Information Production in the Process of Securitization

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Abstract

This paper studies the information production by banks when there is uncertainty about the distribution of the returns on individual loans. In choosing how much information to purchase, banks face a trade-off: a more precise signal reduces the risks investors face, increasing their willingness to pay; but it also increases the volatility of asset prices as a result of investors' Bayesian updating. The incentive is exacerbated when banks securitize instead of issuing

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single loans, because the diversification implied by securitization decreases the marginal benefit of information. When faced with loans of different qualities, banks tend to originate more loans with riskier return under securitization. The findings have bearing on policy issues hotly debated in the aftermath of the crisis. It suggests that rating agencies, even in the absence of conflicts of interests, are limited by the extent of banks' information production. Second, even in the world with public information, the securitization implies reduced information production on the basis of profit maximization.

1 Introduction

The recent subprime mortgage crisis has brought a considerable amount of attention to the market of mortgage-backed securities (MBS) and the process of securitization. Due to a reduction in long-term interest rates, the total issuance of MBS including collateralized mortgage obligations (CMO) experienced a surge through 2003. Starting 2004 as the interest rates picked up, the MBS issued by the Government Sponsored Enterprises (GSE) such as Ginnie Mae, Fannie Mae and Freddie Mac subsided. At the same time, however, the private-label MBS continued to grow until 2007. (Figure 1) Accompanying this rapid growth in the non-agency market, a class of loans that were not conforming to the underwriting standard set by the GSEs were demanded with increasing enthusiasm by these non-agency issuers. (Table 1) It is also noticeable in Table 1 that the rate of securitization went up along with the origination of these non-conforming loans. More specifically, these loans include the Jumbo loans, the Alt-As and the subprime mortgages. The Jumbo loans are loans to borrowers who are qualified for a loan from a GSE but need otherwise an original principal balance larger than the conforming limits. The Alt-A loans are originated to borrowers who have good credit but less than full documentation, higher loan-to-values or more investment properties than are allowed to qualify for a prime loan from a GSE. Subprime mortgages are marketed to borrowers who have weakened credit histories, poor credit scores, high debt-to-income ratios or incomplete credit histories. These loans are clearly riskier than prime loans. But the question is how much more risky? If conforming to the underwriting standards of prime loans gives one a reliable measure of risk, how much information about the return does the fact that a loan is *non*-conforming contain? Intuitively, one should expect more information gathering on the non-conforming loans, since these loans are new to the market and little is known about them. But on the contrary, *less* information production took place in this sector of the market, where billions of dollars were spent. Table 2 shows the share of full documentation loans together with the share of non-agency loans in the market. The share of full-doc loans decreased steadily as the market share of non-conforming loans went up. Why did it happen? Did securitization play a role?

This paper develops a simple model of information production that answers these questions. In the model, a loan is represented by a draw from a distribution of returns. The uncertainty in the riskiness of a mortgage is modeled as the uncertainty in the parameters of the distribution. A risk-averse bank, as the originator of the loan, can costly produce *public* information about the parameter in the return distribution. The amount of information that the bank chooses to produce depends on what it will do with the loans. Throughout the paper, two institutions are compared: the bank can either sell the loans individually to risk-averse investors or securitize them and sell pass-through securities backed up by the loans. It will be shown that when it is in the bank's interest to produce information at all, it always produces less information when securitizing than selling the loans.

Intuitively, when the bank decides how much information to produce, it faces a trade-off: more information makes the financial product (be it the loan or the security) less risky to investors, pushing up the price of the financial product; on the other hand, more information makes the price correlate more with the information than with investors' common prior, which leads to increased price volatility. Securitization, as compared with selling loans individually, exacerbates bank's incentive to produce information for two reasons. The diversification implied by the process of securitization guarantees investors a less risky return, which decreases the marginal benefit of information. Secondly, a larger investor base amplifies the downside of information production, making the volatility of prices more sensitive to information, hence reducing the incentive for information production. These statements will be made precise in the basic model where the bank can acquire information about the unknown variance of the returns.

In a generalized set-up, where the bank is faced with loans of different qualities and can produce information about both the mean and variance of the returns on these loans, the proposition that securitization provides disincentive for information production as compared with selling loans by the piece still holds. Moreover, a simple simulation shows that when the bank can choose how many pieces of loans to include in a pool, it tends to choose a big pool with more riskier loans when it securitizes. This captures exactly what happened in the recent crisis. Banks extend credits to more and riskier borrowers without collecting much information. The behavior is fully rationalized by the bank's profit motive. In fact, since the investors in the model always have zero surplus, securitization is welfare-improving. It achieves better risk-sharing by means of diversification which limits the need of producing costly signals. However, in a model where the bank and investors have different priors about the distribution of return, securitization is no longer a magic cure that can make everybody (weakly) happier. I show by numerical examples that securitization can make the bank better off at the cost of investors.

There is an abundance of work on the institution of securitization, which focuses on the comparison between the originate-to-distribute model and the originate-to-hold model (Pennacchi, 1988; Gorton and Pennacchi, 1995; Petersen and Rajan, 2002; Parlour and Plantin, 2008) This model differs from that line of research in that there is no asymmetric information here. In this paper, the bank cannot improve the return by acquiring information per se, in contrast to the delegated monitoring role banks assume in Diamond (1985). Hence the resulting moral hazard problem as is formalized in the aforementioned papers is absent. The motivation of the paper is to start from the existing institution of securitization and examine the implications this institution has on the aggregate information available on these loans. The case of selling loans by the piece serves as a control group that highlights the essential benefit that mortgage-backed-securities are intended for, namely the geographical diversification. Within the scope of the model, the role of information is to re-distribute risk among the bank and investors. As will be clear in the model, it is only when the bank is sufficiently risk tolerant than the investors that it starts producing information, because information introduces volatility in profits so that the bank can in effect retain part of the risk. In fact, in these cases, the bank is better off simply holding the loans[1]. What prevents banks from doing that? In reality the capital requirement often limits the extent of which the bank can engage in making profitable investments. When banks are prevented from holding loans on their balance sheet due to regulation, securitization can help keep the source of profit alive. Securitization has long raised concerns from the regulators as a way of regulatory arbitrage (Calomiris and Mason, 2004). By transferring mortgage loans to the Real Estate Mortgage Investment Conduits (REMIC), banks can effectively structure a mortgage-backed securities offering as a sale of assets rather than debt financing, removing the loans from the balance sheet and bypassing the minimum capital requirement. It is also from this perspective that

[1] A discussion on optimal holding of loans is in the Appendix.

comparing securitization with the case where banks sell loans individually seems a natural thing to do.

In the market of mortgage-backed securities, banks or originators of loans assume a big role in information production. There is in fact very little investors can do to research the loans. This is partly because of practicality–there are many loans in a mortgage pool–and partly because of feasibility. It is common practice among the GSEs to limit the amount of information available to the market. Information at the pool level is disclosed, but specific information that helps identify the particular loans in the pool is withheld. Diamond and Verrecchia (1991), Fishman and Hagerty, (1990) and Glaeser and Kallal (1997) provide theoretical insights into this phenomenon in models with asymmetric information. These observations motivated me to model the information production as the choice of a bank faced with many homogeneous and competitive investors, who do not have the technology to produce information. In the absence of asymmetric information, the bank is driven entirely by its profit maximization motive. There is a time-honored literature on information production by heterogeneous investors in a competitive market with noisy rational expectations. Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982) and Diamond and Verrecchia (1991) are important contributions to that literature. I chose a more stylized set-up to sharpen the economic intuition behind banks' information decision.

In terms of welfare analysis, the paper is also related to the literature on social and private values of information (Hirshleiffer, 1971; Hakansson, Kunkel and Ohlson, 1982). In the basic set-up where the bank and investors have a common prior, securitization is Pareto improving. However, when the bank and investors have different priors, there is a scope for information to have some social value.

Some recent empirical findings (Dymyanyk and van Hemert, 2007; Mayer, Pence and Sherlund, 2009; Keys, Mukherjee, Seru and Vig, 2010; Mian and Sufi, 2008 and Dellariccia Igan and Laeven, 2008) lend support to the idea of this paper that the lack in information production is closely related to the increase in securitization. In the sample from the First American CoreLogic LoanPerformance database used in Demyanyk and van Hemert (2008), there is a clear decline in the share of loans with full documentation. (Table 2) As is shown in Figure 2 in Mayer, Pence and Sherlund (2009), early payment defaults, highly indicative of weak underwriting standards, rose rapidly for the subprime mortgages of the 2004 to 2007 vintage. Keys, Mukherjee, Seru and Vig (2010) find that increased securitization had adverse effect on banks' screening incentives. By exploiting a rule of thumb that loans with FICO score above a threshold of 620 were more easily securitized, they show that the loans with score just above 620 were more likely to default than those just below the threshold.

The paper is organized as follows. The second section contains a basic model that illustrates the key economic intuition. The third section describes a generalized model that captures more elements of reality. Section 4 presents a model with different prior that discusses some welfare issues. Conclusions follow.

2 The Basic Model: Uncertainty in the Variance

This is a static model. There are a single bank and a large number of homogeneous investors. The bank has the constant absolute risk aversion (CARA) utility with risk tolerance $\rho : U(w) = -\exp(-\frac{w}{\rho})$. Investors' preferences are also of CARA type with risk tolerance $r : u(w) = -\exp(-\frac{w}{r})$. The bank is endowed with n loans. The returns on the loans are drawn independently from a common normal distribution parameterized by (μ_0, σ^2). Ex ante, the variance σ^2 is unknown and is (correctly) believed to follow a normal distribution: $\widetilde{\sigma}^2 \sim N(\sigma_0^2, \sigma_{\sigma_0}^2)$. To make the normality assumption more plausible, suppose σ_0^2 is relatively big and $\sigma_{\sigma_0}^2$ relatively small. The normality is assumed for ease of interpretation. I show in the Appendix that the qualitative results also hold in an environment where the standard deviation of the return is distributed normal, but the economic intuition is blurred by the messy algebra. The bank has a technology to produce a public signal $\widetilde{s}^2$ about the unknown variance $\widetilde{\sigma}^2$:

$$\widetilde{s}^2 = \widetilde{\sigma}^2 + \widetilde{\epsilon},$$

where the noise $\widetilde{\epsilon}$ is distributed as $N(0, \sigma_\epsilon^2)$ independently from $\widetilde{\sigma}^2$. Denote the precision of the signal as $\pi = \frac{1}{\sigma_\epsilon^2}$. The cost of signals is defined in terms of the precision, $c : \mathbf{R}_+ \to \mathbf{R}_+$. It is increasing and convex in π, or $c' \geq 0$, $c" \geq 0$.

Consider two scenarios. The bank can either sell the loans individually to the investors or securitizes the loans and sells the securities to the investors. By securitization, I mean the bank issues claims of the return on the entire pool of loans. An example of this is the pass-through mortgage-backed-security. Assume that the bank issues N shares of securities to N investors, each share being a claim of $\frac{1}{N}$ of the total return on the pool. Assume $N \geq n$.

In either scenario, before the asset market opens, the bank chooses the amount of information, as measured by the precision of the signal, to produce. When it produces information at all, the investors and the bank itself observe a realization of the signal with the chosen precision. Investors make inference about the return based on the signal in a Bayesian fashion. Since all investors are homogeneous and competitive, they are charged by their willingness to pay. The focus is to compare the bank's incentive for information provision in these two scenarios.

The solution concept is the standard backward induction. First, given a level of the precision of the signal, I solve investors' inference problem, which determines the price of the loan or security that the bank can fetch in the asset market. Then, I proceed to bank's problem of information provision, taking its effect on the asset price into account.

Suppose the bank chooses a signal with precision $1/\sigma_\epsilon$. I can write the joint distribution of the unknown variance $\widetilde{\sigma}^2$ and its signal $\widetilde{s}^2$ as

$$\begin{bmatrix} \widetilde{\sigma}^2 \\ \widetilde{s}^2 \end{bmatrix} \sim \mathrm{N}(\begin{bmatrix} \sigma_0^2 \\ \sigma_0^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\sigma_0}^2 & \sigma_{\sigma_0}^2 \\ \sigma_{\sigma_0}^2 & \sigma_{\sigma_0}^2 + \sigma_\epsilon^2 \end{bmatrix}.$$

The conditional distribution of $\widetilde{\sigma}^2$ given a particular realization of $\widetilde{s}^2$ is still normal, with the following posterior mean and variance

$$\begin{aligned}\sigma_1^2(s^2) &= E[\widetilde{\sigma}^2|\widetilde{s}^2 = s^2] = \frac{\sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}\sigma_0^2 + \frac{\sigma_{\sigma_0}^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}s^2; \\ \sigma_{\sigma_1}^2 &= var[\widetilde{\sigma}^2|\widetilde{s}^2 = s^2] = \frac{\sigma_{\sigma_0}^2\sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}.\end{aligned}$$

The notation σ_1^2 and $\sigma_{\sigma_1}^2$ for the posterior mean and variance of the variance of the return are chosen to resemble the notations for their counterparts in the prior, σ_0^2 and $\sigma_{\sigma_0}^2$. Note that ex ante the posterior mean is a random variable, $\widetilde{\sigma}_1^2$, which is also normal.

Problem 1

Consider the first scenario, in which the bank sells loans individually. Investors, conditioning on a realization of the signal, s^2, infer that the variance of the return, $\widetilde{\sigma}^2$, has the distribution

$$f_{\widetilde{\sigma}^2|s^2}(\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma_{\sigma_1}}\exp\{-\frac{(\sigma^2 - \sigma_1^2(s^2))^2}{2\sigma_{\sigma_1}^2}\}.$$

Furthermore, conditional on a particular value of the variance σ^2, the return on a loan, $\widetilde{u}_l$, is distributed according to

$$f_{\widetilde{u}_l|\sigma^2}(u) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{(u - \mu_0)^2}{2\sigma^2}\}.$$

Write the distribution of the return on a loan conditional on a signal s^2 as

$$\begin{aligned}f_{\widetilde{u}_l|s^2}(u) &= \int f_{\widetilde{u}_l,\widetilde{\sigma}^2|s^2}(u,\sigma^2)d\sigma^2 \\ &= \int f_{\widetilde{u}_l|\sigma^2}(u)f_{\widetilde{\sigma}^2|s^2}(\sigma^2)d\sigma^2.\end{aligned}$$

It is easily seen that the resulting distribution is not normal. However the willingness to pay has a neat form under investors' CARA utility representation. Let the willingness to pay upon seeing a signal s^2 be $p_l(s^2)$.

$$\begin{aligned}
\exp(-\frac{p_l(s^2)}{r}) &= E[\exp(-\frac{\widetilde{u}_l}{r})|\widetilde{s}^2 = s^2] \\
&= \int\int \frac{1}{2\pi\sigma_{\sigma_1}\sigma} \exp\{-\frac{1}{2}[\frac{2u}{r} + \frac{(u-\mu_0)^2}{\sigma^2} + \frac{(\sigma^2 - \sigma_1^2(s^2))^2}{\sigma_{\sigma_1}^2}]\} d\sigma^2 du \\
&= \int\int \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(u - \frac{\mu_0 r - \sigma^2}{r})^2}{2\sigma^2}\} du \frac{1}{\sqrt{2\pi}\sigma_{\sigma_1}} \exp\{\frac{\sigma^2 - 2\mu_0 r}{2r^2} - \frac{(\sigma^2 - \sigma_1^2(s^2))^2}{2\sigma_{\sigma_1}^2}\} d\sigma^2 \\
&= \int \frac{1}{\sqrt{2\pi}\sigma_{\sigma_1}} \exp\{-\frac{[\sigma^2 - (\sigma_1^2(s^2) + \frac{\sigma_{\sigma_1}^2}{2r^2})]^2}{2\sigma_{\sigma_1}^2}\} d\sigma^2 \exp\{-[\frac{\mu_0}{r} - \frac{\sigma_1^2(s^2)}{2r^2} - \frac{\sigma_{\sigma_1}^2}{8r^4}]\} \\
&= \exp\{-[\frac{\mu_0}{r} - \frac{\sigma_1^2(s^2)}{2r^2} - \frac{\sigma_{\sigma_1}^2}{8r^4}]\}. \\
&\Rightarrow p_l(s^2) = \mu_0 - \frac{\sigma_1^2(s^2)}{2r} - \frac{\sigma_{\sigma_1}^2}{8r^3}.
\end{aligned}$$

The calculation shows that under the conditional normality and CARA utility assumptions, the willingness to pay can be derived by applying the mean-variance argument twice. First, the willingness to pay given σ^2 is

$$p' = \mu_0 - \frac{\sigma^2}{2r}.$$

Second, note that $\widetilde{p}' = \mu_0 - \frac{\widetilde{\sigma}^2}{2r}$ is itself a normal random variable, hence

$$\begin{aligned}
p_l(s) &= E[\widetilde{p}'|\widetilde{s}^2 = s] - \frac{var[\widetilde{p}'|\widetilde{s}^2 = s]}{2r} \\
&= \mu_0 - \frac{\sigma_1^2(s)}{2r} - \frac{\sigma_{\sigma_1}^2}{8r^3} \\
&= \mu_0 - \frac{1}{2r}(\frac{\sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}\sigma_0^2 + \frac{\sigma_{\sigma_0}^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}s^2) - \frac{\sigma_{\sigma_1}^2}{8r^3}. \qquad (1)
\end{aligned}$$

The second term, which is the posterior mean of the variance of the return, is a weighted average of the prior and signal. When the signal is perfectly informative, i.e. $\sigma_\epsilon^2 \to \infty$, investors rely only on the signal, and vice versa. The third term is an adjustment for the risk of imperfectly estimating risks.

Write $\pi = \frac{1}{\sigma_\epsilon^2}$. The bank's problem of information provision is

$$\begin{aligned}
& \max_\pi EU(n\widetilde{p}^L - c(\pi)) \\
= & \max_\pi EU(n\mu_0 - \frac{n\sigma_1^2(\widetilde{s}^2)}{2r} - \frac{n\sigma_{\sigma_1}^2}{8r^3} - c(\pi)) \\
= & \max_\pi n\mu_0 - \frac{nE[\sigma_1^2(\widetilde{s}^2)]}{2r} - \frac{n}{8r^3}\sigma_{\sigma_1}^2 - c(\pi) - \frac{1}{2\rho}\frac{n^2}{4r^2}var[\sigma_1^2(\widetilde{s}^2)] \\
= & \max_\pi n\mu_0 - \frac{n\sigma_0^2}{2r} - \frac{n}{8r^3}\frac{\sigma_{\sigma_0}^2}{\pi\sigma_{\sigma_0}^2 + 1} - \frac{n^2}{8\rho r^2}\frac{\pi\sigma_{\sigma_0}^4}{\pi\sigma_{\sigma_0}^2 + 1} - c(\pi).
\end{aligned}$$

When $\rho > nr$, the objective function is strictly concave, therefore the first order condition (FOC) is both sufficient and necessary. The solution $\overline{\pi}$ is given by

$$\frac{n(\rho - nr)}{8\rho r^3}\sigma_{\sigma_0}^4 = (\overline{\pi}\sigma_{\sigma_0}^2 + 1)^2 c'(\overline{\pi}). \tag{2}$$

Otherwise, the bank optimally chooses not to produce information at all.

Problem 2

Now consider the second scenario, where the bank sells N shares of pass-through securities. For a given σ^2, the return on a share of the security is distributed normal, $\widetilde{u}_s \sim \mathrm{N}(\frac{n}{N}\mu_0, \frac{n\sigma^2}{N^2})$. The willingness to pay upon seeing s^2 can be derived similarly,

$$p_s(s^2) = \frac{n}{N}\mu_0 - \frac{n\sigma_1^2(s^2)}{2rN^2} - \frac{n^2\sigma_{\sigma_1}^2}{8r^3N^4}. \tag{3}$$

The bank's problem now becomes

$$\begin{aligned}
& \max_\pi EU(N\widetilde{p}_s - c(\pi)) \\
= & \max_\pi n\mu_0 - \frac{n\sigma_0^2}{2rN} - \frac{n^2}{8r^3N^3}\frac{\sigma_{\sigma_0}^2}{\pi\sigma_{\sigma_0}^2 + 1} - \frac{n^2}{8\rho r^2N^2}\frac{\pi\sigma_{\sigma_0}^4}{\pi\sigma_{\sigma_0}^2 + 1} - c(\pi).
\end{aligned}$$

When $\rho > Nr$, the solution $\widehat{\pi}$ is fully characterized by

$$\frac{n^2(\rho - Nr)}{8\rho r^3 N^3}\sigma_{\sigma_0}^4 = (\widehat{\pi}\sigma_{\sigma_0}^2 + 1)^2 c'(\widehat{\pi}).$$

Otherwise, the bank optimally chooses not to produce any information at all.

The following proposition establishes that the bank tends to produce less information when it securitizes the loans than when it sells the loans individually.

Proposition 1 *Let $\overline{\pi}$ be the optimal precision the bank chooses when it sells loans individually and $\widehat{\pi}$ be the optimal precision when it securitizes. We have*

(1) when $\rho > Nr$, $\overline{\pi} > \widehat{\pi} > 0$.

(2) when $nr < \rho \leq Nr$, $\overline{\pi} > \widehat{\pi} = 0$.

(3) when $\rho \leq nr$, $\overline{\pi} = \widehat{\pi} = 0$.

Proof. *Recall that $N \geq n$. The last two statements in the proposition are immediate from the previous analysis. When $\rho > Nr$,both problems have interior solutions. Note that the common RHS of the FOCs has*

$$\frac{d}{d\pi}(\pi\sigma_{\sigma_0}^2 + 1)^2 c'(\pi) = 2\sigma_{\sigma_0}^2(\pi\sigma_{\sigma_0}^2 + 1)c'(\pi) + (\pi\sigma_{\sigma_0}^2 + 1)^2 c"(\pi) > 0.$$

The LHS in Problem 1 is unambiguously greater than that in Problem 2. Therefore, $\overline{\pi} > \widehat{\pi} > 0$. ■

Discussion

In the context of the model, the bank's incentive to produce information is limited by the volatility of asset prices that is introduced by the signals. The trade-off is there in both problems. It can be most easily seen from the formula of the willingness to pay, (1). When the bank produces signals at a higher precision, this has two effects on prices. One, it decreases the posterior variance of the estimate, $\sigma^2_{\sigma_0}$, which reduces the risk that investors face. This translates into an increased certainty equivalent that the bank can extract from the investors. Two, it increases the weight investors assign to the signal relative to the prior, which makes the ex ante willingness to pay more volatile. To the bank, the first effect is positive and the second is negative. In other words, the bank is trading off his own risk premium with the investors' risk premia. When the bank is not sufficiently risk tolerant relative to the investors, the second effect dominates and leaves the bank with no incentive to produce information. Another way of looking at this is that by producing information, the bank effectively retains part of the risk. Information provision here can be thought of a means to redistribute risks. The fact that the bank loads off all of the loans is a bad risk sharing schedule when the bank is more risk tolerant than the investor. If possible, the bank would want to keep the loans to itself. As discussed in the introduction, if it is kept from doing that by the capital requirement and it does not want to let go the opportunity of profit from originating the loans, it will choose to securitize loans.

Now how would the institution of securitization affect the trade-off I have just formalized? Compared to the case where it sells loans individually, the process of securitization exacerbates the bank's incentive for information production. To see the intuition, suppose $N = n$ for now. The key lies in the diversification implied by the process of securitization. While a single loan has return distributed $N(\mu_0, \sigma^2)$ conditional on σ^2, a share of the security has a less risky return distributed $N(\mu_0, \frac{\sigma^2}{n})$. This results in a willingness to pay, $\widetilde{p}_s = \frac{n}{N}\mu_0 - \frac{\widetilde{\sigma}_1^2}{2rn} - \frac{\sigma_{\sigma_1}^2}{8r^3n^2}$,that is less sensitive to signals than $\widetilde{p}_l$. The marginal benefit of information under securitization is reduced by a factor of n^2, therefore the bank chooses a lower precision, if it decides to produce information at all. It is also clear that the bank achieves a higher utility level when securitizing than selling individual loans for any given amount of precision. Since investors are always indifferent, securitization is welfare-improving in this setting.

The presence of N amplifies the reduction in the marginal benefit of information under securitization. Securitization makes the investment projects divisible, hence making the investment opportunity available to a larger number of smaller investors. From the willingness to pay in (3), it is apparent that subdividing the returns on the pool makes each share of security even less risky, hence the marginal benefit of information is abated even more. When N is big, it is likely that the bank does not provide information at all. In fact, the bank would want to make N as big as possible. For N very big, each investor is effectively risk neutral, which eliminates both the investors' and the bank's risk premium, leaving the bank the highest possible utility level $n\mu_0$. This is reminiscent of a result by Peress (2010), where he showed that the bigger a given stock's investor base, the less investors engage in information production.

In the next section, I show how the intuition in this basic model is brought into play in a more realistic setting, where the loans come from distributions with different means and variances and all of those parameters are unknown ex ante.

3 A Model with Loans of Different Qualities

In this section, I present a model where loans come from different distributions and consider the bank's problem of choosing both the information provision and the number of loans, hence, the riskiness of its portfolio. The results bear a clear resemblance to what happened in the market of mortgage-backed-securities up to the subprime mortgage crisis.

Imagine there are loans of different qualities, that is the returns of the loans come from different distributions. Order the distributions so that

$$\begin{aligned}\mu_{01} &< \mu_{02} < ... < \mu_{0n} < ...;\\ \sigma_{01}^2 &< \sigma_{02}^2 < ... < \sigma_{0n}^2 < ....\end{aligned}$$

Distributions with lower indices represent loans whose expected returns are lower but safer. For example, returns on prime mortgages have smaller indices than returns on subprime mortgages, because the former generate lower interest payments with a lower probability of default. There is a fixed supply of loans whose return are drawn from each distribution, which is normalized to 1. Suppose that the bank starts with lower-indexed loans and then move to higher-indexed ones. If originating loans are costly, it faces a choice as to where to stop as it moves up the index.

Formally, for each loan indexed i, the mean is ex ante distributed $\widetilde{\mu}_i \sim N(\mu_{0i}, \sigma^2_{\mu_0})$ and the variances $\widetilde{\sigma}^2_i \sim N(\sigma^2_{0i}, \sigma^2_{\sigma_0})$. For ease of exposition, denote $\overline{\mu} = \frac{1}{n}\sum_{i=1}^{n} \mu_{0i}$ and $\overline{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} \sigma^2_{0i}$. The bank can produce signals about each of the random parameters:

$$
\begin{aligned}
\widetilde{m}_i &= \widetilde{\mu}_i + \widetilde{\varepsilon}_{1i}, \forall i = 1, ..., n; \\
\widetilde{s}^2_i &= \widetilde{\sigma}_i + \widetilde{\varepsilon}_{2i}, \forall i = 1, ..., n.
\end{aligned}
$$

where $\widetilde{\varepsilon}'_{ji}s$ are noises distributed independently across i and j according to $N(0, \sigma^2_{j\epsilon})$, $j = 1, 2$. The information choice is formulated to be two-dimensional: one level of precision for all the signals about the mean and another for the signals about the variance. The interpretation is that the bank is committed to a particular mean-screening or variance-screening technology that guarantees a particular level of accuracy about the relevant parameter without regard to the quality of the loan per se. The cost of information is $C(\pi_1, \pi_2)$, which has the properties that C is increasing in both arguments and strictly convex. Let the cost of originating n loans be captured by an increasing and convex function $K(n)$. The cost structure effectively separates the decision of information acquisition from the decision of originating loans. Let's fix n for now.

Problem 1

When the bank sells the loans individually, assume there is no asymmetric information. Given the signals $(m_{1,}..., m_n, s^2_{1,}..., s^2_n)$, the price for the ith loan is

$$
p_i(m_i, s^2_i) = \frac{\sigma^2_{\epsilon_1}\mu_{0i} + \sigma^2_{\mu_0} m_i}{\sigma^2_{\mu_0} + \sigma^2_{\epsilon_1}} - \frac{1}{2r}\frac{\sigma^2_{\epsilon_2}\sigma^2_{0i} + \sigma^2_{\sigma_0} s^2_i}{\sigma^2_{\sigma_0} + \sigma^2_{\epsilon_2}} - \frac{\sigma^2_{\mu_1}}{2r} - \frac{\sigma^2_{\sigma_1}}{8r^3}.
$$

Let the expected profit to the bank given (π_1, π_2, n) be $\Psi^1(\pi_1, \pi_2; n)$. For a fixed n, the bank's problem is

$$\begin{aligned}
\Psi^1(n) &= \max_{\pi_1,\pi_2} EU(\sum_{i=1}^{n} \widetilde{p}_i - C(\pi_1, \pi_2) - K(n)) \\
= & \max_{\pi_1,\pi_2} \sum_{i=1}^{n} \mu_{0i} - \frac{1}{2r} \sum_{i=1}^{n} \sigma_{0i}^2 - \frac{1}{2\rho}\left(\frac{n\pi_1\sigma_{\mu_0}^4}{\pi_1\sigma_{\mu_0}^2 + 1} + \frac{n}{4r^2}\frac{\pi_2\sigma_{\sigma_0}^4}{\pi_2\sigma_{\sigma_0}^2 + 1}\right) \\
& -\frac{n}{2r}\frac{\sigma_{\mu_0}^2}{\pi_1\sigma_{\mu_0}^2 + 1} - \frac{n}{8r^3}\frac{\sigma_{\sigma_0}^2}{\pi_2\sigma_{\sigma_0}^2 + 1} - C(\pi_1, \pi_2) - K(n).
\end{aligned}$$

The optimal information choice $(\overline{\pi}_1, \overline{\pi}_2)$ is $(0, 0)$ when $\rho \leq r$; and otherwise satisfies the FOCs:

$$\frac{n(\rho - r)}{2\rho r}\sigma_{\mu_0}^4 = (\overline{\pi}_1\sigma_{\mu_0}^2 + 1)^2 C_1(\overline{\pi}_1, \overline{\pi}_2); \tag{4}$$

$$\frac{n(\rho - r)}{8\rho r^3}\sigma_{\sigma_0}^4 = (\overline{\pi}_2\sigma_{\sigma_0}^2 + 1)^2 C_2(\overline{\pi}_1, \overline{\pi}_2). \tag{5}$$

The result contrasts with those in all other cases in that the bank starts to produce information as soon as $\rho > r$. The reason is intuitive. Here the bank produces information about every piece of the loans. Hence, the volatility of profit is the *addition* of the volatility of prices on different pieces of loans. In the basic model, however, there is only one signal, prices of different loans are perfectly correlated. Therefore, volatility in asset prices aggregate into volatility in profit at the rate n^2, so the volatility in profit becomes very sensitive to the amount of information. Consequently, when the negative effect from price volatility is large, the bank has less incentive to produce information. Interestingly, I will show in the following that in the case of securitization, even if the bank produces information about each loan, the volatility of security price adds up at the rate n^2.

Problem 2

When the bank securitizes, the return on the security depends on the return on the entire pool. To keep the model simple, suppose $N = n$. The return per share of security is $\widetilde{u}_s \sim N(\widetilde{\mu}_s, \frac{\widetilde{\sigma}_s^2}{n})$, where $\widetilde{\mu}_s = \frac{1}{n}\sum_{i=1}^{n}\widetilde{\mu}_i$ and $\widetilde{\sigma}_s^2 = \frac{1}{n}\sum_{i=1}^{n}\widetilde{\sigma}_i^2$..The following lemma shows that the investors' inference problem only depends on the signals through the means: $\widetilde{m} = \frac{1}{n}\sum_{i=1}^{n}\widetilde{m}_i$ and $\widetilde{s}^2 = \frac{1}{n}\sum_{i=1}^{n}\widetilde{s}_i^2$. Hence without loss of generality, let the bank do the calculation for the investors and announce the summary statistics $(\widetilde{m}, \widetilde{s}^2)$..This is consistent with the fact that in the market of mortgage-backed securities, the information that banks release is usually pool-level aggregated statistics such as the loan-to-value ratio, weighted-average coupon (WAC) and weighted-average maturity (WAM) of the pool. Information about the individual loans is withheld. For other interpretations of this phenomenon, see for example Glaeser and Kallal (1997).

Lemma 1 *Investors' posterior beliefs about* $(\widetilde{\mu}_s, \widetilde{\sigma}_s^2)$ *depend on* $(\widetilde{m}_{1,\ldots}, \widetilde{m}_n, \widetilde{s}^2_{1,\ldots}, \widetilde{s}_n^2)$ *only through* $(\widetilde{m}, \widetilde{s}^2)$*, where* $\widetilde{m} = \frac{1}{n}\sum_{i=1}^{n}\widetilde{m}_i$ *and* $\widetilde{s}^2 = \frac{1}{n}\sum_{i=1}^{n}\widetilde{s}_i^2$.

Proof. *See Appendix.* ■

Let the expected profit to the bank given (π_1, π_2, n) be $\Psi^2(\pi_1, \pi_2; n)$. For a fixed

n, the bank's problem is

$$\begin{aligned}
&\Psi^2(n) = \max_{\pi_1,\pi_2} EU(n\widetilde{p} - C(\pi_1,\pi_2) - K(n)) \\
= \quad & \sum_{i=1}^{n}\mu_{0i} - \frac{1}{2rn}\sum_{i=1}^{n}\sigma_{0i}^2 - \frac{1}{2\rho}\left(\frac{n\pi_1\sigma_{\mu_0}^4}{\pi_1\sigma_{\mu_0}^2+1} + \frac{1}{4r^2n}\frac{\pi_2\sigma_{\sigma_0}^4}{\pi_2\sigma_{\sigma_0}^2+1}\right) \\
& -\frac{1}{2r}\frac{\sigma_{\mu_0}^2}{\pi_1\sigma_{\mu_0}^2+1} - \frac{1}{8r^3n^2}\frac{\sigma_{\sigma_0}^2}{\pi_2\sigma_{\sigma_0}^2+1} - C(\pi_1,\pi_2) - K(n).
\end{aligned}$$

When there is an interior solution, $(\widehat{\pi}_1, \widehat{\pi}_2)$ satisfies the FOCs,

$$\frac{\rho - nr}{2\rho r}\sigma_{\mu_0}^4 = (\widehat{\pi}_1\sigma_{\mu_0}^2+1)^2 C_1(\widehat{\pi}_1,\widehat{\pi}_2); \tag{6}$$

$$\frac{\rho - nr}{8\rho r^3 n^2}\sigma_{\sigma_0}^4 = (\widehat{\pi}_2\sigma_{\sigma_0}^2+1)^2 C_2(\widehat{\pi}_1,\widehat{\pi}_2); \tag{7}$$

otherwise, $(\widehat{\pi}_1, \widehat{\pi}_2) = (0,0)$.

Proposition 2 *Assume $C_{12} \leq 0$. Fix n. Let $(\overline{\pi}_1, \overline{\pi}_2)$ be the optimal precision the bank chooses when it sells loans individually and $(\widehat{\pi}_1, \widehat{\pi}_2)$ be the optimal precision when it securitizes. We have*

(1) when $\rho > nr$, $(\overline{\pi}_1, \overline{\pi}_2) > (\widehat{\pi}_1, \widehat{\pi}_2) > (0,0)$.

(2) when $r < \rho \leq nr$, $(\overline{\pi}_1, \overline{\pi}_2) > (\widehat{\pi}_1, \widehat{\pi}_2) = (0,0)$.

(3) when $\rho \leq r$, $(\overline{\pi}_1, \overline{\pi}_2) = (\widehat{\pi}_1, \widehat{\pi}_2) = (0,0)$.

Proof. *Suppose $\rho > nr$. Consider the following system of equations in (π_1, π_2),*

$$\begin{cases} (\pi_1\sigma_{\mu_0}^2+1)^2 C_1(\pi_1,\pi_2) = y_1 \\ (\pi_2\sigma_{\sigma_0}^2+1)^2 C_2(\pi_1,\pi_2) = y_2 \end{cases} \tag{8}$$

From the first equation,

$$\frac{d\pi_1}{d\pi_2} = -\frac{(\pi_1\sigma_{\mu_0}^2+1)C_{12}}{2\sigma_{\mu_0}^2 C_1 + (\pi_1\sigma_{\mu_0}^2+1)C_{11}} \in (0, -\frac{C_{12}}{C_{11}}). \tag{9}$$

Differentiate both sides of the second equation with respect to y_2,

$$\frac{d\pi_2}{dy_2} = \frac{1}{(\pi_2\sigma^2_{\sigma_0}+1)[2\sigma^2_{\sigma_0}C_2 + (\pi_2\sigma^2_{\sigma_0}+1)(C_{21}\frac{d\pi_1}{d\pi_2} + C_{22})]} > 0.$$

The last inequality follows by convexity of the cost function: $C_{21}\frac{d\pi_1}{d\pi_2} + C_{22} > -\frac{C_{12}^2}{C_{11}} + C_{22} > 0$. *Moreover,* $\frac{d\pi_1}{dy_2} = \frac{d\pi_1}{d\pi_2}\frac{d\pi_2}{dy_2} > 0$. *By symmetry, it is also true that* $\frac{d\pi_1}{dy_1} > 0$ *and* $\frac{d\pi_2}{dy_1} > 0$. *Notice that Problem 1 corresponds to higher* (y_1, y_2)*, therefore* $(\overline{\pi}_1, \overline{\pi}_2) > (\widehat{\pi}_1, \widehat{\pi}_2)$. ■

The proof requires the cost of signals to be complementary in the sense that the marginal cost of the signal about one parameter decreases with the precision of the signal about the other. However, it is a sufficient condition, not necessary. It is clear from the proof that when the two signals are mildly substitutes, the result still holds. I tend to think of it as a natural assumption. In order to assess the quality of the loan, banks need to invest in collecting and verifying documentations that are informative about both the mean and variance of the return and build statistic models that help predict both parameters.

A complete characterization of the optimal choice of n seems to be analytically intractable. A simple numerical exercise suggests that the bank tends to originate a larger number of loans that involve riskier returns when it securitizes loans than when it sells loans by the piece. The numerical example is parametrized as follows:

$$\rho = 20; \ r = 1.$$

$$\sigma^2_{\mu_0} = 1; \ \sigma^2_{\sigma_0} = 4.$$

$$C(\pi_1, \pi_2) = \pi_1^2 + \pi_2^2;$$

$$\mu_i = 3i^{0.4}, \ \sigma^2_{0i} = i, \ \text{for } i = 1, 2, ..., 30.$$

The results are consistent with the predictions of the model. Figure 2 shows that in Problem 1, the precision of each signal is increasing in n, since the LHS's of (4) and (6), which represent roughly the marginal benefit of information are increasing in n. The opposite is true for Problem 2 (Figure 3). As n exceeds $\frac{\rho}{r}$, the bank stops producing information all together. Finally, Figure 4 clearly shows that under securitization, the bank chooses the biggest number of loans possible, that is 30, while it settles down at 18 pieces of loans in Problem 1. Furthermore, it is true in general that the bank achieves a higher utility level under securitization.

Proposition 3 *The bank achieves a higher utility level when selling securities backed by the pool of loans than selling loans individually.*

Proof. *Given n, it is immediate the bank in Problem 2 achieves a higher utility level pointwise at each (π_1, π_2) than it does in Problem 1. Hence $\Psi^2(n) > \Psi^1(n)$, $\forall n$. Therefore,* $\max_n \Psi^2(n) > \max_n \Psi^1(n)$. ■

The last proposition shows that in the current environment the process of securitization is welfare-improving in that it makes the bank strictly better off while leaving investors always indifferent. Information helps the bank to retain some risk. With securitization, the bank trades off risk for higher price brought by the diversification, which limits the need for costly information production. In other words, securitization is a free way to redistribute wealth and risk among all agents in the economy, by which the use of costly measures such as information is reduced. In the next section, however, I will show that little information production can be problematic. For example, imagine the investors have a biased prior and the bank is aware of that. In that case, reduced information production under securitization can reduce the ex post social welfare.

4 Different Priors

Adopt the set-up of the basic model and retain the assumption that $N = n$. The old notation is used to model bank's beliefs. The investors have beliefs

$$\begin{bmatrix} \widetilde{\sigma}^2 \\ \widetilde{s}^2 \end{bmatrix} \sim N(\begin{bmatrix} \overline{\sigma}_0^2 \\ \overline{\sigma}_0^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\sigma_0}^2 & \sigma_{\sigma_0}^2 \\ \sigma_{\sigma_0}^2 & \sigma_{\sigma_0}^2 + \sigma_\epsilon^2 \end{bmatrix},$$

and $\overline{\sigma}_0^2 \neq \sigma_0^2$.

It can be derived that the FOC in **Problem 1** now looks like the follows. If $\overline{\sigma}_0^2 > \sigma_0^2 - \frac{\rho - nr}{4r^2\rho}\sigma_{\sigma_0}^2$,

$$n(\frac{\rho - nr}{4r^2\rho}\sigma_{\sigma_0}^2 - \sigma_0^2 + \overline{\sigma}_0^2) = \frac{2r}{\sigma_{\sigma_0}^2}(\pi\sigma_{\sigma_0}^2 + 1)^2 c'(\pi); \quad (10)$$

Otherwise, the bank optimally chooses $\pi = 0$.

In order for the bank to produce information, it must believe that the investors are sufficiently pessimistic, in the sense that the investors prior $\overline{\sigma}_0^2$ must be sufficiently big. The condition for an interior solution is easier to satisfy when $\rho > nr$. When $\rho > nr$, the net benefit of signal is positive under the common prior. Hence, even if the bank believes the investors are a little bit more optimistic than itself, it would still produce some information. When $\rho < nr$, the only source of benefit of signaling is to exploit the investors' biased beliefs. Hence their prior should be sufficiently bigger than the bank's to support information production.

The counterpart in **Problem 2** is that when $\overline{\sigma}_0^2 > \sigma_0^2 - \frac{\rho - nr}{4r^2 n\rho}\sigma_{\sigma_0}^2$, the optimal choice π solves

$$\frac{\rho - nr}{4r^2 n\rho}\sigma_{\sigma_0}^2 - \sigma_0^2 + \overline{\sigma}_0^2 = \frac{2r}{\sigma_{\sigma_0}^2}(\pi\sigma_{\sigma_0}^2 + 1)^2 c'(\pi);$$

Otherwise $\pi = 0$.

An immediate observation is that when $\rho > nr$, it is easier for bank in Problem 1 to provide information than the bank in Problem 2. This is because under securitization, the marginal benefit of information is decreased. The opposite is true when $\rho < nr$. The bank in the second problem has smaller negative marginal benefit, which makes it more willing to produce information. When $\rho = nr$, bank in both problems produces information if it believes that the investors' belief $\overline{\sigma}_0^2$ is greater than σ_0^2.

Proposition 4 *Let $\overline{\pi}$ be the optimal precision the bank chooses when it sells loans individually and $\widehat{\pi}$ be the optimal precision when it securitizes. We have*

(1) Suppose $\rho \geq nr$:

(1a) when $\overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2$, $\overline{\pi} = \widehat{\pi} = 0$.

(1b) when $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$, $\overline{\pi} > \widehat{\pi} = 0$.

(1c) when $\overline{\sigma}_0^2 > \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$, $\overline{\pi} > \widehat{\pi} > 0$.

(2) Suppose $\rho < nr$ *:*

(2a) when $\overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$, $\overline{\pi} = \widehat{\pi} = 0$.

(2b) when $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2$, $0 = \overline{\pi} < \widehat{\pi}$.

(2c) when $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{(\rho - nr)(n+1)}{4\rho r^2 n}\sigma_{\sigma_0}^2$, $0 < \overline{\pi} < \widehat{\pi}$.

(2d) when $\overline{\sigma}_0^2 > \sigma_0^2 - \frac{(\rho - nr)(n+1)}{4\rho r^2 n}\sigma_{\sigma_0}^2$, $\overline{\pi} > \widehat{\pi} > 0$.

Proof. *(1) Suppose* $\rho > nr$*. Clearly* $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$*. This proves the (1a) and (1b). Suppose* $\overline{\sigma}_0^2 > \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$*. Take the difference between the two LHS from the FOCs:*

$$
\begin{aligned}
& n\frac{\rho - nr}{4r^2\rho}\sigma_{\sigma_0}^2 - n\sigma_0^2 + n\overline{\sigma}_0^2 - \left(\frac{\rho - nr}{4r^2 n\rho}\sigma_{\sigma_0}^2 - \sigma_0^2 + \overline{\sigma}_0^2\right) \\
= \; & (n^2 - 1)\frac{\rho - nr}{4r^2\rho n}\sigma_{\sigma_0}^2 - (n-1)\sigma_0^2 + (n-1)\overline{\sigma}_0^2 \\
= \; & (n-1)\left[(n+1)\frac{\rho - nr}{4r^2\rho n}\sigma_{\sigma_0}^2 - \sigma_0^2 + \overline{\sigma}_0^2\right] > 0.
\end{aligned}
$$

It is easily checked that $\frac{d}{d\pi}[(\pi\sigma_{\sigma_0}^2 + 1)^2 c'(\pi)] > 0$*. Hence,* $\overline{\pi} > \widehat{\pi} > 0$*.*

(2) can be proved analogously, once one notices that under $\rho > nr$, $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2 < \sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \sigma_0^2 - \frac{(\rho - nr)(n+1)}{4\rho r^2 n}\sigma_{\sigma_0}^2$*. The case of* $\rho = nr$ *can be verified easily.* ■

The case that bears resemblance to what happened during the recent crisis is when $\rho > rn$ and $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$. The bank, as the originator of the loans, has an unbiased view of the risk of those loans. But how do investors, who have never dealt with securities backed up by subprime mortgages, form their prior about the distribution of the return? Rajan, Seru and Vig (2010) empirically suggests that as the level of securitization increases, lenders tend to originate loans that rate high based on characteristics known to the investors and ignore other credit-relevant information. This gives rise to a mildly optimistic opinion about the returns among the investors. In this model, it translate into a prior about the variance that is slightly smaller than the prior of the bank: $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$. Then the investors unknowingly pay a higher price for the security than what it is actually worth. In a stationary environment, where the issuance of the securities repeats over time and the underlying distribution of the parameter is invariant, the investors will eventually come to realize that their priors have been biased. Although investors form expected utility of their future consumption plan using possibly biased prior, after all the realized lifetime utility is derived from the actual consumption plan that is generated by the true underlying parameters of the environment. I can compare the result in this section with that in the basic model to gauge the loss investors suffer from a biased prior. In Problem 1, an investor pays $\frac{\sigma_0^2 - \overline{\sigma}_0^2}{2r(\overline{\pi}\sigma_{\sigma_0}^2 + 1)}$ more than what the loan is worth, where $\overline{\pi}$ is the optimal information provision described by (10). In Problem 2, he pays $\frac{\sigma_0^2 - \overline{\sigma}_0^2}{2rn}$ more than what the security is worth and there is no information production. Intuitively, for a information level high enough in Problem 1, investors lose more under securitization. If the social welfare function gave sufficiently big weights to the investors, the social planner would not choose securitization.

Proposition 5 *Assume $\rho > nr$ and $\sigma_0^2 - \frac{\rho - nr}{4\rho r^2}\sigma_{\sigma_0}^2 < \overline{\sigma}_0^2 \leq \sigma_0^2 - \frac{\rho - nr}{4\rho r^2 n}\sigma_{\sigma_0}^2$. The sufficient and necessary condition, under which investors are worse off when the bank securitizes loans than when it sells loans individually, is*

$$\frac{\rho - nr}{4r^2\rho}\sigma_{\sigma_0}^2 - \sigma_0^2 + \overline{\sigma}_0^2 > \frac{2nr}{\sigma_{\sigma_0}^2} c'(\frac{n-1}{\sigma_{\sigma_0}^2}).$$

Proof. *In Problem 1, the optimal information level $\overline{\pi}$ solves (10). Given the signal and the precision of the signal, investors' expected utility from the loan under the correct belief, $E\exp(-\frac{\widetilde{u}}{r})$, is given by (1). Then the utility of an investor who is endowed with w_0 wealth is*

$$\begin{aligned}
& Eu(w_0 - \widetilde{p} + \widetilde{u}) \\
= & -\exp(-\frac{w_0}{r})E\exp(\frac{\widetilde{p} - \widetilde{u}}{r}) \\
= & -\exp(-\frac{w_0}{r})E_{\widetilde{s}^2}\{\exp(\frac{\widetilde{p}}{r}E[\exp(-\frac{\widetilde{u}}{r})|s^2]\} \\
= & -\exp[-\frac{1}{r}(w_0 - \frac{\sigma_0^2 - \overline{\sigma}_0^2}{2r(\overline{\pi}\sigma_{\sigma_0}^2 + 1)})].
\end{aligned}$$

Observe that $Eu(w_0 - \widetilde{p} + \widetilde{u}) < Eu(w_0)$. So investors incur loss from the trade (unknowingly at the time of the trade for them).

Repeat the same exercise for Problem 2, where $\widehat{\pi} = 0$. $Eu(w_0 - \widetilde{p} + \widetilde{u}) = -\exp[-\frac{1}{r}(w_0 - \frac{\sigma_0^2 - \overline{\sigma}_0^2}{2nr})]$. Hence, the necessary and sufficient condition for investors to have lower utility level in Problem 2 is

$$\begin{aligned}
n &< \overline{\pi}\sigma_{\sigma_0}^2 + 1, \text{ or} \\
\overline{\pi} &> \frac{n-1}{\sigma_{\sigma_0}^2}.
\end{aligned}$$

Since the RHS of (10) is increasing in π. The above condition is equivalent to the inequality in the statement of the proposition. ■

In general, there are many ways to satisfy the above condition. One trivial way is to make the marginal cost of the signal sufficiently low. But I will discuss two other economically more meaningful scenarios. In order to make comparisons, let me also fix some of the parameters throughout these scenarios. In particular,

$$\rho = 10; \; n = 20; \; \sigma_0^2 = 9; \; \overline{\sigma}_0^2 = 8.5;$$

$$\text{and } c(\pi) = \pi^2/20.$$

Numerical Examples

Scenario 1: Very risk averse investors: $r = 0.25$; $\sigma_{\sigma_0}^2 = 5$.

Very risk averse investors are sensitive to information. A marginal increase in the precision of the information increases their willingness to pay much more than it increases the price volatility. This induces the bank to produce very precise signals in Problem 1, $\overline{\pi} = 8.99$. Here the utility of a single investor in Problem 1 is -1.0909, while his utility under securitization is -1.2214.

Scenario 2: Very imprecise prior: $\sigma_{\sigma_0}^2 = 100$; $r = 0.48$.

When investors and the bank have very rough idea about the variance of the distribution of the return ex ante, for a given level of investors' risk tolerance, the marginal benefit of information is high. Here $\overline{\pi} = 1.9934$. A single investor's utility when the bank sells loans is -1.0054, while his utility when the bank securitizes is -1.0558.

The parameters are chosen such that if we swap the value of r in one scenario with that in the other, the result that investors are worse-off under securitization goes away. Basically when the prior belief about the variance is very imprecise, in order to dissuade the bank from producing information under securitization, the investors cannot be overly risk averse. Similar intuition goes through when investors are very risk averse.

Here I have outlined two cases in which securitization with zero information production does *not* lead to Pareto improvement. In either case, the bank is better at the cost of investors. In one scenario, investors are much more risk averse than the bank. In the other, agents have very rough idea about the riskiness of return ex ante.

5 Conclusion

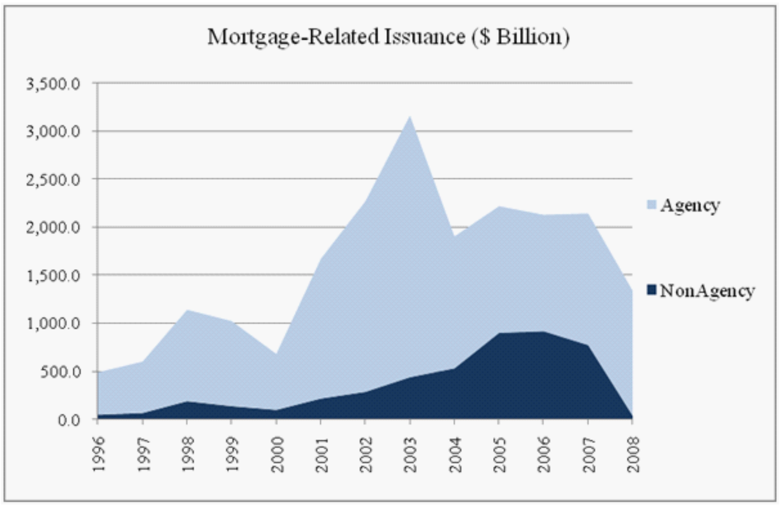
In this paper, I propose a simple model of banks' information production to understand the trade-off banks face when deciding how much information to produce. The diversification inherent in the process of securitization decreases banks' incentive to produce information about the loans they originate. Essentially, the bank weighs gains from increased asset price against losses from increased price volatility. The securitization, by eliminating the idiosyncratic component of risk, promises a less risky return to all investors, diminishing the marginal benefit of information, hence decreasing the bank's incentive of information production.

In a variant of the model, I show that under investors' mild optimism about the return, by securitizing loans and limiting information production, the bank makes itself better off at the cost of the investors. In a more general set-up, the bank faces loans that have different qualities and each in a fixed supply: the loans are indexed such that both the expected return and the variance are increasing in the index. When the bank can produce signals about each of those parameters, I show that (1) the bank is indifferent whether to release all signals or to release summary statistics such as the mean of the signals about the mean and the mean of the signals about the variance; (2) securitization decreases information production. Furthermore, if the bank can choose how many loans it originates, the numerical example suggests that it tends to choose a bigger pool with riskier loans under securitization. This captures exactly what happened in the crisis: following the popularity of securitization, the banks originated more and riskier loans with higher interest rates (subprime) without producing much information (Keys, Mukherjee, Seru and Vig, 2010). It is also consistent with the fact that the bank only disclosed some summary statistics about the loans.

The paper contributes to the literature by formalizing a mechanism of information production within the organization of the bank itself. The model has incomplete but public information and the result is implied purely by the bank's profit motive. This stands in contrast to most existing work which focuses on banks' behavior in a world of asymmetric information, plagued by the moral hazard and adverse selection problems.

Figures and Tables

Figure 1 Agency and Non-agency Issued MBS and CMOs: U.S. 1996-2008



Source: SIFMA website.

Notes: Agency: includes GNMA, FNMA and FHLMC MBS, CMOs.

Non-agency: private-label MBS and CMOs. Data from GSEs,Thomson Reuters, Bloomberg

Table 1 Origination of Mortgages, Issuance of MBS and the Ratio by Type: Subprime, Alt-A, Jumbo and Agency Loans. U.S. 2001-2006

	Subprime			Alt-A		
Year	Origination	Issuance	Ratio	Origination	Issuance	Ratio
2001	190.00	87.10	46%	60.00	11.40	19%
2002	231.00	122.70	53%	68.00	53.50	79%
2003	335.00	195.00	58%	85.00	74.10	87%
2004	540.00	362.63	67%	200.00	158.60	79%
2005	625.00	465.00	74%	380.00	332.30	87%
2006	600.00	448.60	75%	400.00	365.70	91%
	Jumbo			Agency		
Year	Origination	Issuance	Ratio	Origination	Issuance	Ratio
2001	430.00	142.20	33%	1,433.00	1,087.60	76%
2002	576.00	171.50	30%	1,898.00	1,442.60	76%
2003	655.00	237.50	36%	2,690.00	2,130.90	79%
2004	515.00	233.40	45%	1,345.00	1,018.60	76%
2005	570.00	280.70	49%	1,180.00	964.80	82%
2006	480.00	219.00	46%	1,040.00	904.60	87%

Source: Ashcraft and Schuermann (2008), Table 1

Notes: Jumbo origination includes non-agency prime. Agency origination includes conventional/ conforming and FHS/VA loans. Agency issuance GNMA, FHLMC and FNMA.

Table 2 Share of Full-doc Loans and Share of Non-agency Issued Mortgage Related Products

Year	2001	2002	2003	2004	2005	2006	2007
Share of Full Documentation (%)	76.5	70.4	67.8	66.4	63.4	62.3	66.7
Share of Non-agency MBS/CDO (%)	13.1	12.7	13.9	27.9	40.6	43.0	36.1

Source: share of full-doc is from Table 1 in Demyanyk and van Hemert (2008); share of non-agency issued MBS and CMO is from SIFMA.

Figure 2 A Numerical Example: Information Production in Problem 1

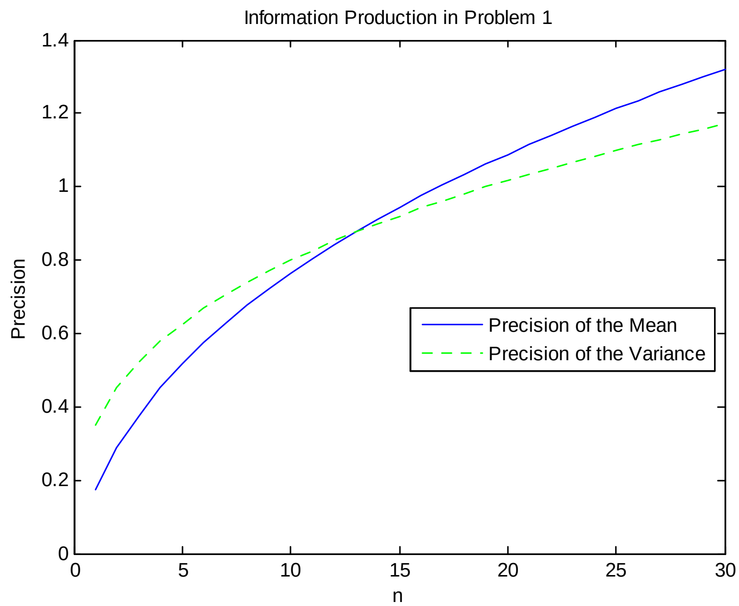


Figure 3 A Numerical Example: Information Production in Problem 2

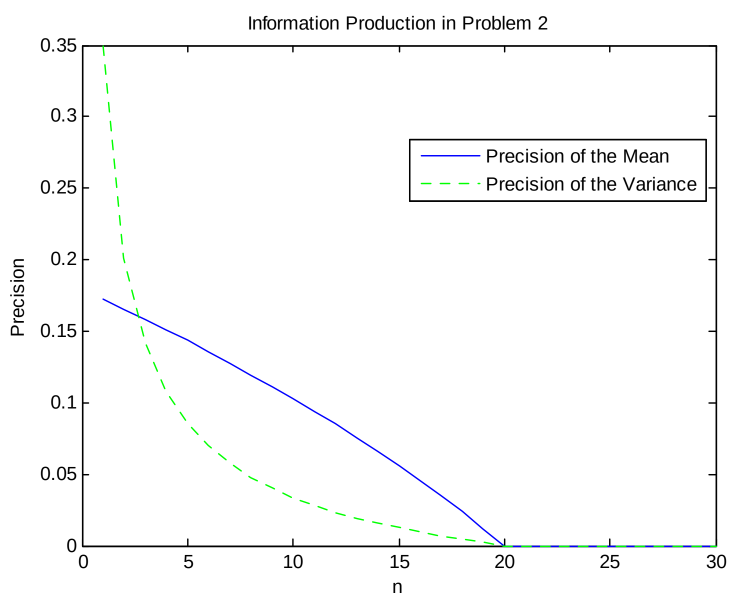
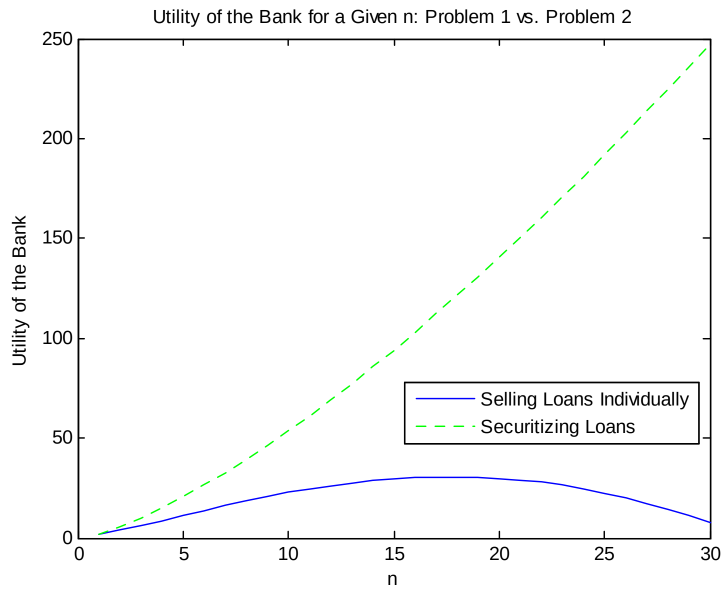


Figure 4 A Numerical Example: Bank's Utility: Problem 1 vs. Problem 2



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Appendix

Optimal Decision of Holding/Selling Loans

Within the framework of the basic model. Consider the problem where the bank chooses the fraction of assets to sell. Given that the bank forms N shares of securities, it chooses to sell αN shares to investors. The bank's problem is

$$\max_{\alpha,\pi} EU(\alpha N\widetilde{p}^I + (1-\alpha)\widetilde{p}^B - c(\pi)),$$

where

$$\begin{aligned}
\widetilde{p}^I &= \frac{n}{N}\mu_0 - \frac{n\widetilde{\sigma}_1^2}{2rN^2} - \frac{n^2\sigma_{\sigma_1}^2}{8r^3N^4}; \\
\widetilde{p}^B &= \frac{n}{N}\mu_0 - \frac{n\widetilde{\sigma}_1^2}{2\rho N^2} - \frac{n^2\sigma_{\sigma_1}^2}{8\rho^3N^4}.
\end{aligned}$$

Hence the FOC with respect to α is

$$(\frac{1}{r}-\frac{1}{\rho})[\frac{n\sigma_0^2}{2N} + \frac{n^2}{4\rho N^2}\frac{\pi\sigma_{\sigma_0}^4}{\pi\sigma_{\sigma_0}^2+1}(\frac{\alpha}{r}+\frac{1-\alpha}{\rho})] + \frac{n^2}{8N^3}\frac{\pi\sigma_{\sigma_0}^4}{\pi\sigma_{\sigma_0}^2+1}(\frac{1}{r^3}-\frac{1}{\rho^3}) = 0.$$

When $\rho < r$, it is optimal to set $\alpha = 1$. When $\rho > r$, it is optimal to set $\alpha = 0$. In all the interesting cases, when the bank chooses to produce a positive amount of information, $\rho > r$, that is it is also optimal for it to actually keep the loans if it can.

Normally Distributed Standard Deviation and a Risk Neutral Bank

In this section, I sketch the intuition for an alternative set-up, in which the standard deviation of the return to loans is normally distributed and the bank is risk neutral. The economic environment and problems are otherwise the same as the Common Prior case.

Denote the return to an individual loan as $\widetilde{u} \sim N(\mu_0, \sigma^2)$, where the standard deviation is ex ante random, $\widetilde{\sigma} \sim N(\sigma_0, \sigma_{\sigma_0}^2)$ The signal structure is

$$\widetilde{s} = \widetilde{\sigma} + \widetilde{\epsilon}, \widetilde{\epsilon} \sim N(0, \sigma_\epsilon^2) \text{ independent from } \widetilde{\sigma}.$$

The Bayesian posterior given a signal $\widetilde{s} = s$ is

$$\widetilde{\sigma} | \widetilde{s} = s \sim N\left(\frac{\sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}\sigma_0 + \frac{\sigma_{\sigma_0}^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}s, \frac{\sigma_{\sigma_0}^2 \sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}\right).$$

Denote $\mu_1(s) = \frac{\sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}\sigma_0 + \frac{\sigma_{\sigma_0}^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}s$ and $\sigma_1^2 = \frac{\sigma_{\sigma_0}^2 \sigma_\epsilon^2}{\sigma_{\sigma_0}^2 + \sigma_\epsilon^2}$. In this set-up, the normalized variance $(\frac{\widetilde{\sigma} - \mu_1(s)}{\sigma_1})^2$ is distributed $\chi^2(1)$.

Assume $\sigma_{\sigma_0}^2 < r^2$. This implies that $\sigma_1^2 < r^2$. We will see from what follows that this assumption guarantees well-defined normal densities. Now consider the investors' willingness to pay in the first problem.

$$\begin{aligned}
\exp(-\frac{p(s)}{r}) &= E[\exp(-\frac{\widetilde{u}}{r}) | \widetilde{s} = s] \\
&= \int\int \frac{1}{2\pi\sigma\sigma_1} \exp\{-\frac{1}{2}[\frac{2u}{r} + \frac{(u-\mu_0)^2}{\sigma^2} + \frac{(\sigma - \mu_1(s))^2}{\sigma_1^2}]\} d\sigma du \\
&= \int\int \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(u - \frac{\mu_0 r - \sigma^2}{r})^2}{2\sigma^2}\} du \frac{1}{\sqrt{2\pi}\sigma_1} \exp\{\frac{\sigma^2 - 2\mu_0 r}{2r^2} - \frac{(\sigma - \mu_1(s))^2}{2\sigma_1^2}\} d\sigma \\
&= \int \frac{1}{\sqrt{2\pi}\sigma_1} \exp\{-\frac{(\sigma - \frac{r^2\mu_1(s)}{r^2 - \sigma_1^2})^2}{2\frac{r^2\sigma_1^2}{r^2 - \sigma_1^2}}\} d\sigma \exp\{\frac{r^2\mu_1(s)^2}{2\sigma_1^2(r^2 - \sigma_1^2)} - \frac{\mu_0}{r} - \frac{\mu_1(s)^2}{2\sigma_1^2}\} \\
&= \frac{r}{\sqrt{r^2 - \sigma_1^2}} \exp\{\frac{\mu_1(s)^2}{2(r^2 - \sigma_1^2)} - \frac{\mu_0}{r}\} \\
&\Rightarrow p(s) = \mu_0 - \frac{r\mu_1(s)^2}{2(r^2 - \sigma_1^2)} - r \ln \frac{r}{\sqrt{r^2 - \sigma_1^2}}.
\end{aligned}$$

Now since the bank is risk neutral, he only cares about the expected profit,

$$\overline{\Pi}(\pi) = n\mu_0 - \frac{nr}{2(r^2 - \sigma_1^2)} E\mu_1(s)^2 - nr \ln \frac{r}{\sqrt{r^2 - \sigma_1^2}} - c(\pi),$$

where

$$\begin{aligned}
E\mu_1(\widetilde{s})^2 &= E(\frac{\sigma_\epsilon^2}{\sigma_{\sigma_0}^2+\sigma_\epsilon^2}\sigma_0+\frac{\sigma_{\sigma_0}^2}{\sigma_{\sigma_0}^2+\sigma_\epsilon^2}\widetilde{s})^2 \\
&= \frac{\sigma_\epsilon^4}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}\sigma_0^2+\frac{2\sigma_\epsilon^2\sigma_{\sigma_0}^2}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}\sigma_0^2+\frac{\sigma_{\sigma_0}^4}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}E\widetilde{s}^2 \\
&= \frac{\sigma_\epsilon^4}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}\sigma_0^2+\frac{2\sigma_\epsilon^2\sigma_{\sigma_0}^2}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}\sigma_0^2+\frac{\sigma_{\sigma_0}^4}{(\sigma_{\sigma_0}^2+\sigma_\epsilon^2)^2}(\sigma_0^2+\sigma_{\sigma_0}^2+\sigma_\epsilon^2) \\
&= \sigma_0^2+\frac{\sigma_{\sigma_0}^4}{\sigma_{\sigma_0}^2+\sigma_\epsilon^2}\text{ decreasing in }\sigma_\epsilon^2,
\end{aligned}$$

as is the variance of the price in Section 2. Hence, the second-order effect of signals manifests itself through the expected value of the posterior variance, in contrast to the variance of the posterior variance in the basic common prior model. The fact that we are modeling the standard deviation here allows $var(\mu_1(\widetilde{s}))$ to affect $E\mu_1(\widetilde{s})^2$, and therefore the expected profit directly. The mechanism remains the same as that in the main text, but with somewhat less clear-cut interpretation. An increase in the precision of the signal decreases posterior mean of the variance, σ_1^2, which tends to increase the expected profit; on the other hand, it also increases the expectation of the posterior variance $E\mu_1(\widetilde{s})^2$, which tends to decrease the expected profit. The optimal level of information balances these two forces.

Now compare Problem 1 and Problem 2. Assume $N = n$.In Problem 2, $\widetilde{u} \sim$

$N(\mu_0, \frac{\sigma^2}{n})$, where ex ante $\widetilde{\sigma} \sim N(\sigma_0, \sigma_{\sigma_0}^2)$. The willingness to pay is derived as follows,

$$
\begin{aligned}
\exp(-\frac{p(s)}{r}) &= E[\exp(-\frac{\widetilde{u}}{r})|\widetilde{s}=s] \\
&= \int\int \frac{1}{2\pi\frac{\sigma}{\sqrt{n}}\sigma_1}\exp\{-\frac{1}{2}[\frac{2u}{r}+\frac{n(u-\mu_0)^2}{\sigma^2}+\frac{(\sigma-\mu_1(s))^2}{\sigma_1^2}]\}d\sigma du \\
&= \int\int \frac{1}{\sqrt{2\pi}\frac{\sigma}{\sqrt{n}}}\exp\{-\frac{(u-\frac{\mu_0 rn-\sigma^2}{rn})^2}{2\frac{\sigma^2}{n}}\}du\frac{1}{\sqrt{2\pi}\sigma_1}\exp\{\frac{\sigma^2-2\mu_0 rn}{2r^2n}-\frac{(\sigma-\mu_1(s))^2}{2\sigma_1^2}\}d\sigma \\
&= \int \frac{1}{\sqrt{2\pi}\sigma_1}\exp\{-\frac{(\sigma-\frac{r^2n\mu_1(s)}{r^2n-\sigma_1^2})^2}{2\frac{r^2n\sigma_1^2}{r^2n-\sigma_1^2}}\}d\sigma\exp\{\frac{r^2n\mu_1(s)^2}{2\sigma_1^2(r^2n-\sigma_1^2)}-\frac{\mu_0}{r}-\frac{\mu_1(s)^2}{2\sigma_1^2}\} \\
&= \frac{r}{\sqrt{r^2-\frac{\sigma_1^2}{n}}}\exp\{\frac{\mu_1(s)^2}{2n(r^2-\frac{\sigma_1^2}{n})}-\frac{\mu_0}{r}\} \\
&\Rightarrow p = \mu_0-\frac{r\mu_1(s)^2}{2n(r^2-\frac{\sigma_1^2}{n})}-r\ln\frac{r}{\sqrt{r^2-\frac{\sigma_1^2}{n}}}.
\end{aligned}
$$

The expected profit in this case is

$$
\widehat{\Pi}(\pi) = n\mu_0-\frac{r}{2n(r^2-\frac{\sigma_1^2}{n})}E\mu_1(s)^2-nr\ln\frac{r}{\sqrt{r^2-\frac{\sigma_1^2}{n}}}-c(\pi).
$$

One can show that $\overline{\pi} \geq \widehat{\pi}$, for pretty much the same reason, that is the diversification implied by the securitization decreases the marginal benefit of information.

Proof of Lemma 1 Suppose the bank discloses all signals. Given $(m_{1,...,}m_n, s_{1,...,}^2 s_n^2)$, investors' posterior beliefs are

$$
\begin{aligned}
\widetilde{\mu}_s|m_{1,...,}m_n &\sim N(\frac{1}{n}\sum_{i=1}^{n}(\frac{\sigma_{\epsilon_1}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}\mu_{0i}+\frac{\sigma_{\mu_0}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}m_i), \frac{\sigma_{\mu_0}^2\sigma_{\epsilon_1}^2}{(\sigma_{\mu_0}^2+\sigma_{\epsilon_1}^2)n}) \\
&= N(\frac{\sigma_{\epsilon_1}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}\overline{\mu}+\frac{\sigma_{\mu_0}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}m, \frac{\sigma_{\mu_0}^2\sigma_{\epsilon_1}^2}{(\sigma_{\mu_0}^2+\sigma_{\epsilon_1}^2)n}); \\
\widetilde{\sigma}_s^2|s_{1,...,}^2 s_n^2 &\sim N(\frac{1}{n}\sum_{i=1}^{n}(\frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}\sigma_{0i}^2+\frac{\sigma_{\sigma_0}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}s_i^2), \frac{\sigma_{\sigma_0}^2\sigma_{\epsilon_2}^2}{(\sigma_{\sigma_0}^2+\sigma_{\epsilon_2}^2)n}) \\
&= N(\frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}\overline{\sigma}^2+\frac{\sigma_{\sigma_0}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}s^2, \frac{\sigma_{\sigma_0}^2\sigma_{\epsilon_2}^2}{(\sigma_{\sigma_0}^2+\sigma_{\epsilon_2}^2)n}).
\end{aligned}
$$

Now suppose the bank discloses $m = \frac{1}{n}\sum_{i=1}^{n} m_i$ and $s^2 = \frac{1}{n}\sum_{i=1}^{n} s_i^2$. The joint distribution of the truths and the signals $(\widetilde{\mu}_s, \widetilde{m}, \widetilde{\sigma}_s^2, \widetilde{s}^2)$, where $\widetilde{\mu}_s = \frac{1}{n}\sum_{i=1}^{n}\widetilde{\mu}_i$ and $\widetilde{\sigma}_s^2 = \frac{1}{n}\sum_{i=1}^{n}\widetilde{\sigma}_i^2$:

$$\begin{bmatrix} \widetilde{\mu}_s \\ \widetilde{m} \\ \widetilde{\sigma}_s^2 \\ \widetilde{s}^2 \end{bmatrix} \sim N(\begin{bmatrix} \overline{\mu} \\ \overline{\mu} \\ \overline{\sigma}^2 \\ \overline{\sigma}^2 \end{bmatrix}, \begin{bmatrix} \frac{\sigma_{\mu_0}^2}{n} & \frac{\sigma_{\mu_0}^2}{n} & 0 & 0 \\ \frac{\sigma_{\mu_0}^2}{n} & \frac{\sigma_{\mu_0}^2+\sigma_{\epsilon_1}^2}{n} & 0 & 0 \\ 0 & 0 & \frac{\sigma_{\sigma_0}^2}{n} & \frac{\sigma_{\sigma_0}^2}{n} \\ 0 & 0 & \frac{\sigma_{\sigma_0}^2}{n} & \frac{\sigma_{\sigma_0}^2+\sigma_{\epsilon_2}^2}{n} \end{bmatrix}),$$

and the posteriors are

$$\begin{aligned} \widetilde{\mu}_s|\widetilde{m} &= m \sim N(\frac{\sigma_{\epsilon_1}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}\overline{\mu} + \frac{\sigma_{\mu_0}^2}{\sigma_{\epsilon_1}^2+\sigma_{\mu_0}^2}m, \frac{\sigma_{\mu_0}^2\sigma_{\epsilon_1}^2}{(\sigma_{\mu_0}^2+\sigma_{\epsilon_1}^2)n}); \\ \widetilde{\sigma}_s^2|\widetilde{s}^2 &= s^2 \sim N(\frac{\sigma_{\epsilon_2}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}\overline{\sigma}^2 + \frac{\sigma_{\sigma_0}^2}{\sigma_{\epsilon_2}^2+\sigma_{\sigma_0}^2}s^2, \frac{\sigma_{\sigma_0}^2\sigma_{\epsilon_2}^2}{(\sigma_{\sigma_0}^2+\sigma_{\epsilon_2}^2)n}). \end{aligned}$$

Q.E.D.