Price Dynamics with Customer Markets

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Abstract

We study a model of firm price setting with customer markets and empirically evaluate its predictions. Our framework captures the dynamics of customers in response to a change in the price set by firms, describes the behavior of optimal prices in the presence of customer retention concerns, and delivers a general equilibrium model of price and customer dynamics. We exploit micro data on purchases from a large U.S. retailer by a panel of households to quantify the model and compare it to the counterfactual benchmark of the monopolistic competition setting. We show that our model with customer markets has markedly different implications in terms of the equilibrium price distribution, and better fits the available empirical evidence on retail prices. Moreover, the dynamic of the response of demand to policy relevant shocks is also distinctive. Our results suggest that inertia in customer reallocation across firms increases the persistence in the response of firms’ demand to these shocks.

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1 Introduction

In this paper we develop and quantify a model of firm price-setting in the presence of customer markets. The customer base of a firm, that is, the set of customers buying from it at a given point in time, is an important determinant of firm performance and survival. Its effects are long lasting, as customer-supplier relationships are subject to a certain degree of stickiness (Hall (2008)). Existing theories suggest that the price is an important instrument to attract and retain customers (Phelps and Winter (1970)). Therefore firms actively seek to maintain and grow their customer base, and this effort impacts their pricing.\footnote{Survey evidence of this can be found in Blinder et al. (1998) for U.S firms, and in Fabiani et al. (2007) for firms in the Euro area.}

We build a micro founded model of firm pricing with customer markets, which we discipline exploiting data on consumers’ and firms’ behavior. We use the estimated model to confront its predictions with data about the price distribution and customer turnover, and to perform policy experiments. As product market frictions are increasingly used to explain a wide array of phenomena, from business cycle dynamics (Kaplan and Menzio (2013)) to the movements in international relative prices (Drozd and Nosal (2012)) and firm performance (Gourio and Rudanko (2014)), advancing our understanding of the micro foundations of pricing with customer markets is of paramount importance.

We study a model where firms set prices responding to idiosyncratic productivity shocks and taking into account the effects of a price change on the dynamics of their customer base. Customers respond to price changes but face search frictions that reduce their ability to reallocate across suppliers of the same good. In particular, customers start each period matched to the same firm from which they bought in the previous period. Once firms have drawn the new productivity level and posted a price for the period, customers can search for a new supplier. To search, customers must pay an idiosyncratic search cost, drawn every period from an i.i.d. distribution. After the search cost is paid, the customer gets to observe the state of another randomly selected firm; she compares it to that of her old supplier, and decides where to buy (extensive margin of demand). After search and matching decisions have been made, each customer decides her purchased quantity of the good (intensive margin of demand).

While being tractable, the model provides a rich laboratory to study how the relationship between customer and price dynamics is shaped, in equilibrium, by idiosyncratic production and search costs. In our framework, firms face a trade-off between charging a higher price and extracting more surplus from customers, versus posting a lower price to extract a lower surplus but from a larger mass of customers. Since customers are hard to win back, losses in the customer base persistently reduce firms’ demand. Similarly, since customers face
frictions when searching for a new firm, gains in the customer base persistently increase firms’
demand. Therefore, unlike the standard monopolistic competition setting, the customer base
is treated as an asset by the firm, making the firm’s price-setting problem a dynamic one. The
equilibrium of our model features both price dispersion and customer dynamics. This is an
important property given our goal to relate the model with data, and since both the dispersion
of prices and the reallocation of customers across competing firms are acknowledged empirical
facts.

We complement our modeling effort with an empirical analysis that relies on novel micro
data documenting pricing and customer base evolution for a large retail firm. We take
advantage of scanner data from a major U.S. retailer recording purchases for a large sample of
households between 2004 and 2006. Household-level scanner data are particularly well suited
to study customer base dynamics. First, we observe a wealth of details on all the shopping
trips each household makes to the chain (list of goods purchased, prices, quantities, etc.).
More importantly, we can infer when customers leave the retailer by looking at prolonged
spells without purchases at the chain. These data allow us to study the relation between a
customer’s decision to abandon the firm and the price of the good—or rather, in this case,
bundle of goods—she consumes there. We show that customer dynamics are indeed affected
by variation in the price: a 1% change in the price of the customer’s typical basket of grocery
goods would raise the firm yearly customer turnover from 14% to 21%.

We use the estimated price elasticity of the customer base, jointly with moments from
the process generating the prices posted by the chain, to identify the key objects of the
model: the distribution of search costs and the properties of the productivity process. We
assess the relevance of customer markets for price dynamics by comparing our model to
a counterfactual economy where only an intensive margin of demand is present. This is an
interesting benchmark, as the pricing problem of the firm in such economy is similar to the one
captured in standard macro models where competition comes only from the downward-sloping
demand of each customer, and the customer base is constant. We design the experiment so
that the counterfactual economy is observationally equivalent to the economy with customer
markets with respect to average demand elasticity, price persistence, and dispersion.

The most obvious difference between our model and those that do not feature an extensive
margin of demand is that the former predicts equilibrium customer dynamics whereas the
latter imply no customer reallocation. Quantitatively, our model delivers a yearly average
turnover of 6%, nearly half of what we measure in the data (14%). This implies that customer
dynamics triggered by variation in prices due to idiosyncratic cost shocks can explain a large
fraction of the overall turnover.

Customer markets also have significant impact on firms’ pricing. We find that customer
markets introduce an element of strategic complementarity in price setting, leading firms to set prices closer to each other. This generates a high mass of prices clustered around the mean that results in a price distribution with higher kurtosis and smaller dispersion than the distribution of production cost. We find this result interesting for two reasons. First, a large fraction of prices bunched around the mean characterizes the distribution of prices in our data and is consistent with the findings of a recent literature documenting the pricing of homogeneous goods (Kaplan and Menzio (forthcoming)). Second, through the lens of our model, the discrepancy between the estimated price and cost distributions is associated to variable markups and incomplete pass-through. The model without customer markets instead does not capture the distinctive characteristics of our pricing data as it delivers a price distribution that mimics closely that of the underlying production cost.

Finally, we use our model to explore the effects of customer markets for the response of demand and prices to shocks that cause aggregate dynamics. In particular, our model represents an interesting setting to study the response of demand to shocks that, by hitting firms asymmetrically, affect price dispersion. This, in fact, influences the incentives of customers to search, generating persistent dynamics in demand, as it takes time for customers to reallocate across firms.

In our model demand responds to variation in prices through two channels. The first one is the intensive margin: customers can adjust the quantity of the good they purchase from their supplier. This channel is static and its adjustment happens immediately. The second channel is the extensive margin: customers can decide to leave the firm. This channel is dynamic because of the presence of a search friction. Therefore, the adjustment is delayed, as it takes time for customers to relocate to firms with lower prices, and persistent because once customers have moved, it is costly for them to change firm again. The shape of the response of demand depends on the strength of these two components. We find that the extensive margin dominates: the long-run response of demand to a cost shock is substantially larger than the short-run one. In the model without customer markets, where only the intensive margin is at work as in standard models of monopolistic competition, the response of demand on impact of the same shock is much larger and decays monotonically as the effect of the shock on prices vanishes.

Related Literature. Our paper relates to the seminal work by Phelps and Winter (1970) who study the pricing problem of a firm facing customer retention concerns. In their paper,
the response of the firm’s customer base to a change in the firm’s price is modeled with an ad hoc function. We instead endogenize customer dynamics which arise as the outcome of customers’ optimal search decisions in response to firms’ pricing. Fishman and Rob (2003), Alessandria (2004), and Menzio (2007) also study the firm price-setting problem in models where search costs prevent customers from freely moving to the lowest price supplier. Fishman and Rob (2003) study the implications of customer markets for firm dynamics. Alessandria (2004) shows that such a model can generate large and persistent deviations from the law of one price, consistent with the empirical evidence on international prices. Menzio (2007) looks at the role of asymmetric information and commitment in the optimal pricing decision of the firm. Differently from our paper, in these papers customers face an homogeneous search cost and, as a result, optimal pricing is such that no endogenous customer dynamics occur in equilibrium.

Unlike the literature cited above, we exploit micro data to discipline our model and provide a quantitative assessment of the relevance of customer markets for pricing. This relates our findings to contributions that aim at documenting empirical stylized facts. Our evidence on the shape of the price distribution ties in to the recent empirical work by Kaplan and Menzio (forthcoming). While their focus is on customers and the price they pay for the same good (or bundle of goods), we are interested in the point of view of sellers and the price they charge. Our finding on the dynamics of demand is consistent with studies documenting the short-run and long-run elasticity of demand. In particular, it fits the evidence on the short-vs. long-run Armington elasticity in the international economics literature (Ruhl (2008)).

Another set of related contributions uses customer markets to address questions different from the ones we study here. Gourio and Rudanko (2014) explore the relationship between the firm’s effort to capture customers and its performance. They show that customer markets have nontrivial implications for the relationship between investment and Tobin’s q. Drozd and Nosal (2012) introduce in a standard international real business cycle model the notion that, when producers want to increase sales, they must exert effort to find new customers. This extension help to rationalize a number of empirical findings on the dynamics of international prices and trade. Dinlersoz and Yorukoglu (2012) focus on the importance of customer markets for industry dynamics in a model where firms use advertising to disseminate information to uninformed customers. Shi (2011) studies a model where firms cannot price discriminate across customers and use sales to attract new customers. Kleshchelski and Vincent (2009) examine the impact of customer markets on the pass-through of idiosyncratic cost shocks to prices in a symmetric equilibrium that does not allow us to study the relationship between customer dynamics and the price distribution. Burdett and Coles (1997) study the role of firm size for pricing when firms use the price to attract new customers. Their work
complements ours: price and customer dynamics in their setting are shaped by the heterogeneity in firm size (age). For us, the driving force is the heterogeneity in productivity. The industrial organization literature has also studied the implications of customer markets for a variety of subjects. For instance, Foster et al. (2013) stress their role in affecting firm survival and Einav and Somaini (2013) and Cabral (2014) focus on their effect on the competitive environment.

Finally, a stream of studies analyzes the implications of product market frictions for business cycle fluctuations. In Petrosky-Nadeau and Wasmer (2011), Bai et al. (2012) and Kaplan and Menzio (2013), aggregate shocks alter the opportunity cost of searching, influencing markup and demand dynamics over the business cycle. Researchers have also explored aggregate dynamics in models where habit preferences lead to persistence in demand (Ravn et al. (2006)). While we abstain from this type of analysis, our quantified model could be extended to allow for the search opportunity-cost to vary with the aggregate state and be used as a tool to answer similar questions.

The rest of the paper is organized as follows. In Section 2 we lay out the model, and in Section 3 we characterize the equilibrium. Section 4 presents the data and descriptive evidence of the relationship between customer dynamics and prices. In Section 5 we discuss identification and estimation of the model. In Section 6 we present some quantitative predictions of the model, contrast them with the outcomes from a model without customer markets, and compare them empirical evidence from our data. In Section 7 we introduce an application of the model, which we use to study the implications of customer markets for the dynamics of demand. Section 8 concludes.

2 The model

The economy is populated by a measure one of firms producing an homogeneous good and a measure $\Gamma$ of customers who consume it.

Customers. We use the index $i$ to denote customers. Let $d(p)$ and $v(p)$ denote, respectively, the static demand and customer surplus as a function of the price $p$ of the good. We assume that: (i) $d(p)$ is continuously differentiable with $d'(p) < 0$, and bounded below with $\lim_{p \to \infty} d(p) = 0$; and (ii) $v(p)$ is continuously differentiable with $v'(p) < 0$, and bounded above with $\lim_{p \to 0^+} v(p) < \infty$. These properties are satisfied in standard models of consumer demand.
Firms. Firms produce a homogenous good and are indexed by $j$. The only choice firms make is to set the price $p$ of the good they produce each period. We assume a linear production technology $y^j = z^j \ell^j$ where $\ell$ is the production input, and $z^j$ is the firm-specific productivity. Idiosyncratic productivity is distributed according to a conditional cumulative distribution function $F(z'|z)$ with bounded support $[\underline{z}, \bar{z}]$. We also assume that $F(z'|z_h)$ first order stochastically dominates $F(z'|z_l)$ for any $z_h > z_l$ to induce persistence in firm productivity. Heterogeneity in firm productivity will be the driver of price dispersion in the type of equilibrium we will focus on.\(^3\) The profit per customer accrued to the firms are $\pi(p,z) \equiv d(p)(p - w/z)$, where the constant $w > 0$ denotes the marginal cost of the input $\ell$. We assume that profits per customer are single-peaked in $p$. Finally, we denote by $m_{t-1}^j$ the customer base of firm $j$, consisting of the mass of customers who bought from firm $j$ in period $t-1$. As we will show later, the state of the firm $j$ in period $t$ is the pair $\{z^j_t, m_{t-1}^j\}$.

Search, matching, and exit from the customer base. Each customer starts period $t$ matched to the firm she bought from in period $t-1$. The customer observes perfectly the state of the firm she is matched to (i.e. $z^j_t$ and $m_{t-1}^j$), which allows her to assess the probability distribution of the path of prices of that firm. Later we will be more specific about the mapping from the state of the firm to the path of prices.

After observing the state of her current match, the customer decides if she is incurring a search cost to draw another firm. In particular, each customer $i$ is characterized by an idiosyncratic random search cost $\psi^i \geq 0$ measured in units of customer surplus, which is drawn each period from the same distribution with density $g(\psi)$, and associated cumulative distribution function denoted by $G(\psi)$. For tractability, we restrict our attention to density functions that are continuous on all the support.

Heterogeneity, albeit transitory, in search costs allows us to study firms’ pricing decisions that are not necessarily knife-edge in the trade-off between maximizing demand and markups. The customer can search at most once per period. Search is random, with the probability of drawing a particular firm $j'$ being proportional to its customer base, i.e. $m_{t-1}^{j'}/\Gamma$. This assumption captures the idea that consumers search new suppliers not by randomly sampling firms but by randomly sampling other consumers and following their behavior.\(^4\) On the technical side, this assumption implies that firms will gain customers proportionally to their customer base. This simplifies the characterization of the firm problem and implies that firm growth is independent of firm size consistent with Gibrat’s law (Steindl (1965)).

Conditional on searching, the customer takes then another decision concerning whether

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\(^3\)For tractability we will abstract from (possible) equilibria where symmetric firms charge different prices as in Burdett and Judd (1983).

\(^4\)This behavior is known as preferential attachment in the extensive literature on network formation.
to exit the customer base of her initial firm and match to the new firm. In particular, the
customer compares the distribution of the path of current and future prices at the two firms
and buys from the firm offering higher expected value. Finally, we assume for simplicity no
recall in the sense that the customer cannot go back to a firm she was matched with in the
past or whose price offer she rejected unless she randomly draws it again when searching.

**Timing of events.** The timing of events is as follows: (i) productivity shocks are realized
for all firms and each firm \( j \) posts a price \( p^j_t \); (ii) each customer draws her search cost \( \psi^i_t \) and
observes the price \( p^j_t \) as well as the relevant state of the firm she is matched with (\( z^j_t \) and
\( m^j_{t-1} \)); (iii) each customer decides whether to search for a new firm or remain matched to
her current one; (iv) if the customer decides to search, she pays the search cost and draws
the (potential) new supplier \( j' \) with probability \( m^j_{t-1}/\Gamma \). The customer perfectly observes
the state of the prospective match and decides whether to exit the customer base of the current
supplier to join that of the new match; if not, she stays with the current match. Finally, (v)
customer surplus, \( v(p^j_t) \), and firm profits, \( m^j_t \pi(p^j_t, z^j_t) \), realize.

**Equilibrium.** A firm and its customers play an anonymous sequential game. We look
for a stationary Markov Perfect equilibrium where strategies are a function of the current
state. There are no aggregate shocks. Although the relevant state for the pricing decision
of the firm could include both the stock of customers and the idiosyncratic productivity, we
conjecture and later show the existence of an equilibrium where optimal prices only depend
on productivity, and we denote by \( P(z) \) the equilibrium pricing strategy of the firm.

The relevant state for the search decision of a customer includes the expectations about
the path of current and future prices of the firm she is matched to, as well as the idiosyn-
cratic search cost. Given the Markovian equilibrium we study, the current realization of
idiosyncratic productivity is a sufficient statistic for the distribution of future prices. As a
result, the search strategy of the customer depends on the current price and productivity
of the firm she is matched to, and on her own search cost. We denote the search decision
as \( s(p, z, \psi) \in \{0, 1\} \), where \( s = 1 \) means that the customer decides to engage in search.
Conditional on searching, the exit decision depends on the continuation value associated to
the firm the customer starts matched to (the outside option), which is fully characterized by
posted price and productivity, as well as on productivity of the firm she has drawn upon the
search, \( z_{\text{new}} \), which determines the continuation value associated to the new firm. We denote
the exit decision as \( e(p, z, z_{\text{new}}) \in \{0, 1\} \), where \( e = 1 \) means that the customer decides to
exit the customer base of her original firm.
2.1 The problem of the customer

Let $V(p^j_t, z^j_t, \psi^i_t)$ denote the value function of a customer $i$ who has drawn a search cost $\psi^i_t$ and is matched to firm $j$, which has current productivity $z^j_t$ and posted price $p^j_t$. This value function solves the following problem,

$$V(p^j_t, z^j_t, \psi^i_t) = \max \left\{ \bar{V}(p^j_t, z^j_t), \tilde{V}(p^j_t, z^j_t) - \psi^i_t \right\},$$

where $\bar{V}(p, z)$ is the customer’s value if she does not search, and $\tilde{V}(p, z) - \psi$ is the value if she does search. The value in the case of not searching is

$$\bar{V}(p^j_t, z^j_t) = v(p^j_t) + \beta \int_{\bar{z}}^{\infty} \int_{\bar{z}}^{z} V(P(z'), z', \psi') dF(z'|z^j_t) dG(\psi').$$

We notice that the state of the firm problem depends on the productivity $z$ because the pricing function $P(\cdot)$ mapping future productivity into prices in the Markov equilibrium makes productivity $z$ a perfect statistic for the distribution of future prices at the firm. The value when searching is given by

$$\tilde{V}(p^j_t, z^j_t) = \int_{-\infty}^{+\infty} \max \left\{ \bar{V}(p^j_t, z^j_t), x \right\} dH(x),$$

where the customer takes expectations over all possible draws of potential new firms, and where $H(\cdot)$ is the equilibrium cumulative distribution of continuation values from which the firm draws a new potential match when searching. For instance, $H(\bar{V}(p^j_t, z^j_t))$ is the probability of drawing a potential match offering a continuation value smaller than or equal to the current match. The following lemma describes the customer’s optimal search and exit policy rules.

**Lemma 1** The customer matched to a firm with productivity $z^j_t$ charging price $p^j_t$: i) searches if she draws a search cost $\psi_t \leq \hat{\psi}(p^j_t, z^j_t)$, where $\hat{\psi}(p, z) \equiv \int_{\bar{V}(p, z)}^{\infty} (x - \bar{V}(p, z)) dH(x) \geq 0$ is the threshold to search; ii) conditional on searching, exits if she draws a new firm promising a continuation value $\bar{V}_{\text{new}}$ larger than the current match, i.e. $\bar{V}_{\text{new}} \geq \bar{V}(p^j_t, z^j_t)$.

The proof of the lemma follows immediately from equations (1)-(3). The lemma states that, as search is costly, not all customers currently matched to a given firm exercise the search option, only those with a low search cost do so. Notice that the threshold $\hat{\psi}(p, z)$

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5We also notice that the state of the firm problem includes the current price $p$, despite in equilibrium productivity is enough to determine the current price, as this notation is needed to study the game between the firm and its customers where the firm could, in principle, deviate from the equilibrium price.
depends both on the price of the firm, \( p \), and its productivity, \( z \). The dependence on the price is straightforward, following from its effect on the surplus \( v(p) \) that the customer can attain in the current period. The intuition behind the dependence on the firm’s productivity is that, as searching is a costly activity, the decision of which firm to patronize is a dynamic one, and involves comparing the value of remaining in the customer base of the current firm with the value of searching. Because of the Markovian structure of prices, the customer’s expectation about future prices is completely determined by the firm’s current productivity.

The next lemma discusses some properties of the continuation value function \( \bar{V}(p, z) \) and, as a consequence, of the threshold \( \hat{\psi}(p, z) \).

**Lemma 2** The value function \( \bar{V}(p, z) \) (the threshold \( \hat{\psi}(p, z) \)) is strictly decreasing (increasing) in \( p \). If \( \hat{V}(z) \equiv \hat{V}(P(z), z) \) is increasing in \( z \), the value function \( \bar{V}(p, z) \) (the threshold \( \hat{\psi}(p, z) \)) is increasing (decreasing) in \( z \).

The proof of Lemma 2 is in Appendix A.1. The lemma states that customers obtain strictly higher value from firms offering a lower current price and, if \( \hat{V}(z) \) is increasing in \( z \), also from firms characterized by higher current productivity. Notice that, under persistence in the productivity process, a sufficient condition for the latter is that equilibrium prices are decreasing in productivity. As a result, customers are not only more likely to search and exit from firms charging higher prices, but they are also more likely to do so from firms with lower productivity if \( \hat{V}(z) \) is increasing in \( z \).

### 2.2 The problem of the firm

In this section we describe the pricing problem of the firm. We start by discussing the dynamics of the customer base as a function of price and productivity, given the optimal search and exit strategy of the customers. Then, we move to the characterization of the firm optimal pricing strategy.

The customer base of firm \( j \) evolves as follows:

\[
\begin{align*}
m_i^j & = m_{i-1}^j - m_{i-1}^j G(\hat{\psi}(p_i^j, z_i^j))\left(1 - H(\hat{V}(p_i^j, z_i^j))\right) + \frac{m_{i-1}^j}{\Gamma} Q(\hat{V}(p_i^j, z_i^j)),
\end{align*}
\]

(4)

where \( G(\hat{\psi}(p_i^j, z_i^j)) \) is the fraction of customers searching firm \( j \)’ customer base, a fraction \( 1 - H(\hat{V}(p_i^j, z_i^j)) \) of which actually finds a better match and exits the customer base of firm \( j \). The ratio \( m_{i-1}^j / \Gamma \) is the probability that searching customers in the whole economy draw firm \( j \) as a potential match. The function \( Q(\hat{V}(p_i^j, z_i^j)) \) denotes the equilibrium mass
of searching customers currently matched to a firm with continuation value smaller than \( \bar{V}(p_t^j, z_t^j) \). Therefore, the product of the two amounts to the mass of new customers entering the customer base of firm \( j \). We can express the dynamics of the customer base as
\[
m_t^j = m_{t-1}^j \Delta(p_t^j, z_t^j),
\]
where the function \( \Delta(\cdot) \) denotes the growth of the customer base and is given by
\[
\Delta(p, z) \equiv 1 - G(\hat{\psi}(p, z)) \left( 1 - H(\bar{V}(p, z)) \right) + \frac{1}{\Gamma} Q(\bar{V}(p, z)).
\]
(5)

Notice that the growth of a firm is independent of its customer base and, therefore, of its size. This result is known as Gibrat’s Law and is consistent with existing empirical evidence on the distribution of firms’ size (see Luttmer (2010)), and depends on our assumption that customers draw new firms with probability proportional to their share of customers. The next lemma discusses the properties of the customer base growth with respect to prices and productivity.

**Lemma 3** Let \( \bar{p}(z) \) solve \( \bar{V}(\bar{p}(z), z) = \max_z \{ V(\mathcal{P}(z), z) \} \); \( \Delta(p, z) \) is strictly decreasing in \( p \) for all \( p > \bar{p}(z) \), and constant for all \( p \leq \bar{p}(z) \). If \( \hat{V}(z) \equiv V(\mathcal{P}(z), z) \) is increasing in \( z \), then \( \Delta(p, z) \) is increasing in \( z \).

The proof of Lemma 3 follows directly from Lemma 2. The growth of the customer base is decreasing in the current price because a higher price reduces the current surplus and therefore the value of staying matched to the firm. When the price is low enough that no firm in the economy offers a higher value to the customer, the customer base is maximized and a further decrease in the price has no impact on the customer growth. If \( \hat{V}(z) \) is increasing in \( z \), the growth of the customer base increases with firm productivity, as a larger \( z \) is associated to higher continuation value which increases the value of staying matched to the firm.

We next discuss the pricing problem of the firm. The firm pricing problem in recursive form solves
\[
\tilde{W}(z_t^j, m_{t-1}^j) = \max_p \ m_t^j \pi(p, z_t^j) + \beta \int_{z_t^j}^{z_t^j} \tilde{W}(z', m_t^j) \ dF(z' | z_t^j),
\]
subject to equation (4), where \( \tilde{W}(z_t^j, m_{t-1}^j) \) denotes the firm value at the optimal price and \( \pi(p, z_t^j) = d(p) (p - w/z_t^j) \) are profits per customer. We study equilibria where the pricing decision of the firm only depends on productivity. Thus, we conjecture that in this equilibrium the value function of a firm is homogeneous of degree one in \( m \), i.e., \( W(z, m) = m \tilde{W}(z, 1) \).
\( m \) \( W(z); W(z) \) solves

\[
W(z) = \max_p \Delta(p, z) \left( \pi(p, z) + \beta \int_z^\infty W(z')dF(z' | z) \right), \tag{6}
\]

and where we used equation (4) and we dropped time and firm indexes to ease the notation. We assume that the discount rate \( \beta \) is low enough so that the maximization operator in equation (6) is a contraction. Therefore, by the contraction mapping theorem we can conclude that our conjecture about homogeneity of \( \tilde{W}(z, m) \) is verified.

We can express the objective of the firm maximization problem as the product of two terms. The first term is the growth in the customer base, \( \Delta(p, z) \), which according to Lemma 3 is strictly decreasing in the price for all \( p > \bar{p}(z) \) and is maximized at any price \( p \leq \bar{p}(z) \). The second term is the expected present discounted value of each customer to the firm, which we denote by \( \Pi(p, z) \). The function \( \Pi(p, z) \) is maximized at the static profit maximizing price, \( p^*(z) \equiv \frac{\varepsilon_d(p)}{\varepsilon_d(p) - 1} \frac{w}{z} \). (7)

It follows that setting a price above the static profit maximizing price is never optimal. Moreover, if \( \bar{p}(z) \leq p^*(z) \), the optimal price will not be below \( \bar{p}(z) \), because in that region profit per customer increase with the price but the customer base is unaffected. Hence, \( \hat{p}(z) \in [\bar{p}(z), p^*(z)] \). If instead \( \bar{p}(z) \geq p^*(z) \), then the optimal price is the static profit maximizing price, \( \hat{p}(z) = p^*(z) \), as at this price both the customer base and the profits per customer are maximized. The following proposition collects these results.

**Proposition 1** Let \( \bar{p}(z) \) solve \( \bar{V}(\bar{p}(z), z) = \max_{z \in [\underline{z}, z]} \{ \bar{V}(\mathcal{P}(z), z) \} \), and let \( p^*(z) \) expressed in equation (7) be the price that maximizes static profits. Denote by \( \hat{p}(z) \) a price that solves the firm problem in equation (6). We have \( \hat{p}(z) \in [\bar{p}(z), p^*(z)] \) if \( \bar{p}(z) < p^*(z) \), and \( \hat{p}(z) = p^*(z) \) otherwise.

A proof of the proposition can be found in Appendix A.2.

### 3 Equilibrium

In this section we define an equilibrium, discuss its existence, and characterize its general properties. We start by defining the type of equilibrium we study.
Definition 1 Let $\hat{V}(z) \equiv \hat{V}(\mathcal{P}(z), z)$ and $p^*(z)$ be given by equation (7). We study stationary Markovian equilibria where $\hat{V}(z)$ is non-decreasing in $z$, and for all $z \in [\hat{z}, \bar{z}]$ the firm pricing strategy $\hat{p}(z)$ solves the first order condition to the firm problem in equation (6) given by

$$\frac{\partial \Pi(p, z)}{\partial p} \cdot \frac{p}{\Pi(p, z)} = -\frac{p}{\Delta(p, z)} \frac{\partial \Delta(p, z)}{\partial p} \geq 0 .$$

(8)

A stationary equilibrium is then

(i) a search and an exit strategy that solve the customer problem for given equilibrium pricing strategy $\mathcal{P}(z)$, as defined in Lemma 1;

(ii) a firm pricing strategy $\hat{p}(z)$ that solves equation (8) for each $z$, given customers’ strategies and equilibrium pricing policy $\mathcal{P}(z)$, and is such that $\hat{p}(z) = \mathcal{P}(z)$ for each $z$;

(iii) two distributions over the continuation values to the customers, $H(x)$ and $Q(x)$, that solve $H(x) = K(\hat{z}(x))$ and $Q(x) = \Gamma \int_{\hat{z}(x)}^{x} G(\hat{\psi}(\hat{p}(z), z)) dK(z)$ for each $x \in [\hat{V}(\hat{z}), \hat{V}(z)]$, where $\hat{z}(x) = \max\{z \in [\hat{z}, \bar{z}] : \hat{V}(z) \leq x\}$, and $K(z)$ solves

$$K(z) = \int_{\hat{z}}^{z} \int_{\hat{z}}^{x} \Delta(\hat{p}(x), x) dF(s|x) dK(x) ds ,$$

for each $z \in [\hat{z}, \bar{z}]$ with boundary condition $\int_{\hat{z}}^{\bar{z}} dK(x) = 1$.

We study equilibria where the continuation value to customers is non-decreasing in productivity, implying that customers’ rank of firms coincides with their productivity. This is a natural outcome as more productive firms are better positioned to offer lower prices and therefore higher values to customers. The first order condition in equation (8) illustrates the trade-off the firm faces when setting the price in a region where customer retention is a concern. When $\hat{p}(z) < p^*(z)$, the optimal price balances the marginal benefit of an increase in price (more profit per customer) with the cost (decrease in the customer base). The requirement that the solution to the firm problem must satisfy the first order condition implies that we study equilibria where the firm objective, and in particular $\Delta(p, z)$, is differentiable in $p$. The next proposition states conditions under which such an equilibrium exists and characterizes its properties.

Proposition 2 Let productivity be i.i.d. with $F(z'|z_1) = F(z'|z_2)$ continuous and differentiable for any $z'$ and any pair $(z_1, z_2) \in [\hat{z}, \bar{z}]$, and let $G(\psi)$ be differentiable for all $\psi \in [0, \infty)$, with $G(\cdot)$ differentiable and not degenerate at $\psi = 0$. There exists an equilibrium as defined in Definition 1 where $\hat{p}(z)$ satisfies equation (8), and
(i) \( \hat{p}(z) \) is strictly decreasing in \( z \), with \( \hat{p}(\bar{z}) = p^*(\bar{z}) \) and \( \hat{p}(z) < \hat{p}(z) < p^*(z) \) for \( z < \bar{z} \), implying that \( \hat{V}(z) \) is strictly increasing. Moreover, the optimal markups are given by

\[
\mu(p, z) \equiv \frac{p}{w/z} = \frac{\varepsilon_{d}(p)}{\varepsilon_{d}(p) - 1 + \varepsilon_{m}(p, z) \Pi(p, z)/(d(p)p)}.
\] (10)

where \( \varepsilon_{d}(p) \equiv \partial \log(d(p))/\partial \log(p) \), \( \varepsilon_{m}(p, z) \equiv \partial \log(\Delta(p, z))/\partial \log(p) \), and \( p = \hat{p}(z) \) for each \( z \).

(ii) \( \hat{\psi}(\hat{p}(z), z) \) is strictly increasing in \( z \), with \( \hat{\psi}(\hat{p}(\bar{z}), \bar{z}) = 0 \) and \( \hat{\psi}(\hat{p}(z), z) > 0 \) for \( z < \bar{z} \), implying that \( \Delta(\hat{p}(z), z) \) is strictly increasing, with \( \Delta(\hat{p}(\bar{z}), \bar{z}) > 1 \) and \( \Delta(\hat{p}(z), z) < 1 \).

Monotonicity of optimal prices follows from an application of Topkis’ theorem. In order to apply the theorem to the firm problem in equation (6) we need to establish increasing differences of the firm objective \( \Delta(p, z) \Pi(p, z) \) in \((p, -z)\). Under the standard assumptions we stated on \( \pi(p, z) \), it is easy to show that \( \Pi(p, z) \) satisfies this property. The customer base growth does not in general verify the increasing difference property. However, under the assumption of i.i.d. productivity, \( \Delta(p, z) \) is independent of \( z \), which, together with Lemma 3, is sufficient to obtain the result. While the results of Proposition 2 refer to the case of i.i.d. productivity shocks, numerical results in Section 6 show the properties of Proposition 2 extend to persistent productivity processes. Finally, differentiability of the distribution of productivity \( F \) is not needed for the existence of an equilibrium. We assume it to ensure that \( H(\cdot) \) and \( Q(\cdot) \) are almost everywhere differentiable so that equation (8) is a necessary condition for optimal prices. However, even when \( F \) is not differentiable and the first order condition cannot be used to characterize the equilibrium, an equilibrium with the properties of Proposition 2 exists where \( \hat{p}(z) \) and \( \hat{\psi}(\hat{p}(z), z) \) are monotonic in \( z \) but not necessarily strictly monotonic for all \( z \). More details on the proof of the proposition can be found in Appendix A.3.

We now comment on the properties of the equilibrium highlighted in the Proposition. The equilibrium is characterized by price dispersion: more productive firms charge lower prices and, therefore, offer higher continuation value to customers. This is important, as price dispersion is what motivates customers to search for lower prices. As in Reinganum (1979), price dispersion hinges on the fact that there is heterogeneity in productivity. If all the firms had the same productivity, Proposition 2 would imply a unique equilibrium where the price is that maximizing static profits, \( p^*(\bar{z}) \), and as a result the customer base of every firm would be constant.\(^6\) The equilibrium is also characterized by dispersion in customer base growth:

\(^6\)This special case is useful to understand our relation to Diamond (1971), which shows in a simple search model that the resulting equilibrium when search cost is positive exhibits no price dispersion and
more productive firms grow faster, and there is a positive mass of lower productivity firms that have a shrinking customer base and a positive mass of higher productivity firms that are expanding their customer base.

Optimal markups in equation (10) depend on three distinct terms: \( \varepsilon_d(p) \), \( \varepsilon_m(p, z) \), and \( x(p, z) \equiv \Pi(p, z)/(d(p)p) \). The terms \( \varepsilon_d(p) \) and \( \varepsilon_m(p, z) \) represent the price elasticities of quantity purchased (per-customer) and of customer growth, respectively. We notice that the elasticity of total firm demand to the price, i.e. \( m \Delta(p, z) d(p) \), is given by \( \varepsilon_d(p) + \varepsilon_m(p, z) \). An increase in price reduces total current demand both because it reduces quantity per customer (intensive margin effect) and because it reduces the number of customers (extensive margin effect). Moreover, the optimal markup solves a dynamic problem as a loss in customers has persistent consequences for future demand due to the inertia in the customer base. This dynamic effect is captured by the term \( x(p, z) \), which measures the firm present discounted value of a customer scaled by the current revenues. It follows that active customer markets are associated with a strictly lower markup than the one that maximizes static profit; the lower, the larger the product \( \varepsilon_m(p, z) x(p, z) \).

The dynamic component of the optimal markup, \( x(p, z) \), amplifies the effect of a given extensive margin elasticity \( \varepsilon_m(p, z) \) on the optimal markup. In fact, everything else being equal, a change in \( \varepsilon_m(p, z) \) has a larger impact on the markup than an equal change in \( \varepsilon_d(p) \). Intuitively, a loss in demand associated to a loss in customers is a persistent loss and, therefore, has a larger impact on the firm’s value, inducing it to charge lower markups with respect to a firm that operates in a market where the customer base is less elastic, even if the overall elasticity of demand, i.e. \( \varepsilon_m(p, z) + \varepsilon_d(p) \), is the same for all \( p \) and \( z \).

Finally, the next remark explores two interesting limiting cases of our model and showcases the effect of the search friction on price dispersion.

**Remark 1** Let search costs be scaled as \( \psi \equiv n \tilde{\psi} \), where \( n > 0 \). That is, let the value function in equation (1) be

\[
V(p, z, \psi) = \max \left\{ \hat{V}(p, z), \tilde{V}(p, z) - n\tilde{\psi} \right\}.
\]

Two limiting cases of the equilibrium stated in Definition 1:

1. Let \( n \to \infty \). Then, in equilibrium: (i) the optimal price maximizes static profits, i.e. \( \hat{p}(z) = p^*(z) \) for all \( z \in [\underline{z}, \bar{z}] \), and (ii) there is no search in equilibrium. Furthermore, the equilibrium is unique.

firms behaving as monopolists. Our model delivers a different outcome because we allow firms to differ in idiosyncratic productivity but can generate the Diamond (1971) results if heterogeneity in productivity is shut down.

\[7\] More details are available in Appendix A.4.
Let \( \pi(p^*(\bar{z}), \bar{z}) > 0 \) and let the assumptions of Proposition 2 be satisfied. Then, \( \hat{p}(\bar{z}) = p^*(\bar{z}) \) and \( \max \{ \hat{p}(z) \} = \hat{p}(\bar{z}) \) approaches \( p^*(\bar{z}) \) as \( n \to 0 \). As a result, in the limit, there is no price dispersion in equilibrium and customers do not search.

A proof of the remark can be found in Appendix A.5. The first limiting case explores the resulting equilibrium when we let search costs diverge to infinity. The model then reduces to one where customer base concerns are not present. Because the customer base is unresponsive to prices, the firm problem reverts to the standard price-setting problem under monopolistic competition widely explored in the macroeconomics literature: the firm sets the price \( p \), taking into account only its impact on static demand \( d(p) \). Not surprisingly, the equilibrium is unique, optimal prices maximize static profits, i.e. \( \hat{p}(z) = p^*(z) \) for all \( z \in [\bar{z}, \bar{z}] \), there is price dispersion, and there is no search in equilibrium. The second limiting case explores the resulting equilibrium when search costs become arbitrarily small. We restrict attention to the model that satisfies the assumptions of Proposition 2, so that the first order condition presented in equation (8) is necessary for optimality. In this case, as the scale of search costs becomes arbitrarily small, equilibrium prices approach the lowest price in the economy, \( p^*(\bar{z}) \). As a result, there is no price dispersion and customers do not search.

4 Data and descriptive evidence

We complement the theoretical analysis with an empirical investigation that relies on cashier register data from a large U.S. supermarket chain. In this section we provide descriptive evidence that the price posted by a firm influences its customers’ decision to leave the firm and measure the size of this effect. This exercise delivers statistics that we will use in Section 5 to estimate our model so to quantify the importance of customer markets in shaping firm price setting.

4.1 Data sources and variable construction

The supermarket chain shared with us scanner data detailing purchases by a panel of households carrying a loyalty card of the chain. The chain operates over a thousand stores across 10 states, and the data reflect this geographical dispersion. For every trip made at the chain between June 2004 and June 2006 by customers in the sample, we have information on the

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8 The assumption \( \pi(p^*(\bar{z}), \bar{z}) > 0 \) is purely technical, and it ensures that the first derivative of the profit function is bounded in the relevant range.

9 The chain is able to associate the loyalty cards belonging to different members of a same family to a single household identifying number, which is the unit of observation in our data. Therefore, in the analysis we use the terms “customer” and “household” interchangeably.
date of the trip, store visited, and list of goods purchased (as identified by their Universal Product Code, UPC), as well as quantity and price paid. The customers in our sample make an average of 150 shopping trips at the chain over the two years; if those trips were uniformly distributed, that would imply visiting a store of the chain six times per month. The average expenditure per trip is $69 for the average household. There is a great deal of variation (the 10th percentile is $29; the 90th is $118) explained, among other things, by income and family size of the different households.\footnote{Our data do not include information on purchases by customers not carrying a loyalty card of the chain. The focus of this study, however, is on “regular” customers who can be meaningfully said to be part of the customer base of a firm. This seems to be more the case for individuals who sign up for a loyalty card than for occasional shoppers who do not do that.}

In the theoretical model we studied the behavior of customers buying from firms producing a single homogeneous good; our application documents the exit decisions of customers from supermarket stores where they buy bundles of goods.\footnote{The choice of focusing on the customer base of the store rather than that of one of the branded product it sells is data driven. With data from a single chain we cannot track the evolution of the customer base of a single brand. In fact, if we observed customers no longer buying a particular brand we could only infer that they are not buying it at the chain we analyze, but we could not rule out that they are buying it elsewhere.} However, under the assumption that customers’ behavior depends on the price of the whole basket of goods they typically buy at the supermarket, we can focus on the resulting price index of the customer basket in order to retrieve the response of the customer base to variation in prices.\footnote{Note that since customers baskets are in large majority composed of package goods, which are standardized products, the assumption that the basket is a homogenous good is not unwarranted.} In particular, to measure the comovement between the customer’s decision to exit the customer base and the price of her typical basket of goods posted at the chain, we need to construct two key variables: (i) an indicator signaling when the household is exiting the chain’s customer base, and (ii) the price of the household basket. Below we briefly describe the procedure followed to obtain them; the details are left to Appendix B.

We consider every customer shopping at the retailer in a given week as belonging to the chain’s customer base in that week. We assume that a household has exited the customer base when she has not shopped at the chain for eight or more consecutive weeks, and that the decision to exit occurred the last time the customer visited the chain. Although brief spells without purchases can be justified with alternative explanations (e.g. consuming inventory or going on vacation), the typical customer is unlikely to experience a eight-week spell without shopping for reasons other than having switched to a different chain. In fact, for the average household in our sample, four days elapse between consecutive grocery trips and the 99th percentile of this statistic is 28 days, half the length of the absence we require before inferring that a household is buying its groceries at a competing chain. This suggests that the eight-
week window is a conservative choice.\textsuperscript{13}

We construct the price of the basket of grocery goods usually purchased by the households in the following fashion. We identify the goods belonging to a household’s basket using scanner data on items the household purchased over the two years in the sample. In a particular week $t$, the price paid by customer $i$, shopping at store $j$ for its basket, represented by the collection of UPC’s in $K_i$, is

$$p_{it} = \sum_{k \in K_i} \omega_{ik} p_{kt}, \quad \omega_{ik} = \frac{\sum_t E_{ikt}}{\sum_{k \in K_i} \sum_t E_{ikt}},$$

(11)

where $p_{kt}$ is the price of UPC $k$ in week $t$ at the store where customer $i$ shops, and $E_{ikt}$ is the expenditure (in dollars) by customer $i$ in UPC $k$ in week $t$. Note that the price of the basket is household specific because households differ in their choice of grocery products ($K_i$) and in the weight such goods have in their budget ($\omega_{ik}$). We face the common problem that household scanner data only contain information on prices and quantities of UPCs when they are actually purchased. Therefore, we complement them with store level data on weekly revenues and quantities sold.\textsuperscript{14} This data allows us to back out weekly prices of each UPC in the sample by dividing total revenues by total quantity sold as in Eichenbaum et al. (2011).

### 4.2 Evidence on customer base dynamics

The availability of individual level scanner data allows us to study the determinants of a customer’s decision to exit the customer base of the firm she is currently shopping from. We estimate a linear probability model where the dependent variable is an indicator for whether the household has left the customer base of the chain in a particular week. Our aim is to capture the effect of the price posted by the chain for the basket of goods purchased by the customer on her decision to exit.

We are interested in the effect of price variation induced by cost shifts idiosyncratic to a firm. Aggregate cost shocks do not change the relative price and, therefore, should not trigger exit from the customer base. To isolate idiosyncratic price variations, we control for the prices posted by the competitors of the chain using the IRI Marketing data set. This database includes weekly UPC’s prices for 30 major product categories for a representative sample of chain stores across 64 markets in the United States.\textsuperscript{15} Thanks to this data, we

\textsuperscript{13}We experimented with 4 weeks and 12 weeks as alternative lengths of the period of absence required to infer the exit from the customer base. In both cases the results are qualitatively similar. However, in the 12-week case the number of exit events becomes too small and we do not have the power to detect significant effects.

\textsuperscript{14}The retailer changes the price of the UPCs at most once per week, hence we only need to construct weekly prices to capture the entire time variation.

\textsuperscript{15}A detailed description of the data can be found in Bronnenberg et al. (2008). All estimates and analyses
can observe the weekly price of a specific UPC at every chain-store sampled by IRI in the Metropolitan Statistical Area of residence of a customer. This allows us to construct the price of the basket bought by the customer at each store in the MSA (at least for the part pertaining to product categories sampled by IRI) in the same fashion described for the price of the basket at our chain. We take the average of such prices across all stores, weighted by market share, to compute the average price of the basket in a market ($p^{mkt}$). To further control for sources of aggregate variation, we include in the regression year-week fixed effects that account for time-varying drivers of the decision of exiting the customer base common across households (e.g., disappearances due to travel during holiday season).

The coefficient on the retailer price of the basket is identified by $UPC$-$chain$ specific shocks as those triggered, for example, by the expiration of a contract between the chain and a manufacturer of a UPC. Within the chain, the price of a same good moves differently in different stores, for instance due to variation in the cost of supplying the store linked to logistics (e.g. distance from the warehouse). Since these shocks can hit differently goods with different intensity in delivery cost (e.g. refrigerated vs. nonrefrigerated goods), $UPC$-$store$ specific shocks also contribute to our identification. For our model to be identified, we do not need to observe shocks that make a supermarket uniformly more expensive than the competition. Shocks that affect the convenience of a chain with respect to a subset of goods on sale suffice to induce the customers who particularly care about those goods to leave. Kaplan and Menzio (forthcoming) use independent scanner data to provide ample evidence for this source of variation. They report that the bulk of price dispersion arises not from the difference from high-price and low-price stores but from dispersion in the price of a particular good (or product category) even among stores with similar overall price level.

In our empirical specification we acknowledge that, unlike posited in the model, customers are heterogenous in more dimensions than their cost to search. We include observable characteristics (age, income, and education) matched from Census 2000 and also consider location as a potential driver of the decision to exit. We control for the number of supermarket stores in the zip code of residence of the customer and factor her convenience in shopping by calculating the distance in miles between her residence and the closest store of the chain and the closest alternative supermarket. To pick up the heterogeneity in the type of goods different customers include in their basket, we control for the price volatility of the customer-specific basket and for its price in the first week in the sample, as a scaling factor. Finally, we calculate customer tenure, defined as the number of consecutive weeks the customer has spent in the customer base of the chain, and include it in the regression to account for the fact that in this paper based on Information Resources Inc. data are by the authors and not by Information Resources Inc.
long-term customers of the chain may be less willing to leave it *ceteris paribus*.

In Table 1, we report results of regressions of the following form,

\[
\text{Exit}_{it} = b_0 + b_1 \log(p_{it}) + b_2 \log(p_{mkt}^{it}) + b_3 \text{tenure}_{it} + X_i'c + \varepsilon_{it} .
\]

(12)

The retailer price in equations (12) can be endogenous if the chain conditions to variables unobserved to the econometrician that also influence the customer’s decision to leave. We use a measure of cost provided by the retailer along with the store price data to instrument for the price of the basket.\(^{16}\) The cost of the basket is obtained as the weighted average of the replacement cost of the UPCs included in it. Its calculation is analogous to that described in equation (11) to obtain the price of the basket.

Table 1: Effect of the price of the basket on the probability of exiting the customer base

<table>
<thead>
<tr>
<th>Exiting: Missing at least 8 consecutive weeks</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\log(p)</td>
<td>0.14**</td>
<td>-0.01</td>
<td>0.16*</td>
<td>0.15**</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.030)</td>
<td>(0.089)</td>
<td>(0.070)</td>
</tr>
<tr>
<td>Walmart entry</td>
<td></td>
<td>0.019*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\log(p_{mkt})</td>
<td>0.001</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Tenure</td>
<td>-0.002***</td>
<td>-0.003***</td>
<td>-0.004***</td>
<td>-0.002***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Observations</td>
<td>52,670</td>
<td>52,670</td>
<td>66,182</td>
<td>52,101</td>
</tr>
</tbody>
</table>

Notes: An observation is a household-week pair. The results reported are calculated through two-stages least squares where we use the logarithm of the cost of the basket (constructed based on the replacement cost provided for each UPC by the retailer) as instrument for the logarithm of the price of the basket. In column (2), the price of the household basket is substituted with a price index for the store overall. In column (4), the exit of the customer is assumed to have occurred in the first week of absence in the eight (or more) weeks spell without purchase at the chain rather than the week of the last shopping trip before the hiatus. We trim from the sample households in the top and bottom 1% in the distribution of the number of trips over the two years. Coefficients on a series of variables are not reported for brevity: demographic controls matched from Census 2000 (ethnicity, family status, age, income, education, and time spent commuting) as well as distance from the closest outlet of the supermarket chain and distance from the closest competing supermarket (provided by the retailer). The logarithm of the price of the household basket in the first week in the sample and the standard deviation of changes in the log-price of the household basket over the sample period are included as a controls in all specifications. Week-year fixed effects are also always included. Robust standard errors are in parenthesis. ***: Significant at 1% **: Significant at 5% *: Significant at 10%.

The results are reported in Table 1. The main specification in column (1) shows that the basket price posted by the retailer significantly impacts the probability of leaving. The

\(^{16}\)See Eichenbaum et al. (2011) for a discussion of this cost measure
effect is also quantitatively important. The average probability of exiting the customer base (0.3% weekly) implies a yearly turnover of 14%; if the retailer’s prices were 1% higher, its yearly turnover would jump to 21%. The coefficient on the competitors’ price, which we would expect to enter with a negative sign, is not significant. This may be due to the fact that the IRI data only allow us to imperfectly capture competitors’ behavior. First, the IRI dataset contains price information only on a subset of the goods included in a customer’s basket, although it arguably covers all the major product categories. Furthermore, the IRI data do not contain detailed information on the location of the outlets. This introduces measurement error in our construction of the set of stores a customer considers as options for her shopping. The negative coefficient on tenure confirms the intuition that the longer the relationship between a firm and a customer, the less likely they are to be interrupted. Among the several individual characteristics we control for it is worth mentioning that distance from stores of the chain and distance from the closest competing store enter with the expected sign. Customers living in proximity of a store of the chain are less likely to leave it, and those living closer to competitors’ stores are more inclined to do so.

In columns (2)-(4) we assess the robustness of these findings. We start by replacing the price of the individual basket with a price index for the store basket, defined as the average of the prices of the UPC’s sold by the store where the customer buys, weighted for their sales. This price is, by construction, equal for all the customers shopping in the same store. Column (2) shows that this results in a price coefficient that is negative and not significant. This confirms the importance of being able to construct individual specific baskets in order to make inference on customer’s behavior.

In column (3), we experiment with an alternative way to control for the effect of competition: we exploit episodes of entry by Walmart, a major retailer with which our chain is in direct competition. We use data from Holmes (2011) to identify the date of entry by a Walmart supercenter, i.e. a store selling groceries on top of general discount goods-in a zip code where our retailer also operates a supermarket. The resulting event study allows us to measure the effect of the retailer price on the probability of exit controlling for the most relevant change in the competitive environment. The coefficient obtained falls in the same ballpark as the estimate in the main specification, which reassures on the effectiveness of the IRI price in measuring the competitors’ behavior.

In column (4), we change the assumption on the imputation of the date of exit. Rather than assuming that the customer left on the occasion of her last trip to the store, we posit that the exit occurred in the first week of her absence. This alternative assumption matches more closely our model where the customer leaves after having seen the prices of her current supplier and decided not to buy there. Even in this case, the main result stays unaffected.
Finally, we performed a placebo test to investigate whether it is possible to obtain results with the same level of significance of our main specification out of pure chance. We estimated our main specification 1,000 times each time with a different dependent variable where exits from the customer base, while kept constant in number, are randomly assigned to customers. We find that only in 2.8% of the cases the simulation yields and price coefficient are positive and significant at 5%.

5 Parametrization and analysis of the model

In this section we discuss the procedure followed to estimate the model. We need to choose the discount factor ($\beta$) as well as four functions: the demand function, $d(p)$, the surplus function $v(p)$, the distribution of search costs $G(\psi)$, and the conditional distribution of productivity $F(z'|z)$. We assume that a period in the model corresponds to a week to mirror the frequency of our data. We fix the firm discount rate to $\beta = 0.995$. In the set of parameters that we consider, this level of $\beta$ ensures that the max-operator in equation (6) is a contraction.\footnote{This level of $\beta$ reflects that the effective discount rate faced by the firm is the product of the usual time preference discount factor and a rescaling element which takes into account the time horizon of the decision maker, as for instance the average tenure of CEOs in the retail food industry reported in Henderson et al. (2006).}

We assume that customers have logarithmic utility in consumption. Consumption is defined as a composite of two types of goods $c \equiv (d^{\theta-1} + n^{\theta-1})^{\frac{1}{\theta-1}}$, with $\theta > 1$.\footnote{Moving from these assumptions we can derive a demand function, $d(p)$, and a customer surplus function, $v(p)$, consistent with the assumptions made in Section 2.} The first good (that we label $d$) is supplied by firms facing product market frictions as described in Section 2; the other good ($n$) acts as a numeraire and it is sold in a frictionless centralized market. The sole purpose of good $n$ is to microfound a downward sloping demand $d(p)$ and, therefore, to allow for an intensive margin of demand. The parameter $\theta$ is chosen so that the implied average intensive margin elasticity of demand $\varepsilon_d(p)$ is 7, a value in the range of those used in the macro literature. The customer budget constraint is given by $p d + n = I$, where $I$ is the agent’s nominal income, which we normalize to one.\footnote{In Appendix C we show that $I$ can be derived based on a model of the labor markets.} We normalize the nominal wage equal to the price of the numeraire good, so that $w = 1$.\footnote{This is equivalent to assume that the numeraire good $n$ is produced by a competitive representative firm with linear production function and unitary labor productivity. See Appendix C for details.}

While we fix the parameters listed above using external sources, our data allow us to estimate the key parameters of the model: those characterizing the idiosyncratic productivity process and the search cost distribution using a minimum-distance estimator. The productivity process influences the variability of prices, which is necessary for customers to
obtain any benefit from search. The parameters of the search cost distribution, on the other hand, directly determine how costly it is to search. Below, we select functional forms for these objects, and explain the moments we choose in our data to identify the associated parameters. The discussion on identification is provided only for the sake of intuition; given the nonlinearity of the model, all the moments contribute to the identification of all the parameters.

We assume that the productivity evolves according to a process of the following form:

$$
\log(z_j^t) = \begin{cases} 
\log(z_{j-1}^t) & \text{with probability } \rho \\
\log(z') \sim N(0, \sigma) & \text{with probability } 1 - \rho.
\end{cases}
$$

Our theoretical model describes how persistence and volatility of productivity ($\rho$ and $\sigma$, respectively) determine autocorrelation and volatility of the resulting firm prices.\(^{21}\) We therefore estimate $\rho$ and $\sigma$ by matching the autocorrelation and the volatility of the logarithm of firm prices to those measured in the data, using a store-level price index.\(^{22}\) We find that the autocorrelation of log-prices in the data is equal to 0.58, while the unconditional measure of standard deviation is 0.02, on a weekly basis.

We assume that customers draw their search cost from a Gamma distribution with shape parameter $\zeta$, and scale parameter $\lambda$. The Gamma is a flexible distribution and fits the assumptions we made over the $G$ function in the specification of the model. In particular, for $\zeta > 1$, we obtain that the distribution of search costs is differentiable at $\psi = 0$.\(^{23}\)

To identify the parameters of the search cost distribution we exploit the estimates of the relationship between the price and the probability of exiting the customer base discussed in Section 4. We identify the scale parameter $\lambda$ by matching the average effect of log-prices on the exit probability predicted by the model to its counterpart in the data, measured by the parameter $b_1$ in equation (12).\(^{24}\) The parameter $\zeta$ measures the inverse of the coefficient of variation of the search cost distribution. In the model, higher dispersion of search costs

\(^{21}\)When solving the model numerically, we approximate the normal distribution on a finite grid, using the procedure described in Tauchen (1986).

\(^{22}\)We perform the analysis at the store level and focus on the property of a price index for the store ($p_{jt}$). This index is computed in a fashion analogous to the customer price index. It is the average of the price of all the UPCs sold in store $j$, weighted for the share of revenues they represent. To estimate persistence and volatility of prices in the data, we exploit the first year of the sample span to obtain the store-level average of the price index and use it to demean the variables so to remove store fixed effects. We then estimate on the second year of data the equation $(\log(p_{jt}) - \log(p_{jt-1})) = k_0 + k_1(\log(p_{jt-1}) - \log(p^j_j)) + \tau_t + \epsilon_{jt}$ pooling all stores. Time fixed effects are included to purge the data from aggregate effects and isolate the variation in price driven by the idiosyncratic component. The estimate of $k_1 = 0.58$ provides us a measure of the autocorrelation of prices. The estimate of the standard deviation of $\epsilon$, $\sigma(\epsilon) = 0.0168$, gives us an estimate of price volatility.

\(^{23}\)In our estimation procedure we do not impose any constraints on the values the parameter $\zeta > 1$ can take. Our unconstrained point estimate lies in the desired region.

\(^{24}\)See Appendix D for more details.
(i.e., lower $\zeta$) implies more mass on the tails of the distribution of search costs. The latter is associated to larger variation in the sensitivity of the exit probability to the price. In the data, we measure this variation by fitting a spline to equation (12), allowing for the marginal effect of price on the probability of exit to vary for different terciles of price levels. We find that higher prices are associated to higher value of $b_1$ as predicted by the model. The dispersion in the estimates of $b_1$ is 0.03. The parameter $\zeta$ is estimated by matching this number to an equivalent statistic generated by the model.25

<table>
<thead>
<tr>
<th>Table 2: Parameter estimates</th>
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<td>Estimates</td>
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<td>Persistence of productivity process, $\rho$</td>
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<td>Scale parameter of search cost distribution, $\lambda$</td>
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<td>Shape parameter of search cost distribution, $\zeta$</td>
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Notes: 95% confidence intervals reported in parenthesis are computed by block bootstrap.

We define $\Omega \equiv [\zeta \ \lambda \ \rho \ \sigma]'$ as the vector of parameters of interest and estimate it with a minimum-distance estimator. Denote by $v(\Omega)$ the vector of the moments predicted by the model as a function of parameters in $\Omega$, and by $v_d$ the vector of their empirical counterparts. Each $n^{th}$ iteration of the estimation procedure unfolds according to the following steps:

1. Pick values for the parameters $\rho_n$, $\sigma_n$, $\lambda_n$ and $\zeta_n$ from a given grid,
2. Solve the model and obtain the vector $v(\Omega_n)$,
3. Evaluate the objective function $(v_d - v(\Omega_n))'\Xi (v_d - v(\Omega_n))$. Where $\Xi$ is a weighting matrix that we assume to be the identity matrix.

We select as estimates the parameter values from the proposed grid that minimize the objective function. Implementing step 2 requires solving a fixed point problem in equilibrium prices. In particular, given our definition of equilibrium and the results of Proposition 2, we look for equilibria where prices are in the interval $[p^*(\bar{z}), p^*(\bar{z})]$. In principle, our model could have multiple equilibria; however, numerically we always converge to the same equilibrium despite starting from different initial conditions. In Appendix D we provide more details on

25See Appendix D for more details.
the numerical solution and estimation of the model. The estimation results are summarized in Table 2.

6 Price and customer dynamics

In this section we use the parameter estimates reported in Table 2 to solve for the equilibrium pricing and search policies implied by our model (henceforth “baseline economy”). Our goal is to highlight how predictions from our model with customer markets differ from those that originate from models where competition for customers does not play a role. The quantification also allows us to compare these predictions with empirical evidence and check whether the implications of our model are borne in the data.

To assess the relevance of customer markets for equilibrium dynamics we contrast our results with those implied by the limiting case where competition for customers is shut down by raising the scale of search costs ($\lambda$) to infinity. We refer to this benchmark as “counterfactual economy.” Since in the counterfactual economy search costs are infinite, customers do not change firm and firms do not compete for customers. It follows that in our baseline specification firms face both an intensive and an extensive margin elasticity of demand; whereas only the latter is present in the counterfactual economy. To make the comparison meaningful, we fix $\theta$ in the counterfactual economy so that the resulting average total elasticity of demand is the same as in our baseline economy (i.e. $\varepsilon_q = \varepsilon_d^{baseline} + \varepsilon_m^{baseline} = \varepsilon_d^{counterfactual}$). We also choose $\sigma$ and $\rho$ targeting the same volatility and autocorrelation of prices as in the baseline estimates. Hence, the two economies are observationally equivalent with respect to the (average) price elasticity of demand and the equilibrium price process.

Customer dynamics. A first order difference between our model and the counterfactual economy is the presence of customer dynamics in equilibrium, as illustrated in Figure 1. In the counterfactual economy, a firm’s customer base is constant; whereas in our model firms with high productivity experience positive net growth of their customers base and lower productivity firms are net losers of customers.\textsuperscript{26} On this dimension, the comparison with the empirical evidence is obviously partial to our baseline model since the fact that customers move across competing firms is observed in virtually every industry. Quantitatively, the model predicts a yearly customer turnover of about 6% in front of an unconditional frequency of exit from the customer base in our data of 14% on a yearly basis. Thus, price variation

\textsuperscript{26}It can be noticed that net customer base growth increases in productivity at a decreasing pace. This is dictated by the asymmetry between the retention and attraction margins in our model. For instance, allowing for an advertising technology would enable the firm to affect the mass of customers arriving and reinforce the link between the arrival rate of customers and firm productivity.
arising from idiosyncratic cost shocks explains almost half of customer dynamics observed in the data.

Figure 1: Customer base growth and productivity

Notes: The figure plots net customer base growth as a function of a firm’s idiosyncratic productivity, for the baseline and the counterfactual economy. The baseline economy is simulated using the parameter estimates in Table 2. The counterfactual economy’s productivity process is obtained matching the same moments (autocorrelation and volatility of the prices of store baskets) as in the baseline estimation but search is shut down (\(\lambda \rightarrow \infty\)). Therefore, in the counterfactual economy there is no extensive margin of demand. The parameter governing the intensive elasticity of demand is chosen for the counterfactual economy so that it matches the same overall elasticity of demand (intensive plus extensive margin) featured by the baseline economy.

The distribution of prices. As we have shown in Section 2, the presence of customer markets also affects firms’ pricing strategies. Here we document this fact analyzing the distribution of prices implied in equilibrium by our model. Since data on prices are typically easily available to researchers, we believe that assessing the predictions of the model in this respect makes for a particularly relevant empirical test. The same would not be true, for instance, if we focused on markups, on which data are harder to come by.\(^{27}\) This choice puts

\(^{27}\text{Our data provides a measure of cost but only for one retailer so that we cannot distinguish between aggregate and idiosyncratic cost shocks.}\)
us in relation with a recent literature that has explored the features of the price distribution with the explicit goal to provide evidence against which the empirical relevance of price setting models can be tested (Kaplan and Menzio (forthcoming)).

**Figure 2:** The distribution of standardized prices: model and data

Notes: In the figure, we plot the distribution of the ratio between the price set by a firm and the average price in the market, $\tilde{p}_j \equiv \log(p_j/p_{mkt})$. The price ratio is standardized (i.e. reported in deviations from its mean and divided by its standard deviation) to make the outcomes from the baseline and the counterfactual models comparable. The blue solid line refers to our baseline economy with customer markets at parameters estimated in Section 5. The counterfactual economy (dashed red line in the plot) features parameters of the productivity process chosen targeting the same moments (autocorrelation and volatility of the prices, and the intensive margin demand elasticity) as in the baseline estimation, but search is shut down ($\lambda \rightarrow \infty$). The green histogram portrays the empirical distribution of the ratio of the price index of each store of the retail chain to the average price index in the Metropolitan Statistical Area where the store is located. Both the numerator and the denominator of this ratio are normalized by their respective averages. Stores whose coefficient of variation for the ratio exceeds 1 are trimmed.

In Figure 2, we compare the distribution of prices in the baseline economy with that of the counterfactual economy and with the empirical distribution emerging from the data.\textsuperscript{28}

In order to compare the model prediction with the data we construct a price index for each supermarket store in our data and analyze the distribution of this object.\textsuperscript{29} We compute

\textsuperscript{28}In the stationary equilibrium of the model, the market price $(\bar{p}^j_{t,mkt})$ does not vary over time, and there is only one market. However, we perform the normalization to make the output of the model comparable with the data, where the market price can vary through time and controlling for it is necessary to isolate the price variation driven by idiosyncratic shocks.

\textsuperscript{29}Notice that in the case of Section 4 we used the individual customer’s basket price as our main regressor.
the price index for store $j$ in each week $t$ as the average of the prices of the UPCs sold by the outlet, weighted for the share of total revenues they generate in the entire sample. We then use the IRI data to obtain the average market price, $p_{t, \text{mkt}}^j$, given by the period $t$ average price index for the same basket of goods posted by retailers operating in the same Metropolitan Statistical Area where store $j$ is located. The statistics plotted in the histogram in Figure 2 is the (standardized) ratio of the store and the market price index where we remove permanent cross-market heterogeneity normalizing both the numerator and the denominator of the ratio by their averages computed over time.

The shape of the price distribution generated by the model with customer markets matches quite closely the empirical distribution of prices in our data. Like in the data, the baseline model shows a high concentration of prices around the mean: the fraction of prices within half a standard deviation is 46% (45.5% in the data), and excess kurtosis is 4.2 (4.6 in the data). Evidence in favor of the shape of the price distribution emerging from our baseline model is not confined to our data. Kaplan and Menzio (forthcoming) report a similar shape for the price distribution of homogeneous goods in the grocery sector; a finding we replicate in Appendix E.

According to our model, the high clustering of prices around the mean is the result of the relatively higher extensive margin elasticity faced by firms in this region. In fact, a given variation in production cost is associated to smaller variation in prices because higher customer retention concerns reduce the pass-through of cost shocks more in this region. High productivity firms instead face weaker competition for customers and therefore decrease their price more in response to a reduction in production cost, contributing to a fatter left tail of the price distribution. Low productivity firms have small markups and hence do not have room to absorb cost variation in their markup, so they change their price more in response to a cost increase, contributing to a fatter right tail of the price distribution.

Finally, it is worth reminding that our model does not obtain a good fit of the price

\begin{footnotesize}
\begin{itemize}
  \item \textsuperscript{30} The procedure requires the price index of the store to be computed on the subset of UPCs for which we have price information both in the retailer’s data and for each store in the IRI data for every week in the sample. This substantially reduces the size of the store basket: in our data, the average store price index is computed using about 1,000 UPCs. On the upside, the procedure naturally selects the best-selling products (for which price information is more likely to appear continuously for all stores).
  \item \textsuperscript{31} Kaplan and Menzio (forthcoming) use Nielsen data to document the features of the price distribution of both single UPCs and bundles of goods bought by the consumers. Whereas the former displays high kurtosis; for the latter they find that the distribution is nearly normal. Our finding does not contrast with theirs. When analyzing bundles, they look at the ratio of the grocery expenditure by a household and the expenditure she would have incurred in had she purchased each item at the average market price. As such, the figure at the numerator can derive from a bundle of goods bought in different stores. Our leptokurtik distribution refers instead to the ratio between a bundle of goods sold at a given store and the average market price of that same bundle.
\end{itemize}
\end{footnotesize}
distribution by targeting the shape of the price distribution in our estimation or by making
ad-hoc assumptions to introduce excess kurtosis: the underlying productivity innovations in
the model are drawn from a normal distribution. In fact the counterfactual economy, which
is characterized by prices roughly equal to a constant markup over marginal cost, displays a
nearly normal price distribution, and therefore cannot explain the excess kurtosis found in
the data.

**Pass-through of cost shocks.** An alternative way to compare the model predictions on
pricing with the empirical evidence is to analyze the pass-through of idiosyncratic shocks.
Our calculations for the baseline model imply an average pass-through of idiosyncratic cost
shocks equal to 13%, well below the 79% predicted in the counterfactual economy.\(^{32}\) In the
presence of competition for customers, a price increase leads to a persistent loss of customers.
Instead, dynamic in the customer base is not a factor in the counterfactual economy. This
implies that, when experiencing an increase in production cost, a firm in a customer market
economy has an extra incentive to reduce the pass-through to price by compressing its margin.

The literature in this case does not offer readily available evidence on the pass-through of
idiosyncratic shocks as firm costs are seldom observed. We use the measure of cost provided
by the retailer which, though imperfect, allows to check the model’s predictions regarding
pass-through. Once again we find that data and theory seem to align. The results from this
exercise are reported in Appendix F: the pass-through measured in the data is in line with
the predictions of a customer markets model and much lower than what the counterfactual
economy would imply.\(^{33}\)

### 6.1 Illustrating the role of search costs

To further illustrate our model, in Figure 3 we plot the results from a comparative static
exercise where we simulate it for different values of the scale of the search cost (\(\lambda\)). Unlike
in Figure 2, we are not forcing dispersion and persistence of the price process to be the same
across the different simulations; therefore all the parameters, except \(\lambda\), are kept constant at

\(^{32}\)Note that the pass-through is incomplete even in the counterfactual economy because, with CES prefer-
ences, the demand of good \(i\) depends on the relative price \(p_i/P\). With a finite number of goods in the basket
of the customer, an increase in \(p_i\) also increases the price of the basket, \(P\), thus reducing the overall increase
in \(p_i/P\) and effect on demand. The effect on \(P\) is larger, the higher the weight of good \(i\) in the basket, that
is the lower the price \(p_i\) and the higher its demand. Therefore, the elasticity of demand \(\varepsilon_d(p)\) increases in \(p\).

\(^{33}\)This result is not inconsistent with evidence of complete pass-through presented by Eichenbaum et al.
(2011) using the same data. First, they measure pass-through conditional on price adjustment; whereas
we look at the unconditional correlation between prices and costs. Second, they deal with UPC-level pass-
through while we measure pass-through of a basket of goods. If retailers play strategically with the pricing
of different products, for example lowering margins on some UPC to compensate the cost increase they
experienced on others, we can obtain both high UPC-level pass-through and low basket-level pass-through.
the values reported in Table 2. This implies that the distribution of production cost is not changing and therefore any change in the distribution of prices is explained by an equivalent variation in the distribution of markups.

Figure 3: Equilibrium prices as a function of the scale of search costs, $\lambda$

Optimal prices

Price distribution

Notes: In the left panel, we plot the optimal log-prices as a function of productivity. In the right panel we plot the cross-sectional distribution of log-prices. The blue solid line refers to our baseline economy with customer markets at parameters estimated in Section 5 ($\lambda = 0.03$). The black dotted line refers to our baseline economy where we set $\lambda = 0.08$. The red dashed line refers to our baseline economy where we raise $\lambda$ to 0.12.

When competition for customers is higher (lower $\lambda$) there is a greater risk for a firm to lose customers to competitors that are more productive. This increases the incentive for each firm to price closer to firms with higher productivity, resulting in a shift of mass from the right to the left of the price distribution. The incentive to reduce prices in response to increased competition for customers is weaker for more productive firms. In fact, in the extreme, the firm with the highest productivity always charges the same price (i.e. the price maximizing profits per customer), independently of $\lambda$. As displayed in the left panel of Figure 3, lower search costs imply a flatter pricing policy as a function of productivity, and therefore lower pass-through of productivity shocks on average.
7 The dynamics of demand: short vs. long run

So far we have analyzed the dynamics of prices and customers in response to idiosyncratic shocks in the presence of customer markets. In this section we broaden our scope, exploring the relevance of customer markets for the propagation of shocks that cause aggregate dynamics in prices and demand. To this purpose we perform a simple but telling experiment where a cost shock affects a subset of the firms in the economy. This exercise is interesting for two reasons. First, from a theoretical perspective, exploring cost shocks that affect firms asymmetrically is useful to highlight one of the main differences between our setup and the standard monopolistic competition models. In fact, such shocks affect price dispersion directly and, therefore, incentivize customers to search for a new supplier. Second, a shock to the effective cost of a subset of the players in the economy is the salient characteristic of a number of real world scenarios. For instance, variations in state sales tax would affect local sellers but not online ones located out-of-state, as they cannot be compelled to collect it. A similar effect is generated by a shock to the real exchange rate, such as a tariff, affecting the competitiveness of foreign producers, or by the introduction of size-contingent employment protection legislation.

The specifics of the experiment we perform are as follows. We consider the economy calibrated in Section 5 in steady state at period $t_0$. We assume that 10% of the firms in the economy are hit by an unexpected and unforeseen shock to production cost which we represent as a scaling factor $\tau_t$: the marginal cost of production of a firm hit by the shock goes from $w_t/z_t^i$ to $\tau_t w_t/z_t^i$. While the choice of the share of firms hit by the shock is arbitrary, we set it close to the share of imports in U.S. consumer basket, so that one could interpret our quantitative results as informative about the propagation of an exchange rate shock affecting the production cost of importers. The cost shock is realized after the firm has learned about idiosyncratic productivity $z_t$, but before pricing and customer’s exit decisions are taken, and dies out according to an AR(1) process, $\tau_t = \rho \tau_{t-1}$ for $t > t_0$.

In the left panels of Figure 4 we plot the response of demand of firms hit by the shock and of firms not hit by the shock. In the right panels of Figure 4 we plot the average price of firms hit by the shock and of firms not hit by the shock. The top row of Figure 4 reports impulse responses simulated in the baseline economy using the parameter estimates in Table 2. The second row reports impulse responses simulated in the counterfactual economy featuring parameters of the productivity process chosen targeting the same moments (autocorrelation

34 Shocks to the dispersion of idiosyncratic productivity represent another example.

35 Since the shock implies aggregate dynamics, we augment our economy with a simple equilibrium model of the labor market to capture the general equilibrium effects of the shock on wages and income. More details on this extension are provided in Appendix C.
Figure 4: The responses of demand and prices to a 1% cost shock hitting 10% of firms

Demand Response: Baseline  

Price Response: Baseline

Demand Response: Counterfactual  

Price Response: Counterfactual

Notes: The left panels plot the impulse responses of total demand (i.e. $d(p)m$) of the firms being hit by a 1% decrease in cost, and of the remaining firms not hit by the shock. The right panels plot the (average) price of the firms hit by a 1% decrease in cost, and of the remaining firms not hit by the shock. The top row reports impulse responses simulated in the baseline economy using the parameter estimates in Table 2. The bottom row reports impulse responses simulated in the counterfactual economy featuring parameters of the productivity process chosen targeting the same moments (autocorrelation and volatility of the prices, and the intensive margin demand elasticity) as in the baseline estimation, but search is shut down ($\lambda \to \infty$).
and volatility of the prices, and the intensive margin demand elasticity) as in the baseline estimation, but search is shut down ($\lambda \to \infty$). The plot refers to a parameterization where the shock leads to a 1% increase in productivity with a persistence parameter $\rho_\tau = 0.9$, so that the half-life of the aggregate shock is approximately a quarter. The qualitative results of this analysis do not depend on the parameters choice. The solid line plots the impulse responses of average demand and price of the firms hit by the cost shocks, while the dotted line refers to the impulse responses of the firms not hit by the cost shocks.

The difference in the propagation of the same shock in the two economies is striking. In the baseline economy, the effect of the shock on demand is persistent and the long-run impact is twice as large as the short-run one. Firms hit by the productivity shock gain more and more market shares as the time elapses, despite the effect of the productivity shock on prices dies out, because it takes time for customers to reallocate from high to low price firms. The response of the price of firms hit by the shock is relatively small, as these firms do not fully pass-through the decrease in marginal cost because the higher productivity reduces the customer retention concerns increasing their market power. Firms that do not experience the increase in productivity respond nevertheless to the shock by decreasing their price in an attempt to retain their customers.

Notice that if we were to only consider the effect of the price on demand through the intensive margin, we would predict a path of demand that would follow the path of prices: as prices revert to steady state, demand per customer reverts to steady state. We instead find that the long-run response of demand is larger than the short-run one. This implies that the action of prices on demand dynamics through the effects on the sticky customer base dominates the effects of prices on demand through the intensive margin in the short run. Evidence on this type of dynamics have been reported in the international macro literature. Ruhl (2008) argues that models of international trade need long-run elasticities of demand larger than short-run ones to explain the pattern in the response of the market share of imports to trade liberalizations. Price and demand dynamics with customer markets may provide an explanation for these patterns.36

The response of demand and prices in the counterfactual economy helps appreciating the importance of customer markets. In absence of customer dynamics, the counterfactual economy predicts a very different shape of the response of demand: the response of demand is larger in magnitude because of the large pass-through of the cost shock to prices and its dynamics follows the dynamics of prices. The different shape of the response of demand is explained by the fact that in this model all the adjustment in demand takes place through

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36See Drozd and Nosal (2012) for an application of customer markets to international real business cycle models.
the intensive margin, so that the response of demand follows the decaying response of prices. Finally, in absence of customer markets there is no strategic complementarity in price setting so that firms not hit by the cost shock do not change their price.

The bottom line of our experiment is that the model of demand and price dynamics with customer markets predicts smaller but more persistent response of demand to cost shocks inducing price dispersion than the counterfactual economy characterized by a model of monopolistic competition. The small pass-through of cost shocks is indicative of a strong response of desired markups to the cost shocks. The persistence in the response of demand is instead due to the inertia in the reallocation of customers. As argued by Blanchard (2009), these features may have important consequences for quantifying the sources of business cycle fluctuations in modern macro models when compared to the standard model of monopolistic competition.

8 Concluding remarks

Across a broad range of industries, being able to retain customers and attract new ones is key to firms’ success and survival. This is increasingly recognized by scholars, as witnessed by the growing number of studies attempting to model this feature and explore its implications. In this paper, we combined a formal model and novel data to study and quantify the consequences of the presence of competition for customers on firms’ pricing and demand dynamics.

Our first contribution is the introduction of a rich yet tractable model of customer markets. We generate stickiness in the customer base of a firm by positing that customers must pay a search cost when they wish to look for a new supplier. Empirically, we exploited detailed data from a retail chain to document a relationship between a firm’s pricing and its customers’ decision to leave its customer base. Retrieving this elasticity and using the data to form other appropriate empirical moments, we were able to estimate the key parameters of the model: those pinpointing the distribution of the search costs and those governing the productivity process. This allowed us to highlight two important implications of our model. First, the incentive to attract and retain customers introduces strategic complementarities pushing firms to set similar prices. This results in a price distribution where a large fraction of prices lies close to the mean, generating a shape in line with the one documented by the available empirical evidence. Second, we presented an application that exemplifies how a customer market model may have important implications for the response of demand to shocks that generate aggregate dynamics. In particular, our counterfactual exercise showed that frictions in customer reallocation contribute to magnify the size and the persistence of the long-run
effects of shocks.

Our study relies on a number of simplifying assumptions, whose relaxation seems of interest for future research. First, for tractability we refrain from explicitly modeling persistent heterogeneity in customers search/opportunity costs (although we control for these factors in the empirical analysis) and we do not allow for price discrimination. The presence of customers heterogeneity in shopping behavior is well documented (Aguiar and Hurst (2007)), which makes studying its implications for optimal pricing and customer dynamics an important topic. Due to lack of data, we do not consider the role of advertising in generating demand dynamics (Hall (2014)). While our conjecture is that the analysis of the pricing incentives presented in this paper would still apply, we think that extending the analysis to advertising, as well as to other strategies to attract and retain customers, and confront it with direct firm level evidence, could provide new insights about firms behavior. Finally, pricing with customer markets has been suggested as a potentially important source of real price rigidities which, combined with nominal rigidities, can substantially affect the propagation of aggregate shocks (Blanchard (2009)). Our framework can accommodate the introduction of nominal rigidities and exploring such setup appears a promising avenue for future work.

References


Appendix

A  Proofs

A.1  Proof of Lemma 2

The proof of Lemma 2 follows from the assumption of \(v(p)\) being strictly decreasing in \(p\) so that \(\hat{V}(p, z)\) is decreasing in \(p\) because of the assumptions that the productivity process is persistent. Finally, notice that

\[
\frac{\partial \hat{p}(p, z)}{\partial p} = -v'(p) (1 - H(V(p, z))) \geq 0 \quad \text{and} \quad \frac{\partial \hat{p}(p, z)}{\partial z} = -\frac{\partial V(p, z)}{\partial z} (1 - H(V(p, z))) \leq 0 .
\]

A.2  Proof of Proposition 1

Let \(\bar{p}(z)\) be the level of price at which no customer matched with a firm with productivity \(z\) searches. Then \(\bar{p}(z)\) satisfies \(\bar{V}(\bar{p}(z), z) = \max_{z \in [\underline{z}, \overline{z}]} \{\hat{V}(P(z), z)\}\). First, given the definition of \(p^*(z)\) and the fact that \(\Delta(p, z)\) is strictly decreasing in \(p\) for all \(p > \bar{p}(z)\), and constant otherwise, it immediately follows that \(\bar{p}(z) \in [\bar{p}(z), p^*(z)]\) if \(\bar{p}(z) < p^*(z)\), and \(\bar{p}(z) = p^*(z)\) otherwise. Next, we show that \(\bar{p}(z) < p^*(z)\) if \(\bar{p}(z) < p^*(z)\). Given the definition of \(\bar{p}(z)\) and the assumptions made on \(G\), at \(p = \bar{p}(z)\): i) \(W(p, z)\) is strictly decreasing in \(p\); ii) \(\Delta(p, z)\) is strictly decreasing in \(p\). Therefore; \(p = \bar{p}(z)\) cannot be a maximum and the result follows.

A.3  Proof of Proposition 2

Monotonicity of prices. We first show that optimal prices \(\bar{p}(z)\) are non-increasing in \(z\). Given, that productivity is i.i.d. and that we look for equilibria where \(\bar{p}(z) \geq p^*(z)\), we have that \(\bar{p}(z) = p^*(z)\) for each \(z\). From Proposition 1 we know that, for a given \(z\), the optimal price \(\bar{p}(z)\) belongs to the set \([p^*(\bar{z}), p^*(\overline{z})]\). Over this set, the objective function of the firm,

\[
W(p, z) = \Delta(p, z) (\pi(p, z) + \beta \text{constant}) ,
\]

is supermodular in \((p, -z)\). Notice the i.i.d. assumption implies that future profits of the firm do not depend on current productivity as future productivity, and therefore profits, are independent from it. Similarly, \(\Delta(p, z)\) does not depend on \(z\), as the expected future value to the customer does not depend on the productivity of the current match as future productivity is independent from it. Abusing notation, we replace \(\Delta(p, z)\) by \(\Delta(p)\). To show that \(W(p, z)\) is supermodular in \((p, -z)\) consider two generic prices \(p_1, p_2\) with \(p_2 > p_1 > 0\) and productivities \(z_1, z_2 \in [\underline{z}, \overline{z}]\) with \(-z_2 > -z_1\). We have that \(W(p_2, z_2) - W(p_1, z_2) \leq W(p_2, z_1) - W(p_1, z_1)\)
if and only if

\[ \Delta(p_2)d(p_2)(p_2 - w/z_2) - \Delta(p_1)d(p_1)(p_1 - w/z_2) \leq \Delta(p_2)d(p_2)(p_2 - w/z_1) - \Delta(p_1)d(p_1)(p_1 - w/z_1), \]

which, using \( \Delta(p_2)d(p_2) < \Delta(p_1)d(p_1) \) as \( d(p) \) is strictly decreasing and \( \Delta(p) \) is non-increasing, is indeed satisfied if and only if \( z_2 < z_1 \). Thus, \( W(p, z) \) is supermodular in \( (p, -z) \). By application of the Topkis Theorem we readily obtain that \( \hat{\rho}(z) \) is non-increasing in \( z \).

Existence of equilibrium. Next we prove existence of an equilibrium. The fixed point problem is a mapping from candidate functions of equilibrium prices, \( P(z) \), to the firm’s optimal pricing strategy, \( \hat{\rho}(z) \), where an equilibrium is one where \( \hat{\rho}(z) = P(z) \) for each \( z \). Notice that \( W(p, z) \) in equation (13) is continuous in \( (p, z) \). By the theorem of maximum, \( \hat{\rho}(z) \) is upper hemi-continuous and \( W(\hat{\rho}(z), z) \) is continuous in \( z \). Given that \( \hat{\rho}(z) \) is non-increasing in \( z \) it follows that \( \hat{\rho}(z) \) has a countably many discontinuity points. We thus proceed as follows. Let \( \hat{P}(z) \) be the set of prices that maximize the firm problem. Whenever a discontinuity arises at some \( \tilde{z} \) (so that \( \hat{P}(\tilde{z}) \) is not a singleton), we modify the optimal pricing rule of the firm and consider the convex hull of the \( \hat{P}(\tilde{z}) \) as the set of possible prices chosen by the firm with productivity \( \tilde{z} \). The constructed mapping from \( P(z) \) to \( \hat{P}(z) \) is then upper-hemiconcious, compact and convex valued. We then apply Kakutani’s fixed point theorem to this operator and obtain a fixed point. Finally, notice that since the convexification procedure described above has to be applied only a countable number of times, the set of convexified prices has measure zero with respect to the density of \( z \). Hence, they do not affect the fixed point.

Necessity of the first order condition. We show that \( Q \) and \( H \) are almost everywhere differentiable, so that Proposition 1 implies that equation (8) is necessary for an optimum. We guess that \( \hat{\rho}(z) \) is strictly decreasing and almost everywhere differentiable. It immediately follows that \( \hat{V}(z) \) is strictly increasing in \( z \) and almost everywhere differentiable. Then, given the assumption that \( F \) is differentiable, we have that \( K \) is differentiable. From \( H(x) = K(\hat{V}^{-1}(x)) \) it follows that \( H \) is also almost everywhere differentiable. Given that \( G \) and \( H \) are differentiable, so is \( Q \). Then the first order condition in equation (8) is necessary for an optimum, which indeed implies that \( \hat{\rho}(z) \) is strictly decreasing and differentiable in \( z \) in any neighborhood of the first order condition. Finally, given that \( \hat{\rho}(z) \) has a countably many discontinuity points, it has countably many points where it is not differentiable, and the first order condition does not apply at those points, but applies everywhere else. These points have measure zero with respect to the density of \( z \) and therefore \( \hat{\rho}(z) \) is almost everywhere differentiable.

Point (i). We already proved that \( \hat{\rho}(z) \) is non-increasing in \( z \). The proof that \( \hat{\rho}(z) \) is strictly decreasing follows by contradiction. Consider that \( \hat{\rho}(z_1) = \hat{\rho}(z_2) = \tilde{\rho} \) for some
\(z_1, z_2 \in [\tilde{z}, \bar{z}]\). Also, without loss of generality, assume that \(z_1 < z_2\). Given that we already established the necessity of the first order condition presented in equation (8) when prices are monotonic, suppose that it is satisfied at the duple \(\{z_2, \hat{p}\}\). Notice that, because of the assumed i.i.d. structure of productivity shocks together with \(\pi_z(p, z) < 0\), it is not possible that the first order condition is also satisfied at the duple \(\{z_1, \hat{p}\}\). Moreover, because the first order condition is necessary and we already established that \(\hat{p}(z)\) cannot be increasing at any \(z\), we conclude that the optimal price at \(z_1\) is strictly larger than at \(z_2\). That is, \(\hat{p}(z_1) > \hat{p}(z_2)\). Notice that this verifies the conjecture used to prove the necessity of the first order condition, which in turn validates the use of equation (8) here.\(^{37}\)

Notice that, because \(\hat{p}(z)\) is strictly decreasing in \(z\), the fact that \(v'(p) < 0\) together with i.i.d. productivity, implies, through an application of the contraction mapping theorem, that \(\hat{V}(z) = \hat{V}(\hat{p}(z), z)\) is increasing in \(z\).

**Point (ii).** \(\hat{\psi}(p, z) \geq 0\) immediately follows its definition. The fact that \(\hat{V}(z)\) is strictly increasing in \(z\), together with Lemma 2, immediately implies that \(\hat{\psi}(\hat{p}(\tilde{z}), \tilde{z}) = 0\) and that \(\hat{\psi}(\hat{p}(z), z)\) is strictly increasing in \(z\). Finally, Lemma 3 implies that \(\Delta(\hat{p}(z), z)\) is increasing in \(z\). Because of price dispersion, some customers are searching, which guarantees that \(\Delta(\hat{p}(\tilde{z}), \tilde{z}) > 1\). Likewise, \(\Delta(\hat{p}(\tilde{z}), \tilde{z}) < 1\).

### A.4 Thought experiment of Section 3

We show that \(\mu(p, z)\) is increasing in \(\varepsilon_q(p, z)\). Notice that equation (10) can be rewritten as

\[
\mu(p, z) = \frac{\varepsilon_q(p, z) + \varepsilon_m(p, z)\bar{x}(p, z)}{\varepsilon_q(p, z) - 1 + \varepsilon_m(p, z)\bar{x}(p, z)},
\]

where \(\bar{x}(p, z) \equiv \Pi(p, z)/\pi(p, z)\). From the equation above we obtain

\[
\frac{\partial \mu(p, z)}{\partial \varepsilon_m(p, z)} = \frac{\bar{x}(p, z)}{\varepsilon_q(p, z) - 1 + \varepsilon_m(p, z)\bar{x}(p, z)}(1 - \mu(p, z)).
\]

A direct implication of nonnegative prices is that \(\varepsilon_q(p, z) - 1 + \varepsilon_m(p, z)\bar{x}(p, z) \geq 0\), so that sign \([\partial \mu(p, z)/\partial \varepsilon_m(p, z)] = \text{sign}[(\bar{x}(p, z))(1 - \mu(p, z))]\). There are two cases to consider. The

\(^{37}\)If prices are not strictly decreasing, this argument cannot be used as the first order condition is not necessary. However, it is possible to prove that \(\hat{p}(z)\) is strictly decreasing in \(z\) for some region of \(z\). The argument follows by contradiction. Suppose that \(\hat{p}(z)\) is everywhere constant in \(z\) at some \(\hat{p}\). Then \(\hat{p}(z) = \hat{p}\) for all \(z\). If \(\hat{p} > p^*(\tilde{z})\), then \(\hat{p}\) would not be optimal for firm with productivity \(\tilde{z}\), which would choose a lower price. If \(\hat{p} = p^*(\tilde{z})\), then continuous differentiability of \(G\) together with \(H = G = Q = 0\) at the conjectured constant equilibrium price imply that the first order condition is locally necessary for an optimum, and a firm with productivity \(z < \tilde{z}\) would have an incentive to deviate according to equation (8), and set a strictly higher price than \(\hat{p}\). Finally, the result that \(\hat{p}(z) < p^*(z)\) for all \(z < \tilde{z}\) and that \(\hat{p}(\tilde{z}) = p^*(\tilde{z})\) follows from applying Proposition 1, and using that \(\hat{p}(z) \geq \hat{p}(\tilde{z})\) and \(\hat{p}(z) = \hat{p}(\tilde{z})\) for all \(z\).
first one is when $\pi(p, z) > 0$, which occurs if and only if $\mu(p, z) > 1$. It implies $\bar{x}(p, z) > 0$ and, therefore, $\partial \mu(p, z)/\partial \varepsilon_m(p, z) < 0$. The second case is when $\pi(p, z) < 0$, which occurs if and only if $\mu(p, z) < 1$. It implies $\bar{x}(p, z) < 0$ and, therefore, $\bar{x}(p, z) < 0$. As a result, $\partial \mu(p, z)/\partial \varepsilon_m(p, z) < 0$.

**A.5 Proof of Remark 1**

**Part (1).** Start by noticing that, because the mean of $G(\psi)$ is positive, the expected value of searching diverges to $-\infty$ as $n$ diverges to infinity. Because prices are finite for all $z \in [\bar{z}, \bar{z}]$, the value of not searching is bounded. As a result, customers do not search so that firms do not face customer base concerns. Formally, $\bar{p}(z) \to \infty$ for all $z \in [\bar{z}, \bar{z}]$. Because $p^*(z)$ is finite for all $z \in [\bar{z}, \bar{z}]$, it follows immediately that $p^*(z) < \bar{p}(z)$ for all $z \in [\bar{z}, \bar{z}]$. Then, using Proposition 1 we obtain that $\hat{p}(z) = p^*(z)$ for all $z \in [\bar{z}, \bar{z}]$.

**Part (2).** From Proposition 2 we have that, in equilibrium, the highest price is $\hat{p}(z)$. Moreover, under the assumptions of Proposition 2, the first order condition is a necessary condition for optimality of prices. We use this to show that, as $n$ approaches zero, $\hat{p}(z)$ has to approach $\hat{p}(\bar{z})$.

In equilibrium, it is possible to rewrite equation (8), evaluated at $\{\hat{p}(z), \bar{z}\}$, as $LHS(\hat{p}(z), n) = RHS(\hat{p}(z), n)$, where

\[
LHS(\hat{p}(z), n) \equiv G' \left( \frac{\hat{\psi}(\hat{p}(z), \bar{z})}{n} \right) \frac{\hat{\psi}_p(\hat{p}(z), \bar{z})}{n} \\
+ \left( G \left( \frac{\hat{\psi}(\hat{p}(z), \bar{z})}{n} \right) H'(\hat{V}(\hat{p}(z), \bar{z})) + \frac{1}{\Gamma} Q'(\hat{V}(\hat{p}(z), \bar{z})) \right) \hat{V}_p(\hat{p}(z), \bar{z}),
\]

\[
RHS(\hat{p}(z), n) \equiv -\frac{\pi_p(\hat{p}(z), \bar{z})}{\Pi(\hat{p}(z), \bar{z})} \left( 1 - G \left( \frac{\hat{\psi}(\hat{p}(z), \bar{z})}{n} \right) \right),
\]

given that $H(\hat{V}(\hat{p}(z), \bar{z})) = Q(\hat{V}(\hat{p}(z), \bar{z})) = 0$.

Suppose that as $n \downarrow 0$, $\hat{\psi}(\hat{p}(z), \bar{z})$ does not converge to zero. Then, $G \left( \frac{\hat{\psi}(\hat{p}(z), \bar{z})}{n} \right) \uparrow 1$ as $n \downarrow 0$. This implies that $\lim_{n \downarrow 0} RHS(\hat{p}(z), n) > 0$.

Consider now the function $LHS(\hat{p}(z), n)$. Again, suppose that as $n \downarrow 0$, $\hat{\psi}(\hat{p}(z), \bar{z})$ does not converge to zero. Notice that the second term of the function approaches a finite number as $\hat{V}_p(\hat{p}(z), \bar{z})$ is bounded by assumptions on $\psi(p)$ and $H'(\hat{V}(\hat{p}(z), \bar{z}))$ and $Q'(\hat{V}(\hat{p}(z), \bar{z}))$ being bounded as a result of Proposition 2. Moreover, as long as $\hat{p}(z) > \bar{p}(z) = p^*(\bar{z})$, we have that $\hat{\psi}_p(\hat{p}(z), \bar{z}) > 0$ so that $\hat{\psi}_p(\hat{p}(z), \bar{z})/n$ diverges as $n$ approaches zero. This means that $G' \left( \frac{\hat{\psi}(\hat{p}(z), \bar{z})}{n} \right) \frac{\hat{\psi}_p(\hat{p}(z), \bar{z})}{n}$ is divergent, and therefore the first order condition cannot be satisfied.

41
This analysis concluded that, if \( \hat{\psi}(\hat{p}(z), z) \) does not converge to zero as \( n \) becomes arbitrarily small, the first order condition, i.e. equation (8), cannot be satisfied. This occurs because \( LHS(\hat{p}(z), n) \) would diverge to infinity, while \( RHS(\hat{p}(z), n) \) would remain finite. It then follows that, as \( n \) approaches zero, a necessary condition is that \( \hat{\psi}(\hat{p}(z), z) \) also approaches zero. This condition can be restated as requiring that \( \hat{p}(z) \) approaches \( \bar{p}(z) \) as \( n \) approaches zero. Moreover, given the assumptions of Proposition 2, \( \bar{p}(z) = \hat{p}(z) = p^*(\bar{z}) \).

In the end, if \( \hat{p}(z) \) approaches \( p^*(\bar{z}) \) as \( n \) becomes arbitrarily small (so that \( \hat{\psi}(\hat{p}(z), z) \to 0 \) and \( \hat{\psi}_p(\hat{p}(z), z) \to 0 \)), we have that \( \lim_{n \to 0} LHS(\hat{p}(z), n) < \infty \) and \( \lim_{n \to 0} RHS(\hat{p}(z), n) < \infty \) as \( \pi_p(p^*(\bar{z}), z) \) is bounded as \( \pi(p^*(\bar{z}), z) > 0 \). However, if \( \hat{p}(z) \) does not approach \( p^*(\bar{z}) \) as \( n \) becomes arbitrarily small, we have that \( LHS(\hat{p}(z), n) \) diverges as \( n \) approaches zero, while \( RHS(\hat{p}(z), n) \) remains finite. As the first order condition has to be satisfied in equilibrium, a necessary condition is that, as \( n \) approaches zero, the highest price in the economy, i.e. \( \hat{p}(z) \), has to approach the lowest price in the economy, i.e. \( p^*(\bar{z}) \).

B Data sources and variables construction

B.1 Data and selection of the sample

The empirical evidence presented in Section 4 is based on two data sources provided by a large supermarket chain that operates over 1500 stores across the United States. This implies that we can observe our agents behavior only when they shop with the chain; on the other hand, cash register data contain significantly less measurement error than databases relying on home scanning (Einav et al. (2010)).

The main data source contains information on grocery purchases at the chain between June 2004 and June 2006 for a panel of over 11,000 households. For each grocery trip made by a household, we observe date and store where the trip occurred, the collection of all the UPCs purchased with quantity and price paid. The data include information on the presence and size of price discounts but do not generally report redemption of manufacturer coupons. Data are collected through usage of the loyalty card; purchases made without using the card are not recorded. However, the chain ensures that the loyalty card has a high penetration by keeping to a minimum the effort needed to register for one. Furthermore, nearly all promotional discount are tied to ownership of a loyalty card, which provides a strong incentive to use it.

Household-level scanner data report information on the price paid conditional on a certain item having been bought by the customer. Therefore, if we do not observe at least one household in our sample buying a given item in a store in a week, we would not be able
to infer the price of the item in that store-week. This has important implications as our
definition of basket requires us to be able to attach a price to each of the item composing it
in every week, even when the customer does not shop. The issue can be solved using another
dataset with information on weekly store revenues and quantities between January 2004 and
December 2006 for a panel of over 200 stores. For each good (identified by its UPC) carried by
the stores in those weeks, the data report total amount grossed and quantity sold. Exploiting
this information, we can calculate unit value prices every week for every item in stock in a
given store, whether or not that particular UPC was bought by one of the households in our
main data. Unit value prices are computed using data on revenues and quantities sold as

$$UVP_{stu} = \frac{TR_{stu}}{Q_{stu}},$$

where $TR$ represent total revenues and $Q$ the total number of units sold of good $u$ in week $t$
in store $s$.

As explained in Eichenbaum et al. (2011), this only allows us to recover an average price
for goods that were on promotion. In fact the same good will be sold to loyalty card carrying
customers at the promotional price and at full price to customers who do not have or use
a loyalty card. Without information on the fraction of these two types of customers it is
not possible to recover the two prices separately. Furthermore, since prices are constructed
based on information on sales, missing values can originate even in this case if no unit of a
specific item is sold in a given store in a week. This is, however, an unfrequent circumstance
and involves only rarely purchased UPCs, which are unlikely to represent important shares
of the basket for any of the households in the sample. For the analysis, we only retain UPCs
with at most two nonconsecutive missing price observations and impute price for the missing
observation interpolating the prices of the contiguous weeks.

On top of reporting revenues and quantities for each store-week-UPC triplet, the store-
level data also contain a measure of cost. This variable is constructed on the basis of the
estimated markup imputed by the retailer for each item and includes more than the simple
wholesale cost of the item (the share of transportation cost, etc.). Eichenbaum et al. (2011)
suggest to think about it as a measure of replacement cost, i.e. the cost of placing an item
on the shelf to replace an analogous one just grabbed by a consumer. We use this measure
to construct our instrument of the basket price.

It is important to notice that the retail chain sets different prices for the same UPC in
different geographic areas, called “price areas.” The retailer supplied store-level information
for 270 stores, ensuring that we have data for at least one store for each price area. In
order to use unit value prices calculated from store-level data to compute the price of the
basket of a specific household, we need to determine to which price area the store(s) at which she regularly shops belong. This information is not supplied by the retailer that kept the exact definition of the price areas confidential. A possible solution is to infer in which price areas the store(s) visited by a household are located by comparing the prices contained in the household panel with those in the store data. In principle the household data should give information on enough UPC prices in a given week to identify the price area representative store whose pricing they are matching. However, even though two stores belonging in the same price area should have the same prices, they may not have the same unit value prices if the share of shoppers using the loyalty card differs in the two stores. Therefore, we choose to restrict our analysis to the set of customers shopping predominantly (over 80% of their grocery expenditure at the chain) in one of the 270 stores for which the chain provided complete store-level data. This choice is costly in terms of sample size: Only 1,336 households (or 12% of the original sample) shop at one of the 270 stores for which we have store-level price data. However, since the 270 representative stores were randomly chosen, the resulting subsample of households should not be subject to any selection bias.

A final piece of the data is represented by the IRI-Symphony database. We use store-level data on quantities and revenues for each UPC in 30 major product categories for a large sample of stores (including small and mom & pop ones) in 50 Metropolitan Statistical Areas in the United States. The data allow to construct unit value prices for all the stores competing with the chain who provided the main dataset. However, the coarse geographic information prevents us from matching each customer with the stores closer to her location (in the same zip code, for instance) and forces us to adopt the MSA as our definition of a market.

### B.2 Variables construction

**Exit from customer base.** The dependent variable in the regression presented in equation (12) is an indicator for whether a customer is exiting the customer base of the chain. With data on grocery purchases at a single retail chain it is hard to definitively assess whether a household has abandoned the retailer to shop elsewhere or is simply not purchasing groceries in a particular week, for instance because she is just consuming its inventory. In fact, we observe households when they buy groceries at the chain but do not have any information on their shopping at competing grocers. Our choice is to assume that a customer is shopping at some other store when she has not visited any supermarket store of the chain for at least eight consecutive weeks. The Exit dummy is then constructed so that it takes value of one in correspondence to the last visit at the chain before a spell of eight or more weeks without shopping there. Table 3 summarizes shopping behavior for households in our sample. It is
immediate to notice that an eight-week spell without purchase is unusual, as customers tend to show up frequently at the stores. This strengthens our confidence that customers missing for an eight-week period have indeed switched to a different retailer.

Table 3: Descriptive statistics on customer shopping behavior

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.dev.</th>
<th>25th pctile</th>
<th>75th pctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of trips</td>
<td>150</td>
<td>127</td>
<td>66</td>
<td>200</td>
</tr>
<tr>
<td>Days elapsed between consecutive trips</td>
<td>4.2</td>
<td>7.5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Expenditure per trip ($)</td>
<td>69</td>
<td>40</td>
<td>40</td>
<td>87</td>
</tr>
<tr>
<td>Frequency of exits</td>
<td>0.003</td>
<td>0.065</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Composition of the household basket and basket price. The household scanner data deliver information on all the UPCs a household has bought through the sample span. We assume that all of them are part of the household basket and, therefore, the household should care about all of those prices. Some of the items in the household’s basket are bought regularly, whereas others are purchased less frequently. We take this into account when constructing the price of the basket by weighting the price of each item by its expenditure share in the household budget. The price of household $i$’s basket purchased at store $j$ in week $t$ is computed as:

$$
p_{kjt} = \sum_{k \in K_i} \omega_{ik} p_{kjt}, \quad \omega_{ik} = \frac{\sum_t E_{ikt}}{\sum_k \sum_t E_{ikt}},
$$

where $K_i$ is the set of all the UPCs ($k$) purchased by household $i$ during the sample period, $p_{kjt}$ is the price of a given UPC $k$ in week $t$ at the store $j$ where the customer shops. $E_{ikt}$ represents expenditure by customer $i$ in UPC $k$ in week $t$ and the $\omega_{ik}$’s are a set of household-UPC specific weights. There is the practical problem that the composition of the consumer basket cannot vary through time; otherwise basket prices for the same customer in different weeks would not be comparable. This requires that we drop from the basket all UPCs for which we do not have price information for every week in the sample. However, the price information is missing only in instances where the UPC registered no sales in a particular week. It follows that only low market-share UPCs will have missing values and, therefore, the UPCs entering the basket computation will represent the bulk of each customer’s grocery expenditure. The construction of the cost of the basket follows the same procedure where we
substitute the unit value price with the measure of replacement cost provided by the retailer.

We choose to calculate the weights using the total expenditure in the UPC by the household over the two years in the sample. This can lead to some inaccuracy in identifying the goods the customer cares for at a given point in time. For example, if a customer bought only Coke during the first year and only Pepsi during the second year of data, our procedure would have us give equal weight to the price of Coke and Pepsi throughout the sample period. If we used a shorter time interval, for example using the expenditure share in the month, we would correctly recognize that she only cares about Coke in the first twelve months and only about Pepsi in the final 12 months. However, weights computed on short time intervals are more prone to bias induced by pricing. For example, a two-weeks promotion of a particular UPC may induce the customer to buy it just because of the temporary convenience; this would give the UPC a high weight in the month. The effect of promotion is instead smoothed when we compute weights using expenditure over the entire sample period.

The construction of the price of the competitors occurs in two steps. First, we use the IRI data and the same procedure described above to obtain a price for the basket of each consumer at every store located in her same MSA. Next, we average those prices across stores to obtain the average market price of the consumer basket. In particular, the price is computed as:

\[
p_{mkt}^{it} = \sum_{z \in M^i} s_z \sum_{k \in K^i} \omega_{ik} p_{kzt}, \quad \omega_{ik} = \frac{\sum_{t} E_{ikt}}{\sum_{k} \sum_{t} E_{ikt}}, \quad s_z = \frac{\sum_{t} R_{zt}}{\sum_{z'} \sum_{t} R_{z't}}
\]

where \(M^i\) is the MSA of residence for customer \(i\) and \(R_{zt}\) represents revenues of store \(z\) in week \(t\). In other words, in the construction of the competitors’ price index, stores with higher (revenue-based) market shares weight more.

**Composition of the store basket and basket price.** The construction of the price (and cost) index for the store is conceptually analogous to that described above for the household basket. In principle, we would want to compute the store price index including all the UPCs sold at a store throughout the sample period, weighted by the share of revenues they generated. However, to keep the composition of the store basket constant through time, we must restrict ourselves to the UPCs for which we have no missing price information in any of the weeks in the sample span. This severely reduces the size of the store basket. At the same time, goods without missing information are the best sellers, which are those the store is likely to care more about.
C A simple model of the labor market

In this appendix we provide details on how the model can be extended to use it to evaluate the role of aggregate shocks.

We assume that each household is divided into a mass $\Gamma$ of shoppers/customers and a representative worker. The preferences of the household are given by

$$E_t \left[ \int_0^\Gamma V_t(p_i^t, z_i^t, \psi_i^t) \, di - J_t \right],$$

where $V_t(p_i^t, z_i^t, \psi_i^t)$ is defined as in equation (1) and it is the value function that solves the customer problem in Section 2.1. We denote as $J_t \equiv \phi \sum_{T=t}^{\infty} \beta^{T-t} \ell_T$ with $\phi > 0$ the disutility from the sequence of labor $\ell_T$. The aggregate state for the household includes the distribution of prices, the distribution of customers over the different firms and the level of income, the wage, and their laws of motion. Given that we allow for aggregate shocks, we have to consider the possibility that the aggregate state varies over time. We index dynamics in the aggregate state through the time subscript $t$ for the value function.

The worker chooses the path of $\ell_t$ that maximizes household preferences in equation (14). The search problem of each customer is as described in Section 2.1. As for the consumption decision, each customer allocates her income across consumption of the good sold in the local market, the demand of which we denote by $d$, and another supplied in a centralized market by a perfectly competitive firms, the demand of which we denote by $n$, to solve the following problem

$$v_t(p_t) = \max_{d, n} \frac{d^{\theta-1} + n^{\theta-1}}{1 - \gamma} \left( \frac{p_t^{\theta-1}}{\theta} \right)^{(1-\gamma)}$$

s.t. $p_t d + q_t \, n \leq I_t,$

where $\theta > 1$ and $I_t \equiv (w_t \ell_t + D_t)/\Gamma$ is nominal income, which the customer takes as given. Nominal income depends on the household labor income ($w_t \ell_t$) and dividends from firms ownership ($D_t$). The first order condition to the problem in equations (15)-(16) delivers the following standard downward sloping demand function for variety $d$

$$d_t(p_t) = I_t \frac{p_t^{-\theta}}{p_t^{1-\theta} + q_t^{1-\theta}}.$$ 

Without loss of generality we use the price $q_t$ as the numeraire of the economy. From the first order conditions for the household problem, we obtain that the stochastic discount factor is
given by $\beta \Lambda_{t+1}/\Lambda_t$, where $\Lambda_{t+s} = \int_0^\Gamma (c^i_{t+s})^{-\gamma}/P^i_{t+s} di$ is the household marginal increase in utility with respect to nominal income; $c^i_{t+s}$ denotes customer $i$’s consumption basket in period $t+s$, and $P^i_{t+s} = ((p^i_{t+s})^{1-\theta} + (q_{t+s})^{1-\theta})^{1/\theta}$ is the associated price.

The production technology of the perfectly competitively sold good (good $n$) is linear in labor, so that its supply is given by $y^n_t = Z_t \ell^n_t$, where $Z_t$ is aggregate productivity, and $\ell^n_t$ is labor demand by this firm. The production technology of the other good (good $d$) is also linear in labor, so that its supply is given by $y^d_t = Z_t \ell^d_t$, where $Z_t$ is aggregate productivity, and $\ell^d_t$ is labor demand by this firm, where $j$ indexes one particular producer. Perfect competition in the market for variety $n$ and in the labor market implies that workers are paid a wage equal to the marginal productivity of labor so that $w_t = q_t Z_t$. Equilibrium in the labor markets requires $\ell_t = \ell^n_t + \int_0^1 \ell^d_t dj$.

There are two exogenous driving processes in our economy: aggregate productivity $Z$ and the numeraire $q$. We consider an economy in steady state at period $t_0$ where expectations are such that $Z_t = 1$ and $q_t = 1$ for all $t \geq t_0$. Notice that in this case the economy coincides with the economy described in Section 2.

D Numerical solution of the model

In order to solve the model, we start by setting the parameters. The parameters $\beta, w, q, \text{ and } I$ are constant throughout the numerical exercises. For the set of estimated parameters $\Omega_n = [\lambda_n, \zeta_n, \rho_n, \sigma_n]'$, we set a search grid. The grid is different for each parameter, as they differ both in their levels and in the sensitivity of the statistics of interest to their variation. We consider a grid with an interval of 0.01 for $\sigma$, 0.05 for $\rho$, 0.5 for $\zeta$, and 0.01 for $\lambda$. Each $\Omega_n$ corresponds to a particular combination of parameters among these grids. For each $\Omega_n$ we set $\theta$ to obtain $E[\varepsilon_d(z)] = 7$.

We next describe how we solve for the equilibrium of the model for a given combination of parameters. We start by discretizing the AR(1) process for productivity to a Markov chain featuring $N = 25$ different productivity values. We then conjecture an equilibrium function $P(z)$. Given our definition of equilibrium and the results of Proposition 2, we look for equilibria where $P(z) \in [p^*(z), p^*(z)]$ for each $z$, and $P(z)$ is decreasing in $z$. Our initial guess for $P(z)$ is given by $p^*(z)$ for all $z$. We experiment with different initial guesses and found that the algorithm always converges to the same equilibrium.

Given the guess for $P(z)$, we can compute the continuation value of each customer as a function of the current price and productivity, i.e. $V(p, z)$, and solve for the optimal search and exit thresholds as described in Lemma 1. Given $P(z)$ and the customers’ search and exit thresholds we can solve for the distributions of customers $Q(\cdot)$ and $H(\cdot)$ as defined in
Definition 1. Notice that the latter also amounts to solve for a fixed point in the space of functions. Here, standard arguments for the existence of a solution to invariant distribution for Markov chains apply. Therefore, the assumption that $F(z'|z) > 0$ and $\Delta(p(z), z) > 0$ ensure the existence of a unique $K(z)$ that solves equation (9). Finally, given $Q(\cdot)$, $H(\cdot)$, $P(z)$ and $\tilde{V}(p, z)$, we solve the firm problem and the obtain optimal firm prices given by the function $\hat{p}(z)$. We use $\hat{p}(z)$ to update our conjecture about equilibrium prices $P(z)$, and iterate this procedure until convergence to a fixed point where $P(z) = \hat{p}(z)$ for all $z \in [z, \bar{z}]$.

Once we have solved for the equilibrium of the model at given parameter values, we construct the statistics to be matched to their data counterpart as follows.

- Log-price dispersion:
  $$\hat{\sigma}_p \equiv \sqrt{\sum_j K(z_j)(\log(\hat{p}(z_j)) - M_p)^2}$$
  where $M_p = \sum_j K(z_j) \log(\hat{p}(z_j))$ and $K(z_j)$ is the equilibrium fraction of customers buying from firms with productivity $z_j$.

- Average comovement between the probability of exiting the customer base and the price:
  $$\hat{b}_1 = \frac{Cov(E(z), \log(\hat{p}(z))))}{(\sigma_p)^2}$$
  where $E(z) \equiv G(\psi(\hat{p}(z), z))(1 - H(\tilde{V}(\hat{p}(z), z)))$, and $Cov(E(z), \log(\hat{p}(z))) = \sum_i K(z_j)(\log(\hat{p}(z_j)) - M_p)(E(z_j) - M_E)$ and $M_E = \sum_j K(z_j) E(z_j)$.

- Dispersion in the marginal effect of the price on the probability of exiting the customer base:
  $$\hat{\sigma}_{b_1} = \sqrt{\sum_j K(z_j)(\hat{b}_1(z_j) - M_{b_1})^2}$$
  where $\hat{b}_1(z_j) = G'(\psi(\hat{p}(z), z))/G(\psi(\hat{p}(z), z))(1 - H(\tilde{V}(\hat{p}(z), z)))^2$ and $M_{b_1} = \sum_i K(z_j)\hat{b}_1(z_j)$.

The autocorrelation of prices, $\hat{p}_p$ coincides with the parameter $\rho$ governing the persistence and autocorrelation of productivity. Thus the model-predicted statistics used to estimate the parameters are given by the vector $v(\Omega_n) = [\hat{\rho}_p, \sigma_p, \hat{b}_1, \hat{\sigma}_{b_1}]'$. We then evaluate the objective function $(v_d - v(\Omega_n))'(v_d - v(\Omega_n))$ at each iteration. We select as estimates the parameter values from the proposed grid that minimize the objective function and check that the optimum in the interior of the assumed grid.

E Price distribution of individual UPCs

Kaplan and Menzio (forthcoming) perform a thorough study of the properties of the distri-
bution of prices in the grocery sector which is highly related to ours. However, our analysis in section Section 6 focuses on a normalized store-level price index; whereas theirs considers an index of dispersion of households’ expenditure in grocery stores (not necessarily at a same store or chain). As such, our results on store baskets and their evidence on price distribution and dispersion for bundles of goods cannot be directly compared.

However, Kaplan and Menzio (forthcoming) also present evidence at the single good (UPC) level. Although this is not the relevant level of observation for our study, we use our data to replicate their findings and establish that any difference between our and their results on bundles of goods comes from the choice of a different object of interest, rather than from some dishomogeneity in the underlying data.

In Figure 5 we plot a distribution comparable to the one Kaplan and Menzio (forthcoming) report in their Figure 2, panel (a). In particular, we take the set of all the UPCs used to compute the store-level price index whose distribution we depict in Figure 2. For each UPC \( k \) sold in store \( j \) belonging to Metropolitan Statistical Area \( m \), we take the price posted by the store in week \( t \) \( (P_{j(m)}^{kt}) \) and normalize it dividing it by the mean of the prices posted in the same week for the same UPC by the stores active in the same MSA \( (P_{m}^{kt}) \). In computing the MSA average, we weight the different stores by their market shares. Formally, we define the normalized price as follows

\[
P_{j(m)}^{kt} = \frac{P_{j(m)}^{kt}}{P_{m}^{kt}}
\]

In Figure 5 we plot the distribution of the normalized price across UPCs, stores and weeks. Just as the comparable figure in Kaplan and Menzio (forthcoming), the distribution exhibits excess kurtosis. It is unimodal, has a peak close to the mean, and thicker tails than a normal distribution with the same mean and variance.

F Pass-through of idiosyncratic shocks

To measure pass-through of idiosyncratic shocks, we regress the log-price index of each store in a given week on its log-cost index. The price index \( p^{j(m)kt} \) is constructed as described in Section 6 and the cost index is analogously computed using the data on replacement cost provided by the retailer. To avoid inflating the short-term (weekly) pass-through due to the persistence of both price and cost variables, we include in the specification lagged values of the independent variable. We experiment with an alternative way to deal with the persistence of the dependent variable by measuring the short-term pass-through using first differences. Finally, we include time and market fixed effects to control for aggregate trend (e.g. demand
Figure 5: Distribution of normalized UPC prices

Notes: The histogram plots the distribution of normalized prices across UPCs, stores, and weeks. The normalized price of a UPC in a week is defined as the ratio of the weekly price of the UPC at a store of the chain that shared data with us to the average price of the same UPC in the Metropolitan Statistical Area where the store is located. The latter is computed using the IRI Marketing database. The set of UPCs considered is that used to compute the store-level price index whose distribution is presented in Figure 2; we discard UPCs whose coefficient of variation is larger than 1. The solid line plots the density of a normal with the same mean and variance as the empirical distribution of the normalized prices.

shocks) that can move prices independently from cost shifts. The results are reported in Table 4 and deliver a consistent picture. The weekly pass-through ranges between 13% and 24%, in line with the customer markets model predictions.
Table 4: Pass-through of idiosyncratic shocks

<table>
<thead>
<tr>
<th>Dep. variable</th>
<th>(1) log($p_j^t$)</th>
<th>(2) log($p_j^t$)</th>
<th>(3) $\Delta$ log($p_j^t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($cost_j^t$)</td>
<td>0.17***</td>
<td>0.24***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>log($cost_{j-1}^t$)</td>
<td>0.04</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>log($cost_{j-2}^t$)</td>
<td>-0.01</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>log($cost_{j-3}^t$)</td>
<td>0.06***</td>
<td>0.05*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>log($cost_{j-4}^t$)</td>
<td>0.07</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>$\Delta$ log($cost_j^t$)</td>
<td></td>
<td></td>
<td>0.13*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.07)</td>
</tr>
</tbody>
</table>

Observations | 12,915 | 8,295 | 8,295 |
MSA f.e. | No | Yes | Yes |
Time f.e. | No | Yes | Yes |

Notes: An observation is a store($j$)-week($t$) pair. The dependent variable is the price index of the store and the independent variables are the cost index of the store and its lags. Standard errors are in parenthesis and are clustered at the store level. ***: Significant at 1% **: Significant at 5% *: Significant at 10%.