EIEF WORKING PAPER SERIES

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Einaudi Institute for Economics and Finance

EIEF Working Paper 06/09

June 2009

Precautionary Demand for Money in a Monetary Business Cycle Model

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June 9, 2009

Abstract

We investigate quantitative implications of precautionary demand for money for business cycle dynamics of velocity and other nominal aggregates. Accounting for such dynamics is a standing challenge in monetary macroeconomics: standard business cycle models that have incorporated money have failed to generate realistic predictions in this regard. In those models, the only uncertainty affecting money demand is aggregate. We investigate a model with uninsurable *idiosyncratic* uncertainty about liquidity need and find that the resulting precautionary motive for holding money produces substantial qualitative and quantitative improvements in accounting for business cycle behavior of nominal variables, at no cost to real variables.

*For extensive discussions and helpful comments, we thank Jonathan Chiu, Makoto Nakajima, Fernando Alvarez, Manuel Amador, Paul Beaudry, Miquel Faig, Belén Jerez, Allen Head, Amartya Lahiri, Fernando Martin, Martin Schneider and Shouyong Shi as well as seminar participants at UCLA, UCSD, UC Irvine, UC Riverside, Stanford, U. Carlos III Madrid, U. of Chicago, UBC, SFU, and conference participants at EERTEC Florence, 2009 Philadelphia Fed Workshop on Monetary and Macroeconomics, LAEF Payments and Networks Conference at UCSB, CMSG Annual Meetings 2008, Chicago Fed Summer Workshop on Money, Banking and Payments, NES Moscow and IHS Vienna. We completed this paper while visiting the Einaudi Institute for Economics and Finance.

1 Introduction

In this paper, we study, theoretically and quantitatively, aggregate business cycle implications of precautionary demand for money. It is an outstanding challenge in the literature to account for business cycle behavior of nominal aggregates and their interaction with real aggregates. Business cycle models that have tried to incorporate money through, for example, cash-in-advance constraints, have done so while assuming that agents face only aggregate risk, which has resulted in the demand for money being largely deterministic, in the sense that the cash-in-advance constraint almost always binds. Such models have unrealistic implications for the dynamics of nominal variables, as well as for interaction between real and nominal variables, when compared to data (see, e.g., Cooley and Hansen, 1995).

Yet precautionary motive for holding liquidity seems to be strong in the data, and its nature suggests that *idiosyncratic* risk may play a key role for money demand, as shown in Telyukova (2009). In that paper, it is documented, for example, that the median household has about 50% more liquidity than it spends on average per month, and that controlling for observables, liquid consumption exhibits volatility consistent with the presence of significant idiosyncratic risk. Thus, aggregate implications of idiosyncratic risk and resulting precautionary money demand are important to investigate, especially given the unresolved questions regarding monetary business cycles. The goal of this paper is to conduct such an investigation. The set of questions we want to answer is: What are the aggregate implications of precautionary demand for money? Can it help account for business cycle dynamics of velocity of money, interest rates and inflation, and their interaction with real variables?

Existing monetary business cycle models that incorporate money demand via a cash-in-advance constraint, such as cash-credit good models, calibrated to aggregate data, cannot account for aggregate facts such as variability of velocity of money, correlation of velocity with output growth or money growth, correlation of inflation with nominal interest rates, and others, as Hodrick, Kocherlakota and Lucas (1991) have shown. The reason is that in such models, the only type of uncertainty the households face in these models is aggregate uncertainty. The magnitude of this uncertainty in the data is not large enough to generate significant precautionary motive for holding money in the model, so that the cash-in-advance constraint almost always binds. Then, money demand in the model is made equivalent to cash-good consumption, tightly linking volatility of money demand to volatility of aggregate consumption. Aggregate consumption, in turn, is not volatile enough in the data to generate observed volatility of money demand (or its inverse, velocity) or other nominal aggregates.

We show that incorporating precautionary demand for money generated by unpredictable idiosyncratic variation, in combination with aggregate uncertainty, makes a crucial difference in the ability of the model to account for monetary facts mentioned above, by breaking the link between money demand and aggregate consumption. Agents generally hold more money than they spend, and money demand is no longer linked to average aggregate consumption, but rather to consumption of agents whose preference shock realizations make them constrained (i.e. they spend all of their balances) in trade. We show that velocity of money can be significantly more volatile in this heterogeneous-agent setting, thanks to the *unconstrained* agents, who are absent in previous models with only aggregate risk. The presence of both constrained and unconstrained households is key to the qualitative and quantitative results. In other words, idiosyncratic risk in this context does not average out in a way that can be adequately captured by a representative agent model, as Hodrick et al (1991) in fact anticipated in the discussion of their results (p. 380). In addition, the magnitude of idiosyncratic volatility is much higher than aggregate volatility: the standard deviation of aggregate consumption is 0.5%; we will measure the standard deviation of idiosyncratic consumption shocks to be around 19%.

We study this link qualitatively and quantitatively in a model that combines, in each period, two types of markets in a sequential manner, and where both aggregate and idiosyncratic uncertainty are present. The first-subperiod market is a standard Walrasian market, which we will term, somewhat loosely, the "credit market". The second market is also competitive, but characterized by anonymity and the absence of barter possibility, which makes a medium of exchange - money - essential in trade. We term this market the "cash market". This setup is consistent with both cash-credit good models a la Lucas and Stokey (1987) and monetary search models in the style of Lagos and Wright (2005).[1] The credit market is much like a standard real business cycle model, with the production function being subject to aggregate productivity shocks. Two features distinguish this market from the RBC framework. First, households have to decide how much money to carry out of this market for future cash consumption. Second, part of the output in the credit market is carried into the cash market by retail firms, who buy these goods on credit and subsequently

[1]Indeed, idiosyncratic preference shocks could be reinterpreted in the theoretical context as idiosyncratic matching shocks (Wallace 2001). This parallel is less applicable when we think about empirical counterparts of preference shocks.

transform them into cash goods. This introduces an explicit link between the real and monetary sectors of the economy, as credit-market capital becomes indirectly productive in the cash market.

At the start of the second-subperiod cash market, agents are subject to uninsurable idiosyncratic preference shocks which determine how much of the cash good they want to consume, but the realization of the shock is not known at the time that agents make their portfolio decisions. This generates precautionary motive for holding liquidity. In our model, we are able to show analytically how the idiosyncratic shocks, and the resulting heterogeneity of households with respect to being constrained in the cash market, result in amplified dynamics of velocity of money. We also show that absent idiosyncratic shocks, the model produces counterfactual price and other nominal dynamics for values of the coefficient of relative risk aversion in the standard range in RBC literature.

Another contribution of our work is the calibration of the model. To our knowledge, all the existing models of the types mentioned above that have looked at aggregate behavior of nominal variables have been calibrated to aggregate data. Instead, we also use micro survey data on liquid consumption from the Consumption Expenditure Survey, like in Telyukova (2009), to calibrate idiosyncratic preference risk in our cash market. Using these data, we are able to discipline our calibration further than is commonly the case. In general, preference risk of the type that creates precautionary liquidity demand has not been measured in calibration of other aggregate models, and in the few contexts where precautionary liquidity demand has appeared, it has been treated as a free parameter (e.g. Faig and Jerez, 2007). Our use of micro data allows us to be very disciplined in our approach.

Once calibrated, we solve the model computationally to investigate the effects of real productivity shocks and monetary policy shocks. We find that precautionary demand for money makes a dramatic difference for the model in terms of helping it account for a variety of dynamic moments related to nominal aggregates in the data. We test these results by also computing a version of the model where we shut down the idiosyncratic risk, and find that without it, the model is incapable of reproducing any of the key nominal moments in the data, much as previous literature has suggested.

Our results lead us to conclude that in many monetary contexts, especially those aimed at accounting for aggregate data facts, it is important not to omit idiosyncratic uncertainty that gives rise to precautionary demand for money. As one example, omitting this empirically relevant mechanism may cause the standard practice of calibrating monetary models to the aggregate money demand equation, as has been done in many cash-in-advance models and monetary search models, to produce misleading results for parameters and incorrect quantitative implications. We demonstrate this by calibrating a version of the model *without* idiosyncratic shocks to target some data properties of aggregate money demand. With this targeting, we find that, first, the model requires some parameter values well outside the standard range in macroeconomics (e.g. a very low risk aversion parameter), and second, even when the model targets money demand, its quantitative performance is still far inferior to the model *with* precautionary demand, along both nominal and, importantly, real dimensions.

This paper is related to several strands of literature. On the topic of precautionary demand for liquidity,[2] the key mechanism in our model is close to Faig and Jerez (2007), Telyukova and Wright (2008) and Telyukova (2009). In Telyukova and Wright (2008) and Telyukova (2009), the idiosyncratic uncertainty about liquidity need is shown, respectively theoretically and quantitatively, to be relevant for household portfolio decisions to hold liquid assets and credit card debt simultaneously. Faig and Jerez (2007) look at the behavior of velocity and nominal interest rates over the long run. They find that with precautionary liquidity demand, the simulated time series of velocity over the last century, interpreted as a series of steady states, fits the empirical series well. Lagos and Rocheteau (2005) study steady state properties of a monetary economy with idiosyncratic preference shocks. [3] On the broad subject of accounting for aggregate behavior of nominal variables, a recent paper is Wang and Shi (2006). In their model, however, search intensity is the key mechanism behind velocity fluctuations over the business cycle.

The paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 demonstrates analytically the impact of precautionary demand for money on the dynamic behavior of money, velocity and interest rates. Section 4 describes our calibration strategy, and section 5 details the solution algorithm. Section 6 presents our results and discusses the quantitative role

[2]The subject of precautionary money demand goes back to at least Keynes(1936), who defined its reason as "to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases". Precautionary demand for money is often modeled in Baumol-Tobin-style inventory-theoretic models, from Whalen (1966) to fully dynamic stochastic models such as Alvarez and Lippi (2009). Uninsurable idiosyncratic liquidity shocks are also an essential element of models based on Diamond and Dybvig (1983). Lucas (1980) studies the equilibrium in a cash-in-advance model with precautionary demand for money.

[3]In another paper on the broad subject of precautionary money demand, Hagedorn (2008) shows that that strong liquidity effects can arise when precautionary demand for money is taken into account in a cash-credit good model with banking.

of precautionary liquidity demand. We then discuss how omission of precautionary demand may lead model calibration and implications astray (section 7) and show how precautionary demand affects welfare costs of inflation (section 8). Section 9 concludes.

2 Model

The economy is populated by a measure 1 of households, who live infinitely in discrete time. The households rent labor and capital to firms, consume goods bought from the firms, and save. There are two types of markets open sequentially during the period. In the first subperiod, a Walrasian market is open, in which all parties involved in transactions are known and all trades can be enforced; thus, intertemporal trade and asset trading are possible. In the second subperiod, the market is competitive, in the sense that all agents are price takers, but we assume that money is essential in trade.[4] Since in the first-subperiod market households pay with either cash or credit for consumption, and as we discuss below, retail firms buy on credit, we will refer to this as the "credit market", while the second subperiod - where payment takes place using money only - will be termed the "cash market".

There are two types of firms in the economy. Production firms use capital and labor as inputs in production, and sell their output in the first subperiod. This output is used for consumption and capital investment in the credit market. However, part of the output is also bought up in the credit market by retail firms, who then transform the goods one-for-one into retail goods to be sold in the cash market.[5]

2.1 Households

Households maximize lifetime expected discounted utility,

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \Big(U(c_t) - Ah_t + \vartheta_t u(q_{\vartheta,t}) \Big) \right] \tag{1}$$

where $0 < \beta < 1$. Utility achieved in each period, depends on consumption c_t and time spent working h_t in the first subperiod, and in the second subperiod, consumption q_t and the preference shock ϑ_t.[6] First-subperiod utility follows the

[4] Temzelides and Yu (2004) derive sufficient conditions under which money is essential in competitive markets. See also Levine (1991) and Rocheteau and Wright (2005).

[5] Our retailers are not meant to correspond one-for-one to the retail sector in the data: some retailers in the data are better characterized as selling in the credit market, whereas the cash sector includes firms that are not retailers.

[6] When subscripting our variables by t, we mean that each variable at time t is chosen conditional on the entire history up to t; expectations are taken accordingly.

Hansen-Rogerson specification of indivisible labor with lotteries. The taste shock ϑ_t realizes when the credit market is already closed and money holdings can no longer be adjusted, as described below. This will lead to precautionary demand for money. The taste shock comes from a distribution with finite support.

We normalize the household's money holdings by the aggregate money holdings. Households start the period with normalized money holdings m_t and choose $\tilde{m}_t$ normalized money to bring into the cash market, before ϑ realizes. Households also own capital k_t and nominal bonds b_t, sold to them by retail firms, as detailed below.[7] We normalize the price of the credit good to one. Let wage, capital rent, real value of one unit of normalized money, and the return on nominal bonds be $w_t, r_t, \phi_t, i_{t-1}$. The budget constraint, expressed in real terms, is

$$\phi_t m_t + (1 + r_t - \delta)k_t + w_t h_t + \phi_t b_t(1 + i_{t-1}) = c_t + \phi_t \tilde{m}_t + k_{t+1} + \phi_t b_{t+1} \quad (2)$$

Given price ψ_t of the cash good, consumption q_ϑ in the cash market, conditional on the preference shock realization ϑ, has to satisfy

$$\psi_t q_{\vartheta,t} \leq \tilde{m}_t. \quad (3)$$

Finally, hours worked are constrained,

$$h \in [0, 1]. \quad (4)$$

At the beginning of the period the government makes a lump sum transfer $\varpi_t M_t$, where M_t is the aggregate money stock; thus, in normalized terms,

$$m_{t+1} = \tilde{m}_t - \psi_t q_{\vartheta,t} + \varpi_t. \quad (5)$$

We can equivalently formulate the problem recursively under standard assumptions, which we do below.

2.2 Production Firms

The problem of the production firm is static and completely standard – to maximize its profits in each period. Given a constant returns to scale production function $y_t = e^{z_t} f(k_t, h_t)$, where z_t is the stochastic productivity level described in more detail below, the problem is: $\max_{k_t,h_t} \left\{e^{z_t} f(k_t, h_t) - w_t h_t - r_t k_t\right\}$. The solution is characterized by the usual first-order conditions.

[7] In principle, households can hold shares of firms as well. We will see that in our formulation all firms make zero profits, share holding is irrelevant. Alternatively, we can formulate the economy with firms selling shares instead of bonds; this leads to equivalent allocations of resources, but involves more notation.

2.3 Retail Firms

Retail firms exist for two periods: they buy the goods in the credit market, selling nominal bonds to households to do so, sell the goods in the subsequent cash market, and settle their debt in the following credit market, before disbanding. Free entry in the retail market yields the following condition, expressed in nominal terms at time t:

$$\Pi_{rt} = \max_{q_t} \frac{\psi_t q_t}{1+i_t} - \frac{q_t}{\phi_t} = 0. \tag{6}$$

All cash receipts from retail sales go towards repayment in the following credit market; the value of this repayment is discounted to the current period using the nominal interest rate. The repayment equals the nominal value in the current period for the q_t goods that were purchased in the credit market by the retailers. Since the cash market is competitive, retail firms will sell all their goods in equilibrium.[8]

2.4 Monetary Policy and Aggregate Shocks

The monetary authority follows an interest rate feedback rule

$$\frac{1+i_{t+1}}{1+\bar{i}} = \left(\frac{1+i_t}{1+\bar{i}}\right)^{\xi_{ii}} \left(\frac{1+\pi_t}{1+\bar{\pi}}\right)^{\xi_{i\pi}} \left(\frac{y_t}{\bar{y}}\right)^{\xi_{iy}} \exp(\varepsilon^{mp}_{t+1}). \tag{7}$$

The variables with bars denote central bank's long-run target levels of output, inflation and the nominal interest rate. The term ε^{mp} denotes a stochastic monetary policy shock which realizes at the beginning of the period. Consistent with the movement in interest rates, the rate of money supply growth ϖ_t adjusts, thus determining the lump-sum injections paid out to the households.

Independent from the monetary policy shock ε^{mp}, the second aggregate shock process is on the productivity level z_t. As is standard in business cycle literature, the level of productivity follows an AR(1) of the form

$$z_{t+1} = \xi_{zz} z_t + \varepsilon^{z}_{t+1}.$$

2.5 Recursive Formulation of the Household Problem

From now on, we will conserve notation by omitting time subscripts, and using primes to denote $t+1$. The aggregate state variables in this economy are the aggregate capital stock, K, the technology shock z, the previous interest rate in the economy i_{-1}, current interest rate i, and the term $(1+\varpi_{-1})\phi_{-1}$, which denotes the previous period's post-injection real value of money, and which households need to

[8] We assume that goods are storable, so even at zero expected real interest rate, this is without loss of generality.

know in order to determine the current rate of inflation in the economy. We will denote these as $S = (K, z, i_{-1}, i, (1 + \varpi_{-1})\phi_{-1})$. The individual state variables at the beginning of the centralized market are normalized money holdings m, capital holdings k, and bond holdings b. Recall that individual money holdings m are defined relative to total money stock M. This renders the money holdings stationary.

Instead of writing this as a problem with separate value functions for centralized and decentralized subperiods, we can formulate the household's problem as a more transparent full-period problem. This means that in the first subperiod the household can make the choices for the second subperiod, contingent on its information at the start of the second subperiod, which is the realization of its preference shock ϑ_t. In sum, we have the following recursive maximization problem:

$$
\begin{aligned}
V(k, m, b, S) = & \max_{c,h,\tilde{m},k',b',\{q_\vartheta\}} \Big\{ U(c) - Ah \\
& + \mathbb{E}_\vartheta \left[\vartheta u(q_\vartheta) + \beta \mathbb{E}_{z',i'} V(k', m', \frac{b'}{1+\varpi}, S') \right] \Big\} && (8) \\
\text{s.t. } c + \phi\tilde{m} + k' + \phi b' = & \ \phi m + \phi b(1 + i_{-1}) + (1 + r - \delta)k + wh && (9) \\
\psi q_\vartheta \leq & \ \tilde{m} && (10) \\
\pi = & \ \frac{(1 + \varpi_{-1})\phi_{-1}}{\phi} && (11) \\
1 + \varpi = & \ \Omega(S) && (12) \\
m' = & \ \frac{\tilde{m}}{1+\varpi} - \frac{\psi q_\vartheta}{1+\varpi} + \frac{\varpi}{1+\varpi} && (13) \\
z' = & \ \xi_{zz} z + \varepsilon_1' && (14) \\
(\widetilde{1 + i'}) = & \ \xi_{ii}(\widetilde{1 + i}) + \xi_{i\pi}\tilde{\pi} + \xi_{iy}\tilde{y} + \varepsilon_2' && (15)
\end{aligned}
$$

The interest rate rule here is given in short hand, with $\tilde{x}$ referring to log-deviations of the variable x from its target level. Given today's aggregate states, and in particular the nominal interest rate i between today and tomorrow, the central bank will adjust the money growth rate to make i arise as an equilibrium price. As a result, we can write the money growth rate as a function of the current aggregate state, as in equation (12).

Denote the household state variables as $s = (k, m, b)$. Denote the policy functions of the household's problem by $g(s, S)$, with $g_x(.)$ as the policy function for the choice variable x.

2.6 Equilibrium

Definition 1. *A Symmetric Stationary Monetary Equilibrium is a set of pricing functions $\phi(S)$, $\psi(S)$, $w(S)$, $r(S)$; law of motion $K'(S)$, value function $V(s,S)$ and policy functions $g_c(s,S)$, $g_h(s,S)$, $g_k(s,S)$, $g_b(s,S)$, $g_m(s,S)$, $\{g_{q,\vartheta}(s,S)\}$, all ϑ, such that:*

1. *The value function solves the household optimization, in* (8)*, with associated policy functions, given prices and laws of motion;*
2. *Production and retail firms optimize, given prices and laws of motion, as in sections 2.2 and 2.3.*
3. *Free entry of retailers:* $\Pi_r = 0$.
4. *Consistent expectations: the aggregate law of motion follows from the sum of all individual decisions (index individual households by i) –*

$$K'(S) = \int_0^1 g_k^i(s,S)di$$

5. *All markets clear:*

$$\int_0^1 g_m^i(s,S)di = 1$$
$$\int_0^1 \phi(S)g_b^i(s,S)di = \int_0^1 \left[\mathbb{E}_\vartheta g_{q,\vartheta}^i(s,S)\right] di$$
$$\int_0^1 g_h^i(s,S)di = H(S)$$
$$(1-\delta)K + e^z f(H(S),K) = \int_0^1 g_c^i(s,S)di + K'(S) + \int_0^1 \left[\mathbb{E}_\vartheta g_{q,\vartheta}^i(s,S)\right] di$$

2.7 Walrasian Market creates Homogeneity

For general utility functions, different realizations of the idiosyncratic preference shock would lead to a nontrivial distribution of wealth (with, for example, those who have recently experienced a sequence of high ϑs now being poorer on average). In turn, households with different wealth could make different portfolio decisions, and hence the distribution across individual state variables would be relevant for the equilibrium prices.

However, the quasi-linear specification of the problem allows equilibria in which all heterogeneity created in the second subperiod washes out in the centralized market.[9] This occurs if the boundary conditions of h are never hit, which we assume

[9] This result has been used extensively in models that combine Walrasian markets with bilateral trade and idiosyncratic *matching* risk, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). Here we use it to combine Walrasian markets with cash markets and idiosyncratic *preference* risk.

to be the relevant case below. Our quantitative strategy later is to solve the problem assuming that the optimal choice of h is interior, and check in our calibrated equilibrium whether this is indeed the case.

After substituting the budget constraint for h into the household's value function, we can split the value function in two parts

$$\begin{aligned} V(s,S) = \\ & A\left(\frac{\phi m + (1+r-\delta)k + \phi b(1+i_{-1})}{w}\right) \\ & + \max_{\cdots}\left\{U(c) - A\left(\frac{c+\phi\tilde{m}+k'+\phi b'}{w}\right) + \mathbb{E}_{\vartheta}\left[\vartheta u(q_{\vartheta}) + \beta\mathbb{E}_{z',i'}[V(s',S')]\right]\right\}. \end{aligned} \tag{16}$$

The following result is immediate, under the assumption of interiority on h.

Result 1. *The choice of $c, \tilde{m}, k', b'$ only depends on the aggregate states S.*

Household wealth differs at the beginning of the Walrasian market, due to heterogeneous trading histories in the previous cash market, but households adjust their hours worked to be able to get the same amount of $c, \tilde{m}, k', b'$. The value function $V(.)$ is differentiable in k, m, b, and the envelope conditions are independent of the individual state variables. Hence, the expectation over ϑ does not matter for intertemporal choice variables, for example:

$$\mathbb{E}_{\vartheta}[\mathbb{E}_{z',i'}V_m(s,S)] = \mathbb{E}_{z',i'}V_m(s,S) = \mathbb{E}_{z',i'}\left[\frac{A\phi(S)}{w(S)}\right].$$

The problem is weakly concave in capital, labor and bond holdings, and the solution is interior, as long as h is interior. The Euler equations with respect to capital and bonds, and the first-order condition with respect to labor thus look as follows:

$$U'(c(S)) = \beta\mathbb{E}_{z',i'}[U'(c(S'))(1+r(S')-\delta)] \tag{17}$$

$$U'(c(S)) = \frac{A}{w(S)} \tag{18}$$

$$\phi U'(c(S)) = \beta\mathbb{E}_{z',i'}\left[\frac{\phi'}{1+\varpi}U'(c(S'))\right](1+i) \tag{19}$$

For future reference, we introduce the following notation, using marginal utilities defined in terms of the marginal productivity of labor (18):

$$\mathbb{E}\left[\frac{w(S)}{w(S')}\right] \equiv \widetilde{\mathbb{E}},\ \mathbb{E}\left[\frac{\phi'(S')}{1+\varpi'}\frac{w(S)}{w(S')}\right] \equiv \widetilde{\mathbb{E}\phi'}.$$

Note that $\frac{\phi}{\beta\widetilde{\mathbb{E}\phi'}} = 1+i$.

2.8 The Choice of Money Holdings and Cash Market Consumption

Above we have discussed the Euler equations that link consumption, capital investment and bond investment between periods. Taking as given the (17)-(19), money holdings solve the following problem

$$\max_{\tilde{m},\{q_\vartheta\}} \left\{ -\phi\tilde{m} + \sum_\vartheta \mathbb{P}(\vartheta)\Big(\frac{\vartheta u(q_\vartheta)}{U'(c)} - \beta\widetilde{\mathbb{E}\phi'}\psi q_\vartheta\Big) + \beta\widetilde{\mathbb{E}\phi'}\tilde{m}\right\} \tag{20}$$

subject to

$$\psi = \frac{1+i}{\phi} \tag{21}$$

$$\psi q_\vartheta \leq \tilde{m} \ \ \forall \ \ \vartheta \tag{22}$$

where we substitute in the equilibrium retailer zero-profit condition. Substitute out (21), noting that $\beta\widetilde{\mathbb{E}\phi'}\psi = 1$. Denote by μ_ϑ as multipliers of (22), and by $\mathbb{P}(\vartheta)$ the probability of a particular shock ϑ realizing. First-order conditions then give

$$\mathbb{P}(\vartheta)\left(\frac{\vartheta u'(q_\vartheta)}{U'(c)} - 1\right) - \psi\mu_\vartheta = 0 \tag{23}$$

$$-\phi + \sum_\vartheta \mu_\vartheta + \beta\widetilde{\mathbb{E}\phi'} = 0, \tag{24}$$

with the appropriate complementary slackness conditions (see characterizing equations below). It is immediate that in this model cash balances are not spent in full for realizations of ϑ that are low enough. Since a social planner would equate $U'(c)$ to $\vartheta u'(q_\vartheta)$, the following conclusions can be drawn:

Result 2. *If a shock ϑ results in a nonbinding constraint, then q_ϑ is the efficient quantity. Moreover, as long as the cash constraint does not bind, the quantity q_ϑ does not respond to the interest rate.*

Moreover, also observe that if some $\hat{\vartheta}$ leads to a binding constraint, then for every $\vartheta > \hat{\vartheta}$, the cash constraint will bind. If $\hat{\vartheta}$ leads to a slack cash constraint, any $\vartheta < \hat{\vartheta}$ will lead to a nonbinding constraint. A binding cash constraint leads to underconsumption of the cash good relative to the social optimum.

2.9 Characterizing Equations

We summarize the above discussion in the system of first-order conditions and Euler equations that characterize the equilibrium of the problem:

$$
\begin{aligned}
U'(c) &= \beta\mathbb{E}[U'(c')(1+r'-\delta)] \qquad (25)\\
U'(c) &= \frac{A}{w}\\
\psi &= \frac{1+i}{\phi}\\
\tilde{\mu}_\vartheta &= \mathbb{P}(\vartheta_i)\left(\frac{\vartheta u'(q_\vartheta)}{U'(c)\psi} - \frac{\phi}{1+i}\right);\ \tilde{\mu}_\vartheta(\tilde{m}-\psi q_\vartheta) = 0\ \forall\vartheta\\
\phi &= \sum_\vartheta \tilde{\mu}_\vartheta + \frac{\phi}{1+i}\\
\frac{\phi}{1+i} &= \frac{\beta\widetilde{\mathbb{E}\phi'}}{1+\varpi}\\
y+(1-\delta)k &= c+k'+\sum_\vartheta \mathbb{P}(\vartheta)q_\vartheta\\
z' &= \xi_{zz}z+\varepsilon_1'\\
(\widetilde{1+i'}) &= \xi_{ii}(\widetilde{1+i})+\xi_{i\pi}\tilde{\pi}+\xi_{iy}\tilde{y}+\varepsilon_2'
\end{aligned}
$$

3 The Impact of Idiosyncratic Uncertainty on Nominal Dynamics

In this section, we demonstrate analytically that there are at least three ways in which idiosyncratic shocks to cash-good preferences can improve the quantitative performance of the model. First, the dynamic behavior of the value of money and prices varies significantly with the probability that the marginal dollar is spent, i.e. that the cash constraint binds. As a result, we show that the model with idiosyncratic shocks can accommodate values of the intertemporal elasticity of substitution (IES) parameter in the standard RBC calibration range ($\sigma \in [1,4]$), whereas the model without shocks would require $\sigma < 1$ ($IES = 1/\sigma > 1$) to produce realistic dynamics of prices. Second, part of the velocity fluctuation is now generated in the cash market, thus increasing the overall magnitude of velocity volatility, and velocity now depends in an intuitive way on nominal interest rates. Third, the standard general-equilibrium substitution channel in cash-credit good models between cash and credit good consumption is now dampened, because cash consumption will now only adjust for the binding realization of the shock.

3.1 The Dynamic Behavior of the Value of Money and Prices

The dynamic behavior of money will be an essential input for relating velocity to interest rates. It is also, however, empirically relevant in itself: one uncontroversial empirical regularity is the degree of persistence of interest rates, prices, and real balances, before and after detrending, over the business cycle. Nominal interest rates have an autocorrelation at quarterly frequency of 0.932; for real balances, it is 0.951.[10] It seems a minimally necessary requirement that a monetary business cycle model can replicate the sign of these autocorrelations. This requirement turns out to imply, in the absence of precautionary money demand, stringent restrictions on the coefficient of relative risk aversion (RRA) σ. In fact, a standard range of parameters for the RRA coefficient used in real business cycle calibration by and large violates this restriction. With precautionary demand, it becomes possible to use parameters in this range.

For the sake of exposition, assume two preference shock realizations ϑ_i, where ϑ_h leads to a binding cash constraint, and ϑ_l to a nonbinding constraint.[11] We write p for the probability of the high shock ϑ_h. Note that if we set $p = 1$, we are back to the case with no idiosyncratic shocks. We simplify the utility function in the credit market to be fully linear, $U(c) = c$. (We later generalize this). Reworking the characterizing equations (25) for the binding case, we find that $q_h = \beta\widetilde{\mathbb{E}\phi'} = (\beta\phi')/(1+\varpi)$, with $(1+i) = \phi/(\beta\widetilde{\mathbb{E}\phi'})$, and so

$$\phi = p\vartheta_h u'(\beta\widetilde{\mathbb{E}\phi'})\beta\widetilde{\mathbb{E}\phi'} + (1-p)\beta\widetilde{\mathbb{E}\phi'}. \tag{26}$$

Now we can calculate what happens to the value of money today, ϕ, in response to a change in the discounted value of money tomorrow, $\widetilde{\mathbb{E}\phi'}$.

Lemma 1. *The elasticity of real balances today with respect to real balances tomorrow, $\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}}$, evaluated at an equilibrium $\phi, \beta\widetilde{\mathbb{E}\phi'}$, is given by*

$$\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}} = \left(1 - \frac{1-p}{1+i}\right)(1-\sigma) + \frac{1-p}{1+i} \tag{27}$$

Proof. The derivative of ϕ with respect to $\beta\widetilde{\mathbb{E}\phi'}$, using (26), (19), (22) and $\tilde{m} = 1$, is

$$\frac{d\phi}{d(\beta\widetilde{\mathbb{E}\phi'})} = p\vartheta_h(u''(\beta\widetilde{\mathbb{E}\phi'})\beta\widetilde{\mathbb{E}\phi'} + u'(\beta\widetilde{\mathbb{E}\phi'})) + (1-p)$$

[10] BP-filtered, nominal interest rate from three-month treasury bonds, real money balances from M2 and the GDP deflator (source: FRED2).

[11] We assume, in this example only, but not in computational work below, that as i changes, the number of binding shocks does not. Generically, for small enough fluctuations, this assumption will hold.

Divide both sides by $\phi/(\beta\widetilde{\mathbb{E}\phi'})$, and using (26), we find

$$\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}} = p\frac{\vartheta(u''(\beta\widetilde{\mathbb{E}\phi'})\beta\widetilde{\mathbb{E}\phi'} + u'(\beta\widetilde{\mathbb{E}\phi'}))}{p\vartheta_h u'(\beta\widetilde{\mathbb{E}\phi'}) + (1-p)} + (1-p)\frac{\beta\widetilde{\mathbb{E}\phi'}}{\phi}.$$

Rewriting this as a function of the interest rate $(\phi/\beta\widetilde{\mathbb{E}\phi'})$, this elasticity then becomes equation (27). □

If this elasticity is negative, then a lower expected value of the money stock tomorrow translates into a higher value of the money stock today. Suppose we are in the steady state and have an *expected* one-time injection of money $\varpi > 0$ one period from now. Then ϕ' will be unaffected in this stationary equilibrium, since tomorrow's prices adjust immediately; this means that $\beta\widetilde{\mathbb{E}\phi'}$ falls in proportion to the injection $1+\varpi$. However, this injection will raise the cash market prices today; moreover, if $\varepsilon_{\phi,\widetilde{\mathbb{E}\phi'}} < 0$, ϕ - the real value of money in today's credit market - will go up. This means that when $P_c' = (1+\varpi)/\phi'$ - the non-normalized credit market price - goes up, $P_c = 1/\phi$ goes down. If we had an expected one-time injection two periods from now (and no trend money growth), P_c'' would rise, P_c' would fall, and P_c would rise. Then, calculating the resulting nominal interest rates, we would get a similar pattern. As a result, we get high volatility of prices and interest rates, and counterfactual responses, in a zigzag pattern, to an expected decline in the value of money in the future.[12] Even if, with $\varepsilon_{\phi,\widetilde{\mathbb{E}\phi'}} < 0$ and this volatile zigzag pattern, the model could produce the right order of magnitude of velocity fluctuations, the fundamental forces behind these fluctuations would be empirically invalid.

When is this elasticity negative? If $p = 1$ (no idiosyncratic uncertainty), this happens when the coefficient of relative risk aversion is $\sigma > 1$. To avoid the counterfactual behavior of prices and interest rates, we thus have to set $\sigma < 1$ in a model with no idiosyncratic risk.[13] However, this is smaller than the standard RRA coefficient on consumption in the real business cycle literature.

With idiosyncratic uncertainty, we are able to put in a much higher σ: in our setup with $p < 1$, the counterfactual price pattern sketched above only occurs if

$$\sigma > \frac{1+i}{p+i}, \tag{28}$$

[12] In our calibration, we will have a nominal interest rate rule with persistence. Analysis of this is a bit trickier: in a setting with $p = 1$ (no idiosyncratic shocks) and $\sigma > 1$, persistence in the nominal interest rate could be achieved by alternating expansions and contractions of the money supply. Again, this would be counterfactual.

[13] $\sigma = 1$ produces $\varepsilon_{\phi,\widetilde{\mathbb{E}\phi'}} = 0$, i.e. ϕ is unresponsive to changes in i, and as a result, real money balances and consumption velocity fluctuates extremely little. We confirm this quantitatively as well.

which can be significantly higher than one. The intuition for the less tight bound is the following: the marginal unit of money is only used with probability p, while with probability $(1-p)$ it is not used. The value of money today is a weighted average of the value of money in use, with probability weight p, and the value of money when not used. Thus, a drop in the value of money tomorrow will, with probability $1-p$, contribute to a drop in the value of money today. If $\sigma > 1$, then the marginal value of a unit of money *in use* will go up – the marginal utility of consumption in the cash market increases faster than the increase in its price. However, if (28) holds, this is now more than offset by the drop in the value of money when not used.[14]

3.2 The Dynamic Behavior of Velocity

The consumption velocity of money in the above example with two idiosyncratic shocks is given by

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p\frac{q_h(1+i)}{\phi},$$

while output velocity is

$$V_y = \frac{PY}{M} = \frac{(y-(1-p)q_l - pq_h)}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p\frac{q_h(1+i)}{\phi}.$$

Since $\frac{q_h(1+i)}{\phi} = \psi q_h = \tilde{m} = 1$, we have

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p$$

$$V_y = \frac{PY}{M} = \frac{(y-(1-p)q_l - pq_h)}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p$$

Let us look at consumption velocity. From the above equations, observe that, as in standard cash-in-advance and cash-credit-good models, the constrained part of the cash market always contributes 1 to the level of consumption velocity, and nothing to velocity fluctuations. If $p = 1$, then the entire cash market does not contribute anything directly to velocity dynamics, so that all velocity movement has to come from the credit market - i.e. from c or ϕ. Instead, in our model, velocity fluctuations are created in the cash market in addition to the credit market, thanks to the low shocks where the cash constraint does not bind.[15]

[14] This effect occurs in general in models where, with some probability, not all money is spent, including models with search frictions. In most calibrations in search models, nevertheless, $0 < \sigma < 1$ is chosen; this avoids issues with negative utility when doing e.g. Nash Bargaining.

[15] Models with variable search intensity also create velocity fluctuations in the cash market (Wang and Shi 2006). Standard search models with fixed match probabilities do not.

One can also see this by looking at marginal rates of substitution between cash and credit market consumption. For the binding case, the MRS is

$$\frac{\vartheta_h u'(q_h)}{U'(c)} = 1 + \frac{i}{p}. \tag{29}$$

Recall from Result 2 that the MRS in the non-binding case is

$$\frac{\vartheta_h u'(q_h)}{U'(c)} = 1. \tag{30}$$

Without preference shocks ($p = 1$), cash market consumption thus always depends on nominal interest rates ($MRS = 1 + i$). Preference shocks, however, add agents who are not constrained ($p < 1$), and whose cash market consumption does not depend on i ($MRS = 1$). Since the unconstrained agents do not adjust their consumption in response to changes in i, and are able to adjust their money spending, they are the only ones who contribute to fluctuations in velocity. For the constrained agents, the response to price changes is in consumption only, and this adjustment exactly offsets the price change, so that the total amount of money spent does not move.

The elasticity of consumption velocity with respect to interest rates can be derived as

$$\varepsilon_{V_c,1+i} = s_c(\varepsilon_{c,1+i} - \varepsilon_{\phi,1+i}) + s_{\text{cash, nb}} \cdot \varepsilon_{\psi,1+i}, \tag{31}$$

where s_c is the share of the credit market in total consumption expenditure, and likewise $s_{cash,nb}$ is the share of cash consumption under non-binding preference shocks. The last term captures the velocity fluctuations in the cash market, which we study first.

Lemma 2. *Elasticity of the cash market price with respect to the interest rate is always positive, and is given by*

$$\varepsilon_{\psi,1+i} = \varepsilon_{\frac{1+i}{\phi},1+i} = \frac{1+i}{\sigma(p+i)} > 0. \tag{32}$$

Thus, the less risk-averse the household is, or the smaller the probability of a binding constraint is, the more of the velocity fluctuations originate in the cash market, ceteris paribus.

Proof. One can derive that $\varepsilon_{\frac{1+i}{\phi},1+i} = 1 - \varepsilon_{\phi,1+i}$. Substituting in

$$\varepsilon_{\phi,1+i} = \frac{1}{\varepsilon_{1+i,\phi}} = \frac{1}{1 - \varepsilon_{\beta\widetilde{\mathbb{E}\phi'},\phi}} = \frac{\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}}}{\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}} - 1}, \tag{33}$$

we find

$$\varepsilon_{\psi,1+i} = \varepsilon_{\frac{1+i}{\phi},1+i} = 1 - \varepsilon_{\phi,1+i} = \frac{-1}{\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}} - 1}$$

Putting the last equation together with (27) yields (32). □

The remaining terms in (31) are also affected by the presence of idiosyncratic shocks. To conclude our analytical discussion, let us now incorporate fully the response of credit-good consumption c to changes in prices and interest rates. To capture all effects, we drop the assumption that $U(c)$ is linear; then there is also a general equilibrium feedback effect linking nominal interest rates and velocity, through the substitution channel between cash and credit market consumption.

The only assumption that we need for analytical tractability is that capital is constant - while this shuts down one equilibrium effect, and also removes the wedge between consumption and output velocity, it does not alter the other effects that we focus on. (Of course, capital fluctuations are an important ingredient of the computed model in the subsequent sections.)

Proposition 1. *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money injection consistent with a given steady state level of* $1+i$*), is*

$$\varepsilon_{V_c,1+i} = s_c \left(\frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash,nb} \left(\frac{1+i}{p+i} \right) \left(\frac{1}{\varepsilon_{U'(c),q_h} + \sigma} \right) \tag{34}$$

This elasticity is an equilibrium object: it tells us how the nominal interest rate and velocity vary when both variables move as a result of a one-time fully anticipated money injection.

Proof. With abuse of notation, let us call $\widetilde{\mathbb{E}\phi'} \overset{def}{=} \mathbb{E}U'(c')\phi'/(1+\varpi)$ in this proof. From the free entry condition $U'(c)q = \beta\mathbb{E}U'(c')\phi'/(1+\varpi) = \beta\widetilde{\mathbb{E}\phi'}$, we find that $-\varepsilon_{q_h,1+\varpi} = (\varepsilon_{U'(c),q_h} + 1)^{-1}$, since $U'(c')\phi'$ is unaffected by an anticipated one-time money injection. Then, relating ϕ to $\widetilde{\mathbb{E}\phi'}$, from $U'(c)\phi = p\vartheta_h U'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}$, we find

$$\varepsilon_{U'(c)\phi,\beta\widetilde{\mathbb{E}\phi'}} = \frac{d\ln U'(c)\phi}{d\ln \beta\widetilde{\mathbb{E}\phi'}} = \left(\frac{p+i}{1+i}(1-\sigma) \right) \frac{1}{\varepsilon_{U'(c),q_h} + 1} + \frac{1-p}{1+i}. \tag{35}$$

Then

$$\begin{aligned} \varepsilon_{V_c,\beta\widetilde{\mathbb{E}\phi'}} = s_c & \left\{ \left[\left(-\frac{1}{\sigma} + 1 \right) \varepsilon_{U'(c),q_h} - \frac{p+i}{1+i}(1-\sigma) \right] \frac{1}{\varepsilon_{U'(c),q_h} + 1} - \frac{1-p}{1+i} \right\} \\ & - s_{cash,nb} \frac{1}{\varepsilon_{U'(c),q_h} + 1}, \end{aligned}$$

where we can calculate $\varepsilon_{c,\beta\widetilde{\mathbb{E}\phi'}}$ as $\frac{\varepsilon_{U'(c),q_h}}{\sigma(\varepsilon_{U'(c),q_h}+1)}$ and $\varepsilon_{q_h,\beta\widetilde{\mathbb{E}\phi'}} = -\varepsilon_{\psi,\widetilde{\mathbb{E}\phi'}}$ as $(\varepsilon_{U'(c),q_h} + 1)^{-1}$; $\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}}$ follows from (35). With $\varepsilon_{\beta\widetilde{\mathbb{E}\phi'},1+i} = (\varepsilon_{U'(c)\phi,\beta\widetilde{\mathbb{E}\phi'}} - 1)^{-1}$ and $\varepsilon_{U'(c)\phi,\beta\widetilde{\mathbb{E}\phi'}}$ from (35), we find (34). A detailed proof is in appendix A. $\square$

Note that the only difference between (35) and (27) is the presence of the term $(\varepsilon_{U'(c),q_h} + 1)^{-1}$, which captures the general equilibrium feedback of movements of q_h on c, taking into account the optimal labor supply decision. As less is sold in the decentralized market, less has to be produced in the labor market. This improves the marginal productivity of labor and raises first-subperiod consumption. As before, in equation (35), if $p = 1$, we would need $\sigma \leq 1$ to get a positive sign for the autocorrelations of prices, real money stock and interest rates, and again, this constraint is relaxed if $p < 1$.

In (34), we can recognize the different channels through which idiosyncratic uncertainty works: (i) credit market effects through the leftmost term; (ii) cash market channel through the right-most $(1 + i)/(p + i)$ term; and (iii) the general equilibrium channel, through $\varepsilon_{U'(c),q_h}$.

We show these components graphically as a function of σ, in figure 1. We see that idiosyncratic shocks raise the elasticity of velocity with respect to interest rates dramatically, as signified by the vertical difference between the second (grey dashed) and third (black dashed) lines in the graph, and allow for a positive elasticity for a much larger range of σ (here presented with $p = 0.5$). We also observe, in the difference between the top dotted line and the top solid line, that the general equilibrium effect is small, but works to raise the elasticity of velocity with respect to the nominal interest rate. Keeping the size of the cash market the same, and lowering p, it can be shown that the sensitivity of velocity to interest rates through the cash market channel is raised. Finally note that, without idiosyncratic shocks, the elasticity of velocity to interest rate is negative if $\sigma > 1$.

4 Calibration

The model period is a quarter. The functional forms that we choose are as follows:

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad u(q) = \frac{x_1 q^{1-\sigma}}{1-\sigma} \qquad f(k,h) = k^{\theta} h^{1-\theta}.$$

In total, we need to calibrate the following parameters, given our functional form choices. The parameters β, σ, A, θ and δ are standard. x_1 and the process for the idiosyncratic shock ϑ determine preferences in the cash market. Finally, the parameters of the exogenous driving processes $\{\xi\}$ and the standard deviations σ_{ε_1} and σ_{ε_2} have to be calibrated.

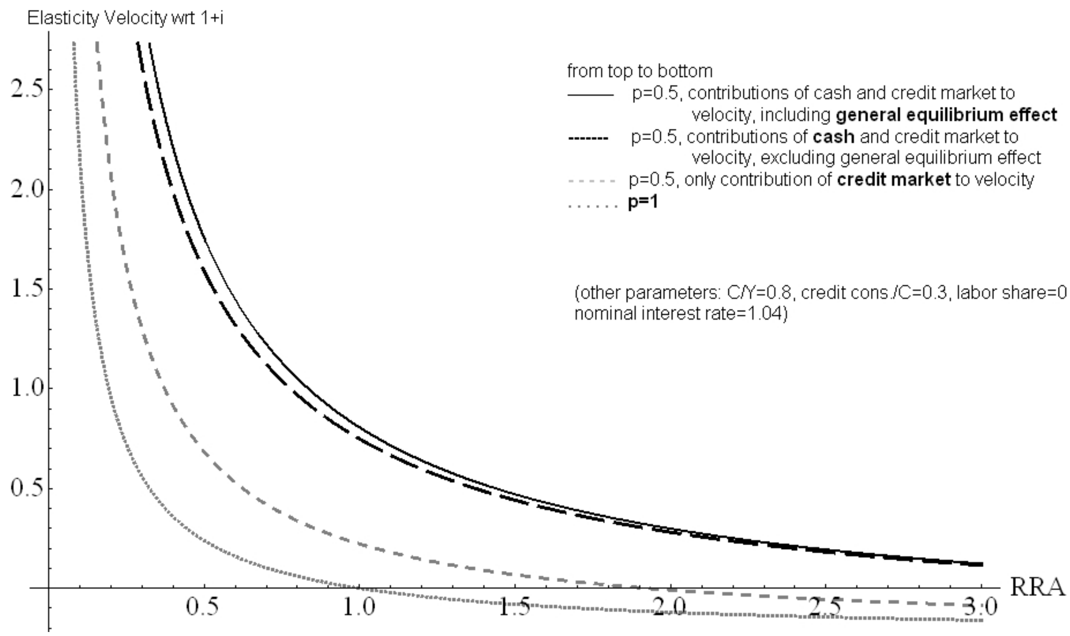

Figure 1: Contributions of Idiosyncratic Uncertainty.

4.1 Preference and Production Parameters

We calibrate the preference and production parameters of the model as follows. $\beta = 0.9901$ matches the annual capital-output ratio of 3. $\sigma = 2$ is chosen within, and on the lower side of, the standard range of 1-4 in the literature. $A = 34$ is chosen to match aggregate labor supply of 0.3. The capital share of output is measured in the data to give $\theta = 0.36$. Quarterly depreciation rate of 2%, consistent with estimates in the data, gives $\delta = 0.02$.

The constant x_1 is calibrated here in two ways. This parameter gives us the size of the retail (cash) market. It also affects the overall level of velocity in the model. As a first alternative, we choose x_1 to target the size of the retail market, given other parameters; we choose this size to be 72% of total consumption in the model, consistent with the aggregate fact, documented in Telyukova(2009), that roughly 75% of the total value of consumer transactions in 2001 took place using liquid payments methods - cash, checks, and debit cards. This number was quoted at 82% in 1986 in Wang and Shi (2006), based on a consumer survey. We remain close to the 2001 target. This produces the parameter $x_1 = 6$.

Alternatively, in a second calibration, we choose x_1 to target the average level of M2 output velocity (V_y) in our data sample (1984-2007, as detailed below). This level of velocity is 1.897, and it gives us $x_1 = 1.042$, with the associated size of

Table 1: Two Alternative Calibrations

	β	σ	A	x_1	θ	δ
Calib. 1 (x_1 targets cash market size)	0.99	2	34	6	0.36	0.02
Calib. 2 (x_1 targets $\mathbb{E}(V)$)	0.99	2	12	1.04	0.36	0.02

the cash market of 50% of total consumption. This calibration also changes the A parameter given our labor supply target. We will show that the dynamic results are not very sensitive to the magnitude of x_1. Table 1 presents the two alternative calibrations discussed so far.

4.2 Idiosyncratic Preference Shock Process

We pose the log of the preference shock to be i.i.d. $N(0, \sigma_\vartheta)$.[16] We interpret our preference shocks as causing fluctuations in household liquid consumption *beyond* expected (e.g. seasonal or planned) fluctuations in the data. To calibrate the process for this shock, we use the same methodology as Telyukova (2009), which estimates a similar process, except of a persistent nature and at monthly frequency, by matching time series properties of survey data on liquid household expenditures. We use quarterly data from the Consumer Expenditure Survey (CEX), and restrict attention to the period 2000-2002. We thus bias the target against our model: before the mid-1990's, credit cards were not ubiquitous, so that many more goods could be considered cash goods than would be today, and these would likely contribute to a higher volatility estimate.

The key measurement that we need in order to calibrate the shock process is the unpredictable component of volatility of cash-good consumption in the data. We take this component of volatility to reflect optimal responses by households to unexpected preference shocks.[17] We will adopt this volatility measure as a calibration target, and use simulated method of moments (SMM) to estimate the standard deviation of the shock process σ_ϑ such that standard deviation of cash-good consumption in the model matches the data target.

The process of this measurement of the unpredictable component of liquid con-

[16]Because of quasilinearity and credit markets in our setting, a shock process of the form $\vartheta' = \vartheta^\rho \varepsilon'$ with $\rho > 0$ would not change aggregate implications of our model, as can be shown from the first-order conditions of the problem.

[17]The preference shocks reflect any situation from being locked out of one's house to a significant household repair that requires fairly quick payment by cash or check, e.g. In these situations, not having the money to meet the expense is very costly, which is well captured by a parameter that shifts (marginal) utility.

sumption is described in Telyukova (2009) in detail; here we recap the essential details. The first step is to separate out cash goods in the CEX data. As our measure, based on the American Bankers Association's 2004 survey of consumer payment methods, we use the following group: food, alcohol, tobacco, rents, mortgages, utilities, household repairs, childcare expenses, other household operations, property taxes, insurance, public transportation, and health insurance. Even in 2004, consumers reported paying for these types of goods with liquid assets (primarily cash and check) in 90% or more of transactions. This proportion would clearly be higher over our longer period of inquiry, 1984-2007. This measure is also conservative, along some other dimensions, from the standpoint of measuring unexpected expenses. First, volatility of expenses could be driven by seasonality (e.g. Christmas gift shopping), and to control for that in part, we remove any expenses made as gifts, which is observable in the CEX; below we also remove seasonality in our regression analysis. Second, the cash-good category excludes many situations that may be reflections of emergencies that require liquid payment, such as an emergency purchase of (or downpayment on) a durable to replace - rather than repair - a broken durable, such as a car or an appliance. Similarly, medical payments, which include co-pays or other out-of-pocket expenses, some of which are unpredictable and may require a liquid payment - are not included either; the decision here was driven by the fact that medical expenses may be payable by credit card today, even though historically this would not be the case. Thus, in measuring the volatility of cash-good consumption, using a lot of the "smooth" good categories while excluding many that may reflect other types of emergencies besides repairs, may understate the measurement of the uncertainty that households face, against which they may hold liquid assets.

Using the above definition of cash goods, we take a number of steps to separate out the idiosyncratic uncertainty component. On the liquid consumption series, we estimate the following fixed-effect model with AR(1) innovations:

$$\begin{aligned} \log(c_{it}^{liq}) &= \beta \mathbf{X_{it}} + u_i + \varepsilon_{it} \\ \varepsilon_{it} &= \rho \varepsilon_{i,t-1} + \eta_{it}. \end{aligned} \tag{36}$$

The vector $\mathbf{X}$ includes, depending on specification, household observables, such as age (a cubic), education, marital status, race, earnings, family size, homeownership status, as well as seasonal effects (a set of month and year dummies). Several specifications including different sets of these observables all produced nearly identical results. u_i is the household fixed effect. The residual ε_{it} is the idiosyncratic com-

Table 2: Unpredictable Volatility of Liquid Consumption, Quarterly CEX Data

	Standard Deviation of η_{it} (%)
Benchmark	19.6
Excluding food	27.5
Excluding food and property taxes	29.4

Table 3: Discretized Log-Preference Shock Process

Log-shock standard deviation: $\sigma_{\vartheta} = 0.405$
Standard deviation of log-liquid consumption (data/model): 0.196/0.196

$\ln\vartheta_1$	$\ln\vartheta_2$	$\ln\vartheta_3$	$\ln\vartheta_4$	$\ln\vartheta_5$	$\mathbb{P}(\vartheta_1)$	$\mathbb{P}(\vartheta_2)$	$\mathbb{P}(\vartheta_3)$	$\mathbb{P}(\vartheta_4)$	$\mathbb{P}(\vartheta_5)$
-0.81	-0.40	0	0.40	0.81	0.07	0.24	0.38	0.24	0.07

ponent of liquid consumption, and it further consists of a persistent component and a transitory component. Since our preference shock is assumed to be i.i.d., we consider the autoregressive component above as predictable, and the innovation η_{it} as reflecting household response to the preference shocks. Table 2 presents the standard deviation of η_{it} based on our benchmark cash-good measure above, as well as two alternatives that exclude some of the more predictable expense groups. We will take the benchmark standard deviation of 19.6% to do our estimation in the model, clearly the most conservative measure.

The estimate of the standard deviation of the log-preference shock that results from our SMM procedure is $\sigma_{\vartheta} = 0.4045$. We discretize our i.i.d. shock under the assumption of Gaussian distribution using the Tauchen (1986) method, and approximate the distribution by 5 discrete shock states, with shocks at maximum two standard deviations away from their mean. Table 3 presents the discretized states, where we denote the probability of a discrete shock state ϑ_i by $\mathbb{P}(\vartheta_i)$. Below, we check robustness of our calibration by discretizing the i.i.d. shocks into 11 shock states, rather than 5, and find the results to be robust.

Finally, to convince the reader that we do not overstate the amount of uncertainty in expenses through our shock calibration, we plot in figure 2 the steady-state distribution of the log of liquid consumption in the model, and compare it to the empirical distribution of the log-consumption residual (η_{it}), with bins centered at the same states as in the model. What is key for the quantitative performance of the model is that we capture the probability of binding shocks correctly; in our 5-state calibration, this is reflected in the top consumption state, as only the top

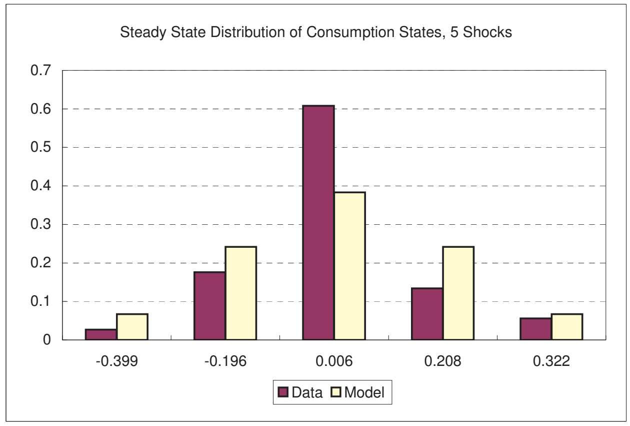

Figure 2: Log-Consumption Distribution in Model and Data

Table 4: Estimates of the Aggregate Shock Processes, 1984-2007

ξ_{zz}	ξ_{ii}	$\xi_{i\pi}$	ξ_{iy}	σ_{ε_1}	σ_{ε_2}
0.944	0.780	0.133	0.011	0.006	0.001

shock binds in our calibrated equilibrium. From the figure, it is apparent that our calibration captures the probability of the top state accurately.

4.3 Aggregate Shock Processes

Finally, we calibrate technology and monetary policy shocks. We model these as two separate processes, as described above. We estimate in our data sample the following two regressions:

$$\begin{aligned} z_t &= \xi_{zz} z_{t-1} + \varepsilon_1 \\ \ln\left(\frac{1+i_t}{1+\bar{i}}\right) &= \xi_{ii} \ln\left(\frac{1+i_{t-1}}{1+\bar{i}}\right) + \xi_{i\pi} \ln\left(\frac{1+\pi_{t-1}}{1+\bar{\pi}}\right) + \xi_{iy} \ln\left(\frac{y_{t-1}}{\bar{y}}\right) + \varepsilon_2 \end{aligned}$$

z_t is the Solow residual measured in the standard way, and we take out the linear trend from both the Solow residual and the output series. The variables with bars over them capture long-term averages of the respective variables in our sample period, as is standard in estimating central banks' targets in policy rules. The sample of data on which we estimate this process is from 1984 until 2007, to capture the period when the Federal Reserve is perceived to have begun using (implicit) inflation targeting. Notice that our interest rate rule depends on endogenous variables. We use the Federal Funds rate as the measure of choice of interest rates in the data. The resulting coefficients are in table 4.

5 Computation

We employ the Parameterized Expectations Approach (PEA) to solve the model. The method approximates the expectations terms in our Euler equation system (25) - two in total - by polynomial functions of the state variables, and the coefficients of the approximation and solved for. We choose the following forms:

$$\mathbb{E}\left[(c')^{-\sigma}(1+e^{z'}\theta(k')^{\theta-1}(h')^{1-\theta}-\delta)\right] = \psi^1(\chi,;\gamma^1) \tag{37}$$
$$\mathbb{E}\left[\frac{1}{w'}\right] = \frac{\widetilde{\mathbb{E}}}{w} = \psi^2(\chi;\gamma^2)$$

where,

$$\psi^j(\chi;\gamma^j) = \gamma_1^j \exp(\gamma_2^j \log k + \gamma_3^j \log z + \gamma_4^j \log i_{-1} + \gamma_5^j \log i + \gamma_6^j \log[\phi_{-1}(1+\varpi_{-1})])$$

The accuracy of approximation can be increased by raising the degree of approximating polynomials above; we have experimented with several forms and found the results robust to them. We find that the convergence properties of our model are good: convergence is monotone and very robust. In order to compute moments from the model, we re-run the model solution 100 times given parameters of the model, and the simulations within each run are for 10,000 periods, where we discard the first 1,000 in computing the moments.

6 Results

In this section, we describe our results with respect to the dynamics of nominal and real variables, and show how the presence of precautionary demand affects the quantitative performance of our model. We also show sensitivity results for our different calibration strategies, described above, as well as examine robustness of preference shock discretization with more states.

6.1 The Role of Precautionary Money Demand

We choose M2 as the basis for analysis of the data, to follow Hodrick et al (1991) and Wang and Shi (2006), among others. Another reason of using this aggregate is that it exhibits much more stationarity over time than M1. Finally, in our micro data work, we think of liquid payment methods as not only cash, but also checks and debit cards, which implies inclusion in the monetary aggregate of checking and savings accounts.[18]

[18]Strictly speaking, this makes our model's money concept fall between M1 and M2, however, for the reasons mentioned, we think of M2 as the appropriate approximation.

To highlight the role of precautionary demand for money in generating the quantitative results that we present, we compute, throughout this section, two versions of our model: one is the benchmark as presented above; the other is the benchmark with the preference shocks shut down. Specifically, in this version, we assign to everyone the highest preference shock with probability 1; this means that everyone's cash constraint binds at all times. We refer to this version of the model as the no-shock model; we think of it as closely replicating standard cash-credit good models with only aggregate risk.

Table 5 summarizes the results concerning the dynamic properties of some key nominal variables. The first column of the table presents results in the data. The second column presents the results for the no-shock model. The last two columns show the results in our benchmark model with the two alternative calibrations for x_1 - one targeting the cash-market size of 72% of consumption, the other - targeting $\mathbb{E}(V_y)$. Besides this moment, we are not targeting any of our result moments in calibration, so that the model is left free in terms of its dynamic performance.

Table 5: Dynamic Properties of the Model

Moment	Data	No-Shock Model	Full Model (Calibration 1)	Full Model (Calibration 2)
$\mathbb{E}(V_y)$	1.897	1.812	1.357	1.898
$\mathbb{E}(V_c)$	1.120	1.380	1.033	1.445
$\sigma(V_y)$	0.017	0.010	0.014	0.014
$\sigma(V_c)$	0.014	0.0002	0.012	0.011
$\sigma(y)$	0.009	0.012	0.012	0.012
$\sigma(1+i)$	0.0026	0.001	0.002	0.002
$corr(V_y, y)$	0.638	0.993	0.585	0.602
$corr(V_y, g_y)$	0.059	0.289	0.142	0.145
$corr(V_c, g_y)$	-0.094	0.110	-0.071	-0.070
$corr(V_y, g_c)$	0.127	0.539	0.233	0.262
$corr(V_c, g_c)$	-0.027	0.176	-0.155	-0.139
$corr(V_y, 1+i)$	0.714	-0.210	0.645	0.638
$corr(V_c, 1+i)$	0.690	-0.896	0.897	0.897
$\varepsilon_{V_y,1+i}$	5.072	-1.747	4.546	4.469
$\varepsilon_{V_c,1+i}$	4.158	-0.123	5.072	4.994
$corr(1+\pi, 1+i)$	0.529	0.768	0.361	0.358

All data logged and BP filtered. Data period: 1984-2007. Velocity moments calculated based on M2. Interest rate is the Fed Funds rate. Inflation measured based on GDP deflator. g_y refers to output growth. "No-Shock" model is the version of the model with idiosyncratic preference shocks shut down. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods each. Calibration 1 sets parameter x_1 to target cash-market size of 72% of total consumption. Calibration 2 sets x_1 to target $\mathbb{E}(V_y)$.

As is clear from the table, precautionary motive for holding money makes an enormous difference for the performance of the model: without it, the model is not able to capture almost any of the moments in the data, while introducing precautionary demand makes the model align quite successfully on nearly all of the dimensions listed. When we do not target mean output velocity, we underpredict the level of both velocity measures by a bit. In our model, as in other monetary business cycle models, money turns over only once a quarter, so it is not surprising that when we do not target mean velocity, we do not get the level high enough. It is also not surprising that in the no-shock model, mean velocity is higher than in the model with shocks; when the cash constraint is always binding, the cash market contributes exactly 1 to velocity level every period. In the model with shocks, that contribution is less than 1 for all the non-binding shock realizations; all but 7% of the households do not spend all of their money, and hence contribute less than 1 to velocity. When we do target output velocity level, we overpredict consumption velocity level slightly.

In terms of volatility of velocity, as our analytical results suggested, we do much better in the model with the preference shocks than in the model without. Even with our relatively low risk aversion parameter (a parameter that needed to be high in both Hodrick et al (1991) and Wang and Shi (2006) to begin to get significant velocity volatility), the model with the preference uncertainty gets close to the data in terms of the volatility of velocity. For output velocity, the model with the preference shocks produces 40% higher volatility than the no-shock model; for consumption velocity, the benchmark produces 60 *times* the volatility of the no-shock model. As discussed in our theory section, the reason is that with the introduction of preference shocks, the cash market contributes significantly to volatility of velocity, whereas it would contribute nothing if the cash constraint were always binding. Notice also that in the model with precautionary demand, we get the proportion of consumption velocity to output velocity volatility right, while it is very far off target in the no-shock model. Since consumption velocity is the major component of output velocity, consumption expenditure being 75% of GDP, the no-shock model is an unsatisfying theory of velocity dynamics because these dynamics come from the wrong source in that model.

Due to the properties of our exogenous driving process, we overpredict output volatility slightly and equally in both the benchmark and the no-shock model. The latter, however, underpredicts volatility of velocity substantially, leading us to conclude that the overprediction of output volatility is immaterial in creating excess

Table 6: Properties of Velocity Volatility

Moment	Data	No-Shock Model	Full Model (Calibration 1)	Full Model (Calibration 2)
$\sigma(M/P)$	0.013	0.003	0.012	0.012
$\mathrm{corr}(V_y, V_{y,-1})$	0.941	0.898	0.898	0.899
$\mathrm{corr}(V_c, V_{c,-1})$	0.937	0.896	0.898	0.899
$\mathrm{corr}(\pi, \pi_{-1})$	0.901	0.870	0.844	0.844
$\mathrm{corr}(M/P, M/P_{-1})$	0.944	0.921	0.898	0.898

All data logged and BP filtered. Model: average of 100 runs.

velocity volatility.

Continuing down the list, the model with preference shocks replicates most moments very well, and much better than the model without shocks. As we analyzed in the model section, we expect that for the risk aversion parameter $\sigma > 1$, as it is here, the correlation of velocity with nominal interest rates (and the respective elasticity) will be negative in the no-shock model, counter to the data. The relevant rows in the table confirm this: with the preference shocks, the relationship between velocity and nominal interest rates has the correct sign and magnitude; without shocks, it is significantly negative. The correlations of output with growth of output and consumption also flip signs relative to data if the model has no idiosyncratic preference risk; with the shocks, the signs are correct, and the magnitudes are close to the data for the most part. Finally, it is apparent that either calibration of the benchmark produces almost the same results with respect to all of the moments listed.

Table 6 presents the dynamic behavior of real balances (M/P), as well as the autocorrelations of velocity, inflation and real balances. These moments, all close to the data in the benchmark model, show that velocity volatility does not come from excessive volatility in the value of the money stock or from volatility at the wrong frequency. Note also that in the no-shock model, volatility of real balances is extremely low relative to the data, again implying that it is unable to reproduce dynamics of the sources of velocity fluctuations.

6.2 Some Other Aggregate Facts

We now assess the performance of our model according to an additional set of facts, listed by Cooley and Hansen (1995) as some of the more significant monetary features of business cycles. A first set of these facts is presented in table 7. The first two

Table 7: More Monetary Business Cycle Facts

Moment	Data	Model	CH data	CH
$corr(V, y)$	0.64	0.58	0.37	0.948
$corr(p, y)$	-0.13	-0.28	-0.57	-0.22
$corr(g_m, y)$	-0.11	-0.15	-0.12	-0.01
$corr(g_m, \pi)$	-0.06	-0.13	-0.29	0.92

All data logged and BP filtered. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods. Benchmark calibration 1.

columns list the performance of our model (Calibration 1) against the data, while the second two show the comparable moments from the Cooley and Hansen sample, and their own performance along this dimension. The facts are that velocity is procyclical, prices are countercyclical, and that correlation of output and inflation with the growth of money supply is negative. We match these facts in the model and get the magnitudes of the correlations about right.

Finally, we want to highlight some aspects of the data that we are not so successful in capturing. As is clear from table 8, our model, as the previous models in its class, misses the liquidity effect, i.e. the negative correlation of nominal interest rates with money growth, and prices and inflation are too flexible relative to data. All of this produces the series of moments replicated in this table. The Cooley-Hansen cash-in-advance model is again presented as a point of comparison and similarly misses these moments. In general, it is not surprising that our prices are completely flexible - we do not build in any frictions to change the speed and magnitude of price adjustment - and that we miss the liquidity effect, partly as a result of this. Sluggish adjustment of prices, as for example in Alvarez, Atkeson and Edmond (2008), is difficult to incorporate without a mechanism like market segmentation. We do not target this mechanism and hence did not expect to get the liquidity effect right.

Appendix B shows additional dynamic results from the model via cross-correlations of some real and nominal variables with output.

6.3 Sensitivity Analysis: 11 Preference Shock States

We also test for sensitivity of our results to how we discretize the idiosyncratic preference shock. In our benchmark case, we estimate the continuous AR(1) process with 5 discrete shock states. We also tried alternatives with 3 to 11 discrete states.

Table 8: Liquidity Effect

Moment	Data	Model	CH data	CH
$corr(g_m, i)$	-0.7(M1)/0.07(M2)	0.79	-0.27	0.72
$corr(y, i)$	0.54	-0.13	0.40	-0.01
$corr(y, \pi)$	0.37	-0.25	0.34	-0.14
$corr(g_m, p)$	0.03	0.61	-0.16	0.43

All data logged and BP filtered. All model moments are computed from 100 repetitions of model solution, with simulations of 10,000 periods.

Table 9: Discretized Log-Preference Shock Process, 11 States

$\ln \vartheta_i$	-0.81	-0.65	-0.49	-0.32	-0.16	0.00	0.16	0.32	0.49	0.65	0.81
$\mathbb{P}(\vartheta_i)$	0.036	0.045	0.078	0.116	0.146	0.159	0.146	0.116	0.078	0.045	0.036

The only thing that can change under alternatives with more than five discrete states is the number of binding shocks and the probability of these: with just five, only the top shock always binds, with constant probability; with eleven, the top two, and sometimes the top three, can bind. Regardless, we have found the results to be very robust. The alternative calibration of the shocks, and the respective results, in a comparison with data and the benchmark model with five shock states, are presented in tables 9 and 10. As is clear from the results table, under this parameterization, the dynamic properties of the model are extremely similar to those of the model with five discrete shock states.

7 The Pitfalls of Standard Calibration under Omission of Precautionary Demand

If precautionary demand for money is omitted from the model, the standard practice of calibrating monetary models to aggregate money demand, which may have a significant precautionary component, would likely produce misleading parameter values and thus affect the quantitative performance of the model. To demonstrate this here, we perform the following test. We take, again, a version of our benchmark model with the preference shocks shut down, so that the cash constraint always binds. But instead of fixing the calibration to the benchmark, as we did above, we will now calibrate the model to two standard targets in the monetary literature: the expected value of velocity, $\mathbb{E}(V_y)$, and the elasticity of inverse velocity with respect

Table 10: Sensitivity: Dynamic Properties of the Model, 11 Shock States

Moment	Data	5 Shock States (benchmark)	11 Shock States
$\mathbb{E}(V_y)$	1.897	1.357	1.395
$\mathbb{E}(V_c)$	1.120	1.033	1.063
$\sigma(V_y)$	0.017	0.014	0.015
$\sigma(V_c)$	0.014	0.012	0.012
$\sigma(y)$	0.009	0.012	0.012
$\sigma(1+i)$	0.0026	0.002	0.002
$corr(V_y, y)$	0.638	0.585	0.569
$corr(V_y, g_y)$	0.059	0.142	0.163
$corr(V_c, g_y)$	-0.094	-0.071	-0.039
$corr(V_y, g_c)$	0.127	0.233	0.247
$corr(V_c, g_c)$	-0.027	-0.155	-0.130
$corr(V_y, 1+i)$	0.714	0.645	0.629
$corr(V_c, 1+i)$	0.690	0.897	0.863
$\varepsilon_{V_y,1+i}$	5.072	4.546	4.448
$\varepsilon_{V_c,1+i}$	4.158	5.072	4.940
$corr(1+\pi, 1+i)$	0.529	0.361	0.288

Both models calibrated to target cash market size of 72% of total consumption.

to the nominal interest rate, $-\varepsilon_{V,1+i}$. This exercise produces different values for parameters x_1, A and, most importantly, σ - the value of relative risk aversion. These alternative values are presented in table 11. The model without preference shocks can only reproduce the monetary targets with the value of $\sigma = 0.15$.[19] This is extremely low relative to the standard range of σ used in calibrated real business cycle models, and very low relative to our benchmark calibration for the model with preference shocks, which reproduces the same targets with $\sigma = 2$. But it is not surprising that such a low value is needed: we showed analytically that the no-shock model cannot get the sign of the elasticity of velocity with respect to nominal interest rates right unless $\sigma < 1$.

The dynamic properties of this model are presented in table 12, where we compare our benchmark model calibrated to match $\mathbb{E}(V_y)$ (Calibration 2) with the no-shock model with the same target. It is apparent that even when targeting $\mathbb{E}(V)$ and $\varepsilon_{V,1+i}$, the model does badly along other nominal dimensions, even when it comes to the same quantities pertaining to *consumption* velocity, and now the quantitative

[19] Values of σ very close to this commonly arise in monetary models without precautionary money demand, from sticky-price models (Rotemberg and Woodford, 1997) to monetary search models (Lagos and Wright, 2005).

Table 11: Targeting Money Demand in Data without Precautionary Money Demand in Model

β	σ	A	x_1	θ	δ
0.990	0.15	3.10	0.508	0.36	0.02

implications on the real side of the model suffer noticeably as well. For instance, here the no-shock model overstates volatility of output, consumption and investment significantly, doubling these volatilities relative to data, while the benchmark gets these standard deviation measures fairly close to the data. In other words, even if we were willing to accept the calibrated parameters that this exercise requires, the results that the model produces are far inferior to the performance of our benchmark with preference shocks. This is true along both nominal and real dimensions, even though the model is rigged, in how it is parameterized, to do well quantitatively on the nominal side. Without precautionary demand in this setting, it seems to be impossible to match both real and nominal facts at the same time. Modeling precautionary demand for money solves this problem.

All of this is relevant, because it suggests that omitting precautionary money demand from monetary models may produce inaccurate results in not only matching data facts, but also conducting policy experiments and drawing normative conclusions.

8 Welfare Costs of Inflation

In our model, changes in steady state inflation distort cash-good consumption only for the binding shock realizations, while at the same time, households value this consumption more than average, and are therefore more sensitive to these distortions. Thus, incorporating idiosyncratic uncertainty adds two opposing forces to the welfare costs of inflation relative to a model without precautionary demand. It could depend on the values of the parameters which of the two forces dominates. Comparing steady states, we calculate the welfare cost of inflation as the percentage reduction in consumption under the Friedman Rule (or zero inflation) that would make a household indifferent between the Friedman Rule and a higher-inflation state. This measure is $1 - \Delta$, with

$$U(\Delta c(\pi_b)) + \mathbb{E}[\vartheta u(\Delta q_\vartheta(\pi_b))] = U(c(\pi)) + \mathbb{E}[\vartheta u(q_\vartheta(\pi))],$$

Table 12: No-Shock Model Targeting Money Demand in the Data

Moment	Data	Benchmark (Calibration 2)	No-Shock Model (Target Money Demand)
$\mathbb{E}(V_y)$	**1.897**	**1.898**	**1.895**
$\mathbb{E}(V_c)$	1.120	1.445	1.442
$\sigma(V_y)$	0.017	0.014	0.028
$\sigma(V_c)$	0.014	0.011	0.003
$\sigma(y)$	0.009	0.012	0.022
$\sigma(c)$	0.005	0.003	0.012
$\sigma(inv)$	0.050	0.044	0.114
$\sigma(1+i)$	0.0026	0.002	0.002
$corr(V_y, y)$	0.638	0.602	0.905
$corr(V_y, g_y)$	0.059	0.145	0.411
$corr(V_c, g_y)$	-0.094	-0.070	-0.185
$corr(V_y, g_c)$	0.127	0.262	0.323
$corr(V_c, g_c)$	-0.027	-0.139	0.256
$corr(V_y, 1+i)$	0.714	0.638	0.333
$corr(V_c, 1+i)$	0.690	0.897	0.999
$\varepsilon_{V_y,1+i}$	**5.072**	4.469	**5.030**
$\varepsilon_{V_c,1+i}$	4.158	4.994	1.763
$corr(1+\pi, 1+i)$	0.529	0.358	0.657

Benchmark with five shock states, calibrated to target $\mathbb{E}(V_y)$. Bolded quantities represent calibration targets.

where the argument in $c(.), q(.)$ denotes the corresponding level of inflation - either π_b, the benchmark cases of Friedman rule or zero inflation, or π, some level of inflation greater than π_b. Manipulating the expression, we get

$$\Delta = \left(\frac{c(\pi)^{1-\sigma} + \sum_{\vartheta} \mathbb{P}(\vartheta)\vartheta q_{\vartheta}(\pi)^{1-\sigma}}{c(\pi_b)^{1-\sigma} + \sum_{\vartheta} \mathbb{P}(\vartheta)\vartheta q_{\vartheta}(\pi_b)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}. \tag{38}$$

We are able to derive the steady state quantities in closed form (see appendix C), and graph the welfare cost as a function of the yearly inflation rate. The dashed (red) line in figure 3 is the portion of welfare cost coming from consumption distortion only under the top idiosyncratic shock, which always binds, while the solid (blue) line graphs the total welfare cost. The dashed line is meant to parallel a model in which the total proportion of households who are at the binding cash constraint is constant regardless of the inflation level, as would be the case in a model without idiosyncratic uncertainty. The solid line incorporates the other margin - that as the level of inflation rises, so does the share of constrained households - which is a margin present in our model, and one that makes a significant quantitative difference, as is shown below. In the left panel of figure 3, we can see that the welfare cost of 10%

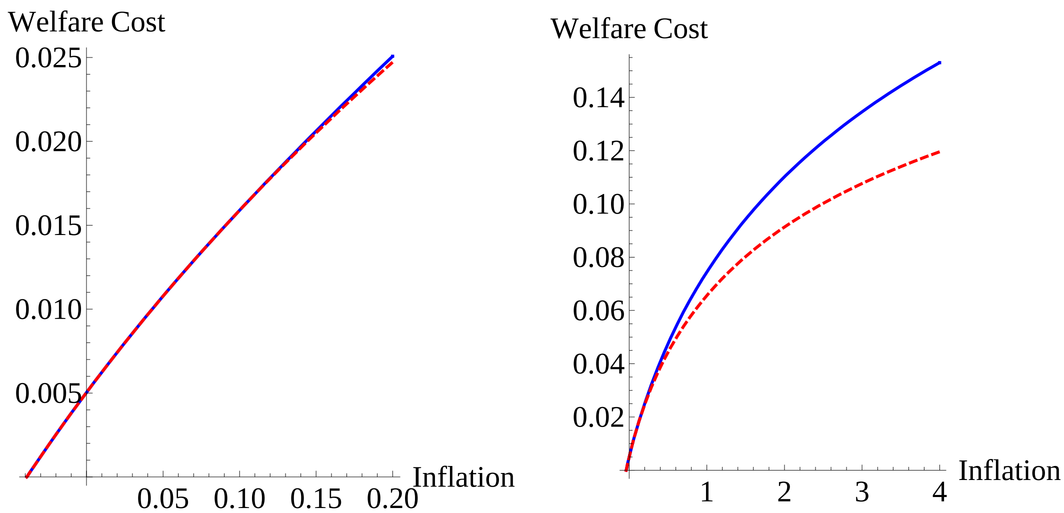

Figure 3: Welfare Cost of Inflation (Benchmark: Friedman Rule)

(yearly) inflation compared with the Friedman rule is 1.58% of the efficient level of consumption, and about 1.2% of output. Compared with 0% inflation, the welfare cost is about 1.46% of consumption, and about 1.1% of output. These numbers are somewhat larger than the numbers in Cooley and Hansen (1989) who report, in a slightly different calculation, welfare costs of about 0.5%, but very similar to numbers in Lucas (2000) and Lagos and Wright (2005).[20]

In the right panel we see where the changing proportion of households at a binding cash constraint starts to matter for the magnitude of the welfare cost. At higher levels of inflation, between 100% and 400%, it is evident that extrapolating the welfare cost from low-inflation states, where in our calibration only the highest shock binds, leads to an underestimate. At 400% inflation, where in our calibration three out of five shocks bind, thus making the share of constrained households much higher than in low-inflation states, this underestimate is significant: 3.5 percentage points out of a total of 15.3% of consumption. As the precautionary motive seems prevalent in the data, a model with a full distribution of idiosyncratic shocks allows us to give a better approximation of welfare costs at higher levels of inflation than models which assume that the cash constraint always binds, and in which the share of consumption subject to this constraint is calibrated using velocity in low-inflation

[20] Lucas (2000) derives a welfare cost of 1.5% for 10% inflation for the log-log money demand with elasticity 0.5, and Lagos and Wright (2005) calculate 1.4% of consumption relative to zero inflation, and 1.6% of consumption relative to the Friedman rule, in their annual calibration, with take-it-or-leave-it pricing.

data.

9 Conclusion

In this paper, we study the aggregate implications of precautionary demand for money. We highlight the importance of modeling uninsurable idiosyncratic risk in preferences as a cause of precautionary motive for holding liquidity. By incorporating this idiosyncratic risk into a standard monetary model with aggregate risk, and by carefully calibrating the idiosyncratic shocks to data, we find that the model matches many dynamic moments of nominal variables well, and greatly improves on the performance of existing monetary models that do not incorporate such idiosyncratic shocks. We show that our results are robust to multiple possible ways of calibrating the model. We show also that omitting precautionary demand while targeting, in calibration, data properties of money demand - a standard calibration practice - produces inferior performance in terms of matching the data, potentially misleading implications for parameters of the model, and may therefore adversely affect the model's policy implications as well.

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A Detailed Proof of Proposition 1.

Proposition 1. *The implicit elasticity of consumption velocity with respect to the nominal interest rate, caused by a one-time fully anticipated money injection (in addition to the constant rate of money injection consistent with a given steady state level of $1+i$), is*

$$\varepsilon_{V_c,1+i} = s_c \left(\frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash,nb} \left(\frac{1+i}{p+i} \right) \left(\frac{1}{\varepsilon_{U'(c),q_h} + \sigma} \right) \tag{39}$$

Proof. The elasticity of velocity with respect to a change in the interest rate (caused by a one-time anticipated additional injection of money $1+\varpi$) is

$$\varepsilon_{V_c,1+i} = \frac{d \ln V_c}{d \ln 1+\varpi} \Big/ \frac{d \ln 1+i}{d \ln 1+\varpi}$$

Velocity is given by

$$V_c = \frac{PC}{M} = \frac{c}{\phi} + (1-p)\frac{q_l(1+i)}{\phi} + p\frac{q_h(1+i)}{\phi},$$

hence,

$$\varepsilon_{V_c,1+\varpi} = s_c(\varepsilon_{c,1+\varpi} - \varepsilon_{\phi,1+\varpi}) - s_{cash,nb}\varepsilon_{q_h,1+\varpi},$$

using $\varepsilon_{\psi,1+\varpi} = -\varepsilon_{q_h,1+\varpi}$.

Since a one-time fully anticipated money injection does not affect tomorrow's ϕ or $U'(c')$, we formulate the elasticities in terms of $\widetilde{\mathbb{E}\phi'}$, which for the duration of the proof we define as $\widetilde{\mathbb{E}\phi'} \overset{def}{=} U'(c')\phi'/(1+\varpi)$; then $\varepsilon_{1+\varpi,\beta\widetilde{\mathbb{E}\phi'}} = -1$, and

$$\varepsilon_{V_c,1+i} = \varepsilon_{V_c,\widetilde{\mathbb{E}\phi'}} / \varepsilon_{1+i,\widetilde{\mathbb{E}\phi'}}.$$

To derive $\varepsilon_{c,\widetilde{\mathbb{E}\phi'}}$, let us derive how h varies with c in the equilibrium. From equation (18), the elasticity is

$$\varepsilon_{c,h} = -\frac{\alpha}{\sigma}, \quad \varepsilon_{U'(c),h} = \alpha \tag{40}$$

Now, from the household budget constraint, we use (40) to derive

$$\varepsilon_{c,q_h} = -\frac{s_{q_h}}{s_c + \frac{1-\alpha}{\alpha}\sigma}, \quad \varepsilon_{U'(c),q_h} = \frac{s_{q_h}\sigma}{s_c + \frac{1-\alpha}{\alpha}\sigma} > 0, \tag{41}$$

where s_{q_h} is the share of total output going to q_h consumption, $s_{q_h} = (pq_h)/Y$; likewise $s_c = c/Y$. Moreover, $\varepsilon_{c,q_h} = -\frac{1}{\sigma}\varepsilon_{U'(c),q_h}$.[21]

[21] Equation (41) captures the general equilibrium effect from changes in q_h: a shift away from cash consumption will lead to an increase in credit market consumption. This effect is proportional to the share of cash consumption under the binding shock in total consumption. In case of idiosyncratic uncertainty, s_{q_h} is smaller (by a factor smaller than p) than total cash market consumption, and hence the elasticity in equation (41) is smaller.

The free entry condition now allows us to link tomorrow's value of money $\widetilde{\mathbb{E}\phi'}$ to today's movements in q_h. Free entry gives $U'(c)q_h = \beta\widetilde{\mathbb{E}\phi'}$ (which is consistent with $q_h = \beta\widetilde{\mathbb{E}\phi'}$ in the old definition), which means that

$$\varepsilon_{q_h,\beta\widetilde{\mathbb{E}\phi'}} = \frac{1}{\varepsilon_{U'(c),q_h}+1}, \tag{42}$$

leading to

$$\varepsilon_{c,\beta\widetilde{\mathbb{E}\phi'}} = -\frac{\varepsilon_{U'(c),q_h}}{\sigma(\varepsilon_{U'(c),q_h}+1)}. \tag{43}$$

To calculate $\varepsilon_{\phi,\beta\widetilde{\mathbb{E}\phi'}}$, use

$$U'(c)\phi = p\vartheta u'(q_h)\frac{\beta\widetilde{\mathbb{E}\phi'}}{U'(c)} + (1-p)\beta\widetilde{\mathbb{E}\phi'} = p\vartheta u'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}, \tag{44}$$

derived from the FOCs of m, q_h, to find

$$\begin{aligned}\frac{d\ln\phi}{d\ln\beta\widetilde{\mathbb{E}\phi'}} = {} & \frac{p\vartheta_h u'(q_h)q_h}{p\vartheta_h u'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}}\frac{d\ln(u'(q_h)q_h)}{d\ln q_h}\frac{d\ln q_h}{d\beta\widetilde{\mathbb{E}\phi'}} \\ & + \frac{(1-p)\beta\widetilde{\mathbb{E}\phi'}}{p\vartheta_h u'(q_h)q_h + (1-p)\beta\widetilde{\mathbb{E}\phi'}} - \frac{d\ln U'(c)}{d\ln q_h}\frac{d\ln q_h}{d\ln\widetilde{\mathbb{E}\phi'}},\end{aligned} \tag{45}$$

Combining (45) and (44), we find

$$\varepsilon_{\phi,\widetilde{\mathbb{E}\phi'}} = \left(\frac{p+i}{1+i}(1-\sigma) - \varepsilon_{U'(c),q_h}\right)\frac{1}{\varepsilon_{U'(c),q_h}+1} + \frac{1-p}{1+i}. \tag{46}$$

Likewise, we can calculate

$$\varepsilon_{U'(c)\phi,\widetilde{\mathbb{E}\phi'}} = \frac{d\ln U'(c)\phi}{d\ln\beta U'(c')\phi'} = \left(\frac{p+i}{1+i}(1-\sigma)\right)\frac{1}{\varepsilon_{U'(c),q_h}+1} + \frac{1-p}{1+i}. \tag{47}$$

With $\varepsilon_{\beta\widetilde{\mathbb{E}\phi'},1+i} = (\varepsilon_{U'(c)\phi,\beta\widetilde{\mathbb{E}\phi'}} - 1)^{-1}$ and $\varepsilon_{U'(c)\phi,\beta\widetilde{\mathbb{E}\phi'}}$ from (47), we find

$$\varepsilon_{1+i,\widetilde{\mathbb{E}\phi'}} = -\left(\frac{1+i}{p+i}\cdot\frac{\varepsilon_{U'(c),q_h}+1}{\varepsilon_{U'(c),q_h}+\sigma}\right). \tag{48}$$

Now we are able to calculate the elasticity of velocity with respect to $\widetilde{\mathbb{E}\phi'}$.

$$\begin{aligned}\varepsilon_{V_c,\beta\widetilde{\mathbb{E}\phi'}} &= \frac{d\ln V_c}{d\ln\widetilde{\mathbb{E}\phi'}} = -\frac{d\ln V}{d\ln 1+\varpi} = -\varepsilon_{V_c,1+\varpi} \\ &= s_c\Big(\frac{d\ln c}{d\ln\widetilde{\mathbb{E}\phi'}} - \frac{d\ln\phi}{d\ln\widetilde{\mathbb{E}\phi'}}\Big) + s_{cash,nb}\frac{d\ln\psi}{d\ln\widetilde{\mathbb{E}\phi'}} \\ &= s_c\left(\Big(\Big(-\frac{1}{\sigma}+1\Big)\varepsilon_{U'(c),q_h} - \frac{p+i}{1+i}(1-\sigma)\Big)\frac{1}{\varepsilon_{U'(c),q_h}+1} - \frac{1-p}{1+i}\right) \\ &\quad - s_{cash,nb}\frac{1}{\varepsilon_{u'(c),q_h}+1}.\end{aligned} \tag{49}$$

From (49) it follows that, for $p = 1$, this elasticity is negative if $\sigma < 1$; for $p < 1$, a larger σ will also lead to a negative elasticity. Combining (48) and (49) yields

$$\varepsilon_{V_c,1+i} = \left(\frac{1+i}{p+i} \cdot \frac{\varepsilon_{U'(c),q_h} + 1}{\varepsilon_{U'(c),q_h} + \sigma} \right) \times \tag{50}$$
$$\left(s_c \left\{ \left[\left(\frac{1}{\sigma} - 1 \right) \varepsilon_{U'(c),q_h} + \frac{p+i}{1+i}(1-\sigma) \right] \frac{1}{\varepsilon_{U'(c),q_h} + 1} + \frac{1-p}{1+i} \right\} \right.$$
$$\left. + \, s_{cash,nb} \frac{1}{\varepsilon_{u'(c),q_h} + 1} \right),$$

which simplifies to

$$\varepsilon_{V_c,1+i} = s_c \left(\frac{1}{\sigma} \frac{1+i}{p+i} - 1 \right) + s_{cash,nb} \left(\frac{1+i}{p+i} \right) \left(\frac{1}{\varepsilon_{U'(c),q_h} + \sigma} \right) \tag{51}$$

□

B Cross-Correlations of Aggregate Variables with Output in Benchmark Model

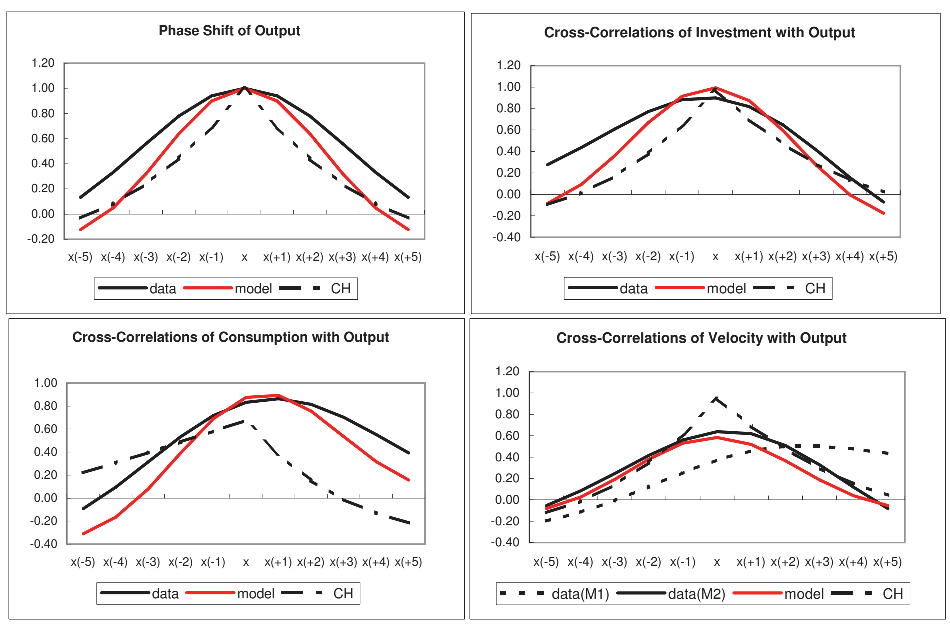

Figure 4: Cross-Correlations of Endogenous Variables with Output

To analyze our performance further with respect to facts highlighted by Cooley and Hansen, we present cross-correlations of several endogenous variables with output in graphical form, in figures 4 - 6. Where possible, we also graph the cross-correlations presented by Cooley and Hansen from their model.[22] With respect to the correlations of real variables with output, we do as well as the Cooley-Hansen model, or better. A notable improvement in our model relative to Cooley and Hansen concerns the dynamic pattern of output velocity (bottom right panel): we match the data for M2 velocity a lot more closely than they did. This last fact is the product of adding precautionary demand for money into the model; we further demonstrate this in figure 5 which shows the same cross-correlations, but comparing our benchmark to our no-shock model. In the bottom right panel, it is clear that the model with preference shocks does a lot better at matching the data than the model without. The other three panels of that figure also show that the improvement in dynamic patterns of real variables relative to Cooley-Hansen results in large part from our driving processes, rather than from preference shocks: we use an interest rate rule, while Cooley and Hansen used a money growth rule.

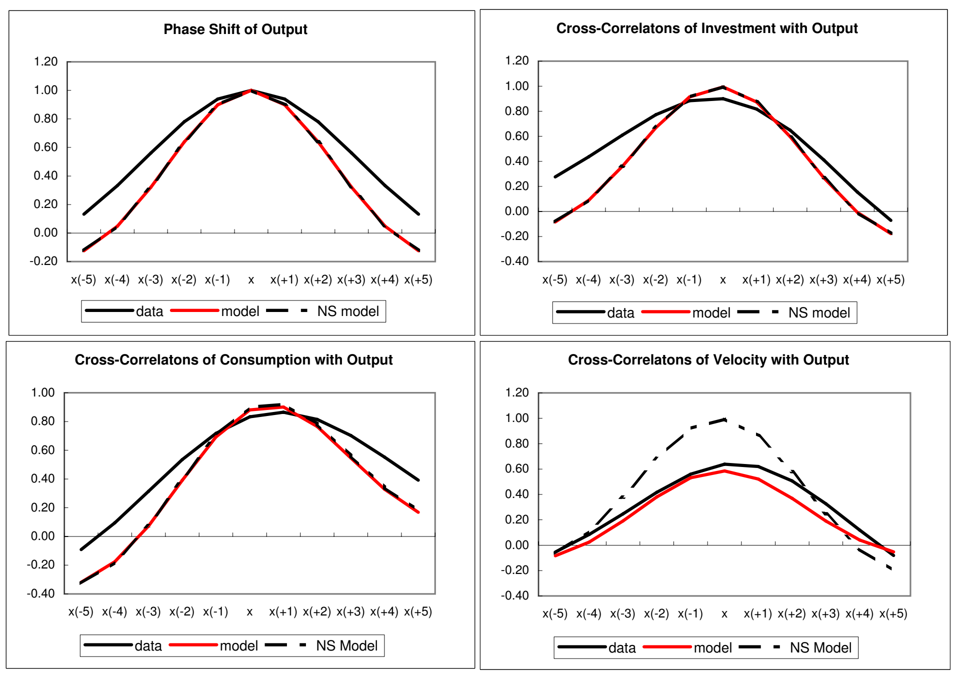

Figure 5: Cross-Correlations of Endogenous Variables with Output, Benchmark vs No-Shock Model

Finally, in figure 6 we present some further cross-correlations that we get less well. While we get the dynamic pattern of money supply half-right (although our cross-correlation bottoms out later than the data suggest), and we improve on Cooley

[22]Obviously, this comparison is limited in that their model is calibrated to a different time period; we do not present their data for space reasons.

and Hansen's cross-correlation of nominal interest rates, we get neither these two, nor the dynamic patterns of prices and inflation. Our performance on the bottom two panels is fairly close to Cooley and Hansen's. Again, we do not expect to get the patterns of prices to replicate the data with a price adjustment mechanism that is as flexible and frictionless as ours.

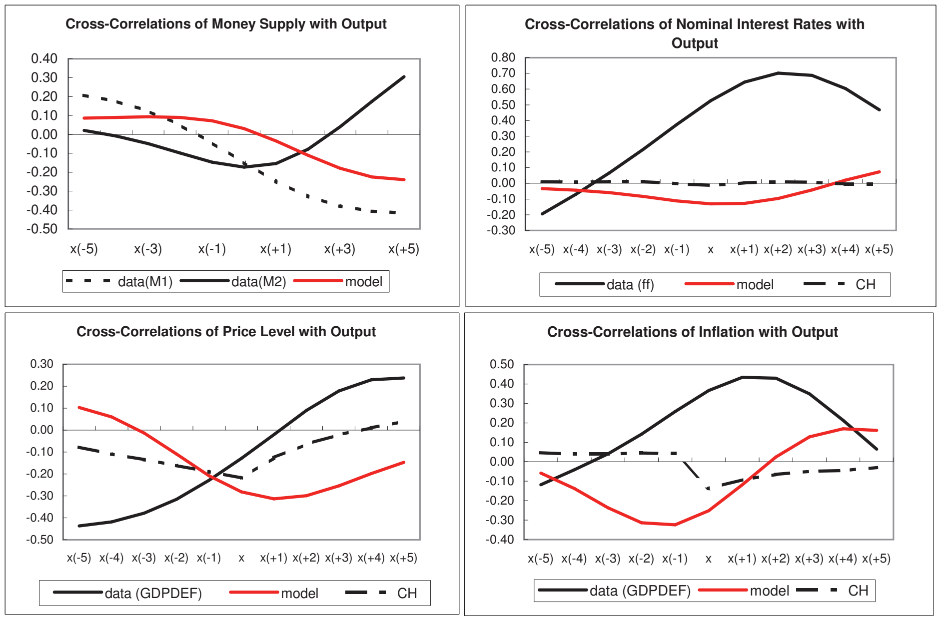

Figure 6: Cross-Correlations of Endogenous Variables with Output

C Steady State Consumption in Closed Form

We have to solve both for credit market and cash market consumption in order to conduct the welfare cost experiment. From the characterizing equation system (25), we get steady-state credit market consumption, after substituting in the capital-labor ratio, from

$$\bar{c} = \left[\frac{A}{1-\theta} \left(\frac{1}{\theta} \left(\frac{1}{\beta} - 1 + \delta \right) \right)^{\frac{\theta}{1-\theta}} \right]^{-\frac{1}{\sigma}}.$$

To get cash market consumption we again appeal to the system (25). The issue for the welfare-cost analysis is that as inflation rate increases, more of the discrete shocks cause the cash constraint to bind. Thus, we solve in closed form here for the general case: suppose that the total number of discrete shock states is n and k of these shocks, from ϑ_{n-k+1} to ϑ_n, bind. For any binding shock, the following system holds, given our functional forms:

$$\begin{aligned}
\bar{\mu}_{\vartheta_i} &= \mathbb{P}(\vartheta_i)\left(\vartheta_i x_1 \bar{q}_{\vartheta_i}^{1-\sigma}\bar{c}^{\sigma} - \frac{\bar{\phi}}{1+\bar{i}}\right) \;\; \forall\, i \in \{n-k+1, k\} \\
\bar{\phi} &= \left(\frac{1+\bar{i}}{\bar{i}}\right) \sum_{i=n-k+1}^{k} \bar{\mu}_{\vartheta_i} \\
\bar{q}_{\vartheta_i} &= \frac{\bar{\phi}}{1+\bar{i}} \;\; \forall\, i \in \{n-k+1, k\}.
\end{aligned}$$

From this, one can solve for the relevant μ_{ϑ_i}, which then determine ϕ, and finally q in all the binding states, which is a function of the nominal interest rate but not of the binding shock level, as expected. Instead, in the remaining (non-binding) states, consumption is given simply by

$$\bar{q}_{\vartheta_i} = (\vartheta_i x_i)^{\frac{1}{\sigma}}\bar{c} \;\; \forall \;\; i \in \{1, n-k\},$$

and is not a function of the nominal interest rate, but does change with the level of the non-binding shock.