Sectoral Shocks and the Beveridge Curve*

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Abstract

The slow recovery of the US labor market and the observed shift in the Beveridge curve has prompted speculation that sector-specific shocks may be responsible for the current recession. We document a significant correlation between shifts in the US Beveridge curve in postwar data and periods of elevated sectoral shocks, relying on a factor analysis of sectoral employment to derive our sectoral shock index. Our index is distinct from the celebrated Lilien (1982) measure of employment dispersion, uncorrelated with various measures of the business cycle, and elevated in the current recession. We show that sector-specific shocks in a multisector model augmented with labor market search can generate shifts in the Beveridge curve. We calibrate a two-sector version of our model using JOLTs data on goods-producing and service-providing sectors and show that sector-specific productivity shocks calibrated to match employment shares can fully account for the upward shift in the Beveridge curve. Consistent with empirical evidence, our model also generates cyclical movements in aggregate matching function efficiency and mismatch across sectors. We augment our standard multisector model with financial frictions to demonstrate that the financial shocks act like sectoral productivity shocks and provide an alternative calibration where a rise in firm borrowing costs generates a shift in the Beveridge curve.

Keywords: sectoral shocks, Beveridge curve, labor reallocation.

JEL Classification: E24

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1 Introduction

You can’t change the carpenter into a nurse easily, and you can’t change the mortgage broker into a computer expert in a manufacturing plant very easily. Eventually that stuff will work itself out . . . Monetary policy can’t retrain people. Monetary policy can’t fix those problems.

Charles Plosser, President of the Federal Reserve Bank of Philadelphia

Even though the Great Recession ended in the middle of 2009, the US labor market has yet to experience a sustained recovery with the unemployment rate remaining above 8%. The weak recovery in the US labor market, despite the depth of the recession, has led some to speculate that a slow recovery is inevitable as the labor force must be reallocated from housing-related sectors to the rest of the economy. Proponents of this view point to the protracted weakness in the housing market and cite the shift in the US Beveridge curve as evidence for sector-specific shocks. Figure 1 displays the US Beveridge curve since 2000 using vacancy data from the Job Openings and Labor Turnover Survey (JOLTs). As the figure shows, vacancy rates have risen without a commensurate fall in the unemployment rate. The vacancy rate prevailing in 2010 and 2011 was consistent with an unemployment rate of less than 6% before the recession.

Figure 1: US Beveridge curve, 2000-2012

Several papers examining the recent shift in the Beveridge curve have highlighted an important sectoral component. Using JOLTs data on vacancies and hires, Barnichon et al. (2010) decompose

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1See Kocherlakota (2010), and Plosser (2011).
the shift in the Beveridge curve into labor force components and sectoral components. Their analysis find an unexpected decline in hires per vacancy that is particularly pronounced in housing-related sectors such as construction, retail trade, and leisure and hospitality. Similarly, Davis et al. (2012) examine the contribution of various sectors to movements in the aggregate job-filling rate and a measure of recruiting intensity (the component of the job-filling rate no explained by changes in tightness). The authors show that housing-related sectors like construction and leisure and hospitality have disproportionately large effects (relative to their employment shares) on the aggregate movements of these variables.

The sectoral shock view of the Great Recession carries important implications for monetary policy. Statements from Kocherlakota (2010) and Plosser (2011) suggest that, if sectoral shocks require labor reallocation and that process is costly and prolonged, then monetary policy may be unable to further reduce the unemployment rate. Indeed, further monetary easing may be counterproductive by delaying needed adjustment in the labor market. Therefore, questions on the role of sector-specific shocks and its implications for both the unemployment rate and level of output are critical in assessing the proper course for monetary policy.

In this paper, we examine the relationship between sector-specific shocks and movements in the Beveridge curve. We show that shifts in the US Beveridge curve in postwar data are correlated with the dispersion of sector-specific shocks, where the latter is measured by a sector-specific shock index obtained by a factor analysis of sectoral employment. Our sector-specific shock index takes the sum of the absolute values of sector-specific shocks obtained from a factor analysis of sectoral employment in the US going back to 1950. Our measure is distinct from the Lilien (1982) measure of employment dispersion and addresses the Abraham and Katz (1986) critique that asymmetric sectoral responses may be due to differing sectoral elasticities to aggregate shocks. Moreover, the sector-specific shock index is elevated in the recovery period of the current recession supporting the view that sector-specific shocks are an important feature of the macro environment in recent years.

We augment a multisector model with labor market search to examine whether sector-specific shocks can account for shifts in the Beveridge curve. Only under a fairly restrictive set of conditions will sectoral shocks leave the aggregate Beveridge curve unchanged. Using estimates of job-filling rates and separation rates from Davis et al. (2010) - hereafter DFH - we calibrate a simple two-sector version of the model and show that a negative productivity shock to the goods-producing sector can fully account for the observed shift in the Beveridge curve in the current recession. The magnitude of the productivity shock to the goods-producing sector is calibrated to match employment shares
of each sector at the recession trough. Our results also show that the link between Beveridge curve shifts and the level of economic activity - the level of output or the unemployment rate - are at best tenuous. A change in the former need not have important implications for the latter. Moreover, our two-sector model generates cyclical movements in aggregate matching function efficiency and labor market mismatch consistent with recent empirical evidence and in contrast to standard one-sector search models.

To better relate the sectoral productivity shocks and matching function differences across sectors that drive our results in the two-sector example, we extend our model to include financial constraints. Our extension demonstrates how aggregate demand shocks, such as a shock that increases households saving, trace out the same aggregate Beveridge curve in a multisector model as aggregate TFP shocks. Likewise, we show how an increase in borrowing rates or a fall in collateral values are equivalent to sector-specific productivity shocks and therefore can account for the shift in the Beveridge curve when a subset of firms is credit constrained. We calibrate our model based on data on small firms versus large firms, where size is used as a proxy for financial constraints, and smaller firms have lower search costs due to a higher job-filling rate. Under our calibration, a shock to borrowing costs generates a shift in the Beveridge curve of comparable magnitude to the shift observed in the data.

Our work is related to literature examining the role of sector-specific shocks for aggregate fluctuations, but differs in its primary focus on accounting for shifts in the Beveridge curve. Sahin et al. (2010) construct a measure of labor market mismatch and argue that mismatch can only account for a relatively small fraction of the rise in unemployment, but do not focus on whether sectoral shocks can account for the shift in the Beveridge curve. Our model shows that time-variation in mismatch can be present even in the absence of sector-specific shocks. Garin et al. (2010) argue that labor reallocation may play a significant role in recent recessions which have been accompanied by jobless recoveries. Among other indicators, the authors demonstrate that the cyclicality and lag structure of labor productivity has changed in recent recessions, and their model includes multiple sectors but does not include labor market search. Using sectoral and geographic dispersion in employment growth and unemployment rates, Valletta and Kuang (2010) argue that structural unemployment is unlikely to be worse in this recession relative to previous recessions and argue that substantial shifts in the Beveridge curve in the past have not been accompanied by large changes in the natural rate of unemployment. Our results support this view that a shift in the Beveridge curve need not imply a rise in structural unemployment.
Our paper is organized as follows. Section 2 describes our method for constructing a long-run sector-specific shock index and its correlation with historic shifts in the Beveridge curve. Section 3 lays out a multisector model augmented with labor market search and summarizes conditions under which sectoral shocks do not shift the aggregate Beveridge curve. Analytical results are described in Section 4 while Section 5 describes our calibration strategy and shows the effect of sectoral productivity shocks in a two-sector model. Section 6 extends the multisector model to incorporate financial frictions and illustrate how financial frictions can account for both movements along the Beveridge curve and shifts in the Beveridge curve. Section 7 concludes.

2 Empirical Evidence on Sectoral Shocks and the Beveridge Curve

As observed in Valletta and Kuang (2010) and Bleakley and Fuhrer (1997), the US Beveridge curve has shifted several times in the postwar era. Since vacancies data from the JOLTs survey is only available after 2000, the Conference Board’s Help-Wanted Index is frequently used as a proxy for the vacancy rate prior to 2000. Figure 2 displays the unfiltered Beveridge curve using the Help-Wanted Index (HWI) normalized by the labor force as a proxy for the vacancy rate. After 1996, the HWI is the composite index derived in Barnichon (2010) and updated to 2011, which adjusts for the shift away from newspaper advertising of vacancies to online advertising. As Figure 2 shows, the historic Beveridge curve is also characterized by periods when the vacancy-unemployment relationship is stable and periods when the curve appears to shift.

Historic shifts in the US Beveridge curve are documented in Bleakley and Fuhrer (1997) and similar periods are highlighted in the figure. Importantly, shifts in the Beveridge curve are not a
business cycle phenomenon with some recessions accompanied by shifts but other shifts occurring during expansions - the behavior of vacancies and unemployment in the mid 1980s provides a good example. Like the Beveridge curve obtained using JOLTs data, the composite HWI Beveridge curve exhibits an upward shift since 2009. In this section, we examine whether historic shifts in the Beveridge curve can be related to sector-specific shocks.

2.1 Existing Measures of Sector-Specific Shocks

The Lilien (1982) measure of dispersion in sectoral employment growth was one of the first measures proposed for sector-specific shocks. Lilien argued that sector-specific shocks could be measured by changes in the dispersion of employment growth across sectors, and argued that sector-specific shocks were an important driver of the business cycle given the strong countercyclical behavior of his measure. Figure 3 plots the Lilien measure using monthly sectoral employment data. The figure demonstrates the strongly countercyclical behavior of the series including in the most recent recessions that have featured a slower recovery in the labor market in comparison to past recessions. In the current recession, the Lilien measure peaks in the summer of 2009 at the recession trough.

Abraham and Katz (1986) questioned whether the Lilien measure was an accurate measure of sector-specific shocks by arguing that increases in the dispersion of employment growth could be attributed to differences in the elasticity of sectoral employment to aggregate shocks. As an alternative, Abraham and Katz argued that sector-specific shocks should result in periods in which vacancies and unemployment are both rising and showed that the Lilien measure does not comove with vacancies. As we argue, our sector-specific shock index addresses the Abraham and Katz critique by allowing loadings on an aggregate factor to differ across sectors. Moreover, by constructing an independent measure of sectoral shocks, we do not assume that Beveridge curve shifts are necessarily indicative of sectoral shocks.

The contribution of sector-specific shocks in aggregate fluctuations has been studied extensively with various factor analyses of prices, industrial production, and employment. Most of these studies attempt to measures the relative contribution of aggregate versus idiosyncratic shocks in highly disaggregated subsector or industry data. Foerster et al. (2011), using quarterly data on industrial production, conduct a structural factor analysis and find that the contribution of idiosyncratic shocks has increased considerably during the Great Moderation relative to earlier periods. Boivin et al. (2009) document that much of the variability of sectoral prices is due to sector-specific shocks, and sectoral prices adjust quickly to these shocks. Malysheva and Sarte (2009) conduct a
similar variance decomposition for subsector and industry employment in the US using data since 1990. They show that common factors explain relatively little of the variation in employment at the disaggregated level but document a high degree of heterogeneity across industries in their loading on common factors. Together, these studies have mixed implications for the role for sectoral shocks in explaining variation at the disaggregated level. While we also conduct a factor analysis of employment data, we differ from these studies by focusing on recovering a long-run series to compare to historic movements in the Beveridge curve, and by constructing a shock index that measures periods in which sectoral shocks are particularly elevated rather than examining the typical contribution of shocks to employment fluctuations.

Somewhat separately, several recent papers have focused on empirical measures mismatch in the labor market or other indicators of labor market reallocation rather than sector-specific shocks. Garin et al. (2010) argue that the last three recessions which feature weak labor market recoveries may be due to costly reallocation. The authors cite the changing properties of labor productivity over the business cycle and changes in the rate at which sectoral employment shares change over the business cycle. Sahin et al. (2010) do not measure sector-specific shocks in the labor market but define a measure of labor market mismatch that measures the degree to which labor market tightness is not equalized across sectors. The authors use vacancy and unemployment data to argue that this measure of mismatch suggests that only a relatively small fraction of the rise in US unemployment can be attributed to labor market mismatch. Our sector-specific shock index is distinct from these papers by remaining agnostic about the importance of reallocation. As we show in our model, sector-specific shocks may shift the Beveridge curve even if labor can be costlessly
reallocated across sectors; a Beveridge curve shift need not imply any mismatch in the labor market.

2.2 Constructing Sector-Specific Shock Index

To derive a long-run measure of sector-specific shocks, we conduct a factor analysis of sectoral employment. By allowing sectoral loadings to differ across sectors, our factor analysis addresses the Abraham and Katz critique. The factor analysis implicitly identifies the sector-specific shock by assuming that loadings on the aggregate factor are invariant over time; that is, sectoral employment responds in a similar manner over the business cycle to aggregate fluctuations. While this assumption is one we would like to relax in future work, the loadings obtained from the factor analysis and listed in Table 1 are reasonable in terms of its ordering of sectors by sensitivity.

To obtain this series, we use long-run US data on sectoral employment. This data is available for the US from January 1950 to February 2012 on a monthly basis for 14 sectors that represent the first level of disaggregation for US employment data. Due to its relatively small share of employment, we drop the mining and natural resources sector. The sectoral data is taken from Bureau of Labor Statistics establishment survey, and the remaining sectors are listed in Table 1. While, in principle, we could use sectoral data on variables like real output, relative prices, or relative wages, employment data offers the longest available history at the highest frequency and is presumably measured with the least error. The principal concern with this data set is the small number of cross-sectional observation relative to the number of observations in the time dimension. While traditional factor analyses draw on highly disaggregated price, output, or employment data, these series are not available before the 1970s. Given our aim of investigating shifts in the Beveridge curve and the relative infrequency of these events, we try to construct the longest possible series for sector-specific shocks.

The log of monthly sectoral employment is detrended to obtain a mean-zero stationary series and the variance of each series is normalized to unity. A single business cycle factor is extracted from the stationary sectoral employment series by means of principal components:

\[ n_{it} = \epsilon_{it} + \lambda_i F_t \text{ for } \forall i \epsilon \{1, \ldots, K\} \]

where \( \epsilon_{it} \) is the mean zero sector-specific shock. While the paucity of cross-sectional data over a long-term does not permit us to choose a higher number of factors or assign standard errors to the factor loadings, principal components remains a valid data summary technique. In future work,
we plan to estimate a Bayesian factor model in order to verify our results and obtain confidence intervals for the factor loadings. Greater structure on the empirical model may also address issues of common trends across sectoral employment, though, it is worth noting that these series exhibit big differences in trends.

The sectoral residual $\epsilon_{it}$ represents the sector-specific shock and we construct a sector-specific shock index to observe the variation over time in sector-specific shocks as the sum of the absolute values of $\epsilon_{it}$ for all sectors:

$$S_t = \sum_{i=1}^{K} |\epsilon_{it}|$$

This measure of sector-specific shocks is always positive and weights all sectors equally. Given that variances are normalized to unity before estimating, the sector specific shocks need not be weighted by their employment shares.

We detrend employment in each sector by means of a cubic deterministic trend. The underlying trend in sectoral employment differs substantially among sectors, and employment shares are nonstationary over the postwar period. For example, manufacturing employment falls as a share of total employment over the whole period, but even falls in absolute terms starting in the 1980s. Sectors, such as construction and information services show a general upward trend characterized by very large and long swings in employment that are longer than simple business cycle variation. In particular, construction is characterized by 8-10 year swings - variation at a lower frequency than typical business cycle frequencies. Higher-order deterministic trends fit certain sectors much better than a simple linear or quadratic trend. Moreover, most of the sectoral employment series obtained by removing a linear or quadratic trend fail a Dickey-Fuller test at standard confidence levels.

### 2.3 Sectoral Shock Index and the Beveridge Curve

Figure 4 displays the sector-specific shock index and Table 1 shows the properties of our factor analysis of sectoral employment.

The sector-specific shock index shows a high degree of persistence and has risen rapidly beginning in 2009. Importantly, the rise in the shock index occurs at the beginning of the recovery, not at the beginning of the recession. Thus, the index is rising while the Lilien measure is falling, and its rise matches the timing of the shift in the Beveridge curve. Also, the sector-specific shock index is not a business cycle measure. Its correlation with various monthly measures of the business
cycle is given in Table 2 and demonstrates this low correlation. Given that the sector-specific shock index is not a business cycle measure, it also exhibits a low correlation with the Lilien measure. For comparison, we also show the business cycle correlations of a sector-specific shock index obtained from a quartic detrending of sectoral employment.

Table 1: Factor analysis of sectoral employment

<table>
<thead>
<tr>
<th>Sector</th>
<th>Loading $\lambda$</th>
<th>Variance SS Shock</th>
<th>Sector</th>
<th>Loading $\lambda$</th>
<th>Variance SS Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>0.31</td>
<td>37.6%</td>
<td>Transportation and Utilities</td>
<td>0.35</td>
<td>20.1%</td>
</tr>
<tr>
<td>Durables Manufacturing</td>
<td>0.36</td>
<td>16.8%</td>
<td>Financial Services</td>
<td>0.18</td>
<td>78.6%</td>
</tr>
<tr>
<td>Nondurables Manufacturing</td>
<td>0.24</td>
<td>62.9%</td>
<td>Leisure and Hospitality</td>
<td>0.34</td>
<td>24.9%</td>
</tr>
<tr>
<td>Education &amp; Health</td>
<td>0.10</td>
<td>93.6%</td>
<td>Information Services</td>
<td>0.29</td>
<td>43.6%</td>
</tr>
<tr>
<td>Professional &amp; Business Services</td>
<td>0.32</td>
<td>33.2%</td>
<td>Government</td>
<td>0.15</td>
<td>83.6%</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>0.28</td>
<td>49.1%</td>
<td>Other Services</td>
<td>0.19</td>
<td>76.2%</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>0.31</td>
<td>37.4%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The level of our sector-specific shock index also rises in the second half of the sample coinciding with the Great Moderation period. This behavior is consistent with the behavior of sectoral employment documented in Garin et al. (2010). Likewise, the rise in the index in the recovery period after the Great Recession is also consistent with the elevated dispersion in labor market conditions highlighted by Barnichon and Figura (2011).

Just as the current shift in the Beveridge curve coincides remarkably with a rise in the sector-specific shock index, historic shifts in the Beveridge curve are also correlated with elevated levels of sector-specific shocks. We illustrate this correlation between shifts in the Beveridge curve and the sector-specific shock index by plotting the shock index against the intercept of a 5 year rolling regression of unemployment on vacancies. Absent any shifts in the Beveridge curve, the intercept
Table 2: Correlation of shock index with business cycle measure

<table>
<thead>
<tr>
<th>Correlation (1/51 - 2/12)</th>
<th>Ind. Prod. (yoy)</th>
<th>Emp. Growth (yoy)</th>
<th>Unemployment Rate</th>
<th>Lilien Measure</th>
<th>Beveridge Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS Shock Index (Cubic Detrend)</td>
<td>-0.026</td>
<td>0.106</td>
<td>0.044</td>
<td>-0.156</td>
<td><strong>0.413</strong></td>
</tr>
<tr>
<td>SS Shock Index (Quartic Detrend)</td>
<td>-0.034</td>
<td>0.075</td>
<td>0.036</td>
<td>-0.053</td>
<td><strong>0.203</strong></td>
</tr>
</tbody>
</table>

should be constant. Therefore, variation in the intercept series captures movements in the Beveridge curve. Figure 5 shows a clear correlation between movements in the intercept of the Beveridge curve and the sector-specific shock index. This correlation in monthly data calculated from 1956-2012 is 0.413 and is shown in the last column in Table 2. This result is robust to the use of a 4th order trend, though somewhat weaker. Our evidence is consistent with the mechanism described by Abraham and Katz confirming their intuition that sector-specific shocks can shift the Beveridge curve.

Figure 5: Correlation of Beveridge curve shifts and sector-specific shocks

3 Model

Our model incorporates labor market search into a standard multisector model as described in Aoki (2001) or Carvalho and Lee (2011). The chief complication in the model is the treatment of labor reallocation, where we consider two extreme cases: no reallocation or costless reallocation. In the former, the labor force has some initial distribution across sectors that cannot be altered and firms hire from these pools subject to a sector-specific matching function. In the latter, the workers in the representative household are free to search in any sector implying that the household surplus to searching in any given sector will be equalized across all sectors. This condition is analogous
to equating the marginal product of labor across sectors in a multisector model with labor on the intensive margin and similar to the generalized Jackman-Roper condition derived in Sahin et al. (2010).

3.1 Final Goods Firms

The consumption good is sold by final good firms who purchase the intermediate output good produced by sectoral firms. We assume a finite set of sectors that produce an intermediate good that is transformed into the final good using a constant elasticity of substitution aggregator.

\[
\Pi_f^t = \max P_t Y_t - \sum_{i=1}^{K} P_{it} Y_{it} \\
s\text{subject to } Y_t = \left( \sum_{i=1}^{K} \phi_{it} Y_{it}^{-\eta} \right)^{-\frac{1}{\eta-1}}
\]

where \( \phi_{it} \) represents a relative demand shock. Optimization by final good firms provides demand functions for each intermediate good and an aggregate price index:

\[
Y_{it} = \phi_{it} Y_t \left( \frac{P_{it}}{P_t} \right)^{-\eta} \\
P_t = \left\{ \sum_{i=1}^{K} \phi_{it} P_{it}^{1-\eta} \right\}^{\frac{1}{1-\eta}}
\]

The price of the final good is normalized to unity, and \( P_{it}/P_t \) represents the relative price of each intermediate good. For \( \eta = 1 \), the CES aggregator is Cobb-Douglas and intermediate goods are neither complements nor substitutes. If \( \eta < 1 \), intermediate goods are complements while if \( \eta > 1 \), intermediate goods are substitutes.

3.2 Intermediate Good Firms

Intermediate goods are produced by competitive firms in each sector who hire labor and post vacancies subject to a linear production function and a law of motion for firm employment. Firms in each sector face sectoral productivity shocks with wages, vacancy posting costs, and a probability
of hiring that may be unique to the sector. The firm’s problem is given below:

\[ \lambda_{it} = \max_{E_t} \sum_{T=0}^{\infty} Q_{t,T} \left( \left( \frac{P_{IT}}{P_t} \right) Y_{tT} - W_{iT} N_{iT} - \kappa_i V_{iT} \right) \] (5)

subject to \( N_{it} = (1 - \delta_i) N_{it-1} + q_{it} V_{it} \)  (6)

\[ Y_{it} = A_t A_{it} N_{it} \] (7)

where \( q_{it} \) is the vacancy yield or job-filling rate. The firm’s vacancy posting condition is given below:

\[ P_{it} A_{it} = W_{it} + \kappa_i \frac{q_{it}}{q_{it+1}} - E_t Q_{t,t+1} (1 - \delta_i) \frac{\kappa_i}{q_{it+1}} \] (8)

where \( Q_{t,t+1} \) is the stochastic discount factor of the representative household between period \( t \) and \( t + 1 \).

Hiring is mediated by a sectoral matching function that depends on the level of vacancies and unemployment in each sector. We allow sectoral matching functions to differ in matching function productivity and elasticity to vacancies, but require the matching function to display constant returns to scale:

\[ q_{it} = H_{it} V_{it} = \vartheta_i \left( \frac{V_{it}}{U_{it}} \right)^{-\alpha_i} \] (9)

### 3.3 Households

Household supply labor across \( K \) distinct sectors and invest in a full-set of state-contingent securities. While hiring in each sector is subject to search frictions, we consider two extreme cases for the household’s ability to allocate labor across sectors. In the case of no reallocation, the household’s available pool of labor in each sector is exogenously given while, alternatively, in the case of costless reallocation, the household is free to have unemployed workers search in any sector.

In the case of no reallocation, the household has no active labor supply or labor allocation decision, but the value function is needed to determine the household surplus from having an additional worker employed. This household surplus determines the Nash-bargained wage:

\[ V (N_{it-1}, \ldots, N_{Kt-1}) = \max \{ u (C_t, N_t) + \beta E_t V (N_{it}, \ldots, N_{Kt}) \} \] (10)

subject to \( C_t = \sum_{i=1}^{K} (W_{it} N_{it} + \Pi_{it}) + B_t - E_t Q_{t,t+1} B_{t+1} \)  (11)

\[ N_{it} = (1 - \delta_i) N_{it-1} + p_{it} U_{it} \forall i \in \{1, \ldots, K\} \] (12)

\[ L_i = N_{it-1} + U_{it} \forall i \in \{1, \ldots, K\} \] (13)
where the labor force is normalized to unity, \( L_i \) is the distribution of labor across sectors (i.e. \( \sum_{i=1}^{K} L_i = 1 \)) and \( \delta_i \) is the separation rate in each sector. As in Monacelli et al. (2010), the pool of available workers is simply those workers not employed in the previous period. The household takes the job-finding rate in each sector \( p_{it} \) as exogenous along with profits from intermediate goods firms \( \Pi_{it} \). The household surplus from an additional worker employed in sector \( i \) is given by the Lagrange multiplier on the law of motion for sectoral employment converted into consumption units by dividing by the household marginal utility (the multiplier on the budget constraint):

\[
J_{it} = W_{it} - U_{nt} + E_t Q_{t,t+1} (1 - \delta_i - p_{it+1}) J_{it+1}
\]

with \( U_{nt} = -\frac{u_n(C_t, N_t)}{u_c(C_t, N_t)} \)

and \( Q_{t,t+1} = \beta \frac{u_c(C_{t+1}, N_{t+1})}{u_c(C_t, N_t)} \)

where \( N_t = \sum_{i=1}^{K} N_{it} \).

In the case of costless reallocation, the household has an active margin of adjustment by reallocating the pool of available workers across sectors irrespective of the initial distribution. The household’s value function is needed again to determine the household surplus for the Nash-bargained wage:

\[
V(N_{1t-1}, \ldots, N_{Kt-1}) = \max \{ u(C_t, N_t) + \beta E_t V(N_{1t}, \ldots, N_{Kt}) \}
\]

subject to \( C_t = \sum_{i=1}^{K} (W_{it} N_{it} + \Pi_{it}) + B_t - E_t Q_{t,t+1} B_{t+1} \)

\( N_{it} = (1 - \delta_i) N_{it-1} + p_{it} U_{it} \forall i \in \{1, \ldots, K\} \)

\( U_t = \sum_{i=1}^{K} U_{it} \)

\( 1 = N_{t-1} + U_t \)

The household chooses the quantity \( U_{it} \) subject to the constraint that labor allocated to each sector not exceed the pool of available workers. As before, the household surplus from an additional worker employed in sector \( i \) is the same as in the case of no reallocation but the capacity of the household to reallocate labor across sectors ensures that the household’s expected surplus from hiring is constant:

\[
p_{it} J_{it} = p_{ht} J_{ht} \text{ for } \forall i, h \in \{1, \ldots, K\}
\]
In other words, the household allocates labor such that the marginal value of an additional worker in a given sector weighted by the job-finding probability is equalized. The job-finding probability is taken as exogenous by the household and is determined in equilibrium by the sectoral matching function and the level of vacancies and unemployed persons in each sector:

\[
p_{it} = \frac{H_{it}}{U_{it}} = \vartheta_i \left( \frac{V_{it}}{U_{it}} \right)^{1-\alpha_i}
\]

Wages are determined via Nash bargaining in each sector. The firm’s surplus is equal to the cost of hiring a new worker, which is the cost of posting a vacancy scaled by the probability of filling the vacancy:

\[
J_{it}^f = \frac{\kappa}{q_{it}}
\]

Nash-bargaining implies that the sectoral wage satisfies the following condition:

\[
\nu J_{it}^f = (1 - \nu)J_{it} \Rightarrow \frac{\nu \kappa}{1 - \nu q_{it}} = J_{it}
\]

Substituting into the dynamic equation for the household surplus, we can solve for the sectoral wage to eliminate the household surplus as an additional variable:

\[
W_{it} = U_{nt} + \frac{\nu}{1 - \nu} \kappa \left( \frac{1}{q_{it}} - E_t Q_{t,t+1} (1 - \delta_i - p_{it+1}) \frac{1}{q_{it+1}} \right)
\]

Under the assumption of costless reallocation, the restriction that the household surpluses, weighted by the job-finding rate, are equalized across sectors implies the Jackman-Roper condition that labor market tightness must be equalized across sectors. To see this, observe that for any two sectors, household optimality and Nash-bargaining imply:

\[
\kappa \frac{\nu p_{jt}}{1 - \nu q_{jt}} = \frac{\nu}{1 - \nu} \kappa \frac{V_{jt}}{U_{jt}} = \frac{\nu}{1 - \nu} \kappa \frac{V_{ht}}{U_{ht}}
\]

This result requires bargaining power and flow vacancy costs to be equalized across sectors but places no restriction on the parameters of the matching function (though the assumption of constant
returns is important). More generally, if bargaining power or vacancy costs differ across sections, a generalized Jackman-Roper condition will obtain where sectoral tightness will be equalized up to a wedge term reflecting differences in bargaining power and vacancy costs. This condition is analogous to the generalized Jackman-Roper condition derived in Sahin et al. (2010). When reallocation is no longer costless, the household surplus weighted by the job-finding rate will generally fail to be equalized across sectors and the household will have an incentive to transfer workers to sectors with a higher surplus or a greater job-finding rate. As a result, labor market tightness across sectors will generally fail to be equalized across sectors.

3.4 Equilibrium

Market-clearing in the labor and in the asset market implies a standard resource constraint augmented with the real costs of posting vacancies for all sectors:

\[ Y_t = C_t + \sum_{i=1}^{K} \kappa V_{it} \]  
\[ N_t = \sum_{i=1}^{K} N_{it} \]  

We define an equilibrium for the economy with costly reallocation and costless reallocation respectively.

**Definition 1.** An equilibrium for the costly reallocation economy is a set of aggregate allocations \( \{Y_t, N_t, C_t\} \), sectoral allocations \( \{Y_{it}, N_{it}, U_{it}, V_{it}\}_{i=1}^{K} \) , a set of prices \( \{W_{it}, P_{it}\}_{i=1}^{K} \) , a set of job-finding and job-filling rates \( \{p_{it}, q_{it}\}_{i=1}^{K} \) , and initial values of sectoral employment and unemployment \( \{N_{i,-1}, U_{i,-1}\}_{i=1}^{K} \) that jointly satisfy:

1. Final good firm production and demand functions (2) - (3) for K sectors
2. Intermediate good production and vacancy posting condition (7) - (8) for K sectors
3. Job-filling and job-finding rates (9) and (20) for K sectors
4. Labor market flows (12) and (13) for K sectors
5. Wage equation (21) for K sectors
6. Market-clearing condition (22) - (23)
Equilibrium in the costless reallocation economy is similar to that of the costly reallocation case with an additional $K$ variables for allocating the pool of available labor:

**Definition 2.** An equilibrium for the costly reallocation economy is a set of aggregate allocations \( \{Y_t, N_t, U_t, C_t\} \), sectoral allocations \( \{Y_{it}, N_{it}, U_{it}, V_{it}\}_{i=1}^K \), a set of prices for \( \{W_{it}, P_{it}\}_{i=1}^K \), a set of job-finding and job-filling rates \( \{p_{it}, q_{it}\}_{i=1}^K \), and initial values of sectoral employment \( \{N_{i,1}\}_{i=1}^K \) and unemployment \( U_{-1} \) that jointly satisfy:

1. Final good firm production and demand functions (2) - (3) for $K$ sectors
2. Intermediate good production and vacancy posting condition (7) - (8) for $K$ sectors
3. Job-filling and job-finding rates (9) and (20) for $K$ sectors
4. Labor market flows (16) for $K$ sectors
5. Aggregate unemployment (17) - (18)
6. Jackman-Roper conditions (19) for $K-1$ sectors
7. Wage equation (21) for $K$ sectors
8. Market-clearing condition (22) - (23)

4 Analytical Results

Though the model we have described is a fully dynamic model, we limit our analytical and numerical analysis to the steady state version of the model as the most natural setting to think about the Beveridge curve. Given the size of flows in the labor market and high rates of separation and hiring on a monthly basis, the labor market quickly converges to a steady state where hiring equals separation. In a single-sector model (i.e. $K = 1$), the steady state Beveridge curve is the relationship between unemployment and vacancies given by the steady state of the labor market flow equation (14):

\[
\delta(1 - U) = \vartheta U^\alpha V^{1-\alpha}
\]

Only changes in the separation rate $\delta$ and the matching function productivity $\vartheta$ shift the Beveridge curve, while other shocks like aggregate productivity simply move unemployment and vacancies along the locus of points traced out by this equation. In a multi-sector model, such an analytical
relationship between $U$ and $V$ does not exist, and the aggregate steady state Beveridge curve is an equilibrium object that we must characterize either analytically or numerically.

A solution for the steady state of the multisector model will express the endogenous variables in terms of aggregate productivity and sector-specific demand and productivity terms. As we have argued, the Beveridge curve is typically treated as a steady state object: the relationship between unemployment and vacancies when flows into and out of unemployment are equalized. However, if the labor market quickly converges to a new steady state based on the underlying state of the demand and productivity shocks that drive the model, then aggregates like output and the unemployment rate will also quickly converge to their steady state levels and the effect of persistent sector-specific shocks will be well-approximated by a comparison of steady states.

4.1 Characterization and Discussion of Static Model

Given that our model has a finite number of sectors with nonzero mass, the distinction between aggregate and sector-specific shocks is not entirely obvious. For any given initial steady state, a log-linear approximation of the CES aggregator and sectoral production functions shows that any sector-specific shock has two components: a direct effect on output by raising, for example, the productivity of sector $i$, and an indirect effect from the induced change in equilibrium labor supplied to sector $i$ and every other sector.

$$y_t = \sum_{i=1}^{K} \gamma_i \left( \frac{1}{\eta-1} \phi_{it} + a_t + a_{it} + n_{it} \right)$$

with $\gamma_i = \frac{-1/\eta}{\phi_i} \left( \frac{\bar{Y}_i}{\bar{Y}} \right)^{\frac{\eta-1}{\eta}}$

and $1 = \sum_{i=1}^{K} \gamma_i$

This expression suggest a simple distinction between aggregate and sector-specific shocks:

**Definition 3.** Sector-specific productivity shocks and sector-specific demand shocks are any linear combination that satisfy the following conditions:

$$\sum_{i=1}^{K} \gamma_i a_{it} = 0$$

$$\sum_{i=1}^{K} \gamma_i \phi_{it} = 0$$
where \( \gamma_i \) is the steady-state real share of each input in aggregate output.

This definition of sector-specific shocks ensures that total output is left unchanged if aggregate productivity and sectoral employment are unchanged; sectoral shocks only affect aggregate output via their endogenous effect on sectoral employment. Using this definition, we can solve the steady-state version of the model and derive expressions for aggregate output and sectoral employment. While we keep the time subscripts in these expressions, these approximations ignore the expectations effects that would be present in the fully dynamic model and assume that sector-specific shocks are sufficiently persistent for the economy to converge to a new steady state.

Solving for sectoral employment under no reallocation \( n_{it}^{nr} \) and costless reallocation \( n_{it}^{r} \) respectively and assuming that disutility of labor supply is constant (as in the canonical search models such as Shimer (2005)), we have:

\[
\begin{align*}
n_{it}^{nr} &= \lambda_i ( (\phi_{it} - (1 - \eta) a_{it}) + y_t - (1 - \eta) a_t ) \\
n_{it}^{r} &= (\phi_{it} - (1 - \eta) a_{it}) + y_t - (1 - \eta) a_t - \eta (s_i \varphi_i + (1 - s_i) \alpha_i) \theta_t
\end{align*}
\]

where \( \lambda_i = \frac{1}{1 + \eta (s_i \varphi_i + (1 - s_i) \alpha_i) \frac{\bar{L}_i/\bar{U}_i}{1 - \alpha_i}} \)

where \( \varphi_i \) is a macro Frisch elasticity that reflects the dependence of the Nash-bargained sectoral wages on labor market tightness and \( s_i \) is the steady state shares of wages in marginal product\(^2\). This parameter is a function of steady-state job-finding rates and vacancy-filling rates along with parameters of the model, including the sectoral separation rate, etc. These expressions for sectoral employment are not materially changed by allowing for wealth effects or convex disutility of labor supply, which would simply add linear functions of \( y_t \) and \( n_t \) to each expression.

As the expressions show, the sector-specific shock enters as a composite shock. In the case of costless reallocation, sectoral employment is a function of the composite sector-specific shock and three aggregate variables (output and tightness and productivity). In the case of no reallocation, aggregate labor market tightness no longer appears. In both cases, it is easy to see that differences in the behavior of sectoral employment across sectors can be driven both by the Lilien channel (i.e. sector-specific shocks) and the Abraham and Katz channel (i.e. differential loads on aggregate variables). Moreover, in the case of no reallocation, since there is little reason a priori in assuming \( \lambda_i \) is equated across sectors, sector-specific shocks will not integrate out when this expression is substituted into the log-linearized CES aggregator.

\(^2\)Specifically, \( s_i = \frac{W_i}{P_i A A_i} \)
By substituting the expressions for sectoral employment into the CES aggregator and the labor market clearing conditions, the effect of sector-specific shocks on aggregate output can be determined analytically up to a log-linear approximation. In the case of no reallocation, sector-specific shocks generically affect the level output:

$$y_{nr}^t = \frac{1 - (1 - \eta)}{1 - \tilde{\lambda}} a_t + \sum_{i=1}^K \gamma_i \frac{\lambda_i}{1 - \tilde{\lambda}} \left( \phi_{it} - (1 - \eta) a_{it} \right)$$

where $$\tilde{\lambda} = \sum_{i=1}^K \gamma_i \lambda_i$$

Aggregate output depends on aggregate productivity as in a standard one-sector model but also depends on sector-specific shocks. Sector-specific shocks lower aggregate output if a negative demand shock or positive productivity shock (assuming $$\eta < 1$$) impacts a sector with a larger value of $$\lambda_i$$ than the input-weighted average $$\tilde{\lambda}$$, which intuitively corresponds to a sector where employment is more responsive to changes in the wage. Thus, sector-specific shocks that impact sectors with more flexible employment - broadly inclusive of hiring frictions - result in larger changes in employment than the corresponding change in other sectors. A negative productivity shock to a sector with flexible employment reduces employment more than the corresponding increase in employment in the sector with stickier employment. Only in the special case where $$\lambda_i$$ are equalized across sectors will sector-specific shocks integrate out and have no affect on aggregate output. This case does not seem particularly realistic given the multiple structural parameters that enter the expression for $$\lambda_i$$.

If reallocation is costless, the solution for output will differ from the no reallocation case but sector-specific shocks will, in general, still be non-neutral with respect to output:

$$y_{r}^t = \left\{ (1 - \eta) + \left( \frac{\overline{U}(1 - \tilde{\alpha}) + \eta \tilde{\varphi}}{\overline{\sigma}} \right) a_t - \sum_{i=1}^K L_i (\phi_{it} - (1 - \eta) a_{it}) \right\}$$

where $$1 - \tilde{\alpha} = \sum_{i=1}^K \frac{U_i}{\overline{U}} (1 - \alpha_i)$$
$$\tilde{\varphi} = \sum_{i=1}^K L_i (s_i \varphi_i + (1 - s_i) \alpha_i)$$
$$\tilde{\sigma} = \sum_{i=1}^K \gamma_i (s_i \varphi_i + (1 - s_i) \alpha_i)$$

A sufficient condition to recover the neutrality of sector-specific shocks is that the input shares $$\gamma_i$$
are equal to the labor force shares $L_i$. This would be true in a model where sectors are identical in matching technology and productivity, but might also hold under less restrictive conditions.

Though a Beveridge curve cannot be recovered, aggregate unemployment and vacancies can be expressed in terms of sectoral employment, which, in turn, are functions of both sector-specific shocks and aggregate shocks:

$$u_t = -\frac{N}{U} \sum_{i=1}^{K} \frac{N_i}{N} u_{it}$$

$$v_t = \sum_{i=1}^{K} \frac{V_i}{V} \frac{1 + \alpha_i \frac{N_i}{U_i}}{1 - \alpha_i} n_{it}$$

Even without substituting in the expressions for sectoral employment, the expressions for aggregate unemployment and vacancies suggest that sectoral shocks are unlikely to integrate out. The distribution of steady-state vacancies $V_i/V$ and employment $N_i/N$ must be equal to the distribution of real inputs $\gamma_i$ given our definition of sector-specific shocks, and the term premultiplying sectoral employment in the expression for aggregate vacancies must also be equalized across sectors or happen to cancel out the sector-specific component in sectoral employment.

### 4.2 Neutrality Results

While our characterization suggests that sectoral shocks are likely to be non-neutral, we can still summarize conditions under which sector-specific shocks do not shift the Beveridge curve and conditions under which aggregate output is not affected by sector-specific shocks.

**Proposition 1.** If labor reallocation is costless across sectors and $\delta_i = \delta$, $\alpha_i = \alpha$, $\vartheta_i = \vartheta$, then sector-specific shocks do not shift the Beveridge curve. If labor allocation is costless across sectors and $V_i/V = N_i/N$ in an initial steady state, then, to a log-linear approximation, sector-specific shocks do not shift the Beveridge curve.

**Proof.** Under costless labor reallocation, the Jackman-Roper condition holds and labor market tightness across sectors must be equalized: $V_{it}/U_{it} = V_{ht}/U_{ht}$ for all $i, h \in \{1, \ldots, K\}$. Summing over the steady state sectoral Beveridge curves (steady state version of (14)):

$$\sum_{i=1}^{K} N_{it} = \sum_{i=1}^{K} \frac{\vartheta}{\delta} \frac{\delta^{-\alpha} V_{it}}{V_t}$$

$$\Rightarrow 1 - U_t = \frac{\vartheta}{\delta} \left( \frac{V_t}{U_t} \right)^{-\alpha} V_t$$
By assumption, $\frac{N_i}{N} = \frac{V_i}{V}$ and a log-linear approximation to the sectoral Beveridge curves and aggregate tightness implies:

$$n_{it} = -\alpha \theta_t + v_{it}$$
$$\theta_t = v_t - u_t$$

Summing over employment shares, we obtain a log-linear approximation to the aggregate Beveridge curve.

$$\frac{U}{N} u_t = \alpha (u_t - v_t) + v_t$$

In each case, the inverse relationship between vacancies and unemployment is given by these expressions and sector-specific productivity shocks do not shift the locus of unemployment, vacancies pairs implied by these equations.

Sufficient conditions under which sector-specific shocks are neutral for output are summarized in the following proposition:

**Proposition 2.** If $\eta = 1$ - sector-specific shocks have no affect on the level of output. In the case of no reallocation, if $\lambda_i = \lambda$ for $\forall i \in \{1, \ldots, K\}$, sector-specific shocks have no affect on the level of output. In the case of full reallocation, if labor shares $L_i = \gamma_i$ input shares for $\forall i \in \{1, \ldots, K\}$, sector-specific shocks have no affect on the level of output.

**Proof.** The proof of these statements follows directly by inspection of the equations for output under no reallocation and full reallocation respectively.

It is important to emphasize that the neutrality results in one case do not generically imply the neutrality results in the other case. For example, symmetry in matching function parameters may imply that sector-specific shocks do not shift the Beveridge curve, but a sector-specific shock may still shift the level of output. In this case, the sector-specific shock represents a movement along the existing Beveridge curve just as aggregate shocks shift labor market tightness. In this sense, the presence of sector-specific shocks cannot be ruled out by simply observing that vacancies/unemployment relationship is stable as suggested by the analysis of Abraham and Katz.

Conversely, neutrality of sector-specific shocks with respect to output need not imply that the Beveridge curve is stable. For example, in the case of no reallocation, if $\lambda_i = \lambda$ for all sectors, then aggregate output is only a function of aggregate productivity. However, the log-linearized equation
for aggregate unemployment and aggregate vacancies demonstrate that the sector-specific shock need not integrate out. Differences in matching function elasticities and steady state distribution of vacancies could shift the Beveridge curve due to composition effects. As we will show in our calibration results, we can obtain a fairly large shift in the Beveridge curve without any significant corresponding change in output due to the sector-specific shock.

5 Calibration

To examine whether sector-specific shocks can explain the magnitude of the shift observed in the Beveridge curve, we calibrate a two-sector version of our model and separate the economy into a goods-producing and service-providing sector. We also present an alternative calibration where one sector is considered a “housing-sensitive” sector while the other sector is classified as a “housing-insensitive sector”. In both cases, we consider the effect of a permanent negative productivity shock and trace out the steady state Beveridge curves by varying aggregate productivity before and after the sector-specific shock. The size of the sector-specific shock is chosen to match the observed distribution of employment across the sectors we have defined at the trough of the recession in late 2009. In the goods vs. services calibration, a negative productivity shock that matches this distribution of employment at the recession trough generates a shift in the model-generated Beveridge curve that matches the observed shift the Beveridge curve data. Alternatively, in the housing vs. non-housing calibration, a similarly calibrated negative shock to housing generates a shift in the model-generated Beveridge curve that explains about 25% of the observed shift.

5.1 Calibration Strategy and Targets

We calibrate a two-sector version of the model assuming no reallocation across sectors and perfect substitutability in the product market (i.e. $\eta \to \infty$). While the latter assumption is certainly a strong one, it ensures that relative prices are fixed at unity and eliminates the need to select values for input shares, $\phi$. The model is calibrated to monthly labor market rates and therefore the discount rate $\beta$ is set to match an annual real interest rate of 4%. Other important aggregate variables are the flow cost of posting vacancies $\kappa$ and the bargaining power of households $\nu$. These values are chosen to deliver a low value of firm accounting profits as in the calibration in Hagedorn and Manovskii (2008) so that small aggregate productivity shocks can trace out a large Beveridge curve. The flow cost of vacancy posting $\kappa = 0.5$ is in line with the calibration in Monacelli et
al. (2010) and household bargaining power is set to zero. These values ensure that aggregate productivity shocks have a large effect on labor market tightness.

To calibrate sectoral matching functions and labor market shares, we draw heavily from the moments calculated in DFH. Monthly separation rates $\delta_i$ are set to match the employment-weighted separation rates for the sectors that compose the goods vs. services breakdown and the housing vs. non-housing breakdown from Table 1 in DFH\(^3\). Monthly vacancy-filling probabilities $q_i$ are set using the daily job-filling rates calculated in Table 3 in DFH\(^4\) and will be used to calibrate $\vartheta_i$, the matching function productivity of each sector. Sectoral matching function elasticities are equalized across sectors and set to 0.5 in line with Monacelli et al. (2010). Steady state employment shares are chosen to match the observed distribution of employment prior to the recession from 2000-2006. Likewise, steady state unemployment shares are chosen to match the observed distribution of unemployment by sector from 2000-2006, where workers are assigned sectors based on the sector in which they were most recently employed. Relative productivity is set to match the distribution of gross output across sectors as reported in the US input-output tables by the Bureau of Economic Analysis. Wages in each sector are fixed at the household disutility for supplying labor in each sector (by the assumption of zero bargaining power), and wages as a percentage of the marginal product of firms in each sector are reported in Table 1. The rigidity of real wages allows for small aggregate productivity shocks to trace out large Beveridge curves.

The labor force is normalized to unity and the steady state unemployment rate is set at 5.5% implying $N = 0.945$ and $U = 0.055$. The level and distribution of vacancies is determined by the other calibration targets. While the distribution of vacancies is comparable to the distribution of

\begin{table}[h]
\centering
\caption{Calibration Targets}
\begin{tabular}{ccccccccc}
\hline
 & Model & Targets & $\delta_i$ & $q_i$ & $N_i/N$ & $U_i/U$ & $P_i/Y_i$ & $V_i/V$ & $V_i/V$ (data) \\
\hline
Calibration 1: & & & & & & & & & \\
Goods-Producing & 3.48 & 0.786 & 17.1\% & 25.0\% & 27.5\% & 15.1\% & 0.409 & 7.8\% & 0.985 & 11.1\% \\
Service-Providing & 3.17 & 0.617 & 82.9\% & 75.0\% & 72.5\% & 84.9\% & 0.602 & 5.0\% & 0.967 & 88.9\% \\
\hline
Calibration 2: & & & & & & & & & \\
Housing related & 4.59 & 0.787 & 34.0\% & 44.9\% & 24.2\% & 39.9\% & 0.597 & 7.1\% & 0.950 & 36.2\% \\
Non-housing related & 2.51 & 0.555 & 66.0\% & 55.1\% & 75.8\% & 60.1\% & 0.517 & 4.6\% & 0.969 & 63.8\% \\
\hline
\end{tabular}
\end{table}

\(^3\)Goods-producing sectors are construction, durables manufacturing, nondurables manufacturing, and natural resources and mining. Housing-related sectors are defined as construction, retail trade, wholesale trade, transportation and utilities, and leisure and hospitality. This definition of housing-related sectors follows closely the discussion in Mian and Sufi (2012) of sectors that are disproportionately exposed to a housing price collapse. While the inclusion of construction is obvious, the other sectors are included due to their sensitivity to consumer spending.

\(^4\)Assuming 20 working days per month, $q_i = 1 - (1 - f_i^{20})$ where $f_i$ is the daily job filling rate.
vacancies observed in JOLTs (as shown in the rightmost column of Table 1), the level of vacancies in
the model is much higher than its direct counterpart in the data, vacancies divided by labor force.
This difference is due to time-aggregation issues in the monthly data, where vacancies are reported
at the end of the month and, therefore, vacancies filled during the month are missed. To ensure
that the job-finding probability and vacancy-filling probability are actually probabilities (rather
than rates), our calibration targets these values while missing the average aggregate and sectoral
vacancy rates in JOLTs. We adjust the model-generated Beveridge curves by dividing vacancies
by a constant so that, at an unemployment rate of 5.5%, the vacancy rate is 2.6% corresponding
to vacancy rate observed in the data at this level of unemployment. All model generated vacancy
rates are adjusted by the same constant; thus this adjustment just shifts the level of the Beveridge
curve leaving its shape unchanged.

5.2 Model-Implied Beveridge Curves

We consider the effect of a permanent negative productivity shock to either the goods-producing
or housing-related sector of sufficient magnitude to match the employment shares of each sec-
tor respectively at the trough of the recession in 2009-2010. For the goods-producing sector, its
employment share fell from a pre-recession average of 17% of total employment to 14% of total
employment by 2010. For the housing-related sector as we have defined it, its employment share
fell from a pre-recession average of 34% of total employment to about 33% of total employment by
2010. While aggregate productivity shocks do cause endogenous changes in employment shares, it
turns out that sector-specific shocks are needed to match the shifts observed in the data. We choose
a sector-specific productivity shock of sufficient magnitude to match employment shares when the
unemployment rate is 9%.

The left panel of Figure 6 shows the model-implied aggregate Beveridge curves before and after
a 1% negative productivity shock that impacts only the goods-producing sector. The nonlinear
steady state Beveridge curve before and after are traced out by varying aggregate productivity to
reach different levels of unemployment. The actual path of unemployment and vacancies would
differ from the steady state Beveridge curve due to transition dynamics, but, as argued in Shimer
(2005), since labor market flows are sufficiently large, actual unemployment and vacancies are
always likely to be near their steady state values. At a 9% unemployment rate, the model-implied
vacancy/labor force rate (adjusted for time-aggregation as described earlier) increases from 1.7% to
2.2%, a 29% increase that matches the magnitude of the shift observed in the data. Vacancy rates
in JOLTs averaged 1.8% for the last 8 months of 2009, while the vacancy/labor force ratio averaged 1.5% over the same period. For the 18 months starting in January 2010, the vacancy rate averaged 2.2% and the vacancy/labor force ratio averaged 1.9%, meaning that these rates rose about 25% over a period in which the unemployment rate averaged 9.3%. Our model-implied 29% shift in the vacancy/labor force ratio closely matches the Beveridge curve shift observed in this recession.

The right panel of Figure 6 shows the Beveridge curve before and after a 1.5% negative productivity shock that impacts only the housing-related sector. As can be seen, this shock produces a much smaller shift of the Beveridge curve, particularly at relatively high unemployment rates. Model-implied vacancy to labor force rates rise from 1.5% to a little over 1.6%, well below the 1.9% vacancy/labor force ratio observed in the data. In other words, the second calibration captures under 25% of the shift in the Beveridge curve. Nevertheless, both calibration exercises generate Beveridge curves that realistically match the pre-recession data and exhibit relative stability in employment shares despite large employment fluctuations consistent with the data.

### 5.3 Sources of the Shift in the Beveridge Curve

The case of two-sector model with perfect substitutability in goods and no labor reallocation readily lends itself to an analysis of the factors that cause the Beveridge curve to shift. Under no reallocation, tightness in each sector is determined by combining the firms’ vacancy posting condition and
the Nash-bargained wage in the sector:

$$A_i = W_i + \frac{\kappa}{q_i} (1 - \beta (1 - \delta_i))$$

$$W_i = z + \frac{\nu}{1 - \nu q_i} (1 - \beta (1 - \delta_i - p_i))$$

where the job-filling rate $q_i$ and job-finding rate $p_i$ are functions solely of sectoral tightness. Exogenous shocks to productivity affect sectoral tightness $\theta_i$ by determining the overall surplus: $A_i - z$; the difference between the marginal product of labor and the household’s marginal rate of substitution (assumed to be constant as is standard in canonical search models). In the absence of labor reallocation, sectors can differ in either their tightness or the various exogenous parameters that determine the cost of search frictions such as matching function efficiency or separation rates. Differences across either dimension will be sufficient to cause a sector-specific shock to shift the Beveridge curve.

If the household bargaining power is set to zero $\nu = 0$ and sectors are identical in search parameters (i.e. matching function efficiency, elasticities and separation rates), we obtain a simple expression relating sectoral tightness to the size of the sectoral surplus:

$$\frac{A_A - z}{A_B - z} = \left( \frac{\theta_A}{\theta_B} \right)^\alpha$$

If sector A has a slack labor market relative to sector B and matching functions are identical, it must be the case that sector A has a smaller surplus than sector B. Because of the nonlinear relationship of the surplus and tightness, productivity shocks to sector A have a larger effect on sectoral tightness than productivity shocks to sector B. The Beveridge curve shift seen in our calibration exercise results from the fact that the labor market in the goods-producing sector is more slack than the labor market in the service-providing sector. In that calibration, the differences in the job-filling rate across sectors is driven by differences in tightness rather than differences in matching function efficiency. With fixed marginal rates of substitution and no reallocation, even fairly small productivity shocks to the sector with a smaller surplus shifts the aggregate Beveridge curve since vacancy posting is dominated by the sector with a larger surplus but unemployment is split more evenly between both sectors.

Alternatively, if we assume an initial state where labor market tightness is equalized across sectors but allow for differences in matching function efficiency across sectors, it follows that the
sector with higher matching function efficiency has a lower surplus than the sector with a lower matching function efficiency:

\[
\frac{A_A - z}{A_B - z} = \frac{\vartheta_B}{\vartheta_A}
\]

If labor market tightness is equalized across sectors, the sector with a higher matching function efficiency has a lower cost of filling a vacancy implying that the wedge between the marginal product of labor and the household’s marginal rate of substitution must be smaller. Even though sectors have the same initial level of tightness, both aggregate and sectoral productivity shocks have a larger effect on the sector with a smaller surplus. As a result, a negative productivity shocks to sector A result in a greater fall in tightness and a shift in the Beveridge curve as aggregate vacancies are dominated by the sector with a relatively larger surplus.

It is worth emphasizing that the assumption of zero bargaining power is not essential to the conclusions obtained here or the results in our calibrated exercise. With nonzero bargaining power, sectoral tightness will continue to be a function of the sectoral surplus \(A_i - z\), and productivity shocks to sectors with a smaller surplus will have a larger effect on sectoral tightness. Sectoral shocks are more likely to shift the aggregate Beveridge curve if search costs are small consistent with the evidence and calibration in Hagedorn and Manovskii (2008). Based on the mechanism described here, the assumption of no reallocation and a constant household marginal rate of substitution are central to generating a shift in the Beveridge curve. When goods display only limited substitutability or reallocation is perfect across sectors, numerical experiments suggest only small shifts in the Beveridge curve. Under perfect reallocation, relative prices adjust so that the ratio of sectoral surpluses remains roughly constant. For example, under costless reallocation, when search parameters are identical across sectors, relative prices adjust to hold constant the ratio of the surplus in each sector:

\[
\frac{P_A A_A - z}{P_B A_B - z} = \frac{\vartheta_B}{\vartheta_A}
\]

5.4 Mismatch and Aggregate Matching Function Efficiency

The model and calibration discussed here also carry important implications for related work on labor market mismatch and the behavior of matching function efficiency. Sahin et al. (2010) construct indices of labor market mismatch using observed vacancies and unemployment. Their indices measure the deviation of observed labor market tightness from the socially-optimal level of labor market tightness as given by a generalized Jackman-Roper condition. As shown in earlier, our model
implies that under costless reallocation, labor market tightness will be equalized across sectors. However, in the case of costless reallocation, the model will feature some level of mismatch that will vary with both aggregate and sector-specific productivity shocks. As Figure 7 illustrates, even with only aggregate productivity shocks, our model generates endogenous variation in mismatch. To the extent that aggregate productivity shocks drive the business cycle, mismatch will be countercyclical, consistent with the behavior of the mismatch indices in Sahin et al. (2010)\(^5\). Moreover, if there are only aggregate productivity shocks, there is a unique aggregate Beveridge curve and variations in mismatch need not be indicative of shifts in the Beveridge curve. Since a sector-specific shock does increase mismatch, the model suggests that shifts in the Beveridge curve should be accompanied by an increase in the elasticity of mismatch to the business cycle rather than simply an increase in mismatch. The countercyclicality of mismatch stems from the same differences in firm surpluses across sectors - labor market tightness in sectors with a relatively smaller surplus is more responsive to productivity shocks heightening differences in labor market tightness across sectors. This effect is asymmetric since tightness is a nonlinear function of the firm surplus.

As with countercyclical mismatch, the model also generates countercyclical movements in aggregate matching function efficiency. Aggregate matching function efficiency varies with both vacancy

\(^5\)Model generated mismatch is computed as:

\[
M^h_t = 1 - \sum_{i=1}^{K} \left( \frac{U_{it}}{U_t} \right)^\alpha \left( \frac{V_{it}}{V_t} \right)^{1-\alpha}
\]
shares and mismatch across sectors:

\[ H = \sum_{i=1}^{K} H_i = U^\alpha V^{1-\alpha} \sum_{i=1}^{K} \left( \vartheta_i \left( \frac{\theta_i}{\theta} \right)^\alpha \frac{V_i}{V} \right) \]

As Figure 7 shows, the countercyclical increase in mismatch coincides with procyclical movements in matching function efficiency. Sedlacek (2011) documents substantial time variation in matching function efficiency and finds procyclical aggregate matching function efficiency. Likewise, Barnichon and Figura (2011) also show that aggregate matching function efficiency varies over time and find a particularly sharp fall in aggregate matching function efficiency during the Great Recession. Sector-specific shocks further reduce aggregate matching function efficiency reflecting the shift in the Beveridge curve. In short, our two-sector model illustrates that neither time-variation in mismatch or aggregate matching function efficiency is sufficient to generate a shift in the Beveridge curve; both phenomenon are properties of aggregate labor markets in presence of imperfect reallocation.

5.5 Relationship of Output and the Unemployment Rate

The degree to which a Beveridge curve shift reflects a change in the natural rate of unemployment depends on the behavior of vacancies. If a sectoral shock shifts the Beveridge curve, the vacancy rate may be higher at any given level of the unemployment rate, but the shock may also raise vacancies sufficiently to keep unemployment unchanged or even reduce unemployment. Though our model has only productivity shocks and, therefore, no concept of aggregate demand, we can use the ratio of output and unemployment to illustrate what effect a Beveridge curve shift has on the relationship between the two variables. As Figure 8 shows, the output-unemployment ratio may behave quite differently from the aggregate Beveridge curve. As the left panel shows, a negative shock to the goods-producing sector leaves this ratio essentially unchanged; the level of output at different unemployment rates is largely unchanged after a sector-specific shock. In other words, once we account for the aggregate component of the sector-specific shock, the output unemployment relationship is left largely unchanged.

The right panel of Figure 8 shows largely the same effect for the housing calibration. At somewhat higher unemployment rates, the output fall is less steep after a sector-specific shock, but the difference is not particularly large. However, the underlying shift in the Beveridge curve is also small under this calibration. Nevertheless, the goods-producing calibration makes clear that Beveridge curve shifts need not involve significant changes in the relationship between output and
unemployment rates.

6 Extension with Financial Frictions

While our calibration in the previous section matches the observed shift in the Beveridge curve with an aggregate and sector-specific productivity shock, the financial nature of the current recession and the behavior of labor productivity in the current recession raise questions about the suitability of a productivity based explanation. In this section, we extend our model to demonstrate how aggregate and sector-specific shocks can be replaced by financial shocks, which can account for both movements along the Beveridge curve and shifts in the Beveridge curve.

6.1 Sticky Prices and Working Capital Constraint

A financial shock that impairs the flow of funds from savers to borrowers is likely to manifest itself as both an aggregate demand shock (such as a monetary policy or discount rate shock that shifts the intertemporal IS curve) and as a shock that raises the cost of firm borrowing. To allow for aggregate demand effects, we posit a set of monopolistically competitive retailers that costlessly differentiate the final good and set prices periodically. To show that aggregate demand shocks trace out the same Beveridge curve as aggregate productivity shocks, it is not necessary to specify the details of the retailers price-setting problem, the exact nature of the aggregate demand shock, or specify a monetary policy rule. Instead, with sticky prices, the real price of the good produced by the final good firm is no longer constant. Input demand conditions and the price of the final good
is slightly altered from equations (3) and (4):

\[ Y_{it} = \phi_{it} Y_t \left( \frac{P_{it}}{P_{ft}} \right)^{-\eta} \]  
(24)

\[ \frac{P_{ft}}{P_t} = \left\{ \sum_{i=1}^{K} \phi_{it} \left( \frac{P_{it}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \]  
(25)

The real price of the final good is \( P_{ft}/P_t \) and \( P_t \) is the nominal price of the final consumption good. This price is time-varying when prices are sticky. Let \( \mu_t^{-1} = P_{ft}/P_t \), where \( \mu_t \) reflects time-variation in mark-ups due to sticky prices and shocks to aggregate demand\(^6\). We can show that variation in mark-ups will trace out the same aggregate Beveridge curve in a multisector model as variation in aggregate TFP shocks.

**Proposition 3.** Let the disutility of labor supply be independent of output \( Y \) (that is, no wealth effect on labor supply). In a steady state, for any value of the markup \( \mu \) and aggregate productivity \( A \), let \( V(A, \mu) \) represents steady state vacancies and let \( U(A, \mu) \) represent steady state unemployment. Then, for any \( A = \bar{A} \), there exists a \( \bar{\mu} \) such that \( V(\bar{A}, 1) = V(1, \bar{\mu}) \) and \( U(\bar{A}, 1) = U(1, \bar{\mu}) \).

**Proof.** See Appendix. \( \square \)

This proposition shows that aggregate demand shocks and aggregate productivity shocks trace out the same Beveridge curves. In a one-sector model, the steady state Beveridge curve is given by the condition equating separations and hires; since this relation is independent of productivity or aggregate demand shocks, these shocks trace out the same Beveridge curve. Our proposition shows that this result generalizes to a multisector model under the assumption of no wealth effects on labor supply (as in Shimer (2005) or Hagedorn and Manovskii (2008). Intuitively, TFP shocks and changes in the markup enter the vacancy posting condition in exactly the same manner while leaving

\(^6\)One possible way to obtain time-varying markups is by assuming a Calvo price-setting for the retailers. The retailers problem becomes:

\[ \max V_{it} = \mathbb{E}_t \sum_{T=t}^{\infty} Q_{t,T} \alpha^{T-t} \left\{ p_t(i) - P_{ft} \right\} y_t(i) \]

subject to \( y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} \)

\[ Q_{t,T} = \beta^{T-t} \left( \frac{C_{st}}{C_{st}} \right)^{-\frac{\theta}{2}} \frac{P_t}{P_T} \]

Optimal price-setting by the retailers along with aggregate price index imply processes for \( P_{ft}/P_t \) - the price of the final good, which is the inverse of the markup.
relative prices and wages unchanged. As a result, equilibrium sectoral and aggregate employment, vacancies, and unemployment are left unchanged.

To model the production side effect of financial shocks, we assume that some sectors face a working capital constraint of the form considered in Christiano et al. (2005). In the financially constrained sectors, firms have to borrow to pay wages and the cost of posting vacancies. For financially constrained firms, the problem of the intermediate goods firms is slightly modified from the baseline model by introducing a borrowing rate $i_r$:

$$
\Lambda_{it} = \max E_t \sum_{T=0}^{\infty} Q_{t,T} \left( \left( \frac{P_{i,t}}{P_t} \right) Y_{tT} - \left( 1 + i^b_T \right) (W_{iT}N_{iT} - \kappa V_{iT}) \right)
$$

subject to

$$
N_{it} = (1 - \delta_i) N_{it-1} + q_{it} V_{it}
$$

$$
Y_{it} = A_t N_{it}
$$

The vacancy posting condition now includes the borrowing rate and changes in expected future borrowing rates:

$$
\frac{P_{it}}{P_t} \frac{A_t}{1 + i^b_t} = W_{it} + \frac{\kappa}{q_{it}} - E_t Q_{t,t+1} (1 - \delta_i) \frac{\kappa}{q_{it+1}} \frac{1 + i^b_{t+1}}{1 + i^b_t}
$$

In steady state, the second term with expected future borrowing rates drops out and changes in the borrowing rate are isomorphic to a negative sector-specific productivity shock as in equation (8). We show in the appendix that a collateral constraint as opposed to a borrowing rate would imply the exact same vacancy posting condition. In Curdia and Woodford (2010), the borrowing rate is the sum of the deposit rate - the instrument of monetary policy - and a credit spread that increases with a financial shock. Therefore, a financial shock would drive up borrowing rates if monetary policy cannot reduce rates sufficiently to offset the rise in credit spreads. A situation in which the zero lower bound is binding represents a good example of a situation where monetary policy cannot offset the effects of a financial shock. To the extent that monetary policy can reduce the borrowing rate for firms, monetary policy has an effect on the production side of the economy and can undo the upward shift in the Beveridge curve. Thus, in a model with financial frictions, both movements along the Beveridge curve and shifts in the Beveridge curve would not be invariant to the conduct of monetary policy.
Table 4: Calibration Targets

<table>
<thead>
<tr>
<th>Model Targets</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ_1/0.595</td>
<td>49.6%</td>
</tr>
<tr>
<td>q_i</td>
<td>49.6%</td>
</tr>
<tr>
<td>N_i/N</td>
<td>38.3%</td>
</tr>
<tr>
<td>U_i/U</td>
<td>49.8%</td>
</tr>
<tr>
<td>P_i/Y</td>
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</tr>
<tr>
<td>V_i/V</td>
<td>0.959</td>
</tr>
<tr>
<td>p_i</td>
<td>5.5%</td>
</tr>
<tr>
<td>U_i/L_i</td>
<td>0.973</td>
</tr>
<tr>
<td>W_i/A_i</td>
<td></td>
</tr>
</tbody>
</table>

6.2 Calibration

6.2.1 Targets and Results

As an alternative to the sectoral productivity shock explanation for the shift in the Beveridge curve, we show here that a financial shock could also account for a shift in the Beveridge curve. In particular, if some subset of firms are financially constrained, an increase in borrowing rates or tightening of collateral constraints would act as a sector-specific shock for this subset of firms. In our calibration, we use hiring and employment data for small versus large businesses where small business are assumed to be financially constrained. Though size is, at best, an imperfect proxy for financial constraints, the presence of financial constraints is unobserved and hiring and employment data by firm age, which may be a better proxy, is unavailable.

In our calibration of a two-sector model with small firms and large firms, we split firms into those with over 500 employees and those with less than 500 employees. Using 2007 data from the Statistics on US Businesses, firms with fewer than 500 employees accounted for 49.6% of total employment. Monthly separation rates are averages from statistics in DFH Table 1, and job-filling rates are computed using DFH Table 3 in the same manner as in Section 5. Relative productivity in each sector is set to match the share of total sales by firms in the two size classes as reported in the Statistics on US Businesses. While we continue to assume no reallocation between the sectors, but unemployment rates are assumed to be the same in each sector and our calibration ensures the labor market tightness is equalized across sectors at the steady state with 5.5% unemployment. Parameters not elsewhere specified are the same as in the calibration in Section 5. Table 4 summarizes our calibration targets and model-generated values.

Figure 9 below shows the effect of a shock that raises borrowing rates by 10 percentage points.

---

7Haltiwanger et al. (2011) document a sharp fall in job creation by startups in the current recession using data from Business Dynamics Statistics. Likewise, Sahin et al. (2011) show that small firms suffered disproportionate declines relative to large firms in employment in the Great Recession.

8While DFH data is for establishments instead of firms, summary data on separation rates and job-filling rates is not available by firm size.
annually and affects only the small firm sector. Since a financial crisis involves both a rise in borrowing rates and various nonprice measures to restrict lending, a shock of this magnitude seems plausible. At an unemployment rate of 9%, the vacancy to labor force ratio (adjusted for time-aggregation as in Section 5) rises from 1.5% to 1.8% explaining over half the shift in the Beveridge curve. A 12 percentage point increase in annualized borrowing rates would fully match the observed shift in the Beveridge curve. Moreover, unlike the earlier calibrations based on sectoral productivity shocks, a sector-specific shock also generates a shift of the Beveridge curve at more moderate unemployment rates.

6.2.2 Discussion

So long as the economy is populated by constrained and unconstrained firms, the financial shock will operate as a sector-specific productivity shock that favors the unconstrained sector. This sector-specific shock accounts for the shift in the Beveridge curve. However, the reduction in aggregate demand due to falls in consumption or investment induced by the financial shock represent movements along the Beveridge curve both before and after the shift. In order for a financial shock that affects firms to shift the Beveridge curve, our calibration in the previous section suggests that financially constrained firms would need to have higher steady-state productivity and/or faster vacancy-filling rates due to higher matching function productivities. As a result, a negative shock to the financially constrained sector would induce relatively greater vacancy posting in the unconstrained sector which fills vacancies more slowly. This composition effect would raise vacancies at any given unemployment rate.
Importantly, without specifying several elements needed to close the model (i.e. monetary policy, financial intermediation, aggregate IS curve), it’s difficult to say whether a financial shock would explain the timing of the movements in unemployment and vacancies observed during the current recession - an initial movement along the old Beveridge curve followed by a shift upwards beginning at the recession trough. In order to match this timing, the effect of the financial shock on the production side (aggregate supply effect) would need to occur at the recession trough rather than at the beginning of the recession. If financial constraints are only binding in periods of expansion, this would account for the delay in the shift in the Beveridge curve until late 2009.

7 Conclusion

Recent discussions of the slow recovery in the US following the recent recession have raised the possibility of sectoral shocks. Proponents of this view have cited the disproportionate impact of the recession on housing-related industries and the shift in the Beveridge curve as evidence of sector-specific shocks. We investigate the role of sector-specific shocks and their impact on the Beveridge curve empirically and theoretically.

On the empirical side, a factor analysis of sectoral employment in the postwar data is used to isolate sector-specific shocks while remaining mindful of the Abraham and Katz critique by allowing sectoral employment to load differentially on a single business cycle factor. We derive a sector-specific shock index by summing the non-aggregate component of sectoral employment, and show that this index is highly persistent, elevated in the current period, and distinct from the business cycle or the Lilien measure of sectoral shocks. Moreover, we show that this measure of sector-specific shocks is elevated in those periods when the Beveridge curve appears to shift.

On the theoretical side, we build a multisector model with labor market search to investigate how sector-specific shocks affect equilibrium variables like the aggregate Beveridge curve and the level of output. Our model shows that sector-specific shocks should, except in special cases, shift the Beveridge curve irrespective of whether labor can be costlessly reallocated across sectors or if labor is fixed in each sector. Likewise, sector-specific shocks will in general be non-neutral for the level of output.

We calibrate a two-sector version of our model and show that a negative productivity shock to the goods-producing sector that matches the distribution of employment shares at the trough of the recession generates a shift in the Beveridge curve that matches the magnitude of the shift.
observed in the data. While we can generate an empirically plausible shift in the Beveridge curve, we show that this sector-specific shock does not greatly alter the relationship between output and employment. We also show that a multisector model can match empirical regularities regarding the cyclicality of mismatch and aggregate matching efficiency even in the absence of sectoral shocks.

In a model with sticky prices, monetary policy would be able to affect the level of aggregate demand and shift unemployment and vacancies along the Beveridge curve. We show that financial shocks act like sector-specific shock and can also generate a shift in the Beveridge curve if a subset of firms is financially constrained. If our flexible price calibration results carry over, this analysis suggests that a shift in the Beveridge curve would not change the natural rate of unemployment and the Phillips curve would be unchanged. As a result, the link between sectoral shocks and the natural rate of unemployment may be misguided and lead to incorrect conclusions about monetary policy.

Given recent evidence provided by DFH that expanding firms have very high vacancy yields, the shift in the Beveridge curve in the current recession may be intimately related to the effect of the crisis on expanding firms that depend heavily on outside credit. As Barnichon et al. (2010) note, the vacancy shortfall is evident in each sector but is particularly pronounced in industries that operate with smaller scale and therefore are more subject to credit frictions. Our model accounts for this fact in a stylized way by positing a set of financially constrained firms with high vacancy-filling rates; this sector is a stand-in for the key characteristics of expanding firms and understanding the behavior of the Beveridge curve in models with firm dynamics is a subject for future research.
References


Curdia, Vasco and Michael Woodford, “Credit spreads and fiscal policy,” Journal of Money, Credit and Banking, 2010, 42 (6), 3–35.


A Additional Proofs

A.1 Proof of Proposition 3

Proof. In the absence of wealth effects on the labor supply, we show that markups trace out the same aggregate Beveridge curve in a multisector model as aggregate productivity shocks. We define the steady state equilibrium as aggregate output $Y$ and sectoral quantities and prices \{\(Y_i, N_i, U_i, V_i, P_i, W_i, p_i, q_i\)\}_{i=1}^{K}$ that satisfy the following steady equilibrium conditions. Implicitly, we are considering only the case of no reallocation.

\[
Y = \left\{ \sum_{i=1}^{K} \phi_i^n Y_i^{\eta-1} \right\}^{\frac{\eta}{\eta-1}} \tag{30}
\]

\[
\frac{P_f}{P} = \left\{ \sum_{i=1}^{K} \phi_i \left( \frac{P_i}{P} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \tag{31}
\]

\[
Y_i = \phi_i Y \left( \frac{P_i}{P_f} \right)^{-\eta} \tag{32}
\]

\[
Y_i = AA_i N_i \tag{33}
\]

\[
\frac{P_i}{P} AA_i = W_i + \frac{\kappa}{q_i} (1 - \beta (1 - \delta_i)) \tag{34}
\]

\[
W_i = z_i + \frac{\nu}{1 - \nu q_i} (1 - \beta (1 - \delta_i - p_i)) \tag{35}
\]

\[
\delta_i N_i = \vartheta_i U_i^{\alpha_i} V_i^{1-\alpha_i} \tag{36}
\]

\[
q_i = \vartheta_i \left( \frac{V_i}{U_i} \right)^{-\alpha_i} \tag{37}
\]

\[
p_i = \vartheta_i \left( \frac{V_i}{U_i} \right)^{1-\alpha_i} \tag{38}
\]

In the flexible price case, $P_f/P = \mu^{-1} = 1$, and for an value of aggregate productivity $A$, there exists an equilibrium $Y$ and \{\(Y_i, N_i, U_i, V_i, P_i/P, W_i, p_i, q_i\)\}_{i=1}^{K}$ that is a function of $A$. The bar superscript indicates that this allocation and prices satisfies these equilibrium conditions for a particular value of $A = \bar{A}$. We now show that if $A = 1$, there exists a value of $P_f/P$ that implies the same values \{\(N_i, U_i, V_i, W_i, p_i, q_i\)\}_{i=1}^{K}$ and possibly different values of $Y$ and $Y_i$ and $P_i/P$. Set $P_f/P = \bar{A}$. It is clear that equations 35-38 are satisfied by the equilibrium allocations and prices \{\(N_i, U_i, V_i, W_i, p_i, q_i\)\}_{i=1}^{K}$.

We can rewrite the vacancy posting condition and input demands as
follows:

\[
\frac{P_i}{P_f} A_i = W_i + \frac{K}{q_i} (1 - \beta (1 - \delta_i))
\]

\[
\left( \frac{P_i}{P_f} \right)^{-\eta} = \frac{Y_i}{\phi_i Y} = \frac{A_i N_i}{\phi_i Z}
\]

\[
Z = \left\{ \sum_{i=1}^{K} \phi_i^\eta (A_i N_i)^{\eta - 1} \right\}^{\frac{1}{\eta - 1}}
\]

By setting \( P_f / P = \bar{A} \), both the vacancy posting condition and the input demands are satisfied by the equilibrium allocation and prices under \( \bar{A} \). However, it is clear that both aggregate output \( Y \) and sectoral outputs \( Y_i \) will not be the same if \( A = 1 \) and \( P_f / P = \bar{A} \). However, under the assumption of no wealth effects, these quantities do not feedback into the other equilibrium conditions. Let \( P_i / P = \bar{A} P_i / P \). This satisfies (32) since, under \( \bar{A} \), \( P_f / P = 1 \). Therefore, for any given \( \bar{A} \), setting \( A = 1 \) and \( P_f / P = \bar{A} \) implies the same allocations of sectoral employment, unemployment and vacancies. Therefore, aggregate unemployment \( U = \sum_{i=1}^{K} U_i \) and vacancies \( V = \sum_{i=1}^{K} V_i \) are unchanged, and aggregate demand shocks that change the markup \( P_f / P \) trace out the same Beveridge curve as aggregate TFP shocks.

In the case of costless reallocation, the same result continues to hold since the Jackman-Roper condition under \( \bar{A} \) is satisfied when \( A = 1 \) and \( P_f / P = \bar{A} \).

\[\square\]

A.2 Collateral Constraint

Our result demonstrating an equivalence between sector-specific shocks and shocks to the borrowing rate in a model with a working capital constraint can be generalized to other types of financial shocks. A common shock considered in the literature is a Kiyotaki and Moore type shock to the value of collateral. We modify the problem of the intermediate goods producer to include a time-varying collateral constraint that limits the ability of the firm to borrow to finance the wage bill and the cost of posting vacancies:

\[
\Lambda_{it} = \max E_t \sum_{T=0}^{\infty} Q_{i,T} \left( \left( \frac{P_{iT}}{P_t} \right) Y_{iT} - \left( 1 + i^h_T \right) (W_{iT} N_{iT} - \kappa V_{iT}) \right)
\]

subject to \( N_{it} = (1 - \delta_i) N_{it-1} + q_i V_{it} \)

\[
Y_{it} = A_t N_{it}
\]

\[
\lambda_t K \geq W_{it} N_{it} + \kappa V_{it}
\]
Fluctuation in $\lambda_t$ can represent a tightening of lending standards by financial institutions or a fall in the value of collateral like real estate or other forms of capital. For simplicity, we continue to assume that labor is the only variable factor of production and that constrained firms have some fixed endowment of capital. The vacancy posting condition in this setting is identical to the vacancy posting condition (29):

$$\frac{P_{it}}{P_t} \frac{A_t}{1 + \varphi_t} = W_{it} + \frac{\kappa}{q_{it}} - E_t Q_{t,t+1} (1 - \delta_i) \frac{\kappa}{q_{it+1}} \frac{1 + \varphi_{t+1}}{1 + \varphi_t}$$

where $\varphi_t$ is the Lagrange multiplier on the collateral constraint and replaces the interest rate on borrowed funds. In steady state, the Lagrange multiplier on the constraint enters as a sector-specific productivity shock for any sector that faces a working capital constraint. A decrease in the value of $\lambda_t$ tightens the constraint and raises the Lagrange multiplier. Therefore, our choice of modeling the financial shock as an interest rate shock instead of a shock to collateral values has no qualitative effects on the behavior of firms.