The Optimal Design of a Bank Stress Test *

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Abstract

In order to ease the tensions in financial markets caused by uncertainties about banks’ solvency, supervisory authorities have recently stress tested banks and revealed the results. Especially in Europe, scholars and commentators questioned the credibility of the stress test exercise and criticized the policies in favor of weak banks. In this paper, I show that the credibility of a bank stress test and the policies to deal with weak banks are interrelated and must not be seen in isolation. Stress tests can remove information asymmetries only if the supervisory authorities implement policies to fix undercapitalized banks. The intuition is that revealing bad information about banks might cause socially inefficient bank runs, but would reduce the adverse selection premium for good banks and give them incentives to take the most efficient investment choice. If there are no policies to fix weak banks, I get that a welfare maximizing regulator would never find it optimal to eliminate adverse selection completely because of the bank run inefficiency. It is optimal to remove adverse selection completely if there are policies fixing undercapitalized banks at no cost for taxpayers. Full disclosure gives the regulator commitment to enforce these policies.

1 Introduction

The disclosure of banks stress test results represents an innovation in bank supervision. Regulators were usually keeping supervisory reports confidential, and disclosing at most formal enforcement actions. Following the recent financial crisis, uncertainties about banks’ solvency and the consequent worsening of financing conditions led regulators to switch to a more transparent approach to bank supervision. Starting with the US in 2009 and Europe in 2010, regulators conducted simultaneous examinations of the largest banks and revealed which of

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1Formal enforcement actions represent the outcome of bank supervision. Examples are cease and desist orders. Public announcement of formal enforcement actions was required in the US in 1990.
them would be undercapitalized in a possible adverse macroeconomic scenario. Contrary to Europe, the US also provided explicit guarantees that undercapitalized banks would be forced to raise capital. The widespread view that US stress tests were more successful than in Europe suggests that the disclosure of stress test results and policies in favor of undercapitalized banks must complement each other. In this paper I provide an explanation for this intuition. I show that disclosure per se is no panacea, unless the regulator implements policies in favor of undercapitalized banks.

The credibility of stress tests and the need of policies to fix undercapitalized banks are the issues that dominated the debate following bank stress tests in the U.S. and Europe. While some commentators and scholars argued that stress tests succeeded in sorting out the good from the weak banks \( ^2 \), others were skeptical about the credibility of stress tests especially in Europe. First, the bailout of the Allied Irish Bank few weeks after passing the stress test completely discredited the 2010 exercise. Second, there are concerns that some banks used loopholes to avoid disclosing relevant information in the second round of European stress tests. For example, “UniCredit did not disclose market-by-market data relating to its operations in eastern European countries within the EU, appearing instead to lump it into its Austrian numbers” \( ^3 \); Lloyds Banking Group suffered large losses from its Irish business and “revealed its European credit exposures only in the UK in this years tests, saying exposures in other countries fell below the EBA’s threshold for disclosure, at 5 per cent of total exposures” \( ^4 \). The lack of policies to deal with undercapitalized banks was the other major criticism to European bank stress tests. While in the U.S. the regulator guaranteed that weak banks would raise capital, in Europe there were concerns about the capacity to fix banks’ weaknesses \( ^5 \). The sovereign debt crises in peripheral European countries made state intervention not feasible, whereas tensions in equity markets increased the costs of raising capital in the market. This raised the concern that stress tests would make problems worse rather than restore confidence in banks.

In this paper, I argue that the credibility of stress tests and policies to fix undercapitalized banks must be seen as two sides of the same coin. Regulators must consider their interplay in order to make bank stress tests effective supervisory tools. The intuition is that stress tests aim at restoring market discipline, but market discipline can be costly for the society. Revealing bad news about a bank might trigger a run and lead the bank to sell assets at fire sales prices to satisfy creditors’ claims. Revealing which banks are bad yields efficiency gains because it reduces the adverse selection premium for good banks and weakens their incentives to take excessive risk. If the costs of market discipline outweigh

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\( ^2 \) “Quality of stress test disclosures a mixed bag”, FT 17/07/2011; Freixas [2010]; Peristian et al. [2010] about the U.S. bank stress test

\( ^3 \) “Quality of stress test disclosures a mixed bag”, FT 17/07/2011.

\( ^4 \) “Quality of stress test disclosures a mixed bag”, FT 17/07/2011.

\( ^5 \) “Power to the regulators”, FT 12/12/2011
the benefits, the regulator will be reticent about the severity of banks’ problems and investors will not take the stress test seriously. It is necessary to reduce the cost of market discipline in order to make the stress test informative. If market discipline leads to restructuring bad banks at no cost for taxpayers, there would be welfare gains from revealing bad banks because early liquidation would not occur. The regulator would remove completely adverse selection. As bad banks might not have the right incentives to restructure, the regulator must penalize bad banks not restructuring. Disclosure makes regulator’s threat credible if the social costs of early liquidation are larger than the costs of imposing a penalty on bad banks.

I build a model with two types of banks, competitive and risk neutral investors, and a regulator. Banks have to rollover a predetermined amount of debt $D$ at time 0 and 1, that is before and after a macroeconomic scenario realizes. The macroeconomic scenario can be favorable or adverse. There is a fraction $\beta$ of banks that would be well capitalized (type G) and a fraction $1 - \beta$ that would be undercapitalized (type B) in the adverse scenario. All banks are well capitalized in the favorable scenario. At time 0 banks can alter the riskiness of their assets by shifting risk. For example, banks could stop monitoring their borrowers. Risk shifting is socially inefficient. Letting banks choose risk allows me to model an endogenous reaction of banks to the stress test. Stress tests usually assume a static framework, but it seems reasonable that investors’ reaction to the stress test might affect the behavior of banks. I restrict the analysis to the case where undercapitalized banks shift risk, whereas well capitalized banks do not as long as debt repayment is not too large. This captures a situation of distress where equity is so low that even well capitalized banks might make inefficient choices if, for example, debt repayment is too large due to adverse selection. Investors decide whether to lend or not to the bank. If they do not lend, there is costly bank liquidation in the sense that assets are sold to inefficient users as in Acharya and Yorulmazer [2008]. Investors know the fraction of good and bad banks, but not their type. There is adverse selection in the adverse scenario because some banks are undercapitalized. To ease information asymmetries, the regulator carries out a stress test revealing which banks would be undercapitalized in a given macro scenario. Stress tests rely on the information disclosed by banks, which the supervisory authority checks and scrutinizes before revealing to the public. I model the strictness in carrying out this process through the effort choice $e$, which I define as the probability that the supervisory authority catches an undercapitalized bank in the pool of banks. The supervisory authority chooses effort maximizing aggregate welfare, which is the sum of banks’ and investors’ wealth net of bankruptcy and stress test costs.

I consider two cases: in the first the regulator just discloses the stress test results, whereas in the second it can also force restructuring of undercapitalized banks. In the first case, the regulator does not completely eliminate information asymmetries. The reason is that undercapitalized banks cannot rollover their debt in the adverse scenario and sell their assets to inefficient users in order to
satisfy creditors’ claims. The regulator reveals just the fraction of undercapitalized banks that makes well capitalized banks choose not to shift risk. If it revealed less undercapitalized banks, the adverse selection premium would be so high that even well capitalized banks would shift risk. In the second case, the regulator has the additional choice of whether to close banks in case of no restructuring. The regulator bears a cost B in case of bank bankruptcy or closure. I focus on the case where closure is socially optimal only if there is early bank liquidation in the adverse state. If investors are willing to lend in the adverse state, the regulator has incentives to forbear because the expected bankruptcy costs are lower than the bankruptcy costs it would bear at time 0. This captures a situation where early liquidation is so inefficient to give the regulator incentives to close down the bank at time 0 before the scenario realizes. I obtain that the regulator completely removes adverse selection because it can commit to close undercapitalized banks in case of no restructuring. Revealing undercapitalized banks becomes socially optimal because restructuring is socially efficient. Commitment is credible because forbearing would lead to the early liquidation of undercapitalized banks in the adverse state, which is more costly than closure. These results are consistent with the different experience with bank stress tests in the U.S. and Europe. The U.S. regulator guaranteed that weak banks would be recapitalized and, according to commentators, managed to restore confidence in banks better than the European regulator.

There are few papers about bank stress tests. Peristian et al. [2010] find that markets used the information from US stress tests to revalue banks. Hirtle et al. [2009] argue that the US stress test was successful because it combined a micro and macro prudential approach and provided a guarantee that weak banks would be recapitalized. Looking at the reaction of CDS and equity prices, Greenlaw et al. [2012] conclude that stress tests in the US were more successful than in Europe. My paper is closely related to the literature about banking supervision. It is important to highlight the differences between stress testing and traditional supervision. Stress tests aim at easing information asymmetries in financial markets. They focus on a relevant fraction of the banking system, and they are forward looking in that they simulate what would happen in a negative and plausible macro scenario. Supervision is typically targeted to individual banks and aims at countervailing the moral hazard incentives coming from the deposit insurance guarantee. A crucial difference is transparency: stress test results are disclosed, whereas supervisory information usually is not. My paper contributes to the debate about the disclosure of supervisory information. Jordan et al. [1999] find evidence that the market learns from the announcement of formal enforcement actions and argue in favor of disclosure. Prescott [2008] provides a theoretical argument against disclosure pointing out that it would reduce the incentives of regulators to collect information from banks. My paper suggests that disclosure is effective only together with policies in favor of

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6See for example Greenlaw et al. [2012]
7For example, Rochet [2004], Rochet [1992], Thakor [1996], and Furlong and Keeley [1990]
undercapitalized banks. My paper is related also to the literature about bank closure, and especially to Mailath and Mester [1994]. They show that closure is a credible threat only if the costs of closing down a bank are lower than the expected continuation costs. I have a similar tradeoff, but what matters in my model is the possibility of a bank run in case of continuation rather than the choice of the bank to do a safe project. Finally, it is important to mention the large literature about the methodological aspects of stress testing\(^8\).

The paper is structured as follows. Section 2 outlines the model setup. Section 3 presents the complete information equilibrium. Section 4 outlines the equilibrium with asymmetric information and without policies to fix undercapitalized banks. Section 5 presents the policies in favor of undercapitalized banks and Section 6 derives the equilibrium when they are implemented. Section 7 points out the role of disclosure. Section 8 concludes the paper.

2 The model setup

I consider a two periods game with a regulator, a continuum of risk neutral and competitive investors, and a measure of banks. Banks start out with a predetermined stock of debt \(D\) and a measure of assets. The return on assets depends on banks’ actions, banks’ type and an aggregate shock. There are two types of banks that differ in terms of the return on their assets: the high return \((A_H)\) and the low return \((A_L)\) in proportion \(\beta\) and \(1-\beta\). Banks’ return is private information. The probability of success and the return depend on an aggregate shock that realizes at time 1 and is public information. In the favorable (F) state (with probability \(1-p\)), banks will succeed for sure at time 2; in the adverse (A) state (with probability \(p\)) they will succeed only with probability \(\alpha_S\). The return in the favorable state is larger than in the adverse state for both types, that is \(A^{F}_j > A^{A}_j\) for \(j = \{H, L\}\). Banks can alter the return distribution by shifting risk at time 0. Risk shifting raises the return on assets by a factor \(\gamma\), but decreases the probability of success to \(\alpha_R < \alpha_S\) in the A state. Table 1 summarizes the payoff distribution.

<table>
<thead>
<tr>
<th>Return ((j = {H, L}))</th>
<th>Safe</th>
<th>Risky</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A^A_j) (\gamma A^{F}_j)</td>
<td>(A^A_j) (\gamma A^{F}_j)</td>
<td>(A^A_j) (\gamma A^{F}_j)</td>
</tr>
<tr>
<td>Probability of success</td>
<td>(\alpha_S)</td>
<td>(\alpha_R)</td>
</tr>
</tbody>
</table>

The interpretation of this payoff structure is in terms of stress scenarios.

\(^8\)See Quagliariello [2009] for an excellent survey
Stress tests simulate what the value of bank equity would be in future adverse and favorable scenarios. I model the adverse scenario as a situation where default risk increases since assets yield a lower return and only with some probability. For example, firms are less likely to repay their loans in a recession than in good times. As in the recent rounds of stress tests in Europe, an adverse scenario might feature a sovereign default which reduces the value of bonds in banks’ portfolio. Since I am interested in the adverse scenario, I simplify the favorable scenario assuming the bank has a zero probability of default. A common assumption in stress test exercises is the static portfolio assumption. The bank is supposed to replace assets coming to maturity in the period considered by the stress test with assets of the same characteristics. They rule out the possibility that the bank reacts to the disclosure of the stress test by changing the composition of its assets. I allow this possibility in my model giving banks the choice of risk at time 0. Investors condition the cost of funds to the stress test results, but banks might react by changing the asset risk. Especially under-capitalized banks might have incentives to take more risk because the disclosure of bad information negatively affects their funding conditions. I am going to make the following assumptions about the payoff structure:

**Assumption 1.** $A_i^L < A_i^H \leq \gamma A_i^L < \gamma A_i^H$ in both states $i = \{A, F\}$

Assumption 1 states that the return on assets of a type L bank doing the risky project is at least as large than the return on assets of type H doing the safe project.

A continuum of risk neutral and competitive investors decide to rollover $D$ at the beginning of period 1 and 2. Investors’ decision is based on the results of the stress test, the state of the world, and depends on the choice of risk. If investors do not rollover, the bank is forced to liquidate its assets at a price $L$.

I make the following assumptions about $L$:

**Assumption 2.** Assume that:

1. $L$ does not depend on the choice of risk
2. $L$ does not depend on banks’ type and $L \leq \alpha \gamma A_L^a$

The first of Assumption 2 is with no loss of generality because there will be liquidation in equilibrium only when banks do the risky project. The second of Assumption 2 represents a shortcut for fire sales. Liquidation will happen in my model only at time 1 when the A state realizes. It would be optimal to allocate the assets of distressed banks to other banks, but state A is so bad that these banks have no cash available. Assets must be sold to outside investors which can extract at most L out of them. Outside investors are inefficient users of banks’ assets because L is lower than what assets are worth in the hand of type L banks.

The regulator maximizes aggregate welfare attributing equal weights to investors and banks. I assume a cost $B$ in case of bank bankruptcy. This could
capture the direct costs of liquidation and the monetary outlay of the deposit insurance fund\textsuperscript{9}. I will consider two cases in the paper:

1. the regulator can only run a stress test;

2. the regulator can run a stress test and force recapitalization.

I model the stress test as a signaling technology that allows the regulator to detect a type H bank with no error and a type L bank only with probability $e$. The pool of banks considered as types H, that is the banks passing the stress test, is composed of all types H ($\beta$) and the fraction $(1 - \beta)(1 - e)$ of types L that are not detected. The stress test costs $K$. Costs include the time and resources banks and their supervisors have to spend in doing a stress test. This way of modeling the stress test captures the procedures followed by the European Banking Authority (EBA) and national central banks. Banks had to report their exposure to risk according to the criteria defined by the EBA. The EBA would then review the reports, check their consistency with the criteria, call for additional reporting or modifications, and finally disclose the results. I use the effort choice $e$ as a shortcut to model the accuracy of the review process. An example of regulators’ effort is the additional guidance that the EBA issued in June 2011 after banks presented preliminary stress test results. The EBA realized that some banks assumed too low losses on sovereign debt exposures, and no rise in interest rates in case of market turmoil. The new guidance imposed banks to consider tougher scenarios. The assumption that the stress test perfectly identifies type H banks but not type L banks captures the fact that review process is more important for weak banks, which are more likely to use loopholes to avoid disclosing information truthfully.

In case (1), the regulator chooses $e$ and whether to run a stress test before investors and banks play. In case (2), the regulator might also force banks to recapitalize threatening closure. Besides effort, the regulator chooses whether to close down a bank that did not recapitalize. The choice happens at $t = \frac{1}{2}$ after banks’ recapitalization choice and before the state of the world realizes. The assets of a closed bank are worth $C$. I make the following assumptions:

Assumption 3. Assume that:

1. $p \alpha \gamma A_L^A - p(1 - \alpha_R)B + (1 - p)\gamma A_L^F > C - B > p(L - B) + (1 - p)\gamma A_L^F$

2. $p(\alpha_S - \alpha_R)B \geq (1 - p)(\gamma - 1)A_L^F - p(\alpha_S - \alpha_R)A_L^A$

The first of Assumption 3 means that it is socially optimal to close a bank only if there is a bank run in state $A$. As Mailath and Mester [1994] pointed out, the decision to close a bank depends on the size of current and future expected monetary outlays by the regulator. As in Mailath and Mester [1994], in

\textsuperscript{9}I do not model explicitly insured depositors. If I had insured depositors together with enough uninsured investors, the only difference in the welfare function would be the monetary outlay by the deposit insurance fund in case of default. The bank would still be subject to the discipline of uninsured depositors.
my model forbearing allows the regulator to save on bankruptcy costs since the bank will not default with some probability. Nonetheless closure is optimal if a bank run happens in state A because liquidation at time 1 is highly inefficient. Forbearing is optimal only if investors are willing to lend to the bank in state A. Assumption 3 defines a scenario in which the costs of a bank run in state A are so large to give the regulator incentives not to forbear. This is the case when bankruptcy costs B are not too large. The second of Assumption 3 means that type L banks doing the safe project is socially optimal. The gain in terms of lower expected bankruptcy costs overweight the gain in terms of expected returns. This assumption will make it optimal for the regulator to give type L incentives to do the safe project.

The structure and timing of the game is the following:

\[
\begin{array}{cccccc}
-\frac{1}{2} & 0 & \frac{1}{2} & 1 & 2 \\
\text{Regulator chooses to run the stress test} & \text{Banks choose risk} & \text{Regulator chooses to close undercapitalized banks} & \text{State F/A realizes} & \text{Payoffs realize} \\
\text{Regulator chooses effort} & \text{First debt rollover} & \text{State F/A realizes} & \text{Second debt rollover} & \\
\text{Regulator discloses the stress test} & \text{Banks raise capital} & & & \\
\end{array}
\]

The regulator chooses whether to run the stress test and effort at the beginning of the game. The regulator also discloses the results in case it decides to run the stress test. Banks choose risk and investors decide the first rollover at time 0. The choice of banks and investors is simultaneous and conditional on knowing some type L banks and the composition of the pool of banks that pass the stress test. This captures the fact that investors use the information provided by the stress test for their lending decisions, and the feedback effect of the stress test on banks' assets value through the risk choice. This point is typically neglected by regulators, which use a static framework in which the bank is not supposed to react to the release of information. At time 0, banks raise capital if required by the regulator. Conditional on observing the recapitalization choice, and before the state of the world realizes, the regulator decides whether to close the banks that were required to raise capital and did not. At time 1 the state of the world realizes and investors decide the second rollover. This is meant to capture the information value of the stress test. Sorting out the solvent from insolvent banks is necessary to guarantee that investors lend to banks. Finally, payoffs realize at time 2.

3 Full Information Benchmark

In this section I will outline the full information equilibrium with no policy and present the benchmark case I will focus on. At time 1, investors decide the debt rollover conditional on the state of the world, bank’s type and bank’s monitoring choice. Consider the adverse state (state A). Investors are willing
to lend to type $j = \{H, L\}$ if

$$
\alpha_s \min\{A_j^A, DR_0^{S, j}, R_{A, 1}^{S, j}\} = DR_0^{S, j},
$$

given type $j$ has chosen the safe project and

$$
\alpha_r \min\{\gamma A_j^A, DR_0^{R, j}, R_{A, 1}^{R, j}\} = DR_0^{R, j},
$$

given type $j$ has chosen the risky project. In case this conditions do not hold, investors run the bank and recover $L$. In state $F$, both projects succeed with probability one. Investors are willing to lend if

$$
A_F^j \geq DR_0^{S, j}
$$

In state $A$, debt is riskless at time 0 because investors expect to break even in both states of the world. The time 0 break even interest rate is $R_0^i$ for both project choices $i = \{R, S\}$. In case investors anticipate a bank run in state $A$, they are willing to lend at time 0 if

$$
pL + (1-p) \min\{A_j^F, DR_0^{S, j}\} = DR_0^{S, j},
$$
in case type $j$ does the safe project$^{10}$. The value of debt is $L < DR_0^{S, j}$ because investors run in state $A$.

Banks choose risk at time 0 taking into account investors’ lending decisions. Type $j$ banks choose the safe project if

$$
p\alpha_s (A_j^A - DR_0^{S, j} R_{A, 1}^{S, j}) + (1-p) (A_j^F - DR_0^{S, j} R_{F, 1}^{S, j}) \geq
p\alpha_r (\gamma A_j^A - DR_0^{R, j} R_{A, 1}^{R, j}) + (1-p) (\gamma A_j^F - DR_0^{R, j} R_{F, 1}^{R, j}).
$$

Given investors beliefs, type $j$ banks choose the safe project if it yields a profit larger than the risky project. Note that profits in state $A$ are nil if investors run the bank. Banks make profits only in state $F$. Since the risky project gives the largest profits in state $F$, type $j$ banks choose the risky project if they anticipate a run by investors in state $A$. Banks choose the safe project only if they expect investors to lend in state $A$. Condition (4) can be rewritten as

$$
pA_j^A (\alpha_s - \alpha_r \gamma) + (1-p)A_j^F (\gamma - 1) \geq p(\alpha_s - \alpha_r) \frac{D}{\alpha_s}.
$$

Type $j$ does the safe project if the marginal gain in terms of expected return outweighs the marginal cost in terms of debt repayment.

The interesting case is when investors run type $j$ banks in case they choose the risky project but not when they choose the safe$^{11}$. and type $j$ has incentives to do the safe project (equation (4) holds). The choice of banks is governed by

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$^{10}$Replace $A_j^F$ with $\gamma A_j^F$ to have the equivalent condition when type $j$ does the risky project.

$^{11}$If investors would run type $j$ independently of the project choice, type $j$ would always do the risky project.
investors’ beliefs in this case. If investors believe type j chooses the safe project, they will charge an interest rate such that type H finds the safe project optimal. If investors hold the opposite belief, they will charge a higher interest rate such that type j banks choose the risky project because investors do not lend in state A. The game between investors and banks has multiple equilibria.

Models featuring multiple equilibria cannot predict agents’ behavior with certainty. To get rid of this unattractive feature, I assume that a fraction $\phi$ of investors believes type j chooses the safe project (S investors) whereas the complementary fraction $1 - \phi$ holds the opposite belief (R investors). The intuition behind this assumption is the following. Given the average of the lending choices by the two types of investors, type H will prefer either the risky or the safe project. As a result, either S or R investors have wrong beliefs which they must adjust according to the optimal choice of type H. As a result, the equilibrium becomes unique.

Consider an equilibrium in which type j does the safe project and there are $\phi$ type S investors and $1 - \phi$ type R investors. R investors do not lend in state A because they believe type j has chosen the risky project. Type j is forced to liquidate a fraction $\lambda$ of assets such that

$$\lambda L = (1 - \phi) DR_{0}^{S,j}. \quad (6)$$

This equation states that liquidation proceeds (the fraction of liquidated assets $\gamma$ times the liquidation price L) must satisfy the total claims of R investors. S investors believe type L is solvent, but see R investors running the bank. S investors lend to type j in state A if

$$\alpha_{s} \min\{(1 - (1 - \phi)DR_{0}^{S,j})A_{j}^{r}, \phi DR_{0}^{S,j} R_{A,1}^{S,j}\} = \phi DR_{0}^{S,j}. \quad (7)$$

Given the previous two equations hold, both S and R investors break even at time 1 and charge the risk free interest rate at time 0. Type j will do the safe project if it yields the maximum profit, that is if the following inequality holds:

$$p\alpha_{s} \left( A_{j}^{s} \left( 1 - (1 - \phi)D \frac{1}{L} - \phi D \frac{1}{\alpha_{s}} \right) + (1 - p) \left( A_{j}^{r} - D \right) \right) \geq$$

$$p\alpha_{r} \left( \gamma A_{j}^{s} \left( 1 - (1 - \phi)D \frac{1}{L} - \phi D \frac{1}{\alpha_{s}} \right) + (1 - p) \left( \gamma A_{j}^{r} - D \right) \right). \quad (8)$$

The left hand side is the profit from the safe project. In state A, R investors run the bank which is left with $1 - (1 - \phi)DR_{0}^{S,j}$ assets. There is no bank run in state F because banks are well capitalized. The right hand side is the profit from the risky project keeping investors beliefs fixed. If this inequality holds, type j chooses the safe project even if type R investors do not lend in state A. Type R
investors adjust their beliefs, and no bank run occurs in equilibrium in state A\textsuperscript{12}.

In the rest of the paper, I will restrict the analysis to the scenario defined in Case 1.

**Case 1.** Consider a scenario where:

1. Conditions (1) and (2) do not hold for type L banks;
2. Condition (1) holds and (2) does not for type H banks;
3. Condition (3) holds for both types;
4. \[
\Delta ER_H \equiv pA^A_H (\alpha_S - \alpha_H \gamma) - (1 - p)A^F_H (\gamma - 1) \geq 0
\]
   \[
\Delta ER_L \equiv pA^A_L (\alpha_S - \alpha_H \gamma) - (1 - p)A^F_L (\gamma - 1) < 0
\]
5. Condition (8) holds for type H banks.

Points (1) and (2) mean that type L banks are undercapitalized in state A, whereas types H are well capitalized in both states unless they do the risky project. Type L banks are those that should not pass the stress test. In this scenario, type L banks have incentives to do the risky project because it yields more in case of success. Point (3) makes sure that both types of banks are solvent at time 0. This captures the forward-looking nature of bank stress tests.

The aim is to check whether banks will be solvent in a plausible future scenario. Evidence following bank stress tests suggests that undercapitalization in the adverse scenario usually implies an increase in financing costs but not a run by investors. Point (4) refers to the banks' incentive constraint (condition (5)). I focus on a case where type H banks might prefer doing the safe project, whereas type L banks strictly prefer the risky project\textsuperscript{13}. Policies should be designed to give type L incentives to do the safe project, which is socially optimal by Assumption (3). Point (5) means that type H banks choose the safe project.

Adding asymmetric information to such a scenario, a tradeoff arises for the regulator. The choice of type L bank will create adverse selection in the market for funds. The adverse selection premium might make the interest rate so large that type H banks prefer choosing the risky project. The regulator should reveal type L banks in order to mitigate adverse selection, but this is costly because investors run type L banks in state A. Policies should aim at recapitalizing type L banks and give them incentives to do the safe project. Since type L strictly prefer the risky project, policies should define a cost in case no capital is raised.

\textsuperscript{12}Note also that condition (4) can hold only if \( A^A_j (1 - (1 - \phi)DR_{S,j}) - \phi D_S > 0 \), which validates the statement that S investors are willing to lend in state A.

\textsuperscript{13}The condition in point (4) is the left hand side of condition (5). If this is negative, the bank has no incentive to do the safe project.
The threat of bank closure might represent a way to force type L banks to recapitalize and give the regulator incentives to completely remove adverse selection.

A further assumption is necessary before proceeding to the asymmetric information case.

**Assumption 4.**

\[
A_H < \frac{p_0 D^{\alpha_S - \alpha_R}}{\Delta E R_H - p(\alpha_S - \alpha_R \gamma) \frac{(1 - \phi) D}{L}}
\]

Assumption 7 makes sure that the optimal effort level chosen by the regulator is such that \(e \in [0, 1]\).

## 4 The Asymmetric Information Equilibrium with a stress test

In this section I consider the case of asymmetric information about banks’ type. The regulator carries out a stress test, which consists in the disclosure of a public signal about banks’ type. The regulator chooses \(e\), which is the probability that a type L bank fails the stress test. Since there is a continuum of banks of measure 1, \(e\) also determines the number of banks that fail the stress test. The stress test reveals \(e(1 - \beta)\) type L banks out of \(1 - \beta\). The remaining \((1 - \beta)(1 - e)\) type L banks pool together with type H banks. The stress test is the only policy tool available to the regulator. At time 1 investors decide whether to rollover conditional on this information, the state of the world and banks’ monitoring choice. By By Case (1), type L banks choose the risky project. Lemma 5 summarizes investors’ decision.

**Lemma 1.** Investors do not lend to the banks that fail the stress test in state A and lend to them at the risk free rate in state G. Investors lend to the pool of banks that pass the stress test at the risk free rate in state F. In state A, they lend to the pool of banks that pass the stress test only if type H chooses the safe project, type L the risky, and \(e \geq \frac{D R_{S,p}^A - (\beta \alpha_S + \alpha_R (1 - \beta)) A_H}{(1 - \beta)(D R_{S,p}^A - \alpha_R A_H)} \equiv \mathcal{E}_{end}\). They charge the rate \(R_{A,1}^S = \frac{\beta + (1 - \beta)(1 - e)}{\beta + (1 - \beta)(1 - e)}\) in this case.

**Proof.** The result for type L banks comes from Section 2. As regards the pool of banks that pass the stress test, consider the state A and the case where type H does the safe and type L the risky. The break even condition is the following:

\[
\frac{\beta}{\beta + (1 - \beta)(1 - e)} \alpha_S \min \{ A_H^A, DR_{S,p}^A, R_{A,1}^S \} +
\frac{(1 - \beta)(1 - e)}{\beta + (1 - \beta)(1 - e)} \alpha_R \min \{ \gamma A_L^A, DR_{S,p}^A, R_{A,1}^S \} = DR_{S,p}^L
\]
At time 1 the bank has to rollover $DR^S_0$, that is the face value of debt plus the time 0 interest rate. Investors know there is adverse selection in the pool of banks that pass the stress test. There is probability $\frac{\beta}{\beta + (1-\beta)(1-e)}$ of lending to a type H bank. Investors are willing to lend to the pool of bank if $A^A_H \geq DR^S_0 R^S_{A,1}$, that is if type H banks are able to pay back. If not, the only borrowers in the pool would be type L banks. Since they do not monitor, type L banks would not be able to pay back and investors would not lend to the pool of banks. Substituting $A^A_H \geq DR^S_0 R^S_{A,1}$ in the break even condition, the interest rate in the Lemma follows. The condition on effort results from the assumption that type H banks are able to pay back investors.

In case type L does the safe and type H the risky, investors run the pool in state A independently of the effort choice because both banks are undercapitalized. The same is true when both types choose the risky project. When both types choose the safe project, the break even condition is

$$\frac{\beta}{\beta + (1-\beta)(1-e)}\alpha_s \min\{A^A_H, DR^S_0 R^S_{A,1}\} + \frac{(1-\beta)(1-e)}{\beta + (1-\beta)(1-e)}\alpha_s \min\{A^L_H, DR^S_0 R^S_{A,1}\} = DR^S_0$$

Type L banks are insolvent if they do the safe project. They cannot promise to repay the interest rate $R = \frac{1}{\alpha_s}$. This implies that the only type borrowing in state A will be type H. Type H will reveal himself and there would be no pooling.

If a bank fails the stress test, investors learn that the bank must be of type L. Since type L banks choose the risky project, investors run the bank in state A and lend at the risk free rate in state F as in the complete information case. Investors still face adverse selection in the pool of banks that pass the stress test. Investors are willing to lend to the pool of banks only if type H chooses the safe project and the regulator puts enough effort. The reason is that type L banks are insolvent independently of the project choice. Investors are willing to lend to the pool of banks that pass the stress test only if the regulator removes enough of them from the pool by putting effort. If effort is too little, investors run the pool of banks in state A unless the condition on $\beta$ in Corollary 1 holds.

**Corollary 1.** In case $\beta \geq \frac{DR^S_0 - \alpha_s A^A_H}{(\alpha_s - \alpha_m)A^A_H}$, investors lend to the pool of banks that pass the stress test even if $e = 0$ when type H chooses the safe project and type L the risky.

**Proof.** Thresholds on $\beta$ are obtained imposing that $\Omega_{end} \geq 0$.

As long as type H banks choose the safe project, asymmetric information does not prevent investors from lending to the pool of banks that pass the stress
test if $\beta$ is large enough. Intuitively, adverse selection is not a problem if enough banks in the pool are solvent.

At time 0 banks make the project choice. At the same time, investors decide whether to lend conditional on the results of the stress test and on banks’ choice. Investors also condition on their equilibrium action at time 1. Lemma 2 summarizes investors’ behavior at time 0.

**Lemma 2.** Investors lend to the banks that fail the stress test at the rate $R_{0}^{\alpha, L} = \frac{D-pL}{(1-p)D}$. They charge the same rate to the pool of banks that pass the stress test if they expect not to rollover in state A. In case they do, investors lend to the pool of banks that pass the stress test at the risk free rate.

**Proof.** Failing the stress test perfectly reveals type L banks. As regard the pool of banks, investors charge the risk free rate at time 0 when they know they will break even in both states of the world. If they run the pool of banks in state A, the interest rate at time 0 contains a risk premium because investors will not be able to fully recover the face value of debt.

The decision to lend to banks that fail the stress test is the same as in the complete information case because a stress test perfectly reveals type L banks. Investors face the risk of losing part of their money in case they do not rollover in state A. As a consequence, investors charge a risk premium. Investors lend at the risk free rate if they anticipate they will lend to the pool of banks that pass the stress test even in state A.

At time 0 banks choose the project taking into account the interest rates charged by investors. Lemma 3 summarizes banks’ choice.

**Lemma 3.** Type L banks choose the risky project independently of the stress test result. Type H banks choose the safe project if:

$$e \geq 1 - \frac{\beta}{1-\beta} \frac{\alpha_s \left[ A_{R} \Delta ER_{H} - p(\alpha_s - \alpha_r \gamma) \left(1 - \phi D\right)\right] - p(\alpha_s - \alpha_r) \phi D}{\alpha_s \left[ A_{N} \Delta ER_{H} - p(\alpha_s - \alpha_r \gamma) \left(1 - \phi D\right)\right]} \equiv \xi_{M},$$

with $\Delta ER_{H} \equiv pA_{R}^{\delta} (\alpha_s - \alpha_r \gamma) - (1-p)A_{N}^{\delta} (\gamma - 1)$. Type H banks choose the risky project if $e < \xi_{M}$.

**Proof.** Type L banks that fail the stress test choose the risky project by Case 1. Case 1, and the fact that the pooling interest rate is larger than the interest rate in case banks do the safe project, imply that type L banks that pass the stress test also choose the risky project. The proof of the result on type H banks is the following. Assume that type H chooses the safe project, but only $\phi$ investors (S investors) believe so. The complementary fraction of investors (R investors) believes type H does the risky. Type R investors run the pool of banks in state
A and force each of them to liquidate the fraction of assets satisfying equation (6). Type S investors are willing to lend to the pool of banks in state A if

\[
\frac{\beta}{\beta + (1 - \beta)(1 - e)} \alpha_S \min\{(1 - \frac{(1 - \phi)D}{L})A^S_H, \phi DR_0^{S,p} R_{A,1}^{S,p}\} + \\
\frac{(1 - \beta)(1 - e)}{\beta + (1 - \beta)(1 - e)} \alpha_R \min\{\gamma A^R_L (1 - \frac{(1 - \phi)D}{L}), \phi DR_0^{S,p} R_{A,1}^{S,p}\} = \phi DR_0^{S,p}
\]

Given this holds, the incentive constraint for type H is the same as in equation (8) except that the interest rate is \(R_{A,1}^{S,p}\) and not \(\gamma\). Rewriting the incentive constraint in terms of effort gives the result. The threshold \(e_M\) is lower than 1 by Assumption 4.

The result for type L banks that pass the stress test follows from Case 1, which states that type L banks choose the risky project in the complete information case. This results implies that type L banks that pass the stress test also choose the risky project. The pooling interest rate is larger than the interest rate type L would be charged when doing the safe project because some of the banks passing the stress test are doing the risky project. Since type L banks choose the risky project when investors believe they do the safe, type L banks still choose the risky project if they pass the stress test and pool with type H banks. Type L banks that pass the stress test exert an externality on type H banks. By Case 1, type H banks do the safe project in the complete information case. In the asymmetric information case, Lemma 3 states that this happens only if \(e \geq e_{lend}\). If effort is lower, type H banks choose the risky project, unless the condition in Corollary 2 holds. The intuition is that type H banks have to pay an adverse selection premium which is decreasing in regulator’s effort. When effort is too low, the debt repayment gets so large that type H banks prefer to choose the risky project. Note that the choice of the safe project by type H implies that investors are willing to lend to the pool in state A, that is \(e \geq e_{lend}\). The intuition is that type H chooses the safe project if the profits in state A are large enough, given the profits in state F are always smaller than for the risky project. High profits in state A guarantee the solvency of type H.

**Corollary 2.** Type H banks do the safe project even when \(e = 0\) if

\[
\beta \geq \frac{p(\alpha_s - \alpha_n)\phi D - \alpha_n}{\alpha_s - \alpha_n} \left[\Delta ER_n - p(\alpha_s - \alpha_n)\frac{(1 - \phi)D}{L}\right] \equiv \beta
\]

**Proof.** The threshold on \(\beta\) comes from imposing \(e_M \leq 0\).

Intuitively, the adverse selection premium is decreasing in the fraction of banks that monitor their assets (\(\beta\)). When this fraction is large enough, the debt repayment is so low that type H banks have incentives to monitor even if
there is asymmetric information.

At time 0, the regulator chooses effort maximizing aggregate welfare. The regulator cannot affect investors’ beliefs, but internalizes the externality that type L banks exert on type H banks. This implies that the welfare function has a discontinuity in $e = e_M$. If effort is larger than this threshold, the welfare function is the following:

$$W = e(1 - \beta) \left[ p(L - B) + (1 - p)\gamma A_L^F \right] + \beta [\rho \alpha_s A_H - p(1 - \alpha_s)B + (1 - p)A_H] + (1 - \beta)(1 - e) \left[ p\rho \gamma A_L^A - p(1 - \alpha_n)B + \gamma A_L^A (1 - p) \right] - K$$

(9)

Note that $e(1 - \beta)$ type L banks fail the stress test and choose the risky project. Investors do not rollover their loans to type L banks in state A. The assets of type L banks are liquidated and become worth L. In state F, type L banks get a return $\gamma A_L^F$. In the pool of banks that pass the stress test, type H banks choose the safe project and type L banks choose the risky. Contrary to those failing the stress test, types L passing the stress test can borrow in state A because investors break even on average. The regulator bears a cost B in case the bank defaults. If effort is lower than $e_M$, aggregate welfare is given by:

$$W = e(1 - \beta) \left[ p(L - B) + (1 - p)\gamma A_L^F \right] + (1 - \beta)(1 - e) \left[ p\rho \gamma A_L^A - p(1 - \alpha_n)B + \gamma A_L^A (1 - p) \right] - K$$

(10)

Since type H banks choose the risky project when $e < e_M$, banks that pass the stress test cannot rollover their debt in state A. In both welfare functions K denotes the cost of the stress test. The regulator chooses the optimal level of effort maximizing aggregate welfare. Proposition 1 illustrates the result.

**Proposition 1.** The optimal effort level is $e_M$.

**Proof.** Maximization of the welfare function (9) subject to $e \geq e_M$ yields the following first order condition:

$$p(1 - \beta)(L - \alpha_n\gamma A_L^A - \alpha_n B) + \mu = 0$$

where $\mu$ is the multiplier of the constraint on effort. The solution features $p(1 - \beta)(\alpha_n\gamma A_L^A - L) = \mu$, and hence $e^* = e_M$.

The welfare function (10) does not depend on effort. Any effort level $e \in [0, e_M)$ is optimal.

The condition in Proposition 1 comes from rearranging the inequality $W[e^* = e_M] \geq W[e < e_M]$. The condition is
\[ \beta \left[ p(\alpha_r A_h^A - L) - (1 - p)A_h^F(\gamma - 1) + p\alpha_s B \right] + p\beta \left[ \alpha_h(\gamma A_h^A + B) - L \right] \Gamma \geq 0, \]

where \( \Gamma \equiv \frac{\sigma_s[A_h^A \Delta NPV - p(\alpha_r - \alpha_h)\gamma L - p(\alpha_r - \alpha_h)\phi D - \alpha_h A_h^A \Delta NPV - p(\alpha_r - \alpha_h)\gamma L]}{p(\alpha_r - \alpha_h)\phi D - \alpha_h}. \) Note that the first term is positive because it is socially optimal that type H does the safe project. The second term is positive by Assumption (2) and the fact that \( \Gamma > 0 \) (see Lemma (3)).

The regulator chooses the minimum level of effort when \( e \in [e_M, 1] \), but any level of effort is optimal when \( e \in [0, e_M) \). In the first case, the higher effort the larger the number of type L banks that are inefficiently liquidated. In the second case, effort does not affect the number of banks that are inefficiently liquidated because investors run also the pool of banks that pass the stress test. Proposition 8 states that the regulator chooses \( e = e_M \) because \( W[e^* = e_M] \geq W[e < e_M] \). Such a level of effort guarantees that banks that pass the stress test can rollover their debt in state A because type H banks choose the safe project. This is optimal because the gains from preventing a run on the pool of banks that pass the stress test overweight the costs from a run on a fraction of type L banks.

In the first stage of the game the regulator decides whether to run a stress test. The regulator runs a stress test if

\[ \beta \left[ p(\alpha_r A_h^A - L) - (1 - p)A_h^F(\gamma - 1) + p\alpha_s B \right] + p\beta \left[ \alpha_h(\gamma A_h^A + B) - L \right] \Gamma \geq K. \]

Condition (11) states that the welfare gains from running the stress test are larger than the costs\(^{14}\). If the regulator does not run a stress test, investors will not be willing to lend to the pool of banks in state A, unless \( \beta \geq \beta \). A stress test prevents the inefficient liquidation of a fraction of banks in state A. The regulator runs a stress test for reasonable values of K, as the welfare gains are likely to be much larger than the costs of a stress test. Note that this is not the case if \( \beta \geq \beta \). By Corollary 2, banks manage to rollover their debt in state A because type H banks choose the safe project even if there is asymmetric information. A stress test is not necessary and there is no reason for the regulator to bear its costs.

This result is consistent with the European stress test experience. Investors’ solvency concerns worsened the financing conditions of European banks after the financial crisis. To avoid repeating a market freeze as in 2007, the regulator decided to run a stress test and provide information to investors. Investors reacted skeptically to the stress test. According to my model, the reason is that investors took into account regulator’s incentives to withhold bad information. In Section 6 I will introduce policies to fix undercapitalized banks in my model

\(^{14}\)See the proof of Proposition 11 for the welfare gains
and provide an explanation for the different success of stress tests in Europe and the US.

5 The policies for undercapitalized banks

The equilibrium when stress tests are the only policy tools is inefficient. Additional policies must complement the stress test. These policies should aim at recapitalizing type banks and give them incentives to do the safe project. This will make sure that investors will not run and the regulator will not face any cost in revealing banks’ type. Policies complementary to the stress test must also be costless to taxpayers. If they are not, the regulator would have weak incentives to reveal type L banks because of the costs in terms of taxpayers’ money rather than inefficient bank liquidation. In this section I consider two policies that satisfy these criteria:

1. Forcing banks to raise capital from the private market: the regulator tells the bank that if it does not raise capital it will be closed.
2. Debt for equity swaps: bank creditors exchange their debt claims for equity claims

Both policies work through an increase in the value of bank’s equity. Capital raising plans force the bank to raise equity directly, whereas debt for equity swaps increase the value of equity by reducing the value of debt. It is important that both policies define the amount of additional equity to be raised, in order to make sure type L banks have incentives to do the safe project and become solvent.

5.1 Raising capital from private markets

It is socially optimal that type L banks choose the safe project. The regulator can only give incentives through capital because it cannot control investors beliefs. Investors’s belief that type L is choosing the risky project might be confirmed in equilibrium if recapitalization is not sufficiently large. In order to determine the capital need, consider an equilibrium in which type L raises capital and does the safe project. As in Section 3, assume there are φ S investors and 1 − φ R investors, and that capital is such R investors run the bank in state A. Type L is forced to liquidate a fraction λ of assets such that

\[
\lambda L = \max\{(1 - \phi)DR_0 - E, \ 0\}.
\]

This equation states that liquidation proceeds (the fraction of liquidated assets γ times the liquidation price L) must satisfy the total claims of R investors. The bank liquidates assets only if the additional capital it raised is not enough to
meet the obligations towards R investors. S investors believe type L is solvent, but see R investors running the bank. S investors lend to type L in state A if

\[
\alpha_s \min\{1 - \frac{\max\{(1 - \phi)DR_0 - E, 0\}}{L}\}A_L + \max\{E - (1 - \phi)DR_0, 0\}, \phi DR_0 R_1\} \ni \\
(1 - \alpha_s) \min\{\max\{E - (1 - \phi)DR_0, 0\}, \phi DR_0 R_1\} = \phi DR_0.
\]

To understand this equation, note that the bank does not need to liquidate its assets if the equity the bank has raised is larger than the claims by R investors. Type S investors would be left with the return on the total bank assets and the fraction of additional capital left after the run by R investors. Given the previous two equations hold, both S and R investors break even at time 1 and charge the risk free interest rate at time 0. Type H will do the safe project if it yields the maximum profit, that is if

\[
p\alpha_s \left( A_L (1 - \frac{\max\{(1 - \phi)D - E, 0\}}{L}) + \max\{E - (1 - \phi)D, 0\} - \phi DR_1 \right) \\
(1 - p) (A_L + E - D) \geq \\
p\alpha_s \left( \gamma A_L (1 - \frac{\max\{(1 - \phi)D - E, 0\}}{L}) + \max\{E - (1 - \phi)D, 0\} - \phi DR_1 \right) \\
+ (1 - p) (\gamma A_L + E - D).
\]

The left hand side is the profit from the safe project. In state A, R investors run the bank which is left with \(1 - \frac{\max\{(1 - \phi)D - E, 0\}}{L}\) assets. There is no bank run in state F because banks are well capitalized. The right hand side is the profit from the risky project keeping investors beliefs fixed. If the regulator requires an amount of equity such that this inequality holds, type L does the safe project in equilibrium and R investors adjust their beliefs. No bank run will occur in equilibrium in state A. Investors will be willing to lend at the rate

\[
R_s^{A,1} = \frac{D - (1 - \alpha_s)E}{\alpha_s D}.
\]

This policy can be implemented only if there are new shareholders willing to invest E in the bank and if shareholders of type L banks are willing to raise capital. Since the amount of capital raised is such that type L does the safe project, the value of type L banks after recapitalization is

\[
V = \rho \alpha_s (A^{A}_L - DR_{A,1} + E) + (1 - p)(A^{A}_L - D + E - E) \\
\rho \alpha_s A^{A}_L + (1 - p)A^{r}_L - D + E.
\]

The total value V is split between old and new shareholders. New shareholders get E, so that they are indifferent between investing and not investing. Old shareholders get \(P\), which equals the bank value after the capital injection net
of the capital injection. It holds that

\[ P = p\alpha_s (A_L - DR_{A,1} + E) + (1 - p)(A_L - D + E) - E \]
\[ \rho \alpha_s A_L + (1 - p)A_L - D. \]

Old shareholders are willing to raise capital:

\[ P = p\alpha_s A_L + (1 - p)A_L - D \geq \Omega, \]  \hspace{1cm} (15)

where \( \Omega \) is the outside option of old shareholders. If shareholders decide not to raise capital, and the regulator does not force them, they choose the risky project. Investors do not lend in state A and charge a risk premium at time 0. The outside option for old shareholders equals the profit from the risky project, that is \( \Omega = pL + (1 - p)\gamma A_L^L \). Condition (15) is not satisfied in the scenario I am considering in this paper (see Case (1)). If the regulator can commit to close the bank in case no capital is raised, \( \Omega = 0 \) and recapitalization occurs in equilibrium.

5.2 Debt for equity swaps

Debt for equity swaps are contracts for which bank creditors exchange their debt claims for equity claims. The bank raises capital indirectly through the reduction in the face value of debt. As recapitalization from private markets, debt for equity swaps must be designed in a way that type L banks have an incentive to participate and choose the safe project, and type L creditors are willing to participate.

Define \( E = D - D' \) the capital raised through the reduction of the face value of debt from \( D \) to \( D' \). The incentive constraint and the participation constraint of type L banks is still given by condition (14) and (15). Bank creditors are willing to exchange debt for equity claims if it yields no losses to them. As new shareholders in case of recapitalization from private markets, bank creditors get \( E = (D - D') \) in case they become shareholders. In case bank creditors do not accept the debt for equity swap, their debt claim is worth \( E = D - D' \) since type L banks are solvent at time 0. Bank creditors accept the debt for equity swap because they are indifferent between becoming shareholders and remaining creditors.

6 The equilibrium with policies for undercapitalized banks

Consider now the case where the regulator runs and discloses a stress test and might force type L banks to raise capital by threatening closure\footnote{Type L banks can raise capital either through recapitalization from private markets or debt for equity swaps. The analysis is exactly the same for both policies.}. Consider a
situation where the stress test reveals $e(1 - \beta)$ type L banks out of $1 - \beta$. The remaining $(1 - \beta)(1 - e)$ type L banks pool together with type H banks. At time 1 investors decide whether to rollover debt conditional on the stress test result, the state of the world, and banks’ project and recapitalization choices. Lemma 1 characterizes investors’ lending choice to the banks that pass the stress test and those that fail the stress test and do not raise capital. Banks that fail the stress test and raise capital choose the safe project. Investors lend at the interest rate 

$$R^{L,0}_{A,1} = \frac{DR^L_S - (1 - \alpha_S)E}{(1 - p)B}. \tag{16}$$

At time $\frac{1}{2}$, the regulator chooses whether to close type L banks that failed the stress test and did not raise capital. The following Lemma states the optimal choice.

**Lemma 4.** The regulator closes all banks that failed the stress test and did not raise capital.

**Proof.** By Assumption 3, $C - B \geq p(L - B) + (1 - p)\gamma A_L^{e}$. \hfill \Box

The reason why the regulator closes all banks failing the stress test and not raising capital is that bank runs in state $A$ are so inefficient that the regulator is willing to bear the utility cost due to bank closures.

At time 0, investors decide the first debt rollover, banks choose the project and whether to recapitalize. Since closure represents a credible threat by Lemma 4, type L banks that fail the stress test have incentives to recapitalize. Investors lend at the risk free rate to these banks because they become solvent. Lemma 2 characterizes investors’ lending choice to the banks that pass the stress test. Banks that fail the stress test and raise capital choose the safe project because it is incentive compatible. The choice by the banks that pass the stress test is described in Lemma 3.

At time $-\frac{1}{2}$, the regulator chooses effort anticipating its optimal closure policy and banks’ project and recapitalization choices. As in the case without policies, the welfare function has a discontinuity in $e = \xi_M$. Type H banks choose the safe project and prevent investors’ run on the pool of banks that pass the stress test only if effort is larger than this threshold. In the case $e \geq \xi_M$, the welfare function is

$$W = e(1 - \beta) [p\alpha_A A_L - p(1 - \alpha_S)B + (1 - p)A_L] + \beta [p\alpha_A A_H - p(1 - \alpha_S)B + (1 - p)A_H] + \beta \left[p\alpha_S \gamma A_L^e - p(1 - \alpha_S)B + \gamma A_L(1 - p)\right] - K \tag{16}$$

As in the case where stress tests are the only policy tool, $e(1 - \beta)$ type L banks fail the stress test. The difference is that now the regulator gives them incentives to do the safe project forcing them to raise capital through the threat of closure.
The additional capital $E$ does not appear in the welfare function because it is just a transfer from capital holders to banks. It appears indirectly through the choice of the safe project by type L banks. The part of the welfare function regarding the pool of banks that pass the stress test is unchanged because policies do not affect them. If effort is lower than $e_M$, aggregate welfare is

$$ W = e(1 - \beta) [p(\alpha_S A_L - p(1 - \alpha_S)B + (1 - p)A_L] + \beta [p(L - B) + (1 - p)\gamma A_H] + + (1 - \beta)(1 - e) [p(L - B) + \gamma A_L (1 - p)] - K $$

(17)

Since type H banks do not monitor when $e < e_M$, investors always run the pool of banks in state A. The banks that fail the stress test are still forced to raise capital. In both welfare functions $K$ denotes the cost of the stress test. The regulator chooses the optimal level of effort subject to $e \geq e_M$ and $e < e_M$. The regulator compares the two and chooses the effort level that maximizes aggregate welfare. Proposition 2 illustrates the result.

**Proposition 2.** The optimal effort level is $e^* = 1$.

**Proof.** Maximization of the welfare function (16) subject to $e \geq e_M$ yields the following first order condition:

$$ (1 - \beta)(A_L \Delta ER + p(\alpha_S - \alpha_H)B) + \mu - \delta = 0 $$

where $\mu$ and $\delta$ are the multipliers of the constraints on $e \geq e_M$ and $e \leq 1$. The solution features $(1 - \beta)(A_L \Delta ER + p(\alpha_S - \alpha_H)B) = \delta$, and hence $e^* = 1$.

Maximization of the welfare function (17) subject to $e < e_M$ yields the following first order condition:

$$ (1 - \beta)(p(\alpha_S A_L - L) - (1 - p)(\gamma - 1)A_L + p\alpha_S B) > 0 $$

The solution in this case would be the highest effort level such that $e < e_M$. The result in the Proposition follows from the fact the inequality $W[e^* = 1] \geq W[e = e_M]$ always holds.

The regulator chooses $e^* = 1$ because revealing undercapitalized banks is efficient since they raise capital and do the safe project. In contrast, the regulator had incentives to hide undercapitalized banks in order to avoid inefficient bank runs in state A. The welfare gains from running a stress test are larger than in the case where there are no policies in favor of type L banks. As a result, the incentives for the regulator to run a stress test are stronger. The regulator prefers running the stress test if

$$ \beta [p(\alpha_S A_H - L) - (1 - p)A_H (\gamma - 1)] + (1 - \beta) [p(\alpha_S A_L - L) - (1 - p)A_L (\gamma - 1)] + + p\alpha_S B \geq K. $$

(18)
Note that if the regulator runs a stress test when there are no policies in favor of undercapitalized banks, it does also when there are policies because welfare gains are larger.

These results, together with those in Section 3, are consistent with the US and European stress test experience. Commentators agree that stress tests in the US managed to restore investors’ confidence in banks better than in Europe. According to my model, the reason is that the US regulator had stronger incentives to make the stress test informative because, differently than the European regulator, it provided a guarantee that banks failing the stress test would be recapitalized.

7 The value of disclosure

In Section 6 I have showed that the regulator completely removes adverse selection if it complements disclosure with policies to fix undercapitalized banks. In this Section I will point out the role of disclosure in obtaining this result. Assume now that the regulator runs a stress test but does not disclose the results to investors. The regulator knows $e(1 - \beta)$ type L banks but investors do not. Type L banks know whether the regulator has identified them. Neither investors nor the regulator can distinguish types L and types H among the pool composed of $\beta + (1 - \beta)(1 - e)$ banks. I will show that, differently from the case of disclosure, the type L banks that the regulator has identified might not raise capital because the threat of closure is not credible.

As usual, I solve the model by backward induction. At time 1 investors know the state of the world and which banks raised capital at time 0. They decide whether to rollover conditional on this information and on banks’ project choice. The type L banks identified by the regulator can rollover their debt in state A if they raise capital. Investors choose to lend by the same argument as in Section 5.1. Investors’ lending choice to the other banks is the same as in Lemma 1. If the type L banks identified by the regulator do not raise capital, the pool is composed of all banks. Investors choose not to lend to the pool and the assets of all banks get liquidated, unless adverse selection is not so severe (the condition in Corollary (1) holds). The pool of banks shrinks if the type L banks identified by the regulator raise capital. Investors choose to lend to the pool if $e \geq E_{lend}$.

At time $\frac{1}{2}$, the regulator choose whether to close the $e(1 - \beta)$ type L banks it has identified in case they did not raise capital. The following Lemma states the optimal choice.

Lemma 5. The regulator closes the type L banks it has identified if and only if they did not raise capital and investors choose not to lend to the pool in state A.
Proof. Consider the case where all banks do the risky project and there is a run on all banks. If the regulator does not close the type L banks it has identified in case they did not raise capital, welfare is 

$$W = e(1 - \beta) \left[ p(L - B) + (1 - p)\gamma A_L^r \right] + \beta \left[ p(L - B) + (1 - p)\gamma A_H^r \right] + (1 - \beta)(1 - e) \left[ p(L - B) + \gamma A_L^r (1 - p) \right] - K$$

If the regulator closes those type L banks, welfare is 

$$W = e(1 - \beta)(C - B) + \beta \left[ p(L - B) + (1 - p)\gamma A_H^r \right] + (1 - \beta)(1 - e) \left[ p(L - B) + \gamma A_L^r (1 - p) \right] - K$$

It holds that welfare in the case of closure is larger than in the case of forbearance because $ C - B > p(L - B) + \gamma A_L^r (1 - p)$. The same result holds true for any other project choice.

Consider now the case where investors choose to lend to the pool of banks that do not raise capital. This is possible in equilibrium only if type H banks do the safe project. Consider this case and that type L banks choose the risky project. If the regulator does not close the type L banks he has identified in case they do not raise capital, welfare is 

$$W = e(1 - \beta) \left[ p\alpha \gamma A_L^s - p(1 - \alpha)B + \gamma A_L^r (1 - p) \right] + \beta \left[ p\alpha \gamma A_H^s + (1 - p)A_H^r \right] + (1 - \beta)(1 - e) \left[ p\alpha \gamma A_L^s - p(1 - \alpha)B + \gamma A_L^r (1 - p) \right] - K.$$

If the regulator closes those type L banks, welfare is 

$$W = e(1 - \beta)(C - B) + \beta \left[ p\alpha \gamma A_H^s + (1 - p)A_H^r \right] + (1 - \beta)(1 - e) \left[ p\alpha \gamma A_L^s - p(1 - \alpha)B + \gamma A_L^r (1 - p) \right] - K.$$

It holds that welfare in the case of forbearance is larger than in the case of closure because $ C - B < p\alpha \gamma A_L^s - p(1 - \alpha)B + \gamma A_L^r (1 - p)$.

The intuition behind Lemma 5 is that bank closure is more efficient than liquidation but less than continuation. As a result, the regulator closes the types L it has identified only if it expects investors not to lend in state A otherwise. In case investors lend, the regulator prefers to forbear. In this case closure does not represent a credible threat to types L not raising capital.

Investors lend at the risk free rate at time 0 in case they expect to rollover in state A. They charge a risk premium otherwise. Banks choose risk and recapitalization at time 0. In case they raise capital, type L banks choose the
safe project because it is incentive compatible. The choice of the banks in the pool is the same as in Lemma 3. The closure policy affects the incentives of banks to raise capital at time 0. Contrary to the case of disclosure, closure is a threat only when investors do not lend in state A in case of no recapitalization. An interplay arises between capital raising choices by type L banks and choice of risk by type H. Lemma 6 characterizes the result.

**Lemma 6.** If $e < e_M$, the type L banks identified by the regulator raise capital and do the safe project, whereas the other banks do the risky project. If $e \geq e_M$, there is a probability $\theta \equiv \frac{p\alpha R(\gamma A_L - DR) + (1 - p)(\gamma A_L - D)}{p\alpha (\gamma A_L^F - DR) + (1 - p)(\gamma A_L^F - D)}$ that the type L banks identified by the regulator raise capital and do the safe project, type H banks do the safe project and type L banks in the pool do the risky project. With the complementary probability, type L identified by the regulator does not raise capital and gets closed, type H banks do the safe project and type L banks in the pool do the risky project.

**Proof.** By Lemma 3, type H banks choose the risky project if $e < e_M$. Type L banks known to the regulator anticipate there will be a run in state A and that the regulator will close them. This gives them incentives to raise capital. Type H banks do the safe project if $e \geq e_M$. This prevents a run on the pool of banks that do not raise capital. Type L banks anticipate the regulator does not close them. Consider an equilibrium where type L banks known to the regulator do not raise capital. In equilibrium no bank raise capital, and this will determine a run on all the banks (unless $e_M \leq 0$) and hence give the regulator incentives to close down types L. This cannot be an equilibrium. Consider an equilibrium where type L banks known to the regulator raise capital. Investors will be willing to lend in state A. This would give types L incentives not to raise capital because condition (15) does not hold. If they do, they choose the safe project which yields old shareholders a value $p(\alpha A_L^A - D) + (1 - p)(A_L^F - D)$. If they do not raise capital, types L known to the regulator can pool with the others and do the risky project because they anticipate they will not be closed since there is no bank run on the pool in state A. This will yield old shareholders a value $p\alpha R(\gamma A_L^F - DR) + (1 - p)(\gamma A_L^F - D)$. The payoff from deviation is larger than the equilibrium payoff, so that there is no equilibrium in which types L known to the regulator raise capital. There exists only equilibrium in mixed strategy. Following Osborne and Rubinstein [1998], I interpret a mixed strategy equilibrium as a situation in which any player is indifferent among its strategies given the beliefs about other players’ strategy. Equilibrium beliefs must be such that types L known to the regulator are indifferent between raising and not raising capital, that is

$$p(\alpha A_L^A - D) + (1 - p)(A_L^F - D) = \theta \left[p\alpha R(\gamma A_L^A - DR) + (1 - p)(\gamma A_L^F - D)\right] + (1 - \theta)0.$$ 

The left hand side is the payoff from raising capital and doing the safe project. The right hand side is the payoff from not raising capital. If other type L banks known to the regulator do not raise capital (with probability $1 - \theta$), a type
L bank known to the regulator will be closed and get 0 because there will be a run on all the banks. If the others raise capital, a type L bank known to the regulator will do the risky project and not get closed because investors are willing to lend to the pool of banks not raising capital.

To understand Lemma 6, note that the only difference from the case of disclosure is that now the regulator removes adverse selection forcing types L to raise capital. What matters for the choice of type H is still the composition of the pool. If adverse selection is severe, that is if \( e < \varepsilon_M \), the debt repayment is so high that type H prefers choosing the risky project. The fraction \( e(1-\beta) \) of type L banks identified by the regulator will be closed by Lemma because all banks would be liquidated in state A otherwise. Consider now the case where the regulator identifies more than \( \varepsilon_M (1-\beta) \) type L banks. Suppose these banks reveal themselves by raising capital. Type H in the pool of banks that do not raise capital will do the safe project and no bank run will occur in state A. A type L has now incentives not to raise capital because it could pool with the other banks and do the risky project. The regulator would not close it down because investors choose to lend in state A in equilibrium. Suppose now types L identified by the regulator do not raise capital and pool together with the other banks. Type H will do the risky project (unless \( e_M = 0 \)) and all the banks will be liquidated in state A. The Types L identified by the regulator have now incentives to raise capital because they anticipate the regulator will close them down. As a result, the types L that the regulator has identified will randomize their choice to raise capital in equilibrium. The lack of disclosures creates uncertainty over the choice to raise capital by type L banks. This is due to the incentives to forbear that the regulator has when a bank run does not occur in state A if types L raise capital.

The regulator chooses effort anticipating its closure policy. As usual, the welfare function has a discontinuity in \( e = \varepsilon_M \). If \( e \geq \varepsilon_M \), welfare is

\[
W = e(1-\beta)\theta \left[ p\alpha_s A^A_L - p(1-\alpha_s)B + (1-p)A^r_L \right] + e(1-\beta)(1-\theta)C + (1-\beta)(1-e) \left[ p\alpha_r \gamma A^A_L - p(1-\alpha_r)B + \gamma A^r_L (1-p) \right] + 
+ \beta \left[ p\alpha_s A^A_H - p(1-\alpha_s)B + (1-p)A^r_H \right] - K
\]

(19)

The regulator has identified \( e(1-\beta) \) type L banks. With probability \( \theta \) they raise capital and do the safe project, whereas with the complementary probability they do not and get closed. Investors lend to the pool of remaining banks because they identify types L either through the recapitalization or the closure choice. If effort is lower than \( \varepsilon_M \), the welfare function is as in equation (14) since types L identified by the regulator raise capital and the pool of remaining banks gets liquidated. The optimal effort level can be found along the lines of Proposition 2. The following Proposition contains a more interesting result.
Proposition 3. Welfare in the case of non disclosure is always lower than in the case of disclosure.

Proof. Suppose the optimal effort level would still be $e^* = 1$ in the case of non disclosure. Welfare is lower than in the case of disclosure because recapitalization of type L banks is efficient ($p\alpha_s A_L - p(1 - \alpha_s) B + (1 - p) A_L > C$) but happens only with some probability.

Disclosing the stress test results is optimal because it eliminates the incentives to forbear and forces types L to raise capital. In case there is no disclosure, types L raise capital only with some probability because they anticipate that the threat of closure is not credible unless a bank run occurs in state A. Disclosure represents a commitment not to forbear by regulators. The policy implication follows that regulators should adopt a fully transparent approach to stress testing if there are policies to fix undercapitalized banks.

8 Conclusions

Following the recent turmoil, uncertainties about banks' solvency have created tensions in financial markets. Supervisory authorities reacted with an unprecedented disclosure of information about banks. Starting with the U.S. in 2009 and Europe in 2010, they required banks to run stress tests and revealed which of them would be undercapitalized in a possible adverse macroeconomic scenario. Contrary to the European authorities, the U.S. supervisory authorities guaranteed that undercapitalized banks would be forced to raise capital. The widespread opinion that the U.S. managed to restore confidence in banks better than Europe suggests that the credibility of stress tests might depend on the policies in favor of undercapitalized banks. In this paper, I provide a model delivering such a result and propose a menu of policies that should complement information disclosure in order to make bank stress tests effective supervisory instruments.

The trade-off in my model is that revealing bad information about banks might cause socially inefficient bank runs, but would reduce the adverse selection premium for good banks and give them incentives to take the most efficient investment choice. If there are no policies to fix weak banks, I get that a welfare maximizing regulator would never find it optimal to eliminate adverse selection completely because of the bank run inefficiency. It is optimal to remove adverse selection completely if there are policies fixing undercapitalized banks at no cost for taxpayers. I consider recapitalization and debt for equity swaps. As bad banks might not have incentives to undertake these policies, the regulator must threaten penalties. I consider the threat of closure in case bad banks do not undertake the policies designed for them. I show that disclosure of stress test results is optimal because it gives the regulator commitment to enforce
these policies.
References


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