A large literature documents that several economic decisions occur infrequently. For instance, individual investors adjust their portfolios sporadically even though the prices of many assets experience large fluctuations at high frequency. Similarly, firms do not reset the price every time the costs of inputs change. These infrequent adjustments at the micro level are a potential source of sluggish behavior at the aggregate level and have thus attracted the interest of macroeconomists (see Stokey 2008). One hypothesis that has been studied is that inaction results from the presence of observation costs, i.e., costs related to the information gathering process, such as those due to the monitoring of the value of equity (in the case of a consumer/investor) or the monitoring of production costs (in the case of a firm). The optimality of economizing attention when information gathering is costly is what we refer to as the “rational inattention” hypothesis.1

Besides the intuitive appeal of observation costs, an important methodological reason that makes it interesting is that the nature of the optimal adjustments implied by this friction is different from the one generated by standard fixed cost, and this translates into different implications for aggregate behavior. Duffie and Sun (1990); Gabaix and Laibson (2002); and Abel, Eberly, and Panageas (2007) show that with observation costs the optimal rule implies time-dependent adjustments, as opposed to state-dependent adjustments that are typical in the standard fixed-cost literature. Abel, Eberly, and Panageas (2009) show that a time-dependent rule may be optimal even in an environment with both observation and transactions costs. Understanding the nature of the decision rule matters because the aggregation of agents following time-dependent rules is different from the one of agents following state-dependent rules as argued by Gabaix and Laibson (2002) and Alvarez, Atkeson, and Edmond (2009) in the consumption-savings and price-setting literature, respectively. Finally,
the modeling of inattentive investors, whose trading in financial markets is only sporadic, may be important to understand the dynamics of assets risk premia, as argued by Duffie (2010), and its volatility, as argued by Chien, Cole, and Lustig (2010).

In spite of the theoretical developments that rational inattention has inspired, micro evidence on rational inattention lags behind. Empirical evidence on the most immediate consequence of observation costs—the infrequent observation of state variables—is not available in standard datasets. We contribute to filling the gap with two novel household surveys that record the frequency with which investors observe the value of their financial investments, as well as the frequency with which they trade assets and durable goods. We consider models with both observation and transaction cost, since the latter are a standard explanation for infrequent adjustments. We use these data to test key predictions of existing rational inattention models and to quantify the relative importance of the observation cost relative to standard transactions costs. We find that to match important patterns in the data and to distinguish between both types of cost we need to introduce a model that shifts the focus from nondurable to durable consumption. This new model implies a mixture of time-dependent and state-dependent rules, where the “importance” of each rule depends on the relative magnitude of the observation and transaction costs.

Our starting point is the models developed by Duffie and Sun (1990); Gabaix and Laibson (2002); Reis (2006); and Abel, Eberly, and Panageas (2007, 2009). These seminal contributions explore the consequences of observation costs in the context of an investor’s optimal savings/portfolio problem that includes the optimal management of a low return liquid asset, required to pay for transactions, analogous to the monetary models with a cash in advance constraint. In Section I we introduce two original datasets that are tailor made to provide detailed evidence on the patterns of nondurable consumption, management of liquid assets, information collection, and trade in financial assets (purchase or liquidation of assets). We find a robust pattern consistent with the assumption that a component of adjustment costs is information gathering, namely that the frequency of trading, the frequency of observation, and the time spent collecting financial information are strongly correlated across investors. However, we find evidence against two specific mechanisms operating in several of the models that focus on nondurable goods and liquid assets. In particular, the models of Duffie and Sun (1990); Gabaix and Laibson (2002); and Abel, Eberly, and Panageas (2007) predict that the observed frequency of observation and financial transactions should coincide, and also predict that the household liquid asset holdings (e.g., the average M1 or M2 balances) should decrease with the frequency of trades in assets. Our analysis shows that both predictions are poorly borne out in the data: the frequency of information acquisition is at least three times larger than the frequency of portfolio trades. Moreover, the data do not display a negative correlation between the household liquid assets and the frequency of asset trades.

Motivated by both the encouraging evidence on the presence of observation cost featured by these models, as well as by the empirical shortcomings specific to the mechanism involving nondurable consumption and liquid assets, Section II develops a new model that preserves a role for costly observations while abstracting from nondurable consumption and liquid assets. We consider a model with both observation and transaction costs and depart from the previous literature by focusing on durable, as opposed to nondurable, consumption goods. This shift has two direct
implications. First, in the model with durable consumption, goods purchases will be large and occur at discrete intervals, and, thus, they will require to hold essentially no liquid assets. Second, and more important, in the model with durable consumption with both observation and transactions costs, the observation frequency is always larger than the trading frequency, as it is in the data. The reason for this result is that durable goods and transactions costs give rise to an inaction region, just as in Grossman and Laroque (1990) and Stokey (2009), where the agent tolerates moderate deviations of the stock of durable goods from the frictionless benchmark. Thus, every time the agent observes her wealth and finds it to be in the inaction region, the model produces an observation without a trade.

In Section III we use numerical simulations of the model to gauge the order of magnitude of observation and transactions costs that are consistent with observed investors’ behavior. This exercise shows that very small observation costs are sufficient to reproduce the frequency of observation that is found in the data. Section IV tests two novel predictions of the durable good model using panel data from two household surveys and one administrative dataset. First, that the frequencies of assets transactions and that of durable adjustment should be positively correlated, both across investors and across time for a given investor. Second, that since more risk tolerant individuals invest more in volatile assets, they value information more and thus observe more frequently. The data lend support to both predictions. Additionally we document that the frequency and size of sales of financial assets spike just before house purchases.

Section V concludes with a discussion of our quantitative findings, and a comparison with the findings in Alvarez, Lippi, and Paciello (2011) on firms’ price setting behavior in a model with observation and menu costs. We also discuss the role of labor income, the possibility that some portfolio observations are available at no cost, and other issues for future research.

I. Observations, Trades, and Liquidity: Theory versus Data

This section reviews some evidence related to a class of models that use the rational inattention hypothesis to study consumption, savings, portfolio theory, and liquidity. In these models the relevant decisions concern the rate of consumption—or savings—and the portfolio composition; the costs are those associated with keeping track of the information about financial variables. Examples of these models are Duffie and Sun (1990); Gabaix and Laibson (2002); Sims (2005); Reis (2006); and Abel, Eberly, and Panageas (2007, 2009) among others.

These models carry neat implications on at least two potentially observable behaviors: first, since investors do their best to avoid collecting information when it is not needed, they will choose to keep the frequency of observations as close as they can to the frequency of financial trades. In models with an observation cost only, such as Duffie and Sun (1990); Gabaix and Laibson (2002); and Abel, Eberly, and Panageas (2007), the two frequencies actually coincide, providing a clear prediction. In Abel, Eberly, and Panageas (2009), where both information and transaction costs are present, the authors show that this prediction holds in finite time with probability one, provided that the transaction cost is sufficiently small. While the authors do not characterize the decision rules for the model with larger transaction cost, these
predictions are surely violated for large enough values of the transaction cost, in which case the model in Abel, Eberly, and Panageas (2009) implies a higher frequency of observation than transactions. The second prediction is implied by the presence of a liquidity-in-advance constraint for the purchase of nondurable goods. This assumption, together with the presence of an opportunity cost on the illiquid assets and of transaction costs between the savings and liquid account, gives rise to a Baumol-Tobin-type management of liquid assets, where the investors who trade more frequently have lower average holdings of liquid assets. In the online Appendix B we provide a description of such a model. In this section we bring both of these predictions to bear with a novel set of data that contains information on how frequently people choose to observe the value of their financial assets and trade them, as well as data on the value and composition of their liquid and financial assets.

A. Data Sources

Our empirical evidence relies on two different sources. The first is a sample of about 1,800 Italian investors with an account at Unicredit Bank, one of the largest banking groups in Europe (for some statistics we use a sample of 40,000 investors), and we refer to this source as UCS. We based most of the analysis on the first survey wave, run in 2003, but occasionally we also rely on data from the 2007 wave. The novel and original feature of this survey is the wealth of information that it has on the frequency with which people gather information on their financial investments and make financial transactions, as well as on investors’ risk preferences, assets, and demographics, which provide an ideal setting for testing predictions of models that emphasize information and transaction costs in household savings and financial decisions. We complement the survey with a 35-month panel of administrative records of 26 different accounts for the same investors. The second source is the 2004 Survey of Households Income and Wealth—a widely used survey on a sample of about 8,000 Italian households managed by the Bank of Italy. This dataset has two useful features: first, unlike UCS, it collects detailed data on durable purchases; this particular wave also has information on the frequency with which investors make financial trades. Both will prove important in Section IV to test the predictions of the model we develop in Section II. Second, while UCS is representative of the population of Unicredit customers, it is not of the Italian population; but SHIW is, and this allows us to make sure that our findings with UCS are not the reflection of sample selection (see the online Appendices A and F and the dataset by Alvarez, Guiso, and Lippi (2012) for a detailed description of the data sources and replication codes).

B. Key Variables Description

We now describe in detail two key variables for our analysis: the frequency with which investors observe investments and the frequency with which they trade financial assets. More details on the two surveys and the variables that we use are given in the online data Appendix.

To our knowledge, UCS is the first large scale survey to collect information on how frequently people check their financial investments and make financial trades. In the 2003 wave sample participants were asked: “How often do you check the
value of your financial investments?” They could answer: (a) every day; (b) at least once a week; (c) every 15 days; (d) once a month; (e) about every three months; (f) about every six months; (g) about once in a year; (h) less than once a year; (i) never check; (l) I have no investments. To obtain information on the frequency of financial trades they were asked: “How often do you change the composition of your financial portfolio and sell or buy financial assets?” The options are: (a) every day; (b) at least once a week; (c) about every two weeks; (d) about every month; (e) about every three months; (f) about every six months; (g) about every year; (h) less than once a year; (i) at maturity; (l) never; (m) I have no investments. Notice that in principle the answer to the question on trading might involve trades that do not give rise to net liquidation of assets, but only to a portfolio “rebalancing.” Below we use an original set of actual transactions data from a sample of Unicredit Bank customers to document the relevance of “rebalancing trades.”

Similar questions were asked in the 2007 UCS wave while the trade frequency question, with the same wording, was also asked in the 2004 SHIW. The only difference with respect to the UCS question is that the first two answers are lumped together as “at least once a week.” Obviously, questions apply only to active investors, implying that some observations (316 in UCS 2003 out of 1,834 participants) will be lost. Next, we use these data to confront two predictions of the rational inattention models.

C. Patterns on Portfolio Trades and Observations

Table 1 shows the joint distribution of the frequency of observing and that of asset trading among the 2003 UCS investors. The table documents several noteworthy features. First, the large mass of observations on the main diagonal of the table shows that there is a strong positive correlation between the frequency with which agents observe their investments and the frequency of asset trading; the correlation is 0.45 with a p-value smaller than 1 percent. Those who observe the portfolio more often also tend to trade more often. This evidence is consistent with the idea, at the core of costly information models, that the trading and information gathering
activity are related. Second, in only a handful of cases (6 percent of the observations), investors trade more frequently than they observe. The fact that few investors trade more often than they observe may be due to minor measurement errors (reassuring about the quality of our indicators of observing and trading frequencies) or reflect rare cases where investors trade blindly.2

Table 2 reports summary statistics on the frequency of observing and trading for different groups of investors both from the UCS 2003 and the SHIW 2004 surveys. Consistently with models that stress information gathering costs and assets trading costs, investors observe their investments and trade assets infrequently. The median number of portfolio observations per year in the sample of UCS investors (as well as for stockholders) is 12, while the median number of asset trades is 2. Smaller asset trading frequencies are estimated for the investors in the SHIW (in the lower panel of the table). The SHIW statistics, computed on a sample that is representative of the Italian investors, are comparable to those observed for US households.3

The table also reports an estimate for the median number of observations in the SHIW sample, imputed from a regression estimated on the UCS data (see the note

2 The August 2010 version of Abel, Eberly, and Panageas (2009) introduces a notion of automatic transfers (transfers that take place without observations) which is able to explain cases where the number of trades is larger than the number of observations. In a related price-setting problem Alvarez, Lippi, and Paciello (2011) show that multiple adjustments between observations may be optimal if the drift of the state variable is above a threshold.

3 ICI (2005) reports information on US equity investors. Figure 33 shows that for 1998, 2001, and 2004 the fraction of equity investors who make no equity trades during a year is 0.58, 0.6, and 0.6 respectively. Assuming a Poisson distributed number of trades with constant intensity this implies an average of about 0.51 trades per year. A somewhat higher statistic is computed by Bonaparte and Cooper (2009) who, using the Survey of Consumer Finances, report that the fraction of households owning stocks that do not adjust their portfolio in one year is about 0.3, which implies an average of 1.2 trades per year.
to the table for more details). The frequency of observation for the equity investor in the SHIW sample is about one-half that for the UCS sample of equity investors.

Though this evidence is consistent with the costly observation hypothesis, it also departs from it in one dimension that is featured in the models with only an observation cost: the data show that investors do not trade every time they observe the value of their investments. It appears that the frequency of trading and the frequency of observation coincide for only 28 percent of the investors (those along the diagonal in Table 1); for 67 percent of the investors the observation frequency is higher than the trading frequency. Thus, only a minority of the investors in the sample conform to the prediction that every observation triggers a trade. Table 2 confirms that investors observe the value of their investments more frequently than they trade, with a ratio between the two average frequencies around three. This pattern holds across investors type, asset levels, and trade frequency. We will argue below that in order to account for this empirical fact, we need to supplement the costly-observation models with a transaction cost, along the lines of Abel, Eberly, and Panageas (2009).

We stress that these patterns are unlikely to be the reflection of some particular feature of the survey wave. To address this concern we reproduced Table 1 using the 2007 UCS. The new joint distribution (not reported) has the same features as the one based on the 2003 wave: only 3.5 percent of the investors trade more frequently than they observe their investments; 24 percent equally frequently and 72.5 percent less frequently, while the average number of observations stays in a ratio of three to one to the number of trades.

One may also worry about measurement errors in the survey data on the frequency of observation and trading. To assess the quality of the observation frequency, we rely on an independent measure available in the 2003 UCS of the amount of financial information investors collect from various sources (such as newspapers, the web, their advisors or the companies’ accounting statements) before making an investment decision. This is a broad measure of the time investors devote to gathering financial information. One would expect that investors who collect more financial information also observe the value of their investments more frequently. We find it reassuring that this correlation appears clearly in the data, as shown in Figure A2 in the online Appendix A.

Concerning the frequency of trading one may question whether the survey measure is a good proxy of the theoretical notion: in the inventory models we described above trading means transferring resources from a savings to a liquid account; in the data, given the wording of the question, some of the trades may involve portfolio rebalancing with no transfers to or from the liquid account. Notice that the presence of rebalancing trades would make the positive gap between observing and trading frequency measured above an underestimate of the theoretically relevant one. Since no information is available in the survey on the nature of the trades, we have resorted to administrative data available for the 2007 Unicredit sample to get a sense of the importance of rebalancing trades. To this end we defined two notions of “rebalancing” trades. The first is a broad notion estimated by assuming that a rebalancing occurs any time there is net sale of one of the financial assets in the investments portfolio and at least one net purchase of another asset. Hence, the number of rebalancing trades in each month is equal to the minimum between the number of net sales and the number of net purchases. In the whole sample the number of trades with
rebalancing is 1.13 per year (median 0.34). Stockholders rebalance almost twice as often as nonstockholders, a reasonable feature. The second notion is narrower, as it considers as “rebalancing trades” all those that involve a simultaneous purchase and sales of two different investment classes but no net liquidation/purchase of investments. For these trades the value of asset sales matches the value of asset purchases. The mean number of rebalancing trades for this measure is smaller than the previous one (0.09 per year and around 0.14 for stockholders). Depending on whether we use the broad or narrow measure of rebalancing frequency, this evidence shows that the share of rebalancing to total trades ranges between 2 percent to 20 percent of total trades (see Tables F5 and F6 in the online Appendix F).4

D. Patterns on Portfolio Trades and Household Liquidity

If portfolio trades are mostly aimed at transferring resources from the asset account to the liquid account, then those agents who trade more frequently should, on average, hold fewer liquid assets. Indeed, the inventory-type models in the costly information literature, such as Duffie and Sun (1990); Gabaix and Laibson (2002); and Abel, Eberly, and Panageas (2007, 2009), predict a negative unitary elasticity between liquid assets holdings (scaled by nondurable consumption) and the number of financial trades.

We test this prediction using both UCS data and SHIW data. Liquid assets are defined as the sum of cash holdings, checking and savings accounts—a measure close to M2—but results are unaffected if we use a narrower definition that includes only cash and checking accounts. Since UCS collects no consumption data we impute it using the 2004 SHIW to estimate nondurable consumptions (see the online Appendix A for details). We then construct the ratio between liquid assets and consumption. Figure 1 plots both the empirical relation between the average level of liquid assets ratio in the sample and the number of trades and its theoretical counterpart as predicted by the inventory models. Panel A uses the SHIW data, and Panel B uses the UCS data. Contrary to the model prediction we find a weak correlation between liquid assets holdings and the number of trades. In both surveys the unconditional correlation is, if anything, slightly positive. This finding is quite robust as we discuss next.

Table 3 shows that the lack of correlation also emerges from multivariate regressions that condition on the cross-sectional variation in income, household size, age, and the importance of labor income over total income. The reason for adding the latter variable is that one might be concerned that the match with the model is far from perfect, and that modeling labor income would change the results on the relation between the level of liquid asset and the frequency of transactions. As a preliminary control

4Specifically, for the sample interviewed in 2006 we have information on the stock and net trading flows of 26 assets categories these investors have at Unicredit. One of these categories is the liquidity (checking) account. These data are available at a monthly frequency for 36 months beginning in December 2006. Since for each asset category we observe the net trades separately from the end of period stocks, we can compute measures of trading frequency. We first obtain a measure of total trades. For this we define a trade in a month as a situation where at least one investment class out of 25 (thus excluding liquid asset) has a net positive or negative flow. Implicit is the idea that a trade is a trip to the bank/broker to buy and/or sell one or more assets. The average annual number of total trades is then obtained. The online Appendix F discusses these estimates and how they relate to the frequency of trading reported in the survey analogous to the one in Table 2. The mean number of total trades is 4.5 (median 3.4); stockholders trade more frequently, and these measures are positively correlated with self-reported measures of trading frequency.
for this we include the ratio of labor income to nondurable consumption in the regressions: the household liquidity remains uncorrelated, if anything slightly positively correlated, with the frequency of trades in assets (see footnote 7 for more details).

One could argue that if individuals vary according to the uninsurable idiosyncratic risks they face, omitting this variable could be responsible for the lack of correlation between liquid assets holdings and trading frequency. To address this concern we add proxies for idiosyncratic labor income risk, namely, a dummy for self-employment and a dummy for government employees; in the Unicredit sample we also insert a specific measure of background risk by adding a dummy if the

Table 3—Liquidity (M2) versus Trade Frequency (Asset transactions)

<table>
<thead>
<tr>
<th></th>
<th>SHIW data</th>
<th>Unicredit data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bivariate</td>
<td>Multivariate</td>
</tr>
<tr>
<td>All investors</td>
<td>(2,808)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Trade frequency</td>
<td>0.005</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Equity investors</td>
<td>(1,535)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Trade frequency</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Notes: Based on the 2004 SHIW and 2003 Unicredit surveys. All variables in logs. All regressions include a constant; standard errors in parentheses. $M/c$ is the liquid asset–to–consumption ratio; $M$ is M2 (similar results obtained for M1); $c$ is nondurable consumption (same results if total consumption is used); for the Unicredit survey consumption is imputed from a regression in the SHIW data using income and other demographics.

a Regression coefficient of bivariate OLS.
b The regressions include the following controls (all in logs): household income, percent of labor income over nondurable consumption, age of household head, number of adults.
c Multivariate regression that includes three measures of background risk: two occupational dummies for self-employed and government employee and a measure of background risk using an indicator of income risk that is available in the survey (see the online Appendix G).
respondent reports that he/she is unable to predict whether his/her income will fall significantly, increase significantly, or remain unchanged in the five years following the interview. Adding these controls leaves results unchanged, as shown in the last column of Table 3. The correlation between liquid assets holdings and trading frequency is essentially the same if we estimate it on the sole sample of government employees who face little or no income risk.

Finally, similar results are obtained if we use a narrow measure of liquid assets (M1) instead of a broad one, if we look at median liquid assets rather than means, and if we scale liquid assets with total financial assets (in this case the simple correlation is somewhat negative both in the UCS sample and in the SHIW, but the elasticity is far from the predicted unit value).5

Summing up, two predictions of the model with nondurable consumption and information cost only—a one-to-one relation between the frequency of observing and that of trading financial assets and a one-to-one (negative) relation between liquid assets and the number of trades—find only partial support in the data. We think that one reason behind the weak evidence on this mechanism lies in the reliance on trades between financial assets and transaction accounts to derive a theory of liquid assets holdings to finance nondurable consumption. To further this view we contrast the prediction between the transactions frequency and demand for liquidity with the one, of identical nature, that emerges in the realm of currency demand models. We use information on the average currency holdings and the average number of cash withdrawals by Italian households taken from the SHIW survey.6 Table 4 reports the (log) correlation between the average currency balance and the frequency of transactions, measured by the number of cash withdrawals. The correlation is always negative and statistically different from zero (between $-0.2$ and $-0.5$). Despite the presence of large measurement error, the inventory theory of cash management finds strong support in the data, while the inventory theory for liquid assets management implicit in the costly information models does not.7 In the next section we show that introducing transaction and observation costs, and shifting the focus from nondurable to durable consumption, helps reconcile the theory with the data.

II. A Model with Observation and Transaction Costs

This section presents a model that is consistent with two empirical facts documented above: first, that the frequency of portfolio trading is smaller than the

5 To further check whether the lack of correlation is due to the number of trades capturing some unobservable determinant of liquid assets holdings, we regressed (unreported) the (log) liquid assets scaled by consumption (or total assets) on the (log) number of trades, adding demographic controls as well as controls for (log) consumption and assets. We have estimated this regression on several subsamples of investors: all investors, all stockholders, direct stockholders. In all cases we found a small elasticity of liquid assets to the number of financial trades, often positive (estimates in the UCS sample range between 0.02 and 0.10 depending on sample and specification) and always quite far from the negative one-to-one correspondence predicted by the model.

6 See Attanasio, Guiso, and Jappelli (2002) and Alvarez and Lippi (2009) for an analysis of these data.

7 One may think that the omission of labor income, which is credited directly into their liquid account, may be responsible for this empirical shortcoming of the model (just like direct cash transfers, e.g., wages paid in cash, might impinge on the Baumol-Tobin theory of the demand for currency). If this was the case, then controlling for the labor income should reveal a negative relation between average liquidity and the frequency of assets transactions. Instead, the lack of correlation between average liquidity and trades’ frequency persists even in the multivariate regressions where the share of labor income over total nondurable consumption is controlled for (see Table 3).
frequency of portfolio observation. Second, that the frequency of trading is uncorrelated with the liquidity of agents. The main feature of this model is to solve the investor’s problem in the presence of two distinct fixed costs: one for observing the value of the portfolio, another for adjusting the stock of durable goods. One difference compared to the previous literature with costly observation is that the model focuses on durable, as opposed to nondurable, consumption.

The focus on the durable goods creates a disconnect between the household liquid asset holdings and the durable expenditures. This is because the fixed cost gives rise to discrete adjustments, so that even when liquid assets are required to pay for the durable good, the liquidity used for this purchase will be withdrawn and spent immediately. The fact that liquid assets used to pay for expenditures have an infinite velocity implies that durable purchases will generate zero average holdings of liquidity (even though these purchases will affect the average size of liquidity withdrawals). The lack of correlation between the household average liquid asset holdings and the frequency of portfolio transactions is consistent with the evidence of Section I. To generate nonzero money holdings, the model would need to be augmented by including a nondurable consumption component, as in standard inventory models. The optimal inventory model in Alvarez and Lippi (2011) combines large infrequent expenditures with small continuous ones and shows the conditions under which both nonzero average holdings of liquid asset and no-relationship between the frequency of financial asset sales (withdrawals) are obtained.

Here we consider the problem of a household who consumes only durable goods. She derives utility proportional to the stock $h$ of durables, which depreciates at rate $\delta$. Her preferences are given by discounted expected utility and a CRRA period utility $U(h) = h^{1-\gamma}/(1-\gamma)$. The agent’s source of funds to buy/sell durables is her financial wealth $a$, a fraction $\alpha$ of which can be invested in risky securities and the remaining in riskless bonds. Let $s$ be a standard Normal random variable, and $R(s, \tau, \alpha)$ be the gross return during a period of length $\tau$ with portfolio $\alpha$ when the innovation to the return is $s$; we have

$$R(s, \tau, \alpha) \equiv e^{(\alpha \mu + (1-\alpha)r - \frac{\alpha^2 \sigma^2}{2})\tau + \alpha \sigma s \sqrt{\tau}}.$$
This is the gross return to a portfolio that is continuously rebalanced to have fraction \( \alpha \) in the risky asset with instantaneous return with mean \( \mu \) and variance \( \sigma^2 \) per unit of time, i.e., as \( \tau \downarrow 0 \), we have 
\[
\frac{1}{\tau} \mathbb{E}[R(s, \tau, \alpha) - 1] \to \alpha \mu + (1 - \alpha) r \quad \text{and} \quad \frac{1}{\tau} \text{Var}[R(s, \tau, \alpha) - 1] \to \alpha^2 \sigma^2.
\]

There are two frictions in our model. The first one is a fixed cost parameter \( \phi_T \) of trading the durable good, as in Grossman and Laroque (1990). So, if there is an adjustment in the stock of durables, say from \( h \) to \( h' \), the agent loses \( \phi_T h \). We assume that this fixed cost applies to either a change in the stock of durables or a change in the share of risky assets \( \alpha \). This assumption differs from Grossman and Laroque (1990), who assume that the transaction cost applies only to durable adjustments but not to adjustments of the portfolio composition. In our setup their assumption implies that every observation gives rise to a portfolio adjustment, a prediction that is inconsistent with the data of the previous section, in particular with the small number of portfolio rebalancing trades for households. The second friction is a fixed cost parameter \( \phi_o \) that is paid by the agent for observing the value of her financial wealth. To preserve homogeneity and conserve on the state space, the observation cost is assumed to be proportional to the asset value: \( \phi_o a \).

We let \( V(a, h, \alpha) \) denote the value function for an agent who, after paying the observation cost, has a durable stock \( h \), financial wealth \( a \) with a fraction \( \alpha \) invested in risky assets. She decides \( \tau \), the length of time until the next observation date, and whether to pay the cost \( \phi_T \), transfer resources, and adjust both the portfolio share \( \alpha \) and her durables stock to \( h' \). These decisions are subject to the budget constraint
\[
a' + h' + h \phi_T I_{\{h', \alpha' \neq (h, \alpha)\}} = h + a,
\]
where \( I_{\{\cdot\}} \) is an indicator of adjustment (of either the durable stock or the portfolio composition). The Bellman equation is then
\[
V(a, h, \alpha) = \max_{a', h', \alpha', \tau} \int_0^\tau e^{-\rho t} U(h'e^{-\delta t}) \, dt
\]
\[
+ e^{-\rho \tau} \int V(a'(1 - \phi_o)R(s, \tau, \alpha'), h'e^{-\delta \tau}, \alpha') \, dN(s),
\]
subject to the budget constraint (2), where \( N(\cdot) \) is the CDF of the standard Normal distribution, and \( R(s, \tau, \alpha) \) is the gross return during a period of length \( \tau \) of the portfolio with share \( \alpha \) as defined in (1). It is convenient to write the value function as
\[
V(a, h, \alpha) = \max \{ \overline{V}(a, h, \alpha), \hat{V}(a, h) \}.
\]

---

8Moreover, since the value function is locally convex near the inaction boundaries, the agent is locally risk-seeking, and would choose very risky portfolios when the durable stock gets close to the boundaries, as indeed happens in Grossman and Laroque (1990).

9Either one or both of the costs can be written in utility terms, as in Abel, Eberly, and Panageas (2009).
This function picks the best of two conditional value functions: the first one, \( \bar{V}(a, h, \alpha) \), gives the value of leaving \( a, h, \alpha \) unchanged and observing again \( \tau \) periods from now:

\[
\bar{V}(a, h, \alpha) = \max_{\tau} \int_0^\tau e^{-\rho t} U(he^{-\delta t}) \, dt + e^{-\rho \tau} \int_{-\infty}^\infty V(a(1 - \phi_o)R(s, \tau, \alpha), he^{-\delta \tau}, \alpha) \, dN(s).
\]

The policy allows the agent not to pay the adjustment cost \( h \phi_T \), at the cost of keeping the durables stock and the portfolio composition unadjusted. The second conditional value function is

\[
\check{V}(a, h) = \max_{a', \tau, \alpha'} \int_0^\tau e^{-\rho t} U([a + h(1 - \phi_T) - a']e^{-\delta t}) \, dt + e^{-\rho \tau} \int_{-\infty}^\infty V(a'(1 - \phi_o)R(s, \tau, \alpha'), [a + h(1 - \phi_T) - a']e^{-\delta \tau}, \alpha') \, dN(s),
\]

which gives the value of the policy where the agent, upon observing her wealth, adjusts her durable stock so that by equation (2) the postadjustment initial stock of durables is \( h' = a + h(1 - \phi_T) - a' \). She also decides a new observation date \( \tau \) and the share of risky assets \( \alpha' \).

### A. Characterization of the Optimal Policy

This section outlines the nature of the optimal policy for the problem in the presence of both transaction and observation costs (see the online Appendix C for an analysis of the special cases where none, or only one, of the costs is present). We show that in this case there is no perfect synchronization between portfolio observations and portfolio adjustments. A numerical illustration of the workings of the model is given in the next section.

The optimal decision rule for trading-transferring resources and adjusting the durable goods is of the \( sS \) type. This is due to the homogeneity of the value function and to the fact that \( \alpha \) is not a state in problem (5). Notice that for any fixed value of \( \alpha \) the value function \( V(a, h, \alpha) \) and the associated functions \( \check{V}(a, h) \) and \( \bar{V}(a, h, \alpha) \) are all homogenous of degree \( 1 - \gamma \) on \( (a, h) \). The homogeneity follows from the assumptions of homogeneity of \( U(\cdot) \), from the specification of the fixed cost of adjustment as proportional to the value of the current state \( (\phi_o a \) and \( \phi_T h) \), and from the linearity of the budget constraint.

Let \( \hat{H}(a + h(1 - \phi_T), a', \alpha, \tau) \) denote the objective function to be maximized on the right-hand side of the Bellman equation for \( \check{V} \) in (5), given the wealth \( a + h(1 - \phi_T) \)
after paying the trade cost. Notice that for fixed values of $\alpha$ and $\tau$ the function $\hat{H}(\cdot, \cdot, \alpha, \tau)$ is homogenous of degree $1 - \gamma$. Then we can consider the maximization:

$$\left\{1 - \hat{\theta}, \hat{\alpha}, \hat{\tau}\right\} = \arg \max_{\theta, \alpha, \tau} \hat{H}(1, 1 - \theta, \alpha, \tau)$$

subject to $0 \leq \theta \leq 1$, $\alpha, \tau \geq 0$.

The homogeneity implies that the optimal decision rules for a generic state $(a, h)$ if the agent trades and adjusts the durable stock are given by

$$a' = \left(1 - \hat{\theta}\right)(a + h(1 - \phi_T)), \quad h' = \hat{\theta}(a + h(1 - \phi_T)),$$

and that the optimal choices of $\tau$ and $\alpha$ are independent of $(a, h)$. For notation convenience we use $a \equiv a/h = (1 - \theta)/\theta$ to denote the normalized ratio of assets to durables. Let $\bar{H}(a, h, \alpha, \tau)$ be the objective function to be maximized on the right-hand side of the Bellman equation for $\bar{V}$ in equation (4) given the state $(a, h, \alpha)$. Notice that for fixed $(\alpha, \tau)$ the function $\bar{H}(\cdot, \cdot, \alpha, \tau)$ is homogenous of degree $1 - \gamma$. Consider the problem

$$\bar{\tau}(a, \alpha) = \arg \max_{\tau \geq 0} \bar{H}(a, 1, \alpha, \tau),$$

where $\bar{\tau}$ is a function of $a/h$ because of the homogeneity of $\bar{H}$. Now consider an agent with $\alpha = \hat{\alpha}$ and durable stock $h$, who pays the observation cost and discovers her financial wealth (net of the observation cost) to be $a$. In this case, using $a \equiv a/h$, the agent will trade and adjust if $\bar{V}(a, 1, \hat{\alpha}) < \hat{V}(a, 1)$, where we have used the homogeneity of $\hat{V}(\cdot)$ and $\bar{V}(\cdot, \hat{\alpha})$. Let

$$\bar{I} \subset \mathbb{R}_+ \equiv \{a : \bar{V}(a, 1, \hat{\alpha}) > \hat{V}(a, 1)\};$$

then the optimal policy is of the form

$$a \in \bar{I} \Rightarrow a' = a, \quad \bar{\tau} = \bar{\tau}(a, \hat{\alpha}), \quad a \not\in \bar{I} \Rightarrow a' = \hat{a} = \frac{1 - \hat{\theta}}{\hat{\theta}}, \quad \tau = \hat{\tau}. $$

In the online Appendix D we prove that the inaction region $\bar{I}$ includes an interval that contains the target asset-to-durable ratio—this happens because in the interior of the interval $\bar{V}(a, 1, \hat{\alpha}) > \hat{V}(a, 1, \hat{\alpha})$. Indeed, we will assume from now on, which we verified numerically in all examples, that the set $\bar{I}$ is given by an interval $[\underline{a}, \overline{a}]$ for values of the normalized state variable $a$ where it is optimal for the agent not to trade and not to adjust the stock of durables$^{10}$

$^{10}$In general, $\bar{I}$ can be composed from the union of disjoint intervals. Proposition 3 in Alvarez, Lippi, and Paciello (2011) shows that, for a stylized version of this model, the inaction set is indeed an interval. The model in this paper does not satisfy all the assumptions used in Proposition 3: namely, it lacks the required symmetry on the period return function, and it has nonzero drift. See the online Appendix D for a thorough discussion of this issue.
It is immediate that \( \hat{a} \in I \), i.e., that the optimal return point \( \hat{a} \) is in the range of inaction. The size of the inaction interval depends on the fixed cost \( \phi_T \), among other determinants. Thus, if at the time of observing the state \( a \) falls in the interval \([a, \bar{a}]\), the agent will find it optimal not to pay the fixed cost, to leave \( h \) unaltered, and to set a new observation date \( \tau(a) \) periods from now. Otherwise, if at the time of observing the state \( a \) falls outside the interval \([a, \bar{a}]\), then the agent will pay the cost \( \phi_T h \), adjust the stock of durables, and set the new ratio of financial assets to durables to \( \hat{a} \). She will also set a new observation date \( \hat{\tau} \) periods from now. The analysis shows that along an optimal path there will be instances where the agent will pay the observation cost but will not trade, as in the data for Italian investors displayed in Table 1 and Table 2.

III. Quantitative Analysis

In this section we analyze some quantitative implications of the model by means of numerical solutions. First, we describe the decision rule, then we solve the model for different parameters values and develop some comparative statics to illustrate its workings. Finally, we use the numerical solution to relate the model to moments taken from the Italian investors’ data.

A. Decision Rules

The horizontal axis of Figure 2 displays the values of the ratio of the financial assets to durable goods \( a = \frac{a}{h} \), right after the agent has observed the value of financial assets, and before deciding whether to trade and to adjust the durable stock. As discussed above, the optimal decision rule about adjusting the durables is of the \( sS \) type. The two vertical bars at \( a \) and \( \bar{a} \) denote the threshold values that delimit the inaction region. The vertical bar at \( \hat{a} \), inside the inaction region, denotes the optimal return point after an adjustment. For small values of the frictions the value of the optimal return point is very close to the one from the frictionless model.\(^{11}\)

The optimal decision after an observation is made of two rules: the first rule is whether (and by how much) to adjust the durable stock; the second rule gives a date for the next observation. The adjustment decision, after observing the value of the assets, depends on the location of the state \( a \). The middle panel contains the range of inaction, where the agent chooses not to adjust her financial asset and durable stock. Outside of this region, the agent will pay the adjustment cost \( \phi_T h \), trade, and adjust her financial asset and stock of durables to the values \((a', h')\) that satisfy \( \frac{a'}{h'} = \hat{a} \). The adjustments that occur to the left of \( a \) involve a sale of part of the durable stock \( h \), while the ones to the right of \( \bar{a} \) involve purchases that add to the durable stock \( h \). The discontinuous solid line in Figure 2 displays the optimal time until the next observation. This rule is made of two functions. One is the optimal time until the next observation contingent on trading, which is given by the constant value \( \hat{\tau} \). This is analogous to the optimal rule in Duffie and Sun (1990) and

\(^{11}\) For the values used in Figure 2, comparing the ratio of durables to total wealth \( \hat{\theta} = h/(a' + h) \) defined in equation (6), with the corresponding ratio in the frictionless model where \( \phi_o = \phi_T = 0 \), denoted by \( \theta \), shows that \( \hat{\theta} = 0.238 \), while \( \theta = 0.237 \).
in Abel, Eberly, and Panageas (2007), where each observation is also a trade. The other function gives the optimal time until the next observation contingent on not trading. This function depends on the state $a$ and is denoted by $\tau(a/h)$ in the figure. Thus, the optimal decision rule (solid line) is given by $\tau(\cdot)$ in the inaction region, and by $\hat{\tau}$ in the adjustment regions.

We notice several properties of the optimal time until the next observation:

(i) the function $\tau(\cdot)$ is hump shaped

(ii) the value of $\tau(\hat{a}) = \hat{\tau}$

(iii) the values of $\tau(a)$ and $\tau(\bar{a})$ are strictly positive

(iv) the function $\tau(\cdot)$ reaches zero for values of $a/h$ strictly inside the adjustment regions

(v) the maximum of $\tau(\cdot)$ is larger than $\hat{\tau}$ and it occurs for $a < \hat{a}$.

The reason point (i) holds is that when the agent is inside the inaction region but close to the borders, she realizes that the state $a$ is likely to reach the adjustment region shortly, and, hence, it is optimal to revise the information soon. In the middle of the inaction region, instead, the expected time before reaching the adjustment region is greater, and, thus, the optimal time to the next revision is longer. The reason point (ii) holds is that if, upon observing, the value of $a/h$ happens to coincide with the optimal return point $\hat{a}$, then the objective function with regard to $\tau$ is the same as the one when adjustment is considered (i.e., $\hat{V}$), and, thus, the optimal time
to the next review is \( \hat{\tau} \). Point (iii) holds because if \( \tau \) was zero on the boundary, the agent would be paying the observation cost an arbitrarily large number of times in a short period of time. Point (iv) holds because if \( a/h \) is large, and the agent is forced not to trade (that is the assumption underlying the definition of the function \( \bar{\tau} \)), she will choose to review immediately and trade. Finally, the reason point (v) holds is that in our parametrization the ratio \( a(t)/h(t) \) has a drift to the right (when it is not controlled by adjustments), approximately equal to the sum of the expected return on the financial asset plus the depreciation rate. Thus when \( a/h \) is close to \( \hat{a} \), but to its left, the agent forecasts to be in the inaction for a time longer than \( \hat{\tau} \).12

**B. Calibration Exercises**

Given the nature of the decision rule it is immediate to see that \( a(t)/h(t) \) follows a stationary Markov process with a unique invariant measure. Table 5 shows how the number of observations and trades varies for different combinations of the observation and transaction costs \( \phi_o \) and \( \phi_T \). In this exercise the parameters \( \gamma, r, \mu, \sigma, \rho \) are set to values that are common in the literature, see, e.g., Abel, Eberly, and Panageas (2009), to facilitate comparison of results. The depreciation parameter is set to 10 percent annual (\( \delta = 0.10 \)), so that the half life of the durable good is between six and seven years, which seems a reasonable value for the kind of durable goods for which we have data (see Table 8). The comparative statics for some of these parameters is discussed below. The first two columns in the table report the values of \( \phi_o \) and \( \phi_T \) used in the computation of each row. The columns Observations and

<table>
<thead>
<tr>
<th>( \phi_o ) (bp)</th>
<th>( \phi_T ) (bp)</th>
<th>Observations</th>
<th>Trades</th>
<th>( \phi_T/\phi_o )</th>
<th>( \hat{a} )</th>
<th>( \alpha_o/\alpha_T )</th>
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<tr>
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<td>12.0</td>
<td>4.0</td>
<td>0.32</td>
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<td>0.31</td>
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</tbody>
</table>

**Notes:** The variables \( \phi_o \) and \( \phi_T \) are measured in basis points (bp). The other parameters are \( \gamma = 4, \delta = 0.10, \rho = 0.02, r = 0.03, \mu = 0.06, \sigma = 0.16 \) per year. The variable \( \hat{a} \equiv 1 - \theta \) is the optimal return point, given by the ratio of the value of assets (net of observation cost) to that of durables (net of transaction cost).

<table>
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<tr>
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12 In Alvarez, Lippi, and Paciello (forthcoming) we study a stylized version of this problem, for which we derive analytically the properties of the decision rules discussed here. In particular, the shape displayed in Figure 2 follows from the combination of two limit cases studied in that paper: the case with uncertainty and no-drift on the state, and the case of no-uncertainty and drift on the state.
Trades report, respectively, the expected number of observations and the expected number of trades per year, under the invariant measure. The fifth column reports the ratio between these frequencies, and the sixth column gives the ratio between the transaction over the observation cost. Since these costs apply to different aggregates ($a$ and $h$, respectively), the next column reports the optimal return point $\hat{a} = a/h$ which is used, in the last column, to compute the ratio between the transaction and observation costs properly scaled.

We use the numerical results of Table 5 to illustrate three quantitative properties of the model with respect to the costs $\phi_o$ and $\phi_T$. First, the model generates substantial inaction with relatively small observation and transactions cost. For instance, in the parameterization used in the third-to-last row of Table 5 the observation cost is $\phi_o = 1/10,000$ of the postobservation financial wealth (i.e., 1/100 of one basis point), and the trade cost is $\phi_T = 20/10,000$ of the preadjustment stock of durables. Yet, in spite of these small costs, the expected number of observations per year is 0.9, and the average number of trades is 0.6.

Second, we notice that keeping all the other parameters fixed, the ratio between the frequency of trades and the frequency of observations is a monotone decreasing function of the ratio of the two costs: $\phi_T / \phi_o$. This can be seen by inspecting the rows in each of the four panels separated by a horizontal line in the table. Three values of the ratio $\phi_T / \phi_o$ (20, 250, and 1,000) appear in the lines of each panel. Note that as the $\phi_T / \phi_o$ ratio increases, the ratio between the frequency of trades to the frequency of observation decreases. As is intuitive, an increase in the relative cost of an adjustment reduces the number of adjustments per observation.13

Third, the workings of our model are quantitatively consistent with the findings of Stokey (2009). She builds a model with both durable and nondurable goods calibrated to interpret the durables as housing stock. She focuses on physical transaction costs (and no information gathering costs). If we parameterize our model according to her baseline parameters, which uses a housing transaction cost of 8 percentage points in line with the high transaction cost for housing estimated by Smith, Rosen, and Fallis (1988), we obtain similar results. In particular, the high transaction cost, and the small depreciation for housing ($\delta = 0.03$), produce a frequency of housing transactions of 0.089 trades per year, which accords well with the cross sectional average house tenure of 11.3 years that she reports in the paper.14 The main difference of this parameterization compared to our setup is the smaller depreciation rate (3 versus 10 percent) which is appropriate for housing versus the more perishable durables which we have in our dataset (see the next section) and, as a consequence, induces less frequent adjustment than we get. The bottom panel of Table 5 shows that when a comparably high transaction cost is fed to our parameterization (last line of the table), the frequency of durables trading is 0.20 per year, or approximately one trade every five years.

13 The result that the ratio of the number of trades to observations is a function of the ratio $\phi_T / \phi_o$ and of other variables, such as the absolute size of the costs, differs from the analytical characterization in Alvarez, Lippi, and Paciello (2011), where the number of trades per observation depends only on $\phi_T / \phi_o$. The latter result is obtained as an approximation for the case of a quadratic period return function and no drift. Both of these assumptions are violated in the durable good model.

14 The other parameters, taken from Table 1 in Stokey (2009), are $\gamma = 3.5$, $\mu = 0.07$, $\sigma = 0.1655$, $\delta = 0.031$, $r = 0.025$, to which we add an observation cost of one basis point.
Next, we calibrate the model to quantify the observation and transaction costs using the data for the Italian investors. As a reference we take two sets of data from Table 2 corresponding to direct stockholders from the UCS and another from all investors for SHIW. We think of the UCS stockholder as representative of a pool of more sophisticated investors, while the data in SHIW are representative of the typical Italian investors. The sophisticated investor trades about four times a year and observes about 12 times a year. The typical investor trades about 0.4 times a year and observes about 3.6 times a year. The first and fourth rows in the upper panel of Table 5 report calibrations that match the behavior of these two types of investors. For an observation cost of $\phi_o = 0.005$ basis point and a transaction cost of $\phi_T = 0.1$ basis point the model predicts 14 observations and four trades per year. This parameterization is close to the behavior of the sophisticated investor, matching the levels and the ratio of the observation and trade frequency. This suggests that the level of both costs is small and that the transaction cost is about six times bigger than the observation costs, i.e., $(\phi_T / \phi_o) \times (h/a) \approx 6$. The calibration in the fourth line uses an observation cost of $\phi_o = 0.005$ basis point and a transaction cost of $\phi_T = 100$ basis points. This produces 4.5 observations and 0.4 trades a year, with roughly 0.1 trades per observation. These figures are close to those of the typical Italian investor. In this case, which by the nature of the SHIW survey is more representative of Italian investors’ , the ratio of the level of the transaction cost is much bigger, several orders of magnitude bigger than the level of the observation cost.

Altogether, the calibration shows that the magnitude of the observation cost $\phi_o$ that is necessary to match observed patterns of behavior is small. For instance, if we use 1 percent of one basis point for a financial wealth of a 130,000 euros (which is about the median for the UCS sample of direct equity holder), the observation cost is about 13 euro cents. The adjustment cost for trading durables is also small in the simulation that we associate with the sophisticated investors, and larger (around 1 percent) for the simulation that we associate with the typical Italian investor.

To quantify the welfare consequences of these costs at the household level Table 6 compares the outcomes of this model with the ones produced by three special cases: the frictionless benchmark, the observation-cost only and the transaction-cost only (see the online Appendix C for analytical solutions of these special cases). The analysis shows that transaction costs in the order of 1 percent of the durable stock are not negligible for the investor. Notice that even though the cost is paid on average once every 2.5 years its impact on welfare, measured by the compensating variation in the annual consumption flow, is about 3.5 percent. For the median investors, to which the model was calibrated, the welfare effect of the observation cost is negligible, a consequence of the small size of the level of the observation cost. From Table 6 we conclude that, at the values of the transaction and observation costs that we focus on, the behavior of the model with both costs is very similar to one with the transaction cost only. We reach this conclusion by comparing the number of trades per year, as well as the posttrade ratio of financial assets to durables, for the two benchmark cases displayed in the two panels at the bottom of Table 6. In particular, a comparison of the “Transaction cost only” panel with the “Observation and transaction

---

15Recall that the observation data for the SHIW investors are noisy as they are imputed from a regression on UCS data; see the note to Table 2.
The "cost" panel shows that the number of trades per year is essentially unaltered by the presence of the small observation cost. Moreover, the consumption loss with regard to the frictionless benchmark is very small, on the order of a few basis points of the annual consumption flow. Alternatively, these figures suggest that for the parameterizations considered in these panels, observing between four and 12 times per year provides almost the same information as observing continuously.

To understand the effect in our estimates of having both frequency of trade and observation, the second and third row of the "Observation cost only" panel in Table 6 matches the model to our sophisticated and typical investors's measured trading frequency setting \( \phi_T = 0 \). The implied observation costs are \( \phi_o = 0.075 \text{ (bp)} \) for sophisticated investors and \( \phi_o = 25 \text{ (bp)} \) for the typical Italian investor, which, applied to the median financial wealth of each group, give a per observation cost of about 1 euro for the sophisticated investor and about 62 euros for the typical Italian investor. These costs are much larger than the ones obtained when we calibrate the model to both observation and trading frequency, especially so for the typical Italian investor.

We conclude the section with a remark on the asymmetry of the \( \bar{\tau}(a/h) \) function discussed in point (v). Notice that if the process for \( a(t)/h(t) \) has a strong drift, i.e., if the return on financial assets plus depreciation is large, then most adjustments will happen in the right adjustment region, and, hence, they will involve a liquidation of assets and a purchase of durables. The frequency with which \( a(t)/h(t) \) hits the "sale" region relative to the "buy" region depends on the strength of the drift, which is approximately equal to \( \alpha \mu + (1 - \alpha) r + \delta \), relative to the variability of this ratio, which is about \( (1 - \theta) \alpha \sigma \approx (\mu - r)/(\gamma \sigma) \). In Table 7 we show how this frequency varies with \( \delta \) and \( \sigma \). As expected from the direct effect on the drift, larger values of \( \delta \) lead to a larger fraction of adjustments being purchases. For a larger value of \( \sigma \), the

<table>
<thead>
<tr>
<th>( \phi_o ) (bp)</th>
<th>( \phi_T ) (bp)</th>
<th>Observations</th>
<th>Trades</th>
<th>( \hat{a} )</th>
<th>Consumption loss w.r.t. frictionless*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>3.22</td>
<td>0</td>
</tr>
<tr>
<td>0.005</td>
<td>0.00</td>
<td>19.6</td>
<td>19.6</td>
<td>3.21</td>
<td>0.02 (percent)</td>
</tr>
<tr>
<td>0.075</td>
<td>0.00</td>
<td>4.0</td>
<td>4.0</td>
<td>3.17</td>
<td>0.14 (percent)</td>
</tr>
<tr>
<td>25.00</td>
<td>0.00</td>
<td>0.4</td>
<td>0.4</td>
<td>2.82</td>
<td>4.10 (percent)</td>
</tr>
<tr>
<td>0.00</td>
<td>0.10</td>
<td>( \infty )</td>
<td>4.1</td>
<td>3.19</td>
<td>0.05 (percent)</td>
</tr>
<tr>
<td>0.00</td>
<td>100.00</td>
<td>( \infty )</td>
<td>0.4</td>
<td>2.80</td>
<td>3.50 (percent)</td>
</tr>
<tr>
<td>0.005</td>
<td>0.10</td>
<td>12.2</td>
<td>4.0</td>
<td>3.19</td>
<td>0.07 (percent)</td>
</tr>
<tr>
<td>0.005</td>
<td>100.00</td>
<td>4.6</td>
<td>0.4</td>
<td>2.81</td>
<td>3.51 (percent)</td>
</tr>
</tbody>
</table>

Notes: The variables \( \phi_o \) and \( \phi_T \) are measured in basis points (bp). The other parameters are \( \gamma = 4, \delta = 0.10, \rho = 0.02, r = 0.03, \mu = 0.06, \sigma = 0.16 \) per year. The variable \( \hat{a} \equiv \frac{1}{\gamma} \) is the optimal return point, given by the ratio of the value of assets (net of observation cost) to that of durables (net of transaction cost).

* This loss gives the compensating variation in the annual flow of durable consumption (in %) that is needed to equate the welfare level to the one obtained in the frictionless model (both value functions are evaluated at the \( \hat{a} \) of the model with frictions).
net effect is to decrease the exposure to risk, so that the variability of $a(t)/h(t)$ actually decreases. Thus, for low $\sigma$, the variability of $a(t)/h(t)$ is high, and, hence, the process reaches both thresholds more often, which explains why the fraction of sales is smaller for $\sigma = 0.06$ than for $\sigma = 0.16$. In our benchmark numerical example 98 percent of adjustments are purchases of durables (see Table 7). This comes close to the comparable figure for Italian investors which is 95 percent (see Table 8).

### IV. Some Evidence on the Model Predictions

This section contrasts two predictions of the durable goods model with data from the UCS and the SHIW survey. The first prediction relates to the frequency of durable purchases and that of assets transactions. The second pertains to the relation between the investor risk aversion and the frequency of observations.

#### A. The Correlation between Trades in Assets and in Durables

The first “test” we consider in this section is specific to our durable goods model. By abstracting from the nondurable purchases, agents in our model trade only to adjust their purchase of durables; hence, we expect that empirically investors who are more active in purchasing durables are also more active in assets transactions, as well as that they concentrate their trading around the times where they adjust durables.

We test this prediction using information in the SHIW 2004 survey on the frequency of durable purchases and the frequency of asset transactions across Italian households. In particular, the survey registers whether the household bought or...
sold durable goods in each of three categories: housing, vehicles, and jewelry in the year prior to the interview. It also records whether it has bought house appliances (furniture, electrical equipment, etc.). Thus, even though we do not have the total number of purchases or sales, we have information on whether the household engaged in trading durables in any of these categories. The upper panel of Table 8 reports the fraction of investor/households that were active in each of these four categories. For instance, the fraction of investors who bought a durable in the “Cars and other transportation” category in 2004 is 15 percent. As a first check of the plausibility of the model prediction that asset trades and durable trades are correlated, the lower panel of the table shows that the fraction of investors who purchased durables in 2004 is higher among those who traded financial assets more often. The same pattern is found by running logit regressions for the purchase of durables, on trade frequencies dummies, nondurables consumption, and demographics. Notice that if we condition on the sample of investors who traded more often (i.e., at least once in a year) the proportion of those who bought a car rises from 15 to 20 percent. Notice that this same pattern is found in each of the various categories for which we have information on the durable purchases. This pattern is consistent with the model outlined in Section IIA if investors differ in the costs $\phi_T$ and $\phi_o$.

Based on the survey data, we also construct a “durable trade frequency” proxy as the sum of the 0/1 dummies for the entries “Jewelry and Antiques,” “Cars,” “Furniture and appliances” (we exclude housing because there are few observations available in a given year). The proxy variable ranges from zero (no purchases or sales across categories) to five (at least one purchase and one sale in each of the first two categories, and one purchase in the Furniture category)

Figure 3 shows that the asset trade frequency and this proxy for durable purchase frequency are strongly correlated. Table 9 shows that the bivariate correlation remains strong and statistically significant in a multivariate regression analysis that includes household income and demographics. The correlation is also visible if the sample is restricted...
Finally, we gathered evidence on the time series pattern of portfolio trades and house purchases by relying on the Unicredit administrative data. A detailed description of the data and further comments are available in the online Appendix F. In sum, we identify a subset of investors who, over the 36 months for which we observe their assets with Unicredit, obtained a mortgage and the month they got it. Typically, the purchase of the house is settled as soon as the mortgage is obtained. We then compute the fraction of investors who liquidate investments and the value of the liquidations on the same month of the house purchase and on the previous and subsequent months. Figure 4 shows that the frequency of liquidation starts increasing in the three months following the purchase of the house.

Table 9—Durable versus Portfolio Trade Frequency

<table>
<thead>
<tr>
<th></th>
<th>All investors</th>
<th>Equity investors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2,808 observations)</td>
<td>(1,535 observations)</td>
</tr>
<tr>
<td>Trade frequency (log)</td>
<td>0.27</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Bivariatea</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td>Multivariateb</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>

Notes: Based on the 2004 SHIW survey. All regressions include a constant; standard errors in parentheses. The durable adjustment frequency is an estimate of the average number of durable purchases per year (see the text for a detailed definition).

a Regression coefficient of bivariate OLS.
b This regression includes the following controls (all in logs): household income, age of household head, number of adults.

Figure 4. Frequency of Financial Asset Sales around the Time of a House Purchase

Notes: The vertical axis measures the fraction of households that liquidate their investment t months after (before when negative) the house purchase. The data come from Unicredit administrative records; see the online Appendix F. The dotted lines denote one–standard error bands around the mean.

Source: Large sample, monthly administrative records (35 months) of 26 accounts for 40,000 investors.
or four months before purchase, jumps up in the month of the house purchase, and drops substantially after the purchase. Moreover, the average value of assets liquidation is around 48,000 euros in the month of purchase but drops to 14,000 in the subsequent month. The online Appendix shows that both findings are statistically significant and robust to controls in probit and Tobit regressions.

B. Observation Frequency and Portfolio Riskiness versus Risk Aversion

An increase in the degree of relative risk aversion has two effects in the model: first, it induces the investor to hold a safer portfolio; second, it increases the value of consumption smoothing. The first effect, whose strength depends on the attractiveness of the risky asset, as measured by its Sharpe ratio \((\mu - r)/\sigma\), lowers the value of information and implies that a more risk-averse investor chooses to observe her investments less frequently. This effect is akin to the one first studied by Verrecchia (1982) (see Corollary 1) and Peress (2004) (Theorem 2) who show that agents with a higher risk aversion have weaker incentives to obtain precise signals (i.e., information) about the return of the risky asset.\(^{16}\) In our context this channel relates to the frequency at which one gathers information about the value of the investments rather than to the quality of the signal received. The second effect raises the value of information and, thus, through this channel, more risk-averse investors should observe more frequently.

Which of the two effects prevails in our model depends on parameters values. For large enough yet realistic values of the Sharpe ratio \((\mu - r)/\sigma\), and small values of the cost \(\phi_o, \phi_T\), the portfolio effect dominates the consumption-smoothing effect over a reasonable range of values of the degree of risk aversion. Hence, the frequency of observing one’s investments is lower for more risk-averse investors. Table 10 shows that the share invested in the risky asset and the frequency of portfolio observations both decrease as the degree of relative risk aversion increases, for a parameterization of the costs that was shown to produce a reasonable match of the behavior of the investor from the UCS survey (see Section IIIB).

<table>
<thead>
<tr>
<th>(\gamma)</th>
<th>Share of risky asset (\alpha)</th>
<th>(\hat{a})</th>
<th>Observations</th>
<th>Trades</th>
<th>Trades Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.15</td>
<td>3.20</td>
<td>6.3</td>
<td>4.1</td>
<td>0.65</td>
</tr>
<tr>
<td>7</td>
<td>0.17</td>
<td>3.20</td>
<td>6.4</td>
<td>3.9</td>
<td>0.62</td>
</tr>
<tr>
<td>6</td>
<td>0.20</td>
<td>3.18</td>
<td>6.8</td>
<td>3.8</td>
<td>0.56</td>
</tr>
<tr>
<td>5</td>
<td>0.23</td>
<td>3.18</td>
<td>6.8</td>
<td>3.6</td>
<td>0.52</td>
</tr>
<tr>
<td>4</td>
<td>0.29</td>
<td>3.18</td>
<td>7.7</td>
<td>3.6</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>3.22</td>
<td>8.4</td>
<td>3.6</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>0.58</td>
<td>3.39</td>
<td>10.3</td>
<td>4.1</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Note: The other model parameters are \(\rho = 0.02, r = 0.03, \mu = 0.06, \sigma = 0.16, \delta = 0.10\) (all per year), \(\phi_o = 1/100\) (bp), \(\phi_T = 1/10\) (bp).

\(^{16}\)Verrecchia assumes the utility function displays constant absolute risk aversion in wealth. Peress shows that the same result obtains if absolute risk aversion is decreasing in wealth, so that wealthier individuals invest more into risky assets and, hence, have greater incentive to acquire information.
We can test this prediction of the model because the UCS survey has an indicator of risk aversion patterned after the Survey of Consumer Finance. Investors are asked: “Which of the following statements comes closest to the amount of financial risk that you are willing to take when you make your financial investment: (i) a very high return, with a very high risk of loosing the money; (ii) high return and high risk; (iii) moderate return and moderate risk; (iv) low return and no risk.” Only 19 percent choose “low return and no risk,” so most are willing to accept some risk if compensated by a higher return. A recent literature on eliciting preferences from survey data shows that direct questions on risk aversion are informative and have predictive power.17 Consistent with this literature, the Unicredit data show that the portfolio share invested in risky assets (direct and indirect stocks) is monotonically decreasing with our index of risk aversion (see the working paper version); the correlation and its significance are confirmed in (unreported) regressions that control also for measures of investors assets, income, and demographic characteristics, reassuring us about the reliability of our risk aversion indicator.

To analyze the model implications Table 11 shows regressions for different groups of the (log) number of times an investor observes his investments and the risk aversion indicator while controlling for endowment (log consumption) and demographic characteristics. We use three dummies for risk aversion, excluding the group with the highest risk aversion. Irrespective of which sample we use (the whole sample or that of the stockholders, total or direct) we find that more risk-tolerant individuals observe their investments more frequently than less risk-tolerant ones, consistent with the prediction of the model.

We notice that the predicted negative relation between risk aversion and the frequency of observations is a property of the model of assets trades and durable goods with attention costs, but it is not specific to it. In fact, the same prediction obtains in models with nondurable goods, assets trades, and attention costs and can, thus, be viewed as a general test of models of assets trades with attention costs.

17 See, among others, Barsky et al. (1997) and Guiso and Paiella (2008).
V. Concluding Remarks

This article provides a quantitative analysis of the rational inattention hypothesis by studying how investors manage their financial assets, liquidity, and consumption under the assumption that they face a cost to observe the value of their assets. First, we present direct empirical evidence from a cross-section of individual investors that is consistent with key features of costly observation models: investors collect information about the value of their investments and trade in assets only infrequently. To the best of our knowledge, this is the first direct evidence that is brought to bear on the issue of infrequent portfolio observations/trades and costly observation of the relevant state.

The second contribution is to modify one single feature of existing rational inattention models of asset management in a way that allows the theory to get closer to matching the data. The modification consists in shifting the focus from nondurable to durable consumption choices. As discussed in the introduction, most models based on nondurable consumption yield two counterfactual predictions: an equal frequency of observing and trading, and a negative link between the frequency of trading and the investor’s liquidity. Our model of durable goods adjustment and asset management reconciles the theory with the data, as the model predicts that the frequency of observing must be greater than the frequency of asset trading and is consistent with the empirical absence of correlation between liquid assets holdings and assets trading frequency.

Two predictions of the durable-goods model are supported by the data. First, a positive correlation is detected between trades in assets and in durables. Second, part of the heterogeneity in the frequency with which investors observe their portfolio can be explained by heterogeneous risk attitudes: because more risk-tolerant individuals invest more in volatile assets, they value information more and, thus, gather information more frequently.

A quantitative assessment of the consequences of observation and transaction costs is developed using numerical simulations of the model to match the number of observations and trades of the typical Italian investor (about four and 0.4 per year, respectively) for a financial wealth of 25,000 euros. The analysis shows that a model with durable goods and no transaction costs implies that to reproduce the low trading frequency observed in the data for the typical Italian investor, the observation cost needs to be on the order of 60 euros per observation. Considering the narrow notion of information gathering used by our article, namely, observing the value of one’s financial wealth, these costs seem unrealistically high. The model with both costs can reproduce the observed frequency of portfolio observations and asset trading with small observation costs (about 1 euro cent per observation) and transaction costs of about 1 percent of the value of durables for the typical Italian investor. For wealthier direct and stockholder investors, who trade and observe more often, the observation cost is about 7 cents, while the transaction cost is smaller, about 0.1 percent of the value of durables. Even though these small observation costs help explain infrequent observations, the patterns of consumer choices and frequency of trades that are produced is very close to the one obtained when the observation cost is absent and consumers face only trading costs. Based on this finding we conclude that assets observation costs, and the inattention they induce, have small impact on
the *individual* investors’ decisions. Trading costs of the classical nature emphasized in the literature carry instead much larger losses to investors. One open question is whether the staggering of decisions that the small observation cost implies at the individual level can aggregate to imply a large effect for the whole economy, as argued in related contexts at least since Taylor (1979) and Mankiw (1985).

The small value of observation costs that we end up estimating is not inherent to the model we propose but follows by matching the model with the investors’ data. In Alvarez, Lippi, and Paciello (2011) we study the price setting problem of a firm facing both an observation cost and an adjustment (menu) cost. While the context is different the nature of the problem is similar, and the firm’s frequency of observation and adjustment (i.e., price changes) depends, as in this article, on the relative costs of each of these actions. Using data on price reviews and price changes from a sample of European firms we find that large observation costs and small menu costs are more appropriate to account for the price-review and price-adjustment frequencies, opposite to the investors’ data. This is due to a smaller observation frequency (about two times per year) and a smaller gap between the frequency of price review and that of price adjustment (the ratio of adjustments to observations is about $\frac{1}{2}$).

We see these findings as reasonable: the observation cost for the investor involves a rather simple task, namely, checking the value of her portfolio. Instead, the observation cost for firms is plausibly larger, as it involves finding out the value of marginal cost or the demand curve. Also the adjustment cost for firms, the menu cost, is likely small compared with the typically large costs involved in the buying (and selling) of durables.

The analysis in this paper assumes that all observations are costly. In the context of the portfolio and savings model, where several financial shocks are likely correlated across agents, households might be able to learn the realization of some shocks without paying the observation cost, a possibility that is not allowed for in our model. In principle, the model might be extended to allow for the random arrival of some free information, along the lines of Alvarez and Lippi (2009). Two remarks are in order. First, while the risky portfolio in the model is a single asset, in reality households’ portfolios are far from being perfectly correlated, so that the “observation” activity may be more involved than the mere reading of the news headlines. Second, and more importantly, when we confront the model with simple statistics from the data we find that the implied observation cost is very small: the model without cost is almost equivalent to that with the calibrated cost. Hence, even without having agents learning some feature of the value of assets exogenously and without cost, we find that the data, as interpreted by our model, point to already extremely small observation cost.

We think that several variations and extensions of the model are worth exploring next. In particular, our model assumes that assets are the only income source for the investor and that all consumption is in the form of durable goods. While our empirical analysis concentrated on a sample of investors, assets are not the only income source for these households. Adding nondurable consumption should be interesting too, partly because it accounts for a large part of total expenditures, and

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18 That is why we also included data on the share of labor income as control in the empirical analysis.
partly because it is complementary to the use of liquid assets. We think that fully incorporating labor income and nondurable consumption is interesting to broaden the applicability of the analysis, but it involves several challenges. Some are conceptual, such as modeling the observation of one’s labor income process, others are technical, such as the proliferation of states in the problem, and finally, other challenges are empirical, such as locating relevant datasets for the measurement of observation frequencies and the action that it triggers related to labor income and nondurable consumption.\footnote{See Sims (2003) and Reis (2006) for a model of information gathering about the properties of labor income and the adjustment of nondurable consumption.}

On the theoretical side, we also find it interesting to study how the decisions rules of models that combine both state dependent and time dependent types of adjustments aggregate and how these types of rules affect the response to aggregate shocks. We leave these tasks for future research.

REFERENCES


