Debt deleveraging and the exchange rate

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Abstract

Deleveraging from high debt can provoke deep recession with significant international side effects. Swings in the nominal exchange rate and large variations in consumption, output, and terms of trade can happen during the adjustment. All these movements are inefficient and interesting trade-offs emerge from the perspective of global welfare. The optimal adjustment to global imbalances should not necessarily require large movements in the nominal exchange rate. A global liquidity trap can be desirable when countries are more open to trade.

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1. Introduction

The decade leading up to the financial crisis was marked by divergences and disequilibria. Global imbalances have been at the center of the debate, with several economists warning against the unsustainability of the US external position. The euro area has experienced internal current account divergences, producing an enormous accumulation of debt. The crisis was most severe in the economies that had piled up too much private or public debt in one form or another. It is still being debated whether the divergences of the past actually caused the crisis or merely reflected other underlining problems.\textsuperscript{1} In any case, the general tendency is for the crisis-ridden countries to reduce debt. In this deleveraging process, exchange-rate policies have been often placed under scrutiny, as in the case between the US and China or in reference to the choice of irrevocably fixing exchange rates in the eurozone.

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Debt deleveraging raises interesting questions on macroeconomic adjustment. A recent literature, spurred by the works of Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2010) and Philippon and Midrigan (2011), has studied the mechanism of adjustment to debt deleveraging but in closed economies. So far the international consequences have been neglected. This is a gap that this work aims at filling given the importance of the aforementioned debate on global and European imbalances. There are two main contributions of this paper. First, to understand the international transmission mechanism of debt deleveraging. Second, to discuss its welfare consequences by asking how monetary and exchange-rate policies should be designed to better accommodate from a global perspective the ensuing macroeconomic adjustment.\textsuperscript{2}

The transmission mechanism of a reduction in one country’s external debt presents some familiar features with that of the old transfer problem, as discussed among others in Keynes (1929). Deleveraging forces creditor countries to cut spending sharply and depress demand. Spending should increase in the rest of the world. But international relative prices might not be in tune with the adjustment.\textsuperscript{3} If the fall in demand is sharper for domestic goods, which is the case when there is home bias in consumption, the excess supply of these goods globally lowers their prices.

\textsuperscript{1} An interesting discussion is in Obstfeld (2011) and Obstfeld and Rogoff (2010).

\textsuperscript{2} It should be noted that none of the papers of Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2010) and Philippon and Midrigan (2011) deals with the welfare consequences of debt deleveraging.

\textsuperscript{3} In the current debate on the unwinding of global imbalances, Feldstein (2011) and Krugman (2011a,b) have stressed the importance of exchange rate movements in correcting global imbalances.
relative to foreign prices and expands overall demand for them, thus easing the depressive impact of deleveraging. These changes in relative prices are achieved by depreciation of the deleveraging country’s currency. In the longer run, a country that has paid down part of its debt is richer than at first, since there is less debt to serve, so the demand for domestic goods is relatively higher. The exchange rate swings from short-term depreciation to appreciation in the long run. But, without home bias, deleveraging does not produce any movement in the exchange rate in both the short and long run.

Following the propagation mechanism described above, we could be tempted to conclude that the exchange rate, and other international relative prices, should move substantially to mitigate the costs of deleveraging, but only when there is home-bias in goods consumption. Otherwise a fixed-exchange rate would be desirable. However, this conclusion is completely misleading if viewed from the perspective of a benevolent planner maximizing welfare in the global economy. Indeed, this planner dislikes any large variations of consumption, output and relative prices and would prefer, instead, to accommodate the adjustment in a smoother way. This is not feasible and interesting trade-offs emerge between output, consumption and terms-of-trade stabilization.

There are three available channels through which the global economy can absorb a better way a deleveraging shock. The first channel is a pure domestic one, already emphasized by the closed-economy literature as in Eggertsson and Krugman (2012). The real interest rate in the deleveraging country should fall to reduce its borrowing costs while adjusting to a lower level of debt. To this end, a policy in which the interest rate of the deleverager stays at the zero-lower bound is desirable. The other two channels have instead an international dimension: the expenditure-switching channel and the fall in the real interest rate in the non-deleveraging countries.

The expenditure-switching channel driven by movements of the exchange rate is clearly desirable from a global perspective to the extent that can mitigate the output recession in the deleveraging country by shifting the burden of adjustment to other countries. However, it leads to costs in terms of movements in the terms of trade, which are unjustified by efficient shocks. In general, the benevolent planner dislikes large variations of the exchange rate and other international relative prices. Indeed, when the expenditure-switching effect is stronger because domestic and foreign goods are more substitutable, the optimal movements in the exchange rate are small. On the contrary, when the expenditure-switching effect is too weak, a depreciation of the currency can adversely reduce the real income of the country, making it even poorer. Also in this case, the exchange rate should not depreciate much.

A fall in the real interest rate in the non-deleveraging countries is also desirable to the extent that can raise foreign consumption to offset the recession in the deleverager. However, the rise in consumption in the rest of the world is also unjustified by efficient shocks and therefore brings inefficiencies. When the expenditure-switching channel is more effective, the fall in the foreign real rate is less needed. On the contrary, when the expenditure-switching channel is weak, the real rate should fall substantially in the rest of the world. In this case a global liquidity trap can be desirable as when countries are more open to trade.

There are some earlier works related to our framework. Obstfeld and Rogoff (2001, 2005, 2007) also studied the exchange-rate implications of a sudden improvement in one country’s current account balance, conducting some comparative-static experiments without analyzing the welfare consequences. Our focus here is on dynamic adjustment, on the role of monetary policy taking into account the zero lower bound and on optimal monetary policy from a global perspective.

Policies at the zero lower bound, in an open economy, have been explored by Svensson (2001, 2003), Jeanne (2009) and Fujiwara et al. (2010, 2011), but in different models without debt deleveraging. There is also substantial literature on open economies analyzing credit-constrained economies and the implications of relaxing or restricting credit access for the equilibrium economy: see among others Aghion et al. (2001), Aoki et al. (2010) and Mendoza (2010) and more recently Devereux and Yetman (2010). In an open-economy model, Cook and Devereux (2011) have studied the optimal response to preferences’ shocks which bring one country to the zero lower bound while appreciating its currency. In a recent work, Fornaro (2012) studies also international debt deleveraging emphasizing similar mechanisms of adjustment as in our framework. He is not concerned with welfare implications but analyzes mostly the occurrence of liquidity traps under a monetary union. Bhattarai et al. (2013) study instead optimal monetary policy in a currency-area model with financial frictions.

This paper is organized as follows. Section 2 describes a deleveraging shock in a simple two-country open-economy endowment model. Section 3 extends the basic model to include nominal rigidities and endogenous output. Section 4 discusses optimal policy from a global perspective. Section 5 performs some robustness analysis by varying the degrees of home bias and the elasticity of substitution between traded goods. Section 6 analyzes the case in which debt deleveraging concerns debt denominated in foreign currency. Section 7 concludes. An online appendix reports the main equations of the model and the solution method.6

2. A simple model

We adopt a simple two-country endowment economy to study how debt deleveraging in one country spreads to the rest of the world economy. The two countries are Home, denoted by H, and Foreign, denoted by F. Each country has an endowment of a good. The two goods, H and F respectively, are traded frictionlessly. The representative agent of country H maximizes utility from consumption

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $\beta$ is the discount factor with $0 < \beta < 1$. The consumption index $C$ is a Cobb--Douglas aggregator of the consumption of the two goods, $C_H$ (denoting Home goods) and $C_F$ (denoting Foreign goods):

$$C = \left(\frac{C_H}{\alpha}\right)^\alpha \left(\frac{C_F}{1-\alpha}\right)^{1-\alpha},$$

where $0 < \alpha < 1$ represents the share of consumption of goods $H$ in the overall consumption basket, for a consumer of country $H$. Given the prices for the two goods, $P_H$ and $P_F$, expressed in the currency of country $H$, the consumption-based price index of the Home country, $P$, is

$$P = P_H^{\alpha} P_F^{1-\alpha}.$$

Consumers in the Foreign country maximize their utility from consumption

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

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4 An efficient shock that could justify such movements is a productivity shock.

5 This channel has a parallel in the closed-economy literature where a fall in the real rate raises consumption of savers.

6 The online appendix is available via the journal's website.
where the consumption basket $C^*$ is:

$$C^* = \left( \frac{C_H}{1-\alpha^*} \right)^{1-\alpha^*} \left( \frac{C_F}{\alpha^*} \right)^{\alpha^*},$$

and now $\alpha^*$, with $0 < \alpha^* < 1$, is the weight given to goods $F$. The general price index in country $F$ is:

$$p^* = p_H(1-\alpha^*)p^*_F,$$

where $p_H$ and $p^*_F$ are the prices of goods $H$ and $F$ in the currency of country $F$.

The two goods are traded with no friction, and the law of one price holds

$$P_F = SP^*_F, \quad P_H = SP^*_H,$$

where $S$ is the nominal exchange rate, defined as units of Home currency per unit of Foreign currency. Preferences are biased toward domestic goods under the assumption that $\alpha = \alpha^* > 1/2$. In this case, our model generates deviations from purchasing power parity (PPP), in which the real exchange rate ($Q$) is proportional to the terms of trade $T = P_F / P_H$

$$Q = \frac{P^*_F}{P^*_H} = \left( \frac{p_H}{p^*_F} \right)^{1-2\alpha} = T^{2\alpha-1}.$$

Given preferences and prices, demands for the goods are:

$$C_H = \alpha \left( \frac{p_H}{P^*_F} \right)^{-1} C, \quad C_F = (1-\alpha) \left( \frac{p_F}{P^*_F} \right)^{-1} C,$$

$$C_H^* = (1-\alpha^*) \left( \frac{p_H}{P^*_F} \right)^{-1} C, \quad C_F^* = \alpha^* \left( \frac{p_F}{P^*_F} \right)^{-1} C^*.$$

Consumers in the Home country receive in every period $t$ an endowment $Y_{Ht}$ of good $H$, which they can sell at the price $P_H t$; they consume a bundle $C_t$ of goods $H$ and $F$ at price $P_t$; borrow or lend resources $D_{t-1} / (1+i)$, in units of currency of country $H$, and pay back or receive the face value of the funds lent in the previous period $D_t$. A positive value for $D_t$ denotes nominal debt. $D_t$ is the only asset traded internationally and $1+i$ is the one-period risk-free nominal interest rate on domestic currency.\(^7\) As a result, the flow budget constraint for consumers in the Home country is:

$$P_t C_t = P_{Ht} Y_{Ht} + D_{t-1} / (1+i) - D_t.$$

There is a limit on the amount of real debt that the agent can take in each period

$$\frac{D_t}{P_t} \leq k,$$

where $k > 0$. Similar constraints have been used in other open-economy models, such as Aoki et al. (2010), Devereux and Yetman (2010) and Mendoza (2010). They are justified in terms of the guarantees that international creditors require when borrowers have limited commitment. As in Eggertsson and Krugman (2012), we do not model the source of this constraint but interpret it as the maximum size of the debt that can be considered safe and that international investors are willing to lend to country $H$ at each point in time. A change in this limit – in particular its reduction over time – constitutes the debt-deleveraging experiment analyzed here.\(^8\) This drop can happen just for a change in confidence triggered by an internal banking or financial crisis – not modeled here – so that international investors are more reluctant to lend to country $H$. In the equilibrium that we are going to analyze, consumers in country $H$ will be at the corner and Eq. (5) limits their borrowing capacity.

Looking now at country $F$, the flow budget constraint is:

$$P^*_F C^*_t = P^*_{Ft} Y^*_{Ft} + \frac{D^*_{t+1}}{S_t (1+i)} - \frac{D^*_t}{S^*_t}$$

where $Y^*_F$ represents the endowment of good $F$ and $D^*_t$ the holding of nominal debt in units of currency $H$. Consumers in country $F$ face a similar borrowing limit in units of their consumption basket:

$$\frac{D^*_t}{P^*_F / S^*_t} \leq k.$$

for a positive $k^*$. In the equilibrium that we are going to analyze, consumers in country $F$ will be creditors in international markets and the limit in Eq. (7) is not binding.

The optimal intertemporal allocation of consumption in country $H$ is governed by the following Euler equation:

$$U_t(C_t) \geq \beta(1+r_t)U_{t+1}(C_{t+1}),$$

where the home-country real interest rate is defined as

$$1 + r_t = \frac{(1 + i_t) P_t}{P_{t+1}}.$$

Similarly, the Euler equation for consumers in country $F$ is:

$$U_t(C^*_t) \geq \beta(1 + r^*_t)U_{t+1}(C^*_{t+1}).$$

where the foreign-country real interest rate is connected to the home-country real rate through

$$(1 + r_t) = (1 + r^*_t) \frac{Q_{t+1}}{Q_t}.$$

Both Euler equations hold with equality when the borrowing limit is not binding.

Equilibrium in goods and asset markets implies

$$Y_{Ht} = T_t^{-\alpha} [\alpha C_t + (1-\alpha) Q_t C_t^*],$$

$$Y^*_{Ft} = T_t^{-\alpha} [(1-\alpha) C_t + \alpha Q_t C_t^*],$$

$$D_t + D^*_t = 0.$$

Combining the equilibrium in the goods market, the terms of trade can be written as

$$T_t = \frac{Y_{Ht}}{Y^*_{Ft}} \left( \frac{(1-\alpha) C_t + \alpha Q_t C_t^*}{\alpha C_t^* + (1-\alpha) Q_t C_t^*} \right),$$

while the real exchange rate follows from

$$Q_t = T_t^{2\alpha-1}.$$
in what follows, we use terms of trade, real and nominal exchange rates interchangeably.\(^9\)

Second, and more important, home bias in consumption is crucial in order for deleveraging to influence the exchange rate. In fact, if preferences are identical across countries (\(\alpha = 1 / 2\)), the terms of trade is independent of the distribution of wealth and just proportional to the ratio of the endowments of the two goods.\(^{10}\) Instead, when there is home bias, the distribution of wealth and debt across countries can also affect relative prices through the demand channel. Imagine that deleveraging in the Home country reduces Home consumption. Since Home consumers demand more of their own goods, the fall in Home consumption depresses the demand for Home goods more than that for Foreign goods. The price of the Home goods relative to Foreign falls, worsening the Home terms of trade and depreciating the Home currency. In these cases, exchange rate management is a factor in the debt-deleveraging transmission mechanism.

2.1. Steady state

A deleveraging shock produced by a lowering of the debt limit \(k\) in the Home country requires some time to be absorbed. In this section we abstract from the adjustment process and compare the initial and final steady-state equilibria. We start from an initial steady state in which the distribution of wealth is such that consumers in the Home country can afford to save more with an inequality because of the borrowing limit. Specularly, it will be lower in the Foreign country. Steady-state Home and Foreign real interest rates (\(T\) and \(T^*\)) are tied to the subjective discount factor \(\beta\)

\[
(1 + r) = (1 + \bar{r}) = \frac{1}{\beta^*},
\]

where an upper bar denotes variables at their steady-state levels. Debt in the Home country is at the borrowing limit (Eq. (5)), and the steady-state level of consumption follows from the budget constraint (Eq. (4))

\[
C = T^{\alpha - 1}Y_H - (1 - \beta)k.
\]

Combining Eqs. (3), (6) and (12) consumption in the Foreign country is given by

\[
QC^* = T^*Y_F + (1 - \beta)k.
\]

The steady-state terms of trade can be simply obtained by appropriately incorporating Eqs. (15) and (16) into Eq. (13)

\[
T = \frac{Y_H}{Y_F} \left[ 1 + (1 - \beta) \left( \frac{2\alpha - 1}{1 - \alpha} \right) \frac{k}{T^{\alpha - 1}Y_H} \right].
\]

Interestingly, the terms of trade and the real exchange rate depend on the level of debt and the distribution of wealth, but only when there is home bias in consumption, i.e. when \(\alpha > 1 / 2\). When we move from a high- to a low-debt equilibrium (\(k\) falls), Eq. (17) shows that the terms of trade improves in the long run. Indeed, consumption for the constrained borrowers is higher in the final than in the initial steady state, since they have less debt and can service it at less real cost. On the contrary, Foreign consumers have to lower consumption. Since there is home bias, the demand for Home goods increases relative to that of Foreign goods in the long run, the terms of trade of country \(H\) improves and the real exchange rate rises. The interesting part of the exercise, however, is the short-run adjustment, which is completely different in form, actually swinging from a short-run currency depreciation to a long-run appreciation.

2.2. Adjustment to a deleveraging shock in country \(H\)

We now study the dynamic adjustment to a deleveraging shock that hits country \(H\). Let us say that for exogenous reasons, there is a fall in the maximum amount of external debt that can be considered risk-free: the debt ceiling \(k\) drops from \(k_{\text{high}}\) to \(k_{\text{low}}\). The adjustment takes place in two periods, the short run and the long run.

In the long run, denoted by a bar, the results of Section (2.1) apply. The real interest rate follows from Eq. (14) while \(T, C\), \(C^*\) and \(Q\) solve Eqs. (3), (13), (15) and (16). With respect to the initial steady state, consumption in the Home country will be higher in the long run, since there is less debt to serve. Specularly, it will be lower in the Foreign country. The terms of trade of the Home country improves if there is home bias. Otherwise, it will be unaffected.

In the short run, the flow budget constraint of the Home country implies:

\[
\frac{1}{C} = \frac{1}{C^*} \beta(1 + r').
\]

where we have assumed log utility, while the Euler equation of the Home consumer holds with an inequality because of the borrowing limit. In the short run, the Home and Foreign rates are related to the changes in the real exchange rate between the short and the long run

\[
1 + r = (1 + r') \frac{Q}{Q^*}.
\]

Using short- and long-run consumption in the Euler equation (Eq. (20)) of country \(F\), we obtain an expression for the short-run real interest rate

\[
(1 + r) = \frac{1}{\beta} \left[ \frac{T^*Y_F + k_{\text{low}}}{T_{\text{low}}Y_F + k_{\text{high}}} \right].
\]

The short-run real rate depends on movements in the terms of trade and debt positions between the short and the long run for a given path of output, which is exogenous and can be considered constant through the exercise. Eq. (22) determines \(r, T\) given that \(T\) is also a function of \(k_{\text{low}}\) as discussed in the previous section. The additional equilibrium condition comes from combining Eqs. (13), (18) and (19) into

\[
T = \frac{Y_H}{Y_F} \left[ 1 + \frac{2\alpha - 1}{1 - \alpha} \frac{k_{\text{low}}}{T^*Y_F + k_{\text{high}}} \right].
\]

Some qualitative implications for the short-run terms of trade can be inferred already from this equation, again assuming home bias in consumption, which is necessary in order for the dynamic and the distribution of debt to affect the terms of trade. When country \(H\) is deleveraging with respect to the world, then it is easy to see that the terms of trade in the short run, \(T\), will move to a higher level. Therefore,
the exchange rate of the Home country will depreciate in the short run but appreciate in the long run. In a world without home bias, the terms of trade will be completely insulated from the deleveraging shock. As in the closed-economy model of Eggertsson and Krugman (2012), a deleveraging shock produces a fall in the Home real interest rate, as shown in Eq. (22), which is enhanced by the short and long-run movements of the terms of trade under the assumption of home bias. In this case, the real rate in the Home country falls more than the real rate of the Foreign country, as shown in Eq. (21).

For a first assessment of the magnitude of the impact of deleveraging on the world economy, we calibrate the model assuming that each period corresponds to one year. In the next section, we consider a more general environment in which deleveraging is spread endogenously over several periods, but in a quarterly model. Here, in a yearly model, considering a steady-state real rate of 2.5% per year we can calibrate \( \beta = 0.9756 \). We set \( \alpha = 0.76 \), which is consistent with the share of the US non-durable consumption spending that goes to US-made products, as shown in Hale and Hobijn (2011). We set \( k_{bgh} = 0.4 \) to match the 40% of the US net external position in debt securities over GDP that Gourinchas et al. (2010) report for the year 2008. We imagine alternative scenarios in which external domestic debt over GDP, defined as \( d_{hg} \), is reduced from 40% to 30%, to 25% and 20%, respectively. According to Gourinchas et al. (2010), the net external debt position of the US before the crisis was around 20% in 2002 and around 30% in 2006.

Fig. 1 shows the adjustment of Home and Foreign consumptions, Home and Foreign real interest rates, the terms of trade and Home external debt as a fraction of GDP after a deleveraging shock. As discussed in Section 2.1, the terms of trade improves in the long run because the Home country reduces its debt exposure and so has more resources available to buy goods. Since there is home bias in preferences, the demand for domestic goods rises together with their relative price. In quantitative terms, the figure shows that all the variables display a negligible difference between the initial and final steady states.

In the short run, the adjustment takes different direction and magnitude. Home consumers must reduce their consumption drastically in order to repay debt. Because of home bias, aggregate demand for goods \( H \) holds more sharply, so the terms of trade worsens, implying a sharp depreciation of the Home currency close to 15% for severe shocks. Since in the short run deleveraging borrowers reduce their demand for goods more than in the long run, the real interest rate falls, an offsetting factor that generates more consumption by consumers in country \( F \). The real interest rate falls more in \( H \) than in \( F \), as is shown in Eq. (21), since the terms of trade (the real exchange rate) rises in the short run before falling in the long run. Notice that starting from a real interest rate of 2.5% the deleveraging shock drives both Home and Foreign rates below zero; and when deleveraging is severe far below zero, to \(-20%\) or more. Consumption in country \( H \) can fall even up to \( 15% \) while that of country \( F \) speculatively rises with the same magnitude.

### 2.3. Efficiency

The simple model shows that consumption, real interest rates and the terms of trade move significantly following a deleveraging shock. But, are these movements efficient? To address this question, we should define the efficient allocation in our model which critically depends on the efficient distribution of wealth. Since the latter changes during the deleveraging experiment, there is not an obvious choice. To sharpen our analysis, we can think at our deleveraging experiment as one that brings the world economy from an inefficient distribution of wealth to an efficient one. The Home country, for un-modeled reasons, has accumulated too much external debt and suddenly is forced to repay part of it to reach the efficient level.

To define the efficient allocation, we solve the maximization of the aggregate welfare

\[
\sum_{t=0}^{\infty} \beta^t \left( \xi \ln(C_t) + (1-\xi) \ln(C_{\text{F}}) \right)
\]

for some Pareto weight \( \xi \) given the two resource constraints (10) and (11). In particular the Pareto weight \( \xi \) is chosen in such a way that in the final steady state the ratio of the marginal utilities of consumption is proportional to the real exchange rate

\[
\frac{\xi}{1-\xi} = \frac{E_{\text{H}}}{E_{\text{F}}}.
\]

As shown in the appendix, by taking a second-order approximation of the above objective function with respect to the final steady state and combining it with a second-order approximation of the resource constraints (10) and (11), a quadratic loss function follows

\[
l_t = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( \xi C_t^2 + (1-\xi) \left( C_{\text{F}}^2 + \alpha(1-\alpha) \right)^2 \right) \tag{23}
\]

which appropriately penalizes deviations of the target variables with respect to the final steady state, and through which it is possible to evaluate the costs of deleveraging. In the loss function (23), a variable with a tilde denotes a deviation of that variable with respect to the final efficient steady state. Any departure of consumption, Home and Foreign, and the terms of trade from their final steady-state values is costly. In particular, the terms of trade is a distinct objective since in a model with multiple goods misalignments of relative prices with respect to their efficient levels are costly because they produce a misallocation of real resources across different uses.

The world economy would be better off by forgiving the inefficient part of the Home-country external debt in a way to immediately achieve the efficient allocation. Obviously, this could entail non-negligible costs – not considered in our framework – which make this option not viable. However, even an orderly deleveraging process, with the large swings shown in Fig. 1, can be costly. In particular, the costs can be as high as a 0.036% permanent reduction in the steady-state consumption of both countries when considering the worst scenario in which external debt drops from 40% to 20%. These costs are not huge but it should be noted that we are evaluating them in terms of a permanent reduction in steady-state consumption, while the adjustment process lasts only two periods. Instead, if we evaluate the costs in terms of a temporary fall in steady-state consumption for an already long ten-year horizon, the drop is around 0.16% in the worst scenario. For a five-year horizon it is around 0.31%.

It is important to add that even in the case in which there is no home bias, and therefore the terms of trade does not move, there are costs of deleveraging according to Eq. (23). Whether the terms of trade (or the exchange rate) moves or does not move should not be interpreted as a symptom of a correct or wrong adjustment to global imbalances.

Some lessons can be drawn from this simple open-economy model. In addition to the channel identified in the closed-economy literature – i.e. that deleveraging produces a fall in the real interest rate – there is an additional mechanism acting through the terms of trade but only if there is home bias in preferences. In this case, the exchange rate of the deleveraging country depreciates on impact and then appreciates in the long run. Consumption of the deleveraging country falls while that of the foreign economy increases given the low real rates. Moreover the real interest rate of the deleverager falls more than that of the other country, again under the assumption of home bias.

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11 We normalize \( H_t = 1 \) so that \( \bar{X} = k_{bgh} / \bar{X} = 0.4 \).
We have also shown that the adjustment process is in general inefficient, if we take the perspective that the final distribution of wealth reached after deleveraging is instead efficient. This observation opens room for policy options to mitigate the adjustment. However, the simple model presented in this section is of a rather limited use. Three options would be available: 1) debt forgiveness; 2) default; and 3) transfers across countries. While the model is not suitable to evaluate the costs of the two former options, the latter one is also hard to enforce for completely disjoint political entities. In what follows, we analyze the role of monetary policy in a framework in which the above three options are not available or used.

There is a further limitation of the model presented above, namely the assumption of endowment economies which limits the extent to which the exchange rate can be an important element in the adjustment process. Indeed, relative-price movements driven by variations in the exchange rate can produce expenditure-switching effects across countries which can mitigate the costs for the deleverager. To this end, in the next section, we extend the model to allow for endogenous production. We also assume nominal rigidities, consistent with the empirical evidence on the real effects of monetary policy shocks, and study the relevance of different exchange-rate regimes and monetary policies in the adjustment to international deleveraging.

3. A model with endogenous production and nominal rigidities

The model used in this section closely follows those of the open-economy macro literature, such as Obstfeld and Rogoff (2001, 2005) and Benigno (2009). The new elements here with respect to the simple model of the previous section are nominal rigidities, endogenous output, debt deleveraging on a longer horizon and more general preference specifications.

Three factors can delay the adjustment to a deleveraging shock and create interesting dynamics. First, nominal rigidity slows the response of relative prices and can lead to a contraction in real output. Second, the zero lower bound on the nominal interest rate can prevent real rates from falling, depressing aggregate demand and output. Finally, the exchange-rate regime may either attenuate or amplify the response of real and nominal exchange rates.

Households in country $H$, a continuum of measure one, have preferences over consumption and work hours as follows:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ c_j^{1-\rho} \left( \frac{1}{1-\rho} \int_0^1 \frac{L_j(j)^{1+\eta}}{1+\eta} d\eta \right) \right] \right\},$$

where $L_j(j)$ is hours worked of variety $j$; $\eta \geq 0$ the inverse of the labor-supply elasticity and $\rho > 0$ the inverse of the intertemporal elasticity of substitution in consumption. Every household can supply all varieties of labor; $C$ is a consumption bundle of goods $H$ and $F$ given by

$$C = (\alpha c_H^{1-\theta} + (1-\alpha) c_F^{1-\theta})^{1/\theta},$$

where $\alpha$, with $\alpha \geq 1/2$, represents still the weight given to home-produced goods in the consumption bundle while $\theta$, with $\theta > 0$, is the intertemporal elasticity of substitution between Home and Foreign-produced goods. The Cobb–Douglas case (Eq. (1)) of the previous section is nested when $\theta = 1$. Given this preference, the consumption-based price index is given by

$$P = \left( \alpha P_H^{1-\theta} + (1-\alpha) P_F^{1-\theta} \right)^{1/\theta}.$$

Differently from the previous section, we now assume that $C_H$ is composed of a continuum of goods $c(h)$ of measure one all produced in country $H$, while $C_F$ is a continuum of goods, $c(f)$, produced in country $F$.

$$C_H = \left[ \int_0^1 c(h)^{1-\theta} dh \right]^{1/\theta} \quad \text{and} \quad C_F = \left[ \int_0^1 c(f)^{1-\theta} df \right]^{1/\theta},$$

13 A currency union can be an exception. Fornero (2013) studies debt-relief policy, in the form of a transfer of wealth from creditors to debtors, in a closed economy.
where $\sigma > 0$ is the elasticity of substitution across goods produced within a country. The price indices $P_h$ and $P_f$ are:

$$P_h = \left[ \int_0^1 p(h)^{1-\alpha} \, dh \right]^{1\over 1-\alpha} \quad P_f = \left[ \int_0^1 p(f)^{1-\alpha} \, df \right]^{1\over 1-\alpha},$$

where $p(h)$ and $p(f)$ are the prices of the goods $h$ and $f$ in the currency of country $H$. Households in country $H$ face the following flow budget constraint:

$$P_h C_t - \int_0^1 W_t(j) L_t(j) \, df + \Psi^*_t - D_t \Psi^*_t - D_{t-1} - k_t P_t - \frac{d_t D_t}{k_t} \frac{d_t}{D_t} \frac{\tilde{Y}_t}{C_t} = 0,$$

where $W_t(j)$ is the nominal wage for the variety of work $j$ and $\Psi^*_t$ are firms' profits, which are distributed to the households in equal proportion. In the flow budget constraint (24) we have added a function $\tilde{X}(\cdot, \cdot)$, appropriately normalized, capturing costs of adjusting the debt position. The function depends on the real debt of the individual households, $d_t = D_t / P_t$, with respect to a threshold $k_t$ and of the country's aggregate real debt, given by $D_t$, again with respect to the same threshold. Excess borrowing, above a certain limit, is costly and may reflect intermediation frictions related to the monitoring that lenders perform when exerting too much credit. We assume that the function $\tilde{X}(\cdot, \cdot)$ is always non-negative, $\tilde{X}(\cdot, \cdot) \geq 0$ since it reflects only costs and not benefits. Moreover, the derivatives of the function with respect to individual debt, $\tilde{X}_d(\cdot, \cdot)$, and to aggregate debt, $\tilde{X}_D(\cdot, \cdot)$, are non-negative $\tilde{X}_d(\cdot, 1) \geq 0$ and $\tilde{X}_D(\cdot, \cdot) \geq 0$. In particular, we assume that $\tilde{X}_D(1, 1)$ is a sort of optimality condition for individual borrowing saying that at the risk-free level of debt the marginal cost of increasing the borrowing capacity is zero. However, increases in aggregate debt above the risk-free threshold are costly at the margin, $\tilde{X}_D(1, 1) > 0$.14 The assumption that the individual cost of borrowing depends also on aggregate debt is not only convenient for technical reasons, as it will be explained later, but also because it might capture several features of the recent financial crisis. For the same characteristics of the individual borrowers, financial intermediaries might charge a different premium on lending depending on the aggregate conditions if these reflect different degrees of vulnerability of the financial system to systemic risk. The aggregate level of debt might be an important signal of this vulnerability since it might imply or predict a worsening of the balance sheets of intermediaries in the case in which more non-performing loans materialize when macroeconomic conditions worsen. Moreover, given the interdependence of the financial system, an overall higher level of aggregate debt might facilitate the contagion of a poor creditworthiness of some sectors of the economy to others, and therefore exacerbate adverse-selection problems in exerting credit to individual borrowers. In a nutshell, during the financial crisis, even reliable borrowers faced a worsening in their credit conditions because of the weakening of the overall financial system due to the high level of debt and leverage accumulated in the past.

The deleveraging experiment that we consider in this section involves a one-time reduction in the threshold $k_t$, which, given the structure of the economy, produces a dynamic adjustment of debt and other aggregate variables. In particular, the zero-lower bound constraint is mainly responsible of the dynamic adjustment. We assume that $k_t$ changes from $k_{max}$ to $k_{min}$.15 This might capture a banking or financial crisis, or just a change in confidence, such that excess borrowing above the lower threshold is now costly. Therefore, borrowers need to deleverage. It should be noted that through the cost function the steady-state debt position of households is determined in our model, a device often used in the literature. Moreover, as it will be shown later, the fact that the cost function depends on individual debt is going to be reflected, through optimality conditions of households, into a borrowing premium.16

Similarly, preferences of households in country $F$ are:

$$E_t \left\{ \sum_{\eta=0}^{\infty} \beta^\eta \left[ \frac{P_f^X(i)}{P_f^{X(i)}} - \int_0^1 \frac{X_t(i)}{1+\eta} \, df \right] \right\},$$

where $C_t$ is now given by

$$C^* = \left(1 - \alpha \right) \left( P_{H1}^{1-\theta} + \alpha P_{F1}^{1-\theta} \right) \frac{1}{1-\alpha},$$

and the related consumption-based price index is

$$P^* = \left(1 - \alpha \right) \left( P_{H1}^{1-\theta} + \alpha P_{F1}^{1-\theta} \right) \frac{1}{1-\alpha}.$$

$L_t(i)$ represents hours worked of type $i$ in foreign firms. The consumption bundles $C_t$ and $C^*$ and the appropriate consumption-based price indices $P_t$ and $P^*$ have the same structure as those of country $H$, whereas $P^*_t(h)$ and $P^*_t(f)$ are now the prices of the goods $h$ and $f$ expressed in the currency of country $F$. The law of one price holds for each traded good (i.e., $P_t(h) = SP_t^H(h)$ and $P_t(f) = SP_t^F(f)$) but, as explained in Section 2, there can be deviations from PPP because of Home bias. Households in country $F$ face a flow budget constraint:

$$P_f C_t = \int_0^1 W_t^I(i) L_t^I(i) \, di + \Psi^*_t + TR^F_t + B_t - \frac{D_t}{(1+\tilde{t})} S_t - \frac{D_t}{S_t},$$

where $W_t^I(i)$ is nominal wage for the variety of work $i$. $\Psi^*_t$ are foreign profits. In writing the flow budget constraint of the foreign consumers, we are assuming that they can borrow and lend also in bonds denominated in their own currency, $B_t$, at the interest rate $1 + \tilde{t}$. However, we assume that these securities are in zero-net supply within the country. The only asset traded internationally is denominated in the currency of country $H$, and consumers of country $F$ hold $D_t$ units of it.17 In writing the budget constraint for consumers in country $F$, we are neglecting the costs of adjusting their debt position because in the equilibrium that we are going to analyze these consumers will be creditors in the international financial markets, and therefore they are not subject to costs of excessive borrowing. Moreover, the borrowing costs of consumers in country $H$ are transferred in terms of profits of intermediation to the consumers in country $F$. Indeed these profits are given by $TR^F_t = P_t k_t \tilde{X}_D(d_t/k_t, \tilde{A}_t/k_t) / S_t$.18

Turning to the consumer's optimality conditions, the stochastic version of the Euler equation (Eq. (9)) still describes the intertemporal allocation of consumption in country $F$ and holds with equality at an interior optimum

$$(C^*_t)^{-\rho} = \beta(1 + \tilde{t}) E_t \left\{ \left( C^*_t+1 \right)^{-\rho} S_{t+1} / P_{t+1} \right\}.$$

14 We further assume that the second derivatives are such that $\tilde{X}_d(d, 1) + \tilde{X}_D(d, 1) > 0$.

15 The one-period deleveraging experiment of the previous section can also be seen as a limiting case of the modeling device used in this section, when the cost of adjustment with respect to the threshold is infinite. Notice also that, as in the model of the previous section, any level of initial debt $d$ such that $d < k_{max}$ is consistent with the steady-state equilibrium, since $\tilde{X}(\cdot, \cdot) \geq 0$, $\tilde{X}_d(1, 1) > 0$ and partial derivatives are non-negative. We set $d = k_{min}$. The analysis further shows that when $k$ falls to $k_{min}$ the level of debt $d$ is adjusted gradually to $k_{max}$ from above.

16 In a log-linear approximation, the model will be isomorphic to one in which the interest-rate is assumed to be elastic with respect to the individual and aggregate debt.

17 In equilibrium $D_t + D_t^\prime = 0$.

18 The welfare analysis of the next section simplifies substantially under this assumption. Otherwise, goods market equilibria will be affected by the costs of intermediation and bring additional effects in the quadratic approximation of the objective function.
while the Euler equation of the Home country changes to
\[
(C_t)^{\phi} \{ 1 - (1 + i_t) \psi \left( \frac{d_t}{k_t} \right) \} = \beta (1 + i_t) E_t \left\{ (C_{t+1})^{\phi} \frac{p_t}{p_{t+1}} \right\},
\]
where \( \psi(d_t/k_t) = k_t \phi(d_t/k_t, \tilde{d}_t/k_t) \) since in equilibrium \( d_t = \tilde{d}_t \). The cost of excessive borrowing for households in country \( H \) endogenously implies a premium in addition to the interest rate paid. Note that when \( d_t = k_t, \phi(\cdot) = 0 \) and we retrieve the standard Euler equation.\(^{19}\)

The Euler equation of households in countries \( F \) with respect to holdings of securities denominated in their currency reads as
\[
(C_t)^{\phi} \{ 1 - (1 + i_t) \psi \left( \frac{d_t}{k_t} \right) \} = \beta (1 + i_t) E_t \left\{ (C_{t+1})^{\phi} \frac{p_t}{p_{t+1}} \right\}.
\]

By combining the two Euler equations for the households in country \( F \), we get that the excess return of investing in foreign versus domestic currency is orthogonal to the stochastic discount factor of the foreign household
\[
E_t \left\{ \frac{\beta (1 + i_t) E_t \left\{ (C_{t+1})^{\phi} \frac{p_t}{p_{t+1}} \right\}}{\left( 1 + i_t \right) - (1 + i_t) \frac{S_t}{C_{t+1}} \right\} = 0,
\]
which in a log-linear approximation delivers the standard uncovered-interest-rate-parity condition.

In both countries there is a continuum of firms, each producing one of the goods. Firms use all the varieties of labor offered in the country, combining them through the following technologies
\[
y(h) = \left[ \int \frac{H}{0} h^\lambda (j)^{\lambda - \gamma} \, dj \right]^{\frac{1}{\lambda - \gamma}}, \quad y'(f) = \left[ \int \frac{1}{0} f^\lambda (i)^{\lambda - \gamma} \, di \right]^{\frac{1}{\lambda - \gamma}},
\]
where \( \tau \) is the elasticity of substitution across varieties of labor, with \( \tau > 1 \). We assume that firms operate under monopolistic competition, setting their prices in a flexible way. It follows that \( p_h(h) = p_{h(t)} = \mu W_t \) for each \( h \) and \( p_f(f) = p_{F(t)} = \mu W_t \) for each \( f \); where \( W_t \) and \( W_t \) are aggregate nominal wages in the respective currencies and the price markup is \( \mu \approx \sigma/(\sigma - 1) \). While prices are flexible, wages adjust in a staggered way following Calvo’s model in which unions, grouping work of the same variety, have monopolistic power in setting wages. In each period, in country \( H(\tau) \), only a fraction \( 1 - \lambda (1 - \lambda^\tau) \) of the varieties of labor, with \( \lambda > 1 \), can have their wages reset according to the macroeconomic conditions and independently of the last adjustment. The remaining fraction of varieties of labor, of measure \( \lambda (1 - \lambda^\tau) \), can only index their wages to the current inflation target, \( \Pi^H(\Pi^H) \), which does not necessarily coincide with actual inflation. It is clear that wage rigidity translates directly into price rigidity, since we do not have productivity shock. The resulting aggregate-supply equations are standard for this kind of model. The set of equilibrium conditions is presented in detail in the online appendix.

4. Optimal adjustment to international deleveraging

In this section we ask how a benevolent central planner, maximizing the utility of the world economy, would optimally react to a deleveraging shock hitting country \( H \). A natural objective of policy is the maximization of the weighted average of the utility of the consumers in the world economy, which is given by
\[
U_t = E_t \left\{ \sum_{t=0}^\infty \left[ \left( \frac{C_t^{1-\mu}}{1-\mu} - \int_0^1 \left( \frac{U(t)^{1-\eta}}{1-\eta} \right) \, dt \right) + (1 - \mu) \left( \frac{C_{t+1}^{1-\mu}}{1-\mu} - \int_0^1 \left( \frac{U(t)^{1-\eta}}{1-\eta} \right) \, dt \right) \right] \right\}
\]
(25)

In particular, consistent with the previous analysis, we choose appropriately the weight \( \xi \) so that the steady state reached after deleveraging is efficient. As before, our experiment entails a reduction of debt that brings the economy from an inefficient allocation to an efficient one. To this end, we assume that there are appropriate subsidies which eliminate the monopolistic distortions in the labor markets. The final steady state is described by the following set of equilibrium conditions
\[
\begin{align*}
\mathcal{C} &= p_H T_H (1-\beta) T^{-1} k_{min}, \\
\mathcal{Q} &= p_H T_F (1-\beta) T^{-1} k_{min}, \\
\mathcal{V}_H &= p_H^\phi \left[ \alpha C + (1-\alpha |\mathcal{Q}|^\mathcal{C} \right] \mathcal{C} \\
\mathcal{V}_F &= p_F^\phi \left[ (1-\alpha |\mathcal{C}|^\mathcal{C} + \alpha |\mathcal{Q}|^\mathcal{C} \right] \mathcal{C} \\
\mathcal{V}_H' &= p_H^\phi \mathcal{C}^\phi \\
\mathcal{V}_F' &= p_F^\phi \mathcal{C}^\phi \\
Q &= \left( (1-\alpha |\mathcal{C}|^\mathcal{C} + \alpha |\mathcal{Q}|^\mathcal{C} \right) \mathcal{C}^\phi
\end{align*}
\]
which clearly determine the allocation of \( \mathcal{C}, \mathcal{C}', \mathcal{V}_H, \mathcal{V}_F, \mathcal{Q}, \mathcal{P}_H, \mathcal{P}_F \) given the level of debt \( k_{min} \), reached after deleveraging and the steady-state inflation rate in country \( H, T, T^H \). It is also easy to show that an efficient allocation should satisfy the condition
\[
\frac{\xi}{1-\xi} \left( \frac{C}{Q} \right)^{-\rho} = \frac{1}{Q}
\]
which indeed is the one determining the weight \( \xi \) given the above derived \( C, C' \) and \( Q \).

The fact that the new steady state is efficient simplifies a lot the analysis. Indeed, by taking a second-order approximation of Eq. (25) around the efficient steady state and combining it with the resource constraints, we obtain an expression containing only quadratic terms which can be correctly evaluated through a first-order approximation of the equilibrium conditions. Details are left to the online appendix. Maximization of the above utility function corresponds to minimization of the following loss function
\[
L_t = \frac{1}{2} \left\{ \sum_{i=1}^n \left[ \phi_i \left( \mathcal{C}_{it} - \mathcal{C}_{it}^* \right)^2 + \phi_i \left( \mathcal{C}_{it}^* \right)^2 + \phi_i \left( \mathcal{Q}_{it} - \mathcal{Q}_{it}^* \right)^2 + \phi_i \left( \mathcal{Q}_{it}^* \right)^2 + \phi_i \left( \ln(\mathcal{Q}_{it}^*) - \ln(\mathcal{Q}_{it}^*) \right)^2 + \phi_i \left( \ln(\mathcal{Q}_{it}^*) - \ln(\mathcal{Q}_{it}^*) \right)^2 \right] \right\}
\]
(26)
for some non-negative parameters \( \phi_i, \phi_i, \phi_i, \phi_i, \phi_i, \phi_i \), and \( \phi_i \) discussed in the online appendix; the variables \( \mathcal{C}, \mathcal{C}, \mathcal{V}_H, \mathcal{V}_F, \mathcal{T} \) represent log deviations with respect to the final steady state of the respective variables, while \( \mathcal{P}_{it}^H \) and \( \mathcal{P}_{it}^F \) are the Home and Foreign producer inflation rates and \( \mathcal{F}^H \) and \( \mathcal{F}^F \) are their respective targets such that \( \mathcal{F} = h \mathcal{F}^H \) and \( \mathcal{F} = h \mathcal{F}^F \).

According to the loss function (Eq. (26)), the benevolent central planner dislikes deviations of the producer inflation rates in each country from their respective targets. This captures the costs of wage dispersion due to misallocation of labor demand across varieties which have the same level of efficiency. Moreover deviations of output, consumption in each country

\(^{19}\) The assumptions on the cost function \( \bar{h}(\cdot) \) imply that \( \psi(d_t) > 0 \). Instead, if the cost function depends only on the first argument, it would be the case that \( \psi(d_t) = 0 \). In this case it follows that, in a first-order approximation of the equilibrium conditions, there will be no effect, both in the Euler equation of borrowers and in their budget constraint, of assuming costs of portfolio adjustment.

\(^{20}\) Notice that one equation is redundant. We have also defined \( p_H = P_H / P \) and \( p_F = P_F / P \).
and the terms of trade from their efficient levels are also penalized. It is optimal to keep the GDP inflation rates at their targets and at the same time to achieve immediate stabilization of output, consumption and the terms of trade at the efficient levels.

However, the efficient allocation can only be reached in the long run when deleveraging ends, while it is not feasible in the short run mainly for two reasons: first, as shown in the simple model of Section 2, an adjustment to a deleveraging shock brings about movements in output, consumption and relative prices whose direction contrasts with the efficient movements built into the objective (Eq. (26)), creating therefore important trade-offs; second, a policy of keeping GDP inflation rates at their targets \( \pi \) and \( \pi' \) at all times may not be feasible because, as shown again in Section 2, a deleveraging shock produces negative real interest rates which, with constant inflation rates, require the nominal interest rates to go below zero and violate the zero-lower bound constraint.21

Given the loss function (Eq. (26)) we solve for the linear–quadratic optimal-policy problem taking into account the zero-lower bound constraints.22 We analyze the effects of an unanticipated deleveraging shock that moves \( k \) immediately from \( k_{\text{max}} \) to \( k_{\text{min}} \). The shock produces a dynamic path of deleveraging which depends endogenously on policy. In particular we calibrate \( k_{\text{max}} \) and \( k_{\text{min}} \) such that the Home country external debt with respect to GDP moves from an initial level of 40% to finally 30% at the end of deleveraging. In particular \( k_{\text{max}} \) is set at 1.6095 while \( k_{\text{min}} \) at 1.2054. The model is calibrated quarterly. We set \( \beta = 0.9938 \) to imply a 2.5% real annual return on a yearly basis. The steady-state inflation rates are set to \( \pi T - \pi = 1.005 \) to imply a 2% inflation rate on a yearly basis in each country. This imply that \( \pi - \pi' = 2\% \) at annual rates. We set the parameter \( \alpha = 0.76 \) as in previous section and calibrate the parameters \( \sigma \) and \( \tau \) to 7.66, implying steady-state mark-ups in goods and labor market equal to 15%. The inverse of the elasticity of substitution in consumption, \( \rho \), is set to 2, consistent with a number of studies, and the inverse of the labor supply elasticity, \( \eta \), is set to 1.5, which is in the range of the estimates of De Walque et al. (2005) in a two-country model of the euro area and the US. The degree of wage rigidities is also taken from De Walque et al. (2005); \( \lambda \) and \( \lambda^* \) are set equal to 0.8, which is consistent with their estimates and implies a duration of wages of 5 quarters in both countries (this is also in line with other micro studies). Finally the elasticity of substitution across Home and Foreign goods is set at a unitary value consistent with what often assumed in previous studies, \( \theta = 1 \). In the next section, we are going to perform robustness analysis along different assumptions on \( \theta \) and \( \alpha \). Finally, the borrowing cost creates, in a log-linear approximation, a spread between the interest rate faced by the borrowers in the Home country and the risk-free rate. This spread depends on the distance between the level of borrowing and what is considered the risk-free threshold of debt

\[ i_t^0 - i_t \equiv e_t \left( d_t - k_t \right). \]

In particular, as shown in the online appendix, \( i_t^0 \), the effective borrowing rate, is the relevant nominal interest rate entering the log-linear approximation of the Euler equation and the external budget constraint of country \( H \). The parameter \( e_t \) is set at 0.047 in such a way that on impact the drop in \( k \), considering a constant debt \( d \), produces a 4.5% spread at annual rates which is consistent with the peak of the TED spread observed during the US financial crisis.

Figs. 2 and 3 show the optimal adjustment following the deleveraging shock under the benchmark calibration compared with a policy in which both countries aim at targeting GDP inflation at 2%, \( \pi_{H1} = \pi_{F2} = 2\% \). These inflation-targeting policies are of particular interest since they have been often found to be the welfare-maximizing policies under cooperation in open-economy models. Indeed, they are the optimal cooperative policies in our model in the absence of deleveraging shocks and under an efficient distribution of wealth across countries.23 Moreover, the adoption of such inflation-targeting policies by many developed countries before the crisis makes them an interesting benchmark of comparison to discuss how optimal policy changes when there is a deleveraging shock. As previously discussed, it should be noted that the latter couple of policies is not feasible since at the beginning of deleveraging they would imply a nominal interest rate for country \( H \) below the zero-lower bound. Considering such a constraint, the GDP inflation rate in country \( H \) needs to fall under the target, as shown in Fig. 3, while the economy stays in the “liquidity trap” for eight quarters. On the contrary, the zero-lower bound is not a constraint for country \( F \).

As shown in Fig. 2, inflation-targeting policies are quite costly for both economies in particular in terms of a contraction in output for several quarters. Moreover, the consumption recession in country \( H \) is particularly deep and counteracted only in part by an expansion in the consumption of Foreign households. The short-run depreciation of the deleverager’s nominal exchange rate is sizeable and around 9%. External debt reaches slowly the efficient level after more than fifteen quarters.

As shown in the same figure, optimal policy improves substantially with respect to inflation-targeting policies. First, it should be noted that country \( F \)’s external debt converges to the new steady state level of 30% of GDP earlier but still after four years. Even under optimal policy, the contraction of the deleverager’s consumption is inevitable, although mitigated. There is now a larger increase in the other country’s consumption. Most important, the output recession is milder in both countries and also shorter in country \( H \). The better adjustment is achieved with less variations of the nominal exchange rate and the terms of trade. The improvements in the real economies are explained by the different monetary policies followed under optimal policy. Interest rate in the deleveraging country should be at the zero-lower bound for a longer horizon, up to three years.24 In country \( F \), the interest rate is also low but remains above the zero-lower bound. The fact that the real interest rates substantially fall in countries \( H \) and \( F \) mitigates the costs of deleveraging. The GDP inflation rates should fluctuate around their target: an increase in foreign inflation is needed at the beginning of deleveraging while inflation in country \( H \) initially falls below target, and then rises afterward. Interestingly, sub-optimal inflation-targeting policies even undershoot their inflation targets, in country \( H \), because of the zero-lower bound constraint. This implies a global disinflation measured by world inflation, \( \pi^w \). Instead, under optimal policy, world inflation stays above the 2% target, in particular at the beginning of the deleveraging episode.

The figures show three channels though which the benevolent planner can cope with a deleveraging shock in country \( H \). First, it can lower the real interest rate in the deleverager to reduce its borrowing costs, mainly through policies in country \( H \) of low or zero nominal interest rate and inflation above target. Second, it can mitigate the consumption and output recession in country \( H \) by expanding consumption in country \( F \) through a lowering of the real rates in country \( F \), again using policies of low or zero nominal interest rate and inflation above target in country \( F \). Third, it can mitigate the output recession in country \( H \) through a worsening of the Home terms of trade and a depreciation of its nominal exchange rate.25

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21 Even in this context debt forgiveness or appropriately-defined transfers can achieve the efficient allocation provided that each monetary policy follows its inflation target.

22 See the online appendix for the details.

23 This is true in models in which there is producer–currency pricing as in the framework of this paper (see Benigno and Benigno, 2006). Engel (2011) shows that with local-currency pricing it is optimal to stabilize CPI inflation rates.

24 The results that under optimal policy the stay at the zero-lower bound is longer than under inflation targeting are in line with the findings of Eggertsson and Woodford (2003) in a simple closed-economy model.
exchange rate to switch expenditure from foreign to domestic goods. However, all the identified channels imply costs in terms of the loss function (Eq.(26)) which should be appropriately weighted.

We now turn to investigate how alternative international transmission mechanisms change the way in which the benevolent planner uses the three channels identified above.

Fig. 2. Impulse responses following the deleveraging shock under optimal policy in comparison with the policy in which both countries follow inflation-targeting policies $\pi_H = 2\%$ and $\pi_F^* = 2\%$, but can be constrained by the zero-lower bound. Variables are: Home and Foreign consumptions ($C, C^*$), Home and Foreign outputs ($Y_H, Y_F^*$), terms of trade ($T$), and the level of external debt of country $H$ as a percentage of its GDP ($d_{gdp}$). All variables, except for $d_{gdp}$, are in percentage deviations from the steady state.

Fig. 3. Impulse responses following the deleveraging shock under optimal policy in comparison with the policy in which both countries follow inflation-targeting policies $\pi_H = 2\%$ and $\pi_F^* = 2\%$, but can be constrained by the zero-lower bound. Variables are: Home and Foreign nominal interest rates ($i, i^*$), Home and Foreign producer inflation rates ($\pi_H, \pi_F^*$), world inflation ($\pi^w$), defined as $\pi^w = 1/2 \cdot \pi + 1/2 \cdot \pi^*$ where $\pi$ and $\pi^*$ are the Home and Foreign CPI inflation rates, and the level of the nominal exchange rate ($S$); inflation and interest rates are in percent and annual rates.
5. Alternative international transmission mechanisms

How do the results of Figs. 2 and 3 change under alternative international transmission mechanisms? We address this question through different assumptions on the elasticity of substitution between Home and Foreign goods, θ, and the degree of home bias in goods consumption, captured by α.

The parameter θ measures the elasticity of substitution of consumption between Home and Foreign goods with respect to variations in their relative price. When θ is high, it suffices a small depreciation of the Home currency to create significant expenditure-switching effects from country F’s goods to those of country H. The classical expenditure-switching channel is clearly stronger when the elasticity θ is larger. However, this is not all that matters for the international transmission mechanism. It should be noted, indeed, that real income depends critically on the position of θ with respect to the unitary value. The real income of country H in units of its own consumption can be written as

$$\frac{P_{HY_{HT}}}{P_{t}} = (\frac{P_{HY}}{P_{t}})^{1-\theta} \left[ \alpha_{1} + (1-\alpha)Q_{t}^{H}C_{t} \right]$$

from which it follows that values of θ below the unitary value might imply that a worsening of the Home terms of trade or a depreciation of the Home currency, i.e. a fall in $P_{HY}/P_{t}$, can have adverse effects on the Home country’s real income making it harder to deleverage. This reminds phenomena of “immsimeterizing” growth in which a depreciation of the currency can increase production of a country but at the same time make it poorer with a reduction in its real income.

Figs. 4 and 5 show how optimal policy changes for alternative values of θ around the benchmark value of one, namely we plot impulse responses under optimal policy for different values of θ in the nominal interest rate. The stay at the zero-lower bound is discouraged by a deleveraging shock under home bias mechanically produces an initial worsening of the Home-country terms of trade and of its nominal exchange rate. However, as shown in the objective function (Eq. (26)), these movements are inefficient and the optimal policy should aim to reduce them by weighing them appropriately with the trade-offs implicit in the loss function. Indeed, when θ = 6, the expenditure-switching channel is strong enough that a smaller depreciation of the currency is sufficient to substantially shift production from foreign to home goods. On the contrary, when θ = 0.5, this channel is weaker and moreover real income of the deleverager’s country is adversely hit. Even in this case, the optimal policy requires a smaller short-run depreciation of country H’s currency and now a substantial appreciation in the long run.

Interestingly, Fig. 5 shows that the stay at the zero-lower bound depends on the alternative assumptions on θ. For low values of θ, the stay at the zero-lower bound for country H is exactly five years. Even country F is now forced to stay at the zero-lower bound and for a long period of two years. This is because the contraction in country H’s consumption is larger and requires more expansion in country F’s consumption, which can be stimulated by a larger fall in its real interest rate and therefore in the nominal interest rate. The stay at the zero-lower bound is completely avoided for higher values of θ where the nominal interest rate of country F stays above the constraint.

Speaking in terms of the three channels identified above, the expenditure-switching channel cannot be used when θ is low. Therefore, the benevolent planner has to rely more on the real interest-rate channels by lowering more the policy rates. A global liquidity trap may be optimal in this case. On the contrary when θ is high, the expenditure-switching channel is more effective, hence there is less need to lower the real interest rates, in

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**Fig. 4.** Impulse responses under optimal policy for different values of θ = 0.5, 1 and 6. Variables are: Home and Foreign consumptions ($C, C^*$), Home and Foreign outputs ($Y_H, Y_F$), terms of trade ($T$), and the level of external debt of country H as a percentage of its GDP ($d_{gdg}$). All variables, except for $d_{gdg}$, are in percentage deviations from the steady state.
particular in the foreign country. The liquidity trap is shorter for the deleverager and absent in country F.

The other dimension through which the international transmission mechanism of the shock could be different is along alternative assumptions on the degree of home bias, captured by the parameter $\alpha$. We have already discussed, in the simple model of Section 2, that the responses of terms of trade and nominal exchange rate depend on this assumption. The higher is the home bias, the more the terms of trade of the deleveraging country worsens implying a nominal exchange rate depreciation. However, according to Eq. (26), all these movements are costly and optimal monetary policy should be directed to mitigate them. Figs. 6 and 7, fixing now $\theta$ at the benchmark unitary value, show how optimal policy changes for alternative values of $\alpha$ around the benchmark of 0.76, plotting also the impulse

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**Fig. 5.** Impulse responses under optimal policy for different values of $\theta = 0.5, 1$ and 6. Variables are: Home and Foreign nominal interest rates ($i, i^*$), Home and Foreign producer inflation rates ($\pi_H, \pi_F^*$), world inflation ($\pi^w$), defined as $\pi^w = \frac{1}{2} \cdot \pi + \frac{1}{2} \cdot \pi^*$ where $\pi$ and $\pi^*$ are the Home and Foreign CPI inflation rates, and the level of the nominal exchange rate ($S$); inflation and interest rates are in percent and annual rates.

**Fig. 6.** Impulse responses under optimal policy for different values of $\alpha = 0.5, 0.76$ and 0.95. Variables are: Home and Foreign consumptions ($C, C^*$), Home and Foreign outputs ($Y_H, Y_F^*$), terms of trade ($T$), and the level of external debt of country $H$ as a percentage of its GDP ($d_{gdp}$). All variables, except for $d_{gdp}$, are in percentage deviations from the steady state.
responses when there is no home bias, i.e. $\alpha = 0.5$, and for a high degree of home bias, $\alpha = 0.95$. As Fig. 7 shows, the nominal exchange rate depreciates only slightly in the absence of home bias. However, there is still an important international transmission of the shock since Home and Foreign real interest rates are more interconnected in this case.\(^{25}\)

The fall in Foreign real rates stimulates consumption in country $F$ to compensate for the fall in country $H$. Not surprisingly, given the fall in both real rates, foreign country is now forced to stay at the zero-lower bound and for a long time. When instead the home-bias parameter is large, the two economies behave like closed economies. Indeed, while the nominal exchange rate moves a lot without much ability to switch expenditure across goods, consumption and output in the Foreign economy are only marginally affected by the deleveraging of country $H$. In this case, the nominal interest rate in country $F$ does not go to the zero-lower bound while the liquidity trap is longer in country $H$. With a high degree of home bias, the large depreciation of the exchange rate is helpless and the entire burden of adjustment is borne by the deleverager.

In terms of the three channel identified above, the only one available to cope with the shock, when $\alpha$ is high, is the fall in the real rate in the deleverager. Indeed, in this case, the economies are closed. For intermediate values of $\alpha$, economies become more open and therefore the other two channels become more relevant. When $\alpha$ is close to 0.5, the ex-16

The Home-country exchange rate depreciates on impact the more, the higher the degree of home bias and the closer to the unitary value the elasticity of substitution is. Higher values of $\theta$ or values below one imply a smaller depreciation and even a short-run appreciation in the absence of home bias. The length of the stay at the zero-lower bound becomes shorter as the elasticity of substitution $\theta$ increases. A low value of $\alpha$ reduces the stay for the deleveraging country at the expenses of a longer stay for the other economy. Finally, the bottom chart of Fig. 8 shows the costs of deleveraging in terms of a permanent reduction, in percentage, in the steady-state consumption levels of both countries. The costs are particularly-sizeable when the international transmission mechanism is weaker. In our model, this weakness depends on two factors: 1) a low elasticity of substitution, since it implies a weak expenditure-switching channel and may cause phenomena of “immiserizing” growth and 2) a high degree of home bias, since economies are closed and the shock can only be absorbed in the Home country.

6. Deleveraging and the original sin

In this section, we study how the transmission mechanism of international debt deleveraging and its efficient adjustment change when the external debt is denominated in foreign currency. Indeed, the analysis of previous sections might be appropriate for countries like the US which has the exorbitant privilege of being able to borrow in its own currency, but not for emerging-market economies which are usually affected by the original sin of borrowing in foreign currency. We make few changes to our model to accommodate this case. In particular the flow budget constraint of households in country $H$ is now written as:

$$P_t \delta_t = \int_0^1 W_t(j) L_t(j) dj + \Pi_t + \frac{S_{Dt}}{1+\delta_t} - S_{Dt-1} - f_tF_t \cdot \frac{S_{D_t}}{P_t} \cdot \frac{1}{S_{F_t}} \cdot \frac{1}{F_t}$$

where indeed the currency denomination of debt is that of country $F$ and the interest-rate paid on debt is the foreign interest rate $1 + i^*$. We have also changed the arguments of the cost function to reflect the new denomination of debt where now $f_t$ represents the risk-free

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\(^{25}\) When $\alpha = 0.5$, the Home and Foreign real interest rates associated with the consumption baskets $C$ and $C^*$ are equal across countries, since $C = C^*$.

\(^{26}\) As discussed in Benigno (2009), in a similar class of models, there is no solution for low values of $\theta$.\(^{26}\)
level of external debt that can be held without costs. Given this budget constraint, the Euler equations change appropriately. Details are left to the online appendix.

Figs. 9 and 10 compare the results of optimal policy when debt of the deleveraging country is denominated in foreign currency with the benchmark case of domestically-denominated debt of Section 4.

Results are in some way surprising. The striking difference is in the response of the policy rates. Under the benchmark case of debt denominated in domestic currency, the liquidity trap is mainly affecting the deleverager’s nominal interest rate. On the opposite, when debt is denominated in foreign currency, it is the foreign interest rate that should be forced to the zero-lower bound and for long time, almost
three years as shown in Fig. 10. This is intuitive since the borrowing cost for the deleverager depends now on foreign interest rates. To ease the costs of deleveraging, the benevolent planner tries to lower at most the foreign interest rate. Indeed, the domestic nominal interest rate is left to rise. The fact that now the foreign interest rate stays longer at the zero-lower bound implies also that the real rate in country $F$ is lower for a long period which pushes up consumption in country $F$ to a larger extent. On impact it rises by 7.5% as opposed to the 4% of the benchmark case. As a consequence, output expands more in country $F$.

It is also surprising to see that the exchange rate depreciates much more when debt is denominated in foreign currency rather than under the benchmark case. Indeed, a depreciation of the nominal exchange rate is even more costly in this case since it under the benchmark case. Indeed, a depreciation of the nominal exchange rate is even more costly in this case since it "inflates" the real resources needed by country $H$ to pay back debt. The overall external debt to GDP initially rises and then falls at a slower pace toward the new steady-state value. However, these costs are outweighed by the benefits. The central planner, by worsening the Home-country terms of trade, can tilt production from the Foreign economy to the Home country. This is needed to mute the expansion in the Foreign economy, caused by a too low real rate. Otherwise, the overheating in country $F$ would be even larger bringing more inefficiencies.

7. Conclusion

We have examined the international implications of debt deleveraging in one country within the world economy and studied how monetary policy should be set at the global level, focusing in particular on the reaction of the nominal exchange rate and policy rates. Deleveraging reduces aggregate demand and may lead to recession, as economic agents save to repay the debt. There are interesting international spillovers through trade and the exchange rate. The adjustment to a deleveraging shock naturally requires movements in two relative prices: namely the exchange rate and the real interest rate. The exchange rate, which is an international relative price, moves in such a way as to rebalance resources across countries. The deleveraging country’s currency will depreciate in the short run and appreciate in the long-run. This depends critically on home bias in consumers’ preferences. Since in the short run agents who are paying down their debt have less resources for consumption, the price of home goods should fall relative to the foreign, and a fall in the exchange rate will assist this adjustment. Once the debt has been repaid, however, agents have more resources to spend and in particular on domestic goods. These exchange rate movements produce expenditure-switching effects which favor production in the deleveraging country at the expenses of the rest of the world. The other important relative price in the adjustment, the real interest rate, will come down in both countries and fall more sharply in the deleveraging country. This fall in the real rates stimulates foreign consumption in order to mitigate the overall impact of the shock on the deleverager and the world economy.

The interesting and surprising result of this paper is that all these large variations in relative prices and quantities are inefficient, when seen from the perspective of a benevolent planner, who cares about world utility and sees the new level of external debt reached after deleveraging as the efficient one. This planner dislikes all the adjustment process described above and would like to immediately achieve the new equilibrium allocation characterized by lower debt. Therefore, important trade-offs emerge between stabilizing consumption, output and relative prices. The desirability of the expenditure-switching channel versus the Home and Foreign real-interest-rate channels depends on the elasticity of substitution in consumption between domestic and foreign goods and on the degree of home bias. Only for elasticities of substitution around the unitary value, the nominal exchange rate of the deleverager is left to depreciate in a sizeable way in the short run. Otherwise, it should move less or even appreciate when the elasticity of substitution is very low. For low degrees of home bias, the real interest rate should fall in a substantial way in the foreign economy and the burden of adjustment is more shared across countries. High degrees of home bias imply that all the burdens are on the deleveraging country because economies are more closed. It is true that the nominal exchange rate and terms of trade vary substantially in these cases, but they are less effective in generating spillovers to the rest of the world.
In this study, we have concentrated on the role of monetary policy and alternative exchange-rate regimes in mitigating or amplifying the costs of debt deleveraging. The zero lower bound on nominal interest rates is a significant constraint in our analysis, because the natural rate of interest falls substantially. We have shown that whether zero-lower bound policies should be coordinated or not depends also on the international transmission mechanism. When the elasticity of substitution between foreign and domestic goods and/or the degree of home bias are low, a global liquidity trap should emerge as an optimal policy of a benevolent planner.

We have analyzed a very simple two-country open-economy model. The consequent limitations are essentially the price paid for the simplifications used. First, in the real world debt deleveraging affects a variety of agents in the economy: households, banks, firms and governments. Distinguishing them in the model would enhance realism and possibly enable us to differentiate the effects of deleveraging on the economy according to which agents are paying down their debt. It is likely that, however, the qualitative results implied by our simple framework would hold also in a more complex context. Second, the asset market structure has been kept very simple — only one asset traded internationally. This is a significant limitation, since the portfolio position of a country is much more complex and diversified involving assets and liabilities, in different currencies and instruments ranging from equity to debt. Finally, we have focused on the response of a benevolent policymaker maximizing welfare of the global economy and abstracted from a possible lack of international monetary policy coordination which might change in a substantial way the equilibrium allocation. In particular, among the three channels identified in this paper to cope with the deleveraging shock, the fall in the real rate of the non-deleveraging country and the depreciation of the nominal exchange rate require some cooperation at the international level. Non-coordinated policies might result in sub-optimal equilibria with higher costs for the world economy. This is an interesting area of analysis which we leave to future research.²⁷

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jinteco.2014.03.001.

References


²⁷ Fujiwara et al. (2011) discuss cooperative versus non-cooperative solutions in the emergence of global liquidity traps.