepsilon^j_{i,t-1}$, by plugging the $\hat{c}_{it}$ and $\hat{c}_{kt}$ computed here into (27).