Liquidity-Based Security Design: The Case of Uninformed Sellers*

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Abstract

Rational uninformed investors facing future liquidity shocks determine primary market prices for asset-backed securities. A competitive liquidity provider stands ready to buy tendered securities. Liquidity provision is distorted by a speculator who receives a private signal regarding the continuous cash flow distribution. This paper considers whether and how the underlying cash flows should be repackaged to maximize uninformed valuation. Uninformed investors hit with liquidity shocks sell (hold) a security if carrying costs exceed (fall below) expected trading losses. The optimal security design is shown to be a liquid senior debt tranche and illiquid junior tranche. A tradeoff theory of optimal leverage arises since increases in senior face value result in lower illiquidity discounts on the junior tranche but higher speculator effort and trading loss discounts on the senior tranche. Optimal senior face value induces junior illiquidity and is lowered marginally to reduce speculator effort. Endogenous speculator effort is U-shaped in senior face value, falling discretely once senior face is raised sufficiently to render the junior tranche illiquid. Consequently, the intensity of informed speculation is non-monotone in claim information-sensitivity. Finally, the privately optimal security design is socially suboptimal, as the desire to reduce speculator rents results in excessive illiquid claims.

1 Introduction

Tranched debt claims are ubiquitous in asset-backed securities (ABS) markets. In prescient papers, DeMarzo and Duffie (1999) and Biais and Mariotti (2005) show such structures can be understood

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as an optimal response by liquidity-motivated primary market issuers who design claims before observing private information and before choosing the quantity of retained claims. While these theories of security design surely offer a partial explanation of observed structures, the financial crisis of 2007/2008 brought to the fore the value *investors* attach to *secondary* market liquidity. How easily can an investor sell, and, more importantly, at what price? Moreover, while issuer private information is surely a concern, many issuers appear to be nearly as hapless as their investor base. Frequently, it was not issuers but *informed speculators* who were the first to identify problems and exploit superior information in trading against the uninformed.[1] This suggests the need for a theory of security design based upon investor demand for secondary liquidity, where the primary concern is the superior information of other investors.

The objective of this paper is to determine optimal security design when security sellers are themselves uninformed, but trade in competitive markets with informed speculation. I consider the following setting. There is a continuum of uninformed investors who either own the rights to a stream of cash flows or who determine primary market prices of securities backed by the cash flows, as in Holmström and Tirole (1993) and Maug (1998). The cash flows are drawn from a continuous distribution with support on the positive real line. The objective of security design is to maximize the uninformed valuation of the cash flows. After securities are designed, an unobservable fraction ($\underline{\gamma}$ or $\overline{\gamma}$) of uninformed investors is hit with a private liquidity shock biasing them towards selling in a competitive secondary market. The liquidity shock takes the form of a proportional carrying cost (C) if a security is held until maturity rather than sold. An uninformed competitive liquidity provider stands ready to buy tendered securities. However, liquidity provision is hindered by the existence of a speculator endowed with a private noisy signal of the true cumulative distribution function (good or bad) governing cash flow. The good ($\overline{F}$) and bad ($\underline{F}$) distribution functions have common support with densities satisfying a standard monotone likelihood ratio condition.

An uninformed investor hit with a liquidity shock expects to sell below fundamental value. Rather incur such a trading loss, an uninformed investor can simply hold until maturity and bear the carrying cost. That is, optimizing behavior by uninformed investors implies that the primary market discount on a given security is equal to the minimum of expected trading losses and carrying costs. Thus, the ex ante uninformed valuation of the total securitized cash flow stream is equal to expected cash flow less a discount for expected trading losses incurred on endogenously liquid securities less a further discount for expected carrying costs borne on endogenously illiquid securities. Within the set of payoff mappings respecting limited liability and having monotone payoffs, this paper determines the security design that minimizes the total endogenous discount.

[1] See Lewis (2010) for specific examples.

It is first shown that in order to have any effect on the total uninformed valuation of cash flows, some claim must be rendered illiquid. After all, if all claims are liquid, the speculator will capture trading gains across all markets, with total uninformed trading losses equal to those under a pass-through structure. Consequently, security design is shown to be irrelevant if the limiting likelihood ratio is sufficiently low in relation to the uninformed demand for immediacy as captured by the proportional carrying cost C.

If cash flow risk is sufficiently high in relation to C, it is feasible to induce the illiquidity of some claim and security design is relevant. Here it is optimal to induce illiquidity by splitting cash flow into two claims: a liquid senior ("debt") tranche having relatively low trading loss-information sensitivity and an illiquid junior ("equity") tranche having relatively high trading loss information-sensitivity. The trading loss information-sensitivity of both claims is increasing in senior debt face value, and the optimal face value is necessarily above the level required to induce junior illiquidity.

The optimal face value on the risky senior debt emerges from an intuitive tradeoff. As the face value on senior debt is raised, the carrying cost discount on the junior tranche falls since the junior tranche itself is smaller. However, the trading loss discount on the senior tranche rises as its value become more sensitive to the speculator's private information. At the optimal face value, marginal carrying costs on the illiquid tranche are equated with marginal trading losses on the liquid tranche. Riskless debt is not optimal since the marginal trading loss discount on such a senior claim is zero, below the marginal carrying cost on the junior claim. Further, at an optimal tradeoff, the senior debt face value is increasing in carrying costs and decreasing in the speculator's signal precision.

Since all agents are rational, the model allows one to analyze the social welfare. It is shown that the privately optimal security design differs from the social optimum. In particular, under the privately optimal securitization of cash flows, carrying costs are above the social optimum, i.e. liquidity as measured by uninformed selling volume is socially suboptimal. Intuitively, the privately optimal security design attempts to reduce speculator gains in the liquid senior market by adopting a lower senior debt face value. This increases the size of the illiquid tranche, implying larger deadweight carrying costs. In contrast, a neutral social planner would always opt for a higher senior face value since speculator gains are of no consequence in terms of aggregate social welfare. In fact, if the underlying cash flows have sufficiently low trading loss information-sensitivity, the planner would achieve first-best social welfare by packaging the cash flows as a liquid pass-through security. In contrast, the private optimum in this case deliberately creates an illiquid junior claim with the goal of reducing speculator gains.

In an extension, it is assumed the speculator has the ability to increase her signal precision at a convex cost. Here again tranching is optimal, with the optimal senior face value weighing carrying costs on the illiquid junior tranche against uninformed trading losses on the liquid senior tranche. However, here the trading loss function is shaped by endogenous changes in the speculator's incentive compatible signal precision. When the senior debt face value is sufficiently low, both securities markets are liquid and speculator effort incentives are maximal. At the threshold where the junior claim becomes illiquid, the incentive compatible signal precision falls discretely, leading to a discrete increase in the uninformed valuation. In some cases the optimal senior debt face value is the illiquidity threshold. In other cases, the optimal face value is above the illiquidity threshold, reflecting an equalization or marginal carrying costs on the illiquid junior tranche and trading losses on the liquid senior tranche, as in the baseline setting. However, the optimal senior debt face value is necessarily lower than in the baseline setting since high senior face value leads to high speculator effort and large uninformed trading losses.

The model extension also reveals an interesting relationship between securitization structure and the degree of intensity of informed speculation. At low senior debt face values, both senior and junior securities markets should be liquid, implying maximal incentive compatible speculator signal precision. At intermediate levels of senior debt face value, the junior security market becomes illiquid, implying lower speculator signal precision. The incentive compatible signal precision is then increasing in senior debt face value, converging to that obtained under a pass-through structure at maximal face value. That is, speculator effort is U-shaped in senior face value. This implies a non-monotone relationship between a security's information-sensitivity and the intensity of informed trading it attracts.

Since the informal folk-theorem of Myers and Majluf (1984), a number of papers have formally derived debt as an optimal funding source for a liquidity-motivated issuer holding private information. Within this literature, the present paper is most closely related to that of DeMarzo and Duffie (1999). They consider optimal pre-design (under ignorance) of a security by an issuer who becomes a monopolistic informed seller in a competitive primary market. In contrast, the model here features a continuum of uninformed sellers facing an informed speculator in a competitive secondary market. They confine attention to one marketed claim. I do not limit the number of securities, but find that it is optimal to split cash flow into two claims. In their model, debt is the optimal monotone source of funding since it minimizes the monopolistic seller's price impact in a fully-revealing separating equilibrium. That is, in the event that issuers signal via retentions, as in Leland and Pyle (1977), debt induces the least-costly separating equilibrium. In the present model, debt is the optimal liquid security since it minimizes uninformed trading losses per unit funding, with underpricing arising from non-revealing order flows. Finally, in their model there is no divergence between the private and socially optimal security design. Despite these differences, the present paper can be viewed as showing that DeMarzo and Duffie's argument is more general. In particular, liquidity motives of either issuers or investors can rationalize debt whether it is the issuer or an informed speculator who holds private information in either fully-revealing or partially-revealing trading stage equilibria.

Biais and Mariotti (2004) also consider optimal pre-design of a security by a prospective monopolistic informed seller. In contrast to DeMarzo and Duffie (1999), Biais and Mariotti assume liquidity providers can pre-commit to an issuance and transfer schedule, allowing for the possibility of cross-subsidies across seller types. Further, they consider that liquidity providers may have market power. In their setting too debt is an optimal security, with debt having the added advantage of reducing liquidity provider rents. In contrast to their paper, I do not consider the issue of liquidity provider market power, nor do I allow for pre-commitment to transfer schedules. Rather, I consider a competitive secondary market à la Kyle (1985).

Nachman and Noe (1994) analyze optimal security design when the issuer is privately informed at the time the security is designed. They consider a setting with fixed issuer liquidity demand (investment scale) so the equilibrium entails pooling of issuers. They show that under technical conditions, including monotonicity, debt is the optimal funding source. Intuitively, debt minimizes cross-subsidies from high to low type issuers, as informally argued by Myers and Majluf (1984).

Dang, Gorton and Holmström (2011) consider an exchange economy with two agents who can benefit from bilateral intertemporal trade. Here debt emerges as an optimal security in that it minimizes the private incentive to become informed–with information production limiting risk-sharing possibilities. Gorton and Pennacchi (1990) analyze the supply of riskless claims when uninformed investors have a demand for safe storage when facing informed speculators. Uninformed investors exercise effective control over an intermediary's financial structure and carve out a safe debt claim. The models share the prediction that debt is an optimal source of liquidity for the uninformed. However, in the setting considered here, riskless debt is not optimal since it does not equate marginal uninformed trading losses and carrying costs.

Boot and Thakor (1993) and Fulghieri and Lukin (2001) also consider security design by a privately informed issuer. As in this paper, they allow for the possibility of informed speculation. High type issuers benefit from informed speculation since this drives prices closer to fundamentals. They show tranching of cash flows can promote information production by relaxing speculator wealth constraints as they trade against pure noise traders in the levered equity market. I deliberately rule out this causal mechanism by considering a speculator with unlimited outside wealth. The most important difference between the models is that these two papers move uninformed trading across securities exogenously. The causal mechanism at the core of my model, endogenous uninformed trading, is necessarily absent from their models.

Axelson (2007) considers a setting where an uninformed issuer sells securities to informed investors using a uniform price sealed bid auction. His model differs from the rest of the literature by considering that the market's valuation of the asset-in-place is informative about the value of the firm's growth options. He shows that if there is sufficient competition amongst informed investors, an information-sensitive equity claim can be an optimal marketed security as it provides the firm with high internal resources when growth options are most valuable. There is no real allocative role to fundraising in the present paper.[2]

The remainder of the paper is organized as follows. The first section considers optimal security design with fixed speculator signal precision. The following section considers optimal security design when the speculator can exert costly effort in order to increase her signal precision. Proofs are in the appendix.

2 Baseline Setting

This section considers optimal security design in the simplest possible setting in which the information structure is fixed.

2.1 Baseline Assumptions

There is one asset and four periods: 1, 2, 3, and 4. The asset generates a single verifiable cash flow $x \in \mathcal{X} \equiv [\underline{x}, \overline{x}] \subseteq \mathbb{R}_{++}$ accruing in period 4. The c.d.f. governing cash flow is determined by a latent state variable $\omega \in \{\underline{\omega}, \overline{\omega}\}$. The two states are equiprobable. If the state is $\underline{\omega}$ ($\overline{\omega}$), cash flow is distributed according to the continuously differentiable c.d.f. $\underline{F}$ ($\overline{F}$) with strictly positive density $\underline{f}$ ($\overline{f}$) on $\mathcal{X}$. The densities satisfy the following Monotone Likelihood Ratio (MLR) condition.

$$\textbf{Assumption 1 (MLR):} \frac{\overline{f}(x_2)}{\underline{f}(x_2)} \geq \frac{\overline{f}(x_1)}{\underline{f}(x_1)} \quad \forall \quad \{(x_1, x_2) \in \mathcal{X} \times \mathcal{X} : x_1 < x_2\}.$$

For the analysis below it is useful to recall MLR implies first-order stochastic dominance (FOSD) and monotone hazard rates (MHR):

$$\begin{aligned} \textbf{FOSD} &: \overline{F}(x) \leq \underline{F}(x) \quad \forall \quad x \in \mathcal{X}. \\ \textbf{MHR} &: \frac{\overline{f}(x)}{1-\overline{F}(x)} \leq \frac{\underline{f}(x)}{1-\underline{F}(x)} \quad \forall \quad x \in \mathcal{X}. \end{aligned}$$

For an arbitrary measurable mapping $a : \mathcal{X} \rightarrow \mathbb{R}$, let:

$$\begin{aligned} \overline{\mu}_a &\equiv \int_{\underline{x}}^{\overline{x}} a(x)\overline{f}(x)dx \\ \underline{\mu}_a &\equiv \int_{\underline{x}}^{\overline{x}} a(x)\underline{f}(x)dx. \end{aligned}$$

[2] Hennessy (2008) allows for such an informational role of prices.

There are three classes of agents: Uninformed Investors (UI), the Liquidity Provider (LP), and an informed speculator (S). All agents are risk-neutral and apply a discount factor of one to consumption at different points in time. S differs from the other agents in that she receives a noisy signal of ω at zero cost, while other agents cannot observe ω. The speculator's signal is correct with probability $\sigma \in (1/2, 1)$.

The UI have measure one and are identical ex ante. The UI are the original owners of the asset. The asset is initially structured as a pass-through security with one share outstanding. One option is to maintain the pass-through structure. Alternatively, in period 1 the UI can split the claims to cash flow into two securities, I and L. Anticipating, the restriction to two securities will be shown to be without loss of generality. The respective payoffs to securities I and L are determined by measurable mappings $i : \mathcal{X} \to \mathbb{R}_+$ and $l : \mathcal{X} \to \mathbb{R}_+$. Letting Φ be the indicator function on $\mathcal{X}$ we have:

$$i + l = \Phi x \quad \forall \quad x \in \mathcal{X}.$$

Security payoffs must respect limited liability (LL) and monotonicity (MN) constraints:

$$\textbf{Assumption 2 (LL)} \quad : \quad 0 \leq l(x) \leq x \quad \forall \quad x \in \mathcal{X}.$$

$$\textbf{Assumption 3 (MN)} \quad : \quad i \text{ and } l \text{ are non-decreasing on } \mathcal{X}.$$

Let $\mathcal{A}$ denote the set of admissible security payoff mappings. From Assumptions 1 and 3 it follows:

$$\overline{\mu}_a \geq \underline{\mu}_a \quad \forall \quad a \in \mathcal{A}. \tag{1}$$

An extant literature in security design including Innes (1990), Nachman and Noe (1994), DeMarzo and Duffie (1999), and DeMarzo (2005) considers that decreasing securities may be inadmissible. One can understand this demand as arising from concerns over two forms of ex post moral hazard. First, agents may engage in unobservable sabotage to reduce cash flow. Second, agents may be able to make unobservable contributions to increase cash flow. If either form of ex post moral hazard is feasible, there will be an equilibrium demand for claim payoffs to be non-decreasing. To see this, note that if a party's claim is decreasing, the party gains from sabotage. Further, if a party's claim is decreasing, the counterparty gains from making an unobservable contribution.

After securities are designed, they are allocated across the asset's original owners (UI) in a pro rata fashion in period 1. UI choose security designs in period 1 in order to maximize the expected net payoffs they derive from their claims on the cash flow. This ex ante uninformed valuation is denoted V. An alternative motivation, yielding the same results, assumes some other party initially owns the asset and chooses security designs to maximize the amount UI are willing to pay to own the marketable claims on cash flow. In either set-up, uninformed investors are the marginal investors determining period 1 ("primary market") valuations as in Holmström and Tirole (1993) and Maug (1998).

The UI face correlated liquidity shocks leading to fluctuations in aggregate UI trading volume, as in Kyle (1985). In particular, in period 2, each UI privately learns about his preference for selling or holding securities. A fraction $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$ of the UI are *impatient* in that they face a carrying cost that will absorb a fraction $C \in (0,1)$ of a security's payoff if held until maturity. This creates a bias for selling in period 3. To fix ideas, the proportional cost C is labeled a *carrying cost.* The carrying cost captures in a tractable reduced-form other shocks creating a preference for selling, e.g. pressing consumption needs, a positive NPV outside investment opportunity, or a shock to idiosyncratic discount rates. The remaining fraction $1-\gamma$ of the UI are *patient* in that they face a fully absorbing cost if they sell in period 3, rather than holding until maturity. The probability of $\gamma = \overline{\gamma}$ is ψ. The behavior of a patient UI is identical to a pure noise-trader since their trading decision is trivial–they always hold until maturity. If one also assumed $C = 1$, then the model would become a pure noise-trading model since UI trading would be purely exogenous. In fact, the analysis below demonstrates security design is irrelevant for uninformed valuation if $C = 1$.

In period 2, the state ω is drawn, with S then observing her noisy signal. In period 3, securities can be sold to the competitive Liquidity Provider (LP). LP and his competitors have deep pockets and are invulnerable to cost shocks, so can buy tendered securities and hold them until maturity. As in Kyle (1985), S and the UI submit market orders at the start of period 3. There is no market segmentation, so LP observes orders across markets. After observing the aggregate sell order, LP competes with other deep-pocketed investors à la Bertrand in bidding for tendered securities. With the threat of competition, LP prices securities at their conditional expected payoff.

Throughout the paper, the variable β plays a critical role as it denotes the updated belief of LP (and his competitors) regarding the probability of the state being $\overline{\omega}$. LP sets his bid price for securities according to:

$$P_a = \beta\overline{\mu}_a + (1-\beta)\underline{\mu}_a \quad \forall \quad a \in \mathcal{A}. \tag{2}$$

In period 4 cash flow is verified and investors are paid.

2.2 The Pass-Through Security

It is most convenient to break down the analysis into two cases according to whether the underlying cash flows have high or low trading loss information-sensitivity (relative to carrying costs). This subsection and the next assume that the underlying cash flows have *Low Sensitivity.*

$$\textbf{Low Sensitivity} \quad : \quad \frac{\overline{\mu}_x}{\underline{\mu}_x} < \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}$$

$$\Updownarrow$$

$$C \quad > \quad \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{\overline{\mu}_x - \underline{\mu}_x}{\overline{\mu}_x + \underline{\mu}_x}.$$

The equilibrium concept is perfect Bayesian equilibrium (PBE) in pure strategies. Consider first equilibrium if cash flow is bundled as a pass-through security. Table 1 depicts order configurations, conjecturing each impatient UI finds it optimal to sell while the speculator masks her trading accordingly.

The aggregate UI order is either $-\underline{\gamma}$ or $-\overline{\gamma}$. Given this trading pattern, the only profitable trading strategy for S is to short-sell $(\overline{\gamma}-\underline{\gamma})$ units if she observes a negative signal. This order size confounds the LP regarding the speculator's signal since the aggregate sell order $-\overline{\gamma}$ can arise from either: negative signal and low UI selling volume or positive signal and high UI selling volume. The posited PBE can be supported with LP (and his competitors) forming the belief that the speculator observed the negative signal in response to any off-equilibrium sell order configurations, i.e. $\beta = 1-\sigma$ off the equilibrium path.

Upon observing the aggregate sell order of size $-\underline{\gamma}$, LP knows S observed the signal $\overline{\omega}$ and so forms the belief $\beta = \sigma$. Upon observing the aggregate sell order of size $-(2\overline{\gamma}-\underline{\gamma})$, LP knows S observed the signal $\underline{\omega}$ and so forms the belief $\beta = 1-\sigma$. In each of these cases, the order flow fully reveals the signal observed by S, implying securities are priced at their signal-contingent expected payoff. That is, there is no mispricing if order flow is fully-revealing. In contrast, LP is confounded upon observing the aggregate sell order of size $-\overline{\gamma}$. When confounded in this way, LP uses Bayes' rule to form the belief:

$$\beta_c \equiv 1-\sigma-\psi+2\sigma\psi. \tag{3}$$

To confirm the conjectured equilibrium it must be verified each impatient UI indeed finds it optimal to sell the pass-through security. Consider then the selling decision of an individual impatient UI. He will sell if and only if his conditional expectation of the price, given personally impatient, exceeds the expected security payoff net of carrying costs. The conditional expectation of the sell price can be computed as the sum over conditional probabilities of each aggregate UI demand state, given personally impatient, times the expected price in each aggregate UI demand state. Letting χ

be an indicator for impatience, we have:

$$\begin{aligned} E[P_x|\chi &= 1] = \left[\frac{\psi\overline{\gamma}}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\right]\frac{1}{2}\left[\overline{\mu}_x+\underline{\mu}_x-(1-\psi)(2\sigma-1)(\overline{\mu}_x-\underline{\mu}_x)\right] \\ &+\left[\frac{(1-\psi)\underline{\gamma}}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\right]\frac{1}{2}\left[\overline{\mu}_x+\underline{\mu}_x+\psi(2\sigma-1)(\overline{\mu}_x-\underline{\mu}_x)\right] \\ &= \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}\left(\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\right)(\overline{\mu}_x-\underline{\mu}_x) \end{aligned} \tag{4}$$

Equation (4) shows each impatient UI is aware of exposure to mispricing since his conditional expectation of the sell price is below fundamental value. Moreover, it can be seen that expected underpricing is increasing in the speculator's signal precision. Intuitively, an impatient UI knows he is personally more likely to face a liquidity shock when $\gamma=\overline{\gamma}$. With higher UI selling volume, he expects a lower price given the LP views high selling volume as indicative of a negative speculator signal.

It follows that an impatient UI will sell if and only if

$$\begin{aligned} \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})(\overline{\mu}_x-\underline{\mu}_x)}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} &\geq \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)(1-C) \\ &\Updownarrow \\ C &\geq \lambda_x \end{aligned} \tag{5}$$

where

$$\lambda_a \equiv \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_a-\underline{\mu}_a}{\overline{\mu}_a+\underline{\mu}_a} \quad \forall \quad a\in\mathcal{A}. \tag{6}$$

The variable λ_a plays a central role, measuring the conditional expectation of security underpricing as a percentage of expected payoff. If the speculator had only an uninformative signal ($\sigma=1/2$), there would be no adverse selection problem and each impatient UI would sell all securities regardless of payoff structure. However, with $\sigma>1/2$, impatient UI are willing to sell only if the carrying cost C exceeds the trading loss measure λ_a. As a short-hand, the variable λ_a is labeled the *trading loss information-sensitivity* of a security. From the perspective of each UI, it is the relevant measure of a security's sensitivity to the latent state variable ω determining the true probability distribution for cash flow, for which the speculator has a noisy signal. With this in mind, the pass-through security market will be liquid if cash flows have Low Sensitivity, as assumed presently.

Having verified each UI will sell as conjectured, consider now the expected trading gain of the speculator. From Table 1 it follows her expected gain is:

$$\begin{aligned} G_x &= \frac{\overline{\gamma}-\underline{\gamma}}{2}\left[\begin{array}{c} (1-\sigma)(1-\psi)[\beta_c\overline{\mu}_x+(1-\beta_c)\underline{\mu}_x-\overline{\mu}_x]+(1-\sigma)\psi[(1-\sigma)\overline{\mu}_x+\sigma\underline{\mu}_x-\overline{\mu}_x] \\ +\sigma(1-\psi)[\beta_c\overline{\mu}_x+(1-\beta_c)\underline{\mu}_x-\underline{\mu}_x]+\sigma\psi[(1-\sigma)\overline{\mu}_x+\sigma\underline{\mu}_x-\underline{\mu}_x] \end{array}\right] \\ &= \frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\mu}_x-\underline{\mu}_x)(\overline{\gamma}-\underline{\gamma}). \end{aligned} \tag{7}$$

In general, the uninformed valuation V is equal to expected cash flow less expected trading losses (speculator trading gains) less expected carrying costs. The analysis above implies the uninformed valuation of a pass-through structure is:

$$V_{PT} = \frac{1}{2}(\overline{\mu}_x + \underline{\mu}_x) - \frac{1}{2}(2\sigma - 1)\psi(1-\psi)(\overline{\mu}_x - \underline{\mu}_x)(\overline{\gamma} - \underline{\gamma}). \tag{8}$$

The next subsection considers whether and how higher uninformed valuations can be achieved by moving away from the pass-through structure. Before doing so, it is useful to consider social welfare under the pass-through structure. Social welfare is calculated from the perspective of a social planner placing an equal weight of one on each agent's utility. Under symmetric information (first-best), LP would simply buy tendered securities from impatient UI at their expected payoff. Ignoring outside endowments, first-best social welfare is just equal to expected cash flow:

$$SW_{FB} = \frac{1}{2}(\overline{\mu}_x + \underline{\mu}_x). \tag{9}$$

First-best social welfare is attained under the pass-through security since

$$SW_{PT} = V_{PT} + G_x = SW_{FB}. \tag{10}$$

2.3 Optimal Security Design: Low Sensitivity

This section considers whether and how the uninformed valuation V can be increased via security design when cash flow has Low Sensitivity. Recall, the trading loss information-sensitivity measure λ_x defined in equation (6) determined the willingness of each impatient UI to sell the pass-through security. With this in mind, suppose instead cash flow is split into two securities L and I with corresponding payoff mappings $(l, i) \in \mathcal{A} \times \mathcal{A}$. Following the same argument as that leading to equation (5), an impatient UI finds it optimal to sell security L if $\lambda_l \leq C$ and to hold security I if $\lambda_i \geq C$. That is, the decision to sell a security again entails a comparison of expected trading loss with carrying costs. Indeed, this simple causal mechanism is central throughout the model: Security design redistributes trading loss information-sensitivity of total cash flow across claims, altering interim-stage trading by the UI.

The following identity is useful:

$$\sum_j a_j = A \Rightarrow \sum_j \left(\frac{\overline{\mu}_{a_j} + \underline{\mu}_{a_j}}{\overline{\mu}_A + \underline{\mu}_A} \right) \lambda_{a_j} = \lambda_A; \quad \{a_j \in \mathcal{A}\}. \tag{11}$$

Identity (11) states that the weighted average of the trading loss information-sensitivities of a basket of securities is just equal to the trading loss information-sensitivity of a single security combining them. This identity leads to the following useful lemma.

Lemma 1 *Any uninformed valuation (V) attainable with three or more securities is attainable with no more than two securities.*

Lemma 1 follows directly from identity (11). If there are three or more securities, they can be sorted into two baskets, those that are liquid and those that are illiquid. The basket of liquid (illiquid) securities can be combined to form a single liquid (illiquid) security. Total carrying costs and speculator trading gains are unchanged, leaving the uninformed valuation V unchanged.

In light of the preceding lemma, the remainder of the analysis confines attention to two securities without loss of generality. The value attainable with two securities will then be compared with that attainable with one pass-through security. It follows from equation (11) that with two securities:

$$l + i = \Phi x \Rightarrow \left(\frac{\overline{\mu}_l + \underline{\mu}_l}{\overline{\mu}_x + \underline{\mu}_x} \right) \lambda_l + \left(\frac{\overline{\mu}_i + \underline{\mu}_i}{\overline{\mu}_x + \underline{\mu}_x} \right) \lambda_i = \lambda_x. \tag{12}$$

The identity in equation (12) states that if cash flow is split into two securities, the weighted average trading loss information-sensitivity is equal to the trading loss information-sensitivity of a single claim to total cash flow. That is, security design cannot change total trading loss information-sensitivity. Rather, security design serves to redistribute trading loss information-sensitivity across claims.

To analyze optimal security design, consider first structures such that both securities remain liquid. If both securities are liquid, in order to mask her trades the speculator shorts both securities upon observing a negative signal. It follows that:

$$\forall \quad (i,l) \in \mathcal{A} \times \mathcal{A} : \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}} \max \left\{ \frac{\overline{\mu}_l - \underline{\mu}_l}{\overline{\mu}_l + \underline{\mu}_l}, \frac{\overline{\mu}_i - \underline{\mu}_i}{\overline{\mu}_i + \underline{\mu}_i} \right\} \leq C, \quad V(i,l) = V_{PT}. \tag{13}$$

The demonstration of equation (13) is as follows. Under the stated inequality, both securities are liquid in the sense that each impatient UI sells them. The speculator will then mask herself by shorting both securities upon observing a negative signal. Order flow is fully revealing and non-revealing in the same manner as in Table 1. And further, total speculator trading gains are the same as under the pass-through security. Since the uninformed valuation is equal to expected cash flow less expected trading losses less carrying costs it follows that all structurings satisfying the condition in (13) are neutral permutations of the pass-through structure.

It follows from equation (13) that a necessary condition for the uninformed valuation to exceed V_{PT} is for one of the securities, say security I, to be structured such that it will be illiquid. Further, trading loss information-sensitivity is bounded above, with

$$\lambda_a < \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}}. \tag{14}$$

Thus, from equation (13) it follows:

$$C \geq \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \Rightarrow V(i,l) = V_{PT} \quad \forall \quad (i,l) \in \mathcal{A}\times\mathcal{A}. \tag{15}$$

In light of equation (15), the rest of this subsection considers optimal security design for the remaining case where C is sufficiently low such that it is possible to induce illiquidity into some security market. In searching for the optimal security design it is worth noting that with Low Sensitivity cash flows, illiquidity of security I implies liquidity of security L:

$$\text{Low Sensitivity: } \lambda_i \geq C \Rightarrow \lambda_l < C. \tag{16}$$

In light of the preceding results, the security design problem reduces to finding the optimal liquid and illiquid securities. The uninformed valuation under a mix of liquid and illiquid securities is:

$$V(i,l) \equiv \underbrace{\frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)}_{\text{Expected Cash Flow}} \underbrace{-\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\mu}_l-\underline{\mu}_l)(\overline{\gamma}-\underline{\gamma})}_{\text{Trading Losses Liquid Security}} \underbrace{-\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}](\overline{\mu}_i+\underline{\mu}_i)C}_{\text{Carrying Costs Illiquid Security}}. \tag{17}$$

Dropping constants from equation (17), the problem for the original uninformed owners is to find a payoff mapping l to solve:

$$\max_{l\in\mathcal{A}} \quad \int_{\underline{x}}^{\overline{x}} [\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{f}(x) - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{f}(x)]l(x)dx \tag{18}$$

where

$$\begin{aligned}
\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma}) &\equiv (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}] \\
\overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma}) &\equiv (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]
\end{aligned}$$

subject to the constraint that the residual security I will not be sold by impatient UI, or

$$NSI: \frac{\psi(1-\psi)(2\sigma-1)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{(\overline{\mu}_x-\overline{\mu}_l)-(\underline{\mu}_x-\underline{\mu}_l)}{(\overline{\mu}_x-\overline{\mu}_l)+(\underline{\mu}_x-\underline{\mu}_l)} \geq C. \tag{19}$$

It is worth noting the NSI constraint can also be written as

$$NSI: \frac{\overline{\mu}_i}{\underline{\mu}_i} \geq \frac{\psi(1-\psi)(2\sigma-1)(\overline{\gamma}-\underline{\gamma}) + C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{\psi(1-\psi)(2\sigma-1)(\overline{\gamma}-\underline{\gamma}) - C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}. \tag{20}$$

The second formulation of the NSI constraint given in equation (20) illustrates that in order for security I to be illiquid, there must be a sufficiently high ratio of state-contingent valuations. Moreover, the required ratio is increasing in C. Intuitively, if C is high, impatient UI have a strong demand for immediacy. To deter them from selling then requires exposing them to higher expected trading losses on the posited illiquid security.

It is also worth noting the NSI constraint can be written as:

$$NSI : \underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{\mu}_l - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{\mu}_l \geq \underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{\mu}_x - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{\mu}_x. \tag{21}$$

This last form of the NSI constraint is of interest for two reasons. First, substituting equation (21) into the uninformed valuation equation (17) reveals

$$V(i,l) = V_{PT} + \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}](\overline{\mu}_i + \underline{\mu}_i)(\lambda_i - C) \tag{22}$$

From the preceding equation it follows that the two security structure dominates the pass-through structure if the NSI constraint is slack.

The second point worth noting is that the left side of equation (21) for the NSI constraint is the objective function itself. Thus, the optimal security design can be determined in two steps. We can first solve a relaxed program (denoted RP1) which ignores the NSI constraint. If the solution to RP1 satisfies NSI then it is the optimal structure. Otherwise, it is not possible to induce illiquidity into any security (NSI cannot be satisfied) implying the uninformed valuation is V_{PT} for all security designs, i.e. security design is then irrelevant.

In solving RP1 we begin by forming the Lagrangian:

$$\mathcal{L}(l) = \int_{\underline{x}}^{\overline{x}} \left[\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{f}(x) - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{f}(x)\right] l(x)dx. \tag{23}$$

From the LL and MN constraints it follows any $l \in \mathcal{A}$ is absolutely continuous. Further, the derivative

$$l' \equiv \delta$$

is well-defined, with $\delta \in [0,1]$, except on a subset of $\mathcal{X}$ with Lebesgue measure zero. Using integration by parts, the Lagrangian can be rewritten as:

$$\mathcal{L}(l) = \int_{\underline{x}}^{\overline{x}} \left[\overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{F}(x) - \underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{F}(x)\right] \delta(x)dx + 2l(\overline{x})[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]C. \tag{24}$$

The optimal contract in RP1 entails maximizing $\mathcal{L}(l)$ with respect to l. Solving the optimal control problem above, the appendix establishes the following lemma.

Lemma 2 *The optimal liquid security in Relaxed Program 1 is a senior debt claim.*

Next it must be verified whether the solution to RP1 satisfies the neglected NSI constraint.

Lemma 3 *The trading loss information-sensitivity of a senior security (λ_{sr}) with face value θ and residual junior security (λ_{jr}) are increasing in θ. Further, the trading loss information-sensitivity of the junior claim exceeds that of total cash flow (λ_x) while that of the senior claim is less than that of total cash flow.*

Since the solution to RP1 is liquid senior debt, and since the information-sensitivity of the junior claim is increasing in senior debt face value, a necessary and sufficient condition for satisfaction of NSI to be feasible, and for the combination of liquid senior debt and illiquid residual equity to be optimal in the full program, is that the trading loss information-sensitivity of the junior claim exceeds C as face value tends to $\overline{x}$. From L'Hôpital's rule it follows the limiting information-sensitivity of a residual junior security is:

$$\lim_{\theta \uparrow \overline{x}} \lambda_{jr}(\theta) = \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]} \frac{\overline{f}(\overline{x}) - \underline{f}(\overline{x})}{\overline{f}(\overline{x}) + \underline{f}(\overline{x})}. \tag{25}$$

Therefore, it is possible to induce the illiquidity of some claim (and to satisfy the NSI constraint) if and only if

$$\frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]} \frac{\overline{f}(\overline{x}) - \underline{f}(\overline{x})}{\overline{f}(\overline{x}) + \underline{f}(\overline{x})} > C \Leftrightarrow \frac{\underline{f}(\overline{x})}{\overline{f}(\overline{x})} < \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}. \tag{26}$$

We then have the following proposition.

Proposition 1 *[Low Sensitivity] Security design is irrelevant if*

$$\frac{\underline{f}(\overline{x})}{\overline{f}(\overline{x})} \geq \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}.$$

Otherwise, the privately optimal security design entails splitting cash flow into an illiquid junior claim and a risky yet liquid senior claim with face value θ^* such that

$$\frac{1 - \overline{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}. \tag{27}$$

The intuition for Proposition 1 is as follows. The first part of the proposition states that if the uninformed demand for immediacy, as captured by the parameter C, is extremely high then they will sell any security they hold at the interim date, even if facing maximal adverse selection. In such cases, the uninformed valuation is necessarily unaffected by security design. Conversely, if C is sufficiently low it is possible to design a residual junior claim that will be illiquid at the interim date, so security design can influence the ex ante uninformed valuation. Here the optimal face value on the senior claim reflects the fundamental tradeoff at the core of the model: expected uninformed trading losses on the liquid senior tranche are weighed against expected carrying costs on the illiquid junior tranche. By raising the face value of the senior debt claim, carrying costs on the illiquid claim fall, but expected trading losses on the liquid claim rise.

Applying the implicit function theorem to equation (27) one obtains:

$$\frac{\partial\theta^*}{\partial\sigma} = -\frac{4C\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[\psi\overline{\gamma}+(1-\psi)\underline{\gamma})]}{[(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})]^2}\frac{[1-\underline{F}(\theta^*)]^2}{[1-\overline{F}(\theta^*)]\underline{f}(\theta^*)-[1-\underline{F}(\theta^*)]\overline{f}(\theta^*)}<0$$

$$\frac{\partial\theta^*}{\partial C} = \frac{2(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{[(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})]^2}\frac{[1-\underline{F}(\theta^*)]^2}{[1-\overline{F}(\theta^*)]\underline{f}(\theta^*)-[1-\underline{F}(\theta^*)]\overline{f}(\theta^*)}>0$$

These comparative statics are consistent with the posited fundamental tradeoff between trading losses on the liquid tranche against carrying costs on the illiquid tranche. For higher σ values, uninformed investor trading losses on the liquid senior debt are a more important concern and so the optimal face value on the senior debt decreases. For higher C values, carrying costs on the illiquid junior tranche are a more important concern and so the optimal face value on the senior debt increases, reducing the value of the illiquid residual claim.

Further intuition for Proposition 1 is provided by way of a simple argument. Suppose one were to take for granted that senior debt is the optimal liquid security (Lemma 2) and confine attention to determining the optimal senior face value. The optimal leveraging of the underlying cash flow solves:

$$\max_{\theta\in[0,\overline{x}]} \quad V(\theta)=\frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[\overline{\mu}_{sr}(\theta)-\underline{\mu}_{sr}(\theta)]-\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]C[\overline{\mu}_{jr}(\theta)+\underline{\mu}_{jr}(\theta)]. \tag{28}$$

Since V is continuous and θ is chosen from a closed bounded interval, it follows from Weierstrass' Theorem a maximum point exists. The first-order condition for the above problem illustrates the posited fundamental tradeoff between trading losses and illiquidity:[3]

$$V'(\theta^*) = 0 \Leftrightarrow \underbrace{\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[\overline{\mu}'_{sr}(\theta^*)-\underline{\mu}'_{sr}(\theta^*)]}_{\text{Marginal Trading Loss Liquid Security}} = \underbrace{-\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]C[\overline{\mu}'_{jr}(\theta^*)+\underline{\mu}'_{jr}(\theta^*)]}_{\text{Marginal Carrying Cost Illiquid Security}}$$

$$\Updownarrow$$

$$\frac{1-\overline{F}(\theta^*)}{1-\underline{F}(\theta^*)} = \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}. \tag{29}$$

2.4 Optimal Security Design: High Sensitivity

This subsection considers optimal security design if the underlying cash flows have a high trading loss information-sensitivity.

$$\textbf{High Sensitivity} \quad : \quad \frac{\overline{\mu}_x}{\underline{\mu}_x} > \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}$$

$$\Updownarrow$$

$$C < \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_x-\underline{\mu}_x}{\overline{\mu}_x+\underline{\mu}_x}.$$

[3] The second-order condition follows from the monotone hazard rate property.

Consider first the privately optimal security design. Without loss of generality attention can be confined to two securities, one liquid and the other illiquid, since equation (11) implies baskets of liquid (illiquid) securities can be combined to form a single liquid (illiquid) security, resulting in the same uninformed valuation. From equation (12) we have the following implication:

$$\text{High Sensitivity: } \lambda_l \leq C \Rightarrow \lambda_i > C. \tag{30}$$

With this implication in mind, the program involves a minor variation on the program of the previous subsection. Specifically, the objective is to maximize the uninformed valuation in equation (17), which reduces to the objective function in equation (18), subject to ensuring the respective securities are liquid and illiquid as posited. Based on the implication in equation (30), instead of imposing the prior subsection's NSI constraint, we now demand that the impatient UI prefer to sell the posited liquid security. This SL constraint takes the form:

$$SL : \lambda_l \leq C \Leftrightarrow \underline{\kappa}(C, \sigma, \psi, \underline{\gamma}, \overline{\gamma})\underline{\mu}_l - \overline{\kappa}(C, \sigma, \psi, \underline{\gamma}, \overline{\gamma})\overline{\mu}_l \geq 0. \tag{31}$$

The feasible set for this program is non-empty since any riskless senior debt claim with face value not greater than $\underline{x}$ satisfies the SL constraint. Intuitively, issuing riskless debt is a ready means of reducing carrying costs while avoiding trading losses. But riskless debt does not maximize the uninformed valuation. To see this, note the left side of the SL constraint is just the objective function for this program, so any security improving upon risk-free debt necessarily satisfies the SL constraint. It follows that in the present setting, the full security design program is identical in form to RP1. And we have the following proposition.

Proposition 2 *[High Sensitivity] The privately optimal security design entails splitting cash flow into an illiquid junior claim and a risky yet liquid senior debt claim with face value θ^* such that*

$$\frac{1 - \overline{F}(\theta^*)}{1 - \underline{F}(\theta^*)} = \frac{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}.$$

2.5 Socially Optimal Security Design

Although the primary focus of this paper is on determining the privately optimal security design, the model demonstrates a divergence between private and social preferences in security design. Specifically, consider again a social planner placing equal weight on all agents. Social welfare under the privately optimal security design cum tranching is equal to the uninformed valuation plus the speculator's expected trading gain, or:

$$SW = SW_{FB} - \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]C[\overline{\mu}_i + \underline{\mu}_i]. \tag{32}$$

In light of the preceding equation, consider first the Low Sensitivity case. Here the socially optimal security design achieves zero carrying costs by bundling cash flow as a pass-through security. That is, the first-best social welfare would be attained by a pass-through structure. However, the privately optimal structuring does not include the speculator's informational rent in the objective function. Hence, as described in Proposition 1, whenever feasible, uninformed owners adopt a structuring that induces illiquidity in order to reduce speculator informational rents.

Consider next the social planner's preferred security design in the High Sensitivity case. The objective of the planner is to maximize total social welfare, or equivalently, to minimize deadweight carrying costs, subject to the constraint that each impatient UI indeed finds it optimal to sell the posited liquid claim. The planner's program can be written as

$$\max_{l \in \mathcal{A}} \quad \int_{\underline{x}}^{\overline{x}} \left[\underline{f}(x) + \overline{f}(x)\right] l(x)dx$$

subject to the SL constraint in equation (31).

In solving the planner's program we begin by forming the Lagrangian with the multiplier m here associated with the SL constraint:

$$\mathcal{L}(l,m) = \int_{\underline{x}}^{\overline{x}} \left[\underline{f}(x) + \overline{f}(x)\right] l(x)dx + m\left[\int_{\underline{x}}^{\overline{x}} \left(\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{f}(x) - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{f}(x)\right) l(x)dx\right]. \tag{33}$$

Using integration by parts, the Lagrangian can be rewritten as:

$$\mathcal{L}(l,m) = -\int_{\underline{x}}^{\overline{x}} \left[(1+m\underline{\kappa})\underline{F}(x) + (1-m\overline{\kappa})\overline{F}(x)\right] \delta(x)dx + 2[1 + mC(\psi\overline{\gamma} + (1-\psi)\underline{\gamma})]l(\overline{x}).$$

Solving the optimal control problem above, the appendix establishes the following proposition which contrasts private and social objectives in security design.

Proposition 3 *If cash flows have low trading loss information-sensitivity, the socially optimal security design is a pass-through structure ensuring all impatient investors sell. The privately optimal design creates an illiquid junior claim (if feasible). If cash flows have high trading loss information-sensitivity, the socially optimal security design splits cash flow into an illiquid junior claim and liquid senior claim with face value θ_s^* satisfying, $\lambda_{sr}(\theta_s^*) = C$, implying impatient investors are just willing to sell it. The socially optimal liquid tranche size exceeds the private optimum.*

The above proposition illustrates that the privately optimal level of liquidity (uninformed security sales) is socially suboptimal. This arises from an inherent conflict between private and social objectives in security design. The privately optimum design sacrifices liquidity in order to reduce the speculator's informational rent. However, from a social welfare perspective, transfers from the uninformed to the informed are of no consequence since they net to zero. Thus, the socially optimal security design maximizes liquidity.

3 Extension: Discretionary Speculator Effort

This section extends the Baseline Setting to determine how optimal security design changes if the speculator can exert costly effort in order to increase her signal precision. We consider that the speculator is endowed with a noisy signal correct with probability $\underline{\sigma} \in (1/2, 1)$. The speculator can increase her signal precision by exerting effort at cost $E : \Sigma \to \mathbb{R}_+$, where $\Sigma \equiv [\underline{\sigma}, 1]$. We adopt

$$\textbf{Assumption 4} \quad : \quad E(\sigma) \equiv \frac{1}{2}\xi(\sigma - \underline{\sigma})^2$$
$$: \quad \xi \geq \frac{(\overline{\gamma} - \underline{\gamma})(\overline{\mu}_x - \underline{\mu}_x)}{2(1 - \underline{\sigma})}.$$

The assumed lower bound on the effort cost parameter ξ ensures the incentive compatible speculator effort is responsive to marginal increases in prospective trading gains.

Before considering specific security designs, it is worth listing key conclusions that carry over from the Baseline Setting. First, Table 1 continues to describe order combinations for any liquid security. Second, equations (8) and (17) still measure the uninformed valuation V, but now σ varies endogenously across alternative structures. Third, the argument demonstrating Lemma (1) remains valid, so without loss of generality attention can be confined to no more than two securities. Finally, the λ-contingent selling rule of each impatient UI remains the same as in the previous section.

3.1 Discretionary Speculator Effort: Low Sensitivity

This subsection considers optimal security design when the underlying cash flows have low trading loss information-sensitivity. To this end, let σ_{pt} denote the speculator's incentive compatible signal precision in the event that the speculator faces a liquid pass-through security market. For this subsection, assume the following inequality is satisfied.

$$\textbf{Low Sensitivity} \quad : \quad \frac{\overline{\mu}_x}{\underline{\mu}_x} < \frac{(2\sigma_{pt} - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{(2\sigma_{pt} - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}$$
$$\Updownarrow$$
$$C \quad > \quad \frac{(2\sigma_{pt} - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})}{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}} \frac{\overline{\mu}_x - \underline{\mu}_x}{\overline{\mu}_x + \underline{\mu}_x}.$$

Consider first equilibrium strategies and payoffs if the cash flows are bundled as a pass-through security. When cash flows have Low Sensitivity, impatient UI are willing to sell a pass-through security. Consider next the incentive compatible (IC) signal precision for the speculator when facing a liquid pass-through market (σ_{pt}). Momentarily, let σ denote the signal precision used by the LP to price securities. In choosing her signal precision, the speculator treats LP beliefs (and prices) as fixed as she chooses amongst $\widetilde{\sigma} \in \Sigma$. From Table 1 it follows that for any liquid security ($l = x$ under the pass-through structure) the expected trading gain for the speculator is:

$$\begin{aligned} G(\widetilde{\sigma}, \sigma) &= \frac{\overline{\gamma} - \underline{\gamma}}{2} \left[\begin{array}{c} (1-\widetilde{\sigma})(1-\psi)[\beta_c \overline{\mu}_l + (1-\beta_c)\underline{\mu}_l - \overline{\mu}_l] + \widetilde{\sigma}(1-\psi)[\beta_c \overline{\mu}_l + (1-\beta_c)\underline{\mu}_l - \underline{\mu}_l] \\ +(1-\widetilde{\sigma})\psi[(1-\sigma)\overline{\mu}_l + \sigma\underline{\mu}_l - \overline{\mu}_l] + \widetilde{\sigma}\psi[(1-\sigma)\overline{\mu}_l + \sigma\underline{\mu}_l - \underline{\mu}_l] \end{array} \right] \\ &= \frac{1}{2}(\overline{\gamma} - \underline{\gamma})(\overline{\mu}_l - \underline{\mu}_l)\left[\psi(1-\psi)(2\sigma - 1) + \widetilde{\sigma} - \sigma\right] \end{aligned} \tag{34}$$

The IC signal precision (σ_{ic}) equates marginal expected trading gains with marginal effort costs:

$$G_1(\sigma_{ic}, \sigma) = E'(\sigma_{ic}) \Rightarrow \sigma_{ic} = \underline{\sigma} + \frac{(\overline{\mu}_l - \underline{\mu}_l)(\overline{\gamma} - \underline{\gamma})}{2\xi}. \tag{35}$$

Under the pass-through security the incentive compatible signal precision is:

$$\sigma_{pt} = \underline{\sigma} + \frac{(\overline{\mu}_x - \underline{\mu}_x)(\overline{\gamma} - \underline{\gamma})}{2\xi}. \tag{36}$$

In equilibrium, LP correctly infers the signal precision and the speculator's expected trading gain is:

$$G(\sigma, \sigma) = \frac{1}{2}(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})(\overline{\mu}_l - \underline{\mu}_l). \tag{37}$$

It follows that the uninformed valuation obtained under the pass-through structure is:

$$V_{PT} = \frac{1}{2}(\overline{\mu}_x + \underline{\mu}_x) - \frac{1}{2}(2\sigma_{pt} - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})(\overline{\mu}_x - \underline{\mu}_x). \tag{38}$$

Having considered the valuation attained under a pass-through structure, the overall optimal security design can be determined in two steps. In the first step, we consider the optimal security design for inducing specific feasible $\sigma \in \Sigma$. In the second step, the optimal σ is chosen from the feasible set, here denoted Σ_F. It is readily verified that σ_{pt} defined in equation (36) constitutes an upper bound on feasible σ.

Consider then an arbitrary $\sigma \in [\underline{\sigma}, \sigma_{pt})$. The objective is to choose a payoff mapping for the liquid security $l \in \mathcal{A}$ to maximize the objective function specified in equation (18) subject to the constraints that: the signal precision σ is incentive compatible (IC); the posited illiquid security satisfies $\lambda_i(\sigma) \geq C$ (NSI); and the posited liquid security satisfies $\lambda_l(\sigma) \leq C$ (SL). Since here $\lambda_x(\sigma) < \lambda_x(\sigma_{pt}) < C$, it follows from equation (12) that the satisfaction of NSI constraint implies satisfaction of the SL constraint. So the relevant constraints are NSI and the following incentive constraint:

$$IC : \overline{\mu}_l - \underline{\mu}_l = 2\xi(\sigma - \underline{\sigma})(\overline{\gamma} - \underline{\gamma})^{-1}. \tag{39}$$

Once again, the third specification of the NSI constraint (equation (21)) shows this constraint can be satisfied if and only if it is satisfied by the solution to a relaxed program which ignores it. So we begin by solving a relaxed program (denoted RP2) ignoring the NSI constraint and subsequently check whether the neglected constraint is satisfied. If the solution to RP2 satisfies the neglected constraint then it is optimal for the implementation of the specified σ. Otherwise, the specified σ is not feasible.

In solving RP2 we begin by forming the following Lagrangian, with the multiplier m here corresponding to the IC constraint:

$$\begin{aligned}\mathcal{L}(l,m) &= \int_{\underline{x}}^{\overline{x}} \left[\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\underline{f}(x) - \overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma})\overline{f}(x)\right] l(x)dx \\ &\quad + m\left[\int_{\underline{x}}^{\overline{x}} \left[\overline{f}(x) - \underline{f}(x)\right] l(x)dx - 2\xi(\sigma-\underline{\sigma})(\overline{\gamma}-\underline{\gamma})^{-1}\right].\end{aligned} \tag{40}$$

From the LL and MN constraints it follows any $l \in \mathcal{A}$ is absolutely continuous. Further, the derivative $l' \equiv \delta$ is well-defined, with $\delta \in [0,1]$, except on a subset of $\mathcal{X}$ with Lebesgue measure zero. Using integration by parts, the Lagrangian can be rewritten as:

$$\begin{aligned}\mathcal{L}(l,m) &= \int_{\underline{x}}^{\overline{x}} [(\overline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma}) - m)\overline{F}(x) - (\underline{\kappa}(C,\sigma,\psi,\underline{\gamma},\overline{\gamma}) - m)\underline{F}(x)]\delta(x)dx \\ &\quad + 2l(\overline{x})C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}] - 2m\xi(\sigma-\underline{\sigma})(\overline{\gamma}-\underline{\gamma})^{-1}.\end{aligned} \tag{41}$$

The objective is to maximize $\mathcal{L}(l,m)$ over $l \in \mathcal{A}$. Solving this optimal control problem we obtain the following lemma which again suggests tranching is optimal, with senior debt an optimal liquid security.

Lemma 4 *In Relaxed Program 2, the optimal liquid security for implementing $\underline{\sigma}$ is risk-free debt with face value $\underline{x}$. The optimal liquid security for implementing $\sigma \in (\underline{\sigma}, \sigma_{pt})$ is senior debt with face value $\widehat{\theta}$ solving*

$$\int_0^{\widehat{\theta}} [\underline{F}(x) - \overline{F}(x)]dx = 2\xi(\sigma-\underline{\sigma})(\overline{\gamma}-\underline{\gamma})^{-1}. \tag{42}$$

Applying the implicit function theorem to equation (42), it follows that senior debt face value is increasing in signal precision:

$$\widehat{\theta}'(\sigma) = \frac{2\xi}{(\overline{\gamma}-\underline{\gamma})[\underline{F}(\theta) - \overline{F}(\theta)]} \quad \forall \quad \sigma \in (\underline{\sigma}, \sigma_{pt}).$$

Intuitively, once the NSI constraint is satisfied, marginal increases in the face value lead to increases in incentive compatible signal precision as they increase prospective informed trading gains in the liquid senior debt market.

Having characterized the solution to RP2, it is necessary to determine whether the candidate optimal security design satisfies the neglected NSI constraint. If the neglected constraint is satisfied, the candidate design is indeed optimal for implementing the respective σ. If it is not satisfied, then the σ under consideration is not feasible. With this in mind, we rewrite the NSI constraint as:

$$NSI : \lambda_{jr}\left(\widehat{\theta}(\sigma),\sigma\right) = \lambda_{jr}(\underline{x},\underline{\sigma}) + \int_{\underline{\sigma}}^{\sigma}\left[\lambda_1^{jr}\left(\widehat{\theta}\left(\widetilde{\sigma}\right),\widetilde{\sigma}\right)\widehat{\theta}'\left(\widetilde{\sigma}\right) + \lambda_2^{jr}\left(\widehat{\theta}\left(\widetilde{\sigma}\right),\widetilde{\sigma}\right)\right]d\widetilde{\sigma} \geq C. \tag{43}$$

Since the integrand in the preceding equation is positive, it follows that in order for the NSI constraint is to be satisfied, the posited σ must be sufficiently high in relation to C. For example, implementing $\underline{\sigma}$ requires liquid senior debt face value no greater than $\underline{x}$. But in such a case C would need to be very low, otherwise the junior tranche would remain liquid and the incentive compatible effort would be σ_{pt}. Conversely, if C is sufficiently low, the NSI constraint can be satisfied by the combination of risk-free debt and minimal signal precision ($\theta = \underline{x}, \sigma = \underline{\sigma}$).

Since λ_{jr} is increasing in the senior debt face value, the highest possible trading loss information-sensitivity on the junior claim is:

$$\lim_{\theta\uparrow\overline{x}}\lambda_{jr}(\theta,\sigma_{pt}) = \frac{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{f}(\overline{x})-\underline{f}(\overline{x})}{\overline{f}(\overline{x})+\underline{f}(\overline{x})}. \tag{44}$$

Therefore, a necessary and sufficient condition for $\Sigma_F \neq \{\sigma_{pt}\}$ is for NSI be slack in the limit, or:

$$\begin{aligned}
\frac{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{f}(\overline{x})-\underline{f}(\overline{x})}{\overline{f}(\overline{x})+\underline{f}(\overline{x})} &> C \\
&\Updownarrow \\
\frac{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]} &> \frac{\underline{f}(\overline{x})}{\overline{f}(\overline{x})}.
\end{aligned}$$

The above analysis yields the following lemma.

Lemma 5 *[Low Sensitivity] The set of feasible incentive compatible signal precisions is $\Sigma_F = \{\sigma_{pt}\}$ if*

$$\frac{\underline{f}(\overline{x})}{\overline{f}(\overline{x})} \geq \frac{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}.$$

Otherwise, Σ_F is the compact interval $[\sigma^{\min},\sigma_{pt}]$, where

$$\sigma^{\min} \equiv \min_{\sigma\in\Sigma} : \lambda_{jr}\left(\widehat{\theta}(\sigma),\sigma\right) = C.$$

Having characterized the feasible set and the optimal securities for implementing each $\sigma \in \Sigma_F$ we turn now to the question of the overall optimal structure. Attention is confined to the interesting case where $\Sigma_F \neq \{\sigma_{pt}\}$, since otherwise security design cannot influence the uninformed valuation and a pass-through structure suffices for optimality. Rather than optimize directly over $\sigma \in \Sigma_F$, it is more convenient to optimize over senior debt face values, recognizing that now the incentive compatible signal precision is defined by the function:

$$\sigma(\theta) \equiv \underline{\sigma} + \frac{[\overline{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta)](\overline{\gamma} - \underline{\gamma})}{2\xi}. \tag{45}$$

The optimal face value with discretionary effort is

$$\begin{aligned} \theta^{**} &\in \arg\max_{\theta \in \Theta_F} \quad V(\theta) \\ \Theta_F &\equiv \left[\widehat{\theta}(\sigma^{\min}), \overline{x}\right] \\ V(\theta) &\equiv \frac{1}{2}(\overline{\mu}_x + \underline{\mu}_x) - \frac{1}{2}[2\sigma(\theta) - 1]\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})[\overline{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta)] - \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]C[\overline{\mu}_{jr}(\theta) + \underline{\mu}_{jr}(\theta)]. \end{aligned} \tag{46}$$

Since V is continuous and Θ_F a closed bounded interval, it follows from Weierstrass' Theorem a maximum point exists. Differentiating the objective function yields:

$$\begin{aligned} V'(\theta) \;=\; & -\frac{1}{2}\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})\left[(2\sigma(\theta) - 1)(\overline{\mu}'_{sr}(\theta) - \underline{\mu}'_{sr}(\theta)) + (\overline{\mu}_{sr}(\theta) - \underline{\mu}_{sr}(\theta))\sigma'(\theta)\right] \\ & -\frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]C[\overline{\mu}'_{jr}(\theta) + \underline{\mu}'_{jr}(\theta)] \end{aligned} \tag{47}$$

If the optimum is interior, it must satisfy the first-order condition:

$$\begin{aligned} & \underbrace{\frac{1}{2}\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})\left[[2\sigma(\theta^{**}_{int}) - 1][\overline{\mu}'_{sr}(\theta^{**}_{int}) - \underline{\mu}'_{sr}(\theta^{**}_{int})] + 2[\overline{\mu}_{sr}(\theta^{**}_{int}) - \underline{\mu}_{sr}(\theta^{**}_{int})]\sigma'(\theta^{**}_{int})\right]}_{\text{Marginal Trading Loss Liquid Security}} \\ = & \underbrace{-\frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]C[\overline{\mu}'_{jr}(\theta^{**}_{int}) + \underline{\mu}'_{jr}(\theta^{**}_{int})]}_{\text{Marginal Carrying Cost Illiquid Security}}. \end{aligned} \tag{48}$$

Comparison of the first-order condition from the Baseline Setting, stated as equation (29), with the above condition is instructive. In particular, we see that endogenous speculator effort raises the marginal cost to increasing senior debt face value. Intuitively, increasing the senior debt face value raises the incentive compatible signal precision, which raises expected trading losses on the liquid tranche. Computing σ' and rearranging terms, the first-order condition can be written as:

$$\frac{1 - \overline{F}(\theta^{**}_{int})}{1 - \underline{F}(\theta^{**}_{int})} = \frac{\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})(4\sigma - 1 - 2\underline{\sigma}) + C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}{\psi(1-\psi)(\overline{\gamma} - \underline{\gamma})(4\sigma - 1 - 2\underline{\sigma}) - C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]}. \tag{49}$$

Once again, the optimal senior debt face value is increasing in C, since increasing the senior debt face value reduces the carrying cost discount. Consistent with the argument that speculator effort raises the marginal cost of increasing senior debt face value, it can be verified that any solution to the extended model's first-order condition (equation (49)) is strictly less than that in the Baseline Setting (equation (27)).

If the optimum is not interior, it must be that $\theta^{**} = \widehat{\theta}(\sigma^{\min})$ since $V(\overline{x}) = V_{PT}$ and $V(\theta) > V_{PT}$ for all other points in Θ_F. A necessary condition for $\widehat{\theta}(\sigma^{\min})$ to be optimal is that the right derivative of the value function V be negative at this point. This will be the case when the marginal adverse selection cost arising from increasing senior debt face value swamps the marginal reduction in carrying costs, e.g. when the incentive compatible σ is very responsive to marginal increases in prospective trading gains.

The appendix establishes the following proposition.

Proposition 4 *[Low Sensitivity] If*

$$\frac{\underline{\underline{f}}(\overline{x})}{\overline{\overline{f}}(\overline{x})} \geq \frac{\underline{f}(\overline{x})}{\overline{f}(\overline{x})} \geq \frac{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\sigma_{pt}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}$$

security design is then irrelevant for the ex ante value obtained by the uninformed owners. Otherwise, the optimal security design entails splitting cash flow into an illiquid junior claim and a liquid senior debt claim with face value strictly less than the optimal face value when signal precision is fixed at $\underline{\sigma}$.

Before concluding this subsection, it is worthwhile to briefly discuss some predictions regarding the determinants of informed trading intensity, as captured by the incentive compatible signal precision. Consider first how incentive compatible signal precision varies with the face value of senior debt. First, for senior face value sufficiently low, both security markets will be liquid and the incentive compatible signal precision will be maximal, equal to that attained under a pass-through structure. However, there will be a critical senior face value at which the junior claim becomes illiquid. At this threshold, the incentive compatible signal precision will fall by a discrete amount, reflecting that fact that the informed speculator can no longer profit from trading in the junior security market. Then, as senior face value is increased incrementally, informed trading gains in that market increase, leading to marginal increases in signal precision. In the limit, as senior face value tends to $\overline{x}$, incentive compatible signal precision tends to the (maximal) level induced under a pass-through structure. Thus, informed speculator gains and signal precision are both U-shaped in senior claim information-sensitivity.

Alternatively, one can consider ranking claims by standard measures of riskiness or information-sensitivity. Consider the following paired ABS capital structures (with the junior claim labeled equity): (1) low risk debt paired with low levered equity; (2) medium risk debt paired with medium levered equity; and (3) high risk debt paired with high levered equity. The proposed theory would predict: highly informed speculation in the first case (as both markets are liquid); poorly informed speculation in the liquid debt market and none in the illiquid equity market in the second case;

and highly informed speculation in the liquid debt market and none in the illiquid equity market in the latter case. Note, this implies a non-monotonic relationship between a security's information-sensitivity and the intensity of informed trading it attracts.

An interesting empirical question is whether measures of informed trading and ABS tranche liquidity are consistent with these predictions.

3.2 Security Design with Discretionary Effort: High Sensitivity

This subsection examines the case where the underlying cash flows have a high trading loss information-sensitivity.

$$\textbf{High Sensitivity} \quad : \quad \frac{\overline{\mu}_x}{\underline{\mu}_x} > \frac{(2\underline{\sigma}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}{(2\underline{\sigma}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - C[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]}$$

$$\Updownarrow$$

$$C \quad < \quad \frac{(2\underline{\sigma}-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{\overline{\mu}_x - \underline{\mu}_x}{\overline{\mu}_x + \underline{\mu}_x}.$$

As in the previous subsection, the overall optimal security design can be determined in two steps. In the first step, we consider the optimal security design for inducing specific feasible $\sigma \in \Sigma$. In the second step, the optimal σ is chosen from the feasible set, a set now denoted Σ_F'.

Consider an arbitrary σ conjectured to be in the feasible set. The objective is to choose a payoff mapping for the liquid security $l \in \mathcal{A}$ to maximize the objective function specified in equation (18) subject to the constraints that: the signal precision σ is incentive compatible (IC); the posited illiquid security satisfies $\lambda_i(\sigma) \geq C$ (NSI); and the posited liquid security satisfies $\lambda_l(\sigma) \leq C$ (SL). Since here $\lambda_x(\sigma) \geq \lambda_x(\underline{\sigma}) > C$, it follows from equation (12) that satisfaction of the SL constraint implies satisfaction of the NSI constraint. So the relevant restrictions for the program are the SL constraint in equation (31) and the incentive constraint in equation (39). But note that the left side of equation (31) is just the objective function for this program. So, we can solve for the optimal structuring by instead solving a relaxed program (again RP2) that accounts for the IC constraint but ignores the SL constraint. If the solution to RP2 satisfies the neglected SL constraint then it is indeed the optimal contract for inducing the posited σ level. Otherwise, the posited σ is not feasible.

From Lemma 4 we know the optimal liquid security in RP2 is senior debt with face value set according to equation (42), which ensures the posited σ is incentive compatible. But to ensure the posited σ is indeed feasible, it is necessary to determine whether the candidate optimal security design satisfies the neglected SL constraint. If the neglected constraint is satisfied, the candidate design is indeed optimal for implementing the respective σ. If it is not satisfied, then the σ under consideration is not feasible. With this in mind, we rewrite the SL constraint as:

$$SL : \lambda_{sr} \left(\widehat{\theta}(\sigma), \sigma\right) = \int_{\underline{\sigma}}^{\sigma} \left[\lambda_1^{sr} \left(\widehat{\theta}\left(\widetilde{\sigma}\right), \widetilde{\sigma}\right) \widehat{\theta}'\left(\widetilde{\sigma}\right) + \lambda_2^{sr} \left(\widehat{\theta}\left(\widetilde{\sigma}\right), \widetilde{\sigma}\right)\right] d\widetilde{\sigma} \leq C. \tag{50}$$

Since the integrand in the preceding equation is positive, it follows that in order for the SL constraint is to be satisfied, the posited σ must be sufficiently low in relation to C.

The above analysis yields the following lemma.

Lemma 6 *[High Sensitivity] The set of feasible incentive compatible signal precisions is the compact interval* $[\underline{\sigma}, \sigma^{\max}]$, *where*

$$\sigma^{\max} \equiv \max_{\sigma \in \Sigma} : \lambda_{sr} \left(\widehat{\theta}(\sigma), \sigma\right) = C.$$

Rather than optimize directly over $\sigma \in \Sigma_F'$, it is more convenient to optimize over senior debt face values. The optimal face value satisfies:

$$\begin{aligned} \theta^{**} &\in \arg\max_{\theta \in \Theta_F'} \quad V(\theta) \\ \Theta_F' &\equiv \left[\underline{x}, \widehat{\theta}(\sigma^{\max})\right]. \end{aligned}$$

Since V is continuous and Θ_F' a closed bounded interval, it follows from Weierstrass' Theorem a maximum point exists. The maximum point cannot be $\underline{x}$ since the value function is increasing at this point. Thus, the maximum point is either $\widehat{\theta}(\sigma^{\max})$ or interior (satisfying equation (48)). The appendix establishes the following proposition.

Proposition 5 *[High Sensitivity] The optimal security design entails splitting cash flow into an illiquid junior claim and a liquid senior debt claim with face value strictly less than the optimal face value when signal precision is fixed at* $\underline{\sigma}$.

Taken together, Propositions 4 and 5 confirm that intuition that with endogenous speculator effort, debt remains the optimal liquid contract, but in order to maximize the uninformed valuation of securities senior face value is reduced with the goal of reducing speculator effort and concomitant uninformed trading losses. It is also worth observing that the uninformed valuation is lower when discretionary speculator effort is feasible. Further, if the speculator could be prevented from acquiring any informative signal, the uninformed valuation would equal expected cash flow, with no discount for illiquidity or trading losses. These observations are consistent with the ignorance-is-bliss argument of Dang, Gorton and Holmström (2011).

4 Conclusions

This paper analyzed optimal security design when the objective is to maximize the valuation uninformed investors attach to a stream of cash flows. Uninformed investors have a demand for liquidity, but liquidity provision is hindered by informed speculation. This exposes the uninformed to underpricing in the event they sell securities. Consequently, the uninformed may prefer to hold rather than sell. A novel theory of optimal security design emerges as one recognizes that alternative structures can influence the interim trading of the uninformed as they weigh trading losses and carrying costs on a security by security basis.

Maximizing the uninformed valuation is equivalent to minimizing the sum their expected trading losses and carrying costs. In general, the optimal structuring entails the creation of senior debt as the optimal liquid claim and residual junior equity as the optimal illiquid claim. The optimal face value on the senior debt is set to ensure the illiquidity of the junior claim, and also ensures marginal trading losses on the liquid claim are equated with marginal carrying costs on the illiquid claim. If the speculator can exert effort to increase signal precision, optimal senior debt face value is lowered in order to reduce effort and expected uninformed trading losses.

At a more general level, the contribution of this paper is to show that the folk-theorem of Myers and Majluf (1984) deriving debt as an optimal source of funding extends well beyond settings where it is the issuer-seller who has a demand for liquidity and the issuer-seller who holds private information. A methodological contribution of the paper is to suggest that it may be productive to move microstructure-based models of corporate finance and security design away from assuming the uninformed are pure noise-traders. Indeed, novel and interesting theories of corporate finance may emerge if one considers that all agents optimize.

PROOFS

Lemma 2: Relaxed Program 1

The objective is to maximize $\mathcal{L}(l)$ over $l \in \mathcal{A}$. Consider first $l \in \mathcal{A}$ that are piecewise continuously differentiable. The Hamiltonian is:

$$H(x,l,\delta,\pi) = \left[\overline{\kappa}\overline{F}(x) - \underline{\kappa}\underline{F}(x)\right]\delta(x) + \pi(x)\delta(x). \tag{51}$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi_l' = -H_l = 0. \tag{52}$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = 2C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}]. \tag{53}$$

Substituting this back into the Hamiltonian we obtain

$$H(x,l,\delta,\pi^*) = \left[\overline{\kappa}\overline{F}(x) - \underline{\kappa}\underline{F}(x) + 2C(\psi\overline{\gamma} + (1-\psi)\underline{\gamma})\right]\delta(x). \tag{54}$$

Maximizing H over admissible δ, a candidate optimal control policy is:

$$\delta^* = 1_{\{x\in\mathcal{X}:\overline{\kappa}\overline{F}(x)-\underline{\kappa}\underline{F}(x)+2C(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})\geq 0\}}. \tag{55}$$

From the transversality condition it follows LL binds at $\underline{x}$ so the state variable l can be expressed as

$$l^*(x) = \underline{x} + \int_{\underline{x}}^{x} \delta^*(\widetilde{x})d\widetilde{x}. \tag{56}$$

By construction, the Hamiltonian is linear in (l,δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l)$ on the space of piecewise continuously differentiable functions in $\mathcal{A}$. It follows from the Stone-Weierstrass Theorem this space is dense in $\mathcal{A}$ endowed with the sup norm. And we know $\mathcal{L}(l)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the optimal liquid security is senior debt. Consider that $\delta^* = 1$ (and zero otherwise) for all $x \in \mathcal{X}$ such that:

$$\begin{aligned} \overline{\kappa}\overline{F}(x) - \underline{\kappa}\underline{F}(x) + 2C(\psi\overline{\gamma} + (1-\psi)\underline{\gamma}) &\geq 0 \\ &\Updownarrow \\ \frac{1-\overline{F}(x)}{1-\underline{F}(x)} &\leq \frac{\underline{\kappa}}{\overline{\kappa}}. \end{aligned} \tag{57}$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0 at θ^* solving:

$$\frac{1-\overline{F}(\theta^*)}{1-\underline{F}(\theta^*)} = \frac{\underline{\kappa}}{\overline{\kappa}}. \blacksquare \tag{58}$$

Lemma 3: Information-Sensitivity of Claims

The information-sensitivity of the junior security is:

$$\lambda_{jr}(\theta) \equiv \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{\overline{\mu}_{jr}(\theta)-\underline{\mu}_{jr}(\theta)}{\overline{\mu}_{jr}(\theta)+\underline{\mu}_{jr}(\theta)}.$$

The information-sensitivity of the junior claim increases in the face value of senior riskless debt since

$$\theta \leq \underline{x} \Rightarrow \lambda_{jr}(\theta) \equiv \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{\overline{\mu}_x-\underline{\mu}_x}{\overline{\mu}_x+\underline{\mu}_x-2\theta}.$$

Consider next information-sensitivity for $\theta \in (\underline{x}, \overline{x})$. Differentiating with respect to θ one obtains:

$$\lambda'_{jr}(\theta) = \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}} \frac{2[\underline{\mu}_{jr}\overline{\mu}'_{jr}-\overline{\mu}_{jr}\underline{\mu}'_{jr}]}{[\overline{\mu}_{jr}+\underline{\mu}_{jr}]^2}. \tag{59}$$

Thus

$$\begin{aligned} \lambda'_{jr}(\theta) &\geq 0 \\ &\Updownarrow \\ \frac{\overline{\mu}_{jr}(\theta)}{\overline{\mu}'_{jr}(\theta)} &\leq \frac{\underline{\mu}_{jr}(\theta)}{\underline{\mu}'_{jr}(\theta)} \\ &\Updownarrow \\ \frac{\int_\theta^{\overline{x}}(x-\theta)\overline{f}(x)dx}{[1-\overline{F}(\theta)]} &\geq \frac{\int_\theta^{\overline{x}}(x-\theta)\underline{f}(x)dx}{[1-\underline{F}(\theta)]} \\ &\Updownarrow \\ \int_\theta^{\overline{x}} x\left(\frac{\overline{f}(x)}{1-\overline{F}(\theta)}\right)dx &\geq \int_\theta^{\overline{x}} x\left(\frac{\underline{f}(x)}{1-\underline{F}(\theta)}\right)dx. \end{aligned} \tag{60}$$

The last inequality follows from the fact that the conditional densities in brackets also have the MLR property.

Consider next the information-sensitivity of the senior security. For $\theta \in (\underline{x}, \overline{x})$ we have:

$$\begin{aligned}
\lambda_{sr}'(\theta) &\geq 0 \\
&\Updownarrow \\
\frac{\overline{\mu}_{sr}(\theta)}{\overline{\mu}_{sr}'(\theta)} &\leq \frac{\underline{\mu}_{sr}(\theta)}{\underline{\mu}_{sr}'(\theta)} \\
&\Updownarrow \\
\frac{\int_{\underline{x}}^{\theta} x\overline{f}(x)dx + \theta[1-\overline{F}(\theta)]}{[1-\overline{F}(\theta)]} &\leq \frac{\int_{\underline{x}}^{\theta} x\underline{f}(x)dx + \theta[1-\underline{F}(\theta)]}{[1-\underline{F}(\theta)]} \\
&\Updownarrow \\
\int_{\underline{x}}^{\theta} x\left(\frac{\overline{f}(x)}{1-\overline{F}(\theta)}\right)dx &\leq \int_{\underline{x}}^{\theta} x\left(\frac{\underline{f}(x)}{1-\underline{F}(\theta)}\right)dx
\end{aligned} \tag{61}$$

To establish the last inequality above it is sufficient to prove that for arbitrary $x \in (\underline{x}, \theta)$

$$\frac{\overline{f}(x)}{1-\overline{F}(\theta)} \leq \frac{\underline{f}(x)}{1-\underline{F}(\theta)}.$$

Consider then that from MLR we have

$$\underline{x} < x_0 < \theta < x_1 < \overline{x} \Rightarrow \underline{f}(x_0)\overline{f}(x_1) \geq \underline{f}(x_1)\overline{f}(x_0).$$

And thus

$$\begin{aligned}
\underline{f}(x_0)\int_{\theta}^{\overline{x}} \overline{f}(x_1)dx_1 &\geq \overline{f}(x_0)\int_{\theta}^{\overline{x}} \underline{f}(x_1)dx_1 \\
&\Updownarrow \\
\underline{f}(x_0)[1-\overline{F}(\theta)] &\geq \overline{f}(x_0)[1-\underline{F}(\theta)].
\end{aligned}$$

Finally, the last statement in the lemma follows from the identity (11). Rearranging we obtain:

$$\lambda_{sr} = \lambda_x + [\lambda_{jr} - \lambda_x]\left[\frac{\overline{\mu}_{jr} + \underline{\mu}_{jr}}{\overline{\mu}_{sr} + \underline{\mu}_{sr}}\right] \leq \lambda_x. \blacksquare$$

Proposition 3: Social Planner's Problem under High Sensitivity Cash Flows

The Hamiltonian for the social planner's problem is:

$$H(x, l, \delta, \pi, m) = -\left[(1 + m\underline{\kappa})\underline{F}(x) + (1 - m\overline{\kappa})\overline{F}(x)\right]\delta(x) + \pi(x)\delta(x). \tag{62}$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi' = -H_l = 0.$$

Thus π^* is a constant, with the transversality condition demanding

$$\pi^* = 2 + 2mC(\psi\overline{\gamma} + (1-\psi)\underline{\gamma}).$$

Substituting this back into the Hamiltonian we have:

$$H(x, l, \delta, \pi^*, m) = [2 + 2mC(\psi\overline{\gamma} + (1-\psi)\underline{\gamma}) - (1 + m\underline{\kappa})\underline{F}(x) - (1 - m\overline{\kappa})\overline{F}(x)]\delta(x).$$

Thus, a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: 2+2mC(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})-(1+m\underline{\kappa})\underline{F}(x)-(1-m\overline{\kappa})\overline{F}(x)\geq 0\}}.$$

From the transversality condition it follows LL binds at $\underline{x}$ so the implied state variable is:

$$l^*(x) = \underline{x} + \int_{\underline{x}}^{x} \delta^*(\widetilde{x})d\widetilde{x}.$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l, m)$ on the space of piecewise continuously differentiable functions in $\mathcal{A}$. It follows from the Stone-Weierstrass Theorem this space is dense in $\mathcal{A}$ endowed with the sup norm. And we know $\mathcal{L}(l, m)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l, m)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the socially optimal liquid security is debt. Consider that $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$(1 + m\underline{\kappa})[1 - \underline{F}(x)] \geq -(1 - m\overline{\kappa})[1 - \overline{F}(x)] \tag{63}$$

The term on the right side of the preceding inequality must be positive otherwise the inequality would always hold, and the optimal liquid security would be a pure pass-through. But this contradicts the fact that a pass-through security would be illiquid in the present setting. Thus, $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$\frac{1 - \overline{F}(x)}{1 - \underline{F}(x)} \leq -\frac{1 + m\underline{\kappa}}{1 - m\overline{\kappa}}.$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0, implying debt is the optimal liquid security. Finally, note the objective function is increasing in debt face value, but so too is the information-sensitivity of the posited liquid debt claim. Thus, the socially optimal face value is such that the SL constraint is binding.■

Lemma 4: Relaxed Program 2

Consider first implementation of $\underline{\sigma}$. From the IC constraint it follows that the liquid security must satisfy $\overline{\mu}_l = \underline{\mu}_l$. Within this set, carrying costs on the illiquid claim are minimized by setting the face value of the liquid senior claim to $\underline{x}$.

Consider next the program RP2 for $\sigma \in (\underline{\sigma}, \sigma_{pt})$. Consider first $l \in \mathcal{A}$ that are piecewise continuously differentiable. The Hamiltonian is:

$$H(x, l, \delta, \pi, m) = [(\overline{\kappa} - m)\,\overline{F}(x) - (\underline{\kappa} - m)\,\underline{F}(x)]\delta(x) + \pi(x)\delta(x).$$

From Pontryagin's Maximum Principle, an optimal control policy δ^* maximizes the Hamiltonian, with the co-state variable π^* being a piecewise continuously differentiable function satisfying the following co-state equation at all points where the control δ^* is continuous:

$$\pi_l' = -H_l = 0.$$

Thus π^* is a constant, with the transversality condition implying

$$\pi^* = 2C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}].$$

Substituting this back into the Hamiltonian we obtain

$$H(x, l, \delta, \pi^*, m) = [2C(\psi\overline{\gamma} + (1-\psi)\underline{\gamma}) + (\overline{\kappa} - m)\,\overline{F}(x) - (\underline{\kappa} - m)\,\underline{F}(x)]\delta(x) + \pi(x)\delta(x).$$

and maximizing H over admissible δ, a candidate optimal control policy is:

$$\delta^* = 1_{\{x \in \mathcal{X}: 2C[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}] + (\overline{\kappa}-m)\overline{F}(x) - (\underline{\kappa}-m)\underline{F}(x) \geq 0\}}.$$

Further, from the transversality condition it follows LL binds at $\underline{x}$ so the implied state variable is:

$$l^*(x) = \underline{x} + \int_{\underline{x}}^{x} \delta^*(\widetilde{x})d\widetilde{x}.$$

By construction, the Hamiltonian is linear in (l, δ) implying Mangasarian's sufficiency conditions are satisfied. Thus, the proposed δ^* is an optimal control and l^* maximizes $\mathcal{L}(l, m)$ on the space of piecewise continuously differentiable functions in $\mathcal{A}$. It follows from the Stone-Weierstrass Theorem this space is dense in $\mathcal{A}$ endowed with the sup norm. And we know $\mathcal{L}(l, m)$ is continuous in l in this topology. Thus, the proposed state variable l^* maximizes $\mathcal{L}(l, m)$ amongst all $a \in \mathcal{A}$.

We next claim the proposed control policy implies the optimal liquid security is debt. Consider that $\delta^* = 1$ (and 0 otherwise) for all $x \in \mathcal{X}$ such that:

$$(\overline{\kappa} - m)\overline{F}(x) - (\underline{\kappa} - m)\underline{F}(x) + 2C(\psi\overline{\gamma} + (1-\psi)\underline{\gamma}) \geq 0$$

$$\frac{1-\overline{F}(x)}{1-\underline{F}(x)} \leq \frac{\underline{\kappa}-m}{\overline{\kappa}-m}.$$

Since MLR implies MHR, the left side of the preceding equation is non-decreasing so the optimal control switches from 1 to 0 at θ^* solving:

$$\frac{1-\overline{F}(\theta^*)}{1-\underline{F}(\theta^*)} = \frac{\underline{\kappa}-m}{\overline{\kappa}-m}.\blacksquare$$

Proposition 4: Optimal Face Value with Speculator Effort (Low Sensitivity)

We claim that the possibility of endogenous increases in effort result in strictly lower senior debt face value so long as NSI can be satisfied by some debt claim. We know that with endogenous effort the optimal face value is either at the interior solution θ^{**}_{int} or at $\widehat{\theta}(\sigma^{\min})$. In the former case, the result follows from the fact that any solution to the extended model's first-order condition (equation (48) is strictly less than that in the Baseline Setting with σ fixed at $\underline{\sigma}$. In the latter case, the result is necessarily true if $\widehat{\theta}(\sigma^{\min}) = \underline{x}$ since the optimal debt face value in the Baseline Setting with fixed σ is always strictly greater $\underline{x}$. If $\widehat{\theta}(\sigma^{\min}) > \underline{x}$ the result follows from the fact that it must then be the case that $\sigma^{\min} > \underline{\sigma}$ and the point at which the NSI constraint is just satisfied is strictly less than the point at which the NSI constraint is satisfied when $\sigma = \underline{\sigma}$.$\blacksquare$

Proposition 5: Optimal Face Value with Speculator Effort (High Sensitivity)

We claim that the possibility of endogenous increases in effort result in strictly lower senior debt face value. We know that with endogenous effort the optimal face value is either at the interior solution θ^{**}_{int} or at $\widehat{\theta}(\sigma^{\max})$. In the former case, the result follows from the fact that any solution to the extended model's first-order condition (equation (48) is strictly less than that in the Baseline Setting with σ fixed at $\underline{\sigma}$. In the latter case, the result follows from the fact that if the solution is at a corner, the value function must be increasing in θ at the corner. But then the value function with σ fixed at $\underline{\sigma}$ would be increasing in θ at this point as well. So in the Baseline Setting the optimal face value would exceed $\widehat{\theta}(\sigma^{\max})$.$\blacksquare$

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Table 1: Order Flow Possibilities

State	Signal	S Order	UI Order	Total Order	Probability	LP Belief
$\overline{\omega}$	$\overline{\omega}$	0	$-\underline{\gamma}$	$-\underline{\gamma}$	$\frac{\sigma(1-\psi)}{2}$	σ
$\overline{\omega}$	$\overline{\omega}$	0	$-\overline{\gamma}$	$-\overline{\gamma}$	$\frac{\sigma\psi}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\overline{\omega}$	$\underline{\omega}$	$-(\overline{\gamma}-\underline{\gamma})$	$-\underline{\gamma}$	$-\overline{\gamma}$	$\frac{(1-\sigma)(1-\psi)}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\overline{\omega}$	$\underline{\omega}$	$-(\overline{\gamma}-\underline{\gamma})$	$-\overline{\gamma}$	$-(2\overline{\gamma}-\underline{\gamma})$	$\frac{(1-\sigma)\psi}{2}$	$1-\sigma$
$\underline{\omega}$	$\underline{\omega}$	$-(\overline{\gamma}-\underline{\gamma})$	$-\underline{\gamma}$	$-\overline{\gamma}$	$\frac{\sigma(1-\psi)}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\underline{\omega}$	$\underline{\omega}$	$-(\overline{\gamma}-\underline{\gamma})$	$-\overline{\gamma}$	$-(2\overline{\gamma}-\underline{\gamma})$	$\frac{\sigma\psi}{2}$	$1-\sigma$
$\underline{\omega}$	$\overline{\omega}$	0	$-\underline{\gamma}$	$-\underline{\gamma}$	$\frac{(1-\sigma)(1-\psi)}{2}$	σ
$\underline{\omega}$	$\overline{\omega}$	0	$-\overline{\gamma}$	$-\overline{\gamma}$	$\frac{(1-\sigma)\psi}{2}$	$1-\sigma-\psi+2\sigma\psi$