Leisure Preferences, Long-Run Risks, and Human Capital Returns

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August 17, 2015

Abstract

We analyze the contribution of leisure preferences to a model of long-run risks in leisure and consumption growth. The marginal utility of consumption is affected by short- and long-run risks in leisure under nonseparable and recursive preferences. Our model matches equity risk premia and macroeconomic moments with plausible coefficients of relative risk aversion. Further, the incorporation of leisure in utility allows us to examine the optimal tradeoff between labor and leisure and derive model implications for the price of and return on human capital. Human capital exhibits returns that are significantly less volatile than and positively correlated with stock returns, implies expected returns that are between 25% and 50% of the equity premium, and a Sharpe ratio that can be 60% higher than that of the equity return.

*We thank Kyung Hwan Shim, Stijn Van Nieuwerburgh, Amir Yaron, and participants at the SFS Cavalcade 2011, the FIRS Meeting 2011, the Michigan Finance brownbag, and the AFA 2015 Meeting for helpful comments and suggestions.

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1 Introduction

A long-standing practice in the analysis of consumption, portfolio choice, and asset pricing in the endowment economy of Lucas (1978) is the measurement of the representative agent’s utility over consumption of nondurable goods and services. This practice, popularized in Hansen and Singleton (1982) and Mehra and Prescott (1985) is justified on the basis of the assumption that intratemporal preferences are separable over consumption of the basket of nondurables and services and other sources of utility. This assumption can be justified in the standard framework of power utility, implying that asset prices are affected only by consumption of nondurable goods and services and not directly by other potential sources of utility. However, as noted in Uhlig (2010), this assumption is no longer valid under recursive preferences, such as those analyzed in Epstein and Zin (1989). With recursive preferences, the marginal utility of consumption depends not only on current consumption, but also on continuation utility. If agents derive utility from quantities other than consumption of nondurables and services, the marginal utility of consumption, and thus asset prices, will depend on these quantities through the continuation utility.\footnote{Implications of preferences over consumption outside of the standard bundle of nondurables and services have been explored previously in the literature. Eichenbaum, Hansen and Singleton (1988) examine implications of preferences over leisure in the context of a non-separable utility function. Yogo (2006) derives a model with non-separable preferences over durable goods and examines implications for the equity premium puzzle. Yang (2011) considers the contribution of preference over durable goods to the long-run risk model.}

The issue of sources of marginal utility of consumption is particularly germane in the context of recent advances in asset pricing that rely on recursive preferences to generate implications for aggregate asset risk premia. In particular, Bansal and Yaron (2004) derive a model with persistent means of consumption growth and volatility that generates asset market phenomena consistent with the observed data under the assumption of recursive preferences. Persistence in these moments is also generated endogenously in general equilibrium economies with recursive preferences by Kaltenbrunner and Lochstoer (2010) and Croce (2014). These frameworks rely on measurement of marginal utility of consumption with respect only to consumption of nondurable goods and services. An open question is the degree to which preferences over quantities other than nondurable goods and services affect equilibrium asset prices. In this paper, we address this question through the analysis of the impact of preferences over the consumption of leisure on equilibrium in asset markets.

We concentrate on the impact of leisure in marginal utility for a number of different reasons. In endowment economy models, asset prices are traditionally determined by agents' allocation of wealth to consumption and investment. Allocating more wealth to investment results in a higher flow of future dividends available for consumption. Agents can also consume income derived through the provision of labor, but there is no explicit tradeoff between provision of work hours and utility. Consequently, agents will optimally provide all available work hours to maximize consumption, and...
the labor-leisure tradeoff will not affect marginal utility, nor, as a result, asset prices. Empirically, however, we observe considerable variation in the provision of labor hours, which is frequently modeled in general equilibrium by introducing leisure preferences, resulting in elastic labor supply. The implication in our context is that agents assess the tradeoff between provision of labor resulting in income flow for consumption, and the consumption of leisure. We analyze the importance of this tradeoff in determining equilibrium asset prices.

An additional benefit of considering preferences over consumption and leisure is in analyzing the return on human capital. The importance of human capital in asset pricing has generated significant attention in the recent literature, including Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Lustig and Nieuwerburgh (2006), and Bansal et al. (2014). In these papers, labor income is viewed as a dividend to human wealth, but the portfolio choice decision in the allocation of the endowment of hours is not explicitly modeled. As a result, an equilibrium price of human capital is not endogenously determined, and the interaction between financial wealth, human wealth, and consumption of resources cannot be fully analyzed. By introducing utility over leisure into the model, we are able to provide an analysis of the risk and price of human capital and its resulting impact on equilibrium financial asset pricing. This analysis also contributes to a growing literature examining the impact of labor and asset pricing, including Favilukis and Lin (2015), Li and Palomino (2014), and Petrosky-Nadeau, Zhang and Kuehn (2013).

Last, introduction of preference for leisure generates implications for equilibrium dividends from firms in the economy. Endowment economy asset pricing models generally specify dividends and consumption as different exogenous processes, with dividend growth dynamics that generate more volatility than consumption growth. By introducing the resource constraints that state that consumption is funded by dividends and labor income with limits to the amount of labor that can be provided, we are able to derive an endogenous dividend growth process. We do not explicitly use this process as it links total dividends to consumption and labor income, rather than the dividends per share of equity ownership typically investigated in the literature. However, the endogenous process can be utilized to better understand the relation between consumption, dividends, and labor income, and the resulting relation between these quantities and asset prices.

We examine financial asset and human capital pricing through the lens of a long-run risk model with non-separable preferences between leisure and consumption. This framework allows us to analyze different degrees of substitutability of leisure and consumption, and resulting implications for macroeconomic and financial asset quantities. We calibrate the model to key moments of the data, guided by an empirical analysis of the joint dynamics of consumption, leisure and wages. In order to compare the impact of including leisure in preferences, we compare our calibrated model to a baseline calibration in Bansal, Kiku and Yaron (2007) in which agents derive utility only from consumption of nondurable goods and services. Additionally, we use the model calibrated to the
moments of macroeconomic and financial market data to generate new implications for the riskiness of investment in human capital and its resulting excess return.

Our empirical analysis indicates that consumption, (detrended) leisure, and wages share persistent common components with explanatory power for dividend dynamics. One of these sources of long-run risk has opposite effects on consumption growth and leisure, driving the negative correlation in these two variables observed in the data. Additionally, the dynamics of these series display common time variation in volatility, supporting the modeling of asset prices with exposures to persistent risk in aggregate economic uncertainty.

In calibration, we find that the model incorporating preference over leisure performs about as well as the nondurable goods and services consumption-only model in matching the aggregate moments of asset returns and macroeconomic quantities, with some marginal improvements and additional insights. Like the calibrations in Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007), our model is able to match the equity risk premium with a reasonable degree of risk aversion. This coefficient of risk aversion is substantially lower when computed relative to gambles over nondurable goods and services than when computed relative to aggregate wealth. The difference in these results is attributable to the fact that with leisure preferences, claims to the consumption bundle reflect only a fraction of total wealth.

We also find that the price-dividend ratio lacks predictive power for leisure, labor income and wage growth, in addition to consumption growth. However, the price-dividend ratio has predictive power for the volatility of these series. These results corroborate the calibration of Bansal, Kiku and Yaron (2007) in emphasizing the conditional volatility, rather than conditional mean as a source of long-run risk. Finally, we find that incorporating leisure preferences reduces the negative slope of the term structure of real interest rates relative to the consumption-only model. This alleviates, but does not eliminate, the criticism of Beeler and Campbell (2012) of negative long-term real yields implied by the long-run risk framework.

In addition to these comparisons with the existing long-run risk calibrations, we document novel implications for the price of human capital risk and the relation between the excess return on human capital and equity. We find that human capital claims to both labor income and wages are much less volatile than those of equities, resulting in a risk premium that is 25-50% of the risk premium on equity. However, while the risk premium is reduced, the reduction in volatility is even greater, such that the Sharpe ratio associated with human capital claims can be 60% larger than that associated with stock market investment. Further, we find that excess returns to human capital claims are positively correlated with excess returns on equities, consistent with the evidence in Bansal et al. (2014) and contrary to that in Lustig and Nieuwerburgh (2006).

The remainder of this paper is organized as follows. In Section 2, we discuss the construction
and sample moments of the data to which we calibrate the model parameters. Additionally, we investigate the joint dynamics of leisure and consumption and wage growth, and these variables' relation to aggregate dividend growth, in order to understand sources of risk and provide parameter estimates for model calibration. In Section 3, we present model solutions for prices of risk and financial asset prices. Calibration of the model and analysis relative to existing long-run risk frameworks is presented in Section 4, with implications for the returns to human capital. Concluding remarks are provided in Section 5.

2 Empirical Analysis

There are three economic primitives in the framework that we examine: consumption growth, wage growth, and leisure. We have several goals in the empirical analysis of these series. First, we document the dynamic properties of the leisure and wage series used in our analysis. While the properties of consumption dynamics are well known, less attention has been paid to leisure and the implied series of wage growth. Second, we want to identify how many independent sources of variation are present in these series and how persistent these sources of variation are. This analysis will guide our model construction in identifying the number and sources of long-run risk in the data. Finally, we examine the relation between dividend growth and any sources of long-run risk in these data. This analysis allows us to parameterize our dividend process for the purpose of calibrating the model.

2.1 Data Description and Construction

We use annual observations for consumption, leisure, labor income, and dividends from 1929-2011. Consumption is measured as per capita real consumption of nondurables and services, as in Bansal and Yaron (2004). Labor income is calculated as in Lettau and Ludvigson (2005) as per capita real after-tax labor income. Specifically, pretax labor income is calculated as wages and salaries, plus personal current transfer receipts, plus employer contributions for employee pension and insurance funds, less the difference in domestic contributions for government social insurance and employer contributions for government social insurance. Taxes are calculated as wage and salary income times personal current taxes, divided by the sum of wage and salary income, proprietors income, rental income, and income receipts on assets. Data are sampled at the annual frequency from 1929 through 2011 and converted to real using the Personal Consumption Expenditure (PCE) deflator. These data are obtained from the National Income and Product Account (NIPA) tables at the Bureau of Economic Analysis (BEA).

The leisure series is the series used in Ramey and Francis (2009b) from the Bureau of Labor
Statistics (BLS), and obtained from Valerie Ramey’s website.\textsuperscript{2} The series is constructed as the ratio of leisure hours to the total number of hours available for work and leisure activities. We assume that the total number of hours is $16 \times 7 = 112$ hours per week.\textsuperscript{3} Wages are inferred using the labor income series described above and hours worked. Specifically, wages are calculated by dividing the real per capita labor income series by number of hours worked to produce a measure of real per capita annual wages.

Asset market data are obtained from CRSP. Dividends per share are computed using the CRSP value-weighted index. We first compute the dividend yield as the difference in the monthly cum-dividend return on the index and the ex-dividend return on the index. The dividend per share is then calculated by multiplying the dividend yield by the lagged value of the cumulative capital gain on the index. Monthly data are summed to the annual frequency and converted to real using the PCE deflator. We use this per-share dividend series and the cumulative capital gain on the index to compute the price-dividend ratio. The real risk-free rate is computed using a simplified version of the procedure in Pflueger and Viceira (2011) and Beeler and Campbell (2012). This rate is obtained by subtracting an estimate of expected inflation from the nominal risk-free rate (one-month T-Bill rate). Expected inflation is measured by regressing future inflation on the current nominal rate and the current and lagged values of monthly inflation for one year.

\textit{Properties of Leisure}

Our leisure series is plotted in Figure 1. As shown in the plot, the time series of leisure is dominated by two episodes. The first is an extremely volatile period from 1929 to 1950, where leisure displays dramatic increases and decreases, punctuated by a large increase in the period immediately following World War II. The second episode is in the period from 1950 to 2011, in which leisure exhibits a steady upward trend. The overall trend of decreasing work hours, and consequently increasing leisure hours, is documented in\textsuperscript{?}, in the context of manufacturing hours, and Ramey and Francis (2009\textsuperscript{a}) using these data.

There are a number of potential explanations for these trends. Manufacturing work hours may have exhibited a secular decline due to the reduced importance of manufacturing in the American economy over the past century. Second, once one accounts for schooling and household production, Ramey and Francis (2009\textsuperscript{a}) show that the trend in work hours (and hence leisure) is dramatically less pronounced. Regardless, the trends present an empirical challenge for our framework, which is

\textsuperscript{2}We thank Valerie Ramey for making the data available at her website, \texttt{http://www.econ.ucsd.edu/~vramey/research.html}.

\textsuperscript{3}In an earlier version of this paper, we utilized a leisure series from Ramey and Francis (2009\textsuperscript{a}). These data differ from the standard measures of labor and leisure by accounting for hours spent in household production and education. The resulting leisure series exhibits less of an upward trend in the post-war data than alternative measures such as the measure used in this paper. We utilize the more standard series since our model does not incorporate household production and the data are available only through 2005.
a standard endowment economy with no household production. Specifically, the trends in work and leisure generate nonstationarity in the leisure series. This problem is not only empirical. Since leisure is necessarily constrained by the number of hours available in a day, it cannot grow at an unconstrained rate.

Due to these issues, we detrend the leisure series using the Hodrick-Prescott filter. As suggested in Hodrick and Prescott (1997), we use a smoothing parameter of 100 for annual data. The trend and cycle component of these series are also depicted in Figure 1. As shown in the figure, the filter implies a nonlinear trend in the leisure series and a stationary cyclical component. The first-order autocorrelation of the cyclical component is 0.67, and the augmented Dickey-Fuller test statistic of -4.96 rejects the null of non-stationarity at the 1% critical level. The autocorrelation function suggests some oscillatory behavior, but the partial autocorrelation function indicates that the series is reasonably well described as an AR(1) process. We utilize the cyclical component of the leisure series in the remainder of our analysis.

Summary Statistics

Summary statistics for growth in consumption, wages, and dividends, and the level of detrended leisure are presented in Table 1. Moments of consumption and dividend growth are familiar to readers of this literature; the mean of consumption growth is approximately 2% per annum, has low volatility of 2.25%, and is positively autocorrelated at the annual frequency, with a first-order autocorrelation of 0.44. Dividend growth has a somewhat lower mean at 1.38% per annum, but is substantially more volatile at 10.82% per annum. Dividends are also less autocorrelated, with first- and second-order autocorrelations of 0.21 and -0.22, respectively.

By construction, the mean level of the leisure cycle component is zero. The mean of the raw series is -0.40, corresponding to an average seven-day week divided into approximately 37 hours of work and 75 hours of leisure time. Leisure exhibits substantial first-order autocorrelation of 0.67, suggesting that time spent in leisure is fairly persistent, and also exhibits fairly high second-order autocorrelation of -0.30. Wage growth over the full sample has a larger mean and volatility than consumption growth; average wage growth is approximately 2.70% with a standard deviation of 3.46% per annum. However, growth in wages is considerably less persistent than either consumption growth or leisure, with first- and second-order autocorrelations of 0.18 and 0.11, respectively.

We display unconditional correlations of the variables in our analysis in Panel B of Table 1. Most of the variables exhibit positive correlation. Consumption and dividend growth exhibit the highest correlation (0.62), followed by consumption and wage growth (0.58). Wage growth is modestly correlated with leisure (0.27) and dividend growth (0.40). Finally, leisure is virtually uncorrelated

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4The first-order autocorrelation in the leisure series is 0.926, and the augmented Dickey-Fuller test fails to reject the null of non-stationarity with a test statistic of -2.25 (5% critical value of -2.92).
with consumption growth (-0.10) and dividend growth (0.06).

### 2.2 Joint Dynamics of Consumption, Leisure, and Dividends

Similar to Bansal and Yaron (2004), we posit a set of joint dynamics for consumption, wages, leisure, and dividends where the variables have potentially time-varying conditional means and volatility. Specifically, the framework that we have in mind allows for the following generalized dynamics:

\[
\begin{pmatrix}
\Delta c_{t+1} \\
\Delta w_{t+1} \\
l_{t+1} \\
\Delta d_{t+1}
\end{pmatrix} =
\begin{pmatrix}
\mu_c \\
\mu_w \\
0 \\
\mu_d
\end{pmatrix} +
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
\phi_c & \phi_w & \phi_l
\end{pmatrix}
\begin{pmatrix}
x_{c,t} \\
x_{w,t} \\
x_{l,t}
\end{pmatrix} +
\begin{pmatrix}
\sigma_{c,t}\eta_{c,t+1} \\
\sigma_{w,t}\eta_{w,t+1} \\
\sigma_{l,t}\eta_{l,t+1}
\end{pmatrix} \\
\varphi_d \left( \sigma_{c,t} + \sigma_{w,t} + \sigma_{l,t} \right) \eta_{d,t+1}
\]

(1)

In this framework, the unconditional mean of leisure is constrained to zero, as discussed in the previous section. The conditional mean of dividend growth potentially depends on the conditional means of consumption growth, wage growth, and leisure.

Identification of the conditional means of consumption growth, wage growth, and leisure is complicated by the fact that the means are latent processes. In order to identify these conditional means and estimate loadings of dividend growth on the conditional means, we speculate that consumption growth, wage growth, and leisure can be represented by a three factor structure,

\[
q_{t+1} =
\begin{pmatrix}
\Delta c_{t+1} \\
\Delta w_{t+1} \\
l_{t+1}
\end{pmatrix} =
\begin{pmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{pmatrix}
\begin{pmatrix}
y_{1,t+1} \\
y_{2,t+1} \\
y_{3,t+1}
\end{pmatrix} = B y_{t+1},
\]

(2)

where \(y_{1,t+1}, y_{2,t+1},\) and \(y_{3,t+1}\) are orthogonal random variables. The conditional means of the observed variables are constructed by regressing them on lags of the principal components,

\[
q_{t+1} = a + A y_t + \eta_{q,t+1},
\]

(3)

and the conditional means of consumption growth, wage growth, and leisure can be represented as \(\{x_{c,t}, x_{w,t}, x_{l,t}\} = A B y_t\). We identify \(y_{t+1}\) and the matrix \(B\) through principal components analysis of the covariance matrix of consumption growth and leisure. We then use the resulting conditional means of consumption growth and leisure to estimate \(\phi_c, \phi_w,\) and \(\phi_l\), the loadings of dividend growth on the conditional means.

Our analysis is equivalent to a transformation of a vector autoregression of consumption growth, wage growth, leisure, and dividend growth, where consumption growth, wage growth, and leisure
do not depend on dividend growth. The addition of the layer of complexity resulting from the principal components analysis is to isolate the conditional mean processes $x_{c,t}$, $x_{w,t}$, and $x_{l,t}$, which are potentially dependent on $\Delta c_t$, $\Delta w_t$, and $l_t$. The principal components analysis above will generate loadings of consumption growth on the extracted $x_{c,t}$ of one and on the extracted $x_{w,t}$ and $x_{l,t}$ of zero by construction. Similarly, wage growth and leisure will load only on their own extracted conditional means with a coefficient of one.

Results of the principal components analysis on the covariance matrix of consumption growth, wage growth, and the cycle portion of leisure from the Hodrick-Prescott filter are presented in Table 2. The analysis suggests that there are three distinct sources of variation in the data. The first principal component explains approximately 53% of the variation in the data, and all three variables have positive loadings. Consumption growth and wage growth load most strongly on this principal component, with less impact of leisure. The second principal component explains approximately 36% of the variation in the three variables. Leisure loads with a coefficient of nearly one (0.91) on this component, while consumption loads negatively and wage growth has a loading close to zero. Finally, the third principal component, explaining approximately 11% of the variation in the three variables, generates negative loadings for consumption growth and leisure and a positive loading for wage growth.

In Panel B of Table 2, we present the results of a reduced-form VAR of the three principal components. The purpose of this VAR is simply to get a feel for the joint dynamics of the principal components and the degree of persistence, if any, in the components. The results indicate that all three principal components appear to have statistically significant coefficients on their own lags. The most persistent principal component is the second component, with a coefficient on its own lag of 0.67 (SE=0.08). This principal component also depends on lagged values of the first principal component with a negative coefficient of -0.14 (SE=0.07). The remaining two principal components depend only on their own lags and have somewhat more subdued persistence. Principal component one has a coefficient of 0.29 (SE=0.10) on its own lag and principal component three has a coefficient of 0.33 (SE=0.08) on its own lag. Thus, the evidence argues in favor of three sources of persistent variation in the data.

We next regress consumption growth, wage growth, and detrended leisure on lags of the principal components in order to construct conditional means of these variables, and regress dividend growth on the constructed conditional means to ascertain which sources of long-run risk are relevant for pricing. Results of this analysis are presented in Table 3. The table indicates that the conditional mean of consumption growth depends significantly positively on the first principal component and negatively on the third principal component. The leisure conditional mean depends significantly on the second principal component, and not on the remaining principal components. Interestingly, wage growth does not depend on any of the principal components, suggesting little evidence of
a persistent conditional mean of wage growth. We conclude that two sources of long-run risk, a conditional mean of consumption growth and a conditional mean of leisure, are likely to describe the data.

In the final row of the table, we regress dividend growth on the estimated conditional means of consumption growth, wage growth, and leisure. Dividend growth is positively and significantly exposed to the conditional means of consumption growth and leisure. The point estimates for consumption exposure of 6.15 (SE=1.77) are larger than the leverage value used in Bansal and Yaron (2004). The point estimate for leisure of 6.26 (SE=2.29) is of similar magnitude. Dividends are negatively exposed to the conditional mean of wage growth, with a point estimate of -4.26 (SE=2.68). However, the exposure is not statistically significant and, as mentioned above, there is little evidence to support wage growth depending on the lagged latent variables.

2.3 Conditional Variance of Consumption, Leisure, and Wage Growth

We next focus on the conditional variance of innovations to consumption growth, wage growth, and leisure. Using the residuals from the projection of consumption onto the lagged values of principal components, equation (3), we analyze variance ratios for the absolute value of the residuals,

$$VR_k = \frac{Var \left( \sum_{j=0}^{J-1} \eta_{k,t+j} \right)}{J \cdot Var \left( \left| \eta_{k,t} \right| \right)}$$

for $k = \{\eta_c, \eta_w, \eta_l\}$. Under the null that variances of innovations are constant, the variance ratio should be close to one and flat with respect to the horizon. We compute variance ratios for horizons $J = 2, 5$, and 10 years.

Variance ratio results are tabulated in Panel A of Table 4. Beneath each statistic, we present the 95% critical values of 10,000 bootstrapped distributions. As shown in the table, evidence for persistence in volatility of consumption growth innovations is borderline. The variance ratio increases with the horizon, but the statistic surpasses the 95% critical value only for the 10-year horizon. Evidence is stronger for the innovation in wage growth and in leisure. The variance ratio for wage growth increases from 1.19 at the 2-year horizon to 2.30 at the 10-year horizon; all three ratios exceed the 95% critical value. Similarly, the variance ratio for leisure rises from 1.33 at the 2-year horizon to 3.29 at the 10-year horizon, which each ratio above the 95% critical value. Thus, the evidence suggests at least borderline evidence of persistence in volatility in each of the innovations.

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5Stock and Watson (2002) present evidence of changes in the volatility of a set of macroeconomic variables over time, and potential explanations. Justiniano and Primiceri (2008) provide an estimation an equilibrium model that supports the importance of investment shocks for these changes in volatility.
As an alternative look at time-varying volatility in the innovations, we fit GARCH(1,1) models to the innovations. Results of this estimation are shown in Panel B of Table 4. The table again suggests stronger evidence in favor of time-variation in the volatility of leisure and wage growth than in consumption growth. The GARCH coefficients for all three variables are statistically significantly different than zero; the point estimates for consumption growth, wage growth, and leisure are 0.85, 0.84, and 0.74, respectively. ARCH coefficients for wage growth and leisure are statistically significantly different than zero, but the coefficient for consumption growth is not.

As in the case of conditional means, we ask how many independent sources of persistent variance are present in the data. We first simply examine the correlation matrix of the volatilities to get a sense of how much commonality is present in the three volatility series. The volatilities of consumption growth and wage growth are highly correlated, with a correlation coefficient of 0.94. Consumption growth volatility is less correlated with leisure volatility, with a correlation coefficient of 0.75. Finally, volatility of wage growth is also highly correlated with leisure with a correlation coefficient of 0.86. This evidence suggests that there is strong commonality in the volatility of the three series.

More formally, we perform a principal components analysis on the volatilities implied by the GARCH(1,1) estimation for the innovations as above. The first principal component dominates, explaining 90% of the variation in the volatilities of the three series. The coefficients suggest that each volatility loads similarly and positively on this principal component. The second principal component explains approximately 9% of variation in volatilities; consumption and wage growth load negatively on this principal component, while leisure loads positively. Finally the last principal component explains only 1% of variation, indicating that two sources of economic uncertainty are likely to characterize the data.

3 Economic Model

The economic environment in which we model consumption, leisure, and portfolio decisions is very similar to that of Bansal and Yaron (2004), but incorporating felicity for leisure into preferences. The framework is an endowment economy with exogenous processes for consumption, leisure, and dividend growth. In this environment, we derive the equilibrium prices of risk, wages, and returns on various claims to the endowment.
3.1 Preferences on Consumption and Leisure

A representative agent maximizes lifetime utility given by Epstein and Zin (1989) preferences:

\[ V_t = \left( (1 - \beta)A_t^{1 - \frac{1}{\psi}} + \beta Q_t^{1 - \frac{1}{\psi}} \right)^{\frac{1}{1 - \frac{1}{\psi}}}, \tag{5} \]

where \( \beta \) is a subjective time discount factor, and \( \psi \) is the elasticity of intertemporal substitution of consumption. \( A_t \) represents the total consumption bundle, defined over consumption of nondurable goods and services, \( C_t \), and leisure, \( L_t \), as

\[ A_t = \left( (1 - \alpha)C_t^{1 - \frac{1}{\rho}} + \alpha (\zeta_t L_t)^{1 - \frac{1}{\rho}} \right)^{\frac{1}{1 - \frac{1}{\rho}}}, \tag{6} \]

where \( \zeta_t \) is a “preference shock” to be defined later in this section. The role of the preference shock is to ensure that utility derived from leisure does not vanish as consumption of non-durable goods and services grows over time. We refer to the total consumption bundle as “total consumption.” \( Q_t \) is the certainty equivalent defined as

\[ Q_t = \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \]

where \( \gamma \) captures risk aversion.

Leisure is measured as the fraction of time \( L_t \equiv 1 - N_t \), where \( N_t \) is labor supplied by households to the production sector. The parameter \( \rho \) captures the elasticity of substitution between consumption of nondurables and services and leisure. To make comparisons with the nondurables and services consumption-only case, we define the fraction of total consumption relative to nondurables and services consumption \( Z_t \equiv A_t/C_t \), such that

\[ Z_t = \left( 1 - \alpha + \alpha \left( \frac{\zeta_t L_t}{C_t} \right)^{1 - \frac{1}{\rho}} \right)^{\frac{1}{1 - \frac{1}{\rho}}}. \tag{7} \]

Notice that the consumption aggregator implies, in general, non-separability in nondurables and services consumption and leisure. Three particular cases are worth noting. The case \( \alpha = 0 \) corresponds to utility from non-durable and services consumption only, the case \( \rho = 1 \) corresponds to the Cobb-Douglas aggregator where \( Z_t \) reduces to \((\zeta_t L_t/C_t)^{\alpha}\), and the case \( \rho = \psi \) implies separable intra-temporal preferences in nondurables and services consumption and leisure.
The representative agent faces the intertemporal budget constraint

\[ \mathbb{E}_t \left[ \sum_{s=0}^{\infty} M_{t,t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[ \sum_{s=0}^{\infty} M_{t,t+s} (W_{t+s} N_{t+s} + D_{t+s} + G_{t+s}) \right], \quad (8) \]

where \( M_{t,t+s} \) is the pricing kernel that discounts cashflows in units of nondurable and services consumption from \( t+s \) to time \( t \), \( W_t \) is the wage earned from supplying a unit of labor to productive activities, \( D_t \) are the dividends from owning the production sector, and \( G_t \) captures other sources of income such as government transfers.

Maximization of utility with respect to the budget constraint yields the intertemporal marginal rate of substitution of consumption

\[ M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{Z_{t+1}}{Z_t} \right)^{\frac{1}{\psi} - \frac{1}{\rho}} \left( \frac{V_{t+1}}{Q_t} \right)^{\frac{1}{\psi} - \gamma}, \quad (9) \]

which represents the pricing kernel for the economy. It also can be expressed as

\[ M_{t,t+1} = \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{Z_{t+1}}{Z_t} \right)^{\frac{1}{\psi} - \frac{1}{\rho}} \right]^{\theta} \left[ \frac{1}{R_{a,t+1}^c} \right]^{1-\theta}, \quad (10) \]

where \( \theta = (1 - \gamma)/(1 - 1/\psi) \), and \( R_{a,t+1}^c \) is the return of the wealth portfolio in units of nondurable and services consumption. The wealth portfolio is a claim on all future total consumption, which includes the opportunity cost of leisure. The price of the wealth portfolio in units of nondurable and services consumption is defined recursively as

\[ S_{a,t} = \mathbb{E}_t \left[ M_{t,t+1} (F_t A_t + S_{a,t+1}) \right], \quad (11) \]

where \( F_t \) is the price of total consumption in units of nondurable and services consumption. The wealth portfolio becomes a claim only on non-durable and services consumption when \( \alpha = 0 \), as in Bansal and Yaron (2004).

Preference for leisure has two effects on the pricing kernel. The first effect is on its CRRA component, when \( \gamma = 1/\psi \). This component is affected by the ratio \( Z_t \) as long as \( \psi \neq \rho \). This is a result of the non-separability of nondurables and services consumption and leisure in preferences. An increase in the ratio \( Z_t \) increases (decreases) the marginal utility of nondurables and services consumption if \( \psi > \rho \ (\psi < \rho) \). This additional term can be written in log form as\(^6\)

\[ \left( \frac{1}{\rho} - \frac{1}{\psi} \right) \Delta z_t = \left( \frac{1}{\rho} - \frac{1}{\psi} \right) (\Delta a_t - \Delta c_t). \]

\(^6\)Throughout the paper, we use lower case to denote the log of a variable and \( \Delta \) to denote the difference operator.
If $\psi > \rho$, this component is positive as long as $\Delta a_t > \Delta c_t$. A total consumption growth higher than nondurables and services consumption growth is a state of high marginal utility if the elasticity of substitution between nondurables and services consumption and leisure is low enough (nondurables and services consumption and leisure tend to be complements), but it is a state of low marginal utility if this elasticity is high enough (nondurables and services consumption and leisure tend to be substitutes).

The second effect of leisure preferences on the pricing kernel is the result of the preference for resolution of uncertainty, when $\gamma \neq \frac{1}{\psi}$. In this case, the marginal rate of substitution of consumption also depends on the difference between the value function $V_{t+1}$ and the certainty equivalent $Q_t$. This difference is captured by the return on the wealth portfolio, $R_{a,t+1}^c$. In the absence of leisure preferences, $R_{a,t+1}^c = R_{c,t+1}^c$. More generally, the riskiness of $R_{a,t+1}^c$ depends not only on nondurables and services consumption but also on the value of leisure. To see this, consider an approximation of the pricing kernel similar to that in Piazzesi and Schneider (2007) under the assumption of log-normality and constant volatility. The recursive utility term can be approximated as

$$\log \left( \frac{V_{t+1}}{Q_t} \right) \approx \text{constant} + \sum_{i=1}^{\infty} \beta^{i-1}(\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta a_{t+1+i}].$$

That is, the marginal utility of consumption depends on revisions on expectations of all future total consumption growth. Leisure preferences make the pricing kernel depend not only on the nondurables and services consumption growth process but also on the evolution of expectations of the value of leisure over time.

A useful alternative representation of the pricing kernel is

$$M_{t,t+1} = M_{t,t+1}^a \left( \frac{F_{t+1}}{F_t} \right)^{-1} \cdot \text{where} \quad M_{t,t+1}^a = \left[ \beta \left( \frac{A_{t+1}}{A_t} \right)^{-\frac{1}{\psi}} \right]^\theta \left[ \frac{1}{R_{a,t+1}^c} \right]^{1-\theta},$$

is the pricing kernel in units of total consumption. $R_{a,t+1}^c \equiv R_{a,t+1}^c F_t/F_{t+1}$ is the return of the wealth portfolio in units of total consumption. Dividends and labor income in the economy are paid in terms of units of consumption of nondurable goods and services. Since households care about total consumption, rather than simply consumption of nondurable goods and services, the riskiness of dividend and labor income cash flows is affected by the evolution of the relative price of total consumption, $F_t$, over time.

It is worth noting that the presence of multiple goods in the consumption aggregator alters the measurement of several quantities of interest relative to the case in which preferences are defined over a single good. These quantities, such as the elasticity of intertemporal substitution and relative risk aversion coefficient, are defined relative to total consumption, rather than simply consumption.
of nondurables and services. As a result, empirical measurements of these quantities are altered relative to the case in which agents derive utility only through consumption of nondurables and services. Uhlig (2007) and Swanson (2012) examine differences in the elasticity of intertemporal substitution and measures of risk aversion, respectively, in models with leisure. In this model, the elasticity of intertemporal substitution of total consumption is given by \( \psi \), and the coefficient of relative risk aversion relative to wealth is \( R^a = \gamma \).\(^7\) An alternative measure of risk aversion, relative to gambles on non-durables and services consumption only, can be computed as

\[
R^c = \gamma \frac{C_t}{F_t A_t} < \gamma.
\]  

(12)

For comparison purposes, we compute both measures of risk aversion in our calibrations.

### 3.2 Consumption, Leisure, and Dividend Growth

We motivate the relation between leisure, consumption, and dividend growth from the analysis in Section 2. Specifically, we assume that all three processes are affected by two sources of long-run (conditional mean) risk and a source of time-varying uncertainty.

The processes for consumption growth, leisure, and dividend growth are specified as

\[
\begin{align*}
\Delta c_{t+1} & = \mu_c + x_t + \phi_{cu} u_t + \sigma_{c,t} \varepsilon_{c,t+1} + \sigma_{cl} \sigma_{l,t} \varepsilon_{l,t+1}, \\
l_{t+1} & = \bar{l} + \phi_{lx} x_t + u_t + \sigma_{lc} \sigma_{c,t} \varepsilon_{c,t+1} + \sigma_{l,t} \varepsilon_{l,t+1}, \\
x_{t+1} & = \phi_x x_t + \sigma_{xt} \varepsilon_{x,t+1}, \\
u_{t+1} & = \phi_u u_t + \sigma_{ut} \varepsilon_{u,t+1}, \\
\Delta d_{t+1} & = \mu_c + \phi_{dx} x_t + \phi_{du} u_t + \sigma_{dc} \sigma_{c,t} \varepsilon_{c,t+1} + \sigma_{dl} \sigma_{l,t} \varepsilon_{l,t+1} + \sigma_{d,t} \varepsilon_{d,t+1},
\end{align*}
\]  

(13)

where \( x_t \) and \( u_t \) characterize the time-varying components in the conditional mean of nondurables and services consumption growth and leisure.\(^8\) Innovations \( \varepsilon_{k,t} \) are i.i.d. \( \sim \mathcal{N}(0,1) \), for \( k = \{c, l, x, u, d\} \). The processes for consumption and dividend growth are similar to those used in Bansal and Yaron (2004), but extended to incorporate sensitivity to a second source of conditional mean risk. Appendix C shows that the dividend growth process in (13) can be obtained endogenously from the resource constraint for the economy, where dividends are linked to choices

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\( ^7 \)In this particular model, the elasticity of intertemporal substitution of total consumption, \( -\frac{\partial \log (A_{t+1}/A_t)}{\partial \log M_{t,t+1}} \), and the elasticity of substitution of consumption of non-durables and services, \( -\frac{\partial \log (C_{t+1}/C_t)}{\partial \log M_{t,t+1}} \), are both equal to \( \psi \).

\( ^8 \)Notice that the process for \( l_t \) allows for positive values of this variable, and therefore does not preclude the possibility of values for \( L_t \) greater than one. We verify in the calibration and simulation of the model that positive values for \( l_t \) are significantly infrequent. Alternatively, the process for leisure can be specified as the negative of a nonnegative process. We tried several of these specifications and found them not flexible enough to capture the joint dynamics of consumption growth and leisure.
of consumption and leisure. However, the endogenous link is not directly useful for our analysis as it relates to total dividends rather than dividends per share, which is our object of interest.

Conditional volatilities in our framework are specified as
\[
\sigma_{k,t} = \sigma_k (1 - I_k + I_k \nu_t)^{1/2},
\]
for \(k = \{c, l, x, u, d\}\), where \(\nu_t\) captures time variation in economic uncertainty. We assume that it follows an autoregressive gamma process with parameters \((\delta, \phi, \varsigma)\). The indicator \(I_k\) is 1 if the process \(k\) is affected by time-varying uncertainty, and 0 otherwise. Specifying the process in this manner allows us to quantify the contribution of time-varying volatility in each process to the results. We note also that this volatility process is different than the approximate square root process in Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007) specifically in that the volatility of our volatility process is also time varying. Shocks to volatility are denoted by \(\varepsilon_{\nu,t+1} \equiv \nu_{t+1} - \mathbb{E}_t[\nu_{t+1}]\).

### 3.3 Wage and Labor Income Growth

The processes for wage and labor income growth are implied by the household’s optimality conditions. These processes allow us to compute and characterize the returns on human capital implied by the model. In an economy with frictionless labor markets, optimality implies that wages are determined by the marginal rate of substitution between leisure and consumption of nondurables and services,

\[
MRS_{cl} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{C_t}{L_t}\right)^{\frac{1}{\rho}} \zeta_t^{1-\frac{1}{\rho}},
\]

Frictions in the labor market such as market power, wage rigidities, or unemployment can generate deviations from this rate. We exogenously capture these deviations by introducing a “wedge” process, \(\xi_t\), such that the wage is

\[
W_t = MRS_{cl} e^{\xi_t} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{C_t}{L_t}\right)^{\frac{1}{\rho}} \zeta_t^{1-\frac{1}{\rho}} e^{\xi_t},
\]

We assume that the wedge is stationary, has zero unconditional mean, and follows the process

\[
\xi_{t+1} = \phi_{\xi} x_t + \phi_{\xi u} u_t + \sigma_{\xi c} \sigma_{c,t} e^\zeta_{c,t} + \sigma_{\xi l} \sigma_{l,t} e^\zeta_{l,t} + \sigma_{\xi u} \xi_{t+1} + \sigma_{\xi t} \xi_{t+1},
\]

---

9 The autoregressive gamma process is the exact discrete-time counterpart of the Cox, Ingersoll and Ross process and avoids the possibility of negative values. This process allows us to obtain tractable approximate closed-form expressions for the model solution. Its properties are described in Jasiak and Gourieroux (2006). Hsu and Palomino (2015) present a general solution for rational equilibrium models were uncertainty is described by Gaussian and autoregressive gamma processes. Le, Singleton and Dai (2010) apply autoregressive gamma process to the analysis of the term structure of interest rates.
where $\sigma_{\xi,t} = \sigma_{\xi}(1 - I_{\xi} + I_{\nu}r_{t})^{1/2}$. The wage equation (15) is affected by the preference shock, $\zeta_t$. For parsimony, we define this shock as
\begin{equation}
\zeta_t \equiv C_t. \tag{17}
\end{equation}

The specification for $\zeta_t$ ensures balanced growth in the economy.\(^{11}\) To see this, we can rewrite equation (15) as
\begin{equation}
\frac{W_t}{C_t} = \left(\frac{\alpha}{1 - \alpha}\right) L_t^{-\frac{1}{\rho}} e^{\xi_t}. \tag{18}
\end{equation}

Notice that consumption of nondurables and services and wages share the same trend under the assumption that leisure and the wedge are stationary.

From equation (18) and the fact that log-labor income is $y_t \equiv \log(W_t(1 - L_t))$, wage and labor income growth can be approximated as
\begin{equation}
\Delta w_t = \Delta c_t + b_{wl}\Delta l_t + \Delta \xi_t, \quad \text{and} \quad \Delta y_t = \Delta c_t + b_{yl}\Delta l_t + \Delta \xi_t, \tag{19}
\end{equation}
respectively, where
\begin{equation*}
b_{wl} = -\frac{1}{\rho}, \quad \text{and} \quad b_{yl} = b_{wl} - \frac{e^l}{1 - e^l}.
\end{equation*}

### 3.4 Prices of Risk

Prices of risk in the economy are represented by the coefficients on innovations in the stochastic discount factor. To obtain analytical expressions for these coefficients, we first approximate equation (7) as
\begin{equation}
z_t = \mu_z + b_{zt}\hat{l}_t, \tag{20}
\end{equation}

\(^{10}\)An alternative specification is $\zeta_t = C_t e^{(1-1/\rho)\xi_t}$. In this case, the process $\xi_t$ has the interpretation of a preference shock that affects the marginal rate of substitution of consumption and leisure, and then the pricing kernel. This specification makes less clear and more difficult to describe the effects of leisure on prices of risk, without improvements in the calibration.

\(^{11}\)Although $\zeta_t$ depends on consumption, we assume that this shock is “external” to the household, such that it is taken as given. This assumption ensures that the elasticity of substitution between consumption and leisure is\(^{16}\)
\begin{equation*}
-\frac{d \log(L_t/C_t)}{d \log W_t} = \rho.
\end{equation*}

A specification where the shock is “internal,” generates a time-varying elasticity. An alternative specification that delivers a constant elasticity is $\zeta_t = C_0 \exp(\mu_z t)$. This specification involves a less parsimonious model with no clear improvement in performance.
where \( \hat{l}_t \equiv l_t - \bar{l} \),

\[
\mu_z = \left( 1 - \frac{1}{\rho} \right)^{-1} \log a_z, \quad b_{zl} = \frac{\alpha e\left(1 - \frac{1}{\rho}\right)^{\bar{l}}}{a_z},
\]

and \( a_z = 1 - \alpha \alpha e\left(1 - \frac{1}{\rho}\right)^{\bar{l}} \). Given this approximation, we show in Appendix A that the innovation in the log pricing kernel can be expressed as

\[
m_{t,t+1} - E_t[m_{t,t+1}] = -\lambda_c \sigma_{c,t+1} - \lambda_l \sigma_{l,t+1} - \lambda_x \sigma_{x,t+1} - \lambda_u \sigma_{u,t+1} - \lambda_{\nu} \epsilon_{\nu,t+1},
\]

where

\[
\lambda_c = \gamma + \left[ \frac{1}{\psi} - \frac{1}{\rho} + \left( \frac{1}{\psi} - \gamma \right) \left( 1 - \eta_a \right) \right] b_{zl} \sigma_{lc},
\]

\[
\lambda_l = \gamma \sigma_{cl} + \left[ \frac{1}{\psi} - \frac{1}{\rho} + \left( \frac{1}{\psi} - \gamma \right) \left( 1 - \eta_a \right) \right] b_{zl},
\]

\[
\lambda_x = \left( \frac{\gamma - \frac{1}{\psi}}{1 - \eta_a \phi_x} \right) \left[ 1 + (1 - \eta_a) b_{zl} \phi_{lx} \right] \eta_a,
\]

\[
\lambda_u = \left( \frac{\gamma - \frac{1}{\psi}}{1 - \eta_a \phi_u} \right) \left[ \phi_{cu} + (1 - \eta_a) b_{zl} \right] \eta_a,
\]

\[
\lambda_{\nu} = (1 - \theta) \eta_a p_{a,\nu},
\]

where the approximation constant \( \eta_a \), and the sensitivity of the wealth-consumption ratio to volatility, \( p_{a,\nu} \), are defined in Appendix A.

There are several similarities and differences in the prices of risk relative to the single consumption good model of Bansal and Yaron (2004). First, the prices of the short- and long-run consumption risks in Bansal and Yaron (2004), \( \lambda_c \) and \( \lambda_x \), respectively, are affected by leisure preferences. The contribution of these preferences to the price of short-run risk \( \lambda_c \) depends on intra-temporal separability between leisure and consumption, as well as on the recursive nature of the preferences. The effect of leisure on this price of risk is proportional to the weight \( b_{zl} \approx \alpha > 0 \). For a correlation component \( \sigma_{lc} > 0 \), a negative shock to consumption also decreases leisure. In this case, a high elasticity \( \rho > \psi \) increases the marginal utility of consumption and then the price of risk, while the opposite occurs if \( \rho < \psi \). Under preferences for the early resolution of uncertainty, \( \gamma > \frac{1}{\psi} \), the effect of leisure on the price of risk is amplified. However, this effect tends to be small since the impact on leisure growth is transitory, as captured by the term \( 1 - \eta_a \). Similarly, the price of long-run risk \( \lambda_x \) is affected by leisure, but only if \( \gamma \neq \frac{1}{\psi} \). For \( \gamma > \frac{1}{\psi} \), a negative loading \( \phi_{lx} \) decreases the price of this risk since lower consumption is compensated with a higher leisure that reduces marginal utility. However, this effect is small since leisure growth is not significantly affected by this shock. Second, the economic interpretation of the prices of risk \( \lambda_l \) and \( \lambda_u \) exactly
mirrors that of \( \lambda_c \) and \( \lambda_x \), respectively. Positive loadings of consumption, \( \sigma_{cl} \) and \( \phi_{cu} \) on these sources of risk increases these prices of risk. The price of the long-run risk \( \lambda_u \) increases for a higher autocorrelation in \( u_t \). Third, as in Bansal and Yaron (2004), time-variation in economic uncertainty \( \nu_t \) is a priced factor as long as \( \gamma \neq \psi \). In this case, however, the price of this risk \( \lambda_{\nu} \) is affected by leisure preferences, and the implied volatility premium is time-varying since there is time-variation in the volatility of economic uncertainty.

3.5 Risk-Free Rate in Units of Consumption

The risk-free asset in the economy is an asset that pays a unit of total consumption with certainty. If agents have preference for leisure and \( \rho \neq 1 \), the risk-free asset will not be equivalent to an asset that pays a unit of nondurables and services consumption, since movements in the relative price of total consumption will make a risk-free bond issued in units of nondurables and services consumption risky. Since zero-coupon real Treasury debt pays a unit of nondurables and services consumption, it will not generally be a risk-free security. However, in accordance with past literature, we refer to this security as the risk-free asset.

The risk-free rate in units of consumption of nondurables and services, \( r_t \), is the conditional expectation of the pricing kernel,

\[
\exp(-r_t) = \mathbb{E}_t[M_{t,t+1}],
\]
given by

\[
r_t = \bar{r} + \left[ \frac{1}{\psi} + \left( \frac{1}{\psi} - \frac{1}{\rho} \right) b_{zl} \phi_{lx} \right] x_t + \left[ \frac{1}{\psi} \phi_{cu} + \left( \frac{1}{\psi} - \frac{1}{\rho} \right) b_{zl} \right] u_t - \left( \frac{1}{\psi} - \frac{1}{\rho} \right) b_{zl} \hat{l}_t + \left[ \Gamma_{\nu} - \frac{\lambda_{\nu}^2 \phi_{\nu} \varsigma_{\nu}}{1 + \lambda_{\nu} \varsigma_{\nu}} - q_{r,\nu} \right] \nu_t
\]

where expressions for \( \bar{r}, \Gamma_{\nu}, \) and \( q_{r,\nu} \) are provided in Appendix B. The sensitivity of the risk-free rate to long-run risks depends on the effect of \( x_t \) and \( u_t \) on expected consumption and total consumption. This sensitivity is not only affected by the elasticity of substitution \( \psi \) and expectations of future nondurables and services consumption growth, but also by non-separability in consumption and leisure preferences. Its intertemporal smoothing effects on the risk-free rate depend on the difference between the elasticities \( \rho \) and \( \psi \), and the sensitivities \( \phi_{lx} \) and \( \phi_{cu} \). For a high elasticity \( \rho > \psi \) a negative \( \phi_{lx} \) decreases expected consumption growth and the risk-free rate after a negative shock on \( x_t \). A negative \( \phi_{cu} \) has a similar effect after a negative shock on \( u_t \). Non-separability also links leisure directly to the risk-free rate. A higher than average leisure (\( \hat{l}_t \)) decreases the risk-free rate for a high elasticity \( \rho > \psi \), but has the opposite effect if leisure and consumption tend to be complements (\( \rho < \psi \)). Finally, the sensitivity of the risk-free rate to volatility is affected by leisure.
preferences through its effects on the wealth-consumption ratio, $p_{a,\nu}$, and the precautionary savings term $q_r$.

### 3.6 Asset Returns

We price and compute expected returns of claims on all future consumption of nondurables and services, dividends, labor income, and wages. The claims on all future labor income and wages allow us to quantify the return on human capital. In models with no leisure preferences, $L = 0$, and the returns on labor income and wage claims are the same. In the presence of leisure preferences, claims on labor income do not depend only on wages but also on the household willingness to work in different states of the world. Therefore, the riskiness and expected returns of claims on wages and labor income can be different.

Our claims have cashflows $K_t = \{C_t, D_t, W_t, W_t^N\}$. From equations (13) and (19), growth in these cashflows follow the process

$$\Delta k_t = \mu_k + \phi_{kx}x_{t-1} + \phi_{ku}u_{t-1} + \phi_{kl}\hat{l}_{t-1} + \phi_{k\xi}\xi_{t-1} + \sigma_{kc}\varepsilon_{c,t-1} + \sigma_{kl}\varepsilon_{l,t-1} + \sigma_{k\xi}\varepsilon_{\xi,t-1} + \sigma_{k}\varepsilon_{k,t-1},$$

for appropriate coefficients defined in Appendix B. The appendix shows that log-returns for these claims can be approximated as

$$r_{k,t+1} = \bar{\eta}_k + \eta_k p_{k,t+1} + \Delta k_{t+1} - p_{k,t},$$

where the price-cashflow ratio has the form

$$p_{k,t} = \bar{p}_k + p_{k,x}x_t + p_{k,u}u_t + p_{k,l}\hat{l}_t + p_{k,\nu}\nu_t + p_{k,\xi}\xi_t.$$

Therefore, innovations in asset returns can be expressed as

$$r_{k,t+1} - E_t[r_{k,t+1}] = r_{k,c}\sigma_{c,t}\varepsilon_{c,t+1} + r_{k,l}\sigma_{l,t}\varepsilon_{l,t+1} + r_{k,\xi}\sigma_{\xi,t}\varepsilon_{\xi,t+1} + r_{k,x}\sigma_{x,t}\varepsilon_{x,t+1} + r_{k,u}\sigma_{u,t}\varepsilon_{u,t+1} + r_{k,\nu}\varepsilon_{\nu,t+1},$$

In the presence of normal and autoregressive shocks, Hsu and Palomino (2015) show that expected excess returns on these claims are

$$\log E_t[\exp(xr_{k,t+1})] = \lambda_x r_{k,c}\sigma_{c,t}^2 + \lambda_l r_{k,l}\sigma_{l,t}^2 + \lambda_x r_{k,x}\sigma_{x,t}^2 + \lambda_u r_{k,u}\sigma_{u,t}^2 + \lambda_{\nu} r_{k,\nu}\sigma_{\nu,t}^2$$

$$+ \delta_{\nu} \log \left[ \frac{1 + (\lambda_{\nu} - r_{\nu})\varsigma_{\nu}}{(1 + \lambda_{\nu}\varsigma_{\nu})(1 - r_{\nu}\varsigma_{\nu})} \right] + \phi_{\nu} \left[ \frac{(\lambda_{\nu} - r_{\nu})}{1 + (\lambda_{\nu} - r_{\nu})\varsigma_{\nu}} - \frac{\lambda_{\nu}}{1 + \lambda_{\nu}\varsigma_{\nu}} + \frac{r_{\nu}}{1 - r_{\nu}\varsigma_{\nu}} \right] \nu_t,$$

where $xr_{k,t+1} \equiv r_{k,t+1} - r_t$. In the absence of volatility shocks, the expected excess return equation...
reduces to the familiar $-\text{cov}(m_{t,t+1},r_{k,t+1})$. Notice that expected excess returns are time varying as a result of time-varying volatility. The last two terms capture the volatility premium. This premium is time-varying since there is time-varying volatility in the volatility process.

4 Analysis

Given the solutions to quantities of interest in Section 3, we calibrate the model to the data to highlight the contribution of leisure preferences to the price of risky claims in the economy. Our calibration provides insight into the marginal contribution of leisure preferences to the pricing of financial claims. Further, the presence of leisure preferences allows us to analyze the impact of aggregate quantities on the expected returns to human capital.

4.1 Calibration

We solve the model using the analytical approximations presented above as in Bansal and Yaron (2004) and Beeler and Campbell (2012).\textsuperscript{12} We assume a monthly frequency, and simulate and aggregate the monthly dynamics to annual frequency to match select macroeconomic and asset pricing statistics of the United States annual data from 1930 to 2011 described in Section 2.\textsuperscript{13} The aggregation procedure from monthly to annual frequency for consumption, dividends, labor income, price-dividend and wealth-consumption ratios is identical to that described in Bansal and Yaron (2004). We describe here the aggregation procedure for annual leisure and wages. Since leisure is defined as a fraction of time, we compute annual leisure $L^α_t \equiv 1 - N^α_t$ as a weighted average of monthly leisure during the year. To compute the weights, notice that the annual wage $W^α_t$ and the annual labor income $Y^α_t = W^α_tN^α_t$ are

$$W^α_t = \sum_{i=0}^{11} W_{t-i}, \quad \text{and} \quad Y^α_t = \sum_{i=0}^{11} W_{t-i}N_{t-i}. $$

It follows that $L^α_t \equiv 1 - N^α_t$ is

$$L^α_t = \sum_{i=0}^{11} \frac{W_{t-i}}{W^α_t} L_{t-i}. $$

For comparison purposes, we present six different calibrations. The first is a baseline calibration\textsuperscript{12}The approximations for price-cashflow ratios are around their unconditional means. We compute these means using a fixed-point algorithm. These approximations are highly accurate even in the presence of autoregressive gamma shocks.\textsuperscript{13}The data series for labor income is adjusted to be consistent with detrended leisure (and labor) in the model. That is, labor income is $Y_t = W_tN_t$, where $W_t$ is obtained as described in Section 2, $N_t = 1 - L_t$, and $L_t$ is detrended leisure.
that corresponds to a model with preferences in consumption only \((\alpha = 0)\), as in Bansal and Yaron (2004). The baseline calibration is similar to the one presented in Bansal, Kiku and Yaron (2010), updated to include data for recent years. This calibration highlights the contribution of the stochastic volatility channel for understanding asset returns. Beeler and Campbell (2012) show that this calibration improves the predictability properties of the long-run risk model. We then present five representative calibrations for the model with leisure preferences. The main difference between these calibrations is that they use five different values for the elasticity of substitution between consumption and leisure, \(\rho = \{0.5, 1, 1.5, 5, 50\}\). This elasticity parameter is difficult to estimate directly from the available data. Comparisons across calibrations allow us to quantify the importance for the results of leisure preferences, the degree of substitution between consumption and labor, and the presence of a wedge between wages and the marginal rate of substitution of consumption and leisure. The case \(\rho = 0.5\) captures complementarity between leisure and consumption. The case \(\rho = 1\) implies a constant share of (monthly) consumption relative to total consumption. In this setting, the return \(R_{at}^c\) in the pricing kernel, equation (11), is equal to the return on the consumption claim, \(R_{c,t}\). The case \(\rho = 1.5\) implies separability in the intra-temporal utility of consumption and leisure since we set \(\psi = 1.5\). The cases \(\rho = 5\) and \(\rho = 50\) capture low and high degrees of substitutability, respectively, between leisure and consumption.

Table 5 presents the parameter values that are common across calibrations. We set the parameter values of \(\psi\) and \(\gamma\) to 1.5 and 10, respectively, to be consistent with Bansal and Yaron (2004). We set \(\phi_x = 0.975\), the value used in Bansal, Kiku and Yaron (2010). The set of volatility parameters \(\delta_\nu, \phi_\nu, \) and \(\varsigma_\nu\) are chosen as follows. We assume that all variables have stochastic volatility except for the independent volatility component of dividend growth. That is, \(I_k = 1\) for \(k = \{c, l, x, u, \xi\}\), and \(I_d = 0\). We normalize \(\varsigma_\nu = \delta_\nu^{-1}(1 - \phi_\nu)\) such that \(\mathbb{E}[^nu_t] = 1\). The persistence of the volatility process is set at \(\phi_\nu = 0.995\). This value and the low value for \(\delta_\nu\) imply significant volatility in the volatility process that increases the volatility premium. We choose these values to match in our baseline calibration the equity premium in the data given the risk aversion parameter of \(\gamma = 10\). Given these volatility parameters, the sets \(\{\mu_c, \sigma_c, \sigma_x\}\) and \(\{\mu_c, \mu_l, \sigma_c, \sigma_{cl}, \sigma_l, \sigma_{lc}, \sigma_x, \sigma_u, \phi_{lx}, \phi_{cu}, \phi_u\}\) are chosen for the baseline and leisure models, respectively. For the baseline model, the parameters are chosen to match the average, volatility, and first-order autocorrelation of consumption growth. For the leisure models, in addition, the same moments are matched for leisure, as well as the loadings of one-period ahead predictive regressions of consumption growth and leisure on their principal components (presented in Table 8). Notice that \(\sigma_c\) and \(\sigma_x\) differ across the baseline and leisure models, given the existence of a second factor driving the conditional mean of consumption growth in the latter. In particular, \(\sigma_c\) is lower and \(\sigma_x\) is higher in the leisure model relative to the baseline one. For the leisure models, the negative correlation between leisure and consumption growth in the data is captured by negative values for \(\phi_{lx}\) and \(\phi_{cu}\), consistent with the empirical analysis. In addition, the high value of \(\phi_u = 0.995\) plays a fundamental role capturing the predictive regression
loadings of leisure on the principal components.

The calibration of the dividend process differs across the baseline and leisure models. For comparison purposes, we set the loading of dividend growth on \( x_t \) to be \( \phi_{dx} = 2.5 \). For the baseline model, the remaining parameters are chosen to match the volatility of dividend growth and the correlation of this variable with consumption growth. For the leisure model, we set values for \( \{\phi_{du}, \sigma_{dc}, \sigma_{dl}, \sigma_d\} \) to capture the volatility of dividend growth and the correlations of dividend growth with leisure, and consumption and leisure growth. Notice that the loading on \( u_t, \phi_{du} = -0.06 \) is slightly negative.

Table 6 presents the parameter values that are specific to the model calibrations with leisure preferences. In order to match the average labor income - consumption ratio, \( \alpha \) is constrained to be the function of \( \rho \),

\[
\alpha = \frac{1}{1 + \frac{1}{W/C \bar{L} \rho}},
\]

where \( W/C \) and \( \bar{L} \) are the average \( W_t/C_t \) and \( L_t \) in the data, respectively. The parameter values describing the wedge process, \( \{\phi_{\xi x}, \phi_{\xi u}, \sigma_{\xi c}, \sigma_{\xi l}, \sigma_{\xi}\} \), are selected given a value for \( \rho \) to simultaneously match the volatility and first-order autocorrelations of labor income and wage growth, as well as their correlations with consumption growth. Quantitatively, the wedge process is justified by the fact that potential calibrations with no wedge imply a volatility for wage growth that is too low relative to the volatility of labor income growth. The parameter value for \( \beta \) is chosen to match the average level of the risk-free rate as well as possible. The risk aversion parameter \( \gamma \) is chosen to match the equity premium in the data. Notice that models with leisure preferences require a level of risk aversion that is lower than 10 to match the premium. On one hand, the negative correlation between leisure and consumption growth reduces the price of risk of \( x_t \) and its contribution to the equity premium since \( \phi_{dx} > 0 \). On the other hand, this reduction is compensated with a positive premium for \( u_t \), since both, its price of risk \( \lambda_u \) and the loading \( \phi_{du} \), are negative. The table also shows that when risk aversion is measured relative to consumption gambles as in equation (12), the coefficient of risk aversion \( R^c \) is around 3.

4.2 Pricing Financial Claims

We first examine the implications of the model for the pricing of financial assets. In Table 7, we present calibrated means, volatilities, first-, and second-order autocorrelations of macroeconomic variables common to the leisure and nondurables and services-only models. The first column

\[\text{For simplicity, this average corresponds to the no-uncertainty case. The deviations from this ratio generated by the effect of uncertainty on the average are minor.}\]
presents the data moments, the second column presents moments implied by the baseline calibration with preferences only over consumption of nondurables and services, and the third column presents moments implied by the model calibrations with leisure preferences. The baseline and leisure model calibrations reproduce the consumption and dividend growth moments in the data. The table also shows that all models capture the correlation of consumption and dividend growth, and the models with leisure preferences also capture the correlation of leisure and dividend growth in the data. In addition, the models with leisure are also calibrated to match moments of leisure, wage, and labor income growth, discussed below.

Table 9 shows that leisure preferences, depending on the elasticity $\rho$, can have a significant impact on properties of the risk-free rate, but minor differences in those of the equity premium relative to the baseline model. Except in the case where $\rho = 0.5$, the model can generate a mean risk-free rate that is identical to that in the baseline calibration and in the data. However, the volatility and autocorrelation of this rate are highly sensitive to the elasticity of substitution between consumption and leisure. For the separable utility case $\rho = \psi$, these properties are very similar to those of the baseline model: lower volatility and higher autocorrelation than in the data. As the elasticity $\rho$ deviates from the intertemporal elasticity $\psi$, the volatility of the risk-free rate increases, while its autocorrelation decreases dramatically. These results provide support for preference specifications with small deviations from $\rho = \psi$. On the contrary, the properties of the price-dividend ratio and equity premium are less sensitive to $\rho$ and similar to those of the baseline model. Although the volatility of the price-dividend ratio and equity premium are less sensitive to $\rho$ and similar to those of the baseline model. Although the volatility of the the price-dividend ratio modestly increases with $\rho$, it is still approximately half of that observed in the data. Finally, incorporating leisure preferences generates an equity premium consistent with both the data and the baseline calibration, and generates slightly more volatility in the equity return as $\rho$ increases than in the baseline case. This has minor implications on the Sharpe ratio of stock returns.

Some insight into how the incorporation of leisure preferences affects the pricing of financial claims can be obtained by decomposing the prices of risk into the fraction attributable to each component. We report the results of this decomposition in Table 10. The baseline model implies compensations for one source of short-run risk, one source of variation in the conditional mean of consumption growth, and one source of variation in its volatility, with prices of risk $\lambda_c$, $\lambda_x$, and $\lambda_\nu$, respectively. The model with leisure preferences adds an additional source of short-run risk, and another source of variation in conditional means, with prices of risk $\lambda_l$ and $\lambda_u$, respectively. The price of risk $\lambda_c$ is smaller than in the baseline model for a low elasticity $\rho$, but increases and becomes comparable in magnitude as the elasticity increases. The percentage contribution of this risk to the equity premium, however, is smaller under leisure preferences. The additional short-run risk innovation has a positive price of risk, $\lambda_l$, smaller in magnitude than $\lambda_c$ and less sensitive to different values of $\rho$. The contribution to the equity premium of this risk tends to be positive and ranges from 0% to almost 12% as the elasticity $\rho$ increases. The prices of risk of variation
in the conditional means of consumption growth (and leisure) are also different from those in the baseline model. The price of risk $\lambda_x$ is smaller since the negative sensitivity of leisure $\phi_{lx}$ implies a lower marginal utility of consumption after a negative shock. The price of risk $\lambda_u$ is negative mainly because a positive shock to $u_t$ reduces consumption growth ($\phi_{cu} < 0$) and then increases marginal utility. The price of this risk, in magnitude, is around one-fifth of the price of risk $\lambda_x$, is not significantly affected by the elasticity $\rho$, and has a positive contribution of around 8% to the equity premium since dividend growth has a negative sensitivity to this risk ($\phi_{du} < 0$). Finally, volatility shocks have a negative price of risk $\lambda_\nu$ as in the baseline model. These shocks generate a positive equity premium with a percentage contribution to the premium of 57% when $\rho = 0.5$, which decreases to around 26% when $\rho = 50$. That is, for low elasticity values, volatility risk is a more significant determinant of the equity premium than in the baseline model.

As in Bansal and Yaron (2004), the introduction of time-varying uncertainty generates time-varying expected excess asset returns. In turn, this time variation affects the predictability of macroeconomic variables and asset returns. Our modeling choice for volatility in equation (14) allows us to determine which source of time variation contributes most to this predictability by setting $I_k = 0$ for $k = \{c, l, x, u, d\}$. In untabulated results, available from the authors upon request, we show that the most significant contribution to the volatility premium arises from time-varying volatility in conditional means ($I_x = I_u = 1$), which combined with time variation in innovations to nondurable and services consumption growth ($I_c = 1$) improves results for predictability. In contrast, time-varying volatility in leisure growth ($I_l = 1$) results in a small deterioration of the predictability of macroeconomic variables and excess stock returns. However, we set $I_l = 1$ because it improves the predictability results for the volatility of leisure and wage growth. We also find that setting $I_d = 0$ slightly improves the lack of predictability of dividend growth.

Finally, we examine the implications of the model for the predictability of levels and volatility of growth in nondurables and services consumption, dividends, leisure, and wages and excess stock returns by the price-dividend ratio. Again, the model with leisure preferences has similar implications for the predictability as the baseline calibration without leisure for consumption and dividend growth. The same pattern is observed in the predictability of leisure and wage growth in models with leisure preferences. That is, the models imply more predictability of the levels of macroeconomic variables and less predictability of excess stock returns and volatility of macroeconomic variables than observed in the data. These predictability results are improved for all variables as the stochastic volatility channel becomes more important in the calibration. Because of the similarity of these results to those presented earlier in the literature, we do not tabulate them, but the tables are available upon request.

In summary, incorporating leisure preferences into the long-run risks model has little quantitative impact on the model’s ability to match the moments of financial asset data, unless the
elasticity of substitution between leisure and consumption is significantly different than the elasticity of intertemporal substitution of consumption. However, the decomposition of risks and their contributions to the equity premium are different given the additional sources of short- and long-run risks. In addition, although the degree of risk aversion in the baseline and leisure models is similar, risk aversion relative to consumption gambles in the leisure model is significantly smaller than in the baseline one. Like the original calibration, the model generates a low risk-free rate, a high equity premium, and time-varying expected returns that are, to some degree, predictable by price-dividend ratios. However, the model continues to have difficulty matching the degree of predictability of excess returns, especially at long horizons, and the predictability of the volatility of macroeconomic variables. One possible remedy is the incorporation of an additional source of volatility, as in Zhou and Zhu (2015). Our principal conclusion is that while a model calibrated to leisure moments does not improve upon the pricing of financial assets, it does not hurt the model’s ability to price financial assets either, and provides additional structure to price other claims such as human capital.

4.3 The Term Structure of Interest Rates

The yield on a bond that pays a unit of nondurable and services consumption at time $t + n$, $r_t^{(n)}$, is obtained from the conditional expectation of the pricing kernel,

$$\exp\left(-r_t^{(n)}\right) = E_t[M_{t,t+n}].$$

This yield depends linearly on contemporaneous and lagged sources of long-run risk and economic uncertainty, such that

$$r_t^{(n)} = \frac{1}{n} \left[A_n + B_{n,x}x_t + B_{n,u}u_t + B_{n,\nu}\nu T + B_{n,\nu}\nu T + B_{n,\nu}\nu T\right],$$

with coefficients $A_n$, $B_{n,x}$, $B_{n,u}$, and $B_{n,\nu}$ described in appendix B. The dependence on lagged sources of risks is captured by leisure, and arises from the stationary nature of this variable. Also, as noted above, in the presence of preferences for leisure, these bonds are not actually risk free if held to maturity since they pay a unit of nondurable and services consumption rather than total consumption.

Beeler and Campbell (2012) report a significantly downward sloping term structure of real rates implied by the long-run risks model. The authors note that United States Treasury inflation protected securities (TIPS) have never exhibited a negative average term slope. While the TIPS average term structure is upward sloping, spreads between 10- and 5-year UK inflation protected bonds have been negative for sustained periods of time, and the average spread is close to zero according to data from Global Financial Data. While our model is calibrated to U.S. rather than U.K. data, these yields suggest that
the term structure is also downward sloping, but not as severely as in the baseline long-run risks model. Figure 3 shows the term structure for annual maturities from 1 to 10 years for our calibrations. Our baseline calibration reproduces the Beeler and Campbell (2012) finding, with a spread between the 10-year bond yield and the 1-month risk-free rate of -1.09%. The model with leisure preferences and $\rho = 0.5$ exacerbates this problem, and implies a spread of -1.54%. Increasing the elasticity parameter results in a less steeply downward sloping term structure, and falls to -0.88% when $\rho = 5$. Preferences with a high substitution between consumption and leisure reduce the sensitivity of long-term bonds to sources of long-run risk and thus the hedging properties of these bonds.

4.4 Substitution of Consumption and Leisure and the Wealth Portfolio

A key parameter in our calibrations is the parameter of intra-temporal substitution between consumption of nondurables and services consumption and leisure. The asset pricing effects of $\rho$ can be understood from the set of equations (21) and their impact on the return on the portfolio of aggregate wealth. As in the baseline model of Bansal and Yaron (2004), recursive preferences ($\gamma \neq \frac{1}{\psi}$) result in the dependence of the pricing kernel on the return of the wealth portfolio, $R_{t,t+1}^c$, as in equation (10). However, the wealth portfolio is no longer a claim only on nondurable and services consumption, but rather a claim on total consumption $F_tA_t$. In this framework, the prices of risk depend on $\frac{1}{\psi} - \frac{1}{\rho}$. This component captures the dependence of the pricing kernel on leisure. For values of $\rho \neq \psi$, the effect of leisure in prices of risk is amplified.

It can be shown that\textsuperscript{16}

\[ F_tA_t = \frac{1}{1 - \alpha} C_tZ_t^{\frac{1}{\rho}}. \]

When $\rho = 1$, nondurables and services consumption is a constant fraction of total consumption (at monthly frequency) and the return on the wealth portfolio is the same as the return on nondurables and services consumption claims ($R_{t,t+1}^c = R_{c,t+1}$), as in the baseline case. However, for values of $\rho \neq 1$, consumption as a fraction of total consumption varies over time. The volatility of the wealth portfolio return can be higher or lower than the volatility of consumption claim returns, affecting the volatility of the pricing kernel. Table 11 shows that, unless the elasticity $\rho$ is low, excess returns on the wealth portfolio are higher in models with leisure preferences than in the baseline model, suggesting that leisure in the utility function increases the riskiness of the wealth portfolio. The expected value and volatility of excess returns on the wealth portfolio increase as the substitutability between leisure and consumption increases. It also can be seen that the Sharpe

\textsuperscript{16}Specifically, if $W_t = MRS_{c,t}$, the relation follows from equation (18), since it implies $F_tA_t = C_t + W_tL_t$. In the presence of deviations of $W_t$ from $MRS_{c,t}$, the same relation is obtained under the additional assumption that the representative household absorbs any profits or losses resulting from these deviations.

a downward sloping average real term structure is at least feasible empirically.
ratio of the wealth portfolio can be significantly higher than that of the stock market for values of $\rho$ close to $\psi$.

4.5 Human Capital Returns

The principal contribution of our model is that it allows us to quantify the risk in human capital and its associated expected returns. We analyze two claims on human capital: claims on all future labor income, and claims on all future wages. In a model with no leisure preferences, households provide labor inelastically and the two claims are the same. In the presence of leisure preferences, households have the ability to adjust leisure over time, affecting their labor income stream. Measuring human capital returns using claims on wages (per unit of time) provides an idea of the value and riskiness of human capital for a fixed amount of labor in the economy. Measuring human capital returns using claims on labor income is more appropriate for understanding portfolio choice decisions. Consequently, it is of interest to analyze the riskiness of both types of claims.

We first examine the calibration to correlations and moments of macroeconomic variables related to leisure preferences, specifically wages, labor income, and leisure. Results are presented in Table 12. As shown in the table, most moments are captured fairly well; the model captures means, volatilities, and autocorrelations of all variables with the exception of lower means for wage and labor income growth, and a slightly low autocorrelation of labor income growth. The difficulty in capturing these means is due to the fact that in the model, leisure is stationary, and so wages and labor income share the same trend with consumption. Correlations are also captured reasonably well. All correlations are of the right sign and relatively close in magnitude to those in the data with one important exception: The correlations of leisure with wage and labor income growth are positive in the data and negative in the model. Different attempts to capture these correlations suggest the importance of a third factor, as found in the empirical analysis, to match them.

Table 13 shows properties of excess returns on labor income and wage claims. For all values of $\rho$, these claims have excess returns that are significantly less volatile than excess stock returns. For values of $\rho \geq 1$, relative to stocks their expected returns are lower and their Sharpe ratios higher. In particular, these Sharpe ratios are significantly higher than that of stocks for values of $\rho$ close to $\psi$, despite the fact that all calibrations match the volatility of labor income and wage growth in the data. Across our five calibrations, expected excess returns on labor income claims range between 1.42% to 2.39%, and those of claims on wages range around 1.25% to 2.29%, in comparison to an expected excess stock return of 5.13%. The ability of households to adjust their labor supply increases the riskiness of labor claims relative to wage claims. Periods of high marginal utility coincide with periods not only of low wages but also of low labor provision, given the negative correlation between consumption and leisure.
Lustig and Nieuwerburgh (2006) obtain an estimate of the return on human capital and find that it is negatively correlated with the return on equity claims. However, Bansal, Tallarini and Yaron (2008) provide a model in which the correlation in human capital and equity returns is zero, and Bansal et al. (2014) document a positive correlation between human capital and equity returns. We investigate the correlation in the returns on dividend and human capital claims and tabulate the results in Table 14. The table also documents the correlations of returns to total consumption claims and nondurables and services consumption claims with excess returns on equity. As shown in the table, the correlation of excess returns on equity with both measures of the return on human capital are positive and increase with the elasticity $\rho$. A similar pattern is observed for correlations of excess equity returns with total and nondurables and services consumption, consistent with the baseline calibration of Bansal, Kiku and Yaron (2007).

We compare the contribution of different sources of risk in the risk premium on human capital claims in Table 15. All sources of risk have a positive contribution to expected excess returns, with the exception of the short-run shocks $\varepsilon_c$ and $\varepsilon_l$, when $\rho$ is low. The contribution of short-run relative to long-run risks increases with this elasticity. As in the case of stocks, time-varying volatility is the main source of risk premia for low elasticities between leisure and consumption. Different to stocks, the contribution of the conditional-mean risk $\varepsilon_u$, is comparable in magnitude to the contribution of $\varepsilon_x$. This result emphasizes the difference in the nature of risk of labor and financial claims.

In summary, human capital claims have expected returns that are between 25% and 50% of expected stock returns with Sharpe ratios that can be 60% higher than that of the market, consistent with Bansal, Tallarini and Yaron (2008). Excess returns on these claims are positively correlated with the excess returns on equity and are significantly affected by the presence of leisure in the utility function and the degree of substitutability of leisure and consumption of nondurables and services. These correlations are supportive of the evidence in Bansal et al. (2014), and run counter to the evidence of negative correlation in Lustig and Nieuwerburgh (2006).

5 Conclusion

Under time recursive preferences, quantities that provide utility such as leisure will matter for agents’ marginal utility of consumption, even if intratemporal preferences are separable. We model asset prices in a framework with persistent moments in consumption, leisure, and wage growth, and recursive preferences, as in the long-run risk model of Bansal and Yaron (2004). We find that the model delivers similar results for financial asset prices as the calibrations of Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007). In particular, equity risk premia, risk-free rates and volatility of financial assets consistent with that observed in the data can be generated using plausible risk aversion parameters, while matching macroeconomic moments. The model reproduces results on
predictability of asset returns, macroeconomic variables, and volatility of macroeconomic variables in Bansal, Kiku and Yaron (2007). The model is able to generate a real term structure with a less pronounced negative slope than long-run risk calibrations with only consumption data. Finally, the model provides support for specifications where the elasticity of substitution between leisure and consumption is close to the intertemporal elasticity of substitution of consumption.

A novel contribution of our analysis is the endogenous generation of the risk premium and price of human capital. The tradeoff between labor and leisure allows us to determine the price of a human capital claim in equilibrium. Similar to Bansal et al. (2014), but in contrast to Lustig and Nieuwerburgh (2006), returns on human capital are positively correlated with returns on financial assets. These human capital returns have a lower risk premium than returns on dividend and consumption claims, with lower loadings on shocks to the conditional mean and larger loadings on contemporaneous consumption shocks. Although the risk premia are lower, Sharpe ratios to human capital can be 60% higher than those to dividend claims. These results provide important guidance to the growing literature investigating the relation between labor and asset prices.
References


A Model Solution

The pricing kernel in equation (10) depends on the portfolio return \( R_{a,t} \), which in turn satisfies the pricing equation (11). The return on this portfolio can be written in terms of the wealth-consumption ratio \( p_{a,t} \equiv \log S_{a,t} - \log F_{a,t} \). Since \( F_{a,t} \propto C_tZ_t^{1-\frac{1}{\rho}} \), this return can be written as

\[
R_{a,t+1} = (1 + e^{p_{a,t+1}}) \left( \frac{C_{t+1}}{C_t} \right) \left( \frac{Z_{t+1}}{Z_t} \right)^{1-\frac{1}{\rho}} e^{-p_{a,t}}.
\]

The equation above can be approximated around \( \bar{\ell}_a = \mathbb{E}[p_{a,t}] \) to obtain

\[
r_{a,t+1} = \eta_a + \eta_ap_{a,t+1} + \Delta c_{t+1} + \left( 1 - \frac{1}{\rho} \right) \Delta z_{t+1} - p_{a,t},
\]

where \( \eta_a = \frac{\exp(\bar{\ell}_a)}{1+\exp(\bar{\ell}_a)} \), and \( \bar{\eta}_a = \log \left[ 1 + \exp(\bar{\ell}_a) \right] - \bar{\ell}_a \eta_a \). Notice that the solution for \( \bar{\ell}_a \) involves a fixed point problem.

Assuming a preference shock \( \zeta \equiv C_t e^{(1-1/\rho)^{-1} k_m \zeta \xi^2 \bar{\ell}_t} \), the ratio \( Z_t \) in equation (7) can be approximated as

\[
z_t = \mu_z + b_{zl} \hat{\ell}_t + b_{zx} \xi \hat{\ell}_t,
\]

where \( \hat{\ell}_t \equiv l_t - \bar{\ell} \),

\[
\mu_z = \left( 1 - \frac{1}{\rho} \right)^{-1} \log a_z, \quad b_{zl} = \frac{\alpha c \left( 1 - \frac{1}{\rho} \right) \xi}{a_z}, \quad b_{zx} = \frac{\alpha c \left( 1 - \frac{1}{\rho} \right) \xi}{\left( 1 - \frac{1}{\rho} \right) a_z} \kappa_m \xi,
\]

and \( a_z = 1 - \alpha + \alpha c \left( 1 - \frac{1}{\rho} \right) \). Let \( W_t \equiv MRS \theta e^{\phi \xi \zeta} \), such that

\[
\frac{W_t}{C_t} = \left( 1 - \frac{\alpha}{1 - \alpha} \right) L_t^{-\frac{1}{\rho}} e^{\kappa \xi \zeta},
\]

where \( \kappa \xi = \kappa_m \xi + \kappa_{\theta} \xi \). Notice that equations (18) and (20) correspond to the case \( \kappa_m = 0 \) and \( \kappa_{\theta} = 1 \).

The pricing equation (11) and the approximations above imply a solution for the price-total consumption ratio given by

\[
p_{a,t} = \bar{p}_a + p_{a,x} x_t + p_{a,u} u_t + p_{a,l} \hat{\ell}_t + p_{a,x} \xi \hat{\ell}_t + p_{a,v} v_t,
\]

where the coefficients satisfy

\[
\bar{p}_a = \frac{1}{1 - \eta_a} \left[ \log \beta + \left( 1 - \frac{1}{\psi} \right) \mu_c + \bar{\eta}_a + \bar{\xi}_a - \frac{\delta_c}{\beta} \log (1 - \theta \eta a p_{a,v} u) \right],
\]

\[
p_{a,x} = \frac{1 - \frac{1}{\psi}}{1 - \eta_a \phi_x} \left[ 1 + (1 - \eta_a) (b_{zl} \phi \xi + b_{zx} \phi \xi) \right],
\]

\[
p_{a,u} = \frac{1 - \frac{1}{\psi}}{1 - \eta_a \phi_u} \left[ \phi_{cu} + (1 - \eta_a) (b_{zl} + b_{zx} \phi u) \right],
\]

\[
p_{a,l} = - \left( 1 - \frac{1}{\psi} \right) b_{zl},
\]

\[
p_{a,v} = q_{a,v} + \frac{\eta_a \phi_{v} p_{a,v}}{1 - \theta \eta a p_{a,v} u},
\]

for \( \bar{q}_a = \frac{1}{\bar{\xi}} \left[ \sigma_{a,v} \sigma_{a,v} (1 - I_v) + \sigma_{a,v} \sigma_{a,v} (1 - I_v) + \sigma_{a,v} \sigma_{a,v} (1 - I_v) + \eta_a p_{a,u} \sigma_{a,u} (1 - I_u) + \eta_a p_{a,u} \sigma_{a,u} (1 - I_u) \right], \) and \( q_{a,v} = \frac{1}{\bar{\xi}} \left[ \sigma_{a,v}^2 I_v + \sigma_{a,v}^2 I_v + \sigma_{a,v}^2 I_v \right] \).
where \( \sigma_{ac} = \left(1 - \frac{1}{\psi}\right) (1 + (1 - \eta_a)(b_{xl}\sigma_{lc} + b_{x\xi}\sigma_{\xi})) \), \( \sigma_{al} = \left(1 - \frac{1}{\psi}\right) (\sigma_{cl} + (1 - \eta_a)(b_{zl} + b_{x\xi}\sigma_{\xi})) \), and \( \sigma_{a\xi} = \left(1 - \frac{1}{\psi}\right) (1 - \eta_a) b_{x\xi} \). The coefficient on volatility \( p_{a,\nu} \) solves a quadratic equation. The solution is the one that makes \( p_{a,\nu} = 0 \) if \( I_k = 0 \) for all \( k \).

## B Asset Prices and Expected Returns

### B.1 Expected Returns

The log-pricing kernel \( m_{t,t+1} \equiv \log M_{t,t+1} \) can be expressed as

\[
-m_{t,t+1} = \Gamma_0 + \Gamma_x x_t + \Gamma_u u_t + \Gamma_l l_t + \Gamma_{\xi} \xi_t + \lambda_c \sigma_{c,t} e_{c,t-1} + \lambda_t \sigma_{t} e_{t-1} + \lambda_x \sigma_{x,t} e_{x,t-1} + \lambda_{\xi} \sigma_{\xi,t} e_{\xi,t-1} + \lambda u \sigma_{u,t} e_{u,t-1} + \lambda \nu e_{\nu,t-1},
\]

where

\[
\begin{align*}
\Gamma_0 &= -\theta \log \beta + \gamma \mu + (1 - \theta) [\eta_a - (1 - \eta_a) \check{p}_a + \eta_a p_{a,\nu} \delta_{a,\nu}], \\
\Gamma_x &= \frac{1}{\psi} + \left(\frac{1}{\psi} - \frac{1}{\rho}\right) (b_{xl}\phi_{lx} + b_{x\xi}\phi_{lx}), \\
\Gamma_u &= \frac{1}{\psi} \phi_{cu} + \left(\frac{1}{\psi} - \frac{1}{\rho}\right) (b_{zl} + b_{x\xi}\phi_{lu}), \\
\Gamma_l &= -\left(\frac{1}{\psi} - \frac{1}{\rho}\right) b_{xl}, \\
\Gamma_{\xi} &= -\left(\frac{1}{\psi} - \frac{1}{\rho}\right) b_{x\xi}, \\
\Gamma_{\nu} &= -(1 - \theta)(1 - \eta_a \phi_u) p_{a,\nu}.
\end{align*}
\]

and the prices of risk are

\[
\begin{align*}
\lambda_c &= \gamma + \left[\frac{1}{\psi} - \frac{1}{\rho} + \left(\frac{\gamma - 1}{\psi}\right) (1 - \eta_a)\right] (b_{xl}\sigma_{lc} + b_{x\xi}\sigma_{\xi}), \\
\lambda_t &= \gamma \sigma_{cl} + \left[\frac{1}{\psi} - \frac{1}{\rho} + \left(\frac{\gamma - 1}{\psi}\right) (1 - \eta_a)\right] (b_{zl} + b_{x\xi}\sigma_{\xi}), \\
\lambda_x &= \frac{\gamma - 1}{1 - \eta_u \phi_u} [1 + (1 - \eta_a)(b_{zl}\phi_{lu} + b_{x\xi}\phi_{lu})] \eta_u, \\
\lambda_u &= \frac{\gamma - 1}{1 - \eta_u \phi_u} [\phi_{cu} + (1 - \eta_a)(b_{zl} + b_{x\xi}\phi_{lu})] \eta_u, \\
\lambda_{\xi} &= \left[\frac{1}{\psi} - \frac{1}{\rho} + \left(\frac{\gamma - 1}{\psi}\right) (1 - \eta_a)\right] b_{x\xi}, \\
\lambda_{\nu} &= (1 - \theta) \eta_a p_{a,\nu},
\end{align*}
\]

Consider a claim on all future cashflows \( K_t \). We are interested in pricing claims on cashflows \( K_t = \{C_t, D_t, W_t, W_t N_t\} \). Growth in these cashflows can be expressed as

\[
\Delta k_t = \mu_k + \phi_k x_{t-1} + \phi_k u_{t-1} + \phi_k l_{t-1} + \phi_k \xi_{t-1} + \phi_k \sigma_{c,t-1} e_{c,t} + \phi_k \sigma_{t-1} e_{t} + \phi_k \sigma_{x,t-1} e_{x,t} + \phi_k \sigma_{\xi,t-1} e_{\xi,t} + \phi_k \sigma_{u,t-1} e_{u,t} + \phi_k \sigma_{\nu,t-1} e_{\nu,t}.
\]  

(23)

34
A claim on all future cashflows has the no-arbitrage price

$$S_{k,t} = \mathbb{E}_t[M_{t,t+1}(K_{t+1} + \tilde{S}_{k,t+1})],$$

and return

$$e^{r_{k,t+1}} = \left(1 + \frac{S_{k,t+1}}{K_{t+1}}\right)\left(\frac{K_{t+1}}{K_t}\right)\left(\frac{K_t}{S_{k,t}}\right).$$

Denote the price-cashflow ratio by $p_{k,t} \equiv \log S_{k,t} - \log K_t$, and approximate the equation above around $\tilde{t}_k = \mathbb{E}[p_{k,t+1}]$ to obtain

$$r_{k,t+1} = \eta_k + \eta_k p_{k,t+1} + \Delta k_{t+1} - p_{k,t},$$

where $\eta_k = \frac{\exp(\tilde{t}_k)}{1 + \exp(\tilde{t}_k)}$, and $\tilde{\eta}_k = \log(1 + \exp(\tilde{t}_k)) - \tilde{t}_k \eta_k$. Notice that the solution for $\tilde{t}_k$ involves a fixed point problem.

The solution for the price-cashflow ratio has the form

$$p_{k,t} = \tilde{p}_k + p_{k,x} x_t + p_{k,u} u_t + p_{k,\xi} \xi_t + p_{k,\nu} \nu_t,$$

with coefficients

$$\tilde{p}_k = \left(\frac{1}{1 - \eta_k}\right) \left\{-\Gamma_0 + \lambda_c \phi_0 \delta_0 + \tilde{\eta}_k + \mu_k + \tilde{\eta}_k - \delta_0 \log(1 + (\lambda_\nu - \eta_k \phi_0) \nu_t)\right\},$$

$$p_{k,x} = \frac{\phi_{kx} - \Gamma_x + \eta_k (\phi_{ux} p_{k,t} + \phi_{xu} \tilde{p}_k)}{1 - \eta_k \phi_x},$$

$$p_{k,u} = \frac{\phi_{ku} - \Gamma_u + \eta_k (\phi_{xu} p_{k,t} + \phi_{u} \tilde{p}_k)}{1 - \eta_k \phi_u},$$

$$p_{k,\xi} = \phi_{k\xi} - \Gamma_\xi,$$

$$p_{k,\nu} = q_{k,\nu} - \frac{(\lambda_\nu - \eta_k \phi_0) \phi_\nu}{1 + (\lambda_\nu - \eta_k \phi_0) \nu_t},$$

for $\tilde{p}_k = \frac{1}{2} \left[\lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c) + \lambda_{k,\xi} \xi^2 (1 - I_c)\right]$, and $q_{k,\nu} = -\Gamma_\nu + \lambda_\nu \phi_\nu + \frac{1}{2} \left[\lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c) + \lambda_{k,\nu} \nu^2 (1 - I_c)\right]$, where $\lambda_{k,\xi} = \lambda_c - \sigma_{k,\xi} - \eta_k p_{k,\xi} \sigma_{k,\xi} - \eta_k \phi_{k,\xi} \sigma_{k,\xi}$, $\lambda_{k,\nu} = \lambda_{k,\nu} - \sigma_{k,\nu} - \eta_k p_{k,\nu} \sigma_{k,\nu} - \eta_k \phi_{k,\nu} \sigma_{k,\nu}$, and $\lambda_{k,\xi} = \lambda_{k,\xi} - \eta_k p_{k,\xi} \sigma_{k,\xi} - \eta_k \phi_{k,\xi} \sigma_{k,\xi}$, $\lambda_{k,\nu} = \lambda_{k,\nu} - \eta_k p_{k,\nu} \sigma_{k,\nu} - \eta_k \phi_{k,\nu} \sigma_{k,\nu}$. Notice that the coefficient on volatility $p_{k,\nu}$ solves a quadratic equation. The solutions is the one that makes $p_{k,\nu} = 0$ if $I_j = 0$ for all $j$, respectively.

Innovations in expected excess log-returns are

$$r_{k,t+1} - E_t[r_{k,t+1}] = r_{c,e} \sigma_{c,t+1} + r_{s,\xi_{c,t+1}} + r_{x,\xi_{c,t+1}} + r_{c,\xi_{c,t+1}} + r_{s,\xi_{t+1}} + r_{x,\xi_{t+1}} + r_{u,\xi_{t+1}},$$

where $r_{k,c} = \sigma_{c,k} + \eta_k (p_{k,c} \sigma_{c,k} + p_{k,c} \sigma_{c,k})$, $r_{k,\xi} = \sigma_{k,\xi} + \eta_k (p_{k,\xi} \sigma_{k,\xi} + p_{k,\xi} \sigma_{k,\xi})$, $r_{k,\xi} = \sigma_{k,\xi} + \eta_k (p_{k,\xi} \sigma_{k,\xi} + p_{k,\xi} \sigma_{k,\xi})$, and $r_{k,\nu} = \eta_k p_{k,\nu}$, and $r_{k,\xi} = \eta_k p_{k,\xi}$.

Hsu and Palomino (2015) show that expected excess returns on these claims are

$$\log \mathbb{E}_t[\exp(x r_{k,t+1})] = \lambda_c r_{c,e} \sigma_{c,e}^2 + \lambda_{r_c} \sigma_{r_c}^2 + \lambda_{x,k} \sigma_{x,k}^2 + \lambda_{s,k} \sigma_{s,k}^2 + \lambda_{u,k} \eta_k p_{k,u} \sigma_{u,k}^2 + \lambda_{x,k} \eta_k p_{k,x} \sigma_{x,k}^2,$$

$$+ \delta_\nu \log \left[\frac{1 + (\lambda_\nu - r_\nu) \nu_t}{1 + (\lambda_\nu - r_\nu) \nu_t - 1 + \lambda_\nu \phi_\nu} - \frac{\nu_t}{1 + (\lambda_\nu - r_\nu) \nu_t} \right].$$

From equations (23), for claims on consumption $\phi_{c,k} = 1$, and $\phi_{c,\xi} = \phi_{c,\xi} = \sigma_{c,\xi} = \sigma_{c,\xi} = 0$. For claims on dividends, $\phi_{d,\xi} = \phi_{d,\xi} = \sigma_{d,\xi} = \sigma_{d,\xi} = 0$. For claims on wages and labor income, $\phi_{w,k} = 1 + b_{k_l} \phi_{w,k} + b_{k_l} \phi_{w,k} + \phi_{w,u} + b_{k_l} + b_{k_l} \phi_{w,u}$, $\phi_{w,k} = b_{k_l} \phi_{w,k} = b_{k_l} \phi_{w,k} = b_{k_l} \phi_{w,k}, \sigma_{k,c} = 1 + b_{k_l} \sigma_{c,k} + b_{k_l} \sigma_{c,k}, \sigma_{k,\xi} = \sigma_{c,k} + b_{k_l} + b_{k_l} \sigma_{c,k}, \sigma_{k,\xi} = b_{k_l} \sigma_{c,k}$, and $\sigma_{k,\xi} = 0$, where $k = \{w, y\}$. 35
B.2 The Term Structure of Interest Rates

The risk-free rate is
\[
    r_t = \Gamma_0 + \delta_v \log(1 + \lambda_v \omega_v) - \lambda_\omega \phi_v \delta_v - \bar{q}_t + \Gamma_x x_t + \Gamma_u u_t + \Gamma_i \xi_t + \left[ \Gamma_v - \lambda_v \phi_v + \lambda_\omega \phi_v - q_{r,v} \right] v_t.
\]
where \( \bar{q}_t = \frac{1}{2} \left[ \lambda^2 \sigma^2 I_c(1 - I_c) + \lambda^2 \sigma^2 I_c(1 - I_c) + \lambda^2 \sigma^2 I_c(1 - I_c) \right], \) and \( q_{r,v} = \frac{1}{2} (\lambda^2 \sigma^2 I_c + \lambda^2 \sigma^2 I_c + \lambda^2 \sigma^2 I_c). \)

The yield of a bond with maturity at \( t + n, r^{(n)}_t \) is obtained from
\[
    \exp\left(-r^{(n)}_t\right) = E_t[M_{t,t+n}] = E_t\left[M_{t,t+1} \exp\left(-r^{(n-1)}_{t+1}\right)\right].
\]

It can be shown that
\[
    r^{(n)}_t = \frac{1}{n} \left[ A_n + B_{n,x} x_t + + B_{n,u} u_t + B_{n,\xi} \xi_t + B_{n,v} v_t \right],
\]
where the coefficients are obtained recursively as
\[
    A_n = A_{n-1} + \Gamma_0 - \lambda_\omega \phi_v \delta_v + \delta_v \log(1 + \lambda_v B_{n-1,v}) \omega_v - \bar{q}^{(n)}_t,
\]
\[
    B_{n,x} = \Gamma_x + B_{n-1,x,\phi_x} + B_{n-1,u} \phi_v + B_{n,i} = \Gamma_i,
\]
\[
    B_{n,u} = \Gamma_v - \lambda_v \phi_v + \frac{(\lambda_v + B_{n-1,u}) \phi_v}{1 + (\lambda_v + B_{n-1,u}) \omega_v} - \bar{q}^{(n)}_{r,v},
\]
where \( \bar{q}^{(n)}_t = \frac{1}{2} [\sigma^2 (1 - I_c) + (\lambda_t + B_{n-1,t})^2 \sigma^2 (1 - I_c) + (\lambda_t + B_{n-1,t})^2 \sigma^2 (1 - I_c) + (\lambda_t + B_{n-1,t})^2 \sigma^2 (1 - I_c)], \) and \( q_{r,v} = \frac{1}{2} [(\lambda_t + B_{n-1,t})^2 \sigma^2 I_c + (\lambda_t + B_{n-1,t})^2 \sigma^2 I_c + (\lambda_t + B_{n-1,t})^2 \sigma^2 I_c + (\lambda_t + B_{n-1,t})^2 \sigma^2 I_c], \) with initial conditions \( A_0 = B_{0,x} = B_{0,u} = B_{0,\xi} = B_{0,v} = 0. \)

C Endogenous Dividend Growth

The dividend growth process in the set of equations (13) can be rationalized from a resource constraint
\[
    C_t = W_t N_t + D_t + G_t,
\]
where \( G_t \) captures sources of income that are not distributed as labor income or dividends, such as debt payments or government transfers. This constraint can be written as
\[
    \frac{D_t}{C_t} = 1 - \frac{W_t N_t}{C_t} - \frac{G_t}{C_t}. \tag{24}
\]

From the wage equation (18), an approximation to the labor income-consumption ratio \( y_t - c_t \equiv \log(W_t N_t/C_t) \) can be written as
\[
    y_t - c_t = \mu_{yc} + b_{y\ell}(l_t - \bar{l}) + b_{y\xi} \xi_t,
\]
where
\[
    \mu_{yc} = \log\left(\frac{\alpha}{1 - \alpha}\right) - \frac{1}{\rho} \bar{l} + \log\left(1 - e^\ell\right), \quad b_{y\ell} = -\left(\frac{1}{\rho} + \frac{e^\ell}{1 - e^\ell}\right), \quad \text{and} \quad b_{y\xi} = \kappa_\xi.
\]

We can assume that the unconditional mean of \( \frac{\Delta G_t}{C_t} \) is \( \bar{G} \) and define the process for \( \Delta G_t \equiv \log G_t - \log G_{t-1} \) as
\[
    \Delta G_t - \Delta c_t = b_{yc}(\Delta c_t - \mu_c) + b_{y\ell} \Delta l_t + b_{y\xi} \Delta \xi_t + \sigma_{y0} \sigma_{d,t-1} \epsilon_{d,t}.
\]
It can be shown that an approximation of the constraint (24) can be expressed as
\[ d_t - c_t = b_{dy}(y_t - c_t) + b_{dg}(g_t - c_t), \]
where
\[ b_{dy} = -\frac{e^{\mu y_c}}{1 - e^{\mu y_c} - G}, \quad \text{and} \quad b_{dg} = -\frac{\bar{G}}{1 - e^{\mu y_c} - G}. \]

The dividend growth equation in (13) follows by making \( \phi_{dx} = 1 + b_{dy}b_{yl}\phi_{yl} + b_{dy}b_{y\xi}\phi_{y\xi}, \phi_{du} = \phi_{cu} + b_{dy}b_{yl} + b_{dy}b_{g\xi}\phi_{g\xi} \),
\[ \phi_{dl} \equiv 0 = -b_{dy}b_{yl} - b_{dy}b_{yl}, \quad \phi_{d\xi} \equiv 0 = -b_{dy}b_{y\xi} - b_{dy}b_{y\xi}, \quad \sigma_{dc} = 1 + (b_{dy}b_{yl} + b_{dy}b_{yl})\sigma_{yl} + (b_{dy}b_{g\xi} + b_{dy}b_{g\xi})\sigma_{g\xi}, \]
\[ \sigma_{dl} = \sigma_{yl} + b_{dy}b_{yl} + b_{dy}b_{yl} + (b_{dy}b_{y\xi} + b_{dy}b_{y\xi})\sigma_{y\xi}, \quad \sigma_{d\xi} \equiv 0 = b_{dy}b_{y\xi} + b_{dy}b_{y\xi}, \quad \text{and} \quad \sigma_{d} = b_{dy}\sigma_{yl}. \]
In Table 1, we present summary statistics for consumption, wage, and dividend growth, as well as the cyclical component of leisure from the Hodrick-Prescott filter. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours \((16 \times 7 = 112)\) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey’s website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Dividends per share are constructed using the CRSP value-weighted portfolio cum- and ex-dividend return series. Dividends are the difference in the cum- and ex-dividend return multiplied by a cumulative capital gain index. Return data are sampled at the monthly frequency, summed to annual quantities, and deflated to real. Consumption and labor income are converted to per capita quantities using midperiod estimates of total population from the BEA. All variables are converted to real quantities using the Personal Consumption Expenditure (PCE) deflator. Data cover the period 1930-2011.

### Table 1: Summary Statistics

#### Panel A: First and Second Moments

<table>
<thead>
<tr>
<th></th>
<th>(\Delta c_t)</th>
<th>(l_t)</th>
<th>(\Delta w_t)</th>
<th>(\Delta d_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.99</td>
<td>0.00</td>
<td>2.70</td>
<td>1.38</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.25</td>
<td>1.20</td>
<td>3.46</td>
<td>10.82</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.44</td>
<td>0.67</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>AC(2)</td>
<td>0.14</td>
<td>0.30</td>
<td>0.11</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

#### Panel B: Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>(\Delta c_t)</th>
<th>(l_t)</th>
<th>(\Delta w_t)</th>
<th>(\Delta d_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta c_t)</td>
<td>1.00</td>
<td>-0.10</td>
<td>0.58</td>
<td>0.62</td>
</tr>
<tr>
<td>(l_t)</td>
<td>1.00</td>
<td>0.27</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>(\Delta w_t)</td>
<td>1.00</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta d_t)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Principal Components of Consumption Growth, Wage Growth, and Leisure

Table 2 presents results of a principal components decomposition of the covariance matrix of consumption growth, wage growth, and leisure. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Wages are the real per capita annual wages implied by real labor income divided by annual hours worked. Leisure is the cyclical component of the fraction of non-sleeping hours \((16 \times 7 = 112)\) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work, filtered using the Hodrick-Prescott filter. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey’s website and employed in Ramey and Francis (2009b). Data are sampled at the annual frequency and cover the period 1930-2011.

| Panel A: Principal Component Analysis |
| Loadings | \(y_{1t}\) | \(y_{2t}\) | \(y_{3t}\) |
| \(\Delta c_t\) | 0.65 | -0.41 | -0.63 |
| \(\Delta w_t\) | 0.72 | 0.11 | 0.68 |
| \(l_t\) | 0.21 | 0.91 | -0.37 |
| % Explained | 53.44 | 35.92 | 10.64 |

| Panel B: Vector Autoregression |
| \(pc_{1,t+1}\) | \(pc_{2,t+1}\) | \(pc_{3,t+1}\) |
| \(pc_{1,t+1}\) | 0.29 | 0.18 | -0.34 |
| SE | (0.10) | (0.12) | (0.23) |
| \(pc_{2,t+1}\) | -0.14 | 0.67 | -0.07 |
| SE | (0.07) | (0.08) | (0.15) |
| \(pc_{3,t+1}\) | -0.08 | -0.08 | 0.33 |
| SE | (0.05) | (0.05) | (0.10) |
Table 3: Dividend Loadings on Macroeconomic Variables

<table>
<thead>
<tr>
<th></th>
<th>$p_{c_{1t}}$</th>
<th>$p_{c_{2t}}$</th>
<th>$p_{c_{3t}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t+1}$</td>
<td>0.67</td>
<td>-0.23</td>
<td>-0.92</td>
</tr>
<tr>
<td>SE</td>
<td>(0.17)</td>
<td>(0.20)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>$\Delta w_{t+1}$</td>
<td>0.50</td>
<td>0.51</td>
<td>-0.10</td>
</tr>
<tr>
<td>SE</td>
<td>(0.30)</td>
<td>(0.36)</td>
<td>(0.67)</td>
</tr>
<tr>
<td>$\ell_{t+1}$</td>
<td>-0.04</td>
<td>0.79</td>
<td>-0.30</td>
</tr>
<tr>
<td>SE</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>$\Delta d_{t+1}$</td>
<td>6.15</td>
<td>-4.26</td>
<td>6.26</td>
</tr>
<tr>
<td>SE</td>
<td>(1.77)</td>
<td>(2.68)</td>
<td>(2.29)</td>
</tr>
</tbody>
</table>
Table 4: Conditional Variance of Consumption, Leisure, and Wage Growth

Table 4 presents results of the analysis of time variation in the volatility of consumption, leisure, and wage growth. Panel A presents variance ratios of residuals from a first stage VAR of consumption, leisure, and wage growth,

\[ q_{t+1} = a + Ay_t + \eta_{q,t+1}, \]

where \( q_{t+1} = \{ \Delta c_t, \Delta w_t, l_t, \} \), log consumption growth, log wage growth, and log detrended, respectively. The variables \( y_t \) are principal components of the covariance matrix of consumption growth, wage growth, and detrended leisure. Variance ratios are computed as

\[ VR_k = \frac{\text{Var}\left(\sum_{j=0}^{J-1} |\eta_{k,t+j}|\right)}{J \cdot \text{Var}(|\eta_{k,t}|)} \]

for \( k = \Delta c, \Delta w, l \) and \( J = 2, 5, \) and 10 years. We present 95\% bootstrapped critical values of the statistics in parentheses. Panel B presents estimates of a GARCH (1,1) model for the conditional variance of the innovations,

\[ \sigma_{k,t+1}^2 = \kappa + \nu_k \sigma_{k,t}^2 + \theta \eta_{k,t-1}^2. \]

Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours \( (16 \times 7 = 112) \) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey’s website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Data are sampled at the annual frequency and cover the period 1930-2011.

### Panel A: Variance Ratios

<table>
<thead>
<tr>
<th>( J )</th>
<th>( \eta_{c,t} )</th>
<th>( \eta_{w,t} )</th>
<th>( \eta_{l,t} )</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>1.03</td>
<td>1.19</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td>(1.19)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>5</td>
<td>1.37</td>
<td>1.73</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(1.37)</td>
<td>(1.36)</td>
</tr>
<tr>
<td>10</td>
<td>1.51</td>
<td>2.30</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>(1.51)</td>
<td>(1.52)</td>
<td>(1.53)</td>
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</tbody>
</table>

### Panel B: GARCH Estimates

<table>
<thead>
<tr>
<th>( \eta_{c,t} )</th>
<th>( \eta_{w,t} )</th>
<th>( \eta_{l,t} )</th>
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</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>SE</td>
<td>(0.08)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>SE</td>
<td>(0.06)</td>
<td>(0.05)</td>
</tr>
</tbody>
</table>
Table 5: Common Parameter Values Across Model Calibrations

Common parameter values for six different model calibrations. The “Baseline” column corresponds to the case of $\alpha = 0$. The “Leisure” column corresponds to the five model calibrations with leisure preferences ($\alpha > 0$). For all specifications, $I_k=1$ for $k = \{c, l, x, u\}$, and $I_d = 0$.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter Model</th>
<th>Baseline</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity parameter of intertemporal consumption</td>
<td>$\psi$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Average consumption growth</td>
<td>$\mu_c \times 10^3$</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td>Loading of consumption growth on $u_t$</td>
<td>$\phi_{cu}$</td>
<td>-0.041</td>
<td></td>
</tr>
<tr>
<td>Volatility parameter of consumption growth</td>
<td>$\sigma_c \times 10^3$</td>
<td>6.31</td>
<td>4.10</td>
</tr>
<tr>
<td>Volatility parameter of consumption growth</td>
<td>$\sigma_{cl} \times 10^3$</td>
<td>0.493</td>
<td></td>
</tr>
<tr>
<td>Average log-leisure</td>
<td>$\bar{l}$</td>
<td>-0.4</td>
<td></td>
</tr>
<tr>
<td>Loading of leisure on $x_t$</td>
<td>$\phi_{lx}$</td>
<td>-3.24</td>
<td></td>
</tr>
<tr>
<td>Volatility parameter of leisure</td>
<td>$\sigma_l \times 10^3$</td>
<td>0.995</td>
<td></td>
</tr>
<tr>
<td>Volatility parameter of leisure</td>
<td>$\sigma_{lc} \times 10^3$</td>
<td>4.48</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation parameter of $x_t$</td>
<td>$\phi_x$</td>
<td>0.975</td>
<td>0.975</td>
</tr>
<tr>
<td>Volatility parameter of $x_t$</td>
<td>$\sigma_x/\sigma_c$</td>
<td>0.0488</td>
<td>0.0695</td>
</tr>
<tr>
<td>Autocorrelation parameter of $u_t$</td>
<td>$\phi_u$</td>
<td>0.995</td>
<td></td>
</tr>
<tr>
<td>Volatility parameter of $u_t$</td>
<td>$\sigma_u/\sigma_c$</td>
<td>0.3243</td>
<td></td>
</tr>
<tr>
<td>Autocorrelation parameter of time-varying volatility</td>
<td>$\phi_\nu$</td>
<td>0.995</td>
<td>0.995</td>
</tr>
<tr>
<td>Parameter of time-varying volatility</td>
<td>$\delta_\nu$</td>
<td>6.05</td>
<td>6.05</td>
</tr>
<tr>
<td>Parameter of time-varying volatility</td>
<td>$\varsigma_\nu \times 10^4$</td>
<td>8.26</td>
<td>8.26</td>
</tr>
</tbody>
</table>

Table 6: Model Specific Parameter Values for the Calibrations

Specific parameter values for five different model calibrations. The common parameter values across models are presented in table 5. Dividend growth is specified as

$$\Delta d_{t+1} = \mu_c + \phi_{dx} x_t + \phi_{du} u_t + \sigma_{dc} \sigma_{ct} \varepsilon_{c,t+1} + \sigma_{dl} \sigma_{lt} \varepsilon_{l,t+1} + \sigma_{dt} \varepsilon_{d,t+1}.$$  

The baseline model corresponds to the case of $\alpha = 0$. The parameter values for the baseline model calibration are $\beta = 0.99964$, $\phi_{dx} = 2.5$, $\sigma_d = 4.75 \sigma_c$, $\sigma_{dc} = 3.31$, and $\phi_{du} = \sigma_{dl} = I_d = 0$.

Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. The parameter values for these models are $\phi_{dx} = 2.5$, $\phi_{du} = -0.06$, $\sigma_d = 7.1780 \sigma_c$, $\sigma_{dc} = 5$, $\sigma_{dl} = 0.87$, and $I_d = 0$.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter Model</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.9997</td>
<td>0.99974</td>
<td>0.99954</td>
<td>0.99926</td>
<td>0.99912</td>
</tr>
<tr>
<td>Risk aversion parameter</td>
<td>$\gamma$</td>
<td>9.3</td>
<td>8.9</td>
<td>9.1</td>
<td>9.15</td>
<td>9.2</td>
</tr>
<tr>
<td>Loading of leisure in the utility function</td>
<td>$\alpha$</td>
<td>0.5605</td>
<td>0.6552</td>
<td>0.6846</td>
<td>0.7233</td>
<td>0.7375</td>
</tr>
<tr>
<td>Elasticity parameter of leisure and consumption</td>
<td>$\rho$</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>Loading of $\xi_t$ on $x_t$</td>
<td>$\phi_{\xi_x}$</td>
<td>-31.45</td>
<td>11.10</td>
<td>-7.18</td>
<td>-14.99</td>
<td>-1.55</td>
</tr>
<tr>
<td>Loading of $\xi_t$ on $u_t$</td>
<td>$\phi_{\xi_u}$</td>
<td>-3.16</td>
<td>-4.13</td>
<td>-5.56</td>
<td>-5.51</td>
<td>-5.97</td>
</tr>
<tr>
<td>Volatility parameter of $\xi_t$</td>
<td>$\sigma_{\xi_c}$</td>
<td>10.46</td>
<td>9.59</td>
<td>6.31</td>
<td>7.75</td>
<td>8.41</td>
</tr>
<tr>
<td>Volatility parameter of $\xi_t$</td>
<td>$\sigma_{\xi_l}$</td>
<td>3.04</td>
<td>3.44</td>
<td>3.23</td>
<td>-1.37</td>
<td>-0.92</td>
</tr>
<tr>
<td>Volatility parameter of $\xi_t$</td>
<td>$\sigma_{\xi} \times 10^3$</td>
<td>5.49</td>
<td>9.46</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>Coeff. of risk aversion (consumption gambles)</td>
<td>$R^c$</td>
<td>3.18</td>
<td>3.04</td>
<td>3.11</td>
<td>3.13</td>
<td>3.15</td>
</tr>
</tbody>
</table>
Table 7: Moments of Growth Rates in Consumption and Dividends

This table contains data and model means, standard deviations, and autocorrelations for growth in log consumption of nondurables and services and dividends per share. \( AC(\cdot, j) \) denotes the autocorrelation of order \( j \). The model statistics are the median of 1,000 simulations of 984 months each, aggregated to the annual frequency. The “Baseline” column corresponds to the case of \( \alpha = 0 \). The “Leisure” column corresponds to the model calibrations with leisure preferences (\( \alpha > 0 \)). Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>Data 1930-2011</th>
<th>Model Baseline</th>
<th>Model Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E[\Delta c] )</td>
<td>1.99</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>( \sigma(\Delta c) )</td>
<td>2.25</td>
<td>2.25</td>
<td>2.26</td>
</tr>
<tr>
<td>( AC(\Delta c, 1) )</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>( AC(\Delta c, 2) )</td>
<td>0.15</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>( E[\Delta d] )</td>
<td>1.38</td>
<td>1.98</td>
<td>2.03</td>
</tr>
<tr>
<td>( \sigma(\Delta d) )</td>
<td>10.82</td>
<td>10.81</td>
<td>10.82</td>
</tr>
<tr>
<td>( AC(\Delta d, 1) )</td>
<td>0.21</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>( AC(\Delta d, 2) )</td>
<td>-0.22</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>( \text{corr}(\Delta c, \Delta d) )</td>
<td>0.62</td>
<td>0.62</td>
<td>0.59</td>
</tr>
<tr>
<td>( \text{corr}(\Delta c, l) )</td>
<td>-0.10</td>
<td>-</td>
<td>-0.10</td>
</tr>
<tr>
<td>( \text{corr}(l, \Delta d) )</td>
<td>-0.06</td>
<td>-</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Table 8: Loadings of Predictive Regressions of Consumption Growth and Leisure on Their Principal Components

This table contains data and model statistics of predictive regressions for growth in log consumption of nondurables and services and detrended leisure on their two principal components. \( AC(pc_i, j) \) denotes the autocorrelation of order \( j \) of the principal component \( i \). The model statistics are the median of 1,000 simulations of 984 months each, aggregated to the annual frequency. The “Model” column corresponds to the model calibrations with leisure preferences (\( \alpha > 0 \)). Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>p(c_{1t} )</th>
<th>p(c_{2t} )</th>
<th>p(c_{1t} )</th>
<th>p(c_{2t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta c_{t+1} )</td>
<td>0.44</td>
<td>-0.16</td>
<td>0.47</td>
<td>-0.16</td>
</tr>
<tr>
<td>t-stat</td>
<td>4.67</td>
<td>-3.94</td>
<td>4.79</td>
<td>-3.99</td>
</tr>
<tr>
<td>( l_{t+1} )</td>
<td>0.21</td>
<td>0.64</td>
<td>-0.11</td>
<td>0.62</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.17</td>
<td>8.07</td>
<td>-0.58</td>
<td>7.72</td>
</tr>
<tr>
<td>( AC(pc_i, 1) )</td>
<td>0.47</td>
<td>0.66</td>
<td>0.49</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Table 9: Annualized Time Average Statistics for Financial Asset Variables
This table contains data and model means, standard deviations, and autocorrelations for financial asset pricing variables, specifically the risk-free rate, the price-dividend ratio, and the equity claim on dividends. $AC(\cdot,j)$ denotes the autocorrelation of order $j$. $SR_b$ denotes the Sharpe Ratio for claims on cashflows $b$. The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1930-2011</td>
<td>Baseline</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{E}[r]$</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
<td>3.40</td>
<td>1.02</td>
</tr>
<tr>
<td>$AC(r,1)$</td>
<td>0.66</td>
<td>0.80</td>
</tr>
<tr>
<td>Price-dividend ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{E}[pd]$</td>
<td>3.33</td>
<td>3.2</td>
</tr>
<tr>
<td>$\sigma(pd)$</td>
<td>40.69</td>
<td>18.46</td>
</tr>
<tr>
<td>$AC(pd,1)$</td>
<td>0.86</td>
<td>0.72</td>
</tr>
<tr>
<td>Claim on dividends</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{E}[xr_d]$</td>
<td>5.13</td>
<td>5.15</td>
</tr>
<tr>
<td>$\sigma(xr_d)$</td>
<td>19.37</td>
<td>15.82</td>
</tr>
<tr>
<td>$SR_d$</td>
<td>0.37</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 10: Prices of Risk and Equity Risk Premia
This table contains the market prices of risk for the four shocks affecting the model economy and the percentage contribution of each shock to the equity premium for different calibrations. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices of Risk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>10</td>
<td>5.40</td>
<td>7.93</td>
<td>9.11</td>
<td>10.54</td>
<td>11.12</td>
</tr>
<tr>
<td>$\lambda_l$</td>
<td>-</td>
<td>3.71</td>
<td>4.17</td>
<td>4.49</td>
<td>4.82</td>
<td>4.96</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>369.67</td>
<td>338.57</td>
<td>324.76</td>
<td>330.55</td>
<td>329.72</td>
<td>330.10</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>-</td>
<td>-64.17</td>
<td>-62.95</td>
<td>-62.53</td>
<td>-60.44</td>
<td>-59.47</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>-2.33</td>
<td>-3.42</td>
<td>-2.89</td>
<td>-2.65</td>
<td>-2.24</td>
<td>-2.10</td>
</tr>
<tr>
<td>Contribution to the Equity Premium (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>26.06</td>
<td>1.96</td>
<td>10.22</td>
<td>14.31</td>
<td>20.76</td>
<td>23.35</td>
</tr>
<tr>
<td>$\varepsilon_l$</td>
<td>-</td>
<td>-0.02</td>
<td>5.17</td>
<td>7.28</td>
<td>10.41</td>
<td>11.61</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>44.59</td>
<td>34.04</td>
<td>32.45</td>
<td>32.39</td>
<td>31.98</td>
<td>31.51</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
<td>-</td>
<td>7.18</td>
<td>7.94</td>
<td>7.88</td>
<td>7.81</td>
<td>7.63</td>
</tr>
<tr>
<td>$\varepsilon_{nu}$</td>
<td>29.35</td>
<td>56.84</td>
<td>44.21</td>
<td>38.13</td>
<td>29.05</td>
<td>25.91</td>
</tr>
</tbody>
</table>
Table 11: The Portfolio of Aggregate Wealth

This table contains data and model means, standard deviations, and autocorrelations for the portfolio of aggregate wealth and the ratio of aggregate wealth to consumption of nondurables and services. $AC(\cdot, j)$ denotes the autocorrelation of order $j$. $SR_b$ denotes the Sharpe Ratio for claims on cashflows $b$. The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>Wealth-consumption ratio</td>
<td>$E[p_a]$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(p_a)$</td>
</tr>
<tr>
<td></td>
<td>$AC(p_{a, 1})$</td>
</tr>
<tr>
<td>Wealth portfolio</td>
<td>$E[xr_a]$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(xr_a)$</td>
</tr>
<tr>
<td></td>
<td>$SR_a$</td>
</tr>
</tbody>
</table>

Table 12: Moments of Leisure, and Growth Rates in Wages and Labor Income

This table contains data and model means, standard deviations, and autocorrelations for macroeconomic variables. $AC(\cdot, j)$ denotes the autocorrelation of order $j$. The model statistics are the median of 1,000 simulations of 984 months each, aggregated to the annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>Data 1930-2011</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 0.5$</td>
<td>$\rho = 1$</td>
</tr>
<tr>
<td></td>
<td>$E[l]$</td>
<td>-0.40</td>
</tr>
<tr>
<td></td>
<td>$\sigma(l)$</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>$AC(l, 1)$</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>$E[\Delta w]$</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>$\sigma(\Delta w)$</td>
<td>3.46</td>
</tr>
<tr>
<td></td>
<td>$AC(\Delta w, 1)$</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$E[\Delta y]$</td>
<td>2.64</td>
</tr>
<tr>
<td></td>
<td>$\sigma(\Delta y)$</td>
<td>3.88</td>
</tr>
<tr>
<td></td>
<td>$AC(\Delta y, 1)$</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>$E[y - c]$</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>$\sigma(y - c)$</td>
<td>9.01</td>
</tr>
<tr>
<td></td>
<td>$AC(y - c, 1)$</td>
<td>0.95</td>
</tr>
<tr>
<td>corr$(\Delta c, l)$</td>
<td>-0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>corr$(\Delta c, \Delta w)$</td>
<td>0.59</td>
<td>0.65</td>
</tr>
<tr>
<td>corr$(\Delta c, \Delta y)$</td>
<td>0.68</td>
<td>0.56</td>
</tr>
<tr>
<td>corr$(l, \Delta d)$</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>corr$(l, \Delta w)$</td>
<td>0.27</td>
<td>-0.06</td>
</tr>
<tr>
<td>corr$(l, \Delta y)$</td>
<td>0.06</td>
<td>-0.26</td>
</tr>
<tr>
<td>corr$(\Delta d, \Delta w)$</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>corr$(\Delta d, \Delta y)$</td>
<td>0.48</td>
<td>0.33</td>
</tr>
<tr>
<td>corr$(\Delta w, \Delta y)$</td>
<td>0.86</td>
<td>0.87</td>
</tr>
</tbody>
</table>
### Table 13: Annualized Time Average Statistics for Human Capital Returns

This table contains data and model means, standard deviations, and Sharpe Ratios for returns on claims to human capital, where $SR_b$ denotes the Sharpe Ratio for claims on cashflows $b$. The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th>Claim on labor income</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[x_{r_y}]$</td>
<td>1.42</td>
<td>1.84</td>
<td>2.02</td>
<td>2.20</td>
<td>2.39</td>
</tr>
<tr>
<td>$\sigma(x_{r_y})$</td>
<td>5.22</td>
<td>3.08</td>
<td>3.63</td>
<td>5.02</td>
<td>5.76</td>
</tr>
<tr>
<td>$SR_{y}$</td>
<td>0.27</td>
<td>0.60</td>
<td>0.56</td>
<td>0.44</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Claim on wages</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[x_{r_w}]$</td>
<td>1.25</td>
<td>1.73</td>
<td>1.91</td>
<td>2.11</td>
<td>2.29</td>
</tr>
<tr>
<td>$\sigma(x_{r_w})$</td>
<td>5.04</td>
<td>2.87</td>
<td>3.46</td>
<td>4.92</td>
<td>5.67</td>
</tr>
<tr>
<td>$SR_{w}$</td>
<td>0.25</td>
<td>0.60</td>
<td>0.55</td>
<td>0.43</td>
<td>0.40</td>
</tr>
</tbody>
</table>

### Table 14: Correlations of Human Capital Claims

This table contains data and model correlations for macroeconomic and asset pricing variables. The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th>Excess Returns</th>
<th>Baseline</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(x_{r_{a}}, x_{r_d})$</td>
<td>0.73</td>
<td>0.23</td>
<td>0.61</td>
<td>0.70</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>$corr(x_{r_{e}}, x_{r_d})$</td>
<td>0.73</td>
<td>0.22</td>
<td>0.61</td>
<td>0.70</td>
<td>0.73</td>
<td>0.75</td>
</tr>
<tr>
<td>$corr(x_{r_{y}}, x_{r_d})$</td>
<td>-</td>
<td>0.27</td>
<td>0.60</td>
<td>0.68</td>
<td>0.73</td>
<td>0.75</td>
</tr>
<tr>
<td>$corr(x_{r_{w}}, x_{r_d})$</td>
<td>-</td>
<td>0.25</td>
<td>0.60</td>
<td>0.69</td>
<td>0.73</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Table 15: **Human Capital Risk Premia**
This table contains the market prices of risk for the four shocks affecting the model economy and the percentage contribution of each shock to human capital premia for different calibrations. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 1.5$</th>
<th>$\rho = 5$</th>
<th>$\rho = 50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices of Risk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>5.40</td>
<td>7.93</td>
<td>9.11</td>
<td>10.54</td>
<td>11.12</td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>3.71</td>
<td>4.17</td>
<td>4.49</td>
<td>4.82</td>
<td>4.96</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>338.57</td>
<td>324.76</td>
<td>330.55</td>
<td>329.72</td>
<td>330.10</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>-64.17</td>
<td>-62.95</td>
<td>-62.53</td>
<td>-60.44</td>
<td>-59.47</td>
</tr>
<tr>
<td>$\lambda_\nu$</td>
<td>-3.42</td>
<td>-2.89</td>
<td>-2.65</td>
<td>-2.24</td>
<td>-2.10</td>
</tr>
<tr>
<td>Contribution to the Premium in Labor Income Claims (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>-20.26</td>
<td>0.17</td>
<td>8.64</td>
<td>21.14</td>
<td>24.90</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>-10.83</td>
<td>7.23</td>
<td>12.55</td>
<td>19.33</td>
<td>20.85</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>25.84</td>
<td>22.42</td>
<td>19.73</td>
<td>17.04</td>
<td>16.09</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
<td>27.95</td>
<td>22.33</td>
<td>21.77</td>
<td>19.23</td>
<td>18.18</td>
</tr>
<tr>
<td>$\varepsilon_\nu$</td>
<td>77.30</td>
<td>47.84</td>
<td>37.31</td>
<td>23.26</td>
<td>19.98</td>
</tr>
<tr>
<td>Contribution to the Premium in Wage Claims (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>-22.88</td>
<td>0.22</td>
<td>9.22</td>
<td>22.15</td>
<td>26.04</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>-12.21</td>
<td>7.74</td>
<td>13.33</td>
<td>20.24</td>
<td>21.79</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>29.30</td>
<td>23.66</td>
<td>20.70</td>
<td>17.57</td>
<td>16.47</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
<td>28.59</td>
<td>21.96</td>
<td>20.93</td>
<td>18.08</td>
<td>16.90</td>
</tr>
<tr>
<td>$\varepsilon_\nu$</td>
<td>77.20</td>
<td>46.41</td>
<td>35.82</td>
<td>21.96</td>
<td>18.80</td>
</tr>
</tbody>
</table>
Figure 1: Time Series of Log Leisure

Figure 1 plots the time series of the log level of leisure, expressed as a fraction of a 112 hour week. We extract trend and cycle components from the leisure level using the Hodrick and Prescott (1997) filter with a smoothing parameter of 100. Data are from the Bureau of Labor Statistics and are sampled at the annual frequency from 1929 through 2011.
Figure 2: Time Series of Conditional Volatility

Figure 2 presents time series of the conditional volatility of consumption, leisure, and wage growth. Volatility is estimated using a GARCH (1,1) model on VAR residuals,

\[ y_t = \mathbf{Py}_{t-1} + u_t, \]
\[ \sigma_{k,t}^2 = \kappa + \nu_k \sigma_{t-1}^2 + \vartheta u_{t-1}^2, \]

where \( y_t = \{\Delta c_t - \bar{\Delta c_t}, \Delta l_t - \bar{\Delta l_t}, \Delta w_t - \bar{\Delta w_t}\} \), demeaned log consumption, leisure, and wage growth, respectively and \( k = \Delta c, \Delta l, \Delta w \). Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours (16 \( \times \) 7 = 112) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey’s website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita after-tax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Data are sampled at the annual frequency and cover the period 1930-2011.
Figure 3: **Term Structure of Real Yields**

This figure plots real yields for the model calibrations for annual maturities from 1 to 10 years. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “$\rho = j$” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 5 and 6.