Deterministic components in the CVAR

A graduate course in the Cointegrated VAR model: Special topics in Rome

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November 2011

Dummy variables in the CVAR

Table: Various dummy variables in Δx_t and x_t

Δx_t $D_{s,t}$	x_t $\sum D_{s,t} = trend$	Δx_t $D_{p,t}$	x_t $\sum D_{p,t} = D_{s,t}$	Δx_t $D_{tr,t}$	x_t $\sum D_{tr,t} = D_{p,t}$	Δx_t $D_{dtr,t}$	$\sum D$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
0	0	0	0	0	0	0	
0	0	0	0	0	0	0	
1	1	1	1	1	1	1	
1	2	0	1	-1	0	-2	
1	3	0	1	0	0	1	
1	4	0	1	0	0	0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

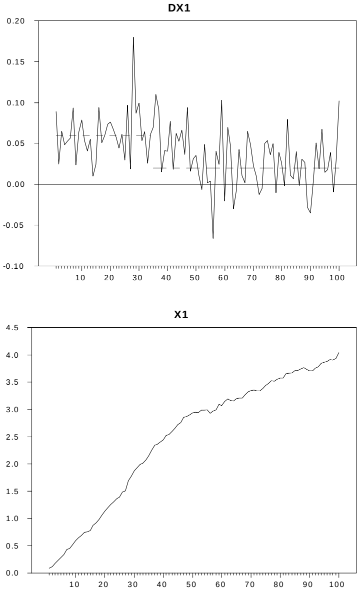


Figure: Illustrating a mean shift in $\Delta x_{1,t}$ and its effect on $x_{1,t}$ (left hand side

Distinguishing between different shocks

- ordinary (normally distributed) random shocks,
- (extra)ordinary large shocks due to permanent interventions ($|\hat{\varepsilon}_{i,t}| > 3.3\hat{\sigma}_{\varepsilon}$) with a delayed dynamic effect in the data, to be described by a blip dummy in the model,
- transitory large innovational outliers with a delayed dynamic effect in the data, to be described by a $+/-$ blip dummy in the model,
- additive transitory outliers (typing mistakes, etc.) with no delayed dynamic effect in the data, to be removed prior to modelling.

Dummy variables in the CVAR

Using dummies to account for extraordinary mean-shifts, permanent blips, and transitory shocks, the cointegrated VAR model is reformulated as:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \Phi_s D_{s,t} + \Phi_p D_{p,t} + \Phi_{tr} D_{tr,_t} + \mu_0 + \varepsilon_t, \\ \varepsilon_t \sim NI(0, \Omega\ ),\ t = 1, ..., T \tag{1}$$

where $D_{s,t}$ is $d_1 \times 1$ vector of mean-shift dummy variables (...0,0,0,1,1,1,...), $D_{p,t}$ is a $d_2 \times 1$ vector of permanent blip dummy variables (...0,0,1,0,0,...) and $\mathbf{D}_{tr,t}$ is a $d_3 \times 1$ vector of transitory shock dummy variables (...0,0,1,-1,0,0,...).

Decomposing the dummy effects

It is useful to partition the dummy effects into an α and a $\beta_\perp$ component:

$$\Phi_s = \alpha\delta_0 + \delta_1, \tag{2}$$

$$\Phi_p = \alpha\varphi_0 + \varphi_1, \tag{3}$$

$$\Phi_{tr} = \alpha\psi_0 + \psi_1. \tag{4}$$

Rewritng the CVAR (without Γ_i):

$$\Delta x_t = \alpha\tilde{\beta}'\tilde{x}_{t-1} + \delta_1 D_{s,t} + \varphi_1 D_{p,t} + \psi_1 D_{tr,t} + \gamma_0 + \varepsilon_t,$$

where $\tilde{\beta}' = [\beta', \beta_0', \delta_0', \varphi_0', \psi_0']$ and $\tilde{x}_{t-1}' = [x_t', 1, D_{s,t}', D_{p,t}', D_{tr,t}',]$. The expected value of Δx_t and $\beta' x_t$ are:

$$\begin{aligned} E\Delta x_t &= \gamma_0 + \delta_1 D_{s,t} + \varphi_1 D_{p,t} + \psi_1 D_{tr,t} \\ E\beta' x_t &= \beta_0 + \delta_0 D_{s,t} + \varphi_0 D_{p,t} + \psi_0 D_{tr,t}. \end{aligned} \tag{5}$$

The moving average form with dummy effects

$$
\begin{aligned}
x_t &= C\sum_{i=1}^{t-1}\varepsilon_i + C\mu_0\sum_{i=1}^{t-1}1+ \\
&\quad C\Phi_s\sum_{i=1}^{t-1}D_{s,i} + C\Phi_p\sum_{i=1}^{t-1}D_{p,i} + C\Phi_{tr}\sum_{i=1}^{t-1}D_{tr,i}+ \\
&\quad C^*(L)(\varepsilon_t + \mu_0 + \Phi_s D_{s,t} + \Phi_p D_{p,t} + \Phi_{tr} D_{tr,t}) + \tilde{X}_0
\end{aligned}
\tag{6}
$$

where

$$C = \beta_\perp(\alpha'_\perp\Gamma\beta_\perp)^{-1}\alpha'_\perp \tag{7}$$

and $\mathbf{C}^*(L)$ is an infinite polynomial in the lag operator L.

Using

$$\begin{aligned} \mu_0 &= \alpha\beta_0 + \gamma_0 \\ \Phi_s &= \alpha\delta_0 + \delta_1 \\ \Phi_p &= \alpha\varphi_0 + \varphi_1 \\ \Phi_{tr} &= \alpha\psi_0 + \psi_1 \end{aligned} \quad \text{and } C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp$$

it is easily shown that the $\boldsymbol{\alpha}$ components will disappear in the summations in (6), so that

$$\begin{aligned} C\Phi_s &= C\delta_1 \\ C\Phi_p &= C\varphi_1 \\ C\Phi_{tr} &= C\psi_1. \end{aligned} \tag{8}$$

and:

$$\begin{aligned} x_t &= C\sum_{i=1}^{t-1}\varepsilon_i + C\gamma_0\sum_{i=1}^{t-1}1 + C\delta_1\sum_{i=1}^{t-1}D_{s,i} + C\varphi_1\sum_{i=1}^{t-1}D_{p,i} \\ &\quad + C\psi_1\sum_{i=1}^{t-1}D_{tr,i} + C^*(L)(\varepsilon_t + \mu_0 + \Phi_s D_{s,t} + \Phi_p D_{p,t} + \Phi_{tr} D_{tr,t}) - \end{aligned} \tag{9}$$

Are the observed outliers additive or innovational?

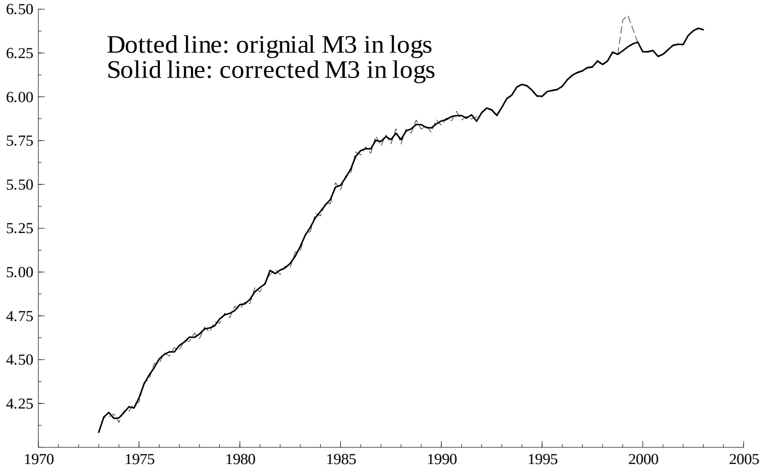
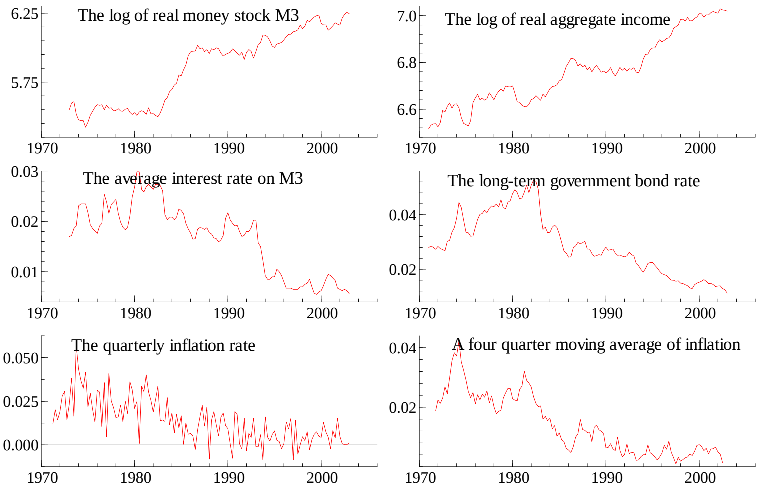
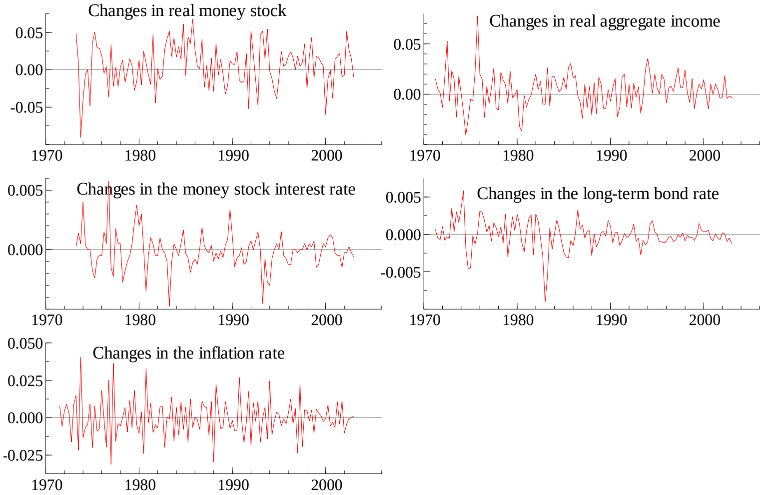


Figure: The original and corrected M3 in logs.

Illustration: the Danish money demand data in levels



The Danish data in differences



The definition of the intervention dummies

- $D_s 831_t = 1$ for $t =$ 1983:1,.....,2003:4, 0 otherwise,
- $D_{tr} 754_t = 1$ for $t =$ 1975:4, -0.5 for 1976:1 and 1976:2, 0 otherwise,
- $D_p 764_t = 1$ for $t =$ 1976:4, 0 otherwise,

The shift dummy, $D_s 831_t$ is restricted to lie in the cointegration space and its difference $\Delta D_s 831_t$ (i.e. $D_p 831_t$) should be included as an unrestricted permanent blip dummy in the VAR equations. In some cases it is not enough to include just the first difference but also the lagged dummy $\Delta D_s 831_{t-1}$.The latter turn out to be significant in the short-term interest rate equation, probably picking up a lagged policy reaction to the large drop in the long-term bond rate.

The VAR model to be estimated

$$
\begin{aligned}
\Delta x_t &= \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \alpha\beta_0 + \alpha\beta_1 t + \alpha\delta_0 D_s 831_t \\
&\quad + \Phi_{p.1} D_p 831_t + \Phi_{p.2} D_p 831_{t-1} \\
&\quad + \Phi_{tr} D_{tr} 754_t + \Phi_{p.3} D_p 764_t + \gamma_0 + \varepsilon_t \\
&= \Gamma_1 \Delta x_{t-1} + \alpha\tilde{\beta}' \tilde{x}_{t-1} + \Phi_{p.1} D_p 831_t + \Phi_{p.2} D_p 831_{t-1} \\
&\quad + \Phi_{tr} D_{tr} 754tr_t + + \Phi_{p.3} D_p 764_t + \gamma_0 + \varepsilon_t,
\end{aligned}
$$

where

$$
\tilde{\beta} = \begin{bmatrix} \beta \\ \beta_0 \\ \beta_1 \\ \delta_0 \end{bmatrix} \text{ and } \tilde{x}_{t-1} = \begin{bmatrix} x_{t-1} \\ 1 \\ t \\ D_s 831_{t-1} \end{bmatrix}.
$$

The estimated results with dummies

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \alpha' \beta \tilde{x}_{t-1} + \Phi D_t + \varepsilon_t$$

$$\alpha'\beta = \begin{bmatrix} m^r_{t-1} & y^r_{t-1} & \Delta p_{t-1} & R_{m_{t-1}} & R_{b,t-1} & D_s831_{t-1} & t-1 \\ -\mathbf{0.26} & \mathbf{0.14} & -0.52 & \mathbf{2.74} & -\mathbf{3.29} & \mathbf{0.032} & \mathbf{0.0004} \\ 0.03 & -\mathbf{0.15} & -0.28 & -\mathbf{2.07} & 0.54 & -0.005 & 0.0002 \\ -0.00 & 0.01 & -\mathbf{0.84} & -0.48 & 0.24 & -0.006 & -\mathbf{0.0002} \\ -0.00 & 0.00 & 0.03 & -\mathbf{0.13} & 0.06 & \mathbf{0.002} & -0.0000 \\ 0.00 & 0.00 & 0.01 & 0.07 & -\mathbf{0.10} & -0.001 & -0.0000 \end{bmatrix}$$

$$\Phi D_t = \begin{bmatrix} D_{tr}754_t & D_p764_t & D_p831_t & D_p831_{t-1} & const \\ 0.01 & -0.02 & 0.03 & -0.02 & 0.64 \\ \mathbf{0.03} & 0.01 & -0.01 & 0.01 & \mathbf{0.88} \\ -\mathbf{0.01} & 0.00 & -0.01 & 0.01 & -0.01 \\ 0.00 & \mathbf{0.01} & 0.00 & -\mathbf{0.00} & -0.01 \\ -0.00 & 0.00 & -\mathbf{0.01} & -0.00 & -0.02 \end{bmatrix}$$

The robustness of the result to the assumption of normal errors

Without controlling for reforms and interventions that have produced extraordinary large residuals the normality assumption is often violated. There are cases when:

1. The linear relationship of the VAR model does not hold for large shocks: market reacts differently to ordinary and extraordinary shocks.
2. The linear relationship of the VAR model holds approximately, but the properties of the VAR estimates are sensitive to the presence of extraordinary large shocks. Ordinary and extraordinary shocks are drawn from different distributions.
3. The estimates of the VAR model are robust to deviations from normality.