Equilibrium Bank Runs Revisited

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August 2011
This version: September 2011

Abstract

In a Diamond and Dybvig (1983) environment, Green and Lin (2003) take a mechanism design approach and show that a bank run equilibrium cannot exist. Peck and Shell (2003) generalize their economic environment and show that it can. The bank run, however, does not emerge because of modifications to the economic environment but rather because the mechanism that implements allocations is not an optimal one. When an optimal mechanism is used, the Peck and Shell (2003) bank run equilibrium disappears.

1 Introduction

Even though Diamond and Dybvig's (1983) seminal article is famously associated with bank runs, bank runs 'don't come easy' in their environment. For example, in the first part of their article, where there is no aggregate risk, they demonstrate that a bank run equilibrium cannot exist when the optimal deposit contract is in place. In the second part of their article, where there is aggregate risk, their analysis ignores the sequential service constraint, (Wallace (1988)). If there is no sequential service constraint, then there is no bank run equilibrium associated with the optimal deposit contract. Subsequently, Green and Lin (2003), GL, take a mechanism design approach, fully account for the sequential service constraint and demonstrate that the optimal bank contract does not admit a bank run equilibrium. Peck and Shell (2003), PS, make two modifications to the GL model and produce a bank run equilibrium.

A few words about the modifications. First, PS alter the preferences of depositors so that incentive constraints bind—incentive constraints do not bind for GL's preferences. Second, PS assume that depositors do not know their positions in the service queue—GL assume they do—which means that GL's powerful backward induction argument does not apply. If anything, PS's modifications generalize the GL environment. PS adopt the preferences of Diamond and Dybvig (1983), where patient and impatient depositors can have different utility functions. GL restrict these functions to be the same. If depositors do not know their positions in the service queue, then the mechanism can choose to either inform or not inform depositors regarding their positions. GL can be interpreted as restricting the mechanism to always inform depositors about their positions in the queue.

Independent of how one views the GL environment vis-à-vis the PS environment, GL use a mechanism that is optimal for their economic environment. Without loss in generality, they restrict their attention to direct revelation mechanisms, where each depositor makes an announcement regarding his private information or type. PS also use a direct revelation mechanism. But for their more general economic environment, a direct revelation mechanism is not an optimal one. I construct an indirect mechanism for the PS environment that *uniquely* implements the best implementable allocation. In other words, the indirect mechanism does not admit a bank run equilibrium. This result reinforces an earlier observation: Bank runs are hard to come by in the Diamond-Dybvig environment.

Cavalcanti and Monteiri (2011) examine the use of indirect mechanisms in a GL environment. Their exercise is relevant because a bank run equilibrium can arise in a GL environment when a direct mechanism is used and depositors' types are correlated, see Ennis and Keister (2009).[1] By expanding the message space for depositors and appropriately designing 'off equilibrium' payoffs so that the planner can learn the true type of each depositor, Cavalcanti and Monteiri (2011) demonstrate that the best implementable allocation can be uniquely implemented by using a backward induction argument. This backward induction argument, however, will not work for the more general PS environment, where depositors do not know their positions in the queue.[2] The indirect mechanism proposed in this paper is sufficiently general to uniquely implement the best implementable allocation for both the GL- and PS-type environments.

The paper is organized as follows. The next section describes the economic environment. Section 3 characterizes the best implementable allocation. Sections 4 and 5 construct mechanisms that uniquely implements it. A concluding comment is offered in the final section.

2 Environment

There are three dates: 0, 1 and 2. The economy is endowed with $Y > 0$ units of date-1 goods. A constant returns to scale technology transforms y units of date-1 goods into $yR > y$ units of date-2 goods.

[1] GL assume that depositor types are identically and independently distributed.

[2] Cavalcanti and Monteiri (2011) propose an alternative indirect mechanism when they examine a PS environment. In one example, they show that their indirect mechanism uniquely implements the best implementable allocation. In another example, their indirect mechanism has a bank run equilibrium.

There are N *ex ante* identical agents. An agent is one of two types $t \in T = \{1, 2\}$: patient, $t = 1$, or impatient, $t = 2$. The utility function for an impatient agent is $u(c^1)$ and the utility function for a patient agent is $v(c^1 + c^2)$, where c^1 is date-1 consumption and c^2 is date-2 consumption. u and v are increasing, strictly concave, and twice continuously differentiable. Agents maximize expected utility.

The number of patient agents in economy is drawn from the probability distribution $\pi = (\pi_0, \ldots, \pi_N)$, where $\pi_n > 0$, $n \in \{1, \ldots N\} \equiv \mathbb{N}$, is the probability that there are n patient agents. A *queue* is the vector $t^N = (t_1, \ldots, t_N) \in T^N$, where $t_k \in T$ is the type of agent that occupies the k^{th} position/coordinate in the queue. Let $P_n = \{t^N \in T^N | \#2 \in t^N = n\}$ and $Q_n = \{j | t_j = 2 \text{ for } t^N \in P_n\}$, where '#2' is the number of patient agents. P_n is the set of queues with n patient agents and Q_n is the queue positions of the n patient agents in $t^N \in P_n$. The probability that $t^N \in P_n$ is $\pi_n / \#P_n = \pi_n / \binom{N}{n}$, where $\#P_n$ is the number of queues $t^N \in P_n$. This specification implies that all potential queues with n patient agents are equally likely. Agents are randomly assigned a position in the queue, where the (unconditional) probability that an agent is assigned to position k is $1/N$. For convenience, call the agent assigned to position k agent k.

The queue realization, t^N, is observed by no one: not by any of the agents nor the planner. Each agent, however, privately observes his type $t \in T$.

The timing of events and actions is as follows. At date 0, the planner constructs a mechanism that determines how date-1 and date-2 consumption are allocated among the N agents, and queue t^N is realized. A mechanism is a set of announcements, M and A, and a allocation rule, $c = (c^1, c^2)$ where $c^1 = (c^1_1, \ldots c^1_N)$ and $c^2 = (c^2_1, \ldots c^2_N)$. At date 1, agents sequentially meet the planner, starting with agent 1. In a meeting with agent k, the planner announces $a_k \in A$ and agent k responds with $m_k \in M$. Only agent k and the planner can directly observe a_k and m_k. (But the planner can reveal (a_k, m_k) to agent $j \geq k$ via announcement a_j, if he wishes.) There is a sequential service constraint at date 1, which means the planner allocates date-1 consumption to agent $k \in \mathbb{N}$ based on the announcements of agents $j \leq k$, i.e., $c^1_k(m^{k-1}, m_k)$, where $m^{k-1} = (m_1, \ldots, m_{k-1})$. Agents consume the date-1 good at their date-1 meetings with the planner. After all agents have met the planner, the planner simultaneously allocates the date-2 consumption good to each agent based on all of the date-1 announcements made by the agents, i.e., agent k receives $c^2_k(m^N)$, where $m^N = (m_1, \ldots, m_N) \in M^N$.

3 Best Weakly Implementable Allocation

An allocation is weakly implementable is if it is an outcome to some equilibrium of the mechanism; it is strongly (or uniquely) implementable if it is an outcome to every equilibrium of the mechanism. Among the set of weakly implementably allocations, the best weakly implementable allocation provides agents with the highest expected utility. To characterize the best weakly implementable allocation, it is without loss of generality to restrict the planner to use a direct revelation mechanism, where agents make truthful announcement, $m_k = t_k \in M^D = \{1, 2\}$. The economy-wide welfare—which is the expected utility of an agent before he learns his type—associated with allocation rule c when agents use strategies $m_k \in M^D$ is

$$\sum_{n=0}^{N} \frac{\pi_n}{\binom{N}{n}} \sum_{t^N \in P_n} \sum_{k=1}^{N} U\left[c_k^1\left(m^{k-1}, m_k\right), c_k^2\left(m_1^N\right), t_k\right], \tag{1}$$

where

$$U\left[c_k^1\left(m^{k-1}, m_k\right), c_k^2\left(m_1^N\right), t_k\right] = u\left[c_k^1\left(m^{k-1}, m_k\right)\right] \text{ if } t_k = 1$$

and

$$U\left[c_k^1\left(m^{k-1}, m_k\right), c_k^2\left(m_1^N\right), t_k\right] = v\left[c_k^1\left(m^{k-1}, m_k\right) + c_k^2\left(m^N\right)\right] \text{ if } t_k = 2$$

The allocation rule c is feasible, i.e., there exists sufficient resources to pay for c for all $m_k \in M^D$, $k \in \mathbb{N}$, if

$$R\left(Y - \sum_{k=1}^{N} c_k^1\left(m^{k-1}, m_k\right)\right) \geq \sum_{k=1}^{N} c_k^2\left(m^N\right). \tag{2}$$

Allocation rule c must be incentive compatible in the sense that agent k has no reason to announce $m_k \neq t_k$. Since impatient agent k only values date-1 consumption, he always announces $m_k = 1$.[3] When $A = \emptyset$,[4] patient agent k has no incentive to depart from the strategy $m_k = 2$, assuming that all other agents j announce $m_j = t_j$, if

$$\begin{aligned} &\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} v\left[c_k^1\left(t^{k-1}, 2\right) + c_k^2\left(t^{k-1}, 2, t_{k+1}^N\right)\right] \geq \\ &\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} v\left[c_k^1\left(t^{k-1}, 1\right) + c_k^2\left(t^{k-1}, 1, t_{k+1}^N\right)\right], \end{aligned} \tag{3}$$

where $x_i^j = (x_i, \ldots, x_j)$ and

$$\hat{\pi}_n = \frac{\pi_n / \binom{N}{n}}{\sum_{n=1}^{N} \pi_n / \binom{N}{n}}$$

[3] This anticipates the result that the best weakly implementable allocation provides zero date-1 consumption to patient agents.

[4] To characterize the best weakly implementable allocation, one wants to choose from the largest possible set of incentive compatible allocations. This occurs when $A = \emptyset$. In particular, when $A = \emptyset$, there is only one incentive compatibility constraint for all patient agents, (3). When $A \neq \emptyset$, there will be distinct incentive constraints for agents k who receive information a_k from the mechanism. Since an appropriately weighted average of these distinct incentive constraints reduces to the single incentive constraint (3), the set of incentive compatible allocations when $A \neq \emptyset$ is a subset of the set of incentive compatible allocations when $A = \emptyset$.

is the conditional probability that agent k is in a specific queue that has n patient agent. The $1/n$ terms that appear in (3) reflect that a patient agent has a $1/n$ chance of occupying each of the patient queue positions in Q_n.

The best weakly implementable allocation, denoted as $c^* = (c^{1*}, c^{2*})$, is given by the solution to

$$\max_{c} \ (1) \text{ subject to (2) and (3),} \tag{4}$$
$$\text{where } m_k = t_k \text{ for all } k \in \mathbb{N} \text{ in (1) and (2).}$$

When agents use truth-telling strategies, c^* has the feature that impatient agents consume at date 1 and patient agents consume only at date 2. The allocation rule c^* corresponds to the analysis contained in PS's Appendix B.

Both PS and Ennis and Keister (2009) demonstrate, by example, that mechanism $\left(M^D, c^*\right)$ can have two equilibria: one where agents play truth-telling strategies, $m_k = t_k$ for all $k \in \mathbb{N}$, and another where agents play bank run strategies, $m_k = 1$ for all $k \in \mathbb{N}$.[5] The bank run equilibria arise in these examples because the direct revelation mechanism $\left(M^D, c^*\right)$ is not an optimal mechanism. An optimal mechanism may be a direct mechanism with $A \neq \emptyset$ or an indirect mechanism, (or both).

4 Direct Mechanisms with $A \neq \emptyset$

When c^* cannot be uniquely implemented by the direct mechanism $\left(M^D, c^*\right)$, the optimal mechanism may be a direct mechanism with $A \neq \emptyset$, i.e., $\left(A, M^D, c^*\right)$. Consider first the example provided by Ennis and Keister (2009). Ennis and Keister (2009) assume the preference specification of GL, which implies that incentive constraint (3) does not bind for allocation c^*. In addition, we know from GL that when $A = \mathbb{N}$ and $a_k = k$, i.e., the planner announces the agent's position in the queue, none of the N incentive compatibility constraints for patient agents bind. This means that mechanism $\left(A = \mathbb{N}, M^D, c^*\right)$ can weakly implement the best allocation in c^*. And the main result of GL implies that mechanism $\left(A = \mathbb{N}, M^D, c^*\right)$ can strongly implement c^*. Therefore, $\left(A = \mathbb{N}, M^D, c^*\right)$ is an optimal mechanism; $\left(M^D, c^*\right)$ admits a bank run equilibrium only because it is a suboptimal mechanism.

Consider now the example provided by PS in their Appendix B. Nosal and Wallace (2009) show that the best weakly implementable allocation, c^*, is not weakly implementable if the direct mechanism is characterized by $A = \mathbb{N}$ and $M = \{1, 2\}$. This implies that the mechanism used by PS, $\left(M^D, c^*\right)$, is an optimal *direct* mechanism. But the optimal mechanism may not be a direct mechanism.

[5] The Ennis and Keister (2009) example that I refer to is their bank run example in section 4.2 of their paper, where agents do not know their position in the queue, as in PS, but where the utility functions of patient and impatient agents are the same, as in GL.

5 Indirect Mechanisms

Suppose that mechanism $\left(M^D, c^*\right)$ weakly, but not strongly, implements the best allocation in c^*, and that mechanism $\left(A, M^D, c\right)$, where $A \neq \emptyset$, cannot weakly implement the best allocation in c^*. Since a direct mechanism cannot uniquely implement the best allocation in c^*, I construct an indirect mechanism that can.

The indirect mechanism $\left(M^I, c\right)$ has $M^I \in \{1, 2, (2, r)\}$. The mechanism exploits the fact that each agent knows when all other agents are playing bank run strategies. One can interpret the message $m_k = (2, r)$ as agent k telling the planner he is patient and other (patient) agents are playing bank run strategies. The mechanism rewards agent k if evidence supports that notion that other agents are playing bank run strategies, and punishes otherwise. The evidence supports a bank run if all agents $j \in \mathbb{N} \backslash \{k\}$ announce either $m_j = 1$ or $m_j = (2, r)$, and does not support a bank run if some agent $j \in \mathbb{N} \backslash \{k\}$ announces $m_j = 2$. For convenience of presentation, I assume that agents are restricted to play pure strategies, (the appendix examines the case where agents can play randomized strategies). Since agents do not know their position in the queue, there are only two possible pure strategy equilibria: one where agents play truth-telling strategies and another where they play bank run strategies.

The following notation is needed. The allocation rule for the indirect mechanism is $c = (c^1, c^2)$, and the date-s payoff to agent j who announces m_j is denoted as $c_j^s|_{m_j}$. Define Z as the set of agents who announce $(2, r)$, i.e., $Z = (2, r) = \{j | m_j = (2, r)\}$. Define $\hat{m}^{k-1}$ as the message vector of length $k-1$ where for each $j \leq k-1$, $\hat{m}_j = 1$ if either $m_j = 1$ or $m_j = (2, r)$, and $\hat{m}_j = 2$ if $m_j = 2$.

I now specify the allocation rule c for the indirect mechanism $\left(M^I, c\right)$. If agent j announces $m_j = (2, r)$, then

$$\begin{aligned} c_j^1|_{(2,r)} &= 0 \\ c_j^2|_{(2,r)} &= \begin{cases} c_j^{1*}\left(\hat{m}^{j-1}, 1\right)(1+\varepsilon) & \text{if} \quad m_i \in \{1, (2, r)\} \text{ for all } i \neq j, \\ 0 & \text{if} \quad m_i = 2 \text{ for some } i \neq j, \end{cases} \end{aligned} \tag{5}$$

where $R > 1 + \varepsilon$, and $\varepsilon > 0$. One can interpret $\varepsilon c_j^{1*}\left(m^{j-1}, 1\right)$ as a reward that agent j receives for announcing $m_j = (2, r)$ when the evidence supports a bank run. If the evidence does not support a bank run, then agent j receives zero. The mechanism is able to gather evidence regarding bank runs since an agent who announces $(2, r)$ receives his consumption payment at date 2.

If agent j announces $m_j = 1$, then

$$\begin{aligned} c_j^1|_1 &= \begin{cases} c_j^{1*}\left(\hat{m}^{j-1}, 1\right) & \text{if} \quad j < N \\ c_j^{1*}\left(\hat{m}^{j-1}, 1\right) + \Delta & \text{if} \quad j = N, \text{ for all } k, m_k \neq 2 \\ c_j^{1*}\left(\hat{m}^{j-1}, 1\right) + \Delta_2 & \text{if} \quad j = N, \; m_k = 2 \text{ for some } k \end{cases}, \\ c_j^2|_1 &= 0. \end{aligned} \tag{6}$$

Allocation rule (5) has the feature that if some agents announce $m_k = (2, r)$, then the planner accumulates "excess goods." That is, instead of making a date-1 payment of $c^{1*}(\hat{m}^{j-1},1)$ to an agent j that announces $m_j = (2,r)$, the planner gives either $c^{1*}(\hat{m}^{j-1},1)(1+\varepsilon)$ or zero at date 2, where $R > 1+\varepsilon$. If no agent j announces $m_j = 2$, then the total amount of this excess, denoted as Δ, is

$$\Delta = \sum_{z\in Z} \frac{(R-1-\varepsilon)\, c_z^{1*}(\hat{m}^{z-1},1)}{R};$$

if some agent j announces $m_j = 2$, then the total excess, denoted as Δ_2, is

$$\Delta_2 = \sum_{z\in Z} c_z^{1*}\left(\hat{m}^{z-1},1\right).$$

According to (6), agents who announce $m_k = 1$ and occupy the first $N-1$ positions in the queue receive the consumption payoff that they would get under the direct revelation mechanism $\left(M^D, c^*\right)$, assuming that $\hat{m}^{k-1}$ is used as the announcement vector. Agent N who announces $m_N = 1$ receives an additional consumption payment of either Δ or Δ_2.

Finally, if agent j announces $m_j = 2$, then

$$\begin{aligned} c_j^1|_2 &= 0, \\ c_j^2|_2 &= \begin{cases} c_j^{2*}\left(\hat{m}^N\right) & \text{if } j < N \\ c_j^{2*}\left(\hat{m}^N\right) + \Delta_2 R & \text{if } j = N \end{cases}. \end{aligned} \tag{7}$$

The structure of the payments associated with c_j^2 resembles that of c_j^1. However, Δ does not appear in c_j^2 because agent j announces $m_j = 2$, and in this situation any agent k who announces $m_k = (2,r)$ will receive zero. Note that the allocation rule (5)-(7) has the planner sometimes throwing away goods. This happens when agent N announces $(2,r)$.

Proposition 1 *The indirect mechanism $\left(M^I, c\right)$ uniquely implements in pure strategies the best weakly implementable allocation in c^*.*

Proof. First, there does not exist an equilibrium where all patient agents j announce $m_j = 1$. Suppose that such an equilibrium exists. Suppose that patient agent k defects from proposed play and announces $m_k = (2,r)$. Since all agents $j \in \mathbb{N}\backslash\{k\}$ are playing $m_j = 1$ in the equilibrium, (5) specifies a payment of $c_k^{1*}\left(m^{k-1},1\right)(1+\varepsilon)$ to agent k which strictly exceeds the equilibrium payment of $c_k^{1*}\left(m^{k-1},1\right)$; a contradiction.

Second, there does not exist an equilibrium where all patient agents j announce $m_j = (2,r)$. Suppose such an equilibrium exists. Then, the equilibrium expected utility to patient agent k is

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N\in P_n} \frac{1}{n}\left\{\sum_{k\in Q_n} v\left[c_k^{1*}\left(\hat{m}^{k-1},1\right)(1+\varepsilon)\right]\right\}. \tag{8}$$

Suppose that patient agent k defects from the proposed equilibrium and announces $m_k = 1$. Using (6), his expected utility is

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \{ \sum_{k \in Q_n} v[c_k^{1*}\left(\hat{m}^{k-1}, 1\right) + \phi \sum_{\substack{j \in Q_n \\ j \neq N}} \left(R - 1 - \varepsilon\right) c_j^{1*}\left(\hat{m}^{j-1}, 1\right)], \tag{9}$$

where

$$\phi = \begin{cases} 1 & \text{if } k = N, \\ 0 & \text{otherwise} \end{cases}.$$

Note that as $\varepsilon \to 0$, the difference between (9) and (8) is

$$\sum_{n=2}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \{ \sum_{k \in Q_n} v[c_k^{1*}\left(\hat{m}^{k-1}, 1\right) + \phi \sum_{\substack{j \in Q_n \\ j \neq N}} \left(R - 1\right) c_j^{1*}\left(\hat{m}^{j-1}, 1\right)] - v\left[c_k^{1*}\left(\hat{m}^{k-1}, 1\right)\right]\} > 0.$$

Hence, for any given N, π, and $R > 1$, the mechanism can choose $\varepsilon > 0$ sufficiently small so that the value of (9) strictly exceeds that of (8), a contradiction.

Finally, consider a truth-telling equilibrium, where $m_j = t_j$ for all j. Constraint (3) implies that patient agent k does not have a strict incentive announce $m_k = 1$. If patient agent k strictly prefers to announce $m_k = 1$ to $m_k = (2, r)$ when all agents $j \in \mathbb{N} \backslash \{k\}$ are playing $m_j = t_j$, then he has no incentive to announce $m_k = (2, r)$ when all agents $j \in \mathbb{N} \backslash \{k\}$ are playing $m_j = t_j$. Suppose patient agent k announces $m_k = (2, r)$. If $t^N \in P_1$, i.e., agent k is the only patient agent in the queue, then his consumption payment will be $c_k^{1*}\left(\hat{m}^{k-1}, 1\right)(1 + \epsilon)$, compared to $c_k^{1*}\left(\hat{m}^{k-1}, 1\right)$ if he announces $m_k = 1$. If, however, $t^N \in P_n$, $n \geq 2$—an event that occurs with strict positive probability—then agent k's consumption will be zero. For $\varepsilon > 0$ arbitrarily small, the expected utility associated with announcing $m_k = (2, r)$ is strictly less than announcing $m_k = 1$. Therefore, agent k strictly prefers to announce $m_k = 2$ to $m_k = (2, r)$.

All of this implies that when $\varepsilon > 0$ is arbitrarily small, the best weakly implementable allocation in c^* is uniquely implemented by the mechanism $\left(M^I, c\right)$. ■

Agents do not know their positions in the queue for the indirect mechanism $\left(M^I, c\right)$. Suppose that the economic environment is modified so agents not only learn their type, but they also (somehow) learn their position in the queue. Proposition 1 and its proof remains valid for the modified economic environment, where agents know their positions in the queue.[6]

[6] Of course, the 'c^*' for the modified environment may be different than the solution to (4). The best weakly implementable allocation for the new environment is given by $\max_c$ (1) subject to (2) and N incentive compatibility constraints, one for each patient agent in position $j \in \mathbb{N}$ in the queue.

6 Final Comment

The insights from the first part of the Diamond and Dybvig (1983) paper are as relevant today as they were almost 30 years ago: a poorly designed deposit contract can invite bank runs, and a well designed one can prevent them.

7 Appendix: Randomized Strategies

Consider first a slightly modified version of incentive constraint (3),

$$\begin{aligned}&\sum_{n=1}^{N}\hat{\pi}_n \sum_{t^N\in P_n}\frac{1}{n}\sum_{k\in Q_n} v\left[c_k^1\left(t^{k-1},2\right)+c_k^2\left(t^{k-1},2,t_{k+1}^N\right)\right] \geq \\ &\sum_{n=1}^{N}\hat{\pi}_n \sum_{t^N\in P_n}\frac{1}{n}\sum_{k\in Q_n} v\left[c_k^1\left(t^{k-1},1\right)+c_k^2\left(t^{k-1},1,t_{k+1}^N\right)\right]+\delta,\end{aligned} \tag{10}$$

where $\delta \geq 0$. When $\delta = 0$, (10) is identical to (3). If $\delta > 0$, then any patient agent j *strictly* prefers to announce $m_j = 2$ to $m_j = 1$, assuming all other agents $i \in \mathbb{N}\backslash\{j\}$ announce $m_i = t_i$. The best δ-weakly implementable allocation, denoted by $c^*(\delta) = (c^{1*}(\delta), c^{2*}(\delta))$, is given by the solution to

$$\max_{c(\delta)} \text{ (1) subject to (2) and (10),}$$
$$\text{where } m_k = t_k \text{ for all } k \in \mathbb{N} \text{ in (1) and (2).}$$

Note that as $\delta \to 0$, $c^*(\delta) \to c^*$ and $c^*(0) \equiv c^*$.

Agents can play randomized strategies. Denote the indirect mechanism as $\left(M^I, c^I\right)$, where $M^I \in \{1, 2, (2, r)\}$ and $c^I = \left(c^{1I}, c^{2I}\right)$. The basic structure of the allocation rule c^I is similar to c presented in text, (5)-(7), but the construction of c^I uses $c^*(\delta)$ and not $c^*(0) = c^*$. To reduce notational clutter I will suppress the 'δ' when using allocations in $c^*(\delta)$ to describe c^I. If agent j announces $m_j = (2, r)$, then

$$\begin{aligned} c_j^{1I}|_{(2,r)} &= 0, \\ c_j^{2I}|_{(2,r)} &= c_j^{1*}\left(\hat{m}^{j-1}, 1\right)(1+\varepsilon) \text{ for all } j,\ \varepsilon > 0. \end{aligned}$$

Note that, unlike $c_j^2|_{(2,r)}$ in the text, (5), agents are never "penalized" for announcing $m_j = (2, r)$. If agent j announces $m_j = 1$, then

$$\begin{aligned} c_j^{1I}|_1 &= \begin{cases} c_j^{1*}\left(\hat{m}^{j-1}, 1\right) & \text{if } j < N \\ c_j^{1*}\left(\hat{m}^{j-1}, 1\right) + \Delta & \text{if } j = N \end{cases}, \\ c_j^{2I}|_1 &= 0. \end{aligned}$$

Since agents are not penalized for announcing $(2, r)$, the Δ_2 term is now irrelevant and does not appear in $c_j^{1I}|_1$, (compared to $c_j^1|_1$ in the text, (6).) Finally, if agent j announces $m_j = 2$, then

$$\begin{aligned} c_j^{1I}|_2 &= 0, \\ c_j^{2I}|_2 &= \begin{cases} c_j^{2*}\left(\hat{m}^N\right) & \text{if } j < N \\ c_j^{2*}\left(\hat{m}^N\right) + \Delta R & \text{if } j = N \end{cases}. \end{aligned}$$

Here, as in the text, (7), an agent who announces $m_j = 2$, where $j = N$, will collect an additional payment, ΔR, where $\Delta R > 0$ if $Z \neq \emptyset$.

Proposition 2 *The indirect mechanism $\left(M^I, c^I\right)$ uniquely implements an allocation that is arbitrarily close to the best weakly implementable allocation in c^*.*

Proof. First, there does not exist an equilibrium where patient agents j announce $m_j = 2$ with probability σ_1 and $m_j = 1$ with probability $1 - \sigma_1$, where $\sigma_1 < 1$. Suppose that such an equilibrium exists. Suppose that patient agent k defects from proposed play and announces $m_k = (2, r)$ with probability one. His consumption will be $c_j^{1*}(\hat{m}^{j-1}, 1)(1 + \varepsilon)$ which is strictly greater than the equilibrium consumption $c_j^{1*}(\hat{m}^{j-1}, 1)$; a contradiction.

Second, there does not exist an equilibrium where patient agents j announce $m_j = 1$ with probability σ_2 and $m_j = (2, r)$ with probability $1 - \sigma_2$, where $\sigma_2 < 1$. (The case where $\sigma_2 = 1$ is covered in the first step, above.) Suppose that such an equilibrium exists. Suppose that patient agent k defects from proposed play and announces $m_k = 1$ with probability one. Note that as $\varepsilon \to 0$, $c_j^{1*}(\hat{m}^{j-1}, 1)(1 + \varepsilon) - c_j^{1*}(\hat{m}^{j-1}, 1) \to 0$ for all $j < N$. *Conditional on patient agent j not occupying the N^{th} position in the queue*, the difference in the expected utility of patient agent j announcing $m_j = 1$ and announcing $m_j = (2, r)$, by continuity, tends to zero as $\varepsilon \to 0$. Patient agent $j = N$ receives a consumption payoff of $c_j^{1*}(\hat{m}^{j-1}, 1) + \Delta$ if he announces $m_j = 1$ and $c_j^{1*}(\hat{m}^{j-1}, 1)(1 + \varepsilon)$ if he announces $m_j = (2, r)$. Hence, as $\varepsilon \to 0$,

$$c_j^{1*}\left(\hat{m}^{j-1}, 1\right) + \Delta - c_j^{1*}\left(\hat{m}^{j-1}, 1\right)(1 + \varepsilon) \to \sum_{z \in Z} \frac{(R-1)\, c_z^{1*}(\hat{m}^{z-1}, 1)}{R} > 0 \text{ if } Z \neq \emptyset.$$

In the proposed equilibrium, the event $Z \neq \emptyset$ occurs with a probability that is strictly greater than zero and $\min_{Z \neq \emptyset}\{\Delta\}$ is not "arbitrarily small." For any given N, π, R and $\sigma_2 < 1$, the mechanism can choose an $\varepsilon > 0$ sufficiently small so that the expected utility of patient agent k announcing $m_k = 1$ strictly exceeds that of announcing $m_k = (2, r)$; a contradiction. This discussion implies that patient agent k can never be indifferent between announcing $m_k = 1$ and $m_k = (2, r)$ when $\varepsilon > 0$ is arbitrarily small; he strictly prefers to announce $m_k = 1$ if other agents j announce $m_j = (2, r)$ with positive probability.

Third, suppose there is an equilibrium where patient agents announce $m_j = 2$ with probability σ_3 and $m_j = (2, r)$ with probability $1 - \sigma_3$, where $\sigma_3 < 1$. Since patient agents are asked to randomize in the equilibrium, it must be the case that patient agent k is indifferent between announcing $m_k = 2$ and $m_k = (2, r)$. But the above discussion, in the second step, implies that patient agent k will defect from proposed equilibrium play and announce $m_k = 1$ with probability one, since he strictly prefers announcing $m_k = 1$ to $m_k = (2, r)$ (or $m_i = 2$); a contradiction. There also cannot be an equilibrium where patient agents j randomize over announcing $m_j = 1$, $m_j = 2$, and $m_j\,(2, r)$; a patient agent k strictly prefers announcing $m_k = 1$ with probability one. The first three steps imply that the indirect mechanism does not admit equilibria with randomization.

Finally, consider an equilibrium where agents of type t_j announce $m_j = t_j$. Suppose that $\delta > 0$. Then patient agent j strictly prefers announcing $m_j = 2$ to $m_j = 1$. Patient agent k strictly prefers announcing $m_k = (2, r)$ to $m_k = 1$ when all other agents $j \in \mathbb{N} \setminus \{k\}$ announce $m_j = t_j$. But for any $\delta > 0$, there exists an $\varepsilon > 0$ sufficiently small so that patient agent k strictly prefers announcing $m_k = 2$ to $m_k = (2, r)$ since $\delta > 0$ implies that agent k strictly prefers announcing $m_k = 2$ to $m_k = 1$. Therefore, for $\delta > 0$ arbitrarily small, $c^*(\delta) \approx c^*$, and the unique equilibrium for mechanism $\left(M^I, c^I\right)$ is characterized by $m_j = t_j$ for all $j \in \mathbb{N}$. ■

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