

# Why Rent When You Can Buy?

## A Theory of Repurchase Agreements\*

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### Abstract

In a model with matching frictions, we provide conditions under which repurchase agreements (or repos) co-exist with asset sales. In a repo, the seller agrees to repurchase the asset at a later date at the agreed price. Absent bilateral trading frictions, repos have no role despite uncertainty about future valuations. Introducing pairwise meetings, we show that agents prefer to sell (or buy) assets whenever they face little uncertainty regarding the future use of the asset. As agents become more uncertain of the value of holding the asset, repos become more prevalent. We show that while the total volume of repos is always increasing with the uncertainty, the total sales volume is hump-shaped. In other words, pairwise matching alone is sufficient to explain why repo markets exist and there is no need to introduce random matching, search frictions, information asymmetries or other market frictions.

## 1. Introduction

Many financial securities, including sovereign and corporate bonds, are traded via repurchase agreements (a.k.a. repos) or on securities lending markets where the seller agrees to repurchase the asset at a later date at a given price.<sup>1</sup> Financial repos are usually associated

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<sup>1</sup>This is also true of many non-financial assets such as cars, houses or airplanes, in which case we talk of leasing or renting.

with collateralized cash loans driven by the financing needs of the borrower. Therefore they appear at first sight only very remotely related – if at all – to the lending (or renting) of a security. However, as we explain below, repos are a key instrument for market participants in search of a specific security. But why do these markets exist? In other words, while repo and securities lending markets coexist with the market for assets, why do market participants engage in repos, i.e. rent the asset, rather than just buying the asset to resale it later if they have the means? Also, what is the impact of the repo market on asset sales? Are asset sale and repo transactions two substitute activities, and hence should we expect the repo and sales volumes to co-move negatively for a given set of traders,?

In this paper, we show that utility maximizing agents do better by combining the repo and asset markets, rather than using the asset market alone. The essential elements are 1) pairwise meeting and trading<sup>2</sup> and 2) uncertain valuation of the asset. We assume that agents receive some preference shocks on the current utility from holding the asset, which may be more or less persistent. Once the shock hits, agents meet and trade in pairs. Then they have to wait until the next trading session (say the next day) to change their position. This is the extent to which trading frictions prevent the emergence of a Walrasian market outcome. Absent these frictions, we show that agents trading in a Walrasian market would consider repos and asset sales as one same instrument.

The explanation for this result is rather intuitive. Pairwise meeting implies that agents have different valuations for the asset today or in the future. An efficient allocation will equate both the current and the future marginal valuations of the asset across agents, which is only possible if there are two instruments: The outright purchase of assets and repo. But why is there a wedge between the present and the future valuations of the asset with pairwise trade, while there is none in a Walrasian market? Suppose you can only trade in pair. Your future valuation depends directly on your asset holdings because it will possibly affect who you meet in the future, your reservation value for the trade, and so the outcome of your match. In a Walrasian environment, the value of your asset is set by its price, and this price is the same independently of who you are and who you meet, of your asset holdings, or the asset holdings of others. Therefore, in pairwise meetings, your current and future valuations will typically differ from the ones of your trading partner, and this is where combining repos and outright purchase is useful. Repos allow agent to attain a level of consumption which depends on their current valuation of the asset, independent of the uncertainty about their

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<sup>2</sup>More generally, we assume that agents are unable to trade within a group with a aggregate valuation identical to the Walrasian market valuation.

future valuation.

Our results are *not* due to *random* matching. Indeed, with pairwise matching, the analysis is complicated by the fact that agents' asset holdings depend on their history of match. So in an environment where agents can have two valuations for the asset, we consider random as well as directed matching, in the sense of Corbae, Temzelides and Wright (2003). With directed matching, we impose a matching rule that, in equilibrium, maximizes the possible surplus from trade. Although agents have a good idea of whom they will meet in the future, they still find repo useful above and beyond the mere acquisition of assets. Also under this matching rule we show that an invariant distribution of assets has a two point support: There is one asset holding for each valuation. The difference in these two points is increasing in the persistence of valuation shocks: As the probability to switch valuation decreases, agents' asset holdings diverge. Inversely, as switching becomes more likely, agents tend to hold the same amount of the asset.

Our results are *not* due to *search frictions* either. Contrary to other papers in the related literature, we do not introduce search frictions: Agents always meet somebody they can trade with. Rather, the friction in this paper is the fact that matching and trading is bilateral. To convince the reader of this, we compute the equilibrium with increased matching speed, as matching speed may be seen as similar to a search friction. We show that agents repo less and less as the time to the next match decreases. However, as we drive the time to the next match to zero, we find that agents still use repos. The reason is that the outcome of the match is still depending on the asset holdings of the agents in the match, hence the wedge between present and future valuation is still there, which gives rise to the simultaneous usefulness of asset sale and repo.

Interestingly, the total volume of asset sales is directly linked with the range of the support: As the difference in asset holdings increases, the sale of asset in a match is also increasing. This is intuitive: With directed matching, agents who just switched their valuation from high to low are matched with those agents who switched valuation from low to high. Therefore, as the difference in their asset holdings grows, also does the gains from trade, so that they trade a larger amount of the asset. However, since types are more persistent, fewer agents switch types so that the total volume of sales can either increase or decrease. We show that the effect of type persistence on sales volume is hump-shaped. Similarly, as the future valuation becomes uncertain, i.e. types are not persistent, agents are unwilling to change their position through asset sales, but they are willing to engage in repos. Therefore, the total volume of repo is decreasing with persistence and it is higher than total sales volume

when the uncertainty is high (or persistence is low).

### **Related Literature**

We first describe the market for securities lending. There are two ways traders can acquire a security for a short term use: They can either engage in securities lending or conduct “specials” repo. “Specials” are described in detail in Duffie (1996). Securities lending and “specials” have different legal and fiscal characteristics, so that a trader may prefer one over the other, but their economic function is the same: They allow a trader to acquire a specific security temporarily.<sup>3</sup> The reasons why a trader needs to borrow a particular security vary, but generally the securities lent are needed to support a trading strategy or a settlement obligation. These motivations are further analyzed in CACEIS (2010), Duffie (1996) or Vayanos and Weill (2008), but for our purpose, it suffices to say that the security provides a service to the borrower that he values above and beyond its mere cash flows.<sup>4</sup>

To induce the lender of a security to trade, he usually obtains a lower repo rate than the prevailing money market rates, and invests the funds in money markets for a profit. The rights of the holder of a security acquired through a repo or securities lending are very similar: In a repo transaction, the buyer owns the collateral asset, he can re-use them during the term of the repo by selling the asset outright, “repoing” them or pledging them to a third party.<sup>5</sup> In a securities-lending transaction, the borrower gains the ownership title to the securities lent while the lender gains full ownership of the title to the securities (or cash) pledged as collateral.<sup>6</sup> Finally, both the repo and the securities lending markets involve trades negotiated bilaterally out of electronic trading platforms and their clearing is executed without the help of a central counterparty.<sup>7</sup> Therefore, repo markets for the purpose of getting access to a securities and the securities lending markets are very similar, and they look a lot like a market where borrowers are just renting the asset for a short period of time. From now on, we will refer to the securities lending market or the repo market as

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<sup>3</sup>See CACEIS (2010) for a description of the different characteristics of repos and securities lending.

<sup>4</sup>Borrower may need to cover a failed transaction in the course of their trading activity, or a short position, or they may need to deliver securities they have not yet purchased against the exercise of a derivatives contract, or they want to raise specific collateral, perhaps for another securities lending transaction.

<sup>5</sup>See Monnet (2011) for the economics of rehypothecation.

<sup>6</sup>As explained in CACEIS (2010), the borrower can re-sell the securities borrowed, voting rights are transferred along with the title. Although the borrower, as owner of the securities, is entitled to the possible economic benefits associated with ownership such as dividends and coupons, he is under the contractual obligations to make equivalent payments in all distributions paid during the terms of the trade to the lender.

<sup>7</sup>See Koepl and Monnet (2010), Carapella and Monnet (2011), Carapella and Mills (2011) or Acharya and Bisin (2010) for some economic analysis of the role of central counterparties.

simply the repo markets.

The importance of the securities lending market is highlighted in the empirical work of Fleming and Garbade (2007), who study the behavior of dealers at the Fed's securities lending facility. This facility was initiated by the Fed to solve settlement fails (when dealers fail to deliver promised securities). As described in the paper, the Fed auctions securities at noon, after the period of greatest liquidity in the over-the-counter securities lending market, to give dealers access to an additional source of securities. Dealers can bid to borrow particular Treasury securities, while providing other Treasuries as collateral. Also, Fleming, Hrungrung and Keane (2010) study the effects of the Term Securities Lending Facility. This facility was instituted by the Fed in March 2008 to alleviate the financing strains of some securities dealers. By enabling dealers to swap less liquid assets for Treasury securities, the Fed allowed dealers to conduct their business at a time when some securities lost their liquidity.

There are few papers explaining the usefulness of repo markets. As Koepl and Chiu (2011) show, private information on the quality of assets can be a factor. The very fact that the seller is willing to repurchase the asset is a guarantee that the asset is of good quality. However, this is hard to apply to Treasury securities or in a dynamic setting where agents learn the quality of the asset. In an environment with no commitment, Mills and Reed (2008) have argued that repos are useful in order to cover counterparty risk. However the difference between repos and collateralized loans is then tenuous and it is not clear why agents do not sell the collateral to obtain fundings.<sup>8</sup> Finally, Duffie (1996) argue that selling an asset involves different costs than the one when conducting repos. In some sense, we would like to have a deeper understanding of the origin of these transaction costs, without necessarily resorting to different fiscal treatments or the inability of some agents to own some class of assets (such as market mutual funds), although we acknowledge the importance of such frictions. In this paper, we show that agents do better by using repos and outright purchase, rather than one of the two alone, even when there is full commitment, the quality of the asset is known, there is no risk exposure, and no differential fiscal treatments across types of trade.

Our paper also builds on several strands of the literature. First and foremost, it is related to the recent literature on over the counter market initiated by Duffie et. al. (2005) and generalized by Lagos and Rocheteau (2009). In this literature, traders face search frictions that they circumvent by contacting intermediaries (dealers). Dealers have access to a centralized interdealers market where they can trade their asset holdings at the market price. In our

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<sup>8</sup>See Lacker (2001) and Kehoe and Levine (2006) for models of collateralized debt.

paper, we abscond from dealers and we only consider the problem of traders facing search frictions, but as in Lagos and Rocheteau, we allow for arbitrary asset holdings.<sup>9</sup> Another important difference is that we do not introduce matching frictions: Our agents will meet for sure, but they will meet in pairs. This allows us to characterize an equilibrium distribution of asset holdings, although holdings can be arbitrary. As in Lagos and Rocheteau (2009), we obtain results on the distribution of assets. They find that more severe search frictions are associated with less dispersion in the equilibrium asset distribution. We find that, as it becomes more likely that traders will have to readjust their portfolios (i.e. increased uncertainty about future valuation) the distribution of asset holdings also becomes less dispersed. Duffie, et. al. (2002) extends Duffie (1996) to study securities lending rates in a dynamic context, where traders have different opinions regarding the underlying value of the security. In their model, borrowing takes place as traders can take short position, i.e. traders who believe that the security's price will decline in the future will borrow and sell it today with the plan of purchasing it cheaper later to reimburse the loan. The option to short a security will naturally bring the price of this security higher, but more surprisingly, higher than the value of the most optimistic trader. This effect however, as well as lending rates are decreasing through time. In our paper, we do not study the dynamics of the lending rate, but rather focus on the reasons why lending could be optimal when short selling is not allowed. In our model, it is the current and future divergence in benefits from holding the security that explains the usefulness of securities lending. In equilibrium, securities lending would be redundant if only one of these margins was different.

Our paper is also related to the literature on the liquidity of capital markets in general. Lagos and Rocheteau (2008) study an economy where agents can pay with money (an intrinsically useless object) or capital that can be used in production. To give a role for cash, they assume that agents are anonymous. But this prevents lending (or renting) from taking place, as agents do not know who they would lend capital to. Independently, Ferraris and Watanabe (2008) consider a very similar framework, but assume that agents can pledge capital as collateral instead of paying with it. Both sets of authors find a very related result: When there is a lack of liquidity or when agents are credit constraint, capital carries a liquidity premium and agents tend to accumulate too much capital relative to its first best level. This effect is absent from our framework because agents can always pay and there is no liquidity problem.

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<sup>9</sup>We suspect that if traders have to trade through dealers with access to an interdealer market, then traders will use lending to extract some of the dealer's surplus.

Our paper also relates to the literature on leasing capital goods and more precisely to the recent paper by Gavazza (2011). There the author studies the leasing and secondary markets for aircrafts. He shows (and empirically confirms) that operators that face more volatile productivity shocks are more likely to lease aircrafts than those with less volatile shocks. Similarly, we find that the repo volume is a decreasing function of the degree of shock persistence. However, our two models differ substantially: Gavazza (2011) uses a model that is closely linked to the airline industry, where frictions are monitoring and transaction costs. In contrast, our only friction is that agents meet in pair. Our model is also related to the literature on leasing by financially constrained firms (for instance Eisfeldt and Rampini, 2009). However, we want to stress that in our context, agents are not financially constraint and in some cases they still prefer to rent than to buy.

The paper proceeds as follows. In Section 2 we describe the environment. In Section 3 we characterize Walrasian allocations as the benchmark. In section 4, we provide two examples to illustrate the importance of pairwise matching and uncertainty about future valuation for the coexistence of asset sale and repo. In section 5, we describe general allocations attainable under pairwise meeting and bargaining. In section 6, we solve for the equilibrium when there is random matching, in the extreme cases when there is full persistence of the preference shocks and no persistence at all. Section 7 analyzes the case with directed matching and solve for the equilibrium distribution and volumes in general. Section 8 concludes.

## 2. The Model

The model is based on Koepl, Monnet, and Temzelides (2012) and Lagos and Rocheteau (2009). Time is discrete and the horizon is infinite. Each period has two sub-periods: A trading stage, followed by a settlement stage.<sup>10</sup> There is a continuum of agents. In each period, there is a measure 1/2 of two types of agents, type  $h$  and type  $\ell$ . The type of an agent switches randomly and with probability  $1 - \pi \in [1/2, 1]$  at the start of the transaction stage. The law of large numbers then guarantees that there is the same measure of each type in each period. Agents are anonymous in the trading stage and their type is private information.

There is a long-lived asset in fixed supply  $A$ . As in Lagos and Rocheteau (2009), we associate this asset to a Lucas-tree: One unit of the asset yields one unit of some fruit in the settlement stage. Agents of type  $i \in \{h, \ell\}$  derive utility  $u_i(a)$  from holding  $a$  units of the

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<sup>10</sup>This two stages could be merged in one, but it helps the exposition to consider two separate stages.

asset.<sup>11</sup> For simplicity, we impose the following condition,

**Assumption 1.**  $u'_h(a) \geq u'_\ell(a)$  for all  $a$ .

Therefore, for a given level of asset holding, the agent with the high type has a higher marginal utility than the agent with the low type. To be concise, we will refer to agents of type  $h$  as agents  $h$  and to agents of type  $\ell$  as agents  $\ell$ .

In the trading stage, agents can agree to trade the asset, in which case the seller transfers the assets *and* the fruits in the settlement stage. Or agents can only agree to trade the fruit of the asset: Then the seller only transfers the fruits that it yields, while he maintains ownership over the asset. We interpret this second trade as a repo trade, as the buyer surrenders the asset back to the seller once he enjoyed the benefits of holding it this period (that is consuming the fruits).

While the trading stage can be seen as a market, there is no market in the settlement stage. There, agents are endowed with a production technology for the numeraire good. It costs them one unit of disutility to produce one unit of this good so that the numeraire good is akin to transferable utility. Agents also consume the numeraire good and derive one unit of utility for each unit they consume. If utility is transferable, the settlement stage does not generate any net utility gains. The numeraire good will be the settlement asset. In the settlement stage, agents settle the terms of the trade that were agreed upon in the previous trading stage.

### 3. Benchmark Walrasian Market

We first consider the case where the trading stage is a Walrasian market. A repo trades at price  $p^r$  while the asset sales at price  $p$ . We consider only stationary equilibrium so that these prices are the same in each period. An agent  $i = h, \ell$  with asset holdings  $a$  has a value

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<sup>11</sup> There are several interpretations for this formulation: Lagos and Rocheteau argue that this is the utility derived from the tree's fruit. Duffie, Garleanu and Pedersen (2009) explain that these are preferences from liquidity, hedging or other benefits that holding the assets may yield.

$W_i(a)$  of holding the asset, where  $W_i(a)$  is defined recursively as

$$\begin{aligned} W_i(a) &= \max_{a_i, q_i^r} u_i(c_i) - d + \beta E_{k|i} W_k(a_i) \\ \text{s.t. } c_i &= a_i + q_i^r \\ d + pa &= pa_i + p^r q_i^r \end{aligned}$$

where the agent repos  $q_i^r$  and purchases an amount  $a_i$  of the asset and  $c_i$  is current consumption of the security's service. Naturally the quantity of repos  $q_i^r$  does not enter in the continuation valuation but only in the momentary utility  $u_i(\cdot)$ . The first order and envelope conditions yield

$$\begin{aligned} u'_i(c_i) + \beta E_{k|i} W'_k(a_i) &= p \\ u'_i(c_i) &= p^r \\ W'_i(a) &= p \end{aligned}$$

Notice that all agents value an additional unit of the asset in the same way when they enter the Walrasian market, independent of their type or of their asset holdings. There are two reasons for this: First, the utility is linear in the numeraire good such that there is no wealth effect in this model and, second, agents are playing against the whole market. In the next section, we will modify the latter. For the time being, the equilibrium prices and quantities satisfy,

$$\begin{aligned} (1 - \beta)p &= p^r \\ u'_h(c_h) = u'_\ell(c_\ell) &= p^r \\ c_h + c_\ell &= 2A \end{aligned}$$

The first equation is a no-arbitrage condition: Agents have to be indifferent between conducting a repo, in which case they have to pay the price  $p^r$  in terms of the numeraire good, and buying the asset at price  $p$  and reselling it in the next period at price  $\beta p$ . These two schemes are payoff equivalent and so should be their cost. As a consequence, anything goes for repos, and in particular  $q_h^r = q_\ell^r = 0$ . In other words, in a Walrasian market, there is no difference between conducting a repos or buying and selling the asset: Repos and outright purchases are perfect substitute. Therefore, absent any additional frictions, the Walrasian benchmark is not helpful to study the structure of the repo and other rental markets. In

the following section, we depart from the Walrasian benchmark by assuming that agents can only meet in pair in which case they bargain over the allocation.

## 4. An Example

In this example, we want to illustrate how *future* pairwise matching can explain a mix repos/sales today, even though 1) agents trade on a Walrasian market *today* and 2) agents know who they will meet tomorrow.<sup>12</sup> Consider a two period economy, where agents are endowed with asset holding  $A$  in the first period. Also in the first period, agents have access to a Walrasian market where they can both sell and/or repo the asset. Agent  $i = h, \ell$  derives utility from consuming fruits in both periods according to the utility

$$U_i(c_1, c_2) = u_i(c_1) + \frac{\beta}{1 - \beta} u_i(c_2),$$

where  $u_i(c)$  satisfies Assumption 1. The utility in period 2 is scaled by  $1/(1 - \beta)$  as we want to compare this environment with our Walrasian benchmark. Since the world ends at  $t = 2$ , there is no difference between repo and sales then, as only the consumption of the fruits matters. However, we will show that pairwise matching in the future can explain the mix repo/sales *today*.

Let us first consider the problem of a type  $i$  agent with asset holding  $A$  in the first period with access to the asset market at price  $p^s$  and repo market at price  $p^r$ . Moreover, let's assume the persistence of type to be  $\pi \in [1/2, 1]$ . Then denoting the expected value for agent  $i$  of holding asset  $a$  at date 2 by  $V_i(a_i)$ , the problem of an agent  $i$  is

$$\begin{aligned} \max_{q^s, q^r} \quad & u_i(c_i) - d + \frac{\beta}{1 - \beta} V_i(a_i) \\ \text{s.t.} \quad & p^r q^r + p^s q^s \leq d \\ & c_i = a + q^s + q^r \\ & a_i = a + q^s \end{aligned}$$

Note that  $q^s, q^r$  and  $d$  can all get positive or negative values. Using the first order conditions

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<sup>12</sup>In the Appendix, we consider another example to illustrate how pairwise matching today (rather than tomorrow) can explain the usefulness of repos.

for the two types we obtain:

$$p^r = u'_\ell(c_\ell^*) = u'_h(c_h^*) \quad (1)$$

$$p^r = p^s - \frac{\beta}{1-\beta} V'_i(a_i) \quad (2)$$

If  $V_i$  is concave, there is a unique combination of asset holdings for both types  $a_h^*$  and  $a_\ell^*$  that can satisfy (1) and (2), with  $a_h^* + a_\ell^* = 2A$ . Equation (2) guarantees that agent  $i$  is indifferent between repo and sales *when holding*  $a_i^*$ .

In period 2, we assume that an agent  $h$  is matched with an agent  $\ell$  who just switched. Given the distribution of asset holdings has a two-point support at the end of period 1, this matching rule implies that a type  $i$  agent with asset holding  $a$ , will be matched with a type  $j \neq i$  agent with asset holding  $2A - a$ . Therefore, in equilibrium, agents know exactly who they will meet.

First, we derive the second period value of asset holding  $a$  for agent  $i$ , denoted by  $v_i(a)$ , so that  $V_i(a) = \pi v_i(a) + (1 - \pi)v_j(a)$ , where  $j \neq i$ . Naturally  $v_i(a) = u_i(c_i) - d$  for  $i = h, \ell$ , where  $c_i$  and  $d$  are the solutions to the symmetric bargaining problem at  $t = 2$  between an agent  $h$  with asset holdings  $a_h$  and an agent  $\ell$  with asset holding  $a_\ell$ :

$$\max [u_h(c_h) - d - u_h(a_h)]^{1/2} [u_\ell(c_\ell) + d - u_\ell(a_\ell)]^{1/2}$$

subject to  $c_h + c_\ell = a_h + a_\ell = 2A$ . The first order conditions give us  $u'_h(c_h^*) = u'_\ell(c_\ell^*)$  where  $c_h^* + c_\ell^* = 2A$ , and  $d$  satisfies

$$d = \frac{1}{2} [u_h(c_h^*) - u_h(a_h)] + \frac{1}{2} [u_\ell(c_\ell^*) - u_\ell(a_\ell)].$$

Using these, we obtain the usual expression for  $v_i(a)$  for  $i = h, \ell$ , as the value of the current asset holding plus a share of the trade surplus,

$$v_h(a) = u_h(a) + \frac{1}{2} S(a, a_\ell) \quad (3)$$

$$v_\ell(a) = u_\ell(a) + \frac{1}{2} S(a_h, a) \quad (4)$$

where  $S(a_h, a_\ell) = u_h(c_h^*) + u_\ell(c_\ell^*) - u_h(a_h) - u_\ell(a_\ell)$  is the match surplus from trade. Notice that, contrary to the case with a Walrasian market, the payoff is not linear in asset holdings, but naturally depends on both agents' asset holdings. In particular, a quick inspection of (3) and (4) and the bargaining problem reveals that the marginal payoffs at  $t = 2$  are pinned

down by how the asset allocation affects the agents' outside options. Clearly, the marginal payoffs are sensitive to asset holdings since  $V_i''(a) < 0$ , which confirms our initial guess that (2) was giving a two-point support distribution for the asset holding at the end of the Walrasian market.

Now (2) implies that  $V_h'(a_h^*) = V_\ell'(a_\ell^*)$  so that agents  $h$  and  $\ell$  equate the period 2-marginal value of asset holdings. However, using (4) it is easy to see that  $V_i'(a)$  is a scaled weighted average of the marginal utility of each agent's type, or<sup>13</sup>

$$V_i'(a) = \frac{1}{2}u'_i(c_i^*) + \frac{1}{2} [\pi u'_i(a) + (1 - \pi)u'_j(a)]$$

Since  $u'_h(c_h^*) = u'_\ell(c_\ell^*) \equiv u'^*$ , if  $\pi < 1$  then (1) implies that

$$\frac{1}{2} [u'_h(a_h^*) + u'^*] > V_h'(a_h^*) = V_\ell'(a_\ell^*) > \frac{1}{2} [u'_\ell(a_\ell^*) + u'^*]$$

so that at  $(a_h^*, a_\ell^*)$  there is a wedge between the intertemporal marginal utility and the intratemporal marginal utility. Hence  $u'_h(a_h^*) \neq u'_\ell(a_\ell^*)$  so that  $a_i^* \neq c_i^*$  for  $i = h, \ell$  and as a consequence both the repo market and the sales market are active at  $t = 1$ . If  $\pi = 1$ , there is no wedge in the sense that  $V_i'(c_i^*) = u'_i(c_i^*)$  and in this case  $a_i^* = c_i^*$ : the repo market is inactive.

This example shows the importance for the argument of an agent's outside option in bargaining: It is this outside option that determines the marginal value of holding some asset. Holding too little asset would give a bad outside option and, as  $V_i''(a) < 0$ , holding too much may cost too much relative to the additional benefits. This explains why agents do not want to take extreme positions, where they would repo or sell all their assets. Also, this example illustrates that it is pairwise trade that matters for the result: Indeed, agents

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<sup>13</sup>For example,

$$V_h'(a) = \pi \left\{ \frac{1}{2}u'_h(a) + \frac{1}{2}[u'_h(\cdot) \frac{\partial c_h^*}{\partial a} + u'_\ell(\cdot) \frac{\partial c_\ell^*}{\partial a}] \right\} + (1 - \pi)v'_\ell(a)$$

and since  $\partial c_h^*/\partial a + \partial c_\ell^*/\partial a = 1$  we can use the first order condition of the bargaining problem  $u'_h(c_h^*) = u'_\ell(c_\ell^*)$  to obtain

$$\begin{aligned} V_h'(a) &= \pi \left\{ \frac{1}{2}u'_h(a) + \frac{1}{2}u'_\ell(c_\ell^*) \right\} + (1 - \pi) \left\{ \frac{1}{2}u'_\ell(a) + \frac{1}{2}u'_h(c_h^*) \right\} \\ &= \frac{1}{2}u'(c_h^*) + \frac{1}{2} [\pi u'_i(a) + (1 - \pi)u'_j(a)] \end{aligned}$$

are not randomly matched in this example. Below, we will show that this result holds true in the more general set-up.

## 5. Pairwise Meeting and Bargaining

We now assume that each agent  $h$  is matched with exactly one agent  $\ell$  in the trading stage. We describe several matching technologies later. We consider allocations that are the solution to a bargaining game between both agents. We will consider a generic meeting between an agent  $h$  holding a generic amount of the asset  $a_h$  and an agent  $\ell$  holding a generic amount of the asset  $a_\ell$ . An allocation is a triple  $\{q^s(a_h, a_\ell), q^r(a_h, a_\ell), d(a_h, a_\ell)\}$  where  $q^s$  denotes the quantity of the asset that the agent  $h$  buys from the agent  $\ell$  (sells if negative),  $q^r$  is the quantity of the asset that the agent  $h$  buys or repo from the agent  $\ell$  (sells or reverse repo if negative) and  $d$  is the numeraire transfer that the agent  $h$  makes in the settlement stage to the agent  $\ell$  (receives if negative). We only focus on stationary and symmetric allocations. An allocation is feasible if

$$\begin{aligned} q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \\ q^r(a_h, a_\ell) + q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \end{aligned}$$

Notice that we do not allow short-selling. We will denote by  $(\mathbf{q}^s, \mathbf{q}^r, \mathbf{d})$  the feasible allocations for all possible matches such that  $(\mathbf{q}^s, \mathbf{q}^r, \mathbf{d})$  defines invariant distributions of asset holdings for agents  $h$  and  $\ell$ . We denote these distributions by  $\mu_i(a)$  for  $i \in \{h, \ell\}$ , where we have dropped the reference to the allocation for convenience. If they exist, a property of any invariant distribution is that

$$\frac{1}{2} \int a d\mu_h(a) + \frac{1}{2} \int a d\mu_\ell(a) = A$$

Then we can define recursively the expected value for agent  $i \in \{h, \ell\}$  of holding asset  $a$ , before entering the trading stage,  $V_i(a)$ , as

$$\begin{aligned} V_h(a) &= \pi \int [u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell) \\ &+ (1 - \pi) \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h) \end{aligned} \quad (5)$$

where  $c_h(a, a_\ell) = a + q^s(a, a_\ell) + q^r(a, a_\ell)$  is the consumption of the security's service of a type  $h$  with  $a$  units of the security matched with a type  $\ell$  holding  $a_\ell$  units of the security. Similarly,  $c_\ell(a_h, a) = a - q^s(a_h, a) - q^r(a_h, a)$  is the consumption of the security's service of a type  $\ell$  with  $a$  units of the security matched with a type  $h$  holding  $a_h$  units of the security. With probability  $\pi$ , an agent  $h$  remains an agent  $h$ . Then he meets an agent  $\ell$  with asset  $a_\ell$  according to the distribution  $\mu_\ell$ . Since he remains an agent  $h$ , he enjoys instant utility  $u_h(\cdot)$  from his asset holdings  $a + q^s(a, a_\ell) + q^r(a, a_\ell)$  at the end of the settlement stage. However, he only carries  $a + q^s(a, a_\ell)$  over to the next period since repos do not involve the transfer of the asset but only of fruits. The agent values this portfolio according to  $\beta V_h(a + q^s(a, a_\ell))$ . With probability  $1 - \pi$  the agent  $h$  becomes an agent  $\ell$ . In this case, he meets an agent  $h$  according to the distribution  $\mu_h$  and he enjoys instant utility  $u_\ell(\cdot)$  from his asset holdings  $a - q^s(a_h, a) - q^r(a_h, a)$ . He values his remaining portfolio according to  $\beta V_\ell(a - q^s(a_h, a))$ . Similarly for agents  $\ell$ ,

$$\begin{aligned} V_\ell(a) &= \pi \int [u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a))] d\mu_h(a_h) \\ &+ (1 - \pi) \int [u_h(c_h(a, a_\ell) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell))] d\mu_\ell(a_\ell) \end{aligned} \quad (6)$$

We assume that agents cannot commit to participate ex-ante and an allocation  $(\mathbf{q}^s, \mathbf{q}^r, \mathbf{d})$  is individually rational if all agents prefer the allocation to being in autarky this period. That is, for any portfolio  $a$ , an agent  $h$  matched with an agent  $\ell$  with a portfolio  $a_\ell$  prefers the allocation than not trading today, i.e.

$$u_h(c_h(a, a_\ell)) - d(a, a_\ell) + \beta V_h(a + q^s(a, a_\ell)) \geq u_h(a) + \beta V_h(a),$$

and similarly for an agent  $\ell$  matched with an agent  $h$  with portfolio  $a_h$ ,

$$u_\ell(c_\ell(a_h, a)) + d(a_h, a) + \beta V_\ell(a - q^s(a_h, a)) \geq u_\ell(a) + \beta V_\ell(a).$$

From now on, for concision and whenever there is no risk of confusion, we will drop references to the agents' portfolios in an allocation.

With general Nash bargaining where the agent  $h$  has bargaining power  $\theta \in [0, 1]$ , the allocation of an agent  $h$  with portfolio  $a_h$  matched with an agent  $\ell$  with portfolio  $a_\ell$  solves

the following problem:

$$\begin{aligned} & \max_{q^s, q^r, d} [u_h(c_h) - d + \beta V_h(a_h + q^s) - u_h(a_h) - \beta V_h(a_h)]^\theta \\ & \quad \times [u_\ell(c_\ell) + d + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta V_\ell(a_\ell)]^{1-\theta} \end{aligned}$$

subject to the allocation being feasible. The first order conditions for an interior solution<sup>14</sup> are

$$V'_h(a_h + q^s) = V'_\ell(a_\ell - q^s) \tag{7}$$

$$u'_h(c_h) = u'_\ell(c_\ell) \tag{8}$$

$$\begin{aligned} d(a_h, a_\ell) = & (1 - \theta)[u_h(c_h) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta V_h(a_h)] \\ & - \theta[u_\ell(c_\ell) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta V_\ell(a_\ell)] \end{aligned} \tag{9}$$

Equations (7) and (8) characterize the allocations  $q^s(a_h, a_\ell)$  and  $q^r(a_h, a_\ell) = c_\ell - [a_\ell + q^s(a_h, a_\ell)]$ . Inspecting (7) and (8) agents use repos whenever  $c_h \neq a_h + q^s$  and  $c_\ell \neq a_\ell - q^s$ . Also notice that (8) together with  $c_h + c_\ell = a_h + a_\ell$  uniquely defines  $c_h$  and  $c_\ell$ . The transfer  $d(a_h, a_\ell)$  redistributes the surplus from the trade according to the bargaining weights. Finally, notice that the allocation depends on the distributions of asset holdings  $\mu_i$  for  $i = h, \ell$  as they affect the value functions  $V_i$ . Therefore, to fully characterize the equilibrium with an invariant distribution, we need to specify how agents are matched. In the next section, we assume that agents are randomly matched. Then we assume that agents are matched in a more sophisticated way.

## 6. Random Matching: Special Cases

Here, we study two extreme cases with either  $\pi = 1/2$  or  $\pi = 1$  and an agent  $h$  is randomly matched with an agent  $\ell$ . In the case with  $\pi = 1/2$ , preference shocks have no persistence and current preferences do not give any information on future preferences. In the case with  $\pi = 1$  preference shocks are fully persistent as they are fixed forever.

With no persistence and random matching, we obtain the following result.

**Proposition 2.** *With random matching and  $\pi = 1/2$ , there is a unique invariant equilibrium characterized by a distribution of asset holdings for each type that are degenerate at some level  $\bar{a} = A$  with  $q^s(\bar{a}, \bar{a}) = 0$ , and  $q^r(\bar{a}, \bar{a}) > 0$ .*

<sup>14</sup>In the case where  $q^s = a_\ell$ , (7) becomes  $V'_h(a_h + q^s) > V'_\ell(a_\ell - q^s)$ , while in the case where  $q^s + q^r = a_\ell$ , (8) becomes  $u'_h(a_h + a_\ell) \geq u'_\ell(0)$ .

In the case without persistence, (5) and (6) imply that  $V_h(a) = V_\ell(a)$  for all  $a$ , such that agents  $h$  and  $\ell$  enjoy the same value of holding the asset. In this case, (7) implies that  $a_h + q^s(a_h, a_\ell) = a_\ell - q^s(a_h, a_\ell)$  with  $q^s(a_h, a_\ell) > 0$  if and only if  $a_\ell > a_h$  and  $q^s(a_h, a_\ell) < 0$  otherwise. That is, agents leave the match holding the same quantity of asset. Hence, the unique invariant equilibrium is one where the distribution of asset holding is degenerate at  $\bar{a} = A$  and  $q^s(\bar{a}, \bar{a}) = 0$ . This is very intuitive: Since all agents give the same value to future returns, they extinguish all surplus from trading the asset by averaging their asset holding (i.e. once an agent holding  $a_h$  trade with an agent holding  $a_\ell$ , they both end up with  $(a_h + a_\ell)/2$ ) and in equilibrium they hold the same amount of the asset. Then (8) together with Assumption 1 imply that  $q^r(\bar{a}, \bar{a}) > 0$ : While agents value future asset returns the same way, they differ in their valuation of current return. Therefore, there is a benefit from repos, where only the current return is traded.

With full persistence however, there is an equilibrium with neither asset sales nor repo in equilibrium.

**Proposition 3.** *With random matching and  $\pi = 1$ , there is an equilibrium with a degenerate distribution of asset holdings for each type at some level  $\bar{a}_h$  and  $\bar{a}_\ell$  with  $\bar{a}_h > \bar{a}_\ell$  where  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$  and  $q^r(\bar{a}_h, \bar{a}_\ell) = 0$ .*

We will first verify that the proposed allocation is an equilibrium. Since  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$  and  $q^r(\bar{a}_h, \bar{a}_\ell) = 0$ , equation (9) implies that  $d(\bar{a}_h, \bar{a}_\ell) = 0$ . Using (5) and (6), we then have for  $i = h, \ell$ ,

$$V_i(\bar{a}_i) = \frac{u_i(\bar{a}_i)}{1 - \beta} \quad (10)$$

and (7) and (8) imply that  $\bar{a}_h$  and  $\bar{a}_\ell$  are uniquely given by

$$u'_h(\bar{a}_h) = u'_\ell(\bar{a}_\ell)$$

with  $\bar{a}_h = 2A - \bar{a}_\ell$ . This verifies that there is no asset sales or repos in equilibrium. Also, combining the last equation with Assumption 1, we can verify that  $\bar{a}_h > \bar{a}_\ell$ . This equilibrium is unique whenever endowments are symmetric across all agents (and no constraint binds – which may happen if some agents  $\ell$  are endowed with too many securities in the first place) so that all agents  $\ell$  hold the same amount  $a_\ell$  and all agents  $h$  holds  $a_h$ . To see this notice that if an agent  $h$  endowed with  $a_h$  meets an agent  $\ell$  endowed with  $a_\ell$ , then the bargaining solution imposes that they trade so that (8) holds. But the unique solution is  $a_h + q^s(a_h, a_\ell) = \bar{a}_h$

and  $a_\ell - q^s(a_h, a_\ell) = \bar{a}_\ell$ . Since  $a_h + a_\ell = 2A$ , such a  $q^s$  exists and takes the agents directly to the equilibrium distribution of asset holdings.

For general levels of persistence  $\pi \in (0, 1)$  and random matching, we are unable to determine analytically the total volume of sales and repos as we cannot solve analytically for the invariant equilibrium distribution of asset holdings.<sup>15</sup> We suspect that as an agent  $\ell$  is better endowed, he will sell more to agent  $h$ , and as agent  $h$  is less endowed, he will buy more from agent  $\ell$ . This would hint to more trade as agents valuations differ and we would expect that the distributions of asset holdings become more spiked around their respective mean  $\bar{a}_h$  and  $\bar{a}_\ell$  as  $\pi$  increases, where the means are diverging as  $\pi$  increases. However, since agents can switch randomly from one type to the other, it is difficult to fully characterize the equilibrium without resorting to numerical simulations. In the next section we impose directed search and we basically confirm this intuition.

## 7. Directed search

We now describe a more sophisticated matching technology than just matching traders  $h$  and  $\ell$  at random (notice that our matching technology was already fairly sophisticated as we could match agents  $h$  with agents  $\ell$ ). Following Corbae, Temzelides and Wright (2003), we use directed search: The matching function now specifies that agents who switched types are matched together. Therefore a “new” agents  $h$  will be matched with a “new” agent  $\ell$ .<sup>16</sup> Below we verify that this matching function is an equilibrium matching rule (where such a term is precisely defined). We devote the rest of this section to the following result.

**Proposition 4.** *With directed search, there is an equilibrium characterized by a degenerate distribution of asset holdings for each type at some level  $\bar{a}_i$  with  $i = h, \ell$  with  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ ,  $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$  and  $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$  where  $q^r$  solves  $u'_h(\bar{a}_h + q^r) \geq u'_\ell(\bar{a}_\ell - q^r)$  (with equality if  $q^r < \bar{a}_\ell$ ).*

In words, each type of agents is holding a specific portfolio, either  $\bar{a}_h$  or  $\bar{a}_\ell$  for type  $h$  and  $\ell$  respectively. Agents who just switched type adjust their asset holdings so that they hold

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<sup>15</sup>This is a usual problem in models with pairwise trade and arbitrary asset holdings. Agents in Kiyotaki and Wright (1989) or Duffie et. al. (2005) trade an indivisible asset with a unit upper bound. Lagos and Wright (2005) introduces a Walrasian market with quasi-linear preferences so that agents can level their asset holdings, thus giving a degenerate distribution of assets. In a Lagos-Wright environment, there is no role for repos.

<sup>16</sup>It turns out that in equilibrium this matching function will maximize the gains from trade in each match, so that if agents could choose, they would actually want to be matched in this way, as we show below.

their type's portfolio. Then they conduct repo as if they never switched. Agents who did not switch type just engage in repo. Loosely speaking, there is a sense in which agents first access the asset market and then engage in repo. To verify that this is an equilibrium we need to verify that an agent would not prefer to be matched with a different agent than the one he is assigned to, or that no agent would prefer to interact with him to trading with his assigned agent. In the terminology of Corbae, Temzelides and Wright (2003), the proposed matching rule is an equilibrium matching if no coalition consisting of 1 or 2 agents can do better (in the sense that the discounted lifetime utility of all agents in the coalition increases) by deviating in the following sense: An individual can deviate by matching with himself (i.e. being in autarky this period) rather than as prescribed by the matching rule; and a pair can deviate by matching with each other rather than as prescribed by the matching rule.

It should be clear that the bargaining solution is always better than autarky (although not in a strict sense). Therefore we only need to check deviations by a coalition of 2 agents. An agent  $\ell$  with  $\bar{a}_h$  could decide to form a coalition with an agent  $\ell$  with  $\bar{a}_\ell$  or an agent  $h$  with  $\bar{a}_h$ . It is a property of the bargaining solution that an agent  $\ell$  will obtain a lower payoff being matched with an agent  $h$  with a higher amount of asset (he can extract less since the marginal utility of obtaining more of the asset is lower for this agent). Hence, an agent  $\ell$  with  $\bar{a}_h$  prefers to be matched with an agent  $h$  with  $\bar{a}_\ell$ . Also, it is a property of the bargaining solution that, given he has to meet an agent holding  $a$ , an  $\ell$  agents prefer to be matched with the agent with the highest marginal utility (so agent  $h$ ).<sup>17</sup> We now turn to agents  $h$ . An agent  $h$  with  $\bar{a}_\ell$  could decide to form a coalition with an agent  $h$  with  $\bar{a}_h$  or an agent  $\ell$  with  $\bar{a}_\ell$ . As above, however, it is a property of the bargaining solution that an agent  $h$  payoff matched with an agent  $\ell$  will get a higher utility whenever the agent  $\ell$  is holding more asset. Hence, the agent  $h$  will not want to be matched with an agent  $\ell$  holding  $\bar{a}_\ell$ . Also, an agent  $h$  with  $\bar{a}_h$  prefers to be matched with the agent holding  $\bar{a}_\ell$  with the lowest marginal utility, i.e. with an  $\ell$  agent. Therefore there is no 2-agents coalition where both agents would do better than under the prescribed matching technology, which shows that, combined with the distribution over  $\{\bar{a}_h, \bar{a}_\ell\}$ , it is an equilibrium.

We now derive the properties of this equilibrium. We first consider the terms of trade for a repo and the sale of the asset. The “price” of the repo trade between an agent  $h$  holding  $\bar{a}_h$  and an agent  $\ell$  holding  $\bar{a}_\ell$  is  $d(\bar{a}_h, \bar{a}_\ell)$  since in this trade agents  $h$  and  $\ell$  keep the same asset holding. Now, the “price” of the repo combined with the asset sale transaction is  $d(\bar{a}_\ell, \bar{a}_h)$  as

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<sup>17</sup>It is easy to show this with  $u_h(a) = \alpha u_\ell(a)$  with  $\alpha > 1$ : Compute the bargaining solution and show that  $\partial[u_\ell(a_\ell - q^r - q^s) + \beta V(a - q^s) + d]/\partial\alpha > 0$ .

the agent  $h$  and  $\ell$  now change their asset position, but also trade fruits. In the Appendix, we show that

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1 - \theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)] \\ d(\bar{a}_\ell, \bar{a}_h) &= d(\bar{a}_h, \bar{a}_\ell) + \bar{u} + \beta(1 - \theta)[V_h(\bar{a}_h) - V_h(\bar{a}_\ell)] + \beta\theta[V_\ell(\bar{a}_h) - V_\ell(\bar{a}_\ell)] \end{aligned}$$

where

$$\bar{u} = (1 - \theta)[u_h(\bar{a}_h) - u_h(\bar{a}_\ell)] + \theta[u_\ell(\bar{a}_h) - u_\ell(\bar{a}_\ell)],$$

so that, as expected,  $d(\bar{a}_\ell, \bar{a}_h) > d(\bar{a}_h, \bar{a}_\ell)$ , and the price of the repo combined with the asset sale is composed of the repo price, plus the weighted present and discounted lifetime gains of switching position for agents  $h$  and  $\ell$ .

The directed matching technology specifies that an agent  $h$  with  $\bar{a}_\ell$  meets an agent  $\ell$  with  $\bar{a}_h$  and an agent  $h$  with  $\bar{a}_h$  meets an agent  $\ell$  with  $\bar{a}_\ell$ . Given  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ ,  $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$  and  $q^r(\bar{a}_h, \bar{a}_\ell) = q^r(\bar{a}_\ell, \bar{a}_h) = q^r$ , we obtain the following value functions,

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Notice that we need to specify the value of the outside option in order to solve for the bargaining solution in equilibrium, i.e.  $V_h(\bar{a}_\ell)$  and  $V_\ell(\bar{a}_h)$ . To simplify the analysis we assume that the matching technology then specifies that an agent  $h$  holding  $\bar{a}_\ell$  (respectively  $\bar{a}_h$ ) who did not trade in the previous period is matched with an agent  $\ell$  holding  $\bar{a}_h$  (respectively  $\bar{a}_\ell$ ), and reversely.<sup>18</sup> Therefore we obtain,

$$\begin{aligned} V_h(\bar{a}_\ell) &= \pi[u_h(\bar{c}_h) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{c}_\ell) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{c}_\ell) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{c}_h) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Using these value functions we find that

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \frac{\bar{u}}{1 - \beta}$$

so that the value of selling the asset is just  $\bar{u}/(1 - \beta)$ , the lifetime discounted surplus from

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<sup>18</sup>Notice that in equilibrium, there is always trade unless  $\pi = 1$  in which case agents never switch type so that this ad-hoc rule is never used.

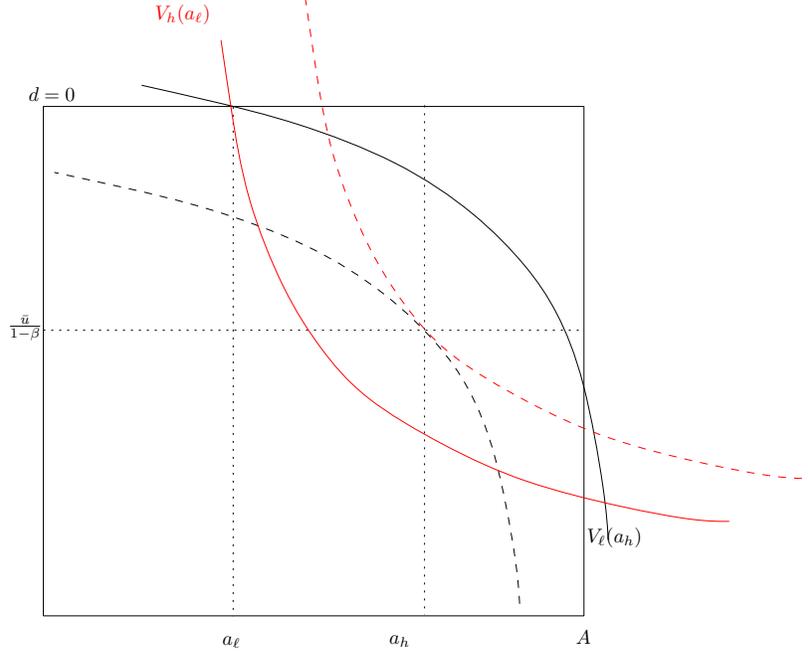


Figure 1: Intertemporal gains from trade

adjusting portfolios. Notice that as  $\pi \rightarrow 1/2$  we have  $\bar{a}_h \rightarrow \bar{a}_\ell$  so that  $\bar{u} \rightarrow 0$  and there is no value of selling the asset. Figures 1 and 2 illustrate this equilibrium.

Figure 1 shows the indifference curves for  $V_i(a) + d$  for two agents: One agent  $h$  endowed with  $a_\ell$  and one agent  $\ell$  endowed with  $a_h$ . Indifference curves are tangent at the stationary distribution points  $(\bar{a}_h, \bar{a}_\ell)$  (the axis for  $d$  is reversed). We can also scale the continuation utility by  $1 - \beta$  to compare it with present utility. Then assumption 1 and the likelihood that an agent  $h$  reverts to an agent  $\ell$  in the future imply that  $(1 - \beta)V'_h(a) \leq u'_h(a)$ . By the same argument, notice that  $(1 - \beta)V'_\ell(a) \geq u'_\ell(a)$ . Therefore, as illustrated in Figure 2, the indifference curves for  $u_i(a) + d$  for both agents will be tangent at a point south-east of the  $(\bar{a}_h, \bar{a}_\ell)$ . This explains why repos are useful: They exploit intratemporal gains from trade.

In the Appendix, we characterize the bargaining solution with directed matching  $q^r$ ,  $\bar{a}_h$  and  $\bar{a}_\ell$ .

**Proposition 5.** *The degenerate supports  $\bar{a}_h$  and  $\bar{a}_\ell$  of the two distributions with bargaining*

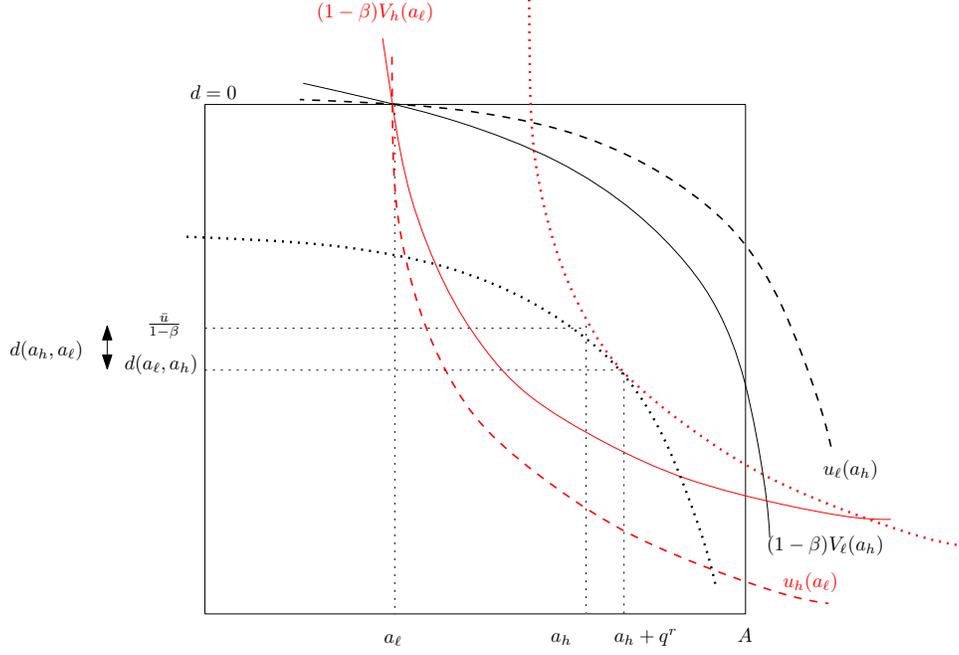


Figure 2: Intratemporal gains from trade

are fully characterized by the following equations,

$$\begin{aligned}
u'_h(\bar{c}_h) &= u'_l(\bar{c}_l) \\
u'_h(\bar{c}_h) &= \frac{[\pi - (2\pi - 1)\beta][\theta u'_l(\bar{a}_l) - (1 - \theta)u'_h(\bar{a}_h)] - (1 - \pi)[\theta u'_l(\bar{a}_h) - (1 - \theta)u'_h(\bar{a}_l)]}{(2\pi - 1)(1 - \beta)(2\theta - 1)} \\
\bar{c}_h + \bar{c}_l &= \bar{a}_h + \bar{a}_l = 2A
\end{aligned}$$

Not surprisingly, the solution with directed matching is similar to the solution with random matching in the case of no persistence,  $\pi = 1/2$ , or full persistence,  $\pi = 1$ . Indeed, notice from the second equation that when  $\pi = 1/2$ , we must have

$$\theta[u'_l(\bar{a}_l) - u'_l(\bar{a}_h)] = (1 - \theta)[u'_h(\bar{a}_h) - u'_h(\bar{a}_l)]$$

and the unique solution is  $\bar{a}_l = \bar{a}_h = A$ . In this case,  $q^s = 0$  and  $q^r > 0$ . Also, if  $\pi = 1$  we have

$$u'_h(\bar{c}_h) = \frac{\theta u'_l(\bar{a}_l) - (1 - \theta)u'_h(\bar{a}_h)}{(2\theta - 1)}$$

with solution  $q^r = 0$  and  $u'_l(\bar{a}_l) = u'_h(\bar{a}_h)$ . In this case  $q^s = 0$ . We also obtain the result on markets volumes as a function of persistence.

**Corollary 6.**  $\bar{a}_h - \bar{a}_\ell \geq 0$  is increasing in  $\pi$ . Sales volume is hump-shaped in  $\pi$  while repos volume is strictly decreasing in  $\pi$ .

The intuition for this result is straightforward. When  $\pi = 1/2$ , agents future type is independent of their current type. Therefore, two agents' future value of the asset is the same. Since the bargaining solution equates the marginal benefit of holding the asset, all agents hold the same quantity of assets. Therefore with  $\pi = 1/2$  we obtain  $\bar{a}_h = \bar{a}_\ell = A$ : There is no outright purchase, but only some repo to allocate the fruits to those agents  $h$  who like it the most. As  $\pi$  increases, it is more likely that an agent  $h$  becomes once again an agent  $h$  next period. Therefore his valuation for the asset increases, and starting from  $a_h = a_\ell = A$ , there are gains from trade when an agent  $h$  meets an agent  $\ell$ . In this case,  $\bar{a}_h > A > \bar{a}_\ell$ . In equilibrium only those agents who switch types have gains to trade the asset and so the volume of asset sales is increasing. Also repos are decreasing as agents hold more of the asset they like. Finally, when  $\pi = 1$ , agents know their type for sure. Hence in equilibrium, all gains from trades (be it asset trade or fruit trade) are extinguished, so that there is neither sales nor repos.

Also, we can find the price for repo and asset sales.

**Corollary 7.** Let  $p^r$  be the price of a repo and  $p^s$  the price of a sale. Then

$$\begin{aligned} p^r &= \frac{(1 - \theta)[u_h(\bar{c}_h) - u_h(\bar{c}_h - q^r)] + \theta[u_\ell(\bar{c}_\ell + q^r) - u_\ell(\bar{c}_\ell)]}{q^r} \\ p^s &= \frac{(1 - \theta)[u_h(\bar{c}_h - q^r) - u_h(\bar{c}_\ell + q^r)] + \theta[u_\ell(\bar{c}_h - q^r) - u_\ell(\bar{c}_\ell + q^r)]}{(1 - \beta)(\bar{a}_h - \bar{a}_\ell)} \end{aligned}$$

From the transfers  $d(a_h, a_\ell)$  we have  $d(\bar{a}_h, \bar{a}_\ell) = p^r q^r$  as the pair of agents who did not switch types only conduct repos. Therefore,

$$p^r q^r = (1 - \theta)[u_h(\bar{c}_h) - u_h(\bar{a}_h)] + \theta[u_\ell(\bar{a}_\ell) - u_\ell(\bar{c}_\ell)].$$

Also, since the pair of agents that switched conducts both an asset sale  $q^s$  to adjust their position, and then a repo. Therefore  $d(\bar{a}_\ell, \bar{a}_h) = p^r q^r + p^s q^s$ . Since

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \frac{\bar{u}}{1 - \beta}$$

we obtain that

$$p^s q^s = \frac{\bar{u}}{1 - \beta}$$

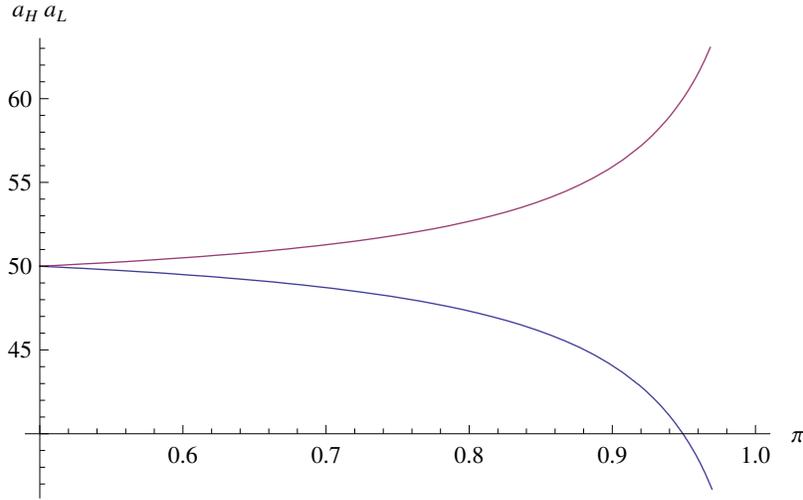


Figure 3: Asset holdings.

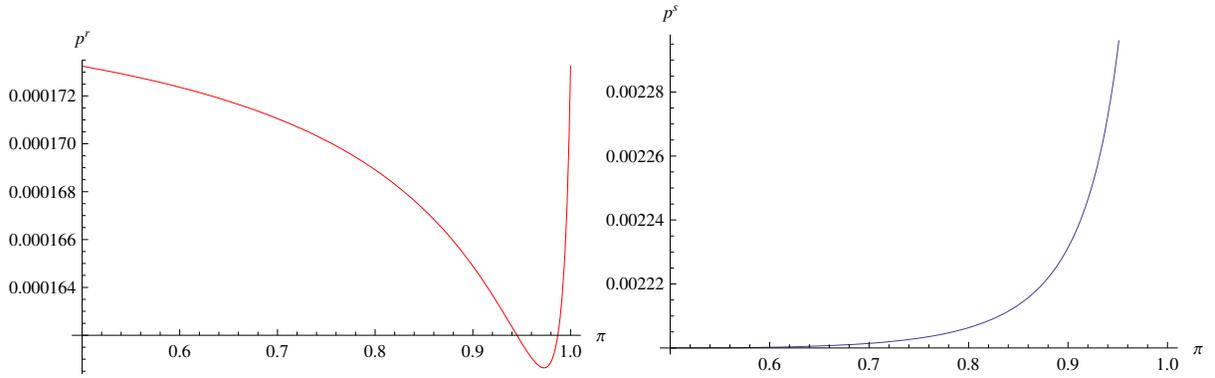


Figure 4: Repo prices (left) and outright purchase price (right)

and using the expression for  $\bar{u}$ , with  $q^s = \bar{a}_h - \bar{a}_\ell$ , we get the result.

Figure 3 shows how  $\bar{a}_h$  (red curve) and  $\bar{a}_\ell$  (blue curve) evolve as  $\pi$  varies from  $1/2$  to  $1$ . The parameters chosen are  $\theta = 0.5$ ,  $\lambda = 0.1$ ,  $\sigma = 2$ ,  $\beta = 0.9$ , and  $A = 50$ . Interestingly, the rate of divergence increases as types become more persistent. Hence, as  $\pi$  becomes large, we should expect some wide movements in prices and quantities.

This intuition is confirmed by Figure 4 that shows prices for repo  $p^r$  and asset sales  $p^s$ .

Similarly total repo volume  $q^r$  and total sales volume  $(1 - \pi)q^s$  display very different pattern, as illustrated in Figure 5. At  $\pi = 0.9$ , the total volume of repo is approximately 20% of the outstanding securities, while total sales are only 1% of outstanding securities.

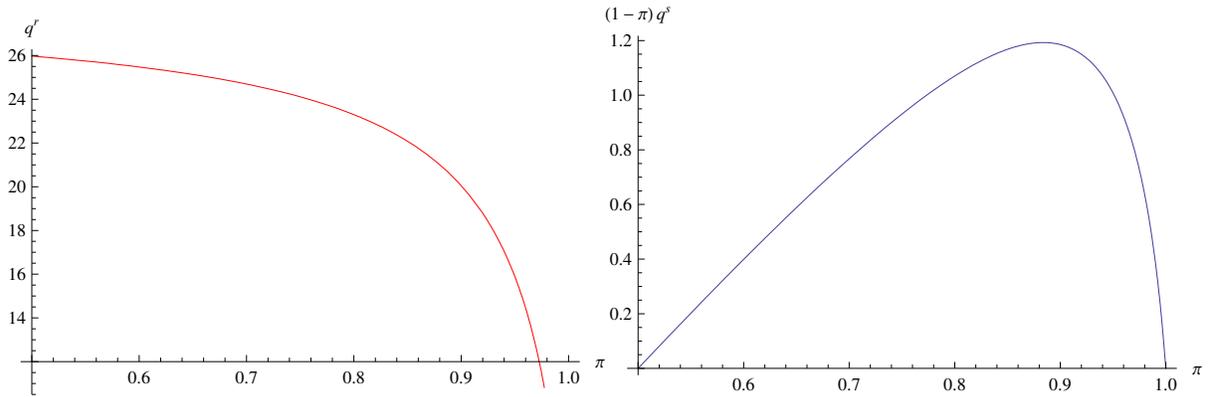


Figure 5: Repo volumes (left) and outright purchase volumes (right)

Interestingly, the coefficient of risk aversion  $\sigma$  is the one with the most impact on asset volumes and values. For the sake of illustration and without any intention of being a serious numerical exercise, with  $A = 50$ , we can match the observations on repos and sales of Treasury securities, with  $\sigma = 0.5$  (close to risk neutrality) and quite high persistence,  $\pi = 0.9$ .<sup>19</sup>

Next we will study what happens when agents become more patient. Unlike higher persistence, which increases the level of reallocation of assets via sale, higher patience results in more reallocation of assets via repos.

## 7.1. Patience

It is clear that in general those prices in Corollary 7 are different from their Walrasian equivalent, and in particular that  $(1 - \beta)p^s$  is different from  $p^r$ . However, an interesting case to consider is when agents become very patient. Then it is legitimate to guess that the allocation will converge to the Walrasian one, as it is in some sense equivalent to agents trading with each other very frequently. However, it is also as if agents were also changing type very often and although we have the illusion that they can trade very fast when  $\beta$  converges to one, they are also bargaining a lot to readjust their portfolio and this friction remains. Indeed, as  $\beta$  tends to one, the solution to the bargaining problem is characterized by  $\bar{a}_h, \bar{a}_\ell \rightarrow A$ , so that asset sales converge to zero. Hence, we obtain

$$\lim_{\beta \rightarrow 1} (1 - \beta)p^s = (1 - \theta)u'_h(A) + \theta u'_\ell(A).$$

<sup>19</sup>The average daily volume of Treasury repos is approximately twice the one for Treasury sales in the US according to ICAP, see <http://www.icap.com/investor-relations/monthly-volume-data.aspx>.

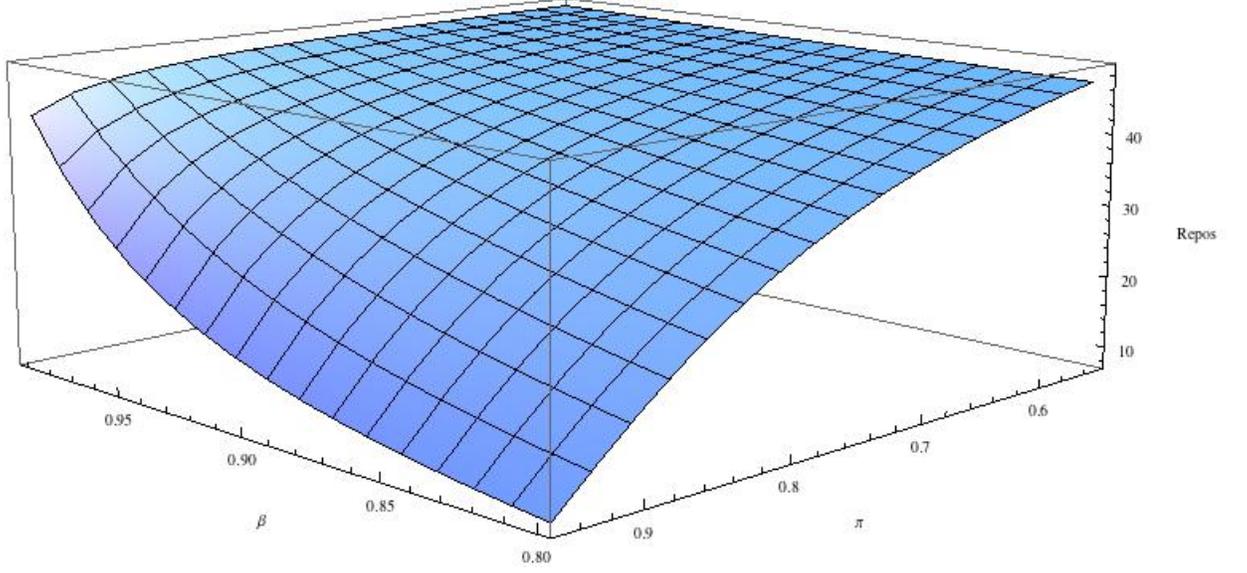


Figure 6: Repo volume in the  $(\beta, \pi)$ -space.

However in the limit  $q^r$  satisfies  $u'_h(A + q^r) = u'_\ell(A - q^r)$  and Assumption 1 guarantees that  $q^r > 0$  is bounded away from zero. Since  $q^r > 0$  and  $u_i(\cdot)$  is concave,

$$\begin{aligned} q^r u'_h(A + q^r) &< u_h(A + q^r) - u_h(A) < q^r u'_h(A) \\ q^r u'_\ell(A) &< u_\ell(A) - u_\ell(A - q^r) < q^r u'_\ell(A - q^r) \end{aligned}$$

and in general  $p^r \neq (1 - \beta)p^s$ . For illustration, we use the following utility function:  $u_h(a) = \frac{a^{1-\sigma}}{1-\sigma}$  and  $u_\ell(a) = \lambda u_h(a)$  where  $\lambda \in (0, 1)$ . Then we obtain

$$q^r = \frac{\lambda^{-\frac{1}{\sigma}} \bar{a}_\ell - \bar{a}_h}{1 + \lambda^{-\frac{1}{\sigma}}}$$

so that

$$\bar{a}_h + q^r = \lambda^{-\frac{1}{\sigma}} \frac{2A}{1 + \lambda^{-\frac{1}{\sigma}}} \quad \text{and} \quad \bar{a}_\ell - q^r = \frac{2A}{1 + \lambda^{-\frac{1}{\sigma}}}$$

Figure 6 shows how repo volume moves along the  $(\beta, \pi)$ -dimension.

As we have argued above,  $a_\ell$  and  $a_h$  tends to  $A$  whenever  $\beta \rightarrow 1$ . Therefore in this case,

$$\lim_{\beta \rightarrow 1} (1 - \beta)p^s = (1 - \theta + \lambda\theta)A^{-\sigma},$$

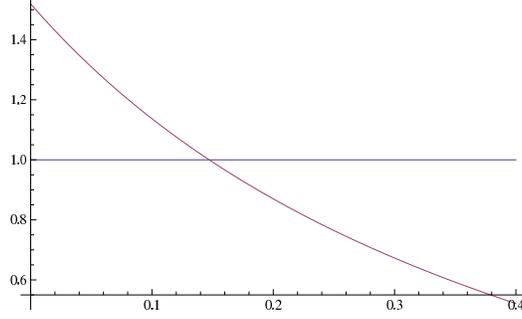


Figure 7: Ratio of repo and purchase prices

while

$$\lim_{\beta \rightarrow 1} p^r = \frac{A^{-\sigma} \left(\lambda^{\frac{1}{\sigma}} + 1\right)^{\sigma}}{1 - \sigma \left(1 - \lambda^{\frac{1}{\sigma}}\right)} \left[2^{1-\sigma} - \left(\lambda^{\frac{1}{\sigma}} + 1\right)^{1-\sigma}\right] \left\{1 - \theta + \lambda \theta \frac{\left(\lambda^{\frac{1}{\sigma}} + 1\right)^{1-\sigma} - \left(2\lambda^{\frac{1}{\sigma}}\right)^{1-\sigma}}{\lambda 2^{1-\sigma} - \lambda \left(\lambda^{\frac{1}{\sigma}} + 1\right)^{1-\sigma}}\right\}.$$

With  $\lambda = 0.1$  and  $\sigma = 2$ , we plot the ratio  $\lim_{\beta \rightarrow 1}(1 - \beta)p^s / \lim_{\beta \rightarrow 1} p^r$  as a function of  $\theta \in [0, 0.4]$ .

As Figure 7 shows,  $\lim_{\beta \rightarrow 1}(1 - \beta)p^s > \lim_{\beta \rightarrow 1} p^r$  for low values of  $\theta$  and the inequality is reversed otherwise. In the next section we correct for the frequency with which agents change type as they can trade more often.

## 7.2. Frequent Trades

In this subsection we study the consequences of agents meeting more frequently. More specifically, what are the consequences of reducing the time until the next meeting from one unit to  $\Delta < 1$  units? And, what happens when  $\Delta \rightarrow 0$ ? If  $\beta$  denotes discounting over a period of unit length, and  $\pi$  denotes the probability of maintaining the same type over a period of unit length, we assume agents discount future at rate  $\beta_{\Delta} = 1 - \Delta(1 - \beta)$  and the probability of maintaining the same type is  $\pi_{\Delta} = 1 - \Delta(1 - \pi)$  over a period of  $\Delta < 1$  unit length. Clearly the level of consumption by the type  $h$  and  $\ell$  agents will remain the same, however, the share of asset reallocation via repo and sale changes. We denote the repo level when  $\Delta < 1$  units of time elapses until the next meeting by  $q_{\Delta}^r$ , and we show in the Appendix,

**Proposition 8.** *For any  $\Delta < 1$ , we have  $q_{\Delta}^r < q^r$ . And as  $\Delta \rightarrow 0$ ,  $q_{\Delta}^r$  decreases to  $q_0^r > 0$ .*

That is even with very frequent trades agents use repo. Although agents can trade more often they still face the friction that trading has to be bilateral and this gives a role for repo.

### 7.3. Outside Option

An intuitive explanation for our results, reminiscent of the intuition from our earlier example, is that agents may prefer to use repos (to acquiring an asset), because they do not want to lock in a position that may be difficult to undo later at an agreeable price. When they engage in repos, agents are not locked into a position. To make this intuition more precise, we modify the environment slightly and assume that agents' outside option is to access a Walrasian market from next period onward. Then the outside option for an agent  $i$  holding  $a$  units of the asset is  $u_i(a) + \beta \tilde{W}_i(a)$  where  $\tilde{W}_i(a) = \pi W_i(a) + (1 - \pi)W_{-i}(a)$  and  $W(a)$  has been defined in Section 3. The possibility to trade on a Walrasian market would make the “lock-in” problem a little less severe, as agents could sell their securities on the walrasian market next period. Therefore we would expect the repo trade to decrease relative to the economy where agents do not have the option to unload their asset holdings on a Walrasian market. Still, agents are locked-in for one period and we would still expect repo to have a role. Indeed, let  $\bar{q}^r$  be the equilibrium level of repo taking place in the economy where agents have the option to trade at Walrasian price in the next period, and let  $q^r$  be the equilibrium level of repo when they do not have this option. Then, in the Appendix, we show

**Proposition 9.** *With directed search and bargaining, there is an equilibrium where  $q^r > \bar{q}^r > 0$ .*

The equilibrium with the option to trade on a Walrasian market displays the same features as the one in our original set-up. That is, whether agents switched types or not, they always repo  $\bar{q}^r$ . In addition, those agents who switched types trade  $\bar{q}^s = \bar{a}_h - \bar{a}_\ell$  and zero otherwise, where  $\bar{a}_h$  and  $\bar{a}_\ell$  are given by some equilibrium conditions. The important result is that, although agent's outside option is the Walrasian price (next period), agents will still use repos, but less so than if they did not have the option to trade at Walrasian price the following period. Therefore, the repo volume declines as we take the economy “closer” to its Walrasian benchmark.

## 8. Conclusion

This paper presents a simple environment with trading frictions where agents trade both in repo and asset markets. Repos are useful because agents can enjoy the service from holding the asset today, without changing their future portfolio. We find this is important for agents to maintain an appropriate outside option in bilateral trade: Holding too few assets (selling too much) would weaken the agent's future position, while holding too many (selling too few) would diminish the marginal value of the asset, which would then be relinquished "at a low price." With directed matching and two valuations, we characterize an equilibrium as a two-point supports. These two equilibrium asset holdings are converging as the valuation shock becomes more persistent. We find that the volume for repos is always decreasing in the persistence of the valuation shock, while the volume of asset sales is hump-shaped. This hump is explained by two interacting margins: On the one hand, less agents are switching valuation when it becomes more persistent, but on the other hand they trade a larger quantity each time they switch valuation in order to hold the equilibrium amount.

This has interesting implications for the organization of the repo market. In particular our theory predicts that the repos market will be thinner when there is little uncertainty about one's future preferences. Although we leave it for future research, we suspect that monetary policy (which is operated in the repo market) will have a higher impact then, as a lower quantity of repos can affect the market. Similarly, starting from a situation where agents know their future preferences, as uncertainty is growing, so is the volume of asset sales. Therefore, more sales have to be conducted in order to move the market. If we associate "normal times" with times when agents have a good idea about their future preferences, then monetary policy should be conducted with repos. However, with uncertainty growing overly large, monetary policy will be more effective in moving markets price if it is conducted via asset sales/purchases.

## 9. Appendix

### 9.1. Example

This example seeks to illustrate the role of pairwise matching *today* to explain the usefulness of repos. It also indicates how linear utilities from holding the asset affects the results. We suppose that an agent  $i = h, \ell$  derives utility from consuming fruits in both periods according to the utility  $U_i(c_1, c_2) = u_i(c_1) + \beta\lambda_i c_2$ , where  $u_i(c)$  satisfies Assumption 1 while  $\lambda_h \geq \lambda_\ell$ ,

$c_1$  is the quantity of fruits consumed at  $t = 1$  and  $c_2$  is the quantity consumed at  $t = 2$ . We assume  $\pi = 1$  so that agents do not switch type. An agent  $h$  is matched with an agent  $\ell$  and they bargain over the allocation. There is no further trade in period 2. We assume that  $a_h + a_\ell = 2A$ . Given there is no trade at  $t = 2$ , the bargaining problem at  $t = 1$  between agent  $h$  with bargaining power  $\theta \in [0, 1]$  and agent  $\ell$  with bargaining power  $1 - \theta$  is

$$\max_{c_h, c_\ell, d} [u_h(c_h) - d + \beta\lambda_h q^s - u_h(a_h)]^\theta [u_\ell(c_\ell) + d - \beta\lambda_\ell q^s - u_\ell(a_\ell)]^{1-\theta}$$

subject to  $c_h + c_\ell = a_h + a_\ell$  and  $q^s \in [-a_h, a_\ell]$ . The solution to the bargaining problem is then

$$\begin{aligned} u'_h(c_h^*) &= u'_\ell(c_\ell^*) \\ (a_\ell - q^s)(\lambda_h - \lambda_\ell) &= 0 \\ d(q^s, q^r) &= (1 - \theta)[u_h(c_h^*) + \beta\lambda_h q^s - u_h(a_h)] + \theta[u_\ell(c_\ell^*) - \beta\lambda_\ell q^s - u_\ell(a_\ell)] \end{aligned} \quad (11)$$

where  $c_h^* = a_h + q^r + q^s$  and  $c_\ell^* = a_\ell - q^r - q^s$ . Clearly, Assumption 1 implies  $c_h^* > c_\ell^*$ . Also these optimal consumption levels are uniquely pinned down by the first order conditions together with  $c_h^* + c_\ell^* = a_h + a_\ell$ . Agents just choose the mix between repo and asset sales to achieve those levels. If  $\lambda_h > \lambda_\ell$  then the solution is  $q^s = a_\ell$ , and  $q^r = -c_\ell^*$ . In words, agent  $\ell$  sells all his assets to agent  $h$  but repo some to achieve the desired level of consumption at  $t = 1$ .

Under which conditions would agent  $\ell$  be indifferent between repo and asset sales? To be indifferent, agent  $\ell$  should obtain the same payoff by keeping some amount  $\tilde{a}_\ell > 0$  into period 2 instead of selling everything at  $t = 1$ . If he keeps  $\tilde{a}_\ell$ , he can lower the amount of security he rents to  $\tilde{q}^r = -c_\ell^* + \tilde{a}_\ell$ , while he only sells  $\tilde{q}^s = a_\ell - \tilde{a}_\ell$ . In turn, his new total transfer is  $d(a_\ell - \tilde{a}_\ell, \tilde{q}^r)$  instead of  $d(a_\ell, -c_\ell^*)$ . Therefore agent  $\ell$  would be indifferent keeping some assets or selling it all if and only if

$$u(c_\ell^*) + d(a_\ell, -c_\ell^*) = u(c_\ell^*) + d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) + \beta\lambda_\ell \tilde{a}_\ell$$

where the last term on the right hand side is the payoff from carrying over asset into the next period. This expression can be simplified to

$$d(a_\ell, -c_\ell^*) - d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) = \beta\lambda_\ell \tilde{a}_\ell.$$

However, using (11) we obtain that

$$d(a_\ell, -c_\ell^*) - d(a_\ell - \tilde{a}_\ell, \tilde{q}^r) = \theta\beta\lambda_\ell\tilde{a}_\ell + (1 - \theta)\beta\lambda_h\tilde{a}_\ell \geq \beta\lambda_\ell\tilde{a}_\ell$$

In words, agent  $\ell$  prefers to sell his entire portfolio to agent  $h$  today as he is able to extract some of agent  $h$ 's higher valuation of the asset tomorrow for his own benefit today (which is a similar effect to Lagos and Rocheteau). Notice that if  $\lambda_h = \lambda_\ell$  then both agents would have the same valuation for the asset at  $t = 2$  and agent  $\ell$  would be indifferent between repo and asset sales at  $t = 1$ , as in the Walrasian benchmark. Also, if  $\theta = 1$  then agent  $\ell$  would be unable to extract some of the future gain from agent  $h$  and so agent  $\ell$  would be indifferent between repo and asset sales. In all other cases, i.e. whenever  $\lambda_h > \lambda_\ell$  and  $\theta < 1$ , repos are useful to equate the marginal utilities at  $t = 1$  and to achieve efficiency. In the general model, we endogenize the marginal utility  $\lambda_h$  and  $\lambda_\ell$ , but the intuition remains the same.

## 9.2. Proof of Proposition 4

We need to show that no 1 or 2 agent(s) wish(es) to form a coalition and be better off. It should be clear that no 1 agent wants to form a coalition (this option is already embedded in the bargaining problem).

Now, an agent  $\ell$  with  $\bar{a}_h$  could decide to form a coalition with an agent  $\ell$  with  $\bar{a}_\ell$  or an agent  $h$  with  $\bar{a}_h$ . It is a property of the bargaining solution that an agent  $\ell$  will obtain a lower payoff being matched with an agent  $h$  with a higher amount of asset (he can extract less since the marginal utility of obtaining more of the asset is lower for this agent). Hence, an agent  $\ell$  with  $\bar{a}_h$  prefers to be matched with an agent  $h$  with  $\bar{a}_\ell$ . Also, it is a property of the bargaining solution that, given he has to meet an agent with asset holdings  $a$ , an  $\ell$  agents prefer to be matched with the agent with the highest marginal utility (so agent  $h$ ).<sup>20</sup>

Also, an agent  $h$  with  $\bar{a}_\ell$  could decide to form a coalition with an agent  $h$  with  $\bar{a}_h$  or an agent  $\ell$  with  $\bar{a}_\ell$ . As above, however, it is a property of the bargaining solution that an agent  $h$  payoff matched with an agent  $\ell$  will get a higher utility whenever the agent  $\ell$  is holding more asset. Hence, the agent  $h$  will not want to be matched with an agent  $\ell$  holding  $\bar{a}_\ell$ . Also, an agent  $h$  with  $\bar{a}_h$  prefers to be matched with the agent holding  $\bar{a}_\ell$  with the lowest marginal utility, i.e with an  $\ell$  agent.

Hence there are no 2-agents coalition where both agents would do better than under the

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<sup>20</sup>It is easy to show this with  $u_h(a) = \alpha u_\ell(a)$  with  $\alpha > 1$ : Compute the bargaining solution and show that  $\partial[u_\ell(a_\ell - q^r - q^s) + \beta V(a - q^s) + d]/\partial\alpha > 0$ .

prescribed matching technology, which shows that it, together with the distribution over  $\{\bar{a}_h, \bar{a}_\ell\}$  is an equilibrium.

## Proof of Proposition 5

The value functions are

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Adding both equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1 - \beta} \quad (12)$$

Also

$$\begin{aligned} V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] \end{aligned}$$

and adding both equations, we obtain also

$$V_h(\bar{a}_\ell) + V_\ell(\bar{a}_h) = V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) \quad (13)$$

From the first order conditions of the bargaining problem, we then can compute  $d(\bar{a}_h, \bar{a}_\ell)$  as

$$d(\bar{a}_h, \bar{a}_\ell) = (1 - \theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_h)] - \theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell)] \quad (14)$$

where we have used (12)-(13) and the fact that  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ . Therefore, using (14) we obtain

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h) &= \\ u_h(\bar{a}_h) + \beta V_h(\bar{a}_h) + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell)] \end{aligned}$$

Also

$$u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) = u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) + (1 - \theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell)]$$

In a similar fashion, using  $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$  we can rewrite  $d(\bar{a}_\ell, \bar{a}_h)$  as

$$d(\bar{a}_\ell, \bar{a}_h) = d(\bar{a}_h, \bar{a}_\ell) + \bar{u} + \beta(1 - \theta)[V_h(\bar{a}_h) - V_h(\bar{a}_\ell)] + \beta\theta[V_\ell(\bar{a}_h) - V_\ell(\bar{a}_\ell)] \quad (15)$$

where

$$\bar{u} = (1 - \theta)[u_h(\bar{a}_h) - u_h(\bar{a}_\ell)] + \theta[u_\ell(\bar{a}_h) - u_\ell(\bar{a}_\ell)]$$

Therefore, using (13) and (15) and simplifying we obtain

$$u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell) = u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h) + (1 - \theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]$$

Similarly

$$u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) = u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell) + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)]$$

Hence, combining all these expressions, we obtain

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + \pi\theta S + (1 - \pi)(1 - \theta)\tilde{S} \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + \pi(1 - \theta)S + (1 - \pi)\theta\tilde{S} \\ V_h(\bar{a}_\ell) &= \pi[u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_\ell)] + (1 - \pi)[u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)(1 - \theta)S + \pi\theta\tilde{S} \\ V_\ell(\bar{a}_h) &= \pi[u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_h)] + (1 - \pi)[u_h(\bar{a}_h) + \beta V_h(\bar{a}_h)] + (1 - \pi)\theta S + \pi(1 - \theta)\tilde{S} \end{aligned}$$

where

$$\begin{aligned} S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell) \\ \tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h) \end{aligned}$$

Solving for  $V_h(\bar{a}_h)$  we obtain

$$(1 - \beta)V_h(\bar{a}_h) = \frac{(1 - \pi)[u_\ell(\bar{a}_h) + (1 - \theta)\tilde{S}] + [\pi - (2\pi - 1)\beta][u_h(\bar{a}_h) + \theta S]}{1 - (2\pi - 1)\beta}$$

And taking the derivative, we have

$$(1 - \beta)V'_h(\bar{a}_h) = \frac{u'_\ell(\bar{a}_h)(1 - \pi) + [\pi - (2\pi - 1)\beta]u'_h(\bar{a}_h) + (1 - \pi)(1 - \theta)\frac{\partial \tilde{S}}{\partial \bar{a}_h} + \theta\frac{\partial S}{\partial \bar{a}_h}[\pi - (2\pi - 1)\beta]}{1 - (2\pi - 1)\beta}$$

Using the first order condition for  $q^r$  we obtain after some simplifications,

$$(1 - \beta)(1 - (2\pi - 1)\beta)V'_h(\bar{a}_h) = u'_\ell(\bar{a}_h)\theta(1 - \pi) + u'_h(\bar{a}_h)(1 - \theta)[\pi - (2\pi - 1)\beta] \\ + u'_h(\bar{a}_h + q^r)[1 - \pi + (2\pi - 1)(1 - \beta)\theta]$$

Since (12) holds, we use the first order condition for  $q^r$  and simplify to obtain

$$(1 - \beta)(1 - (2\pi - 1)\beta)V'_\ell(\bar{a}_\ell) = u'_\ell(\bar{a}_\ell - q^r)[\pi - (2\pi - 1)(\beta + (1 - \beta)\theta)] + (1 - \pi)(1 - \theta)u'_h(\bar{a}_\ell) \\ + \theta[\pi - (2\pi - 1)\beta]u'_\ell(\bar{a}_\ell)$$

The first condition for  $q^s$  imposes that  $V'_h(\bar{a}_h) = V'_\ell(\bar{a}_\ell)$ . Using the fact that  $u'_\ell(\bar{a}_\ell - q^r) = u'_h(\bar{a}_h + q^r)$  and simplifying, we obtain

$$u'_h(\bar{a}_h + q^r) = \frac{[\pi - (2\pi - 1)\beta][\theta u'_\ell(\bar{a}_\ell) - (1 - \theta)u'_h(\bar{a}_h)] - (1 - \pi)[\theta u'_\ell(\bar{a}_h) - (1 - \theta)u'_h(\bar{a}_\ell)]}{(2\pi - 1)(1 - \beta)(2\theta - 1)}$$

Together with the first order condition on asset sales and the feasibility constraint, this completes the proof.

### 9.3. Proof of Corollary 6

The equilibrium allocation is given by

$$u'_h(\bar{c}_h) = u'_\ell(\bar{c}_\ell) \\ u'_h(\bar{c}_h) = \frac{[\pi - (2\pi - 1)\beta][\theta u'_\ell(\bar{a}_\ell) - (1 - \theta)u'_h(\bar{a}_h)] - (1 - \pi)[\theta u'_\ell(\bar{a}_h) - (1 - \theta)u'_h(\bar{a}_\ell)]}{(2\pi - 1)(1 - \beta)(2\theta - 1)} \quad (16) \\ \bar{c}_h + \bar{c}_\ell = \bar{a}_h + \bar{a}_\ell = 2A$$

Let

$$\alpha_1(\pi) = \frac{\pi - (2\pi - 1)\beta}{2\pi - 1} = \frac{\pi}{2\pi - 1} - \beta$$

and

$$\alpha_2(\pi) = \frac{1 - \pi}{2\pi - 1}$$

then  $\alpha'_1(\pi) = \alpha'_2(\pi) = \frac{-1}{(2\pi-1)^2} < 0$ . And we can rewrite (16) as

$$u'_h(\bar{c}_h)(1 - \beta)(2\theta - 1) = \alpha_1(\pi)[\theta u'_\ell(\bar{a}_\ell) - (1 - \theta)u'_h(\bar{a}_h)] - \alpha_2(\pi)[\theta u'_\ell(\bar{a}_h) - (1 - \theta)u'_h(\bar{a}_\ell)]$$

Notice that  $\bar{c}_h$  is not a function of  $\pi$ , so that using  $\bar{a}_h + \bar{a}_\ell = A$  and the implicit function theorem, we have

$$0 = \alpha'_1(\pi)[\theta u'_\ell(\bar{a}_\ell) - (1 - \theta)u'_h(\bar{a}_h)]d\pi + \alpha_1(\pi)[- \theta u''_\ell(A - \bar{a}_h) - (1 - \theta)u''_h(\bar{a}_h)]d\bar{a}_h \quad (17)$$

$$- \alpha'_2(\pi)[\theta u'_\ell(\bar{a}_h) - (1 - \theta)u'_h(\bar{a}_\ell)]d\pi - \alpha_2(\pi)[\theta u''_\ell(\bar{a}_h) + (1 - \theta)u''_h(A - \bar{a}_h)]d\bar{a}_\ell \quad (18)$$

which we can simplify as

$$\frac{d\bar{a}_h}{d\pi} = \frac{\alpha'_1(\pi) \{ \theta [u'_\ell(\bar{a}_\ell) - u'_h(\bar{a}_h)] + (1 - \theta) [u'_h(\bar{a}_\ell) - u'_h(\bar{a}_h)] \}}{\alpha_1(\pi)[\theta u''_\ell(A - \bar{a}_h) + (1 - \theta)u''_h(\bar{a}_h)] + \alpha_2(\pi)[\theta u''_\ell(\bar{a}_h) + (1 - \theta)u''_h(A - \bar{a}_h)]} \quad (19)$$

Since  $u'_i(\bar{a}_\ell) > u'_i(\bar{a}_h)$  for both  $i$  and  $\alpha'_i(\pi) < 0$ , the numerator is negative. Concavity of the utility function implies that the denominator is also negative. Therefore we have  $d\bar{a}_h/d\pi > 0$ .

Given  $\pi$  the volume of repo in this economy is given by  $q^r$  (since all agents use repo) while the volume of asset sales is given by  $(1 - \pi)q^s = (1 - \pi)(\bar{a}_h - \bar{a}_\ell)$ . Clearly, the sales volume is hump shaped as when  $\pi = 1/2$  we have  $\bar{a}_h = \bar{a}_\ell$  so that  $q^s = 0$  while when  $\pi = 1$ ,  $q^s = 0$  as well. However,  $(1 - \pi)q^s > 0$  for all other values of  $\pi$ . Since the problem is continuous, sales volume is hum-shaped. Also, the fact that  $\bar{c}_h = \bar{a}_h + q^r$  is a constant implies that total volume of repo (i.e.  $q^r$  since all agents engage in repo) is declining in  $\pi$ . Since there are no repo when  $\pi = 1$ , the volume of repo is declining to zero.

## 9.4. Proof of Proposition 8

Given the time to the next meeting is  $\Delta < 1$ , we denote the asset holdings and repo level by  $\bar{a}_{\ell,\Delta}$ ,  $\bar{a}_{h,\Delta}$  and  $q^r_\Delta$ . Using this notation, we have  $c^*_\ell = \bar{a}_\ell - q^r = \bar{a}_{\ell,\Delta} - q^r_\Delta$  and  $c^*_h = \bar{a}_h + q^r =$

$\bar{a}_{h,\Delta} + q_\Delta^r$ . Using the equilibrium condition of Proposition 5, we get

$$\begin{aligned}
& (2\theta - 1)u'_h(c_h^*) \\
= & \frac{[\pi - (2\pi - 1)\beta][\theta u'_\ell(c_\ell^* + q^r) - (1 - \theta)u'_h(c_h^* - q^r)] - (1 - \pi)[\theta u'_\ell(c_h^* - q^r) - (1 - \theta)u'_h(c_\ell^* + q^r)]}{(2\pi - 1)(1 - \beta)} \\
= & \frac{[\pi_\Delta - (2\pi_\Delta - 1)\beta_\Delta][\theta u'_\ell(c_\ell^* + q_\Delta^r) - (1 - \theta)u'_h(c_h^* - q_\Delta^r)]}{(2\pi_\Delta - 1)(1 - \beta_\Delta)} \\
& - \frac{(1 - \pi_\Delta)[\theta u'_\ell(c_h^* - q_\Delta^r) - (1 - \theta)u'_h(c_\ell^* + q_\Delta^r)]}{(2\pi_\Delta - 1)(1 - \beta_\Delta)}
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
& LH(q^r) + \left( \frac{(1 - \pi)}{(1 - 2(1 - \pi))(1 - \beta)} \right) (LH(q^r) - RH(q^r)) \\
= & LH(q_\Delta^r) + \left( \frac{(1 - \pi_\Delta)}{(1 - 2(1 - \pi_\Delta))(1 - \beta_\Delta)} \right) (LH(q_\Delta^r) - RH(q_\Delta^r)) \tag{20}
\end{aligned}$$

where

$$LH(q^r) = \theta u'_\ell(c_\ell^* + q^r) - (1 - \theta)u'_h(c_h^* - q^r)$$

and

$$RH(q^r) = \theta u'_\ell(c_h^* - q^r) - (1 - \theta)u'_h(c_\ell^* + q^r).$$

For  $q \leq (c_h^* - c_\ell^*)/2$ , we have

$$u'_\ell(c_h^* - q) < u'_\ell(c_\ell^* + q) < u'_\ell(c_\ell^*) = u'_h(c_h^*) < u'_h(c_h^* - q) < u'_h(c_\ell^* + q)$$

therefore we have  $LH(q) > RH(q)$ . Moreover, concavity of  $u_h$  and  $u_\ell$  implies

$$\frac{d}{dq}LH(q) < 0 < \frac{d}{dq}RH(q).$$

Now, notice that for  $\Delta < 1$

$$\begin{aligned}
\frac{(1 - \pi_\Delta)}{(1 - 2(1 - \pi_\Delta))(1 - \beta_\Delta)} &= \frac{(1 - \pi)}{(1 - 2\Delta(1 - \pi))(1 - \beta)} \\
&< \frac{(1 - \pi)}{(1 - 2(1 - \pi))(1 - \beta)}
\end{aligned}$$

therefore (20) implies  $q_\Delta^r < q^r$  for  $\Delta < 1$ . Notice that as  $\Delta \rightarrow 0$ , the share of reallocation via repo decreases to  $q_0^r > 0$  which is determined by

$$(2\theta - 1)u'_h(c_h^*) = LH(q_0^r) + \frac{(1 - \pi)}{(1 - \beta)} (LH(q_0^r) - RH(q_0^r)).$$

This completes the proof.

## 9.5. Proof of Proposition 9

We still assume that agents who did not switch are matched together, while those agents who just switched are matched with each other. With Nash bargaining, the allocation of an agent  $h$  with portfolio  $a_h$  matched with an agent  $\ell$  with portfolio  $a_\ell$  solves the following problem:

$$\begin{aligned} \max_{q^s, q^r, d} & [u_h(a_h + q^s + q^r) - d + \beta V_h(a_h + q^s) - u_h(a_h) - \beta \tilde{W}_h(a_h)]^\theta \\ & \times [u_\ell(a_\ell - q^s - q^r) + d + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta \tilde{W}_\ell(a_\ell)]^{1-\theta} \end{aligned}$$

with first order conditions

$$\begin{aligned} V'_h(a_h + q^s) &= V'_\ell(a_\ell - q^s) \\ u'_h(a_h + q^s + q^r) &= u'_\ell(a_\ell - q^s - q^r) \\ d(a_h, a_\ell) &= (1 - \theta)[u_h(a_h + q^s + q^r) + \beta V_h(a_h + q^s) - u_h(a_h) - \beta \tilde{W}_h(a_h)] \\ &\quad - \theta[u_\ell(a_\ell - q^s - q^r) + \beta V_\ell(a_\ell - q^s) - u_\ell(a_\ell) - \beta \tilde{W}_\ell(a_\ell)] \end{aligned}$$

We still assume that agents who did not switch types are matched together while those agents who just switched are matched together. We first solve for  $\tilde{W}_i(a)$ . By definition,  $\tilde{W}_i(a) = \pi W_i(a) + (1 - \pi)W_j(a)$  with  $i \neq j \in \{h, \ell\}$  and where  $W_i$  denotes the value of participating in the Walrasian market as a type  $i$ . From the problem of agents in the Walrasian market, it should be clear that  $W_i(a) = pa + W_i(0)$ , where  $W_i(0)$  is given by

$$\begin{aligned} W_i(0) &= u_i(a_i^w) - pa_i^w + \beta E_{k|i} W_k(a_i^w) \\ &= u_i(a_i^w) - p^r a_i^w + \beta E_{k|i} W_k(0) \end{aligned}$$

where  $a_i^w$  is the solution to  $u'_i(a_i^w) = p^r$  and  $u'_h(a_h^w) = u'_\ell(a_\ell^w)$  with  $a_\ell^w + a_h^w = 2A$ . Solving for  $W_i(0)$  we have

$$\begin{aligned} W_h(0) &= u_h(a_h^w) - p^r a_h^w + \beta\pi W_h(0) + \beta(1 - \pi)W_\ell(0) \\ W_\ell(0) &= u_\ell(a_\ell^w) - p^r a_\ell^w + \beta\pi W_\ell(0) + \beta(1 - \pi)W_h(0) \end{aligned}$$

so that

$$(1 - \beta)W_h(0) = \alpha[u_h(a_h^w) - p^r a_h^w] + (1 - \alpha)[u_\ell(a_\ell^w) - p^r a_\ell^w]$$

where  $\alpha = \frac{1 - \beta\pi}{1 + \beta - 2\beta\pi} \in [0, 1]$ . Similarly,

$$(1 - \beta)W_\ell(0) = \alpha[u_\ell(a_\ell^w) - p^r a_\ell^w] + (1 - \alpha)[u_h(a_h^w) - p^r a_h^w]$$

Therefore,

$$\begin{aligned} (1 - \beta)\tilde{W}_h(0) &= \pi(1 - \beta)W_h(0) + (1 - \pi)(1 - \beta)W_\ell(0) \\ &= u(a_h^w) - p^r a_h^w + [\pi + \alpha - 2\pi\alpha][u_\ell(a_\ell^w) - p^r a_\ell^w - u_h(a_h^w) + p^r a_h^w] \end{aligned}$$

and

$$\begin{aligned} (1 - \beta)\tilde{W}_\ell(0) &= (1 - \pi)(1 - \beta)W_h(0) + \pi(1 - \beta)W_\ell(0) \\ &= u(a_\ell^w) - p^r a_\ell^w + [\pi + \alpha - 2\pi\alpha][u_h(a_h^w) - p^r a_h^w - u_\ell(a_\ell^w) + p^r a_\ell^w] \end{aligned}$$

Notice that

$$\tilde{W}_h(0) + \tilde{W}_\ell(0) = \frac{u(a_h^w) - p^r a_h^w + u(a_\ell^w) - p^r a_\ell^w}{1 - \beta}$$

In this environment the first order condition of the bargaining problem gives us

$$u'_h(\bar{a}_h + q^r) = u'_\ell(\bar{a}_\ell - q^r)$$

so that

$$\begin{aligned} \bar{a}_h + q^r &= a_h^w, \\ \bar{a}_\ell - q^r &= a_\ell^w. \end{aligned}$$

The value functions are

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell)] \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h)] \end{aligned}$$

Adding both equations, we obtain

$$V_h(\bar{a}_h) + V_\ell(\bar{a}_\ell) = \frac{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r)}{1 - \beta} = \tilde{W}_h(a_h^w) + \tilde{W}_\ell(a_\ell^w) \quad (21)$$

From the bargaining first order condition, we obtain

$$\begin{aligned} d(a_h, a_\ell) &= (1 - \theta)[u_h(a_h + q^s + q^r) - u_h(a_h) + \beta V_h(a_h + q^s) - \beta \tilde{W}_h(a_h)] \\ &\quad - \theta[u_\ell(a_\ell - q^s - q^r) - u_\ell(a_\ell) + \beta V_\ell(a_\ell - q^s) - \beta \tilde{W}_\ell(a_\ell)] \end{aligned}$$

so that the transfer  $d(\bar{a}_h, \bar{a}_\ell)$  is (using the fact that  $q^s(\bar{a}_h, \bar{a}_\ell) = 0$ ),

$$\begin{aligned} d(\bar{a}_h, \bar{a}_\ell) &= (1 - \theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_h) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_h)] \\ &\quad - \theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_\ell)] \end{aligned}$$

Therefore, we obtain (using the relation between  $\bar{a}_i$  and  $a_i^w$  as well as equation (21)):

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_h, \bar{a}_\ell) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_h) + \beta \tilde{W}_h(\bar{a}_h) \\ &\quad + \theta \{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\} \end{aligned}$$

Also

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_h, \bar{a}_\ell) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_\ell) + \beta \tilde{W}_\ell(\bar{a}_\ell) \\ &\quad + (1 - \theta) \{u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_\ell) - u_h(\bar{a}_h)\} \end{aligned}$$

In a similar fashion, we obtain (using  $q^s(\bar{a}_\ell, \bar{a}_h) = \bar{a}_h - \bar{a}_\ell$ )

$$\begin{aligned} d(\bar{a}_\ell, \bar{a}_h) &= (1 - \theta)[u_h(\bar{a}_h + q^r) - u_h(\bar{a}_\ell) + \beta V_h(\bar{a}_h) - \beta \tilde{W}_h(\bar{a}_\ell)] \\ &\quad - \theta[u_\ell(\bar{a}_\ell - q^r) - u_\ell(\bar{a}_h) + \beta V_\ell(\bar{a}_\ell) - \beta \tilde{W}_\ell(\bar{a}_h)] \end{aligned}$$

Therefore,

$$\begin{aligned} u_\ell(\bar{a}_\ell - q^r) + d(\bar{a}_\ell, \bar{a}_h) + \beta V_\ell(\bar{a}_\ell) &= u_\ell(\bar{a}_h) + \beta \tilde{W}_\ell(\bar{a}_h) \\ &\quad + (1 - \theta)[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)] \end{aligned}$$

and similarly

$$\begin{aligned} u_h(\bar{a}_h + q^r) - d(\bar{a}_\ell, \bar{a}_h) + \beta V_h(\bar{a}_h) &= u_h(\bar{a}_\ell) + \beta \tilde{W}_h(\bar{a}_\ell) \\ &\quad + \theta[u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h)] \end{aligned}$$

Using the above calculations, we obtain

$$\begin{aligned} V_h(\bar{a}_h) &= \pi[u_h(\bar{a}_h) + \beta \tilde{W}_h(\bar{a}_h)] + (1 - \pi)[u_\ell(\bar{a}_h) + \beta \tilde{W}_\ell(\bar{a}_h)] + \theta\pi S + (1 - \theta)(1 - \pi)\tilde{S} \\ V_\ell(\bar{a}_\ell) &= \pi[u_\ell(\bar{a}_\ell) + \beta \tilde{W}_\ell(\bar{a}_\ell)] + (1 - \pi)[u_h(\bar{a}_\ell) + \beta \tilde{W}_h(\bar{a}_\ell)] + \pi(1 - \theta)S + (1 - \pi)\theta\tilde{S} \end{aligned}$$

where

$$\begin{aligned} S &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_h) - u_\ell(\bar{a}_\ell) \\ \tilde{S} &= u_h(\bar{a}_h + q^r) + u_\ell(\bar{a}_\ell - q^r) - u_h(\bar{a}_\ell) - u_\ell(\bar{a}_h) \end{aligned}$$

And taking the derivative, we have

$$V'_h(\bar{a}_h) = \pi[u'_h(\bar{a}_h) + \beta p] + (1 - \pi)[u'_\ell(\bar{a}_h) + \beta p] + (1 - \pi)(1 - \theta)\frac{\partial \tilde{S}}{\partial \bar{a}_h} + \theta\pi\frac{\partial S}{\partial \bar{a}_h}$$

and using the first order condition for  $q^r$  we obtain

$$\begin{aligned} V'_h(\bar{a}_h) &= \beta p + (1 - \pi)u'_\ell(\bar{a}_h) + \pi u'_h(\bar{a}_h) \\ &\quad + (1 - \pi)(1 - \theta)[u'_h(\bar{a}_h + q^r) - u'_\ell(\bar{a}_h)] \\ &\quad + \theta\pi[u'_h(\bar{a}_h + q^r) - u'_h(\bar{a}_h)] \end{aligned}$$

Also, using the first order condition for  $q^r$  we obtain

$$\begin{aligned} V'_\ell(\bar{a}_\ell) &= \beta p + \pi u'_\ell(\bar{a}_\ell) + (1 - \pi)u'_h(\bar{a}_\ell) \\ &\quad + \pi(1 - \theta)[u'_\ell(\bar{a}_\ell - q^r) - u'_\ell(\bar{a}_\ell)] + (1 - \pi)\theta[u'_\ell(\bar{a}_\ell - q^r) - u'_h(\bar{a}_\ell)] \end{aligned}$$

The first condition for  $q^s$  imposes that  $V'_h(\bar{a}_h) = V'_\ell(\bar{a}_\ell)$ . Using the fact that  $u'_\ell(\bar{a}_\ell - q^r) = u'_h(\bar{a}_h + q^r)$  and simplifying, we obtain

$$u'_h(\bar{a}_h + q^r) = \frac{\pi\theta u'_\ell(\bar{a}_\ell) + (1-\pi)(1-\theta)u'_h(\bar{a}_\ell) - (1-\pi)\theta u'_\ell(\bar{a}_h) - \pi(1-\theta)u'_h(\bar{a}_h)}{(1-2\pi)(1-2\theta)}$$

Therefore the equilibrium is given by

$$\begin{aligned} u'_h(\bar{a}_h + q^r) &= u'_\ell(\bar{a}_\ell - q^r) \\ \bar{a}_h + \bar{a}_\ell &= 2A \\ u'_h(\bar{a}_h + q^r) &= \frac{\pi\theta u'_\ell(\bar{a}_\ell) + (1-\pi)(1-\theta)u'_h(\bar{a}_\ell) - (1-\pi)\theta u'_\ell(\bar{a}_h) - \pi(1-\theta)u'_h(\bar{a}_h)}{(1-2\pi)(1-2\theta)} \end{aligned}$$

Notice that  $\beta$  does not impact the equilibrium allocation.

Also, suppose that  $q^r = 0$  is an equilibrium. Then  $u'_h(\bar{a}_h) = u'_\ell(\bar{a}_\ell)$  and

$$u'_h(\bar{a}_h)(1-2\theta) = (1-\theta)u'_h(\bar{a}_\ell) - \theta u'_\ell(\bar{a}_h)$$

or

$$\theta [u'_\ell(\bar{a}_h) - u'_h(\bar{a}_h)] = (1-\theta) [u'_h(\bar{a}_\ell) - u'_h(\bar{a}_h)]$$

However, since  $\bar{a}_\ell \leq \bar{a}_h$  the RHS is positive, while the LHS is negative by assumption. Therefore,  $q^r = 0$  cannot be an equilibrium.

Now we show that the amount of repo is actually lower under this arrangement. The equilibrium allocations under the benchmark and Walrasian outside-option, can be summarized by

$$\begin{aligned} &(1-2\pi)(1-\beta)(1-2\theta)u'_h(c_h^*) \\ &= \pi(1-\beta) [\theta u'_\ell(c_\ell^* + \bar{q}^r) - (1-\theta)u'_h(c_h^* - \bar{q}^r)] \\ &\quad - (1-\pi)(1-\beta) [\theta u'_\ell(c_h^* - \bar{q}^r) - (1-\theta)u'_h(c_\ell^* + \bar{q}^r)] \end{aligned}$$

where  $u'_h(c_h^*) = u'_\ell(c_\ell^*)$  with  $c_h^* + c_\ell^* = 2A$  and  $q^{r*}$  and  $\bar{q}^*$  are the repo amounts under the benchmark and Walrasian outside-option respectively. Define

$$LH(q^r) = \theta u'_\ell(c_\ell^* + q^r) - (1-\theta)u'_h(c_h^* - q^r)$$

and

$$RH(q^r) = \theta u'_\ell(c_h^* - q^r) - (1 - \theta)u'_h(c_\ell^* + q^r).$$

Note that for  $q^r \leq (c_h^* - c_\ell^*)/2$ , we have

$$u'_\ell(c_h^* - q^r) < u'_\ell(c_\ell^* + q^r) < u'_\ell(c_\ell^*) = u'_h(c_h^*) < u'_h(c_h^* - q^r) < u'_h(c_\ell^* + q^r)$$

therefore we have  $LH(q^r) > RH(q^r)$ , which implies for all  $q^r$ ,

$$[\pi(1 - \beta) + (1 - \pi)\beta] LH(q^r) - (1 - \pi)RH(q^r) > \pi(1 - \beta)LH(q^r) - (1 - \pi)(1 - \beta)RH(q^r)$$

Finally concavity of  $u_h$  and  $u_\ell$  implies

$$\frac{d}{dq^r} LH(q^r) < 0 < \frac{d}{dq^r} RH(q^r),$$

hence  $[\pi(1 - \beta) + (1 - \pi)\beta] LH(q^r) - (1 - \pi)RH(q^r)$  and  $\pi(1 - \beta)LH(q^r) - (1 - \pi)(1 - \beta)RH(q^r)$  are both decreasing in  $q^r$ . This means we have

$$\begin{aligned} & (1 - 2\pi)(1 - \beta)(1 - 2\theta)u'_h(c_h^*) \\ = & [\pi(1 - \beta) + (1 - \pi)\beta] [\theta u'_\ell(c_\ell^* + q^{r*}) - (1 - \theta)u'_h(c_h^* - q^{r*})] \\ & - (1 - \pi) [\theta u'_\ell(c_h^* - q^{r*}) - (1 - \theta)u'_h(c_\ell^* + q^{r*})] \\ > & \pi(1 - \beta) [\theta u'_\ell(c_\ell^* + q^{r*}) - (1 - \theta)u'_h(c_h^* - q^{r*})] \\ & - (1 - \pi)(1 - \beta) [\theta u'_\ell(c_h^* - q^{r*}) - (1 - \theta)u'_h(c_\ell^* + q^{r*})] \end{aligned}$$

therefore  $q^{r*} > \bar{q}^r$ . Note that the change in  $\beta$  does not affect  $\bar{q}^r$ , but as  $\beta$  approaches to zero, then  $q^{r*} \rightarrow \bar{q}^r$ .

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