

Spontaneous Discrimination

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Abstract

This paper considers a dynamic economy where agents are repeatedly matched with one another and they decide whether to form a profitable partnerships. Each agent has a physical colour and a *social colour*. The social colour of an agent is a signal about the physical colour of agents in his partnership history. Before an agent makes a decision, he observes the other's physical and social colours. Neither the physical colour, nor the the social colour is payoff-relevant.

We identify environments where, in some equilibria, agents condition their decisions on the colours of their partners, that is, they discriminate. The main result of the paper is that every *stable* equilibria involve discrimination in these environments.

1 Introduction

Consider a white community in the south where some people are members of the Ku Klux Klan. They dislike and are willing to punish anybody who is not white, and even those who are *associated* to non-whites. The rest of the community has no bias against people of other skin colour. People observe, perhaps imperfectly, each others' interactions in the community. Suppose now that a non-white individual seeks employment in this community. Of course, he will not be hired by Klan members. But even the unbiased people might be reluctant to give him a job because they are afraid of being punished by the Klan. At the end, the non-white individual could remain unemployed, that is, the whole community might end up discriminating against non-whites. Crucial in this story is that individuals observe some information about the interactions of others, for otherwise, unbiased individuals would not be afraid of hiring. But how large does the Klan have to be in order to sustain discrimination? This paper shows that even if nobody belongs to the Klan, the community might end up discriminating against non-whites.

This paper puts forward a new theory of racial discrimination. This theory is based on the assumption that each individual carries information about the physical colour of those to whom

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he is associated through past interactions. Individuals can condition their decisions on these information. The main result of this paper is that individuals might prefer to interact with others of their own colour and with those who are not associated to other colours. In other words, being associated to one's own physical colour becomes valuable through the equilibrium play.

To give an intuition for this result, let us revisit the story above and consider an unbiased member of the community. He might not want to interact with anybody who employed a non-white worker because he is afraid of being punished in the future for being indirectly associated to non-whites. In fact, for the same reason, he might not want to interact with those who are only indirectly associated to non-whites but never employed them. As a result, employing non-whites and being associated to them is punished not only by the Klan, but by the unbiased community members who are concerned about their associations. This concern can be self-enforcing and independent of the Klan. If unbiased individual are reluctant to interact with those who are associated with non-whites then unbiased individuals are better off not being associated to non-whites.

In the specific model analysed in this paper, agents are repeatedly matched with each others. After being matched, agents have to decide whether or not to enter into a profitable relationship. Each agent maximizes the discounted present value of expected monetary payoffs. Every relationship generates positive payoffs for both parties. Each agent has a physical colour which is either black or white. Before an agent makes a decision about forming a relationship, he observes the physical colour of his potential partner and an additional piece of information about his history of relationships. We model this information as a binary signal, either black or white, and refer to it as the *social colour* of the agent. If an agent decides to enter into a partnership with another one, his social colour becomes the physical or social colours of his partner with positive probability.

In our model, there always exists a *colour-blind* equilibrium where agents ignore the physical and social colours of each others. We show that, in general, there are other equilibria where the decisions of the agents are conditioned on colours, that is, agents discriminate. We then investigate the stability properties of the equilibria. An equilibrium is called *stable* if, after perturbing the equilibrium strategies slightly, a myopic best-response dynamics converges back to the equilibrium. The main result of the paper is that, under certain conditions, each stable equilibrium involves discrimination. In particular, the colour-blind equilibrium is unstable.

We emphasize that both the physical and social colours of the agents are payoff-irrelevant. Each agent cares only about monetary payoffs, and have no intrinsic preferences for the colours of his partners. Therefore, the equilibrium discrimination in our model is not *taste-based*. In addition, agents of different types are identical in terms of payoff-relevant characteristics both from the ex ante and the ex-post points of view. That is, the colours of the agents reveal no information regarding the profitability of his partnership. Therefore, the discrimination in our model is not a statistical one.

There is a large literature on taste-based discrimination, see Becker (1971) and Schelling (1971).

These theories explain racial discrimination by assuming that individuals derive disutility from interacting with members of another race. Such preferences may be the result of group selection if a group does well relative to other groups if its members cooperate only with other group members. Alternatively, they might be outcomes of group formation processes. Similar people tend to have similar backgrounds which equips them with similar tastes, values, and attitudes. This facilitates making collective choices (Baccara and Yariv (2008), see also Alesina and Ferrara (2005)).

A common critique of tased based theories is that employers who do not discriminate make larger profits than those who discriminate and, hence, they would overtake competitive markets. In our model, an employer who does not discriminate also has higher instantenous profits. However, these short-term gains from colour-blind hiring policy is offset by being boycotted by members of his own race in the future. That is, it is precisely the profit-maximizing behaviour which leads to equilibrium descrimination.

According to statistical discrimination theories, employers believe that observable physical attributes of the workers are correlated with unobservable but payoff-relevant characteristics. For an overview of statistical discrimination theories, see Fang and Moro (2010). Phelps (1972) explains differences in wages of black and white workers by assuming that the unobservable productivity of a worker is correlated with his colour. Employers use the colour as a signal about the productivity of a worker.

Arrow (1973) shows that discrimination can be a result of self-fulfilling expectations even if all agents are identical ex ante. Workers in his model can decide how much to invest in human capital. These decisions are not observable. Employers expect black workers to invest less than white workers and, hence, they offer lower wages to black workers. Anticipating this, black workers rationally invest less in human capital than white workers. As a result, workers of diffent colours are different ex-post. Coate and Loury (1993) places Arrow's arguments in an equilibrium model but treats wages exogenously, like our model does. This assumption is relaxed in Moro and Norman (2004). Rosén (1997) offers another explanation for self-fullfilling statistical discrimination. In this model, workers privately observe their idiosyncratic productivity prior to applying for a job. If black workers apply for jobs with lower productivities, firms rationally expect white applicants to be more productive. Therefore, firms prefer to hire white workers which results in a lower value for unemployed black workers. As a consequence, black workers rationally apply for jobs even if they are less productive. In Mailath, Samuelson, and Shaked (2000), employers perfectly observe the worker's productivities. However, the employers may decide not to search for workers among black workers in anticipation of low skill-investment.

The workers in our model are identical both ex-ante and ex-post. A notable difference between statistical discrimination and our theories is that white and black individuals might mutually discriminate each others in our model. Such a phenomenon is inconsistent with statistical discrimination because the signal value of the colour must be the same for an employer regardless of his own colour.

Lang, Manove, and Dickens (2005) shows that small taste-based or statistical discriminations can have large effects. They show that even if employers have only lexicographic preferences towards hiring white workers, or white workers only slightly more productive than black ones, white workers might end up with significantly higher wages than black ones.

There are a few models where discrimination arises without any payoff-relevant differences between agents of different colours. Eeckhout (2006) considers a dynamic marriage market where individuals are randomly matched. Once a marriage is formed, the parties repeatedly play the Prisoner's Dilemma. If either party defects, both individuals return to the market and are re-matched. In order to induce some cooperation, the equilibrium play must involve defection with positive probability at the beginning of a marriage. Otherwise, agents would defect and search for a new partner. The author shows that any colour-blind equilibrium is Pareto dominated by strategies where the probability of defection depends on the colour of the partner.

A white agent discriminates against black workers in our model because of the threat of not being hired by other white agents in the future. The punishment by peers against acting differently from the peers' is a well-known phenomenon in sociology as well as in economics, see Austen-Smith and Jr. (2005) and the references therein.

In the model of Mailath and Postlewaite (2006), there is a population of men and women who are matched and produce offspring in each period. Agents differ in their non-storable endowments and care about the consumptions of their descendants. In addition, some agents have a physical attribute such as blue eye-colour. This attribute is inherited by the offspring. There exist equilibria where the attribute has a value, that is, agents with the attribute are better off than agents without it. This is because agents with high endowments and no attribute prefer to match with the agents with low endowment and attribute rather than with agents with high income and no attribute. By doing so, they forego present consumption but increase the expected consumption of their offsprings by equipping them with the attribute. In other words, the biological attribute is used to transfer wealth to future generations.¹ The social colour in our model is payoff-irrelevant but has a value in some equilibria, much like the biological attribute in Mailath and Postlewaite (2006).

The social colour in our model plays a role which is similar to the *labels* in Kandori (1992). Kandori considers a model where members of two communities interact with each others repeatedly. Each member of a community is randomly matched with a member of the other community and plays a game in every period. Players only observe the actions played in their past matches. However, each player carries an observable information, a label, about his past history of actions. A label of an individual is determined by his previous label, his partner's label, and the action he takes. The author proves a Folk Theorem for this setting. In the light of this paper, the fact that against condition their actions on payoff-irrelevant information in some equilibria is perhaps less surprising. We emphasize that our main result is that every stable equilibrium involves discrimination in some environment.

¹A similar explanation has been proposed to explain the evolution of peacock tails Ridley (1993)

2 The Model

There is a unit mass of population of agents. Each agent lives forever and is risk-neutral. Time is continuous and the common discount rate is r .

Agents randomly receive opportunities to participate in production. These opportunities arrive independently across agents and time according to a Poisson distribution with arrival rate δ . Agents with opportunities are matched with one another instantaneously. In each match, one of the agents becomes an *employer* and the other one becomes a *worker* with equal probabilities.² The two agents observe a match specific shock, s , which is exponentially distributed, that is, $G(s) = 1 - e^{-\lambda s}$. Then, the employer decides whether or not to employ the worker. If the employer employs the worker, he receives a payoff of s and the worker receives a constant wage, $M (> 0)$.³ Otherwise, both agents receive a payoff of zero. Each agent maximizes the discounted present value of monetary payoffs.

Each agent has a two-dimensional type. The first coordinate is the physical colour of the agent and the second one is the *social colour*. The physical colour is either black (b) or white (w) and never changes. A fraction of μ_w of the population is white and fraction of μ_b is black. The social colour is also either black or white and evolves as follows. The social colour of a worker does not change. If an employer employs a worker with type (c_1, c_2) then the social colour of the employer remains the same with probability $1 - \gamma$, changes to c_1 with probability $\gamma\alpha$ and becomes c_2 with probability $\gamma(1 - \alpha)$. If the employer decides not to employ, his social colour remains the same with probability $(1 - \gamma)$ and becomes his physical colour with probability γ .

Note that the social colour of an agent carries information about his past employees. An agent's social colour is more likely to be white if he hired white workers, or workers with white social colour in the past.

Notice that the types of the agents are payoff irrelevant in the sense that the payoff of an agent only depends on the history of shock realizations and employment decisions but not on his type and the types with whom he interacts. If there were no types, there was a unique equilibrium in this model where employers always employ the workers. In fact, this is true even if agents have physical colours but no social colours. This is because an employer receives a positive payoff if he employs the worker and in the absence of social colour, such a decision cannot affect his future employment.

In this model, only the employers make decisions. A strategy of an employer is a mapping from his past history, his type, and the type of the worker into an employment decision. In what follows, we restrict attention to *steady state equilibria*, that is, we characterize equilibria where the agents' strategies depend neither on time, nor on history but can depend on his type.

²Following the convention of the literature on racial discrimination, we adopt the employer-employee terminology. However, we interpret a partnership as any mutually beneficial social or economic interaction.

³That is, the total surplus generated in a relationship is $s + M$.

3 Best Responses

If an employer is better off employing a worker given a certain realization of the shock then he is also better off employing him if the realization of the shock was higher. Therefore, the employment decision can always be characterized by a cutoff. Of course, these cutoffs can depend on the type of the employer and the worker. Next, we characterize these cutoffs in terms of the value functions. To be more specific, we fix a steady-state population strategy and derive the best-response cutoffs of each agent. To this end, let V_{c_1, c_2} denote the value function of an agent with type $(c_1, c_2) \left(\in \{b, w\}^2 \right)$ at a moment when the agent does not have a production opportunity. That is, V_{c_1, c_2} is the maximum discounted present value of the payoffs what an agent can achieve with type (c_1, c_2) .

Consider, for example, the decision problem of a white employer with social colour c who faces a worker with type (b, w) and observes a shock s . If he employs the worker, he receives a payoff of s . His social colour remains c with probability $(1 - \gamma)$ and changes to b or w with probabilities $\gamma\alpha$ and $\gamma(1 - \alpha)$, respectively. Hence, if the worker is employed, the discounted present value of the employer's payoffs is

$$s + (1 - \gamma) V_{w, c} + \gamma\alpha V_{w, b} + \gamma(1 - \alpha) V_{w, w}. \quad (1)$$

If he does not employ the worker, his value would be

$$(1 - \gamma) V_{w, c} + \gamma V_{w, w}. \quad (2)$$

Therefore, the agent employs the worker whenever (1) is larger than (2). The cutoff, above which the employer hires the worker, is the shock s where (1) and (2) are equal, that is, $\gamma\alpha (V_{w, w} - V_{w, b})$. Since the shock is always positive, having a negative cutoff is equivalent to having a zero cutoff. Therefore, one can restrict attention to weakly positive cutoffs, in which case, the best-response cutoff is uniquely defined by $\max \{0, \gamma\alpha (V_{w, w} - V_{w, b})\}$.

Notice that this cutoff does not depend on the social colour of the employer, c . In both (1) and (2), the only term which depends on c is $(1 - \gamma) V_{w, c}$, and therefore, c cancels out in the computation of the cutoff. In fact, the cutoff of an agent never depends on his social colour in equilibrium. This is because the social colour of an employer only affects his payoff in the event when his new social colour remains his old one, and this event is independent of his decision. Therefore, the equilibrium cutoff of an agent can only depend on his physical colour but not on his social colour.

Let x_{c_1, c_2}^c denote the cutoff value of an employer with physical colour c if the type of the worker is (c_1, c_2) . We denote the colour which is not c by $-c$ for $c \in \{w, b\}$. Above, we have shown that $x_{b, w}^w = \max \{0, \gamma\alpha (V_{w, w} - V_{w, b})\}$. The other cutoffs can be computed similarly and they are summarized by the following

Lemma 1 *The following equations establish the relationship between best-response cutoffs and the value functions:*

$$\begin{aligned} x_{-c,-c}^c &= \max \{0, \gamma (V_{c,c} - V_{c,-c})\}, \\ x_{c,-c}^c &= \max \{0, \gamma (1 - \alpha) (V_{c,c} - V_{c,-c})\}, \\ x_{-c,c}^c &= \max \{0, \gamma \alpha (V_{c,c} - V_{c,-c})\}, \\ x_{c,c}^c &= 0. \end{aligned}$$

If an employer with physical colour c is considering hiring a worker, he is concerned about his new social colour. Having a social colour c instead of $-c$ provides the agent with an additional value of $V_{c,c} - V_{c,-c}$. This difference can be interpreted as a *bias* of the agent towards his own physical colour.⁴ The lemma says that the best-response cutoffs are proportional to this bias up to the requirement that the cutoffs are non-negative. The coefficients of the bias corresponding to various cutoffs are determined by the probabilities of the social colour becoming c and $-c$, which in turn, depends on the type of the worker.

Let $x^c = x_{-c,-c}^c$ and notice that

$$x_{c,-c}^c = (1 - \alpha) x^c, \quad x_{-c,c}^c = \alpha x^c, \quad \text{and} \quad x_{c,c}^c = 0. \quad (3)$$

Observe that the value functions of two agents with the same type are identical. Therefore, this lemma implies that any stationary equilibrium is symmetric. That is, employers with the same physical colour use the same strategies. Also notice that an equilibrium strategy of an employer with physical colour c can be identified with x^c . In what follows, we refer to the cutoff x^c as a strategy (or cutoff) while keeping in mind that the cutoffs against different types of workers are defined by (3).

Suppose that each employer with physical colour c uses cutoff x^c . In order to show that this strategy profile is an equilibrium, x^c must be a best-response against (x^c, x^{-c}) for $c \in \{b, w\}$. Next, we explicitly characterize the best-response cutoffs. In fact, we characterize best responses against any stationary distribution of strategies as long as these strategies satisfy the equations in (3). This turns out to be useful when we examine the stability properties of the equilibria. Let X^c denote the random variable corresponding to the distribution of cutoffs in the population. We shall compute $V_{c,c} - V_{c,-c}$ for $c \in \{b, w\}$ and, by Lemma 1, these objects identify the best-response cutoffs.

Let Π_{c_1, c_2}^l and Π_{c_1, c_2}^e denote the value functions of a worker and employer with type $(c_1, c_2) \in \{b, w\}^2$, respectively. The heuristic equation describing the relationship between V_{c_1, c_2} , Π_{c_1, c_2}^l and Π_{c_1, c_2}^e is:

$$V_{c_1, c_2} = (1 - \delta dt)(1 - r dt) V_{c_1, c_2} + \delta dt \left(\frac{1}{2} \Pi_{c_1, c_2}^s + \frac{1}{2} \Pi_{c_1, c_2}^e \right).$$

To see this, notice that an agent does not receive an opportunity with probability $1 - \delta dt$ in the next dt time, and hence his value remains V_{c_1, c_2} . This is discounted at the rate r . With the remaining

⁴This bias may well be negative, that is, an agent is better off if his physical colour does not coincide with his social colour.

probability, the agent receives an opportunity and becomes an employer or a worker with equal probabilities. Dividing through by dt and taking the limit as dt goes to zero:

$$V_{c_1, c_2} = \frac{\delta}{\delta + r} \left(\frac{1}{2} \Pi_{c_1, c_2}^l + \frac{1}{2} \Pi_{c_1, c_2}^e \right). \quad (4)$$

A worker with type (c, c) is employed whenever he is matched with an employer with physical colour c , which happens with probability μ_c . He also gets employed whenever he is matched with an employer with physical colour $-c$ whose cutoff is x^{-c} and $s \geq x^{-c}$. This happens with probability $\mu_{-c}(1 - EG(X^{-c}))$, where the expectation is taken according to the distribution of the cutoff X^{-c} . Finally, the worker's value changes to $V_{c,c}$ and gets M whenever he is employed, therefore,

$$\Pi_{c,c}^l = M(\mu_c + \mu_{-c}(1 - EG(X^{-c}))) + V_{c,c}. \quad (5)$$

. Similarly,

$$\Pi_{c,-c}^l = M(\mu_c(1 - EG((1 - \alpha)X^c)) + \mu_{-c}(1 - EG(\alpha X^{-c}))) + V_{c,-c}. \quad (6)$$

Using (4), (5), and (6) we can express $V_{c,c} - V_{c,-c}$ as follows:

$$\begin{aligned} V_{c,c} - V_{c,-c} &= \frac{\delta}{\delta + r} \left[\frac{1}{2} (\Pi_{c,c}^l - \Pi_{c,-c}^l) + \frac{1}{2} (\Pi_{c,c}^e - \Pi_{c,-c}^e) \right] \\ &= \frac{\delta}{\delta + r} \frac{1}{2} M [\mu_c EG((1 - \alpha)X^c) + \mu_{-c} (EG(\alpha X^{-c}) - EG(X^{-c}))] \\ &\quad + \frac{\delta}{\delta + r} [V_{c,c} - V_{c,-c}] \end{aligned}$$

That is,

$$V_{c,c} - V_{c,-c} = \frac{M\delta}{2r} [\mu_c EG((1 - \alpha)X^c) + \mu_{-c} (EG(\alpha X^{-c}) - EG(X^{-c}))].$$

Recall from Lemma 1 that the best-response cutoff of an employer with physical colour c against a worker with type $(-c, -c)$ is $\gamma(V_{c,c} - V_{c,-c})$. Let $K = M\delta\gamma/2r$. Then substituting in from the previous displayed equality:

$$K [\mu_c EG((1 - \alpha)X^c) + \mu_{-c} (EG(\alpha X^{-c}) - EG(X^{-c}))]. \quad (7)$$

As we pointed out earlier, agents with the same physical colour use the same cutoffs in every equilibrium. Suppose that each employer with physical colour c uses x^c , that is, $X^c \equiv x^c$. Then the best response of an agent can be written as

$$\tilde{b}^c(x^c, x^{-c}) = K [\mu_c G((1 - \alpha)x^c) + \mu_{-c} (G(\alpha x^{-c}) - G(x^{-c}))]. \quad (8)$$

Recall that since the shocks are always positive, one can restrict attention to weakly positive cutoffs, in which case, the best-response correspondence is uniquely identified by

$$b^c(x^c, x^{-c}) = \max \left\{ 0, \tilde{b}^c(x^c, x^{-c}) \right\}. \quad (9)$$

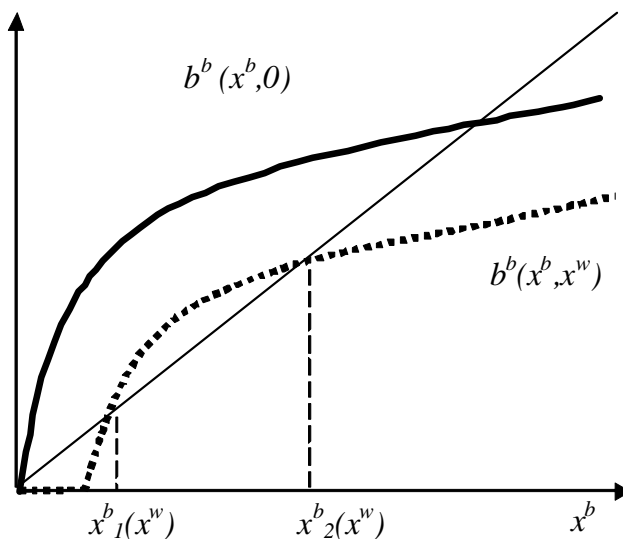


Figure 1: Best Responses

Then (x^c, x^{-c}) defines an equilibrium if and only if

$$(x^c, x^{-c}) = (b^c(x^c, x^{-c}), b^{-c}(x^{-c}, x^c)). \quad (10)$$

A notable feature of the best response function is that it does not depend on the distribution of social colours. The next two lemmas describe some properties of the best-response curves.

Lemma 2 *The function b^c satisfies the following properties:*

- (i) *for each x^{-c} , if $b^c(x^c, x^{-c}) > 0$ then b^c is locally concave in x^c ,*
- (ii) *$b^c(0, x^{-c}) = 0$ for all x^{-c} ,*
- (iii) *for all $\bar{x}^{-c} > 0$, $b^c(x^c, 0) = \lim_{x^{-c} \rightarrow \infty} b^c(x^c, x^{-c}) \geq b^c(x^c, \bar{x}^{-c})$.*

Part (ii) implies that the function $b^c(x^c, 0)$ intersects with the 45 degree line at $x^c = 0$. Whether or not there is another intersection has great importance at characterizing the type of equilibria.

Lemma 3 *Let $\lambda_0 = 1/(K(1-\alpha)\mu_c)$. Then,*

- (i) *if $\lambda > \lambda_0$, then there exists a unique $x^c > 0$ such that $b^c(x^c, 0) = x^c$, and*
- (ii) *if $\lambda \leq \lambda_0$, then $b^c(x^c, 0) < x^c$ for all $x^c > 0$.*

Figure 1 plots $b^b(x^b, 0)$ and $b^b(x^b, x^w)$ ($x^w > 0$) for the case of a large λ . The function $b^c(x^c, x^{-c})$ is identical to $\tilde{b}^c(x^c, x^{-c})$ when $x^{-c} = 0$. For $x^{-c} > 0$, $b^c(x^c, x^{-c})$ is a downwards shift of $b^c(x^c, 0)$ except it is zero whenever the shifted curve was negative. Since \tilde{b}^c is concave in x^b , $b^b(x^b, x^w)$ is also locally concave in x^c whenever it is positive (part (i) of Lemma 2). Part (ii) of Lemma (2) says that if the cutoff of each agent with colour c is zero then the best response cutoff

of an agent with colour c is also zero. To see this, notice that if $x^c = 0$ then agents with physical colour c are better off having a social colour $-c$ than c . This is because their social colours have no impact on their employments if the employer has colour c ($x^c = 0$) but they will be more likely to be employed by agents with physical colour $-c$ if their social colour is $-c$. Therefore, an employer with physical colour c always employs a worker with type $(-c, -c)$, that is, the best-response cutoff is zero.

Part (iii) of Lemma (2) states that the best response cutoff of an agent with physical colour c is the same when agents with physical colour $-c$ do not discriminate ($x^{-c} = 0$) or they totally discriminate ($x^{-c} = \infty$). To see this note that an agent with physical colour c is always employed by colour $-c$ agents if ($x^{-c} = 0$) and is never employed by them if ($x^{-c} = \infty$). In both cases, the social colour of an agent with colour c has no impact on his employment if he meets an employer with colour $-c$. Therefore, in either case, the best-response is determined solely by the cutoff x^c . Part (iii) also says that b^c decreases if x^{-c} becomes larger than zero. The intuition is as follows. As x^{-c} becomes positive, a worker with colour c benefits from having a social colour $-c$ whenever he meets an employer of colour $-c$. Therefore, holding x^c fixed, an agent with colour c will discriminate less against workers with type $(-c, -c)$, that is, b^c goes down.

The slope of $b^c(., 0)$ is large if and only if λ is large. In addition, this slope converges to zero as x^c goes to infinity. Therefore, the function $b^c(., 0)$ intersects with the 45-degree line at a strictly positive value if and only if λ is large (Lemma 3).

4 Equilibrium Characterization

An immediate implication of part (ii) of Lemma 2 is that if the cutoff of each agent is zero, then the best-response is also zero. That is,

Remark 1 *The cutoff profile defined by $x^c = x^{-c} = 0$ is an equilibrium.*

Next, we show that if λ is large, exists at least one other equilibrium which involves discrimination. Suppose for a moment that agents with physical colour $-c$ are non-strategic and their cutoff is zero, and consider our model as a game played by only agents with colour c . Part (i) of Lemma 3 implies that there exists an x^c such that $b^c(x^c, 0) = x^c$, that is, the best response of an agent is x^c whenever every other agent with colour c uses cutoff x^c . In other words, the cutoff x^c is an equilibrium in the game where only colour- c agents are strategic. Since this cutoff is positive, agents discriminate others with physical and social colours $-c$. What if agents of colour $-c$ become strategic? Part (ii) of Lemma 2 implies that zero is a best response of an agent with colour $-c$ as long as other agents with colour $-c$ have cutoffs zero. That is, if the cutoff profile is $(x^c, 0)$ then each agent best responds. We can conclude the following

Remark 2 *If $\lambda > 1/(K(1 - \alpha)\mu_c)$ then for each $c \in \{b, w\}$, there is a unique $x^c > 0$ such that the cutoff profile $(0, x^c)$ is an equilibrium.*

In general, if (x_*^c, x_*^{-c}) is an equilibrium cutoff profile, then it satisfies (10), and, in particular, $x_*^c = b^c(x_*^c, x_*^{-c})$ for $c \in \{b, w\}$. This means that the function $b^c(\cdot, x_*^{-c})$ intersects with the 45-degree line at x_*^c . Hence, it is useful to investigate the intersection of the curve $b^c(\cdot, x^{-c})$ and the 45-degree line for each x^{-c} . As we pointed out (see part (ii) of Lemma 2), these curves intersect at zero. The next lemma investigates the strictly positive points of intersections.

Lemma 4 *If $\lambda \geq 1/(K(1-\alpha)\mu_c)$ then either*

(i) *for all $x^{-c} > 0$ there exist $x_1^c(x^{-c}), x_2^c(x^{-c})$ such that $x_i^c(x^{-c}) = b^c(x^c(x^{-c}), x^{-c})$ and $0 < x_1(x^{-c}) < x_2(x^{-c})$, or*

(ii) *there exist $\underline{x}^{-c}, \bar{x}^{-c} \in \mathbb{R}_{++}$, $\underline{x}^{-c} \leq \bar{x}^{-c}$, such that for all $x^{-c} \in (\underline{x}^{-c}, \bar{x}^{-c})$: $b^c(x^c, x^{-c}) < x^c$, and for all $x^{-c} \in \mathbb{R}_{++} \setminus [\underline{x}^{-c}, \bar{x}^{-c}]$ there exist $x_1^c(x^{-c}), x_2^c(x^{-c})$ such that $x_i^c(x^{-c}) = b^c(x_i^c(x^{-c}), x^{-c})$, $0 < x_1(x^{-c}) < x_2(x^{-c})$, and*

$$\lim_{x^{-c} \rightarrow \underline{x}^{-c}} x_1^c(x^{-c}) = \lim_{x^{-c} \rightarrow \bar{x}^{-c}} x_1^c(x^{-c}) = \lim_{x^{-c} \rightarrow \bar{x}^{-c}} x_2^c(x^{-c}) = \lim_{x^{-c} \rightarrow \underline{x}^{-c}} x_2^c(x^{-c}).$$

In addition, $x_1^c(x^{-c})$ is increasing first, then is decreasing, and $x_2^c(x^{-c})$ is decreasing first, then is increasing. Finally, $\lim_{x^{-c} \rightarrow 0} x_1^c(x^{-c}) = 0$.

As we explained before, when x^{-c} increases from zero the curve $b^c(\cdot, x^{-c})$ is shifted downwards and it intersects with the 45-degree line at two positive values. For $c = b$, these intersections, $x_1^b(x^w)$ and $x_2^b(x^w)$, are depicted on Figure 1. As x^{-c} increases more, the curve will be shifted down even more. However, at some point, a further increase in x^{-c} results an upwards shift in $b^c(\cdot, x^{-c})$, and as x^{-c} goes to infinity, $b^c(\cdot, x^{-c})$ converges back to $b^c(\cdot, 0)$ (see part (iv) of Lemma 2). Two things can possibly happen to the positive intersections with the 45-degree line: either $b^c(\cdot, x^{-c})$ intersects with the 45-degree line even when it is shifted down the most (part (i)), or this curve is pushed below the 45-degree line on an interval (part (ii)).⁵ In the latter case, the best-response curve does not intersect with the 45-degree line on this interval. Figure 2 illustrates the statement of the lemma for the case described in (i). Since, $\lim_{y \rightarrow 0} x_1^c(y) = 0$, we can define $x_1^c(0)$ to be zero.

Recall that part (ii) of Lemma 2 states that $b^c(\cdot, y)$ always intersects with the 45-degree line at zero. Let $x_0^c(y) \equiv 0$. Now, we can define equilibria in terms of the intersections of the curves $\{x_i^b\}_{i=0}^2$ and $\{x_i^w\}_{i=0}^2$. Formally, (x_*^c, x_*^{-c}) is an equilibrium cutoff profiles if and only if there exist $i, j \in \{0, 1, 2\}$ such that

$$x_*^c = x_i^c(x_*^{-c}) \text{ and } x_*^{-c} = x_j^{-c}(x_*^c). \quad (11)$$

Notice that by (8) the best-response cutoff of an agent with colour c is largest if $x^c = \infty$ and $x^{-c} = 0$. In this case, the best-response cutoff is $K\mu_c$. This implies that the equilibrium cutoff of an agent with colour c can never exceed $K\mu_c$. Let $x_{\max}^c = K\mu_c$.

⁵There is a non-generic third case where the curve $b^c(\cdot, x^{-c})$ is tangent to the 45-degree line when it is shifted down the most.

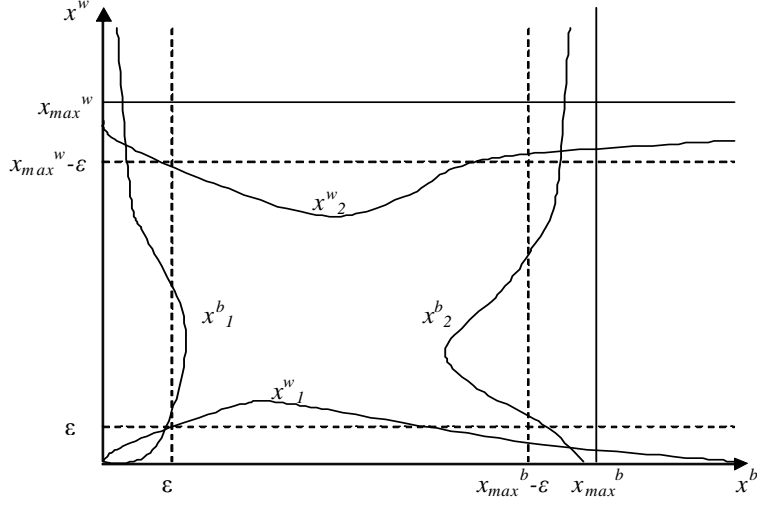


Figure 2: Equilibria

Proposition 1 For all K, μ_c, α and $\varepsilon (> 0)$ there exists a λ_0 such that if $\lambda \geq \lambda_0$, then if x_*^c is an equilibrium cutoff then either:

- (i) $x_*^c = 0$, or
- (ii) $x_*^c \in (0, \varepsilon)$, or
- (iii) $x_*^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$.

This proposition states if λ is large enough then in every equilibrium an agent either does not discriminate at all ($x_*^c = 0$), or weakly discriminates ($x_*^c < \varepsilon$), or strongly discriminates ($x_{\max}^c - \varepsilon < x_*^c$). Proposition 1 is illustrated on Figure 3. Intuitively, strong discrimination of agents with colour c corresponds to the curve x_2^c , weak discrimination corresponds to x_1^c , and x_0^c implies no discrimination.

The statement of the proposition allows any combination of these possibilities in equilibrium. That is, there might be an equilibrium where the white agents strongly discriminate against the black agents while the black agents do not discriminate against the whites at all. It is easy to see, however, that the intersection of x_0^c and x_2^{-c} is unique (for $c \in \{b, w\}$). In addition, since $x_1^c(0) = 0$, the intersection of x_1^c and x_0^{-c} is $(0, 0)$, that is, there is no equilibrium where one colour weakly discriminates and the other does not discriminate at all. The proposition neither implies the existence, nor the uniqueness of any of the other types of equilibria. The next section introduces a stability concept and we shall fully characterize those equilibria which are stable.

5 Stability

Next, we introduce a fairly standard notion of stability⁶. It is based on the requirement that a simple myopic best-response dynamics converges to the equilibrium if the strategies of the agents are slightly perturbed around the equilibrium. We model the myopic best-response dynamics by assuming that initially agents best-respond to some stationary population strategy which might be different from the actual one. Then each agent stochastically receives an opportunity to update his strategy. Whenever an agent has this opportunity, he myopically adjusts his strategy to the current environment. That is, he best-responds to the current population strategy as if it was never to change.

Formally, suppose that initially the distribution of cutoffs of black and white agents are described by a pair of random variables (X^b, X^w) and the strategy of each agent satisfies the statement of Lemma 1.⁷ Agents receive opportunities to update their strategies according to a Poisson process with an arrival rate normalized to be one.⁸ If an agent has this opportunity at time t , he best-responds to the cutoff distribution at t as if it was constant over time. Let $x_t^c(X^c, X^{-c})$ denote the best-response cutoffs of an agent with colour c at time t if the initial distribution of cutoffs was (X^c, X^{-c}) .

Definition 1 *The equilibrium cutoff vector (x_*^b, x_*^w) is said to be stable if there exists an $\varepsilon > 0$, such that if $|X^c - x_*^c| < \varepsilon$ almost surely for $c \in \{b, w\}$ then $\lim_{t \rightarrow \infty} x_t^c(X^c, X^{-c}) = x_*^c$ for $c \in \{b, w\}$.*

In what follows we describe the equation governing the best-response dynamics. Fix (X^b, X^w) and let (X_t^b, X_t^w) denote the distribution of population cutoffs at time t . We shall denote the best response of an agent with colour c by x_t^c suppressing its argument (X^b, X^w) . By (7), the best-response of an agent with colour c at time t is

$$x_t^c = K [\mu_c EG((1 - \alpha) X_t^c) + \mu_{-c} (EG(\alpha X_t^{-c}) - EG(X_t^{-c}))].$$

Next, we approximate x_{t+dt}^c by assuming that between t and $t + dt$ each agent changes his strategy to the time t best-response cutoffs. There is a measure of dt agents who receive an opportunity to change their strategies between t and $t + dt$ and they all switch to x_t^c . Therefore,

$$\begin{aligned} x_{t+dt}^c &= K [\mu_c EG((1 - \alpha) X_{t+dt}^c) + \mu_{-c} (EG(\alpha X_{t+dt}^{-c}) - EG(X_{t+dt}^{-c}))] \\ &= (1 - dt) K [\mu_c EG((1 - \alpha) X_t^c) + \mu_{-c} (EG(\alpha X_t^{-c}) - EG(X_t^{-c}))] \\ &\quad + dt K [\mu_c EG((1 - \alpha) x_t^c) + \mu_{-c} (EG(\alpha x_t^{-c}) - EG(x_t^{-c}))] \\ &= (1 - dt) x_t + dt \tilde{b}^c(x_t^c, x_t^{-c}), \end{aligned}$$

⁶See, for example, Chapter 3 of Fudenberg and Levine.

⁷This latter assumption is satisfied if each agent best responds to *some* population strategy.

⁸This normalization is without the loss of generality because this arrival rate only affects the speed of convergence but not the limits.

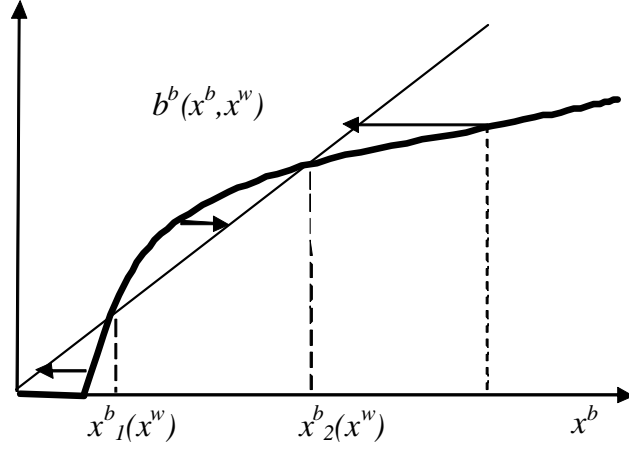


Figure 3: Best-response Dynamics

where the first equality follows because the strategy of $1-dt$ measure of the population is described by (X_t^c, X_t^{-c}) , and the rest uses (x_t^c, x_t^{-c}) . The second equality follows from (9). Taking dt to zero leads to the following differential equation describing the evolution of x_t^c :

$$\frac{dx_t^c}{dt} = \tilde{b}^c(x_t^c, x_t^{-c}) - x_t^c.$$

As we mentioned before, we can restrict attention to non-negative cutoffs. Note that if $x_t^c = 0$ then $\tilde{b}^c(x_t^c, x_t^{-c}) = b(x_t^c, x_t^{-c}) = 0$ by (9) and part (ii) of Lemma 2. Hence, the previous displayed equation implies that $dx_t^c/dt = 0$ whenever $x_t^c = 0$. Therefore,

$$\frac{dx_t^c}{dt} = \begin{cases} 0 & \text{if } x_t^c = 0 \\ \tilde{b}^c(x_t^c, x_t^{-c}) - x_t^c & \text{if } x_t^c > 0 \end{cases}. \quad (12)$$

Figure 3 helps to understand the best-response dynamics derived from (12). Consider $x^b > x_2^b(x^w)$. At this point, the best response curve is below the 45 degree line, that is $b^b(x^b, x^w) < x^b$. In general, if (x^b, x^w) is to the right from the x_2^b curve, the best response of a black agent is smaller than x^b . Equation (12) implies that the best response of a black agent decreases on this region. This is represented by a horizontal arrows pointing to the left. A similar argument shows, that if $x_1^b(x^w) < x^b < x_2^b(x^w)$, then $b^b(x^b, x^w) > x^b$. Hence, by (12), the best response of a black agent increases. This is represented by horizontal a arrow pointing to the right between the points $x_1^b(x^w)$ and $x_2^b(x^w)$. Finally, if $x^b < x_1^b(x^w)$, then $b^b(x^b, x^w) < x^b$ which is represented by a horizontal arrow pointing to the left.

We are ready to state the main theorem of the paper.

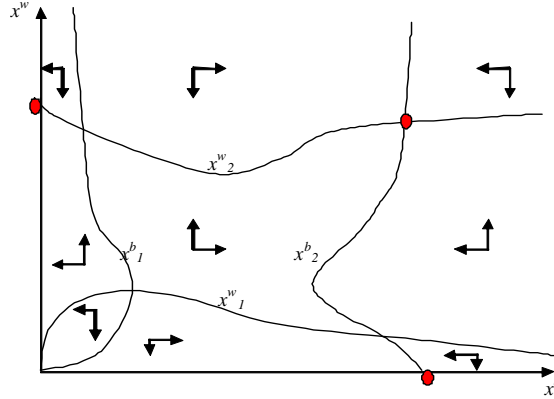


Figure 4: Stable Equilibria

Theorem 1 For all K , μ_c , α and $\varepsilon (> 0)$ there exists a λ_0 such that if $\lambda \geq \lambda_0$, then there are exactly three stable equilibria $(x_*^w, 0)$, $(0, x_*^b)$, and (x_*^{rw}, x_*^{rb}) such that $x_*^c, x_*^{rc} \in (x_{\max}^c - \varepsilon, x_{\max}^c)$ for $c \in \{b, w\}$.

If λ is large enough there are the following three equilibria in our model. First, the white population discriminates strongly against the black one, while the blacks do not discriminate at all. Second, the blacks strongly discriminate against the whites, while the whites do not discriminate at all. And finally, both population strongly discriminate against the other.

Equation (12) implies that the change in best responses at time t depends only on the time- t best responses, (x_t^b, x_t^w) , but not directly on the distribution of strategies, (X_t^b, X_t^w) . In particular, the initial distribution of strategies affects the best-response dynamics only through the initial best-response profile (x_0^b, x_0^w) . Therefore, using (12), we can represent the best-response dynamics by constructing a Phase Diagram, plotted on Figure 4. For each (x^b, x^w) cutoff vector, the horizontal and vertical arrows on this figure indicate the directions of the best responses of black and white agents, respectively.

To explain how the arrows are drawn, recall from Figure 3, that if $x^b > x_2(x^w)$ then $b^b(x^b, x^w) < x^b$ which is presented by an arrow pointing to the left. In general, if (x^b, x^w) is to the right from the x_2^b curve, the best response of a black agent is smaller than x^b . This is represented in Figure 4 by horizontal arrows pointing to the left on the area which is right of the curve x_2^b . Similarly, Figure 3 shows that if $x_1^b(x^w) < x^b < x_2^b(x^w)$, then the best response of a black agent increases. This is why the horizontal arrows are pointing to the right between the curves x_1^b and x_2^b on Figure 4. Finally, if $x^b < x_1^b(x^w)$, then $b^b(x^b, x^w) < x^b$ which is represented by horizontal arrows pointing

to the left on the area which is left of x_1^b . The vertical arrows are constructed in a similar manner representing the best-response dynamics of the white population.

Figure 4 can be used to understand the stability properties of various equilibria. Consider, for example, the colour-blind equilibrium, $(0, 0)$. There are points below the curve x_1^w and right to the curve x_1^b arbitrarily close to $(0, 0)$. At these points, x_t^b increases and x_t^w decreases, and the vector (x_t^b, x_t^w) converges to the intersection of x_2^b and x_0^w . Hence, the colour blind equilibrium is unstable. Similarly, it is easy to see that the intersections of the curves x_1^c and x_1^{-c} and the curves x_1^c and x_2^{-c} are unstable for $c \in \{b, w\}$. On the other hand, from any point close to any of the equilibria described in the statement of Theorem 1, the arrows point towards the equilibrium, and hence, these equilibria are stable.

6 Discussion

In this section, we derive some comparative static results and discuss some of the assumptions of our model.

6.1 Comparative Statics

In what follows we focus on the case of a large λ , that is, where the statement of Theorem 1 is valid. Recall that there are exactly three stable equilibria. Next, we investigate how the cutoffs in these three equilibria change if the parameters of our model change. We emphasize that these comparative static results are only valid as long as the change in the parameters are small enough, so that Theorem 1 holds. The following table summarizes the comparative static results. This table indicates what happens to the cutoffs if a certain parameter increases. It turns out that direction of the change in cutoffs is the same in all three equilibria.

	x_*^w	x_*^b
discount rate (r)	down	down
wage (M)	up	up
shock distribution (λ)	up	up
measure of whites (μ_w)	up	down
matching frequency (δ)	up	up
persistence (γ)	up	up

The proof of these results is straightforward, hence, it is omitted. Below, we provide some intuition behind these observations.

An agent cares about his social colour because it influences his future employment. Therefore, the larger is the part of an agent's payoff due to future wages the more likely it is that employer's condition their decisions on the types of the workers. For example, if the discount rate goes up, agents care more about their current payoffs relative to their future payoffs. Hence, they become

more eager to employ workers no matter what their types are, and as a result, their cutoffs go down. Similarly, if M goes up being an employed worker becomes more important, and the cutoffs go up. When λ goes up, the expected payoff of an employer decreases. In other words, having the option of being a worker becomes more important relative to being an employer. This is why an increase in λ has the same effect as a decrease in M .

When μ_w increases, agents receive a larger fraction of their payoffs from interacting with white agents. This makes it more expensive to discriminate against whites, and cheaper to discriminate against blacks. As a result, x_*^w goes up while x_*^b goes down.

An employer's social colour does not remain the same with probability $1 - \gamma$ independently of his decision. The larger is γ the more likely it is that the worker's type has an impact on the employer's future payoff. Hence, it becomes easier to discriminate.

6.2 Assumptions

Our goal in this paper was to present a simple model which demonstrates that discrimination can arise purely because agents carry information about their past actions. Some of the assumptions are made in order to be able to provide a graphical representation of equilibria and stability. We do not claim that equilibrium discrimination is robust to all the features of our model. In what follows, we discuss some of our assumptions and extensions of the model.

Distribution of shocks.— We have assumed that the distribution of the match-specific shock which determines the surplus of a partnership is exponential. To what extent does our main result depend on this assumption? For general distributions, we have no hope for a full characterization of equilibria such as in Theorem 1. However, whether or not the colour blind equilibrium is stable depends only on the slope of the best-response functions at $(0, 0)$. If this slope is less than one, the colour blind equilibrium is unique and stable. Otherwise, each stable equilibrium involves discrimination. We formally state this result in the following

Theorem 2 *Suppose that s is distributed on \mathbb{R}_+ according to the CDF G . If G is concave on \mathbb{R}_+ then either*

- (i) $(0, 0)$ is the unique equilibrium and is stable, or
- (ii) $(0, 0)$ is not stable, and there exists a stable equilibrium.

Notice that the total surplus of a partnership, $s + M$, is always positive. This assumption makes the socially optimal employment decisions very easy to characterize. Efficiency requires employers to hire whenever they can. This simplifies our analysis. Even if shocks could be negative, the stability of the colour-blind equilibrium only depends on the slope of the best-response curve at zero. However, in this case, there might be stable equilibria different from those described in Theorem 1. In particular, it is possible that white employers prefer to hire black workers and vice versa. That is, a social colour of an agent is more valuable if it is different from his physical colour.⁹

⁹These results are available upon request.

Social colour.— The social colour of an employer who does not hire becomes his physical colour conditional on the social colour changing. This can be motivated by assuming that if a white agent refuses to hire a black employee despite the positive surplus he will be considered loyal to other whites and hostile to blacks. However, the main reason for this assumption is that it enabled us to give a two-dimensional graphical representation of our problem. Recall that a consequence of this assumption is that the best response cutoff of an employer does not depend on his social colour (see Lemma 1 and (3)).

We assume that the social colour is a binary information and its evolution is only determined by the type of the worker and the physical colour of the employer. This is much in the spirit of Kandori (1992). One can model the information an employer observes about a worker in more complicated ways. For example, an employer might randomly draw a sample of the physical colour of the agents in the history of the worker. Then the type of the worker would be his physical colour and his full history. Such modelling would lead to a complex type space but does not alter our main result regarding the instability of the colour-bling equilibrium.

It is easy to construct social colours different from ours which do not lead to discrimination. For example, if this colour is not informative about past decisions then the colour-blind equilibrium is stable and unique. We have not characterized those processes which necessarily lead to discrimination.

More attributes and social colours.— In reality, there are more than one observable physical attributes of individuals. It is also possible that there are several labels attached to an individual conditional on his history. Of course, agents might condition their actions on these other attributes and labels. However, it is not hard to show that as long as the social colour corresponding to the skin colour evolves according to our assumptions a version of Theorem 2 is still valid. That is, provided that λ is large, the colour-blind equilibrium is unstable and there exist stable equilibria.

Constant wage.— Workers receive a constant wage, M , regardless of their types, the types of their employers and the profitability of the partnership. Therefore, any inefficiency due to discrimination is in the form of suboptimal unemployment decisions. In particular, an agent against whom others discriminate is only worse off because he is not employed frequently enough. It would be interesting to allow wages to be endogenous and analyze wage differentials due to racial discrimination. Unfortunately, it is not entirely clear how endogenous wages affect our main results. The problem is that if a black worker is willing to get a paycut in order to be employed by a white employer then there will be more white employers who employ black workers. This would increase the number of white agents with black social colour, which in turn, makes it less costly for a white agent to have black social colour. This would make it less likely that discrimination arises in equilibrium. A potential solution for this problem is to allow the social colour to change as a function of the wage offered to a worker. The lower the wage of a black worker is the more likely it is that the employer's social colour becomes white.

We are currently working on models where wages are set endogenously. Preliminary results

suggest that as long as the wage of a worker cannot fall to zero, the main results of our paper remain valid. There are various theories of wage determination, like efficiency wages and moral hazard problems, that lead to strictly positive wages even if the outside option of a worker is zero.

6.3 Empirical Implications

Our theory is different from taste-based and statistical discrimination theories because agents have no intrinsic preferences for interacting with others of the same colour, and their skin-colour provides no signal about their productivity. Next we discuss empirical predictions of our model which are different from that of these other two theories.

It is not hard to construct models of both statistical and taste-based theories which generate the same comparative static results as ours. The key observation is that past history of a worker affects the likelihood of him being hired. A white employer, for example, has a larger cutoff against a black worker than against a white one. As a result, the profit of a white employer who hires black workers is higher than that of those who hire white workers. This would also be true if discrimination was taste-based, and one can imagine a variation of statistical discrimination which also generates this result. However, a white employer also uses a larger cutoff against other whites with black social colour than against whites with white social colour. Therefore, white employers hiring white workers with black social colours earn more than those who hire whites with white social colours. Note that the social colour of an agent is more likely to be black if there were more blacks in his past history. Hence, our model predicts that the profit of a white employer from hiring a white worker is stochastically increasing in the number of black agents in the history of the worker.

In addition, agents would never discriminate in our model if they knew that their interactions were not observed. Our theory predicts that as the interactions become harder to observe it becomes less likely that discrimination arises. Therefore, people are more likely to discriminate in smaller communities, like villages, where people better observe the actions of others than in larger communities, such as large cities, where individuals have less information about each others. This is in sharp contrast to the predictions of the other two theories.

A notable feature of our model is that there exist stable equilibria where each race discriminates against the other one. This is inconsistent with the theory of statistical discrimination, according to which, the hiring decision of an employer should not depend on his own skin-colour.

7 Conclusion

This paper put forward a new theory of racial discrimination. Individuals discriminate against others because they do not want to be associated to the other race. Although the information about each others' association is not payoff-relevant, it plays a major role in determining the behavior of

economic agents. Indeed, we showed that every stable equilibrium must involve discrimination in some environments.

Our model does not explain why agents use skin colour as a basis of discrimination as opposed to other observable physical attributes. People differ in height, weight, eye-colour, and along many other dimensions. A potential explanation might be that members of a family, or a community are more likely to have the same skin colour than the same height or weight. For a tall individual it would be more costly to discriminate against short ones if many of his relatives are short. Recall that a white agent discriminates against those who are associated to blacks in our model because he is afraid of those whites who are more associated to whites. Since individuals are necessarily associated to short and tall individuals, these attributes cannot be used to sustain discrimination. Another reason for using skin colour is that it is easier to observe than other attributes such as the eye-colour.

We assumed throughout this paper that the surplus generated by a partnership is divided among the worker and employer exogenously. We have excluded the possibility that discrimination results in different wages conditional on employment. Perhaps the most important elaboration of our model would be to allow wages and profits to be determined endogenously.

We have not discussed policy in this paper. Recall that a white employer discriminates against black workers because he is afraid of not being hired by white employers with white social colour in the future. Hence, a policy intervention which would increase the fraction of those whose social colour is different from his physical colour reduces the incentive to discriminate. It is clear that subsidizing employers who hire workers of different physical colour will increase the fraction of those whose physical and social colour are different. This, in turn, decreases the fraction of those who have the same physical and social colours and reduce the incentive to discriminate. Such subsidies must be paid from taxes which might alter the incentives to produce. In order to discuss policy in a meaningful way, one must properly model production and the worker's incentives.

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A Appendix

A.1 Proof of the Lemmas

Proof of Lemma 2. (i) Notice that $b^c(x^c, x^{-c}) = \tilde{b}^c(x^c, x^{-c})$ whenever $b^c(x^c, x^{-c}) > 0$. Hence, it is enough to show that \tilde{b}^c is concave in x^c . By (8)

$$\frac{\partial \tilde{b}^c(x^c, x^{-c})}{\partial x^c} = K\mu_c(1 - \alpha)g((1 - \alpha)x^c),$$

where $g(x) = \lambda e^{-\lambda x}$ for all $x \geq 0$. Therefore, this partial derivative is positive and decreasing.

(ii) By (8),

$$\tilde{b}^c(0, x^{-c}) = K [\mu_{-c} (G(\alpha x^{-c}) - G(x^{-c}))] \leq 0,$$

because $G(\alpha \bar{x}^{-c}) - G(\bar{x}^{-c}) \leq 0$. Hence, (9) implies that $b^c(0, x^{-c}) = 0$.

(iii) Notice that $\lim_{x^{-c} \rightarrow \infty} G(\alpha x^{-c}) - G(x^{-c}) = 0$. Therefore, by (8) and (9),

$$b^c(x^c, 0) = \lim_{x^{-c} \rightarrow \infty} b^c(x^c, x^{-c}) = K \mu_c G((1 - \alpha)x^c).$$

Finally, since $G(\alpha \bar{x}^{-c}) - G(\bar{x}^{-c}) \leq 0$,

$$b^c(x^c, 0) \geq b^c(x^c, \bar{x}^{-c}).$$

■

Proof of Lemma 3. Notice that if $x^{-c} = 0$ then $\tilde{b}^c(x^c, x^{-c}) \geq 0$, and by (9), $\tilde{b}^c(x^c, 0) = b^c(x^c, 0)$. We have showed in the proof of part (i) of Lemma 2 that

$$\frac{\partial b^c(x^c, 0)}{\partial x^c} = K \mu_c (1 - \alpha) \lambda e^{-\lambda(1-\alpha)x^c}.$$

This derivative is $K \mu_c (1 - \alpha) \lambda$ at $x^c = 0$, and converges to zero as x^c goes to infinity.

(i) If $\lambda > \lambda_0$, $K \mu_c (1 - \alpha) \lambda > 1$ and therefore $b^c(x^c, 0) > x^c$ around zero. Since the curve $b^c(x^c, 0)$ is concave (part (i) of Lemma 2) and its derivative goes to zero as x^c goes to infinity, there exists a unique $x^c > 0$ such that $b^c(x^c, 0) = x^c$.

(ii) If $\lambda \leq \lambda_0$, $K \mu_c (1 - \alpha) \lambda \leq 1$. Since the curve $b^c(x^c, 0)$ is concave (part (i) of Lemma 2) $b^c(x^c, 0) < x^c$ for all $x^c > 0$. ■

Proof of Lemma 4. For each x^{-c} consider the following function of x^c :

$$\begin{aligned} B^{x^{-c}}(x^c) &= \tilde{b}^c(x^c, x^{-c}) - x^c \\ &= K \mu_c G((1 - \alpha)x^c) - x^c + K \mu_{-c} (G(\alpha x^{-c}) - G(x^{-c})) \end{aligned}$$

Observe that, by (9), $b^c(x^c, x^{-c}) = x^c$ if and only if x^c is a root of $B^{x^{-c}}$. First, we establish some properties of $B^{x^{-c}}$.

(1) The function $B^{x^{-c}}$ is strictly concave. This follows from the proof of part (i) of Lemma 2.

(2) $\left. \frac{dB^{x^{-c}}}{dx^c} \right|_{x^c=0} > 0$. It follows from the proof of part (i) of Lemma 3.

(3) $\lim_{x^c \rightarrow \infty} B^{x^{-c}}(x^c) = -\infty$. This is because G is a CDF and hence, $\tilde{b}^c(x^c, x^{-c}) \leq K$.

(4) $\lim_{x^c \rightarrow 0} B^{x^{-c}}(x^c) < 0$. This is because $G(\alpha x^{-c}) - G(x^{-c})$ is negative.

(5) Generically, $B^{x^{-c}}$ has either zero or two roots. This follows from (1)-(4).¹⁰

Notice that the part $B^{x^{-c}}(x^c)$ which depends on x^{-c} is additively separable from the rest. Hence, any change in x^{-c} results in a vertical shift of this curve. Whether there are two or zero intersections depends on the size of this shift. Let $H(x^{-c}) = K \mu_{-c} (G(\alpha x^{-c}) - G(x^{-c}))$. Next we establish some properties of this function.¹¹

¹⁰The non-generic case is when this function is tangent to the constant zero line. There can be at most one such an x^{-c} .

¹¹These properties are straightforward consequences of the assumption that $G(s) = 1 - e^{-\lambda s}$.

- (6) $H(x^{-c})$ is strictly negative.
- (7) $\lim_{x^{-c} \rightarrow \infty} H(x^{-c}) = \lim_{x^{-c} \rightarrow 0} H(x^{-c}) = 0$.
- (8) $\arg \min H(x^{-c}) = (-\log \alpha) / [\lambda(1 - \alpha)] = \widehat{x}^{-c}$.
- (9) H is strictly decreasing on $(0, \widehat{x}^{-c})$ and strictly increasing on $(\widehat{x}^{-c}, \infty)$.

We are ready to prove the two parts of the lemma.

(i) Suppose that $\max B^{\widehat{x}^{-c}}(x^c) > 0$. This, together with (8), implies that $\max B^{\widehat{x}^{-c}}(x^c) > 0$ for all $x^{-c} > 0$. By (3) the value of $B^{x^{-c}}$ is sometimes negative. The intermediate Value Theorem implies that $B^{x^{-c}}$ has at least one root. Hence, by (5), has exactly two roots.

(ii) Suppose that $\max B^{\overline{x}^{-c}}(x^c) < 0$. This means that there are values of x^{-c} for which $B^{x^{-c}}$ is always negative. (9) implies that the set of such x^{-c} s is an interval. Let us denote this interval by $(\underline{x}^{-c}, \overline{x}^{-c})$. From (2) and (7) it follows that $\underline{x}^{-c} > 0$ and $\overline{x}^{-c} < \infty$. The argument establishing that $B^{x^{-c}}$ has two roots whenever $x^{-c} \in \mathbb{R}_+ \setminus [\underline{x}^{-c}, \overline{x}^{-c}]$ is analogous to the argument in part (i). From (1) and (9) it follows that

$$\lim_{x^{-c} \rightarrow \underline{x}^{-c}} x_1^c(x^{-c}) = \lim_{x^{-c} \rightarrow \overline{x}^{-c}} x_1^c(x^{-c}) = \lim_{x^{-c} \rightarrow \overline{x}^{-c}} x_2^c(x^{-c}) = \lim_{x^{-c} \rightarrow \underline{x}^{-c}} x_2^c(x^{-c}).$$

It remains to show that $x_1^c(x^{-c})$ is increasing first, then is decreasing, and $x_2^c(x^{-c})$ is decreasing first, then is increasing. On the interval $(0, \widehat{x}^{-c})$ an increase in x^{-c} results a downwards shift of $B^{x^{-c}}$ (see (9)). Hence, by (1), $x_1^c(x^{-c})$ is increasing and $x_2^c(x^{-c})$ is decreasing on this interval. On the interval $\mathbb{R}_+ \setminus (0, \widehat{x}^{-c})$ an increase in x^{-c} results an upward shift of $B^{x^{-c}}$ (see (9)). Hence, by (1), $x_1^c(x^{-c})$ is decreasing and $x_2^c(x^{-c})$ is increasing on this interval. Finally, it follows from (2) and (7) that $\lim_{x^{-c} \rightarrow 0} x_1^c(x^{-c}) = 0$. ■

A.2 Proof of Proposition 1

Before we proceed with the proof of Proposition 1 we prove a few Lemmas about the equilibrium cutoffs. For convenience we introduce a few new notations. We shall denote $\min\{\mu_b, \mu_w\}$ by μ_{\min} . In addition, we define two constants

$$\begin{aligned} \psi_0 &= \frac{1}{\frac{1}{4}K\alpha(1-\alpha)\mu_{\min}}, \\ \psi_1 &= K\mu_{\min}\frac{1}{2}(1-2^{-\alpha})(1-2^{-(1-\alpha)}). \end{aligned} \tag{13}$$

In the proofs of the lemmas we often use the inequality stated in the next

Lemma 5 For all $\xi \leq \lg 2$

$$1 - e^{-\xi} \geq \frac{1}{2}\xi. \tag{14}$$

In what follows (x^c, x^{-c}) denotes an equilibrium cutoff profile.

Lemma 6 There exists a λ_0 such that for all $\lambda \geq \lambda_0$ either $\max\{x^c, x^{-c}\} \leq \psi_0\lambda^{-2}$ or $\max\{x^c, x^{-c}\} \geq \psi_1$.

Proof. First, suppose that both cutoffs are strictly positive, that is, $x^b, x^w > 0$. Then, by (7),

$$\begin{aligned} x^b + x^w &= \sum_{c \in \{b, w\}} K [\mu_c G((1 - \alpha)x^c) + \mu_{-c} (G(\alpha x^{-c}) - G(x^{-c}))] \\ &= \sum_{c \in \{b, w\}} K \mu_c [G((1 - \alpha)x^c) + G(\alpha x^c) - G(x^c)] \\ &= \sum_{c \in \{b, w\}} K \mu_c (1 - e^{-\alpha \lambda x^c}) (1 - e^{-(1-\alpha)\lambda x^c}), \end{aligned} \quad (15)$$

where the first equality follows from rearranging the terms corresponding to the same colour and the second one from $G(x) = 1 - e^{-x}$ and

$$[1 - e^{-\alpha \lambda x^c}] + [1 - e^{-(1-\alpha)\lambda x^c}] + [1 - e^{-\lambda x^c}] = (1 - e^{-\alpha \lambda x^c}) (1 - e^{-(1-\alpha)\lambda x^c}).$$

We consider two cases. If $\max\{x^b, x^w\} \geq (\log 2)/\lambda$, then from the previous equality it follows that

$$\begin{aligned} x^b + x^w &\geq K \mu_{\min} (1 - e^{-\alpha \log 2}) (1 - e^{-(1-\alpha) \log 2}) \\ &= K \mu_{\min} (1 - 2^{-\alpha}) (1 - 2^{-(1-\alpha)}) = 2\psi_1. \end{aligned}$$

Since $\max\{x^b, x^w\} \geq x^b + x^w$, the previous inequality chain implies $\max\{x^b, x^w\} \geq \psi_1$. If $\max\{x^b, x^w\} \leq (\log 2)/\lambda$, then, by Lemma 5,

$$1 - e^{-\alpha \lambda x^c} \geq \frac{1}{2} \alpha \lambda x^c \text{ and } 1 - e^{-(1-\alpha)\lambda x^c} \geq \frac{1}{2} (1 - \alpha) \lambda x^c \quad (16)$$

for each $c \in \{b, w\}$. Equations (15) and inequalities (16) imply that

$$\begin{aligned} \max\{x^b, x^w\} &\geq \sum_{c \in \{b, w\}} \frac{1}{4} K \alpha (1 - \alpha) \mu_c \lambda^2 (x^c)^2 \\ &\geq \frac{1}{4} K \alpha (1 - \alpha) \mu_{\min} \lambda^2 [(x^c)^2 + (x^{-c})^2] \geq \frac{1}{\psi_0} \lambda^2 (\max\{x^b, x^w\})^2. \end{aligned}$$

Hence, $\max\{x^b, x^w\} \leq \psi_0 / (\lambda^2)$.

Second, suppose that one of the cutoffs is zero, and without loss of generality assume that $x^b = 0$ and, hence, $\max\{x^b, x^w\} = x^w$. Then, by (7),

$$x^w = K \mu_w (1 - e^{-(1-\alpha)\lambda x^w}).$$

If $x^w \geq (\log 2)/\lambda$ then

$$x^w \geq K \mu_w (1 - e^{-(1-\alpha) \log 2}) \geq \psi_1.$$

If $x^w \leq (\log 2)/\lambda$ then, by Lemma 5,

$$x^w \geq 2K \mu_w (1 - \alpha) \lambda x^w.$$

If $\lambda > 1/(2K \mu_w (1 - \alpha))$ then the previous inequality implies that $x^w \leq 0$ and hence, $x^w < \psi_0 \lambda^{-2}$.

■

Lemma 7 *There exists a λ_0 such that if $\lambda \geq \lambda_0$ and $x^c \geq \psi_1$ then either $x^{-c} \leq \psi_0 / (\lambda)^2$ or $x^{-c} \geq \psi_1 / 2$.*

Proof. Suppose that $x^c \geq \psi_1$. If $x^{-c} < 0$ then $x^{-c} \geq \psi_1/2$. Suppose now that $x^{-c} > 0$. Then

$$\begin{aligned} x^{-c} &= K\mu_{-c}G((1-\alpha)x^{-c}) + K\mu_c(G(\alpha x^c) - G(x^c)) \\ &\geq K\mu_{-c}\left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) - K\mu_c e^{-\lambda\alpha\psi_1}, \end{aligned} \quad (17)$$

where the equality is just (7) and the inequality follows from $x^c \geq \psi_1$. We consider two cases.

Case 1: $x^{-c} \geq (\lg 2)/\lambda$. If λ is large enough so that $K\mu_c e^{-\lambda\alpha\psi_1} \leq \frac{1}{2}\psi_1$,

$$\begin{aligned} K\mu_{-c}\left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) - K\mu_c e^{-\lambda\alpha\psi_1} &\geq K\mu_{-c}\left(1 - e^{-(1-\alpha)\lg 2}\right) - \frac{1}{2}\psi_1 \\ &\geq K\mu_{-c}\left(1 - 2^{-(1-\alpha)}\right) - \frac{1}{2}\psi_1 \geq \frac{1}{2}\psi_1, \end{aligned}$$

where the last equality follows from $\psi_1 \leq K\mu_{-c}(1 - 2^{-(1-\alpha)})$. The previous inequality chain and (17) imply $x^{-c} \geq \frac{1}{2}\psi_1$.

Case 2: $x^{-c} < (\lg 2)/\lambda$. Then, by Lemma 5,

$$1 - e^{-\lambda(1-\alpha)x^{-c}} \geq \frac{1}{2}(1-\alpha)\lambda x^{-c}. \quad (18)$$

If λ is large enough so that $K\mu_{\max}e^{-\lambda\alpha\psi_1} \leq \psi_0/(\lambda)^2$, the previous inequality implies that

$$K\mu_{-c}\left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) - K\mu_c e^{-\lambda\alpha\psi_1} \geq K\mu_{\min}\frac{1}{2}(1-\alpha)\lambda x^{-c} - \psi_0\lambda^{-2}.$$

This inequality and the inequality chain (17) yields

$$\left(K\mu_{\min}\frac{1}{2}(1-\alpha)\lambda - 1\right)x^{-c} \leq \frac{\psi_0}{\lambda^2}.$$

If λ is large enough so that $K\mu_{\min}\frac{1}{2}(1-\alpha)\lambda - 1 > 1$ then $x^{-c} \leq \psi_0\lambda^{-2}$. ■

Recall that x_{\max} is the largest possible cutoff which can be a best response to a cutoff profile and $x_{\max} = K\mu_c$.

Lemma 8 *For all $\varepsilon > 0$, there exists a λ_0 , such that if $\lambda > \lambda_0$ and $x^c \geq \psi_1/2$ then either $x^{-c} \in (\psi_0/(\lambda)^2, \psi_1/2)$ or $x^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$.*

Proof. Suppose that $x^c \geq \psi_1/2$ and that $x^{-c} \in (\psi_0/(\lambda)^2, \psi_1/2)$. It is enough to show that for all $\varepsilon > 0$ there is a λ_0 such that if $\lambda > \lambda_0$ then whenever $x^c \geq \psi_1/2$ and $x^{-c} \in (\psi_0/(\lambda)^2, \psi_1/2)$ the cutoff x^c must be in the set $(x_{\max}^c - \varepsilon, x_{\max}^c)$.

Notice that from (7) and $x_{\max} = K\mu_c$ it follows that

$$\begin{aligned} x_{\max}^c - x^c &= K\mu_c - \left[K\mu_c\left(1 - e^{-\lambda(1-\alpha)x^c}\right) + K\mu_{-c}\left(1 - e^{-\lambda x^{-c}} - 1 + e^{-\lambda\alpha x^{-c}}\right)\right] \\ &= K\mu_c e^{-\lambda(1-\alpha)x^c} - K\mu_{-c}\left(e^{-\lambda x^{-c}} - e^{-\lambda\alpha x^{-c}}\right) \\ &= K\mu_c e^{-\lambda(1-\alpha)x^c} + K\mu_{-c}e^{-\lambda\alpha x^{-c}}\left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right). \end{aligned} \quad (19)$$

Case 1: $x^{-c} \geq \psi_1/2$. Then

$$\begin{aligned} K\mu_c e^{-\lambda(1-\alpha)x^c} + K\mu_{-c} e^{-\lambda\alpha x^{-c}} \left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) &\leq K\mu_c e^{-\lambda(1-\alpha)x^c} + K\mu_{-c} e^{-\lambda\alpha x^{-c}} \\ &\leq K\mu_c e^{-\frac{1}{2}\lambda(1-\alpha)\psi_1} + K\mu_{-c} e^{-\frac{1}{2}\lambda\alpha\psi_1}, \end{aligned}$$

where the first inequality follows from $1 - e^{-(1-\alpha)\lambda x^{-c}} \leq 1$ and the second one from $x^{-c}, x^c \geq \psi_1/2$. This inequality chain and (19) imply that

$$x_{\max}^c - x^c \leq K\mu_c e^{-\frac{1}{2}\lambda(1-\alpha)\psi_1} + K\mu_{-c} e^{-\frac{1}{2}\lambda\alpha\psi_1}.$$

Notice that for each ε there is a λ_0 such that if $\lambda > \lambda_0$ the right-hand-side of this inequality is smaller than ε and, hence, $x^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$.

Case 2: If $x^{-c} \leq \psi_0/(\lambda)^2$, then,

$$\begin{aligned} K\mu_c e^{-\lambda(1-\alpha)x^c} + K\mu_{-c} e^{-\lambda\alpha x^{-c}} \left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) &\leq K\mu_c e^{-\lambda(1-\alpha)x^c} + K\mu_{-c} \left(1 - e^{-(1-\alpha)\lambda x^{-c}}\right) \\ &\leq K\mu_c e^{-\frac{1}{2}\lambda(1-\alpha)\psi_1} + K\mu_{-c} \left(1 - e^{-\frac{\psi_0(1-\alpha)}{\lambda}}\right), \end{aligned}$$

where the first inequality follows from $e^{-\lambda\alpha x^{-c}} \leq 1$ and the second one from $x^c \geq \psi_1/2$ and $x^{-c} \leq \psi_0/(\lambda)^2$. This inequality chain and (19) imply that

$$x_{\max}^c - x^c \leq K\mu_c e^{-\frac{1}{2}\lambda(1-\alpha)\psi_1} + K\mu_{-c} \left(1 - e^{-\frac{\psi_0(1-\alpha)}{\lambda}}\right).$$

Observe that as λ goes to infinity both $K\mu_c e^{-(1/2)\lambda(1-\alpha)\psi_1}$ and $1 - e^{-\psi_0(1-\alpha)/\lambda}$ converge to zero. Therefore, for each ε there is a λ_0 such that if $\lambda > \lambda_0$ the right-hand-side of this inequality is smaller than ε and $x^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$. ■

We are ready to prove Proposition 1. By Lemma 6, we have to consider two cases: either $\max\{x^c, x^{-c}\} \leq \psi_0\lambda^{-2}$ or $\max\{x^c, x^{-c}\} \geq \psi_1$.

Case 1: $\max\{x^c, x^{-c}\} \leq \psi_0\lambda^{-2}$. Note that for each ε there is a λ_0 such that for all $\lambda > \lambda_0$ the term $\psi_0\lambda^{-2}$ is strictly smaller than ε , and hence, $x^b, x^w < \varepsilon$. Therefore, either (i) or (ii) holds in the statement of Proposition 1.

Case 2: $\max\{x^c, x^{-c}\} \geq \psi_1$. Without loss of generality assume that $\max\{x^c, x^{-c}\} = x^c$. By Lemma 7, we have to consider only the following two subcases: either $x^{-c} \leq \psi_0/(\lambda)^2$, or $x^{-c} \geq (1/2)\psi_1$.

Case 2.a: $x^{-c} \leq \psi_0/(\lambda)^2$. Then for each ε there is a λ_0 such that if $\lambda \geq \lambda_0$ then $x^{-c} \leq \varepsilon$, and by Lemma 8, $x^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$. In this case (i) holds for x^{-c} and (iii) holds for x^c .

Case 2.b: $x^{-c} \geq (1/2)\psi_1$. Since $x^c \geq \psi_1 > (1/2)\psi_1$, Lemma 8 implies that $x^c \in (x_{\max}^c - \varepsilon, x_{\max}^c)$ for $c \in \{b, w\}$. Therefore, (iii) of the statement of Proposition 1 holds for $c \in \{b, w\}$.

A.3 Proof of Theorem 1

By Proposition 1, for all ε there exists a λ_0 such that any equilibria can be classified into one of the cases described by the following table.

	x^c	x^{-c}
Case 1	0	0
Case 2	0	$\in (0, \varepsilon)$
Case 3	0	$\in (x_{\max}^{-c} - \varepsilon, x_{\max}^{-c})$
Case 4	$\in (0, \varepsilon)$	$\in (0, \varepsilon)$
Case 5	$\in (0, \varepsilon)$	$\in (x_{\max}^{-c} - \varepsilon, x_{\max}^{-c})$
Case 6	$\in (x_{\max}^{-c} - \varepsilon, x_{\max}^{-c})$	$\in (x_{\max}^{-c} - \varepsilon, x_{\max}^{-c})$

We shall consider each case separately. We show that equilibria described by Case 2 do not exist and equilibria corresponding to Cases 1, 4, and 5 are unstable. Finally, we prove that equilibria corresponding to Cases 3 and 6 are unique and stable. Note that this accomplishes the proof of Theorem 1. In what follows, we use the notations introduced in the last subsection, see (13).

Case 2.

In order to show that there does not exist an equilibrium described by Case 2, it is enough to prove that if λ is large enough and $x^c = 0$ then $x^{-c} = 0$ or $x^{-c} \geq \psi_1$. By Lemma 6, $x^{-c} \geq \psi_1$ or $x^{-c} \leq \psi_0/(\lambda)^2$. If $x^{-c} \geq \psi_1$, we are done. It remains to be shown that $x^{-c} \leq \psi_0/(\lambda)^2$ implies $x^{-c} = 0$. We prove it by contradiction, and assume that $x^{-c} \in (0, \psi_0/(\lambda)^2]$. Then,

$$x^{-c} = K\mu_{-c}G((1-\alpha)x^{-c}) = K\mu_{-c}\left(1 - e^{-\lambda(1-\alpha)x^{-c}}\right) \geq \frac{1}{2}K\mu_{-c}\lambda(1-\alpha)x^{-c} > x^{-c},$$

where the first equality is just (7) with $x^c = 0$, the first inequality follows from Lemma 5, and the second one from λ being large. Note that the previous inequality chain cannot hold, hence, $x^{-c} = 0$.

Case 1.

We show that the equilibrium cutoff profile $(0, 0)$ is unstable. By Definition 1, it is enough to show that there exists a distribution of cutoff profiles *nearby* $(0, 0)$ such that the best-response dynamics does not converge to $(0, 0)$. To this end, choose X^c and X^{-c} to be deterministic variables such that $X^{-c} = 0$ and $X^c = \delta$, where $\delta \in (0, \lg 2/[\lambda(1-\alpha)])$. Let the best response of an agent with colour c at time t denoted by x_t^c if the initial distribution of cutoffs is (X^c, X^{-c}) . Equations (7) and (12) imply that $x_t^{-c} = 0$ for all t . However, we show that x_t^c does not converge to 0 for sufficiently large λ . Since $x_0^c > 0$, it is enough to prove that $dx_t^c/dt > 0$ whenever x_t^c is small but positive. Suppose that $x_t^c \in (0, \lg 2/[\lambda(1-\alpha)])$. Then

$$\begin{aligned} \frac{dx_t^c}{dt} &= K\mu_c\left(1 - e^{-\lambda(1-\alpha)x_t^c}\right) - x_t^c \\ &\geq \lambda\frac{1}{2}K\mu_c(1-\alpha)x_t^c - x_t^c = \left(\lambda\frac{1}{2}K\mu_c(1-\alpha) - 1\right)x_t^c, \end{aligned}$$

where the first equality is just (12) with $x_t^{-c} = 0$ and the inequality follows from $x_t^c \in (0, \lg 2 / [\lambda (1 - \alpha)])$ and Lemma 5. If λ is large enough then $\lambda K \mu_c (1 - \alpha) / 2 > 1$, and hence, $dx_t^c / dt > 0$.

Cases 4 and 5.

Using the equation describing the best-response dynamics, (12), we construct the Jacobian matrix corresponding to the dynamic system (x_t^c, x_t^{-c}) :

$$J(x_t^c, x_t^{-c}) = \begin{bmatrix} \frac{d\tilde{b}^c(x_t^c, x_t^{-c})}{dx_t^c} - 1, & \frac{d\tilde{b}^c(x_t^c, x_t^{-c})}{dx_t^{-c}}, \\ \frac{d\tilde{b}^{-c}(x_t^c, x_t^{-c})}{dx_t^c}, & \frac{d\tilde{b}^{-c}(x_t^c, x_t^{-c})}{dx_t^{-c}} - 1 \end{bmatrix}. \quad (20)$$

where all the derivatives are taken at (x_0^c, x_0^{-c}) . Since in Cases 4 and 5 $x_0^c, x_0^{-c} > 0$, the Hartman-Grobman Theorem implies that (x_0^c, x_0^{-c}) is not a stable equilibrium if an eigenvalue of $J(x_0^c, x_0^{-c})$ has a positive real part. It is well-known that if $\text{tr } J(x_0^c, x_0^{-c}) > 0$ or $\det D(x_0^c, x_0^{-c}) < 0$, then the real part of at least of the eigenvalues is positive. Therefore, in order to establish that (x_0^c, x_0^{-c}) is unstable it is enough to show that $\text{tr } J(x_0^c, x_0^{-c}) > 0$.

In Case 4, Proposition 1 and Lemma 6 imply that $x^c \in (0, \psi_0 / (\lambda)^2)$ if λ is large enough. In Case 5, Proposition 1 and Lemma 7 imply that $x^c \in (0, \psi_0 / (\lambda)^2)$ if λ is large enough. Also notice that

$$\frac{d\tilde{b}^{-c}(x^{-c}, x^c)}{dx^{-c}} = K \mu_{-c} \lambda (1 - \alpha) e^{-\lambda(1-\alpha)x^{-c}} \geq 0,$$

and for sufficiently large λ ,

$$\begin{aligned} \frac{d\tilde{b}^c(x^c, x^{-c})}{dx^c} &= K \mu_c \lambda (1 - \alpha) e^{-\lambda(1-\alpha)x^c} \geq K \mu_c \lambda (1 - \alpha) \left(1 - e^{-(1-\alpha)\psi_0/\lambda}\right) \\ &\geq \frac{1}{2} K \mu_c \lambda (1 - \alpha), \end{aligned}$$

where the first inequality follows from $x^c < \psi_0 / (\lambda)^2$ and the second one from $e^{-(1-\alpha)\psi_0/\lambda} < 1/2$ if λ is large. Therefore, if λ is large enough,

$$\begin{aligned} \text{tr } J(x_0^c, x_0^{-c}) &= \frac{d\tilde{b}^c(x_0^c, x_0^{-c})}{dx_0^c} - 1 + \frac{d\tilde{b}^{-c}(x_0^c, x_0^{-c})}{dx_0^{-c}} - 1 \\ &\geq \frac{1}{2} \lambda K \mu_{\min} (1 - \alpha) - 2 > 0. \end{aligned}$$

Case 3.

Remark 2 established that, if λ is large, the equilibrium exists and is unique in this case. It remains to show that this equilibrium is stable. Notice that this equilibrium corresponds to the intersection of the x_2^{-c} and x_0^c curves, that is, $(0, x_2^{-c}(0))$. Since the curve x_2^{-c} is continuous, there exist δ_1 and δ_2 such that if $x^c < \delta_1$ then $|x^{-c} - x_2^{-c}(x^c)| < \delta_2$. In addition, we established in Section 5 that if δ_1 and δ_2 is small enough, $x^c < \delta_1$ and $|x^{-c} - x_2^{-c}(0)| < \delta_2$ then

$$\begin{aligned} &> 0 \quad \text{if } x^{-c} < x_2^{-c}(x^c), \\ \tilde{b}^c(x^c, x^{-c}) - x^c < 0 \text{ and } \tilde{b}^{-c}(x^{-c}, x^c) - x^{-c} < 0 &\quad \text{if } x^{-c} > x_2^{-c}(x^c), \\ &= 0 \quad \text{if } x^{-c} = x_2^{-c}(x^c). \end{aligned} \quad (21)$$

Let δ be so small that for any cutoff distribution (X^c, X^{-c}) , if $|X^c| < \delta$ and $|X^{-c} - x_2^{-c}(0)| < \delta$ almost surely then the initial best-response cutoff profile, (x_0^c, x_0^{-c}) , satisfy $x^c < \delta_1$ and $|x^{-c} - x_2^{-c}(0)| < \delta_2$. Then (21) implies that (x_t^c, x_t^{-c}) is in the rectangle

$$\{(x^c, x^{-c}) : x^c \in (0, \delta_1), |x^{-c} - x_2^{-c}(0)| < \delta_2\}$$

for all t .¹² Therefore, $\lim_{t \rightarrow \infty} x_t^c = 0$ by (21). This, together with (21), implies $\lim x_t^{-c} = x_2^{-c}(0)$.

Case 6.

First, we show that if this equilibrium exists it is stable. Recall the matrix introduced in Cases 4 and 5, $J(x_0^c, x_0^{-c})$. Since $x_0^c, x_0^{-c} > 0$, we can apply the Hartman-Grobman Theorem which implies that (x_0^c, x_0^{-c}) is a stable equilibrium if all eigenvalues of $J(x_0^c, x_0^{-c})$ have negative real parts. It is well-known that if $\text{tr} D(x_0^b, x_0^w) < 0$ and $\det D(x_0^b, x_0^w) > 0$ then the eigenvalues indeed have negative real parts. In this case, if λ is large enough then $x^b, x^w > x_{\max} - \varepsilon > \psi_1/2$. In addition, for all $\delta > 0$ there is a λ_0 such that if $\lambda > \lambda_0$,

$$\frac{d\tilde{b}^c(x^c, x^{-c})}{dx^c} - 1 = \lambda K \mu_c (1 - \alpha) e^{-\lambda(1-\alpha)x^c} - 1 \in \left(-1, -1 + \frac{\delta}{2}\right), \quad (22)$$

and

$$\begin{aligned} \frac{d\tilde{b}^c(x^c, x^{-c})}{dx^{-c}} &= \lambda K \mu_{-c} \left(\alpha e^{-\lambda \alpha x^{-c}} - e^{-\lambda x^{-c}} \right) \\ &= \lambda K \mu_{-c} e^{-\lambda \alpha x^{-c}} \left(\alpha - e^{-\lambda(1-\alpha)x^{-c}} \right) \in \left(0, \frac{\delta}{2}\right). \end{aligned}$$

Thus,

$$\text{tr} D(x_0^c, x_0^{-c}) < -2 + \delta < 0 \text{ and } \det D(x_0^c, x_0^{-c}) > 1 - \delta^2 > 0.$$

In order to show the existence of an equilibrium in this case, we show that the curves x_2^c and x_2^{-c} are defined on $[\psi_1/2, \infty)$ and they intersect. By (22), $\tilde{b}^c(x^c, x^{-c}) - x^c$ is strictly decreasing in x^c on this interval. Since \tilde{b} is bounded from above by x_{\max} , $\lim_{x^c \rightarrow \infty} [\tilde{b}^c(x^c, x^{-c}) - x^c] = -\infty$. In addition, Lemma 8 implies that, $\tilde{b}^c(x^c, x^{-c}) \geq x_{\max}^c - \varepsilon = K \mu_c - \varepsilon > \psi_1/2$ if $x^c, x^{-c} \in [\psi_1/2, \infty)$. Therefore, $\tilde{b}^c(x^c, x^{-c}) - x^c$ is strictly decreasing, positive at $x^c = \psi_1/2$, and becomes negative as x^c gets large whenever $x^{-c} \in [\psi_1/2, \infty)$. Therefore, for each $x^{-c} \in [\psi_1/2, \infty)$ there exists exactly one x^c such that $\tilde{b}^c(x^c, x^{-c}) = x^c$. We denote this x^c by $x_2^c(x^{-c})$ (see Lemma 4). Lemma 8 implies that $x_2^c(x^{-c}) \in [x_{\max}^c - \varepsilon, x_{\max}^c]$ for all $x^{-c} \geq \psi_1/2$. Since this argument holds for both c , the mapping $x_2^c \circ x_2^{-c} : [\frac{1}{2}\psi_1, \infty) \rightarrow [x_{\max}^c - \varepsilon, x_{\max}^c]$ is well-defined and clearly continuous. Therefore, there exists an $x_*^c \in [x_{\max}^c - \varepsilon, x_{\max}^c]$ such that

$$x_2^c(x_2^{-c}(x_*^c)) = x_*^c.$$

Define $x_*^{-c} = x_2^{-c}(x_*^c)$. Then, by (11), (x_*^c, x_*^{-c}) is an equilibrium cutoff profile.

¹²This is because $dx_t^c/dt < 0$ whenever $x_t^c = \delta_1$, $dx_t^{-c}/dt < 0$ if $x_t^{-c} = x_2^{-c}(0) + \delta_2$ and $dx_t^{-c}/dt > 0$ if $x_t^{-c} = x_2^{-c}(0) - \delta_2$.

In order to show the uniqueness, consider the mapping $B : [\psi_1/2, \infty)^2 \rightarrow \mathbb{R}^2$ defined by

$$B(x^c, x^{-c}) = (\tilde{b}^c(x^c, x^{-c}) - x^c, \tilde{b}^{-c}(x^{-c}, x^c) - x^{-c}).$$

Note that (x^c, x^{-c}) is an equilibrium if and only if $B(x^c, x^{-c}) = (0, 0)$. Note that the Jacobian matrix of B is just $J(x^c, x^{-c})$. We have concluded above that the determinant of this matrix is strictly positive on $x^c, x^{-c} \in [\psi_1/2, \infty)$. Therefore, B is an injection and there can only be at most one (x^c, x^{-c}) satisfying $B(x^c, x^{-c}) = (0, 0)$.

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