News Trading and Speed*

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May 29, 2013

Abstract

Speed matters: we show that an investor's optimal trading strategy is significantly different when he observes news faster than others versus when he does not, holding the precision of his signals constant. When the investor has fast access to news, his trades are much more sensitive to news, account for a much bigger fraction of trading volume, and forecasts short run price changes. Moreover, in this case, an increase in news informativeness increases liquidity, volume, and the fast investor's share of trading volume. Last, price changes are more correlated with news and trades contribute more to volatility when the investor has fast access to news.

KEYWORDS: Informed trading, news, volatility, volume, price discovery, high frequency trading.

*We thank Terry Hendershott, Jennifer Huang, Leonid Kogan, Pete Kyle, Stefano Lovo, Victor Martinez, Han Ozsoylev, Tarun Ramadorai, Dimitri Vayanos, Xavier Vives, and Mao Ye for their suggestions. We are also grateful to finance seminar participants at Copenhagen Business School, Univ. Carlos III in Madrid, ESSEC, Lugano, IESE, INSEAD, Oxford, Paris Dauphine, and conference participants at the UBC Finance Winter Conference, the NYU Stern Microstructure Meeting, CNMV International Conference on Securities Markets, Central Bank Microstructure Workshop, Market Microstructure Many Viewpoints Conference in Paris, the 5^{th} Paris Hedge Fund Conference, and the High Frequency Trading conference in Paris for valuable comments.

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Authorities are exploring potential holes in the system, including new algorithms referred to as "news aggregation" that search the internet, news sites and social media for selected keywords, and fire off orders in milliseconds. The trades are so quick, often before the information is widely disseminated, that authorities are debating whether they violate insider trading rules.

"FBI joins SEC in Computer Trading Probe," The Financial Times, March 6, 2013.

1 Introduction

In an efficient market, prices should immediately reflect public information. Yet, recent empirical evidence show that information in news is not immediately impounded into prices, so that trading on news is profitable.[1] There are two possible, non-exclusive, explanations for these findings. First, news traders could filter out more precise signals from news because they process information more efficiently. Second, news traders might react faster to news. Existing models of informed trading focus on the first explanation.[2] Does this matter? Does fast access to news significantly alter an investor's behavior relative to the case in which he processes news more efficiently?

These questions are of broad interest. First, understanding the respective effects of speed and accuracy on news traders' optimal behavior is required to empirically assess the source of their profits. Moreover, some market participants now trade on news at the very high frequency using highly computerized trading strategies.[3] Their profits derive *both* from an efficient processing of and a fast access to the virtually continuous flow of messages generated by the trading process (quote updates, trades, cancellations

[1]See, for instance, Busse and Green (2002), Tetlock, Saar-Tsechansky, and Macskassy (2008), Engelberg (2008), Tetlock (2010), and Engelberg, Reed, Ringgenberg (2012).

[2]For instance, Kim and Verrecchia (1994) assume that when news about the payoff of an asset is released, some traders ("skilled information processors") are better able to interpret their informational content and therefore form more accurate forecasts than dealers. However, in Kim and Verrecchia (1994) all traders receive news at the same time.

[3]See the opening quotation and "Computers that trade on the news," the New York Times, May 22, 2012.

etc.).[4] Models incorporating both advantages are therefore needed to understand high frequency news trading and its effects. Last, such models can also shed light on the process by which news gets impounded in prices, a question of interest for various areas of finance.[5]

In order to study news trading, we consider a dynamic model similar to Kyle (1985), but in which new information about the payoff of a risky security arrives before each trading round.[6] One investor ("the speculator") and dealers observe signals ("news") about this information. In the baseline model, the speculator processes news more efficiently (i.e., his signal is more precise) but he receives news at the same time as dealers do. We then consider the case in which dealers receive news with a lag of one period relative to the speculator. Our central finding is that the speculator's optimal trading strategy is very different in each case.

In the absence of a speed advantage, the speculator's optimal trade in each period is proportional to dealers' forecast "error" (that is, the difference between the speculator's and dealers' forecast of the asset payoff), as in Kyle (1985). Thus, the news affects the speculator's trading strategy only insofar as it affects his forecast of the asset payoff. In contrast, when the speculator has a speed advantage, the news affects his trading strategy over and above this forecast effect. That is, the news becomes a distinct determinant of his strategy.

The intuition is as follows. Suppose that the speculator just received good news.

[4] For instance, Brogaard, Hendershott, and Riordan (2012) find that high frequency traders react to information contained in limit order book updates, market-wide returns, and macroeconomic announcements. Jovanovic and Menkveld (2012) or Zhang (2012) show that high frequency traders also use index futures price information as a source of information to establish positions in underlying stocks. In order to secure fast access to information, high frequency traders position their computers close to trading platforms' servers (a practice called co-location), or buy direct access to exchanges datafeed.

[5] For instance, it is important to understand the sources of volatility in financial markets (see, e.g., French and Roll (1986)).

[6] Thus, the news arrival rate in our model is commensurate with the trading frequency. This is a reasonable assumption for high frequency news traders. For instance, Hendershott (2011) writes: "*At an HFT firm there is a near infinite amount of financial market data arriving continuously.*" One reason is that each order submitted to the market or quote update is a signal. Hendershott (2011) estimates the number of orders for U.S. equity markets alone at about 100,000 per second.

His forecast of the asset payoff increases, which commands buying shares of the asset whether the speculator gets news faster or not. However, if he gets news faster, the speculator also expects dealers to soon mark up the price of the asset, when receiving the news. To exploit this foreknowledge of the short run quote dynamics, the speculator optimally buys more shares than he would in the absence of a speed advantage. That is, the investor's optimal trading strategy is *more responsive* to news when he reacts faster to news.

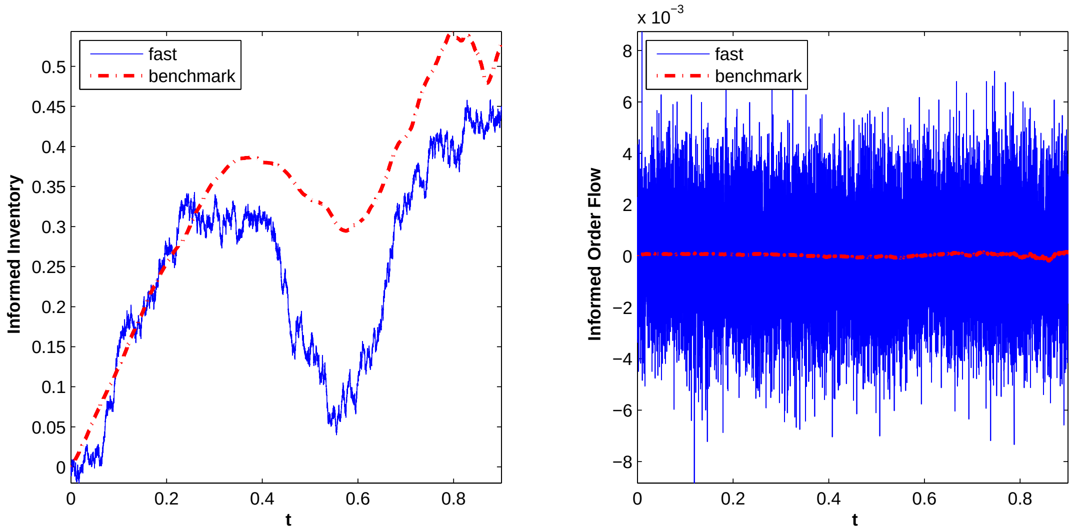
In the continuous time version of the model, these effects lead to a particularly simple characterization for the stochastic process followed by the speculator's optimal position in the risky asset. The drift of this process is proportional to dealers' forecast error and its volatility is proportional to news. However, this volatility is zero if the speculator does not react faster to news, even if he processes news more efficiently than the dealer.[7] In contrast, it is strictly positive when the speculator gets news faster than the dealer. Moreover, in this case, the drift of the speculator's position is less sensitive to dealers' forecast error. Thus, fast access to news significantly alters the speculator's trading strategy: (i) he trades much more aggressively on news, so that his risky position is an order of magnitude more volatile; and (ii) he trades less aggressively on dealers' forecast error.

For this reason, the speculator's trades leave very different "footprints" on market data (volume, price changes, trades) when he has a speed advantage. First, the speculator's share of trading volume is much higher when he reacts faster to news because his optimal trading strategy then calls for much larger adjustments in his portfolio holdings at each point in time, as Figure 1 shows.

Second, fast access to news considerably strengthens the correlation between the speculator's trade at a given point in time and subsequent cumulative price changes, especially when price changes are measured over a short time interval right after the

[7]This finding is standard; see Back (1992), Back and Pedersen (1998), Back, Cao, and Willard (2000) or Chau and Vayanos (2008).

Figure 1: Speculator's trading strategy at 1-second frequency. The figure plots the evolution of the speculator's position (left graph) and the change in this position—the speculator's trade—(right graph), when he has a speed advantage (plain line) and when he has no speed advantage (dot-dashed line), using the characterization of his optimal trading strategy in each case in the continuous time model (derived in Section 3), and aggregated over 1-second intervals. The simulation considers one particular path for news in the model and parameters used for the simulation are $\sigma_u = \sigma_v = \sigma_e = \Sigma_0 = 1$ (see Theorem 1). The liquidation date $t = 1$ corresponds to 1 month.



investor's transaction. The reason is that the speculator's trade anticipates on dealers' quote updates when he reacts fast to news while it does not otherwise.

Last, changes in the speculator's holding of the risky asset are less positively autocorrelated when he reacts fast to news. Indeed, in this case, these changes over short time intervals are predominantly determined by news. As the news is uncorrelated, it does not generate serial correlation in the speculator's trades. In contrast, when the speculator has no speed advantage, changes in his holdings are mainly determined by changes in dealers' forecast error about the asset payoff. As this error is persistent, changes in the speculator's position are also persistent (as in Kyle (1985)).

These footprints of the speculator when he reacts faster to news match well stylized facts about high frequency traders: (a) their trades account for a large fraction of the trading volume (Hendershott, Jones, and Menkveld (2011), Brogaard, Hendershott, and Riordan (2012) or Chaboud, Chiquoine, Hjalmarsson, and Vega (2013)), (b) their aggressive orders (i.e., marketable orders) anticipate very short run price changes (Kirilenko, Kyle, Samadi, and Tuzun (2011) or Brogaard, Hendershott, and Riordan (2012)), and (c) they have a relatively low positive autocorrelation (Hirschey (2013)).

When the speculator reacts to news faster, the model also predicts that an improvement in news informativeness for dealers should trigger a joint increase in trading volume, the speculator's contribution to volume, and liquidity. Indeed, dealers' quote updates are more sensitive to news when this is more precise. Hence, with fast access to news, the speculator can better forecast short run dealers' quote updates when dealers' news is more informative, which induces the speculator to trade even more aggressively on news. As a result, the volatility of his position in the risky asset increases, which means that both trading volume and the fraction of this volume due to the speculator increase with dealers' news informativeness. However, as dealers receive more informative news, they also better forecast the asset payoff, which alleviates their exposure to informed trading. Accordingly, in equilibrium, liquidity improves when dealers' news is more informative, even though informed trading intensifies in this case.[8]

In contrast, when the speculator does not react faster to news, an increase in news informativeness for dealers induces the speculator to trade less aggressively on his private information. Hence, an improvement in news informativeness strengthens liquidity while reducing the speculator's share of trading volume and trading volume. These conflicting implications regarding the effects of news informativeness for dealers on liquidity and volume offer one way to test whether speed plays a role or not in news traders' strategies.

The nature of price discovery also depends on whether the speculator has a faster access to news or not. In the latter case, short-run price changes are more strongly positively correlated with news and less correlated with dealers' forecast error than in

[8] One polar case of the model is when dealers and the (faster) speculator observe the same news. The speculator's responsiveness to news and therefore the volatility of his position in the risky asset are then maximal—yet still finite.

the former. The first effect strengthens informational efficiency while the second weakens it. In equilibrium, they exactly cancel out so that, at any point in time, informational efficiency—measured as the expected squared difference between the transaction price and the speculator's estimate of the asset payoff—is the same whether the speculator has a speed advantage or not.

The speculator's speed advantage also affects the relative contributions of trades and news to short run volatility. Trades move prices more when the speculator has a speed advantage because they are more informative: they contain information on the asset payoff *and* impending news. Hence, upon receiving news, dealers update their quotes by a smaller amount when the speculator has a speed advantage. These two effects exactly offset each other so that volatility is unaffected by whether the speculator gets faster access to news or not. Thus, the model implies that the fraction of volatility due to news is smaller when the speculator is faster. This is another empirical implication of the model that one could test using the methodology of Hasbrouck (1991) to measure the relative contributions of trades and public information to volatility.

Our model builds upon the dynamic version of Kyle (1985). In contrast to Kyle (1985), we allow for news arrival in each trading round, as in Back and Pedersen (1998) (BP(1998)), Chau and Vayanos (2008) (CV(2008)), and Martinez and Roşu (2013) (MR(2013)). In BP(1998) and MR(2013), only the informed investor receives news. This is a special case of our model in which dealers' news are uninformative. In this polar case, as in BP(1998), the investor never aggressively trades on news, whether fast or not. In contrast, in MR(2013), informed investors aggressively trade on news because they have ambiguity aversion about the asset payoff while the speculator is risk neutral in our model. In CV(2008), dealers and the informed investor receive news at the same time. The instantaneous variance of the informed investor's position is zero in their model, as we obtain when the speculator has no speed advantage. In Foster and Viswanathan (1990) (FV(1990)), dealers observe news with a lag as in our model.

However, between news arrivals, the informed investor can trade continuously so that the news arrival rate relative to the trading rate is zero in FV(1990). As a result, and in contrast to our model, in FV(1990) the instantaneous variance of the informed investor's position is nil, just as it is in BP(1998) and CV(2008).

In order to obtain a closed-form solution for the equilibrium, we focus on the continuous time version of the model. The possibility for the speculator to trade continuously is not key for our findings, however. In the Internet Appendix, we show that results are unchanged when news and trades occur at discrete points in time. Our results just require at least two trading rounds, since otherwise dealers' lagged observation of news cannot play a role.

Finally, our paper is related to the growing theoretical literature on high frequency trading since, as mentioned previously, some high frequency trading firms trade on news.[9] Relative to this literature, our main contribution is to offer an equilibrium characterization of the dynamic trading strategy for a speculator who can both react faster to news and process news more efficiently. As explained previously, this is likely to be the case for investors trading on news at the high frequency.[10]

The paper is organized as follows. Section 2 describes the model. In Section 3, we show that the speculator trades significantly more aggressively on news when he gets advanced access to news. Section 4 derives implications for volume, price changes, and trade autocorrelation; and Section 5 studies the effects of a change in dealers' news informativeness. Finally, in Section 6, we study how the speed at which the speculator reacts to news affects price discovery and volatility. Section 7 concludes. The appendix

[9] For theories of high frequency trading, see Jovanovic and Menkveld (2012), Biais, Foucault, and Moinas (2012), Pagnotta and Philippon (2012), Cartea and Penalva (2012), or Hoffmann (2013).

[10] High frequency traders' strategies are heterogeneous (see SEC (2010)). In particular, some HFTs follow market making strategies (see Brogaard, Hendershott, and Riordan (2012) or Menkveld (2013)). This type of strategy is not captured by our model, in which the speculator only submits market orders, as in Kyle (1985). Ours seems to be a reasonable assumption to model high frequency traders exploiting private information since Brogaard, Hendershott, and Riordan (2012) show empirically that only market orders submitted by high frequency traders are a source of adverse selection. Moreover, some HFTs mainly use market orders (see Baron, Brogaard, and Kirilenko (2012) or Hagströmer and Norden (2013)).

contains the proofs for the continuous time version of the model. The Internet Appendix shows the robustness of our findings in the discrete time version.

2 Model

Trading for a risky asset takes place at T trading rounds during the time interval $[0,1]$, with time between trades $\Delta t = \frac{1}{T}$. The liquidation value of the asset is

$$v_T \;=\; v_0 + \sum_{t=1}^{T} \Delta v_t, \tag{1}$$

with all variables normally distributed: $v_0 \sim \mathcal{N}(0, \Sigma_0)$, with $\Sigma_0 > 0$, and $\Delta v_t = v_t - v_{t-1} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_v^2 \Delta t)$. The risk-free rate is assumed to be zero.

In each trading round t, one risk neutral informed speculator ("he") and noise traders submit market orders to a risk neutral competitive dealer ("she"), who sets the price at which trades take place.[11] We denote by Δx_t and Δu_t the market orders of the speculator and noise traders, respectively, with $\Delta u_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_u^2 \Delta t)$. The speculator chooses his trade optimally given his information, as explained in more detail below. Thus, in trading round $t = 1, \ldots, T$, the order flow executed by the dealer is

$$\Delta y_t \;=\; \Delta u_t + \Delta x_t. \tag{2}$$

New information regarding the liquidation value of the asset arrives at the beginning of each trading round t (see Figure 2). Specifically, at t, the speculator observes a signal

$$\Delta s_t = \Delta v_t + \Delta \varepsilon_t, \tag{3}$$

[11] For tractability, we focus on the case in which there is a single speculator. Extending the model to the case with multiple speculators is not straightforward (see, for instance, Holden and Subrahmanyam (1992) or Back, Cao, and Willard (2000) for treatment without news). It is therefore left for future work.

where $\Delta\varepsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2 \Delta t)$. At the same time, the dealer observes a signal

$$\Delta z_{t-\ell} \;=\; \Delta s_{t-\ell} + \Delta e_{t-\ell}, \tag{4}$$

where $\ell = 1$ or $\ell = 0$ (see below) and $\Delta e_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_e^2 \Delta t)$. We refer to Δs_t and Δz_t as the news observed by the speculator and the dealer, respectively. At date 0 only the speculator observes v_0, and subsequently receives more precise news than the dealer if $\sigma_e > 0$. These assumptions reflect the speculator's ability to better process information than the dealer.

When $\ell = 0$, the dealer receives information on innovations in the asset value, Δv_t, without delay relative to the speculator. In contrast, when $\ell = 1$, the dealer always receives information on innovations in the asset value with a lag of one period relative to the speculator. That is, the dealer is slower in getting access to news than the speculator, and ℓ measures the latency with which he obtains information. More specifically, when $\ell = 0$, the dealer's information set before observing the order flow in trading round t is $\mathcal{I}_t^{(\ell=0)} = \{\Delta z_\tau\}_{\tau \le t} \cup \{\Delta y_\tau\}_{\tau \le t-1}$. In contrast, when $\ell = 1$, the dealer's information set is $\mathcal{I}_t^{(\ell=1)} = \{\Delta z_\tau\}_{\tau \le t-1} \cup \{\Delta y_\tau\}_{\tau \le t-1} = \mathcal{I}_t^{(\ell=0)} \setminus \{\Delta z_t\}$.

We refer to the case in which $\ell = 0$ as the *benchmark model*, and to the case in which $\ell = 1$ as the *fast model*. In the latter case, the speculator observes news faster than the dealer, but otherwise the two models are identical. Hence, by contrasting the properties of the two models, we can isolate the effects of speed of access to information while holding the precision of information constant.

Our findings depend on the informativeness of news for the dealer relative to the informativeness of news for the speculator, i.e., on σ_e, rather than on the absolute level of σ_ε. Hence, to simplify notations, we set $\sigma_\varepsilon = 0$, i.e., the speculator observes perfectly the innovation in the asset value. When $\sigma_e = 0$, the speculator and the dealer observe the same news but not necessarily at the same speed. For technical reasons, Σ_0 must be

Figure 2: Timing of events during trading round t

Speculator receives signal Δs_t	Dealer sets quote $q_t = \mathsf{E}\big(v_T \mid \mathcal{I}_t^{(\ell)}\big)$	Traders submit market orders	Dealer sets price $p_t = \mathsf{E}\big(v_T \mid \mathcal{I}_t^{(\ell)} \cup \Delta y_t\big)$
Dealer receives signal $\Delta z_{t-\ell}$		Order flow is realized $\Delta y_t = \Delta x_t + \Delta u_t$	Trading takes place at p_t
$\ell = 0$: Benchmark model $\ell = 1$: Fast model			

strictly positive[12]; however, this parameter can be very close to zero. In this case, when in addition $\sigma_e = 0$, the fast model can be interpreted as a model in which the speculator has *short-lived information*, that is, information that will be observed perfectly by dealers at the beginning of the next trading round, as in Admati and Pfleiderer (1988). This case is rather special, however, since a long-lived information advantage for the speculator only requires $\sigma_e > 0$, no matter how small σ_e in fact is.

We denote by q_t the dealer's expectation of the asset liquidation value just before she observes the aggregate order flow Δy_t, and by p_t the transaction price in trading round t. As the dealer is competitive and risk neutral, she executes the order flow at a price equal to her expectation of the asset liquidation value conditional on her information, including that contained in the order flow in trading round t, as in Kyle (1985). Thus,

$$q_t \;=\; \mathsf{E}\big(v_T \mid \mathcal{I}_t^{(\ell)}\big) \quad \text{and} \quad p_t \;=\; \mathsf{E}\big(v_T \mid \mathcal{I}_t^{(\ell)} \cup \Delta y_t\big), \tag{5}$$

where $\mathcal{I}_t^{(\ell)}$, the information set of the dealer just before trading at t, was defined above.

In each trading round, before choosing his trade, the speculator receives news Δv_t and observes the dealer's quote q_t. Hence, the speculator's information set at date t is $\mathcal{J}_t^{(\ell)} = \mathcal{I}_t^{(\ell)} \cup \{v_\tau\}_{\tau \le t}$.[13] A trading strategy for the speculator is a vector of functions $x = (x_1, x_2, ..., x_T)$ so that x_t is measurable with respect to $\mathcal{J}_t^{(\ell)}$, where x_t specifies

[12] See the discussion at the end of Section 3.

[13] Here we assume that the speculator can perfectly infer the dealer's signal from the quote. This is true in equilibrium, since, as shown in Section 3, the dealer's quote depends linearly on the signal.

the speculator's position at the end of trading round t. The speculator's trade at t is therefore $\Delta x_t = x_t - x_{t-1}$. For a given trading strategy, the speculator's expected profit at date t is

$$\pi_t \;=\; \mathsf{E}\left(\sum_{\tau=0}^{T} (v_T - p_\tau)\Delta x_\tau \;\middle|\; \mathcal{J}_t^{(\ell)} \right). \tag{6}$$

As in Kyle (1985), we focus on sequential equilibria. A sequential equilibrium is such that, at each date: (i) the dealer's pricing policy is given by equation (5) and (ii) the speculator's trading strategy maximizes his expected trading profit (6) given the dealer's pricing policy. Furthermore, as in Kyle (1985), we restrict our attention to linear equilibria.

Let the index B refer to the Benchmark model (with $\ell = 0$), and F refer to the Fast model (with $\ell = 1$). For instance, denote the dealer's information set by

$$\mathcal{I}_t^B \;=\; \mathcal{I}_t^{(\ell=0)} \quad \text{and} \quad \mathcal{I}_t^F \;=\; \mathcal{I}_t^{(\ell=1)}. \tag{7}$$

In a linear equilibrium, the transaction price in each trading round is a linear function of the unanticipated part of the order flow

$$p_t \;=\; q_t + \lambda_t^k \Big(\Delta y_t - \mathsf{E}\big(\Delta y_t | \mathcal{I}_t^k\big) \Big) \qquad \text{for } k \in \{B, F\}. \tag{8}$$

In the benchmark model, the dealer's quote is of the form

$$q_t \;=\; p_{t-1} + \mu_t^B \Delta z_t, \tag{9}$$

while in the fast model it is of the form[14]

$$q_t \;=\; p_{t-1} + \mu_{t-1}^F\Big(\Delta z_{t-1} - \mathsf{E}\big(\Delta z_{t-1}|\mathcal{I}_t^F\big)\Big). \tag{10}$$

The speculator's optimal trade is a linear function of his past and current news and past and current dealer quotes. In the Internet Appendix, we show that, in a linear equilibrium, the speculator's optimal trading strategy necessarily has the following form:

$$\Delta x_t \;=\; \underbrace{\beta_t^k(v_t - q_t)\Delta t}_{\substack{\text{Forecast Error}\\ \text{Component}}} \quad + \quad \underbrace{\gamma_t^k \Delta v_t}_{\substack{\text{News Trading}\\ \text{Component}}} \qquad \text{for } k \in \{B, F\}. \tag{11}$$

Thus, the speculator's optimal trade at date t is a function of the dealer's forecast error, $v_t - q_t$, and of the news received by the speculator, Δv_t, at this date. Intuitively, the speculator should buy (sell) the asset when the dealer underestimates (overestimates) the liquidation value, that is, when the forecast error is positive (negative). This intuition is captured by the first component of the speculator's strategy, and we refer to this component as the *forecast error* component. It is standard in models of trading with asymmetric information such as Kyle (1985), Back and Pedersen (1998), Back, Cao, and Willard (2000), etc.

The forecast error component implicitly depends on his current news Δv_t, since $v_t = v_0 + \sum_{\tau=1}^t \Delta v_\tau$. However, news affects this component *only through* its effect on v_t, the speculator's forecast of the asset liquidation value. If, in addition, $\gamma_t^k \neq 0$, news received at date t affects the investor's trading strategy above and beyond its effect on the forecast. Thus, we refer to the second component of the investor's trading strategy as the *news trading* component, and we say that there is *news trading* when $\gamma_t^k \neq 0$.

[14]In the benchmark case, the dealer cannot forecast news from the past trading history since the news is uncorrelated and the speculator observes news at the same time as the dealer. Thus, $\mathsf{E}\big(\Delta z_t|\mathcal{I}_t^B\big) = 0$ in the benchmark case. In contrast, $\mathsf{E}\big(\Delta z_{t-1}|\mathcal{I}_t^F\big) \neq 0$ in the fast model because the order flow Δy_{t-1}, which is part of $\mathcal{I}_t^F$, might be correlated with Δz_{t-1}, the dealer's (lagged) news received at t. This will be the case in equilibrium.

Actually, the direction and size of news directly affects the informed investor's behavior in this case.[15] In the next section, we show that news trading arises if and only if the speculator has a speed advantage, i.e., $\gamma_t^B = 0$ whereas $\gamma_t^F > 0$.

In the rest of the paper, we focus on the continuous time version of the model. Indeed, as in Kyle (1985), the coefficients that characterize the speculator's optimal trading strategy (e.g., β_t^k and γ_t^k) and the dealer's pricing policy (e.g., λ_t^k) do not have a closed-form expression in the discrete time version. Moreover, the equilibrium obtained in continuous time is more directly comparable to other related extensions of the Kyle (1985) model, e.g., Back and Pedersen (1998) or Chau and Vayanos (2008), as these are set in continuous time. Our findings however are not specific to continuous time trading since, as shown in the Internet Appendix, the qualitative conclusions are identical in the discrete time version of the model.

For completeness, in Appendix A we formally define the continuous time equivalent of the model laid out in this section. Intuitively, one can think of the continuous time version as the case in which Δt, the interval of time between two trading rounds, becomes infinitesimal and is denoted by $\mathrm{d}t$.[16]

3 Optimal News Trading

In this section, we derive the equilibrium of the benchmark model and the fast model when news and trades take place in continuous time. In this case, $\mathrm{d}p_t$ and $\mathrm{d}q_t$ denote the increments of the processes followed by the transaction price and the dealer's quote, while $\mathrm{d}x_t$ denotes the change in the speculator's position. The next theorem provides

[15] For instance, suppose that at date t, $q_t = 100$ and that, after receiving news, the investor forecasts the asset liquidation value to be $v_t = 105$. If $\gamma_t^k = 0$, the investor will buy the asset at date t, whether he just received good or bad news at this date. In contrast, if $\gamma_t^k > 0$, the direction and size of the investor's trade at date t will depend on both the direction and size of the news. In particular, the investor may eventually sell the asset at date t if the news at this date is sufficiently bad, even though $v_t > q_t$.

[16] One must use extra care in describing the continuous time analog of equations (8)–(10), since $t - \mathrm{d}t$ is not well defined in continuous time.

a characterization of the equilibrium coefficients β_t^k, γ_t^k, λ_t^k, μ_t^k in both the benchmark and the fast models, i.e., when $k \in \{B, F\}$. In particular, it shows that there is no news trading in the benchmark case ($\gamma_t^B = 0$), while there is news trading when the speculator reacts faster to news ($\gamma_t^F > 0$). This difference implies that the speculator's trades have very different properties when he is fast and when he is not (see Section 4).

Theorem 1. *In the benchmark model there is a unique linear equilibrium, of the form*[17]

$$\mathrm{d}x_t = \beta_t^B (v_t - p_t)\mathrm{d}t + \gamma^B \mathrm{d}v_t, \tag{12}$$

$$\mathrm{d}p_t = \mu^B \mathrm{d}z_t + \lambda^B \mathrm{d}y_t, \tag{13}$$

with coefficients given by

$$\beta_t^B = \frac{1}{1-t}\,\frac{\sigma_u}{\Sigma_0^{1/2}}\left(1 + \frac{\sigma_v^2\sigma_e^2}{\Sigma_0(\sigma_v^2+\sigma_e^2)}\right)^{1/2}, \tag{14}$$

$$\gamma^B = 0, \tag{15}$$

$$\lambda^B = \frac{\Sigma_0^{1/2}}{\sigma_u}\left(1 + \frac{\sigma_v^2\sigma_e^2}{\Sigma_0(\sigma_v^2+\sigma_e^2)}\right)^{1/2}, \tag{16}$$

$$\mu^B = \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2}. \tag{17}$$

In the fast model there is a unique linear equilibrium, of the form:

$$\mathrm{d}x_t = \beta_t^F (v_t - q_t)\mathrm{d}t + \gamma^F \mathrm{d}v_t, \tag{18}$$

$$\mathrm{d}q_t = \lambda^F \mathrm{d}y_t + \mu^F(\mathrm{d}z_t - \rho^F \mathrm{d}y_t), \tag{19}$$

[17] In the benchmark model we express the forecast error component as a multiple of $v_t - p_t$, rather than of $v_t - q_t$, as in the discrete time version. We do this because in the continuous time benchmark model q_t is not a well defined Itô process (see Footnote 29 in Appendix A). Nevertheless, replacing q_t with p_t has no effect on the trading strategy $\mathrm{d}x_t$ in continuous time, since $(p_t - q_t)\,\mathrm{d}t = \lambda_t\,\mathrm{d}y_t\,\mathrm{d}t = 0$.

with coefficients given by

$$\beta_t^F = \frac{1}{1-t}\,\frac{\sigma_u}{(\Sigma_0+\sigma_v^2)^{1/2}}\,\frac{1}{\left(1+\frac{\sigma_e^2}{\sigma_v^2}g\right)^{1/2}}\left(1+\frac{(1-g)\sigma_v^2}{\Sigma_0}\frac{1+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g}{2+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g}\right), \tag{20}$$

$$\gamma^F = \frac{\sigma_u}{\sigma_v}\,g^{1/2} = \frac{\sigma_u}{(\Sigma_0+\sigma_v^2)^{1/2}}\,\frac{\left(1+\frac{\sigma_e^2}{\sigma_v^2}g\right)^{1/2}(1+g)}{2+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g}, \tag{21}$$

$$\lambda^F = \frac{(\Sigma_0+\sigma_v^2)^{1/2}}{\sigma_u}\,\frac{1}{\left(1+\frac{\sigma_e^2}{\sigma_v^2}g\right)^{1/2}(1+g)}, \tag{22}$$

$$\mu^F = \frac{1+g}{2+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g}, \tag{23}$$

$$\rho^F = \frac{\sigma_v}{\sigma_u}\frac{g^{1/2}}{1+g} = \frac{\sigma_v^2}{\sigma_u(\Sigma_0+\sigma_v^2)^{1/2}}\,\frac{(1+\frac{\sigma_e^2}{\sigma_v^2}g)^{1/2}}{2+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g}, \tag{24}$$

and g is the unique root in $(0,1)$ of the cubic equation

$$g = \frac{\left(1+\frac{\sigma_e^2}{\sigma_v^2}g\right)(1+g)^2}{\left(2+\frac{\sigma_e^2}{\sigma_v^2}+\frac{\sigma_e^2}{\sigma_v^2}g\right)^2}\,\frac{\sigma_v^2}{\sigma_v^2+\Sigma_0}. \tag{25}$$

In both models, when $\sigma_v \to 0$, the equilibrium converges to the unique linear equilibrium in the continuous time version of Kyle (1985).

We first discuss the properties of the process followed by the speculator's optimal position, x_t. In equilibrium, the drift of this process is given by the forecast error component, and its volatility is given by the news trading component. Theorem 1 yields the following corollary.

Corollary 1. *If the speculator reacts faster to news, and if $\sigma_e < +\infty$ and $\sigma_v > 0$, then there is news trading, i.e., $\gamma^F > 0$. Otherwise, there is no news trading. In particular, $\gamma^B = 0$ for all parameter values.*

The speculator's optimal trading strategy is therefore significantly different when he gets access to news faster versus when he does not. In the former case, this strategy consists of repeated small trades in the same direction for a relatively long period of time since the dealer's forecast error, $v_t - q_t$, changes slowly over time. This is still the case *on average* when the speculator reacts faster to news, but then his optimal position becomes much more volatile since $\gamma^F > 0$. Thus, over a short time interval, he trades much more and in larger sizes than when he does not react faster to news (see the next section for a formal statement). To an external observer, the speculator's holdings would appear as only being driven by incoming news when he has a speed advantage, since changes in a stochastic process over a short period of time are predominantly determined by the volatility component of this process, and not by its drift component.

This difference between the fast model and the benchmark model is crucial for all our remaining findings. The intuition for this key result is as follows. Suppose that the speculator receives good news, i.e., $\mathrm{d}v_t > 0$. He then marks up his forecast v_t of the long run value of the asset relative to the dealer's forecast. This leads the investor to buy shares of the asset whether he is slow or fast but at a very slow rate to avoid dissipating his informational advantage. This effect of good news on the investor's trade is captured by the forecast error component of his trading strategy. However, when the dealer receives news with a lag, there is a second effect: the investor expects the dealer to soon receive good news, since $\mathrm{d}z_t = \mathrm{d}v_t + \mathrm{d}e_t$, which will induce her to mark up her quote (see equations (10) and (19)). The speculator optimally exploits this foreknowledge of the short-run quote dynamics by buying shares in addition to those bought based on his update of the long-run value of the asset. This extra motive for buying shares after good news is captured by the news trading component of his strategy, which explains why $\gamma^F > 0$. In contrast, when the dealer reacts to news with no lag, she updates her quote to reflect news before the speculator can exploit his forecast of this update. For this reason, in the benchmark model $\gamma^B = 0$, even when the dealer's news is less precise (i.e., $\sigma_e > 0$).

There are two limit cases in which there is no news trading even when the speculator gets news faster than the dealer. First, there is obviously no news trading when there is no news, that is, when $\sigma_v = 0$. Second, and more interestingly, there is no news trading when σ_e becomes infinite. In this case, the dealer's quote updates become insensitive to news, because the news is completely uninformative for the dealer when σ_e tends to infinity. As a result, the speculator cannot use his news to forecast short-run quote updates, and he stops trading on news, i.e., γ^F tends to zero when σ_e becomes infinite.

Corollary 2. *For all parameter values and at each date, $\beta_t^F < \beta_t^B$.*

This result shows that the two components of the speculator's trading strategy are interdependent. Indeed, the speculator compensates his increased aggressiveness on news in the fast model by optimally trading less aggressively on the dealer's forecast error. Thus, he partially substitutes profits derived from trading on his forecast of the "long run" value of the asset by profits from trading on his foreknowledge of short run quote dynamics. As explained in Section 6, this substitution effect has an impact on the nature of price discovery.[18]

We now turn our attention to the dealer's pricing policy. As in Kyle (1985), we measure market illiquidity by λ, the immediate price impact of a trade.

Corollary 3. *Illiquidity is higher when the speculator has a speed advantage, i.e., $\lambda^F > \lambda^B$.*

When the speculator reacts faster to news, trades move prices more because they contain more information since the speculator trades on news more aggressively. Furthermore, in this case, the dealer can forecast news from past trades since these trades depend on news. Formally, in the fast model, $\mathsf{E}(\mathrm{d}z_t|\mathrm{d}y_t) = \rho^F \mathrm{d}y_t$, where ρ^F is defined in equation (24). Thus, in the fast model, the dealer's quote update depends on the innovation in news, i.e., $\mathrm{d}z_t - \rho^F \mathrm{d}y_t$, (see equation (19)), rather than the news itself,

[18] It can also be shown that β_t^B and β_t^F are increasing in σ_v and σ_u. When σ_v increases, uncertainty on the final payoff of the asset is larger for the dealer, other things equal. This is also the case when σ_u increases, because the order flow becomes noisier. In either case, the speculator optimally reacts by trading more aggressively on the dealer's forecast error. As these effects are standard, we omit the proof of these results for brevity. The effects of σ_e on β_t^k and γ^F are analyzed in Section 5.

dz_t. Moreover, as news can be anticipated from past trades, the dealer's quote updates are less sensitive to news in the fast model, as claimed in the next corollary.

Corollary 4. *Quote updates are less sensitive to news when the speculator has a speed advantage, i.e.,* $\mu^F < \mu^B$.

In Section 6, we show that this finding has important implications for the relative contributions of news and trades to volatility.

Theorem 1 holds for all parameter values, except $\Sigma_0 = 0$. As Σ_0 approaches zero, the dealer's forecast error becomes very small, at least at date $t = 0$. However, the forecast error component of the speculator's trading strategy remains finite, which implies that β_0^k approaches infinity when Σ_0 goes to zero. This precludes the existence of a linear equilibrium when $\Sigma_0 = 0$. However, Theorem 1 and all our results (which are all implications of the theorem) remain unchanged even when Σ_0 is very close to zero. When this is the case and in addition $\sigma_e = 0$, the speculator has almost no long-lived information advantage. In particular, when he is fast, the speculator anticipates that the dealer will receive the same news as he observes with a lag. Thus, the speculator cannot trade on news for long. Yet, the news trading component of his strategy remains finite and is in fact maximal (see Corollary 8 in Section 5).[19]

4 Detecting News Trading

In the previous section, we have shown that the speculator's optimal trading strategy contains a news trading component when the investor is fast but not otherwise. We now show that this feature implies that the speculator's "footprints" (the effects of his trades on volume and prices) depend on whether he trades on news or not. Identifying these footprints is useful to assess the extent to which speculators who trade on news

[19] When $\sigma_e = 0$, one can show that $\gamma^F = \frac{\sigma_u}{\sigma_v}\left(\frac{\sigma_v^2+2\Sigma_0}{\sigma_v^2} + \sqrt{\left(\frac{\sigma_v^2+2\Sigma_0}{\sigma_v^2}\right)^2 - 1}\right)$ using the expression for γ^F in Theorem 1, and solving for g in equation (25).

exploit a speed advantage or are only better at processing news.

4.1 News Trading and Volume

As explained in the previous section and shown in Figure 1, the speculator's position is much more volatile when he reacts faster to news than when he does not. Thus, the fraction of total trading volume due to the speculator's trades is much higher when he has a speed advantage. To see this formally, let the *Informed Participation Rate* (IPR_t) be the instantaneous contribution of the speculator's trade to total trading volume:

$$IPR_t = \frac{\mathsf{Var}(\mathrm{d}x_t)}{\mathsf{Var}(\mathrm{d}y_t)} = \frac{\mathsf{Var}(\mathrm{d}x_t)}{\mathsf{Var}(\mathrm{d}u_t) + \mathsf{Var}(\mathrm{d}x_t)} \tag{26}$$

Corollary 5. *The Informed Participation Rate is higher when the speculator trades on news (i.e., reacts faster to news). Specifically:*

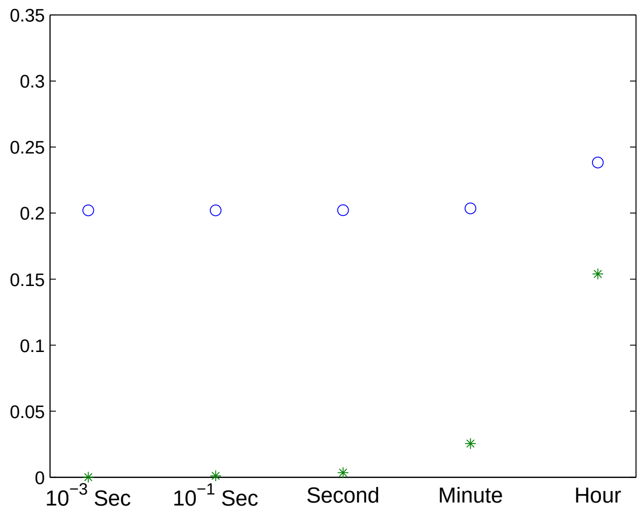
$$IPR^B = 0, \qquad IPR^F = \frac{g}{1+g} > 0, \tag{27}$$

where $g \in (0,1)$ is defined in Theorem 1.

In the benchmark model, the speculator optimally chooses to trade in very small sizes relative to noise traders in order to avoid dissipating his long-run informational advantage too quickly, as in Kyle (1985). Hence, over short time intervals, the speculator's order flow is negligible relative to noise traders' order flow. In contrast, in the fast model, the speculator's order flow over a short time interval is of the same order of magnitude as noise traders' flows because the speculator optimally trades much more aggressively on news.

The expressions for the informed participation rate in Corollary 5 are obtained when the speculator's order flow and trading volume are measured over an infinitesimal time interval. In Appendix B, we show that the Informed Participation Rate remains higher when the speculator reacts faster to news even when trades are aggregated over time

Figure 3: Informed participation rate at various sampling frequencies. The figure plots the fraction of the trading volume due to the speculator when data are sampled over time intervals of various lengths (10^{-3} seconds, 10^{-1} seconds, 1 second, 1 minute, 1 hour) in (a) the benchmark model, marked with "$*$"; and (b) the fast model, marked with "$\circ$". The parameters used for the simulation are $\sigma_u = \sigma_v = \sigma_e = \Sigma_0 = 1$ (see Theorem 1). The liquidation date $t = 1$ corresponds to 1 month.



intervals of arbitrary length. However, in this case, the Informed Participation Rate in the benchmark model is not zero and increases with the length of the time interval over which trades and volume are measured, as shown in Figure 3.

High Frequency Traders (HFTs) have been shown to account for a large fraction of the total trading volume in various financial markets around the world. For instance, Brogaard, Hendershott, and Riordan (2012) find that market orders by HFTs account for about 41% of the trading volume for the Nasdaq stocks in their sample. There are several possible explanations for HFTs' large share of trading volume. For instance, HFTs may have crowded out slow traders (as implied by Biais, Foucault, and Moinas (2013)), or they might intermediate many transactions as market makers. Our model suggests another, non exclusive, explanation: optimal trading on news at the high frequency (that is, on frequent signals) can require frequent and large adjustments in portfolio holdings when speculators have access to news slightly faster than other market participants, despite the higher impact costs associated with this strategy. Furthermore, as our model shows, this strategy can be optimal even when the speculator's informational advantage is long-lived (that is, even if $\sigma_e > 0$).

4.2 News Trading and Trade Autocorrelation

In both the fast and the benchmark model, i.e., for both $k \in \{B, F\}$, the dealer's forecast error component, $\beta_t^k (v_t - q_t)\,\mathrm{d}t$, changes slowly over time because the speculator wishes to preserve his informational advantage, and does not trade aggressively on the forecast error. As a result, the drift component of the speculator's position changes very slowly, which means that the average direction of the speculator's trade is relatively stable. This feature is a source of positive autocorrelation in the speculator's trades. However, when the speculator reacts faster to information, over a short time interval his trades are mainly driven by news. As the news is uncorrelated, they are not a source of autocorrelation in the speculator's trades. For this reason, the autocorrelation of the speculator's order flow is smaller in the fast model. In fact, over very short time intervals, this autocorrelation is zero, as the next corollary shows.

Corollary 6. *The autocorrelation of the speculator's trades over short time intervals is lower when he reacts faster to news. More specifically, for $\tau \in (0, 1-t)$,*

$$\begin{aligned} \mathsf{Corr}(\mathrm{d}x_t^B, \mathrm{d}x_{t+\tau}^B) &= \left(\frac{1-t-\tau}{1-t}\right)^{\lambda^B \beta_0^B - \frac{1}{2}} > 0, \\ \mathsf{Corr}(\mathrm{d}x_t^F, \mathrm{d}x_{t+\tau}^F) &= 0. \end{aligned} \tag{28}$$

In Appendix B, we show that the autocorrelation of the speculator's trades remains smaller in the fast model even when his trades are aggregated over non infinitesimal time intervals, but that it increases when the interval of time over which trades are aggregated gets larger. Actually, over longer time intervals, the net change in the speculator's portfolio holding becomes increasingly determined by the forecast error component of his trading strategy, which as explained before is a source of autocorrelation in the speculator's trades.

Brogaard (2011) and Hirschey (2013) find a positive but small autocorrelation in HFTs' aggregate order flow on Nasdaq. This finding is consistent both with the benchmark and the fast models. Some papers (Menkveld (2013) or Kirilenko, Kyle, Samadi, and Tuzun (2011)) find evidence of mean reverting inventories for HFTs. Mean-reverting positions should induce a negative autocorrelation in HFTs' trades. This may stem from inventory constraints, which are absent from our model. Accounting for these constraints in the speculator's optimization problem is beyond the scope of this paper, but they would naturally lead to mean reversion in the speculator's position. Alternatively, mean reversion in inventories might be characteristic of high frequency market-making, a strategy which is not captured by our model.[20]

4.3 News Trading and Price Changes

As explained in Section 3, the speculator trades more aggressively on news in the fast model because, in this case, news are informative on the short-run dynamics of quotes, in addition to the long-run liquidation value of the asset. For instance, he aggressively buys when receiving good news as he expects the dealer to soon mark up her quote. Intuitively, this behavior implies a positive relationship between the speculator's trade and subsequent price changes. To formalize this relationship, let CPI_t be the covariance between the speculator's trade per unit of time and the subsequent cumulative price change over the time interval $[t, t+\tau]$ for $\tau > 0$:

$$CPI_t(\tau) \;=\; \mathsf{Cov}\left(\frac{\mathrm{d}x_t}{\mathrm{d}t}, p_{t+\tau} - p_t\right). \tag{29}$$

This covariance can be seen as a measure of the *Cumulative Price Impact (CPI)* of the speculator's trade at a given point in time. Thus, it is a measure of trade informativeness.

[20]For instance, Menkveld (2013) shows that the high frequency trader in his dataset behaves very much as a market maker rather than as an informed investor.

Corollary 7. *In the benchmark model, the cumulative price impact is*

$$CPI_t^B(\tau) \;=\; C_1^B\left[1-\left(1-\frac{\tau}{1-t}\right)^{\lambda^B\beta_0^B}\right], \tag{30}$$

while in the fast model it is

$$CPI_t^F(\tau) \;=\; C_0^F + C_1^F\left[1-\left(1-\frac{\tau}{1-t}\right)^{(\lambda^F-\mu^F\rho^F)\beta_0^F}\right], \tag{31}$$

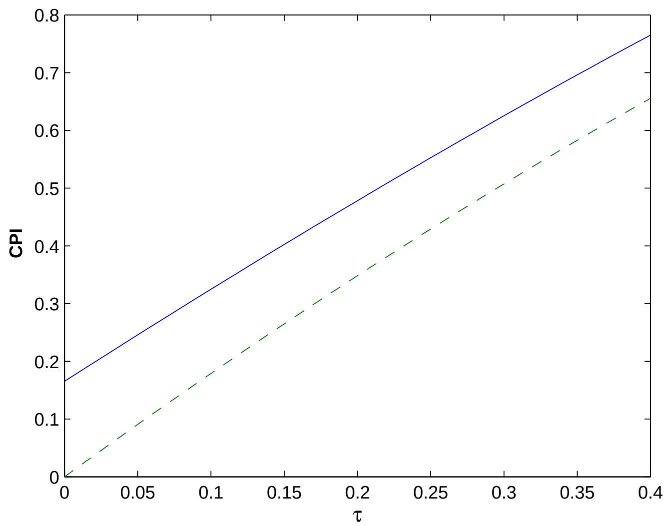
where C_0^F, C_1^B and C_1^F are positive coefficients given in the proof of this corollary.

Figure 4 illustrates the corollary for specific values of the parameters. As expected, the covariance between the speculator's trade and the subsequent cumulative price change is positive, and at short horizons (small τ), it is much larger when the speculator reacts faster to news. This difference shrinks as one measure price changes over longer horizons. Actually, as explained in Section 4.2, the average direction of the speculator's trades is relatively stable over time, in both the fast and the benchmark models. Thus, a speculator's buy (sell) order is followed by additional buy (sell) orders on average, which implies that the immediate price change associated with a trade by the speculator is followed by additional price changes in the same direction, whether he is fast or not.

As Figure 4 shows, measuring CPI for various values of τ could be useful empirically to assess the relative importance of news trading in a speculator's trading strategy. Indeed, a large value of CPI over short horizons (a positive "intercept") is indicative of news trading, since $CPI_t^F(\tau) \approx C_0^F > 0$, while $CPI_t^B(\tau) \approx 0$ for τ small. In contrast, the rate at which CPI increases with τ (the "slope") indicates the magnitude of the forecast error component, i.e., the rate at which the speculator slowly exploits the dealer's forecast error. We are not aware of empirical papers on HFTs reporting CPI.[21]

[21] In their Figure 1, Brogaard, Hendershott, and Riordan (2012) plot the correlation coefficients between aggressive order imbalances of HFTs in their sample and subsequent returns over a 1 second interval. However, they do not show the correlation coefficients between HFTs' order imbalances and

Figure 4: Cumulative Price Impact at Different Horizons. The figure plots the cumulative price impact at $t = 0$, $\mathsf{Cov}\left(\frac{\mathrm{d}x_0}{\mathrm{d}t}, p_\tau - p_0\right)$ against the horizon $\tau \in (0, 1]$ in (a) the benchmark model, with a dotted line; and (b) the fast model, with a solid line. The parameters used are $\sigma_u = \sigma_v = \sigma_e = \Sigma_0 = 1$ (see Theorem 1). The liquidation date $t = 1$ corresponds to 1 month.



However, Brogaard, Hendershott, and Riordan (2012) and Kirilenko, Kyle, Samadi, and Tuzun (2011) find that aggressive orders (that is, marketable orders) by HFTs have a positive correlation with subsequent returns over *a very short horizon*. They interpret this finding as reflecting HFTs' ability to anticipate short-term price movements, which is indeed the source of the correlation between the speculator's trade and short-term price changes in the fast model.

5 News Informativeness, Volume, and Liquidity

We measure the informativeness of news for the dealer by $\nu = \frac{1}{\sigma_e}$, since a smaller σ_e means that the news received by the dealer provides a more precise signal about innovations in the asset value. News vendors (Reuters, Bloomberg, or Dow Jones) now report firm-specific news in real time, assigning a direction and a relevance score to each news (see, for instance, Gross-Klussmann and Hautsch (2011)). One could take as a

subsequent *cumulative returns*.

proxy for ν the average news relevance score provided by these vendors for a firm or a portfolio of firms. Indeed, firms with more relevant news should be firms for which dealers receive more informative signals.[22] More generally, recent advances in textual analysis offer ways to measure newswires informativeness (see, for instance, Boudoukh, Feldman, Kogan, and Richardson (2013)).

Corollary 8. *When the speculator reacts faster to news, the rate γ^F at which he trades on news increases with the news informativeness, i.e., $\frac{\partial \gamma^F}{\partial \nu} > 0$. In contrast, in both the benchmark and the fast models, the speculator trades less aggressively on the dealer's forecast error when the news informativeness increases, i.e., $\frac{\partial \beta^k}{\partial \nu} < 0$ for $k \in \{F, B\}$. If the dealer receives uninformative news ($\nu = 0$), then there is no news trading ($\gamma^F = 0$), and $\beta^F = \beta^B$.*

In the fast model, an increase in news informativeness for the dealer strengthens the speculator's incentive to trade on news for two reasons. First, the speculator can better forecast the dealer's news (since $\mathsf{Var}(\mathrm{d}z_t|\mathrm{d}v_t) = \sigma_e^2$). Second, the dealer's quote update becomes more sensitive to news, other things equal, i.e., μ^k increases with ν. Thus, an increase in news informativeness for the dealer enables the speculator to better forecast short-run quote updates, so that he trades more aggressively on this knowledge in the fast model. This complementarity between the informed investor's trading intensity and the precision of signals received by less informed agents is absent from standard models of informed trading.

In contrast, the speculator trades less aggressively on the dealer's forecast error when the dealer receives more informative news. Actually, as explained previously, the speculator substitutes part of his profits from trading on the dealer's forecast error with profits from trading on news. Moreover, as the dealer receives more informative news, she corrects more rapidly her forecast error, inducing the speculator to trade even less

[22] High frequency trading firms are less likely to rely on relevance scores provided by data vendors, as these are provided with a delay relative to the source news.

aggressively on this error. The first effect operates only when the speculator trade on news, that is, in the fast model. The second effect is standard in models of informed trading, and operates even in the benchmark case, which explains why β^B also declines with ν.

In the limit when the dealer's news is uninformative ($\sigma_e = +\infty$), the model is formally equivalent to the case in which the dealer never receives news, as in Back and Pedersen (1998). In this case, the equilibrium is the same in the benchmark and the fast models, and is identical to that derived in Back and Pedersen (1998). In particular, even if the speculator receives news faster than the dealer, there is no news trading, i.e., γ^F converges to zero when σ_e approaches $+\infty$.

Corollary 9. *In the fast model, an increase in the dealer's news informativeness triggers an increase in (i) the speculator's participation rate ($\frac{\partial IPR^F}{\partial \nu} > 0$), (ii) trading volume ($\frac{\partial \mathsf{Var}(dy)}{\partial \nu} > 0$), and (iii) liquidity ($\frac{\partial \lambda^F}{\partial \nu} < 0$).*

When news informativeness for the dealer increases, the speculator trades more aggressively on news, as shown by Corollary 8. As a result, trading volume increases and the speculator accounts for a larger share of this trading volume. Usually, increased informed trading leads to a less liquid market. This is not the case here. Indeed, as the dealer gets more precise news, she can better forecast the asset payoff and she is therefore less exposed to adverse selection. Hence, in equilibrium, when news informativeness for the dealer increases, the model implies a joint increase in both informed trading and liquidity.

These testable implications of the fast model are in sharp contrast with other models analyzing the effects of public information. These models usually imply that an increase in the precision of public signals for dealers is associated with a *lower* trading volume, as informed investors trade less, and greater market liquidity, as dealers are less exposed to adverse selection (see, for instance, Propositions 1 and 2 in Kim and Verrechia (1994)). The first implication holds in the benchmark model, but not in the fast model.[23]. Corollary 9 also implies that controlling for the precision of dealer's news is important to analyze the effect of news trading on liquidity. Indeed, in our model, variations in the precision of news lead to a positive association between liquidity and news trading. However, this correlation does not mean that news trading causes the market to be more liquid. Instead, as Corollary 3 shows, the opposite is true.

Interestingly, Kelley and Tetlock (2013) find that trading volume and liquidity for a stock are substantially larger on days with Dow Jones news for this stock than on days without DJ news (see their Table II). This fact is consistent with our prediction if one interprets days without DJ news as days in which news for a stock are less precise than on days with DJ news. The model also predicts that the fraction of trading volume due to informed trading should be higher on days with news. These predictions regarding the effects of news informativeness cannot be easily obtained in models with short-lived information such as Admati and Pfleiderer (1988) since these models assume that news are observed perfectly by dealers after one period, that is, they implicitly focus on the case $\sigma_e = 0$.[24]

6 Price Discovery and Volatility

As explained in the introduction, technological advances have enabled some investors to react faster to news in recent years. In this section, we use our model to study how this evolution could affect price discovery and the sources of price volatility. As in Kyle

[23]In the benchmark case, a decrease in σ_e (increase in ν) generates a *decrease* in β^B *and* λ^B (see equations (14) and (16)). A decrease in β^B implies that the speculator trades less over a given time interval. Thus, in the benchmark case, an increase in news informativeness generates an increase in liquidity, but a decrease in volume.

[24]Variations in liquidity trading between days with and without news may explain why volume and liquidity are higher on days with news. Specifically, if liquidity traders trade more on days with news then liquidity and volume might be higher on these days, as implied by Admati and Pfleiderer (1988). In contrast, our predictions do not rely on systematic variations in liquidity trading according to news informativeness.

(1985), we measure price discovery by the average squared pricing error at t, i.e.,

$$\Sigma_t^k \;=\; \mathsf{E}\big((v_t - p_t^k)^2\big), \qquad k \in \{B, F\}, \tag{32}$$

where $p_t^B = p_t$ in the benchmark model and $p_t^F = q_t$ in the fast model.[25] The smaller is Σ_t^k, the higher is informational efficiency at date t. If $\Sigma_t^k = 0$, the market is strong-form efficient at date t: the price at this date is just equal to the speculator's forecast of the asset payoff. The next result shows that informational efficiency is identical in both the fast and the benchmark models, but the *nature* of price discovery is not.

Corollary 10. *For $k \in \{B, F\}$, the change in Σ_t^k is given by*

$$\mathrm{d}\Sigma_t^k \;=\; -2\,\mathsf{Cov}(\mathrm{d}p_t^k, v_t - p_t^k) - 2\,\mathsf{Cov}(\mathrm{d}p_t^k, \mathrm{d}v_t) + (2\sigma_v^2 + \Sigma_0)\mathrm{d}t. \tag{33}$$

When the speculator reacts faster to news, short run changes in prices are more correlated with innovations in the asset value (i.e., $\mathsf{Cov}(\mathrm{d}p_t^k, \mathrm{d}v_t)$ is higher when $k = F$), but less correlated with the dealer's forecast error (i.e., $\mathsf{Cov}(\mathrm{d}p_t^k, v_t - p_t^k)$ is smaller when $k = F$). Overall, $\mathrm{d}\Sigma_t^k$ is identical whether or not the speculator has a speed advantage.

According to equation (33), prices become more quickly strong-form efficient when (i) prices impound the speculator's news more swiftly ($\mathsf{Cov}(\mathrm{d}p_t^k, \mathrm{d}v_t)$ increases), and (ii) the dealer reduces her forecast error more rapidly ($\mathsf{Cov}(\mathrm{d}p_t^k, v_t - p_t)$ increases). Corollary 10 shows that news trading affects these two determinants of price discovery in opposite ways. On the one hand, the speculator trades more aggressively on news when he reacts faster to news, so that the news is more quickly reflected into prices. On the other hand, due to the substitution effect, he trades less aggressively on the dealer's forecast error ($\beta_t^F < \beta_t^B$), as shown in Corollary 2.

[25] This definition guarantees that p_t^k is a well defined Itô process in both the benchmark and the fast model; see Footnote 29 in Appendix A. This is useful for calculations, but innocuous since in both models the difference $p_t - q_t$ is infinitesimal.

In equilibrium, these two effects exactly offset each other, so that the rate at which the market eventually becomes strong-form efficient is the same whether or not the speculator reacts faster to news. As $\Sigma_0^k = \Sigma_0$, the last part of Corollary 10 implies that informational efficiency is the same in both the benchmark and the fast models, i.e., $\Sigma_t^B = \Sigma_t^F$ at all dates t. Hence, news trading does not affect the speed of price discovery. However, in the fast model, price changes are more correlated with news, and less with the dealer's forecast error. Thus, the model predicts that short-term returns (dp_t^k) should become more correlated with news dv_t after shocks enabling some investors to get access to news faster than other investors. As explained in Section 5, techniques from textual analysis could be used to develop a proxy for news (dv_t) and test this prediction.

There is another way in which the nature of price discovery is affected: fast access to news alters the relative contributions of trades and quotes to the overall price variance. To show this, we decompose price variance into two components: (i) the "trade component" that captures the effect of trades on the dealer's forecast of the asset payoff, and (ii) the "news component" that captures the effect of dealer's news on this forecast. Specifically, let σ_p be the instantaneous volatility of the price process p_t^k, where, as explained previously, $p_t^B = p_t$ and $p_t^F = q_t$. We have:

$$\sigma_p^2 \mathrm{d}t \;=\; \mathsf{Var}(\mathrm{d}p_t^k) \;=\; \underbrace{\mathsf{Var}(\mathrm{d}p_{\text{trades},\,t}^k)}_{\text{Trade Component}} + \underbrace{\mathsf{Var}(\mathrm{d}p_{\text{quotes},\,t}^k)}_{\text{News Component}}, \tag{34}$$

where $\mathsf{Var}(\mathrm{d}p_{\text{trades},\,t}^k) = \mathsf{Var}(\lambda^k \mathrm{d}y_t)$, $\mathsf{Var}(\mathrm{d}p_{\text{quotes},\,t}^B) = \mathsf{Var}(\mu^B \mathrm{d}z_t)$, and $\mathsf{Var}(\mathrm{d}p_{\text{quotes},\,t}^F) = \mathsf{Var}\big(\mu^F(\mathrm{d}z_t - \rho^F \mathrm{d}y_t)\big)$.

Corollary 11. *Whether the speculator has a speed advantage or not, the instantaneous volatility of prices is constant, and equal to*

$$\sigma_p^2 \;=\; \sigma_v^2 + \Sigma_0. \tag{35}$$

However, trades contribute to a larger fraction of this volatility when the speculator reacts faster to news.

The volatility of price changes is independent of whether the speculator has a speed advantage or not in getting access to news. However, this speed advantage alters the relative contributions of trades and quote updates to volatility. In the fast model, trades contribute more to volatility since trades are more informative than in the benchmark case (see Section 4.3). The flip side is that the dealer's quote is less sensitive to news, as explained in Section 3 (see Corollary 4). Thus, the contribution of quote revisions to return volatility is lower in the fast model.

Hasbrouck (1991) shows how to estimate the relative contributions of trades and public information to the volatility of the random walk component of prices. Using this approach, one could test Corollary 11 by using exogenous shocks to the speed at which speculators can get access to news. For instance, one could use the first availability of co-location facilities in a country as an instrument for the speed of access to information, as in Boehmer, Fong, and Wu (2012). Corollaries 10 and 11 imply that price changes should be more correlated with news after the introduction of co-location and that trades should contribute relatively more to volatility after this introduction. That is, the introduction of co-location should strengthen the role of trades in impounding news into prices.

7 Conclusion

In this paper, we have compared the optimal trading strategy of an informed investor (the speculator) when he observes news either at the same speed as the dealers or faster than the dealers, holding constant the precision of the signals conveyed by the news. Our main result is that the speculator's optimal trading strategy is very different in each case.

When the investor gets news at the same speed as dealers, his trades are completely determined by the difference between his estimate of the asset payoff and dealers' estimate of this payoff ("dealers' forecast error"), as is typical in models of informed trading. News does not affect the speculator's trade over and above its effect on dealers' forecast error, even if the speculator extracts more precise signals from news than dealers. In contrast, news becomes a distinct determinant of the investor's trading strategy and the speculator's trades are much more sensitive to news when the speculator gets advanced access to news. For instance, in this case, the investor may optimally sell the asset after bad news, even though his forecast of the asset payoff after receiving the news still exceeds the dealers' forecast.

We have shown that the behavior of the speculator when he has advanced access to news matches well some stylized facts about high frequency traders. Moreover, the model yields several testable implications that one could use to test whether speed is a determinant of news traders' profitability. First, an increase in news informativeness should lead to a joint increase in liquidity, volume, and the contribution of informed trading to volume when some investors get faster access to news. Second, differential speed of access to news should strengthen (a) the correlation between price changes and news, and (b) the contribution of trades to volatility. Recent advances in textual analysis, combined with richer news data and technological changes in the dissemination of news, offer opportunities to test these predictions.

A Proofs of Results

Before proving Theorem 1, we define the information sets of the market participants. In the benchmark model, define $\mathcal{I}_t^q = \{z_\tau\}_{\tau \leq t} \cup \{y_\tau\}_{\tau < t}$ the dealer's information set just before trading at t; $\mathcal{I}_t^p = \{z_\tau\}_{\tau \leq t} \cup \{y_\tau\}_{\tau \leq t}$ the dealer's information set just after trading at t; and $\mathcal{J}_t^q = \mathcal{I}_t^q \cup \{v_\tau\}_{\tau \leq t}$ the speculator's information set just before trading at t.

In the fast model, define $\mathcal{I}_t^q = \{z_\tau\}_{\tau<t} \cup \{y_\tau\}_{\tau<t}$ the dealer's information set just before trading at t; $\mathcal{I}_t^p = \{z_\tau\}_{\tau<t} \cup \{y_\tau\}_{\tau\le t}$ the dealer's information set just after trading at t; and $\mathcal{J}_t^q = \mathcal{I}_t^q \cup \{v_\tau\}_{\tau\le t}$ the speculator's information set just before trading at t.

In both models, the dealer sets the quote q_t and the trading price p_t as follows:

$$\begin{aligned} q_t &= \mathsf{E}(v_1 | \mathcal{I}_{t+\mathrm{d}t}^q), \\ p_t &= \mathsf{E}(v_1 | \mathcal{I}_t^p). \end{aligned} \tag{A.1}$$

Then, q_t represents the quote just before the dealer receives the order flow $\mathrm{d}y_t = \mathrm{d}x_t + \mathrm{d}u_t$, and $p_{t+\mathrm{d}t}$ is the price at which this order flow is executed.[26]

A.1 Proof of Theorem 1

Benchmark model: First, we compute the optimal trading strategy of the speculator from the set of strategies of the form $\mathrm{d}x_\tau = \beta_\tau^B(v_\tau - p_\tau)\,\mathrm{d}\tau + \gamma_\tau^B\,\mathrm{d}v_\tau$, $\tau \ge t$, while taking as given the dealer's pricing rule $\mathrm{d}p_\tau = \lambda_\tau^B \mathrm{d}y_\tau + \mu_\tau^B \mathrm{d}z_\tau$. For $t \in [0,1)$, the speculator's expected profit is

$$\pi_t = \mathsf{E}\left(\int_t^1 (v_1 - p_{\tau+\mathrm{d}\tau})\,\mathrm{d}x_\tau \;\middle|\; \mathcal{J}_t^q \right). \tag{A.2}$$

For convenience, we now omit the superscript B for the coefficients β, γ, μ, λ. To simplify the formula for the speculator's expected profit, for $\tau \ge t$ denote by[27]

$$V_\tau = \mathsf{E}\big((v_\tau - p_\tau)^2 \mid \mathcal{J}_t^q \big). \tag{A.3}$$

By the law of iterated expectations, we can replace v_1 in (A.2) by $v_{\tau+\mathrm{d}\tau} = v_\tau + \mathrm{d}v_\tau$. Also, $p_{\tau+\mathrm{d}\tau} = p_\tau + \mu_\tau(\mathrm{d}v_\tau + \mathrm{d}e_\tau) + \lambda_\tau(\mathrm{d}x_\tau + \mathrm{d}u_\tau)$, and $\mathrm{d}x_\tau = \beta_\tau(v_\tau - p_\tau)\,\mathrm{d}\tau + \gamma_\tau\,\mathrm{d}v_\tau$,

[26] Note that, compared to the discrete time model in Section 2, the index of q_t is shifted by the time increment $\mathrm{d}t$. This is done to ensure that q_t is a well defined Itô process in the fast model.

[27] This can be written $V_{t,\tau}$ to indicate dependence on t, but for simplicity we only write V_τ.

hence:

$$
\begin{aligned}
\pi_t \;&=\; \mathsf{E}\left(\int_t^1 \Big(v_\tau - p_\tau \;+\; (1-\mu_\tau)\,\mathrm{d}v_\tau \;-\; \lambda_\tau\,\mathrm{d}x_\tau\Big)\,\mathrm{d}x_\tau \;\Bigg|\; \mathcal{J}_t^q\right) \\
&=\; \int_t^1 \Big(\beta_\tau V_\tau + (1-\mu_\tau-\lambda_\tau\gamma_\tau)\gamma_\tau\sigma_v^2\Big)\,\mathrm{d}\tau.
\end{aligned}
\tag{A.4}
$$

V_τ can be computed recursively:

$$
\begin{aligned}
V_{\tau+\,\mathrm{d}\tau} \;&=\; \mathsf{E}\Big((v_{\tau+\,\mathrm{d}\tau} - p_{\tau+\,\mathrm{d}\tau})^2 \mid \mathcal{J}_t^q\Big) \\
&=\; \mathsf{E}\Big((v_\tau +\, \mathrm{d}v_\tau - p_\tau - \mu_\tau\,\mathrm{d}v_\tau - \mu_\tau\,\mathrm{d}e_\tau - \lambda_\tau\,\mathrm{d}x_\tau - \lambda_\tau\,\mathrm{d}u_\tau)^2 \mid \mathcal{J}_t^q\Big) \\
&=\; V_\tau + (1-\mu_\tau-\lambda_\tau\gamma_\tau)^2\sigma_v^2\,\mathrm{d}\tau + \mu_\tau^2\sigma_e^2\,\mathrm{d}\tau + \lambda_\tau^2\sigma_u^2\,\mathrm{d}\tau - 2\lambda_\tau\beta_\tau V_\tau\,\mathrm{d}\tau.
\end{aligned}
\tag{A.5}
$$

therefore V_τ satisfies the first-order linear ODE:

$$
V_\tau' \;=\; -2\lambda_\tau\beta_\tau V_\tau + (1-\mu_\tau-\lambda_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + \lambda_\tau^2\sigma_u^2, \tag{A.6}
$$

or equivalently $\beta_\tau V_\tau = \frac{-V_\tau' + (1-\mu_\tau-\lambda_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + \lambda_\tau^2\sigma_u^2}{2\lambda_\tau}$. Substitute this into (A.4), and integrate by parts:

$$
\begin{aligned}
\pi_t \;=\; &-\frac{V_1}{2\lambda_1} + \frac{V_t}{2\lambda_t} + \int_t^1 V_\tau\Big(\frac{1}{2\lambda_\tau}\Big)'\,\mathrm{d}\tau \\
&+ \int_t^1 \left(\frac{(1-\mu_\tau-\lambda_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + \lambda_\tau^2\sigma_u^2}{2\lambda_\tau} + (1-\mu_\tau-\lambda_\tau\gamma_\tau)\gamma_\tau\sigma_v^2\right)\,\mathrm{d}\tau.
\end{aligned}
\tag{A.7}
$$

This is essentially the method of Kyle (1985): we have eliminated the choice variable β_τ and replaced it by V_τ. Since $V_\tau > 0$ can be arbitrarily chosen, in order to get an optimum we must have $\big(\frac{1}{2\lambda_\tau}\big)' = 0$, which is equivalent to

$$
\lambda_\tau \;=\; \text{constant} \;=\; \lambda. \tag{A.8}
$$

For a maximum, the transversality condition

$$V_1 \;=\; 0 \tag{A.9}$$

must be also satisfied.

We now turn to the choice of γ_τ. The first order condition with respect to γ_τ in (A.7) is

$$-(1-\mu_\tau-\lambda_\tau\gamma_\tau)+(1-\mu_\tau-\lambda_\tau\gamma_\tau)-\lambda_\tau\gamma_\tau \;=\; 0 \quad\Longrightarrow\quad \gamma_\tau \;=\; 0. \tag{A.10}$$

Thus, there is no news trading in the benchmark model. Note also that the second order condition is $\lambda_\tau > 0$.[28]

Next, we derive the pricing rules from the dealer's zero profit conditions. From (A.1), equations $p_t = \mathsf{E}(v_1|\mathcal{I}_t^p)$ and $q_t = \mathsf{E}(v_1|\mathcal{I}_{t+\mathrm{d}t}^q) = \mathsf{E}(v_1|\mathcal{I}_t^p, \mathrm{d}z_t)$ imply $q_t = p_t + \mu_t\,\mathrm{d}z_t$, where

$$\mu_t \;=\; \frac{\mathsf{Cov}(v_1,\ \mathrm{d}z_t \mid \mathcal{I}_t^p)}{\mathsf{Var}(\ \mathrm{d}z_t \mid \mathcal{I}_t^p)} \;=\; \frac{\mathsf{Cov}\big(v_0+\int_0^1\ \mathrm{d}v_\tau,\ \mathrm{d}v_t+\ \mathrm{d}e_t \mid \mathcal{I}_t^p\big)}{\mathsf{Var}(\ \mathrm{d}v_t+\ \mathrm{d}e_t \mid \mathcal{I}_t^p)} \;=\; \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2} \;=\; \mu. \tag{A.11}$$

Also, equations $q_t = \mathsf{E}(v_1|\mathcal{I}_{t+\mathrm{d}t}^q)$ and $p_{t+\mathrm{d}t} = \mathsf{E}(v_1|\mathcal{I}_{t+\mathrm{d}t}^q, \mathrm{d}y_t)$ imply that $p_{t+\mathrm{d}t} = q_t + \lambda_t \mathrm{d}y_t$, and also $\mathrm{d}p_t = \mu_t^B \mathrm{d}z_t + \lambda_t^B \mathrm{d}y_t$, which proves (13). Furthermore, since $\lambda_t = \lambda$ is constant,

$$\lambda \;=\; \frac{\mathsf{Cov}(v_1,\ \mathrm{d}y_t \mid \mathcal{I}_{t+\mathrm{d}t}^q)}{\mathsf{Var}(\ \mathrm{d}y_t \mid \mathcal{I}_{t+\mathrm{d}t}^q)} \;=\; \frac{\mathsf{Cov}(v_1, \beta_t(v_t-p_t)\,\mathrm{d}t+\ \mathrm{d}u_t \mid \mathcal{I}_{t+\mathrm{d}t}^q)}{\mathsf{Var}(\beta_t(v_t-p_t)\,\mathrm{d}t+\ \mathrm{d}u_t \mid \mathcal{I}_{t+\mathrm{d}t}^q)} \;=\; \frac{\beta_t\Sigma_t}{\sigma_u^2}, \tag{A.12}$$

where $\Sigma_t = \mathsf{E}\big((v_t-p_t)^2|\mathcal{I}_t^p\big) = \mathsf{E}\big((v_t-p_t)^2\big)$. As in the derivation of (A.6), it is straightforward to check that Σ_t satisfies the first-order linear ODE:

$$\Sigma_t' \;=\; -2\lambda\beta_t\Sigma_t + (1-\mu)^2\sigma_v^2 + \mu^2\sigma_e^2 + \lambda^2\sigma_u^2. \tag{A.13}$$

This is the same ODE as (A.6), except in that case $\tau \in [t,1]$ and the initial condition is

[28] The condition $\lambda_\tau > 0$ is also a second order condition with respect to the choice of β_τ. To see this, suppose $\lambda_\tau < 0$. Then if $\beta_\tau > 0$ is chosen very large, equation (A.6) shows that V_τ is very large as well, and thus $\beta_\tau V_\tau$ can be made arbitrarily large. Thus, there would be no maximum.

$V_t = (v_t - p_t)^2$; instead, equation (A.13) is defined for $t \in [0,1]$ and has initial condition Σ_0. By solving explicitly (A.6) and (A.13), one sees that the transversality condition $V_1 = 0$ is equivalent to $\int_t^1 \beta_\tau \, \mathrm{d}\tau = +\infty$, and in turn this is equivalent to $\Sigma_1 = 0$.

Since λ is constant, equation (A.12) implies that $\beta_t \Sigma_t = \lambda \sigma_u^2$ is constant. Equation (A.13) then implies that Σ_t' is constant. From $\Sigma_1 = 0$, we get $\Sigma_t = (1-t)\Sigma_0$, and $\beta_t = \frac{\beta_0}{1-t}$. Then, (A.13) becomes $-\Sigma_0 = -2\lambda^2\sigma_u^2 + (1-\mu)^2\sigma_v^2 + \mu^2\sigma_e^2 + \lambda^2\sigma_u^2$. Since $\mu = \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2}$, we get $\lambda^2\sigma_u^2 = \Sigma_0 + \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2}$, which implies (16). Then, $\beta_0 = \frac{\lambda\sigma_u^2}{\Sigma_0}$ and $\beta_t = \frac{\beta_0}{1-t}$ imply (14).

Fast model: As for the benchmark model, we compute the optimal trading strategy of the speculator, while taking as given the dealer's pricing rules $\mathrm{d}q_\tau = \lambda_\tau^F \mathrm{d}y_\tau + \mu_\tau^F(\mathrm{d}z_\tau - \rho_\tau^F \mathrm{d}y_\tau)$ and $p_{\tau+\mathrm{d}\tau} = q_\tau + \lambda_\tau^F \mathrm{d}y_\tau$. The speculator has the same objective function as in (A.2), but his trading strategy uses the quote q_τ as a state variable, i.e., he chooses among strategies of the form $\mathrm{d}x_\tau = \beta_\tau^F(v_\tau - q_\tau)\,\mathrm{d}\tau + \gamma_\tau^F\,\mathrm{d}v_\tau$.[29]

For convenience, we now omit the superscript F for the coefficients β, γ, μ, λ, ρ, l. Denote by

$$V_\tau \;=\; \mathsf{E}\big((v_\tau - q_\tau)^2 \mid \mathcal{J}_t^q\big). \tag{A.14}$$

As in the case of the benchmark model, we replace v_1 by $v_{\tau+\mathrm{d}\tau} = v_\tau + \mathrm{d}v_\tau$. Also, $p_{\tau+\mathrm{d}\tau} = q_\tau + \lambda_\tau(\mathrm{d}x_\tau + \mathrm{d}u_\tau)$, and $\mathrm{d}x_\tau = \beta_\tau(v_\tau - q_\tau)\,\mathrm{d}\tau + \gamma_\tau\,\mathrm{d}v_\tau$. Hence:

$$\begin{aligned}\pi_t \;&=\; \mathsf{E}\left(\int_t^1 \Big(v_\tau - q_\tau \;+\; \mathrm{d}v_\tau \;-\; \lambda_\tau\,\mathrm{d}x_\tau\Big)\,\mathrm{d}x_\tau \;\middle|\; \mathcal{J}_t^q\right)\\ &=\; \int_t^1 \Big(\beta_\tau V_\tau + (1-\lambda_\tau\gamma_\tau)\gamma_\tau\sigma_v^2\Big)\,\mathrm{d}\tau.\end{aligned} \tag{A.15}$$

By comparing the first equation in (A.15) with the first equation in (A.4), we observe a key difference between the benchmark and the fast model. Indeed, in the benchmark model, an extra $(\mu_\tau \mathrm{d}v_\tau)\,\mathrm{d}x_\tau$ is subtracted from the speculator's objective function. This

[29] This is because p_τ is not a well defined Itô process. Indeed, $\mathrm{d}p_\tau = p_{\tau+\mathrm{d}\tau} - p_\tau = q_\tau - p_\tau + \lambda_\tau^F \mathrm{d}y_\tau$, and $q_\tau - p_\tau$ depends on the lagged signal $\mathrm{d}z_{\tau-\mathrm{d}\tau}$, which is not a well defined Itô increment.

term comes from the dealer's quote adjustment by (the unpredictable part of) $\mu_\tau \mathrm{d}z_\tau$, which in the benchmark model is included in the price paid by the speculator. This lowers the benefit of news trading in the benchmark model compared to the fast model. Since, as we have already proved, the optimal news trading is zero in the benchmark, it is reasonable to expect that there is positive news trading in the fast model. Indeed, we will prove that, in the fast model, $\gamma_\tau > 0$.

To obtain the equation for V_τ, we proceed as in the benchmark model, except that we replace λ_τ by l_τ:

$$\begin{aligned} V_{\tau+\mathrm{d}\tau} &= \mathsf{E}\big((v_{\tau+\mathrm{d}\tau} - q_{\tau+\mathrm{d}\tau})^2 \mid \mathcal{J}_t^q\big) \\ &= V_\tau + (1-\mu_\tau - l_\tau\gamma_\tau)^2\sigma_v^2\,\mathrm{d}\tau + \mu_\tau^2\sigma_e^2\,\mathrm{d}\tau + l_\tau^2\sigma_u^2\,\mathrm{d}\tau - 2l_\tau\beta_\tau V_\tau\,\mathrm{d}\tau, \end{aligned} \tag{A.16}$$

hence V_τ satisfies the first-order linear ODE:

$$V_\tau' = -2l_\tau\beta_\tau V_\tau + (1-\mu_\tau - l_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + l_\tau^2\sigma_u^2, \tag{A.17}$$

or equivalently $\beta_\tau V_\tau = \frac{-V_\tau' + (1-\mu_\tau - l_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + l_\tau^2\sigma_u^2}{2l_\tau}$. Substitute this into (A.15), and integrate by parts:

$$\begin{aligned} \pi_t &= -\frac{V_1}{2l_1} + \frac{V_t}{2l_t} + \int_t^1 V_\tau\Big(\frac{1}{2l_\tau}\Big)'\,\mathrm{d}\tau \\ &\quad + \int_t^1 \bigg(\frac{(1-\mu_\tau - l_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + l_\tau^2\sigma_u^2}{2l_\tau} + (1-\lambda_\tau\gamma_\tau)\gamma_\tau\sigma_v^2\bigg)\,\mathrm{d}\tau. \end{aligned} \tag{A.18}$$

Since $V_\tau > 0$ can be arbitrarily chosen, in order to get an optimum we must have $\big(\frac{1}{2l_\tau}\big)' = 0$, which is equivalent to $l_\tau =$ constant. For a maximum, the transversality condition $V_1 = 0$ must be also satisfied.

We now turn to the choice of γ_τ. The first order condition with respect to γ_τ in (A.18) is

$$-(1-\mu_\tau - l_\tau\gamma_\tau) + (1-\lambda_\tau\gamma_\tau) - \lambda_\tau\gamma_\tau \;=\; 0 \qquad \Longrightarrow \qquad \gamma_\tau \;=\; \frac{\mu_\tau}{2\lambda_\tau - l_\tau} \;=\; \frac{\mu_\tau}{\lambda_\tau + \mu_\tau\rho_\tau}. \tag{A.19}$$

Thus, we obtain a nonzero news trading component. The second order condition is $\lambda_\tau + \mu_\tau\rho_\tau > 0$. There is also a second order condition with respect to β_τ, i.e., $l_\tau > 0$; see Footnote 28.

Next, we derive the pricing rules from the dealer's zero profit conditions. As for the benchmark model, equation (A.1) implies $p_{t+\mathrm{d}t} = q_t + \lambda_t \mathrm{d}y_t$; it also implies $q_{t+\mathrm{d}t} = \mathsf{E}(v_1|\mathcal{I}^p_{t+\mathrm{d}t}, \mathrm{d}z_t) = p_{t+\mathrm{d}t} + \mu_t(\mathrm{d}z_t - \rho_t \mathrm{d}y_t)$, hence $q_{t+\mathrm{d}t} = q_t + \lambda_t \mathrm{d}y_t + \mu_t(\mathrm{d}z_t - \rho_t\mathrm{d}y_t)$, which proves (19). The coefficients are given by

$$\begin{aligned}
\lambda_t \;&=\; \frac{\mathsf{Cov}_t(v_1,\, \mathrm{d}y_t)}{\mathsf{Var}_t(\,\mathrm{d}y_t)} \;=\; \frac{\mathsf{Cov}_t(v_1, \beta_t(v_t - p_t)\,\mathrm{d}t + \gamma_t\,\mathrm{d}v_t + \,\mathrm{d}u_t)}{\mathsf{Var}(\beta_t(v_t-p_t)\,\mathrm{d}t + \gamma_t\,\mathrm{d}v_t + \,\mathrm{d}u_t)} \;=\; \frac{\beta_t\Sigma_t + \gamma_t\sigma_v^2}{\gamma_t^2\sigma_v^2 + \sigma_u^2},\\
\rho_t \;&=\; \frac{\mathsf{Cov}_t(\,\mathrm{d}z_t,\, \mathrm{d}y_t)}{\mathsf{Var}_t(\,\mathrm{d}y_t)} \;=\; \frac{\gamma_t\sigma_v^2}{\gamma_t^2\sigma_v^2+\sigma_u^2},\\
\mu_t \;&=\; \frac{\mathsf{Cov}_t(v_1,\, \mathrm{d}z_t - \rho_t\,\mathrm{d}y_t)}{\mathsf{Var}_t(\,\mathrm{d}z_t - \rho_t\,\mathrm{d}y_t)} \;=\; \frac{-\rho_t\beta_t\Sigma_t + (1-\rho_t\gamma_t)\sigma_v^2}{(1-\rho_t\gamma_t)^2\sigma_v^2 + \rho_t^2\sigma_u^2 + \sigma_e^2}.
\end{aligned} \tag{A.20}$$

By the same arguments as for the benchmark model, $\Sigma_t = (1-t)\Sigma_0$, $\beta_t = \frac{\beta_0}{1-t}$, and $\beta_t\Sigma_t$, λ_t, ρ_t, μ_t are constant. Also, Σ_t satisfies the same ODE (A.17) as V_τ, and $\Sigma_t' = -\Sigma_0$, hence

$$-\Sigma_0 \;=\; -2l_t\beta_t\Sigma_t + (1-\mu_\tau - l_\tau\gamma_\tau)^2\sigma_v^2 + \mu_\tau^2\sigma_e^2 + l_\tau^2\sigma_u^2. \tag{A.21}$$

We now define the following constants:

$$\begin{aligned}
&a \;=\; \frac{\sigma_u^2}{\sigma_v^2}, \quad b \;=\; \frac{\sigma_e^2}{\sigma_v^2}, \quad c \;=\; \frac{\Sigma_0}{\sigma_v^2},\\
&g \;=\; \frac{\gamma^2}{a}, \quad \tilde\lambda \;=\; \lambda\gamma, \quad \tilde\rho \;=\; \rho\gamma, \quad \psi = \frac{\beta_0\Sigma_0}{\sigma_u^2}\gamma, \quad \tilde l \;=\; l\gamma.
\end{aligned} \tag{A.22}$$

With these notations, equations (A.19)–(A.21) become

$$\begin{aligned}\tilde{\lambda} &= \mu(1-\tilde{\rho}), \quad \tilde{\lambda} = \frac{\psi+g}{1+g}, \quad \tilde{\rho} = \frac{g}{1+g}, \quad \mu = \frac{1-\psi}{1+b(1+g)} \\ c &= \frac{2\psi}{g} - (1-\mu-\tilde{l})^2 - \mu^2 b - \frac{\tilde{l}^2}{g}.\end{aligned} \tag{A.23}$$

Substitute $\tilde{\lambda}$, $\tilde{\rho}$, μ in $\tilde{\lambda} = \mu(1-\tilde{\rho})$ and solve for ψ:

$$\psi = \frac{1-(1+b)g-bg^2}{2+b+bg} = \frac{1+g}{2+b+bg} - g. \tag{A.24}$$

The other equations, together with $\tilde{l} = \tilde{\lambda} - \mu\tilde{\rho}$, imply

$$\tilde{\lambda} = \frac{1}{2+b+bg}, \quad \tilde{\rho} = \frac{g}{1+g}, \quad \mu = \frac{1+g}{2+b+bg}, \quad \tilde{l} = \frac{1-g}{2+b+bg}, \tag{A.25}$$

$$1+c = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2}. \tag{A.26}$$

From (A.22), we get

$$\gamma = a^{1/2}g^{1/2}, \quad \beta_0 = \frac{\sigma_u^2}{\Sigma_0\gamma}\psi = \frac{a}{c\gamma}\psi = \frac{a^{1/2}}{cg^{1/2}}\psi. \tag{A.27}$$

From (A.24) and (A.26), we get $\psi = \frac{1+g}{2+b+bg} - g = \frac{g(2+b+bg)}{(1+g)(1+bg)}\big(\frac{(1+g)^2(1+bg)}{g(2+b+bg)^2} - \frac{(1+g)(1+bg)}{2+b+bg}\big) = \frac{g(2+b+bg)}{(1+g)(1+bg)}\big(c+1-\frac{(1+g)(1+bg)}{2+b+bg}\big) = \frac{g(2+b+bg)}{(1+g)(1+bg)}\big(c+(1-g)\frac{(1+b+bg)}{2+b+bg}\big)$. We compute $\beta_0 = \frac{a^{1/2}g^{1/2}(2+b+bg)}{(1+g)(1+bg)}\big(1+\frac{1-g}{c}\frac{1+b+bg}{2+b+bg}\big)$. Using again (A.26), we get

$$\beta_0 = \frac{a^{1/2}}{(1+c)^{1/2}(1+bg)^{1/2}}\left(1+\frac{1-g}{c}\frac{1+b+bg}{2+b+bg}\right). \tag{A.28}$$

Now substitute a, b, c from (A.22) in equations (A.25)–(A.28) to obtain equations (20)–(25). Moreover, the second order conditions $\lambda + \mu\rho > 0$ and $l > 0$ are equivalent to $g \in (-1, 1)$.

Finally, we show that the equation $1 + c = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2}$ has a unique solution $g \in$

$(-1,1)$, which in fact lies in $(0,1)$. This can be shown by noting that

$$F_b(g) \;=\; 1+c, \quad \text{with} \quad F_b(x) \;=\; \frac{(1+bx)(1+x)^2}{x(2+b+bx)^2}. \tag{A.29}$$

One verifies $F_b'(x) = \frac{(x+1)(x-1)(2+b+3bx)}{x^2(2+b+bx)^3}$, so $F_b(x)$ decreases on $(0,1)$. Since $F_b(0) = +\infty$ and $F_b(1) = \frac{1}{1+b} < 1$, there is a unique $g \in (0,1)$ so that $F_b(g) = 1+c$.[30]

For future use, we derive from (A.25) the following formulas:

$$\begin{aligned} \gamma \;&=\; a^{1/2}g^{1/2}, \quad \lambda \;=\; \frac{1}{\gamma}\,\frac{1}{2+b+bg}, \quad \rho \;=\; \frac{1}{\gamma}\,\frac{g}{1+g}, \quad \mu \;=\; \frac{1+g}{2+b+bg}, \\ l \;&=\; \frac{1}{\gamma}\,\frac{1-g}{2+b+bg}, \quad 1-\mu-l\gamma \;=\; \frac{b+bg}{2+b+bg}. \end{aligned} \tag{A.30}$$

A.2 Useful Comparative Statics

To compare the fast and benchmark models, and to do some comparative statics for the coefficients involved in Theorem 1, we prove the following result.

Lemma A.1. *With the notations in Theorem 1, the following inequalities are true:*

$$\mu^F \;<\; \mu^B, \quad \lambda^F \;>\; \lambda^B, \quad \beta_0^F \;<\; \beta_0^B. \tag{A.31}$$

Proof. Recall that in the proof of Theorem 1, we have denoted

$$a \;=\; \frac{\sigma_u^2}{\sigma_v^2}, \quad b \;=\; \frac{\sigma_e^2}{\sigma_v^2}, \quad c \;=\; \frac{\Sigma_0}{\sigma_v^2}. \tag{A.32}$$

We start by showing that

$$\mu^F \;=\; \frac{1+g}{2+b+bg} \;<\; \mu^B \;=\; \frac{1}{1+b}. \tag{A.33}$$

By computation, this is equivalent to $g < 1$, which is true since $g \in (0,1)$.

[30] One can check that $F_b(x) = 1+c$ has no solution on $(-1,0)$: When $b \le 1$, $F_b(x) < 0$ on $(-1,0)$. When $b > 1$, $F_b(x)$ attains its maximum on $(-1,0)$ at $x^* = -\frac{2+b}{3b}$, for which $F_b(x^*) = \frac{(b-1)^3}{b(b+2)^3} < 1$.

We show that

$$\lambda^F = \frac{(1+c)^{1/2}}{a^{1/2}} \frac{1}{(1+bg)^{1/2}(1+g)} > \lambda^B = \frac{c^{1/2}}{a^{1/2}} \left(1 + \frac{b}{c(b+1)}\right)^{1/2}. \tag{A.34}$$

After squaring the two sides, and using $1 + c = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2}$, we need to prove that $\frac{1}{g(2+b+bg)^2} > c + 1 - \frac{1}{1+b}$, or equivalently $\frac{1}{1+b} > \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2} - \frac{1}{g(2+b+bg)^2} = \frac{2+b+g+2bg+bg^2}{(2+b+bg)^2}$. This reduces to proving $1 + b + (1-g)(1+bg) > 0$, which is true, since $b > 0$ and $g \in (0, 1)$.

In the proof of Theorem 1, we have $\psi = \frac{1+g}{2+b+bg} - g = \frac{g(2+b+bg)}{(1+g)(1+bg)}\left(c + (1-g)\frac{(1+b+bg)}{2+b+bg}\right) > 0$. But $\frac{1+g}{2+b+bg} > g$ implies $bg < \frac{1-g}{1+g}$. We now show that

$$\beta_0^F = \frac{a^{1/2}}{cg^{1/2}} \left(\frac{1+g}{2+b+bg} - g\right) < \beta_0^B = \frac{a^{1/2}}{c} \left(c + \frac{b}{1+b}\right)^{1/2}, \tag{A.35}$$

where we use (A.24) and (A.27) for β_0^F, and (14) for β_0^B. Using (A.26), the desired inequality is equivalent to $\frac{1}{g}\frac{(1-g-bg-bg^2)^2}{(2+b+bg)^2} < c + 1 - \frac{1}{1+b} = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2} - \frac{1}{1+b}$, or $\frac{1}{1+b} < \frac{4+3b+bg(2-b)-bg^2(1+2b)-b^2g^3}{(2+b+bg)^2}$. After some algebra, this is equivalent to $bg^2(1+g)^2 + bg(1 + 4g + g^2) < 3 + 2b$. We use $bg < \frac{1-g}{1+g}$ (proved above) to show that $bg^2(1+g)^2 < g(1-g^2)$ and $bg(1 + 4g + g^2) < (1-g)\frac{1+4g+g^2}{1+g} < (1-g)(1+3g)$. Then, it is sufficient to prove that $g(1-g^2) + (1-g)(1+3g) < 3 + 2b$, or $1 + 3g - 3g^2 - g^3 < 3 + 2b$. For this, it is sufficient to prove $1 + 3g - 3g^2 < 3 + 2b$. But $1 + 3g - 3g^2$ attains its maximum value of $1 + \frac{3}{4}$ at $g = \frac{1}{2}$, and $1 + \frac{3}{4} < 3 + 2b$. □

A.3 Proof of Corollary 1

In the last part of the proof of Theorem 1, we have shown that, when $\sigma_e < \infty$, equation (25) has a unique solution $g \in (0, 1]$. Thus, since $\gamma_t^F = \frac{\sigma_u}{\sigma_v} g^{1/2}$, there is news trading when the speculator reacts faster to news. Moreover, when σ_e approaches $+\infty$, or equivalently when $b = \frac{\sigma_e^2}{\sigma_v^2}$ approaches $+\infty$, it is straightforward to show that $g = \frac{(1+bg)(1+g)^2}{(2+b+bg)^2} \frac{1}{1+c}$ converges to zero. Thus, if the dealer receives no news, there is no news trading.

A.4 Proof of Corollary 2

From Theorem 1, we have $\beta_t^B/\beta_t^F = \beta_0^B/\beta_0^F$. Now, Lemma A.1 implies $\beta_0^B/\beta_0^F > 1$, which yields the result.

A.5 Proof of Corollaries 3 and 4

See Lemma A.1.

A.6 Proof of Corollary 5

In the benchmark model, equation (12) and $\gamma_t^B = 0$ imply that $\mathsf{Var}(\mathrm{d}x_t) = (\beta_t^B)^2\Sigma_t \mathrm{d}t^2 = 0$, since $\mathrm{d}t^2 = 0$. Also, $\mathsf{Var}(\mathrm{d}u_t) = \sigma_u^2 \mathrm{d}t$. Thus, $IPR_t^B = \frac{\mathsf{Var}(\mathrm{d}x_t)}{\mathsf{Var}(\mathrm{d}x_t)+\mathsf{Var}(\mathrm{d}u_t)} = 0$.

In the fast model, equation (18) implies $\mathsf{Var}(\mathrm{d}x_t) = (\gamma^F)^2\sigma_v^2\mathrm{d}t$, and equation (21) implies $(\gamma^F)^2\sigma_v^2 = \sigma_u^2 g$. Thus, $IPR^F = \frac{\sigma_u^2 g\mathrm{d}t}{\sigma_u^2 g\mathrm{d}t+\sigma_u^2\mathrm{d}t} = \frac{g}{g+1}$. From Theorem 1, we know that $g \in (0,1)$. Hence, $IPR^F > 0$.

A.7 Proof of Corollary 6

For $k \in \{B,F\}$, if $p_t^B = p_t$ and $p_t^F = q_t$, we write the equilibrium equations

$$\begin{aligned} \mathrm{d}x_t &= \beta_t^k(v_t - p_t^k)\mathrm{d}t + \gamma^k \mathrm{d}v_t, \\ \mathrm{d}p_t &= \mu^k(\mathrm{d}z_t - \rho^k \mathrm{d}y_t) + \lambda^k \mathrm{d}y_t \;=\; \mu^k \mathrm{d}z_t + l^k \mathrm{d}y_t. \end{aligned} \tag{A.36}$$

We first prove the following useful result.

Lemma A.2. *In both the benchmark and the fast models, i.e., if $k \in \{B,F\}$, and for all $s < t \in (0,1)$,*

$$\begin{aligned} \mathsf{Cov}(v_s - p_s^k, v_t - p_t^k) &= \Sigma_s \left(\frac{1-t}{1-s}\right)^{l^k\beta_0^k}, \\ \frac{1}{\mathrm{d}s}\mathsf{Cov}(\mathrm{d}v_s, v_t - p_t^k) &= (1 - l^k\gamma^k - \mu^k)\sigma_v^2 \left(\frac{1-t}{1-s}\right)^{l^k\beta_0^k}, \end{aligned} \tag{A.37}$$

where $l^k = \lambda^k - \mu^k \rho^k$.

Proof. To simplify notation, we omit the superscript k for p_t^k. Fix $s \in (0,1)$. Denote by $X_t = \mathsf{Cov}(v_s - p_s, v_t - p_t)$. For $t \geq s$, $\mathrm{d}X_t = \mathsf{Cov}(v_s - p_s, \mathrm{d}v_t - \mathrm{d}p_t) = -\mathsf{Cov}(v_s - p_s, \mathrm{d}p_t) = -l^k \beta_t^k X_t \mathrm{d}t = -l^k \frac{\beta_0^k}{1-t} X_t \mathrm{d}t$. Then, $\mathrm{d}\ln(X_t) = l^k \beta_0^k \, \mathrm{d}\ln(1-t)$. Also, at $t = s$, we have $X_s = \Sigma_s$. Thus, we have a first order differential equation, with solution given by the first equation in (A.37).

Denote by $Y_t = \frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, v_t - p_t)$. For $t > s$, $\mathrm{d}Y_t = \frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, \mathrm{d}v_t - \mathrm{d}p_t) = -\frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, \mathrm{d}p_t) = -l^k \beta_t^k Y_t \mathrm{d}t = -l^k \frac{\beta_0^k}{1-t} Y_t \mathrm{d}t$. Then, $\mathrm{d}\ln(Y_t) = l^k \beta_0^k \, \mathrm{d}\ln(1-t)$. At $t = s + \mathrm{d}s$, we have $Y_{s+\mathrm{d}s} = \frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, v_s - p_s + \mathrm{d}v_s - \mathrm{d}p_s) = \frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, \mathrm{d}v_s) - \frac{1}{\mathrm{d}s} \mathsf{Cov}(\mathrm{d}v_s, l^k \mathrm{d}y_t + \mu^k \mathrm{d}z_t) = (1 - l^k \gamma^k - \mu^k) \sigma_v^2$. Thus, we have a first order differential equation, with solution given by the second equation in (A.37).

$\square$

We now prove Corollary 6. For the benchmark model, denote by $l^B = \lambda^B$. Then, using the notations from Lemma A.2, we get

$$\mathsf{Corr}(\mathrm{d}x_t^B, \mathrm{d}x_{t+\tau}^B) = \frac{\mathsf{Cov}(v_t - p_t, v_{t+\tau} - p_{t+\tau})}{\mathsf{Cov}(v_t - p_t)^{1/2} \, \mathsf{Cov}(v_{t+\tau} - p_{t+\tau})^{1/2}} = \frac{\Sigma_t \left(\frac{1-t-\tau}{1-t}\right)^{\lambda^B \beta_0^B}}{\Sigma_t^{1/2} \Sigma_{t+\tau}^{1/2}}. \quad \text{(A.38)}$$

Since $\Sigma_s = \Sigma_0(1-s)$, we obtain $\mathsf{Corr}(\mathrm{d}x_t^B, \mathrm{d}x_{t+\tau}^B) = \left(\frac{1-t-\tau}{1-t}\right)^{\lambda^B \beta_0^B - \frac{1}{2}}$.[31]

In the fast model, we use both equations in (A.37) to show that the autocovariance of the informed order flow, $\mathsf{Cov}(\mathrm{d}x_t^F, \mathrm{d}x_{t+\tau}^F)$, is of order $\mathrm{d}t^2$. But the informed order flow variance is of order $\mathrm{d}t$, therefore the autocorrelation is of order $\mathrm{d}t$, which as a number equals zero in continuous time.

A.8 Proof of Corollary 7

If $k \in \{B, F\}$, consider the following notations similar to those from the proof of Lemma A.2: $X_{t,t+\tau} = \mathsf{Cov}(v_t - p_t^k, v_{t+\tau} - p_{t+\tau}^k)$, and $Y_{t,t+\tau} = \frac{1}{\mathrm{d}t} \mathsf{Cov}(\mathrm{d}v_t, v_{t+\tau} - p_{t+\tau}^k)$.

[31] Note that $\lambda^B \beta_0^B = 1 + \frac{\sigma_v^2 \sigma_e^2}{\Sigma_0(\sigma_v^2 + \sigma_e^2)} > 1$.

Denote by $l^k = \lambda^k - \mu^k\rho^k$, and $\alpha^k = \left(\frac{1-(t+\tau)}{1-t}\right)^{l^k\beta_0^k}$. Then, the corollary is proved if we show that $CPI_t^k(\tau) = C_0^k + C_1^k(1-\alpha^k)$, where

$$\begin{aligned} C_0^B &= 0, \quad \text{and} \quad C_1^B = \beta_0^B\Sigma_0, \\ C_0^F &= \left(\mu^F + l^F\gamma^F\right)\gamma^F\sigma_v^2, \quad \text{and} \quad C_1^F = \beta_0^F\Sigma_0 + (1-\mu^F - l^F\gamma^F)\gamma^F\sigma_v^2. \end{aligned} \tag{A.39}$$

For convenience, drop the superscript k. Denote by $n = 1-\mu-l\gamma$. We write $CPI_t(\tau) = \mathsf{Cov}\left(\frac{\mathrm{d}x_t}{\mathrm{d}t}, p_{t+\tau} - p_t\right)$. Since $\mathrm{d}x_t = \beta_t(v_t - p_t)\,\mathrm{d}t + \gamma\,\mathrm{d}v_t$, we obtain $CPI_t(\tau) = \beta_t\,\mathsf{Cov}\left(v_t - p_t, p_{t+\tau} - p_t\right) + \frac{\gamma}{\mathrm{d}t}\,\mathsf{Cov}\left(\mathrm{d}v_t, p_{t+\tau} - p_t\right) = \beta_t\,\mathsf{Cov}\left(v_t - p_t, v_{t+\tau} - p_t\right) + \frac{\gamma}{\mathrm{d}t}\,\mathsf{Cov}\left(\mathrm{d}v_t, v_{t+\tau} - p_t\right) - \beta_t X_{t,t+\tau} - \frac{\gamma}{\mathrm{d}t} Y_{t,t+\tau}$. But $\mathsf{Cov}\left(v_t - p_t, v_{t+\tau} - p_t\right) = \Sigma_t$, and $\mathsf{Cov}\left(\mathrm{d}v_t, v_{t+\tau} - p_t\right) = \sigma_v^2\,\mathrm{d}t$. Hence, using Lemma A.2 to compute $X_{t,t+\tau}$ and $Y_{t,t+\tau}$, we get $CPI_t(\tau) = \beta_t\Sigma_t(1-\alpha) + \gamma\sigma_v^2(1-n\alpha)$. Theorem 1 implies that $\beta_t\Sigma_t$ is constant, hence $\beta_t\Sigma_t = \beta_0\Sigma_0$. We get $CPI_t(\tau) = C_0 + C_1(1-\alpha)$, where $C_0 = \gamma\sigma_v^2(1-n)$ and $C_1 = \beta_0\Sigma_0 + \gamma\sigma_v^2 n$. One can now verify directly, for both $k \in \{B, F\}$, that C_0^k and C_1^k are as in equation (A.39).

Finally, we note that $n^F > 0$. Indeed, denote by $b = \frac{\sigma_e^2}{\sigma_v^2}$, and by $g \in (0,1)$ as in Theorem 1. Then, according to equation (A.30), $n^F = 1 - \mu^F - l^F\gamma^F = \frac{b+bg}{2+b+bg} > 0$.

A.9 Proof of Corollary 8

Remember that, by definition, $a = \frac{\sigma_u^2}{\sigma_v^2}$, $b = \frac{\sigma_e^2}{\sigma_v^2}$ and $c = \frac{\Sigma_0}{\sigma_v^2}$. For the benchmark model, as in Theorem 1 and Lemma A.1, we have

$$\beta_0^B = \frac{\sigma_u}{\Sigma_0^{1/2}}\left(1 + \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2}\right)^{1/2} = \frac{a^{1/2}}{c}\left(c + \frac{b}{1+b}\right)^{1/2}. \tag{A.40}$$

From the second equality, β_0^B is increasing in $b = \frac{\sigma_e^2}{\sigma_v^2}$, hence β_0^B is decreasing in $\nu = \frac{1}{\sigma_e}$. Since β_t^B is proportional to β_0^B, it follows that β_t^B is decreasing in ν.

As in the proof of Theorem 1, let $F(b,x) = \frac{(1+bx)(1+x)^2}{x(2+b+bx)^2}$, with $\frac{\partial F}{\partial b} = -\frac{(1+x)^2(2+bx+bx^2)}{x(2+b+bx)^3}$ and $\frac{\partial F}{\partial x} = -\frac{(1-x)(1+x)(2+b+3bx)}{x^2(2+b+bx)^3}$. Since $g \in (0,1)$ is the solution of $F(b, g(b,c)) = 1+c$, by differentiating with respect to b and c, respectively, we get $\frac{\partial F}{\partial b} + \frac{\partial F}{\partial x}\,\frac{\partial g}{\partial b} = 0$, and

$\frac{\partial F}{\partial x}\frac{\partial g}{\partial c} = 1$. We compute

$$\frac{\partial g}{\partial b} = -\frac{g(1+g)(2+bg+bg^2)}{(1-g)(2+b+3bg)}, \quad \frac{\partial g}{\partial c} = -\frac{g^2(2+b+bg)^3}{(1-g)(1+g)(2+b+3bg)}. \tag{A.41}$$

Thus, g is decreasing in b, hence g is increasing in ν. As $\gamma^F = \frac{\sigma_u}{\sigma_v} g^{1/2}$, it follows that γ^F is increasing with ν as well.

From the proof of Theorem 1, we also have

$$\beta_0^F = \frac{a^{1/2}}{cg^{1/2}}\left(\frac{1+g}{2+b+bg} - g\right). \tag{A.42}$$

Using (A.41), we compute $\frac{\partial \beta_0^F}{\partial b} = \frac{a^{1/2}g^{1/2}(1+g)^2(2+3b+3bg+b^2g+b^2g^2)}{2c(1-g)(2+b+bg)(2+b+3bg)}$. Thus, β_0^F is increasing in b, hence β_0^F is decreasing in ν.

Finally, when $\nu = 0$, $\sigma_e = +\infty$ and $b = +\infty$. Then, as in the proof of Corollary 1, we get $g = 0$ and $\gamma^F = 0$. Moreover, equation (A.40) implies that, in the benchmark model, $\beta_0^B = \frac{a^{1/2}}{c}(c+1)^{1/2}$. In the fast model, equation (A.42) implies that $\beta_0^F = \frac{a^{1/2}}{c}\frac{1+g}{g^{1/2}(2+b+bg)}$, since $g = 0$. But equation (A.26) implies that $(1+c)^{1/2} = \frac{(1+bg)^{1/2}(1+g)}{g^{1/2}(2+b+bg)}$. Since $g = 0$ and $b = +\infty$, by considering only the dominant terms, we show that $bg = 0$, hence $\beta_0^F = \frac{a^{1/2}}{c}(c+1)^{1/2}$. Thus, when $\nu = 0$, $\beta^F = \beta^B$.

A.10 Proof of Corollary 9

In the fast model, denote by $TV^F = \mathsf{Var}(\mathrm{d}y_t)$ the trading volume, and $IPR^F = \frac{\mathsf{Var}(\mathrm{d}x_t)}{\mathsf{Var}(\mathrm{d}y_t)}$ the informed participation rate. Then, by Corollary 5, $TV^F = \sigma_u^2(1+g)$, and $IPR^F = \frac{g}{1+g}$, hence TV^F and IPR^F have the same dependence on σ_e as g. From (A.41), g is decreasing in b, hence also in σ_e. Thus, both TV^F and IPR^F are decreasing in σ_e, i.e., are increasing in $\nu = \frac{1}{\sigma_e}$.

As in equation (A.34), we have $(\lambda^F)^2 = \frac{1+c}{a}\frac{1}{(1+bg)(1+g)^2}$. Using the formula for $\frac{\partial g}{\partial b}$ in (A.41), we compute $\frac{\partial\left((1+bg)(1+g)^2\right)}{\partial b} = -\frac{g(1+g)^3(1+bg)}{1-g} < 0$. Therefore, λ^F is increasing in b, hence in σ_e. Thus, higher precision of the public signal (lower σ_e) implies higher liquidity (lower price impact coefficient λ^F).

A.11 Proof of Corollary 10

We compute $\mathrm{d}\Sigma_t^k = 2\,\mathsf{Cov}(\mathrm{d}v_t - \mathrm{d}p_t^k, v_t - p_t^k) + \mathsf{Cov}(\mathrm{d}v_t - \mathrm{d}p_t^k, \mathrm{d}v_t - \mathrm{d}p_t^k)$. Since the news $\mathrm{d}v_t$ is orthogonal to $v_t - p_t^k$ in both models, $\mathrm{d}\Sigma_t^k = -2\,\mathsf{Cov}(\mathrm{d}p_t^k, v_t - p_t^k) - 2\,\mathsf{Cov}(\mathrm{d}p_t^k, \mathrm{d}v_t) + \mathsf{Var}(\mathrm{d}v_t) + \mathsf{Var}(\mathrm{d}p_t^k)$. But $\frac{1}{\mathrm{d}t}\mathsf{Var}(\mathrm{d}v_t) = \sigma_v^2$; and by Corollary 11, $\sigma_p^2 = \frac{1}{\mathrm{d}t}\mathsf{Var}(\mathrm{d}p_t^k) = \sigma_v^2 + \Sigma_0$. We have just proved (33).

Equation (13) implies that in the benchmark model $\mathrm{d}p_t = \mu^B \mathrm{d}z_t + \lambda^B \mathrm{d}y_t$. Since $\mathrm{d}y_t = \mathrm{d}x_t + \mathrm{d}u_t$, with $\mathrm{d}x_t = O(\mathrm{d}t)$,

$$\mathsf{Cov}(\mathrm{d}p_t, \mathrm{d}v_t) \;=\; \mu^B \sigma_v^2 \mathrm{d}t. \tag{A.43}$$

Equation (19) implies that in the fast model $\mathrm{d}q_t = \lambda^F \mathrm{d}y_t + \mu^F(\mathrm{d}z_t - \rho^F \mathrm{d}y_t)$. Since $\mathrm{d}x_t = \gamma^F\, \mathrm{d}v_t + O(\mathrm{d}t)$, we obtain

$$\mathsf{Cov}(\mathrm{d}p_t, \mathrm{d}v_t) \;=\; \Big(\gamma^F(\lambda^F - \mu^F\rho^F) + \mu^F\Big)\sigma_v^2 \mathrm{d}t. \tag{A.44}$$

Next, we prove that $\gamma^F(\lambda^F - \mu^F\rho^F) + \mu^F > \mu^B$. Using (A.30) and (A.33), we need to show that $\frac{2}{2+b+bg} > \frac{1}{1+b}$, which is equivalent to $1 > g$. But this is true, since $g \in (0,1)$.

Finally, from the proof of Theorem 1, in both models $\Sigma_t^k = \Sigma_0(1-t)$.

A.12 Proof of Corollary 11

Denote $\mathsf{Var}(\mathrm{d}p_t^k) = \sigma_p^2 \mathrm{d}t$ the variance of the instantaneous price changes, and we use Theorem 1 to compute the two components of this variance. In the benchmark model, $\mathsf{Var}\big(\mathrm{d}p^B_{\text{trades},\,t}\big) = (\lambda^B)^2\sigma_u^2 \mathrm{d}t = \Big(\Sigma_0 + \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2}\Big)\mathrm{d}t$. Also, $\mathsf{Var}\big(\mathrm{d}p^B_{\text{quotes},\,t}\big) = (\mu^B)^2(\sigma_v^2+\sigma_e^2)\mathrm{d}t =$

$\frac{\sigma_v^4}{\sigma_v^2+\sigma_e^2}\mathrm{d}t$. We obtain the volatility decomposition in the benchmark model,

$$\sigma_p^2 \;=\; \frac{\mathsf{Var}(\mathrm{d}p_t)}{\mathrm{d}t} \;=\; \left(\Sigma_0 + \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2}\right) \;+\; \frac{\sigma_v^4}{\sigma_v^2+\sigma_e^2} \;=\; \Sigma_0 + \sigma_v^2. \tag{A.45}$$

Similarly, in the fast model, $\mathsf{Var}\big(\mathrm{d}p^F_{\text{trades},\,t}\big) = (\lambda^F)^2\big((\gamma^F)^2\sigma_v^2 + \sigma_u^2\big)\mathrm{d}t$, and using equation (A.30) we compute $\mathsf{Var}\big(\mathrm{d}p^F_{\text{trades},\,t}\big) = \frac{1+g}{g(2+b+bg)^2}\sigma_v^2\,\mathrm{d}t$. Also, $\mathsf{Var}\big(\mathrm{d}p^F_{\text{quotes},\,t}\big) = (\mu^F)^2\big((1-\rho^F\gamma^F)^2\sigma_v^2+\sigma_e^2+(\rho^F)^2\sigma_u^2\big)\mathrm{d}t = \frac{(1+g)(1+b+bg)}{(2+b+bg)^2}\sigma_v^2\mathrm{d}t$. According to (A.26), $\Sigma_0+\sigma_v^2 = \sigma_v^2(1+c) = \sigma_v^2\frac{(1+g)^2(1+bg)}{g(2+b+bg)^2}$, hence we have

$$\sigma_p^2 \;=\; \frac{\mathsf{Var}(\mathrm{d}q_t)}{\mathrm{d}t} \;=\; \frac{1+g}{g(2+b+bg)^2}\sigma_v^2 \;+\; \frac{(1+g)(1+b+bg)}{(2+b+bg)^2}\sigma_v^2 \;=\; \Sigma_0+\sigma_v^2. \tag{A.46}$$

We now show that the volatility component coming from quote updates is larger in the benchmark, i.e., $\frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2} = \frac{1}{1+b} > \frac{(1+g)(1+b+bg)}{(2+b+bg)^2}$. The difference is proportional to $3-g+2b+bg-bg^2 = 2(1+b)+(1-g)(1+bg) > 0$. Since the total volatility is the same, it also implies that the volatility component coming from the trades is larger in the fast model.

B Sampling at Lower Frequencies than the Trading Frequency

In this section, we show that Corollaries 5 and 6 in Section 4 generalize when trades are aggregated over intervals of an arbitrary length $\Delta\tau$. Suppose trading takes place in continuous time, but trades are aggregated over $T>0$ time intervals of equal length $\frac{1}{T}=\Delta\tau$. Then, data are indexed by $t\in\{1,2,\dots,T\}$, which corresponds to calendar time $\tau = t\Delta\tau \in [0,1]$. Denote by $\Delta x_t = x_t - x_{t-1} = \int_{(t-1)\Delta\tau}^{t\Delta\tau}\mathrm{d}x_\tau$ the aggregate informed order flow over the t-th time interval.[32]

[32] This is related, but not equivalent, to the order flow at the t-th trading round in the discrete model of Section 2. In the limit when $\Delta\tau$ approaches zero, it is reasonable to expect that the two notions are equivalent. This depends on whether the coefficients of the discrete time model (as described in

The empirical counterpart of the Informed Participation Rate and the autocorrelation of the speculator's order flow when data are aggregated every $\Delta\tau$ periods of time are, respectively,

$$\begin{aligned} IPR_t &= \frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t) + \mathsf{Var}(\Delta u_t)}, \\ &\mathsf{Corr}(\Delta x_t, \Delta x_{t+s}). \end{aligned} \tag{B.1}$$

Proposition 1. *When the sampling interval $\Delta\tau$ is small, the empirical informed participation rate in the benchmark increases with $\Delta\tau$ and is always below its level in the fast model:*

$$\begin{aligned} IPR_t^B &= \frac{(\beta_t^B)^2 \Sigma_t}{\sigma_u^2} \Delta\tau + o(\Delta\tau), \\ IPR_t^F &= \frac{(\gamma^F)^2 \sigma_v^2}{(\gamma^F)^2 \sigma_v^2 + \sigma_u^2} + \frac{o(\Delta\tau)}{\Delta\tau}. \end{aligned} \tag{B.2}$$

The informed order flow autocorrelation in the fast model increases with the sampling interval $\Delta\tau$ and is always below its level in the benchmark:

$$\begin{aligned} \mathsf{Corr}(\Delta x_t^B, \Delta x_{t+s}^B) &= \left(\frac{1-(t+s)\Delta\tau}{1-t\Delta\tau} \right)^{\lambda^B \beta_0^B - \frac{1}{2}} + \frac{o(\Delta\tau)}{\Delta\tau}, \\ \mathsf{Corr}(\Delta x_t^F, \Delta x_{t+s}^F) &= \frac{\beta_{t+s}^F \Big(\beta_t^F \Sigma_t + \gamma^F \big(1 - l^F \gamma^F - \mu^F\big) \sigma_v^2 \Big)}{(\gamma^F)^2 \sigma_v^2} \\ &\quad \times \left(\frac{1-(t+s)\Delta\tau}{1-t\Delta\tau} \right)^{l^F \beta_0^F} \Delta\tau + o(\Delta\tau). \end{aligned} \tag{B.3}$$

Proof. The aggregate trade over the t-th interval is $\Delta x_t = \int_{(t-1)\Delta\tau}^{t\Delta\tau} \beta_\tau (v_\tau - p_\tau)\,\mathrm{d}\tau + \gamma\,\mathrm{d}v_\tau$. When $\Delta\tau$ is small, β_τ is approximately constant over the interval $[(t-1)\Delta\tau, t\Delta\tau]$. Thus, in the benchmark model we have $\Delta x_t^B = \beta_t^B (v_t - p_t)\Delta\tau + o(\Delta\tau)$, since $\gamma^B = 0$. This implies $\mathsf{Var}(\Delta x_t^B) = (\beta_t^B)^2 \Sigma_t (\Delta\tau)^2 + o\big((\Delta\tau)^2\big)$. Also, $\mathsf{Var}(\Delta u_t) = \sigma_u^2 \Delta\tau$, which yields the informed participation rate in (B.2). Using Lemma A.2 in Appendix A, we obtain

the Internet Appendix) converge to the corresponding coefficients of the continuous time version. We suspect this is true, as in Kyle (1985), although we have not formally proved it.

$\mathsf{Cov}(\Delta x_t^B, \Delta x_{t+s}^B) = \beta_{t+s}^B \beta_t^B \Sigma_t \left(\frac{1-(t+s)\Delta\tau}{1-t\Delta\tau}\right)^{\lambda^B \beta_0^B} (\Delta\tau)^2 + o\big((\Delta\tau)^2\big)$, which proves the first equation in (B.3).

In the fast model, $\Delta x_t^F = \beta_t^F (v_t - p_t)\Delta\tau + \gamma^F \Delta v_t + o(\Delta\tau)$. Then, $\mathsf{Var}(\Delta x_t^F) = (\gamma^F)^2 \sigma_v^2 \Delta\tau + o(\Delta\tau)$, which implies the informed participation rate in (B.2). Using Lemma A.2, $\mathsf{Cov}(\Delta x_t^F, \Delta x_{t+s}^F) = \beta_{t+s}^F \Big(\beta_t^F \Sigma_t + \gamma^F \big(1 - l^F \gamma^F - \mu^F\big)\sigma_v^2\Big) \left(\frac{1-(t+s)\Delta\tau}{1-t\Delta\tau}\right)^{l^F \beta_0^F} (\Delta\tau)^2 + o((\Delta\tau)^2)$, where $l^F = \lambda^F - \mu^F \rho^F$, which proves the second equation in (B.3).

□

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Internet Appendix for "News Trading and Speed" – Not for publication

In this Appendix, we present the discrete time version of the fast and benchmark models in subsections I.1 and I.2.

I.1 Discrete Time Fast Model

We use the notations from Section 2. Denote by $\mathcal{I}_t^q = \{\Delta z_\tau\}_{\tau \leq t-1} \cup \{\Delta y_\tau\}_{\tau \leq t-1}$ the dealer's information set just before trading at t, and by $\mathcal{I}_t^p = \{\Delta z_\tau\}_{\tau \leq t-1} \cup \{\Delta y_\tau\}_{\tau \leq t} = \mathcal{I}_t^q \cup \{\Delta y_t\}$ the information set just after trading at t. The zero profit condition for the competitive dealer translates into the formulas

$$q_t \; = \; \mathsf{E}(v_1 \mid \mathcal{I}_t^q), \qquad p_t \; = \; \mathsf{E}(v_1 \mid \mathcal{I}_t^p). \tag{I.4}$$

We also denote

$$\Omega_t \; = \; \mathsf{Var}(v_t \mid \mathcal{I}_t^p), \qquad \Sigma_t \; = \; \mathsf{Var}(v_t \mid \mathcal{I}_t^q). \tag{I.5}$$

Definition 1. *A pricing rule p_t is called linear if it is of the form $p_t = q_t + \lambda_t \Delta y_t$, for all $t = 1, \ldots, T$.*[33]

The next result shows that if the pricing rule is linear, then the speculator's strategy is also linear, and furthermore it can be decomposed into a forecast error component, $\beta_t(v_t - q_t)\Delta t$, and a news trading component, $\tilde{\gamma}_t \Delta v_t$, where $\tilde{\gamma}_t \equiv \gamma_t - \beta_t \Delta t = \frac{\alpha_t l_t \mu_t}{\lambda_t - \alpha_t l_t^2}$ (see (I.9)).

[33] We could defined more generally, a pricing rule to be linear in the whole history $\{\Delta y_\tau\}_{\tau \leq t}$, but as Kyle (1985) shows, this is equivalent to the pricing rule being linear only in Δy_t.

Theorem 2. *Any equilibrium with a linear pricing rule must be of the form*

$$\begin{aligned}
\Delta x_t &= \beta_t(v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t, \\
p_t &= q_t + \lambda_t \Delta y_t, \\
q_{t+1} &= p_t + \mu_t(\Delta z_t - \rho_t \Delta y_t),
\end{aligned} \tag{I.6}$$

for $t = 1, \ldots, T$*, where* β_t*,* γ_t*,* λ_t*,* μ_t*,* ρ_t*,* Ω_t*, and* Σ_t *are constants that satisfy*

$$\begin{aligned}
\lambda_t &= \frac{\beta_t \Sigma_{t-1} + \gamma_t \sigma_v^2}{\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2}, \\
\mu_t &= \frac{\left(\sigma_u^2 + \beta_t^2 \Sigma_{t-1}\Delta t - \beta_t \gamma_t \Sigma_{t-1}\right)\sigma_v^2}{\left(\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right)\sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1}\Delta t + \sigma_u^2\right)\sigma_v^2}, \\
l_t &= \lambda_t - \rho_t \mu_t \ = \ \frac{\beta_t \Sigma_{t-1}(\sigma_v^2 + \sigma_e^2) + \gamma_t \sigma_v^2 \sigma_e^2}{\left(\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right)\sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1}\Delta t + \sigma_u^2\right)\sigma_v^2}, \\
\rho_t &= \frac{\gamma_t \sigma_v^2}{\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2}, \\
\Omega_t &= \Sigma_{t-1} + \sigma_v^2 \Delta t - \frac{\beta_t^2 \Sigma_{t-1}^2 + 2\beta_t \gamma_t \Sigma_{t-1}\sigma_v^2 + \gamma_t^2 \sigma_v^4}{\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2}\,\Delta t, \\
\Sigma_t &= \Sigma_{t-1} + \sigma_v^2 \Delta t \\
&\quad - \frac{\beta_t^2 \Sigma_{t-1}^2(\sigma_v^2 + \sigma_e^2) + \beta_t^2 \Sigma_{t-1}\Delta t \sigma_v^4 + \sigma_v^4 \sigma_u^2 + \gamma_t^2 \sigma_v^4 \sigma_e^2 + 2\beta_t \gamma_t \Sigma_{t-1}\sigma_v^2 \sigma_e^2}{\left(\beta_t^2 \Sigma_{t-1}\Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right)\sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1}\Delta t + \sigma_u^2\right)\sigma_v^2}\,\Delta t.
\end{aligned} \tag{I.7}$$

The value function of the speculator is quadratic for all $t = 1, \ldots, T$*:*

$$\pi_t \ = \ \alpha_{t-1}(v_{t-1} - q_t)^2 + \alpha'_{t-1}(\Delta v_t)^2 + \alpha''_{t-1}(v_{t-1} - q_t)\Delta v_t + \delta_{t-1}. \tag{I.8}$$

The coefficients of the optimal trading strategy and the value function satisfy

$$
\begin{aligned}
\beta_t \Delta t &= \frac{1 - 2\alpha_t l_t}{2(\lambda_t - \alpha_t l_t^2)}, \\
\gamma_t &= \frac{1 - 2\alpha_t l_t (1 - \mu_t)}{2(\lambda_t - \alpha_t l_t^2)} \;=\; \beta_t \Delta t + \frac{\alpha_t l_t \mu_t}{\lambda_t - \alpha_t l_t^2}, \\
\alpha_{t-1} &= \beta_t \Delta t (1 - \lambda_t \beta_t \Delta t) + \alpha_t (1 - l_t \beta_t \Delta t)^2, \\
\alpha'_{t-1} &= \alpha_t (1 - \mu_t - l_t \gamma_t)^2 + \gamma_t (1 - \lambda_t \gamma_t), \\
\alpha''_{t-1} &= \beta_t \Delta t + \gamma_t (1 - 2\lambda_t \beta_t \Delta t) + 2\alpha_t (1 - l_t \beta_t \Delta t)(1 - \mu_t - l_t \gamma_t), \\
\delta_{t-1} &= \alpha_t \big(l_t^2 \sigma_u^2 + \mu_t^2 \sigma_e^2 \big) \Delta t + \alpha'_t \sigma_v^2 \Delta t + \delta_t.
\end{aligned}
\tag{I.9}
$$

The terminal conditions are

$$
\alpha_T \;=\; \alpha'_T \;=\; \alpha''_T \;=\; \delta_T \;=\; 0. \tag{I.10}
$$

The second order condition is

$$
\lambda_t - \alpha_t l_t^2 \;>\; 0. \tag{I.11}
$$

Given Σ_0, conditions (I.7)–(I.11) are necessary and sufficient for the existence of a linear equilibrium.

Proof. First, we show that equations (I.7) are equivalent to the zero profit conditions of the dealer. Second, we show that equations (I.9)–(I.11) are equivalent to the speculator's strategy in (I.6) being optimal. These two steps prove that equations (I.6)–(I.11) describe an equilibrium. Conversely, all equilibria with a linear pricing rule must satisfy these equations since the trading strategy in (I.6) is the best-response to the linear pricing rule.

Zero profit of dealer: Start with with the dealer's update due to the order flow at $t = 1, \ldots, T$. Conditional on $\mathcal{I}_t^q$, the variables $v_{t-1} - q_t$ and Δv_t have a bivariate normal distribution:

$$\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}^q_{t-1} \;\sim\; \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \right). \tag{I.12}$$

The aggregate order flow at t is of the form

$$\Delta y_t \;=\; \beta_t(v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t + \Delta u_t. \tag{I.13}$$

Denote by

$$\Phi_t \;=\; \mathsf{Cov}\left(\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix}, \Delta y_t \right) \;=\; \begin{bmatrix} \beta_t \Sigma_{t-1} \\ \gamma_t \sigma_v^2 \end{bmatrix} \Delta t. \tag{I.14}$$

Then, conditional on $\mathcal{I}^p_t = \mathcal{I}^q_t \cup \{\Delta y_t\}$, the distribution of $v_{t-1} - q_t$ and Δv_t is bivariate normal:

$$\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}^p_t \;\sim\; \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right), \tag{I.15}$$

where

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \;=\; \Phi_t \, \mathsf{Var}(\Delta y_t)^{-1} \Delta y_t \;=\; \begin{bmatrix} \beta_t \Sigma_{t-1} \\ \gamma_t \sigma_v^2 \end{bmatrix} \frac{1}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2} \, \Delta y_t, \tag{I.16}$$

and

$$\begin{aligned} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} &\;=\; \mathsf{Var}\left(\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \right) - \Phi_t \, \mathsf{Var}(\Delta y_t)^{-1} \Phi_t' \\ = \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2 \Delta t \end{bmatrix} &- \frac{1}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2} \begin{bmatrix} \beta_t^2 \Sigma_{t-1}^2 & \beta_t \gamma_t \Sigma_{t-1} \sigma_v^2 \\ \beta_t \gamma_t \Sigma_{t-1} \sigma_v^2 & \gamma_t^2 \sigma_v^4 \end{bmatrix} \Delta t. \end{aligned} \tag{I.17}$$

We compute

$$p_t - q_t \;=\; \mathsf{E}(v_t - q_t \mid \mathcal{I}^p_t) \;=\; \mu_1 + \mu_2 \;=\; \frac{\beta_t \Sigma_{t-1} + \gamma_t \sigma_v^2}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2} \, \Delta y_t, \tag{I.18}$$

which proves equation (I.7) for λ_t. Also,

$$\begin{aligned}
\Omega_t \;&=\; \mathsf{Var}(v_t - q_t \mid \mathcal{I}_t^p) \;=\; \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 \\
&=\; \Sigma_{t-1} + \sigma_v^2\Delta t - \frac{\beta_t^2\Sigma_{t-1}^2 + 2\beta_t\gamma_t\Sigma_{t-1}\sigma_v^2 + \gamma_t^2\sigma_v^4}{\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2}\,\Delta t,
\end{aligned} \tag{I.19}$$

which proves the formula for Ω_t.

Next, to compute $q_{t+1} = \mathsf{E}(v_t \mid \mathcal{I}_{t+1}^q)$, we start from the same prior as in (I.12), but we consider the impact of both the order flow at t and the dealer's signal at $t+1$:

$$\begin{aligned}
\Delta y_t \;&=\; \beta_t(v_{t-1} - q_t)\Delta t + \gamma_t\Delta v_t + \Delta u_t, \\
\Delta z_t \;&=\; \Delta v_t + \Delta e_t.
\end{aligned} \tag{I.20}$$

Denote by

$$\begin{aligned}
\Psi_t \;&=\; \mathsf{Cov}\left(\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix}, \begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix}\right) \;=\; \begin{bmatrix} \beta_t\Sigma_{t-1} & 0 \\ \gamma_t\sigma_v^2 & \sigma_v^2 \end{bmatrix}\Delta t, \\
V_t^{yz} \;&=\; \mathsf{Var}\left(\begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix}\right) \;=\; \begin{bmatrix} \beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2 & \gamma_t\sigma_v^2 \\ \gamma_t\sigma_v^2 & \sigma_v^2 + \sigma_e^2 \end{bmatrix}\Delta t.
\end{aligned} \tag{I.21}$$

Conditional on $\mathcal{I}_{t+1}^q = \mathcal{I}_t^q \cup \{\Delta y_t, \Delta z_t\}$, the distribution of $v_{t-1} - q_t$ and Δv_t is bivariate normal:

$$\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}_{t+1}^q \;\sim\; \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right), \tag{I.22}$$

where

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \;=\; \Psi_t\,(V_t^{yz})^{-1}\begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix} \;=\; \frac{\begin{bmatrix} \beta_t\Sigma_{t-1}(\sigma_v^2 + \sigma_e^2)\Delta y_t \;-\; \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\Delta z_t \\ \gamma_t\sigma_v^2\sigma_e^2\Delta y_t \;+\; (\beta_t^2\Sigma_{t-1}\Delta t + \sigma_u^2)\sigma_v^2\Delta z_t \end{bmatrix}}{(\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2)\sigma_e^2 + (\beta_t^2\Sigma_{t-1}\Delta t + \sigma_u^2)\sigma_v^2}, \tag{I.23}$$

and

$$\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = \mathsf{Var}\left(\begin{bmatrix} v_{t-1}-q_t \\ \Delta v_t \end{bmatrix}\right) - \Psi_t\,(V_t^{yz})^{-1}\,\Psi_t'$$

$$= \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2\Delta t \end{bmatrix} - \frac{\begin{bmatrix} \beta_t^2\Sigma_{t-1}^2(\sigma_v^2+\sigma_e^2) & \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2 \\ \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2 & (\beta_t^2\Sigma_{t-1}\Delta t+\gamma_t^2\sigma_e^2+\sigma_u^2)\sigma_v^4 \end{bmatrix}}{(\beta_t^2\Sigma_{t-1}\Delta t+\gamma_t^2\sigma_v^2+\sigma_u^2)\sigma_e^2+(\beta_t^2\Sigma_{t-1}\Delta t+\sigma_u^2)\sigma_v^2}\,\Delta t. \tag{I.24}$$

Therefore,

$$\begin{aligned} q_{t+1}-q_t &= \mu_1+\mu_2 \\ &= \frac{\big(\beta_t\Sigma_{t-1}(\sigma_v^2+\sigma_e^2)+\gamma_t\sigma_v^2\sigma_e^2\big)\Delta y_t \;+\; \big(\sigma_u^2+\beta_t^2\Sigma_{t-1}\Delta t-\beta_t\gamma_t\Sigma_{t-1}\big)\sigma_v^2\Delta z_t}{(\beta_t^2\Sigma_{t-1}\Delta t+\gamma_t^2\sigma_v^2+\sigma_u^2)\,\sigma_e^2+(\beta_t^2\Sigma_{t-1}\Delta t+\sigma_u^2)\,\sigma_v^2} \end{aligned} \tag{I.25}$$

$$= l_t\Delta y_t \;+\; \mu_t\Delta z_t \;=\; (\lambda_t-\rho_t\mu_t)\Delta y_t \;+\; \mu_t\Delta z_t, \tag{I.26}$$

which proves equation (I.7) for μ_t, l_t, and ρ_t. Also,

$$\begin{aligned} \Sigma_t &= \sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2 \\ &= \Sigma_{t-1}+\sigma_v^2\Delta t - \frac{\beta_t^2\Sigma_{t-1}^2(\sigma_v^2+\sigma_e^2)+\beta_t^2\Sigma_{t-1}\Delta t\sigma_v^4+\sigma_v^4\sigma_u^2+\gamma_t^2\sigma_v^4\sigma_e^2+2\beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2}{(\beta_t^2\Sigma_{t-1}+(\beta_t+\gamma_t)^2\sigma_v^2+\sigma_u^2)\,\sigma_e^2+(\beta_t^2\Sigma_{t-1}+\sigma_u^2)\,\sigma_v^2}\,\Delta t, \end{aligned} \tag{I.27}$$

which proves the formula for Σ_t.

Optimal Strategy of Speculator: At each $t=1,\ldots,T$, the speculator maximizes the expected profit: $\pi_t = \max\sum_{\tau=t}^{T}\mathsf{E}\big((v_T-p_\tau)\Delta x_\tau\big)$. We prove by backward induction that the value function is quadratic and of the form given in (I.8): $\pi_t = \alpha_{t-1}(v_{t-1}-q_t)^2 + \alpha_{t-1}'(\Delta v_t)^2+\alpha_{t-1}''(v_{t-1}-q_t)\Delta v_t+\delta_{t-1}$. At the last decision point ($t=T$) the next value function is zero, i.e., $\alpha_T=\alpha_T'=\alpha_T''=\delta_T=0$, which are the terminal conditions (I.10).

This is the transversality condition: no money is left on the table. In the induction step, if $t = 1, \ldots, T-1$, we assume that π_{t+1} is of the desired form. The Bellman principle of intertemporal optimization implies

$$\pi_t \;=\; \max_{\Delta x} \; \mathsf{E}\Big((v_t - p_t)\Delta x + \pi_{t+1} \mid \mathcal{I}_t^q, v_t, \Delta v_t\Big). \tag{I.28}$$

The last two equations in (I.6) imply that the quote q_t evolves by $q_{t+1} = q_t + l_t \Delta y_t + \mu_t \Delta z_t$, where $l_t = \lambda_t - \rho_t \mu_t$. This implies that the speculator's choice of Δx affects the trading price and the next quote by

$$\begin{aligned} p_t &= q_t + \lambda_t(\Delta x + \Delta u_t), \\ q_{t+1} &= q_t + l_t(\Delta x + \Delta u_t) + \mu_t \Delta z_t. \end{aligned} \tag{I.29}$$

Substituting these into the Bellman equation, we get

$$\begin{aligned} \pi_t \;=\; & \max_{\Delta x} \; \mathsf{E}\Big(\Delta x(v_{t-1} + \Delta v_t - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t) \\ & + \alpha_t (v_{t-1} + \Delta v_t - q_t - l_t \Delta x - l_t \Delta u_t - \mu_t \Delta v_t - \mu_t \Delta e_t)^2 \;+\; \alpha_t' \Delta v_{t+1}^2 \\ & + \alpha_t''(v_{t-1} + \Delta v_t - q_t - l_t \Delta x - l_t \Delta u_t - \mu_t \Delta v_t - \mu_t \Delta e_t)\Delta v_{t+1} \;+\; \delta_t\Big) \\ =\; & \max_{\Delta x} \; \Delta x(v_{t-1} - q_t + \Delta v_t - \lambda_t \Delta x) \\ & + \alpha_t\Big((v_{t-1} - q_t - l_t \Delta x + (1-\mu_t)\Delta v_t)^2 + (l_t^2 \sigma_u^2 + \mu_t^2 \sigma_e^2)\Delta t\Big) \;+\; \alpha_t' \sigma_v^2 \Delta t \\ & + \; 0 \; + \; \delta_t. \end{aligned} \tag{I.30}$$

The first order condition with respect to Δx is

$$\Delta x \;=\; \frac{1 - 2\alpha_t l_t}{2(\lambda_t - \alpha_t l_t^2)}(v_{t-1} - q_t) \;+\; \frac{1 - 2\alpha_t l_t (1-\mu_t)}{2(\lambda_t - \alpha_t l_t^2)}\Delta v_t, \tag{I.31}$$

and the second order condition for a maximum is $\lambda_t - \alpha_t l_t^2 > 0$, which is (I.11). Thus, the optimal Δx is indeed of the form $\Delta x_t = \beta_t (v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t$, where $\beta_t \Delta t$ and γ_t

are as in (I.9). We substitute Δx_t in the formula for π_t to obtain

$$
\begin{aligned}
\pi_t \;=\;& \Big(\beta_t\Delta t(1-\lambda_t\beta_t\Delta t)+\alpha_t(1-l_t\beta_t\Delta t)^2\Big)(v_{t-1}-q_t)^2 \\
&+\Big(\alpha_t(1-\mu_t-l_t\gamma_t)^2+\gamma_t(1-\lambda_t\gamma_t)\Big)\Delta v_t^2 \\
&+\Big(\beta_t\Delta t+\gamma_t(1-2\lambda_t\beta_t\Delta t)+2\alpha_t(1-l_t\beta_t\Delta t)(1-\mu_t-l_t\gamma_t)\Big)(v_{t-1}-q_t)\Delta v_t \\
&+\alpha_t\big(l_t^2\sigma_u^2+\mu_t^2\sigma_e^2\big)\Delta t \;+\; \alpha_t'\sigma_v^2\Delta t \;+\; \delta_t.
\end{aligned}
\tag{I.32}
$$

This proves that indeed π_t is of the form $\pi_t=\alpha_{t-1}(v_{t-1}-q_t)^2+\alpha_{t-1}'(\Delta v_t)^2+\alpha_{t-1}''(v_{t-1}-q_t)\Delta v_t+\delta_{t-1}$, with α_{t-1}, α_{t-1}', α_{t-1}'' and δ_{t-1} as in (I.9). □

We now briefly discuss the existence of a solution for the recursive system given in Theorem 2. The system of equations (I.7)–(I.9) can be numerically solved backwards, starting from the boundary conditions (I.10). We also start with an arbitrary value of $\Sigma_T > 0$.[34] By backward induction, suppose α_t and Σ_t are given. One verifies that equation (I.7) for Σ_t implies

$$
\Sigma_{t-1} \;=\; \frac{\Sigma_t\big(\sigma_v^2\sigma_u^2+\sigma_v^2(\sigma_u^2+\gamma_t^2\sigma_e^2)\big)-\sigma_v^2\sigma_u^2\sigma_e^2\Delta t}{\big(\sigma_u^2\sigma_e^2+\sigma_v^2(\sigma_u^2+\gamma_t^2\sigma_e^2)+\beta_t^2\Delta t^2\sigma_v^2\sigma_e^2-2\gamma_t\beta_t\Delta t\sigma_v^2\sigma_e^2\big)-\Sigma_t\beta_t^2\Delta t\big(\sigma_v^2+\sigma_e^2\big)}.
\tag{I.33}
$$

Then, in equation (I.7) we can rewrite λ_t, μ_t, l_t as functions of $(\Sigma_t, \beta_t, \gamma_t)$ instead of $(\Sigma_{t-1}, \beta_t, \gamma_t)$. Next, we use the formulas for β_t and γ_t to express λ_t, μ_t, l_t as functions of $(\lambda_t, \mu_t, l_t, \alpha_t, \Sigma_t)$. This gives a system of polynomial equations, whose solution λ_t, μ_t, l_t depends only on (α_t, Σ_t). Numerical simulations show that the solution is unique under the second order condition (I.11), but the authors have not been able to prove theoretically that this is true in all cases. Once the recursive system is computed for all $t = 1, \ldots, T$, the only condition left to do is to verify that the value obtained for Σ_0 is the correct one. However, unlike in Kyle (1985), the recursive equation for Σ_t is not linear, and therefore the parameters cannot be simply rescaled. Instead, one must

[34] Numerically, it should be of the order of Δt.

numerically modify the initial choice of Σ_T until the correct value of Σ_0 is reached.

I.2 Discrete Time Benchmark Model

We use the notations from Section 2. Denote by $\mathcal{I}_t^q = \{\Delta z_\tau\}_{\tau \le t} \cup \{\Delta y_\tau\}_{\tau \le t-1}$ the dealer's information set just before trading at t, and by $\mathcal{I}_t^p = \{\Delta z_\tau\}_{\tau \le t} \cup \{\Delta y_\tau\}_{\tau \le t} = \mathcal{I}_t^q \cup \{\Delta y_t\}$ the information set just after trading at t. The zero profit condition for the competitive dealer translates into the formulas

$$q_t \;=\; \mathsf{E}(v_t \mid \mathcal{I}_t^q), \qquad p_t \;=\; \mathsf{E}(v_t \mid \mathcal{I}_t^p). \tag{I.34}$$

We also denote

$$\Sigma_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^p), \qquad \Omega_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^q). \tag{I.35}$$

The next result shows that if the pricing rule is linear, the speculator's strategy is also linear, and furthermore it only has a forecast error component, $\beta_t(v_t - q_t)\Delta t$.

Theorem 3. *Any linear equilibrium must be of the form*

$$\begin{aligned} \Delta x_t &= \beta_t (v_t - q_t)\Delta t, \\ p_t &= q_t + \lambda_t \Delta y_t, \\ q_t &= p_{t-1} + \mu_{t-1}\Delta z_t, \end{aligned} \tag{I.36}$$

for $t = 1, \ldots, T$, where by convention $p_0 = 0$, and β_t, γ_t, λ_t, μ_t, Ω_t, and Σ_t are constants *that satisfy*

$$
\begin{aligned}
\lambda_t &= \frac{\beta_t \Sigma_t}{\sigma_u^2}, \\
\mu_t &= \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}, \\
\Omega_t &= \frac{\Sigma_t \sigma_u^2}{\sigma_u^2 - \beta_t^2 \Sigma_t \Delta t}, \\
\Sigma_{t-1} &= \Sigma_t + \frac{\beta_t^2 \Sigma_t^2}{\sigma_u^2 - \beta_t^2 \Sigma_t \Delta t}\,\Delta t - \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}\,\Delta t.
\end{aligned}
\tag{I.37}
$$

The value function of the speculator is quadratic for all $t = 1, \ldots, T$*:*

$$
\pi_t = \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}. \tag{I.38}
$$

The coefficients of the optimal trading strategy and the value function satisfy

$$
\begin{aligned}
\beta_t \Delta t &= \frac{1 - 2\alpha_t \lambda_t}{2\lambda_t(1 - \alpha_t \lambda_t)}, \\
\alpha_{t-1} &= \beta_t \Delta t(1 - \lambda_t \beta_t \Delta t) + \alpha_t(1 - \lambda_t \beta_t \Delta t)^2, \\
\delta_{t-1} &= \alpha_t\big(\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2)\big)\Delta t + \delta_t.
\end{aligned}
\tag{I.39}
$$

The terminal conditions are

$$
\alpha_T = \delta_T = 0. \tag{I.40}
$$

The second order condition is

$$
\lambda_t(1 - \alpha_t \lambda_t) > 0. \tag{I.41}
$$

Given Σ_0*, conditions (I.37)–(I.41) are necessary and sufficient for the existence of a linear equilibrium.*

Proof. First, we show that equations (I.37) are equivalent to the zero profit conditions of the dealer. Second, we show that equations (I.39)–(I.41) are equivalent to the speculator's strategy being optimal.

Zero Profit of dealer: Start with with the dealer's update due to the order flow at $t = 1, \ldots, T$. Conditional on $\mathcal{I}_t^q$, v_t has a normal distribution, $v_t | \mathcal{I}_t^q \sim \mathcal{N}(q_t, \Omega_t)$. The aggregate order flow at t is of the form $\Delta y_t = \beta_t (v_t - q_t)\Delta t + \Delta u_t$. Denote by

$$\Phi_t \;=\; \mathsf{Cov}(v_t - q_t, \Delta y_t) \;=\; \beta_t \Omega_t \Delta t. \tag{I.42}$$

Then, conditional on $\mathcal{I}_t^p = \mathcal{I}_t^q \cup \{\Delta y_t\}$, $v_t \sim \mathcal{N}(p_t, \Sigma_t)$, with

$$\begin{aligned}
p_t \;&=\; q_t + \lambda_t \Delta y_t, \\
\lambda_t \;&=\; \Phi_t \,\mathsf{Var}(\Delta y_t)^{-1} \;=\; \frac{\beta_t \Omega_t}{\beta_t^2 \Omega_t \Delta t + \sigma_u^2}, \\
\Sigma_t \;&=\; \mathsf{Var}(v_t - q_t) - \Phi_t \,\mathsf{Var}(\Delta y_t)^{-1} \Phi_t' \;=\; \Omega_t - \frac{\beta_t^2 \Omega_t^2}{\beta_t^2 \Omega_t \Delta t + \sigma_u^2}\,\Delta t \\
&=\; \frac{\Omega_t \sigma_u^2}{\beta_t^2 \Omega_t \Delta t + \sigma_u^2}.
\end{aligned} \tag{I.43}$$

To obtain the equation for λ_t, note that the above equations for λ_t and Σ_t imply $\frac{\lambda_t}{\Sigma_t} = \frac{\beta_t}{\sigma_u^2}$. The equation for Ω_t is obtained by solving for Σ_t in the last equation of (I.43).

Next, consider the dealer's update at $t = 1, \ldots, T$ due to the signal $\Delta z_t = \Delta v_t + \Delta e_t$. From $v_{t-1} | \mathcal{I}_{t-1}^p \sim \mathcal{N}(p_{t-1}, \Sigma_{t-1})$, we have $v_t | \mathcal{I}_{t-1}^p \sim \mathcal{N}(p_{t-1}, \Sigma_{t-1} + \sigma_v^2 \Delta t)$. Denote by

$$\Psi_t \;=\; \mathsf{Cov}(v_t - p_{t-1}, \Delta z_t) \;=\; \sigma_v^2 \Delta t. \tag{I.44}$$

Then, conditional on $\mathcal{I}_t^q = \mathcal{I}_{t-1}^p \cup \{\Delta z_t\}$, $v_t | \mathcal{I}_t^q \sim \mathcal{N}(q_t, \Omega_t)$, with

$$\begin{aligned}
q_t \;&=\; p_{t-1} + \mu_t \Delta z_t, \\
\mu_t \;&=\; \Psi_t \,\mathsf{Var}(\Delta z_t)^{-1} \;=\; \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}, \\
\Omega_t \;&=\; \mathsf{Var}(v_t - p_{t-1}) - \Psi_t \,\mathsf{Var}(\Delta z_t)^{-1} \Psi_t' \;=\; \Sigma_{t-1} + \sigma_v^2 \Delta t - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_e^2}\,\Delta t \\
&=\; \Sigma_{t-1} + \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}\,\Delta t.
\end{aligned} \tag{I.45}$$

Thus, we prove the equation for μ_t. Note that equation (I.45) gives a formula for Σ_{t-1} as a function of Ω_t, and we already proved the formula for Ω_t as a function of Σ_t in (I.37). We therefore get Σ_{t-1} as a function of Σ_t, which is the last equation in (I.37).

Optimal Strategy of Speculator: At each $t = 1, \ldots, T$, the speculator maximizes the expected profit: $\pi_t = \max \sum_{\tau=t}^{T} \mathsf{E}\big((v_T - p_\tau)\Delta x_\tau\big)$. We prove by backward induction that the value function is quadratic and of the form given in (I.38): $\pi_t = \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}$. At the last decision point ($t = T$) the next value function is zero, i.e., $\alpha_T = \delta_T = 0$, which are the terminal conditions (I.40). In the induction step, if $t = 1, \ldots, T-1$, we assume that π_{t+1} is of the desired form. The Bellman principle of intertemporal optimization implies

$$\pi_t \;=\; \max_{\Delta x} \; \mathsf{E}\Big((v_t - p_t)\Delta x + \pi_{t+1} \mid \mathcal{I}_t^q, v_t, \Delta v_t\Big). \tag{I.46}$$

The last two equations in (I.36) show that the quote q_t evolves by $q_{t+1} = q_t + l_t \Delta y_t + \mu_t \Delta z_{t+1}$. This implies that the speculator's choice of Δx affects the trading price and the next quote by

$$\begin{aligned} p_t \;&=\; q_t + \lambda_t(\Delta x + \Delta u_t), \\ q_{t+1} \;&=\; q_t + \lambda_t(\Delta x + \Delta u_t) + \mu_t \Delta z_{t+1}. \end{aligned} \tag{I.47}$$

Substituting these into the Bellman equation, we get

$$\begin{aligned} \pi_t \;&=\; \max_{\Delta x} \; \mathsf{E}\Big(\Delta x(v_t - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t) \\ &\qquad + \alpha_t(v_t + \Delta v_{t+1} - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t - \mu_t \Delta z_{t+1})^2 \;+\; \delta_t\Big) \\ &=\; \max_{\Delta x} \; \Delta x(v_t - q_t - \lambda_t \Delta x) \\ &\qquad + \alpha_t\Big((v_t - q_t - \lambda_t \Delta x)^2 + (\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2))\Delta t\Big) \;+\; \delta_t. \end{aligned} \tag{I.48}$$

The first order condition with respect to Δx is

$$\Delta x = \frac{1 - 2\alpha_t \lambda_t}{2\lambda_t(1 - \alpha_t \lambda_t)}(v_t - q_t), \tag{I.49}$$

and the second order condition for a maximum is $\lambda_t(1 - \alpha_t \lambda_t) > 0$, which is (I.41). Thus, the optimal Δx is indeed of the form $\Delta x_t = \beta_t(v_t - q_t)\Delta t$, where $\beta_t \Delta t$ satisfies equation (I.39). We substitute Δx_t in the formula for π_t to obtain

$$\pi_t = \Big(\beta_t \Delta t(1 - \lambda_t \beta_t \Delta t) + \alpha_t(1 - \lambda_t \beta_t \Delta t)^2\Big)(v_t - q_t)^2 + \alpha_t\big(\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2)\big)\Delta t + \delta_t. \tag{I.50}$$

This proves that indeed π_t is of the form $\pi_t = \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}$, with α_{t-1} and δ_{t-1} as in (I.39). $\square$

Equations (I.37) and (I.39) form a system of equations. As before, it is solved backwards, starting from the boundary conditions (I.40), and so that $\Sigma_t = \Sigma_0$ at $t = 0$.