Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking

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Abstract

What are the effects of unconventional monetary policies during panic-based financial crises? To address this question, I develop a dynamic general equilibrium model of banking. A novel mechanism gives rise to multiple equilibria. In the good equilibrium, all banks are solvent. In the bad equilibrium, many banks are insolvent and subject to runs. The bad equilibrium is also characterized by deflation and by a flight to liquidity (i.e., depositors are willing to hold more money and less deposits in comparison to the good equilibrium).

I consider two types of monetary injections: loans to banks and asset purchases. Both policies counteract deflation and reduce the losses of insolvent banks, but two novel implications are salient. First, for some parameter values, a temporary increase of money supply (implemented using either loans to banks or asset purchases) amplifies the flight to liquidity. Second, asset purchases preclude a crisis only if the central bank is committed to creating inflation in the event of a panic.

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1 Introduction

A peculiar event of the United States' 2007 to 2009 financial crisis was a dramatic increase in the private sector's willingness to hold liquid assets, a "flight to liquidity". The Federal Reserve reacted aggressively, implementing unconventional monetary policies. The flight to liquidity and interventions from the Fed resulted in an approximately constant price level and sizable drop of the money multiplier.[1] The Great Depression included a similar drop of the money multiplier. Friedman and Schwartz (1963) argue that lack of adequate Federal Reserve intervention generated deep deflation, and precipitated what would have otherwise been a regular or deep recession into the Great Depression.

During these crises, several financial institutions became insolvent and were subject to runs. More than one-fifth of the commercial banks in the United States suspended operations during the Great Depression, as reported by Friedman and Schwartz (1963). The collapse of Lehman Brothers in September 2008 has been followed by a "run on repo" and on other institutions without deposit insurance (i.e., the shadow banking system), documented by Gorton and Metrick (2012a,b).

Motivated by these events, the first contribution of this paper is to provide a dynamic general equilibrium model of banking with multiple equilibria. The multiplicity of equilibria is based on a novel channel, compared to the existing literature about banks runs and banking panics. In the good equilibrium, all banks are solvent. In the bad equilibrium, many banks are insolvent and subject to runs. The distress of the banking sector is associated with deflation, drop of asset prices and flight to liquidity (i.e., depositors are willing to hold more money and less deposits in comparison to the good equilibrium). In the model, runs and insolvencies are systemic events, in the sense that many financial institutions are contemporaneously subject to distress. Therefore, the model captures the systemic nature of financial crises.

The second contribution is the analysis of some monetary policies used during the recent financial crisis: loans to banks and asset purchases. Using numerical simulations of the model, I show that both policies counteract deflation and reduce the losses of insolvent banks, but two novel implications are salient. First, for some parameter values, a temporary increase of money supply (implemented using either loans to banks or asset purchases) amplifies the flight to liquidity. Second, loans to banks rule out the bad equilibrium, while asset purchases preclude a crisis only if the central bank is committed to creating inflation in the event of a panic.

[1] The money multiplier is the ratio of broad monetary aggregates, such as M1 or M2, to the monetary base M0.

In the model, households are subject to uninsurable preference shocks that affect the marginal utility of consumption, similarly to Diamond and Dybvig (1983). There is an exogenous supply of two assets in the economy: fiat money and a productive asset (capital). Two trading frictions create a precautionary demand for money. First, the consumption expenditure of households is subject to a cash-in-advance constraint. Second, households cannot sell capital to acquire money after the realization of preference shocks. However, holding money has a cost, which is represented by the return from holding the productive asset.

Banks offer demand-deposit contracts to pool the liquidity risk of households, allowing for withdrawals of money after the realization of preference shocks. In the model, banks are unregulated institutions that perform maturity transformation without deposit insurance. These features capture the primary idea of commercial banks in the 1930s and the shadow banking system in recent years.

Two frictions in the banking sector are crucial to model a crisis. First, demand-deposit contracts are expressed in nominal terms (i.e., specified in terms of money). Second, assets held by banks are hit by idiosyncratic uninsurable shocks.[2] The uninsurability is the result of private information: each bank observes its own shock, but it takes time for other banks and households to observe the shocks. This friction creates asymmetric information: households do not know whether their own bank has been hit by a positive or negative shock.[3]

I consider the effects of one-time unanticipated idiosyncratic shocks to banks. When the shocks hit the economy, a good equilibrium always exists; all banks are solvent (including banks hit by a negative shock) and pool the liquidity risk of households. A bad equilibrium exists for a large subset of the parameter space. The bad equilibrium is characterized by three features. First, the economy experiences deflation and a drop of the nominal price of capital. Second, as a consequence of the drop of the price of capital, banks hit by negative shocks become insolvent. Due to asymmetric information, it is not possible to immediately identify insolvent banks. Eventually, the (in)solvency of banks becomes common knowledge and insolvent banks are subject to runs. Third, anticipating runs, households hold less deposits and more money in comparison to the good equilibrium (flight to liquidity) to self-insure against the risk of having high marginal utility of consumption. This scenario is an equilibrium because there is a feedback from the flight to liquidity to the drop of prices. Due to the flight to liquidity, some money is held only for precautionary reasons and it is not spent. Unspent money is idle and "stored under the mattresses", thus less money is in circulation in the economy for transaction purposes. Due to an argument related to the

[2] A bad shock destroys some assets and a good shock results in an increase in the quantity of assets.

[3] Gorton (2008) emphasizes the uncertainty regarding the identities of the financial institutions that incurred significant losses associated with the housing market during the Great Recession.

quantity-theory of money, the price level is proportional to the amount of money used for transactions. As a result, the bad equilibrium is characterized by deflation (the price level is smaller compared to the good equilibrium and to the pre-crisis level, when all the money is spent). Due to the drop of the price of consumption goods, the asset (capital) that produces such goods is less valuable and its price drops as well.

Within the category of bad outcomes, there are actually multiple bad equilibria, more precisely up to two bad equilibria, depending on parameters. The two bad equilibria arise from strategic complementarity across depositors; if everybody else reduces deposits at banks, an individual depositor wants to do the same. Crucially, the strategic complementarity arises only within the class of bad equilibria, while does not emerge in the good equilibrium.

I analyze two types of monetary injections: 1) the case of a central bank that buys assets on the market and 2) the case of a central bank that offers loans to banks. Both types of monetary interventions reduce the losses of insolvent banks and counteract the deflationary spiral that arises from the flight to liquidity, a result consistent with the Friedman-Schwartz hypothesis regarding the Great Depression.

The first novel result about monetary policy is related to the case of a temporary monetary injection (i.e., money supply reverts to the pre-crisis level when the panic ceases). For some parameter values, both loans to banks and asset purchases decrease the willingness of households to hold deposits, exacerbating the flight to liquidity and reducing further the money multiplier. By exacerbating the flight to liquidity, the effectiveness of a monetary injection reduces in comparison to an economy with exogenous movements in money demand.[4] The possible amplification of the flight to liquidity is the result of two counteracting effects. First, monetary policy pushes the equilibrium outcome closer to what would prevail if agents did not panic, thereby stabilizing the economy and reducing the magnitude of flight to liquidity. Second, money injections increase demand for assets regardless of whether the central bank buys them directly or because private banks want to buy more after receiving loans. This higher demand increases asset prices, reducing the return from holding assets for any given future asset price. Since banks invest part of their deposits in assets, the drop in asset returns implies a drop in the return banks are willing to pay to depositors, thereby reducing the willingness of depositors to hold deposits. This second mechanism counteracts the first stabilizing force, and the total effect on the equilibrium value of deposits is ambiguous.

The second result about monetary policy is a comparison between the ability of loans to banks and of asset purchases to preclude a crisis. A sufficiently large monetary injection

[4]In the model, the flight to liquidity is endogenous and thus not policy-invariant. The importance of focusing on flight to liquidity that is not policy-invariant is noted also by Christiano, Motto, and Rostagno (2003).

creates an inflationary pressure on the price level and on the nominal price of capital, ruling out the the bad equilibrium. However, the central bank might be unable or unwilling to create inflation due to considerations that are not captured by the model (welfare costs of inflation, credibility, etc.). Therefore, I also analyze the ability of non-inflationary monetary injections to rule out panics. The main result is that non-inflationary asset purchases cannot rule out the bad equilibrium. Contrarily, if the policy instrument the central bank chooses is loans to banks, the crisis is ruled out. It is however crucial that loans to banks have the same seniority as deposits,[5] therefore the central bank suffers losses on loans to banks that go bankrupt. Loosely speaking, the equal seniority assumption implies that losses of insolvent banks are borne not only by depositors, but also by the central bank. Consequently, households are willing to hold more deposits and the flight to liquidity is less pronounced. More precisely, loans to banks break the strategic complementarity that gives rise to multiple bad equilibria. If a credible monetary authority commits to this policy, the crisis and losses turn out to be just an off-equilibrium outcome.

Comparison with the literature. The role of bank runs has been analyzed broadly in the literature as a key element of financial crises. For the 2008 financial crisis, runs on the "shadow banking system" are discussed by e.g. Brunnermeier (2009), Duffie (2010), Gorton and Metrick (2012a,b), and Lucas and Stokey (2011)[6]. Ivashina and Scharfstein (2010) documents runs by borrowers who drew down their credit lines, leading to a spike in commercial and industrial loans reported on bank balance sheets. For the Great Depression, bank runs are studied extensively by Friedman and Schwartz (1963). For banking crises during the national banking era (1863 to 1914), see Gorton (1988).

Diamond and Dybvig (1983) formalize the notion of bank runs as "panics", using multiplicity of equilibria. There are several differences between Diamond and Dybvig (1983) and the model I present in this paper. First, all variables in Diamond and Dybvig (1983) are expressed in real terms so it is difficult to define "monetary injections" in such a model and use it for monetary policy. Second, crises and runs in my model are systemic events, while Diamond and Dybvig (1983) does not necessarily give the same prediction. Third, the choice to run in my model is the dominant strategy if a bank is insolvent, while a depositor in Diamond and Dybvig (1983) runs if all other depositors of the same bank are running. In Diamond and Dybvig (1983), the causality between runs and insolvencies goes in both directions. In my

[5]Seniority refers to the order of repayment in the event of bankruptcy. Senior debts are repaid first during bankruptcy, while other, junior debts are repaid thereafter, if residual funds remain.

[6]The importance of the run on repo has been disputed by Krishnamurthy, Nagel, and Orlov (2012). However, Gorton and Metrick (2012b) use a more comprehensive data source and argue that the conclusion of Krishnamurthy, Nagel, and Orlov (2012) is premature since it focuses only on a part of the repo market.

model, looking at one bank in the economy, insolvency causes a run; but runs are responsible for insolvencies through a general equilibrium effect that depresses nominal prices.

Diamond and Rajan (2006) and Allen, Carletti, and Gale (2013) introduce money in models of banking crises, combined with the role of monetary policy responding to exogenous shocks to money demand. Since the shocks to money demand are exogenous in these papers, they are policy-invariant. On the contrary, in my model, movements in money demand during a panic are driven by changes in beliefs of market participants and thus are not policy-invariant.

Allen and Gale (1998), Diamond and Rajan (2006), and Allen, Carletti, and Gale (2013) offer models in which the nominal deposit contract is optimal. If contracts cannot be contingent on the realization of some aggregate variables because of contracting difficulties, the nominal contract is strictly preferred to the real one because it allows reaching the first-best allocation through adjustment of the price level. The justification for the nominal contract that I use, discussed in the Online Appendix[7], follows the same approach of these papers.

A crucial friction I use to produce a panic-based crisis is represented by asymmetric information. One of the views of the Great Recession, emphasized, for example, by Gorton (2008), is uncertainty regarding the identities of the financial institutions that incurred significant losses associated with the housing market. The inability of depositors to sort good and bad banks can be inferred also from indirect evidence and it appears to be a theme of other crises episodes too. Bernanke (2010) and Armantier et al. (2011) emphasize the "stigma" associated with borrowing from the discount window. Financial institutions known to have used the facility of the Fed are perceived as weak and thus might come under pressure by creditors. A similar "stigma" was associated with banks that borrowed from the government-established RFC (Reconstruction Finance Corporation) in 1932, as described by Friedman and Schwartz (1963). The role and management of information are also important for the history of clearinghouses, discussed by Gorton and Mullineaux (1987). Jacklin and Bhattacharya (1988) and Bigio (2012) analyze financial crises in models with informational asymmetries.

A different literature, including Ennis and Keister (2003), Martin, Skeie, and Von Thadden (2011), Gertler and Kiyotaki (2013) and Angeloni and Faia (2013) emerged recently, trying to combine three-periods models of runs with the macro infinite-horizon formulation of the workhorse business cycle model. The paper of Gertler and Kiyotaki (2013) is closely related to mine because crises are systemic and a key role is played by a drop in asset prices. Different from my model, they do not include money, and their variables are expressed in real rather than nominal terms. The drop in asset prices in Gertler and Kiyotaki (2013) is due to fire-

[7]The Online Appendix is available at https://sites.google.com/site/robertorobatto/papers/Robatto_JMP_Online_Appendix.pdf

sales and to a long-lasting disruption of the banking system, while my driving force is the combination of asymmetric information and precautionary demand for liquidity.

The assumptions concerning the structure of trading in the model are very similar to Telyukova and Visschers (2011) and are also analogous to Bianchi and Bigio (2013), Lucas (1990) and to the approach used in some search-theoretic models of money such as Lagos and Wright (2005). The role of the precautionary demand for money is based on the considerations of Lucas and Stokey (2011). I conjecture that a precautionary demand for money and the multiple equilibria mechanism that I describe can arise also in models with a different structure of trading, such as models where agents face transaction costs (e.g. Alvarez, Atkeson, and Kehoe (2002)) or limits on the amount of assets that can be sold in each period (e.g. Kiyotaki and Moore (2012)). I also conjecture that the demand for money can be replaced by a more general demand for liquidity, including not only money but also other liquid assets such as government bonds. This would be consistent with Krishnamurthy and Vissing-Jorgensen (2012), who document that government bonds have lower yields compared to other Aaa-rated corporate bonds, suggesting a demand for safety and liquidity provided by US Treasuries.

In terms of the solution approach, I use the full non-linear model without the need to rely on any approximation: the importance of considering non-linearities in macroeconomic models with financial frictions has been emphasized in other works such as Brunnermeier and Sannikov (2012), though such paper uses a continuous-time approach. The numerical solution method that I use relies on computation of Gröbner bases; Kubler and Schmedders (2010) describes how to use Gröbner bases to find equilibria in economic models.

Other papers focus on similar aspects of financial crises and on the role of policy, but with alternative models. Caballero and Krishnamurthy (2005, 2008) present a model in which Knightian uncertainty is responsible for a flight to quality and analyze the role of the lender of last resort in this situation. Brunnermeier and Sannikov (2011) develop a framework in which a shock to financial intermediaries triggers deflationary pressures and debt-deflation mechanisms similar to Fisher (1933). They also focus on monetary policies that help bank recapitalization, but in their model no financial institution is insolvent. Krishnamurthy (2010) analyzes the role of policy (including monetary policy) to counteract the effects of balance-sheet amplification mechanisms and Knightian uncertainty.

This paper is also related to the literature that incorporates the Friedman-Schwartz hypothesis into quantitative DSGE models to analyze the role of monetary policy and its transmission mechanism during the Great Depression, such as Christiano, Motto, and Rostagno (2003) and Bordo, Erceg, and Evans (2000).

Layout. The rest of the paper is organized as follows. Section 2 introduces the model with a constant supply of money and no monetary policy intervention. The results are presented in Section 4 and monetary policy is analyzed in Section 5. Section 6 discusses some extensions and Section 7 concludes.

2 Model

The economy is populated by a unit mass of banks indexed by $b \in [0,1]$ and by a double continuum of households[8] indexed by $h \in \mathcal{H} = [0,1] \times [0,1]$. There is also a unit mass of bankers (i.e., bank's shareholders), but they play a minor role in the model.

Time is discrete and each period is divided into two parts, day and night. I use capital letters to denote quantities and prices that refer to the day, and lower-case letters to denote quantities and prices at night. Superscripts h and b refer to household h and bank b.

2.1 Households and banks

Household $h \in \mathcal{H}$ enjoys utility from goods c_t^h consumed at night according to the utility function:

$$\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \varepsilon_t^h \log c_t^h$$

where ε_t^h is a preference shock realized at the beginning of the night, and can take two values:

$$\varepsilon_t^h = \begin{cases} \bar{\varepsilon} > 0 & \text{(impatient)} \quad \text{with probability } \kappa \\ \underline{\varepsilon} = 0 & \text{(patient)} \quad \text{with probability } 1-\kappa. \end{cases} \tag{1}$$

and I normalize $\underline{\varepsilon}$ to zero. The preference shock is private information of household h, it is i.i.d. over time and across households, and the law of large numbers applies to each subset of $\mathcal{H}$ with a continuum of households. I impose the normalization:

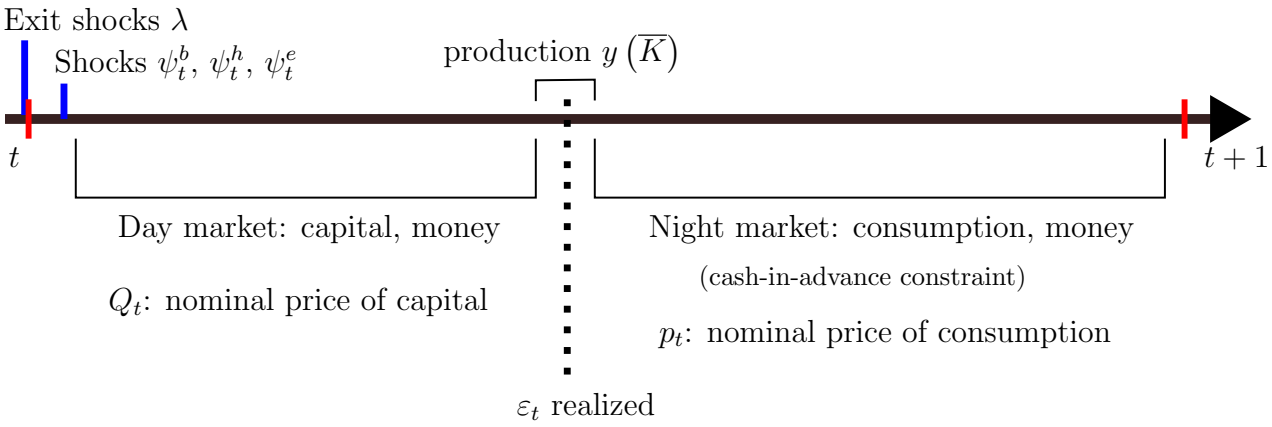
$$\mathbb{E}\left(\varepsilon_t\right) = 1. \tag{2}$$

Therefore, equations (1) and (2) imply $\kappa\bar{\varepsilon} = 1$.

The banking sector is perfectly competitive and the objective of banks is to maximize profits.

[8] The double continuum of households is required because each bank faces a continuum of depositors, and there is a continuum of banks in the economy.

Figure 1: Timing: trading and shocks



2.2 Assets, trading and interaction between banks and households

Assets. There are three assets in the economy: money, capital and deposits. Capital is in fixed supply $\overline{K}$. The supply of money is given by M_t and in this Section with no central bank intervention, I assume that M_t is constant, $M_t = \overline{M}$ for all t (this assumption is relaxed in Section 5 when studying monetary policy). A deposit issued by bank b is a claim that is redeemable on demand at bank b. The supply of deposits is endogenously determined in equilibrium. I impose a particular demand-deposit contract in the economy (rather than deriving it from an explicit contracting problem), the optimal contract is discussed in Section 6 and in the Online Appendix. The key assumption is that the contract is specified in terms of money (in nominal terms).

Markets. Trading takes place in a day market and in a night market, as represented in Figure 1. During the day, there is a Walrasian market in which households and banks trade money, capital and deposits. The price of money is normalized to one and Q_t is the price of one unit of capital (the price of deposits is the same as the price of money). After the day market closes, capital produces output with a linear technology $y(\cdot)$: each unit of capital produces Z units of output (where $0 < Z < \infty$) and there is no depreciation. The output $y\left(\overline{K}\right) = Z\overline{K}$ is the only consumption good in the economy. At night, there is another centralized market through which agents buy consumption goods subject to a cash-in-advance constraint.[9] Let p_t be the price of consumption in terms of money.

[9]Households cannot consume output produced by their own stock of capital, similarly to standard models with a cash-in-advance constraint such as Lucas and Stokey (1987).

State variables and day trading. Each agent $i \in \mathcal{H} \cup [0,1]$ (where i is an index that represents both households and banks) starts the day with a vector of state variables $\boldsymbol{X}_t^i$ given by:

$$\boldsymbol{X}_t^i = \left\{ \left(K_{t-1}^i, m_{t-1}^i, d_{t-1}^i \right), \psi_t^i \right\}$$

where K_{t-1}^i is capital, m_{t-1}^i is money and d_{t-1}^i are deposits. The term ψ_t^i is an idiosyncratic shock to capital whose value is private information of agent i. The initial stock of capital of agent i is thus given by[10] $K_{t-1}^i (1 + \psi_t^i)$. The shocks ψ_t^i are idiosyncratic in the sense that $\int K_{t-1}^i \psi_t^i di = 0$. In addition, the redistributive effects of shocks cancel out within the banking sector and within the household sector:[11]

$$\int_0^1 K_{t-1}^b \psi_t^b db = 0, \qquad \int_{\mathcal{H}} K_{t-1}^h \psi_t^h dh = 0. \tag{3}$$

During the day, agent i has access to the Walrasian market where she can adjust her portfolio of money, deposits and capital. Let M_t^i, D_t^i and K_t^i be the amount of money, deposits and capital that agent i has after leaving the day market.

A crucial restriction on the choices of households is represented by following Assumption.

Assumption 2.1. *Each household $h \in \mathcal{H}$ can hold deposits from at most one bank.*

This assumption is justified by some costs of maintaining banking relationships. Formally, the cost is zero if household h holds deposits from one bank, and infinite if household h holds deposits from two or more banks. The assumption can be relaxed, but it is crucial that households are subject to the risk of facing a run on their own bank(s).[12]

Let $\mathcal{H}(b) \subset \mathcal{H}$ be the set of depositors of bank $b \in [0,1]$, and let $b(h) \in [0,1]$ be the bank of household $h \in \mathcal{H}$.

Due to the dynamic nature of the interaction between households and banks, household h starts period t with preexisting deposits d_t^h and bank b starts period t with preexisting deposits d_t^b. The choice of D_t^h by household $h \in \mathcal{H}(b)$ is thus a decision regarding rolling over the preexisting deposits (fully or partially) and/or buying new deposits. For bank b, the difference $D_t^b - d_t^b$ is the net issuance of deposits. If $D_t^b > d_t^b$, bank b increases its amount

[10] The shock ψ_t^i affect the quantity of capital in the sense that, after the shock is realized, agent i holds a larger or a smaller quantity of capital than before the realization of the shock. This formulation allows me to capture a story where there are shocks to the quality of capital without the need to model heterogeneity in capital. In a sense, having e.g. twice as many units of capital is equivalent to having the same amount of capital and doubling permanently the productivity.

[11] Based on the result of Al-Najjar (2004).

[12] The results are qualitatively unchanged if I impose that a household cannot hold deposits from a continuum of banks or from a finite but large number of banks.

of deposits and thus receives new resources from households. Otherwise, bank b decreases its amount of deposits and must reimburse some preexisting deposits[13].

Night: withdrawals and consumption. At night, households learn the realization of their own preference shock ε_t^h. Then, they decide withdrawals of money w_t^h from their own bank and consumption c_t^h.

Households are served sequentially and, in the event of a run on a bank, the bank might not have enough cash to serve all depositors. Household h can withdraw any amount $0 \leq w_t^h \leq \min\left\{D_t^h, l_t^h\right\}$ where l_t^h is a limit on withdrawals determined by the position in the line. If the household is last in line during a run then $l_t^h = 0$ and thus $w_t^h = 0$. If the bank of household h is not subject to a run or if household h is first in line then $l_t^h = +\infty$ and $0 \leq w_t^h \leq D_t^h$.

The consumption decision c_t^h of household h is subject to a cash in advance constraint. Household expenditure $p_t c_t^h$ are limited by the sum of the money M_t^h chosen during the day and withdrawals w_t^h chosen at night:

$$p_t c_t^h \leq M_t^h + w_t^h.$$

Banks do not take any economic decision at night. From the perspective of bank b, the amount of money withdrawn by depositors is $w_t^b = \int_{\mathcal{H}(b)} w_t^h dh$ and is limited by the feasibility constraint $w_t^b \leq M_t^b$. The money that is distributed at night to depositors cannot exceed the amount M_t^b that bank b held at the end of the day. Banks cannot distribute capital to depositors at night, and there exists no technology or market to convert capital into money at night.

If a bank is subject to a run at night (i.e., if the limit on withdrawals is $l_t^h = +\infty$ for some depositor $h \in \mathcal{H}(b)$), the bank is liquidated at $t+1$ while the day market is open. All assets of the bank are sold on the market and deposits that had not been withdrawn at night are repaid (if the value of assets is insufficient, depositors are repaid pro-rata). If there is some value left after paying depositors, it is distributed to shareholders-bankers.

Return on deposits not withdrawn. During the day, banks promise to pay a return $1 + R_t^D$ (in $t+1$) on deposits that are not withdrawn at night.[14] The promised return

[13] To describe precisely the interaction between banks and depositors, I must specify what happens if many preexisting deposits are not rolled over during the day and the bank does not have enough money m_t^b or capital $K_{t-1}^b\left(1+\psi_t^b\right)$ to immediately repay them. If such circumstances occur, the bank is liquidated immediately (while the day-market is opened), and depositors get pro-rata repayment.

[14] The term $1 + R_t^D$ is the face value of deposit, conditional on not withdrawing the deposit at night.

$1 + R_t^D$ is a market price that is taken as given by both banks and households; the results are unchanged if I allow each bank to post a bank-specific return during the day.

However, banks might not have enough resources to pay the promised return. The actual return on deposits paid by bank b is given by $1 + r_t^b \leq 1 + R_t^D$ and it can be smaller than the promised return. Both l_t^h and $\left\{r_t^b\right\}_{b\in[0,1]}$ are in the information set of household h during the night of time t, but they are not know during the day of time t.

2.3 Exit shocks, dividends and bankers

Between the night of $t-1$ and the day of t, each bank is subject to an exit shock with probability λ as represented in Figure 1. Assuming a law of large numbers, λ is also the fraction of banks hit by the exit shock. Each surviving bank is "split" in $\frac{1}{1-\lambda} > 1$ new banks, so the measure of banks is not affected by the exit shock.[15]

The timing is as follow: between the night of time $t-1$ and the day of t, each bank is either hit by the exit shock or split in $\frac{1}{1-\lambda}$ new banks. Then the shocks to capital are realized.

The vector of state variables of an exiting bank e, at the beginning of time t is:

$$\boldsymbol{X}_t^e = \left\{\left(K_{t-1}^e, m_{t-1}^e, d_{t-1}^e\right), \psi_{t-1}^e\right\}$$

where the superscript "e" denotes an exiting bank. Then, the day market opens and liquidation takes place. The liquidation of an exiting bank is identical to the liquidation of a bank subject to a run, described in Section 2.2. Recall that if there is some value left after paying depositors, it is distributed to bankers. Let π_t be the total value of dividends that is paid to bankers (dividends are paid to bankers using money). Bankers are hand-to-mouth and use the dividends π_t to finance consumption at night: their consumption is $\frac{\pi_t}{p_t}$. The assumption of hand-to-mouth bankers simplify the analysis and the exposition, but the results of the paper are unchanged in a model without bankers, where banks pay dividends to households and households trade claims on the stream of dividends of banks.

[15] The exit shock allows to describe properly the ownership of banks and to obtain a well-defined steady-state.

2.4 Shocks to capital

The support of the shocks to capital is $\left\{\underline{\psi}, 0, \overline{\psi}\right\}$, where $-1 < \underline{\psi} < 0 < \overline{\psi}$. I will analyze the effects of a one-time unanticipated shock. Formally,

$$\Pr\left(\psi_t^h = 0,\, \psi_t^b = 0,\, \psi_t^e = 0 \text{ for all } h,\, b,\, e\right) = 1.$$

The assumption that the shock is unanticipated can be relaxed (see Section 6). When the non-zero shocks hit the economy, a fraction $\alpha^{BAD} \in (0, 1)$ of the capital stock is hit by the bad shock $\underline{\psi}$.

2.5 State of the economy and sunspot

The aggregate state of the economy $\boldsymbol{X}_t$ at the beginning of the day is:

$$\boldsymbol{X}_t = \left\{\Pr_t^{\boldsymbol{X}^b},\, s_t, \{\boldsymbol{X}_t^e\}_{\text{exiting banks}}\right\}$$

where $\Pr_t^{\boldsymbol{X}^b}$ is the probability distribution over the states of banks in the economy[16], s_t is a sunspot, and the last term denotes the state variables of banks hit by exit shocks λ. The sunspot is an exogenous process that determines equilibrium selection, when multiple equilibria arise. The sunspot s_t selects the good equilibrium with probability one, therefore the bad equilibrium is unanticipated.

Knowledge of the aggregate state allows only obtaining information regarding the overall distribution of assets and liabilities of banks, but it does not allow knowing assets and liabilities of particular bank $b \in [0, 1]$. Crucially, the aggregate state $\boldsymbol{X}_t$ does not allow inference about shocks $\left\{\psi_t^b\right\}_{b\in[0,1]}$ that hit banks.

2.6 Information

Finally, I summarize the information structure. There are three sources of private information.

1. Shocks to capital ψ_t^h and ψ_t^b: the realization of the shock ψ_t^b is private information of bank b; similarly, the realization of ψ_t^h is private information of household h.

[16] I only consider cases in which the states of banks take finitely many values.

2. Day market: trading in the day market is anonymous in the sense that it is impossible to observe the amount of money M_t^i, capital K_t^i and deposits D_t^i held by agent $i \in \mathcal{H} \cup [0,1]$ at the end of the day.

3. Preference shocks ε_t^h: the realization of the preference shock ε_t^h is private information of household h.

3 Equilibrium

For future reference, it is useful to define the (expected) nominal return on capital[17]:

$$1 + R_t^K \equiv \frac{Q_{t+1} + Zp_t}{Q_t} \tag{4}$$

To understand this expression, consider the following. With 1\$, you can buy $1/Q_t$ units of capital during the day. Each unit of capital produces Z units of output that can be sold at night at price p_t (generating proceeds $\frac{1}{Q_t} Z p_t$) and the $\frac{1}{Q_t}$ units of capital can be sold tomorrow at price Q_{t+1}. Therefore, every dollar invested gives a gross return $1 + R_t^K$.

3.1 Bank problem

Given the vector of state variables $\boldsymbol{X}_t^b = \left\{ \left(K_{t-1}^b, m_{t-1}^b, d_{t-1}^b \right), \psi_t^b \right\}$ of bank b and the price Q_t at which capital can be traded in the day market, the balance sheet of a bank b at the beginning of the day is:

Assets	*Liabilities*
Nominal value of capital $= K_{t-1}^b \left(1 + \psi_t^b\right) Q_t$	Nominal value of deposits $= d_{t-1}^b$
Money $= m_{t-1}^b$	Net worth N_t^b

where "net worth" is the difference between the value of assets and the value of deposits:

$$N_t^b \equiv K_{t-1}^b \left(1 + \psi_t^b\right) Q_t + m_{t-1}^b - d_{t-1}^b. \tag{5}$$

[17] The definition of $1 + R_t^K$ incorporates the fact that the crisis is a zero-probability event (thus Q_{t+1} is known with probability one) and the idiosyncratic shocks to capital is zero with probability one.

The net worth $N_t^b \in \mathbb{R}$, so it can be either positive or negative. If $N_t^b \geq 0$, the bank is solvent; the value of assets is larger than the value of deposits d_t^b. If $N_t^b < 0$, the bank is insolvent: the value of its assets is less than liabilities toward depositors. Crucially, a bank with a negative net worth can be active in equilibrium because of asymmetric information. Since bank b takes the price Q_t as given, the net worth N_t^b summarizes the vector of state variables $\boldsymbol{X}_t^b$ for the purpose of understanding the choices the bank makes.

The problem of a bank with net worth N_t^b is to maximize the value distributed to shareholders-bankers in the event of liquidation. Since such value is given by the net worth, then the bank wants to maximize its net worth in $t+1$ under limited liability, $\max\left\{0, N_{t+1}^b\right\}$, by choosing deposits D_t^b, money M_t^b and capital K_t^b, taking as given the market return on deposits R_t^D and withdrawals w_t^b by depositors at night. In this Section, I formulate the problem of bank b conditional on bank b surviving to $t+1$ and not being hit by the exit shock, but the Online Appendix shows that including the possibility of exit does not alter the results.

The problem of bank b is:

$$\max_{D_t^b, M_t^b, K_t^b} \mathbb{E}_\psi \max\left\{0, N_{t+1}^b\right\} \tag{6}$$

subject to the budget constraint (7) and the law of motion of net worth (8):

$$K_t^b Q_t + M_t^b \leq D_t^b + N_t^b \tag{7}$$

$$N_{t+1}^b = (1-\lambda) K_t^b \left(1 + \psi_{t+1}^b\right) Q_{t+1} + m_t^b - d_t^b \tag{8}$$

where m_t^b and d_t^b are the nominal values of money and deposits at the end of the night of time t:

$$m_t^b \equiv (1-\lambda)\left[\left(M_t^b - w_t^b\right) + y\left(K_t^b\right) p_t\right] \tag{9}$$

$$d_t^b \equiv (1-\lambda)\left(D_t^b - w_t^b\right)\left(1 + R_t^D\right). \tag{10}$$

The term $1-\lambda$ in equations (8), (9) and (10) captures the fact that bank b is split in $\frac{1}{1-\lambda}$ new banks between the night of t and the day of $t+1$ (see Section 2.3). The expectation $\mathbb{E}_\psi$ is taken with respect to the shock to capital ψ_{t+1}^b. Banks must also satisfy a non-negativity constraint on money $M_t^b \geq 0$, capital $K_t^b \geq 0$ and deposits $D_t^b \geq 0$. The solution to the problem of banks is summarized by the following Proposition, and the proof is provided in the Online Appendix

Proposition 3.1. *Given N_t^b and prices Q_t, R_t^K and R_t^D, the optimal choice of bank b is:*

1. *deposits:*

$$D_t^b = \begin{cases} 0 & \text{if } R_t^D > R_t^K \\ \text{any amount } \geq 0 & \text{if } R_t^D = R_t^K \\ +\infty & \text{if } R_t^D < R_t^K \end{cases}$$

2. *money holding* $M_t^b = \kappa D_t^b$*;*

3. *capital holding* $K_t^b = \frac{N_t^b + D_t^b - M_t^b}{Q_t}$.

To understand the result, I first focus on a bank that starts with zero net worth, $N_t^b = 0$. The law of large numbers implies that a fraction κ of depositors withdraw at night to finance consumption expenditures. Thus banks keep an amount of money $M_t^b = \kappa D_t^b$ that is just enough to finance such withdrawals. The remaining resources $D_t^b - M_t^b = (1-\kappa) D_t^b$ are invested in capital, yielding a net return $(1-\kappa) D_t^b R_t^K$ in $t+1$. Since the bank will have to pay the market return $(1-\kappa) D_t^b R_t^D$ on deposits not withdrawn, the profit of the bank is $(1-\kappa) D_t^b \left(R_t^K - R_t^D\right)$, and the bank chooses $D_t^b = 0$ if $R_t^K < R_t^D$ (otherwise it would make negative profit), $D_t^b = +\infty$ if $R_t^K > R_t^D$ (because it can make strictly positive profits on every dollar of deposit) and it is indifferent between any $D_t^b \geq 0$ if $R_t^K = R_t^D$. In the relevant case in equilibrium, $R_t^K = R_t^D$, and thus banks make zero profits.

If a bank has a positive net worth, $N_t^b > 0$, a similar analysis applies. The bank invests a fraction κ of deposits in money and a fraction $1-\kappa$ in capital. The whole net worth N_t^b is invested in capital to maximize the value of net worth tomorrow.

Finally, I describe the behavior of a bank with negative net worth, $N_t^b < 0$, focusing on relevant case $R_t^K = R_t^D$. A bank b with negative net worth does not earn profit on deposits if $R_t^K = R_t^D$. Therefore, its net worth at $t+1$ remains negative.[18] Consequently, the bank is indifferent among any choices because its payoff will always be zero due to limited liability. I consider the case in which a bank with negative net worth behaves like a good solvent bank. In a richer model, a bad bank has the possibility of being hit by a good shock and become solvent, avoiding runs and liquidation. In this case, the optimal choice for the the bank is to take its decision by maximizing its net worth conditional on becoming solvent and surviving since the payoff in the event of runs and liquidation is zero anyway.

[18] Note also that the bank cannot invest 100% of its deposits in money because $M_t^b < D_t^b + N_t^b$ using the budget constraint (7) and $N_t^b < 0$.

3.1.1 Actual return on deposits

I now define the actual return on deposits r_t^b.

$$r_t^b \equiv \min\left\{R_t^D,\, \widehat{r}_t^b\right\} \tag{11}$$

where $\widehat{r}_t^b$ solves:

$$\mathbb{E}_\psi\left\{K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1}\right\} + ZK_t^b p_t = \left(D_t^b - w_t^b\right)\left(1+\widehat{r}_t^b\right)$$

or, using $\psi_{t+1}^b = 0$ with probability one and rearranging:

$$1+\widehat{r}_t^b = \frac{K_t^b\left(Q_{t+1}+Zp_t\right)}{D_t^b - w_t^b}. \tag{12}$$

The variable $\widehat{r}_t^b$ is the return that can be paid to deposits not withdrawn using proceeds from selling output $ZK_t^b p_t$ and the (expected) value of capital $K_t^b Q_{t+1}$.

3.1.2 Fraction of depositors served during a run

If all depositors of bank b attempt to withdraw money at night, only a fraction f_t^b of depositors is served (f stands for first in line). The fraction of depositors served is:

$$f_t^b = \frac{M_t^b}{D_t^b} \tag{13}$$

From the viewpoint of household h that has deposits at bank $b\,(h)$, $f_t^{b(h)}$ is also the probability of being served during a run. If all depositors of bank $b\,(h)$ want to withdraw all their deposits, then:

$$\Pr_t\left(l_t^h = +\infty \middle|\, \text{bank } b\,(h) \text{ is subject to a run}\right) = f_t^{b(h)}.$$

3.2 Household problem

Given the vector of state variables $\boldsymbol{X}_t^h = \left\{\left(K_{t-1}^h, m_{t-1}^h, d_{t-1}^h\right), \psi_t^h\right\}$ of household h and the price Q_t at which capital can be traded in the day market, the nominal wealth A_t^h of household h is:

$$A_t^h \equiv K_{t-1}^h\left(1+\psi_t^h\right)Q_t + m_{t-1}^h + d_{t-1}^h.$$

Household $h \in \mathcal{H}$ is assigned a bank $b(h) \in [0,1]$. To formalize the utility maximization problem of households, let $n_t^h = \left\{\varepsilon_t^h,\, r_t^{b(h)},\, l_t^h\right\} \in \mathcal{N}$ be the vector of variables whose value is in the information set of household h at night, where:

$$\mathcal{N} = \left\{ n = \{\varepsilon,\, r,\, l\} \middle|\, \varepsilon \in \{\bar{\varepsilon}, \underline{\varepsilon}\},\, r \in \mathbb{R},\, l \in \{0, +\infty\} \right\}$$

(n stands for night). First, household h forms beliefs $\Pr_t^h\left(r_t^{b(h)} = r,\, l_t^h = l\right)$ that, combined with the exogenous process for ε_t^h described in (1), imply a probability distribution over $n \in \mathcal{N}$. Second, during the day, household h chooses money M_t^h, deposits D_t^h and capital K_t^h. Third, at night, household h chooses withdrawals $w^h\left(n_t^h\right)$ and consumption $c^h\left(n_t^h\right)$ conditional on the realization of n_t^h. Let $V_t\left(A_t^h\right)$ be the value of holding nominal wealth. The Bellman equation is:

$$V_t\left(A_t^h\right) = \max_{M_t^h, D_t^h, K_t^h} \mathbb{E}_n \left\{ \max_{w^h\left(n_t^h\right), c^h\left(n_t^h\right)} \left[\varepsilon_t^h \log c^h\left(n_t^h\right) + \beta \mathbb{E}_\psi V_{t+1}\left(A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right)\right) \right] \right\} \tag{14}$$

subject to the budget constraint (15), the limit on withdrawals (16), the cash-in-advance constraint (17) and a non-negativity constraint on money $M_t^h \geq 0$, deposits $D_t^h \geq 0$ and capital $K_t^h \geq 0$:

$$M_t^h + D_t^h + Q_t K_t^h \leq A_t^h \tag{15}$$

$$0 \leq w^h\left(n_t^h\right) \leq \min\left\{D_t^h,\, l_t^h\right\} \tag{16}$$

$$p_t c^h\left(n_t^h\right) \leq M_t^h + w^h\left(n_t^h\right) \tag{17}$$

where the value of wealth $A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right)$ is:

$$A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right) = \left[K_t^h\left(1 + \psi_{t+1}^h\right)\right] Q_{t+1} + d^h\left(n_t\right) + m^h\left(n_t^h\right) \tag{18}$$

and:

$$d^h\left(n_t^h\right) \equiv \left[D_t^h - w^h\left(n_t^h\right)\right]\left(1 + r_t^{b(h)}\right) \tag{19}$$

$$m^h\left(n_t^h\right) \equiv \left[M_t^h + w^h\left(n_t^h\right) - p_t c^h\left(n_t^h\right)\right] + p_t\left(Z K_t^h\right). \tag{20}$$

The term $d^h\left(n_t^h\right)$ represents deposits not withdrawn $D_t^h - w^h\left(n_t^h\right)$, plus the actual return $r_t^{b(h)}$ paid by bank $b(h)$. The term $m^h\left(n_t^h\right)$ is money at the end of the night, which is the sum of unspent money at night (i.e., money held during the day M_t^h plus withdrawals $w^h\left(n_t^h\right)$ minus consumption expenditure $c^h\left(n_t^h\right) p_t$) plus proceeds from selling output $Z K_t^h$ at night at price p_t. The expectation $\mathbb{E}_n$ is taken with respect to the beliefs over $n \in \mathcal{N}$ and the expectation $\mathbb{E}_\psi$ is taken with respect to the shock to capital ψ_{t+1}^h.

Assumption 3.2. *If household $h \in \mathcal{H}$ is indifferent among several quantities of deposits, the household selects the smallest D_t^h that maximizes her utility.*

When households are indifferent among several deposit choices, they use banks only to insure against liquidity risk, and invest directly in capital all the wealth they want to carry to $t+1$. This assumption simplifies the derivation of the good equilibrium.

The next Proposition states the solution to (14), focusing on the relevant case $R_t^D = R_t^K$. The proof is provided in the Online Appendix.

Proposition 3.3. *Given beliefs $Pr_t^h(\cdot)$ and prices Q_t, R_t^K and $R_t^D = R_t^K$, household h chooses:*

$$M_t^h = \eta_t^M A_t^h$$

$$D_t^h = \eta_t^D A_t^h$$

$$Q_t K_t^h = \eta_t^K A_t^h$$

$$w_t^h = w^h\left(n_t^h\right) = \begin{cases} D_t^h & \text{if } \varepsilon_t^h = \bar{\varepsilon}, \quad r_t^{b(h)} \in \mathbb{R}, \quad \text{and } l_t^h = +\infty \\ 0 & \text{if } \varepsilon_t^h = \bar{\varepsilon}, \quad r_t^{b(h)} \in \mathbb{R}, \quad \text{and } l_t^h = 0 \\ D_t^h & \text{if } \varepsilon_t^h = 0, \quad r_t^{b(h)} < 0, \quad \text{and } l_t^h = +\infty \\ 0 & \text{if } \varepsilon_t^h = 0, \quad r_t^{b(h)} < 0, \quad \text{and } l_t^h = 0 \\ 0 & \text{if } \varepsilon_t^h = 0, \quad r_t^{b(h)} \geq 0, \quad \text{and } l_t^h \in \{0, +\infty\} \end{cases}$$

$$c^h\left(n_t^h\right) = \begin{cases} 0 & \text{if } \varepsilon_t^h = 0 \\ \frac{M_t^h + w^h\left(n_t^h\right)}{p_t} & \text{if } \varepsilon_t^h = \bar{\varepsilon} \end{cases}$$

where $\eta_t^M, \eta_t^D, \eta_t^K \in [0,1]$ are independent of A_t^h.

Since the felicity from consumption is log, I guess and verify that household choices during the day are proportional to initial wealth A_t^h. At night, an impatient household ($\varepsilon_t^h = \bar{\varepsilon}$) withdraws deposits if unconstrained ($l_t^h = +\infty$) and uses money $M_t^h = \eta_t^M A_t^h$ and withdrawals w_t^h to finance her consumption expenditures. If the household is patient ($\varepsilon_t^h = 0$), her choice of consumption is zero, but she is nonetheless willing to withdraw if the actual return on deposits is negative ($r_t^{b(h)} < 0$). In this crucial case, the nominal return on money is zero, thus larger than the nominal return on deposits not withdrawn. The household runs on the bank and withdraws all the available deposits D_t^h if the bank still has money while household h is served ($l_t^h = +\infty$). If instead the bank has no money ($l_t^h = 0$), the household is stuck with zero withdrawals and receives a negative return on deposits.

Since M_t^h, D_t^h and K_t^h are proportional to initial wealth A_t^h, the following corollary holds.

Corollary 3.4. *The choices M_t^h, D_t^h and K_t^h of the household sector can be described by a representative household with initial wealth $\overline{A}_t \equiv \int_{\mathcal{H}} A_t^h dh$.*

Consequently, the shocks ψ_t^h hitting the capital owned by the household sector are irrelevant, because they simply modify the distribution of wealth but they do not influence the total value of wealth $\overline{A}_t$. It is however crucial that the idiosyncratic shocks to capital hit the balance sheet of banks, creating heterogeneity among financial intermediaries.

3.3 Dividends

Recall that banks are subject to the exit shock λ. Given the vector of state variables $\boldsymbol{X}_t^e$ of an exiting bank e:

$$\boldsymbol{X}_t^e = \left\{ \left(K_{t-1}^e, m_{t-1}^e, d_{t-1}^e \right), \psi_t^e \right\}$$

the net worth of such bank is:

$$N_t^e = K_{t-1}^e \left(1 + \psi_t^e\right) Q_t + m_{t-1}^e - d_{t-1}^e$$

and the total value of dividends paid to bankers is:

$$\pi_t = \int \max\left\{0, N_t^e\right\} de + \int_{\{\text{banks subject to run at } t-1, night\}} \max\left\{0, N_t^b\right\} db. \tag{21}$$

In the relevant equilibrium cases, either no bank is subject to runs (in the good equilibrium) or banks subject to runs have negative net worth (in the bad equilibria). Therefore, the second term in (21) is always zero in equilibrium.

3.4 Market clearing conditions

The following market clearing conditions must hold in equilibrium:

- capital market and money market, day:

$$\int_0^1 K_t^b db + \int_{\mathcal{H}} K_t^h dh = \overline{K} \tag{22}$$

$$\int_0^1 M_t^b db + \int_{\mathcal{H}} M_t^h dh = \left(\overline{M} - \pi_t\right) \tag{23}$$

  since an amount π_t of money is used to pay dividends to bankers;

- deposits, day:

$$\int_0^1 D_t^b db = \int_{\mathcal{H}} D_t^h dh \tag{24}$$

- goods market clearing, night:

$$\int_{\mathcal{H}} c_t^h dh + \frac{\pi_t}{p_t} = Z\overline{K}; \tag{25}$$

where $\frac{\pi_t}{p_t}$ is the consumption of bankers.

3.5 Equilibrium definition

Given the state of the economy $\boldsymbol{X}_t$ (see Section 2.5), the distribution over banks' state $\Pr_t^{\boldsymbol{X}^b}$ and the price of capital Q_t imply the distribution $\Pr_t^N$ over the net worth $\left\{N_t^b\right\}_{b\in[0,1]}$ defined by:

$$\Pr_t^N\left(N_t^b = N; Q_t\right) = \sum_{\left\{\boldsymbol{X}_t^b | K_{t-1}^b\left(1+\psi_t^b\right)Q_t + m_{t-1}^b - d_{t-1}^b = N\right\}} \Pr_t^{\boldsymbol{X}^b}\left(\boldsymbol{X}_t^b\right).$$

Although the probability $\Pr_t^{\boldsymbol{X}^b}$ over $\boldsymbol{X}_t^b$ is given by the state of the economy, the probability $\Pr_t^N$ is an endogenous object because it depends on the price of capital Q_t. For a given $\Pr_t^{\boldsymbol{X}^b}$, the price of capital influences the (in)solvency of banks in the economy. The role of Q_t in the determination of net worth is at the heart of the multiple equilibria mechanism.

Combining $\Pr_t^N$ with the the optimal choice of banks described in Proposition 3.1 and with the expressions for r_t^b and f_t^b in equations (11) and (13), I obtain a probability distribution over the actual return on deposits $r_t^{b(h)}$ and the limits on withdrawals l_t^h of household h denoted by:

$$\Pr_t^{(r,l)}\left(r_t^{b(h)} = r,\ l_t^h = l\right),\ r \in \mathbb{R} \text{ and } l \in \{0, +\infty\} \tag{26}$$

for all $h \in \mathcal{H}$.

Since I require that the promised return on deposits R_t^D is equalized across all banks, I impose a pooling equilibrium in the banking market, similar to Akerlof (1970). The results are unchanged if I allow each bank b to post a bank-specific promised return on deposits. In this case, the equilibrium that arises is still a pooling one because bad banks want to imitate good banks to survive as long as possible.

The next definition formalizes the equilibrium concept. I require that the beliefs of households concerning $r_t^{b(h)}$ and l_t^h are rational in the sense that they reflect the realized probability distribution (26).

Definition 3.5. *Given the initial state of the economy* $\boldsymbol{X}_t$*, an equilibrium is a collection of:*

- *prices* Q_t *and* p_t *and return on capital* R_t^K *and on deposits* R_t^D*;*
- *household beliefs* $Pr_t^h(\cdot)$ *about* $r_t^{b(h)}$ *and* l_t^h*, for all* $h \in \mathcal{H}$*;*
- *household choices* $\left\{M_t^h, D_t^h, K_t^h, \left\{w^h\left(n_t^h\right), c^h\left(n_t^h\right)\right\}_{n_t^h \in \mathcal{N}}\right\}$ *for all* $h \in \mathcal{H}$*;*
- *bank choices* $\left\{D_t^b, M_t^b, K_t^b\right\}$ *for all* $b \in [0,1]$*;*
- *limits on withdrawals* $l_t^h \in \{0, +\infty\}$ *for all* $h \in \mathcal{H}$*;*
- *liquidation returns* r_t^b *and fraction of depositors served during a run* f_t^b*, for all* $b \in [0,1]$*;*
- *dividends* π_t *paid to bankers;*

such that:

- *(banks: optimality, returns and limits on withdrawals) for all* b*, the choices* $\left\{D_t^b, M_t^b, K_t^b\right\}$ *are optimal (Proposition 3.1);* r_t^b *and* f_t^b *are defined, respectively, by equations* (11) *and* (13) *and*[19]*:*

$$l_t^h = 0 \text{ for some } h \in \mathcal{H}(b) \Rightarrow \int_{\mathcal{H}(b)} w^h\left(n_t^h \middle| l_t^h = +\infty\right) dh > M_t^b;$$

- *(households' optimality) for all* h*, the choice* $\left\{M_t^h, D_t^h, K_t^h, \left\{w^h\left(n_t^h\right), c^h\left(n_t^h\right)\right\}_{n_t^h \in \mathcal{N}}\right\}$ *solves problem* (14)*;*
- *(rational expectations) households' beliefs are rational:*

$$Pr_t^h\left(r_t^{b(h)} = r,\, l_t^h = l\right) = Pr_t^{(r,l)}\left(r_t^{b(h)} = r,\, l_t^h = l\right), \quad r \in \mathbb{R} \text{ and } l \in \{0, +\infty\};$$

- *(dividends)* π_t *is defined by* (21)*;*
- *(market clearing) the market clearing conditions hold.*

[19] This condition implies that if a household faces a limit on withdrawals $l_t^h = 0$, then the unconstrained amount of withdrawals $\int_{\mathcal{H}(b)} w^h\left(n_t^h \middle| l_t^h = +\infty\right) dh$ is not feasible).

Restriction on parameters. I impose a restriction on λ, $\underline{\psi}$, β and κ to ensure that there exists a well-defined steady-state (see Section 4.1).

Assumption 3.6. $\lambda = \frac{1-\beta}{\beta+(1-\beta)(1/\kappa)}$.

Assumption 3.7. *The parameters $\underline{\psi}$, β and κ satisfy:*

$$(1-\beta)(1-\kappa) < 1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right].$$

4 Results

In this Section, I present the results of the model. Section 4.1 describes the steady-state with no shocks to capital for all t. Then, starting from the economy in steady-state, I consider the effects of one-time unanticipated idiosyncratic shocks to capital at time t, $\psi_t^i \in \left\{\underline{\psi}, \overline{\psi}\right\}$. At time t, multiple equilibria can arise: a good equilibrium where prices and aggregate quantities are the same as in the steady-state; and up to two bad equilibria described in Section 4.2. If the economy experiences a crisis at time t (bad equilibrium), the crisis lasts one period and the economy is in a good equilibrium from $t+1$ onward.

I impose two restrictions on initial conditions. All banks are alike at the beginning of the day and their holding of capital and money are large enough to guarantee that banks hit by the bad shock $\underline{\psi}$ are solvent in the good equilibrium. More precisely, as a consequence of the next Assumption, banks hit by the bad shock $\underline{\psi}$ have zero net worth in the good equilibrium.

Assumption 4.1. *All banks are identical at the end of the night of time $t-1$: for all $b, b' \in [0,1]$:*

$$\begin{aligned} K_{t-1}^b &= K_{t-1}^{b'} \\ m_{t-1}^b &= m_{t-1}^{b'} \\ d_{t-1}^b &= d_{t-1}^{b'} \end{aligned}$$

and, for all $b \in [0,1]$:

$$K_{t-1}^b\left(1+\underline{\psi}\right)\left[\frac{\beta}{1-\beta}+\left(\frac{1}{\kappa}-1\right)\right]\frac{\overline{M}}{\overline{K}} + m_{t-1}^b - d_{t-1}^b = 0. \tag{27}$$

4.1 Steady-state and good equilibrium

In the steady-state, prices and the nominal interest rate are constant ($Q_t = Q^*$, $p_t = p^*$ and $R_t^K = R^*$). All banks are identical and solvent ($N_t^b = N^* > 0$ for all b), the market return

on deposits is equal to the return on capital ($R_t^D = R_t^K = R^*$), all banks pay the promised return on deposits that are not withdrawn ($r_t^b = R_t^D = R^*$) and there are no runs ($l_t^h = +\infty$ for all h). Therefore, banks pool the liquidity risk of households, insuring them against preference shocks. The representative household holds deposits D^* and no money ($M_t^h = 0$), because there are no runs and thus withdrawals at night are used to finance consumption expenditure. Dividends to bankers are also constant at $\pi_t = \pi^*$.

Proposition A.2 in Appendix A presents the complete characterization of steady-state prices (Q^*, p^* and R^*) and quantities (N^*, D^* and π^*) in closed form, as functions of the parameters.

If the idiosyncratic shocks to capital $\psi_t^i \in \left\{\underline{\psi}, \overline{\psi}\right\}$ hit the economy, a good equilibrium always exists. The idiosyncratic shocks imply a redistribution of capital within the banking sector and within the household sector, but prices and aggregate quantities in the good equilibrium are the same as in the steady-state: price of capital $Q_t = Q^*$, price of consumption goods $p_t = p^*$, nominal returns on capital and on deposits $R_t^K = R_t^D = R^*$, deposits and money of the representative household $D_t^h = D^*$ and $M_t^h = 0$, and profits π^*. Equation (27) implies that all banks are solvent in the good equilibrium, including banks hit by the bad shock $\underline{\psi}$. The good equilibrium is described by Proposition A.4 in Appendix A.

4.2 Bad equilibria

When the unanticipated shocks to capital $\underline{\psi}$ and $\overline{\psi}$ hit the economy, the good equilibrium is not the unique one, for a large subset of the parameter space. There can be up to two bad equilibria, depending on the parameters. I cannot solve for the bad equilibria in closed form, so I compute it numerically. However, I am able to use the full non-linear model without relying on approximations. In this Section, I describe the results of a bad equilibrium, the full set of equations and details regarding the numerical computations are in Appendix B.

A bad equilibrium at time t is characterized by four features.

i. The price level is $p_t < p^*$ and the nominal price of capital is $Q_t < Q^*$: the economy experiences deflation and a drop of (nominal) asset price.

ii. Banks hit by the bad idiosyncratic shock $\underline{\psi} < 0$ are insolvent: $N_t\left(\underline{\psi}\right) < 0$, where $N_t\left(\underline{\psi}\right)$ is the net worth of bank b with shock to capital $\psi_t^b = \underline{\psi}$. Banks hit by $\overline{\psi}$ are solvent, $N_t\left(\overline{\psi}\right) > 0$.

iii. Insolvent banks pay a negative actual return on deposits at night, $r_t\left(\underline{\psi}\right) < 0 < R_t^D$, and are subject to runs at night. Solvent banks pay the promised return R_t^D and are not subject to runs.

iv. The representative household holds deposits $D_t^h < D^*$ and money $M_t^h > 0$ (flight to liquidity).

The insolvency of banks hit by $\underline{\psi}$ (ii) is a direct consequence of the drop of Q_t (i). Recall that deposits are expressed in terms of money and thus the nominal value of liabilities of banks is not affected by prices. Insolvent banks pay actual return on deposits $r_t\left(\underline{\psi}\right) < R_t^D$ (iii) because they do not have enough resources to start with. Such banks are subject to runs because the actual return on deposits is negative, $r_t\left(\underline{\psi}\right) < 0$, while the return from withdrawing and holding money is zero. Therefore, running is the optimal choice of depositors. The flight to liquidity (iv) is a result of fear of runs (iii): anticipating runs, households hold more money and less deposits in order to (partially) self-insure against liquidity needs.

The scenario described in i - iv is an equilibrium because there is a feedback from the flight to liquidity (iv) to the drop of prices (i). Due to the flight to money by all households, some money is held by households whose realized preference shock is $\varepsilon_t^h = 0$. Such money is unspent and "stored under the mattresses", therefore less money is used for transactions in the economy. Multiplying both sides of the market clearing condition for goods (25) by p_t (and ignoring dividends for simplicity):

$$\underbrace{p_t \int_{\mathcal{H}} c_t^h dh}_{\text{consumption expenditure}} = p_t Z \overline{K}.$$

In the good equilibrium, the left-hand side is equal to $\overline{M}$ because all the money is spent and $p_t = p^* \equiv \frac{\overline{M}}{Z\overline{K}}$. In the bad equilibrium, the left-hand side is $< \overline{M}$ because some money in the economy is unspent, therefore $p_t < p^*$. Finally, the real price of asset $\frac{Q_t}{p_t}$ must be (approximately) constant because the bad equilibrium does not influence the productivity of capital. Thus, a drop of p_t associates with a drop of Q_t.

A fraction α^{BAD} of banks is hit by the bad shock $\underline{\psi}$, then α^{BAD} is also the fraction of insolvent banks. During the day, households cannot distinguish solvent and insolvent banks because they do not observe the shocks $\left\{\psi_t^b\right\}_{b\in[0,1]}$. At night, household $h \in \mathcal{H}$, holding deposits at bank $b\left(h\right)$, faces one of the following possibilities.

1. With probability $1 - \alpha^{BAD}$, bank $b\left(h\right)$ pays the promised return R_t^D. Therefore, bank $b\left(h\right)$ is not subject to runs and household h can withdraw any amount $\leq D_t^h$.

2. With probability α^{BAD}, bank $b\left(h\right)$ is bankrupt in $t+1$, paying a return $r_t\left(\underline{\psi}\right) < 0$. Therefore the optimal choice is to run and try to withdraw as much as possible:

(a) with probability $f_t^{b(h)}$, household h is first in line ($l_t^h = +\infty$) so she is able to withdraw any amount of money $w_t^h \leq D_t^h$;

(b) with probability $1 - f_t^{b(h)}$, household h is "last in line" ($l_t^h = 0$), so she is unable to withdraw money, $w^h\left(n_t^h \middle| \, l_t^h = 0\right) = 0$. In this case, if household h is impatient ($\varepsilon_t = \bar{\varepsilon}$), she is able to buy some consumption goods only if she chose to hold some money $M_t^h > 0$ during the day.

Therefore, the beliefs of households are:

$$\begin{aligned}
\Pr_t^h\left(r_t^{b(h)} = R_t^D,\ l_t^h = +\infty\right) &= 1 - \alpha^{BAD} \\
\Pr_t^h\left(r_t^{b(h)} = r_t\left(\underline{\psi}\right) < 0\,, l_t^h = +\infty\right) &= f_t^{b(h)}\alpha^{BAD} \\
\Pr_t^h\left(r_t^{b(h)} = r_t\left(\underline{\psi}\right) < 0\,, l_t^h = 0\right) &= \left(1 - f_t^{b(h)}\right)\alpha^{BAD}.
\end{aligned}$$

The welfare loss in the bad equilibrium is a misallocation of consumption across households. Consider households with the same initial wealth: optimality requires the same level of consumption for these households. However, some households are last in line during a run ($l_t^h = 0$) and thus their withdrawals and consumption expenditure is limited by a binding cash-in-advance constraint. Other households are first in line during runs or face no runs on their own bank ($l_t^h = +\infty$), thus their consumption expenditure is unconstrained and, in equilibrium, it is higher than the consumption expenditure of households that are last in line in runs.[20]

In order to solve for a bad equilibrium, I conjecture that households run on banks hit by $\underline{\psi}$, and then I solve for the equilibrium. The conjecture is verified if $r_t\left(\underline{\psi}\right) < 0$ so that "running" is indeed the optimal choice of households. The initial conjecture is confirmed for a wide range of parameters. For some values of the parameters, $0 < r_t\left(\underline{\psi}\right) < R_t^D$ thus there exists only the good equilibrium for such parameters.

4.3 Numerical example

The values of parameters are shown in Table 1. I consider two values of κ (i.e., the probability that household h is hit by the preference shock $\varepsilon_t^h = \bar{\varepsilon} > 0$): $\kappa = 0.5$ and $\kappa = 0.85$. The

[20] There is another welfare difference between the good and bad equilibria, driven by a difference in the evolution of the distribution of wealth across households. This effect is a force that increases welfare in the good equilibrium. The Online Appendix discusses the two welfare differences between the good and bad equilibria (misallocation of consumption and distribution of wealth in $t+1$ across households), and sketches an extension of the model in which welfare is always higher in the good equilibrium.

Table 1: Parameters value

Parameter	Description	Value
β	Discount factor	0.988
Z	Productivity, $y(K) = ZK$	1/3
$\overline{M}$	Money supply	1
$\overline{K}$	Supply of capital	1
$\underline{\psi}$	Bad shock to capital	-0.25
$\overline{\psi}$	Good shock to capital	0.03
α^{BAD}	Fraction of banks hit by the shock $\underline{\psi}$	0.1
κ	$\Pr\left(\varepsilon_t^h = \bar{\varepsilon}\right)$	$\{0.5, 0.85\}$

value of λ is determined using β, κ and Assumption 3.6; the value of $\bar{\varepsilon}$ is determined by κ and the normalization in equation (2). Since $\alpha^{BAD} = 0.1$, 10% of the banks are hit by $\underline{\psi}$ when the shocks to capital hit the economy. The parameters satisfy Assumption 3.7.

For the case $\kappa = 0.5$, the actual return on deposits of insolvent banks in the bad equilibrium is $r_t\left(\underline{\psi}\right) = -0.14 < 0$ and the other key endogenous variables are plotted in Figure 2. The economy is in steady-state in $t = 0$, experiences a crisis in $t = 1$ and then reverts to normal in $t = 2$. The top panel plots the prices p_t, Q_t and the nominal return on capital R_t^K. The bottom left panel of Figure 2 plots some key variables in the money market: deposits, money held by households during the day and a monetary aggregate denoted M1. M1 is defined as the sum of deposits and money held by households, in line with the standard definition of such monetary aggregate. The model captures the flight to liquidity that I mentioned in the Introduction; households hold less deposits and more money in comparison to $t = 0$. As a result, the drop in M1 with constant $\overline{M}$ implies a drop in the money multiplier[21], which is again consistent with empirical evidence. The evolution of the monetary aggregates in the model can be (qualitatively) compared with the data of Friedman and Schwartz (1970) regarding the Great Depression, plotted in the bottom right panel of Figure 2. During the Great Depression, the U.S. economy experienced a drop in the total stock of money (first line from the top) similar to the drop in M1, a drop in total deposits (second line from the top), and an increase in currency held by households (first line from the bottom). Thus the model captures these facts related to the money market.[22]

[21] In this context, the money multiplier is the ratio between M1 and $\overline{M}$.

[22] Results from this version of the model with a fixed money supply $\overline{M}$ cannot be compared with the 2008 financial crisis because of the monetary injections from the Federal Reserve.

Table 2: Comparison between good equilibrium, mild crisis and deep crisis, $\kappa = 0.85$

Variable name		Good Equilibrium	Mild Crisis	Deep Crisis
Price level	p_t	3	2.79	2.59
Price of capital	Q_t	80	73	70.6
Money holding by households	$\int_{\mathcal{H}} M_t^h dh$	0	0.61	0.88
Deposits	$\int_{\mathcal{H}} D_t^h dh$	1.17	0.46	0.13
M1	$\int_0^1 D_t^b db + \int_{\mathcal{H}} M_t^h dh$	1.17	1.07	1.01
Return on capital	R_t^K	0.0125	0.11	0.14
Return on deposits, insolvent banks	r_t^b s.t. $N_t^b < 0$	(n.a.)	-0.13	-0.99

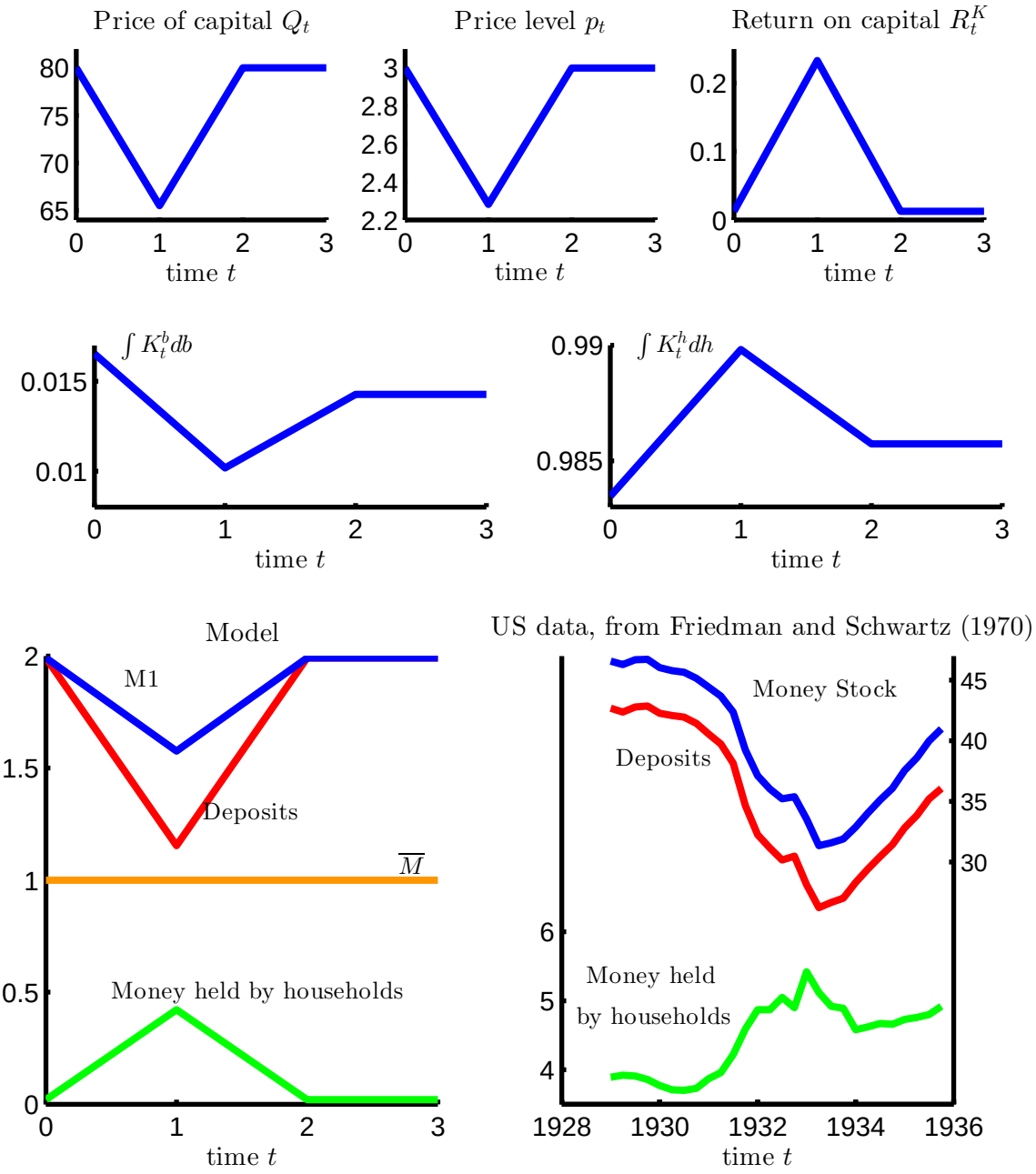
The middle panel of Figure 2 plots the evolution of capital held by banks (left panel) and households (right panel). During the crisis, banks have fewer resources because of the flight away from deposits. Therefore banks reduce holdings of capital with respect to pre-crisis level. Since the supply of capital is fixed, households must increase their holdings of capital in equilibrium. Interpreting the banking sector in the model as the shadow banking system in the U.S. and assuming that commercial banks are part of the household sector in the model, then the result concerning capital holdings is consistent with the data analyzed by He, Khang, and Krishnamurthy (2010).[23]

Table 2 shows the result for the case $\kappa = 0.85$. Under this parameterization, there exist two bad equilibria that I label "mild crisis" and "deep crisis". In the deep crisis equilibrium, in comparison with the mild crisis, the drop in prices is more pronounced, the flight to liquidity and the drop in M1 are larger, and the return on deposits of insolvent banks is much smaller (deposits not withdrawn are worth \$0.86 per dollar in the mild crisis and \$0.01 per dollar in the deep crisis).

The Online Appendix analyzes the sensitivity of the bad equilibria to the value of α^{BAD}. The larger the parameter α^{BAD}, the worse the crisis because there are more bad banks and thus households are more concerned about facing a run on their own bank. Therefore the flight to liquidity is more pronounced.

[23] He, Khang, and Krishnamurthy (2010) finds that, during the recent U.S. financial crisis, securitized assets shifted from sectors dependent on repo financing to commercial banks.

Figure 2: Bad equilibrium: numerical example and comparison with the Great Depression



The top panel plots the evolution of the price level p_t, the nominal price of capital Q_t and the return on capital R_t^K. The economy experiences the crisis in period $t = 1$ and reverts to normal in $t + 1$. The middle panel plots the stock of capital held by banks (left) and households (right) at the end of the day market. The bottom left panel plots the evolution of variables in the money market: money supply $\overline{M}$, deposits, money held by households and $M1$ = deposits + money held by households ($\kappa = 0.5$). The figure on the bottom right panel is based on Table 2 from Friedman and Schwartz (1970). The green line is currency held by the public (left axis), the red line is total deposits (right axis) and the blue line is the "Money Stock" (i.e., the sum of currency held by the public and deposits, right axis). Data are quarterly and seasonally adjusted, in billions of dollars.

4.4 Understanding the multiplicity of bad equilibria

The next Proposition suggests that the driving force of the multiplicity of bad equilibria is a strategic complementarity across depositors. Recall that $\mathcal{H}(b)$ denotes the depositors of bank b and thus $\int_{\mathcal{H}(b)} D_t^h dh = D_t^b$ is the amount of deposits of bank b.

Proposition 4.2. *Fix prices Q_t, p_t and R_t^K. The actual return on deposits r_t^b of bank b with negative net worth ($N_t^b < 0$) satisfies:*

$$\frac{\partial r_t^b}{\partial \left[\int_{\mathcal{H}(b)} D_t^h dh\right]} = \frac{\partial r_t^b}{\partial D_t^b} > 0.$$

The proof is provided in the Online Appendix. To understand the result, recall that an insolvent bank b has preexisting losses borne by depositors that hold deposits from the bank. If households decide to hold many deposits at bank b, each dollar of deposit bears small losses; the opposite is also true. Consequently, the higher deposits $\int_{\mathcal{H}(b)} D_t^h dh$ chosen by other depositors of bank b, the higher the willingness of household h to hold deposits issued by bank b, explaining the strategic complementarity. Such strategic complementarity does not arise in the good equilibrium because all banks are solvent and $r_t^b = R_t^D$ is independent of the choices of other depositors.

The result of Proposition 4.2 is a partial equilibrium exercise in the sense that it is derived fixing prices and analyzing the behavior of only one bank b in the economy. Next, I use general equilibrium analysis to examine feedback between deposits by all households $\int_{\mathcal{H}} D_t^h dh$ and the actual return on deposits $r_t\left(\underline{\psi}\right)$ paid by insolvent banks hit by the bad shock $\underline{\psi}$. In this general equilibrium analysis, prices Q_t, p_t and R_t^K are not held constant. I first fix a value of deposits $\hat{D} \in (0, D^*]$ and then solve for equilibrium prices Q_t, p_t and the actual return $r_t\left(\underline{\psi}\right)$ that arise in an economy in which households are forced to choose deposits $\hat{D}$[24]. Second, I take as given the prices Q_t, p_t and $r_t\left(\underline{\psi}\right)$ just computed and allow the representative household h to make her optimal choices of money M_t^h, deposits D_t^h and capital K_t^h. I obtain a relationship between D_t^h and the value of $\hat{D}$: a fixed point that satisfies $D_t^h = \hat{D}$ is an equilibrium of the model.

The top panel of Figure 3 plots the value of $r_t\left(\underline{\psi}\right)$ as a function of $\hat{D}$ (the left panel uses $\kappa = 0.5$ and the right panel uses $\kappa = 0.85$). The general equilibrium analysis does not alter the result of Proposition 4.2. The return $r_t\left(\underline{\psi}\right)$ of an insolvent bank is increasing in the amount of deposits $\hat{D}$.

[24]To compute prices, I solve for the equilibrium forcing households to choose $\hat{D}$ and dropping the FOC of households with respect to D_t^h.

The bottom panel of Figure 3 plots the choices of deposit D_t^h by the representative household h as a function of $\hat{D}$, taking as given the return $r_t\left(\underline{\psi}\right)$ plotted in the top panel. The choice of deposits D_t^h is increasing in the value of $\hat{D}$ but only for a subset of the horizontal axis. Despite the non-monotonicity, two bad equilibria arise if $\kappa = 0.85$, while there exists only one bad equilibrium if $\kappa = 0.5$. To understand this difference, recall that the maximization problem of banks (6) is subject to the budget constraint (7) and to the non-negativity constraint $K_t^b \geq 0$. The budget constraint combined with $K_t^b \geq 0$ and the decision rule of banks $M_t^b = \kappa D_t^b$ (from Proposition 3.1) implies $D_t^b\left(1-\kappa\right) + N_t^b \geq 0$ or, for bank b with net worth $N_t^b = N_t\left(\underline{\psi}\right) < 0$:

$$D_t^b \geq \frac{-N_t\left(\underline{\psi}\right)}{1-\kappa} > 0. \tag{28}$$

Since I force households to hold deposits $\hat{D}$, the market clearing condition for deposits (24) implies $D_t^b = \hat{D}$. Therefore equation (28) becomes:

$$\hat{D} \geq \frac{-N_t\left(\underline{\psi}\right)}{1-\kappa} > 0. \tag{29}$$

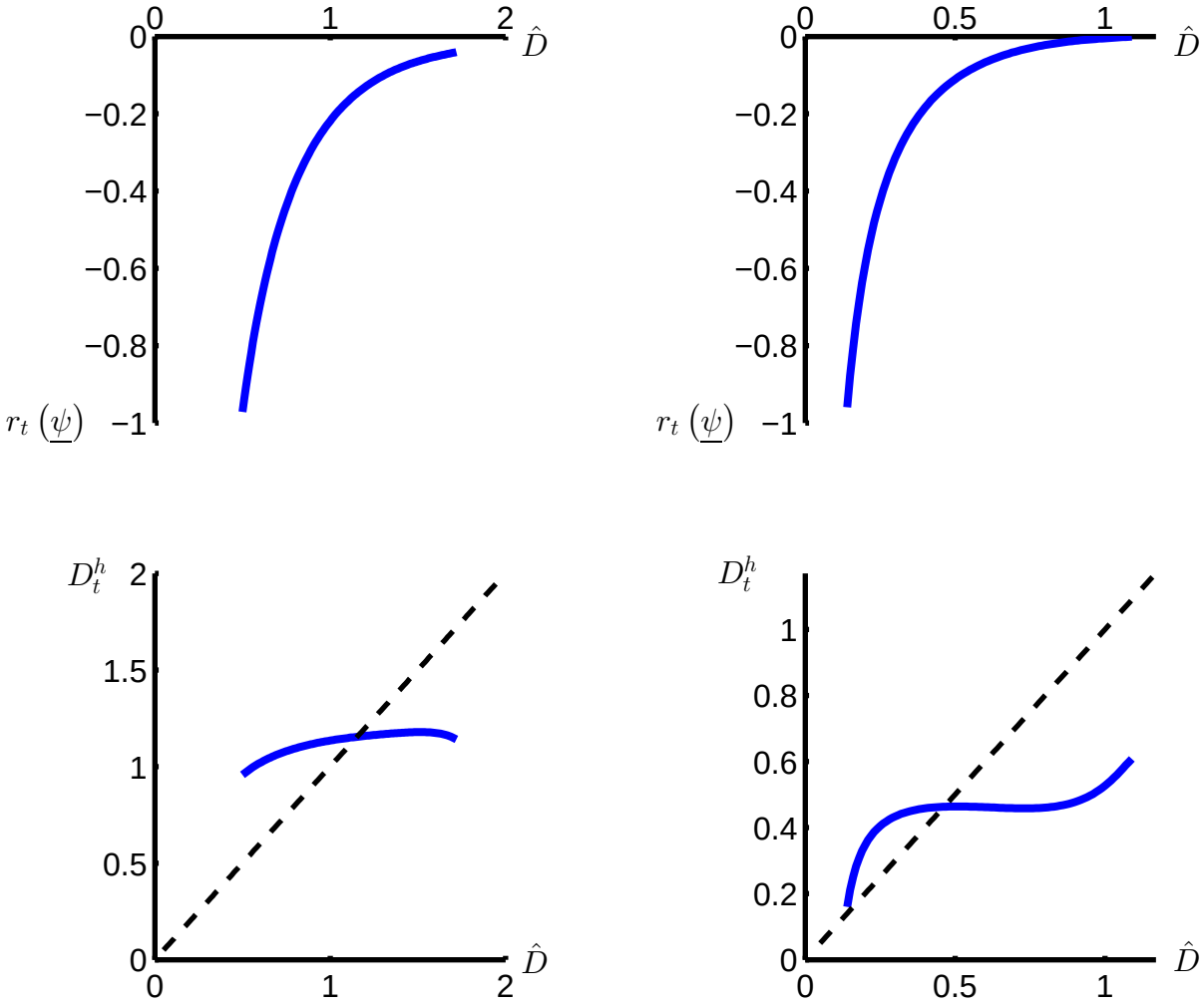
For the case $\kappa = 0.5$, equation (29) is satisfied for $\hat{D} \geq 0.5$ (recall that $N_t\left(\underline{\psi}\right)$ is an equilibrium object which is itself computed after fixing $\hat{D}$). Therefore it is not possible to fix a value of $\hat{D}$ smaller than 0.5 to look for a deep crisis equilibrium. For the case $\kappa = 0.85$, equation (29) is instead satisfied for $\hat{D} \geq 0.135$, and the equilibrium value of deposit in the deep crisis is 0.136.

Since the value of $N_t\left(\underline{\psi}\right)$ is endogenous, it is affected by monetary policy. As I show in Section 5, the right-hand side of equation (29) depends on monetary policy. Monetary injections decrease the (absolute) value of $N_t\left(\underline{\psi}\right)$. For sufficiently large monetary injections, there exist a deep crisis equilibrium even for the case $\kappa = 0.5$.

As an alternative explanation for the lower bound on $\hat{D}$, the actual return on deposits $r_t\left(\underline{\psi}\right)$ is bounded below by -1 because depositors cannot lose more than 100% of deposits. The top panel of Figure 3 shows that the lower bound $r_t\left(\underline{\psi}\right) = -1$ is reached at $\hat{D} = 0.5$ for the case $\kappa = 0.5$ and at $\hat{D} = 0.135$ for the parameterization $\kappa = 0.85$.

I solved the model with several values of the parameters and could find at most two equilibria. It is not possible to exclude that there can be more than two bad equilibria using more general versions of the model (e.g., a model designed for quantitative analysis). But understanding the multiplicity of bad equilibria in such models is outside the scope of this paper.

Figure 3: Actual return on deposits of an insolvent bank and strategic complementarity



Left panel: $\kappa = 0.5$; right panel: $\kappa = 0.85$. The plots are obtained as follow: First, I fix a value of $\hat{D}$ and then solve for the equilibrium prices Q_t, p_t and for the actual return on deposits $r_t\left(\underline{\psi}\right)$ paid by banks hit by shock $\underline{\psi}$, without including the FOC of the household with respect to D_t^h. The second step is a partial equilibrium exercise. Given $r_t\left(\underline{\psi}\right)$, Q_t and p_t, I allow the representative household h to make her optimal choices. The resulting D_t^h is plotted on the bottom panel; the dashed line is the 45 degrees line.

5 Monetary policy

I consider the effects of a monetary authority that injects money into the economy during a crisis. The monetary authority announces a policy that will be implemented in the event of a panic, and can credibly commit to it. The central bank chooses money supply $M_t = \overline{M}(1+\mu_t)$ by choosing the value of μ_t.

I focus on two policies that capture some interventions of the Federal Reserve during the recent financial crisis: asset purchases and loans to banks.

A sufficiently large permanent monetary injection creates an inflationary pressure, increasing the price level and ruling out the bad equilibrium. However, the central bank might be unable or unwilling to create inflation due to considerations that are not captured by the model.[25] Moreover, the increase of money supply implemented during the recent financial crisis are temporary, as suggested by Bernanke (2010). Due to these considerations, I impose two restrictions on monetary policy. First, money supply in $t+1$ (i.e., when the crisis is over) must revert to pre-crisis level $\overline{M}$, therefore prices after the crisis are Q^* and p^*. Second, I restrict the analysis to policies that do not create inflation during the crisis: as a result of a monetary injection, the price of capital and the price level must satisfy $Q_t \leq Q^*$ and $p_t \leq p^*$ in the bad equilibria.[26] I use the term "non-inflationary monetary injection" to denote a policy that satisfies the two restrictions.

The Online Appendix presents the result of monetary injections that do not satisfy non-inflationary restriction.

5.1 Asset purchases

The central bank buys capital on the market during the day of time t and sells it in $t+1$, therefore money supply reverts to $\overline{M}$ after the crisis. The return from holding capital are rebated to households in $t+1$[27]. For a given a monetary injection μ_t, I conjecture that

[25] First, there might be welfare costs of a one-time increase in the price level or costs regarding regaining credibility to keep a low inflation in the long run after the increase in the price level, that are not included in the model. Second, the zero lower bound has been reached in 2008, and new Keynesian models suggest that inflationary policies require credibility to commit to future inflation (see e.g. Krugman (1998), Eggertsson and Woodford (2003) and Werning (2011)).

[26] In a testimony before the Committee on Financial Services of the U.S. House of Representatives, Bernanke (2010) said that "In due course [...] as the expansion matures the Federal Reserve will need to begin to tighten monetary conditions to prevent the development of inflationary pressures. The Federal Reserve has a number of tools that will enable it to firm the stance of policy at the appropriate time.".

[27] In $t+1$, each household h gets a transfer proportional to her wealth A_t^h at time t; this assumption allows me to still be able to guess-and-verify the form of the value function and the choices of the households. See Appendix C for a full description of the model with monetary injections.

there exists a bad equilibrium and I solve for the non-linear system of equations described in Appendix C. The solution of the system of equations is a "candidate bad equilibrium". The candidate outcome is an equilibrium if $Q_t \leq Q^*$ and $p_t \leq p^*$ (in order to satisfy the restriction on the non-inflationary nature of the monetary intervention) and if $r_t\left(\underline{\psi}\right) < 0$ (see Section 4.2).

For each level of non-inflationary monetary injection μ_t implemented with asset purchases, Figure 4 displays the outcome of the most important endogenous variables.[28] When the central bank buys capital on the market, the demand for capital rises and therefore the price Q_t is higher in comparison to an economy without intervention. Due to the monetary injection, more money is in circulation and thus the price level p_t is also higher.

The higher Q_t has two counteracting effects on deposits: the first effect increases the demand of deposits and the second effect decreases it. The consequence of asset purchases on the demand for deposits is ambiguous. First, the higher price of capital reduces losses of insolvent banks. Consequently, the actual return on deposits $r_t\left(\underline{\psi}\right)$ paid by insolvent banks is higher. This effect increases demand of deposits from households since losses on deposits of insolvent banks are smaller. Second, using the fact that the monetary injection is temporary and thus $Q_{t+1} = Q^*$, the nominal return on capital, equation (4), becomes:

$$1 + R_t^K = \frac{Q^* + ZP_t}{Q_t} \tag{30}$$

The increase of Q_t implied by the monetary injections reduces R_t^K. Since solvent banks pay the promised return on deposits $R_t^D = R_t^K$, the return R_t^D decreases with monetary injections as well. Since the return on deposits R_t^D paid by good banks lowers, the demand of deposits by households decreases. The downward pressure on the demand for deposits is a consequence of the temporary nature of the monetary injection.

In the numerical example that I consider, the higher the monetary injection μ_t the lower the deposits.[29] The intervention of the central bank exacerbates the flight to liquidity and the effectiveness of a monetary injection reduces in comparison to an economy with exogenous movements in money demand. The higher price of capital Q_t reduces the losses of insolvent banks and thus it increases $r_t\left(\underline{\psi}\right)$, while the reduction in deposits decreases $r_t\left(\underline{\psi}\right)$ (as a consequence of Proposition 4.2). In the numerical example, $r_t\left(\underline{\psi}\right)$ is increasing in μ_t, but

[28] Figure 4 uses the parameters in Table 1 and $\kappa = 0.5$. The results are robust to the choices of other parameter values with an important caveat. For some value of the parameters, a bad equilibrium does not exist for some μ_t. For instance, there exists no equilibrium using the parameters in Table 1, $\kappa = 0.85$ and $\mu_t \in (0.12, 0.15)$, while there exists an equilibrium if $\mu_t \in [0, 0.12]$ and $\mu_t \in [0.15, 0.17]$. The Online Appendix presents and discusses these results.

[29] In an alternative calibration with $\kappa = 0.2$, the equilibrium value of deposits is increasing for $\mu \in (0, 0.5)$.

the reduction of deposits partially offsets the positive effect of the higher Q_t.

As discussed in Section 4.4, monetary injections affect the value of $N_t\left(\underline{\psi}\right)$, reducing the right-hand side of equation (29). Therefore, for sufficiently large monetary injections, there exists a deep crisis equilibrium for the case $\kappa = 0.5$ (red lines in Figure 4). Recall that the deep crisis equilibrium is characterized by an even higher flight to liquidity, compared to the mild crisis equilibrium. The combination of deep crisis equilibrium and strategic complementarity imply that non-inflationary asset purchases result in at least one crisis equilibrium.[30]

5.2 Liquidity facility: loans to banks

The central bank provides loans to banks during the day at time t. For each dollar borrowed at time t, banks must repay $1 + R_t^K$ dollars during the day in $t+1$. Banks can use funds borrowed from the central bank to hold money or buy capital. The budget constraint (7) of bank b during the day becomes[31]:

$$K_t^b Q_t + M_t^b \leq D_t^b + (\text{loans from central bank})_t^b + N_t^b.$$

In case some banks are insolvent in the economy, I must consider the ability of the central bank to recover in $t+1$ the loans made in the day at time t. At one extreme, consider the case in which loans from the central bank are senior with respect to depositors. The central bank is able to recover the full value of loans, and depositors split the value of assets after the central bank is repaid. The following Proposition states that this case is equivalent to Section 5.1 in which the central bank buys capital on the market.

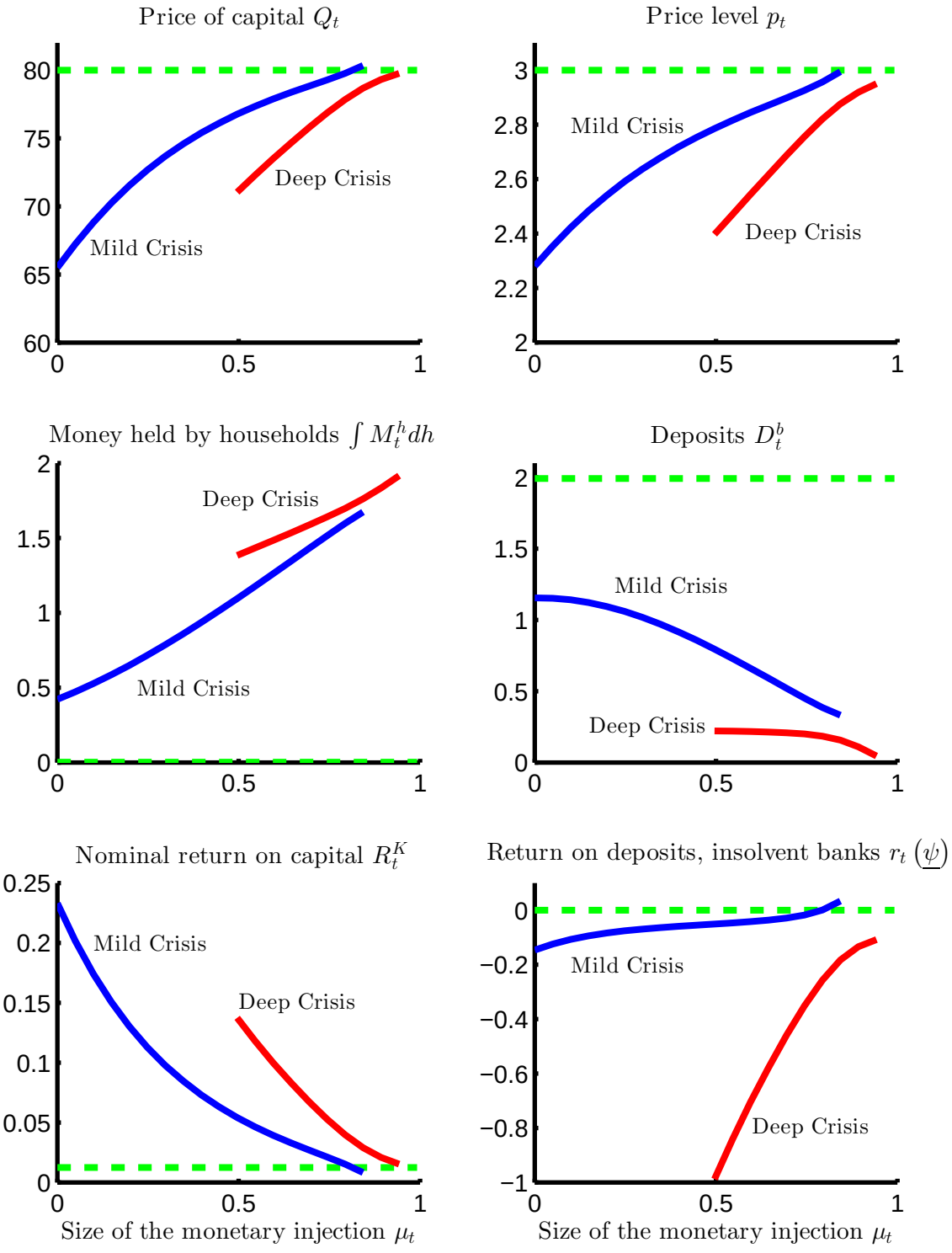
Proposition 5.1. *If there exists an equilibrium in which the central bank buys assets for a value $\mu_t \overline{M}$ and money supply reverts to $M_{t+1} = \overline{M}$ after the crisis, then the same equilibrium exists in an economy in which the central bank offers a total amount $\mu_t \overline{M}$ of loans to banks, with higher seniority compared to deposits, and $M_{t+1} = \overline{M}$.*

Intuitively, the central bank bears no risk in both cases (asset purchases and loans to banks with high seniority), therefore the two cases are equivalent. The proof is provided in the Online Appendix.

[30] A monetary injection μ^* that results in $Q_t = Q^*$ rules out the bad equilibrium because all banks would be solvent, due to Assumption 4.1. However, given a monetary injection $\mu_t = \mu^* - \varsigma$ for an arbitrary small $\varsigma > 0$, there exists a bad equilibrium with $Q_t < Q^*$.

[31] See Appendix C.2 for the full description of the problem of banks.

Figure 4: Effects of monetary policy: asset purchases



The variable on the horizontal axis is the size of the monetary injection μ_t. For each subplot, the blue line represents the mild crisis equilibrium, the red line the deep crisis equilibrium, and the green dotted line the good equilibrium without monetary intervention. Parameter values: see Table 1, $\kappa = 0.5$.

I now focus on the other extreme case: loans from the central bank have the same seniority as deposits. The central bank faces the actual return $r_t\left(\underline{\psi}\right) < 0$ on loans to insolvent banks.[32]

In this case, the central bank breaks the strategic complementarity in the choice of deposits by the household sector. The idea behind this result is that preexisting losses of insolvent banks are borne by both depositors and the central bank. Thus, households are willing to hold more deposits (compared to an economy where the central bank buys capital) and the large flight to liquidity that arises in the deep crisis cannot be an equilibrium.[33] Nevertheless, Figure 5 shows that, for the parameter values in Table 1 and $\kappa = 0.5$, a monetary injection that does not rule out a crisis still reduces deposits and amplify the flight to liquidity.

Overall, loans to banks appear to be a better policy instrument in comparison to asset purchases. First, loans to banks rule out the deep crisis equilibrium, while asset purchases don't. Second, from Figure 5, moderate monetary intervention ($\mu_t > 0.3$) implies return on insolvent banks $r_t\left(\underline{\psi}\right) > 0$. Therefore, there exists no bad equilibrium if $\mu_t > 0.3$ (see Section 4.2) and the central bank is committed to offering loans to banks with the same seniority as deposits. Third, restricting the analysis to the mild crisis equilibrium, loans to banks rule out the mild crisis equilibrium if $\mu_t > 0.3$, while asset purchases requires a much larger monetary injections ($\mu_t > 0.8$, see Figure 4). In a richer model in which there are other mechanisms responsible for generating a financial crisis (e.g., fire-sales that depresses the real price of capital), I conjecture that there are cicumnstances in which non-inflationary asset purchases are not able to rule out the mild crisis equilibrium while loans to banks still rule out the mild crisis equilibrium.

6 Discussion

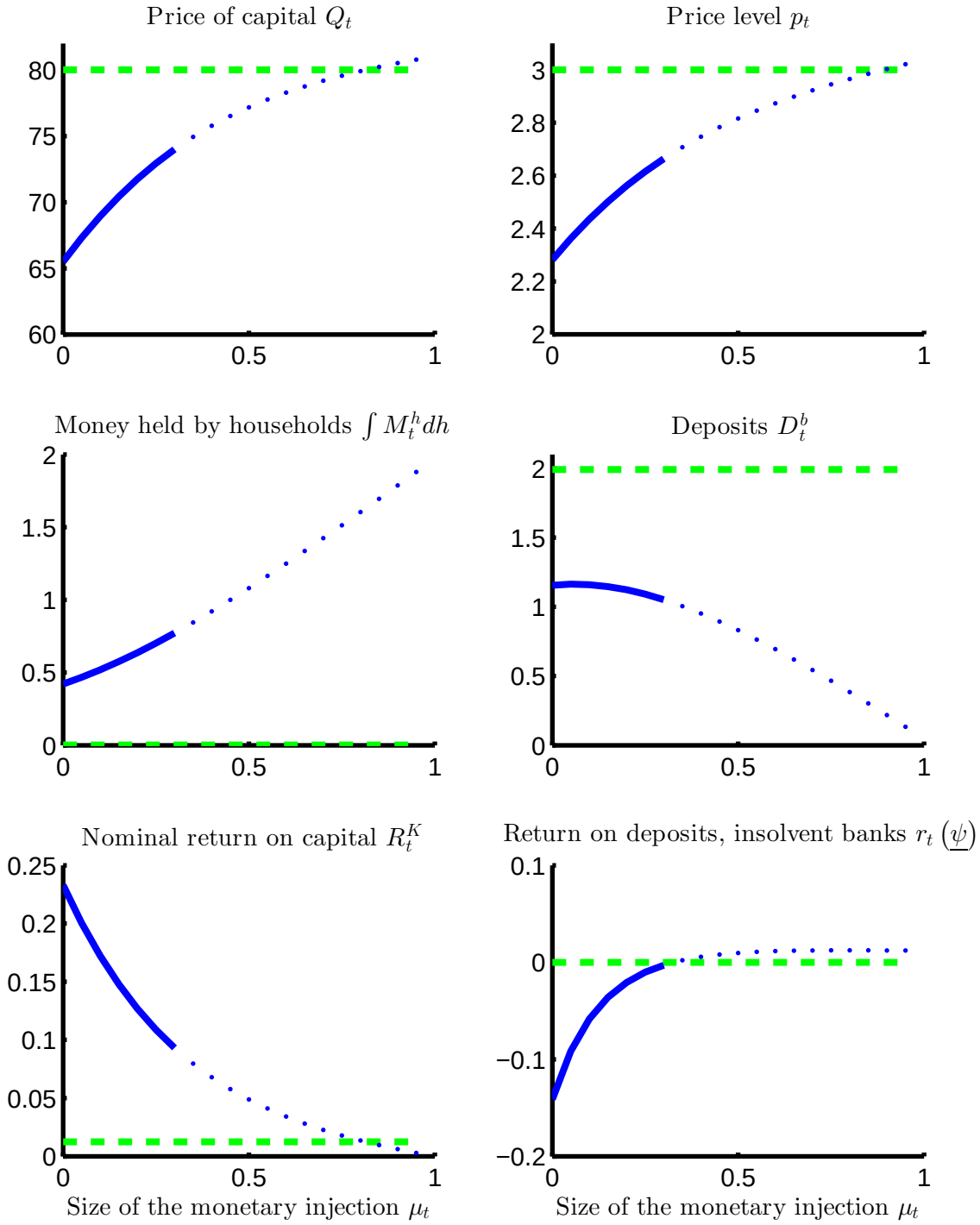
Demand-deposit contract in nominal term. The Online Appendix derives the optimal demand-deposit contract in an extended version of the model, in which productivity of capital Z is subject to aggregate shocks. The nominal contract achieves the optimal ex-ante allocation because the productivity shocks influence the price level p_t through the market

[32]The actual return on deposits of an insolvent bank under the monetary policy intervention analyzed in this Section is similar to the definition in equation (11) (see Appendix C for a complete description of the model with monetary interventions):

$$\mathbb{E}_t\left\{K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1}\right\} + ZK_t^b p_t = \left[D_t^b + (\text{loans from central bank})_t - w_t^b\right]\left[1 + r_t\left(\underline{\psi}\right)\right].$$

[33]More precisely, if there exists two bad equilibria with a constant money supply $M_t = \overline{M}$, the strategic complementarity is weakened by a small monetary injection, but it disappears for a large enough increase in the money supply.

Figure 5: Effects of monetary policy: loans to banks



The variable on the horizontal axis is the size of the monetary injection μ_t. In each subplot, the green dashed line represents the good equilibrium without any monetary intervention, the blue solid line the equilibrium value of the endogenous variables for which the monetary injection μ_t result in $r_t\left(\underline{\psi}\right) < 0$, and the blue dotted line the candidate bad equilibrium for values of μ_t such that $r_t\left(\underline{\psi}\right) > 0$ (see Section 4.2). Parameter values: see Table 1, $\kappa = 0.5$.

clearing condition for goods. Thus, the real value of deposits $\frac{D_t^h}{p_t}$ adjusts and becomes de facto contingent on the realization of the shocks. Following Diamond and Rajan (2006) and Allen, Carletti, and Gale (2013), if it is costly to write state-contingent contracts, the nominal contract is strictly preferred. The Appendix also discusses other contract features I consider such as the sequential service constraint and the return paid to depositors.

Anticipated shock ψ_t^b. I have presented a model with a one-time, unanticipated shock to capital. I argue that the model can be extended, allowing for anticipated shocks. Assume there are two types of capital, capital A and capital B. Capital A is risk-free while capital B is subject to anticipated shocks $\underline{\psi}$ and $\bar{\psi}$ (in the sense that there is a positive probability of non-zero shocks to type-B capital). Both types have the same productivity Z. I want to think of an allocation in which type-B capital is held only by banks, and households hold only type-A. I conjecture that this allocation is an equilibrium because banks are risk neutral, therefore they are willing to pay more than households for type-B capital even if this type is subject to shocks. In this framework, the shock to capital held by banks is anticipated, and the rest of the analysis is unchanged.

Capital requirements and government equity injections. Assumption 4.1 requires that the net worth of banks is "large enough" to represent a buffer against the bad shock $\underline{\psi}$, fixing nominal prices at the good-equilibrium level. Imposing a regulation that requires larger net worth allows banks to avoid insolvency in the bad equilibrium, because the larger net worth buffers against both the bad shock $\underline{\psi}$ and the drop in asset prices (this type of policy is usually referred to as capital requirement). Moreover, when the economy is in the good equilibrium, a Modigliani-Miller argument holds and the composition of deposits and net worth does not influence the choices of banks or the equilibrium. Using the language of Hanson, Kashyap, and Stein (2011), Assumption 4.1 is a "microprudential" regulation in the sense that it is partial equilibrium (it takes the price of capital as given). Contrarily, a financial crises in the model can be avoided with regulation that uses a "macroprudential" approach (recognizing the importance of general equilibrium effects) and establishes a sufficiently large capital requirement.

An additional policy that can be studied in the model is equity injection by the government. If I allow banks to issue equity, I conjecture that the private sector is not willing to buy it during a crisis because of a debt overhang problem (depositors, who are senior with respect to shareholders, benefit from the equity injection). Moreover, due to asymmetric information, the model is consistent with the analysis of Hoshi and Kashyap (2010): banks might refuse

equity assistance fearing having to admit losses and sending a negative signal. A public authority that forces all banks to accept equity might mitigate or even rule out a crisis, rationalizing the intervention of the U.S. government in October 2008 (see Veronesi and Zingales (2010)).

7 Conclusions

I have presented a new framework that includes bank runs in a dynamic, general equilibrium model, and I have used it to study unconventional monetary policy during panic-based financial crises. Restricting the analysis to non-inflationary monetary policies, the main result is that asset purchases cannot rule out a panic, while the central bank can avoid a crisis by offering loans to banks with the same seniority as other deposits. Thus, the central bank must have the (legal) ability to take a loss on a loan to a particular bank, even though this is just an off-equilibrium outcome. Factors that influence credibility become crucial: how is the credibility to take losses established and communicated by the central bank? More importantly, in a more general model in which some banks are insolvent even in the good equilibrium (because of fundamental shocks), does the central bank have to take losses on such fundamentally insolvent banks in order to credibly commit to a policy that rules out a panic? The story suggested by these questions appears consistent with what happened in 2008; the failure of Lehman Brothers in September 2008 might have communicated the (legal) inability of the Federal Reserve to make loans to banks facing risk of insolvency (contrary to what happened in March 2008 regarding Bear Stearns), thus opening up the possibility of a panic-based crisis.

I have also briefly discussed how the model can be used to analyze other policies such as capital requirements and equity injections by the government. Finally, the framework can be extended to analyze precautionary money demand and runs not only by households but also by firms. A richer version of the model can be used for quantitative analysis in order to assess the contribution of panics and fundamental shocks to actual episodes of financial crises.

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Appendix

A Steady-state and good equilibrium

Let:

$$\left(\boldsymbol{X}^b\right)^* \equiv \left\{\left[\left(K^b\right)^*, \left(m^b\right)^*, \left(d^b\right)^*\right], \psi_t^b = 0\right\}$$

be the initial state of bank b, where:

$$\left(K^b\right)^* \equiv \frac{\overline{K}\left(1-\beta\right)\left(1-\kappa\right)}{1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right]} \tag{31}$$

$$\left(m^b\right)^* \equiv \frac{\overline{M}\left(1-\beta\right)\left(1-\kappa\right)}{1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right]} \tag{32}$$

$$\left(d^b\right)^* \equiv \frac{\overline{M}\left(1-\kappa\right)\left[1+\underline{\psi}\left(1-\kappa\right)-\beta\left(1-\kappa+\underline{\psi}-2\kappa\underline{\psi}\right)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]}. \tag{33}$$

Lemma A.1. *The state* $\left(\boldsymbol{X}^b\right)^*$ *satisfies Assumption 4.1 and* $\frac{\left(K^b\right)^*}{\overline{K}} \in (0,1)$.

The first result of the Lemma derives from plugging (31), (32) and (33) into (27). The second result follows from Assumption 3.7 and it implies that banks hold a fraction $\in (0,1)$ of the stock of capital in the economy, therefore the steady-state is well-defined.

The next Proposition describes the steady-state of the economy.

Proposition A.2. *If the state of the economy $\boldsymbol{X}_t$ satisfies $Pr_t^{\boldsymbol{X}^b}\left(\boldsymbol{X}_t^b = \left(\boldsymbol{X}^b\right)^*\right) = 1$ and $\boldsymbol{X}_t^e = \frac{\left(\boldsymbol{X}^b\right)^*}{1-\lambda}$ for all exiting banks, there exists an equilibrium such that:*

- *prices are:*

$$Q_t = Q^* \equiv \left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right]\frac{\overline{M}}{\overline{K}},$$

$$p_t = p^* = \frac{\overline{M}}{Z\overline{K}},$$

$$R_t^K = R_t^D = R^* \equiv \frac{(1-\beta)\,\kappa}{(1-\beta)\,(1-\kappa) + \beta\kappa}$$

- *dividends paid by exiting banks are:*

$$\pi_t = \pi^* \equiv \frac{\overline{M}\left(-\underline{\psi}\right)(1-\beta)\,(1-\kappa)}{1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right]}; \tag{34}$$

- *all banks have same initial net worth:*

$$N_t^b = N^* \equiv \frac{\overline{M}\left(-\underline{\psi}\right)(1-\kappa)\left[1-\kappa-\beta\,(1-2\kappa)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]} \quad \textit{for all } b$$

  and they all make the same choices: $D_t^b = D^* \equiv \frac{\overline{M}-\pi^*}{\kappa}$, $M_t^b = \kappa D_t^b = \overline{M}$ and $K_t^b = \frac{N^* + (1-\kappa)\frac{\overline{M}-\pi^*}{\kappa}}{Q^*}$ for all b;

- *the representative household h with wealth $A_t^h = \overline{A}_t$ has beliefs:*

$$Pr_t^h\left(r_t^{b(h)} = R^*,\, l_t^h = +\infty\right) = 1$$

  and her choice is given by Proposition 3.3 where $\eta_t^M = 0$, $\eta_t^D = \frac{1-\beta}{1-\beta(1-\kappa)}$ and $\eta_t^K = \frac{\beta\kappa}{1-\beta(1-\kappa)}$;

- *actual return on deposits and limits on withdrawals are $r_t^b = R^*$ for all b and $l_t^h = +\infty$ for all h;*

- *the economy is in steady-state: $\boldsymbol{X}_{t+1} = \boldsymbol{X}_t$.*

In order to prove the Proposition, it is useful to state the following result. Proposition A.3 re-states Proposition 3.3 focusing on the choices of households during the day and adding some details about the objective function of households. The proof is identical to the the proof of Proposition 3.3 that is provided in the Online Appendix.

Proposition A.3. *Given beliefs $Pr_t^h(\cdot)$ and prices Q_t, R_t^K and $R_t^D = R_t^K$, the choices of money M_t^h, deposits D_t^h and capital K_t^h are:*

$$\begin{aligned} M_t^h &= \eta_t^M A_t^h \\ D_t^h &= \eta_t^D A_t^h \\ K_t^h &= \eta_t^K A_t^h \end{aligned}$$

where η_t^M, η_t^D and η_t^K solve:

$$\begin{aligned} \max_{\eta_t^M,\eta_t^D,\eta_t^K} \Bigg\{ & Pr_t^h\left(r_t^{b(h)} \in \mathbb{R}, l_t^h = +\infty\right) \kappa \left[\bar{\varepsilon} \log\left(\frac{\eta_t^M + \eta_t^D}{p_t}\right) + \frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right)\right)\right] + \\ & Pr_t^h\left(r_t^{b(h)} \in \mathbb{R}, l_t^h = 0\right) \kappa \left[\bar{\varepsilon} \log\left(\frac{\eta_t^M}{p_t}\right) + \frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right) + \eta_t^D\left(1+r_t^{b(h)}\right)\right)\right] + \\ & Pr_t^h\left(r_t^{b(h)} < 0,\, l_t^h = +\infty\right)(1-\kappa) \left[\frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right) + \eta_t^M + \eta_t^D\right)\right] + \\ & \left[1 - Pr_t^h\left(r_t^{b(h)} < 0,\, l_t^h + \infty\right)\right](1-\kappa) \left[\frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right) + \eta_t^M + \eta_t^D\left(1+r_t^{b(h)}\right)\right)\right] \Bigg\}. \end{aligned} \tag{35}$$

subject to $\eta_t^M + \eta_t^D + \eta_t^K = 1$ and η_t^M, η_t^D, $\eta_t^K \in [0,1]$.

I can now prove Proposition A.2.

Proof. Taking as given the return on deposits $R_t^D = R_t^K = R^*$, the choices of banks follows from Proposition 3.1. Moving to the problem of households, I first show that $M_t^h = 0$ or $\eta_t^M = 0$. Given households beliefs, the problem (35) simplifies to:

$$\begin{aligned} \max_{\eta_t^M,\eta_t^D,\eta_t^K} \Bigg\{ & \kappa\bar{\varepsilon} \log\left(\frac{\eta_t^M + \eta_t^D}{p_t}\right) + \kappa\frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right)\right) + \\ & \left[(1-\kappa)\frac{\beta}{1-\beta} \log\left(\eta_t^K\left(1+R_t^K\right) + \eta_t^M + \eta_t^D\left(1+R^*\right)\right)\right] \Bigg\}. \end{aligned}$$

Using $\eta_t^K = 1 - \eta_t^M - \eta_t^D$ from Proposition 3.3 and imposing $R_t^D = R_t^K = R^*$:

$$\text{FOC } \eta_t^M: \quad \frac{1}{\eta_t^D + \eta_t^M} - \underbrace{\frac{\beta R^* (1-\kappa)}{(1-\beta)\left[(1+R^*)(1-\eta_t^M) + \eta_t^M\right]}}_{\geq 0} - \frac{\beta\kappa}{(1-\eta_t^D-\eta_t^M)(1-\beta)} \tag{36}$$

$$\text{FOC } \eta_t^D: \quad \frac{1}{\eta_t^D + \eta_t^M} - \frac{\beta\kappa}{(1-\eta_t^D-\eta_t^M)(1-\beta)} \tag{37}$$

Since η_t^D solves (37) equalized to zero, the FOC with respect to η_t^M is ≤ 0, so it must be $\eta_t^M = 0$ because of the non-negativity constraint on money. Moreover, using $\eta_t^M = 0$, equation (37) can be solved for η_t^D:

$$\eta_t^D = \frac{1-\beta}{1-\beta(1-\kappa)} \tag{38}$$

and the value of η_t^K is computed using $\eta_t^M + \eta_t^D + \eta_t^K = 1$. Since all banks are solvent and pay the return $r_t^b = R_t^D = R^* > 0$ and $l_t^h = +\infty$, the optimal withdrawal decision at night is to withdraw only if $\varepsilon_t^h = \bar{\varepsilon}$ (Proposition (3.3)). Since only impatient households withdraw and banks hold $M_t^b = \kappa D_t^b$, then there are no runs and $l_t^h = +\infty$.

Using $M_t^h = 0$, the market clearing condition of the day-money market, equation (23) becomes:

$$\overline{M} - \pi_t = \int M_t^b db$$

and using the optimal choice of bank $M_t^b = \kappa D_t^b$:

$$\overline{M} - \pi_t = \int_0^1 M_t^b db = \kappa \int_0^1 D_t^b db. \tag{39}$$

Using the market clearing conditions for deposits, equation (24):

$$\int_0^1 D_t^b db = \int_{\mathcal{H}} D_t^h dh = \eta_t^D \overline{A}_t \tag{40}$$

where the last equality uses the result of Corollary 3.4. Therefore, combining equations (39) and (40):

$$\frac{\overline{M} - \pi_t}{\kappa} = \eta_t^D \overline{A}_t. \tag{41}$$

Plugging the expression for η_t^D from equation (38) into equation (41) and solving for $\overline{A}_t$:

$$\overline{A}_t = \frac{1-\beta(1-\kappa)}{1-\beta} \frac{\overline{M} - \pi_t}{\kappa}. \tag{42}$$

The value of wealth of the household sector $\overline{A}_t$ can also be computed summing the value of capital, money and deposits of households. The stock of capital owned by households is given by the total supply of capital $\overline{K}$ minus the capital owned by active banks $\left(K^b\right)^*$ minus the capital owned by banks under liquidation: each bank under liquidation has $\frac{\left(K^b\right)^*}{1-\lambda}$ and there is a measure λ of such banks. The value of capital of households is:

$$\left(\overline{K} - \left(K^b\right)^* - \lambda\frac{\left(K^b\right)^*}{1-\lambda}\right)Q^* = \left(\overline{K} - \frac{\left(K^b\right)^*}{1-\lambda}\right)Q^*.$$

Similarly, the stock of money held by households is:

$$\overline{M} - \left(m^b\right)^* - \lambda\frac{\left(m^b\right)^*}{1-\lambda} = \overline{M} - \frac{\left(m^b\right)^*}{1-\lambda}$$

and the value of deposits of households is:

$$\left(d^b\right)^* + \lambda\frac{\left(d^b\right)^*}{1-\lambda} = \frac{\left(d^b\right)^*}{1-\lambda}.$$

As a result, the sum of the value of capital, money and deposits is:

$$\overline{A}_t = \left(\overline{K} - \frac{\left(K^b\right)^*}{1-\lambda}\right)Q^* + \left(\overline{M} - \frac{\left(m^b\right)^*}{1-\lambda}\right) + \frac{\left(d^b\right)^*}{1-\lambda}. \tag{43}$$

Combining (42), (43) and using $\pi_t = \pi^*$ and the expression for π^* in (34), the value of λ from Assumption 3.6, the definitions of $\left(K^b\right)^*$, $\left(m^b\right)^*$ and $\left(d^b\right)^*$ in (31), (32) and (33), I solve for Q^*:

$$Q^* = \left[\frac{\beta}{1-\beta} + \left(\frac{1}{\kappa} - 1\right)\right]\frac{\overline{M}}{\overline{K}}. \tag{44}$$

The price of consumption good p^* follows from market clearing in the goods market, using the fact that consumption expenditure is subject to a cash-in-advance constraint and all the money $\overline{M}$ is spent:

$$Z\overline{K}p^* = \overline{M}. \tag{45}$$

Since all banks have the same state variable $\left(\boldsymbol{X}^b\right)^*$, they have the same net worth. The expression for net worth is derived from equation (5) using $\psi_t^b = 0$, $Q_t = Q^*$ and the state $\left(\boldsymbol{X}^b\right)^*$:

$$N^* = \left(K^b\right)^* Q^* + \left(m^b\right)^* - \left(d^b\right)^* = \frac{\overline{M}\left(-\underline{\psi}\right)(1-\kappa)\left[1-\kappa-\beta\left(1-2\kappa\right)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]}.$$

Profits π^* are computed using the state $\boldsymbol{X}_t^e$ of banks hit by the exit shock, using the fact that the mass of exiting banks is λ, and using $\lambda = \frac{1-\beta}{\beta+(1-\beta)\bar{\varepsilon}}$ from Assumption 3.6:

$$\pi^* = \lambda \frac{\left(K^b\right)^* Q^* + \left(m^b\right)^* - \left(d^b\right)^*}{1-\lambda} = \frac{\overline{M}\left(-\underline{\psi}\right)(1-\beta)(1-\kappa)}{1-\beta\left[1-\kappa\left(1+\underline{\psi}\right)\right]}.$$

Using the values of N^* and π^* in the statement of the Proposition, then the state $\boldsymbol{X}_{t+1}^b$ of bank a $b \in [0,1]$, after the splitting process, is given by $\left(\boldsymbol{X}^b\right)^*$ (the law of motion of the state variables of banks is described in the Online Appendix):

$$K_t^b = \frac{N^* + (1-\kappa)\frac{\overline{M}-\pi^*}{\kappa}}{Q^*}(1-\lambda) = \left(K^b\right)^*$$

$$m_t^b = ZK_t^b p^*(1-\lambda) = \left(m^b\right)^*$$

$$d_t^b = \left(D_t^b - w_t^b\right)(1+R^*)(1-\lambda) = \left(d^b\right)^*$$

where the last equalities uses $D_t^b = \frac{\overline{M}-\pi^*}{\kappa}$ and:

$$w_t^b = \int_{\mathcal{H}(b)} w_t^h dh = \kappa D_t^b.$$

Since the economy is in steady-state, $Q_{t+1} = Q_t = Q^*$ and:

$$R_t^K = \frac{Q_{t+1} + Zp_t}{Q_t} - 1 = \frac{Q^* + Zp^*}{Q^*} = \frac{(1-\beta)\kappa}{(1-\beta)(1-\kappa)+\beta\kappa}$$

where the last equality uses the expressions for Q^* and p^* in the statement of the Proposition.

Finally, since $R_t^D = R_t^K$, the market clearing condition for deposits (24) holds because banks are indifferent among issuing any amount of deposits $D_t^b \geq 0$ (Proposition 3.1). □

The next Proposition describes the good equilibrium when the economy is initially in steady-state and it is hit by the one-time unanticipated idiosyncratic shocks to capital $\psi_t^i \in \left\{\underline{\psi}, \overline{\psi}\right\}$.

Proposition A.4. *If the state of the economy $\boldsymbol{X}_t$ satisfies:*

$$Pr_t^{\boldsymbol{X}^b}\left(\boldsymbol{X}_t^b = \left\{\left[\left(K^b\right)^*, \left(m^b\right)^*, \left(d^b\right)^*\right], \underline{\psi}\right\}\right) = \alpha^{BAD}$$
$$Pr_t^{\boldsymbol{X}^b}\left(\boldsymbol{X}_t^b = \left\{\left[\left(K^b\right)^*, \left(m^b\right)^*, \left(d^b\right)^*\right], \overline{\psi}\right\}\right) = 1 - \alpha^{BAD}$$

$$\boldsymbol{X}_t^e = \left\{\left[\frac{\left(K^b\right)^*}{1-\lambda}, \frac{\left(m^b\right)^*}{1-\lambda}, \frac{\left(d^b\right)^*}{1-\lambda}\right], \underline{\psi}\right\} \quad \textit{for a fraction } \alpha^{BAD} \textit{ of banks under liquidation}$$

$$\boldsymbol{X}_t^e = \left\{ \left[\frac{\left(K^b\right)^*}{1-\lambda}, \frac{\left(m^b\right)^*}{1-\lambda}, \frac{\left(d^b\right)^*}{1-\lambda} \right], \overline{\psi} \right\} \quad \textit{for a fraction } 1-\alpha^{BAD} \textit{ of banks under liquidation}$$

there exists an equilibrium such that:

- *banks hit by* $\psi_t^b = \underline{\psi}$ *have* $N_t^b = 0$*; banks hit by* $\psi_t^b = \overline{\psi}$ *have* $N_t^b = N^{**} > N^* > 0$ *where:*

$$N^{**} \equiv Q^* \left(K^b\right)^* \left(1+\bar{\psi}\right) + \left(m^b\right)^* - \left(d^b\right)^* = \frac{\overline{M}\left(\bar{\psi}-\underline{\psi}\right)(1-\kappa)\left[1-\kappa-\beta\left(1-2\kappa\right)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]};$$

- *the choices of bank* b *are given by* $D_t^b = \frac{\overline{M}-\pi^*}{\kappa}$, $M_t^b = \kappa D_t^b = \overline{M} - \pi^*$ *and* $K_t^b = \frac{N_t^b + (1-\kappa)\frac{\overline{M}-\pi^*}{\kappa}}{Q^*}$*;*

- *prices, households' beliefs and choices, dividends, actual returns on deposits and limits on withdrawals are the same as in Proposition A.2.*

The result can be proven using a similar approach as in Proposition A.2. The net worth of banks is computed using (5) and taking into account the shocks $\underline{\psi}$ and $\bar{\psi}$. Due to Assumption 4.1, banks hit by the bad shock have zero net worth.

Proof. The derivation of the choices of banks and of the values of η_t^M, η_t^D and η_t^K are identical to the proof of Proposition A.2.

Since $\pi_t = \pi^*$ and since the idiosyncratic shocks to capital cancels out not only at the aggregate level but also within the banking sector and within the household sector (equation (3)), then equations (42) and (43) still hold and the price of capital is Q^*, see equation (44). Also, all money is spent and thus the price of consumption good is p^*, see equation (45).

Using equation (5), the net worth of a bank b hit by the shock $\psi_t^b = \underline{\psi}$ is:

$$N^* = \left(K^b\right)^* \left(1+\underline{\psi}\right) Q^* + \left(m^b\right)^* - \left(d^b\right)^* = 0$$

where the last equality follows by equation (27) in Assumption (4.1).

Using equation (5), the net worth of a bank b hit by the shock $\psi_t^b = \overline{\psi}$ is:

$$N_t^b = \frac{\overline{M}\left(\bar{\psi}-\underline{\psi}\right)(1-\kappa)\left[1-\kappa-\beta\left(1-2\kappa\right)\right]}{\kappa\left[1-\beta\left(1-\kappa\left(1+\underline{\psi}\right)\right)\right]} \equiv N^{**}$$

and $N^{**} > N^*$ because $\bar{\psi} - \underline{\psi} > -\underline{\psi}$.

The overall net worth of the banking sector is the same as in steady-state, therefore profits are still given by π^* because the exit shocks are i.i.d. across banks.

The states of banks are not constant over time (and thus the economy is not in a steady-state). However, following the same steps as in the Online Appendix that derives the Good Equilibrium After a Crisis, it is possible to show that the net worth of banks is constant over time provided that the price of capital remains at $Q_t = Q^*$. This result implies also $R_t^K = R^*$.

Finally, since $R_t^D = R_t^K$, the market clearing condition for deposits (24) holds because banks are indifferent among issuing any amount of deposits $D_t^b \geq 0$ (Proposition 3.1). □

B Bad equilibrium

The state variables of banks $\left\{\boldsymbol{X}_t^b\right\}_{b\in[0,1]}$ are the same as in Proposition A.4.

Households. Households hold capital and money that is not held by banks, plus deposits. The total wealth of households $\overline{A}_t$ is given by an expression similar to (43), but with some differences:

$$\overline{A}_t = \left[\overline{K} - \left(K^b\right)^* - \lambda\frac{\left(K^b\right)^*}{1-\lambda}\right] Q_t + \left(\overline{M} - \left(m^b\right)^* - \lambda\frac{\left(m^b\right)^*}{1-\lambda}\right) + \\ + \lambda\alpha^{BAD}\left[\frac{\left(K^b\right)^*}{1-\lambda}\left(1+\underline{\psi}\right)Q_t + \frac{\left(m^b\right)^*}{1-\lambda}\right] + \left(1-\lambda\alpha^{BAD}\right)\left[D^*\left(1-\kappa\right)\left(1+R^*\right)\right]. \quad (46)$$

The first term is the value of capital: the total supply $\overline{K}$ minus capital owned by active banks $\left(K^b\right)^*$ minus capital owned by banks under liquidation $\lambda\frac{\left(K^b\right)^*}{1-\lambda}$. The second term is the value of money; similarly to capital, it is given by the total supply of money $\overline{M}$ minus the money owned by active banks $\left(m^b\right)^*$ minus the money held by banks under liquidation $\lambda\frac{\left(m^b\right)^*}{1-\lambda}$. The third term is the value of deposits from banks hit by the exit shock and by $\underline{\psi}$ (such banks are insolvent, therefore all their assets are used to repay depositors). The last term is the value of deposits at other banks.

Using the beliefs described in Section 4.2, equation (35) can be rewritten:

$$\max_{\eta_t^M,\eta_t^D,\eta_t^K}\Bigg\{\left(1-\alpha^{BAD}\right)\Bigg[\kappa\bar{\varepsilon}\log\left(\eta_t^M+\eta_t^D\right)+\frac{\beta}{1-\beta}\kappa\log\left(\eta_t^K\right)+$$
$$+\frac{\beta}{1-\beta}\left(1-\kappa\right)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+R_t^D\right)+\eta_t^M\right)\Bigg]+$$
$$+\alpha^{BAD}f_t^b\Bigg[\kappa\bar{\varepsilon}\log\left(\eta_t^M+\eta_t^D\right)+\frac{\beta}{1-\beta}\kappa\log\left(\eta_t^K\right)+$$
$$+\frac{\beta}{1-\beta}\left(1-\kappa\right)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D+\eta_t^M\right)\Bigg]+$$
$$\alpha^{BAD}\left(1-f_t^b\right)\Bigg[\kappa\bar{\varepsilon}\log\left(\eta_t^M\right)+\frac{\beta}{1-\beta}\kappa\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+r_t^{b(h)}\right)\right)+$$
$$+\frac{\beta}{1-\beta}\left(1-\kappa\right)\log\left(\eta_t^K\left(1+R_t^K\right)+\eta_t^D\left(1+r_t^{b(h)}\right)+\eta_t^M\right)\Bigg]\Bigg\}$$

subject to $\eta_t^M+\eta_t^D+\eta_t^K\leq 1$. Using:

$$\eta_t^K=1-\eta_t^M-\eta_t^D \tag{47}$$

the FOCs with respect to η_t^M and η_t^D are given by:

$$\left(1-\alpha^{BAD}\right)\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}-\frac{\beta(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M)R_t^K+\eta_t^D(R_t^D-R_t^K)}\right]+$$
$$+\alpha^{BAD}f_t^b\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}-\frac{\beta(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M-\eta_t^D)R_t^K}\right]+$$
$$+\alpha^{BAD}\left(1-f_t^b\right)\left[\frac{1}{\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1+R_t^K}{(1-\eta_t^M)(1+R_t)+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}+\right.$$
$$\left.-\frac{\beta(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M)R_t^K+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right]=0 \tag{48}$$

$$\begin{aligned}\left(1-\alpha^{BAD}\right)\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}+-\frac{\beta(1-\kappa)}{1-\beta}\frac{R_t^D-R_t^K}{1+(1-\eta_t^M)R_t^K+\eta_t^D(R_t^D-R_t^K)}\right]+\\ +\alpha^{BAD}f_t^b\left[\frac{1}{\eta_t^D+\eta_t^M}-\frac{\beta}{1-\beta}\kappa\frac{1}{1-\eta_t^M-\eta_t^D}-\frac{\beta(1-\kappa)}{1-\beta}\frac{R_t^K}{1+(1-\eta_t^M-\eta_t^D)R_t^K}\right]+\\ +\alpha^{BAD}\left(1-f_t^b\right)\left[\frac{\beta}{1-\beta}\kappa\frac{r_t^{b(h)}-R_t^K}{(1-\eta_t^M)(1+R_t^K)+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}+\right.\\ \left.+\frac{\beta(1-\kappa)}{1-\beta}\frac{r_t^{b(h)}-R_t^K}{1+(1-\eta_t^M)R_t^K+\eta_t^D\left(r_t^{b(h)}-R_t^K\right)}\right]=0 \qquad (49)\end{aligned}$$

I numerically verify that η_t^M, η_t^D and η_t^K lie in the interval $[0,1]$. The behavior of households is given by equations (46), (47), (48) and (49) that pin down $\overline{A}_t$, η_t^K η_t^M and η_t^D.

Return on capital and return on deposits. The return on capital is given by equation (4) evaluated at $Q_{t+1}=Q^*$ (see the Online Appendix, Section "Good Equilibrium After a Crisis" for a proof of the fact that the price of capital is equal to Q^* after a crisis). The return on deposits is $R_t^D=R_t^K$, satisfying the market clearing condition for deposits. Thus I have two equations that pin down R_t^D and R_t^K.

Banks. The net worth of solvent banks $N_t\left(\bar{\psi}\right)$ and insolvent banks $N_t\left(\underline{\psi}\right)$ is given by equation (5) evaluated at the respective values of ψ_t^b and using the value of capital $\left(K^b\right)^*$, money $\left(m^b\right)^*$ and deposits $\left(d^b\right)^*$ from equations (31), (32) and (33). I then use the budget constraint of banks (7) separately for good and bad banks, evaluated at the optimal choice of money described by Proposition 3.1 and taking as given the demand of deposits by households (using $R_t^K=R_t^D$, banks are indifferent among any amount of deposits). The behavior of banks is given by four equations (the two definitions of net worth and the two budget constraints) that pin down $N_t\left(\bar{\psi}\right)$, $N_t\left(\underline{\psi}\right)$, $K_t\left(\bar{\psi}\right)$ and $K_t\left(\underline{\psi}\right)$.

Actual return on deposits of insolvent banks and depositors served during a run. The actual return on deposits of insolvent banks, $r_t\left(\underline{\psi}\right)$ is given by equation (12), using $Q_{t+1}=Q^*$ and $w_t^b=M_t^b=\kappa D_t^b$. The fraction of depositors served during a run is given by (13) evaluated at $D_t^b=\eta_t^D\overline{A}_t$ and $M_t^b=\kappa D_t^b=\kappa\eta_t^D\overline{A}_t$ that imply $f_t=\kappa$. Therefore I have two equations that pin down $r_t\left(\underline{\psi}\right)$ and f_t.

Dividends. Using the state variable of banks hit by the exit shock described in Proposition A.4 and the fact that there is a mass λ of such banks, the dividends paid to bankers are:

$$\pi_t = \lambda\left(1-\alpha^{BAD}\right)\frac{\left(K^b\right)^*\left(1+\overline{\psi}\right)Q_t+\left(m^b\right)^*-\left(d^b\right)^*}{1-\lambda} = \lambda\left(1-\alpha^{BAD}\right)\frac{N_t\left(\overline{\psi}\right)}{1-\lambda}$$

because a fraction α^{BAD} of banks is hit by $\underline{\psi}$ and it has negative net worth (thus, all the assets of such insolvent banks are used to repay depositors and the banks hit by $\underline{\psi}$ do not contribute to dividends).

Market clearing. The withdrawal and consumption decisions of households (Proposition 3.3) together with the market clearing condition for goods (25) imply that:

$$Z\overline{K}p_t = \pi_t + \kappa\left[\left(1-\alpha^{BAD}\right)\left(M_t^h+D_t^h\right)+\alpha^{BAD}f_t\left(M_t^h+D_t^h\right)+\alpha^{BAD}\left(1-f_t\right)M_t^h\right] \quad (50)$$

where $M_t^h = \eta_t^M A_t^h$, $D_t^h = \eta_t^D A_t^h$ and I consider the wealth $A_t^h = \overline{A}_t$ of the representative household. Equation (50) says that the total consumption expenditure is equal to dividends (bankers spend all their dividends π_t to buy consumption) plus the consumption expenditure of the fraction κ of impatient households. A fraction $1-\alpha^{BAD}$ of impatient households face solvent banks and is able to withdraw D_t^h, thus their consumption expenditure is $M_t^h + D_t^h$. A fraction $\alpha^{BAD}f_t$ of impatient households is first in line during runs, so they can withdraw and they spend $M_t^h + D_t^h$. A fraction $\alpha^{BAD}\left(1-f_t\right)$ is last in line during a run and consume only using money M_t^h. The market clearing condition for money during the day is given by equation (23) where $\int M_t^b db = \kappa\int D_t^b db = \kappa\eta_t^D\overline{A}_t$ and $\int M_t^h dh = \eta_t^M\overline{A}_t$.

The two market clearing conditions for goods and money pin down the price level p_t and the price of capital Q_t.

Solution method. I obtain a non-linear system of 15 polynomial equations in 15 unknowns. Since I take as given the state of the economy $\boldsymbol{X}_t$ and the price of capital $Q_{t+1} = Q^*$ in $t+1$, I am essentially solving a static problem, only for period t. I solve the system in Mathematica using the command NSolve and selecting the real solutions that satisfy all the non-negativity constraints on money, deposits and capital imposed on the maximization problem of households and banks, and the constraints η_t^M, η_t^D, $\eta_t^K \in [0,1]$.

The command NSolve in Mathematica computes the numerical Gröbner bases associated with the system of polynomial equations and then uses eigensystem methods to extract numerical roots, finding all solutions to the system[34].

[34]See Kubler and Schmedders (2010) for an introduction to Gröbner bases applied to the computation of

C Temporary monetary injections

When the central bank injects money in the economy, some details of the model are slightly different from what is described in the main part of the paper. In this Appendix, I provide the details about households, banks, market clearing conditions and I discuss the solution methods in the case of temporary monetary interventions (i.e., under the assumption that the central bank chooses money supply $M_{t+1} = \overline{M}$ at $t+1$).

C.1 Households

The Bellman equation is:

$$V_t\left(A_t^h\right) = \max_{M_t^h, D_t^h, K_t^h} \mathbb{E}_n \left\{ \max_{w^h\left(n_t^h\right), c^h\left(n_t^h\right)} \left[\varepsilon_t^h \log c^h\left(n_t^h\right) + \beta \mathbb{E}_\psi V_{t+1}\left(A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right)\right)\right] \right\} \tag{51}$$

subject to the budget constraint (15), the limit on withdrawals (16), the cash-in-advance constraint (17) and a non-negativity constraint on money $M_t^h \geq 0$, deposits $D_t^h \geq 0$ and capital $K_t^h \geq 0$. Differently from Section 3.2, the value of wealth $A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right)$ is given by:

$$A_{t+1}^h\left(n_t^h, \psi_{t+1}^h\right) = \left[K_t^h\left(1+\psi_{t+1}^h\right)\right] Q_{t+1} + d^h\left(n_t\right) + m^h\left(n_t^h\right) + \frac{A_t^h}{\overline{A}_t} T_{t+1} \tag{52}$$

where $d^h\left(n_t^h\right)$ and $m^h\left(n_t^h\right)$ are defined in equations (19) and (20), $\overline{A}_t = \int_{\mathcal{H}} A_t^h dh$ is the total wealth of the household sector and T_{t+1} are transfers from the monetary authority to households in $t+1$ (defined in Section C.4). The formulation of equation (52) implies that the transfers from the central bank are distributed proportionally to the wealth A_t^h of each household h in period t. This assumption allows me to still be able to guess-and-verify the shape of the value function.

C.2 Banks

Let L_t^b be the amount of loans from the central bank to bank b. Bank b receives funds from the central bank during the day of time t. During the day of $t+1$, bank b has to pay back $L_t^b\left(1+R_t^{CB}\right)$ where R_t^{CB} is the nominal interest rate charged by the central bank for the loan.

equilibria in economic models. For a description of the solution method of NSolve, see Lichtblau (2000).

The problem of bank b with net worth N_t^b is:

$$\max_{D_t^b, L_t^b, M_t^b, K_t^b} \mathbb{E}_t \left(\max \left\{ 0, N_{t+1}^b \right\} \right) \tag{53}$$

subject to the budget constraint (54) and the law of motion of net worth (55):

$$K_t^b Q_t + M_t^b \leq D_t^b + L_t^b + N_t^b \tag{54}$$

$$N_{t+1}^b = (1-\lambda) K_t^b \left(1 + \psi_{t+1}^b \right) Q_{t+1} + m_t^b - d_t^b \tag{55}$$

where m_t^b and d_t^b are the nominal values of money and deposits at the end of the night of time t:

$$m_t^b \equiv (1-\lambda) \left(M_t^b - w_t^b \right) + y \left(K_t^b \right) p_t$$

$$d_t^b \equiv (1-\lambda) \left(D_t^b - w_t^b \right) \left(1 + R_t^b \right) + L_t^b \left(1 + R_t^{CB} \right).$$

Banks must also satisfy a non-negativity constraint on money and capital, $M_t^b \geq 0$ and $K_t^b \geq 0$.

The results of Proposition (3.1) about D_t^b and M_t^b are unchanged; in particular, the bank still wants to hold an amount of money $M_t^b = \kappa D_t^b$ because at night it will only face withdrawals from depositors and none of the funds lent by the central bank will be withdrawn at night. The choice of K_t^b is given by:

$$K_t^b = \frac{N_t^b + L_t^b + D_t^b - M_t^b}{Q_t}$$

and it follows from the budget constraint (54). The choice of L_t^b is similar to the choice of D_t^b:

$$L_t^b = \begin{cases} 0 & \text{if } R_t^{CB} > R_t^K \\ \text{any amount } \geq 0 & \text{if } R_t^{CB} = R_t^K \\ +\infty & \text{if } R_t^{CB} < R_t^K. \end{cases} \tag{56}$$

This result is similar to the choice of D_t^b and can be proven similarly to Proposition 3.1. Intuitively, banks invest all the funds that they get from the central bank in capital (because the central bank does not withdraw any money at night), therefore bank b wants to hold $L_t^b = 0$ if the return R_t^{CB} is larger than R_t^K, $L_t^b = +\infty$ if $R_t^{CB} < R_t^K$ and any amount of loans if $R_t^{CB} = R_t^K$.

In equilibrium $R_t^{CB} = R_t^K$, to make sure that banks are willing to take the amount of loans offered by the central bank.

C.3 Actual return on deposits and fraction of depositors served during a run

Under the relevant case $R_t^D = R_t^{CB}$, the actual return on deposits is given by:

$$r_t^b \equiv \min\left\{R_t^D, \widehat{r}_t^b\right\}$$

If loans to banks are senior compared to deposits, $\widehat{r}_t^b$ solves:

$$\mathbb{E}_\psi\left\{K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1}\right\} + ZK_t^b p_t = \left(D_t^b - w_t^b\right)\left(1+\widehat{r}_t^b\right) + L_t^b\left(1+R_t^{CB}\right) \tag{57}$$

If loans to banks have the same seniority as deposits, $\widehat{r}_t^b$ solves:

$$\mathbb{E}_\psi\left\{K_t^b\left(1+\psi_{t+1}^b\right)Q_{t+1}\right\} + ZK_t^b p_t = \left(D_t^b - w_t^b + L_t^b\right)\left(1+\widehat{r}_t^b\right) \tag{58}$$

Using $\psi_{t+1}^b = 0$ with probability one and rearranging, equations (57) and (58) become, respectively:

$$1+\widehat{r}_t^b = \frac{K_t^b\left(Q_{t+1}+Zp_t\right) - L_t^b\left(1+R_t^{CB}\right)}{D_t^b - w_t^b} \tag{59}$$

and:

$$1+\widehat{r}_t^b = \frac{K_t^b\left(Q_{t+1}+Zp_t\right)}{D_t^b - w_t^b + L_t^b}. \tag{60}$$

The fraction of depositors served during the run is still given by (13).

C.4 Central bank

During the day of time t, the central bank injects money $\mu_t\overline{M}$ in the economy by either offering loans L_t^{CB} to banks and/or buying assets K_t^{CB} on the market at price Q_t, therefore:

$$\overline{M}\mu_t = L_t^{CB} + Q_t K_t^{CB}. \tag{61}$$

In $t+1$, the money supply goes back to the pre-crisis level $\overline{M}$. The central bank obtains a return T_{t+1} from loans and from holding assets, which is distributed to households as described in Section C.1. If loans to banks are senior compared to depositors, then:

$$T_{t+1} = L_t^{CB}R_t^{CB} + Q_t K_t^{CB}R_t^K. \tag{62}$$

Otherwise, if loans to banks have the same seniority as depositors:

$$T_{t+1} = L_t^{CB}\left[\left(1-\alpha^{BAD}\right)R_t^{CB} + \alpha^{BAD} r_t\left(\underline{\psi}\right)\right] + Q_t K_t^{CB} R_t^K. \tag{63}$$

A fraction α^{BAD} of loans was given to banks that were insolvent (because they had been hit by the shock $\underline{\psi}$) and are going bankrupt, therefore the central bank gets a return $r_t\left(\underline{\psi}\right)$ on such loans. The remaining fraction $1-\alpha^{BAD}$ was given to good solvent banks and therefore the return on such loans is R_t^{CB}. The return on capital K_t^{CB} is given by the market return R_t^K.

C.5 Market clearing conditions

Let K_t^{CB} be the amount of capital bought directly by the central bank. The market clearing conditions (22) and (23) are replaced by:

$$\int_0^1 K_t^b db + \int_{\mathcal{H}} K_t^h dh + K_t^{CB} = \overline{K} \tag{64}$$

$$\int_0^1 M_t^b db + \int_{\mathcal{H}} M_t^h dh = \overline{M}\left(1+\mu_t\right) - \pi_t. \tag{65}$$

The market clearing conditions for deposits 24 and for consumption goods 25 are unchanged. Additionally, the following condition must hold:

$$L_t^{CB} = \int_0^1 L_t^b db \tag{66}$$

requiring that the total supply of loans by the central bank is equal to the demand by private banks.

C.6 Equilibrium definition

Definition C.1. *Given the initial state of the economy $\boldsymbol{X}_t$, an equilibrium is a collection of:*

- *prices Q_t and p_t and return on capital R_t^K, on deposits R_t^D and on loans by the central bank R_t^{CB};*
- *household beliefs $Pr_t^h(\cdot)$ about $r_t^{b(h)}$ and l_t^h, for all $h \in \mathcal{H}$;*

- *household choices* $\left\{M_t^h, D_t^h, K_t^h, \left\{w^h\left(n_t^h\right), c^h\left(n_t^h\right)\right\}_{n_t^h \in \mathcal{N}}\right\}$ *for all* $h \in \mathcal{H}$*;*
- *bank choices* $\left\{D_t^b, M_t^b, K_t^b, L_t^b\right\}$ *for all* $b \in [0,1]$*;*
- *limits on withdrawals* $l_t^h \in \{0, +\infty\}$ *for all* $h \in \mathcal{H}$*;*
- *liquidation returns* r_t^b *and fraction of depositors served during a run* f_t^b*, for all* $b \in [0,1]$*;*
- *dividends* π_t *paid to bankers;*
- *central bank: money injected* μ_t*, loans* L_t^{CB}*, asset purchased* K_t^{CB} *and transfers to depositors* T_{t+1}*;*

such that:

- *(banks: optimality, returns and limits on withdrawals) for all* b*, the choices* $\left\{D_t^b, M_t^b, K_t^b, L_t^b\right\}$ *are optimal (Proposition 3.1 and equation (56));* r_t^b *is defined by equation (59) (if loans* L_t^b *are senior compared to depositors) or equation (60) (if loans* L_t^b *have the same seniority as deposits);* f_t^b *is defined by equation (13); the limit on withdrawals* l_t^h *of the depositors of bank* b *satisfy:*

$$l_t^h = 0 \text{ for some } h \in \mathcal{H}(b) \Rightarrow \int_{\mathcal{H}(b)} w^h\left(n_t^h \middle| \, l_t^h = +\infty\right) dh > M_t^b;$$

- *(households' optimality) for all* h*, the choice* $\left\{M_t^h, D_t^h, K_t^h, \left\{w^h\left(n_t^h\right), c^h\left(n_t^h\right)\right\}_{n_t^h \in \mathcal{N}}\right\}$ *solves problem (51);*
- *(central bank) the choices of the central bank satisfy equation (61); transfers* T_{t+1} *are defined by equation (62) (if loans* L_t^b *are senior compared to depositors) or equation (63) (if loans* L_t^b *have the same seniority as deposits);*
- *(rational expectations) households' beliefs are rational:*

$$Pr_t^h\left(r_t^{b(h)} = r,\, l_t^h = l\right) = Pr_t^{(r,l)}\left(r_t^{b(h)} = r,\, l_t^h = l\right), \quad r \in \mathbb{R} \text{ and } l \in \{0, +\infty\};$$

- *(dividends)* π_t *is defined by (21);*
- *(market clearing) the market clearing conditions (Sections C.5) hold.*

Note that there is no need to include transfers T_{t+1} in the state of the economy in $t+1$. That's because, in $t+1$, the supply of money and capital is, respectively, $\overline{M}$ and $\overline{K}$, and I can compute the wealth of households $\overline{A}_{t+1}$ using the total supply of money and capital minus the value of assets held by banks and bankers, that are included in the state $\boldsymbol{X}_{t+1}$.

C.7 Numerical solution

I follow the same approach described in Appendix B.

- The FOC of the household problem (48) and (49) are replaced by the expressions derived after guessing-and-verifying the form of the value function of the problem (51); I still use the guesses $M_t^h = \eta_t^M A_t^h$, $D_t^h = \eta_t^D A_t^h$ and $K_t^h = \eta_t^K A_t^h$.
- The return on capital and on deposits are unchanged.
- Banks: the definition of net worth is unchanged, the budget constraint (7) is replaced by (54) using the fact that all banks get the same amount of loans, $L_t^b = L_t^{b'}$ for all $b \in [0, 1]$ and using (61) and the supply of loans from (66).
- The actual return on deposits of insolvent banks (11) is replaced by (58); the fraction of depositors served during a run f_t is still given by $f_t = \kappa$.
- The expression for dividends is unchanged.
- Equation (50) is unchanged. The market clearing condition (23) is replaced by (65); the market clearing for capital (64) is omitted because of Walras' Law.
- In addition to the equations described in Appendix B, there is one additional equation (equation (63)) that pins down the value of transfers T_{t+1}.

Online Appendix

The Online Appendix is available at: https://sites.google.com/site/robertorobatto/papers/Robatto_JMP_Online_Appendix.pdf.