Durable consumption and asset management with transaction and observation costs*

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Abstract

The empirical evidence on rational inattention lags the theoretical developments: micro evidence on one of the most immediate consequence of observation costs – the infrequent observation of state variables – is not available in standard datasets. We contribute to filling the gap using new household surveys. To match these data we modify existing models shifting the focus from non-durable to durable consumption. The model features both observation and transaction costs and implies a mixture of time-dependent and state-dependent rules. Numerical simulations explain the frequencies of trading and observation of the median investor with small observation costs and larger transaction cost.

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1 Introduction

A large literature documents that several economic decisions occur infrequently.[1] For instance, individual investors adjust their portfolios sporadically even though the prices of many assets experience large fluctuations at high frequency. Similarly, firms do not reset the price every time the costs of inputs change. These infrequent adjustments at the micro level are a potential source of sluggish behavior at the aggregate level and have thus attracted the interest of macroeconomists. One hypothesis that has been studied in the recent literature is that inaction might result from the presence of observation costs, i.e. costs related to the information gathering process, such as those due to the monitoring of the value of equity (in the case of a consumer/ investor) or the monitoring of production costs (in the case of a firm). The optimality of economizing attention when information gathering is costly is what we refer to as the "rational inattention" hypothesis.[2]

Besides the intuitive appeal of observation costs, an important methodological reason that makes it interesting is that the nature of the optimal adjustments implied by this friction is different from the one generated by standard fixed cost, and this translates into different implications for aggregate behavior. Duffie and Sun (1990), Gabaix and Laibson (2001), Abel, Eberly, and Panageas (2007) show that with observation costs the optimal rule implies time-dependent adjustments, as opposed to state-dependent adjustments that are typical in the standard fixed-cost literature. Abel, Eberly, and Panageas (2009) show that a time-dependent rule may be optimal even in an environment with both observation and transactions costs. Understanding the nature of the decision rule matters because the aggregation of agents following time-dependent rules is different from the one of agents following state-dependent rules as argued by Gabaix and Laibson (2001) and Alvarez, Atkeson, and Edmond (2009) in the consumption-savings and price-setting literature, respectively. Finally, the modeling of inattentive investors, whose trading in financial markets is only sporadic, may be important to understand the dynamics of assets risk premia, as argued by Duffie (2010), and its volatility, as argued by Chien, Cole, and Lustig (2010).

In spite of the theoretical developments that rational inattention has inspired, micro evidence on rational inattention lags behind. Empirical evidence on the most immediate consequence of observation costs − the infrequent observation of state variables − is not available in standard datasets. We contribute to filling the gap with two novel household

[1]See Stokey (2008) for a survey of the recent literature.

[2]Two broad areas where this hypothesis is studied are the consumption, savings, and portfolio theory (see e.g. Duffie and Sun (1990), Gabaix and Laibson (2001), Sims (2003) and Reis (2006), Abel, Eberly, and Panageas (2007) and price setting problems (see e.g. Mankiw and Reis (2002) and Woodford (2009)). Our use of the term "rational inattention" is broader than as seminally proposed by Sims, and follows more closely the use by Reis and Abel et al.

surveys that record the frequency with which investors observe the value of their financial investments, as well as the frequency with which they trade assets and durable goods. We consider models with both observation and transaction cost, since the latter are a standard explanation for infrequent adjustments. We use these data to test key predictions of existing rational inattention models and to quantify the relative importance of the observation cost relative to standard transactions costs. We find that to match important patterns in the data and to distinguish between both types of cost we need to introduce a model that shifts the focus from non-durable to durable consumption. This new model implies a mixture of time-dependent and state-dependent rules, where the "importance" of each rule depends on the relative magnitude of the observation and transaction costs.

Our starting point are the models developed by Duffie and Sun (1990), Gabaix and Laibson (2001), Reis (2006), and Abel, Eberly, and Panageas (2007, 2009). These seminal contributions explore the consequences of observation costs in the context of an investor's optimal savings/portfolio problem that includes the optimal management of a low return liquid asset, required to pay for transactions, analogous to the monetary models with a cash in advance constraint. In Section 2 we introduce two original datasets that are tailor-made to provide detailed evidence on the patterns of non-durable consumption, management of liquid assets, information collection, and trade in financial assets (purchase or liquidation of assets). We find a robust pattern consistent with the assumption that a component of adjustment costs is information gathering, namely that the frequency of trading, the frequency of observation, and the time spent collecting financial information are strongly correlated across investors. However, we find evidence against two specific mechanisms operating in several the models that focus on non-durable goods and liquid assets. In particular, the models of Duffie and Sun (1990), Gabaix and Laibson (2001), and Abel, Eberly, and Panageas (2007) predict that the observed frequency of observation and financial transactions should coincide, and also predict that the household liquid asset holdings (e.g. the average M1 or M2 balances) should decrease with the frequency of trades in assets. Our analysis shows that both predictions are poorly born out in the data: the frequency of information acquisition is at least 3 times larger than the frequency of portfolio trades. Moreover, the data do not display a negative correlation between the household liquid assets and the frequency of asset trades.

Motivated by both the encouraging evidence on the presence of observation cost featured by these models, as well as by the empirical shortcomings specific to the mechanism involving non-durable consumption and liquid assets, Section 3 develops a new model that preserves a role for costly observations while abstracting from non-durable consumption and liquid assets. We consider a model with both observations and transaction cost, and depart from the previous literature by focusing on durable, as opposed to non-durable, consumption goods.

This shift has two direct implications. First, in the model with durable consumption goods purchases will be large and occur at discrete intervals, and thus they will require to hold essentially no liquid assets. Second, and more important, in the model with durable consumption with both observation and transactions costs, the observation frequency is always larger than the trading frequency, as it is in the data. The reason for this result is that durable goods and transactions costs give rise to an inaction region, just like in Grossman and Laroque (1990) and Stokey (2009), where the agent tolerates moderate deviations of the stock of durable goods from the frictionless benchmark. Thus, every time the agent observes her wealth and finds it to be in the inaction region the model produces an observation without a trade.

In Section 4 we use numerical simulations of the model to gauge the order of magnitude of observation and transactions costs that are consistent with observed investors' behavior. This exercise shows that very small observation costs are sufficient to reproduce the frequency of observation that is found in the data. Section 5 tests two novel predictions of the durable good model using two household survey and well as an administrative panel data. First, that the frequencies of assets transactions and that of durable adjustment should be positively correlated, both across investors, and across time for a given investor. Second, that since more risk tolerant individuals invest more in volatile assets, they value information more and thus observe more frequently. The data lend support to both predictions. Additionally we document that the frequency and size of sales of financial assets spikes just before house purchases.

Section 6 concludes with a discussion of our quantitative findings, and a comparison with the findings in Alvarez, Lippi, and Paciello (2011) on firms' price setting behavior in a model with observation and menu costs. We also discuss the role of labor income, an ingredient that is absent in the literature on which we build this paper, the possibility that some portfolio observations are available at no cost, and other issues for future research.

2 Observations, Trades, and Liquidity: Theory vs. Data

This section reviews some evidence related to a class of models that use the rational inattention hypothesis to study consumption, savings, portfolio theory and liquidity. In these models the relevant decisions concern the rate of consumption - or savings - and the portfolio composition; the costs are those associated with keeping track of the information about financial variables. Examples of these models are Duffie and Sun (1990), Gabaix and Laibson (2001), Sims (2005), Reis (2006), Abel, Eberly, and Panageas (2007, 2009) among others.

These models carry neat implications on at least two potentially observable behaviors:

first, since investors do their best to avoid collecting information when it is not needed, they will choose to keep the frequency of observations as close as they can to the frequency of financial trades. In models with an observation cost only, such as Duffie and Sun (1990), Gabaix and Laibson (2001), Abel, Eberly, and Panageas (2007), the two frequencies actually coincide, providing a clear prediction. In Abel, Eberly, and Panageas (2009), where *both* information and transaction costs are present, the authors show that this prediction holds in finite time with probability one, provided that the transaction cost is sufficiently small. While the authors do not characterize the decision rules for the model with larger transaction cost, these predictions are surely violated for large enough values of the transaction cost, in which case the model in Abel, Eberly, and Panageas (2009) implies a higher frequency of observation than transactions. The second prediction is implied by the presence of a liquidity-in-advance constraint for the purchase of non-durable goods. This assumption, together with the presence of an opportunity cost on the illiquid assets and of transaction costs between the savings and liquid account, gives rise to a Baumol-Tobin-type management of liquid assets, where the investors who trade more frequently have lower average holdings of liquid assets. In Appendix A-2 we provide a description of such a model. In this section we bring both of these predictions to bear with a novel set of data that contains information on how frequently people choose to observe the value of their financial assets and trade them as well as data on the value and composition of their liquid and financial assets.

2.1 Data sources

Our empirical evidence relies on two different sources. The first is a sample of about 1,800 Italian investors with an account at Unicredit Bank, one of the largest banking groups in Europe (for some statistics we use a sample of 40,000 investors), and we refer to this source as UCS. We based most of the analysis on the first survey wave, run in 2003, but occasionally we also rely on data from the 2007 wave. The novel and original feature of this survey is the wealth of information that it has on the frequency people gather information on their financial investments and make financial transactions, as well as on investors' risk preferences, assets and demographics which provide an ideal setting for testing predictions of models that emphasize information and transaction costs in household savings and financial decisions. We complement the survey with a 35 month panel of administrative records of 26 different accounts for the same investors. The second source is the 2004 Survey of Households Income and Wealth - a widely used survey on a sample of about 8,000 Italian households managed by the Bank of Italy. This dataset has two useful features: first, unlike UCS it collects detailed data on durable purchases; this particular wave has also information on the frequency investors make financial trades. Both will prove important in Section 5 to test the predictions of the model we develop in Section 3. Second, while UCS is representative of the population of Unicredit customers it is not of the Italian population; but SHIW is, and this allows us to make sure that our findings with UCS are not the reflection of sample selection.

2.2 Key variables description

We now describe in detail two key variables for our analysis: the frequency investors observe investments and the frequency they trade financial assets. More details on the two surveys and the variables that we use are given in the data appendix.

To our knowledge, UCS is the first large scale survey to collect information on how frequently people check their financial investments and make financial trades. In the 2003 wave sample participants where asked: "How often do you check the value of your financial investments?". They could answer: a) every day; b) at least once a weak; c) every 15 days; d) once a month; e) about every three months; f) about every six months; g) about once in a year; h) less than once a year i) never check; l) I have no investments. To obtain information on the frequency of financial trades they were asked: "How often do you change the composition of your financial portfolio and sell or buy financial assets?" The options are: a) every day; b) at least once a week; c) about every two weeks; d) about every month; e) about every three months; f) about every six months; g) about every year; h) less than once a year; i) at maturity; l) never; m) I have no investments. Notice that in principle the answer to the question on trading might involve trades that do not give rise to net liquidation of asset, but only to a portfolio "rebalancing" . Below we use an original set of actual transactions data from a sample of Unicredit Bank customers to document the relevance of "rebalancing trades".

Similar questions were asked in the 2007 UCS wave while the trade frequency question, with the same wording, was also asked in the 2004 SHIW. The only difference with respect to the UCS question is that the first two answers are lumped together as "at least once a week". Obviously, questions only apply to active investors, implying that some observations (316 in UCS 2003 out of 1,834 participants) will be lost. Next, we use these data to confront two predictions of the rational inattention models.

2.3 Patterns on portfolio trades and observations

Table 1 shows the joint distribution of the frequency of observing and that of asset trading among the 2003 UCS investors. The table documents several noteworthy features. First, the large mass of observations on the main diagonal of the table shows that there is a strong positive correlation between the frequency with which agents observe their investments and

Table 1: Frequencies of portfolio observation and trade

	Observations per year:									
	365	52	26	12	4	2	1	< 1	–	never
Trades per year:										
365	**29**	5	1	1	0	0	0	0		0
52	14	**24**	2	4	2	0	1	0		0
26	13	18	**13**	7	1	0	0	0		0
12	16	29	35	**65**	17	3	1	3		0
4	18	19	24	97	**103**	6	2	0		2
2	5	20	23	63	84	**53**	2	0		2
1	9	14	8	37	48	33	**16**	1		4
< 1	7	17	12	37	41	29	7	**24**		8
at maturity	11	17	5	48	60	23	27	38	**-**	13
never	4	4	0	9	15	14	4	12		**35**

Source: *Unicredit* survey 2003, All investors. Each entry is the number of investors' observations in each cell; entries on main diagonal highlighted in bold. Summary statistics: Fraction on main diagonal: 24%; Fraction below main diagonal: 70%.

the frequency of asset trading, the correlation is 0.45 with a p-value smaller than 1 per cent. Those who observe the portfolio more often also tend to trade more often. This evidence is consistent with the idea, at the core of costly information models, that the trading and information gathering activity are related. Second, in only a handful of cases (6 percent of the observations) investors trade more frequently than they observe. The fact that few investors trade more often than they observe may be due to minor measurement errors (reassuring about the quality of our indicators of observing and trading frequencies) or reflect rare cases where investors trade blindly.[3]

Table 2 reports summary statistics on the frequency of observing and trading for different groups of investors both from the UCS 2003 and the SHIW 2004 surveys. Consistently with models that stress information gathering costs and assets trading costs, investors observe their investments and trade assets infrequently. The median number of portfolio observations per year in the sample of UCS investors (as well as for stockholders) is 12, while the median number of asset trades is 2. Smaller asset trading frequencies are estimated for the investors in the SHIW (in the lower panel of the Table). The SHIW statistics, computed on a sample that

[3] The August 2010 version of Abel, Eberly, and Panageas (2009) introduces a notion of automatic transfers (transfers that take place without observations) which is able to explain cases where the number of trades is larger than the number of observations. In a related price setting problem Alvarez, Lippi, and Paciello (2011) show that multiple adjustments between observations may be optimal if the drift of the state variable is above a threshold.

is representative of the Italian investors, are comparable to those observed for US households.[4] The table also reports an estimate for the median number of observations in the SHIW sample, imputed from a regression estimated on the UCS data (see the note to the table for more details). The frequency of observation for the equity investor in the SHIW sample is about 1/2 that for the UCS sample of equity investors.

Table 2: Number of portfolio observations and trades per year

	# portfolio observations			# portfolio trades			Assets[a]	
	mean	med.	std	mean	med.	std	mean	med
	Unicredit Survey 2003 (UCS)							
All investors	42	12	98	14	2	56	252	77
Stockholders	56	12	113	18	2	64	331	120
Stockholders (direct)	66	12	122	22	4	71	367	132
	SHIW Survey 2004							
All investors		3.6[b]		2	0.4	7	50	26
Stockholders (direct)		5.4[b]		3	0.7	7	67	36

Note: The upper panel is based on the *Unicredit* survey for 2003. All investors (1,518) are individuals with at least 1,000 euros in bank deposits who also have some other financial investments. This survey is designed to oversample the wealthy. Stockholders (984) includes individuals holding stocks of listed or unlisted firms directly, or through a mutual fund, or a managed investment account. Direct stockholders (736) includes individuals holding stocks of listed or unlisted companies directly. The lower panel is based on *SHIW* 2004 survey, that is designed to be representative of Italian households. It includes 2,808 households with financial assets other than bank or postal account (1,535 of which own stocks directly). See Online Appendix A-1 for a more detailed description and comparison of these surveys. –[a]Thousands, in 2003 euros. –[b]Imputed to the SHIW investors from a regression estimated on the UCS data. The specification (in logs) includes the following regressors: the number of asset trades, the investor's financial assets and controls for the investor age, gender, education and marital status. The R^2 is 0.3 for the UCS full sample and 0.2 for the UCS sample of stockholders. In each SHIW sample the reported quantity is the median of the fitted values produced by the estimated regression.

Though this evidence is consistent with the costly observation hypothesis it also departs from it in one dimension that is featured in the models with *only* an observation cost: the data show that investors do not trade every time they observe the value of their investments. It appears that the frequency of trading and the frequency of observation coincide for only 28% of the investors (those along the diagonal in Table 1); for 67% of the investors the observation frequency is higher than the trading frequency. Thus, only a minority of the investors in the sample conforms to the prediction that every observation triggers a trade.

[4]ICI (2005) reports information on US equity investors. Figure 33 shows that for 1998, 2001 and 2004 the fraction of equity investors who make no equity trades during a year is 0.58, 0.6, and 0.6 respectively. Assuming a Poisson distributed number of trades with constant intensity this implies an average of about 0.51 trades per year. A somewhat higher statistic is computed by Bonaparte and Cooper (2009) who, using the Survey of Consumer Finances, report that the fraction of household owning stocks who do not adjust their portfolio in one year is about 0.3, which implies an average of 1.2 trades per year.

Table 2 confirms that investors observe the value of their investments more frequently than they trade, with a ratio between the two average frequencies around 3. This pattern holds across investors type, asset levels, and trade-frequency. We will argue below that in order to account for this empirical facts, we need to supplement the costly-observation models with a transaction cost, along the lines of Abel, Eberly, and Panageas (2009).

We stress that these patterns are unlikely to be the reflection of some particular feature of the survey wave. To address this concern we reproduced Table 1 using the 2007 UCS. The new joint distribution (not reported) has the same features as the one based on the 2003 wave: only 3.5% of the investors trade more frequently than they observe their investments; 24% equally frequently and 72.5% less frequently while the average number of observations stays in a ratio of 3 to 1 to the number of trades.

One may also worry about measurement errors in the survey data on the frequency of observation and trading. To assess the quality of the observation frequency, we rely on an independent measure available in the 2003 UCS of the amount of financial information investors collect from various sources (such as newspapers, the web, their advisors or the companies' accounting statements) before making an investment decision. This is a broad measure of the time investors devote to gathering financial information. One would expect that investors who collect more financial information, also observe the value of their investments more frequently. We find it reassuring that this correlation appears clearly in the data, as shown in Figure A-6 in Section A-1.5.

Concerning the frequency of trading one may question whether the survey measure is a good proxy of the theoretical notion: in the inventory models we described above trading means transferring resources from a savings to a liquid account; in the data, given the wording of the question, some of the trades may involve portfolio rebalancing with no transfers to or from the liquid account. Notice that the presence of rebalancing trades would make the positive gap between observing and trading frequency measured above an underestimate of the theoretically relevant one. Since no information is available in the survey on the nature of the trades, we have resorted to administrative data available for the 2007 Unicredit sample to get a sense of the importance of rebalancing trades. To this end we defined two notions of "rebalancing" trades. The first is a broad notion estimated by assuming that a rebalancing occurs any time there is net sale of one of the financial assets in the investments portfolio and at least one net purchase of another asset. Hence the number of rebalancing trades in each month is equal to the minimum between the number of net sales and the number of net purchases. In the whole sample the number of trades with rebalancing is 1.13 per year (median 0.34). Stockholders rebalance almost twice as often than non-stockholders, a reasonable feature. The second notion is narrower, as it considers as "rebalancing trades" all those that involve a simultaneous purchase and sales of two different investment classes but no net liquidation/purchase of investments. For these trades the value of asset sales matches the value of asset purchases. The mean number of rebalancing trades for this measure is smaller than the previous one (0.09 per year and around 0.14 for stockholders). Depending on whether we use the broad or narrow measure of rebalancing frequency, this evidence shows that the share of rebalancing to total trades ranges between 2% to 20% of total trades (see Table A-17 in the data appendix).[5]

2.4 Patterns on portfolio trades and household liquidity

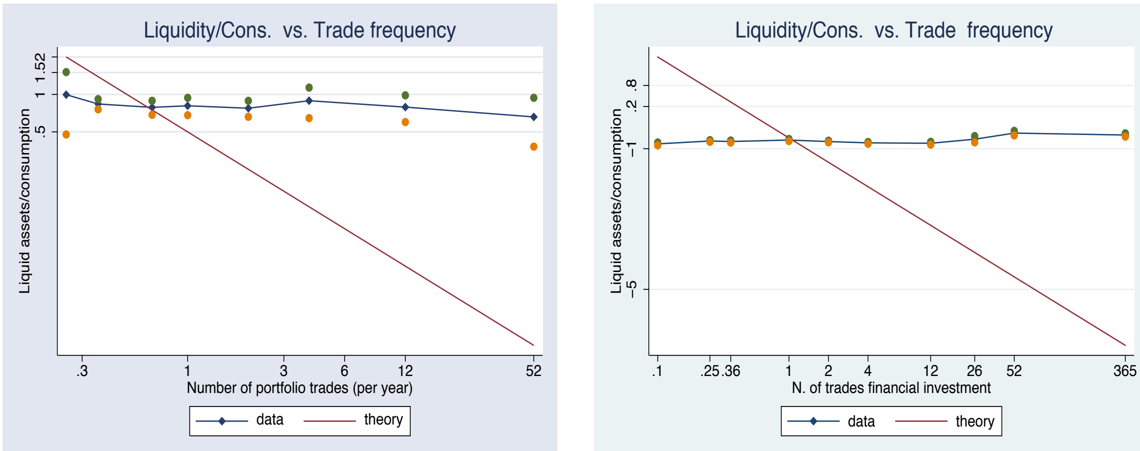
If portfolio trades are mostly aimed at transferring resources from the asset account to the liquid account then those agents who trade more frequently should, on average, hold less liquid assets. Indeed, the inventory-type models in the costly information literature, such as Duffie and Sun (1990), Gabaix and Laibson (2001), Abel, Eberly, and Panageas (2007, 2009), predict a negative unitary elasticity between liquid assets holdings (scaled by non durable consumption) and the number of financial trades.

We test this prediction using both UCS data and SHIW data. Liquid assets are defined as the sum of cash holdings, checking and savings accounts - a measure close to M2 - but results are unaffected if we use a narrower definition that includes only cash and checking accounts. Since UCS collects no consumption data we impute it using the 2004 SHIW to estimate non-durable consumptions (see Online Appendix A-1 for details). We then construct the ratio between liquid assets and consumption. Figure 1 plots both the empirical relation between the average level of liquid assets ratio in the sample and the number of trades and its theoretical counterpart as predicted by the inventory models. Panel A shows uses the SHIW data and Panel B uses the UCS data. Contrary to the model prediction we find a weak correlation between liquid assets holdings and the number of trades. In both surveys the unconditional correlation is, if anything, slightly positive. This finding is quite robust as we discuss next.

[5] Specifically, for the sample interviewed in 2006 we have information on the stock and net trading flows of 26 assets categories these investors have at Unicredit. One of these categories is the liquidity (checking) account. These data are available at a monthly frequency for 36 months beginning in December 2006. Since for each asset category we observe the net trades separately from the end of period stocks, we can compute measures of trading frequency. We first obtain a measure of total trades. For this we define a trade in a month as a situation where at least one investment class out of 25 (thus excluding liquid asset) has a net positive or negative flow. Implicit is the idea that a trade is a trip to the bank/broker to buy, or sell, or buy and sell assets, and that in a trip one can buy or sell more than one asset. The average annual number of total trades is then obtained. The Online Appendix A-5.2.1 discusses these estimates and how they relate to the frequency of trading reported in the survey analogous to the one in Table 2. The mean number of total trades is 4.5 (median 3.4); stockholders trade more frequently and these measures are positively correlated with self reported measures of trading frequency.

Figure 1: Household Liquidity vs. asset trade frequency

A. SHIW 2004 *B*. Unicredit 2003



Note: Log scale. Liquidity is measured by the ratio M/c, where M is $M2$ (similar results obtained for $M1$) and c is non-durable consumption (same results if total consumption is used); in the Unicredit survey consumption is imputed from a regression in the SHIW data using income and other demographics. For each trade frequency bin, the figure plots the mean of M/c (dots denote the standard deviation of the mean statistic) and the model predicted relation, given by $M/c = 1 \ / \ (2 \times \text{Trading Frequency})$.

Table 3 shows that the lack of correlation also emerges from multivariate regressions that condition on the cross-sectional variation in income, household size, age and the importance of labor income over total income. The reason for adding the latter variable is that one might be concerned that the match with the model is far from perfect, and that modeling labor income would change the results on the relation between the level of liquid asset and the frequency of transactions. As a preliminary control for this we include the ratio of labor income to non-durable consumption in the regressions: the household liquidity remains uncorrelated, if anything slightly positively correlated, with the frequency of trades in assets (see footnote 8 for more details).

One could argue that if individuals vary according to the uninsurable idiosyncratic risks they face, omitting this variable could be responsible for the lack of correlation between liquid assets holdings and trading frequency. To address this concern we add proxies for idiosyncratic labor income risk, namely a dummy for self employment and a dummy for government employees; in the Unicredit sample we also insert a specific measure of background risk by adding a dummy if the respondent reports that he/she is unable to predict if his/her income will fall significantly, increase significantly or remain unchanged in the 5 years following the interview. When adding these controls results are unchanged, as shown in the last column of Table 3. The correlation between liquid assets holdings and trading frequency is essentially the same if we estimate it on the sole sample of government employees who face little or no

Table 3: Liquidity (M2) vs. trade frequency (asset transactions)

	Dependent variable: M/c ; Regressor: asset trade frequency				
	Shiw data		*Unicredit data*		
	bivariate[a]	multivariate[b]	bivariate[a]	multivariate[b]	w. risk controls[c]
All investors	*(2,808 obs.)*		*(1,365 obs.)*		
Trade freq.	0.005	0.03	0.10	0.13	0.12
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Equity investors	*(1,535 obs.)*		*(875 obs.)*		
Trade freq.	0.06	0.06	0.08	0.11	0.10
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)

Note: Based on the 2004 SHIW and 2003 Unicredit surveys. All variables in logs. All regressions include a constant; standard errors in parenthesis. M/c is the liquid asset to consumption ratio; M is $M2$ (similar results obtained for $M1$); c is non-durable consumption (same results if total consumption is used); for the Unicredit survey consumption is imputed from a regression in the SHIW data using income and other demographics. −[a]Regression coefficient of bivariate OLS. −[b]The regressions include the following controls (all in logs): household income, % of labor income over non-durable consumption, age of household head, number of adults. −[c] Multivariate regression that includes three measures of background risk: two occupational dummies for self employed and government employee and a measure of background risk using an indicator of income risk that is available in the survey.

income risk.

Finally, similar results are obtained if we use a narrow measure of liquid assets (M1) instead of a broad one, if we look at median liquid assets rather than means, if we scale liquid assets with total financial assets (in this case the simple correlation is somewhat negative both in the UCS sample and in the SHIW but the elasticity is far from the predicted unit value).[6]

Summing up, two predictions of the model with non-durable consumption and information cost only − a one to one relation between the frequency of observing and that of trading financial assets and a one to one (negative) relation between liquid assets and the number of trades − find only partial support in the data. We think that one reason behind the weak evidence on this mechanism lies in the reliance on trades between financial assets and transaction accounts to derive a theory of liquid assets holdings to finance non-durable

[6]To further check whether the lack of correlation is due to the number of trades capturing some unobservable determinant of liquid assets holdings we regressed (unreported) the (log) liquid assets scaled by consumption (or total assets) on the (log) number of trades adding demographic controls, as well as controls for (log) consumption and assets. We have estimated this regression on several sub samples of investors: all investors, all stockholders, direct stockholders. In all cases we found a small elasticity of liquid assets to the number of financial trades, often positive (estimates in the UCS sample range between 0.02 and 0.10 depending on sample and specification) and always quite far from the negative one-to-one correspondence predicted by the model.

consumption. To further this view we contrast the prediction between the transactions frequency and demand for liquidity with the one, of identical nature, that emerges in the realm of currency demand models. We use information on the average currency holdings and the average number of cash withdrawals by Italian households taken from the SHIW survey.[7] Table 4 reports the (log) correlation between the average currency balance and the frequency of transactions, measured by the number of cash withdrawals. The correlation is always negative and statistically different from zero (between −0.2 and −0.5). Despite the presence of large measurement error, the inventory theory of cash management finds strong support in the data, while the inventory theory for liquid assets management implicit in the costly information models does not.[8] In the next section we show that introducing transaction and observation costs, and shifting the focus from non-durable to durable consumption, helps reconciling the theory with the data.

Table 4: Liquidity (Currency) vs. trade frequency (# withdrawals)

	without ATM card		*with ATM card*	
	bivariate	multivariate	bivariate	multivariate
	Dependent Variable $\log(M/c)$			
$\log n$	-0.24*	-0.22*	-0.25*	-0.24*
	(900 obs.)	*(900 obs.)*	*(2,326 obs.)*	*(2,325 obs.)*
	Dependent Variable $\log(W/c)$			
$\log n$	-0.39*	-0.40*	-0.52*	-0.52*
	(2,250 obs.)	*(2,249 obs.)*	*(1,256 obs.)*	*(1,255 obs.)*

All variables in logs. All regressions include a constant. The asterisk denotes that the null hypothesis of a zero coefficient is rejected by a t-test with a 1 per cent confidence level. M = average currency holdings (coins and bills), c = average consumption paid in cash during the year, n = average number of cash withdrawals per year from ATM and bank-branches, W = average size of withdrawal from ATM and bank branches. For the multivariate we include a dummy for self-employed, and the percentage of income paid in cash. Source: SHIW 2004.

[7] See Attanasio, Guiso, and Jappelli (2002) and Alvarez and Lippi (2009) for an analysis of these data.

[8] One may think that the omission of labor income, that is credited directly into their liquid account, may be responsible for this empirical shortcoming of the model (just like direct cash transfers, e.g. wages paid in cash, might impinge on the Baumol-Tobin theory of the demand for currency). If this was the case, then controlling for the labor income should reveal a negative relation between average liquidity and the frequency of assets transactions. Instead, the lack of correlation between average liquidity and trades' frequency persists even in the multivariate regressions where the share of labor income over total non-durable consumption is controlled for (see Table 3).

3 A model with observation and transaction costs

This section presents a model that is consistent with two empirical facts documented above: first, that the frequency of portfolio trading is smaller than the frequency of portfolio observation. Second, that the frequency of trading is uncorrelated with the liquidity of agents. The main feature of this model is to solve the investor's problem in the presence of two distinct fixed costs: one for observing the value of the portfolio, another for adjusting the stock of durable goods. One difference compared to the previous literature with costly observation is that the model focuses on durable, as opposed to non-durable, consumption.

The focus on the durable goods creates a disconnect between the household liquid asset holdings and the durable expenditures. This is because the fixed cost gives rise to discrete adjustments, so that even when liquid assets are required to pay for the durable good, the liquidity used for this purchases will be withdrawn and spent immediately. The fact that liquid asset used to pay for expenditures have an infinite velocity, implies that durable purchases will generate zero average holdings of liquidity (even though these purchases will affect the average size of liquidity withdrawals). The lack of correlation between the household average liquid asset holdings and the frequency of portfolio transactions is consistent with the evidence of Section 2. To generate non-zero money holdings, the model would need to be augmented by including a non-durable consumption component, as in standard inventory models. The optimal inventory model in Alvarez and Lippi (2011) combines large infrequent expenditures with small continuous ones and shows the conditions under which both non-zero average holdings of liquid asset and no-relationship between the frequency of financial asset sales (withdrawals) are obtained.

Here we consider the problem of a household who consumes only durable goods. She derives utility proportional to the stock h of durables, which depreciates at rate δ. Her preferences are given by discounted expected utility, and a CRRA period utility $U(h) = h^{1-\gamma}/(1-\gamma)$. The agent's source of funds to buy/sell durables is her financial wealth a, a fraction α of which can be invested in risky securities and the remaining in risk-less bonds. Let s be a standard Normal random variable, and $R(s, \tau, \alpha)$ be the gross return during a period of length τ with portfolio α when the innovation to the return is s, we have:

$$R(s, \tau, \alpha) \equiv e^{(\alpha\mu+(1-\alpha)r-\frac{\alpha^2\sigma^2}{2})\tau+\alpha\sigma s\sqrt{\tau}}. \tag{1}$$

This is the gross return to a portfolio that is continuously rebalanced to have fraction α in the risky asset with instantaneous return with mean μ and variance σ^2 per unit of time, i.e. as $\tau \downarrow 0$, we have $\frac{1}{\tau}\mathbb{E}[R(s, \tau, \alpha) - 1] \to \alpha\mu + (1-\alpha)r$ and $\frac{1}{\tau}\text{Var}[R(s, \tau, \alpha) - 1] \to \alpha^2\sigma^2$.

There are two frictions in our model. The first one is a fixed cost parameter ϕ_T of trading the durable good, as in Grossman and Laroque (1990). So, if there is an adjustment in the stock of durables, say from h to h', the agent looses $\phi_T\ h$. We assume that this fixed cost applies to either a change in the stock of durables or a change in the share of risky assets α. This assumption differs from Grossman and Laroque (1990) who assume that the transaction cost only applies to durable adjustments but not to adjustments of the portfolio composition. In our setup their assumption implies that every observation gives rise to a portfolio adjustment, a prediction that is inconsistent with the data of the previous section, in particular with the small number of portfolio rebalancing trades for households.[9] The second friction is a fixed cost parameter ϕ_o that is paid by the agent for observing the value of her financial wealth. To preserve homogeneity and conserve on the state space, the observation cost is assumed to be proportional to the asset value: $\phi_o\ a$.[10]

We let $V(a, h, \alpha)$ denote the value function for an agent who, after paying the observation cost, has a durable stock h, financial wealth a with a fraction α invested in risky assets. She decides τ, the length of time until the next observation date and whether to pay the cost ϕ_T, transfer resources and adjust both the portfolio share α and her durables stock to h'. These decisions are subject to the budget constraint:

$$a' + h' + h\ \phi_T\ I_{(h',\alpha')\neq(h,\alpha)} = h + a \tag{2}$$

where $I_{\{\cdot\}}$ is an indicator of adjustment (of either the durable stock or the portfolio composition). The Bellman equation is then:

$$\begin{aligned} V(a,h,\alpha) &= \max_{a',h',\alpha',\tau} \int_0^\tau e^{-\rho t} U(h' e^{-\delta t}) dt \\ &+ e^{-\rho\tau} \int V\left(a'(1-\phi_o) R(s,\tau,\alpha')\ ,\ h' e^{-\delta\tau},\ \alpha'\right) dN(s) \end{aligned}$$

subject to the budget constraint (2), where $N(\cdot)$ is the CDF of the standard Normal distribution, and $R(s, \tau, \alpha)$ is the gross return during a period of length τ of the portfolio with share α as defined in (1). It is convenient to write the value function as:

$$V(a,h,\alpha) = \max\left\{\bar{V}(a,h,\alpha)\ ,\ \hat{V}(a,h)\right\}\ . \tag{3}$$

This function picks the best of two conditional value functions: the first one, $\bar{V}(a, h, \alpha)$, gives

[9] Moreover, since the value function is locally convex near the inaction boundaries, the agent is locally risk lover, and would choose very risky portfolios when the durable stock gets close to the boundaries, as indeed happens in Grossman and Laroque (1990).

[10] Equivalently, one can write one or two of the cost in utility terms, as in Abel, Eberly, and Panageas (2009).

the value of leaving a, h, α unchanged and observing again τ periods from now:

$$\begin{aligned}\bar{V}(a,h,\alpha) &= \max_{\tau} \int_0^{\tau} e^{-\rho t} U(h e^{-\delta t}) dt \\ &+ e^{-\rho\tau} \int_{-\infty}^{\infty} V\left(a(1-\phi_o)R(s,\tau,\alpha)\ ,\ h e^{-\delta\tau}\ ,\ \alpha\right) dN(s)\ . \end{aligned} \tag{4}$$

The policy allows the agent not to pay the adjustment cost $h\phi_T$, at the cost of keeping the durables stock and the portfolio composition unadjusted. The second conditional value function is

$$\begin{aligned}\hat{V}(a,h) &= \max_{a',\tau,\alpha'} \int_0^{\tau} e^{-\rho t} U([a + h(1-\phi_T) - a'] e^{-\delta t}) dt \qquad (5) \\ &+ e^{-\rho\tau} \int_{-\infty}^{\infty} V\left(a'(1-\phi_o)R(s,\tau,\alpha')\ ,\ [a + h(1-\phi_T) - a'] e^{-\delta\tau},\ \alpha'\ \right) dN(s)\ . \end{aligned}$$

which gives the value of the policy where the agent, upon observing her wealth, adjusts her durable stock so that by equation (2) the post-adjustment initial stock of durables is $h' = a + h(1-\phi_T) - a'$. She also decides a new observation date τ and the share of risky assets α':

3.1 Characterization of the optimal policy

This section outlines the nature of the optimal policy for the problem in the presence of both transaction and observation costs (see Section A-3 for an analysis of the special cases where none, or only one of the costs is present). We show that in this case there is no perfect synchronization between portfolio observations and portfolio adjustments. A numerical illustration of the workings of the model is given in the next section.

The optimal decision rule for trading-transferring resources and adjusting the durable goods is of the sS type. This is due to the homogeneity of the value function and to the fact that α is not a state in problem (5). Notice that for any fixed value of α the value function $V(a,h,\alpha)$ and the associated functions $\hat{V}(a,h)$ and $\bar{V}(a,h,\alpha)$ are all homogenous of degree $1-\gamma$ on (a,h). The homogeneity follows from the assumptions of homogeneity of $U(\cdot)$, from the specification of the fixed cost of adjustment as proportional to the value of the current state ($\phi_o a$ and $\phi_T h$), and from the linearity of the budget constraint.

Let $\hat{H}(a + h(1-\phi_T), a', \alpha, \tau)$ denote the objective function to be maximized on the right hand side of the Bellman equation for $\hat{V}$ in (5), given the wealth $a + h(1-\phi_T)$ after paying the trade cost. Notice that for fixed values of α and τ the function $\hat{H}(\cdot,\cdot,\alpha,\tau)$ is homogenous of degree $1-\gamma$. Then we can consider the maximization:

$$\{1-\hat{\theta},\hat{\alpha},\hat{\tau}\} = \arg\max_{\theta,\alpha,\tau} \hat{H}\left(1,1-\theta,\alpha,\tau\right) \quad \text{subject to } 0 \le \theta \le 1,\ \alpha,\tau \ge 0\ . \tag{6}$$

The homogeneity implies that the optimal decision rules for a generic state (a,h) if the agent trades and adjusts the durable stock are given by:

$$a' = (1-\hat{\theta})\left(a + h(1-\phi_T)\right)\ ,\quad h' = \hat{\theta}\ \left(a + h(1-\phi_T)\right),$$

and that the optimal choices of τ and α are independent of (a,h). For notation convenience we use $\mathbf{a} \equiv a/h = (1-\theta)/\theta$ to denote the normalized ratio of assets to durables. Let $\bar{H}(a,h,\alpha,\tau)$ be the objective function to be maximized on the right hand side of the Bellman equation for $\bar{V}$ in equation (4) given the state (a,h,α). Notice that for fixed (α,τ) the function $\bar{H}(\cdot,\cdot,\alpha,\tau)$ is homogenous of degree $1-\gamma$. Consider the problem:

$$\bar{\tau}\left(\mathbf{a},\alpha\right) = \arg\max_{\tau\ge 0} \bar{H}\left(\mathbf{a},1,\alpha,\tau\right)\ .$$

where $\bar{\tau}$ is a function of a/h because of the homogeneity of $\bar{H}$. Now consider an agent with $\alpha = \hat{\alpha}$ and durable stock h, who pays the observation cost and discovers her financial wealth (net of the observation cost) to be a. In this case, using $\mathbf{a} \equiv a/h$, the agent will trade and adjust if $\bar{V}\left(\mathbf{a},1,\hat{\alpha}\right) < \hat{V}\left(\mathbf{a},1\right)$, where we have used the homogeneity of $\hat{V}(\cdot)$ and $\bar{V}(\cdot,\hat{\alpha})$. Let

$$\bar{I} \subset \mathbb{R}_+ \equiv \left\{\mathbf{a} :\ \bar{V}\left(\mathbf{a},1,\hat{\alpha}\right) > \hat{V}\left(\mathbf{a},1\right)\right\}\ ,$$

then the optimal policy is of the form:

$$\mathbf{a}\ \in\ \bar{I} \implies \mathbf{a}' = \mathbf{a}\ ,\ \tau = \bar{\tau}(\mathbf{a},\hat{\alpha})\ \ ,\ \ \mathbf{a} \notin \bar{I} \implies \mathbf{a}' = \hat{\mathbf{a}} = \frac{1-\hat{\theta}}{\hat{\theta}}\ ,\quad \tau = \hat{\tau}\ .$$

In Online Appendix A-3.4 we prove that the inaction region $\bar{I}$ includes an interval that contains the target asset-to-durable ratio –this happens because in the interior of the interval $\bar{V}(a,1,\hat{\alpha}) > \hat{V}(a,1,\hat{\alpha})$. Indeed we will assume from now on, which we verified numerically in all examples, that the set $\bar{I}$ is given by an interval $[\underline{\mathbf{a}},\bar{\mathbf{a}}]$ for values of the normalized state variable $\mathbf{a}$ where it is optimal for the agent not to trade and not to adjust the stock of durables.[11]

[11] In general, $\bar{I}$ can be composed from the union of disjoint intervals. Proposition 3 in Alvarez, Lippi, and Paciello (2011), shows that, for a stylized version of this model, the inaction set is indeed an interval. The model in this paper does not satisfies all the assumptions used in Proposition 3: namely it lacks the required symmetry on the period return function and it has non-zero drift. See section Online Appendix A-3.4 for a

It is immediate that $\hat{\mathbf{a}} \in \bar{I}$, i.e. that the optimal return point $\hat{\mathbf{a}}$ is in the range of inaction. The size of the inaction interval depends on the fixed cost ϕ_T, among other determinants. Thus if at the time of observing the state $\mathbf{a}$ falls in the interval $[\underline{\mathbf{a}}, \bar{\mathbf{a}}]$ the agent will find it optimal not to pay the fixed cost, to leave h unaltered, and to set a new observation date $\tau(\mathbf{a})$ periods from now. Otherwise, if at the time of observing the state $\mathbf{a}$ falls outside the interval $[\underline{\mathbf{a}}, \bar{\mathbf{a}}]$, then the agent will pay the cost $\phi_T\ h$, adjust the stock of durables, and set the new ratio of financial assets to durables to $\hat{\mathbf{a}}$. She will also set a new observation date $\hat{\tau}$ periods from now. The analysis shows that along an optimal path there will be instances where the agent will pay the observation cost but will not trade, as in the data for Italian investors displayed in Table 1 and Table 2.

4 Quantitative Analysis

In this section we analyze some quantitative implications of the model by means of numerical solutions. First, we describe the decision rule, then we solve the model for different parameters values and develop some comparative statics to illustrate its workings. Finally, we use the numerical solution to relate the model to moments taken from the Italian investors' data.

4.1 Decision rules

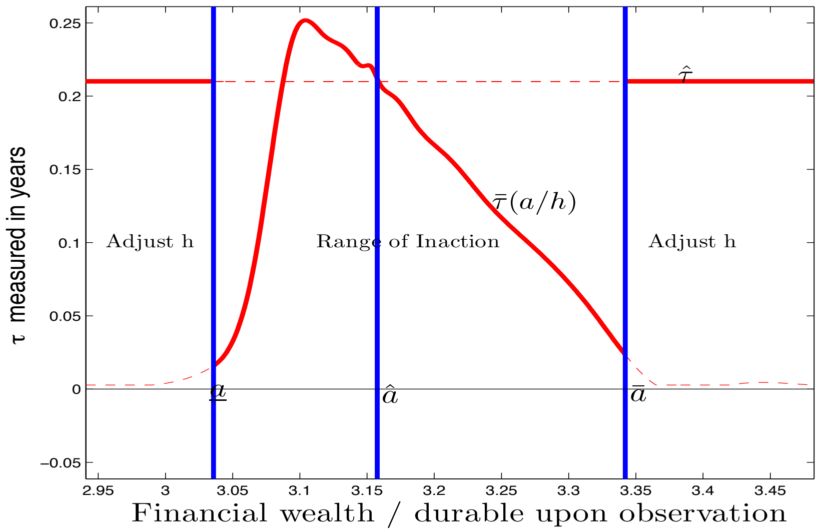
The horizontal axis of Figure 2 displays the values of the ratio of the financial assets to durable goods $\mathbf{a} \equiv a/h$, right after the agent has observed the value of financial assets, and before deciding whether to trade and to adjust the durable stock. As discussed above, the optimal decision rule about adjusting the durables is of the sS type. The two vertical bars at $\underline{\mathbf{a}}$ and $\bar{\mathbf{a}}$ denote the threshold values that delimit the inaction region. The vertical bar at $\hat{\mathbf{a}}$, inside the inaction region, denotes the optimal return point after an adjustment. For small values of the frictions the value of the optimal return point is very close to the one from the frictionless model.[12]

The optimal decision after an observation is made of two rules: the first rule is whether (and by how much) to adjust the durable stock, the second rule gives a date for the next observation. The adjustment decision, after observing the value of the assets, depends on the location of the state $\mathbf{a}$. The middle panel contains the range of inaction, where the agent chooses not to adjust her financial asset and durable stock. Outside of this region, the agent

thorough discussion of this issue.

[12]For the values used in Figure 2, comparing the ratio of durables to total wealth $\hat{\theta} \equiv h'/(a' + h')$ defined in equation (6), with the corresponding ratio in the frictionless model where $\phi_o = \phi_T = 0$, denoted by θ, shows that $\hat{\theta} = 0.238$, while $\theta = 0.237$.

Figure 2: Optimal Decision rule (normalizing $h = 1$)



Note: Benchmark parameter values are $\gamma = 4$, annualized: $\rho = 0.02$, $\mu = 0.06$, $\sigma = 0.16$, $r = 0.03$, $\delta = 0.10$, and $\phi_o = 0.01$ basis points , $\phi_T = 0.5$ basis points.

will pay the adjustment cost $\phi_T\ h$, trade and adjust her financial asset and stock of durables to the values (a', h') that satisfy $a'/h' = \hat{\mathbf{a}}$. The adjustments that occur to the left of $\underline{\mathbf{a}}$ involve a sale of part of the durable stock h, while the ones to the right of $\bar{\mathbf{a}}$ involve purchases that add to the durable stock h. The discontinuous solid line in Figure 2 displays the optimal time until the next observation. This rule is made of two functions. One is the optimal time until the next observation contingent on trading, which is given by the constant value $\hat{\tau}$. This is analogous to the optimal rule in Duffie and Sun (1990) and in Abel, Eberly, and Panageas (2007), where each observation is also a trade. The other function gives the optimal time until the next observation contingent on *not* trading. This function depends on the state $\mathbf{a}$, and is denoted by $\bar{\tau}(a/h)$ in the figure. Thus, the optimal decision rule (solid line) is given by $\bar{\tau}(\cdot)$ in the inaction region, and by $\hat{\tau}$ in the adjustment regions.

We notice several properties of the optimal time until the next observation:

1. the function $\bar{\tau}(\cdot)$ is hump shaped
2. the value of $\bar{\tau}(\hat{\mathbf{a}}) = \hat{\tau}$
3. the values of $\bar{\tau}(\underline{\mathbf{a}})$ and $\bar{\tau}(\bar{\mathbf{a}})$ are strictly positive
4. the function $\bar{\tau}(\cdot)$ reaches zero for values of a/h strictly inside the adjustment regions
5. the maximum of $\bar{\tau}(\cdot)$ is larger than $\hat{\tau}$ and it occurs for $\mathbf{a} < \hat{\mathbf{a}}$

The reason why point 1 holds, is that when the agent is inside the inaction region but close to the borders, she realizes that the state ($\mathbf{a}$) is likely to reach the adjustment region shortly, and hence it is optimal to revise the information soon. In the middle of the inaction region, instead, the expected time before reaching the adjustment region is greater, and thus the optimal time to the next revision is longer. The reason why point 2 holds is that if, upon observing, the value of a/h happens to coincide with the optimal return point $\hat{\mathbf{a}}$, then the objective function w.r.t. τ is the same as the one when adjustment is considered (i.e. $\hat{V}$), and thus the optimal time to the next review is $\hat{\tau}$. Point 3 holds because if τ was zero on the boundary, the agent would be paying the observation cost an arbitrarily large number of times in a short period of time. Point 4 holds because if a/h is large, and the agent is forced not to trade (that is the assumption underlying the definition of the function $\bar{\tau}$), she will choose to review immediately and trade. Finally, the reason why point 5 holds is that in our parametrization the ratio $a(t)/h(t)$ has a drift to the right (when it is not controlled by adjustments), approximately equal to the sum of the expected return on the financial asset plus the depreciation rate. Thus when a/h is close to $\hat{\mathbf{a}}$, but to its left, the agent forecasts to be in the inaction for a time longer than $\hat{\tau}$.[13]

4.2 Calibration exercises

Given the nature of the decision rule it is immediate to see that $a(t)/h(t)$ follows a stationary Markov process with a unique invariant measure. Table 5 shows how the number of observations and trades varies for different combinations of the observation and transaction costs ϕ_o and ϕ_T. In this exercise the parameters $\gamma, r, \mu, \sigma, \rho$ are set to values that are common in the literature, see e.g. Abel, Eberly, and Panageas (2009), to facilitate comparison of results. The depreciation parameter is set to ten percent annual ($\delta = 0.10$), so that the half life of the durable good is between 6 and 7 years, which seems a reasonable value for the kind of durable goods for which we have data (see Table 8). The comparative statics for some of these parameters is discussed below. The first two columns in the table report the values of ϕ_o and ϕ_T used in the computation of each row. The columns # *observ.* and # *trades* report, respectively, the expected number of observations and the expected number of trades per year, under the invariant measure. The fifth column reports the ratio between these frequencies and the sixth column gives the ratio between the transaction over the observation cost. Since these costs apply to different aggregates (a and h, respectively), the next column

[13] In Alvarez, Lippi, and Paciello (2011) we study a stylized version of this problem, for which we derive analytically the properties of the decision rules discussed here. In particular, the shape displayed in Figure 2 follows from the combination of two limit cases studied in that paper: the case with uncertainty and no-drift on the state, and the case of no-uncertainty and drift on the state.

Table 5: Observation and trade frequency (per year) as $\frac{\phi_T}{\phi_o}$ varies

ϕ_o (bp)	ϕ_T (bp)	# observ.	# trades	$\frac{\#trades}{\#observ.}$	$\frac{\phi_T}{\phi_o}$	$\hat{\mathbf{a}}$	$\frac{\phi_T}{\phi_o}\frac{h}{a}$
0.005	0.1	12.0	4.0	0.32	20	3.2	6
0.005	1.3	8.4	1.6	0.18	250	3.1	80
0.005	5.0	7.3	1.0	0.14	1,000	3.1	326
0.005	100.0	4.5	0.4	0.08	20,000	2.8	7,116
0.01	0.2	7.6	2.8	0.38	20	3.2	6
0.01	2.5	5.7	1.3	0.22	250	3.1	81
0.01	10.0	5.1	0.8	0.15	1,000	3.0	331
0.10	2.0	2.5	1.3	0.50	20	3.0	6
0.10	25.0	1.9	0.6	0.31	250	2.9	84
0.10	100.0	1.6	0.4	0.24	1,000	2.8	357
1.00	20	0.9	0.6	0.64	20	2.9	7
1.00	250	0.7	0.3	0.39	250	2.7	92
1.00	$1,000$	0.6	0.2	0.31	1,000	2.5	396

Notes: The variables ϕ_o and ϕ_T are measured in basis points (bp). The other parameters are $\gamma = 4, \delta = 0.10$, $\rho = 0.02$, $r = 0.03$, $\mu = 0.06$, $\sigma = 0.16$ per year. The variable $\hat{\mathbf{a}} \equiv \frac{1-\theta}{\theta}$ is the optimal return point, given by the ratio of the value of assets (net of observation cost) to that of durables (net of transaction cost).

reports the optimal return point $\hat{\mathbf{a}} = a/h$ which is used, in the last column, to compute the ratio between the transaction and observation costs properly scaled.

We use the numerical results of Table 5 to illustrate three quantitative properties of the model with respect to the costs ϕ_o and ϕ_T. First, the model generates substantial inaction with relatively small observation and transactions cost. For instance in the parameterization used in the third-to-last row of Table 5 the observation cost is $\phi_o = 1/10,000$ of the post-observation financial wealth (i.e. 1/100 of one basis point), and the trade cost is $\phi_T = 20/10,000$ of the pre-adjustment stock of durable. Yet, in spite of these small costs, the expected number of observations per year is 0.9 and the average number of trades is 0.6.

Second we notice that keeping all the other parameters fixed the ratio between the frequency of trades and the frequency of observations is a monotone decreasing function of the ratio of the two costs: ϕ_T/ϕ_o. This can be seen by inspecting the rows in each of the 4 panels separated by a horizontal line in the table. Three values of the ratio ϕ_T/ϕ_o (20, 250, and 1,000) appear in the lines of each panel. Note that as the ϕ_T/ϕ_o ratio increases, the ratio between the frequency of trades to the frequency of observation decreases. As intuitive, an increase in the relative cost of an adjustment reduces the number of adjustment per observation.[14]

Third, the workings of our model are quantitatively consistent with the findings of Stokey (2009). She builds a model with both durable and nondurable goods calibrated to interpret the durables as housing stock. She focuses on physical transaction costs (and no information gathering costs). A parametrization of our model according to her baseline parameters, which uses a housing transaction cost of 8 percentage points in line with the high transaction cost for housing estimated by Smith, Rosen, and Fallis (1988). This high transaction costs, and the small depreciation for housing ($\delta = 0.03$), produce a frequency of housing transactions of 0.089 trades per year, which accords well with the cross sectional average house tenure of 11.3 years that she reports in the paper.[15] The main difference of this parametrization compared to our setup is the smaller depreciation rate (3 vs 10 per cent) which is appropriate for housing vs. the more perishable durables which we have in our data set (see the next section) and, as a consequence, induces *less* frequent adjustment than we get. The bottom panel of Table 5 shows that when a comparably high transaction cost is fed to our parametrization (last line of the table) the frequency of durables trading is 0.20 per year, or approximately one trade every 5 years.

Next, we calibrate the model to quantify the observation and transaction costs using the data for the Italian investors. As a reference we take two set of data from Table 2 corresponding to *direct stockholders* from the UCS and another from *all investors* for SHIW. We think of the UCS stockholder as representative of a pool of more *sophisticated investors*, while the data in SHIW are representative of the *typical Italian investors*. The sophisticated investor trades about 4 times a year, and observes about 12 times a year. Instead the typical investor trades about 0.4 times a year and observes about 3.6 times a year.[16] The first and fourth rows in the upper panel of Table 5 report calibrations that match the behavior of these two type of investors. For an observation cost of $\phi_o = 0.005$ basis point and a transaction cost of $\phi_T = 0.1$ basis point the model predicts 14 observations and 4 trades per year. This parametrization is close to the behavior of the sophisticated investor, matching the levels and the ratio of the observation and trade frequency. This suggests that the level of both costs is small and that the transaction cost is about six times bigger than the observation costs, i.e. $(\phi_T/\phi_o) \times (h/a) \approx 6$. The calibration in the fourth line uses an observation cost of $\phi_o = 0.005$

[14]The result that the ratio of the number of trades to observations is a function of the ratio ϕ_T/ϕ_o *and* of other variables, such as the absolute size of the costs, differs from the analytical characterization in Alvarez, Lippi, and Paciello (2011), where the number of trades per observation depends *only* on ϕ_T/ϕ_o. The latter result is obtained as an approximation for the case of a quadratic period return function and no drift. Both of these assumptions are violated in the durable good model.

[15]The other parameters, taken from Table 1 in Stokey (2009), are $\gamma = 3.5$, $\mu = 0.07$, $\sigma = 0.1655$, $\delta = 0.031$, $r = 0.025$, to which we add an observation cost of 1 basis point.

[16]Recall that the observation data for the SHIW investors are noisy as they are imputed from a regression on UCS data, see the note to Table 2.

Table 6: Observations, trades and welfare loss in different models (per year)

ϕ_o (bp)	ϕ_T(bp)	# observ.	# trades	$\hat{\mathbf{a}}$	Consumption loss w.r.t. frictionless*
Model: No frictions					
0.00	0.00	∞	∞	3.22	0
Model: Observation cost only					
0.005	0.00	19.6	19.6	3.21	0.02 (%)
0.075	0.00	4.0	4.0	3.17	0.14 (%)
25.00	0.00	0.4	0.4	2.82	4.10 (%)
Model: Transaction cost only					
0.00	0.10	∞	4.1	3.19	0.05 (%)
0.00	100.00	∞	0.4	2.80	3.50 (%)
Model: Observation & transaction cost					
0.005	0.10	12.2	4.0	3.19	0.07 (%)
0.005	100.00	4.6	0.4	2.81	3.51 (%)

Notes: The variables ϕ_o and ϕ_T are measured in basis points (bp). The other parameters are $\gamma = 4, \delta = 0.10$, $\rho = 0.02$, $r = 0.03$, $\mu = 0.06$, $\sigma = 0.16$ per year. The variable $\hat{\mathbf{a}} \equiv \frac{1-\theta}{\theta}$ is the optimal return point, given by the ratio of the value of assets (net of observation cost) to that of durables (net of transaction cost). $-$*This loss gives the compensating variation in the annual flow of durable consumption (in %) that is needed to equate the welfare level to the one obtained in the frictionless model (both value functions are evaluated at the $\hat{\mathbf{a}}$ of the model with frictions).

basis point and a transaction cost of $\phi_T = 100$ basis points. This produces 4.5 observations and 0.4 trades a year, with roughly 0.1 trades per observation. These figures are close to those of the typical Italian investor. In this case, which by the nature of the SHIW survey is more representative of Italian investors', the ratio of the level of the transaction cost is much bigger, several orders of magnitude bigger than the level of the observation cost.

Altogether, the calibration shows that the magnitude of the observation cost ϕ_o that is necessary to match observed patterns of behavior is small. For instance, if we use one percent of one basis point for a financial wealth of a 130,000 euros (which is about the median for the UCS sample of direct equity holder), the observation cost is about 13 euro cents. The adjustment cost for trading durables are also small in the simulation that we associate with the sophisticated investors, and larger (around one percent) for the simulation that we associate with the typical Italian investor.

To quantify the welfare consequences of these costs at the household level Table 6 compares the outcomes of this model with the ones produced by three special cases: the frictionless benchmark, the observation-cost only and the transaction-cost only (see Section A-3 for analytical solutions of these special cases). The analysis shows that transaction costs in the order of one percent of the durable stock are not negligible for the investor. Notice that even though the cost is paid on average once every 2.5 years its impact on welfare, measured by the compensating variation in the annual consumption flow, is about 3.5 percent. For the median investors, to which the model was calibrated, the welfare effect of the observation cost is negligible, a consequence of the small size of the level of the observation cost. From Table 6 we conclude that, at the values of the transaction and observation costs that we focus on, the behavior of the model with both costs is very similar to one with the transaction cost only. We reach this conclusion by comparing the number of trades per year, as well as the post-trade ratio of financial assets to durables, for the two benchmark cases displayed in the two panels at the bottom of Table 6. In particular, a comparison of the "Transaction cost only" panel with the "Observation & transaction cost" panel shows that the number of trades per year is essentially unaltered by the presence of the small observation cost. Moreover the consumption loss w.r.t. the frictionless benchmark is very small, in the order of a few basis points of the annual consumption flow. Alternatively, these figures suggest that for the parameterizations considered in these panels observing between 4 to 12 times per year provides almost the same information as observing continuously.

To understand the effect in our estimates of having both frequency of trade and observation, the second and third row of the "Observation cost only" panel in Table 6 matches the model to our *sophisticated* and *typical* investors's measured trading frequency setting $\phi_T = 0$. The implied observation cost are $\phi_o = 0.075$ (bp) for *sophisticated* investors and $\phi_o = 25$ (bp) for the *typical* italian investor, which applied to the median financial wealth of each group give a per observation cost of about 1 euro for the sophisticated investor and about 62 euros for the typical italian investor. These cost are much larger than the ones obtained when we calibrate the model to both observation and trading frequency, especially so for the typical Italian investor.

We conclude the section with a remark on the asymmetry of the $\bar{\tau}(a/h)$ function discussed in point 5. Notice that if the process for $a(t)/h(t)$ has a strong drift, i.e. if the return on financial assets plus depreciation is large, then most adjustments will happen in the right adjustment region, and hence they will involve a liquidation of assets and a purchase of durables. The frequency with which $a(t)/h(t)$ hits the "sale" region relative to the "buy" region depends on the strength of the drift, which is approximately equal to $\alpha\mu + (1-\alpha)r + \delta$, relative to the variability of this ratio, which is about $(1-\theta)\alpha\sigma \approx (\mu - r)/(\gamma\sigma)$. In Table 7 we show how this frequency varies with δ and σ. As expected from the direct effect on the drift, larger values of δ lead to a larger fraction of adjustments being purchases. For larger value of σ, the net effect is to decrease the exposure to risk, so that the variability of $a(t)/h(t)$ actually *decreases*. Thus, for low σ, the variability of $a(t)/h(t)$ is high, and hence the process reach

Table 7: Durable trades in the model: Fraction of Purchases

	Depreciation Rate (δ)			Volatility (σ)		
	0.05	0.10	0.15	0.06	0.16	0.26
Fraction of Purchases	0.88	0.98	0.99	0.60	0.98	1.00

Benchmark parameters: $\gamma = 4,\ \mu = 0.06,\ \rho = 0.02,\ r = 0.03,\ \phi_o = 0.01$ (bp), $\phi_T = 0.5$ (bp).

both thresholds more often, which explains why the fraction of sales is smaller for $\sigma = 0.06$ than for $\sigma = 0.16$. In our benchmark numerical example 98% of adjustments are purchases of durables (see Table 7). This comes close to the comparable figure for Italian investors that is 95% (see Table 8).

5 Some evidence on the model predictions

This section contrasts two predictions of the durable goods model with data from the UCS and the SHIW survey. The first prediction relates to the frequency of durable purchases and that of assets transactions. The second pertains to the relation between the investor risk aversion and the frequency of observations.

5.1 The correlation between trades in assets and in durables

The first "test" we consider in this section is specific to our durable goods model. By abstracting from the non-durable purchases, agents in our model trade only to adjust their purchase of durables, hence we expect that empirically investors who are more active in purchasing durables are also more active in assets transactions, as well as that they concentrate their trading around the times where they adjust durables.

We test this prediction using information in the SHIW 2004 survey on the frequency of durable purchases and the frequency of asset transactions across Italian households. In particular, the survey registers whether the household bought or sold durable goods in each of three categories: housing, vehicles and jewelry in the year prior to the interview. It also records whether it has bought house appliances (furniture, electrical apparel, etc). Thus, even though we do not have the total number of purchases or sales, we have information on whether the household engaged in trading durables in any of these categories. The upper panel of Table 8 reports the fraction of investor/households who were active in each of these 4 categories. For instance, the fraction of investors who bought a durable in the "Cars & other transportation" category in 2004 is 15%. As a first check of the plausibility of the

Table 8: Fraction of Investors who adjusted the durables stock in 2004

	Jewelry & Antiques	Cars & other	Furniture & appliances	All	Housing[a]
All investors (2,808 obs.)					
Fraction adjusting	0.09	0.15	0.37	0.47	
% purchases[b]	97	83	100	95	
By Investor type					
Portfolio adjust.					
< 1 per year (824 obs.)	0.07	0.13	0.34	0.42	0.04
≥ 1 per year (1,984 obs.)	0.12	0.20	0.45	0.57	0.06

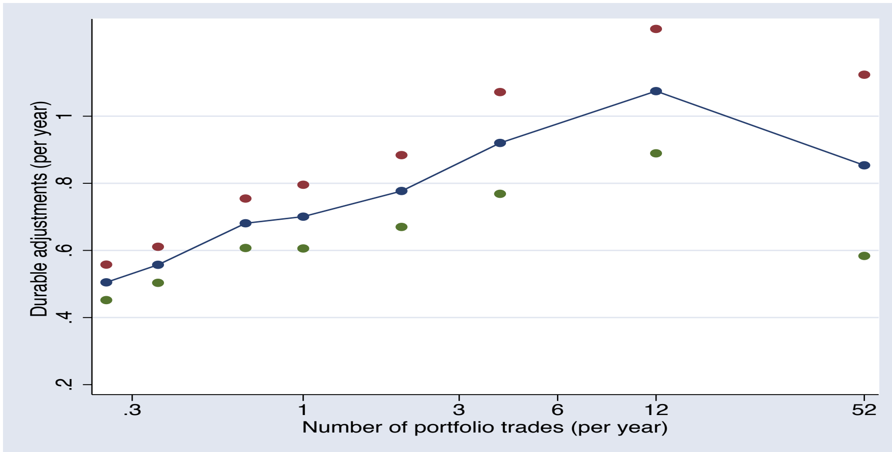
Source: SHIW - Bank of Italy. Notes: Statistics computed for 2,808 households with financial assets other than bank or postal account. For each category of durables, an adjustment means that the household records at least one purchase or one sale in 2004. –[a]Measures the proportion of household who bought or sold a house over a 5-year window. –[b]Percentage of trades that are purchases (by construction it is 100 for the Furniture & Appliances category because sales are not recorded).

model prediction that asset trades and durable trades are correlated, the lower panel of the table shows that the fraction of investors who purchased durable goods in 2004 is higher among those who traded financial assets more often. The same pattern is found by running logit regressions for the purchase of durables, on trade frequencies dummies, non-durables consumption, and demographics. Notice that if we condition on the sample of investors who traded more often (i.e. at least once in a year) the proportion of those who bought a car rises from 15 to 20%. Notice that this same pattern is found in each of the various categories for which we have information on the durable purchases. This pattern is consistent with the model outlined in Section 3.1 if investors differ in the costs ϕ_T and ϕ_o.

Based on the survey data, we also construct a "durable trade frequency" proxy as the sum of the 0/1 dummies for the entries "Jewelry & Antiques", "Cars.", "Furniture & appliances" (we exclude housing because there are few observations available in a given year). The proxy variable ranges from 0 (no purchases or sales across categories) to 5 (at least one purchase and one sale in each of the first two categories, and one purchase in the Furniture category). Figure 3 shows that the asset trade frequency and this proxy for durable purchase frequency are strongly correlated. Table 9 shows that the bivariate correlation remains strong and statistically significant in a multivariate regression analysis that includes household income and demographics. The correlation is also visible if the sample is restricted to equity investors (those for whom, according to the model, the information problem is likely to be more relevant).

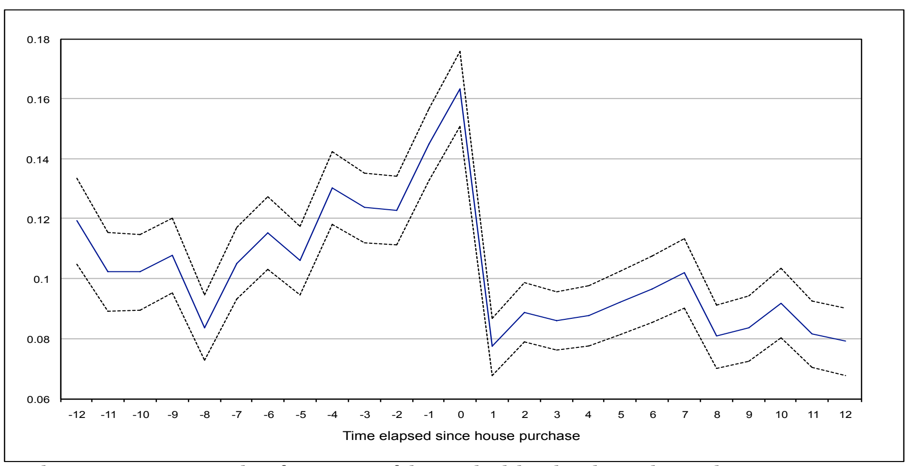
Finally, we gathered evidence on the time series pattern portfolio trades and house pur-

Figure 3: Durable vs. Portfolio trades in SHIW 2004



Note: Portfolio trades is the number of asset transactions in 2004 (categorical); Durable trades is a proxy for the number of durable transactions in 2004 (see the text). The line plots the mean of the durable trades indicator corresponding to each portfolio-trade-frequency bin. The dots denote 2 standard error bands around the mean.

Figure 4: Frequency of financial asset sales around the time of a house purchase



Note: The vertical axis measures the fraction of household who liquidate their investment t months after (before when negative) the house purchase. The data come from Unicredit administrative records, see the Online Appendix A-5.3. The dotted lines denote 1 standard error bands around the mean. Source: large sample, monthly administrative records (35 months) of 26 accounts for 40,000 investors.

chases by relying on the Unicredit administrative data. A detailed description of the data and further comments are available in the Online Appendix A-5.3. In sum, we identify a subset of investors who, over the 36 months for which we observe their assets with Unicredit, obtained a mortgage and the month they got it. Typically, the purchase of the house is settled as soon as the mortgage is obtained. We then compute the fraction of investors who liquidate investments and the value of the liquidations on the same month of the house purchase and on the previous and subsequent months. Figure 4 shows that the frequency of liquidation starts increasing in the the 3-4 months before purchase, jumps up in the month of the house purchase, and drops substantially after the purchase. Moreover, the average value of assets liquidation is around 48,000 euros in the month of purchase but drops to 14,000 in the subsequent month. The online appendix shows that both findings are statistically significant, and robust to controls in probit and tobit regressions.

Table 9: Durable vs. portfolio trade frequency

Dependent variable: (log) durable trade freq. ; Regressor: (log) portfolio trade frequency				
	All investors (2,808 obs.)		*Equity investors (1,535 obs.)*	
	bivariate[a]	Multivariate[b]	bivariate[a]	Multivariate[b]
Trade freq. (log)	0.27	0.16	0.16	0.09
	(0.03)	(0.03)	(0.04)	(0.04)

Note: Based on the 2004 SHIW survey. All regressions include a constant; standard errors in parenthesis. The durable adjustment frequency is an estimate of the average number of durable purchases per year (see the text for a detailed definition). −[a]Regression coefficient of bivariate OLS. −[b]This regression includes the following controls (all in logs): household income, age of household head, number of adults.

5.2 Observation frequency and portfolio riskiness vs. risk aversion

An increase in the degree of relative risk aversion has two effects in the model: first, it induces the investor to hold a safer portfolio; second, it increases the value of consumption smoothing. The first effect, whose strength depends on the attractiveness of the risky asset, as measured by its Sharpe ratio $(\mu - r)/\sigma$, lowers the value of information and implies that a more risk-averse investor chooses to observe her investments less frequently. This effect is akin to the one first studied by Verrecchia (1982) (see Corollary 1) and Peress (2004) (Theorem 2) who show that agents with a higher risk aversion have weaker incentives to obtain precise signals (i.e. information) about the return of the risky asset.[17] In our context this channel relates to the frequency one gathers information about the value of the investments rather than to the quality of the signal received. The second effect raises the value of information and thus, through this channel, more risk averse investors should observe more frequently.

[17]Verrecchia assumes the utility function displays constant absolute risk aversion in wealth. Peress shows that the same result obtains if absolute risk aversion is decreasing in wealth, so that wealthier individuals invest more into risky assets and hence have greater incentive to acquire information.

Which of the two effects prevails in our model depends on parameters values. For large enough, yet realistic values of the Sharpe ratio $(\mu - r)/\sigma$, and small values of the cost ϕ_T, ϕ_o, the portfolio effect dominates the consumption-smoothing effect over a reasonable range of values of the degree of risk aversion. Hence, the frequency of observing ones' investments is lower for more risk averse investors. Table 10 shows that the share invested in the risky asset and the frequency of portfolio observations both decrease as the degree of relative risk aversion increases, for a parametrization of the costs that was shown to produce a reasonable match of the behavior of the investor from the UCS survey (see Section 4.2).

Table 10: Effect of risk aversion (γ) on trade and observation frequency

γ	Share of risky asset α	$\hat{\mathbf{a}}$	# observ.	# trades	$\frac{\#\ trades}{\#\ observ.}$
8	0.15	3.20	6.3	4.1	0.65
7	0.17	3.20	6.4	3.9	0.62
6	0.20	3.18	6.8	3.8	0.56
5	0.23	3.18	6.8	3.6	0.52
4	0.29	3.18	7.7	3.6	0.46
3	0.39	3.22	8.4	3.6	0.43
2	0.58	3.39	10.3	4.1	0.43

Notes: The other model parameters are $\rho = 0.02$, $r = 0.03$, $\mu = 0.06$, $\sigma = 0.16$, $\delta = 0.10$ (all per year), $\phi_o = 1/100$ (bp), $\phi_T = 1/10$ (bp).

We can test this prediction of the model because the UCS survey has an indicator of risk aversion patterned after the Survey of Consumer Finance. Investors are asked: "Which of the following statements comes closest to the amount of financial risk that you are willing to take when you make your financial investment: (1) a very high return, with a very high risk of loosing the money; (2) high return and high risk; (3) moderate return and moderate risk; (4) low return and no risk". Only 19% choose "low return and no risk", so most are willing to accept some risk if compensated by a higher return. A recent literature on eliciting preferences from survey data shows that direct questions on risk aversion are informative and have predictive power.[18] Consistent with this literature, the Unicredit data shows that the portfolio share invested in risky assets (direct and indirect stocks) is monotonically decreasing with our index of risk aversion (see the working paper version); the correlation and its significance are confirmed in (unreported) regressions that control also for measures of investors assets, income and demographic characteristics, reassuring us about the reliability of our risk aversion indicator.

[18]See, among others, Barsky et al. (1997) and Guiso and Paiella (2008).

To analyze the model implications Table 11 shows regressions for different groups of the (log) number of times an investor observes his investments and the risk aversion indicator while controlling for endowment (log consumption) and demographic characteristics. We use three dummies for risk aversion, excluding the group with the highest risk aversion. Irrespective of which sample we use (the whole sample or that of the stockholders, total or direct) we find that more risk tolerant individuals observe their investments more frequently than less risk tolerant ones, consistent with the prediction of the model.

We notice that the predicted negative relation between risk aversion and the frequency of observations is a property of the model of assets trades and durable goods with attention costs but it is not specific to it. In fact the same prediction obtains in models with non-durable goods, assets trades and attention costs and can thus be viewed as a general test of models of assets trades with attention costs.

Table 11: Risk aversion and portfolio observations in the data

Dependent Variable : log of Number of Observations per year				
	All Investors (1456 obs.)		*Direct + Indirect* Stockholders *(944 obs.)*	
Risk Aversion Dummies:				
Very Low γ	1.36***	-	1.14***	-
Low γ	0.66***	-	0.51***	-
Medium γ	0.50***	-	0.35*	-
Share of Risky Assets α		1.67***		0.58**

Three, two or one asterisks denote that the null hypothesis of a zero coefficient is rejected by a t-test with a 1, 5 or 10 per cent confidence level. Source: UCS 2003 survey. All regressions include controls (demographics, consumption, etc). Risk aversion is measured by the answer to a question on risk and expected returns sought. High risk aversion is the excluded category.

6 Concluding remarks

This paper provides a quantitative analysis of the rational inattention hypothesis by studying how investors manage their financial assets, liquidity and consumption under the assumption that they face a cost to observe the value of their assets. First, we present direct empirical evidence from a cross-section of individual investors that is consistent with key features of costly observation models: investors collect information about the value of their investments and trade in assets only infrequently. To the best of our knowledge, this is the first direct evidence that is brought to bear on the issue of infrequent portfolio observations/trades and costly observation of the relevant state.

The second contribution is to modify one single feature of existing rational inattention models of asset management in a way that allows the theory to get closer to matching the data. The modification consists in shifting the focus from non-durable to durable consumption choices. As discussed in the introduction, most models based on non-durable consumption yield two counterfactual predictions: an equal frequency of observing and trading, and a negative link between the frequency of trading and the investor's liquidity. Our model of durable goods adjustment and asset management reconciles the theory with the data, as the model predicts that the frequency of observing must be greater than the frequency of asset trading, and is consistent with the empirical absence of correlation between liquid assets holdings and assets trading frequency.

Two predictions of the durable-goods model are supported by the data. First, a positive correlation is detected between trades in assets and in durables. Second, part of the heterogeneity in the frequency with which investors observe their portfolio can be explained by heterogeneous risk attitudes: because more risk tolerant individuals invest more in volatile assets they value information more and thus gather information more frequently.

A quantitative assessment of the consequences of observation and transaction costs is developed using numerical simulations of the model to match the number of observations and trades of the typical Italian investor (about 4 and 0.4 per year, respectively) for a financial wealth of 25,000 euros. The analysis shows that a model with durable goods and no transaction costs implies that to reproduce the low trading frequency, observed in the data for the typical Italian investor, the observation cost needs to be in the order of 60 euros per observation. Considering the narrow notion of information gathering used by our paper, namely observing the value of one's financial wealth, these costs seem unrealistically high. The model with both costs can reproduce the observed frequency of portfolio observations and asset trading with small observation costs (about 1 euro cents per observation) and transaction costs of about 1% of the value of durables for the typical Italian investor. For wealthier direct and stockholder investors, who trade and observe more often, the observation cost is about 7 cents while the transaction costs is smaller, about 0.1% of the value of durables. Even though these small observation costs help explaining infrequent observations, the patterns of consumer choices and frequency of trades that are produced is very close to the one obtained when the observation cost is absent and consumers only face trading costs. Based on this finding we conclude that assets observation costs, and the inattention they induce, have small impact on the *individual* investors' decisions. Trading costs of the classical nature emphasized in the literature carry instead much larger losses to investors. One open question is whether the staggering of decisions that the small observation cost implies at the individual level can aggregate to imply a large effect for the whole economy, as argued in related contexts at least since Taylor (1979) and Mankiw (1985).

The small value of observation costs that we end up estimating is not inherent to the model we propose, but follows by matching the model with the investors' data. In Alvarez, Lippi, and Paciello (2011) we study the price setting problem of a firm facing both an observation cost and an adjustment (menu) cost. While the context is different the nature of the problem is similar and the firm's frequency of observation and adjustment (i.e. price changes) depend, as in this paper, on the relative costs of each of these actions. Using data on price-reviews and price-changes from a sample of European firms we find that large observation costs and small menu costs are more appropriate to account for the price-review and price-adjustment frequencies, opposite to the investors' data. This is due to a smaller observation frequency (about 2 times per year) and a smaller gap between the frequency of price-review and that of price-adjustment (the ratio of adjustments to observations is about 1/2). We see these findings as reasonable: the observation cost for the investor involves a rather simple task, namely checking the value of her portfolio. Instead, the observation cost for firms is plausibly larger, as it involves finding out the value of marginal cost or the demand curve. Also the adjustment cost for firms, the menu cost, is likely small compared with the typically large costs involved in the buying (and selling) of durables.

The analysis in this paper assumes that all observations are costly. In the context of the portfolio and savings model, where several financial shocks are likely correlated across agents, households might be able to learn the realization of some shocks without paying the observation cost, a possibility that is not allowed for in our model. In principle, the model might be extended to allow for the random arrival of some free information, along the lines of Alvarez and Lippi (2009). Two remarks are in order. First while the risky portfolio in the model is a single asset, in reality households portfolios are far from being perfectly correlated, so that the "observation" activity may be more involved than the mere reading the heading of the news. Second, and more importantly, when we confront the model with simple statistics from the data we find that the implied observation cost is very small: the model without cost is almost equivalent to that with the calibrated cost. Hence, even without having agents learning some feature of the value of assets exogenously and without cost we find that the data, as interpreted by our model, points to already extremely small observation cost.

We think that several variations and extensions of the model are worth exploring next. In particular, our model assumes that assets are the only income source for the investor and that all consumption is in the form of durable goods. While our empirical analysis concentrated on a sample of investors, assets are not the only income source for these households.[19] Adding

[19]That is why we also included data on the share of labor income as control in the empirical analysis.

non-durable consumption should be interesting too, partly because it accounts for a large part of total expenditures, and partly because it is complementary to the use of liquid assets. We think that fully incorporating labor income and non-durable consumption is interesting to broaden the applicability of the analysis, but it involves several challenges. Some are conceptual, such as the modeling the observation of one's labor income process, others are technical, such as the proliferation of states in the problem, and finally other challenges are empirical, such as locating relevant data sets for the measurement of observation frequencies and the action that it triggers related to labor income and non-durable consumption.[20]

On the theoretical side, we also find it interesting to study how the decisions rules of models that combine both state dependent and time dependent type of adjustments aggregate and how these type of rules affect the response to aggregate shocks. We leave these tasks for future research.

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[20] See Sims (2003), Reis (2006) for a model of information gathering about the properties of labor income and the adjustment of non-durable consumption.

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Online Appendices

Durable consumption and asset management with transactions and observation costs

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A-1 Data sources and variables' definitions

A-1.1 The Survey of Household Income and Wealth (SHIW)

The Bank of Italy Survey of Household Income and Wealth (SHIW) collects detailed data on demographics, households' consumption, income, transaction habits and money holdings and household financial and real assets. It started to be run in the mid 1960s but is available on tape only since 1984. Since 1989 it has been conducted biannually and sampling methodology, sample size and broad contents of the information collected is unchanged. It is one of the few household surveys that collects consumption information, separately for non durable expenditures and purchases of durable categories, together with wealth and income data. In this study we rely on the 2004 wave which matches closely the 2003 UCS. Each survey covers about 8,000 households, constituting a representative sample of the Italian resident population. Sampling is in two stages, first municipalities and then households. Households are randomly selected from registry office records. Households are defined as groups of individuals related by blood, marriage or adoption and sharing the same dwelling. The head of the household is conventionally identified with the husband, if present. Brandolini and Cannari (1994) present a detailed discussion of sample design, attrition, response rates and other measurement issues, and comparisons of the SHIW variables with the corresponding aggregates. The 2004 wave interviewed 8,012 households with a response rate of 36%. The SHIW is publicly accessible; an English version of the questionnaire is available and data can be downloaded at www.bancaditalia.it/statistiche/indcamp/bilfait.

A-1.2 The Unicredit Survey (UCS)

The Unicredit Investors Survey (UCS) draws on the population of clients of one of the three largest European banking groups, with over 4 million accounts in Italy. Two waves of the survey, which is proprietary, are currently available, the first was run in 2003, the second in 2007. The first wave interviewed 1,834 individuals (1,716 the second) with a checking account in one of the banks that are part of the Unicredit Group based in Italy. The sample is representative of the eligible population of customers, excluding customers less than 20 years old or older than 80, and those who hold accounts of less than 1,000 euro (less than 10,000 euros in the 2007 wave) or more than 2.5 million euro.

UCS goal is to study retail customers' financial behavior and expectations. The survey has detailed information on households' demographic structure, individuals financial assets holding (both within and outside the bank), real wealth components and income. It has data on attitudes towards saving and financial investment, propensity to take financial risk, retirement saving and life insurance as well as data relevant for financial decision taking such as financial information activity, financial literacy, trading experience and practice, assets knowledge and confidence in markets. Interviews for the first wave have been administered between September 2003 and January 2004 (in the first half of 2007 for the second wave) by an Italian leading poll agency, which also serves the Bank of Italy for the Survey on Household Income and Wealth (SHIW). Most interviewers had substantial experience in administering the Bank of Italy SHIW (see below). The Computer Assisted Personal Interview (CAPI) methodology was employed for all interviews. Before the interview, each customer was contacted by phone.

The sampling design is similar to that of the Bank of Italy SHIW. As in SHIW, the population of account holders is stratified along geographical area of residence (North-East, North-West, Central and Southern Italy), city size (less that 30,000 inhabitants and more), and wealth held with Unicredit (as of December 31, 2003). The use of the same company to run the field and the similar sample design facilitates the comparison between UCS and SHIW. The questionnaire was designed with the help of field experts and academic researchers. It has several sections, dealing with household demographic structure, occupation, propensity to save, to invest and to risk, financial information and literacy, individual and household financial portfolio and investment strategies, real estate, entrepreneurial activities, income and expectations, life insurance and retirement income. The wealth questions match those in the Bank of Italy SHIW, which allows interesting comparison between the wealth distributions in the two surveys.

An important feature of the UCS is that sample selection is based on individual clients of Unicredit. The survey, however, contains detailed information also on the household head – defined as the person responsible for the financial matters of the family – and spouse, if present. Financial variables are elicited for both respondents and household.

A-1.3 UCS assets data and wealth definition

UCS contains detailed information on ownership of real and financial assets, and amounts invested. Real assets refer to the household. Financial assets refer to both the account holder and the household. For real assets, UCS reports separate data on primary residence, investment real estate, land, business wealth, and debt (mortgage and other debt). Real asset amounts are elicited without use of bracketing.

Two definitions of financial wealth are available. One refers to the individual account holder, and the other to the entire household. The two can differ because some customers keep financial wealth also in different banks or financial institutions (multi-banking) and/or because different household members have different accounts. In this study we only rely on household level variables.

Calculation of financial assets amounts requires some imputation. First of all, respondents report ownership of financial assets grouped in 10 categories. Respondents are then asked to report financial assets amounts; otherwise, they are asked to report amounts in 16 predetermined brackets and if the stated amount is closer to the upper or lower interval within each bracket. In the 2003 wave the questions are the same used in the Bank of Italy SHIW.

A-1.4 The distribution of financial wealth in the UCS and SHIW survey

Since the UCS sample only includes banked individuals and there is an asset threshold for participation in the survey, it is no surprise that investors in the two samples are different particularly in terms of average assets holdings. While in the UCS sample people have on average euros 212,000 (median 56,000) in financial assets, the average in SHIW 2004 is only 23,400 (median 7,000). To make a more meaningful comparison between the two dataset we select SHIW and UCS households so as to include only those with positive investments other than in transaction accounts but exclude those with financial assets greater than 1 million euros, to try account for the oversampling of the wealthy in UCS.

Figure A-5: Household distribution of wealth in SHIW and UCS

SHIW 2004 | Unicredit 2003

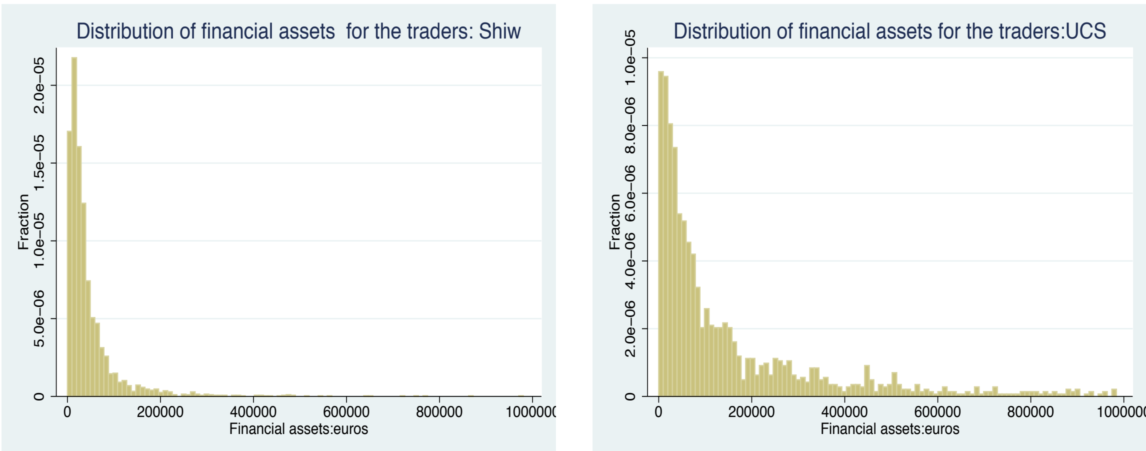


Figure A-5 shows the distribution of financial wealth in the 2003 UCS and the 2004 SHIW; once the comparison is limited to the sample of investors the shapes of the distribution become more similar, though large differences between the two dataset remain mainly because UCS is designed to oversample the financially wealthy, as can be seen from the thicker probability mass in the UCS distribution at higher levels of wealth.

A-1.5 Definition of variables constructed from survey responses

Attention when buying durables . Response to the question in UCS 2007: "Before making a purchase involving a relatively large amount of money (such as a car, a washing machine or furniture), some people tend to visit several shops or dealers in order to compare various prices and try to get a good balance in terms of price/quality ratio. How does this description fits your type?" Possible answers are: "Not at all", "Very little, "Somewhat.", "Close enough", "Very much". We code these answers with numbers between 1 and 5.

Cash holdings. Only available in SHIW which asks people: "What sum of money do you usually have in the house to meet normal household needs?"

Number of withdrawals. Only available in SHIW which asks people: "In 2004 how many cash withdrawals did you or other members of your household make directly at a bank or Post Office on average per month?"

Durable consumption purchases. Only available in SHIW which collects information on purchases of three categories of durables: precious objects, means of transport, and furniture, furnishings, household appliances and sundry articles, by asking: "During 2004 did you (or your household) buy ... (item...)? If "Yes, what is the total value of the objects bought (even if they were not paid for completely)?" Thus, SHIW collects separately whether a purchase took place and its value.

Financial diversification. The ratio of stocks held in mutual funds and other investment accounts to total stocks (direct plus indirect). The index ranges from 0 to 1.

Liquid assets. We use two measures of liquid assets, a narrow one corresponding roughly to M1, and a broader one similar to M2. The first measure is defined as the sum of average cash holdings and checking accounts; the broader measure adds to this savings accounts. Accounts are figures at the household level and figures are variables are defined in the same way in UCS and SHIW. Since UCS has no information on average cash holdings, we impute it from SHIW 2004. Imputation is done by first running a regression on the SHIW sample (retaining only those with a checking account) of average cash holdings (scaled by house value) on a number of variable observed also in UCS: a set of demographics, a fifth order polynomial in income (scaled by house value), a forth order polynomial in age and interactions between demographics and house value and interactions between income and the polynomial in age and income and demographics. The regressions explains 64% of the cross sectional variability. The estimated coefficients are retrieved and used to predict cash holdings in UCS 2003.

Non durable consumption. From the SHIW question: "What was the monthly average spending of your household in 2004 on all consumer goods, in cash, by means of credit cards, cheques, ATM cards, etc?. Consider all spending, on both food and non-food consumption, and exclude only: purchases of precious objects, purchases of cars, purchases of household appliances and furniture, maintenance payments, extraordinary maintenance of your dwelling, rent for the dwelling, mortgage payments, life insurance premiums, contributions to private pension funds". Answers are multiplied by 12 to obtain an annual figure. Since non durable consumption is not available in UCS, it is imputed using SHIW information. This is done by first running a regression on the SHIW sample (retaining only those with a checking account) of non durable consumption (scaled by house value) on a number of variable observed also in UCS: a set of demographics, a fifth order polynomial in income (scaled by house value), a forth order polynomial in age, interactions between demographics and house value and interactions between income and the polynomial in age and income and demographics. The regression explains 91% of the cross sectional variability. The estimated coefficients are retrieved and used to predict cash holdings in UCS 2003.

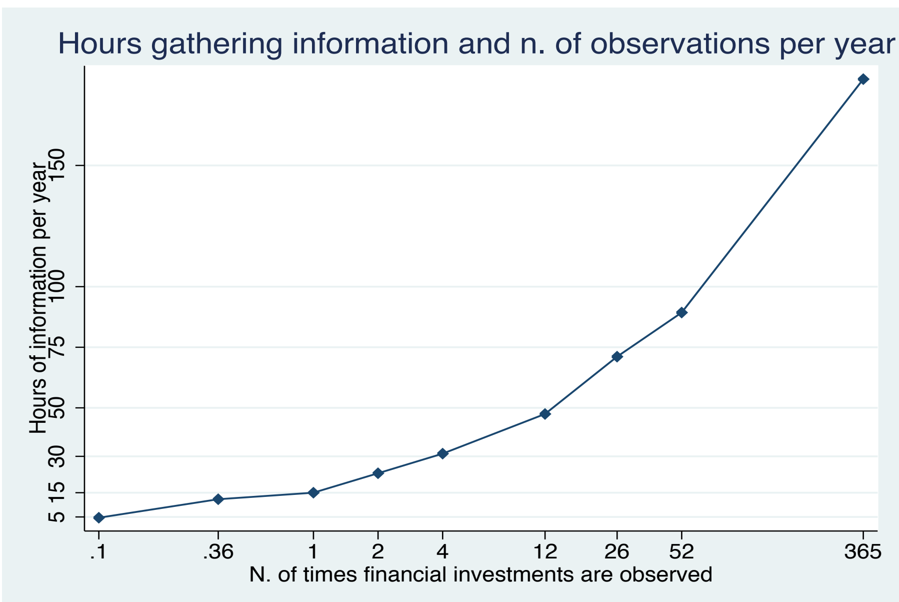
Observing frequency. Response to the question: "How frequently do you check the value of your financial investment?" Coded as: every day; at least once a week; about every two weeks; about every month; about every three months; about every six months; about every year; less than once a year; never. The number of times (frequency) an investor observes her investments is computed from the responses as: every day=365; at least once a week=52; about every two weeks=26; about every month=12; about every three months=4; about every six months=2; about every year=1; less than once a year=0.36; never=0.10. To impute the last category we approximate "never" as meaning once every 10 years.

Stockholders. We construct two identifiers of stockholders: direct and total. Direct stockholders are investors owing stocks of single companies, either listed or unlisted. Total stockholders own stocks either directly or through a mutual fund or a managed investment account.

Risk aversion. Response to the question in UCS: "Which of the following statements comes closest to the amount of financial risk that you are willing to take when you make your financial investment?: (1) a very high return, with a very high risk of loosing the money; (2) high return and high risk; (3) moderate return and moderate risk; (4) low return and no risk."

Time spent in collecting financial information. Response to question: "How much time do you usually spend, in a week, to acquire information on how to invest your savings? (think about time reading newspapers, internet, talk to your financial advisor, etc.)". Coded as: no time; less than 30 minutes; between 30 minutes and 1 hour; 1-2 hours; 2-4 hours; 4-7 hours; more than 7 hours. The hours per year indicator is constructed by coding: no time=0; less than 30 minutes=$0.25 \times 4 \times 12 = 12$ (assuming 15 minutes a week); between 30 minutes and 1 hour=$(45/60) \times 4 \times 12 = 36$ (assuming 45 minutes a week; 1-2 hours = $(90/60) \times 4 \times 12 = 72$ (assuming 90 minutes a week); 2-4 hours=$(180/60) \times 4 \times 12 = 144$ (assuming 3 hours a weak); 4-7 hours=$(330/60) \times 4 \times 12 = 264$ (assuming 5 hours and 30 minuets a weak); more than 7 hours $(450/60) \times 4 \times 12 = 360$ (assuming 7 hours and 30 minutes a weak).

Figure A-6: Time spent gathering financial information



Source: UCS 2003 survey.

Trading. Response to question: "How often do you trade financial assets (sell or buy financial assets)?" Coded as: every day; at least once a week; about every two weeks; about every month; about every three months; about every six months; about every year; less than once a year; at maturity; never. The number of times (frequency) an investor trades her investments is computed from the responses as: every day=365; at least once a week=52; about every two weeks=26; about every month=12; about every three months=4; about every six months=2; about every year=1; less than once a year=0.36; at maturity =0.25; never=0.10. To impute the last two categories we assume that average maturity is 4 years and approximate "never" to mean once every 10 years.

A-2 Inattention, Liquidity and Non-Durable Goods

This section reviews a class of models that use the rational inattention hypothesis to study consumption, savings, portfolio theory and liquidity. In these models the relevant decisions concern the rate of consumption - or savings - and the portfolio composition; the costs are those associated with keeping track of the information about financial variables. Examples of these models are Duffie and Sun (1990), Gabaix and Laibson (2001), Sims (2005), Reis (2006), Abel, Eberly, and Panageas (2007, 2009) among others.

We describe a version of the rational inattention model of Abel, Eberly, and Panageas (2009). Households maximize discounted expected utility derived from the consumption of non-durables c and period utility $u(c) = c^{1-\gamma}/(1-\gamma)$, with CRRA coefficient γ, and discount rate ρ. The purchases of non-durable goods are subject to a cash-in-advance constraint (CIA): agents must pay with resources drawn form a liquid asset account, with real value denoted by m. Non-negative liquid assets represent a broad monetary aggregate, such as M2, and have a low real return r_L. The agent's source for the liquid asset is her financial wealth a, a fraction α of which is invested in risky assets and the remaining in risk-less bonds. The risk-less bond yields $r > r_L$, and the risky asset has continuously compounded normally distributed return, with instantaneous mean μ and variance σ^2 (the portfolio is assumed to be managed so that it stays continuously rebalanced with fraction α of risky asset). There is a fixed cost ϕ_o of observing the value of the agent's financial wealth and a fixed cost ϕ_T of changing α and transferring resources between the investment and the liquid assets accounts. From now on we call *observation* the agent's act of observing the value of the her financial wealth, and *trade* the agent's act of adjusting the portfolio. We assume that the costs are fixed, in the sense that they are incurred regardless of the size of the adjustment, but they are proportional to the current value of the stock variable a. This specification, which is standard in the literature - see for example Grossman and Laroque (1990) and Abel, Eberly, and Panageas (2007) - is adopted for convenience, since it preserves the homogeneity of the value function, and hence reduces the dimensionality of the problem. We remark that the models of Duffie and Sun (1990), Gabaix and Laibson (2001), Abel, Eberly, and Panageas (2007) are obtained as a special case of this model by setting the trade cost $\phi_T = 0$.

We denote by $V(a, m)$ the value of the problem for an agent with current liquid assets m, who has just paid the observation cost discovering that her financial wealth is a. She must decide whether to pay the cost ϕ_T, choose α and transfer some resources to/from the liquid account, as well as τ, the length of the time period until the new observation of her financial assets. The budget constraint at the time of observation is

$$a' + m' + a\ \phi_T\ I_{m' \neq m} = a + m \tag{A-1}$$

where $I_{\{\cdot\}}$ is an indicator of transfers.

Then the value function solves:

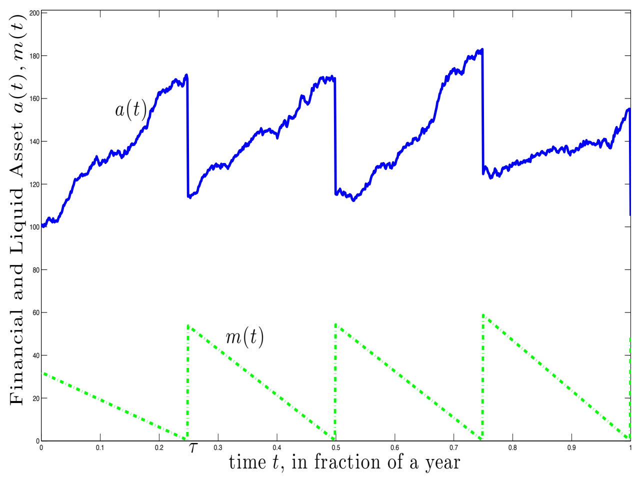
$$V(a, m) \;=\; \max_{m', \alpha, \tau, c(\cdot)} \int_0^{\tau} e^{-\rho t} u(c(t)) dt + e^{-\rho \tau} \int_{-\infty}^{\infty} V\left(a'(1-\phi_o) R(s, \tau, \alpha),\ m(\tau)\right) dN(s)$$

subject to the budget constraint (A-1) at the time of the adjustment, and to the liquid asset-in-advance constraint between adjustment:

$$\mathrm{d}m(t)/\mathrm{d}t = r_L m(t) - c(t) \quad , \quad m(0) = m', \quad m(t) \geq 0, \quad \text{for } t \in [0, \tau]. \tag{A-2}$$

An interesting result common to the models of Duffie and Sun (1990), Gabaix and Laibson (2001), and Abel, Eberly, and Panageas (2007, 2009) is that trades and observations coincide. In particular, consider an agent with $m = 0$, so at this time she must observe and trade. Then she would choose a transfer $m'(0) > 0$ from her financial asset to her liquid asset, a future observation date $\bar{\tau}$, a portfolio share α, and path of consumption between adjustments $c(\cdot)$, with the property that $m(\bar{\tau}) = 0$, i.e. she is planning to run out of liquid asset just at the next observation date. In other words, starting from a state of zero liquid asset, the agent will observe at deterministic equally spaced intervals of length $\bar{\tau}$, and every time she observes the value of her financial assets she will transfer resources to her liquid account.

Figure A-7: Simulated Path of Financial Asset $a(t)$ and Liquid Assets $m(t)$



The nature of the optimal policy is illustrated in Figure A-7, which displays the values for the financial asset and liquid asset in a simulation for an agent following the optimal policy. Notice that the liquid asset $m(t)$ follows a saw-tooth path familiar from the inventory models, such as Baumol-Tobin's classic problem, with withdrawals (and observations) at equally spaced time periods: 0, $\bar{\tau}$, 2 $\bar{\tau}$, etc. One difference compared to Baumol and Tobin is that the value of the withdrawal (the vertical jumps up in $m(t)$, and down for $a(t)$ in Figure A-7), and hence the rate of consumption between successive withdrawals, depends on the level of the financial assets that the agent observes just before the withdrawal.[21] We stress that in this model following the optimal policy implies that every observation coincides with a trade. Also notice, for future reference, that the homogeneity of the problem implies that after an observation and trade, the agent sets the same ratio $m(0)/a(0) \equiv \bar{m}$.

[21] In the example considered in the figure, following the model parametrization by Abel, Eberly, and Panageas (2009), the time variation in the size of withdrawals is small because the variance of the portfolio return is small relative to its mean trend.

The saw-tooth pattern of liquid asset holdings displayed in the figure makes clear that the model predicts a negative correlation between the average liquidity (scaled by non durable consumption) and the number of transactions (e.g. liquidity transfers), as in the standard inventory model of cash holdings.[22] For instance, in the Baumol-Tobin deterministic model the average liquid balance equals half of the liquidity transfer. Together with the identity positing that the product of the number of transactions (n) times the average liquidity transfer ($2\ m$) is equal to the flow of expenditure over a given time period (c), gives us $M/c = 1/(2\ n)$. This implies that the log correlation between M/c and the frequency of transactions is -1. This same prediction holds in this model and can be tested empirically, as we do in Section 2.4.[23]

Interestingly Abel, Eberly, and Panageas (2009) show that the synchronization between observation and trading holds not only when the cost of transferring resources from financial asset to liquid asset is zero, i.e. when $\phi_T = 0$, but also for $\phi_T > 0$, provided that this cost is not too large. For the case when $\phi_T > 0$ this is a surprising result. To see that, notice that it implies that the following deviation is not optimal. Increase the amount of liquid asset withdrawn $m(0)$, keep the same consumption profile, pay the observation cost, and learn the value of the financial assets at the scheduled time $\bar{\tau}$. Note that by construction in the deviation $m(\bar{\tau}) > 0$. At this time consider following a sS type policy: if the ratio $m(\bar{\tau})/a(\bar{\tau})$ is similar to what it would have been after a withdrawal (given by $\bar{m}$), then do not trade. If the ratio is small enough, then pay the trade cost ϕ_T, trade, and set the ratio $m(0)/a(0)$ equal to $\bar{m}$. This deviation has the advantage of saving the fixed trading cost ϕ_T with a strictly positive probability (i.e. the probability that $m(\bar{\tau})/a(\bar{\tau})$ is large). It has the disadvantage that it increases the opportunity cost by holding more of the liquid asset. Abel, Eberly, and Panageas (2009) show that indeed this deviation is not optimal, provided that ϕ_T is not too large. We will return to this when we present our model based on durable goods.

A-3 Special cases

This appendix outlines special cases of the more general problem with both information and transactions costs described in the text.

A-3.1 The frictionless case: $\phi_o = \phi_T = 0$

The first order conditions for the continuous time problem are:

$$\begin{aligned} \theta &: \quad 0 = U'(\theta w)w + v'(w)[-\delta - \alpha\mu - (1-\alpha)r]w - v''(w)(1-\theta)\alpha^2\sigma^2 w^2 \ , \\ \alpha &: \quad 0 = v'(w)[(1-\theta)(\mu - r)]w + [v''(w)(1-\theta)^2\sigma^2 w^2]\alpha \ . \end{aligned}$$

[22]Unlike the classic currency management problem in Baumol and Tobin, the new models seem more appropriately applied to broader notions of liquidity, such as M1 or M2.

[23] Alvarez and Lippi (2009) show that the negative correlation between m/c and n extends to stochastic inventory models.

We can rewrite the foc's as:

$$\theta \;:\; \frac{U'(\theta w)}{v'(w)} + \frac{-v''(w)w}{v'(w)}(1-\theta)\alpha^2\sigma^2 = \delta + r + \alpha(\mu - r) \ , \tag{A-3}$$

$$\alpha \;:\; (1-\theta)\alpha = \frac{1}{-v''(w)w/v'(w)}\left[\frac{\mu - r}{\sigma^2}\right] \ . \tag{A-4}$$

We assume that the utility function is homogenous of degree $\gamma > 0$, $U(h) = \frac{h^{1-\gamma}}{1-\gamma}$. In this case the value function will be homogenous of degree γ too, so we can write it as:

$$v(w) = v(1)\ w^{1-\gamma} \tag{A-5}$$

and foc becomes:

$$\theta \;:\; \frac{\theta^{-\gamma}}{(1-\gamma)\ v(1)} + \gamma(1-\theta)\alpha^2\sigma^2 = \delta + r + \alpha(\mu - r) \ , \tag{A-6}$$

$$\alpha \;:\; \alpha(1-\theta) = \frac{1}{\gamma}\frac{\mu - r}{\sigma^2} \ . \tag{A-7}$$

and the Bellman equation

$$\rho v(1) = \frac{\theta^{1-\gamma}}{1-\gamma} + v(1)(1-\gamma)\left[-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2\alpha^2(1-\theta)^2\right]$$

The foc w.r.t. θ and the Bellman equation can be combined and simplify to:

$$\begin{aligned} \frac{\theta^{1-\gamma}}{v(1)(1-\gamma)} &= -\gamma(1-\theta)\theta\alpha^2\sigma^2 + \theta\left(\delta + r + \alpha(\mu - r)\right) \ , \\ \frac{\theta^{1-\gamma}}{v(1)(1-\gamma)} &= \rho - (1-\gamma)\left(-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2(1-\theta)^2\alpha^2\right) \ . \end{aligned}$$

Equating these two expressions we obtain:

$$\begin{aligned} & -\gamma(1-\theta)\theta\alpha^2\sigma^2 + \theta[\delta + r + \alpha(\mu - r)] \\ = \ & \rho - (1-\gamma)\left[-\theta\delta + (1-\theta)(r + \alpha(\mu - r)) - \frac{1}{2}\gamma\sigma^2(1-\theta)^2\alpha^2\right] . \end{aligned} \tag{A-8}$$

We are looking for a solution for this equation with $\theta > 0$. This will require some assumptions on parameters. Among these assumptions, we will include conditions that guarantee that the problem has a finite solution. We will consider two cases. In the first case α is exogenous, and the second where α is optimized over. In the first case this expression is quadratic equation in θ. In the second case, where we replace α by its solution found above, θ can be written as the zero of a higher order polynomial.

In the case of α endogenous, substituting the first order conditions for $\alpha(1-\theta) = (\mu -$

$r)/(\sigma^2\gamma)$ we obtain:

$$\theta \;=\; \frac{1}{\gamma}\frac{\rho}{(\delta+r)} - \frac{1-\gamma}{\gamma}\frac{\left[r+\frac{1}{2}(\mu-r)^2/(\gamma\sigma^2)\right]}{\delta+r} \; . \tag{A-9}$$

Few comments are in order. First, as $\delta \to \infty$, then $\theta \to 0$. Second, for $\gamma = 1$, the 'log' case, the expression for $\theta = \rho/(\delta+r)$. When $\gamma > 1$ this expression is always positive. When $\gamma < 1$, the expression for θ can be negative. It is non-negative if

$$\rho \geq (1-\gamma)\left[r + \frac{1}{2}(\mu-r)^2/(\gamma\sigma^2)\right] \; . \tag{A-10}$$

This conditions ensures that the discount rate ρ is large enough so that utility is bounded from above.

Coming back to the expression for the value function, replacing the optimal value of α on the foc for θ derived above we obtain:

$$v(1) = \frac{\theta^{-\gamma}}{(1-\gamma)(r+\delta)} \; , \tag{A-11}$$

thus, as long as θ as given in (A-9) is non-negative, which is assured by condition (A-10), the problem is well defined.

A-3.2 The problem with observation cost only

This appendix derives the first order conditions that solve the problem with observation cost (and no transaction cost).

$$\begin{aligned} v(1) \;&=\; \frac{1-e^{-(\rho+(1-\gamma)\delta)\tau}}{(1-\gamma)(\rho+(1-\gamma)\delta)}\;\theta^{1-\gamma} \\ &+\; e^{-\rho\tau}v(1)\int_{-\infty}^{\infty}\left[(1-\theta)\;(1-\phi_o)R(s,\tau,\alpha)+\theta e^{-\delta\tau}\right]^{1-\gamma}dN(s) \end{aligned} \tag{A-12}$$

Letting $\Omega \equiv (1-\theta)\;(1-\phi_o)$ for notation convenience, the foc with respect to τ gives

$$\begin{aligned} \frac{\left(\theta\; e^{-\delta\tau}\right)^{1-\gamma}}{1-\gamma} &= \rho v(1)\int_{-\infty}^{\infty}\left(\Omega R(s,\tau,\alpha)+\theta e^{-\delta\tau}\right)^{1-\gamma}dN(s) + \\ -\; v(1)\int_{-\infty}^{\infty}&(1-\gamma)\left[\Omega R(s,\tau,\alpha)+\theta e^{-\delta\tau}\right]^{-\gamma}\left[\Omega\frac{\partial R(s,\tau,\alpha)}{\partial\tau} - \delta\theta e^{-\delta\tau}\right]dN(s) \end{aligned} \tag{A-13}$$

where

$$\frac{\partial R(s,\tau,\alpha)}{\partial\tau} = \left[\alpha\mu + (1-\alpha)r - \alpha^2\frac{\sigma^2}{2} + \frac{\alpha\sigma s}{2\sqrt{\tau}}\right]R(s,\tau,\alpha)$$

Simple algebraic manipulation of (A-13) yields:

$$\int_{-\infty}^{\infty} \left\{ \frac{\Omega \left(\rho R(s,\tau,\alpha) - (1-\gamma)\frac{\partial R(s,\tau,\alpha)}{\partial \tau} \right) + \theta e^{-\delta\tau}(\rho + (1-\gamma)\delta)}{[\Omega R(s,\tau,\alpha) + \theta e^{-\delta\tau}]^{\gamma}} \right\} dN(s) = \frac{\left(\theta\ e^{-\delta\tau}\right)^{1-\gamma}}{(1-\gamma)\, v(1)} \quad \text{(A-14)}$$

The foc with respect to θ gives:

$$\frac{1 - e^{-(\rho+(1-\gamma)\delta)\tau}}{v(1)(\rho + (1-\gamma)\delta)e^{-\rho\tau}(1-\gamma)} \theta^{-\gamma} = \int_{-\infty}^{\infty} \frac{(1-\phi_o)R(s,\tau,\alpha) - e^{-\delta\tau}}{[(1-\theta)(1-\phi_o)R(s,\tau,\alpha) + \theta e^{-\delta\tau}]^{\gamma}}\ dN(s) \quad \text{(A-15)}$$

Rewrite eq (A-12) as

$$\frac{1 - e^{-(\rho+(1-\gamma)\delta)\tau}}{v(1)(\rho + (1-\gamma)\delta)e^{-\rho\tau}(1-\gamma)}\ \theta^{-\gamma} = \frac{e^{\rho\tau}}{\theta} - \frac{1}{\theta}\int_{-\infty}^{\infty} \left[(1-\theta)\ (1-\phi_o)R(s,\tau,\alpha) + \theta e^{-\delta\tau}\right]^{1-\gamma} dN(s)$$

Equating these two equations gives

$$e^{\rho\tau} = \int_{-\infty}^{\infty} \frac{(1-\phi_o)R(s,\tau,\alpha)}{[(1-\theta)(1-\phi_o)R(s,\tau,\alpha) + \theta e^{-\delta\tau}]^{\gamma}}\ dN(s) \quad \text{(A-16)}$$

The foc with respect to α gives

$$0 = \int_{-\infty}^{\infty} \frac{\frac{\partial R(s,\tau,\alpha)}{\partial \alpha}}{[(1-\theta)(1-\phi_o)\ R(s,\tau,\alpha) + \theta e^{-\delta\tau}]^{\gamma}}\ dN(s) \quad \text{(A-17)}$$

where

$$\frac{\partial R(s,\tau,\alpha)}{\partial \alpha} = \left[(\mu - r - \alpha\sigma^2)\tau + \sigma s\sqrt{\tau}\right] R(s,\tau,\alpha)$$

Equations (A-12), (A-14), (A-16) and (A-17) give a system of four equations in the 4 unknowns $v(1)$, θ, τ and α.

A-3.3 The Grossman - Laroque case: $\phi_o = 0$ and $\phi_T > 0$

In this appendix we provide more details on the model with adjustment cost only. One difference compared to the original GL model is that the portfolio share α can only be adjusted when the stock of durables is revised (so α becomes a state for the problem, which we omit for notation simplicity). The value function is:

$$\rho V(a,h) \geq \frac{d^{1-\gamma}}{1-\gamma} + V_a(a,h)a[r + \alpha(\mu - r)] - V_h(a,h)h\delta + \frac{1}{2}V_{aa}(a,h)a^2\alpha^2\sigma^2\ ,$$

with equality in the inaction region. The possibility of adjustment gives

$$V(a,h) \geq \max_{a'\geq 0,\alpha} V(a', h(1-\phi_T) + a - a')\ ,$$

with equality if adjustment of durables is optimal at that (a, h). The maximization with respect to the post-adjustment value of a' gives the foc:

$$V_a(a', h(1-\phi_T) + a - a') = V_h(a', h(1-\phi_T) + a - a') \ . \tag{A-18}$$

The maximization w.r.t. α gives:

$$\alpha = \frac{\mu - r}{\frac{-V_{aa}(a,h)a}{V_a(a,h)}\sigma^2} \ .$$

We can write all the conditions jointly as:

$$\rho V(a,h) = \max \quad \Bigg(\rho \max_{a' \geq 0, \alpha} V(a', h(1-\phi_T) + a - a') \ , \\ \frac{d^{1-\gamma}}{1-\gamma} + V_a(a,h)a[r + \alpha(\mu - r)] - V_h(a,h)h\delta + \frac{1}{2}V_{aa}(a,h)a^2\alpha^2\sigma^2 \Bigg)$$

Using the homogeneity of V, and $x = a/h$, rewrite this equation in the inaction region as

$$\rho V(x,1) \ = \ \frac{1}{1-\gamma} + V_a(x,1) \ x \ [r + \alpha(\mu - r)] - V_h(x,1) \ \delta + \frac{1}{2}V_{aa}(x,1) \ x^2\alpha^2\sigma^2$$

To write this PDE as an ODE, we use homogeneity to express V_h in terms of V_a,

$$V_h(x,1) \ = (1-\gamma)V(x,1) - x \ V_a(x,1)$$

so that, after replacing into the PDE and collecting terms, gives the following ODE:

$$(\rho + \delta(1-\gamma)) \ \ V(x,1) \ = \ \frac{1}{1-\gamma} + V_a(x,1) \ x \ [\delta + r + \alpha(\mu - r)] + \frac{1}{2}V_{aa}(x,1) \ x^2\alpha^2\sigma^2$$

so that the solution for $V(x,1)$ in the inaction range is

$$V(x,1) = \frac{1}{(1-\gamma)\,(\rho + \delta(1-\gamma))} + \sum_{i=1}^{2} \ A_i \ x^{\eta_i}$$

where the roots η_i solve the characteristic equation

$$0 = \rho + \delta(1-\gamma) - \left(\delta + r + \alpha(\mu - r) - \frac{\alpha^2\sigma^2}{2}\right)\eta - \frac{\alpha^2\sigma^2}{2}\eta^2 \ .$$

The policy rule is characterized by three numbers: the optimal return point $\hat{x}$, and the 2 barriers $(\underline{x}, \bar{x})$ that delimit the inaction region: an agent with $x \in (\bar{x}, \underline{x})$ does not adjust. Outside this region, i.e. for $x \geq \bar{x}$ or $x \leq \underline{x}$, the value function is characterized by paying the fixed cost and adjusting so that the post-adjustment ratio is $\hat{x}$.

The closed form solution for the ODE, up to the two constant of integration A_1, A_2 in the inaction region allows us to then write a system of 5 equations and unknowns. The unknowns are $A_1, A_2, \underline{x}, \bar{x}, \hat{x}$. The five equations are the first order condition for the optimal return point equation (A-19), a pair of value matching conditions at each boundary equations (A-20), and a pair of smooth pasting conditions at each boundary equations (A-21):

$$V_a(\hat{x},1) = V_h(\hat{x},1) \ , \tag{A-19}$$

$$\left(\frac{1+\bar{x}-\phi_T}{1+\hat{x}}\right)^{1-\gamma} V(\hat{x},1) = V(\bar{x},1) \ , \ \left(\frac{1+\underline{x}-\phi_T}{1+\hat{x}}\right)^{1-\gamma} V(\hat{x},1) = V(\underline{x},1) \ , \tag{A-20}$$

$$V_a(\underline{x},1)(1-\phi_T) = V_h(\underline{x},1) \quad , \quad V_a(\bar{x},1)(1-\phi_T) = V_h(\bar{x},1) \ . \tag{A-21}$$

Equation (A-19) follows from equation (A-18). The value matching equations (A-20) use that

$$V(\bar{a}-\tilde{\Delta}^*, \bar{h}(1-\phi_T)+\tilde{\Delta}^*) = \max_{\tilde{\Delta}} V(\bar{a}-\tilde{\Delta}, \bar{h}(1-\phi_T)+\tilde{\Delta}) = V(\bar{a},\bar{h})$$

$$1 = \frac{V_a(\hat{a},\hat{h})}{V_h(\hat{a},\hat{h})} \text{ and } \frac{\bar{a}-\tilde{\Delta}^*}{\bar{h}(1-\phi_T)+\tilde{\Delta}^*} = \frac{\hat{a}}{\hat{h}} \implies \Delta^* = \frac{\bar{x}-\hat{x}(1-\phi_T)}{1+\hat{x}}$$

where $\bar{x} = \bar{a}/\bar{h}$, $\hat{x} = \hat{a}/\hat{h}$ and $\Delta^* = \tilde{\Delta}^*/\bar{h}$, and the homogeneity of degree $(1-\gamma)$ of $V(\cdot)$. The same derivation applies in the lower boundary $\underline{x}$. To derive the smooth pasting conditions equations (A-21) consider pairs (a,h) such that $a + h(1-\phi_T)d = \bar{a} + \bar{h}(1-\phi_T) = \hat{a} + \hat{h}$ and that $a/h > \bar{a}/\bar{h}$ for some fixed $\bar{a}, \bar{h}$ so that after adjustment the agent will go to $\hat{a}, \hat{h}$. Notice that for any such pair (a,h) the agent will also pay the fixed cost and adjust to the same level $\hat{a}, \hat{h}$. Thus we have:

$$V(a,h) = V(\bar{a},\bar{h}) = V(\hat{a},\hat{h})$$

or using $a = \bar{a} + \bar{h}(1-\phi_T) - h(1-\phi_T)$:

$$V(\bar{a} + \bar{h}(1-\phi_T) - h(1-\phi_T), h) = V(\hat{a},\hat{h})$$

and differentiating w.r.t. h we have

$$0 = -V_a(\bar{a} + \bar{h}(1-\phi_T) - h(1-\phi_T), h)(1-\phi_T) + V_h(\bar{a} + \bar{h}(1-\phi_T) - h(1-\phi_T), h)$$

Taking the limit as $h \uparrow \bar{h}$ we have

$$V_a(\bar{a},\bar{h})(1-\phi_T) = V_h(\bar{a},\bar{h})$$

and using homogeneity we obtain equation (A-21). Repeating the same argument for values of $a/h < \underline{a}/\underline{h}$ we obtain the other smooth pasting condition.

For completeness we also describe the expected number of adjustment per unit of time, given the two barriers $\bar{x}$ and $\underline{x}$ and given the optimal return point $\hat{x}$. Let $T(a,h)$ be the expected time until an adjustment takes place, starting from state (a,h). We have

$$0 = 1 + T_a(a,h)a\left(r + \alpha(\mu - r)\right) - T_h(a,h)h\delta + \frac{1}{2}T_{aa}(a,h)a^2\alpha^2\sigma^2$$

with boundaries $T(\bar{a},\bar{h}) = T(\underline{a},\underline{h}) = 0$. We are interested in $T(\hat{a},\hat{h})$, which gives the expected time between successive adjustments. We notice that T is homogeneous of degree zero, and hence we can write:

$$0 = 1 + T_a(x,1)x\left(r + \alpha(\mu - r) + \delta\right) + \frac{1}{2}T_{aa}(x,1)x^2\alpha^2\sigma^2$$

with boundaries $T(\bar{x},1) = T(\underline{x},1) = 0$. The solution to this equation is

$$T(x) = B_0 + B_1 x^\lambda + B_2 \log x$$

where

$$\lambda = 1 - \frac{r + \alpha(\mu - r) + \delta}{\alpha^2\sigma^2/2} \quad , \quad B_2 = \frac{1}{\alpha^2\sigma^2/2 - (r + \alpha(\mu - r) + \delta)} \; .$$

The constants B_0 and B_1 are chosen to satisfy the terminal conditions, namely

$$0 = B_0 + B_1\underline{x}^\lambda + B_2 \log \underline{x} \quad , \quad 0 = B_0 + B_1\bar{x}^\lambda + B_2 \log \bar{x}$$

or

$$B_0 = -B_2 \left[\frac{\log \bar{x} - \log \underline{x}}{\underline{x}^\lambda - \bar{x}^\lambda}\, \bar{x}^\lambda + \log \bar{x}\right] \quad , \quad B_1 = B_2\, \frac{\log \bar{x} - \log \underline{x}}{\underline{x}^\lambda - \bar{x}^\lambda} \; .$$

A-3.4 The Bellman equation when $\phi_o > 0, \phi_T > 0$

Here we use the homogeneity of the value function $V(a,h)$ to reduce the Bellman equation to a function of a single variable by setting $h = 1$.

$$\begin{aligned}
\bar{V}(a,1,\alpha) &= \max_\tau \left\{ \int_0^\tau e^{-\rho t} U(e^{-t\delta}) dt \right.\\
&+ \left. e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left((1-\phi_o)e^{\delta\tau}R(s,\tau,\alpha)a \;, 1, \hat{\alpha}\right) dN(s) \right\}\\
&= \max_\tau \left\{ \frac{1 - e^{-(\rho+(1-\gamma)\delta)\tau}}{(1-\gamma)(\rho + (1-\gamma)\delta)} \right.\\
&+ \left. e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left((1-\phi_o)e^{\delta\tau}R(s,\tau,\alpha)a \;, 1, \hat{\alpha}\right) dN(s) \right\} \qquad \text{(A-22)}
\end{aligned}$$

$$
\begin{aligned}
\hat{V}(a,1) &= \max_{\tau,\ \alpha,\ 0\le a'\le a+1-\phi_T} \left\{ \int_0^\tau e^{-\rho t} U([a+(1-\phi_T)-a']e^{-t\delta}) dt \right. \\
&+ \left. e^{-\rho\tau} \int_{-\infty}^{\infty} V\left(a'(1-\phi_o)R(s,\tau,\alpha)\ ,\ [a+(1-\phi_T)-a']e^{-\delta\tau},\ \hat{\alpha}\right) dN(s) \right\} \\
&= \max_{\tau,\ \alpha,\ 0\le a'\le a+1-\phi_T} [a+(1-\phi_T)-a']^{1-\gamma} \left\{ \int_0^\tau e^{-\rho t} U(e^{-t\delta}) dt \right. \\
&+ \left. e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left(\frac{a'}{[a+(1-\phi_T)-a']}(1-\phi_o)e^{\delta\tau}R(s,\tau,\alpha)\ ,\ 1,\ \hat{\alpha}\right) dN(s) \right\} \\
&= \max_{\tau,\ 0\le a'\le a+1-\phi_T} [a+(1-\phi_T)-a']^{1-\gamma} \left\{ \frac{1-e^{-(\rho+(1-\gamma)\delta)\tau}}{(1-\gamma)(\rho+(1-\gamma)\delta)} \right. \qquad \text{(A-23)} \\
&+ \left. e^{-(\rho+(1-\gamma)\delta)\tau} \max_{\alpha} \int_{-\infty}^{\infty} V\left(\frac{a'}{[a+(1-\phi_T)-a']}(1-\phi_o)e^{\delta\tau}R(s,\tau,\alpha)\ ,\ 1,\ \hat{\alpha}\right) dN(s) \right\}
\end{aligned}
$$

With these expressions it is easy to see that inaction is optimal around the optimal return point. Consider the feasible policy $a' = a$, we have that

$$
\begin{aligned}
\hat{V}(a,1) &= \max_{\tau,\ \alpha} \left\{ \int_0^\tau e^{-\rho t} U((1-\phi_T)e^{-t\delta})\ dt \right. \\
&+ \left. e^{-(\rho+(1-\gamma)\delta)\tau} \int_{-\infty}^{\infty} V\left(ae^{\delta\tau}R(s,\tau,\alpha)\ ,\ (1-\phi_T),\ \hat{\alpha}\right) dN(s) \right\} \ < \ \bar{V}(a,1,\ \hat{\alpha})
\end{aligned}
$$

where the last inequality follows by the fact that utility is increasing in the stock of durables. This prove that the inaction region includes an interval that contains the target asset-to-durable ratio.

In the remainder of this section we given a detailed account of the relevant results of the related applied mathematical literature used to characterize the form of the control and inaction sets. We also relate it to the analytical results in the stylized model in Alvarez et all 2011.

It is useful to first consider the related problem in the applied mathematical literature known as the "Stochastic Cash Balance Problem with Fixed Cost". For our purposes the relevant references are :

- Edwin H. Neave's paper Management Science, Vol 16, No 7, March 1970, pp 72-490, "The Stochastic Cash Balance Problem with Fixed Costs for Increases and Decreases",
- Avner Bar-Ilan's paper in International Economic Review, Vol. 31, No. 1 (Feb., 1990), pp. 229-234, "Trigger-Target Rules Need Not be Optimal with Fixed Adjustment Costs: A Simple Comment on Optimal Money Holding Under Uncertainty" ,
- Xin Chen and David Simchi-Levi's paper in Probability in the Engineering and Informational Sciences, vol 23, 2009, pp 545562. "A New Approach for the stochastic cash balance problem with fixed cost"

In this problem the *one dimensional* state, when left uncontrolled, changes randomly, with iid increments. The problem is set in *discrete time*, so there are no meaningful distinction for the state to have continuous sample path. There is a period convex cost that the agent bears if no action is taken, and which depend on the state as of the beginning of the period. If an action is taking, the state can be reset to any desired value. Resetting the state involves paying a fixed cost, potentially different for increases ($K > 0$) and decreases ($Q > 0$), and in many cases also a proportional cost, also potentially different for increases and decreases (with per unit cost denoted by k and q respectively). The objective is to determine the optimal policy for each value of the state: i.e. whether an action should be taken, and if so where the state should be reseted after the action is taken.

In what follow we will specialize the case to $K = Q > 0$ so the fixed cost is the same for increases and decreases, and assume that there is no variable cost ($q = k = 0$). This special case is the one closer to the set-up in our paper. Following the notation in this literature, we denote x the state and $y(x)$ the optimal policy. The general solution is that (indeed for each iteration of the Bellman equation) one can find 6 numbers: $t \le t^+ \le T \le U \le u^- \le u$ such that:

$$y(x) = \begin{cases} T & \text{if } x \le t \\ \in \{x, T\} & \text{if } x \in (t, t^+) \\ x & \text{if } x \in [t^+, u^-] \\ \in \{x, U\} & \text{if } x \in (u^-, u) \\ U & \text{if } x \ge u \end{cases}$$

Note that if $q = k = 0$ then $U = T$. On can show that if $Q = K > 0$ then $t^+ < u^-$. Definitely in $[t^+, u^-]$ there is inaction. Also, with regularity conditions (such as letting the cost to grow unbounded if the state grows with no bounds) one has that $-\infty < T < U < +\infty$, so there is definitely control for large enough (absolute) value of the state. The issue is that for intermediate values, such $x \in (t, t^+)$ or $x \in (u^-, u)$ it is optimal to *either have control or inaction*. In other words, as x increases (or respectively as it decreases) there can be a sequence of intervals where the optimal policy alternates between control and inaction. Indeed one can construct examples of problems where this is the case (see for instance, the example in Neave 1970, section 3.1 or the example in Bar-Ilan 1990, section 3).

Notice that in continuous time problems where the state follows a diffusion this is not really a concern. The reason is that even though the results holds for discrete time approximation of the problem, the *ergodic set* will consist of the inaction set, $[t^+, u^-]$ in the notation above.

Now, what makes our problem more complicated is that even though it is set in continuous time, as long as there is a strictly positive observation cost, the agent observes the state at (endogenously) discretely sampled periods. Thus, for our purposes, the relevant problem is the discrete time described above. An additional complication is that in our problem the state is two dimensional. We do not emphasize the second, because -given the assumed homogeneity, the problem can be reduced to a one state.

While in general the inaction set can be complicated, Neave also show conditions under which it is not, i.e. where $t = t^+$ and $u^- = u$, or as he called where the police is simple. In section 4 of Neave 1970, there is a Theorem (section 4.2 page 489). The conditions are that if a) the expected value of the period cost function is *quasi-convex* and *symmetric* and b) the distribution of the shocks to the state has *zero mean* and it is *symmetric*, and has a *quasi-concave density*. Under these assumptions the policies are "simple" so that the inaction set is an interval.

Indeed in a simpler related setup ("Optimal Price Setting with Observation and Menu Costs", by Alvarez, Lippi and Paciello, forthcoming in the QJE) we analyze a continuous time model with both fixed costs of observation and of transaction, and a one dimensional state (which follows a Brownian motion). We assume that the period return function is quadratic. For the case where the state has no drift in Proposition 3 of that paper we show a result similar to Neave's, so that the inaction set is an interval. Note that our result deals with the fact that due to the observation cost, the decisions are taken at state-dependent discretely sampled periods. While the set up of the two papers share that there are both observation and transaction cost, the set-up does not satisfies all the assumptions of Proposition 3. In particular the period return function is not symmetric in the state (which in this paper is the ratio of assets to durables) and the state has a non-zero drift. Nevertheless in every numerical computation we have solved we have found the inaction set to be an interval.

A-4 No trade between observations

Notice that between observation dates, with $\sigma > 0$, the agent does not know her wealth. Moreover, there is a strictly positive probability that her wealth can be arbitrarily close to zero. This precludes the strategy where the agent increases her holding of durables between observations dates by withdrawing resources from her financial assets, since she may otherwise violate her budget constraint. Thus, we will only need to consider the case where the agent is allowed to decrease her durable holding, and increase her holding for financial assets in between observation dates. We will like to show that it is not optimal to decrease durable goods, and hence that the restriction on the policies is not binding. But to verify this in the case for $\sigma > 0$ is complicated, so we restrict attention to the case with $\sigma = 0$. We conjecture that the conclusion extends to the case of $\sigma > 0$.

In particular for $\sigma = 0$ we consider the following variational problem taking as given the interval τ, as well as the initial and final wealth $w(0), w(\tau)$, and where during the observation period the agent can sell durables at rate $x(t)$ and invest the proceeds in the financial assets which accumulate at rate r:

$$\max_{\{a(0),h(0),a(\tau),h(\tau),x(t)\}} \int_0^\tau u(h(t))e^{-\rho t}dt$$

subject to :

$$w(0) = a(0) + h(0) \ , \ w(\tau) = a(\tau)(1 - \phi_o) + h(\tau),$$
$$\dot{h}(t) = -\delta h(t) - x(t) \ , \ \dot{a}(t) = r \ a(t) + x(t),$$
$$a(0), h(0), a(\tau), h(\tau), x(t) \geq 0.$$

Proposition 1. If $\rho < r + \delta\gamma$, then the non-negativity constraint binds, and the optimal policy is $x(t) = 0$ for all $t \in [0, \tau]$.

Note that in the case of $\phi_o = 0$ and $\sigma = 0$ (so $\mu = r$), the solution of the continuous time problem gives a value for the fraction of wealth in durables θ equal to $\theta = \frac{r-(r-\rho)/\gamma}{r+\delta}$. Thus the condition for $x(t) = 0$ given in the proposition is equivalent to the condition for $\theta < 1$. This is quite intuitive, for $x(t) = 0$ we require the return on the financial portfolio to be good relative to the discount rate and the depreciation rate. Moreover, if the parameters are such that $\theta < 1$, then when ϕ_o is small but positive, the agent will still choose $x(t) = 0$.

Proof. Let $e^{-\rho t}\lambda(t)$ be the multiplier of the law of motion of a at t, $e^{-\rho t}\mu(t)$ the multiplier of the law of motion of h at t, ν_0 the multiplier of the wealth at $t = 0$ constraint, and ν_1 the multiplier of the wealth constraint at $t = \tau$. The Lagrangean is:

$$\begin{aligned} &\int_0^\tau e^{-\rho t}\left[u(h(t)) + \lambda(t)\left(a(t)r + x(t) - \dot{a}(t)\right) + \mu(t)\left(-h(t)\delta - x(t) - \dot{h}(t)\right)\right] dt \\ +\;\; & \nu_1[(1-\phi)a(\tau) + h(\tau) - w(\tau)] + \nu_0[w(0) - a(0) - h(0)] \end{aligned}$$

Integrating by parts we have:

$$\begin{aligned} &\int_0^\tau e^{-\rho t}\left[u(h(t)) + \lambda(t)\left(a(t)r + x(t) - \rho a(t)\right) + \dot{\lambda}(t)a(t) + \mu(t)\left(-h(t)\delta - x(t) - \rho h(t)\right) + \dot{\mu}(t)h(t)\right] dt \\ &+\nu_1[(1-\phi)a(\tau) + h(\tau) - w(\tau)] + \nu_0[w(0) - a(0) - h(0)] \\ &-\lambda(\tau)e^{-\rho\tau}a(\tau) + \lambda(0)a(0) - \mu(\tau)e^{-\rho\tau}h(\tau) + \mu(0)h(0) \end{aligned}$$

We can then use the current value Hamiltonian, with state state (d, a) and control x is:

$$H(a, d, y) = u(d) + \mu(-\delta d - x) + \lambda(ar + x)$$

and use μ, λ the costates. The FOC for $t \in (0, \tau)$ include:

$$H_x = 0 : -\mu(t) + \lambda(t) \leq 0$$

with equality if $x(t) > 0$, and

$$\begin{aligned} \dot{\lambda} &= \rho\lambda - H_a : \; \dot{\lambda}(t) = \rho\lambda(t) - \lambda(t)r \\ \dot{\mu} &= \rho\mu - H_d : \; \dot{\mu}(t) = \rho\mu(t) - u'(h(t)) + \mu(t)\delta \end{aligned}$$

While there are more first order conditions involving $a(0), h(0), a(\tau), h(\tau)$ the previous ones suffice for our current purpose. By way of contradiction, assume $x(t) > 0$ in an interval between $0 < t_0$ and $t_1 < \tau$. Then $\lambda = \mu$ and $\dot{\lambda}/\lambda = \rho - r$ in this interval:

$$u'(h(t)) = \lambda(\delta + r) = (\delta + r)\;\lambda(t_0)\;e^{(\rho - r)t}$$

for $t \in [t_0, t_1]$. By taking logs and differentiating w.r.t. time, using that $u(h) = h^{1-\gamma}/(1-\gamma)$ we obtain

$$\frac{\dot{h}(t)}{h(t)} = \frac{r - \rho}{\gamma}$$

From the law of motion of h, if $x \geq 0$:

$$\frac{\dot{h}(t)}{h(t)} \leq -\delta.$$

Hence, if

$$\rho < r + \delta\gamma$$

we arrive to a contradiction with $x(t) > 0$ on $[t_0, t_1]$. ■

A-5 Unicredit administrative panel data

In this appendix we use administrative panel data on assets stocks and net flows from investors to a) compute several measures of trading frequency and distinguish between types of assets trades; b) offer more direct evidence on the timing of between assets liquidation and durable goods purchases.

A-5.1 Data description

The Unicredit administrative data contain information on the stocks and on the net flows of 26 assets categories that investors have at Unicredit.[24] These data are available at monthly frequency for 35 months beginning in December 2006. There are two samples. The first is a sample of about 40,000 investors that were randomly drawn from the population of investors at Unicredit and that served as a reference sample from extracting the investors to be interviewed in the 2007 survey. We refer to this as the large sample. We do not have direct access to the administrative records for the large sample; calculations and estimates on this sample were kindly done at Unicredit. The second which we call the "survey sample" has the same administrative information for the investors that actually participated in the 2007 survey. We do have access to the survey-sample data which can additionally be matched with the information from the 2007 survey. A description of the merged data is in Guiso, Sapienza, and Zingales (2010). Since some households left Unicredit after the interview the administrative data are available for 1,541 households instead on the 1,686 in the 2007 survey. Notice that both the large sample and the survey sample are balanced panel data. Since the administrative record registers both the stock of each asset category at the end of the period as well as the net trading flow into that category, we can directly identify trading decisions, which would not be possible if only assets valuation at the end of period were available. One of the 26 assets is the checking account. In what follows we distinguish assets into two categories: liquid assets, which we identify with the checking account, and investments the sum of the remaining 25 assets classes. We also experimented, with no change on the results, with a broader definition of liquid assets, and hence a narrower definition of financial

[24]The list includes: checking accounts, time deposits, deposit certificates, stock mutual funds, money market mutual funds, bond mutual funds, other mutual funds, ETF, linked funds, Italian stocks, foreign stocks, unit linked insurance recurrent premium, unit linked insurance one shot premium, stock market index, life insurance recurrent premium, life insurance one shot premium, pension funds, T-bills short term, T-bonds, indexed T-bonds, other T-bills, managed accounts, own bank bonds, corporate bonds Italy, corporate bonds foreign, other bonds.

assets.[25] For the following it is useful to establish some notation. Let denote the net flow of asset i owned by investor j in month t. Let the first asset be the checking account and let its net flow be $C_{jt} = f_{0jt}$. The net investments flow is the sum over the remaining assets $F_{jt} = \sum_{i=1}^{25} f_{ijt}$.

A-5.2 Measuring trading frequency

We construct several measures of trading frequency from the survey sample. It is useful to start looking simply at the cross sectional distribution of net investment flows F_{jt}: a positive net flow in month t means that over that month the investor has purchased some investments (asset purchase), a negative net flow that he has sold investments (an asset sale). In the first case there must have been a transfer from the checking account i.e. from the liquid asset account; in the second a transfer to the checking account i.e. to the liquid asset account. Later we will return to the relation between investment sales/purchases and flows into/from liquid asset accounts. A zero value of F_{jt} means that the investors made no net trade in that month. Table A-12 tabulates the distribution of net investment flows over the 35 months in the sample both in current value and scaled by the average total financial assets of each investor to remove some of the sample heterogeneity; Table A-13 shows the conditional distribution of asset purchases (in absolute value) and sales. There are three main features. First, there is a lot of inaction. In more than 50% of the investor/month observations net investment flows are zero, which is consistent with infrequent portfolio adjustments. Second, both sales and purchases of assets tend to be lumpy; median asset sales is 7,200 euros and asset sales tends to be larger than purchases (median value 1,000 euros). On average, a liquidation accounts for 19% of the value of the investors assets while a purchase for about 5%. Third, asset liquidations are (almost twice) less frequent than purchases.

To obtain an estimate of the average number of trades due to net asset sales for each investor j in the sample we first define $L_{jt} = 1$ if in month t investor j has sold some investments, that is if the net investment flow $F_{jt} < 0$. We then compute the average annual number of trades with asset sales for household j as $N_j = \sum_{t=1}^{35} L_{jt} \times \frac{12}{35}$. We repeat this exercise but counting as asset sales those only net sales of asset in excess of 500 and 1000 euros. Table A-14 shows summary statistics for the number of trades with asset sales in the whole sample as well as in the sample of stockholders. The latter are defined based on whether the investor owns stocks directly (direct stockholders) and directly or indirectly (total stockholders) in the first month of the sample. In the whole sample there is one asset sale per year in median and 1.4 in mean; stockholders sell assets roughly twice more frequently. If one focuses on trades that are likely to involve durable purchases (asset sales in excess of 1,000 euros) the median is around 0.7 per year and the mean is 1.

[25] There are three other assets that could possibly be included in the liquid assets class: time deposits, deposit certificates, and money market mutual funds. Deposits certificates (pronti contro termini) are really repurchase agreements, with maturities between 1-3 months and 1 year, so for our purposes we do not think of them as liquid. In Italy money market mutual funds do not allow to transfer funds and write checks easily, as in the US; hence we do not think of them as liquid assets either. Time deposits in Italy have no pre-specified maturity, and allow investors to convert them into checking within a short period, typically around a week. For our purposes they can also be considered liquid assets, so we have also experimented defining liquid asset as the sum of both checking accounts and time deposits. We found the results to be essentially the same, so we do not systematically report the results with both definitions.

Table A-12: Distribution of net investment flows

Total Net Investment Flows (F_{jt})		
Percentile	Euro Values	Share of Average Assets
1th	-90,000	-0.63
5th	-10,787	-0.083
10th	-605	-0.004
25th	0	0
50th	0	0
75th	0	0
90th	1,512.8	0.011
95th	10,985	0.072
99th	75786	0.49
Mean	-98.9	-0.005
Sd	9.33	e+080.19

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1400 investors.

Table A-13: Distribution of net investment flows

	Net asset sales: $\max\{F_{jt}, 0\}$		Net asset purchases: $\max\{-F_{jt}, 0\}$	
Percentile	Euro values	Share of avg. asset	Euro values	Share of avg. assets
1th	2	0.00002	10	0.00005
5th	79	0.0005	100	0.0005
10th	245	0.001	100	0.0008
25th	1155.4	0.01	200	0.002
50th	7,323.7	0.05	1,000	0.008
75th	26,506	0.20	10,000	0.06
90th	75,046	0.55	37,8911.4	0.23
95th	120,430	0.83	10,985	0.46
99th	250,872	1.60	69,566	0.98
Mean	26,768.6	0.19	14,526.4	0.08
Sd	55,960	0.42	43,965	0.21
N obs	6,236	6,236	11,142	11,142

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1400 investors.

Table A-14: Summary statistics for the average annual number of asset sales trades

	All Asset Sales N_{Sj}		Asset sales ≥ 500		Asset sales ≥ 1000	
	Median	Mean (sd)	Median	Mean (sd)	Median	Mean (sd)
Total sample	1.03	1.40 (1.29)	1.03	1.17 (1.11)	0.70	1.06 (1.03)
Stockholders (total)	1.71	1.81 (1.28)	1.37	1.53 (1.13)	1.02	1.40 (1.07)
Stockholders (direct)	1.71	1.97 (1.30)	1.37	1.69 (1.19)	1.37	1.55 (1.12)

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

A-5.2.1 Frequency of total trades

Total average number of trades in a year is computed as follows. We first define: *Assets purchase indicator:* $P_{jt} = 1$ if in month $t, f_{ijt} > 0$ for at least one financial asset $i > 0$ and 0 otherwise. *Assets sale indicator:* $S_{jt} = 1$ if in month $t, f_{ijt} < 0$ for at least one $i > 0$ and 0 otherwise. That is, we say there is an asset purchase (sale) if there is a net purchase (sale) for at least one of the 25 investment categories. Notice that in the same month both indicators can take value 1 if the investor sells one type of investment and buys another. We then define $T_{jt} = 1$ if ($P_{jt} = 1$ and $S_{jt} = 0$) or ($P_{jt} = 0$ and $S_{jt} = 1$) or ($P_{jt} = 1$ and $S_{jt} = 1$). Implicit here is the idea that a trade is a trip to the bank/broker to buy or sell assets or to do both and in a trip one can buy and/or sell more than one asset. Finally we compute the average number of total trades per year as

$$N_{Tj} = \sum_{t=1}^{35} T_{jt} \times \frac{12}{35}$$

as well as the average number of trades that involve at least one asset sale and at least one asset purchase respectively:

$$N_{Pj} = \sum_{t=1}^{35} P_{jt} \times \frac{12}{35} \quad \text{and} \quad N_{Sj} = \sum_{t=1}^{35} S_{jt} \times \frac{12}{35}$$

Table A-15 shows summary statistics for the total survey sample and the subsamples of direct and indirect stockholders. We report also a decomposition between trades that are sales of assets and trades that are purchases. On average the total number of trades is 4.5 (median 3.4) per year and there is significant heterogeneity in the sample (standard deviation 3.7). On average 2 trades a year involve assets sales and 3.6 involve assets purchases. Notice that the sum of trades involving assets liquidations and those involving assets sales do not sum to the total number of trades; this is because they also include rebalancing trades and

Table A-15: Summary statistics for the average annual number of total trades (N_{Tj})

	Median		Mean		Std Dev
Liquid Assets =	Checking	Broad	Checking	Broad	Checking
All trades (N_{Tj})	3.4	3.4	4.5	4.5	3.7
Of which asset Sales (N_{Sj})	1.4	1.4	2.0	2.0	2.0
Of which asset Purchases (N_{Pj})	2.4	2.4	3.6	3.5	3.6
Stockholders (N_{Tj}) (direct+indirect)	5.1	5.1	5.8	5.8	3.6
Stockholders (N_{Tj}) (direct)	5.8	5.5	6.0	6.0	3.4

Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors. Several of the statistics are computed for two definitions of Liquid Assets. The baseline case uses "Checking Account", and the "Broad" case includes "Time deposits". See discussion in footnote 25.

they are thus counted twice, one as a sale and one as a purchase.[26] For comparison if we use a broader definition of liquid assets that besides checking accounts includes time deposits (and hence it excludes them from financial assets) we obtain that the results are essentially the same.

[26] The estimated trades frequency from the administrative data is positively correlated with the one reported in the survey by asking directly the investors and, as in the latter, stockholders trade significantly more frequently. However there are also important differences. First he average number of trades using the panel data measure is lower than the self reported one; second, the cross sectional distribution of trades frequency estimated from the panel is highly skewed but not as much as in the one from self reported data. However these differences are to be expected. First, in the panel we cannot identify trades that occur at a frequency higher than the month, whereas in the survey some report that they trade daily, weekly or every two weeks; second, the panel-based measure is likely to underestimate the number of trades of those who trade more frequently because we use information on trades only at Unicredit and thus we miss trades at other banks. This is more of an issue for frequent traders who happen also to be wealthier and thus to have investments at more than one bank. Third, the concept of investments differs in the two measures. In the administrative data we include savings accounts which are likely to be excluded from the self reported definition since the question in the survey mentions T-bills, Bonds, Stocks, Mutual funds, Managed accounts etc. , but does not mention savings accounts explicitly. These differences can produce non-classical measurement error which can explain both the difference in mean number of trades and the imperfect correlation. When we adjust the survey-based estimates to make the two more comparable e.g. setting at 12 the number of trades per year (the maximum the panel-based estimate can take) in the survey when people report a larger number) or look at investors who only have accounts with Unicredit the correlation improves.

A-5.2.2 Rebalancing trades

Next we want to separate rebalancing trades to get a sense of their relative importance. We define two notions of rebalancing trades. The first is a broad notion: we define a rebalancing trade any month where there is net sale of one of the financial assets in the investments portfolio and at least one net purchase of another asset class, or vice versa. Hence the number of rebalancing trades in each month is equal to the minimum between the number of net sales and the number of net purchases. For each investor this measure is then annualized. Table A-16 shows summary statistics of this broad definition. In the whole sample the number of trades with some rebalancing is 1.13 per year (median 0.34). Stockholders rebalance almost twice more frequently than non-stockholders a very reasonable feature. The second indicator of rebalancing trades is narrow as it considers as rebalancing all trades that involve a simultaneous purchase and sales of two different investment classes and no net liquidation/purchase of overall investments. That is trades for which the value of sales matches exactly the value of purchases of assets during that month. The mean number of trades with rebalancing only is much smaller: 0.09 per year and around 0.14 for stockholders (Table A-16).

Table A-16: Summary statistics of estimates of the annual number of rebalancing trades

	Number of trades with some rebalancing			Number of trades with only rebalancing		
	Median	Mean	Std dev	Median	Mean	Std dev
Whole sample	0.34	1.13	1.88	0.00	0.09	0.53
Stockholders (direct+indirect)	1.03	1.78	2.29	0.0	0.14	0.70
Stockholders (direct)	1.03	1.85	2.30	0.00	0.13	0.65

Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

To get a sense of the relative importance of rebalancing trades on total trades we have computed the ratio of rebalancing to total for each household. Table A-17 shows summary statistics for the whole sample and from the sample of stockholders. Depending on whether we use the broad or narrow measure of rebalancing this ratio ranges between 2% and 20% of total trades. Thus overall rebalancing trades are only a small fraction of total trades. Looking among the largest trades, say those on the 95th percentile of the size distribution of trades, only half of these trades involve some rebalancing, and a substantial fraction (38%) never rebalances.

For comparison if we use a broader definition of liquid asset that includes time deposits, and hence excludes them from financial investments, we obtain that the results are essentially

Table A-17: Average ratio of rebalancing trades on total financial trades

	ratio of trades with some rebalancing			ratio of trades with only rebalancing		
	Median	Mean	Std dev	Median	Mean	Std dev
Whole sample	0.13	0.18	0.21	0	0.017	0.07
Stockholders (direct+indirect)	0.21	0.25	0.22	0	0.022	0.09
Stockholders (direct)	0.21	0.25	0.22	0	0.018	0.08

Source: *Unicredit* survey sample, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

the same. With this alternative measure of liquid/financial asset we will have that the average ratio of trades with some rebalancing have a median (mean) of 0.34 (and 1.1), and those with rebalancing only have a median (and mean) of 0 (and 0.08).

A-5.3 Evidence on assets sales and durable purchases

In this section we present two types of evidence on the durables goods / asset liquidations model. The first type of evidence shows that liquidations become more frequent and their size larger around the purchase of a house. The second exploits the time relation between investments liquidations and changes in the liquid assets (checking account).

A-5.3.1 Asset sales around the purchase of a house

To obtain direct evidence on the link between investments liquidations and durable purchases we rely on the large panel. Unicredit has agreed to add to each observation a flag for those households that over the 35 months covered in the sample have obtained a final approval for mortgage in a certain month. In Italy the closing of the house purchase takes place normally as soon as the final approval of the mortgage takes place. Hence we focus on investments liquidations in the months around a house purchase. We found that 875 households out of 40,000 in our sample have obtained a mortgage (and thus bought a house). We first focus on this group of investors and compute the fraction that liquidate investments in the same month they obtain the mortgage and at several months before and after obtaining the mortgage. Figure 4 shows the pattern of liquidations at various leads and lags over a 24 months window centered around the month of the purchase, month 0. The fraction of house purchasers who sell financial investments starts increasing around four months before the purchase and peaks exactly in the same month they settle for the payment. After the mortgage is obtained the fraction of asset sales drops and stays roughly constant. This pattern is fully consistent with investors timing assets liquidations to meet the house payment. We briefly comment on the level of this fraction, as it relates to the prediction of our model. While it is clear that the fraction is much larger in the months before than right after a mortgage is obtained, not all the investors sell assets just before obtaining a final approval of a mortgage by Unicredit. Some reasons why this may be the case are the following: i) some mortgages may be obtain when investors refinance, although this is very rare in Italy, ii) some mortgages may be obtained when investors move to a smaller house, iii) some of the assets can be obtained from a different bank.

To show more formal evidence we run controlled probit regressions for the decision to sale investments and Tobit estimates for the amount of asset sold using the following specification

$$y_{it} = \sum_{j=-6}^{+6} \alpha_j DF_{it-j} + \beta w_{it_0} + \lambda S_{iT_0} + \delta' T + \eta_{it}$$

Where y_{it} is the outcome variable (a dummy = 1 if a sale of investments occurs in month t in the probit, or the amount of liquidated investments in the Tobit estimate), the variable DF_{it} is a dummy equal to 1 if a mortgage is obtained in month t; to model the timing of asset sales we include up to 6 leads and lags in this indicator in our specification; the variable w_{it_0} is the value of the total assets of the investors at the start of the sample (denoted as t_0), the variable S_{iT_0} a dummy equal to 1 if the investors owns stock at the beginning of the sample, the T denote a vector of time dummies and η_{it} a regression error. Results of the estimates as shown in Table A-18; the first column reports the probit estimates and the second column the Tobit estimates for the size of liquidations. Results are rather clear. Investors start selling investments around four months before the purchase of the house. The likelihood of a asset sale and its size both increase markedly as the month of the purchase approaches. In the month when the purchase of the house occurs the probability that the investor liquidates investments is 7 percentage points above the average probability of a liquidation in a generic month (which is 9.6 percent) and the size of liquidation is 34,400 euro above mean. Cumulatively over the month of purchase and the four months before the probability that an investors who purchases a house liquidates assets is 20.9 percentage points above the mean probability and the liquidation size is 103,000 euros above mean. At lags higher than 4 there is no statistically significant difference neither in the probability of liquidating investments nor on their size; the same is true after the purchase. As the estimates shows all the lead coefficients are not statistically different from zero. We find this pattern consistent with investors timing assets liquidations in view of the house purchase.

A-5.3.2 Investments Sales and Liquid Asset

The information on flows of investments liquidation and purchases and on changes in the liquid asset (i.e. checking account) can be used to provide some additional evidence of the link between trades where investments are sold and durables purchased. For this we use the temporal patterns of asset sales and liquid asset changes in our panel data to show that the spending rate of liquid assets that comes from asset sales is at least twice as fast as the one consistent with a model with steady expenditure financed with cash and the observed

Table A-18: Timing of assets sales and house purchases

Regressor	Probit estimates for asset sale decision: Coefficient	Standard error	Tobit estimates of size of asset sold: Coefficient	Standard error
Obtained mortgage:				
α_0 : current	0.070***	0.012	34413.8***	3889.2
α_1 : lag 1	0.053***	0.012	26985.6***	4075. 6
α_2 : lag 2	0.029***	0.011	12338.9***	4437.1
α_3 : lag 3	0.027***	0.011	14200.6***	4457.9
α_4 : lag 4	0.030***	0.011	15696.9***	4489.7
α_5 : lag 5	0.007	0.010	1778.6	4982.8
α_6 : lag 6	0.017	0.011	5840.4	4949.9
α_{-1}: lead 1	-0.010	0.010	-6863.4	5103.3
α_{-2}: lead 2	0.003	0.010	3993.9	4868.3
α_{-3}: lead 3	-0.001	0.010	288.3	5015.2
α_{-4}: lead 4	0.002	0.010	-1205.9	5128.6
α_{-5}: lead 5	0.007	0.010	2277.3	5154.2
α_{-6}: lead 6	0.013	0.011	6592.5	5142.4
Other regressors:				
β : Investor total assets	1.51e-07***	1.24e-08	0.124***	0.006
λ: Stockholder	0.094***	0.003	42174.19***	2294.0
N. observations	31247		31247	
Pseudo R^2	0.07		0.0164	

Coefficients of the probit estimates are marginal effects. ***; **; denote significance at 1% or less and 5% respectively. Source: *Unicredit large sample*, monthly administrative records (35 months) of 26 accounts for each of 40,000 investors, out of which 875 have obtained a mortgage during one of the 35 months.

frequency of asset liquidations.

We run a regression between C_{jt} -the net flow of euros in the checking account in a given month- and the net investments flow F_{jt} distinguishing between the net flow of investment sales F_{jt}^S and investment purchases F_{jt}^P also in euro amounts during the same month, as well as with lags. Empirically four lags are sufficient to characterize the dynamics. We notice that by construction F_{jt}^S and F_{jt}^P are either zero or positive. So for instance $F_{jt}^S = 100$ means that over that month there is a net investments sale of 100 euros. Likewise if $F_{jt}^P = 100$ there is a net purchase of assets for 100 euros of value during the month. Thus F_{jt}^S and F_{jt}^P are never positive in the same month t for an investor j and they are zero if there are no trades with a net cash flow. The regression we run is

$$C_{jt} = \sum_{k=0}^{4} \beta_k F^S_{jt-k} + \sum_{k=0}^{4} \gamma_k F^P_{jt-k} + \delta W_{jt} + h_j + u_{jt}$$

where W_{jt} is investor j total financial assets, h_j is an investor j fixed effect and u_{jt} an error term. Estimated coefficients are shown in Table A-19. To interpret these estimates remember that from Table A-14 the annual number of assets sales for the median household is around 1. If assets sales were used mostly to finance a steady flow of consumption expenditures one would expect that since they occur once a year the euros obtained from the assets sold would be spent out at a rate of roughly 1/12 for each euro of assets sold. Hence in the first month one should see an increase in the cash account of about 0.92 cents per euro of investments liquidation and a negative effect of about 0.08 cents in the subsequent months. Instead we see a smaller increase in liquid assets over the same month of the sale of assets and a much faster decrease in the following two months. In the two month subsequent to one euro of asset sale, approximately 60 cents are spent (the sum of (1-0.703)+0.23+0.16). Instead the pattern that we document can be generated by a model where sales of investments are targeted to large disbursements, such as those involving durables goods purchases.

For comparison if we use a broader definition of liquid asset that includes time deposits (hence excludes them from financial investments) we obtain that the results are essentially the same. In particular the pattern of coefficients of the lags of investments sales on change in liquid asset account, i.e. of the coefficients β_k for $k = 0, 1..., 4$ are $0.70, -0.22, -0.16, 0.002$ and -0.03 respectively.

A-5.4 Background risk controls on Table 3

In this section we include controls for background risk on the regressions of liquid asset relative to consumption on trading frequency. We use the Unicredit sample. In this data set we have the following proxies for background risk: two occupational dummies for self employed and government employees and a measure of background risk using a self-reported measure of income risk. This indicator equals one if the respondent is unable to predict if his or her income will fall significantly, increase significantly, or remain unchanged in the 5 years following the interview. Running regression in the sample of sole self-employed or in the sample of sole government employees leads to a very similar estimate of the slope regression of log liquid assets on the number of trades.

Table A-19: Temporal pattern of changes in the liquid and investments assets

Change in liquid asset in a month		
Regressors	Coefficient	Standard Error
Flow of investment sales:		
β_0: current	0.703***	0.0057
β_1: lag 1	-0.23***	0.0062
β_2: lag 2	-0.16***	0.0065
β_3: lag 3	0.002	0.006
β_4: lag 4	-0.03	0.0065
Flow of investment purchases:		
γ_0: current	-0.65***	0.0065
γ_1: lag 1	0.020***	0.007
γ_2: lag 2	-0.076***	0.007
γ_3: lag 3	0.056***	0.007
γ_4: lag 4	-0.011**	0.006
Investor total assets:		
δ :	0.092***	0.0025
N. observations	31622	
R^2	0.47	

OLS regressions of the net flow into the checking account on the net flow of investments sales and purchases. Estimates include investors fixed effects. *** ** indicate significant at 1% or less and 5% respectively. Source: *Unicredit survey sample*, monthly administrative records (35 months) of 26 accounts for each of 1541 investors.

Table A-20: Liquid assets and portfolio trades

	Table 3 last col. Whole sample	Table 3 last col. stockholders	Whole sample and controls for background risk	Stockholders and controls for background risk
	log liquid assets: log $M2$/cons.	log liquid assets: log $M2$/cons.	log liquid assets:: log $M2$/cons.	log liquid assets: log $M2$/cons.
log N. of trades				
financial invest.	0.1266***	0.1082***	0.1155***	0.0962***
	(0.0241)	(0.0289)	(0.0251)	(0.0301)
log household	-0.5243***	-0.6936***	-0.5600***	-0.6948***
consumption	(0.1404)	(0.1574)	(0.1419)	(0.1598)
log household	0.2882**	0.4457***	0.2529**	0.3439**
income	(0.1191)	(0.1501)	(0.1236)	(0.1569)
Years of education	0.0017	0.0061	0.0155	0.0239
	(0.0104)	(0.0132)	(0.0117)	(0.0147)
Age	0.0180***	0.0200***	0.0202***	0.0228***
	(0.0031)	(0.0041)	(0.0034)	(0.0044)
Resident in the	-0.3876***	-0.4043***	-0.3965***	-0.3840***
North	(0.0850)	(0.1080)	(0.0886)	(0.1120)
Male	0.0379	0.1091	-0.0235	0.0843
	(0.0970)	(0.1282)	(0.1072)	(0.1431)
Married	-0.2410**	-0.1694	-0.1867*	-0.1377
	(0.0944)	(0.1210)	(0.0982)	(0.1258)
Resident in a	0.1960**	0.1717	0.2287***	0.1705
small city	(0.0848)	(0.1070)	(0.0882)	(0.1107)
stockholder	0.1067		0.1016	
	(0.0908)		(0.0940)	
Self employed			0.1814	0.2365
			(0.1243)	(0.1517)
Government			-0.1387	-0.1397
employee			(0.1000)	(0.1255)
Income risk			-0.1903*	-0.3201**
dummy			(0.0989)	(0.1273)
No. Observations	1363	875	1269	822
R^2	0.069	0.073	0.077	0.087

Source: The independent variable in each of the regressions is the log of the ratio of $M2$ to inputed durable consumption. *** ** indicate significant at 1% or less and 5% respectively, standard errors in parenthesis. *Source: Unicredit survey sample*, of monthly administrative records (for 35 months) of 26 accounts augmented by survey questions for 1541 investors.