Insider Trading and Disclosure

Luca Gelsomini*

IÉSEG School of Management (LEM CNRS); CRETA, University of Warwick

Abstract

This study is the first to propose a comprehensive theory of the public disclosure of trades by an insider whose orders have no direct price impact. A reputational model shows that not all insiders manipulate prices with uninformed disclosures. More importantly, it shows that an insider earns at least as much with a disclosure rule as without one, implying that mandatory disclosure is unnecessary because trades are revealed voluntarily. A generalization of the theory analyzes first the disclosure of ex-ante unverifiable inside statements without requiring the insider trader to disclose 'honestly'; to date this has been a limiting assumption in the literature. Numerous policy implications are drawn.

Keywords: Mandatory vs. voluntary trade disclosure, ex-ante unverifiable information disclosure, insider trading, market manipulation, securities regulation.

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There is no doubt that public disclosure by insider traders plays an important role in capital markets. Under semi-strong market efficiency, public disclosure of inside statements may improve the transfer of information from the insider to the market. Yet, public disclosure may be used by the insider to manipulate the market. For example, the insider may disclose ex-ante unverifiable statements that are false in order to move the stock price away from its real value (see Benabou and Laroque 1992, hereafter BL, and van Bommel 2003, henceforth VB). The insider may also intentionally manipulate the market by disclosing statements about his trade, i.e., whether he bought or sold, which are verifiable and mandatory under a number of regulations (see, e.g., Fishman and Hagerty 1995, hereafter FH, and John and Narayanan 1997, hereafter JN). The resulting manipulations are precisely the reason why the design of disclosure policies for insider traders represents a major challenge for regulators.

In this paper, we present a theory of insider trading and disclosure. Our theory is the first to link the mandatory disclosure of trades to the voluntary disclosure of ex-ante unverifiable information. A difference between these two types of disclosure is that the former is easier to interpret. Because trade disclosure is mandatory, the market can be sure of the insider's motivation to produce a statement. Because disclosed trades are verifiable, the market can also be sure of the truthfulness of the insider's statement. In examining mandatory trade disclosure prior to examining ex-ante unverifiable messages, we are able to model either signaling channel in sequence.

By staging our analysis in a simple reputational model of trade disclosure, we establish many novel results. Unlike in the trade disclosure literature, the main results of our model are that: (1) not all insiders manipulate the market price with uninformed disclosures; and (2) an insider earns at least as much with a disclosure rule as without, meaning that mandatory disclosure is unnecessary because informative trades are revealed voluntarily. The second result is key to extend our analysis to the voluntary disclosure of ex-ante unverifiable information that can be sent by the insider, for example through media (see Sobel 2000) or rumors, with predictions in line with the first model result above. These predictions do not rely on the assumption that with positive probability the insider sends messages 'honestly', which is the main limitation in the literature on ex-ante unverifiable information disclosure.[1]

Our main results have different implications. Our first result (i.e., that not all insiders manipulate the market price with uninformed disclosures) could reduce practitioners' concerns about the trade disclosure rule. The effect of disclosures on prices in the first result depends on four predetermined elements: asset value properties; market beliefs; the weight assigned to future profits (i.e., inter-temporal discount factor); and the probability of the insider being informed about the real asset value. Although these elements all play a role, in general the first result tells us that only an insider who is sufficiently likely to be informed does not attempt to destabilize the market with price manipulations. Thus, this result might also stimulate debate on whether to allow only this type of insider to disclose. Our second result (i.e., informative trades are disclosed voluntarily) instead reduces authorities' concerns about making laws against the missed disclosure of trades and about being vigilant against missed disclosure. Our model also enables us first to answer the important question posed by BL (p. 947) regarding whether a rule mandating trade disclosure prevents insiders from manipulating using ex-ante

[1] For what concerns the disclosure of ex-ante unverifiable information, honesty is hard to interpret, and is thus not enforceable (see BL, p. 947). Hence a priori it is difficult to reconcile this moral conduct with that of a profit-maximizing insider.

unverifiable announcements. The model also offers insights on the effects of the US short-swing rule and on the effects of a regulation allowing large insiders to delay their disclosure of trades. Our theory and its implications are explained in the remainder of this introduction.

In our theory, disclosures are the only information released by the insider to the market. This is due to the assumption that the orders of the insider convey no information (and therefore have no direct price impact) at the time of the trade.[2] This assumption is empirically supported by Lakonishok and Lee (2001) who track the trading and trade reporting activities of insiders in the US, and find that the change in stock price around the time of the trading dates is not statistically significant.[3] Further, as in VB, our theory assumes that the insider faces a constraint on asset holdings that may be due to budgetary reasons or because he wishes to control for risk exposure. This constraint adds an inter-temporal dimension to the insider's strategy space. Specifically, the maximum number of shares that the insider can trade today depends on the changes in his asset holdings yesterday. To illustrate this by means of an example, consider an insider who has no asset holdings and therefore no asset exposure. Consider also that this insider can have an asset exposure of 100 shares, meaning that he is limited to buying or selling up to 100 units of the asset today. If the insider buys 20 units today, his asset-exposure constraint limits him to buying up to 80 additional units, or to selling up to 120 units, tomorrow. In essence, if our insider buys more units today, he will be able to buy fewer (or sell more) units tomorrow. Similarly, if he sells more today, he will be able to sell fewer (or buy more) tomorrow.[4]

Armed with this premise, we start by focusing on the special case of an insider who may be privately informed about the real asset value on one occasion only. We consider mandatory trade disclosure, in which each trade is publicly revealed after it is executed and before the next order is placed. One could conjecture that this insider uses public disclosures as a lever to move the asset price to enhance his profits. However, for any properties of the asset value we prove that even public disclosure has no price impact, provided that this insider is constrained in his asset holdings. In fact, in equilibrium the market believes public disclosure by such an insider to be *un*informative. This outcome also holds when modeling the voluntary disclosure of trades and of ex-ante unverifiable information, thereby confirming the result derived by BL and hypothesized by VB. To investigate what happens off the equilibrium path, we consider a standard two-round trading model, and show that if prices were to react to disclosure (or its absence), our strategic insider, if informed, would completely deceive the market in probability. Consequently, the market efficiently anticipates this insider's behavior and in equilibrium ignores disclosures, which makes our insider earn as much as he would do under market anonymity, in which the insider cannot disclose any signal.

It is unusual for an insider to be privately informed just once, however. In most other instances, because of

[2]Instead, most theoretical models of mandatory trade disclosure focus on a large insider who affects the stock price not only through disclosure after the trade, but also directly through the trade itself.

[3]For what concerns insiders' disclosures, Lakonishok and Lee (2001) discuss the difficulty of tracking when insiders' trades were made available to the market. Prior to 2002, the filing information was typically being made available to the public after several days. Hence price responses to disclosures were delayed.

[4]In contrast, the maximum number of shares that the insider can trade per day in other models used in the literature is independent of his asset holdings. For example, in the FH and the JN models the insider can buy or sell up to a fixed number of shares every day, which implies that i) the insider has no budgetary constraint, ii) he is indifferent to infinite increase in his risk exposure and, more importantly, iii) he can only increase his asset exposure gradually. The constraint on asset holdings overcomes these implications.

his specific characteristics, the insider may also become privately informed about the real asset value at some unknown points in the future. Technically speaking, to model this latter form of informational asymmetry, we employ, as in BL, a two-round infinitely repeated framework (henceforth, ‘reputational framework’), in which at the beginning of each repetition, the insider may observe a change in the real asset value that becomes common knowledge in any case at the end of the repetition.[5] In this setting, the market believes current disclosures by the insider to be informative only if, at any repetition in the (recent) past, the disclosures did not move prices away from the real asset value at that time. In particular, if it is believed to be informative, the disclosure of an initial purchase pushes the price upwards just as far as the informative disclosure of an initial sale pushes it downwards.

For a reputational framework with mandatory trade disclosure, if the market believes current disclosures to be informative, four *relevant* insider options can be identified for the current repetition. In detail, if at this repetition our insider turns out to be informed, he may (1) trade up to his maximum cap, aiming to *lead* the price toward the real asset value, earning as much as he would do under anonymity, and subsequently profiting again by reversing his position completely, in the same repetition, if the disclosure causes the price to overshoot the real value. Otherwise he may (2) *mislead* the market, trading in the opposite direction and reversing his position afterwards, in the same repetition.[6] This option allows our insider to earn more than he would from leading the price toward the real value in the current repetition, but only as much as under anonymity in the next future, when the market starts to ignore disclosures. Indeed, as in Allen and Gale’s (1992) study, the market cannot determine whether our investor actually observes inside information in the current repetition. Thus, if uninformed, our insider may (3) *manipulate* the market, pretending to be informed—in jargon, bluffing (Harris 2002)—randomly disclosing that he has initially bought or sold, thereby moving the price up or down respectively, and then reversing his initial position in the same repetition.[7] This option in expectation allows our insider to earn more than he would from not trading in the current repetition; but, if he moves prices by chance away from the real value, expected profits over future repetitions will be reduced because the market will start ignoring disclosures. Hence, if he is uninformed at the current repetition, our insider may prefer (4) *not to trade*.

The solution to the insider’s profit-maximization problem identifies three regions of equilibria. In two regions, disclosures at all (or some) repetitions are believed to be at least partially informative, meaning that the price changes following each of these disclosures. In the third region, disclosures are never informative. In the two regions in which disclosures are informative, the insider assigns high to medium weight to profits in future repetitions, or future profits. In both these informative regions, if the market believes disclosures to be informative at the current repetition, then in equilibrium, if the insider is informed at this repetition, he prefers to lead the price toward the real asset value (rather than to mislead the market), reversing his initial position at the same repetition if the disclosure causes the price to overshoot. However, if the insider is uninformed at this repetition, his behavior is different in the two informative regions: in one region he manipulates, in the other he does

[5]We make no reference to finite repetitions. If our trader received inside information repeatedly with positive probability until a certain moment in time, and he imagined that after this he was certainly not going to be informed any more, then, starting from this future date and solving backwards, the equilibrium in each repetition would coincide with that derived when no repetition occurred.

[6]In JN this may be an equilibrium behavior only if the asset value distribution has unequal mass below and above its mean.

[7]This behavior was first examined in FH, where in equilibrium the insider manipulates *whenever* he is uninformed.

not. Specifically, there exists a threshold in the (predetermined) probability of receiving new inside information. This threshold progressively increases as the (predetermined) weight, or preference, given to future profits relative to current profits decreases from high to medium. For each discount factor associated with this band of inter-temporal preferences (and thus for all interest rates in most world economies[8]), uninformed manipulations occur less often as the probability of receiving new inside information increases, up to this threshold. Beyond this threshold, we enter the informative region in which the insider does not manipulate when uninformed. In rough terms, this means that an insider who is less likely to be informed undertakes uninformed manipulations while an insider who is more likely to be informed does not. With respect to the informative equilibria without manipulation, in those with manipulation the price following any informative disclosure reacts partially rather than fully—in proportion to how likely it is that the insider is informed—as a consequence of semi-strong market efficiency. For completeness, we note that in the third region, i.e., that of uninformative equilibria, the insider assigns a small weight to future profits, which is unrealistic. In this region, prices never shift because if they did the insider would mislead the market.

At each equilibrium in which prices react to trade disclosure, the insider always expects to earn as much or more than he would without disclosure. For this reason, mandating disclosures is unnecessary because informative trades will be advertised voluntarily. In detail, the insider discloses voluntarily not only when he knows that the resulting price will overshoot the privately known asset value (in which case a profitable reversal is possible) but also when it will undershoot this value (and thus no reversal is possible). In fact, by hiding information about price over- or under-shooting at no cost, the insider causes the price following a voluntary disclosure to shift the most, that is, as much as under mandatory disclosure. In causing the price to shift the most, the insider ensures the highest occurrence of price overshooting within each repetition, and the most profitable associated reversal.

Ultimately, price overshooting represents the goal to which end our insider, if mandated to disclose his trades, leads the price toward the real value, and if he is not mandated to disclose his trades, he advertises them voluntarily. Ever since Kyle (1985), a strand of literature has focused on an insider who, with positive probability, leads the price toward its real value, undertaking reversals in case his strategic signal (e.g., the order flow, trade disclosure) causes the price to overshoot the privately known quotation. In principle, the modeling of price overshooting is possible for every class of asset value properties, excluding the class of a random variable with two realizations (assumed, e.g., in BL, FH, and JN). In fact, these two realizations would otherwise systematically bracket equilibrium prices, meaning that the insider, when informed, would have no incentive to disclose voluntarily. For tractability, however, this strand of literature, including the work of Huddart et al. (2001), hereafter HHL, generally assumes normality. Instead, our predictions hold whether or not the asset value distribution is continuous or (to a certain degree) asymmetric, or its support is unbounded. Further, overshooting in our model is not at all due to the lack of precision of the signal. In fact, identical dynamics can be identified on the equilibrium path, whether the insider has to disclose only trade *direction* or trade *size* as well.

We now consider an insider who cannot disclose his trades, but who may send ex-ante unverifiable messages

[8]Discount factors spanning from high to medium translate to interest rates ranging from nearly zero to values well above those seen in most world economies.

to the public. In the reputational framework, when the market interprets some ex-ante unverifiable messages as favorable and some others as unfavorable, in equilibrium the price in response to a favorable/unfavorable message reacts as it would following the informative disclosure of a purchase/sale, which is exactly the transaction made by the insider before he sends the unverifiable message in question. Specifically, the insider's incentive to mislead the market when informed, as well as his incentive to manipulate when uninformed, are unaffected with regards to the case of a trade disclosure. Hence, three analogous regions of equilibria exist and manipulations arise in one of these. As a corollary of the equilibrium correspondence between ex-ante unverifiable messages and undisclosed trades, if an insider who may disclose ex-ante unverifiable information starts to be required to disclose his trades, the additional effect that the revelation of trades has on prices is, in general, identical to the effect observed when only ex-ante unverifiable messages may be sent. Thus, when these latter messages would otherwise manipulate prices, mandating trade disclosure makes the insider indifferent about sending unverifiable messages, but it does not prevent him from manipulating using effect-equivalent trade disclosures. This means that "actions *do not* speak louder than words."

A regulatory concern relates to the tension between two elements implied by public disclosure. Advocates of public disclosure argue that transparency increases price efficiency. Opponents argue that it increases manipulative behaviors. While mere speculations by an insider whose disclosures lead the price toward the real value are in a sense desirable because they promote earlier information release and thus price efficiency (see Hart 1977; Leland 1992), the distortive effect of manipulations on prices is clearly undesirable. Even though all manipulations are illegal (e.g., see SEC 2004, Section 9a2), it is hard to prosecute those who undertake them. For this reason, in the case of a predicted occurrence of manipulations, a regulator who aims to prevent them has no alternative but to refine the market rules. In this case, our model also tells us that such illegal conduct cannot be eliminated by suppressing the trade disclosure rule unless the insider is at the same time forbidden to disclose ex-ante unverifiable information. In the search for a mechanism that boosts price efficiency but deters market manipulation, we carry the analysis a little further by first modeling the US short-swing rule.

The short-swing rule contained in Section 16(b) of the SEC Security Exchange Act—but not contained in any EU Directive—constrains an insider who is obliged to disclose his trades, because the short-swing rule forces him to surrender any profit from reversals if they are undertaken within 6 months from the first trade. In equilibrium, for any properties of the asset value, this extra rule implies fully informative disclosures. On the one hand, it ensures that our insider does not manipulate when he is uninformed. On the other hand, it discourages the insider, when informed, from deceiving the market. The insider trades only when informed and only in the beginning, i.e., before disclosure, leading the price toward the real value. As a corollary, if the short-swing rule were ever to be imposed when trade disclosure was not, our insider would have no incentive to disclose trades or ex-ante unverifiable information voluntarily. To highlight the advantages and disadvantages of the US short-swing rule when imposed unconditionally, we consider the reputational framework. Even though in certain instances the addition of the short-swing rule to the mandatory disclosure rule prevents uninformed manipulation, in other instances Section 16(b) is ineffective because manipulations would never have been attempted in any case. Indeed, provided the inside information is sufficiently long-lived such that even trade reversals may be disclosed before the end of the repetition (that is, provided the repetition is made of three or more trading rounds), then if the insider weighs future profits heavily, this extra rule is not only unnecessary but also prevents reversals whose disclosure (or meaningful missed disclosure) would have shifted prices even closer to the real asset value.

There are various ways to explain why the orders of our insider have no direct price impact. For instance, we can invoke the notion of a large market in which the insider's maximum trading capacity is quantitatively negligible. A negligible trading pressure by a large insider can also be justified, as in BL. They invoke the results of Kyle (1985, 1989) and Laffont and Maskin (1990), who show that in imperfectly competitive markets this insider can limit the leakage of information into prices. In order to do so, he can split each pre-determined order into smaller orders and submit them in sequence. In this case, our predictions hold if the time between the first of a series of relatively small purchases/sales and the public disclosure of a signal is sufficient for the large insider to buy/sell up to the cap on total exposure. Specifically, the availability of sufficient time to disclose ex-ante unverifiable information is always a possibility because the timing of this disclosure is at the discretion of the insider. Conversely, under mandatory trade disclosure, our predictions hold only if the large insider is allowed to report the sequence of trades with sufficient delay.[9]

If a large insider is mandated to disclose *each* trade, small or large, *without delay*, namely before placing a new order, HHL show that this insider finds it necessary to undertake informed dissimulations, that is, to reduce the dissemination of inside information by adding a random component to his informed disclosures. However, because dissimulations are expensive to undertake, this insider earns substantially less than he would do under market anonymity. In the HHL model, the insider is assumed to be always informed and to have unlimited trading capacity, but nonetheless it can be conjectured that for a rule requiring trades to be disclosed without delay, an insider with very large but finite total exposure cap also dissimulates. If this conjecture is correct, our study suggests that, under mandatory trade disclosure, it is the imposition of a very tight deadline to report trades that causes dissimulations. Ceteris paribus, if the large insider has enough time to place a series of relatively small orders, up to his cap, before reporting them, he does not dissimulate. Instead, he behaves as predicted by our theory. In fact, whether the price following the disclosure of the series of orders overshoots the real value or not, his profits are expected to be higher than they are from dissimulation. The disappearance of this deceptive practice (i.e., dissimulation) provides a rationale for allowing large insiders to report trades with delay.

The remainder of this paper is organized as follows. Section I presents the assumptions. Section II studies the effects of a regulation that, following each purchase or sale, mandates public disclosure of *trade direction*. Section III investigates the foundation of mandatory and voluntary trade disclosure, then generalizing the analysis to the case of a voluntary disclosure of ex-ante unverifiable information. Section IV focuses on market beliefs. Section V extends our analysis to, among others, *trade size* disclosure. Section VI evaluates the US short-swing rule. Section VII summarizes the main findings.

[9] Rather than referring to the US regulation that in 2002 drastically reduced the possible delay to a relative tight delay of 2 days, here we refer to the cases of Italy, Belgium, and France, for example, with median delays of 5, 7, and 14 days respectively (see Fidrmuc et al. 2013).

I. Assumptions

There are two agents in the model, an insider ($\mathcal{I}$) who can trade a risky asset, and the market ($\mathcal{M}$) that sets prices.

The real value of the risky asset, $\widetilde{v}\in\mathcal{V}$, is a non-degenerate random variable. The statistical properties of $\widetilde{v}$ are common knowledge. The support $\mathcal{V}\subseteq \Re$ includes its lower-bound $\underline{b}$ (or upper-bound $\bar{b}$), if and only if this boundary value is finite; $E[\widetilde{v}]$, the unconditional mean of $\widetilde{v}$, is normalized to 0. In the beginning, $F(\cdot)$, the cumulative distribution function of $\widetilde{v}$, is absolutely continuous, and $f(\cdot)$, the non-cumulative probability distribution function of $\widetilde{v}$, is symmetric (in Section III, the absolute continuity and symmetry requirements are relaxed).

The timing is as follows: Before the realization of $\widetilde{v}$, denoted v, gets exogenously revealed to the market at the very end of the period, a sequence of two rounds of trading, $n \in \{1, 2\}$, takes place (Section V.A considers any finite sequence of rounds). Round n consists of three steps. In step 1, a public disclosure occurs. In step 2, the insider submits an order or quantity. In step 3, the market fixes the price at which the order is executed.

Two main states of the world are possible: $\widetilde{s}\in \{I, U\}$. State I occurs with commonly known probability q. Only the insider knows the realization of $\widetilde{s}$. When $\widetilde{s}=I$, we say that the insider is *informed*. In round n=1, an informed insider learns whether $\widetilde{v}<0$ or $\widetilde{v}>0$, and then in round n=2, he learns $\widetilde{v}=v$ (for the case of an informed insider who observes $\widetilde{v}=v$ already in round n=1, see Section V). When the insider privately knows that $\widetilde{s}=U$, we say that he is *uninformed*. In state U, the insider does not know anything about the realization of $\widetilde{v}$ at any round.

At round n, the insider trades a quantity x_n that is positive for a purchase, negative for a sale, and 0 otherwise. The insider is constrained in asset holdings, in that he is restricted to *hold* $x_n\in[-x_{\mathcal{I}},x_{\mathcal{I}}]$, where $x_{\mathcal{I}}$, which is the cap on total exposure, is strictly positive and finite, and x_0, or the amount of shares held at the very beginning of the period (i.e., before round n=1 starts), is normalized without loss of generality to 0.[10] Thus, the insider *trades* $x_1\in[-x_{\mathcal{I}}, x_{\mathcal{I}}]$ and $x_2\in [-x_{\mathcal{I}}-x_1, x_{\mathcal{I}}-x_1]$. The insider's trading strategy, denoted X, is the vector of functions $\langle X_1, X_2\rangle$. To account for mixed strategies, X_n assigns to realizations of $\widetilde{s}$ and $\widetilde{v}$ probability distributions defined over round n quantities *traded*.[11] The market's strategy, or pricing rule, is the vector of functions $\langle P_1, P_2\rangle$. Assume an intra-period discount factor equal to 1, and denote, with π_n=$x_n(v - p_n)$, the portion of insider's profits attributable to the round $n \in \{1, 2\}$ trade, where p_n=$P_n(\Omega_n)$ is the price in round n, and Ω_n is $\mathcal{M}$'s information set at round n.

[See Fig. 1.]

Definition 1 *The equilibrium is defined as follows: (i) A strategy by $\mathcal{I}$ that maximizes the overall sum of discounted expected profits over time, given the price setting rule and the information $\mathcal{I}$ has when making each trade, (ii) a strategy that allows $\mathcal{M}$ to set each price equal to the asset's expected value, given $\mathcal{I}$'s strategy and the information publicly available (market efficiency condition). Finally, (iii) each agent's belief about the other agent's strategy is correct in equilibrium.*

[10] Other authors have assumed *symmetric* upper and lower bounds in the change of holdings (e.g., VB, Brunnermeier and Pedersen 2005).

[11] Implicitly, the insider acts strategically when uninformed, as in Allen and Gale (1992), FH, Goldstein and Guembel (2008), and JN, in contrast with the assumption in another important strand of literature, in which the insider, when uninformed, is not present in the trading venue (see Chakraborty and Yilmaz 2004a, 2004b, 2008).

Implicitly following the market efficiency condition, assume that at the very beginning of the period, the asset price (in this case p_0) equals 0,[12] and at the very end of the period, the asset is traded at the real value.

As a distinctive feature of this model, public disclosures are the only signals sent by the insider to the market.[13] In our model, under a trade disclosure rule, the market at round n only learns whether the insider sold or bought or did not trade in round $n-1$. Specifically, in the second round the signal $\tau \in \{-1, 0, 1\}$ gets publicly released: τ=-1 implies that the insider sold during the first round, whereas τ=1 implies a purchase (for post-trade disclosures of submitted *quantities*, see Section V). Finally, τ=0 implies no revelation in the second round about the sale or purchase that the insider undertook during the first round (if trade disclosure is mandated, τ=0 implies inactivity at the first round). Because no disclosure occurs in the first round, we have that Ω_1=$\{\varnothing\}$ and Ω_2=$\{\tau\}$, and consequently P_1: $\{\varnothing\} \to \mathcal{V}$ and P_2: $\{-1, 0, 1\} \to \mathcal{V}$.

II. Markets with mandatory trade disclosure

This section analyzes a regulation mandating disclosures of the trade direction, first by considering the case of a non-repeated sequence of two rounds, then by presenting a multi-period framework in which this sequence is repeated infinite times.

II.A. Single-period uninformative equilibrium

Consider the case in which the insider cannot disclose any information publicly (i.e., the case of *market anonymity*), which we refer to for comparative purposes. In this case, a 'unique price' equilibrium exists, in which absence of any public signal (i.e. Ω_n=$\{\varnothing\}$) implies no price shift. Here, equilibrium price uniqueness refers to a unique component of infinite equilibria, all of which share the same equilibrium prices, p_n= 0, even though these equilibria differ in the insider's strategy. Under market anonymity, the equilibrium trading behavior of an insider aware of $\widetilde{v}$<0 (or $\widetilde{v}$>0) is such that $\sum_n x_n$ equals $-x_{\mathcal{I}}$ (resp., $x_{\mathcal{I}}$) or, said differently, such that at the end of the period this type of insider holds $-x_{\mathcal{I}}$ (resp., $x_{\mathcal{I}}$). To achieve this end-of-period goal, in equilibrium this type of insider can place any probability on all round n=1 trade quantities (x_1=0 included). At the equilibrium, the end-of-period expected profits of an insider aware of $\widetilde{v}$<0 and those of an insider aware of $\widetilde{v}$>0 equal $x_{\mathcal{I}}\xi$, where:

$$\xi = E[\widetilde{v}|\widetilde{v} > 0] = 2\int_0^{\bar{b}} v\,\mathbf{1}(0 < v < \bar{b}) f(v)\, dv, \tag{1}$$

and $\mathbf{1}(\cdot)$ is the indicator function (for brevity, instead of $\int_\imath^\jmath \mathbf{1}(\imath{<}v{<}\jmath) f(v) dv$, from now on we write $\int_\imath^\jmath h(v) dv$, where $h(v)$ equals $f(v)$ for any $\imath{<}v{<}\jmath$ and equals 0 otherwise). For the uninformed type of insider, the equilibrium trading behavior is such that $\sum_n x_n \in [-x_{\mathcal{I}}, x_{\mathcal{I}}]$, an end-of-period goal achieved by placing any probability on all round n trade quantities. In particular, this type of insider is indifferent about whether or not to trade at any

[12] This assumption plays no role in determining the results but facilitates exposition by allowing us to describe whether and how the prices set by $\mathcal{M}$ shift from this initial level.

[13] To reconduct the current structure to that in Kyle (1985), we should introduce noise traders who submit orders, together with $\mathcal{I}$, through a system that instantaneously reveals to the public the round n flow of orders, $\widetilde{u}_n$+x_n, only in aggregate, where $\widetilde{u}_n$, the noise traders' demand in n, is a continuous random variable such that $\widetilde{u}_n$ and $\widetilde{v}$ are independent. Of the two ways, adduced in the introduction to this analysis, to justify a non-informative order flow, the first then could be formalized with any density of $\widetilde{u}_n$ with support $(-\infty, \infty)$. In this structure, as long as $2x_{\mathcal{I}}$ is quantitatively negligible, noise traders provide full camouflage because $E[x_n|\widetilde{u}_n{+}x_n] \approx E[x_n|\widetilde{u}_n]$.

round, because any possible trading behavior in equilibrium causes him to earn 0 expected profits by the end of the period. Finally, notice that all equilibria in which an insider aware of $\tilde{v}<0$ and an insider aware of $\tilde{v}>0$ place a first round order equal respectively to $-x_{\mathcal{I}}$ and to $x_{\mathcal{I}}$ are robust to a probability that the market exogenously learns v in advance, i.e., at the end of the first rather than the second round.

Next consider *mandatory trade disclosure*. As for the case of market anonymity, an initial trade by the insider does not affect the short-run price, p_1. In fact, because $\Omega_1=\{\varnothing\}$, the market sets $p_1=p_0=0$. Although the subsequent public disclosure of the initial trade might alter the long-run price p_2, we prove that in the single period, the only pricing rule that is justified (and thus, part of an equilibrium) is one that makes $\mathcal{M}$ disregard this disclosure.

Proposition 1 *Under mandatory trade disclosure, in the single period, the unique price equilibrium is as follows: $\mathcal{M}$ sets $p_n=0$; an insider aware of $\tilde{v}<0$ and one aware of $\tilde{v}>0$ trade in such a way that $\sum_n x_n=-x_{\mathcal{I}}$ and $\sum_n x_n=x_{\mathcal{I}}$, respectively, provided that they disclose the same signal τ with equal probability (even 0 or 1); and an uninformed insider trades any quantity x_n.*

Proof. See Internet Appendix A. ■

In equilibrium, because p_n equals 0, the payoff that the insider expects to achieve equals the one expected under market anonymity. Specifically, under mandatory trade disclosure, the end-of-period expected profits of an uninformed insider equal 0, whereas those of an insider aware of $\tilde{v}<0$ (or $\tilde{v}>0$) amount to $x_{\mathcal{I}}\xi$.

To see why disclosure of trades (and disclosure of no trade) is not informative, consider any candidate equilibrium pricing rule, such that the signal $\tau=-1$ or $\tau=0$ or $\tau=1$ causes the price p_2 to shift from $p_0=0$. For each of these pricing rules, derive $\mathcal{I}$'s best reply under the assumption that, when informed, the insider already observes $\tilde{v}=v$ in the first round. Holding this optimal trading strategy fixed, notice that the candidate pricing rule in question causes $\mathcal{M}$ to reply to *all* types of insider aware of $\tilde{v}=v<0$ or $\tilde{v}=v>0$ with a price in the opposite partition of the support $\mathcal{V}$.[14] This incorrect price shift follows an identical first round order, x_1 (and thus an identical disclosure of trade direction). This optimal trading behavior is unaffected if the insider only observes whether $\tilde{v}<0$ or $\tilde{v}>0$ in the first round, case in which any of these candidate pricing rules continues to suffer from the same problem. Now recall that, because $f(\cdot)$ is symmetric around 0, the probability of $\tilde{v}$ being greater or smaller than p_0 is the same. It then follows that any of these candidate pricing rules is (in expectation) wrong. In fact, at least half of the time, prices shift in the wrong partition of $\tilde{v}$, regardless of whether, in the first round, the insider knows $\tilde{v}=v$ or $\tilde{v}\gtrless 0$. In conclusion, no pricing rule such that $p_2\neq p_0$ can be an equilibrium one.

[14]To make some off-the-path manipulative attempts by $\mathcal{I}$ explicit, consider the following candidate equilibrium pricing rules and the associated $\mathcal{I}$ best responses. Holding $p_1=0$ unchanged, suppose that $P_2(\tau=-1)$—namely, the price in response to a disclosed sale—is negative, $P_2(\tau=0)$ is non-negative, and $P_2(\tau=1)$ is positive (this is case $C3$ in the proof to Proposition 1). In this case, the round $n=1$ placed orders in response to these prices, as well as the disclosed trade directions, depend on the exact value that $P_2(\tau=-1)$, $P_2(\tau=0)$, and $P_2(\tau=1)$ assume. Specifically, if both $P_2(\tau=1)$ and $\frac{P_2(\tau=0)}{2}$ are not greater than $|P_2(\tau=-1)|$, each type of insider aware of $\tilde{v}=v>0$ prefers to disclose a first round sale, which moves p_2 down (in the wrong direction). If this latter condition on prices is not satisfied, each type of insider aware of $\tilde{v}=v<0$ finds it optimal to purchase or not to trade in $n=1$, depending on whether $P_2(\tau=1)\geq\frac{P_2(\tau=0)}{2}$ or $0<P_2(\tau=1)\leq\frac{P_2(\tau=0)}{2}$, which causes p_2 to shift up (in the wrong direction). Suppose instead that $P_2(\tau=-1)$ is positive, and that $P_2(\tau=0)$ and $P_2(\tau=1)$ are non-positive (this is case $C6$ in the proof to Proposition 1). In this case, each type of insider aware of $\tilde{v}=v<0$ finds it optimal to sell the same quantity in the first round and to continue selling up to the total exposure cap thereafter. The initial sale causes p_2 to shift up (in the wrong direction).

The result in Proposition 1 is in line with the classical result in finitely repeated zero-sum games of incomplete information, in which it is impossible for an informed sender to mislead an uninformed receiver (Aumann and Maschler 1995). Here, $\mathcal{M}$ does not use the signal received, because $\mathcal{I}$'s preferences over actions are completely opposed to what can be roughly defined as $\mathcal{M}$'s preferences to set prices efficiently. If prices reacted to the trade disclosure (or to its absence), the pricing rule would not be justified, and in this sense, $\mathcal{M}$ would be worse off and would deviate. Otherwise, regardless of whether $\mathcal{I}$ is actually informed, with a probability greater than half, prices would shift in the opposite direction with respect to v, and in practice, $\mathcal{M}$ could do better by tossing a coin. This result is due to the position limit assumption (as explained in Section III), but does not rely on $\mathcal{M}$ knowing the effective size of the cap on total exposure, $x_{\mathcal{I}}$.

With respect to the equilibrium trading strategy depicted under market anonymity, the one under mandatory trade disclosure is constrained as follows: The probability that an insider aware of $\widetilde{v}<0$ and one aware of $\widetilde{v}>0$ place on first round purchases is the same. Analogously, the probability that these types of insider place on first round sales is identical, as is the probability placed on x_1=0. In this way, the revelation of information following the first round trade is eliminated. To eliminate information revelation, the insider can, but does not have to, employ mixed strategies, which is why so-called dissimulation is not a driving force for our result. What matters is that all the informed types of insider initially disregard their information and disclose (at least in probability) an identical signal. By contradiction, suppose that the insider(s) aware of $\widetilde{v}>0$ decided to signal τ=0 *less* often than the insider(s) aware of $\widetilde{v}<0$, for example. For each of these types of informed insider, the best trading plan associated with the signaling plan in question implies a payoff that equals the equilibrium payoff. However, the resulting best reply is not of equilibrium, because τ=0 shifts p_2 *down* below 0, a pattern that has been shown *not* to be compatible with that of an equilibrium pricing rule. Finally, notice that even when the uninformed type of insider signals differently from what the informed types signal, the market still does not extract useful information. Therefore, the pricing rule p_n=0 is justified.[15]

II.B. Informative equilibria of the reputational framework

This subsection assesses whether alternative equilibria are possible, for which disclosed trades become relevant. We allow for an infinite repetition of the single period and refer to an equilibrium as a sequence of history-contingent replies that satisfy certain sequential conditions. When analyzing a problem with $t \in \mathbb{N}$ periods, additional assumptions are needed. First, an *inter*-period discount factor, $\delta \in [0, 1)$, is assumed. Second, δ and q do not vary over time. Third, the insider involved in the infinite repetition is always the same, yet at the very beginning of each period, $\widetilde{s}$ and $\widetilde{v}$ are drawn again. In particular, both $\widetilde{s}$ and $\widetilde{v}$ are i.i.d. over periods. Fourth, for any repetition of the period, x_0 is normalized to 0, meaning that the amount of shares held by the insider at

[15] None of the infinite equilibria implied by Proposition 1 is robust to a small probability that $\mathcal{M}$ exogenously learns v in advance, in which case, if p_n=0, an insider aware of $\widetilde{v}<0$ (or $\widetilde{v}>0$) strictly prefers to sell (resp., buy) in the first round. The new trading strategy causes $\mathcal{M}$ to deviate, setting $p_2(\tau{=}-1)<0$, $p_2(\tau{=}0)$=0, and $p_2(\tau{=}1)>0$. At these new prices however, an insider aware of $\widetilde{v}<0$ (or $\widetilde{v}>0$) buys (resp., sells) in the first round, which moves p_2 in the *wrong* direction, then reverses his initial position in the likely event of an exogenous, public revelation of v at the end of the second round. To obtain an equilibrium that is robust to the perturbation in question, we can require, for example, that $\mathcal{M}$ sets prices efficiently, in the *weak* sense, *if* no equilibrium exists that is robust to a small probability that $\mathcal{M}$ learns v in advance, such that p_n equals 0 even though an insider aware of $\widetilde{v}<0$ initially sells and one aware of $\widetilde{v}>0$ initially buys (for the cases in this paper, other than that of a single period with mandatory trade disclosure, this imposition will be unnecessary). At this equilibrium, the insider's expected profits coincide with those realized under market anonymity.

the end of each period does not affect the subsequent-period space of actions. The implicit justification for this assumption is that the insider can always rebalance his holdings at the end of each period, when shares are traded at the real value that has just been exogenously revealed to the public.

For an infinite repetition of the two rounds, consider the following strategy for $\mathcal{M}$:

Conjecture 1 *Suppose $\mathcal{M}$'s strategy is to set p_1=0 and p_2=$P_2^{\mu}(\cdot)$ in the first period, where $P_2^{\mu}(\tau$=$-1)$=$-\mu$, $P_2^{\mu}(\tau$=$0)$=0, and $P_2^{\mu}(\tau$=$1)$=μ, and $\mu \geq 0$ is the magnitude of the second-round price shift. At the second round of the t^{th} period, if the outcome of all $t-1$ preceding periods has been τ=$-1 \wedge v$<0 or τ=0 or τ=$1 \wedge v$>0, then play P_2^{μ}; otherwise, set p_2=0.*[16]

The analysis is now restricted to what, for μ>0, we call a *trigger* strategy, which consists of a generic history-contingent pricing rule with a *punishment* scheme that makes the market ignore subsequent disclosures when the insider *defects*, that is, when at a specific period he causes the price to go in the wrong direction with respect to v. The punishment refers to the expected decrease in per period profits suffered by $\mathcal{I}$ after *defection*, namely, after the market observes vp_2<0. Conjecture 1 implies that, in case period t disclosure moves the price away from period t real value—which unveils a manipulative behavior that is recorded by the market only at the end of period t—then, from period t+1 onward, the market reverts to single-period equilibrium behavior by setting prices equal to 0. In this case, from period t+1, the insider reverts to single-period equilibrium behavior. It will be shown that, depending on δ and q, sub-classes of the trigger strategy in Conjecture 1 are part of an equilibrium.

Here, $\mathcal{M}$ could be thought of as representing the behavior of a semi-strong efficient market as a whole (as in BL) or serving as an intermediary. The pricing rule in this subsection is a *Grim* trigger strategy that applies a punishment consisting of $\mathcal{M}$ reverting to single-period equilibrium behavior *forever* (Friedman 1971). Thus, a legitimate question is whether the notion of a single agent, $\mathcal{M}$, accomplishing its task by selecting this pricing rule is consistent with the idea of more non-cooperative auctioneers, each of whom must set the efficient price at a different round.[17] Section IV explains why the answer to this question is positive, for any equilibrium trigger strategy selected by the single agent. Section IV also accounts for the multiplicity of equilibrium price reactions to a certain sequence of disclosures.

II.B.1. Benchmark case ($\mathbf{q = 1}$)

Given the pricing rule in Conjecture 1, if at a certain period prices shift positively, at that period an insider aware of $\widetilde{v}$<0 (or $\widetilde{v}$>0) confronts a choice between two relevant behaviors. If he decides to incur market punishment, the insider finds it optimal initially to buy (resp., sell) up to the cap on total exposure and undertake a complete reversal of the initial position afterward, in the second round; in this case, he misleads the market, or simply *misleads*. Conversely, if this type of insider decides not to incur the punishment, he finds it optimal to sell (resp., buy) up to the total exposure cap in the first round, completely reversing this position in the second round if he

[16] The symbol $\wedge$ stands for *and*.

[17] Caldentey and Stacchetti (2010, p. 250) also reflect on how inter-round equilibrium dynamics might be affected if the agent setting the efficient price in one round is not active in future rounds. To support the interpretation of the Kyle auctioneer as the reduced form of more bidders competing on price, they suggest imagining an indefinitely large group of bidders, each of whom acts only once and then quits, with the consequence that these bidders avoid multi-round collusion.

is aware of a second-round price overshooting—that is, if v lies between p_2 and p_0—or undertaking no second-round exchange at all otherwise. In this case, he leads the market, or simply *leads*. Having identified these two relevant behaviors, let $\bar{\alpha}\in[0,1]$ be the probability with which this type of insider chooses to mislead rather than lead. Our first lemma follows immediately.

Lemma 1 *Consider mandatory trade disclosure, the pricing rule in Conjecture 1, and a specific period in which μ is greater than 0. After having decided to mislead and lead the market with probabilities $\bar{\alpha}$ and $1-\bar{\alpha}$, respectively, an informed type of insider expects to earn $\bar{\alpha}\cdot M(\mu)+(1-\bar{\alpha})\cdot L(\mu)$ in that period, where:*

$$L(\mu) = x_{\mathcal{I}}\left[\xi + 4\int_0^{\mu}(\mu - v)h(v)dv\right], \tag{2}$$

and

$$M(\mu) = x_{\mathcal{I}}\left[\xi + 4\mu\int_0^{\bar{b}}h(v)dv\right]. \tag{3}$$

Proof. See Internet Appendix A. ■

As long as μ is positive at a certain period, the two-round profits that an informed insider expects from misleading at that period, $M(\mu{>}0)$, are greater than those from leading, $L(\mu{>}0)$. By leading, this type of insider still has a chance to benefit from an additional price differential at the second round, so the profits expected from leading are greater than the equilibrium profits that the insider expects after a defection, in each period in which he is informed—formally, $L(\mu{>}0){>}x_{\mathcal{I}}\xi$.

The next proposition presents the equilibrium for q=1, in which $\mathcal{I}$ is systematically informed and must figure out $\bar{\alpha}^{*I}$, the level of $\bar{\alpha}$ that maximizes his discounted expected profits over periods, where by construction, $\bar{\alpha}^{*I}$ is not only $\mathcal{I}$'s optimal choice when, in the current period, $\mathcal{I}$ is informed and μ is positive, but also $\mathcal{I}$'s optimal planned choice when he is informed again in any future period in which μ is still positive.

Proposition 2 *For mandatory trade disclosure, a reputational framework, and an insider who is informed with certainty at any period, (i) if $\delta\geq\delta_{\nabla}=\frac{M(\mu=\xi)-L(\mu=\xi)}{M(\mu=\xi)-x_{\mathcal{I}}\xi}$, an equilibrium exists in which disclosures affect prices. Specifically, $\mathcal{M}$ undertakes the strategy in Conjecture 1, setting $\mu=\xi$, and the insider systematically leads the market (i.e., $\bar{\alpha}^{*I}=0$), never incurring market punishment. (ii) If $\delta<\delta_{\nabla}$, at each repetition the equilibrium coincides with the single period equilibrium derived under mandatory trade disclosure.*

Proof. See Internet Appendix A. ■

The logic of this result is as follows: Consider a situation in which μ today is greater than 0. If $\delta{>}\delta_{\nabla}$, namely, if an insider who is always informed gives substantial weight to the profits achievable from leading in all periods—which is an alternative to earning even more only once today by misleading, but then earning less forever—he opts for the former option with certainty. Thus, prices are never manipulated at the equilibrium that is in pure strategies. Because disclosures are fully informative, each equilibrium second-round price shift equals ξ. Conversely, if $\delta{<}\delta_{\nabla}$, namely, if this insider does not weigh future profits enough, he always misleads (i.e., chooses $\bar{\alpha}^{*I}=1$). However, $\mathcal{M}$ anticipates such misleading behavior and ignores disclosures by setting μ=0. In this latter case, in equilibrium $\mathcal{I}$ trades as he would if the single period were not repeated. Finally, in the unimportant case with δ=δ_{∇},[18] the insider is indifferent toward misleading or leading, for any value of μ. For brevity, in this case, we refer only to the latter behavior, and therefore only to the most informative equilibrium.

II.B.2. Generalized case (q $\in (0,1]$): Informative equilibria with or without manipulative threat

Consider an insider who is not informed with certainty. Provided that at a certain period prices shift positively as stated in Conjecture 1, at that period the insider, when uninformed, is confronted with choosing between two relevant behaviors. If he decides to risk incurring market punishment, he finds it optimal to either buy or sell initially up to the cap on total exposure, so that the second-round price moves up or down respectively, then reverse completely this position in the second round. In this case, the uninformed insider pretends to be informed, that is, he *bluffs*. Conversely, if he decides not to risk incurring the punishment, he does not disclose any purchase or sale in that period; in this case, he *does not bluff*. Let $\bar{\beta}\in[0,1]$ be the probability with which $\mathcal{I}$ chooses to bluff as opposed to not bluffing when uninformed. The next lemma follows immediately:

Lemma 2 *Consider mandatory trade disclosure, the pricing rule in Conjecture 1, and a specific period in which μ is greater than 0. After having decided to bluff with probability $\bar{\beta}$ (i.e., to defect with probability $\frac{\bar{\beta}}{2}$), an uninformed type of insider expects to earn $\bar{\beta}\cdot B(\mu)$ in that period, where $B(\mu)$=$2\mu x_{\mathcal{I}}$.*

Proof. See Internet Appendix A. ∎

The intuition behind this result is simple. If at a certain period an uninformed type of insider decides not to bluff, he expects to earn 0 profits in that period (no matter what the second round trade is). Conversely, as long as μ is positive, by bluffing he expects to earn $B(\mu{>}0){>}0$. In particular, an insider who bluffs is indifferent about disclosing a purchase or a sale. In either case, because of the symmetry of the pricing rule, he expects to earn the same end-of-period profits. In addition, the choice between a purchase disclosure and a sale disclosure does not even impact on the likelihood that one of these disclosures accidentally causes the price to be wrong—an event that, because of the symmetry of the punishment scheme and of $f(\cdot)$, occurs with probability $\frac{\bar{\beta}}{2}$. Still, we discard any candidate equilibrium bluffing behavior, such that the probability with which $\mathcal{I}$ discloses a purchase differs from the probability with which $\mathcal{I}$ discloses a sale. In fact, the symmetric pricing rule in Conjecture 1 is justified only if beliefs in response to a purchase and to a sale disclosure are restricted to assigning the same probability to the uninformed type of insider. For this reason, if the insider chooses to bluff at the equilibrium, he randomizes with equal probability between a sale disclosure and a purchase disclosure.

When q is not restricted to equal 1, another dimension is added to the insider's problem presented in the previous subsection. At any period in which μ is still expected to be positive, $\mathcal{I}$ now must choose $\bar{\alpha}$ when informed and $\bar{\beta}$ when uninformed, taking into account that this choice causes prices to shift again, in the subsequent period, with probability $1-\bar{\alpha}$ and $1-\frac{\bar{\beta}}{2}$, respectively. In particular, the inter-temporal problem that $\mathcal{I}$ must solve differs,

[18]The case is unimportant because δ_{∇} is a point in the continuum.

depending on whether in the current period, that is, period t=1, he is informed. Thus, let $\bar{\alpha}^{*I}$ and $\bar{\beta}^{*I}$ (or $\bar{\alpha}^{*U}$ and $\bar{\beta}^{*U}$) be the levels of $\bar{\alpha}$ and $\bar{\beta}$ that maximize $E[\Pi^I]$ (resp., $E[\Pi^U]$), equal to the discounted sum of profits that $\mathcal{I}$ expects to earn over time when in period t=1 he is (resp., is not) informed. Specifically, for an insider who is currently uninformed, $\bar{\beta}^{*U}$ represents his optimal choice today if μ at the current period is positive, and also his optimal planned choice when uninformed again in any future period in which μ is expected to be positive. Instead, for a insider who is currently informed (or uninformed), $\bar{\beta}^{*I}$ (resp., $\bar{\alpha}^{*U}$) represents his optimal planned choice tomorrow, if at that future period the insider turns out to be uninformed (resp., informed) and μ is still expected to be positive. The next lemma defines $\mathcal{I}$'s best response.

Lemma 3 *Consider mandatory trade disclosure, a reputational framework, and an insider who receives new information every period with probability q. Given the pricing rule in Conjecture 1, identify the pairs $\bar{\alpha}^{*I}, \bar{\beta}^{*I} = \arg\max_{\bar{\alpha},\bar{\beta}} E[\Pi^I]$ and $\bar{\alpha}^{*U}, \bar{\beta}^{*U} = \arg\max_{\bar{\alpha},\bar{\beta}} E[\Pi^U]$, where:*

$$E[\Pi^I]=\bar{\alpha} \cdot M(\mu) + (1-\bar{\alpha}) \cdot L(\mu) + \bar{\alpha}\frac{\delta}{1-\delta} \cdot qx_{\mathcal{I}}\xi + (1-\bar{\alpha})\,\delta \cdot \mathcal{S}(q, \delta, \mu, \bar{\alpha}, \bar{\beta}), \tag{4}$$

$$E[\Pi^U] = \bar{\beta} \cdot B(\mu) + \Big(1-\frac{\bar{\beta}}{2}\Big)\delta \cdot \mathcal{S}(q, \delta, \mu, \bar{\alpha}, \bar{\beta}) + \frac{\bar{\beta}}{2}\frac{\delta}{1-\delta} \cdot qx_{\mathcal{I}}\xi, \tag{5}$$

and

$$\mathcal{S} = \frac{q[\bar{\alpha} \cdot M(\mu) + (1-\bar{\alpha}) \cdot L(\mu)] + (1-q)\bar{\beta} \cdot B(\mu) + \frac{\delta}{1-\delta}[q\bar{\alpha} + \frac{(1-q)\bar{\beta}}{2}]qx_{\mathcal{I}}\xi}{1 - \delta\,\frac{2(1-q\bar{\alpha})-\bar{\beta}(1-q)}{2}}. \tag{6}$$

If μ is greater than 0 in period t=1, then the optimal trading behavior of an informed (or uninformed) insider in period t=1 translates to the insider leading the market with probability $\bar{\alpha}^{*I}$ (resp., bluffing with probability $\bar{\beta}^{*U}$).

Derivation of $\mathcal{S}$ in Lemma 3. For an infinite horizon, with the market's pricing rule held fixed, by defecting at period t=1, the insider's expected profits from period t=2 on (discounted to t=1) equal $\frac{\delta}{1-\delta}qx_{\mathcal{I}}\xi$; by not defecting in t=1, they equal $\delta\mathcal{S}$, where $\mathcal{S}$ depends on $\bar{\alpha}$, $\bar{\beta}$, q, δ, and μ. In particular, $\mathcal{S}=\sum_{i=0}^{\infty}\delta^i W_{i+1}$, where:

$$W_1 = q[\bar{\alpha} \cdot M(\mu) + (1-\bar{\alpha}) \cdot L(\mu)] + (1-q)\bar{\beta} \cdot B(\mu), \tag{7}$$

and

$$W_{j+1} = \bar{\alpha}q^2 x_{\mathcal{I}}\xi + q(1-\bar{\alpha})W_j + (1-q)(1-\bar{\beta})W_j + \frac{(1-q)\bar{\beta}}{2}W_j + \frac{(1-q)\bar{\beta}}{2}qx_{\mathcal{I}}\xi, \forall j>1, \tag{8}$$

which also can be written as $W_{j+1}=\gamma+\varphi W_j, \forall j>1$, where $\gamma=[q\bar{\alpha}+\frac{(1-q)\bar{\beta}}{2}]qx_{\mathcal{I}}\xi$ and $\varphi=[\frac{2(1-q\bar{\alpha})-\bar{\beta}(1-q)}{2}]$. This is a first-order linear difference equation. Thus, $W_{j+1}=\gamma[\sum_{i=0}^{j-1}\varphi^i]+\varphi^j W_1=\gamma\frac{1-\varphi^j}{1-\varphi}+\varphi^j W_1$. It follows that:

$$\mathcal{S} = \sum_{i=0}^{\infty}\delta^i\Big[\varphi^i W_1 + \gamma\frac{1-\varphi^i}{1-\varphi}\Big] = \frac{W_1 + \frac{\delta}{1-\delta}\gamma}{1-\delta\varphi}. \tag{9}$$

The series converges if $|\delta\varphi|<1$, which is always verified because $0 \leq \delta < 1$ and $0 \leq \varphi \leq 1$. That is, (i) $\varphi \leq 1$ $\therefore -\bar{\beta}(1-q) \leq 2q\bar{\alpha}$, and (ii) $0 \leq \varphi \therefore 0 \leq 2(1-q\bar{\alpha}) - \bar{\beta}(1-q) \therefore q(2\bar{\alpha}-\bar{\beta}) \leq 2-\bar{\beta}$, which hold whenever

$\bar{\alpha}\in[0,1]\wedge\bar{\beta}\in[0,1]\wedge q\in(0,1]$. It is also easy to check that $\frac{\delta}{1-\delta}qx_{\mathcal{I}}\xi<\delta\cdot\mathcal{S}$, $\forall\delta>0\wedge\mu>0$. ■

Intuitively, the function $\mathcal{S}$ embeds several elements. The insider does not know whether he will be informed at each future date but knows that at any date he will have learned whether he is newly informed before signaling. In the decision process, $\mathcal{I}$ accounts for the probability of receiving new information, how much he weighs future profits, and the consequences of each signal on the direction of present and future price shifts. In particular, for an insider who is currently informed (or uninformed), his optimal choice today, $\bar{\alpha}^{*I}$ (resp., $\bar{\beta}^{*U}$), coincides with his optimal planned choice when informed (resp., uninformed) tomorrow, if prices in that future period are still expected to react to disclosure. These choices coincide because of the assumption of an insider who learns only about $\widetilde{v}<0$ or $\widetilde{v}>0$ rather than $\widetilde{v}=v$ in the first round, which simplifies the analysis. Otherwise, the multi-period problem of an insider who is currently informed—but not of one who is currently uninformed—is affected in a non-trivial way, as solved in Section V.B.2.

The next lemma defines the level of μ at which the pricing rule in Conjecture 1 is efficient.

Lemma 4 *For mandatory trade disclosure, a reputational framework, and $\mathcal{I}$ who receives information with probability q in every period and trades optimally given the pricing rule in Conjecture 1, the market efficiency condition holds for $\mu=\mathbf{1}(\bar{\alpha}^{*I}<\frac{1}{2})[1-(1-q)\bar{\beta}^{*U}](1-2\bar{\alpha}^{*I})\xi$.*

Beliefs formed in response to current disclosures account directly for the current and indirectly for the planned choices by an insider aware about prices that move as stated in Conjecture 1. In the present period, disclosures are informative (μ is positive) only if $\bar{\alpha}^{*I}<\frac{1}{2}$, that is, if an insider who is currently informed leads with probability greater than $\frac{1}{2}$. In this case, provided $\bar{\beta}^{*U}=0$, a level of μ equal to $1-2\bar{\alpha}^{*I}$ ensures efficient pricing. This level must be reduced (i.e., multiplied by $1-(1-q)\bar{\beta}^{*U}$) if $\bar{\beta}^{*U}>0$, that is, if the insider bluffs with positive probability when currently uninformed.

The next proposition presents the closed-form solution to the general problem for $q\in(0,1]$ in markets with mandatory trade disclosure. The structure underlying this problem is that of a new, important class of supergame—more precisely, of infinitely repeated games with discounting. In the next section, we extend this critical result and provide some commentary.

Proposition 3 *For mandatory trade disclosure and a reputational framework, three regions over the space in $\delta\in[0,1)$ and $q\in(0,1]$ are identified. They correspond to three classes of equilibria in which $\mathcal{M}$ undertakes the strategy in Conjecture 1. In detail, (1) if $\delta\geq\Delta(q,\mu=\xi)$, in each period, μ equals ξ, and $\mathcal{I}$ leads when informed and does not bluff when uninformed (i.e., $\bar{\alpha}^{*I}=\bar{\beta}^{*U}=0$); (2) if $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$, then up to the j^{th} repetition, μ equals $q\xi$ and $\mathcal{I}$ leads when informed and bluffs when uninformed (i.e., $\bar{\alpha}^{*I}=0,\bar{\beta}^{*U}=1$), where j is the period in which $vp_2<0$, and from period $j+1$ on, the equilibrium coincides with the single period equilibrium; and (3) if $\delta<\nabla(q,\mu=q\xi)$, in each period, μ equals 0, and $\mathcal{I}$ reverts to single-period equilibrium behavior. Specifically, $\Delta(q,\mu)$ equals $\frac{B(\mu)}{B(\mu)+\frac{q}{2}[L(\mu)-x_{\mathcal{I}}\xi]}$ and $\nabla(q,\mu)$ equals $\frac{M(\mu)-L(\mu)}{\frac{1+q}{2}M(\mu)-\frac{1-q}{2}L(\mu)+(1-q)B(\mu)-qx_{\mathcal{I}}\xi}$.*

Proof. See Internet Appendix A. ■

The logic of the result is as follows: Holding fixed any specific value of μ greater than 0, notice that, (1) in case $\delta \geq \Delta(q, \mu{>}0)$, the pairs $\bar{\alpha}^{*I}$=0,$\bar{\beta}^{*I}$=0 and $\bar{\alpha}^{*U}$=0,$\bar{\beta}^{*U}$=0 maximize the functions $E[\Pi^I]$ and $E[\Pi^U]$, respectively. Thus, $\mathcal{I}$'s optimal choice is to lead when informed and not to bluff when uninformed. Given this insider's behavior, a level of μ equal to ξ guarantees price efficiency. At this level of μ, the insider does not deviate from the derived strategy. Consequently, if $\delta \geq \Delta(q, \mu{=}\xi)$, equilibrium disclosures are fully informative and no manipulation arises. (2) In case $\nabla(q, \mu{>}0) \leq \delta \leq \Delta(q, \mu{>}0)$, the pairs $\bar{\alpha}^{*I}$=0,$\bar{\beta}^{*I}$=1 and $\bar{\alpha}^{*U}$=0,$\bar{\beta}^{*U}$=1 maximize $E[\Pi^I]$ and $E[\Pi^U]$, respectively. Hence, $\mathcal{I}$'s best reply is to lead when informed and bluff when uninformed, providing that in the period, prices are known to react to disclosures. Given this trading strategy, a level of μ equal to $q\xi$ guarantees price efficiency. At this level of μ, no deviation by $\mathcal{I}$ occurs. It follows that, (2.a) if q=1 and $\nabla(q{=}1, \mu{=}\xi) \leq \delta \leq \Delta(q{=}1, \mu{=}\xi)$, no manipulation occurs, so equilibrium disclosures are again fully informative. Conversely, (2.b) if $q{<}1$ and $\nabla(q{<}1, \mu{=}q\xi) \leq \delta \leq \Delta(q{<}1, \mu{=}\xi)$, equilibrium disclosures are partially informative until a manipulative attempt causes the price to shift in the wrong direction, an event that occurs by the end of the k^{th} period with probability $1-(\frac{1+q}{2})^k$.[19] After period j, at the end of which $\mathcal{M}$ observes $vp_2{<}0$ and records the manipulation, $\mathcal{I}$ stops leading when informed and bluffing when uninformed, and reverts to single-period equilibrium behavior. (3) In case $\delta \leq \nabla(q, \mu{>}0)$, the arguments maximizing the two functions do not always coincide, but still, $\bar{\alpha}^{*I}$=$\bar{\alpha}^{*U}$=1. This means that, if prices shifted, $\mathcal{I}$ would always mislead when informed. Thus, $\mathcal{M}$ anticipates $\mathcal{I}$'s misleading behavior, which is why in equilibrium $\mathcal{M}$ ignores disclosures and $\mathcal{I}$ trades as he does in the single period. Here and below, if the insider is indifferent about misleading and leading (or about bluffing and non-bluffing), we refer only to the latter behavior. Therefore, for any unimportant pair δ=$\Delta(q{<}1, \mu{>}0), q{<}1$ or δ=$\nabla(q, \mu{>}0), q$, we refer only to the most informative equilibrium.

Note that the functions $\nabla(q, \mu)$ and $\Delta(q, \mu)$ are continuous, and that $\nabla(q{=}1, \mu{=}\xi)$ is equal to δ_∇ (derived in the benchmark case) and smaller than $\Delta(q{=}1, \mu{=}\xi)$. Also note that $\lim_{q\to 0} \Delta\,(q, \mu{=}\xi) \to 1$ and $\frac{\partial(\Delta(q,\mu=\xi))}{\partial q}{<}0$. Figure 2 contains an example with $\widetilde{v}{\sim}$U$[-1, 1]$ to provide a graphical idea of the closed-form solution to the issue.

[See Fig. 2.]

Two final remarks are in order. First, starting from any pair $\delta \simeq 1$ and $q \in (0, 1)$ associated with an informative equilibrium without manipulation, by gradually decreasing δ, at some point a switch in the equilibrium always occurs, to one in which $\mathcal{I}$ still has no incentive to mislead but has an incentive to bluff. This is due to the fact that, for any positive μ, the *overall* incentive that an informed $\mathcal{I}$ has to mislead (rather than lead) today is smaller than the one that the same insider has to bluff (rather than not bluff) today when uninformed. On the one hand, in each period, the *extra*-payoff from misleading, $[M(\mu){-}L(\mu)]$, is smaller than that from bluffing, $[B(\mu) - 0]$.[20] On the other hand, misleading implies a punishment with certainty, whereas bluffing implies a defection only with probability $\frac{1}{2}$. In conclusion, for $q \in (0, 1)$, whenever δ assumes values just below

[19] For k=1, the probability of a defection equals $\epsilon = \frac{1-q}{2}$. For k=2, it equals $\epsilon + \epsilon(1-\epsilon)$, that is, the probability of defection today plus that of a defection in period t=2, provided a punishment has not yet occurred. By the end of period t=k, a defection occurs with probability $\epsilon + \epsilon(1-\epsilon) + ... + \epsilon(1-\epsilon)^{k-1}$=$\epsilon\frac{1-(1-\epsilon)^k}{1-(1-\epsilon)}$=$1-(\frac{1+q}{2})^k$.

[20] Actually, $M(\mu){-}L(\mu){<}B(\mu) \therefore 2x_{\mathcal{I}}[\int_0^\mu 2vh(v)dv{+}\int_\mu^{\bar{b}} 2\mu h(v)dv]{<}2\mu x_{\mathcal{I}} \therefore \int_0^\mu (\mu - v)h(v)dv{>}0$, for all $\mu{>}0$.

$\Delta(q<1, \mu=\xi)$, at the equilibrium $\mathcal{I}$ continues leading when informed but starts bluffing when uninformed. Second, disclosures are both informative and non-manipulative if and only if δ is greater than $\Delta\,(q=1, \mu=\xi)$ and, at the same time, $\mathcal{I}$ is sufficiently likely to become informed (i.e., q is sufficiently high). In fact, because the threshold function $\Delta\,(q, \mu=\xi)$ is decreasing in q, only those pairs δ and q lying on the right of this threshold are associated with informative equilibria without manipulation. In particular, the intuition behind the negative slope of $\Delta\,(q, \mu=\xi)$ is as follows: The higher the probability that $\mathcal{I}$ becomes informed in future periods, the more often $\mathcal{I}$ can lead and earn more than he could under anonymity, if disclosures are informative in these periods, and consequently, the smaller the weight that $\mathcal{I}$ has to assign to future profits to choose to manipulate when uninformed in the current repetition (causing future disclosures not to be informative with a 50% chance).

III. Foundation of mandatory and voluntary disclosure

We start with a focus on mandatory and voluntary trade disclosures, then extend the study to the voluntary disclosure of ex-ante unverifiable information.

III.A. Voluntary vs. mandatory trade disclosure

To study the foundation of mandatory and voluntary *trade* disclosures, and highlight the role of the position limit to which $\mathcal{I}$ is subject, together with the role of the asset value properties, we start with a comparison against the benchmark model of FH. The propositions below refer to our insider, who is constrained in asset holdings.

In FH, for a disclosure to be forthcoming, it must be mandatory, because disclosures reduce the profits of the informed insider. For a single period of $n \in \{1, 2\}$ rounds, assume that an insider, informed with probability q, can trade a (divisible) unit $x_{\mathcal{I}}$ per round, that his orders have no direct price impact, and that $\widetilde{v}\in\{\underline{b},\overline{b}\}$ has equally likely priors. Under mandatory trade disclosure, in equilibrium $p_1=0$, and in response to a sale (or purchase) disclosure, $p_2=q\underline{b}$ (resp., $p_2=q\overline{b}$). At these prices, an insider aware of $\widetilde{v}<0$ sells $x_{\mathcal{I}}$ twice, whereas one aware of $\widetilde{v}>0$ purchases $x_{\mathcal{I}}$ twice. In both cases though these types of insider earn less than they would under market anonymity. Instead, an uninformed insider randomizes with equal probability between trading $x_1=x_{\mathcal{I}}$, $x_2=-x_{\mathcal{I}}$ versus $x_1=-x_{\mathcal{I}}$, $x_2=x_{\mathcal{I}}$, earning an end-of-period expected profit equal to $x_{\mathcal{I}}q\overline{b}>0$. Because the informed insider's loss from disclosure is equal in magnitude to the uninformed insider's gain, the profits that the insider expects to earn before knowing whether he is informed or uninformed are higher under mandatory trade disclosure (compared with market anonymity) only if $q<\frac{1}{2}$.

In contrast, in our model under mandatory trade disclosure, the equilibrium end-of-period expected profits of any type of insider are equal to or greater than those under market anonymity. Thus, if an insider with constrained asset holdings were to choose between trading under a mandatory trade disclosure rule and trading under anonymity, he would always at least weakly prefer the former option. Intuitively, this is why, for disclosure to be forthcoming, it does not have to be mandatory. The next proposition presents the equilibria for the case in which our insider can voluntarily decide whether to disclose each purchase or sale.

Proposition 4 *For voluntary trade disclosure, in the single period, a unique price equilibrium exists in which* $p_n=0$*; an insider aware of* $\widetilde{v}<0$ *and one aware of* $\widetilde{v}>0$ *disclose the same signal* τ *with equal prob-*

ability, trading in such a way that $\sum_n x_n = -x_{\mathcal{I}}$ *and* $\sum_n x_n = x_{\mathcal{I}}$*, respectively; and an uninformed insider attaches any probability to any signal* τ*, trading in such a way that* $\sum_n x_n \in [-x_{\mathcal{I}}, x_{\mathcal{I}}]$*. Within the reputational framework, informative equilibria exist in which* $\mathcal{M}$ *undertakes the strategy in Conjecture 1. Specifically, if* $\delta \geq \Delta(q,\mu=\xi)$*, in each period,* μ *equals* ξ*, and an insider who is aware of* $\widetilde{v}<0$ *(or* $\widetilde{v}>0$*) trades as if under mandatory disclosure, voluntarily revealing his initial trade, whereas an uninformed insider does not disclose. If* $\nabla(q,\mu=q\xi) \leq \delta < \Delta(q,\mu=\xi)$*, in each period up to the* j^{th} *repetition,* μ *equals* $q\xi$*, and the insider, when informed, trades and discloses as before, but when uninformed, he randomizes with equal probability between selling and purchasing up to the maximum cap, then discloses the initial trade, and finally reverses the initial position completely; from period* $j+1$ *on, the equilibrium coincides with the single period equilibrium.*

Proof. See Internet Appendix A. ■

Compared with a mandatory trade disclosure, under a voluntary trade disclosure, the conditions for an equilibrium in which informative trades arise are harder to satisfy. The signal $\tau=0$ is more opaque than when disclosures are mandatory, and therefore, in the single period, no pricing rule with prices that react to voluntarily disclosed trades (or to the absence of disclosure) is part of an equilibrium. To see it, consider any of these pricing rules, and derive the optimal response of an insider who is allowed (but not mandated) to disclose trades and who observes $\widetilde{v}=v$ already in the first round. Given the insider's optimal response, notice that $\mathcal{M}$ turns out to reply to *all* types of insider aware of $\widetilde{v}=v<0$ or $\widetilde{v}=v>0$ with prices that move in the wrong direction. For the same reasons adduced for the case of a mandatory disclosure, the pricing rule in question is not justified, no matter whether an informed $\mathcal{I}$ observes $\widetilde{v} \gtrless 0$ or $\widetilde{v}=v$ in the first round. In equilibrium, the probability that these informed types of insider place on a first round sale (or absence of trade, or purchase) is not necessarily the same. However, the probability that they signal $\tau=-1$ (or $\tau=0$, or $\tau=1$) is identical and can take any value from 0 to 1. In this way, the revelation of inside information is eliminated, and $p_n=0$ is justified. Finally, consider the class of equilibria in which an insider aware of $\widetilde{v}<0$ and an insider aware of $\widetilde{v}>0$, respectively, trade $-x_{\mathcal{I}}$ and $x_{\mathcal{I}}$ in the first round but do not disclose. This class of equilibria is robust to a small probability that $\mathcal{M}$ exogenously learns v in advance.

Within the reputational framework, provided the weight assigned to future periods is sufficient (more precisely, provided $\nabla(q,\mu=q\xi) \leq \delta$), in equilibrium the insider voluntarily discloses trades that are informative, earning at least as much as with marker anonymity, whether informed and uninformed. Specifically, until defection, voluntarily disclosed trades (as a function of the realizations of $\widetilde{s}$ and $\widetilde{v}$) and prices (as a function of disclosures) are identical to those in Proposition 3, because the relevant payoff structure[21] coincides with the one analyzed under mandatory disclosure. Suppose that today μ is positive, and consider first an insider who is currently aware of $\widetilde{v}>0$ (the case in which the insider is aware of $\widetilde{v}<0$ is symmetric). The only case in which this insider incurs a market punishment is when he voluntarily discloses a sale. In this case, by initially selling as much as possible and buying back up to the total exposure cap in the second round, he expects to earn the most, namely, $M(\mu>0)$, in the current period. Alternatively, if this type of insider does not aim to incur the

[21]The term *relevant* refers to the end-of-period payoff that $\mathcal{I}$ achieves from misleading, leading, bluffing, and not bluffing, as well as the implications that the pursuit of one specific payoff or another has on the probability of a punishment.

punishment, he can choose between two main options: not disclosing any trade (which does not prevent him from placing either a buy or a sell order initially) or disclosing a purchase. The latter option is more profitable, provided the insider initially purchases up the maximum and completely reverses the initial position if $v<p_2$. Then the expected profit equals $L(\mu>0)$. Next, consider an insider who is currently uninformed. This type of insider can bluff (disclosing either a purchase or sale), in which case he risks incurring the punishment but expects to earn $B(\mu>0)$ by the end of the period. Alternatively, he can avoid disclosure, and then is certain about not incurring the punishment but expects to earn 0 profits in that period.[22] In conclusion, the four relevant insider options parallel those under mandatory disclosure.

The next proposition highlights the general asset value properties that drive the model's equilibria derived so far, when the insider is constrained on asset holdings. To ease the task of understanding the proposition, consider any symmetric distribution of $\widetilde{v}$ centered around 0 (a realization that, for the time being, is assumed not to be possible), and define, with $r>0$, the realization of $\widetilde{v}$ that is closest to 0 from the right. Three observations regarding the repeated framework are in order. First, for $\delta \geq \Delta(q,\mu{=}\xi)$, at a specific period, the disclosure by an informed insider moves the price in the direction of the real value, whereas the disclosure by an uninformed insider does not take place, provided that μ is positive and $f(\,\cdot\,)$ is such that $r<\xi$, two conditions that ensure an incentive for informed types of insider to lead the price toward the real value. In fact, an insider who learns $-\xi<v<0$ and one who learns $0<v<\xi$ both earn more than under anonymity by reversing their initial position, trading at $P_2(\tau{=}-1){=}-\xi$ and $P_2(\tau{=}1){=}\xi$, respectively. Second, for $\nabla(q,\mu{=}q\xi) \leq \delta<\Delta(q,\mu{=}\xi)$, the uninformed insider also discloses his trade (to manipulate market beliefs), provided μ is positive and $f(\cdot)$ is now such that $r<q\xi$, two conditions that make an insider who learns $-q\xi<v<0$ and one who learns $0<v<q\xi$ both increase their profits by reversing the initial position, trading at $P_2(\tau{=}-1){=}-q\xi$ and $P_2(\tau{=}1){=}q\xi$, respectively. Third, for $\delta \geq \Delta(q,\mu{=}\xi)$ and $r \nless \xi$ (or for $\nabla(q,\mu{=}q\xi) \leq \delta<\Delta(q,\mu{=}\xi)$ and $r \nless q\xi$), the only equilibrium that arises is the one with uninformative disclosures. The result is formalized and generalized as follows:

Proposition 5 *Relax the requirement of a symmetric $f(\,\cdot\,)$ or of a $F(\,\cdot\,)$ being absolutely continuous, and consider the following restrictions:*

$$R1: \ \Pr(\widetilde{v}<0){=}\Pr(\widetilde{v}>0){=}\tfrac{1}{2}, \qquad R2: \ \Pr(-\gamma<\widetilde{v}<0){=}\Pr(0<\widetilde{v}<\gamma)\neq 0,$$

$$R3: \ E[\widetilde{v}\mid \underline{b}\leq \widetilde{v} \leq -\gamma]{=}-E[\widetilde{v}\mid \gamma \leq\widetilde{v}\leq\overline{b}], \qquad R4: \ E[\widetilde{v}\mid -\gamma<\widetilde{v}<0]{=}-E[\widetilde{v}\mid 0<\widetilde{v}<\gamma],$$

where γ equals ξ if $\delta \geq \Delta(q,\mu{=}\xi)$ (or $q\xi$ if $\nabla(q,\mu{=}q\xi) \leq \delta<\Delta(q,\mu{=}\xi)$). Under restrictions R1 to R4, all the preceding equilibrium results still hold. Those in the single period only require R1 to be satisfied.

Notice that $|\underline{b}|$ does not have to equal $\overline{b}$; even $f(\,\cdot\,)$ does not have to be symmetric. In addition, *R1* has two implications. On the one hand, it ensures an equal probability mass below and above $E[\widetilde{v}]$, a restriction that is sufficient to guarantee that the results in the single period hold. For example, the proof of Proposition 1 does not rely on the support of $\widetilde{v}$ being continuous, on the number of realizations of $\widetilde{v}$ below and above 0 being equal, on the distance between each realization of $\widetilde{v}$ and 0, or on whether a realization of $\widetilde{v}$ below or above 0 is more

[22] If an insider who is currently uninformed signals $\tau{=}0$, his current period expected profits do not depend on whether he trades. Thus, for $\delta \geq \Delta(q,\mu{=}\xi)$, the pre-defection equilibrium trading behavior can differ from that in Proposition 3.

likely than another realization lying on the same side of the support. On the other hand, *R1* implicitly tells us that $\widetilde{v}$=0 is either a zero-probability event or simply not possible, depending on whether the support of $\widetilde{v}$ is continuous around the initial price. When informed, the insider thus is clearly aware of whether the real asset value is below or above 0, and no ambiguity arises about whether a trade disclosure pushed the market price in the wrong direction. For the results in Proposition 3 and 4 to hold, *R2–R4* are necessary to ensure that, at the beginning of a period, an insider aware of $\widetilde{v}<0$ and one aware of $\widetilde{v}>0$ expect to achieve the same end-of-period extra-payoff from misleading. In particular, *R2* is necessary to allow for possible equilibrium price overshooting, which gives the insider an incentive to send meaningful signals and to disclose voluntarily. In turn, *R2* implicitly requires the existence of at least four realizations of $\widetilde{v}$, two smaller than 0, and two greater. Specifically, for $\delta \geq \Delta(q,\mu=\xi)$ (or for $\nabla(q,\mu=q\xi) \leq \delta<\Delta(q,\mu=\xi)$), at least one realization of $\widetilde{v}$ must lie below $-\xi$, at least one must lie over both $(-\xi, 0)$ and $(0, \xi)$ (resp., over both $(-q\xi, 0)$ and $(0, q\xi)$), and at least one must be greater than ξ. In contrast, the results in the single period hold even if only two realizations are possible, one smaller and one greater than 0. This means that, for a random variable $\widetilde{v}\in\{\underline{b},\bar{b}\}$ with equally likely priors, systematic manipulation by an uninformed insider in the FH benchmark model is driven by the ability to trade up to a unit per round. If the insider's ability to trade gets constrained on asset holdings, disclosures become no longer informative, and manipulations no longer occur. Furthermore, if the asset value proprieties also change, so that *R1–R4* are satisfied, equilibrium price overshooting becomes possible, and with it, the (voluntary) disclosure of informative and, in some instances, manipulative signals.

III.B. Voluntary disclosure of ex-ante unverifiable information

This subsection considers the voluntary disclosure of ex-ante unverifiable messages when the market interprets these messages in up to three distinctive ways (for more than three distinctive ways, see Section V.A and end of Section V.B.2). Unlike for the case of a trade disclosure—where τ=-1 and τ=1 imply that the insider sold and bought, respectively—ex-ante unverifiable messages are not a priori associated with any trade undertaken. Their disclosure is equivalent to the disclosure of non-necessarily truthful trades. Therefore, ex-ante unverifiable messages are trickier to interpret.

Consider a setting in which trades are unobservable. The insider sends an observable message $\phi_{i,m}\in \Phi_m \subset \Phi$, $m \in \{-1, 0, 1\}$, at the beginning of each period second round, so that P_2: $\Phi \rightarrow \mathcal{V}$, where $\phi_{i,m}$ is a priori not correlated with any trade, Φ_m is non-empty, $\Phi_m \cap \Phi_{\neg m}$=$\varnothing$, and Φ is the universe of non-costly (verbal or non-verbal) messages.[23] In this setting, if the period is not repeated, the insider's messages are not informative at the equilibrium. In fact, for any pricing rule with prices that react somehow to a specific message or another, and the insider's associated best response, the pricing rule in question turns out to be wrong in expectation. Conversely, within the reputational framework, the insider's messages may be informative at the equilibrium. To construct an informative equilibrium, consider the following strategy for $\mathcal{M}$:

Conjecture 2 *Suppose $\mathcal{M}$'s strategy is to set p_1=0 and p_2=$P_2^{\mu'}(\cdot)$ in the first period, where $P_2^{\mu'}(\phi_{\cdot,-1})$=$-\mu'$,*

[23]Inactivity by an insider who does not send any message is a signal per se.

$P_2^{\mu'}(\phi_{\cdot,0})$=0, *and* $P_2^{\mu'}(\phi_{\cdot,1})$=$\mu' \geq 0$. *In the second round of the* t^{th} *period, if the outcome of all* $t-1$ *preceding periods has been* $\phi_{\cdot,-1}\wedge v<0$ *or* $\phi_{\cdot,0}$ *or* $\phi_{\cdot,1}\wedge v>0$, *then play* $P_2^{\mu'}$; *otherwise, set* p_2=0.

The equilibria with ex-ante unverifiable information disclosure are presented in Proposition 6:

Proposition 6 *For ex-ante unverifiable information disclosure, under R1, in the single period a unique price equilibrium exists in which* p_n=0; *an insider aware of* $\widetilde{v}<0$ *and one aware of* $\widetilde{v}>0$ *disclose* $\phi_{i,m}$ *with equal probability, trading in such a way that* $\sum_n x_n$=$-x_{\mathcal{I}}$ *and* $\sum_n x_n$=$x_{\mathcal{I}}$, *respectively; and an uninformed insider attaches any probability to any signal* $\phi_{\cdot,m}$ *and to any trade quantity* x_n. *Under R1–R4, within the reputational framework, informative equilibria exist in which* $\mathcal{M}$ *undertakes the strategy in Conjecture 2. Specifically, if* $\delta \geq \Delta(q,\mu=\xi)$, *in each period,* μ' *equals* ξ, *an insider who is aware of* $\widetilde{v}<0$ *(or* $\widetilde{v}>0$*) trades as under a trade disclosure rule, and signals* $\phi_{\cdot,-1}$ *(resp.,* $\phi_{\cdot,1}$*), whereas an uninformed insider signals* $\phi_{\cdot,0}$. *If* $\nabla(q,\mu=q\xi) \leq \delta<\Delta(q,\mu=\xi)$, *up to the* j^{th} *repetition,* μ' *equals* $q\xi$, *and the insider, when informed, trades and signals as before, whereas when uninformed, randomizes with equal probability between selling up to the maximum and signaling* $\phi_{\cdot,-1}$ *on one side and purchasing up to the maximum and signaling* $\phi_{\cdot,1}$ *on the other, then reverses the initial position completely; from period* j+1 *on, the equilibrium coincides with the single period equilibrium.*

In the single period, $\mathcal{M}$ is unable to extract meaningful information even if the insider is mandated to disclose trades. Therefore, in a setting in which the insider's messages are not tied to a specific trade, the result cannot be other than confirmed. Intuitively, this is why, in all single-period equilibria with ex-ante unverifiable information disclosure, prices do not move. Of all these equilibria, the ones in which an insider aware of $\widetilde{v}<0$ and one aware of $\widetilde{v}>0$ respectively trade $-x_{\mathcal{I}}$ and $x_{\mathcal{I}}$ in the first round display robustness to a small probability that $\mathcal{M}$ exogenously learns v in advance.

Within the reputational framework, for each pair δ and q such that $\nabla(q,\mu=q\xi) \leq \delta$, in equilibrium the ex-ante unverifiable messages that the insider sends before defection are informative and allow him to earn as much or more than he does under anonymity. Specifically, with respect to the reputational equilibrium derived under a trade disclosure rule, before defection the insider signals $\phi_{\cdot,-1}$ (or $\phi_{\cdot,0}$, or $\phi_{\cdot,1}$) instead of τ=-1 (resp., τ=0; τ=1), and the price in response to the ex-ante unverifiable message in question reacts as it would following the disclosure of a sale (resp., absence of disclosure, disclosure of a purchase). The analogy with the reputational equilibrium achieved under a trade disclosure rule is due to the fact that the relevant payoff structures coincide. Suppose that today μ' is positive, and consider an insider who observes $\widetilde{v}>0$, for example. If this insider chooses to incur market punishment, it is optimal for him to sell initially up to the total exposure cap, signal $\phi_{\cdot,-1}$, and then buy back up to the maximum cap, which implies a current period expected profit equal to $M(\mu'$=$\mu>0)$. Instead, if this insider chooses not to incur market punishment, it is optimal for him to purchase initially up to the total exposure cap, signal $\phi_{\cdot,1}$, and reverse completely the initial position if $v<p_2$, which implies a current period expected profit equal to $L(\mu'$=$\mu>0)$. Then consider an insider who is currently uninformed. If this insider chooses to risk the punishment, it is optimal for him to trade and signal either x_1=$-x_{\mathcal{I}}$ and $\phi_{\cdot,-1}$ or x_1=$x_{\mathcal{I}}$ and $\phi_{\cdot,1}$, then reverse completely the initial position, which implies a current period expected profit equal to $B(\mu'$=$\mu>0)$. Conversely, if the uninformed insider chooses to avoid the punishment with certainty, he can only signal $\phi_{\cdot,0}$, which allows him to earn 0 current period expected profits. Finally notice that at each informative equilibrium, all messages $\phi_{\cdot,-1} \in \Phi_{-1}$ sent before defection convey information about a decrease of the real asset value, which is why we classify these messages as *unfavorable*; for the opposite reason, we classify all messages $\phi_{\cdot,1} \in \Phi_1$ sent before defection as *favorable*.

With reference to VB's study, often cited when referring to an insider trader who spreads rumors (see, e.g., Kyle and Viswanathan 2008), the structure proposed herein is more general and allows for several innovative existence results. The two models in VB are more a characterization of a pure strategy equilibrium rather than a proof of existence, and for different reasons, they are not quite right (see Internet Appendix B). The present work contributes to this strand of literature by establishing a firmer foundation for information-based manipulations.

IV. Robustness (Part I): Market beliefs

This section focuses on market beliefs and highlights how the informative equilibria derived above are robust in two regards.

Within the reputational framework, for the same pair δ and q, an unlimited number of alternative trigger strategies can be part of an equilibrium. On the one hand, the way prices shift following the same disclosures can differ. On the other hand, equilibria exist in which prices start shifting again at some point following a defection. As a sort of guided tour through this wide universe of multiple equilibria, Internet Appendix C identifies four minimal restrictions on market beliefs, such that *if* any equilibrium price shift occurs at period t, then the way the price reacts in response to a specific signal, disclosed at period t, is unique and equal to $q\xi$ or ξ in magnitude, depending on whether $\nabla(q, \mu{=}q\xi) \leq \delta{<}\Delta(q, \mu{=}\xi)$ or $\delta \geq \Delta(q, \mu{=}\xi)$ respectively; we term this result *price-shift uniqueness.*

Rather than a unique $\mathcal{M}$, let us now consider more non-cooperative auctioneers, each of whom has to set the efficient price at one or more distinct rounds. The aim of this exercise is to reveal that even under this alternative specification, the equilibrium dynamics are unaffected. An uninformative equilibrium always exists in which the current auctioneer ignores disclosures, irrespective of how future auctioneers are believed to respond. More important, informative equilibria analogous to those achievable with a unique $\mathcal{M}$ also always exist. Consider an informative equilibrium in which $\mathcal{M}$ undertakes a Grim trigger strategy for example. An analogous equilibrium exists in which, along the equilibrium path, the price responses by future auctioneers imply a punishment equivalent to $\mathcal{I}$'s intrinsic misbehavior against past auctioneers. This analogous equilibrium arises because all agents share the awareness that post-defection prices are set by auctioneers who disregard disclosures, which justifies beliefs about pre-defection disclosures being informative. Given the third condition in the standard definition of equilibrium employed herein, it is the awareness about any implied punishment-equivalent price response by those auctioneers acting in the future that supports the response by the current auctioneer, when the latter believes that the history of disclosures currently observed is somehow informative.

V. Robustness (Part II): Inside information arrival and trade size disclosure

This section discusses alternative versions of our model. In particular, we relax two restrictive assumptions: (1)

a quality improvement in the information possessed by an informed insider (from $\widetilde{v}<0$ or $\widetilde{v}>0$ observed in n=1, to $\widetilde{v}$=v observed in n=2) and (2) a public disclosure about the direction of trade but not about its size. We show that several equilibria exist, the outcomes of which are in line with those derived so far.

The first assumption is relaxed by analyzing an insider who, when informed, observes $\widetilde{v}$=v from the first round. The model is such that an equilibrium characterization is possible, both under a regulation about trade disclosure and under voluntary ex-ante unverifiable information disclosure.

When we twist the second assumption, our structure is sufficient to account for the full range of consequences that four regulations (alternatives to the mandatory/voluntary disclosure of trade direction) imply: mandatory trade size disclosure, voluntary trade size disclosure, voluntary trade size disclosure when revelation of trade direction is mandatory, and voluntarily disclosure of trade direction, trade size, or nothing.

V.A. Robustness of the single-period uninformative equilibrium

This subsection examines a period comprised of any finite sequence of rounds, $n \in \{1,..,N\}$, in which $\mathcal{I}$ trades x_1 and $x_{n>1}\in[-x_{\mathcal{I}}-\sum_{i=1}^{n-1}x_i, x_{\mathcal{I}}-\sum_{i=1}^{n-1}x_i]$ according to the strategy $\langle X_1,..,X_N\rangle$. For any possible non-degenerate random variable $\widetilde{v}\in\mathcal{V}$, regardless of whether in the first round an informed insider learns only $\widetilde{v}\gtrless E[\widetilde{v}]$ or $\widetilde{v}$=v, a unique price equilibrium *exists* in which the market ignores disclosures, setting $p_{n\in\{1,..,N\}}$, the price at each round, equal to $E[\widetilde{v}]$. This equilibrium holds for any combination of the provision of order direction and order size disclosures considered in this work, as well as in the case of an insider who sends ex-ante unverifiable messages that the market can interpret in any distinctive way. In equilibrium, any insider aware of $\widetilde{v}<E[\widetilde{v}]$ (or $\widetilde{v}>E[\widetilde{v}]$) trades in such a way that $\sum_n x_n$=$-x_{\mathcal{I}}$ (resp., $\sum_n x_n$=$x_{\mathcal{I}}$), and his disclosure at a specific round is in probability identical to the disclosure that any other type of informed insider would make in the same round, whereas the uninformed insider places any probability on all trade quantities and disclosures.

To understand why this equilibrium exists, suppose that $\mathcal{M}$ believes disclosures to be uninformative and therefore ignores them, setting $p_{n\in\{1,..,N\}}$ equal to $E[\widetilde{v}]$. Holding this pricing rule fixed, note that at any round but the last one, any specific type of insider is indifferent about trading one quantity or another (even 0), provided he trades optimally in round N. Each of these alternative sequences of trades is part of a best reply, because it is not possible for this type of insider to earn more otherwise. It follows that, provided all types of insider signal identically, the pricing rule in question is justified.

V.B. Robustness of the informative equilibria of the reputational framework

This subsection examines an infinitely repeated two-round period. To ease exposition, we refer to a real asset value, $\widetilde{v}$, whose properties are those defined in Section I, and because we deal with Grim trigger strategies, to $\mathcal{I}$'s strategy and $\mathcal{M}$'s pricing rule before defection (if any).

V.B.1. Insider's gradual learning under trade size disclosure

Consider an insider who, when informed, learns $\widetilde{v}\gtrless 0$ in the first round and $\widetilde{v}$=v in the second. Also consider a Grim trigger strategy with the following characteristics: First, the insider is thought of as defecting when, at the end of a certain period, it happens that $p_2 v<0$. Second, at *each* second round before defection, the function P_2 is such that a revelation of a purchase (or of a specific purchased quantity) causes a positive price shift, equal in magnitude to the negative shift following the revelation of a sale (resp., of an identical quantity, when sold); for a regulation that mandates (or allows for) trade size revelation, P_2 is non-decreasing in the disclosed quantity x_1; and absence of any trade disclosure causes the price not to shift.

For any of the four alternative regulations dealing with the trade size disclosure listed in the beginning of Section V, and for each pair δ and q such that $\nabla(q, \mu{=}q\xi) \leq \delta$, at least one equilibrium with informative disclosures exists, whose outcome in terms of traded quantities (as a function of the realizations of $\widetilde{s}$ and $\widetilde{v}$) and prices (as a function of traded quantities) is *identical* to that proposed in Proposition 3. If the regulation allows for the sole voluntary disclosure of trade size, the insider discloses these traded quantities voluntarily. If the regulation allows for a voluntary disclosure of trade size when revelation of the trade direction is mandatory (or for a voluntary disclosure of trade direction, trade size, or nothing), the insider discloses the direction of these trades and, depending on the specification of the function P_2, also their size.[24]

V.B.2. Insider's immediate learning under any disclosure setting (non-trivial)

Here we consider an insider who, when informed, already learns $\widetilde{v}{=}v$ in the first round, and explain that, for any of the alternative signaling channels studied so far, three regions over the space in $\delta \in [0,1)$ and $q \in (0,1]$, characterized by high, intermediate, and low values of δ, can be identified—call them upper, intermediate, and lower regions, respectively. For each pair δ and q lying over the upper (or intermediate; or lower) region, an equilibrium with fully (resp., partially; non-) informative disclosures exists in which the pricing rule and the insider's strategy coincide with those employed for $\delta \geq \Delta(q, \mu{=}\xi)$ (resp., for $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$; for $\delta < \nabla(q, \mu{=}q\xi)$) by the market and by an insider who, when informed, only observes $\widetilde{v} \gtrless 0$ at the first round.

To simplify the exposition, we focus on a regulation that mandates revelation of trade direction but prevents revelation of its size, and consider the strategy in Conjecture 1. Regarding the voluntary disclosure of the sole trade direction (or any regulations dealing with trade size disclosure listed in the beginning of Section V, or the disclosure of ex-ante unverifiable information), the reasoning is analogous, because the relevant payoff structures are equivalent.

As for the case examined in Section II, which differs from the one considered in terms of the rate of arrival of inside information, the insider's expected extra profits from bluffing when uninformed in any period (or from misleading when informed in any *future* period) are identical across periods. However, unlike the case studied in Section II, the insider's extra profits from misleading when informed in the *current* period depend on $\widetilde{v}{=}v$. Thus, for each insider who observes a specific realization $\widetilde{v}{=}v$ at the first round of the current period, the solution to his multi-period profit-maximization problem requires distinguishing his specific optimal choice in period $t{=}1$ from the optimal planned choice when informed again in future periods. Therefore, the equation in Lemma 4 changes

[24] In this case, for each pair δ and q, such that $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$ (or that $\Delta(q, \mu{=}\xi) \leq \delta$), an equilibrium exists in which all types (resp., informed types) of insider disclose trade size and $P_2(^{\tau=1}_{x_1=0}) < P_2(^{\tau=1}_{x_1=x_{\mathcal{I}}}) {=} \gamma$, where P_2 is such that $\{-1,0,1\} \cup [-x_{\mathcal{I}}, x_{\mathcal{I}}] \rightarrow \mathcal{V}$, the signal $\{\tau \neq 0, x_1{=}0\}$ implies no trade size revelation, and γ is defined in Proposition 5. For the same pair δ and q, a second equilibrium exists in which these types of insider only disclose trade direction and $P_2(^{\tau=1}_{x_1=x_{\mathcal{I}}}) < P_2(^{\tau=1}_{x_1=0}) {=} \gamma$. For $\nabla(q, \mu{=}q\xi) \leq \delta$, a third equilibrium also exists in which $P_2(^{\tau=1}_{x_1=x_{\mathcal{I}}}) {=} P_2(^{\tau=1}_{x_1=0}) {=} \gamma$. At this equilibrium, an insider aware of $\widetilde{v}{<}0$ (or $\widetilde{v}{>}0$) reveals the sold (resp., purchased) quantity with probability $\underline{\varsigma}_t \in [0,1]$ (resp., $\overline{\varsigma}_t \in [0,1]$), while with probability $1{-}\underline{\varsigma}_t$ (resp., $1{-}\overline{\varsigma}_t$), he only discloses trade direction. Specifically, for $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$, the uninformed insider also discloses, revealing how much he initially sold or purchased—as opposed to revealing only trade direction—with probabilities $\underline{\varsigma}_t$ and $\overline{\varsigma}_t$ respectively.

so that μ reflects the expectation of the period t=1 optimal choices by all informed types of insider.

In determining the three regions of equilibria, of all types of insider who are currently informed, only the type who observes $|v| \geq |\mu|$ needs to be taken into account. To see why, suppose that μ>0 and define, with $\mathcal{X}(\mu, v)$, the current period extra-payoff that an informed insider earns from misleading rather than leading, where $\mathcal{X}(\mu{>}0, v \in (-\mu, \mu))$=$|2x_{\mathcal{I}}v|$ depends on the specific value of v that the insider currently observes, whereas $\mathcal{X}(\mu{>}0, |v| \geq |\mu|)$=$2x_{\mathcal{I}}\mu$ does not. Two observations are in order. First, because the insiders aware of $|v| \geq |\mu|$ have equal incentives to mislead today, their current period optimal choice is identical. Second, if the latter insiders misled in a candidate informative equilibrium, μ>0 would not be justified, because any second round price shift would be too large. These two observations bring us to the following important remark: A characteristic informative equilibria share is that all the insiders who are currently aware of $|v| \geq |\mu|$ choose to lead today. In addition, because $\mathcal{X}(\mu{>}0, v \in (-\mu, \mu)){<}\mathcal{X}(\mu{>}0, |v| \geq |\mu|)$, all the latter insiders are more tempted to mislead today than is any insider aware of $v \in (-\mu, \mu)$. As a result, if one type (and thus each type) of insider aware of $|v| \geq |\mu|$ chooses to lead today—which we have explained to be necessary for any informative equilibrium to arise—then every type of insider aware of $v \in (-\mu, \mu)$ also chooses to lead today, because the latter insider has a smaller incentive to mislead. This result reveals not only that no informed manipulation occurs at the equilibrium but also that, to identify informative equilibrium behaviors, it is sufficient to restrict attention to the optimal choices by an insider who currently observes $|v| \geq |\mu|$ and by one who is currently uninformed.

For the highest values of δ and any $q \in (0, 1]$, disclosures are fully informative at the equilibrium, so μ equals ξ. In fact, because the insider weighs future profits heavily, at current period he leads when informed and does not to manipulate otherwise. Now, starting from any pair $\delta \simeq 1, q \in (0, 1)$ and gradually shifting the parameter δ down, at some point a first switch in the equilibrium occurs, to one with uninformed manipulations that cause μ to equal $q\xi$. Specifically, in line with Proposition 3, this switch in equilibrium takes place before a further decrease of δ causes the equilibrium to switch again, to one in which no disclosure is informative. The driving force for this result is that, for any pair δ and $q \in (0, 1)$ and a positive μ, the *overall* incentive that an uninformed insider has to bluff (rather than not to bluff) today is greater than the *overall* incentive that an insider aware of $\widetilde{v}$=v has to mislead (rather than lead) today. Consider those insiders currently aware of $|v| \geq |\mu|$, who have the highest incentive to mislead. Because $[B(\mu) - 0]$=$\mathcal{X}(\mu{>}0, |v| \geq |\mu|)$, the end-of-period extra profits that the uninformed insider expects to achieve when bluffing (rather than not bluffing) today equal the ones that an insider aware of $|v| \geq |\mu|$ achieves from misleading (rather than leading) today. Nonetheless, the different intertemporal consequences that these two choices imply are such that, for an insider aware of $|v| \geq |\mu|$ (and thus for any informed type of insider), choosing to mislead today is overall less appealing than it is for an uninformed insider to choose to bluff today. It follows that, between the upper region (which contains all pairs $\delta \simeq 1, q$) and the lower region (which contains all pairs δ=$0, q$), there exists one and only one region of equilibria in which the weight granted by $\mathcal{I}$ to future profits is not high enough to prevent him from manipulating today when uninformed but is still too high to cause him to mislead today, when informed. In particular, the threshold between the upper region and the intermediate region is decreasing in q, for the same reason as the function $\Delta(q{,}\mu{=}\xi)$ (see Section II.B.2). In fact, the smaller the probability that $\mathcal{I}$ becomes informed in future periods, the more he must weigh future profits to avoid putting his reputation at risk when currently uninformed. Hence, only those pairs δ and q lying to the right of this threshold are associated with informative equilibria without bluffing.

One final remark is in order. With regard to public trade disclosure (a similar argument applies to ex-ante unverifiable information disclosure), given the trigger strategy defined in Section V.B.1, the level of information embedded in prices does not increase in equilibrium if we allow for a structural switch in the disclosure setting, from one in which only three signals can be sent (i.e., τ=-1, τ=0, and τ=1) to one in which infinite alternative signals (i.e., exact quantity traded) may be.[25] However, when the latter disclosure setting is taken into account, for some pairs δ and q up to infinite other alternatives, informative equilibria exist in which the level of information reflected in prices is higher, provided the notion of defection triggering the Grim punishment—that is, the first restriction characterizing the trigger strategy in Section V.B.1—is refined. Nonetheless, none of these equilibria is a separating equilibrium in which p_2 systematically equals $\mathcal{I}$'s information.[26]

VI. (Dis)advantages of the US short-swing rule

This section studies the US short-swing rule. In our analysis, this rule forces an insider, subject to mandatory trade disclosure, to give up current period profits if he undertakes trade reversals during the period. To assess the implications of this rule for market quality, we consider both the level of price efficiency and the occurrence of price manipulation. Regulators generally perceive an increase in the former as a possible target; however, the latter harms market integrity. In this respect, no synthetic index of market quality or price-level stability has been generally accepted.

The imposition of the US short-swing rule causes trade reversals to be dominated actions, yet in specific settings, this rule does not eliminate the incentive to manipulate. Consider a real asset value $\widetilde{v} \in \{\underline{b},\bar{b}\}$, and an insider who can trade, as in FH and JN, *a (divisible) unit $x_{\mathcal{I}}$ per round*. With this set of assumptions, a unique two-round equilibrium exists in which, perhaps surprisingly, the insider systematically manipulates with uninformed disclosures. Compared with the case in which the short-swing rule is not added to the mandatory disclosure rule (see beginning of Section III.A), the equilibrium behavior of the informed types of insider and the pricing rule coincide: an insider aware of $\widetilde{v}$=$\underline{b}$ (or $\widetilde{v}$=$\bar{b}$) sells (resp., purchases) $x_{\mathcal{I}}$ twice, and the price following the disclosure of a sale (resp., purchase) equals $q\underline{b}$ (resp., $q\bar{b}$). Instead, the uninformed insider randomizes with equal probability between selling and purchasing $x_{\mathcal{I}}$ in the first round, then places no further order in the second round. Compared with a situation in which he is inactive in either round, the uninformed insider still expects to earn 0 profits, but causes the equilibrium price following trade disclosure to move less; therefore, when informed, the insider earns more.

Holding the assumption of an asset value $\widetilde{v}$ with two equally likely priors unchanged, next consider an insider *with constrained asset holdings* who is subject to the US short-swing rule. Among the different two-round

[25] Even when the number of possible signals is highest, namely, when the regulation allows voluntary disclosures of trade size but mandates revelation of trade direction (or voluntary disclosures of trade size, trade direction, or nothing), the specification of P_2 only determines whether, in equilibrium, $\mathcal{I}$ discloses trade size too (resp., trade direction or trade size), as explained in Section V.B.1.

[26] At any candidate separating equilibrium, if $\mathcal{I}$ does not perfectly reveal his information, punishment occurs. However, $\mathcal{I}$ has no incentive to avoid punishment. Not only is the end-of-period payoff following perfect revelation never greater than what $\mathcal{I}$ achieves, in equilibrium and under anonymity, but it is also smaller than what $\mathcal{I}$ can get from defecting.

equilibria that arise, there exists a class in which disclosures are informative, and an insider who observes $\widetilde{v}=\underline{b}$ (or is uninformed, or observes $\widetilde{v}=\bar{b}$) expects to earn $x_{\mathcal{I}}\xi$ (resp., 0; $x_{\mathcal{I}}\xi$) per period by trading $x_1=-x_{\mathcal{I}}$ (resp., by placing any probability on all first round trade quantities, $x_1=0$ included; by trading $x_1=x_{\mathcal{I}}$) and never trading in the second round. In all but one of the equilibria belonging to this class, the uninformed insider undertakes useless manipulations that do not benefit any type of insider. Motivated by this observation, we refine the notion of equilibrium as follows:

Definition 2 *If a type of insider is indifferent about whether or not to place orders at any round, he opts for no order submission, unless this choice causes another type of insider to earn less.*

Taking Definition 2 into account, for any non-degenerate random variable $\widetilde{v}\in\mathcal{V}$ and a period of N rounds, in equilibrium an insider with constrained asset holdings who is subject to the US short-swing rule never manipulates prices. Specifically, even if multiple equilibria are still possible,[27] only one of these equilibria displays robustness to a small probability that $\mathcal{M}$ exogenously learns v in advance. At this equilibrium, an insider who observes $\widetilde{v}<E[\widetilde{v}]$ (or is uninformed, or observes $\widetilde{v}>E[\widetilde{v}]$) trades $x_1=-x_{\mathcal{I}}, x_{n\geq 2}=0$ (resp., trades $x_{n<N}=0$; trades $x_1=x_{\mathcal{I}}, x_{n\geq 2}=0$), and any price following the disclosure of a first round sale (or no first round disclosure, or the disclosure of a first round purchase) is equal to $E[\widetilde{v}|\widetilde{v}<0]$ (resp., to $E[\widetilde{v}]$; to $E[\widetilde{v}|\widetilde{v}>0]$).

To highlight the advantages and disadvantages implied by the imposition of the US short-swing rule on an insider constrained in asset holdings, consider this latter equilibrium. Using the case of $N=2$, we can draw the following conclusions: With respect to the equilibrium with mandatory trade disclosure presented in Proposition 3, for $\delta<\nabla(q,\mu=q\xi)$, the US short-swing rule makes disclosures informative, and for $\nabla(q,\mu=q\xi)\leq\delta<\Delta(q,\mu=\xi)$, it eliminates uninformed manipulations that otherwise would have occurred. However, for $\Delta(q,\mu=\xi)\leq\delta$, this additional rule neither reduces manipulations (which would have not arisen in any case) nor improves price efficiency (because first round disclosures would have been equally informative). The US short-swing rule even may have *negative* effects. Following the informative disclosure of an initial sale or purchase, this rule, because it prevents trade reversals, indirectly compromises any further revelation of inside information that the disclosure of a reversal (or of a missed reversal) would have conveyed otherwise. This happens for $N>2$, which implies that inside information is sufficiently long-lived, provided δ is sufficiently high. In this case, if the short-swing rule is *not added* to mandatory trade disclosure, a reputational equilibrium exists in which the insider never manipulates (because δ is sufficiently high), and the possible disclosure of reversals implies higher price efficiency (see Internet Appendix D). For example, consider the equilibrium behavior of an insider who is currently aware of $\widetilde{v}>0$ (the behavior of an insider aware of $\widetilde{v}<0$ is symmetric). In the first round, this insider purchases up to the maximum cap to lead the price toward its real value. If the second-round price following the disclosure of a purchase *over*shoots the inside information, this insider undertakes a second-round reversal, whose disclosure at the beginning of the third round causes the price to shift even closer to the real value. Conversely, if the second-round price following the disclosure of a purchase *under*shoots the inside information, though this insider would like

[27] For $N=2$, e.g., with probability $\psi\in[0,1]$, an insider aware of $\widetilde{v}<E[\widetilde{v}]$ (or $\widetilde{v}>E[\widetilde{v}]$) trades $x_1=-x_{\mathcal{I}}$ (resp., $x_1=x_{\mathcal{I}}$) and $x_2=0$, while with probability $1-\psi$, he trades $x_1=0$ and $x_2=-x_{\mathcal{I}}$ (resp., $x_2=x_{\mathcal{I}}$) so that $P_2(\tau=0)=E[\widetilde{v}]$ and $-P_2(\tau=-1)=P_2(\tau=1)=E[\widetilde{v}|\widetilde{v}>0]$.

to purchase again, he will not be able to do so, because the cap on total exposure already had been reached in the first round. Therefore, in the second round he does not trade. Nonetheless, even the absence of trade disclosure at the beginning of the third round shifts the price closer to the real value. These insights suggest some reflections about the unconditional introduction of the short-swing rule, which in this case would be unsuccessful.

The predictions about SEC Section 16(b) are robust, in two regards. First, the results are unaffected if the insider, when informed, already learns $\widetilde{v}=v$ rather than $\widetilde{v}\gtrless E[\widetilde{v}]$ in the first round. Second, traded quantities and price responses do not change in equilibrium if the regulation mandates trade size disclosure.

VII. Conclusions (main findings)

This article considers public disclosure by an insider whose orders have no direct price impact.

First, we focus our analysis on a regulation that mandates such an insider to disclose publicly the trades he undertakes. The analysis brings us to the following main equilibrium result: For any discount factor in line with those in most world economies, if prices are known to react to public disclosures then the insider, when informed, trades and thus discloses with the objective of leading these prices toward the real asset value, reversing his initial position if the disclosure causes price overshooting. When this insider is uninformed, his behavior depends on the probability of being informed in the future. Specifically, an insider who is less likely to be informed tends to manipulate the market with uninformed disclosures while an insider who is more likely to be informed does not. Thus, in showing that not all insiders influence the market with socially undesirable but barely indictable manipulations, the equilibrium result reduces practitioners' concerns about this form of disclosure. In showing that more likely informed insiders tend not to manipulate when uninformed, this result might also stimulate policy debate on whether to allow only these insiders to disclose.

The second main equilibrium result discussed in this article is that it is unnecessary to mandate trade revelation. Intuitively, the insider earns at least as much with a disclosure rule as without. Hence, by changing the regulation and making trade reporting non-compulsory, whenever the price is known to react to disclosures, the insider will trade as under mandatory disclosure, voluntarily revealing his trades immediately after having bought or sold up to his (privately known) maximum cap. This result not only reduces authorities' concerns about enforcing the reporting of trades with punitive laws or expensive oversight. It also highlights a link to the strand of literature on the voluntary disclosure of ex-ante unverifiable information by insider traders, to which we contribute by relaxing the assumption of a honest insider and showing what follows: When the market interprets an ex-ante unverifiable inside statement to be favorable/unfavorable, prices move in the same way that they would following the disclosure of a purchase/sale, which is the trade that the insider actually undertakes before producing the unverifiable statement in question. This third main equilibrium result suggests that, if the insider plans to manipulate prices with ex-ante unverifiable statements when uninformed, the introduction of a rule that requires this insider to disclosure his trades does not prevent price manipulation.

In our theory, manipulations can be eradicated altogether by adding the short-swing rule to the mandatory disclosure rule. In fact, this additional rule indirectly forbids trade reversals and thereby ensures that any otherwise appealing deceptive disclosure is not made. However, if it is imposed on those insiders whose disclosures in the absence of short-swing rule are already purely informative and not manipulative, then this additional rule is not only unnecessary. Provided at the same time inside information is sufficiently long-lived, the short-swing rule is also price inefficient because it prevents the public revelation of undertaken and missed reversals that would otherwise have pushed prices even closer to the real asset value.

Finally we note that, if the disclosure of inside statements can be delayed, even a very large insider will behave exactly as predicted by our theory, limiting the direct impact of orders on prices by breaking down each pre-decided order into small chunks, and disclosing to the public immediately after the full execution of the pre-decided order. Under mandatory trade disclosure, this prediction suggests that one way of preventing such an insider from undertaking the illicit informed dissimulations suggested in the literature is to allow him to delay disclosing his trades.

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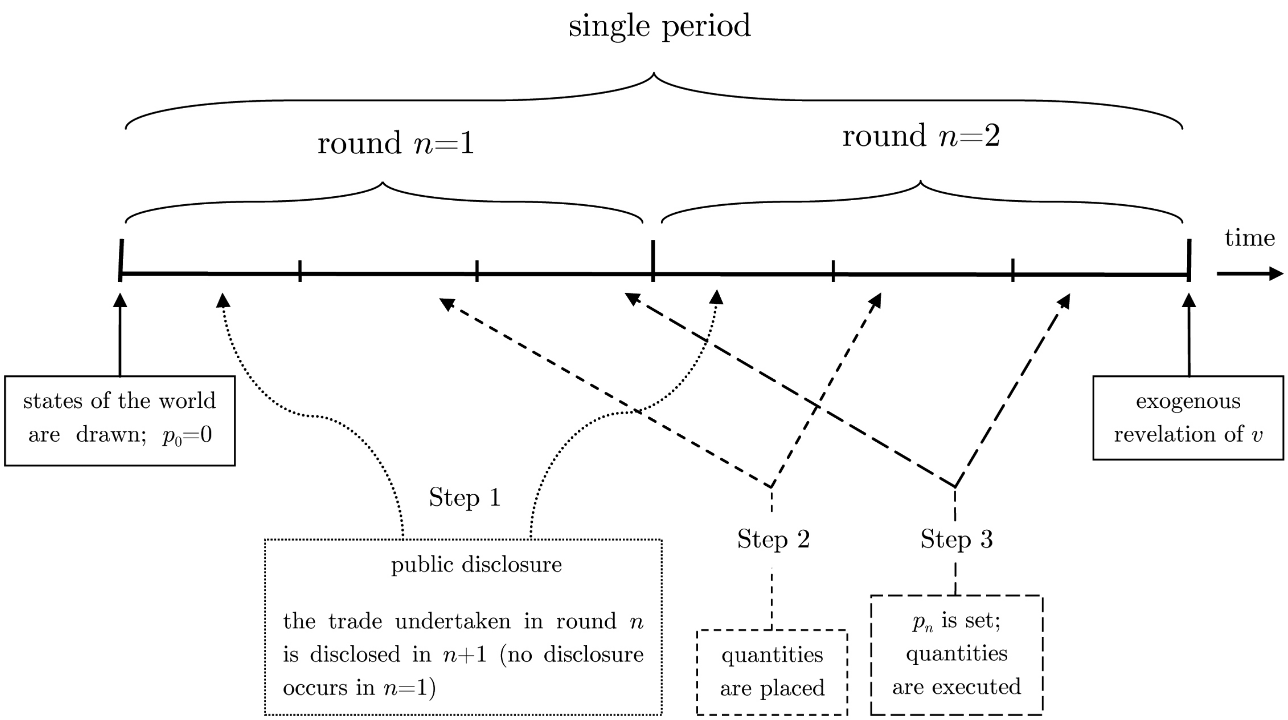

Fig. 1. Time line Timing of events in a single period.

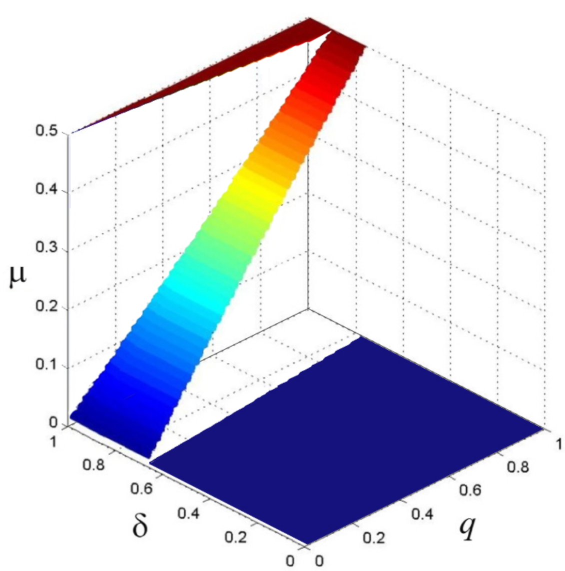

Fig. 2. Equilibria of the reputational framework Behavior of μ (magnitude of the equilibrium price shift following trade disclosure) for each pair δ and q (inter-period discount factor and probability that $\mathcal{I}$ becomes informed in future periods, respectively) in the case of a real asset value $\widetilde{v}\sim$U$[-1,1]$.

Not for Publication: Web-based Technical Appendix to

“Insider Trading and Disclosure”

Internet Appendix A

Proof of Proposition 1. For any possible pricing rule such that, first, p_1=0 and, second, $\exists\tau : P_2(\tau) \neq 0$, under the assumption that $\mathcal{I}$ already observes $\widetilde{v}$=v in the first round, we derive the optimal strategy for the insider, X. Holding X fixed, we show that $\mathcal{M}$ is setting either $P_2(\tau, X_1(\widetilde{s}{=}I,\widetilde{v}{=}v{<}0)){>}0$ in response to the disclosure by each insider aware of $\widetilde{v}{=}v{<}0$ or $P_2(\tau, X_1(\widetilde{s}{=}I,\widetilde{v}{=}v{>}0)){<}0$ in response to the disclosure by each insider aware of $\widetilde{v}{=}v{>}0$, which are what we call contradictions.

Eight cases (from $C1$ to $C8$) representing all possible combinations of second-round price responses can be identified.

$C1$: $P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}{-1}) \geq 0$.[1] Given these second-round price responses, the following sub-cases can be identified. (i) If at least one signal $\tau{=}i$ causes $P_2(\tau{=}i)$ to equal 0, an insider aware of $\widetilde{v}{=}v{<}0$ finds it optimal to trade a quantity $X_1(\widetilde{s}{=}I,\widetilde{v}{=}v{<}0)$ such that $\tau \neq i$. In fact, for $P_2(\tau{=}1)$ (or $P_2(\tau{=}0)$, or $P_2(\tau{=}{-1})$) different from 0, by trading $x_1{=}x_{\mathcal{I}}, x_2{=}{-2}x_{\mathcal{I}}$ (resp., $x_1{=}0, x_2{=}{-x_{\mathcal{I}}}$; $x_1 \lesssim 0, x_2 \lesssim -x_{\mathcal{I}}$) and thus signaling $\tau{=}1$ (resp., $\tau{=}0$; $\tau{=}{-1}$), this insider earns more than $x_{\mathcal{I}}v$; instead, by signaling $\tau{=}i$, this insider cannot earn more than $x_{\mathcal{I}}v$.[2] Holding X_1 fixed, we have that $P_2(\tau \neq i, X_1(\widetilde{v}{=}v{<}0)){>}0$, $\forall v{<}0$, which is a contradiction. (ii) If $P_2(\tau){>}0$, $\forall\tau$, it is straightforward to see that the optimal trading strategy for the insider, X, is such that $P_2(\tau, X_1(\widetilde{v}{=}v{<}0)){>}0$, $\forall v{<}0$, which is again a contradiction.

$C2$: $P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}{-1}) \leq 0$. This case is symmetric to $C1$.

$C3$: $P_2(\tau{=}1){>}0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}{-1}) < 0$. Given these second-round price responses, an insider aware of $\widetilde{v}{=}v{>}\max\{0;\zeta;\epsilon\}$ strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$, where $\zeta{=}P_2(\tau{=}{-1}){+}P_2(\tau{=}1)$ and $\epsilon{=}P_2(\tau{=}{-1}){+}\frac{P_2(\tau=0)}{2}$.[3] Thus, as long as $\max\{\zeta;\epsilon\} \leq 0$, any insider aware of $\widetilde{v}{=}v{>}0$ finds it optimal to trade $X_1(\widetilde{s}{=}I, \{\zeta;\epsilon\} \leq 0{<}v){=}{-x_{\mathcal{I}}}$. Holding

[1] The symbol $\wedge$ stands for *and*.

[2] The symbols $\gtrsim$ and $\lesssim$ stand for *just greater than* and *just smaller than*, respectively.

[3] First, notice that an insider aware of $\widetilde{v}{=}v{>}0$ strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ rather than to trade any alternative pair of quantities $x_1{<}0, x_2{=}\cdot$. Second, an insider who observes $v \geq P_2(\tau{=}1)$ always strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ rather than to trade $x_1{>}0, x_2{=}\cdot$, whereas an insider who observes $0{<}v{<}P_2(\tau{=}1)$ strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ rather than to trade $x_1{>}0, x_2{=}\cdot$ only if $\{-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2(\tau{=}{-1})]\}{>}\{x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}1)]\} \therefore v{>}\zeta$. Third, an insider aware of $v \geq P_2(\tau{=}0)$ always strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ rather than to trade $x_1{=}0, x_2{=}\cdot$, whereas an insider aware of $0{<}v{<}P_2(\tau{=}0)$ strictly prefers to trade $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ rather than to trade $x_1{=}0, x_2{=}\cdot$ only if $\{-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2(\tau{=}{-1})]\}{>}\{-x_{\mathcal{I}}[v - P_2(\tau{=}0)]\} \therefore v{>}\epsilon$.

X_1 fixed, we have that $P_2(\tau{=}-1, X_1(\widetilde{s}{=}I, \{\zeta;\epsilon\} \leq 0{<}v)){<}0$, $\forall v{>}0$, which is a contradiction. Now we show that, even for $\max\{\zeta;\epsilon\}{>}0$, a contradiction arises.

To proceed, the following intermediate results need to be listed: (1) An insider aware of $\widetilde{v}{=}v{<}0$ strictly prefers to trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ (or $x_1{=}0, x_2{=}-x_{\mathcal{I}}$) rather than to trade any other pair of quantities such that $x_1{>}0$ (resp., $x_1{=}0$). (2) An insider aware of $v \leq P_2(\tau{=}-1){<}0$ strictly prefers to trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ (or $x_1{=}0, x_2{=}-x_{\mathcal{I}}$) rather than to trade $x_1{<}0, x_2{=}\cdot$. (3) An insider who observes $P_2(\tau{=}-1){<}v{<}0$ strictly prefers to trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ (or $x_1{=}0, x_2{=}-x_{\mathcal{I}}$) rather than to trade $x_1{<}0, x_2{=}\cdot$ only if $x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}1)]$ (resp., $-x_{\mathcal{I}}[v - P_2(\tau{=}0)]$) is strictly greater than $-x_{\mathcal{I}}v + 2x_{\mathcal{I}}[v - P_2(\tau{=}-1)]$, that is, only if $v{<}\zeta$ (resp., $v{<}\epsilon$). (4) If $P_2(\tau{=}1){>}\frac{P_2(\tau=0)}{2}$ (or $P_2(\tau{=}1){=}\frac{P_2(\tau=0)}{2}$; or $P_2(\tau{=}1){<}\frac{P_2(\tau=0)}{2}$), the profits that an insider aware of $\widetilde{v}{=}v{<}0$ earns from trading $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ are greater than (resp., equal to; smaller than) those from trading $x_1{=}0, x_2{=}-x_{\mathcal{I}}$.

For $\max\{\zeta;\epsilon\}{>}0$, as a consequence of the results at points 1 to 4, the following conclusions can be drawn: (a) Suppose that $P_2(\tau{=}1){>}\frac{P_2(\tau=0)}{2}$. (a.i) In the sub-case of $\zeta{>}0$, for any value of ϵ, the optimal strategy for the insider, X, is such that, when he is informed about $\widetilde{v}{=}v{<}0$, he trades $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$. Holding X fixed, it follows that $P_2(\tau, X_1(\widetilde{v}{=}v{<}0)){>}0$, $\forall v{<}0$, which is a contradiction. (a.ii) The remaining sub-case with $\zeta \leq 0{<}\epsilon$ is not of interest, because it refers to a situation where $P_2(\tau{=}-1){+}P_2(\tau{=}1) \leq 0{<}P_2(\tau{=}-1){+}\frac{P_2(\tau=0)}{2} \therefore P_2(\tau{=}1) {<}\frac{P_2(\tau=0)}{2}$, whereas the case under consideration (i.e., point a) is the one in which $P_2(\tau{=}1) {>}\frac{P_2(\tau=0)}{2}$. (b) Suppose that $P_2(\tau{=}1){=}\frac{P_2(\tau=0)}{2}$ (case in which $P_2(\tau{=}0){>}0$). This condition on prices, together with the condition that $\max\{\zeta;\epsilon\}{>}0$, imply that $\zeta{=}\epsilon{>}0$, in which case each insider aware of $\widetilde{v}{=}v{<}0$ finds it optimal to randomize between trading $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ and trading $x_1{=}0, x_2{=}-x_{\mathcal{I}}$. Holding the optimal trading behavior by all these types of insider fixed, regardless of the probability with which each of them chooses to buy as opposed to not trading in the first round (even probability 0 or 1), the price in response to their disclosure turns out to lie above 0, which is a contradiction. (c) Suppose that $P_2(\tau{=}1){<}\frac{P_2(\tau=0)}{2}$ (case in which $P_2(\tau{=}0){>}0$). (c.i) In the sub-case of $\epsilon{>}0$, for any value of ζ, the optimal strategy for the insider, X, is such that, when he is informed about $\widetilde{v}{=}v{<}0$, he trades $x_1{=}0, x_2{=}-x_{\mathcal{I}}$. Holding X fixed, it follows that $P_2(\tau, X_1(\widetilde{v}{=}v{<}0)){>}0$, $\forall v{<}0$, which is a contradiction. (c.ii) The remaining sub-case with $\epsilon \leq 0{<}\zeta$ is not of interest, because it refers to a situation where $P_2(\tau{=}-1){+}\frac{P_2(\tau=0)}{2}{\leq}\ 0{<}P_2(\tau{=}-1){+}P_2(\tau{=}1) \therefore \frac{P_2(\tau=0)}{2}{<}P_2(\tau{=}1)$, whereas the case under consideration (i.e., point c) is the one in which $P_2(\tau{=}1){<}\frac{P_2(\tau=0)}{2}$.

$C4$: $P_2(\tau{=}1) > 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}{-1}){<}0$. This case is symmetric to $C3$.

$C5$: $P_2(\tau{=}{-1}) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}1) < 0$. Given these second-round price responses, the optimal strategy for the insider, X, is such that, when he is informed about $\widetilde{v}{=}v{>}0$, he trades $x_1 \gtrsim 0; x_2 \lesssim x_{\mathcal{I}}$. Holding X fixed, it follows that $P_2(\tau, X_1(\widetilde{v}{=}v{>}0)){<}0$, $\forall v{>}0$, which is a contradiction.

$C6$: $P_2(\tau{=}{-1}) > 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}1) \leq 0$. This case is symmetric to $C5$.

$C7$: $P_2(\tau{=}0) > 0 \wedge P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}{-1}) \leq 0$. Given these second-round price responses, the following cases can be identified. (i) For $P_2(\tau \neq 0){=}0$, then we are in case $C1$. (ii) For $P_2(\tau \neq 0) \neq 0$, all types of insider aware of $\widetilde{v}{=}v{>}0$ strictly prefer to trade $x_1{=}x_{\mathcal{I}}, x_2{=}0$ rather than to trade $x_1{=}0; x_2{=}\cdot$, meaning that they all signal in a way that pushes p_2 below 0. (iii) For $P_2(\tau{=}1){=}0 \wedge P_2(\tau{=}{-1}){<}0$ (or for $P_2(\tau{=}1){<}0 \wedge P_2(\tau{=}{-1}){=}0$), the optimal trading strategy for the insider, X, is such that, when he is informed about $\widetilde{v}{=}v{>}0$, he trades $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$ (resp., $x_1 \gtrsim 0, x_2 \lesssim x_{\mathcal{I}}$). Holding X fixed, it follows that $P_2(\tau{=}{-1}, X_1(\widetilde{v}{=}v{>}0)){<}0$ (resp., $P_2(\tau{=}1, X_1(\widetilde{v}{=}v{>}0)){<}0$), $\forall v{>}0$, which is a contradiction.

$C8$: $P_2(\tau{=}0) < 0 \wedge P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}{-1}) \geq 0$. This case is symmetric to $C7$. ■

Proof of Lemma 1. For $\mu{>}0$, if an insider who is currently informed decides to lead the market, his end-of-period expected profit equals $L(\mu){=}x_{\mathcal{I}}[2\int_0^{\mu}(2\mu - v)h(v)dv{+}2\int_{\mu}^{\bar{b}}vh(v)dv]$, which can be rearranged to give Equation 2. Instead, if this insider decides to mislead the market, his end-of-period expected profit equals $M(\mu){=}x_{\mathcal{I}}2\int_0^{\bar{b}}(2\mu + v)h(v)dv$, which can be rearranged to give Equation 3. ■

Proof of Proposition 2. Here we only identify the level of $\bar{\alpha}$ that, for $\mu{>}0$, maximizes the discounted expected profits over periods. In detail, $\bar{\alpha}^{*I}{=}\arg\max_{\bar{\alpha}} E[\Pi]$, where $E[\Pi]{=}\mathcal{T}{+}\varkappa\{\mathcal{T}{+}\varkappa\{\mathcal{T}{+}\varkappa\{...\}\}\}{=}\frac{\mathcal{T}}{1-\varkappa}$, $\mathcal{T}{=}\bar{\alpha}{\cdot}M(\mu){+}(1{-}\bar{\alpha})\cdot L(\mu){+}\bar{\alpha}\frac{\delta x_{\mathcal{I}}\xi}{1-\delta}$, and $\varkappa{=}\delta(1{-}\bar{\alpha})$. Because $\delta \gtreqless \delta_{\nabla} \rightarrow \frac{\partial E[\Pi]}{\partial \bar{\alpha}} \lesseqgtr 0$, we have that if $\delta{>}\delta_{\nabla}$, then $\bar{\alpha}^{*I}$ equals 0, whereas if $\delta{<}\delta_{\nabla}$, then $\bar{\alpha}^{*I}$ equals 1. ■

Proof of Lemma 2. Given $\mu{>}0$, if an insider who is currently uninformed decides not to bluff, his end-of-period expected profit equals 0, for any probability placed on all second-round trade quantities ($x_2{=}0$ included). If this insider decides to bluff, his end-of-period expected profit equals $B(\mu){=}\int_v\{x_{\mathcal{I}}(-1)(-\mu)+x_{\mathcal{I}}[v{-}(-\mu)]\}f(v)dv{=}2\mu x_{\mathcal{I}}{>}0$. ■

Proof of Proposition 3. For $\mu{>}0$, we prove that if $\delta \geq \Delta(q, \mu)$, then $\bar{\alpha}^{*I}{=}\bar{\beta}^{*U}{=}0$; if

$\nabla(q,\mu) \leq \delta \leq \Delta(q,\mu)$, then $\bar{\alpha}^{*I}$=0, $\bar{\beta}^{*U}$=1; and if $\delta \leq \nabla(q,\mu)$, then $\bar{\alpha}^{*I}$=1.

In order to identify $\bar{\alpha}^{*I}$ and $\bar{\beta}^{*U}$, we compute $\frac{\partial E[\Pi^I]}{\partial \bar{\alpha}}$, $\frac{\partial E[\Pi^I]}{\partial \bar{\beta}}$, $\frac{\partial E[\Pi^U]}{\partial \bar{\alpha}}$ and $\frac{\partial E[\Pi^U]}{\partial \bar{\beta}}$. The denominator of each of the four resulting fractions is function of $\bar{\alpha}$ and $\bar{\beta}$ and is squared, whereas each numerator is not function of the variable with respect to which we compute the derivative. Thus, both for the case in which $\mathcal{I}$ is currently informed and for the case in which he is currently uninformed, the solution to the maximization problem must be a corner solution.

Consider the function $E[\Pi^I]$. Notice (a) that $E[\Pi^I|\bar{\alpha}=\bar{\beta}=0] \gtreqless E[\Pi^I|\bar{\alpha}=0,\bar{\beta}=1] \Leftrightarrow \delta \gtreqless \Delta(q,\mu)$, (b) that $E[\Pi^I|\bar{\alpha}=\bar{\beta}=0] \gtreqless E[\Pi^I|\bar{\alpha}=1] \Leftrightarrow \delta \gtreqless \frac{M(\mu)-L(\mu)}{M(\mu)-L(\mu)+q[L(\mu)-x_{\mathcal{I}}\xi]}$, and (c) that $E[\Pi^I|\bar{\alpha}=0,\bar{\beta}=1] \gtreqless E[\Pi^I|\bar{\alpha}=1] \Leftrightarrow \delta \gtreqless \nabla(q,\mu)$. Three conclusions can be drawn: First, by focusing on point a and c, because $\Delta(q,\mu) > \nabla(q,\mu) \therefore 2B(\mu) > M(\mu) - L(\mu)$ is always verified, it follows that the pair $\bar{\alpha}=0,\bar{\beta}=0$ maximizes the function $E[\Pi^I]$ if and only if $\delta \geq \Delta(q,\mu)$. Second, by focusing on point b and c, because $\nabla(q,\mu) < \frac{M(\mu)-L(\mu)}{M(\mu)-L(\mu)+q[L(\mu)-x_{\mathcal{I}}\xi]} \therefore 2B(\mu) > M(\mu) - L(\mu)$ is always verified, it follows that the pair $\bar{\alpha}=1,\bar{\beta}=\cdot$ maximizes the function $E[\Pi^I]$ if and only if $\delta \leq \nabla(q,\mu)$. Therefore, third, the remaining pair, $\bar{\alpha}=0,\bar{\beta}=1$, maximizes the function $E[\Pi^I]$ if and only if $\nabla(q,\mu) \leq \delta \leq \Delta(q,\mu)$.

Next, consider the function $E[\Pi^U]$. To identify the condition under which the pair $\bar{\alpha}=0,\bar{\beta}=0$ maximizes $E[\Pi^U]$, notice first that $E[\Pi^U|\bar{\alpha}=\bar{\beta}=0] \gtreqless E[\Pi^U|\bar{\alpha}=0,\bar{\beta}=1] \Leftrightarrow \delta \gtreqless \Delta(q,\mu)$; second, that $E[\Pi^U|\bar{\alpha}=\bar{\beta}=0] \gtreqless E[\Pi^U|\bar{\alpha}=1,\bar{\beta}=0] \Leftrightarrow \delta \gtreqless \frac{M(\mu)-L(\mu)}{M(\mu)-L(\mu)+q[L(\mu)-x_{\mathcal{I}}\xi]}$; and, third, that $E[\Pi^U|\bar{\alpha}=0,\bar{\beta}=1] \gtreqless E[\Pi^U|\bar{\alpha}=1,\bar{\beta}=1] \Leftrightarrow \delta \gtreqless \nabla(q,\mu)$. It is easy to check that the pair $\bar{\alpha}=0,\bar{\beta}=0$ maximizes the function in question if and only if $\delta \geq \Delta(q,\mu)$. Proceeding in this way, it can be shown that either $\bar{\alpha}=1,\bar{\beta}=0$ or $\bar{\alpha}=1,\bar{\beta}=1$ maximizes $E[\Pi^U]$ if and only if $\delta \leq \nabla(q,\mu)$, and that $\bar{\alpha}=0,\bar{\beta}=1$ maximizes the function $E[\Pi^U]$ if and only if $\nabla(q,\mu) \leq \delta \leq \Delta(q,\mu)$. ■

Proof of Proposition 4. Here we consider only the single period. For any possible pricing rule such that, first, p_1=0 and, second, $\exists \tau : P_2(\tau) \neq 0$, under the assumption that $\mathcal{I}$ already observes $\widetilde{v}=v$ in the first round, we derive the optimal behavior for each informed type of insider (given by the triple x_1, τ, x_2). Holding each of these optimal behaviors fixed, we show that $\mathcal{M}$ is setting either $p_2>0$ in response to the signal sent by any insider aware of $\widetilde{v}=v<0$ or $p_2<0$ in response to the signal sent by any insider aware of $\widetilde{v}=v>0$, which are what we call contradictions.

Eight cases (from $C1'$ to $C8'$) representing all possible combinations of second-round price responses can be identified.

$C1'$: $P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}-1) \geq 0$. The analysis of this case is in line with that conducted under mandatory trade disclosure (see proof to Proposition 1, case $C1$).

$C2'$: $P_2(\tau{=}1) \leq 0 \wedge P_2(\tau{=}0) \leq 0 \wedge P_2(\tau{=}-1) \leq 0$. This case is symmetric to $C1'$.

$C3'$: $P_2(\tau{=}1){>}0 \wedge P_2(\tau{=}0) \geq 0 \wedge P_2(\tau{=}-1) < 0$. Given these second-round price responses, an insider aware of $\widetilde{v}{=}v{>}\max\{0;\zeta;\epsilon'\}$ strictly prefers to signal $\tau{=}-1$ and trade $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$, where $\epsilon'{=}P_2(\tau{=}-1){+}P_2(\tau{=}0)$.[4] Therefore, as long as $\max\{\zeta;\epsilon'\} \leq 0$, all types of insider aware of $\widetilde{v}{=}v{>}0$ find it optimal to sell initially up to the cap on total exposure and signal $\tau{=}-1$. Holding this optimal behavior fixed, it turns out that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price smaller than 0, which is a contradiction.

Now we show that, even for $\max\{\zeta;\epsilon'\}{>}0$, a contradiction arises.

To proceed, the following intermediate results need to be listed: (1) An insider aware of $\widetilde{v}{=}v{<}0$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}0$) and trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ rather than to signal $\tau{=}1$ (resp., $\tau{=}0$) and trade any other pair of quantities $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{=}\cdot, x_2{=}\cdot$). (2) An insider aware of $v \leq P_2(\tau{=}-1){<}0$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}0$) and trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ rather than to signal $\tau{=}-1$ and trade $x_1{<}0, x_2{=}\cdot$. (3) An insider who observes $P_2(\tau{=}-1){<}v{<}0$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}0$) and trade $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$ rather than to signal $\tau{=}-1$ and trade $x_1{<}0, x_2{=}\cdot$ only if $x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}1)]$ (resp., $x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}0)]$) is strictly greater than $-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2(\tau{=}-1)]$, that is, only if $v{<}\zeta$ (resp., $v{<}\epsilon'$). (4) If $P_2(\tau{=}1){>}P_2(\tau{=}0)$ (or $P_2(\tau{=}1){=}P_2(\tau{=}0)$, or $P_2(\tau{=}1){<}P_2(\tau{=}0)$), the profits that an insider aware of $\widetilde{v}{=}v{<}0$ earns from signaling $\tau{=}1$ and trading $x_1{=}x_{\mathcal{I}}$, $x_2{=}-2x_{\mathcal{I}}$ are greater than (resp., equal to; smaller than) those from signaling $\tau{=}0$ and trading $x_1{=}x_{\mathcal{I}}, x_2{=}-2x_{\mathcal{I}}$.

For $\max\{\zeta;\epsilon'\}{>}0$, as a consequence of the results at points 1 to 4, the following conclusions can be drawn: (a) Suppose that $P_2(\tau{=}1){>}P_2(\tau{=}0)$. (a.i) In the sub-case of $\zeta{>}0$,

[4] First, notice that an insider aware of $\widetilde{v}{=}v{>}0$ strictly prefers to signal $\tau{=}-1$ and trade $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$ rather than to signal $\tau{=}-1$ and trade any alternative pair of quantities $x_1{<}0, x_2{=}\cdot$. Second, an insider who observes $v \geq P_2(\tau{=}1)$ (or $v \geq P_2(\tau{=}0)$) strictly prefers to signal $\tau{=}-1$ and trade $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$ rather than to signal $\tau{=}1$ (resp., $\tau{=}0$) and trade $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{=}\cdot, x_2{=}\cdot$). Third, an insider aware of $0{<}v{<}P_2(\tau{=}1)$ (or $0{<}v{<}P_2(\tau{=}0)$) strictly prefers to signal $\tau{=}-1$ and trade $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$ rather than to signal $\tau{=}1$ (resp., $\tau{=}0$) and trade $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{=}\cdot, x_2{=}\cdot$) only if $\{-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2(\tau{=}-1)]\}$ is strictly greater than $\{x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}1)]\}$ (resp., than $\{x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}0)]\}$), that is, only if $v{>}\zeta$ (resp., $v{>}\epsilon'$).

for any value of ϵ', each insider aware of $\widetilde{v}=v<0$ finds it optimal to signal $\tau=1$ and trade $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$. Holding this optimal behavior fixed, it follows that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price greater than 0, which is a contradiction. (a.ii) The remaining sub-case with $\zeta \leq 0<\epsilon'$ is not of interest, because it refers to a situation where $P_2(\tau=-1)+P_2(\tau=1) \leq 0<P_2(\tau=-1)+ P_2(\tau=0) \therefore P_2(\tau=1)<P_2(\tau=0)$, whereas the case under consideration (i.e., point a) is the one in which $P_2(\tau=1)>P_2(\tau=0)$. (b) Suppose that $P_2(\tau=1)=P_2(\tau=0)$ (case in which $P_2(\tau=0)>0$). This condition on prices, together with the condition that $\max\{\zeta;\epsilon'\}>0$, imply that $\zeta=\epsilon'>0$, in which case any insider aware of $\widetilde{v}=v<0$ finds it optimal to trade $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$ and randomize between signaling $\tau=1$ and signaling $\tau=0$. Holding the optimal behavior by these types of insider fixed, regardless of the probability with which each of them chooses to signal $\tau=1$ as opposed to signaling $\tau=0$ (even probability 0 or 1), the price in response to their signal turns out to lie above 0, which is a contradiction. (c) Suppose that $P_2(\tau=1)<P_2(\tau=0)$ (case in which $P_2(\tau=0)>0$). (c.i) In the sub-case of $\epsilon'>0$, for any value of ζ, each insider aware of $\widetilde{v}=v<0$ finds it optimal to signal $\tau=0$ and trade $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$. Holding this optimal behavior fixed, it follows that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price greater than 0, which is a contradiction. (c.ii) The remaining sub-case with $\epsilon'\leq 0<\zeta$ is not of interest, because it refers to a situation where $P_2(\tau=-1)+P_2(\tau=0) \leq 0<P_2(\tau=-1)+P_2(\tau=1) \therefore P_2(\tau=0)<P_2(\tau=1)$, whereas the case under consideration (i.e., point c) is the one in which $P_2(\tau=1)<P_2(\tau=0)$.

*C*4$'$: $P_2(\tau=1) > 0 \wedge P_2(\tau=0) \leq 0 \wedge P_2(\tau=-1)<0$. This case is symmetric to *C*3$'$.

*C*5$'$: $P_2(\tau=-1) \geq 0 \wedge P_2(\tau=0) \geq 0 \wedge P_2(\tau=1) < 0$. Given these second-round price responses, notice that the profits that an insider aware of $\widetilde{v}=v<0$ earns from signaling $\tau=-1$ and trading $x_1 \lesssim 0, x_2 \lesssim -x_{\mathcal{I}}$ (or signaling $\tau=0$ and trading $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$) are greater than those from signaling $\tau=1$ and trading $x_1>0, x_2=\cdot$. Notice also that if this type of insider signals $\tau=-1$ (or $\tau=0$), the profits that he earns from trading $x_1 \lesssim 0, x_2 \lesssim -x_{\mathcal{I}}$ (resp., $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$) are not smaller than those from trading any other pair of quantities such that $x_1<0$ (resp., $x_1=\cdot$).

Therefore, in order to derive the optimal signal sent by an insider aware of $\widetilde{v}=v<0$, it is sufficient to check whether this type of insider prefers to signal $\tau=-1$ and trade $x_1 \lesssim 0, x_2 \lesssim -x_{\mathcal{I}}$ rather than to signal $\tau=0$ and trade $x_1=x_{\mathcal{I}}, x_2=-2x_{\mathcal{I}}$. Specifically: (i) If $\frac{P_2(\tau=-1)}{2}>P_2(\tau=0)$, case in which $P_2(\tau=-1)>0$ (or if $\frac{P_2(\tau=-1)}{2}<P_2(\tau=0)$, case in which $P_2(\tau=0)>0$, or if $\frac{P_2(\tau=-1)}{2}=$

$P_2(\tau{=}0){>}0$), each insider aware of $\widetilde{v}{=}v{<}0$ finds it optimal to signal $\tau{=}{-1}$ (resp., to signal $\tau{=}0$; to randomize between signaling $\tau{=}{-1}$ and signaling $\tau{=}0$). Holding the insider's optimal behavior fixed, it follows that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price greater than 0, which is a contradiction. (ii) If $\frac{P_2(\tau=-1)}{2}{=}P_2(\tau{=}0){=}0$, then we are in case $C2'$.

$C6'$: $P_2\left(\tau{=}{-1}\right) > 0 \wedge P_2\left(\tau{=}0\right) \leq 0 \wedge P_2\left(\tau{=}1\right) \leq 0$. This case is symmetric to $C5'$.

$C7'$: $P_2\left(\tau{=}0\right) > 0 \wedge P_2\left(\tau{=}1\right) \leq 0 \wedge P_2\left(\tau{=}{-1}\right) \leq 0$. Given these second-round price responses, an insider aware of $\widetilde{v}{=}v{<}\min\left\{0;\zeta';\epsilon'\right\}$ strictly prefers to signal $\tau{=}0$ and trade $x_1{=}x_{\mathcal{I}}, x_2{=}{-2}x_{\mathcal{I}}$, where $\zeta'{=}P_2(\tau{=}0){+}\frac{P_2(\tau=1)}{2}$.[5] Therefore, as long as $\min\left\{\zeta';\epsilon'\right\} \geq 0$, all types of insider aware of $\widetilde{v}{=}v{<}0$ find it optimal to purchase initially up to the cap on total exposure and signal $\tau{=}0$. Holding this optimal behavior fixed, it turns out that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price greater than 0, which is a contradiction.

Now we show that, even for $\min\left\{\zeta';\epsilon'\right\}{<}0$, a contradiction arises.

To proceed, the following intermediate results need to be listed: (1) An insider aware of $\widetilde{v}{=}v{>}0$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}{-1}$) and trade $x_1{\gtrsim}\, 0, x_2{\lesssim}\, x_{\mathcal{I}}$ (resp., $x_1{=}{-x_{\mathcal{I}}}$, $x_2{=}2x_{\mathcal{I}}$) rather than to signal $\tau{=}1$ (resp., $\tau{=}{-1}$) and trade any other pair of quantities $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{<}0, x_2{=}\cdot$). (2) An insider aware of $v \geq P_2(\tau{=}0){>}0$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}{-1}$) and trade $x_1{\gtrsim}\, 0, x_2{\lesssim}\, x_{\mathcal{I}}$ (resp., $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$) rather than to signal $\tau{=}0$ and trade any pair of quantities. (3) An insider who observes $0{<}v{<}P_2(\tau{=}0)$ strictly prefers to signal $\tau{=}1$ (or $\tau{=}{-1}$) and trade $x_1{\gtrsim}\, 0, x_2{\lesssim}\, x_{\mathcal{I}}$ (resp., $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$) rather than to signal $\tau{=}0$ and trade any pair of quantities only if $x_{\mathcal{I}}[v - P_2(\tau{=}1)]$ (resp., $-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2(\tau{=}{-1})]$) is strictly greater than $x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2(\tau{=}0)]$, that is, only if $v{>}\zeta'$ (resp., $v{>}\epsilon'$). (4) If $P_2(\tau{=}{-1}){>}\frac{P_2(\tau=1)}{2}$ (or $P_2(\tau{=}{-1}){=}\frac{P_2(\tau=1)}{2}$, or $P_2(\tau{=}{-1}){<}\frac{P_2(\tau=1)}{2}$), the profits that an insider aware of $\widetilde{v}{=}v{>}0$ earns from signaling $\tau{=}1$ and trading $x_1{\gtrsim}\, 0$, $x_2{\lesssim}\, x_{\mathcal{I}}$ are greater than (resp., equal to; smaller than) those from signaling $\tau{=}{-1}$ and trading $x_1{=}{-x_{\mathcal{I}}}, x_2{=}2x_{\mathcal{I}}$.

[5] First, notice that an insider aware of $\widetilde{v}{=}v{<}0$ strictly prefers to signal $\tau{=}0$ and trade $x_1{=}x_{\mathcal{I}}, x_2{=}{-2}x_{\mathcal{I}}$ rather than to signal $\tau{=}0$ and trade any alternative pair of quantities. Second, an insider who observes $v \leq P_2\left(\tau{=}1\right)$ (or $v \leq P_2\left(\tau{=}{-1}\right)$) strictly prefers to signal $\tau{=}0$ and trade $x_1{=}x_{\mathcal{I}}, x_2{=}{-2}x_{\mathcal{I}}$ rather than to signal $\tau{=}1$ (resp., $\tau{=}{-1}$) and trade $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{<}0, x_2{=}\cdot$). Third, an insider aware of $P_2\left(\tau{=}1\right){<}v{<}0$ (or $P_2\left(\tau{=}{-1}\right){<}v{<}0$) strictly prefers to signal $\tau{=}0$ and trade $x_1{=}x_{\mathcal{I}}, x_2{=}{-2}x_{\mathcal{I}}$ rather than to signal $\tau{=}1$ (resp., $\tau{=}{-1}$) and trade $x_1{>}0, x_2{=}\cdot$ (resp., $x_1{<}0, x_2{=}\cdot$) only if $\{x_{\mathcal{I}}v - 2x_{\mathcal{I}}[v - P_2\left(\tau{=}0\right)]\}$ is strictly greater than $\{x_{\mathcal{I}}[v - P_2\left(\tau{=}1\right)]\}$ (resp., than $\{-x_{\mathcal{I}}v{+}2x_{\mathcal{I}}[v - P_2\left(\tau{=}{-1}\right)]\}$), that is, only if $v{<}\zeta'$ (resp., $v{<}\epsilon'$).

For min $\{\zeta'; \epsilon'\}<0$, as a consequence of the results at points 1 to 4, the following conclusions can be drawn: (a) Suppose that $P_2(\tau{=}-1)>\frac{P_2(\tau=1)}{2}$ (case in which $P_2(\tau{=}1)<0$). (a.i) In the sub-case of $\zeta'<0$, for any value of ϵ', each insider aware of $\widetilde{v}{=}v{>}0$ finds it optimal to signal $\tau{=}1$ and trade $x_1 \gtrsim 0, x_2 \lesssim x_{\mathcal{I}}$. Holding this optimal behavior fixed, it follows that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price smaller than 0, which is a contradiction. (a.ii) The remaining sub-case with $\epsilon'<0 \leq \zeta'$ is not of interest, because it refers to a situation where $P_2(\tau{=}-1)+P_2(\tau{=}0)<0 \leq P_2(\tau{=}0)+\frac{P_2(\tau=1)}{2} \therefore P_2(\tau{=}-1)<\frac{P_2(\tau=1)}{2}$, whereas the case under consideration (i.e., point a) is the one in which $P_2(\tau{=}-1)>\frac{P_2(\tau=1)}{2}$. (b) For $P_2(\tau{=}-1)=\frac{P_2(\tau=1)}{2}=0$, we are in case $C1'$. (c) Suppose that $P_2(\tau{=}-1)=\frac{P_2(\tau=1)}{2}<0$. This condition on prices, together with the condition that min $\{\zeta'; \epsilon'\}<0$, imply that $\zeta'{=}\epsilon'{<}0$, in which case any insider aware of $\widetilde{v}{=}v{>}0$ finds it optimal to randomize between signaling $\tau{=}1$ and trading $x_1 \gtrsim 0, x_2 \lesssim x_{\mathcal{I}}$ on one side and signaling $\tau{=}-1$ and trading $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$ on the other. Holding the optimal behavior by these types of insider fixed, regardless of the probability with which each of them chooses to signal $\tau{=}1$ as opposed to signaling $\tau{=}0$ (even probability 0 or 1), the price in response to their signal turns out to lie below 0, which is a contradiction. (d) Suppose that $P_2(\tau{=}-1)<\frac{P_2(\tau=1)}{2}$ (case in which $P_2(\tau{=}-1)<0$). (d.i) In the sub-case of $\epsilon'<0$, for any value of ζ', each insider aware of $\widetilde{v}{=}v{>}0$ finds it optimal to signal $\tau{=}-1$ and trade $x_1{=}-x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$. Holding this optimal behavior fixed, it follows that in response to the signal sent by these types of insider, $\mathcal{M}$ is setting a second round price smaller than 0, which is a contradiction. (d.ii) The remaining sub-case with $\zeta'<0 \leq \epsilon'$ is not of interest, because it refers to a situation where $P_2(\tau{=}0)+\frac{P_2(\tau=1)}{2}<0 \leq P_2(\tau{=}-1)+P_2(\tau{=}0) \therefore \frac{P_2(\tau=1)}{2}<P_2(\tau{=}-1)$, whereas the case under consideration (i.e., point d) is the one in which $P_2(\tau{=}-1)<\frac{P_2(\tau=1)}{2}$.

$C8'$: $P_2(\tau{=}0)<0 \wedge P_2(\tau{=}1) \geq 0 \wedge P_2(\tau{=}-1) \geq 0$. This case is symmetric to $C7'$. ■

Internet Appendix B

On post-trade mandatory disclosure: Reconsidering van Bommel (2003). This appendix reconsiders the paper by van Bommel (2003), hereafter VB, which presents a Kyle's model with a risky asset exchanged among an insider, noise traders, $\mathcal{M}$, and competitive followers. The insider, $\mathcal{I}$, is constrained on asset holdings, and his orders have no direct price impact. The insider sends rumors to followers who reveal rumors to $\mathcal{M}$ through a change in asset demand. Two separate one-period models are presented. In the first, $\mathcal{I}$ is commonly known to be of type *"Honest."* By assumption, if an Honest insider observes $\widetilde{v}=v<0$ (or $\widetilde{v}=v \geq 0$), he says *"sell"* (resp., *"buy"*), while if he is uninformed, he does not say anything. In the second model, $\mathcal{I}$ is known to be of type *"Bluffer."* By assumption, if a Bluffer is informed about the real asset value, he behaves like an Honest, while if he is uninformed, he randomizes with equal probability between saying *"buy"* and saying *"sell."*

Under the assumptions that, first, $\widetilde{v}\sim\text{U}[-2,2]$, second, $\widetilde{\mathbf{u}}\sim\mathsf{N}(\mathbf{0},\mathbf{I})$, third, $cov(\widetilde{\mathbf{u}},\widetilde{v})=0$ and, fourth, the period is made of $n \in \{1,..,N\}$ rounds, VB argues that, as $n \to \infty$, in equilibrium the price at round n asymptotically converges to a certain value. However, the equilibrium price dynamic derived from assuming an Honest insider and the one derived from assuming a Bluffer are not appropriate, mainly because $\widetilde{v}$ and the aggregate demand at round n are treated as independent random variables, even though they are indirectly dependent ($\widetilde{v}$ affects $\mathcal{I}$'s rumor; the rumor impacts on followers' demand, which in turn affects the mean of the aggregate demand). Even considering van Bommel's (2008) paper, in which the author tries to justify why in VB the insider does not trade in any round from round $n=2$ to round $n=N-1$, the pricing rule is not justified.

In order to save the conclusion in VB, we propose to change the structure of VB models as follows: First, rather than a period made of infinite rounds, assume a period made of two rounds. Second, assume that $\mathcal{I}$ can spread rumors directly to $\mathcal{M}$. Under this new structure, the (corrected) contribution of VB analysis is the following: In the model with an Honest insider, if the insider says *"sell"* (or does not say anything, or says *"buy"*), then $p_2=-1$ (resp., $p_2=0$; $p_2=1$). In the model with a Bluffer, if the insider says *"sell"* (or *"buy"*), then $p_2=-q$ (resp., $p_2=q$). Note that these equilibria hold for a more general class of distributions than $\widetilde{\mathbf{u}}\sim\mathsf{N}(\mathbf{0},\mathbf{I})$. Also note that followers do not play a role, so there is no need to assume about them any more.

To relax this peculiar notion of type assumption, VB considers the single-period models with an Honest type and with a Bluffer type, and allows the profit-maximizing insider to choose between two alternatives when informed: to signal in accordance with his type or to *"cheat"* (i.e., to spread a so called *"false"* rumor). It is argued that the rumor is not informative any more because, holding fixed $\mathcal{M}$'s best response to an insider forced to play according to his type, the insider cheats when informed. However, this argument only proves that, for this very specific pricing rule, a deviation by the insider occurs.

VB also considers a reputational framework, and a pricing rule with a punishment scheme that makes the market ignore subsequent disclosures when the insider defects. In this framework, the sufficient condition for the sustainability of the so-called *"Honest equilibrium"* proposed in VB consists of an inter-period discount factor δ such that, when $\mathcal{I}$ is uninformed in period t=1, the profits from being Honest forever are greater than the profits from being Bluffer in period t=1 and, provided he does not immediately incur the punishment, Honest from period t=2 on. However, among other points, it is unclear why the insider should consider the opportunity of randomizing between saying *"sell"* and saying *"buy"* when uninformed at a certain date, but not when facing an identical situation in the future. Our methodology and results differ drastically. Specifically, for $\delta\in[0,1)$ and $q\in(0,1)$, we use four choice variables, $\bar{\alpha}^{*I}$, $\bar{\beta}^{*I}$, $\bar{\alpha}^{*U}$, and $\bar{\beta}^{*U}$, to prove the existence of and characterize different informative equilibria. These choice variables become five in case the insider, when informed, learns $\widetilde{v}$=v from the beginning of the period (see Section V).

While an ad hoc Grim trigger strategy for the sustainability of the Honest equilibrium is imposed in VB, we consider a general Grim trigger strategy. Using this strategy, we identify another group of informative equilibria, i.e., those with manipulation, which are similar to the equilibrium presented by VB in the stage game with an imposed Bluffer type. Further, in Internet Appendix C, we assess the existence of alternative informative equilibria.

Three final remarks are in order. First, contrary to what is stated in VB (p. 1502), not all $f(\cdot)$ can be used. Second, it is untrue that the analysis in VB *"uses a special case of the Crawford and Sobel (1982) signaling game"* (VB, p.1500). Unlike in VB one-period models, cheap-talk games allow the sender to choose what to signal; unlike in VB reputational framework, cheap-talk games do not require private information to be exogenously revealed at any time. Third, it is untrue that the equilibria in VB are Nash equilibria.

Internet Appendix C

Price-shift uniqueness. This appendix provides a sort of guided tour through the wide universe of multiple equilibria that arise in the reputational framework, and lists minimal restrictions on beliefs (that is, minimal conditions on $\mathcal{M}$'s trigger strategy) that guarantee *price-shift uniqueness.* To start with, notice that informative equilibria exist, in which $\mathcal{M}$ ignores signals in the first periods. For each pair δ and q associated with an equilibrium in which the pricing rule is such that period t=1 prices may react to public signals, infinite alternative equilibria exist, in which prices start moving according to the same rule from period t>1, as if history started from this period, whereas in the preceding $t-1$ periods, prices do not react to signals. In fact, any time signals are believed not to be informative in a specific period, $\mathcal{I}$ cannot do any better than trading as he does when this period is not repeated, a behavior that confirms $\mathcal{M}$'s initial beliefs. Although no limit can be set to the initial number of periods in which signals are believed not to be informative, in the following analysis there is no loss in generality in assuming that signals are believed to be informative from period t=1.

When selecting among $\mathcal{M}$'s trigger strategies, it seems natural to think of the following minimal conditions:

Condition 1 *In period t, only τ=0 (or $\phi_{\cdot,0}$) is never interpreted as a defection.*

Condition 2 *In period t, $P_2(\tau{=}-1) \gtrless 0 \Leftrightarrow P_2(\tau{=}1) \lessgtr 0$ (or $P_2(\phi_{\cdot,\varpi}) \gtrless 0 \Leftrightarrow P_2(\phi_{\cdot,\varpi'}) \lessgtr 0$, where $\varpi \cdot \varpi' < 0$).*

Condition 1 requires the signal τ=0 (or $\phi_{\cdot,0}$), disclosed at period t, to be the only signal following which no punishment at period t+1 is applied, even if this signal causes the price at period t to move in the wrong direction with respect to v. Condition 2 states that, if $P_2(\tau{=}1)$ (or $P_2(\phi_{\cdot,\varpi})$) shifts from 0, then $P_2(\tau{=}-1)$ (resp., $P_2(\phi_{\cdot,\varpi'})$) *somehow* shifts too, but in the opposite direction, and vice versa.

Even when restricting attention just to Grim trigger strategies, if only the first or second condition is imposed, informative equilibria exist, whose on-path dynamics are alternative to those in Proposition 5 and 6. By simultaneously imposing Condition 1 and 2, these alternative informative equilibria can be discarded. This is shown in three examples below (to simplify the argument, the focus is on the case of mandatory trade disclosure and on an asset value whose properties are those defined in Section I).

(1) The second condition alone is not enough to guarantee price-shift uniqueness. Consider a trigger strategy that differs from the one in Conjecture 1 in the function P_2': $\tau{=}-1 \rightarrow p_2{=}-q\xi$, $\tau{=}1 \rightarrow p_2{=}\xi$, and in the following sequential condition: *At the second round of the t^{th} period, if the outcome of all $t-1$ preceding periods has been $\tau{=}-1$ or $\tau{=}1 \wedge v{>}0$, then play P_2'; otherwise, set* p_2=0. For sufficiently high values of δ and sufficiently small values of q, this alternative trigger strategy is part of an equilibrium in which no punishment ever

takes place. In detail, when the insider observes $\widetilde{v}<0$ (or $\widetilde{v}>0$), he sells (resp., purchases) up to the cap on total exposure in the first round, subsequently trading $x_2=2x_{\mathcal{I}}$ if $-q\xi<v$ (resp., $x_2=-2x_{\mathcal{I}}$ if $v<\xi$) or $x_2=0$ otherwise. Instead, when uninformed, the insider trades $x_1=-x_{\mathcal{I}}$, $x_2=2x_{\mathcal{I}}$. In this way, the uninformed insider expects to earn positive profits by the end of the period without incurring market punishment. This happens because, given the trigger strategy in question, disclosed sales never trigger market punishment.[6] Finally notice that it is irrelevant to specify whether or not inactivity triggers market punishment, because the insider would not signal $\tau=0$ in any case. Under Condition 1, this alternative equilibrium is discarded.

(2) Likewise, the first condition alone is not enough to guarantee price-shift uniqueness. Consider a Grim trigger strategy that satisfies the first condition, with a pre-defection pricing rule such that $P_2''(\tau=-1)=P_2''(\tau=0)=0$ and $P_2''(\tau=1)=\xi$. Given this pricing rule, an insider who is informed about $\widetilde{v}<0$ and one who is uninformed about the real asset value are both indifferent about signaling $\tau=-1$ or $\tau=0$. In either case, by trading optimally, these types of insider expect to earn $x_{\mathcal{I}}\xi$ and 0 profits, respectively, without incurring market punishment. It follows that, for sufficiently high values of δ, an informative equilibrium can be identified, in which these types of insider hide their private information completely by disclosing $\tau=-1$ with probability $\sigma \in [0,1]$ and $\tau=0$ with probability $1-\sigma$, and expect to earn as much as under anonymity. Instead, an insider who is informed about $\widetilde{v}>0$ leads the price towards its real value, expecting to earn $L(\mu=\xi)$ by the end of the period. Under Condition 2, which prevents $P_2''(\tau=-1)$ from equalling 0 if $P_2''(\tau=1)$ differs from 0, this alternative equilibrium is discarded.

(3) Consider a Grim trigger strategy that satisfies the first condition, with a pre-defection pricing rule such that $P_2'''(\tau=-1)$ is *greater* than 0, $P_2'''(\tau=0)$ is equal to 0, and $P_2'''(\tau=1)$ is equal to ξ. For sufficiently high values of δ, it is easy to see that an informative equilibrium exists, in which an insider aware of $\widetilde{v}<0$ or one who is uninformed both signal $\tau=0$, expecting to earn as much as under anonymity. In fact, $\tau=0$ is the only signal that ensures that these types of insider do not incur market punishment. Instead, an insider who is informed about $\widetilde{v}>0$ leads the price toward its real value, expecting to earn $L(\mu=\xi)$ by the end of the period.

[6] For high values of q, this alternative trigger strategy is not part of an equilibrium, because an insider aware of $\widetilde{v}>0$ also prefers to trade $x_1=-x_{\mathcal{I}}, x_2=2x_{\mathcal{I}}$, in this way causing the price to shift in the wrong direction with certainty.

Under Condition 2, which requires $P_2'''(\tau{=}-1)$ to be negative if $P_2'''(\tau{=}1)$ is positive, this counterintuitive equilibrium is discarded.

The next lemma follows as a consequence of Condition 1 and 2.

Lemma 5 *For a reputational framework with trade (or ex-ante unverifiable information) disclosure and beliefs that are restricted to be such that Condition 1 and 2 hold, any trigger strategy is part of an informative equilibrium 'only if' (but not 'if') in each period in which the market conditions on disclosures,* $P_2(\tau{=}1){>}0$ *(resp.,* $P_2(\phi_{\cdot,m\neq0}) \neq 0$*) and* $P_2(\tau{=}0)$ *(resp.,* $P_2(\phi_{\cdot,0})$*) is 'sufficiently close' or equal to* 0.

Proof of Lemma 5. In this proof, we consider a period in which second-round prices react to disclosures. Specifically, in the first part of this proof (part I), we study the cases of mandatory and voluntary trade disclosure. In the second part of this proof (part II), we study the case of ex-ante unverifiable information disclosure.

(I) First, for $P_2(\tau{=}-1) \leq 0 \leq P_2(\tau{=}1)$, we prove that if $P_2(\tau{=}0)$ is 'too far away' from 0, the market efficiency condition is not satisfied. Suppose that $P_2(\tau{=}0){<}0$ (for $P_2(\tau{=}0){>}0$, the argument is symmetric). If an insider informed about $\widetilde{v}{>}0$ decides to disclose $\tau{=}1$—which in this case is the only signal that pushes the second round price toward its real value—this type of insider finds it optimal to buy up to the cap on total exposure in the first round, and subsequently trade $x_2{=}-2x_{\mathcal{I}}$ if $v{<}P_2(\tau{=}1)$ or $x_2{=}0$ otherwise. The expected profits associated with this behavior are equal to $2x_{\mathcal{I}}\{\int_0^{P_2(\tau=1)}[2P_2(\tau{=}1) - v]h(v)dv + \int_{P_2(\tau=1)}^{\bar{b}} vh(v)dv\}$. Instead, if this type of insider decides to disclose $\tau{=}0$—a signal that moves the second round price in the wrong direction without triggering market punishment—under a mandatory (or voluntary) trade disclosure rule, he finds it optimal to trade $x_1{=}0$ and $x_2{=}x_{\mathcal{I}}$ (resp., $x_1{=}-x_{\mathcal{I}}$ and $x_2{=}2x_{\mathcal{I}}$). The expected profits associated with this behavior are equal to $2x_{\mathcal{I}}\int_0^{\bar{b}}[v - P_2(\tau{=}0)]h(v)dv$ (resp., $2x_{\mathcal{I}}\int_0^{\bar{b}}[v - 2P_2(\tau{=}0)]h(v)dv$). It follows that, if $P_2(\tau{=}0)$ is smaller than $4\int_0^{P_2(\tau=1)}[v - P_2(\tau{=}1)]h(v)dv$ (resp., $2\int_0^{P_2(\tau=1)}[v - P_2(\tau{=}1)]h(v)dv$), an insider aware of $\widetilde{v}{>}0$ prefers to signal $\tau{=}0$ rather than to signal $\tau{=}1$ and thus the market efficiency condition is violated.[7]

Second, for $P_2(\tau{=}1) \leq 0 \leq P_2(\tau{=}-1)$, we prove that if $P_2(\tau{=}0) \neq 0$, again the market efficiency condition is not satisfied. To prove the result, it is sufficient to notice that, if

[7] In the case in which $P_2(\tau{=}-1){=}0$, it is easy to see that an insider aware of $\widetilde{v}{>}0$ prefers to signal $\tau{=}0$ rather than to signal $\tau{=}-1$. Therefore, even in this boundary case, this type of insider finds it optimal to move the price away from its real value.

$P_2(\tau{=}0){<}0$, an insider aware of $\widetilde{v}{>}0$ prefers to disclose $\tau{=}0$ (which moves the price in the wrong direction without triggering punishment) rather than to disclose $\tau{=}{-}1$ (which in this case is the only signal that moves the price toward its real value). For $P_2(\tau{=}0){>}0$, the argument is symmetric.

Third, for $P_2(\tau{=}1) \leq 0 \leq P_2(\tau{=}{-}1)$, we prove that if $P_2(\tau{=}0){=}0$, no informed type of insider prefers to push the price toward its real value. Specifically, we show that an insider informed about $\widetilde{v}{>}0$ prefers to signal $\tau{=}0$ rather than $\tau{=}{-}1$ (for a symmetric argument, an insider informed about $\widetilde{v}{<}0$ prefers to signal $\tau{=}0$ rather than $\tau{=}1$). Under mandatory (or voluntary) trade disclosure, if an insider aware of $\widetilde{v}{>}0$ decides to signal $\tau{=}0$, he finds it optimal to trade $x_1{=}0, x_2{=}x_{\mathcal{I}}$ (resp., $x_1{=}\cdot, x_2{=}x_{\mathcal{I}}{-}x_1$), which allows him to earn $x_{\mathcal{I}}\xi$ in expectation. Instead, if this type of insider decides to signal $\tau{=}{-}1$, he expects to earn less than $x_{\mathcal{I}}\xi$. In detail, in case he decides to disclose a sale, this type of insider is confronted with choosing between two relevant options: (i) trading $x_1 \lesssim 0$ and subsequently buying or selling depending on the realization of $\widetilde{v}{=}v$, and (ii) buying up to the cap on total exposure in the first round and subsequently trading $x_2{=}2x_{\mathcal{I}}$ if $P_2(\tau{=}{-}1){<}v$ or $x_2{=}0$ otherwise. The former trading option allows him to earn less than $x_{\mathcal{I}}\xi$ in expectation. The latter trading option allows him to earn $-x_{\mathcal{I}}\xi{+}2x_{\mathcal{I}}\int_{P_2(\tau=-1)}^{\bar{b}}[v - P_2(\tau{=}{-}1)]h(v)dv$ in expectation (which is again less than $x_{\mathcal{I}}\xi$).

(II) For $P_2(\phi_{\cdot,\varpi'}) \leq 0 \leq P_2(\phi_{\cdot,\varpi})$, where ϖ equals -1 (or 1) if ϖ' equals 1 (resp., -1), we show that if $P_2(\phi_{\cdot,0})$ is 'too far away' from 0, the market efficiency condition is not satisfied. Suppose that $P_2(\phi_{\cdot,0}){<}0$ (for $P_2(\phi_{\cdot,0}){>}0$, the argument is symmetric). If an insider aware of $\widetilde{v}{>}0$ decides to signal $\phi_{\cdot,\varpi}$—which in this case is the only message that moves the second round price toward its real value—this type of insider finds it optimal to buy up to the cap on total exposure in the first round, and subsequently trade $x_2{=}{-}2x_{\mathcal{I}}$ if $v{<}P_2(\phi_{\cdot,\varpi})$ or $x_2{=}0$ otherwise. The expected profits associated with this behavior are equal to $2x_{\mathcal{I}}\{\int_0^{P_2(\phi_{\cdot,\varpi})}[2P_2(\phi_{\cdot,\varpi}) - v]h(v)dv{+}\int_{P_2(\phi_{\cdot,\varpi})}^{\bar{b}}vh(v)dv\}$. Instead, if this type of insider decides to signal $\phi_{\cdot,0}$—a message that moves the price in the wrong direction without triggering market punishment—it is optimal for him to trade $x_1{=}{-}x_{\mathcal{I}}, x_2{=}2x_{\mathcal{I}}$. The expected profits associated with this behavior are equal to $2x_{\mathcal{I}}\int_0^{\bar{b}}[v - 2P_2(\phi_{\cdot,0})]h(v)dv$. It follows that, if $P_2(\phi_{\cdot,0})$ is smaller than $2\int_0^{P_2(\phi_{i,\varpi})}[v - P_2(\phi_{\cdot,\varpi})]h(v)dv$, an insider aware of $\widetilde{v}{>}0$ prefers to signal $\phi_{\cdot,0}$ rather than to signal $\phi_{\cdot,\varpi}$ and thus the market efficiency condition is violated.[8] ■

To provide an intuition for this lemma, we refer to the case of mandatory/voluntary trade disclosure (the case of ex-ante unverifiable information disclosure is similar but simpler). Condition 1 and 2 restrict the analysis to two classes of trigger strategies such that, at a specific period, either $P_2(\tau{=}1) \leq 0 \leq P_2(\tau{=}{-1})$ or $P_2(\tau{=}{-1}) \leq 0 \leq P_2(\tau{=}1)$, setting no condition on whether the missed disclosure of a purchase or of a sale moves prices at that period. We consider these two classes in order: In case $P_2(\tau{=}1) \leq 0 \leq P_2(\tau{=}{-1})$, no equilibrium exists in which disclosures are informative. In case $P_2(\tau{=}{-1}) \leq 0 \leq P_2(\tau{=}1)$, (i) if $P_2(\tau{=}0)$ is positive (or negative) and set *too far away* from 0, an insider aware about $\widetilde{v}{<}0$ (resp., $\widetilde{v}{>}0$) finds it optimal to move the price away from its real value, which is why the market efficiency condition is violated. Instead, (ii) if $P_2(\tau{=}0)$ is *sufficiently close* or equal to 0 (more precisely, if $P_2(\tau{=}0)$ is such that $\int_0^{P_2(\tau=1)}[v - P_2(\tau{=}1)]h(v)dv \leq \frac{P_2(\tau=0)}{\varrho} \leq \int_0^{-P_2(\tau=-1)}[-P_2(\tau{=}{-1}) - v]h(v)dv$, where ϱ equals 4 or 2 depending on whether trade disclosures are mandatory or voluntary, respectively), both types of informed insider prefer to lead the market rather than to signal $\tau{=}0$. This means that, under Condition 1 and 2, all trigger strategies such that, at a specific period, $P_2(\tau{=}1)$ is greater than 0 and $P_2(\tau{=}0)$ is sufficiently close to (but different) from 0 satisfy the market efficiency condition only if (but not if) an insider who is uninformed at this specific period finds it optimal to signal $\tau \neq 0$. As a consequence of Condition 1 and 2, *on the equilibrium path*, price shifts can occur only following the disclosure of a purchase or a sale, turning out to be positive or negative, respectively.

To better describe the implications of Lemma 5, we draw attention to the class of Grim trigger strategy such that, before defection, the way in which period t price reacts to one period t disclosure or another is identical *across* periods. Because of (i) restrictions *R1* to *R4* and Condition 1 and 2, (ii) the symmetry of the space of actions, and (iii) the symmetric consequences implied by the misleading behavior of one or the other type of informed insider, it follows that, if $\mathcal{M}$ undertakes a Grim trigger strategy belonging to the class in question, then all on-path pre-defection price shifts equal $q\xi$ or ξ in magnitude, depending on whether $\nabla(q, \mu{=}q\xi) \leq \delta{<}\Delta(q, \mu{=}\xi)$ or $\delta \geq \Delta(q, \mu{=}\xi)$, respectively.[9] Specifically, under a

[8] In the case in which $P_2(\phi_{\cdot,\varpi'}){=}0$, an insider aware of $\widetilde{v}{>}0$ prefers to signal $\phi_{\cdot,0}$ rather than $\phi_{\cdot,\varpi'}$. Hence, even in this boundary case, this type of insider finds it optimal to move the price away from its real value.

[9] Given his multi-period decision problem, if $\mathcal{I}$ turns out to be indifferent about misleading and leading

trade disclosure rule (or ex-ante unverifiable information disclosure), a pre-defection price response to τ=0 (resp., $\phi_{\cdot,0}$) equal to 0 is always admissible in equilibrium; for any pair q, δ such that q=1, $\delta \geq \delta_{\nabla}$ or q<1, $\nabla(q, \mu{=}q\xi) \leq \delta < \Delta(q, \mu{=}\xi)$, all price responses to τ=0 (resp., $\phi_{\cdot,0}$) such that $0 < |P_2(\tau{=}0)| \leq \varrho \int_0^{q\xi} (q\xi - v)h(v)dv$ (resp., $0 < |P_2(\phi_{\cdot,0})| \leq 2\int_0^{q\xi} (q\xi - v)h(v)dv$) are also admissible. Because all these alternative price responses to τ=0 (resp., $\phi_{\cdot,0}$) are off-path responses, there is no loss in generality in setting $P_2(\tau{=}0){=}0$.

Under Condition 1 and 2, even restricting attention only to trigger strategies with a Grim punishment, multiple informative equilibria can be identified such that, before defection, a (finite, time-varying) number of periods in which disclosures are believed to convey information about the insider's information is *alternated* with a (finite, time-varying) number of periods in which disclosures are believed to be uninformative. The next condition on beliefs prevents this intermittent pre-defection price reaction. Without this condition, the insider's incentives to mislead the market when informed, as well as his incentive to manipulate when uninformed, would also depend on the frequency with which $\mathcal{M}$ stops conditioning on disclosures before defection, and this in turn would have consequences on the way pre-defection prices react to disclosures—that is, price shift uniqueness would not be achieved.

Condition 3 *If at a certain period disclosures are believed to be informative, disclosures are also believed to be informative in each subsequent period until a (new) defection is publicly recorded.*

Next, consider any pair δ and q such that an informative equilibrium exists in which pre-defection price responses are supported by a Grim punishment. For the same pair δ and q, other informative equilibria may exist in which the same pre-defection price responses are supported by a less severe punishment (that is, a *non-Grim* punishment) such that prices at some point after defection start reacting again to disclosures. In this regard, Condition 4 constrains beliefs formed in response to a disclosure—and prices set by a market holding these beliefs—as follows:

Condition 4 *Let beliefs be such that: (i) Before a defection, if prices react to disclosures, they move as if, after this defection, a Grim punishment occurs. (ii) At some point after a*

(or about bluffing and non-bluffing), we break the indifference in favour of the latter alternative.

defection, prices can start reacting again to disclosures, provided the implicit market punishment following this defection represents a deterrent to support all past price changes.

To understand the implications of this condition for price-shift uniqueness, consider any pairs δ, q such that $\delta \geq \nabla(q, \mu{=}q\xi)$, and the associated equilibrium pre-defection pricing rule presented in Proposition 3, 4, or 6, which is supported by the Grim punishment. If prices at some point after defection start reacting again to disclosures, and in a way that does not represent a deterrent that is as strong as the Grim punishment, then before this defection, the insider's incentive to mislead the market, as well as his incentive to bluff, are affected. Condition 4 prevents repercussions of a weakening of market punishment following a defection on the way prices react to disclosures before this defection.

Provided that the four conditions on beliefs presented in this appendix are simultaneously fulfilled, if any equilibrium price shift occurs at period t, then the way the price reacts in response to a specific signal, disclosed at period t, is unique. Specifically, the magnitude of the price shift depends on q and δ. This result is presented in more detail in Proposition 7:

Proposition 7 *Consider a reputational framework with either trade disclosure or ex-ante unverifiable information disclosure. Under restrictions R1 to R4 and Condition 1, 2, 3, and 4, at a specific period the equilibrium price shifts downwards (or upwards) 'only if' the insider discloses $\tau{=}-1$ or $\phi_{\cdot,\varpi'}$ (resp., $\tau{=}1$ or $\phi_{\cdot,\varpi}$). Specifically, for $\delta \geq \Delta(q, \mu{=}\xi)$, this price shift equals ξ in magnitude, and occurs in response to the disclosure by an insider aware of $\widetilde{v}{<}0$ (resp., $\widetilde{v}{>}0$). For $\nabla(q, \mu{=}q\xi) \leq \delta{<}\Delta(q, \mu{=}\xi)$, this price shift equals $q\xi$ in magnitude, and occurs in response to the disclosure by an insider aware of $\widetilde{v}{<}0$ (resp., $\widetilde{v}{>}0$) or by an insider who is currently uninformed. For $\delta{<}\nabla(q, \mu{=}q\xi)$, no price shift ever occurs.*

Finally, note that (i) for $\delta{=}\nabla(q, \mu{=}q\xi)$ or $\delta{=}\Delta(q, \mu{=}\xi)$, only one informative equilibrium satisfies Condition 1, 2, 3, and 4. At this equilibrium, any disclosure following a first defection is believed not to be informative. Conversely, (ii) for each pair δ and q such that $\nabla(q, \mu{=}q\xi){<}\delta{<}\Delta(q, \mu{=}\xi)$ or $\delta{>}\Delta(q, \mu{=}\xi)$, many other informative equilibria also satisfy Condition 1, 2, 3, and 4. At each of these equilibria, the pricing rule is such that, at some point after a first defection, prices start reacting again to disclosures. However, (ii.a) while for any pair δ, q such that $\nabla(q, \mu{=}q\xi){<}\delta{<}\Delta(q, \mu{=}\xi)$ and $q{<}1$ an unlimited number of alternative on-path dynamics are possible,[10] (ii.b) for each pair $\delta{>}\delta_{\nabla}, q{=}1$ and each pair $\delta{>}\Delta(q, \mu{=}\xi), q$, all the informative equilibria display identical on-path dynamics, because no defection *actually* occurs in equilibrium.

Internet Appendix D

Informative disclosure of (missed) trade reversals. This appendix considers a situation in which trade disclosure is imposed when the short-swing rule is not, and characterizes a reputational equilibrium in which the insider does not undertake uninformed manipulations and price efficiency is higher than with the short-swing rule. This equilibrium arises under the conditions that the insider repeatedly receives *long-lived* inside information, i.e., $N \geq 3$, and he weights future profits sufficiently. For simplicity's sake, we refer to the case of $N{=}3$ (for $N{>}3$, the argument is similar). Denote, with $\langle P_1, P_2, P_3 \rangle$, the market's pricing rule at a certain period and, with $\tau' {\in} \{-1, 0, 1\}$, the public signal released at the beginning of the third round. Specifically, $\tau'{=}{-1}$ (or $\tau'{=}0$; or $\tau'{=}1$) reveals that in the second round the insider sold (resp., did not trade; bought); hence, because $\Omega_3{=}\{\tau, \tau'\}$, it follows that P_3: $\{-1, 0, 1\}^2 {\to} \mathcal{V}$. Under the assumption that the statistical properties of $\widetilde{v}$ defined in Section I hold, consider a sequential condition, such that prices at period t react to disclosed trades unless disclosure moved prices away from the real asset value in any of the $t-1$ preceding periods. Now, consider an insider who is aware of $\widetilde{v}{>}0$ (for an insider who is aware of $\widetilde{v}{<}0$ the argument is symmetric). In equilibrium, this insider behaves as follows: In the first round, he buys up to the cap on total exposure. In the case in which $\widetilde{v}{=}v \in [0, \xi)$, in the second round he completely reverses his initial position. In this case, if $\widetilde{v}{=}v \in [\underline{\xi}, \xi)$, where $\underline{\xi}{=}E[\widetilde{v}|0 \leq \widetilde{v} \leq \xi]$, in the third round this insider completely reverses his second round position—said differently, the second round reversal is followed by a symmetric third round reversal—while if $\widetilde{v}{=}v \in (0, \underline{\xi})$, he does not trade any more. Conversely, in the case in which $\widetilde{v}{=}v \in [\xi, \overline{b})$, in the second round the insider does not trade. In this case, if $\widetilde{v}{=}v \in [\xi, \overline{\xi})$, where $\overline{\xi}{=}E[\widetilde{v}|\xi \leq \widetilde{v} {<} \overline{b}]$, in the third round this insider completely reverses his first round position, while if $\widetilde{v}{=}v \in [\overline{\xi}, \overline{b})$, he does not trade.

[10]For each of these pairs δ, q, consider the informative equilibrium in which, immediately after a first defection, the price equals 0 for the smallest possible number of periods such that the entire pricing rule following this defection is still sufficient to support all prices set before defection. Infinite other equilibria exist in which, after the first defection *actually* occurs, $\mathcal{M}$ correctly believes that no disclosure is informative for a greater (but finite) number of periods.

For what concerns equilibrium prices, following an initial purchase, at the second round the price response $P_2(\tau{=}1)$ equals ξ, while at the third round $P_3\binom{\tau=1}{\tau'=-1}$ and $P_3\binom{\tau=1}{\tau'=0}$ equal $\underline{\xi}$ and $\overline{\xi}$ respectively; symmetrically, following an initial sale, we have $P_2(\tau{=}-1){=}-\xi$, $P_3\binom{\tau=-1}{\tau'=1}{=}-\underline{\xi}$, and $P_3\binom{\tau=-1}{\tau'=0}{=}-\overline{\xi}$; finally, p_1, $P_2(\tau{=}0)$, and $P_3(\tau{=}0,\cdot)$ are all equal to 0. Absolute continuity of $F(\,\cdot\,)$ and symmetry of $f(\,\cdot\,)$ can be easily relaxed, and a more general set of restrictions that includes *R1–R4* identified.

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