Optimal Monetary Responses to Asset Price Levels and Fluctuations: A Ramsey Primal Approach

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Abstract

Should monetary policy react to asset prices levels and changes in the context of financial frictions? To answer the question, we provide a tractable monetary Ramsey approach for a heterogeneous agents model with conventional policy (interest rate or money growth target) and unconventional policy (purchase of private illiquid assets) as instruments, in which heterogeneous agents' interaction is summarized in one implementability condition. We show that entrepreneurs hold too much liquid asset in a model with equity issuance and resale (liquidity) constraints. In the steady state, optimal policy targets at paying interest on liquid assets or reducing the money supply available, leading to an equivalent increase of 0.4% in permanent consumption compared to the economy under no policy. In responding to adverse liquidity shocks, the paths of macroeconomic variables under no policy and optimal policy are sharply different and suggest the need for buying illiquid assets and raising interest rate, instead of usual response of reducing interest rate. Finally, we prove that the unconventional policy dominates the conventional counterpart, but the welfare difference of them is quantitatively negligible.

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I Introduction

The recent financial crisis has shown that asset market liquidity fluctuations can have huge impacts on the real economy. As financial frictions widened, the economy plunged into a recession from 2007Q4 to 2009Q2. However, if one were to consider that one of the driving forces of the crisis was indeed the liquidity aspect transmitted to the real economy, it is surprising that there is little understanding on how should a central bank react or even whether it should or should not react.

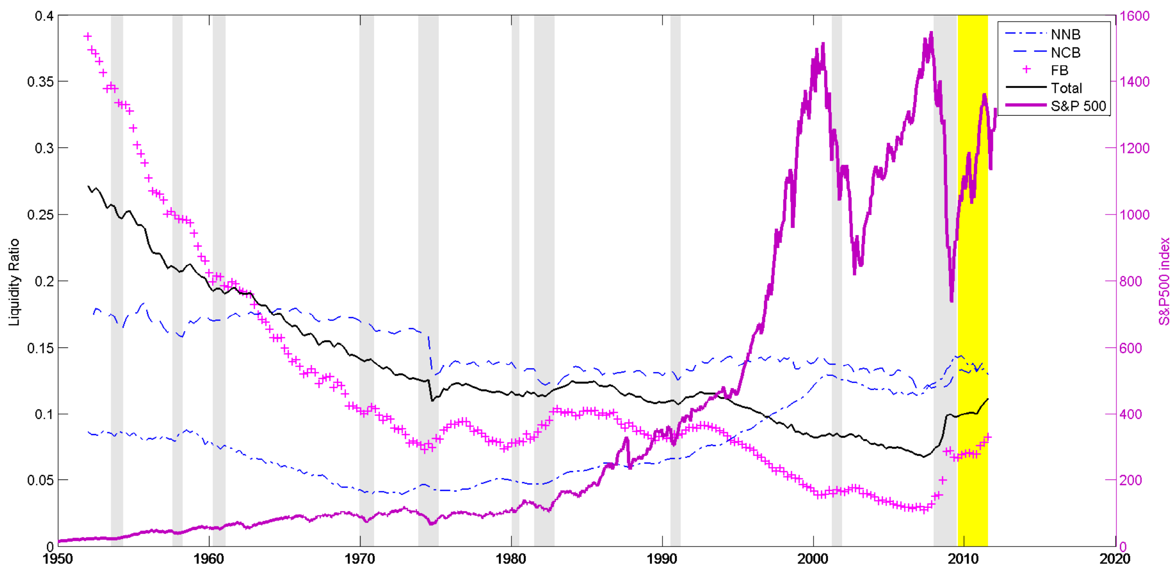

Figure 1: **Liquidity Ratio from 1952Q1** Liquidity ratio, S&P 500 over time. Liquidity ratio is defined as the total liquid assets (check, deposit, tradable receivable and T-Bills while for financial sector with one more source, net over-night interbank lending) over total assets (for financial sectors total assets adjusted by the required reserve in central bank). NNB stands for non-farm and non-corporate business. NCB stands for non-farm and corporate business. FB stands for financial business excluding central bank. Source: 1952Q1-2011Q3 Flow of Funds Table, F102, F103, B102 and B103, Federal Reserve Z1 statistical release. S&P 500 index is obtained from Yahoo Finance. The shaded region depicts the NBER recession period, except that the last (yellow) region depicts the period after 2007-2009 recession until 2011Q3.

Concerns about the interest rate and the degree of asset market liquidity affect the portfolio allocation of low-return-liquid and high-return-illiquid assets (which by itself affect investment and the real economy). One way of seeing this is by plotting liquid vs. illiquid compositions in portfolio as in Figure 1. During the last recession, the liquid share has increased, even though the equity price, as approximated by S&P 500, was already bouncing back in 2009 as seen in the last shaded region. This change of liquidity ratio suggests large rebalancing of portfolio in all sectors during and after the past recession. The question we aim to tackle, therefore, is how should optimal monetary policy behave in a liquidity constrained economy in which portfolio re-balancing in general equilibrium might cause inefficiency.

Our approach emphasizes monetary policy over fiscal policy. Firstly, it has more discretion in the short run and, moreover, we can provide a framework for the recent debate on possible types of monetary instruments. The first monetary instrument targets only on liquid assets, usually called the conventional instrument. Examples of such are steadily increasing or decreasing the money stock, or changing the interest rate of liquid assets. The second instrument is motivated by the FED's large purchases of illiquid assets, which we exemplify through open market operations on private equity and label them as unconventional policies[1]. The main distinction between them is the introduction of the central bank's holding of partially liquid assets.

In our model, entrepreneurs have uninsured investment opportunity risks. They want to finance future investment by saving through holding other's equity or intrinsically valueless liquid assets. However, there are equity financing and resale frictions. The equity financing constraint limits new equity issuance up to a fraction of one's new investment. At the same time, resale (liquidity) frictions set a bound on the fraction of equity that can be sold. Therefore in financing new investment, the equity issuance constraint limits the outside financing while the resale friction limits the internal financing. Hence, the financial frictions reduce the

[1]We will abuse of notation calling the privately owned equity "private equity", but with a different meaning than the usual jargon.

amount of resources for investment that should be transferred from non-productive agents to productive ones, not only reducing the output produced but also limiting consumption smoothing.

To lessen the liquidity frictions, entrepreneurs will thus hold intrinsically valueless fully liquid assets as extra internal savings. Liquid assets in the economy help lubricate funds transfer in the economy. However, by holding liquid assets, they do not internalize their own effect on lowering the return of the assets. In equilibrium, there is too much holding of liquid assets. Importantly, we ask whether, given the liquidity friction in a competitive economy, a constrained planner (who also respects the liquidity friction) can improve the social welfare. Nevertheless, we are not dealing with policy that can entirely eliminate liquidity friction.

We depart from early monetary policy literature such as Woodford (2003) by focusing on the Ramsey problem of optimal monetary policy and use the primal approach, which we can fully solve analytically. We find that the "implementability" condition, that summarizes all the decentralized market conditions, equals the net-worth difference of different types of agents to the total gain if non-resalable capital become resaleable. The implementability condition suggests that, as agents switch back and forth from being productive-type (with investment opportunity) to unproductive-type (without investment opportunity), consumption-smoothing will be harder the larger are financial frictions in the economy. Then, it is worthwhile to consider two extreme cases in which fiat liquid assets disappear

1. If there is no equity issuance friction, idiosyncratic investment opportunity risk is fully insured and equity resale friction does not matter, leading to zero net-worth difference and perfect consumption smoothing;

2. If there is no equity resale friction (the equity is fully liquid), the value gained by transforming non-resaleable into resaleable is zero since all equity is resaleable. Savings

through holding equity will be enough to finance future new investment. Again, we have zero net-worth difference and perfect consumption smoothing.

Finally, the implementability constraint also shows that unconventional policies do weakly dominate conventional ones theoretically, since the latter can be shown to be a subset of the former.

To our best knowledge, we are the first to give an answer to what is the optimal monetary policy in the context of liquidity frictions. Standard New-Keynesian optimal policy uses the second order approximation to the objective function of a representative household, usually finding a balance between output gap and inflation gap[2]. Such strategy is not particularly attractive in the context of liquidity friction, where heterogeneous agents and uninsured risks become the central theme. With the help of the implementability condition, heterogeneity can be summarized in one constraint for the policy maker.

We calibrate and structurally estimate the model, using liquid assets data from U.S. flow of funds jointly with aggregate investment from 1991 to 2007. Especially, we want to estimate the size of the shock to liquidity constraint such that it will induce private sectors to re-balance the asset portfolio as in the data. Such treatment of data is novel and is directly linked to the question posed.

Our result shows that the monetary authority should "deflate" the economy in steady state, since agents have a propensity to over-save in liquid assets. Intuitively, one way of reducing such problem is by constantly shortening the supply of liquid assets, or, equivalently, an annual 4% real interest rate on liquid assets[3]. Once liquid assets earn higher rate of

[2]As we do not follow the strategy of approximating the objective function, we also do not have welfare ranking problems, since in principle one can consider all higher order terms.

[3]Arguing that optimal policy is deflation abstracts from other margins that we do not consider, such as price stickiness. A more correct interpretation is that liquidity margins suggest an increase on the liquid asset. We do not incorporate sticky price consideration and the calculation of deflation could be thought of

return due to policy, those who have investment opportunities and liquid assets will have a better internal financing through liquid assets. More wealth is then transferred from agents with funds but no investment opportunity, to those with investment opportunity but not enough funds. The welfare gains of such policy amount to almost .4% increase on permanent total consumption. Moreover, the optimal level of interest rate is increasing in the liquidity frictions, since higher liquidity frictions deter more resources transfer.

Finally, we examine various shocks estimated from data, including liquidity shocks that lead to a harder resaleability of the illiquid assets, and productivity shocks.[4]

For an unexpected adverse liquidity shock, the policy should aim at help financing investment through increasing the interest rate on liquid assets. Importantly, even though theoretically we prove that the unconventional monetary policy weakly dominates conventional one, quantitatively it is negligibly.

Related Literature The literature on financial frictions is vast, spanning mostly borrowing constraints and, more recently, liquidity frictions [5]. Since we are interested in optimal policy with liquidity problems, we build upon the model of Kiyotaki and Moore (2011), as we view it as an otherwise standard business cycle model in which financial frictions are important. The most important difference is that we embed conventional and unconventional policy instruments in a way that we are able to reach optimal policy solution. Monetary

average liquid asset return after inflation adjustment. The rationale for doing this is exactly to highlight the monetary policy under flexible prices, which is rarely discussed.

[4] As usual in models with only adverse unexpected liquidity shocks (Kiyotaki and Moore (2011)), flight to liquidity increases the net-worth of agents who hold liquid assets and leads to a bigger change in the supply of illiquid assets. The two effects increase illiquid asset price and increase total consumption, which we do not observe in reality. Thus, we span all the possibilities of combination of liquidity shocks and productivity shocks to overcome the difficulties. Especially future expected liquidity shocks with current productivity shock can be thought of as higher degree of financing frictions that leads to capital mis-allocation and lower productivity.

[5] On borrowing constraints, the literature is very vast, but the seminal work of Kiyotaki and Moore (1997) and the recent survey of Brunnermeier, Eisenbach, and Sannikov (In Progress) are good examples of the broad literature that exists.

policy, therefore, is designed to influence on the return on liquid assets. At the same time, we are able to compare conventional and unconventional policies.

The most related literature to our paper is the one that merges monetary policy and financial frictions. On one hand, some have investigated how "unconventional monetary policies", i.e., that change the central bank's balance sheet, could be used and rationalized (Negro, Eggertsson, Ferrero, and Kiyotaki (2011) and Gertler and Karadi (2011)). Another strand has evaluated financial frictions in a New-Keynesian model with price stickiness, as discussed in Woodford (2003) and, more specifically, in Christiano, Motto, and Rostagno (2007). Both strands, however, are silent in the policy optimality in an economy with liquidity frictions.

We depart from the literature, first by discussing optimal policy with financial frictions, but also by bringing the Ramsey approach such as in Chari and Kehoe (1999) to this literature. Monetary policy in our setup is non-super neutral due to distributive effects, and it is not due to the price stickiness usually assumed[6]. Even though we have a somewhat similar result to "Friedman rule", the reason behind is very different. The key is the propensity to over-save, instead of the usual opportunity cost of holding money due to transaction needs. Our purpose, however, is not to explain data-observed inflation targets (which the New-Keynesian literature can do using price-stickiness) but how financial frictions alter the optimal level of real interest rate.

There is a recent literature that studies pecuniary externality from financial friction, in which the competitive market is neither efficient, nor constrained efficient. Bianchi (2009), Bianchi (2010), Korinek (2009) and Lorenzoni (2008) look at how far an decentralized economy with borrowing constraints is from the first best allocation, attainable by government

[6]Following the jargon, neutral means the level of money stock does not matter and super-neutral means that growth of money does not matter either.

policies. We share a similar feature with the literature by introducing prices in the budget constraint, the agent does not take into account her own effect on prices and generate an externality. However, we depart from the literature by restricting attention to constrained efficient allocation, instead of first best. Monetary policy therefore respects liquidity frictions and highlights the over-saving of liquid assets in the decentralized economy.

II A Canonical Model of Financial Frictions

A Set-up

In this section we consider a variant of Kiyotaki and Moore (2011) in which we introduce an inelastic supply of labor and monetary policy. Conventional policies are exemplified by a helicopter drop or drain policy, but the results are entirely equivalent if we were to think about interest rate management on liquid assets if money was thought as a very general liquid asset such as the T-Bill. For unconventional policies, we consider open market operations on purchasing private equity[7]. We try to stick as much as possible to Kiyotaki and Moore (2011) model in order to evaluate the gains from optimal policy in an otherwise standard model, but some changes are needed to accommodate such policies. Therefore, we'll be brief in explaining the set-up used.

Time is discrete and infinite. The economy has two types of agents, entrepreneurs with measure 1 and household with measure L. Each agent has expected utility of

$$E_t \sum_{s=0}^{\infty} \beta^s log\left(c_{t+s}\right)$$

[7] A more detailed explanation of such interpretations is given in Section 4.

at time t. Only entrepreneurs have access to a constant-returns-to-scale technology for producing output from capital and labor. An entrepreneur holding k_t capital at the beginning of period t can employ l_t in a competitive labor market to produce

$$y_t = A_t (k_t)^{\alpha} (l_t)^{1-\alpha}.$$

To produce output, entrepreneurs have to be involved in the production process so that their participation is necessary. Production is completed within period t, during which capital depreciates to be $(1-\delta) k_t$, where $0 < \delta < 1$. $A_t = e^{z_t}$ is common to all entrepreneurs and z_t follows

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z.$$

Entrepreneurs hire each unit of labor at a competitive real wage w_t. Due to constant return to scale technology, profits on capital are linear in individual entrepreneur's capital[8]

$$y_t - w_t l_t = r_t k_t$$

where r_t is the equilibrium profits on capital. The household side is assumed to be supplying L unit of inelastic labor for simplicity[9]. After introducing labor, r_t can now be determined by clearing labor markets. For each entrepreneur with k_t, their decision on hiring labor is

$$(1-\alpha) A_t (k_t)^{\alpha} l_t^{-\alpha} = w_t, \rightarrow l_t = \left[\frac{(1-\alpha) A_t}{w_t}\right]^{\frac{1}{\alpha}} k_t.$$

[8]The return on individual capital is linear in their capital stock level , but decreasing in aggregate capital stock level.

[9]If we specify them to have the same discount rate β in preference and allow them to buy equity for savings, they will not do so because the equilibrium rate of return will be less than β. The use of Lunits for workers is a normalization itself, since we keep the entrepreneurs as being 1 unit throughout.

Therefore, if aggregate capital stock is K_t, the labor demand is $\left[\frac{(1-\alpha)A_t}{w_t}\right]^{\frac{1}{\alpha}} K_t$. Wage is then $w_t = (1-\alpha)\, A_t\, (K_t/L)^{\alpha}$ and the gross profits are $A_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t = \alpha A_t (K_t/L)^{\alpha-1} k_t$. Thus, profits on capital are

$$r_t = \alpha A_t \left(\frac{K_t}{L}\right)^{\alpha-1}. \qquad (1)$$

The arrival of an investment opportunity, i.e., the chance to produce new capital from general output, is independently distributed across entrepreneurs (but not across the household) and through time (we assume a constant fraction at every point in time). At each date t, π fraction of entrepreneurs have the opportunity, while the other $1-\pi$ do not. In later numerical analysis, π will be matched to investment spike in the data. Investment completed in period t will be available as capital in period $t+1$:

$$k_{t+1} = (1-\delta)\, k_t + i_t$$

We assume there is no insurance market against having an investment opportunity, so that the market is incomplete. In order to finance the investment opportunity, an entrepreneur can issue an equity claim to the future output from the investment, but due to the friction only θ fraction of the investment can be issued. Such friction can be motivated in a production process in which entrepreneurs have to participate in the production to produce full amount of future output and outsiders may just be able to get $1-\theta$ of the future output. The other friction that we introduce is the equity resale friction; entrepreneurs have difficulties in selling their capital, as they can sell only up to ϕ fraction of their own equity backed by physical capital each period. Resale friction is common especially when information asymmetry is severe. Since we abstract from different asset category while putting all assets (except fully liquid assets) together, ϕ measures the average degree of resale friction.

Table I: **Balance Sheet of a Typical Entrepreneur**

Liquid Assets (Money)	Own Equity Issued
Holding of Others' Equity	
Own Unmortgaged Capital Stock	Net Worth

An entrepreneur has liquid assets, others equity, and un-mortgaged capital stock in his balance sheet as in Table I. For simplicity, both own equity unmortgaged initially and outside equity can be sold at most ϕ fraction and depreciate at the same rate δ. Therefore, own equity and outside equity are perfect substitutes. Entrepreneurs can remortgage their previously un-mortgaged capital stock up to ϕ_t fraction of that. ϕ_t fluctuates over time to capture the uncertainty of liquidity frictions. Therefore, the exogenous shocks in this economy are summarized by (z_t, ϕ_t).

Let n_t be the equity and let m_t be money held by an individual entrepreneur at the start of period t. The above discussion can be summarized as

$$n_{t+1} \geq (1-\theta)\, i_t + (1-\phi_t)\,(1-\delta)\, n_t \tag{2}$$

$$m_{t+1} \geq 0 \tag{3}$$

The first constraint summarizes the amount of equity held in the next period. The minimum equity held would be the sum of un-mortgaged investment and equity that cannot be resold. The second constraint is just a non-negativity constraint on liquid assets (money). Private agents cannot issue very liquid assets like T-bills. Commercial papers, even some that are very liquid, are still less liquid than assets issued by the government, which are backed up by taxes and government enforcement power. Therefore, any debt that is issued by firms and commercial banks should be thought of as the non-fully resalable assets in the model.

The return on private assets in the model thus should be regarded as average return from equity and bonds in reality.

Now, we introduce conventional policy first and leave the unconventional policy in the next subsection. Let q_t be the price of equity and let p_t be the price of liquid assets, in terms of consumption goods[10]. From now on, we use liquid assets and money interchangeably since money is a case of liquid assets. The flow of funds constraint for an entrepreneur at time t is then given by

$$c_t + i_t + q_t \left(n_{t+1} - i_t - (1-\delta)\, n_t\right) + p_t \left(m_{t+1} - m_t - \tilde{M}_t\right) = r_t n_t$$

where $\tilde{M}_t$ is the new money supply from government at time t. We model the conventional monetary policy as a helicopter drop, but, since what matters is the return of money, one could think of changing the return of liquid assets with equivalent results. The individual entrepreneurs take the new increased supply as given and think that they will not affect the equilibrium. If monetary authorities do not react at all, $\tilde{M}_t$ will always be 0. Importantly, we do not restrict the policy to be helicopter drop, i.e, $\tilde{M}_t \geq 0$. The policy can also be as a money drain, i.e, $\tilde{M}_t < 0$. In that case, the policy is equivalent to a taxation on all entrepreneurs with $p_t \tilde{M}_t$ and the government use the proceeds to pay interest rate on liquid assets since the policy will change the rate of return on liquid assets.

B Recursive Equilibrium

We want to focus on an economy with valued liquid assets or money. Once money is valued, it is used as an alternative source for savings since equity financing is insufficient due to the

[10]The reason to denote consumption goods as the measure is because it is convenient to think about rate of return on liquid assets

resaleability friction. Therefore, productive entrepreneurs will sell up to ϕ_t fraction of equity and their constraints are all binding[11]. To reach this interesting economy equilibrium, we assume the following, as in Kiyotaki and Moore (2011)[12]

$$Assumption : \delta\theta + \pi(1-\delta)\phi < (\beta - 1 + \delta)(1-\pi).$$

Entrepreneurs with investment opportunities, under the above assumption, will borrow to the limit so that constraint (2) will bind and the flow of funds becomes

$$c_t^i + [1-\theta q_t]\, i_t \quad = \quad [r_t + q_t\phi_t(1-\delta)]\, n_t + p_t\left(m_t + \tilde{M}_t\right).$$

Using (2) and (3), the investing entrepreneur's consumption is $1-\beta$ fraction of the net-worth, as we have log-utility. Therefore

$$c_t^i = (1-\beta)\left\{r_t n_t^i + \left[\phi_t q_t + (1-\phi_t)\, q_t^R\right](1-\delta)\, n_t^i + p_t\left(m_t^i + \tilde{M}_t\right)\right\}, \tag{4}$$

where $q_t^R = \frac{1-\theta q_t}{1-\theta} < 1$ as $q_t > 1$. Investment is thus

$$i_t = \frac{[r_t + q_t\phi_t(1-\delta)]\, n_t^i + p_t\left(m_t^i + \tilde{M}_t\right) - c_t^i}{1-\theta q_t}. \tag{5}$$

[11] We will assume that the optimal policy is also respecting the binding constraint, since we are looking only into policies that can be decentralized into a competitive equilibrium market as such.

[12] To understand the assumption, suppose the assumption hold and the steady state capital is K. Note that the following is impossible,

$$[\delta\theta + \pi(1-\delta)\phi]\, K > \delta(1-\pi)\, K.$$

To see this, the right hand side is the saving of non-investing entrepreneurs (with populations $1-\pi$) in steady state; the left hand side is the sum of new equity issued ($\delta\theta K$, which is the investment to compensate depreciation) and existing equity sold ($\pi(1-\delta)\phi K$). Then the inequality says that investing entrepreneurs can transfer all the savings from non-investing entrepreneurs, which is not possible by the assumption. Thus, the first best outcome cannot be achieved by individual savings.

For entrepreneurs without investment opportunity

$$c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t^s + q_t (1-\delta) n_t^s + p_t \left(m_t^s + \tilde{M}_t \right),$$

where the consumption can be solved as

$$c_t^s = (1-\beta) \left\{ r_t n_t^s + q_t (1-\delta) n_t^s + p_t \left(m_t^s + \tilde{M}_t \right) \right\}. \tag{6}$$

Meanwhile, these entrepreneurs choose the portfolio of money and equity. A typical non-investing entrepreneur will be indifferent between money and equity. Therefore, from first order condition,

$$\begin{aligned} u'\left(c_t^s\right) &= \beta E_t \left[\frac{p_{t+1}}{p_t} \left[(1-\pi) u'\left(c_{t+1}^{ss}\right) + \pi u'\left(c_{t+1}^{si}\right) \right] \right] \\ &= \beta (1-\pi) E_t \left[\frac{r_{t+1} + (1-\delta) q_{t+1}}{q_t} u'\left(c_{t+1}^{ss}\right) \right] + \\ &\quad \beta \pi E_t \left[\frac{r_{t+1} + (1-\delta) \phi_{t+1} q_{t+1} + (1-\delta)(1-\phi_{t+1}) q_{t+1}^R}{q_t} u'\left(c_{t+1}^{si}\right) \right]. \end{aligned} \tag{7}$$

where E_t denotes the conditional expectation at date t, and c_{t+1}^{si} and c_{t+1}^{ss} measures the date t non-investing entrepreneur's consumption at date $t+1$ for having or not having investment opportunity respectively (before government transfers and subsidies).

We can do aggregation in the economy easily due to the linearity in equity and liquid assets in these equations. Before aggregation, it is appropriate now to introduce another government instrument, the purchasing and selling of private equity.

More recently, the central banks have implemented a new set of policies in which they buy private equity with partial liquidity, such as mortgage backed securities. We consider, therefore, how open market operations should be used in such context. The coined term "unconventional" for open market operation is due to the fact that the asset that the FED is holding has partial resaleability. Furthermore, it pumps the liquid asset in the economy, which could be thought as money or T-Bills, to inject liquidity in the system. There are possibly indirect instruments for targeted purchases, but we will discuss the direct one for simplicity and also since it was what the Fed had actually done the most. We, therefore, introduce another instrument, which is N_t^g denoting the equity that can be purchased from private sector, as a "quantity" choice variable of the social planner. When the economy is endowed with $K_t - N_t^g$ and M_t at period t, then $\pi\left(K_t - N_t^g\right)$ capital and πM_t money is in the hands of investing entrepreneurs.

Aggregate investment I_t can be derived from (5):

$$\left(1-\theta q_t\right) I_t = \left[r_t + q_t \phi_t \left(1-\delta\right)\right] \pi \left(K_t - N_t^g\right) + p_t \pi \left(M_t + \tilde{M}_t\right) - C_t^i. \tag{8}$$

Goods market clearing gives (subtract labor income and labor consumption on both sides)

$$r_t K_t = C_t + I_t + G_t, \tag{9}$$

where G_t is government consumption and C_t is total consumption. Total consumption is defined as

$$C_t = C_t^i + C_t^s \tag{10}$$

where consumptions of investing and saving entrepreneurs are

$$\begin{aligned} C_t^i &= \left(1-\beta\right)\left\{r_t \pi \left(K_t - N_t^g\right) + \left[\phi_t q_t + \left(1-\phi_t\right) q_t^R\right]\left(1-\delta\right)\pi\left(K_t - N_t^g\right) + p_t \pi \left(M_t + \tilde{M}_t\right)\right\} & (11) \\ C_t^s &= \left(1-\beta\right)\left[r_t\left(1-\pi\right)\left(K_t - N_t^g\right) + q_t\left(1-\delta\right)\left(1-\pi\right)\left(K_t - N_t^g\right) + p_t\left(1-\pi\right)\left(M_t + \tilde{M}_t\right)\right] & (12) \end{aligned}$$

Then we should have an aggregate portfolio choice equation. Define the equity held by entrepreneurs without investment opportunities at the end of period t as N_{t+1}^{s}, where

$$N_{t+1}^{s} = \theta I_t + \left[\phi_t \pi \left(1-\delta\right) + \left(1-\pi\right)\left(1-\delta\right)\right] K_t.$$

Notice that in (7), the aggregate version of c_{t+1}^{ss} and c_{t+1}^{si} is

$$\begin{aligned} C_{t+1}^{ss} &= \left(1-\pi\right)\left[\left(1-\beta\right)\left(r_{t+1} + \left(1-\delta\right) q_{t+1}\right) N_{t+1}^{s} + p_{t+1}\left(M_t + \tilde{M}_t\right)\right] \\ C_{t+1}^{ss} &= \pi\left[\left(r_{t+1} + \phi_{t+1}\left(1-\delta\right) q_{t+1} + \left(1-\phi_{t+1}\right)\left(1-\delta\right) q_{t+1}^{R}\right) N_{t+1}^{s} + p_{t+1}\left(M_t + \tilde{M}_t\right)\right] \end{aligned}$$

from which one can rewrite (7) as

$$\begin{aligned} & \left(1-\pi\right) E_t \left[\frac{\left(r_{t+1} + \left(1-\delta\right) q_{t+1}\right)/q_t - p_{t+1}/p_t}{\left(r_{t+1} + \left(1-\delta\right) q_{t+1}\right) N_{t+1}^{s} + p_{t+1}\left(M_t + \tilde{M}_t\right)}\right] \qquad (13) \\ = \; & \pi E_t \left[\frac{p_{t+1}/p_t - \left[r_{t+1} + \phi_{t+1}\left(1-\delta\right) q_{t+1} + \left(1-\phi_{t+1}\right)\left(1-\delta\right) q_{t+1}^{R}\right]/q_t}{\left[r_{t+1} + \phi_{t+1}\left(1-\delta\right) q_{t+1} + \left(1-\phi_{t+1}\right)\left(1-\delta\right) q_{t+1}^{R}\right] N_{t+1}^{s} + p_{t+1}\left(M_t + \tilde{M}_t\right)}\right]. \end{aligned}$$

When we have open market operations, we can think of the government using the money supply and return from previous equity to buy extra holding of private equity $N_{t+1}^{g} - \left(1-\delta\right) N_t^{g}$, which translates into private sector's holding of equity of $K_t - N_t^{g}$ at each date t. To back out the money spent on open market operations, the government expenditure now has to satisfy:

$$G_t + q_t\left[N_{t+1}^{g} - \left(1-\delta\right) N_t^{g}\right] + \psi\left(N_{t+1}^{g}\right) = r_t N_t^{g} + p_t \bar{M}_t \qquad (14)$$

In future analysis, we tie our hand by setting $G_t = 0$ to abstract from fiscal part. We view that the marginal cost will be small once government step in to buy equity, while the marginal cost will be very high when the government holds very few or zero private equity, but the specifics of the function is discussed when presenting the parameters. Finally, the capital evolution is

$$K_{t+1} - N_{t+1} = (1-\delta)(K_t - N_t) + I_t \tag{15}$$

Therefore, we have the following recursive equilibrium definition:

Definition. A recursive competitive equilibrium is defined as functions z_t, ϕ_t, C_t, I_t, q_t, p_t, r_t, K_{t+1}, N^g_{t+1} of $(z_{t-1}, \phi_{t-1}, K_t, N^g_t)$ that satisfies (8), (10), (10), (11), (12), (13), (14), and (15), given stochastic processes of (z_t, ϕ_t) and given a sequence of money supply rule$\{\tilde{M}_t, \bar{M}_t\}_0^\infty$.

Notice that the definition of equilibrium already imposes capital market clearing and money market clearing through investment and portfolio balancing equation.

III The Optimal Monetary Policy Problem

The approach to reach the optimal policy is in the same spirit of the public finance literature (see Chari and Kehoe (1999) for a survey) on obtaining an "implementability condition", the so-called primal approach. After constructing the equilibrium conditions of a decentralized market, we solve out prices to depend only on allocations. The problem then becomes of a social planner choosing allocations under two constraints: one that defines a competitive equilibrium and the other that defines resources constraint.

To do so, we first describe how one can obtain an implementability condition with conventional and unconventional policies. Then we show that the conventional policy is actually a particular case of an unconventional setup.

A Unconventional and Conventional Policy Together

A.1 Implementability Condition

Let $S_t = K_t - N_t^g$ be the holding of equity in the private sector. One can solve q_t from equations (8) and (11),

$$\frac{\beta}{1-\beta}C_t^i = (1-\theta q_t)\left[I_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta}\pi S_t\right] \tag{16}$$

Note that $\frac{1}{1-\beta}C_t^i$ is the net-worth of the investing agents, so $\frac{\beta}{1-\beta}C_t^i$ is the value of their equity holding (including inside and outside equity). On the left-hand side of equation (16), we have the total equity holding, on the right hand side we sum all the parts that constitute the equity holding: for I_t investment, $\theta q_t I_t$ should be subtracted and, out of πS_t initial equity holding, the investing agents have $(1-\delta)(1-\phi_t)\frac{1-\theta q_t}{1-\theta}\pi S_t$ after depreciation and equity selling (note that the market value for those that cannot be sold is $\frac{1-\theta q_t}{1-\theta}$). Therefore,

$$q_t = \frac{1-d_t}{\theta} \text{ and } q_t^R = \frac{d_t}{1-\theta},$$

where $d_t = \frac{\frac{\beta}{1-\beta}C_t^i}{S_{t+1}-(1-\delta)S_t+\frac{(1-\delta)(1-\phi_t)}{1-\theta}\pi S_t}$. One can interpret d_t as the down-payment rate. ($\frac{\beta}{1-\beta}C_t^i$ is the amount that investors save, while $S_{t+1}-(1-\delta)S_t+\frac{(1-\delta)(1-\phi_t)}{1-\theta}\pi S_t$ is the capital that will be used in production). Hence, the price of capital can be interpreted as one minus down-payment rate over the fraction of the investment (θ) that can be initially issued. To solve p_t, again from equation (11), one can express the price of money $p_t$as

$$p_t = \frac{1}{\pi\left(M_t+\tilde{M}_t\right)}\left\{\frac{C_t^i}{1-\beta} - r_t\pi S_t - \left[\frac{(1-d_t)}{\theta}\phi_t + \frac{d_t}{1-\theta}(1-\phi_t)\right](1-\delta)\pi S_t\right\}$$

Then plug p_t and q_t into equation (12) and it yields:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t)\frac{(1-\theta)-d_t}{\theta(1-\theta)}(1-\delta)S_t \tag{17}$$

To interpret the implementability condition, recall that $\frac{C_t^s}{1-\beta}$ is the net worth of the saving agents and $\frac{C_t^i}{1-\beta}$ is the net worth of the investing agents, so the left hand side is the net worth difference of saving agents and investing agents normalized by the populations. Such difference is determined by the resaleability friction of the equity held after depreciation. This difference occurs since the shadow price for saving agents is $q_t = \frac{1-d_t}{\theta}$ while for investing agents is $q_t^R = \frac{d_t}{1-\theta}$. Hence the implementability condition states that the net-worth difference of two types of agents comes exactly from the price difference on non-resalable capital due to financing friction.

In a nutshell, the implementability condition summarizes the frictions. If we relax the financing friction, $q_t = q_t^R$, there will be no net-worth difference. If we relax the resaleability ($\phi = 1$), the net worth difference will also be equal to zero, as one would expect in a usual business cycle model.

A.2 Set Up Ramsey Problem

Now, suppose one wants to assign equal welfare weight to each agent in the economy. The constrained planner's problem is then given by:

Problem 1.

$$\max_{C_t^i, C_t^s, S_{t+1}, N_{t+1}^g} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \pi \log\left(\frac{C_t^i}{\pi}\right) + (1-\pi)\log\left(\frac{C_t^s}{1-\pi}\right) \\ +L\log\left[(1-\alpha)A_t\left(\frac{S_t+N_t^g}{L}\right)^{\alpha}/L\right] \end{array} \right\}$$

subject to:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t)\frac{(1-\theta)-d_t}{\theta(1-\theta)}(1-\delta)S_t$$

$$C_t^i + C_t^s + \left(S_{t+1} + N_{t+1}^g\right) + \psi\left(N_{t+1}^g\right) = r_t K_t + (1-\delta)\left(S_t + N_t^g\right)$$

where $d_t = \frac{\frac{\beta}{1-\beta}C_t^i}{\left(K_{t+1}-N_{t+1}^g\right)-(1-\delta)\left(K_t-N_t^g\right)+\frac{(1-\delta)(1-\phi_t)}{1-\theta}\pi\left(K_t-N_t^g\right)}$.

Consumption can be solved as a function of S_t, S_{t+1}, N_t^g and N_{t+1}^g, with detailed calculation in the appendix. Not surprisingly we have two instruments, which give rise to two first order conditions and two state variables, one on private equity holding and another on government equity holdings:

$$\left(\frac{\pi}{C_t^i}\frac{\partial C_t^i}{\partial S_{t+1}} + \beta E_t \frac{\pi}{C_{t+1}^i}\frac{\partial C_{t+1}^i}{\partial S_{t+1}}\right) + \left(\frac{(1-\pi)}{C_t^s}\frac{\partial C_t^s}{\partial S_{t+1}} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s}\frac{\partial C_{t+1}^s}{\partial S_{t+1}}\right) + \beta\frac{\alpha L}{S_{t+1}} = 0 \quad (18)$$

$$\left(\frac{\pi}{C_t^i}\frac{\partial C_t^i}{\partial N_{t+1}^g} + \beta E_t \frac{\pi}{C_{t+1}^i}\frac{\partial C_{t+1}^i}{\partial N_{t+1}^g}\right) + \left(\frac{(1-\pi)}{C_t^s}\frac{\partial C_t^s}{\partial N_{t+1}^g} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s}\frac{\partial C_{t+1}^s}{\partial N_{t+1}^g}\right) + \beta\frac{\alpha L}{N_{t+1}^g} = 0 \quad (19)$$

We assume the cost function for government holding private equity is a concave function $(\psi'(.) > 0, \psi''(.) < 0)$ and satisfy that $\psi(0) = 0$, $\psi'(0) = 0$. For small shocks, the deviations from zero open market operation should be small since it is very costly to hold private equity; For large shocks, it becomes necessary for the government to purchase significant amount of private equity, known as unconventional monetary policy to stabilize asset price and enhance liquidity.

A full characterization of each term, as well as some further algebra that simplifies the interpretation of the results, is relegated to the appendix. Finally, the second order condition is checked numerically to ensure a maximum.

B Conventional Policies Only

In this section, we restrict attention to the problem when $N_t^g = 0$, so that the central bank can only change rates of return on money, whether by a helicopter drop of money or changing interest rates paid on reserves.

Recall that the competitive equilibrium is defined by equations (8)-(15) and the additional constraint that $N_t^g = 0$. Then we can solve p_t and q_t from equations (8) and (11) as we did before. By plugging the prices back, with the additional constraint that the government does not buy illiquid assets, the implementability becomes:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t)\frac{(1-\theta)-d_t}{\theta(1-\theta)}(1-\delta)K_t \tag{20}$$

The interpretation is very similar. But now since all asset is privately claimed, we do not need to distinguish between privately and publicly claimed assets. In what regards to the structure of the Ramsey problem, we only have one first order condition, since we have constrained to one instrument.

C The Equivalence and Dominance Result

From the implementability conditions, one can see that conventional policy is a subset of all the allocations that can be attained through unconventional policies. Therefore, we have the following equivalence result.

Proposition. *Suppose the government has both conventional and unconventional instruments. The optimal allocation is the same as having only the unconventional instrument.*

Proof. $\tilde{M}_t$ does not show up and setting $\tilde{M}_t = 0$ will not affect the optimal $S_{t+1}$and, if one has unconventional policies to use. □

Corollary. *Unconventional monetary policies dominate conventional ones.*

To understand the proposition and corollary, one should recall the central friction in this economy: equity resale friction. Intuitively, the imperfection on the selling equity reduces the rate of return on equity and induces a pecuniary externality, since agents do not take into account their own effect on holding the liquid asset. Furthermore, agents tend to hold liquid assets which are intrinsically valueless. Both unconventional and conventional policy are intended to correct this externality. However, unconventional policy is more accurate since it targets directly at the illiquid asset and the dominance result becomes straightforward.

IV Quantitative Examination of Optimal Policy

In this section, we highlight how important is optimal policy through a series of numerical exercises. Our benchmark is a competitive economy with no policy intervention (constant money supply). We discuss steady-state values as well as impulse response functions under no policy, policy with only conventional instruments and with unconventional instruments. The experiment exercise is to demonstrate how should the optimal policy react both qualitatively and quantitatively (or if it should react at all).

A Parameters

Some of the parameters used are standard in the literature such as depreciation rate, capital share in production and discount factor, while more elaboration should be given to π, ϕ, θ and L. $\pi = 6\%$ of the entrepreneurs have investment opportunity each quarter, which is the number to match investment spikes observed from U.S. manufacturing plants (See Doms and Dunne (1998), Cooper, Haltiwanger, and Power (1999) and Negro, Eggertsson, Ferrero, and

Table II: **Parameters for Quantitative Exercises**

α	β	δ	θ	$\bar{\phi}$	L	μ	a	π
Capital Share	Discount Factor	Depreciation Rate	Issuance Friction	Resale Friction	Labor Population	Cost param 1	Cost param 2	Inv Prob.
0.33	0.99	0.025	0.19	0.19	6	0.005	1.11872	0.06

Kiyotaki (2011)). For the financial frictions, previous work by Negro, Eggertsson, Ferrero, and Kiyotaki (2011) has assumed the mean values for θ and ϕ to be 19%, matching total treasury bills outstanding to total assets. We perform another exercise, looking at the ratio of liquid assets to total assets in the economy in the stable period (1991Q1 to 2007Q4) and we confirm these results. Later we will vary ϕ to check robustness, which directly measures the resale friction.

Finally, L should show the ratio of workers to entrepreneurs in the economy. The main difference between workers and entrepreneurs is the access to equity markets to fund the investment opportunity. Therefore, we calibrate this value to be in line with the participation rate of households from SCF in 2009 that we see in the equities market (19%), a number in line with previous studies from Mankiw and Zeldes (1991) and Heaton and Lucas (1999). Such number translates into $L = 6$.

When using unconventional monetary policy, the cost for the government of buying private equities is assumed to be

$$\psi\left(N^g_{t+1}\right) = \mu\left[log\frac{\left(1+N^g_{t+1}\right)}{a}\right]^2$$

Since we look for a cost on holding assets, and not on the changes of purchase, we consider a function well defined in the positive side (log). Therefore, our task is to find μ and a such that the steady state private holding of equity is the same as in the conventional policy. This requirement leads to the above policy[13], together with previous parameters, is summarized in Table II.

Finally, for the evolution of exogenous state variables z_t and ϕ_t , we follow the literature on assuming the productivity an AR(1) process and also take the resaleability as an AR(1). We estimate the two processes to be:

$$z_t = 0.9225 z_{t-1} + \epsilon_t^z$$
$$\phi_t - \bar{\phi} = 0.895 \left(\phi_{t-1} - \bar{\phi}\right) + \epsilon_t^{\phi}$$

where ϵ_t^z are i.i.d zero mean normal random variable with standard deviation 0.0134, ϵ_t^{ϕ} are i.i.d zero mean random variable with standard deviation 0.0052 and $corr\left(\epsilon_t^z, \epsilon_t^{\phi}\right) = 0.495$. The productivity random process is the standard Solow residual process and is taken from estimation of Thomas (2002), in line with previous studies. The AR(1) coefficient of ϕ_t process and its residual come from estimating the model with observed rate of return on liquidly assets. Namely, we could think of not assuming that the government has already taken the optimal policy, and we use the actual rate of return on liquid assets when estimating it. We estimate the process of ϕ_t using observed policy among other direct aggregate variables (investment, liquidity assets value and total asset value) through Bayesian estimates that are detailed in the appendix.

B Steady State

We discuss the steady state in three scenarios: no policy, optimal conventional policy and optimal unconventional policy.

[13] The choice of using this log-function instead of the most common quadratic cost was just to ensure computational tractability to avoid negative values

In Table III, we normalize all the variables to be deviations from the no-policy case. The table allows us to not only evaluate the gains from optimal policy, but also to compare how better is unconventional compared to conventional, since we have already showed that the former dominates the latter.

Table III: **Steady State Value**

Normalizing the quantities in the economy with no policy as 1 and comparing the increase or decrease of each variables. C^i : consumption of investing agents. C^s: consumption of saving agents. C^L: consumption of workers. I: investment. N_g/S: ratio of government held equity over private held. K: capital. Asset price: q. Total money value: M^p. liquidity ratio: the ratio of liquid assets value over total assets. Finally, equivalent consumption gain is how much each agent would increase its consumption permanently by changing to the respective policy

	Conventional Policy	Unconventional Policy
Annual Interest Rate	+3.527%	+3.444%
Total Output	+0.917%	+0.917%
C^i	+5.416%	+5.318%
C^s	-3.395%	-3.395%
C^L	+0.909%	+0.909%
I	+2.797%	2.803%
N_g/S	0	1%
K	+2.797%	+2.803%
Asset Price	-9.249%	-9.056%
Total Money value	+35.642%	+35.22%
Liquidity Ratio	+40.719%	+40.146%
Equivalent Consumption Gains	+0.359%	+0.361%

Firstly, optimal monetary policy plays a role in the steady-state, by changing the rate of return on liquid asset, as one can see in the annualized interest rate, about 3.5% annually. The intuition is that saving entrepreneurs save too much in the liquid asset and use the return to finance investment. However, they create an externality on others because they reduce the return from liquid assets, inducing the others to save even more to finance future investment. A way to overcome this is by reducing the supply of the liquid asset, which increases the rate of return and it leans against the pecuniary externality. By increasing the rate of return on the liquid asset, entrepreneurs will enjoy better return from that for future new investment, as seen by about 35.5% increase in liquid assets value.

The second distinguished feature is the capital stock held in equilibrium. As one would expect, investing agents are constrained due to the financial friction, but due to redistribution policy, the capital stock increases by about 3% and is closer to the first best. Therefore, the asset price q, which implicitly measures the degree of financing and resaleability constraint, is closer to 1, the first best outcome in which either financing friction or resaleability friction is eliminated. Thus, even though it is still constrained, the shadow value of relaxing the constraint decreases after policy intervention, leading to a higher capital and therefore a higher welfare.

The welfare gains computed suggest that the benefits from having an optimal monetary policy in such environment are equivalent to increasing the consumption of each agent, permanently, by .36%, a sizable number since it is a permanent change in the economy.

A final comparison is on unconventional and conventional policy outcome. The quantities and prices are very similar to what conventional policy can achieve. Interest rate need not be that high since the constrained planner has another instrument to achieve the desired allocation.

C Simulations

Now, we examine how monetary policy responds to shocks, through impulse response functions. We consider four cases: a pure productivity shock, a pure liquidity shock, a combination of a productivity and a liquidity shock and a combination of productivity with expected future liquidity shock. Our focus is mainly on comparing optimal policy (both conventional and unconventional policy) with a constant liquid assets supply, which we label as no-policy.

In doing that, we log-linearized the model to solve the rational expectation system[14].

C.1 Active Conventional Monetary Policy and No Policy

- *Pure Productivity Shock*

The first shock that we consider is a pure productivity shock (Figure 2). The shock that we investigate is a negative one standard-deviation shock to productivity.

Without policy, such shock drives the price of the equity and money down under a constant money supply policy because there are less resources for agents to save. To emphasize the price level change, the initial triggering of a pure adverse productivity shock leads to slightly higher interest rate and thus slightly lower price of liquid assets in recession. Therefore, pure productivity shock is at odd with data.

With policy intervention, however, when one considers a helicopter drop (drain) type of policy, the equity price drops while the return becomes very high after the shock. To achieve this, we see a positive rate of return on liquid assets that is even higher than the interest rate under no policy. The gains from having a conventional policy can be seen in the consumption of savers and investors, even though the total consumption is less affected because workers don't have their consumption much affected. Overall, aggregate consumption, investment and output do not change significantly under conventional policy and no policy, when only productivity shocks hit. Importantly, the steady state level is still higher under conventional policy. Given that the response in percentage term is similar, the conventional policy still gives a better allocation of resources.

[14] Detailed computation can be found in the Matlab code available in the authors' website

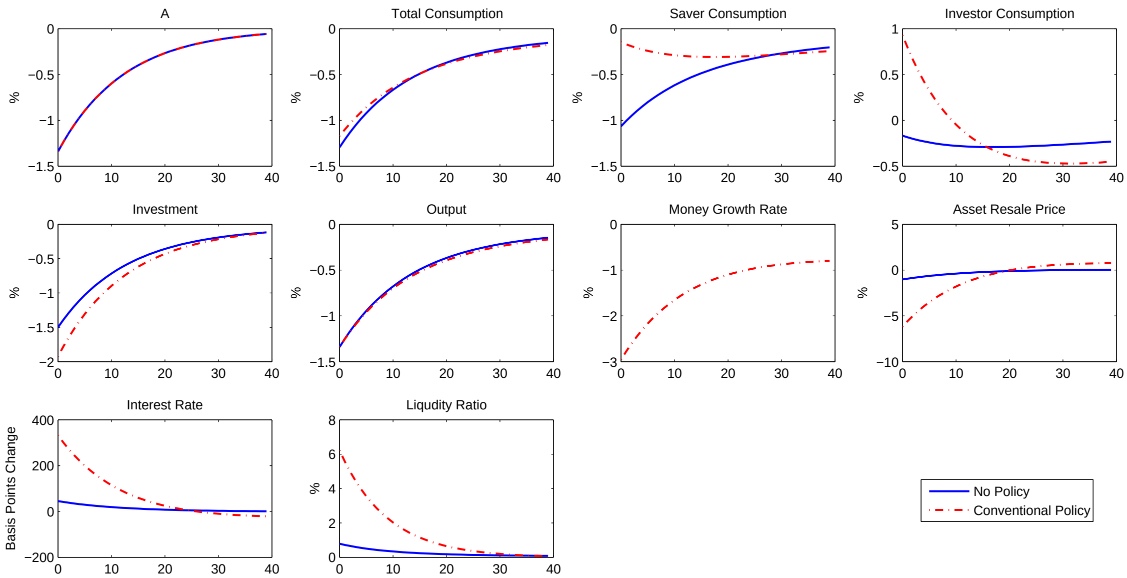

Figure 2: **No Policy and Conventional Policy:** A **shock only** One standard deviation shock to ϕ with correlated shock to lnA. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

- *Pure Liquidity Shocks*

Now we consider a pure adverse liquidity shock (Figure 3). We consider, once more, an autoregressive shock with a one standard deviation shock of 0.0052 obtained from a Bayesian estimation of the model during the "great moderation", so it can be thought as a small shock during regular periods.

As documented in Kiyotaki and Moore (2011), liquidity shocks usually lead to a flight to liquidity as seen in the figure with a lower rate of return on liquid assets. Money price increases so that there is deflation pressure. However, at the same time, the illiquid asset supply drop drastically so that real asset price actually increases even though the nominal price decreases.[15].Therefore, asset prices (q_t, p_t) increase and lead to higher wealth, which

[15]The nominal asset price is lower after pure liquidity shocks, since money's real price (in terms of consumption goods) increases more than real asset price initially due to flight to liquidity. We do not plot

will yield initial higher individual consumption, but lower investment (given that output will be initially the same, investment has to drop). Overall, pure liquidity shock is at odds with the data too.

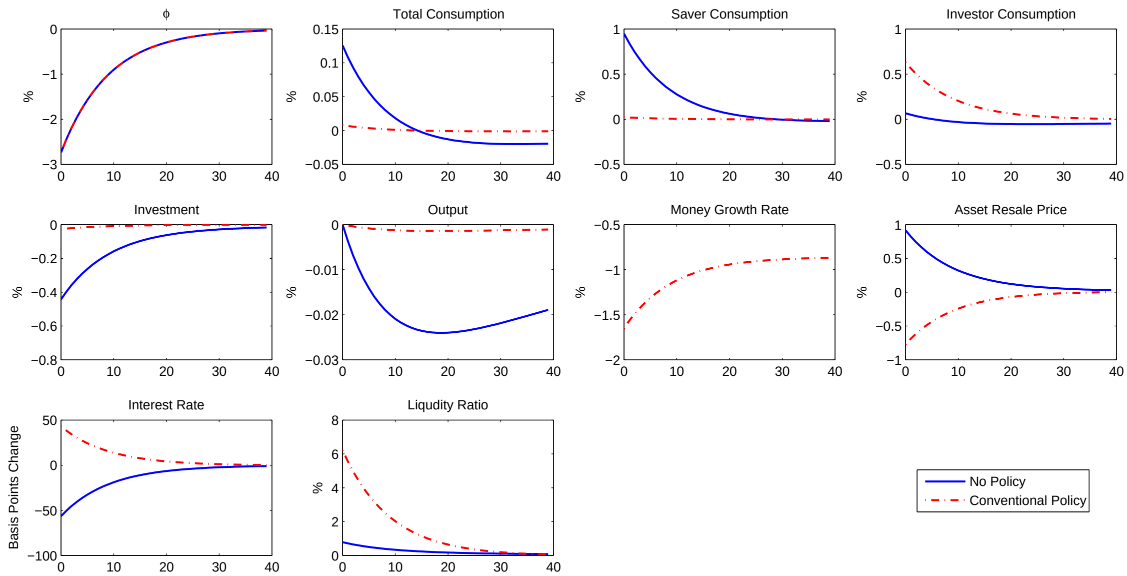

Figure 3: **No Policy and Conventional Policy: ϕ shock only** One standard deviation shock to ϕ with correlated shock to lnA. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

Under optimal policy, there is an apparent trade-off between present and future as we can see from the aggregate consumption graph. Aggregate consumption rarely fluctuates. Not surprisingly, the aggregate investment and output are stable as well. This outcome is achieved by changing the liquidity asset return and then changing the consumption gap between saving and investing entrepreneurs. Note that, the larger increase in total investing entrepreneurs' consumption is a manifestation of more consumption smoothing.

- *Productivity and Liquidity Shocks*

money's real price, since we focus on the return from liquid assets instead of the price.

From the last two experiments, we saw that pure productivity or pure liquidity shock misses some important stylized facts observed in recession in the data, no matter whether there is optimal policy or not (mainly consumption path, rate of return on liquid assets and asset resale price). We overcome the odd behavior by considering a liquidity shock accompanied by a productivity shock and estimate its variance-covariance matrix from data (Figure 4). The shock we investigate is a liquidity shock accompanied by a simultaneous TFP as described before. The correlation between them is also obtained from Bayesian estimates and it comes to be relatively big: 0.49. The economic reason behind such experiment could be, for instance, that financing frictions lead to mis-allocation and reduce TFP, but we do not model it endogenously. Moreover, with such shock we span another possibility which is a liquidity shock that does not translate into an asset price movement in a world without policy.

Without the optimal policy, the adverse ϕ shock will reduce the demand on equity because of liquidity run, but not enough to reduce the asset price. Two forces roughly cancel out each other on this exercise: on one hand, the portfolio rebalancing to liquid assets; on the other, as productivity is auto-regressive, the economy becomes more unproductive today and in the future too, overcoming more consumption and less investment today from the liquidity run discussed before. As a result, investment will decrease drastically, while consumption should also fall because it is accompanied by a TFP loss that reduces the available resources. All these features are in line with stylized facts observed in usual recessions, particularly the portfolio rebalancing in recent years.

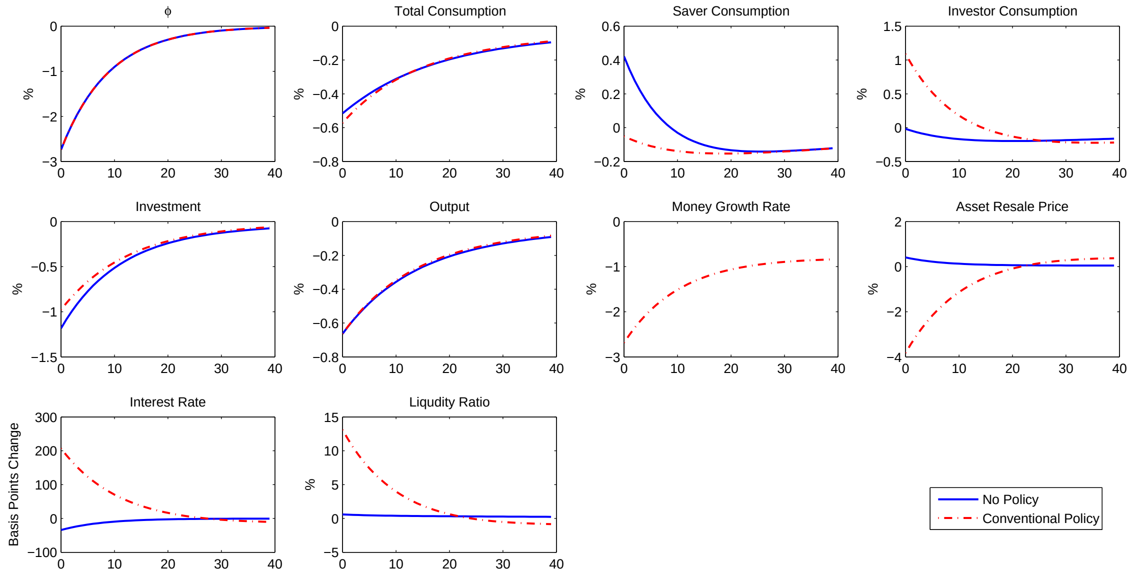

Figure 4: **No Policy and Conventional Policy:ϕ shock and A shock** One standard deviation shock to ϕ with correlated shock to lnA. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

With optimal policy, investment drops, but less than without an optimal policy to counterbalance such effect. Again, the central bank is redistributing wealth through the payment of liquid assets, which helps on consumption smoothing (from the gap between saving and investing entrepreneurs consumptions).

Not surprisingly, optimal policy achieves a more stable result through redistributing resources. This can be seen from the welfare improvement, which shows that the welfare increases after a shock. Note, however, that since we are comparing to the steady-state, the optimal policy was already better than the constant money supply case and it becomes even better.

- *Expected Future Liquidity Shocks and Productivity Shocks*

The previous shocks lead to policy response, but without too much impact on stabilizing macroeconomic real variables. We consider expected future liquidity shock, say 4 quarters later, and current productivity jump and examine how much the policy could achieve (Figure 5).

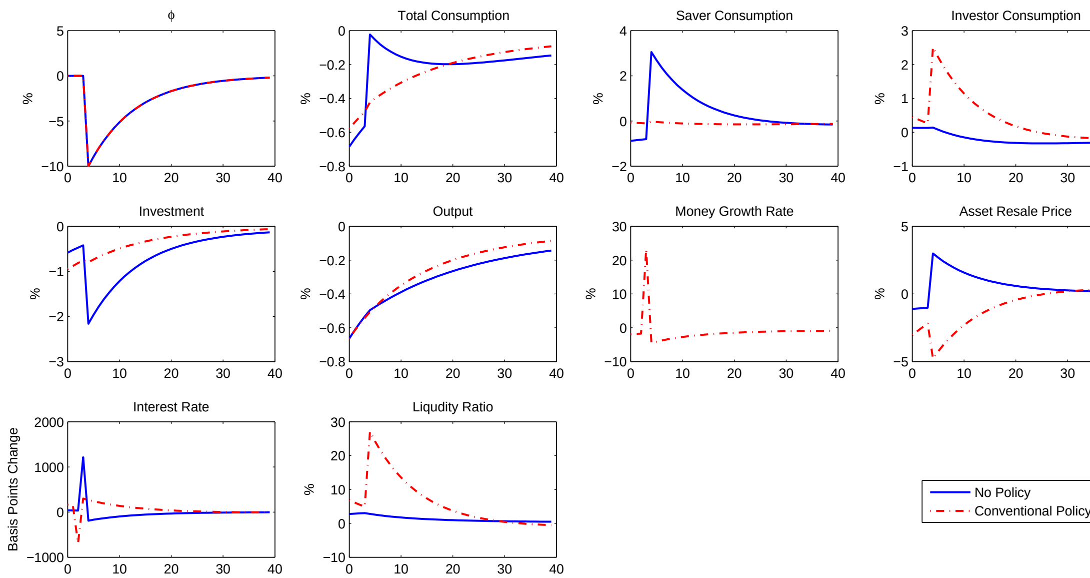

Figure 5: **No Policy and Conventional Policy: future ϕ shock and A shock** One standard deviation shock to ϕ with correlated shock to lnA. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

Such experiment is intended to partially capture the fall of Lehman Brothers in 2008Q3. The fall did not immediately stop all the business. In fact, many previous Lehman related business still ran into 2009. However, the fall may have triggered the expectation that in the near future many assets would be very illiquid, which is captured by a 4 quarters later liquidity shock. At the same time, funding froze from the banking sector, limiting efficient production, reducing total factor productivity in the economy, which is seen in the data computed by most policy paper.

The purpose of this exercise is twofold. First, to discuss under which conditions is policy more relevant, and secondly to discuss a liquidity based shock in which the asset price actually goes down. We still take the same structure discussed before, but with shock on current liquidity known 4 quarters before.

Without policy intervention, the macroeconomic variables are very unstable. For example aggregate consumption decreases by 0.7% initially, then increases a lot, decreases back to a level that is lower than the steady state and stay there persistently. The reason for that can been seen from investment, which decreases initially because of low productivity, slightly increases afterward due to less consumption and then incur a big jump because of expected liquidity shock. Persistent low investment thus leads to persistent future low consumption. Not surprisingly, output is persistently lower.

The role of the government policy is remarkable. Interest is kept almost constant until date three, when the rate reduces greatly so that there is a negative interest rate on holding liquid assets at date 3. By increasing money growth in 25% at date 3, it helps on keeping liquid asset accumulation low and a larger room for policy to increase the rate of return from date 4 to date 5. When the real shock hits on date 4, the liquid asset return was maintained high by the central bank which helps on smoothing funds transfer. This experiment demonstrate significantly that monetary policy should move fast in responding to market price and return fluctuation.

C.2 Unconventional Policy and Conventional Policy

Now we follow the same structure of the previous version where we have discussed productivity, liquidity and joint productivity-liquidity shocks. The difference, henceforth, is that we want to compare the gains from using unconventional policies vs. conventional ones. We have already established that conventional policies lead to a non-negligible increase in welfare compared to a constant money supply case under steady-state. Moreover, we have theoretically established that unconventional policies are weakly better than conventional ones. A further question that one may ask is: why should we bother understanding conventional instruments if we know that unconventional ones dominate them?

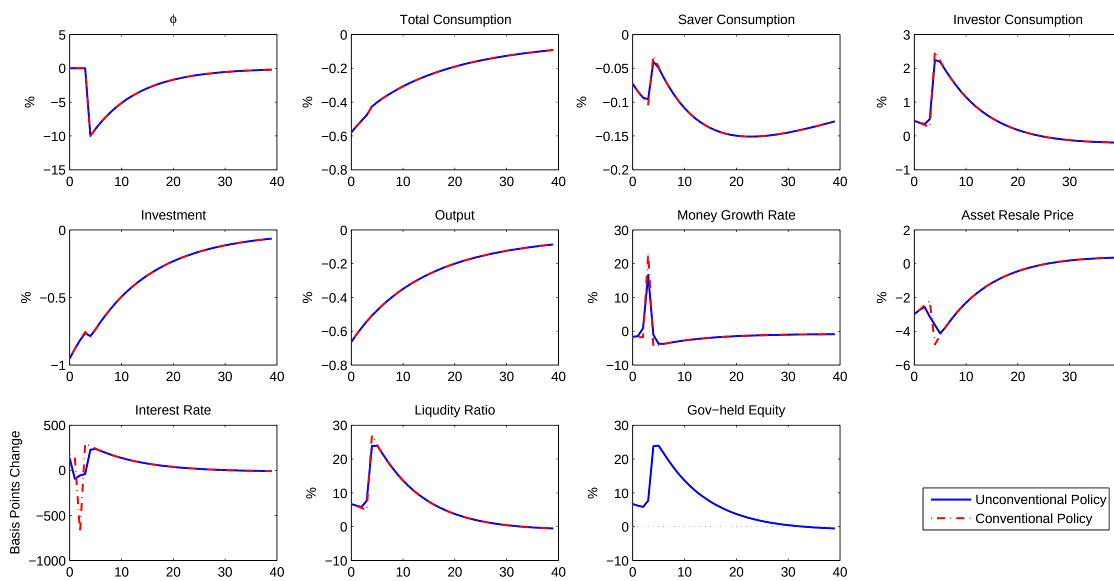

Figure 6: **Unconventional versus Conventional** 1 standard deviation shock to future $\phi$with correlated shock to lnA. Money growth rate path is the level path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state.

The answer can be depicted from comparing conventional and unconventional policy in the *"Expected Future Liquidity Shocks and Productivity Shocks"* experiment (Figure 6): the optimal path under both policies is roughly the same, being robust to any shock. However, we have assumed that helicopter drain is feasible, which may well not be when truly implementing it. If helicopter drain is not possible, unconventional policies may help attain the desired allocations we have before. Conventional and unconventional policy under other experiments are almost exactly the same (including interest rates) and we will not show them here due to space restriction.

Allowing for a new instrument leads to an interest rate increase of about 3%, a much bigger number than the 0.05% that we had under conventional policies. However, such difference leads roughly to the same allocation. There is no significant difference on the path of the variables using unconventional or conventional policies.

Even though the path is indistinguishable, the levels under unconventional policy are higher, since we are comparing to a higher steady state. The results for the liquidity shock case only and simultaneous shocks also give indistinguishable paths between conventional and unconventional policies.

Finally, two comments are worth mentioning. First of all, such results do not depend on the cost function used. The intuition behind it is that the unconventional policy, if possible, almost completely reduces the pecuniary externality by changing the rate of return on liquid assets and illiquid equity. The unconventional policy leaves very few room for improvement. For roughly any concave function tested, our results persist. The changes are even smaller if we use convex cost function since marginal cost becomes higher after purchase. Besides that, these results should not be seen as a case against unconventional policies. On the contrary, even though unconventional policies cannot change the paths of variables, they do change the steady state from which we are comparing to. Therefore, the welfare remains higher during all periods after the shock in an unconventional policy. Besides that if, for instance, helicopter drain policies are not implementable, unconventional policies can substitute them.

C.3 Liquidity Ratio in the Data and the Model

As can be seen in all the exercises, liquidity ratio fluctuates more under active policies. The key reason is that monetary policies enable the liquid assets to be more valuable and to lubricate funds transfer for investment. Without policy, individuals generate a larger degree of externality and thus make the value of liquid assets very low. Importantly, they have a rigid demand on liquid assets in all experiments, since the other assets give unfavorable return due to illiquidity. Illiquidity shows as lower asset prices in the productivity shocks, or as higher fractions of non-resalable asset in liquidity shocks. Therefore, agents in our model do have large rebalancing to liquid assets as in the data. Given the effective low interest rate policy, optimal policy nevertheless suggests that liquidity ratio probably should be even higher so that rebalancing to liquid assets does not hurt its ability to transfer funds.

D Robustness

We compute the steady-state level of capital for different parametrization to draw comparative statics[16]. Importantly, since one can draw a relationship between capital and the rate of return on liquid assets, one can evaluate the relationship between the financial frictions and the return of liquid assets in the steady-state. The optimal rate of return on the liquid asset should be decreasing in steady state ϕ.

Relaxing steady state ϕ from .14 to .25, for instance, the optimal rate of return on the liquid asset would jump from around annual 6% to almost 0% (Figure 7) [17]. A thorough look at how endogenous variables change as one tighten or loosen liquidity friction is in Table IV.

[16]Such results can be derived from the equations, but we present the results numerically for ease of interpretation.

[17]For numbers of ϕ higher than this, the assumption that ensures that the constraints are binding is not satisfied.

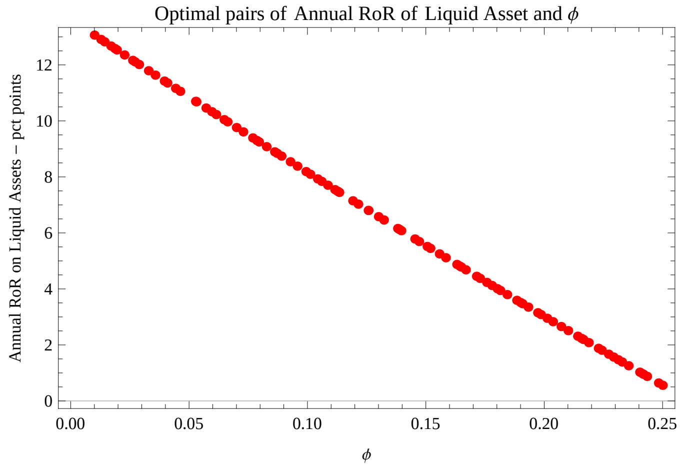

Figure 7: **Steady State Comparison**

Compared to no policy, total capital stock under optimal policy increases less as the friction relaxes. Not surprisingly, the targeted optimal interest rate under policy is smaller, the higher is ϕ (smaller financial friction). Hence, the gains from having optimal policy are reduced as the financial friction is relaxed. Moreover, the competitive equilibrium allocations under optimal policy and no policy become similar as one relaxes the frictions, a result that is expected, since there would be no role for optimal policy if there was no friction[18].

V Conclusion

We study a tractable model of optimal monetary policy instruments dealing with financial frictions, namely equity issuance and resale frictions. We provide an implementability condition that summarizes all the restrictions of a competitive equilibrium allocation in this model, allowing us to derive the social optimal allocation. By doing so, we avoid the usual ambiguous welfare ranking problem in the optimal monetary policy literature.

[18]Due to space restriction, we do not show the impulse response functions for different parameters, but the qualitative results discussed previously are the same, and the policy conclusions remain.

Table IV: **Robustness**

	$\phi = .18$		$\phi = .19$		$\phi = .20$	
	Conventional Policy	Unconv. Policy	Conventional Policy	Unconv. Policy	Conventional Policy	Unconv. Policy
Annual Int. Rate	+4.02%	+4.06%	+3.527%	+3.444%	+3.00%	+2.95%
Total Output	+1.038%	1.043%	+0.917%	0.917%	+0.784%	0.790%
C^i	+6.148%	+6.052%	+5.416%	+5.318%	+4.667%	+4.579%
C^s	-3.837%	-3.837%	-3.395%	-3.395%	-2.949%	-2.949%
C^L	+1.047%	+1.047%	+0.909%	+0.909%	+0.789%	+0.789%
I	+3.197%	3.197%	+2.797%	2.803%	+2.407%	2.803%
K	+3.197%	+3.197%	+2.797%	+2.803%	+2.407%	+2.407%
Asset Price	-10.411%	-10.217%	-9.249%	-9.056%	-8.061%	-7.868%
Total Money value	+35.070%	+34.706%	+35.642%	+35.22%	+36.065%	+35.578%
Liquidity Ratio	+40.800%	+40.273%	+40.719%	+40.146%	+40.465%	+39.822%
Equivalent Permanent Consumption	+0.418%	+0.420%	+0.359%	+0.361%	+0.308%	+0.309%

Both optimal conventional and unconventional monetary policy should target at paying interest rate on liquid assets. Due to the pecuniary externality arising from the liquidity constraint, liquid assets return will be too low and there will always be room for policy to improve welfare. In the steady-state, permanent aggregate consumption increases by almost 0.4% under optimal policy than that under no policy. Moreover, when hit by an adverse liquidity shock, such difference increases even more. Finally, we showed that unconventional policies dominate conventional counterparts. But quantitatively, the difference that it generates on macroeconomic real variables are very small.

Note that we do not assume sticky price but the pecuniary externality on holding liquid assets still need policy intervention. Monetary policy, therefore, acts like a redistribution device transferring resources from non-liquid assets holders to liquid assets ones. Whenever the economy runs into liquidity problem, firms or banks will typically hold excess liquid assets, usually more than they should. A usual policy by lowering interest rate should be reconsidered. Agents will have a rigid demand on liquid assets if other assets market persistently incur resale (liquidity) problems. In that sense, lowering the interest rate only hurts the ability for financing future investment, since it exacerbates the incentive to hold even more liquid assets in a world where they are the only favorable saving choices.

One drawback and potential future work is how the monetary policy will change illiquid asset market resaleability endogenously (how ϕ changes endogenously by monetary policies). We viewed it as an exogenous fluctuation but it could certainly depend on market expectation and asset quality. This possibility is left for future work.

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Appendix

A Ramsey Problem

From the implementability condition and resources constraint, one can express aggregate investing and saving entrepreneurs' consumption as

$$\begin{aligned} C_t^i &= \pi\{r_t K_t - K_{t+1} + (1-\delta) K_t - G_t - \psi\left(N_{t+1}^g\right) \\ &\quad -(1-\pi)(1-\beta)(1-\phi_t)\left(\frac{(1-\theta)-d_t}{\theta(1-\theta)}\right)(1-\delta)(K_t - N_t^g)\} \\ C_t^s &= (1-\pi)\{r_t K_t - K_{t+1} + (1-\delta) K_t - G_t - \psi\left(N_{t+1}^g\right) \\ &\quad +\pi(1-\beta)(1-\phi_t)\left(\frac{(1-\theta)-d_t}{\theta(1-\theta)}\right)(1-\delta)(K_t - N_t^g)\} \end{aligned}$$

Noticing that one can plug in d_t and express C_t^i and C_t^s only in terms of S_t, N_t^g, $S_{t+1}$and N_{t+1}^g:

$$\begin{aligned} C_t^i &= \frac{\pi}{1-B_t}\{r_t\left(S_t + N_t^g\right) - \left(S_{t+1} + N_{t+1}^g\right) + (1-\delta)\left(S_t + N_t^g\right) - G_t - \psi\left(N_{t+1}^g\right) \\ &\quad -(1-\pi)(1-\beta)(1-\phi_t)(1-\delta) S_t/\theta\} \end{aligned} \tag{21}$$

$$C_t^s = r_t\left(S_t + N_t^g\right) - \left(S_{t+1} + N_{t+1}^g\right) + (1-\delta)\left(S_t + N_t^g\right) \quad -G_t - \psi\left(N_{t+1}^g\right) - C_t^i, \tag{22}$$

where $B_t = \beta\pi(1-\pi) S_t / \left\{\theta\left[\frac{1-\theta}{(1-\phi)}\left[\frac{S_{t+1}}{(1-\delta)} - S_t\right] + \pi S_t\right]\right\}$. Therefore, one can rewrite the Ramsey problem as

Problem 2.

$$\max_{S_{t+1}, N_{t+1}^g} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c} \pi\log\left(\frac{C_t^i}{\pi}\right) \\ +(1-\pi)\log\left(\frac{C_t^s}{1-\pi}\right) + L\log\left((1-\alpha) A_t\left(\frac{S_t+N_t^g}{L}\right)^{\alpha}/L\right) \end{array}\right]\right\}$$

subject to (21) and (22).

We now have two instruments, which give rise to two first order necessary conditions (FONCs), one on private equity holding and another on government equity holdings:

$$\begin{aligned} [S_{t+1}] &: \left(\frac{\pi}{C_t^i}\frac{\partial C_t^i}{\partial S_{t+1}} + \beta E_t\frac{\pi}{C_{t+1}^i}\frac{\partial C_{t+1}^i}{\partial S_{t+1}}\right) + \left(\frac{(1-\pi)}{C_t^s}\frac{\partial C_t^s}{\partial S_{t+1}} + \beta E_t\frac{(1-\pi)}{C_{t+1}^s}\frac{\partial C_{t+1}^s}{\partial S_{t+1}}\right) + \beta\frac{\alpha L}{S_{t+1}} = 0 \\ \left[N_{t+1}^g\right] &: \left(\frac{\pi}{C_t^i}\frac{\partial C_t^i}{\partial N_{t+1}^g} + \beta E_t\frac{\pi}{C_{t+1}^i}\frac{\partial C_{t+1}^i}{\partial N_{t+1}^g}\right) + \left(\frac{(1-\pi)}{C_t^s}\frac{\partial C_t^s}{\partial N_{t+1}^g} + \beta E_t\frac{(1-\pi)}{C_{t+1}^s}\frac{\partial C_{t+1}^s}{\partial N_{t+1}^g}\right) + \beta\frac{\alpha L}{N_{t+1}^g} = 0 \end{aligned}$$

We derive the expression for each term in the FONC for S_{t+1}, while leaving the FONC for N_{t+1} since it is very similar and even simpler.

$$\frac{\pi}{C_t^i}\frac{\partial C_t^i}{\partial S_{t+1}} = -\frac{\pi^2}{(1-B_t)\,C_t^i} - \frac{\theta\,(1-\theta)\,B_t^2}{\beta\,(1-\pi)\,(1-\phi)\,(1-\delta)\,(1-B_t)\,S_t}$$
$$\frac{(1-\pi)}{C_t^s}\frac{\partial C_t^s}{\partial S_{t+1}} = \frac{\pi\,(1-\pi)}{(1-B_t)\,C_t^s} + \frac{(1-\pi)\,\theta\,(1-\theta)\,B_t^2}{\beta\pi\,(1-\phi)\,(1-\delta)\,(1-B_t)\,S_t}\frac{C_t^i}{C_t^s} - \frac{(1-\pi)}{C_t^s}.$$

$$\begin{aligned}
\beta E_t\frac{\pi}{C_{t+1}^i}\frac{\partial C_{t+1}^i}{\partial S_{t+1}} &= \beta E_t\{\frac{\pi^2\left[\alpha r_{t+1}+(1-\delta)-(1-\pi)\,(1-\beta)\,(1-\phi_t)\,(1-\delta)\,/\theta\right]}{C_{t+1}^i\,(1-B_{t+1})} \\
&+ \frac{\pi}{(1-B_{t+1})}\frac{B_{t+1}}{S_{t+1}}\left[1-\frac{B_{t+1}}{\beta\pi\,(1-\pi)}\left[\theta\pi-\frac{\theta\,(1-\theta)}{1-\phi_t}\right]\right]\} \\
\beta E_t\frac{(1-\pi)}{C_{t+1}^s}\frac{\partial C_{t+1}^s}{\partial S_{t+1}} &= \beta E_t\frac{(1-\pi)}{C_{t+1}^s}\left[\alpha r_{t+1}+(1-\delta)\right] \\
&- \beta E_t\{\frac{\pi\,(1-\pi)\left[\alpha r_{t+1}+(1-\delta)-(1-\pi)\,(1-\beta)\,(1-\phi_t)\,(1-\delta)\,/\theta\right]}{C_{t+1}^s\,(1-B_{t+1})} \\
&+ \frac{1-\pi}{(1-B_{t+1})}\frac{B_{t+1}}{S_{t+1}}\left[1-\frac{B_{t+1}}{\beta\pi\,(1-\pi)}\left[\theta\pi-\frac{\theta\,(1-\theta)}{1-\phi_t}\right]\right]\frac{C_{t+1}^i}{C_{t+1}^s}.
\end{aligned}$$

If the planner cannot purchase equity, the equivalent problem is by setting $N_{t+1} = 0$ and $S_{t+1} = K_{t+1}$ at all t. In computing the optimal policy, simply by replacing $S_{t+1} = K_{t+1}$ and ignoring the FONC for N_{t+1}

We do not derive the second order conditions for the Ramsey problems (one and two instruments), since the algebra becomes too tedious. Instead, we check all our calculations numerically by ensure that the FONCs give the welfare maximized solution.

B Prices and Policy Instruments

One can simply back out prices including asset price, return on liquid assets and policy instrument from quantity variables. Here, we just explain how prices can be backed out in the steady-state. The steady state version of the portfolio choice equation become

$$(1-\pi)\frac{(r+(1-\delta)\,q)\,/q-x}{(r+(1-\delta)\,q)\,N^s+M^p} = \pi\frac{x-\left[r+\phi\,(1-\delta)\,q+(1-\phi)\,(1-\delta)\,q^R\right]/q}{(r+\phi\,(1-\delta)\,q+(1-\phi)\,(1-\delta)\,q^R)\,N^s+M^p}$$

where $x = \frac{p_{t+1}}{p_t}$ measures the return on liquid assets and $M^p = \frac{C^i}{\pi(1-\beta)} - rK - \left[\frac{(1-d)\phi + d(1-\phi)}{\theta}\right](1-\delta)K$ measures the total value of liquid assets. Rearrange to express x as

$$\begin{aligned} & \left[\frac{1-\pi}{(r+(1-\delta)q)N^s + M^p} + \frac{\pi}{[r+\phi(1-\delta)q+(1-\phi)(1-\delta)q^R]N^s + M^p}\right]x \\ = & \frac{(1-\pi)(r+(1-\delta)q)/q}{(r+(1-\delta)q)N^s+M^p} + \frac{\pi\left[r+\phi(1-\delta)q+(1-\phi)(1-\delta)q^R\right]/q}{[r+\phi(1-\delta)q+(1-\phi)(1-\delta)q^R]N^s+M^p} \end{aligned}$$

where $N^s = \theta I + \phi\pi(1-\delta) + (1-\pi)(1-\delta)K$, and $q = (1-d)/\theta$.

In Kiyotaki and Moore (2011) (constant money supply) economy, the net rate of return on money is always zero in the steady state ($p_{t+1}/p_t - 1 = 0$) given that the money supply does not change.

C Details on the Estimation

We estimate the model using Bayesian methods. The purpose of the exercise is to obtain the distribution of the shock of ϕ and how it correlates with A shocks. In order to do so, we estimate the dynamic stochastic general equilibrium model with measurement errors.

As usual, to have identification, we consider the number of shock/measurement errors to be the same as the number of observed variables. We introduce 5 shocks in the estimation: resaleability, productivity shock (σ_z), resaleability and productivity correlation ($\sigma_{z,\phi}$), as well as measurement errors on the total liquid asset value ($\sigma_{ln(M^P)}$) and the expected interest rate on liquid assets ($\sigma_{ln(x)}$). We have already calibrated the productivity shock from previous literature, leaving 4 shocks to be estimated (with 2 measurement errors).

We consider deviations from the HP trend for aggregate investment. The other variables that we consider are related to portfolio rebalancing. The linkage of portfolio rebalancing and its impact on investment is novel and directly related to our question. For the total liquid asset value, as defined in Figure 1, we considered check, deposit, tradable receivable and T-Bills. Total assets value also come from the Flow of Funds table and is also defined in Figure 1. For the rate of return of liquid assets, we considered the 3-Month Treasury bill rate adjusted for expected inflation from the Michigan survey. The sample period used is from 1991 to 2007, in order to consider a stationary and stable period, that we can be sure to be dealing with "normal" times.

Following the literature on setting priors, we consider inverse gamma for standard error of the structural shocks to have a conjugate prior. Table V summarizes the prior and posterior information. We have tried many different priors and the posteriors are very robust. Interested readers can directly check the code available on the authors' website.

For the standard deviation of ϕ, we consider it small because ϕ itself is already small and we want to have a high probability of staying in the positive domain. The liquid asset expected returns are expected and subject to some errors, and the money value may have

Table V: **Prior and Posterior of the Parameters**

	Prior Distribution			Posterior Distribution			
Parameters	Distribution	Mean	Std	Mean	Mode	5%	95%
σ_ϕ	Inverse Gamma	0.01	1	0.0052	0.0051	0.0046	0.0058
$\sigma_{ln(x)}$	Inverse Gamma	0.01	1	0.0323	0.0314	0.0280	0.0362
$\sigma_{ln(M^P)}$	Inverse Gamma	0.01	1	0.0086	0.0046	0.0026	0.0158
$\sigma_{z,\phi}$	Inverse Gamma	0.3	1	0.4746	0.4950	0.3382	0.5765
ρ_ϕ	Beta	0.90	0.05	0.8929	0.8951	0.8626	0.9201

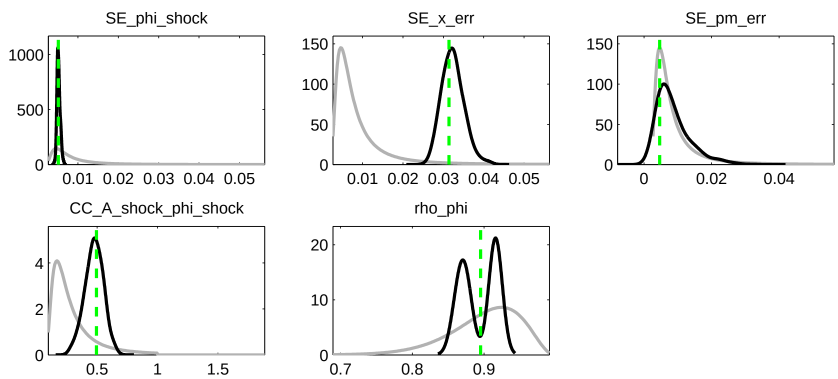

Figure 8: **Prior and Posterior Distributions** Prior distributions are shown in gray color, while posterior distributions are shown in black color. The header of each subplot stands for the parameters estimated. SE_phi_shock: σ_ϕ. SE_x_err: $\sigma_{ln(x)}$. SE_pm_err: $\sigma_{ln(M^P)}$. CC_A_shock_phi_shock: $\sigma_{z,\phi}$. rho_phi: ρ_ϕ

some accounting errors too. So we provide a rather flat prior with a small mean on the measurement errors. We leave the mean of the persistency of liquidity shocks ρ_ϕ, to be somewhat moderate number 0.9. Finally, we start with a somewhat large productivity-liquidity shocks correlation, since financial impact on TFP maybe large. The posterior mode of σ_ϕ, $\sigma_{z,\phi}$ and ρ_ϕ estimated are used in our numerical analysis. The posterior shape of the 3 parameters are very concentrated, given that we have a relatively flat prior.