Income Insurance and the Equilibrium Term-Structure of Equity*

Roberto Marfè†

[This draft includes the Online Appendix]

Abstract

This paper documents that GDP, wages and dividends feature term-structures of variance-ratios respectively flat, increasing and decreasing. Income insurance within the firm from shareholders to workers empirically and theoretically explains these term-structures. Risk sharing smooths wages but only concerns transitory risk and, hence, enhances the short-run risk of dividends. A simple general equilibrium model, where labor rigidity affects dividend dynamics and the price of short-run risk, reconciles standard asset pricing facts with the term-structures of equity premium and volatility and those of macroeconomic variables, at odds in leading models.

Keywords: term structure of equity · income insurance · dividend strips · distributional risk · equilibrium asset pricing

JEL Classification: D51, E21, G12

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Leading asset pricing models describe many features of financial markets but fail to explain the timing of equity risk. These have different rationale (e.g. habit formation, time-varying expected growth, disasters, prospect theory) but share an important feature: priced risk comes from variation in long-run discounted cash-flows.\textsuperscript{1} Instead, van Binsbergen, Brandt, and Koijen (2012); van Binsbergen, Hueskes, Koijen, and Vrugt (2013); van Binsbergen and Koijen (2015) document that the term-structures of equity volatility and premia are downward sloping—that is markets compensate short-run risk. Moreover, standard models usually base on assumptions concerning dividend dynamics which imply an upward-sloping term-structure of dividend risk (i.e. volatilities or variance-ratios of dividends’ growth rates which increase with the horizon). Instead, consistently with Belo, Collin-Dufresne, and Goldstein (2014), this paper documents that dividend risk is strongly downward-sloping, such that many models overestimate long-horizon dividend risk by one order of magnitude.

Why is dividend-risk downward-sloping? Under which conditions, does downward-sloping dividend risk transmit to equity risk and premia? This paper empirically and theoretically addresses these questions by providing a macroeconomic foundation of the timing of risk and by reconciling, in equilibrium, standard asset pricing facts with the new evidence about the term-structures. This is important because the term-structures of both fundamentals and equity provide information about how prices are determined in equilibrium. Hence, a term-structure perspective offers additional testable implications to asset pricing frameworks and can help us to understand the macroeconomic determinants of asset prices.

The paper argues that labor rigidity is at the heart of the timing macroeconomic risk. Danthine and Donaldson (1992, 2002), among others, show that a mechanism of income insurance from shareholders to workers, which takes place within the firm, leads to volatile and pro-cyclical dividends and, hence, explains why equity commands a high compensation.\textsuperscript{2} Beyond such a cyclicality effect of income insurance, I show that also a term-structure effect takes place. Since output, wages and dividends are co-integrated (Lettau and Ludvigson, 2005), income insurance implies that the transitory component of aggregate risk is shared asymmetrically among workers and shareholders, whereas the permanent component is faced by both. Namely, wages are partially insured with respect to transitory risk, whereas dividends load more on transitory risk as a result of operating leverage. Thus, wage and dividend risks shift respectively toward the long and the short horizon and workers and shareholders

\textsuperscript{1}Examples are the seminal works by Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), Bansal and Yaron (2004) and Gabaix (2012), among others.

\textsuperscript{2}The idea that the very role of the firm is that of insurance provider has a long tradition since Knight (1921) as well as Baily (1974), Azariadis (1978), Boldrin and Horvath (1995) and Gomme and Greenwood (1995). They suggest that workers’ remuneration is partially fixed in advance and, hence, shareholders bear most of aggregate risk but, in exchange of income insurance, gain flexibility in labor supply. More recently, Guiso, Pistaferri, and Schivardi (2005), Shimer (2005) and Ríos-Rull and Santaeulália-Llopis (2010) provide empirical support.
bear respectively more long-run risk and more short-run risk. I embed this mechanism of income insurance in an otherwise standard closed-form general equilibrium model and show that, under standard preferences, the term-structure effect of income insurance is inherited by financial markets and leads to downward sloping term-structures of equity risk and premia.

An empirical investigation supports the main model mechanism. First, I document that the timing of risk of macroeconomic variables is heterogeneous. The term-structures of risk of output, wages and dividends are respectively flat, increasing and decreasing. In accord with the model, these term-structures –and the co-integrating relationship among the levels of these variables– support the idea that both workers and shareholders are subject to permanent shocks but they share transitory shocks asymmetrically, as a result of income insurance. Consistently with the model mechanism, Guiso, Pistaferri, and Schivardi (2005) provide evidence that insurance does not concern permanent shocks and Ríos-Rull and Santaéulalia-Llopis (2010) document that the wage-share is stationary and counter-cyclical. Second, I further support the term-structure effect of income insurance by investigating the model prediction that the variance ratios of dividends and the gap between the variance ratios of wages and dividends should be respectively decreasing and increasing with the wage-to-dividends ratios –i.e. the resources devoted to workers’ remuneration relative to shareholders’ remuneration. The model predicts that, after a negative transitory shock, wages are partially insured whereas dividends are hit twice. This implies that, on the one hand, wages are high relative to dividends (the cyclicality effect) and that wages load less than dividends on transitory risk (the term-structure effect). Therefore, the distance between the upward-sloping variance ratios of wages and the downward-sloping variance ratios of dividends should increase when the wage-to-dividends ratio is high. The opposite holds after a positive transitory shock. I provide robust empirical evidence that such a dynamic relation obtains in the actual data. Finally, I show that the remainder of output minus wages features a term-structure of risk which is markedly downward-sloping and essentially recovers the negative slope of dividend risk. This implies that the bulk of the gap between the approximatively flat variance-ratios of output and the decreasing variance-ratios of dividends should be imputed to wages. Thus, alternative channels, such as financial leverage (Belo, Collin-Dufresne, and Goldstein, 2014) or investment (Ai, Croce, Diercks, and Li, 2012; Kogan and Papanikolaou, 2015), likely play a lesser role, at least at the aggregate level.

The model consists of a few simple ingredients and is similar to Greenwald, Lettau, and Ludvigson (2014). Investment is not directly modelled and total resources are shared by workers and shareholders. The former do not invest and consume their wages, and the latter receive dividends and, thus, act as a representative agent on the financial markets. Total resources are subject to both permanent and transitory technological shocks (Bansal,
Income insurance concerns transitory risk only and is driven by the excess risk aversion of workers relative to shareholders, as in Danthine and Donaldson (2002). The insurance mechanism, on the one hand, generates counter-cyclical wage share and downward-sloping dividend risk and, on the other hand, enhances the pricing of short-run risk. Indeed, the equilibrium state-price density equals the shareholders’ marginal utility but is affected by income insurance through the endogenous dynamics of dividends. Differently from Danthine and Donaldson (2002), the model leads to a stationary equilibrium even in presence of permanent shocks (time-varying long-run growth as in Bansal and Yaron (2004)) and recursive preferences. Under standard preferences (i.e. intertemporal substitution larger than wealth effect and preference for the early resolution of uncertainty), the term-structure of equity premia is non-monotone: long-run growth leads to an upward-sloping effect and transitory risk to a downward-sloping one. Indeed, transitory shocks are expected to recover and, hence, are risky in the short-run and safe in the long-run. For income insurance strong enough, operating leverage enhances the transitory risk of dividends and its equilibrium price. In turn, the endogenous negative slope of dividend risk transmits to equity returns.

The model calibration exploits the information from the term-structures of aggregate consumption, wage and dividend risk. On the one hand, they represent additional stylized facts captured by the model; on the other hand, the timing of macroeconomic risk allows to quantitatively measure the strength of income insurance and, hence, the operating leverage on dividends. Moreover, the term-structures of risk provide information about the persistence of latent factors—a main issue in asset pricing models. The baseline calibration reconciles the term-structures of both fundamentals and equity with the traditional asset pricing facts, such as the risk-free rate and equity premium puzzles as well as the the excess-volatility of equity (also at long-horizons, as in Beeler and Campbell (2012)). Moreover, the model leads to upward-sloping real interest rates, consistently with TIPs data. van Binsbergen et al. (2013) refine the findings of van Binsbergen et al. (2012) suggesting that the negative slope of equity premia is an unconditional property of the data, whereas conditional equity premia are slightly increasing in good times and strongly decreasing in bad times. I show that such a dynamics obtains in the model—without deteriorating other results—once fundamentals feature business cycle uncertainty. Time-varying risk premia lead to a pattern of long-horizon predictability consistent with actual data. Most of general equilibrium asset pricing models—such as the

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3The model differs from Greenwald et al. (2014) because: first, the wage-share is not an exogenous shock but derives from income insurance and has counter-cyclical dynamics as in Ríos-Rull and Santaulàlia-Llopis (2010); second, the model does not account for (time-varying) preference shocks—which help to generate a-cyclical variation in equity returns—since the focus of the paper is on the connection between income insurance and the term-structures of dividends and equity; third, the model state-price density is fully endogenous, whereas Greenwald et al. (2014) specify an exogenous and constant risk-free rate; fourth, the model calibration exploits information from the timing of macroeconomic risk, whereas it is different in Greenwald et al. (2014).
long-run risk model of Bansal and Yaron (2004), which obtains as a sub-case— cannot explain simultaneously all of these stylized facts. Closed-form solutions allow to decompose the equity premium and to derive an equilibrium density which captures the relative contribution of discounted cash-flows at each horizon. In the baseline calibration, the horizon that mostly contributes to the equity premium is the immediate future, that is the density is monotone decreasing (whereas it is hump-shaped in a long-run risk model). This suggests that a “short-run” explanation of financial markets is needed to reconcile the timing of macroeconomic risk with the traditional asset pricing facts.

Related literature. A few recent papers suggest alternative and complementary channels which can contribute to explain the term-structure of equity. Belo, Collin-Dufresne, and Goldstein (2014) consider financial leverage, Croce, Lettau, and Ludvigson (2014) focus on beliefs formation, Ai, Croce, Diercks, and Li (2012) and Kogan and Papanikolaou (2015) study investment decisions and Hasler and Marfè (2015) investigate disaster recovery.4

The economic mechanism in Belo et al. (2014) differs from that of this paper: they argue that the negative slope of dividend risk comes out from rigidity in financial leverage. Namely, financial leverage switches upward-sloping EBIT risk to downward-sloping dividend risk. However, I empirically show that the term-structure of risk of EBIT is marked downward-sloping and close to that of dividends. Thus, the financial leverage channel can only have a negligible term-structure effect. Most of the change in the slope obtains earlier than financing decisions and should be imputed to wages.

Croce et al. (2014) study the investors’ beliefs about fundamentals. Namely, they assume the usual long-run risk dynamics for dividends and show that bounded rationality under limited information can lead to downward-sloping dividend risk under the subjective measure of the investors. While this mechanism is complementary to that of this paper, Croce et al. (2014) start from an assumption about dividend dynamics which is strongly inconsistent with the data. Dividend risk is not upward-sloping but it is indeed markedly downward-sloping under the physical measure. Thus, the beliefs formation channel can only have a negligible term-structure effect relative to the bulk of the term-structure effect driven by wages.

Ai et al. (2012) propose a general equilibrium model where heterogeneous investment risk due to vintage capital leads to a downward-sloping term-structure of equity premia. Ai et al. (2012) do not investigate the term-structure of return volatility and require an extreme preference for the early resolution of uncertainty to obtain a sizeable equity premium. The investment channel is complementary to the income insurance mechanism: however, the fact that the term-structure of risk of output minus wages almost recovers the slope of dividend

risk suggests that the investment channel is likely small.

Both Belo et al. (2014) and Kogan and Papanikolaou (2015) are partial equilibrium models: given dividends, a downward-sloping term-structure of equity premia obtains because of the exogenous co-movements between the priced factors and, respectively, financial leverage and investment-specific risk. A general equilibrium approach would produce economic restrictions on those correlations and offer additional insights into the relationship between either financial leverage or investment-specific risk and equity term-structures.

Hasler and Marfè (2015) suggest that the negative slope of dividend risk is due to post-disaster recovery. As long as dividends recover after being hit by a disaster, dividend risk shifts towards the short horizon. While this mechanism is both alternative and complementary to that of income insurance, they interact each other since usually human capital is affected by disasters to a lower extent than physical capital.

A number of recent papers point out the importance of labor relations for asset pricing. This paper complements such a literature investigating the term-structure implications of income insurance in general equilibrium for both dividends and equity. Note that an implicit mechanism of income insurance due to labor markets frictions—such as infrequent wage resettling in Favilukis and Lin (2012), search frictions in Kuehn, Petrosky-Nadeau, and Zhang (2012) and labor mobility in Donangelo (2014)—can lead to a similar term-structure effect. I explicitly model income insurance because it intuitively represents the economic mechanism of risk sharing, beyond the exact combination of frictions which give rise to labor rigidity. While Danthine and Donaldson (1992, 2002), Favilukis and Lin (2012) and Kuehn et al. (2012) propose big macro-finance models and rely on numerical solutions, I consider a simpler and more parsimonious economy and provide analytical solutions.5

The model has also implications for the cross-section of equity returns. Lettau and Wachter (2007, 2011) show in partial equilibrium that a model which captures the downward-sloping term-structure of equity automatically gives rise to a value premium, when value firms are defined as shorter duration equity than growth firms. The term-structure effect of income insurance provides a foundation to the intuition of Lettau and Wachter (2007). Indeed, Marfè (2015) shows that such a result obtains in an extended version of the model and provides empirical support documenting a strong relation between variation in the wage-share and the value premium dynamics. Thus, the model is also consistent with Koijen, Lusting, and van Nieuwerburgh (2014), who empirically document that markets compensate short run-risk and that the value premium dynamics is related to business cycle risk.

The paper is organized as follows. Section 1 provides empirical support to the main

5Even if my model is silent on production and workers saving decisions, Danthine, Donaldson, and Siconolfi (2006) show that a similar form of distributional risk improves the asset pricing implications of a standard real business cycle model without deteriorating business cycle predictions.
model mechanism. Section 2 describes the economy and derives the equilibrium asset pricing predictions. Asset pricing results are in Section 3. A model extension is discussed in Section 4. Section 5 concludes.

1 Empirical Support

The first part of this section documents that the timing of risk is heterogeneous among macroeconomic variables. The second part of the section argues and empirically supports the idea that a mechanism of income insurance between workers and shareholders is at the heart of the timing of risk. The next sections show that, once such an income insurance is embedded in an otherwise standard general equilibrium model, it is possible to reconcile standard asset pricing facts with the new evidence about the term-structure of equity.

I consider data of the aggregate economy (such as real GDP, labour compensations, EBIT and dividends from NIPA tables) as well as from the non-financial corporate sector (such as value added, labour compensations, EBIT and dividends from the Flow of Funds).

1.1 The Timing of Macroeconomic Risk

I define the timing of risk of a given variable as the term-structure of its variance ratios (VR’s hereafter, i.e. the ratio of the growth rates variance at horizon $\tau$ relative to $\tau$ times the variance at horizon one). VR’s capture whether the variance of a given variable increases linearly with the observation interval. Hence, downward-sloping VR’s below unity imply that risk concentrates at short horizons. Vice-versa, upward-sloping VR’s above unity imply that risk concentrates at long horizons.

The upper panels of Figure 1 provide a number of insights. The left and right panels consider the aggregate economy and the non-financial corporate sector. In both cases, we observe that the VR’s of wages are markedly upward-sloping and above unity, whereas the VR’s of EBIT and dividends are markedly downward-sloping and below unity. Instead, the VR’s of total output lie in the middle and are approximatively flat or slightly upward-sloping. These results are consistent with Beeler and Campbell (2012) and Belo et al. (2014). Note that the usual assumptions of asset pricing models about the dynamics of dividends are

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6The VR’s of the generalized payout of Larrain and Yogo (2008) –i.e. the sum of dividends, interest, and net repurchases of equity and debt– have a downward-sloping term-structure similar to EBIT and dividends, as I show in the Online Appendix A.

7The VR’s of aggregate consumption are slightly upward-sloping and lie between those of aggregate wages and those of GDP (see Figure 2). Indeed, consumption is mostly funded by wages. This result is consistent with the theoretical and empirical findings of Berk and Walden (2013): consumption is quite well approximated by the sum of wages and dividends. This approximation holds true in the model of this paper as a result of limited market participation. The Online Appendix A shows that the term-structure of the VR’s of consumption are very close to that of the VR’s of the sum of wages and dividends.
strongly inconsistent with the data. Indeed, time-variation in the conditional moments of a geometric process (e.g. long-run growth, stochastic volatility, time-varying disasters) induce markedly upward-sloping VR’s. Thus, long-horizon dividend risk is usually overestimated by an order of magnitude or more.

1.2 Does Income Insurance Explain the Timing of Risk?

What does explain the heterogeneity in the timing of macroeconomic risk? I argue that the main driver is an explicit or implicit mechanism of income insurance from shareholders to
workers that takes place within the firm. The reasoning is the following.

On the one hand, Danthine and Donaldson (2002) among others suggest that risk-sharing between workers and shareholders (as well as labor markets frictions) leads to smooth wages and, as a result of operating leverage, risky dividends. An implication is that the labor-share is counter-cyclical and the dividend-share is pro-cyclical, as documented by Ríos-Rull and Santaeulàlia-Llopis (2010). Thus, I name this mechanism as the cyclicality effect of income insurance.

On the other hand, Guiso, Pistaferri, and Schivardi (2005) document that such an income insurance exists but does not concern permanent shocks. Consistently, Lettau and Ludvigson (2014) document that macroeconomic variables are subject to both permanent and transitory shocks and Lettau and Ludvigson (2005) document that wages and dividends are co-integrated: i.e. they face the same permanent shocks. I extend this result and provide evidence that all quantities in Figure 1 are co-integrated (see the Online Appendix A).

Therefore, beyond the cyclicality effect of income insurance, also a term-structure effect takes place. Given co-integration, the mechanism of income insurance only concerns transitory shocks. Thus, after a negative transitory shock, the fraction of total resources devoted to workers’ remuneration is partially insured by a decrease of the resources devoted to shareholders’ remuneration. Such an insurance mechanism implies that the labor- and dividend-shares are respectively decreasing and increasing in the transitory component of output. As a consequence, wages and dividends load on transitory risk respectively less and more than output. This produces VR’s of wages and dividends which are respectively increasing and decreasing and respectively above and below the VR’s of output. Such a term-structure effect of income insurance coincides with the empirical VR’s in the upper panels of Figure 1.

The lower panels of Figure 1 provide in an intuitive way a strong empirical support to the term-structure effect of income insurance and its quantitative magnitude. Indeed, the VR’s of the remainder of output minus wages recover the negative slope of the term-structures of both EBIT and dividends. This means that the bulk of the distance between the flat or slightly upward-sloping term-structure of output and the downward-sloping term-structure of dividends is due to wages. Instead, alternative channels such as investment (suggested by Ai, Croce, Diercks, and Li (2012) and Kogan and Papanikolaou (2015)) or financial leverage (suggested by Belo, Collin-Dufresne, and Goldstein (2014)) have a little or negligible impact on the timing of aggregate dividend risk.

In the following, I provide more formal empirical support to the cyclicality and term-structure effects of income insurance.
The Cyclicality Effect of Income Insurance

Accordingly with the above reasoning, the cyclicality effect of income insurance should imply that: i) changes in the labor-share are negatively correlated with changes in output; and ii) variation in the labor-share negatively drives variation in the dividend-share.

Consistently with a number of previous works, I find empirical support for both those stylized facts. In the aggregate economy, the labor-share is counter-cyclical –i.e. the correlation of changes in the labor-share and changes in log GDP is about -.26– and moves negatively with the dividend-share –i.e. the correlation between the labor- and dividend-shares is about -.67. Similarly, the corresponding correlations in the non-financial corporate sector are about -.25 and -.65. These results resemble the empirical findings of Boldrin and Horvath (1995), Shimer (2005) and Ríos-Rull and Santaulàlia-Llopis (2010). Moreover, I provide evidence that the labor-share Granger causes the dividend-share, whereas investment and financing decisions do not (see the Online Appendix A).

The Term-Structure Effect of Income Insurance

If income insurance only concerns transitory shocks, wages and dividends load respectively less and more than output on transitory risk. Given co-integration, this leads to a term-structure effect. Namely, after a negative (positive) transitory shock, workers’ remuneration increases (decreases) relative to shareholders’ remuneration, and the VR’s of wages and dividends respectively increase and decrease (decrease and increase) relative to those of output.

To test this dynamic relation, I build time-series of VR’s of wages, dividends and output and, then, I regress the VR’s of wages, the VR’s of dividends and their difference on the logarithm of the wage-to-dividends ratio:

\[
\begin{align*}
    \text{VR}_W|_{t,\tau} &= a + b_w \log \frac{W}{D_t} + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t, \\
    \text{VR}_D|_{t,\tau} &= a + b_d \log \frac{W}{D_t} + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t, \\
    \text{VR}_W - \text{VR}_D|_{t,\tau} &= a + b_{wd} \log \frac{W}{D_t} + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t,
\end{align*}
\]

for several horizons \(\tau\) ranging from 2 to 7 years. I include the VR’s of output as a control to account for the variation in the VR’s due to permanent shocks. Accordingly with the above reasoning, coefficients \(b_w\) and \(b_{wd}\) should be positive and the coefficient \(b_d\) should be negative. I perform these regressions using data from both the aggregate economy and the non-financial corporate sector. Estimation results are reported in Table 1.

Sign and significance of coefficients strongly support the term-structure effect of income insurance. When resources devoted to workers’ remuneration are high (low) relative to shareholders’ remuneration, wages load less (more) on transitory risk and their VR’s increase (de-
Table 1: The Term-Structure Effect of Income Insurance

The table reports the estimates of the regressions:

\[
\text{VR}_W - \text{VR}_D|_{t,\tau} = a + b_w \log \frac{W}{D}_t + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t, \\
\text{VR}_W|_{t,\tau} = a + b_w \log \frac{W}{D}_t + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t, \\
\text{VR}_D|_{t,\tau} = a + b_d \log \frac{W}{D}_t + b_y \text{VR}_Y|_{t,\tau} + \epsilon_t,
\]

where the dependent variables are either the variance ratios of wages, those of dividends or their difference computed at time \(t\) with horizon \(\tau\) ranging from 2 to 7 years; the independent variables are the time \(t\) logarithm of the wage-to-dividend ratios and the time \(t\) variance ratios of output with horizon \(\tau\). Panel A considers the aggregate economy: wages, dividends and GDP are from NIPA tables on the sample 1947:1-2013:3 at quarterly frequency. Panel B considers the non-financial corporate sector: wages, dividends and value added are from the Flow of Funds on the sample 1951:4-2015:1 at quarterly frequency. The time-series of variance ratios are computed through a rolling window of 10 years centered at \(t\). The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

### Panel A – Aggregate Economy

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<tr>
<td>(b_{wd})</td>
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<td>0.89***</td>
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### Panel B – Non-Financial Corporate Sector

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<td>(b_{wd})</td>
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<td>3.42***</td>
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<td>(5.14)</td>
<td>(6.28)</td>
<td>(7.10)</td>
</tr>
<tr>
<td>adj-R(^2)</td>
<td>0.02</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
<td>0.20</td>
<td>0.31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_w)</td>
<td>0.44***</td>
<td>1.01***</td>
<td>1.08***</td>
<td>0.91***</td>
<td>0.77***</td>
<td>0.63***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(6.95)</td>
<td>(9.98)</td>
<td>(9.27)</td>
<td>(7.43)</td>
<td>(6.64)</td>
<td>(5.83)</td>
</tr>
<tr>
<td>adj-R(^2)</td>
<td>0.76</td>
<td>0.79</td>
<td>0.77</td>
<td>0.69</td>
<td>0.69</td>
<td>0.71</td>
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</table>

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_d)</td>
<td>-1.15</td>
<td>-2.41***</td>
<td>-3.34***</td>
<td>-4.39***</td>
<td>-5.48***</td>
<td>-7.10***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.60)</td>
<td>(-2.77)</td>
<td>(-3.31)</td>
<td>(-4.09)</td>
<td>(-5.18)</td>
<td>(-6.15)</td>
</tr>
<tr>
<td>adj-R(^2)</td>
<td>0.02</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
<td>0.21</td>
<td>0.33</td>
</tr>
</tbody>
</table>
crease). Indeed, the coefficient $b_w$ is positive and highly significant at almost any considered horizon $\tau$. The opposite holds for dividends: the relation between their VR’s and the wage-to-dividends ratio is negative and the coefficient $b_d$ is highly significant. In turn, a positive and highly significant relation obtains between the wage-to-dividends ratio and the distance between the VR’s of wages and dividends (i.e. $b_{wd} > 0$). See the Online Appendix A for the robustness checks: sign and significance of the coefficients are preserved when controlling for time trend as well as financing and investment decisions.

The above results strongly support the idea that income insurance within the firm is the main driver of the timing of dividend risk.

### Income Insurance and Dividend Growth Predictability

The previous analysis documents how income insurance determines the term structure of dividend risk. Now I look at additional testable implications. As long as income insurance concerns transitory risk only, the wage-to-dividends ratios should positively forecast dividend growth because both are decreasing in the current level of the transitory shock. Therefore, I consider long-horizon predictability regressions of dividend growth on the wage-to-dividends ratio. Estimation results are reported in Table 2. The regressions support the mechanism of income insurance: the coefficients are positive and highly significant at all the considered horizons. The explanatory power is large: the adjusted $R^2$ ranges from about 20% to 60%.

The Online Appendix A shows the same regressions when the financial leverage ratio and the investment-share are used as controls for the financing and investment decisions. The sign and the significance of the coefficients for the wage-to-dividends ratio are preserved. Moreover, the adjusted $R^2$ are very close to those of the univariate regressions. These results provide further robustness to the mechanism of income insurance, which is sizeable and leads to dividend growth predictability.

### Table 2: Long-Horizon Predictability of Dividend Growth.

The table reports the coefficient, Newey-West t-statistics and adjusted-$R^2$ of the regressions of cumulative dividend growth rates over the horizons of 3, 5, 10 and 15 years on the current logarithm of the wages-to-dividends ratio. Panel A and B consider respectively the aggregate economy on the sample 1929-2012 and the non-financial corporate sector on the sample 1946-2013 at yearly frequency.

<table>
<thead>
<tr>
<th>Panel A – Aggregate Economy</th>
<th>Panel B – Non-Financial Corporate Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon (years)</td>
<td>3 5 10 15</td>
</tr>
<tr>
<td>coefficient</td>
<td>0.419*** 0.470*** 0.672*** 0.860***</td>
</tr>
<tr>
<td>t-stat</td>
<td>2.80 3.62 8.02 8.91</td>
</tr>
<tr>
<td>adj-$R^2$</td>
<td>0.32 0.42 0.56 0.62</td>
</tr>
</tbody>
</table>
2 Model

2.1 Economy

A representative firm produces an operational cash-flows, which can be interpreted as the total output minus investments: $C = Y - I$. Such an operational cash-flows represents the total resources shared by workers and shareholders: the former receive wages ($W$) and the latter receive dividends ($D$). The resource constraint requires $C = W + D$. To keep the model simple, I assume limited market participation such that workers do not access the financial markets and consume their wages. Consequently, shareholders act as a representative agent on the stock market and consume dividends.\(^8\)

Agents feature recursive preferences in spirit of Kreps and Porteus (1979), Epstein and Zin (1989) and Weil (1989). For the sake of tractability, I assume their continuous time counterpart, as in Duffie and Epstein (1992). These preferences allow for the separation between the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion (RRA). Given a consumption process $C$, the utility at each time $t$ is defined as

$$ J_t = E_t \int_{u \geq t} f(C_u, J_u) du, \quad \text{with} \quad f(C, J) = \beta \chi J \left( \frac{C^{1-1/\psi}}{(1-\gamma J)^{1/\chi}} - 1 \right), \quad (1) $$

where $\chi = \frac{1-\gamma}{1-1/\psi}$, $\gamma$ is RRA, $\psi$ is EIS and $\beta$ is the time-discount rate. Power utility obtains for $\psi \rightarrow 1/\gamma$. Workers’ and shareholders’ risk aversion are denoted by $\gamma_w$ and $\gamma_s$ with $\gamma_w > \gamma_s$ as in Danthine and Donaldson (1992, 2002). Their consumption levels are denoted by $C_{w,t}$ and $C_{s,t}$ respectively.

Total resources are interpreted as a technological shock and their dynamics are modelled as the product of a permanent and a transitory shock. The former features time-variation in expected growth, in spirit of long-run risk literature, and induces an upward-sloping effect on the term-structure of risk. The latter is usually considered in the real business cycle literature and induces a downward-sloping effect. Then, the two shocks jointly lead to flexibility at

---

\(^8\)The endogenous determination of market participants goes beyond the scope of this paper. The assumption of limited market participation allows for tractability and for comparability with most of endowment economy equilibrium models. Recently, Berk and Walden (2013) show that labor markets provide risk-sharing to workers, such that their consumption endogenously equals their wages and limited market participation obtains.
modelling the timing of risk. Namely, total resources $C = XZ$ have dynamics given by:

$$d \log C_t = dx_t + dz_t,$$

$$d \log X_t = dx_t = (\mu_t - \sigma^2 x_t/2) dt + \sigma_x dB_{x,t},$$

$$d \mu_t = \lambda_\mu (\bar{\mu} - \mu_t) dt + \sigma_\mu dB_{\mu,t},$$

$$d \log Z_t = dz_t = \lambda_z (\bar{z} - z_t) dt + \sigma_z dB_{z,t},$$

Those processes feature homoscedasticity and independent Brownian shocks for the sake of exposition and tractability.

2.2 Income Insurance, Wages and Dividends

The standard Walrasian Cobb-Douglas economy leads to a constant labor share and workers paid at their marginal productivity: $W_t = \alpha C_t$, with $\alpha \in (0, 1)$. The opposite extreme scenario would be the case where workers are promised perfect income smoothing, i.e. wages are constant (and potentially equal to the unconditional expected marginal productivity).

Here, I postulate that workers and shareholders agree in advance on a risk sharing rule. A contract $C$ is designed to effect risk sharing between workers and shareholders. The two agent types arrange an agreement such that wages and dividends share the same long-run dynamics of total resources (i.e. $C_t, W_t$ and $D_t$ are co-integrated) but the former feature an insurance to transitory shocks, implying a levered exposition of the latter. Namely, the contingent wage payments and dividend distributions are governed by the following sharing rule:

$$C_t = \left\{ (W_t, D_t) : W_t = X_t \tilde{W}_t(\phi), \quad D_t = X_t \tilde{D}_t(\phi), \quad W_t + D_t = C_t \quad \forall t \right\},$$

where

$$\tilde{W}_t(\phi) = (\alpha Z_t)^{1-\phi} \quad \text{and} \quad \tilde{D}_t(\phi) = Z_t - (\alpha Z_t)^{1-\phi},$$

$\alpha \in (0, 1)$ and $\phi \in (0, 1)$ denotes the degree of income insurance. Hence, under limited market participation in equilibrium,

$$C_{w,t} = X_t \tilde{W}_t(\phi), \quad \text{and} \quad C_{s,t} = X_t \tilde{D}_t(\phi).$$

The degree of income insurance $\phi$ represents the smoothing effect of labor relations on wages and the corresponding leverage effect on dividends. Namely, wages and dividends are respectively a concave and convex function of $Z_t$ for $\phi > 0$. Instead, wages reduce to the Walrasian
case with constant labor-share $\alpha$ for $\phi \to 0$.  

In spirit of Danthine and Donaldson (1992, 2002), income insurance $\phi$ can be interpreted as the outcome of bargaining negotiations between workers and shareholders due to heterogeneity in risk attitudes. I assume that the degree of income insurance from shareholders to workers satisfies the following objective:

$$
\phi = \arg \max_{\phi' \in (0,1)} \mathcal{J}(\phi') = \arg \max_{\phi' \in (0,1)} \left( \mathbb{E}_z[\tilde{W}(\phi')] \right)^{\theta} \left( \mathbb{E}_z[\tilde{D}(\phi')] \right)^{1-\theta}, \quad (9)
$$

Although ad-hoc, the objective $\mathcal{J}(\phi)$ is intended to reflect the relative bargaining power of the two parties and, jointly with the contract rule of Eq. (6), determines simultaneously both their consumption shares. Namely, Eq. (9) represents the aggregate outcome of bargaining in labor negotiations—both individual ones and those accomplished by unions. The bargaining parameter $\vartheta$ is set:

$$
\vartheta = \frac{\alpha \gamma_w}{\alpha \gamma_w + (1-\alpha) \gamma_s} = \frac{\alpha h}{1 + \alpha (h-1)}, \quad (10)
$$

and is determined by both heterogeneity in risk aversion, $h = \gamma_w/\gamma_s$, and the consumption shares in the Walrasian benchmark, $\alpha$—which obtains for $h = 1$. Namely, $\vartheta$ is increasing in $h$: the larger $h$, the larger the insurance component of wages upon the labor productivity level $\alpha C_t$, as in Danthine and Donaldson (2002). The degree of income insurance is increasing in the heterogeneity in risk aversion, $h$, and in the Walrasian benchmark, $\alpha$:

$$
\partial_h \phi > 0, \quad \partial_\alpha \phi > 0, \quad \text{if } \gamma_w > \gamma_s.
$$

Consequently, income insurance leads to smooth wages and risky dividends with the additional feature of counter-cyclical labor-share:

$$
\omega(z_t) = W_t/C_t = \alpha (\alpha e^{z_t})^{-\vartheta}. \quad (11)
$$

9The degree of income insurance is constrained in the interval $(0,1)$ because $\phi < 0$ implies an income insurance from workers to shareholders and a pro-cyclical labor-share, whereas $\phi > 1$ implies wages decreasing in the aggregate shock $Z_t$. Thus $\phi \notin (0,1)$ is implausible.

10Assuming workers and shareholders negotiate directly with each other, a proper formulation of a Nash bargaining solution requires

$$
\phi_t = \arg \max_{\phi \in (0,1)} \left( U(C_{w,t}) \right)^{\tilde{\vartheta}} \left( V(C_{s,t}) \right)^{1-\tilde{\vartheta}}
$$

where $\tilde{\vartheta} \in (0,1)$ is a bargaining power parameter and heterogeneity in risk aversion induces the incentive to bargain, given an outside option standardized to zero. Unfortunately, such a solution is neither tractable nor stationary. Instead, the objective in Eq. (9) guarantees tractability, preserves the economic mechanism by Danthine and Donaldson (2002) and gives rise to a term-structure effect since it does not rule out co-integration. Lifetime utilities are replaced by expectations (over the stationary distribution of $z_t$) of the components of workers’ and shareholders’ consumptions affected by income insurance (i.e. $\tilde{W}(\phi)$ and $\tilde{D}(\phi)$) and preference heterogeneity is embedded in the parameter $\vartheta$ which, with a slight abuse of terminology, can be interpreted as the bargaining power of workers.
The labor-share equals the Walrasian benchmark $\alpha$ times an adjustment factor decreasing and convex in $z_t$. The latter captures the effect of labor relations and reduces to one if $\phi \to 0$. Also a term-structure effect takes place: the endogenous co-integration among $C$, $W$ and $D$ implies that distributional risk due to income insurance is a short-term risk and provides a rationale for downward sloping term-structures of both dividends and equity.

This stylized model of income insurance is intended to capture the key effects of labor rigidity on the dividend dynamics. The smoothing effect on wages due to income insurance can obtain also in an indirect way. For instance, infrequent wage resettling or search frictions can induce a similar operating leverage on dividends. This paper empirically supports and explicitly models income insurance since it represents the general and intuitive economic mechanism which explains the dynamics of dividends, despite the exact friction or combination of frictions in labor markets. As it will be shown, the resulting dividends dynamics reconcile the traditional asset pricing facts with the recent evidence about the term-structures.

Finally, the model is kept simple and parsimonious such that the effect of income insurance on asset prices is fully captured by a unique endogenous parameter, $\phi$. If such a parameter is interpreted as exogenous, all equilibrium asset pricing implications characterized in the paper continue to hold even if alternative or complementary explanations for the dividend dynamics are taken into consideration –such as financial leverage (Belo et al. (2014)) and heterogeneous investment risk (Ai et al. (2012)).

Dividends and wages evolve endogenously at equilibrium. Denote with

$$d \log I = \mu^T dt + \sigma^T_x dB_x + \sigma^T_z dB_z, \quad I = \{C, W, D\}$$

the instantaneous dynamics of total consumption, wages and dividends. Provided $\phi > 0$, a number of results is worth noting. First, long-run growth affects in the same way the drift ($\partial_\mu \mu^W = \partial_\mu \mu^C = \partial_\mu \mu^D = 1$) and volatility ($\sigma^W_x = \sigma^C_x = \sigma^D_x = \sigma_x$) of total consumption, wages and dividends because labor relations do not alter the impact of permanent shocks. Second, expected consumption growth depends on $z_t$: $|\partial_\mu \mu^C| = \lambda_z$. However, the persistence of expected growth of wages and dividends is altered by income insurance: the larger $\phi$, the stronger the persistence of wages (i.e. $|\partial_\mu \mu^W| < \lambda_z$) and the weaker that of dividends (i.e. $|\partial_\mu \mu^D| > \lambda_z$). Third, the volatility of wages and dividends due to transitory shocks is affected by income insurance. Namely, $\sigma^W_x = (1 - \phi)\sigma_x$ is constant, lower than $\sigma^C_x$ and decreasing with $\phi$. Therefore, income insurance induces a smoothing effect on wages at the cost of more

---

$^{11}$Notice that $\omega(z_t)$ is not bounded from above in $(0, 1)$. However, for plausible parameters, the probability that $\omega(z_t) > 1$ is negligible (namely, $P(\omega(z_t) > 1) \approx 1 \times 10^{-43}$ in the baseline calibration of section 3).

volatile dividends. Indeed, $\sigma_z^D$ is larger than $\sigma_z^C$ and endogenously heteroscedastic:

$$\sigma_z = \sigma_z^C < \sigma_z^D = \left(1 + \phi \frac{\omega(z_t)}{1-\omega(z_t)}\right) \sigma_z.$$  

(12)

Fourth, such an excess volatility or leverage, $\sigma_z^D - \sigma_z$, is proportional and increasing with both $\phi$ and the wages-to-dividends ratio: $\omega(z_t)/(1 - \omega(z_t))$; increasing with workers’ bargaining power; and endogenously counter-cyclical (i.e. $\partial z \sigma_z^D < 0$).

The focus now turns on the term structure of growth rates’ volatility. Denote with $C_t(\tau, u) = E_t[C_{\tau+\tau}^u]$, $W_t(\tau, u) = E_t[W_{\tau+\tau}^u]$ and $D_t(\tau, u) = E_t[D_{\tau+\tau}^u]$ the moment generating functions of the logarithm of total consumption, wages and dividends. With this result in hand, the term structure of dividends growth rates’ volatility is computed as in Belo et al. (2014):

$$\sigma_{\tau}^2 = \frac{1}{\tau} \log \left( \frac{D_{\tau+\tau}^u}{D_{\tau+1}^u} \right).$$  

(13)

Note that $\mu_t$ and $z_t$ lead respectively to an upward-sloping ($\partial \mu_t, \tau \sigma_D(t, \tau) > 0$) and a downward-sloping effect ($\partial z_t, \sigma_D(t, \tau) < 0$). Interestingly, the stronger income insurance, the larger the volatility level ($\partial \phi, \tau \sigma_D(t, \tau) > 0$) and the more pronounced the downward-sloping effect: $\partial \phi, \tau \sigma_D(t, \tau) < 0$. Therefore, income insurance enhances the strength of the transitory shock and leads to an excess of short-run risk in dividends distributions with respect to total consumption. The opposite holds for wages: $\partial \phi, \tau \sigma_W(t, \tau) < 0$ and $\partial \phi, \tau \sigma_W(t, \tau) > 0$. Such an effect is the model counterpart of the empirical distance between the term-structures of output and those of dividend risk and wage risk, documented in Figure 1.

While the instantaneous volatility of dividends is increasing with the wage-to-dividends ratio as in Eq. (12), the larger the horizon, the lower the sensitivity of dividend volatility because $z_t$ is a transitory shock. Thus, the variance ratio of dividends,

$$VR_D(t, \tau) = \frac{\sigma_{\tau}^2(t, \tau)}{\sigma_{\tau}^2(t, 1)},$$

is decreasing in the wage-to-dividends ratio, as documented in Table 1. In turn, the distance between the variance ratios of wage and dividends –i.e. the term-structure effect of income insurance– is increasing in the wage-to-dividends ratio, as in the actual data.

Similarly, a number of other interesting quantities can be computed in closed form. Denote with $\sigma_\omega(t, \tau)$ and $\sigma_{1-\omega}(t, \tau)$ the volatilities of the labor-share and dividend-share and with

13This mechanism is similar to that of an economy with preference heterogeneity but without limited market participation: in good times wealth shifts from high risk averse agents towards low risk averse agents. Labor relations lead to a similar counter-cyclical dynamics through the endogenous dynamics of dividends instead of the aggregate risk aversion.
\( \rho_{C,\omega}(t, \tau) \) and \( \rho_{C,(1-\omega)}(t, \tau) \) their correlations with total consumption. For \( \phi > 0 \), we obtain:

\[
0 < \sigma_\omega(t, \tau) < \sigma_{1-\omega}(t, \tau), \\
\rho_{C,\omega}(t, \tau) < 0 < \rho_{C,(1-\omega)}(t, \tau),
\]

such that the labor-share is smoother than the dividend-share, the former is counter-cyclical and the latter is pro-cyclical. These results are due to income insurance, but are also an important feature of the data, as shown in Section 1.

For the sake of exposition and to derive analytical results for asset prices, in the following I make use of a log-linearized dynamics of dividends:

\[
\log D_t \approx \log X_t + \log \bar{D} + \partial_z \log D \bigg|_{z=\bar{z}} (z_t - \bar{z}) = x_t + d_0 + d_z z_t, \tag{14}
\]

where \( \log \bar{D} = d_0 + d_z \bar{z} = \log \left( e^{\bar{z}} - \alpha^1 - \phi e^{(1-\phi)\bar{z}} \right) \) captures the steady-state of \( \bar{D}(\phi) \), and \( d_z \) satisfies:

\[
d_z = 1 + \frac{\phi \alpha}{\alpha e^{\bar{z}} - \alpha}. \tag{15}
\]

Notice that, as usual, the dividend process is still increasing in the states \( x_t \) and \( z_t \) but inherits their homoscedasticity.

### 2.3 Asset Prices

**Equilibrium State-Price Density**

Preferences in Eq. (1) and the log-linearized dividend dynamics of Eq. (14) guarantee a model solution which emphasizes the role of the state-variables \( \mu_t \) and \( z_t \) in the formation of prices. A first order approximation of the shareholders’ consumption-wealth ratio around its endogenous steady state, \( e^{\sigma_t} \), provides closed form solutions for prices and return moments up to such approximation (Benzoni, Collin-Dufresne, and Goldstein (2011)).

From the shareholders’ perspective, the marginal utility evaluated at optimal consumption is the valid state-price density (Duffie and Epstein (1992)):

\[
\xi_{t,u} = e^{\int_t^u f_V(C_{s,\tau},V(C_{s,\tau}))d\tau} \frac{f_C(C_{s,u},V(C_{s,u}))}{f_C(C_{s,t},V(C_{s,t}))}, \quad \forall u \geq t. \tag{16}
\]

Hence, the price of an arbitrary payoff stream \( \{F_u\}_{u \geq t} \) is given by \( \mathbb{E}_t[\int_t^\infty \xi_{t,u} F_u du] \). The equilibrium state price density \( \xi_{0,t} \) satisfies:

\[
C_t = W_t + D_t = I_C[\xi_{0,t}] e^{-d_0 - (d_z - 1)z_t}, \tag{17}
\]

where \( I_C[\xi_{0,t}] = \{ C_{s,t} : \xi_{0,t} = e^{\int_0^t f_j(s)ds} f_C(t) \big| X_t, \mu_t, z_t \} \) denotes the time \( t \) shareholders’
optimal consumption implied by $\xi_{0,t}$.

Although the state price density equals the first order condition of shareholders, it depends on the risk attitudes of both agent types. The left hand side of Eq. (17) is given by the exogenous flows of total resources produced by the firm. The right hand side is given by the product of two terms: the former is the optimality condition for market participants; the latter is a time-varying term which captures the distributional risk due to income insurance. Namely, such a term equals the inverse of the dividend share, i.e. $C_t/D_t$, and its variability increases with $\phi$. Instead, in absence of income insurance, the labor-share is constant and the second term on the right hand side of Eq. (17) reduces to $(1 - \alpha)^{-1}$.

Notice that, even if the equilibrium state-price density is an involved function of the integrated process $C_t$, the economy is characterized by a stationary equilibrium: this is a necessary condition to produce realistic testable implications but usually fails to hold in models of preference heterogeneity. Many real business cycle models circumvent the problem by excluding permanent shocks (e.g. $C_t = Z_t$). Here, allowing for both permanent and transitory shocks (e.g. $C_t = X_tZ_t$) and still obtaining a stationary equilibrium is of crucial importance in order to study the equilibrium implications for the term-structure of equity.

The equilibrium state price density has dynamics given by

$$\frac{d\xi_{0,t}}{\xi_{0,t}} = -r(t)dt - \theta_x(t)dB_{x,t} - \theta_\mu(t)dB_{\mu,t} - \theta_z(t)dB_{z,t},$$

(18)

where the instantaneous risk-free rate satisfies

$$r(t) = r_0 + r_\mu \mu_t + r_z z_t,$$

(19)

with $r_\mu = \frac{1}{\psi}$, $r_z = -\frac{\lambda_z}{\psi} d_z$ and the instantaneous prices of risk are given by

$$\theta_x(t) = -\frac{\partial f_{C}}{f_{C}} X_t \sigma_x = \gamma_s \sigma_x,$$

(20)

$$\theta_\mu(t) = -\frac{\partial f_{\mu}}{f_{\mu}} \sigma_\mu = \frac{\gamma_s - 1/\psi}{\psi e^{\gamma q + \lambda_\mu}} \sigma_\mu,$$

(21)

$$\theta_z(t) = -\frac{\partial f_{z}}{f_{z}} \sigma_z = \left(\gamma_s - \frac{\lambda_z (\gamma_s - 1/\psi)}{\psi e^{\gamma q + \lambda_z}}\right) d_z \sigma_z.$$

(22)

The risk-free rate is affine in $\mu_t$ and $z_t$. The coefficients $r_\mu$ and $r_z$ are respectively positive and negative and both decrease in magnitude with $\psi$, as usual under recursive preferences. Moreover, $r_z$ increases in magnitude with the degree of income insurance $\phi$.

The permanent shock commands a price of risk $\theta_x(t)$ due to the contribution of $x_t$ to the instantaneous volatility of shareholders’ consumption. Long-run growth commands a price of

\footnote{Under CRRA preferences ($\psi \to 1/\gamma_s$), the optimality condition for the shareholders takes the usual power form: $I_C[\xi_{0,t}] = (\xi_{0,t} e^{-\beta t})^{-1/\gamma_s}$ and the term capturing the distributional risk is unchanged. Instead, for $\phi \to 0$, the state-price density reduces to $\xi_{0,t} = e^{-\beta t (1 - \alpha) C_t}^{-\gamma_s}$, as in a Lucas economy.}
risk \theta_\mu(t), which has the usual form in long-run risk models. This is a price for the contribution of \mu_t to the variation in the continuation utility value of shareholders. Hence, \theta_\mu(t) disappears under power utility (\psi \rightarrow \gamma_s^{-1}). Such a price of risk increases with the preferences for the early resolution of uncertainty \gamma_s - 1/\psi and decreases in magnitude with the rate of reversion \lambda_\mu. The transitory shock leads to a price of risk \theta_z(t), which has two components. The first, \gamma_s d_z \sigma_z, is a positive price for the contribution of z_t to the instantaneous volatility of shareholders’ consumption. The second, \theta_z(t) = \gamma_s d_z \sigma_z, is a price for the contribution of z_t to the variation in the continuation utility value of shareholders. Namely, at the opposite than \theta_\mu, this term is negative and increasing in the rate of reversion \lambda_z under preferences for the early resolution of uncertainty. Such a term disappears under power utility. Interestingly, both the components of \theta_z(t) are proportional to the coefficient \psi > 1, which is increasing in \phi. Therefore, for \gamma_s > \psi > 1, income insurance leads to a positive price for its effect on the current dividend volatility and a negative price for its effect on the evolution of shareholders’ utility.

**Equilibrium Dividend Strips**

The equilibrium price of the market dividend strip with maturity \tau is exponential affine in x_t, \mu_t and z_t:

\[ P_{t,\tau} = \mathbb{E}_t [\xi_{t,t+\tau}D_{t+\tau}] = X_t e^{A_0(\tau) + A_\mu(\tau) \mu_t + A_z(\tau) z_t}, \quad (23) \]

Hence, the strip’s price-dividend ratio is stationary. The functions \( A_\mu(\tau) \) and \( A_z(\tau) \) are respectively the semi-elasticity of the price with respect to \mu_t and z_t:

\[ A_\mu(\tau) = \partial_\mu \log P_{t,\tau} = \frac{(1-e^{-\lambda_\mu \tau})(1-1/\psi)}{\lambda_\mu}, \quad (24) \]

\[ A_z(\tau) = \partial_z \log P_{t,\tau} = \left( \frac{1}{\psi} (1 - e^{-\lambda_z \tau}) + e^{-\lambda_z \tau} \right) \left( 1 + \frac{\phi_0}{\alpha e^{\psi \tau} - \alpha} \right). \quad (25) \]

First, \( A_\mu(\tau) \) increases with \psi, whereas \( A_z(\tau) \) decreases with \psi. Second, \( A_z(\tau) \) increases with the degree of income insurance \phi. Therefore, the leverage effect on dividends due to income insurance also affects prices: namely, such an effect is amplified for \psi < 1 and vice-versa. Finally, for \psi \rightarrow 1, the strip’s price-dividend ratio reduces to a state-independent function of the horizon \tau. The instantaneous volatility and premium on the dividend strip with maturity \tau are given by

\[ \sigma_\mu(t,\tau) = \sqrt{\sigma_z^2 + \frac{(1-e^{-\lambda_\mu \tau})^2(\psi-1)^2}{\lambda_\mu^2 \psi^2} \sigma_\mu^2 + \frac{e^{-2 \lambda_z \tau (\psi_{\mu} + \lambda_z \tau - 1)^2}}{\psi^2} d_z^2 \sigma_z^2}, \quad (26) \]

\[ (\mu_v - \gamma)(t,\tau) = \gamma_\mu \sigma_z^2 - \frac{(1-e^{-\lambda_\mu \tau})(1-1/\psi)(1-\psi_{\mu} \psi_{\mu}^2)}{\lambda_\mu (e^{\psi_{\mu} + \lambda_\mu})} \sigma_\mu^2 + \left( \frac{1}{\psi} (1 - e^{-\lambda_z \tau}) + e^{-\lambda_z \tau} \right)^2 \left( \lambda_\mu + \psi_{\mu} \gamma e^{\psi_{\mu}} \right) d_z^2 \sigma_z^2. \quad (27) \]
The three shocks of the model contribute to the return volatility and command a premium. The permanent shock does not lead to excess volatility. Instead, the loadings on $B_{\mu,t}$ and $B_{z,t}$, are proportional to $\sigma_\mu$ and $\sigma_z$, but also depend on the horizon $\tau$, the elasticity of intertemporal substitution and the persistence of the states. Namely, the loading on long-run growth is increasing with $\psi$ and decreasing with $\lambda_\mu$. Instead the loading on the transitory shock is decreasing with $\psi$ and increasing with $\lambda_z$. The latter is also amplified by the leverage effect due to income insurance, $d_z$.

The premium on the dividend strip is given by the sum of the compensations to the three shocks. The compensation for the permanent shock is positive and has the usual form: $\gamma_s \sigma^2_\mu$. Instead, the compensations associated to the states $\mu_t$ and $z_t$ depend also on the horizon $\tau$, the elasticity of intertemporal substitution and the persistence of the states. Long-run growth commands a premium which is increasing with $\psi$ and decreasing with $\lambda_\mu$:

$$\frac{\gamma_s \sigma^2_\mu}{\lambda_\mu (e^{\psi(\gamma_s - 1)} - 1)} A_\mu(\tau) \sigma^2_\mu.$$ (28)

Such a term is a compensation for the contribution of $\mu_t$ to the variability of the continuation utility value of shareholders. If the intertemporal substitution effect dominates the wealth effect and shareholders have preferences for the early resolution of uncertainty - e.g. $\gamma_s > \psi > 1$ - long-run growth leads to a positive premium on the dividend strip. The premium for the exposition to the transitory shock $z_t$ is given by the sum of two terms:

$$\gamma_s A_z(\tau) d_z \sigma^2_z, \quad \text{and} \quad - \frac{\lambda_z (\gamma_s - 1)}{\lambda_z} A_z(\tau) d_z \sigma^2_z.$$ (29)

The former term compensates for the contribution of $z_t$ to the instantaneous volatility of shareholders’ consumption. Such a premium is always positive and decreases with $\psi$. Instead, the latter term is an intertemporal compensation. It is decreasing with $\psi$ and is negative (positive) under the shareholders’ preference for the early (late) resolution of uncertainty. Moreover, both terms depend on the horizon $\tau$ and increase in magnitude with income insurance, $d_z$.

The following is a key result of the paper. The slopes of the term-structures of dividend strips’ volatility and premia are given by

$$\partial_{\tau} \sigma^2_P(t, \tau) = \frac{2(\psi - 1)}{\psi^2} \left( e^{-2\lambda_\mu\tau} (\psi - 1) \lambda_\mu^{-1} \sigma^2_\mu - e^{-2\lambda_z\tau} (\psi + e^{-\lambda_\mu\tau} - 1) \lambda_z d^2_z \sigma^2_z \right),$$ (30)

$$\partial_{\tau} (\mu - \tau)(t, \tau) = \frac{(\gamma_s - 1)}{\psi^2} \left( e^{-\lambda_\mu\tau} \sigma^2_\mu + \lambda_z e^{-\lambda_z\tau} \frac{d^2_z \sigma^2_z}{e^{\psi\tau}} \right).$$ (31)

The slope of the term-structure of volatility depends on two terms, due respectively to $\mu_t$ and $z_t$. The former is always positive and, hence, implies an upward sloping effect. Instead,
the latter term is negative if the intertemporal substitution effect dominates the wealth effect and vice-versa. Therefore, the term-structure of volatility is monotone upward sloping if $\psi < 1$, whereas it is not necessarily monotone if $\psi > 1$. A non-monotone (e.g. U-shaped) term-structure of risk obtains if the leverage effect due to income insurance, $d_z$, outweighs the upward sloping effect due to long-run growth for some horizons $\tau$.

Also the slope of the term-structure of premia depends on two terms, due to $\mu_t$ and $z_t$. The former is positive if the intertemporal substitution effect dominates the wealth effect ($\psi > 1$) and shareholders have preferences for the early resolution of uncertainty ($\gamma_s > 1/\psi$). The latter term is negative if the intertemporal substitution effect dominates the wealth effect ($\psi > 1$) and vice-versa. Thus, the slope of equity premia can be economically interpreted as:

$$
\text{slope} = \left( \frac{\text{intertemporal substitution}}{\psi - 1} \right) \times \left( \frac{\text{resolution of uncertainty}}{\gamma_s - 1/\psi} \right) \times \left( \frac{\text{long-run risk}}{\lambda_{\mu, \sigma_{\mu}}} \right) \times \left( \frac{\text{income insurance}}{\phi} \right) \times \left( \frac{\text{short-run risk}}{\lambda_z, \sigma_z} \right).
$$

Under the usual parametrization $\gamma_s > \psi > 1$, variation in long-run growth leads to an upward-sloping effect, whereas transitory shocks lead to a downward-sloping effect. Therefore, the term-structure of equity premia is not necessarily monotone as long as both permanent and transitory shocks enter the model. Namely, the slope is negative for income insurance strong enough.\(^{15}\)

Such an analytical result clearly explains why the standard long-run risk model cannot capture the recent evidence about dividend strips. Indeed, long-run risk models (i.e. $C_t = X_t$) rule out transitory shock $z_t$ and do a good job at matching a number of asset pricing moments as long as $\psi > 1$. Both the term-structures of volatility and premia are monotone increasing. The alternative scenarios, in which either only the transitory shock enters the model ($C_t = Z_t$) or long-run growth is constant ($C_t = X_tZ_t$ with $\mu_t = \bar{\mu}$), lead to monotone decreasing term-structures of risk and premia for $\psi > 1$. However, in such cases the equity premium is not sizeable since it decreases with $\psi$. Instead, when we account for both $\mu_t$ and $z_t$, the model can accommodate for both a high equity premium and downward sloping term-structures of equity risk and premia in the short-run. Therefore, the endogenous dynamics of dividends due to income insurance allows to reconcile the above stylized facts.

**Equilibrium Market Asset**

The market price is given by the time integral of the dividend strip price, $P_t = \int_0^\infty P_{t, \tau} d\tau$:

$$
P_t = \mathbb{E}_t \left[ \int_t^\infty \xi_{t, u} D_u du \right] = X_t/\beta^{-1}e^{u_0\chi^{-1}+d_0+u_\mu\chi^{-1}\mu_t+(u_z\chi^{-1}+d_z)z_t}
$$

\(^{15}\)A similar result obtains measuring the slopes by means of the term-spreads $\sigma_p^2(t, \infty) - \sigma_p^2(t, 0)$ and $(\mu_{P-r})(t, \infty) - (\mu_{P-r})(t, 0)$. Negative spreads obtain if the leverage effect of dividends due to income insurance is large enough.
where \( u_0, u_\mu \) and \( u_z \) are endogenous constants. The market price dividend ratio is a stationary function of \( \mu_t \) and \( z_t \) and equals the wealth-consumption ratio of shareholders. Prices increase with \( \mu_t \) as long as the intertemporal substitution effect dominates the wealth effect and increase with \( z_t \): \( \partial_\mu P_t \geq 0 \) if \( \psi \geq 1 \) and \( \partial_z P_t > 0 \), \( \forall \psi, \gamma_s \).

The instantaneous volatility and premium on the market asset are given by

\[
\sigma_P(t) = \sqrt{\sigma_x^2 + \left(\frac{1-1/\psi}{e^{\psi q} + \lambda_\mu}\right)^2 \sigma_\mu^2 + \left(1 - \frac{1-1/\psi}{e^{\psi q} + \lambda_z}\right)^2 d_z^2 \sigma_z^2},
\]

(33)

\[
(\mu_P - r)(t) = \gamma_s \sigma_x^2 + \frac{(1-1/\psi)(\gamma_s-1/\psi)}{(e^{\psi q} + \lambda_\mu)^2} \sigma_\mu^2 + \frac{\gamma_s \psi e^{\psi q} + \lambda_\mu)(\psi e^{\psi q} + \lambda_z)}{(e^{\psi q} + \lambda_z)^2 \psi^2} d_z^2 \sigma_z^2.
\]

(34)

The permanent shock \( x_t \) enters the return dynamics exactly as the dividend dynamics, \( \sigma_x \). Instead, the loadings on the shocks \( B_{\mu,t} \) and \( B_{z,t} \) depend on the preference parameters and are respectively increasing and decreasing in \( \psi \). The loading on \( B_{\mu,t} \) always leads to an excess-volatility of market returns over dividends, whereas the loading on \( B_{z,t} \) generates excess-volatility for \( \psi < 1 \) and vice-versa.

The equity premium is given by three components due to the three shocks. The permanent shock \( x_t \) requires the usual positive compensation \( \gamma_s \sigma_x^2 \). Long-run growth leads to a premium, which is positive if the intertemporal substitution effect dominates the wealth effect and shareholders have preference for the early resolution of uncertainty. Instead, \( z_t \) commands a premium which is always positive, decreasing with \( \psi \) and increasing with \( \gamma_s \). Furthermore, such a compensation term is increasing in the degree of income insurance.

Finally, provided \( \gamma_s > \psi > 1 \), the whole equity premium is increasing in \( \gamma_s \) but non-monotone in \( \psi \). Indeed, \( \mu_t \) leads to compensations increasing with the horizon, whereas short-run but persistent uncertainty due \( z_t \) leads to compensations decreasing with the horizon. Once the model is calibrated with realistic parameters, such a term-structure perspective sheds lights on which risks are priced in equilibrium. Using the definition of the market asset price as the time integral of the dividend strip prices, it is possible to write the equity premium as a time integral and, hence, to derive an equilibrium density which describes the contribution of future discounted cash-flows at each horizon:

\[
\mathcal{H}(t, \tau) = \frac{\Pi(t, \tau)}{(\mu_P - r)(t)} \quad \text{with} \quad (\mu_P - r)(t) = \int_0^\infty \Pi(t, \tau)d\tau.
\]

(35)

The shape of the equilibrium density \( \mathcal{H}(t, \tau) \) tells us whence the equity premium comes from. The more the mass of probability concentrates on either short or long horizons, the more the riskiness associated to such horizons deserves a compensation and contributes to the whole equity premium. Therefore, \( \mathcal{H}(t, \tau) \) is natural metric to understand the effect of the term-
structures of equity on an important equilibrium outcome, such as the equity premium.\(^{16}\)

**Equilibrium Bond and Equity Yields**

The equilibrium price of the zero-coupon bond with maturity \(\tau\), \(B_{t,\tau} = \mathbb{E}_t[\xi_{t,t+\tau}]\), is stationary and exponential affine in \(\mu_t\) and \(z_t\). Hence, the bond yield is state-dependent but its volatility inherits the homoscedasticity of the states:

\[
\varepsilon(t, \tau) = -\frac{1}{\tau} \log B_{t,\tau} = \frac{1}{\tau} \left( -K_0(\tau) + (1-e^{-\lambda_\mu \tau}) \mu_t \lambda_\mu^{-1} \mu_t + (1-e^{-\lambda_z \tau}) r_z \lambda_z^{-1} z_t \right),
\]

where \(K_0(\tau)\) is a deterministic function of the maturity. The short- and long-run limits of the term-structure of real yields lead to the steady state term-spread:

\[
\varepsilon(t, \infty) - \varepsilon(t, 0) = -\frac{r_\mu \theta_\mu \sigma_\mu}{\lambda_\mu} - \frac{r_z \theta_z \sigma_z}{\lambda_z} - \frac{r_\mu^2 \sigma_\mu^2}{2\lambda_\mu^2} - \frac{r_z^2 \sigma_z^2}{2\lambda_z^2},
\]

which can be either positive or negative for \(\gamma_s > \psi > 1\): indeed, \(r_\mu \theta_\mu > 0\) and \(r_z \theta_z < 0\).\(^{17}\)

Armed with these results, the focus turns on the equity yields as introduced by van Binsbergen et al. (2013): \(\frac{1}{\tau} \log D_t/P_{t,\tau}\). The model equity yield is given by

\[
p(t, \tau) = \frac{1}{\tau} \left( d_0 - A_0(\tau) + \frac{(1-e^{-\lambda_\mu \tau})(1/\psi-1)}{\lambda_\mu} \mu_t + (1-e^{-\lambda_z \tau})(1-1/\psi)d_z z_t \right)
\]

and, hence, is a stationary function of the states and the maturity. Moreover, it can be decomposed as:

\[
p(t, \tau) = \varepsilon(t, \tau) - g_D(t, \tau) + \varrho(t, \tau).
\]

The equity yield is given by the difference among the yield on the risk-less bond, \(\varepsilon(t, \tau)\), and the dividend expected growth, \(g_D(t, \tau) = \log(D_t(\tau, 1)/D_t) / \tau\), plus a premium, \(\varrho(t, \tau)\). The latter is a state-independent function of the maturity. The transitory shock determines the level of its short-run limit, whereas long-run growth determines the level of its long-run limit. Therefore, the term-spread

\[
\varrho(t, \infty) - \varrho(t, 0) = \frac{\gamma_s^e \psi \lambda_\mu + \varepsilon^{eq}}{(\lambda_\mu + \varepsilon^{eq}) \lambda_\mu^2 \psi} \sigma_\mu^2 - \frac{\lambda_z + \gamma_s^e \psi^{eq}}{(\lambda_z + \varepsilon^{eq}) \psi} d_z^2 \sigma_z^2,
\]

can be either positive or negative depending on the model parameters. Thus, downward-sloping premia on equity yields obtain for income insurance large enough.

---

\(^{16}\)Standardizing \(\Pi(t, \tau)\) by \((\mu_P - r)(t)\) allows to compare the timing decomposition of the equity premium among models and parameter settings even if they produce different magnitudes for the premium.

\(^{17}\)A monotone downward-sloping term-structure of real yields obtains in most of long-run risk models. Here, for income insurance large enough, the real yields are upward-sloping consistently with the actual data about TIPs.
3 Asset Pricing Results

3.1 Model Calibration and the Timing of Dividend Risk

Model parameters are set by choosing cash-flows parameters in order to match some moments from the time-series of consumption, wages and dividends growth rates and by choosing preference parameters to provide a good fit of standard asset pricing moments.

The calibration procedure uses information from the term-structures of cash-flows to assess the equilibrium asset pricing implications. Namely, I exploit analytical solutions to set the cash-flows parameters. The model has eight parameters $\Theta = \{\bar{\mu}, \sigma_x, \lambda_\mu, \sigma_\mu, \lambda_z, \sigma_z, \alpha, h\}$, which characterize the joint dynamics of aggregate consumption, wages and dividends ($\bar{z} = 0$ for simplicity). I choose eight moment conditions: the relative error between the empirical and the model long-run growth of consumption ($g_C$), yearly volatility of consumption ($\sigma_C(1)$) and dividends ($\sigma_D(1)$) growth rates, average level of the dividend-share ($1 - \omega$) and its volatility ($\sigma_{1-\omega}$):

$$m_1(\theta) = \frac{|g_C^{\text{empirical}} - g_C^{\text{model}}|}{g_C^{\text{empirical}}},$$
$$m_2(\theta) = \frac{|\sigma_C(1)^{\text{empirical}} - \sigma_C(1)^{\text{model}}|}{\sigma_C(1)^{\text{empirical}}},$$
$$m_3(\theta) = \frac{|\sigma_D(1)^{\text{empirical}} - \sigma_D(1)^{\text{model}}|}{\sigma_D(1)^{\text{empirical}}},$$
$$m_4(\theta) = \frac{|(1-\omega)^{\text{empirical}} - (1-\omega)^{\text{model}}|}{(1-\omega)^{\text{empirical}}},$$
$$m_5(\theta) = \frac{|\sigma_{1-\omega}^{\text{empirical}} - \sigma_{1-\omega}^{\text{model}}|}{\sigma_{1-\omega}^{\text{empirical}}},$$

and three additional conditions that capture the relative error between the empirical and the model term-structures of variance ratios of consumption, wages and dividends over a ten years horizon:

$$m_6(\theta) = \sum_{\tau=2}^{10} \frac{|VR_{C}^{\text{empirical}}(\tau) - VR_{C}(\tau)|}{VR_{C}^{\text{empirical}}(\tau)},$$
$$m_7(\theta) = \sum_{\tau=2}^{10} \frac{|VR_{W}^{\text{empirical}}(\tau) - VR_{W}(\tau)|}{VR_{W}^{\text{empirical}}(\tau)},$$
$$m_8(\theta) = \sum_{\tau=2}^{10} \frac{|VR_{D}^{\text{empirical}}(\tau) - VR_{D}(\tau)|}{VR_{D}^{\text{empirical}}(\tau)}.$$

The variance-ratios of growth rates are computed as $VR_i(\tau) = \frac{\sigma_i^2(\tau)}{\sigma_i^2(1)}$ for $i = \{C, W, D\}$. The latter three moment conditions capture the timing of the macroeconomic risk and, in particular, the term-structure effect of income insurance.
Finally, I obtain the parameter vector $\Theta$ by minimizing the average-relative-error:

$$\Theta = \arg \min_\theta ARE(\Theta) = \arg \min_\theta \frac{1}{8} \sum_{i=1}^{8} m_i(\theta).$$

The empirical moments are as follows: I set the long-run growth rate of consumption to 2% and the volatility of consumption to 2.5%, which are the usual values from the literature; the volatility of dividends is set to 15%, which is the value reported in Belo et al. (2014); the average value of the dividend-share is set to 6% and its volatility to 1.6%, which are computed using the ratio of net dividends over the sum of net dividends and wages from the US non-financial corporate sector (as in the model). These numbers are close to the values considered in Longstaff and Piazzesi (2004), Lettau and Ludvigson (2005) and Santos and Veronesi (2006). The variance-ratios of wages and dividends from the US non-financial corporate sector are computed as in Section 1: wage risk increases from one to about 2 over a 10 years horizon, whereas dividend risk decreases from one to about 0.2. Finally, the variance-ratios of consumption are computed from the growth rates in Beeler and Campbell (2012) and increase from one to about 1.5 over a 10 years horizon.

Table 3 reports the model parameters and Table 4 reports both the empirical and the model-implied moments of cash-flows. The left panel of Figure 2 shows the model implied term-structures of variance-ratios for both aggregate consumption, wages and dividends, as well as their empirical counterparts. The model accurately captures both rise and decline of respectively wage and dividend risk with the horizon. Therefore, the calibration procedure does a good job at recovering the whole shape of the empirical term-structures and, hence, the timing of consumption, wages and dividend risk. The size of such risks is shown in the right panel of Figure 2, which plots the term-structures of the corresponding volatilities. The decline in the timing of dividend risk is due to the levered exposition of dividends to the transitory component of consumption, $z_t$. Namely, a moderate degree of preference heterogeneity ($h = 1.57$) leads to an income insurance parameter $\phi = 0.36$ which implies an operating

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Table 3: Calibration – Model Parameters
Table 4: Calibration – Cash-Flows Moments

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Variance ratios of consumption  \( V_{RC}(τ) \)

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Variance ratios of wages  \( V_{RW}(τ) \)

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Variance ratios of dividends  \( V_{RD}(τ) \)

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Figure 2: Term-structures of consumption, wages and dividends

Left panel: Variance-ratios of wages (blue), consumption (black) and dividends (red) as a function of the horizon. Dashed lines denote empirical data. Right panel: Volatility of wages (blue), consumption (black) and dividends (red) as a function of the horizon. Cash-flows parameters are from Table 3.

leverage coefficient \( d_z = 5.77 \). Such an operating leverage allows for both i) the correct slope of the term-structure of dividends volatility, and ii) the excess volatility of dividends over consumption in yearly growth rates (i.e. 15% versus 2.5%). At the same time, the model
leads to smooth wage growth rates (i.e. 2%).

Section 1 empirically investigates the term-structure effect of income insurance by recognizing a dynamic relationship between the wage-to-dividend ratios and, respectively, the variance ratios of dividends and the differential in the variance ratios of wages and dividends. The former relationship is negative and the latter is positive, as documented in Table 1. The model leads to the same result: the left and right panels of Figure 3 show that the economic mechanism of income insurance is the driver of heterogeneity in the timing of macroeconomic risk. When wages are high (low) relative to dividends, as a result of income insurance, dividend risk becomes more (less) downward-sloping and the gap between the timing of wage risk and dividend risk increases (decreases). Similarly, income insurance leads to dividend growth predictability. Table 2 documents a positive intertemporal relationship between the current wage-to-dividend ratio and future dividend growth. The same result holds in the model, as shown in Figure 4. Since income insurance only concerns transitory risk, both the wage-to-dividend ratio and expected dividend growth are decreasing in the current level of the transitory shock. Hence, dividend growth predictability obtains.

**Figure 3: The Term-Structure Effect of Income Insurance**

The left and right panels show respectively the variance ratios of dividends and the difference between the variance ratios of wages and dividends as a function of the logarithm of the wage-to-dividends ratio for several horizons (the cases $\tau = 2, 5$ and 10 years are denoted in blue, green and red). Cash-flows parameters are from Table 3.

A number of insights are noteworthy. First, although very parsimonious, the model dynamics are flexible enough to capture the main properties of the empirical data. Thus, the calibration further supports the model assumption of both permanent and transitory shocks, in line with Greenwald et al. (2014) and Lettau and Ludvigson (2014).

Second, the heterogeneity in the term-structures of risk of consumption, wages and dividends obtains from the endogenous income insurance parameter $\phi$. Consistently with the empirical findings of Section 1, matching the positive slope of wage risk allows to fill the
Figure 4: Income Insurance and Dividend Growth Predictability

Expected dividend growth is plotted as a function of the logarithm of the wage-to-dividends ratio for several horizons (the cases $\tau = 2$, 5 and 10 years are denoted in blue, green and red) and for $\mu_t = \bar{\mu}$. Cash-flows parameters are from Table 3.

gap between the approximatively flat term-structure of consumption risk and the downward-sloping one of dividend risk. Thus, the calibration further supports the idea that income insurance is the main determinant of the timing of macroeconomic risk. In absence of income insurance ($h = 1 \Rightarrow \phi = 0$), the three term-structures would be equal and collapse on that of consumption.

Third, the shape of the term-structure of dividend risk is the result of the combination of a downward-sloping effect due to $z_t$ and an upward-sloping effect due to $\mu_t$. One issue with asset pricing models is that often the main model mechanism essentially relies on a latent factor which is difficult to estimate. A calibration procedure which exploits the information implied by the term-structures of risk of macroeconomic variables i) allows to capture additional empirical moments, and ii) offers a way to infer about the persistence ($\lambda_{\mu}, \lambda_z$) and the variance ($\sigma_{\mu}, \sigma_z$) of latent factors.

Fourth, the long-run growth factor $\mu_t$ has moderately persistent and smooth dynamics ($\lambda_{\mu} = .59$ and $\sigma_{\mu} = 1.7\%$). The model does not require an excessive persistence in expected consumption growth, as suggested by Constantinides and Ghosh (2011) and Beeler and Campbell (2012).

Finally, I set shareholders’ preference parameters such that they have preference for the early resolution of uncertainty ($\gamma > 1/\psi$) and the intertemporal substitution effect dominates the wealth effect ($\psi > 1$), as in most of the asset pricing literature. Namely, in the baseline calibration the pair $\gamma = 10$ and $\psi = 1.25$ belongs to the usual range of values. The time-discount rate $\beta$ is set to 4%.
3.2 The Term Structure of Equity

I focus on the standard case in which the intertemporal substitution effect dominates the wealth effect ($\psi > 1$) and shareholders have preference for the early resolution of uncertainty ($\gamma > 1/\psi$). The term structures of both premia and return volatility on the dividend strips are decreasing with the maturity at short and medium horizons—in which the downward-sloping effect due to the transitory shock dominates the upward-sloping effect due to long-run growth—and approximatively flat at long horizons—in which the two effects offset each other. Figure 5 shows the term structures as a function of the horizon.

![Figure 5: Term-structures of equity, consumption, and dividends](image)

Left panel: Equity volatility (blue) and equity premium (red) as a function of the horizon. Right panel: Equity volatility (blue, solid and dashed line denote respectively dividend strip and stock), dividend volatility (red) and consumption volatility (black) as a function of the horizon. Cash-flows parameters are from Table 3 and $\gamma = 10$, $\psi = 1.25$ and $\beta = 4\%$.

Preference heterogeneity ($h$) alters the slope of the term structures. The larger the heterogeneity, the stronger the income insurance ($\phi$) and, hence, the operating leverage effect on dividends ($d_z$). This enhances the price associated to transitory risk and, for $\psi > 1$, induces a downward-sloping effect on the term-structure of equity premia: these are decreasing over a longer horizon and their slopes are larger in magnitude.

Dividend strips feature excess volatility over dividends at any horizon. Volatility approaches a value above fundamentals’ risk in the long-run, as shown in Figure 5. Such a result obtains endogenously and although homoscedastic fundamentals. In particular, the “long-run” excess volatility is given by:

$$\sigma_P^2(t, \infty) - \sigma_D^2(t, \infty) = \frac{(1-2\psi)\sigma_\mu^2}{\lambda^2\psi^2} + \frac{d_z^2\sigma_z^2}{\psi^2}.$$

Long-run excess volatility increases with $\phi$, decreases with $\psi$ and does not depend on risk.
aversion. Long-run growth contributes negatively to the excess-volatility for $\psi > 1/2$ and vice-versa. Instead, $z_t$ always contributes positively to the excess-volatility. Thus, for $\phi$ large enough, a positive excess-volatility can obtain even if $\psi > 1/2$.

The model generates a long-run excess volatility of equity over dividends in line with the recent empirical evidence about the decreasing variance ratios of dividends, documented by Belo et al. (2014) and in Section 1. Instead, long-run risk models imply payouts to shareholders riskier than equity returns, as pointed out by Beeler and Campbell (2012). Namely, a long-run risk model ($C_t = X_t$) produces “long-run” excess volatility only for $\psi < 1/2$ — a parametrization under which standard asset pricing implications fail to obtain.

### 3.3 The Market Asset and the Horizon Decomposition of the Equity Premium

The model reconciles the evidence about the term-structures of both equity and macroeconomic variables with standard asset pricing facts. The baseline calibration ($\gamma = 10, \psi = 1.25$) accurately matches the unconditional level of the equity premium, which is about 5.3%, quite close to the data. Such a result is particularly remarkable since neither stochastic volatility and disasters nor unrealistically high and time-varying risk aversion are required. The return volatility is about 15.5%, which is also close to the data. Moreover, return volatility leads to a large excess-volatility over consumption and dividends on the whole term-structure, including the long-horizon. The model produces a Sharpe ratio (about 35%), which accurately matches the data. Such a result is peculiar and due short-run but persistent risk, amplified by income insurance. The model leads to a risk-free rate (about 1.5%) and a relatively low volatility (about 4.5%), slightly above the data. The model also captures quite well the level and the volatility of the price-dividend ratio (about $\exp(3.3)$ and 22.5%). Table 5 summarizes the empirical and the model-implied moments and reports the asset pricing moments for many pairs ($\gamma, \psi$). A low risk-free rate and high levels of the first two return moments of the market asset obtain for $(\gamma = 10, \psi = 1.25)$, $(\gamma = 7.5, \psi = 1)$ and $(\gamma = 5, \psi = .75)$. Decreasing the elasticity of intertemporal substitution leads to an increase in the equity premium but the price-dividend ratio and the risk-free rate are respectively too smooth and too volatile in comparison with the data. Hence, the choice $(\gamma = 10, \psi = 1.25)$ seems preferable. The operating leverage effect due to income insurance not only is crucial to the modeling of the term-structures but also helps to match the standard asset pricing moments (Danthine and Donaldson, 2002). The premium increases with $\phi$ and decreases with $\psi$, all else being equal. The latter relation has the following rationale. The correct calibration of the timing of macroeconomic risk implies that in equilibrium the price for short-run but persistent risk dominates the price of long-run variations in expected growth.
<table>
<thead>
<tr>
<th>Data sample</th>
<th>$r%$</th>
<th>$\sigma_r%$</th>
<th>$\mu_P - r%$</th>
<th>$\sigma_P%$</th>
<th>$SR%$</th>
<th>$\log P/D$</th>
<th>$\sigma_{log P/D}%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1931-2009</td>
<td>0.60</td>
<td>3.00</td>
<td>6.20</td>
<td>19.8</td>
<td>31.3</td>
<td>3.38</td>
<td>45.0</td>
</tr>
<tr>
<td>1947-2009</td>
<td>1.00</td>
<td>2.70</td>
<td>6.30</td>
<td>17.6</td>
<td>35.8</td>
<td>3.47</td>
<td>42.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model – Baseline calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model – Walrasian benchmark (no income insurance: $h = 1 \Rightarrow \phi = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model – Alternative preference settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1.25</td>
</tr>
<tr>
<td>1.5</td>
</tr>
</tbody>
</table>

| 7.5 | 0.75 | 4 | 0.01 | 7.79 | 9.38 | 24.0 | 39.1 | 3.10 | 36.4 | > 0 |
| 1 | 4 | 1.67 | 5.80 | 6.01 | 18.4 | 32.6 | 3.22 | 3.41 | = 0 |
| 1.25 | 4 | 2.39 | 4.67 | 4.45 | 15.3 | 29.1 | 3.30 | 22.4 | < 0 |
| 1.5 | 4 | 2.81 | 3.89 | 3.50 | 13.1 | 26.8 | 3.36 | 37.5 | < 0 |

| 5 | 0.75 | 4 | 1.45 | 7.79 | 8.00 | 24.0 | 33.4 | 3.08 | 36.4 | > 0 |
| 1 | 4 | 2.65 | 5.80 | 5.07 | 18.6 | 32.2 | 3.22 | 3.41 | = 0 |
| 1.25 | 4 | 3.19 | 4.67 | 3.63 | 15.3 | 23.7 | 3.31 | 22.4 | < 0 |
| 1.5 | 4 | 3.47 | 3.89 | 2.80 | 13.1 | 21.4 | 3.38 | 37.5 | < 0 |

The Walrasian case with no income insurance ($h = 1 \Rightarrow \phi = 0$) and, hence, constant labor-share fails to describe both the risk-free rate and equity premium puzzles as well as the downward-sloping term-structure of equity.

To better understand the equilibrium equity premium, it is instructive to adopt a term-structure perspective. The equilibrium density $H(t, \tau)$ of Eq. (35) represents the relative contribution of each horizon $\tau$ to the whole premium. The horizon decomposition of the equity premium does not depend only on the dynamics of the permanent and transitory shocks, but also on the equilibrium compensations for these risks. Figure 6 shows that, in the baseline calibration, the equilibrium density is monotone decreasing with the horizon. Moreover, an increase (decrease) in preference heterogeneity leads to a stronger (weaker) income insurance mechanism and, hence, to a shift of the density towards the short (long) horizon. Indeed, income insurance leads not only to riskier dividends but also to a larger compensation for the transitory risk. This horizon decomposition sheds lights on the nature of priced risk.
in equilibrium. A “short-run” explanation of market compensations is needed in order to simultaneously describe both the standard asset pricing moments and the term-structures of both fundamentals and equity.

3.4 Bond and Equity Yields

Income insurance affects the equilibrium equity yields and their components: the yields on the risk-less bond, the expected dividend growth and the premium on the equity yields, as defined by van Binsbergen et al. (2013). Figure 7 plots these quantities at the steady-state as a function of the horizon both in presence and absence of income insurance. The left upper panel shows that the equity yield is decreasing with the horizon under income insurance, whereas it is increasing in the Walrasian benchmark ($\phi = 0$). For $\psi > 1$, the downward-sloping effect due to income insurance outweighs the upward-sloping effect due to long-run growth. The right upper panel shows the term-structure of the real bond yields. In absence of income insurance, $\mu_t$ drives the term-structure of interest rates. Similarly to long-run risk models, a downward-sloping term-structure obtains for $\psi > 1$, inconsistently with actual data from TIPS. Instead, in presence of income insurance, $z_t$ induces a stronger upward-sloping effect, which leads to a positive slope for the bond yields. Intuitively, income insurance enhances short-run risk in equilibrium and, hence, bonds with short maturities are a better hedge than equity. Therefore, increasing bond yields obtain. Finally, the right lower panel reports the term-structure of the premia on the equity yields. Such premia increase and decrease with horizon respectively in absence and in presence of income insurance.
In summary, the decomposition of equity yields provides two noteworthy results. First, income insurance helps to understand the term-structures of the equity yield premia, in line with the findings of van Binsbergen et al. (2013). Second, at the same time, income insurance helps to reconcile the term-structure of real interest rates with the equity results.

4 Uncertainty and Time-Varying Slope of Risk Premia

While downward-sloping dividend risk is a very robust feature of the data, the empirical evidence about the term structures of equity is under debate. On the one hand, equity volatility inherits the negative slope of dividend risk; on the other hand, the slope of equity premia depends on how and how much the sources of long-run and short-run risks are priced in equilibrium. Downward-sloping equity premia documented by van Binsbergen et al. (2012)
appear as an unconditional property of dividend strips’ returns. Indeed, van Binsbergen et al. (2013) provide evidence that the term-structure of equity premia is time-varying and switches from flat or slightly increasing in good times to strongly decreasing in bad times.

The model of the previous sections does not allow to study risk premia time-variation neither for dividend strips nor for the market asset. While income insurance generates endogenous heteroscedasticity (see Eq. 12), the log-linearized dynamics of dividends used to solve for the equilibrium rules out time-variation in equity premia. This section shows that a minimal modification of the previous model accounts for the refinements by van Binsbergen et al. (2013). Namely, I just introduce heteroscedasticity in the transitory shock $z_t$:

$$z_t = \bar{z} - \hat{z}_t \quad \text{with} \quad d\hat{z}_t = \lambda_z (\bar{z} - \hat{z}_t) dt + \sigma_z \sqrt{\hat{z}_t} dB_{z,t}. \quad (40)$$

where $\hat{\sigma}_z = \sigma_z / \sqrt{\bar{z}}$ and $\bar{z} > 0$. Thus, $z_t$ is still a zero-mean stationary process but its volatility is decreasing in $z_t$. Such a property captures larger uncertainty during business cycle downturns. Given the affine specification of the transitory shock, the model can be solved with the same methodology of the previous sections (results are derived in the Online Appendix B).

This form of heteroscedasticity in fundamentals leads to two results. First, under income insurance the wage-share is decreasing in $z_t$ and, hence, dividends load more on the transitory shock when the latter is high volatile –i.e. operating leverage is counter-cyclical. Second, the price of risk associated to transitory risk is no more constant but is decreasing with $z_t$ and, hence, higher compensations obtain during business cycle downturns. These two facts lead to a slope of the term-structure of equity premia which is time-varying. Indeed, long-run growth leads to an upward-sloping effect and the transitory shock leads to a downward-sloping effect but the latter depends on the volatility of $z_t$. Therefore, the downward-sloping effect is counter-cyclical. As a result, the term-structure of equity premia can be flat or slightly upward-sloping in good times and strongly downward-sloping in bad times.

To provide an illustration of the equilibrium results, I calibrate the model with the same methodology of the previous section. The model generates steady state moments in line with the data: 0.6% risk-free rate with 3% volatility, an equity premium of 7.6% with return volatility of 15.2% and Sharpe ratio of 50% and log price-dividend ratio of 3.48 with 34% volatility. The left upper panel of Figure 8 shows the term-structures of the model implied VR’s (solid lines) and their empirical counterparts (dashed lines). Similarly to Figure 2, the model matches the upward- and downward-sloping shapes of wage and dividend risk and the flat term-structure of consumption risk. The right upper panel of Figure 8 shows the model implied volatility of dividends at the steady-state (solid line) and the 5-95% probability interval (dot-dashed and dashed lines). Dividend volatility is strongly decreasing with $z_t$ at short
Figure 8: Business Cycle Uncertainty and Term-Structures

Left upper panel: Term-structures of variance ratios of consumption, wages and dividends as a function of the horizon. Dashed lines denote empirical data. Other panels: Term structures of dividend volatility (right upper panel), equity volatility (left lower panel) and equity premia (right lower panel) as a function of the horizon. Solid lines denote the steady state ($z_t = \bar{z}$), dashed and dot-dashed lines denote respectively good and bad states. Cash-flows parameters: $\sigma_x = .001, \bar{\mu} = .02, \lambda_{\mu} = .591, \sigma_{\mu} = .017, \bar{z} = .1, \lambda_z = .3, \sigma_z = .027, \alpha = .911$ and $h = 1.57$. Preference parameters: $\gamma_s = 10, \psi = 1.25$ and $\beta = 4\%$.

horizons, whereas the effect of the transitory shock disappears in the long-run. The lower panels of Figure 8 shows the model volatility (left) and premia (right) of the dividend strip returns at the steady-state (solid line) and the 5-95% probability interval (dot-dashed and dashed lines). Under standard preferences ($\gamma_s > \psi > 1$), equity volatility inherits the negative slope of dividend risk. The transitory shock affects the level of the term-structure, which is counter-cyclical, but does not alter the sign of the slope. Instead, the transitory shock affects both the level and the slope of the term-structure of equity premia. Equity compensations are low and barely flat or slightly increasing in normal and good times. However, compensations increase in size and become markedly downward-sloping in bad times. Business cycle uncertainty helps the model to match the conditional dynamics of the term-structure of equity premia and, hence, to provide further support to the main model mechanism of income.
insurance.

The model of this section leads to time-variation also in the equity premium. Namely, the premium on the market asset is a decreasing function of the transitory shock for $\gamma_s > \psi > 1$. An usual way to assess the time-varying properties of risk premia is to verify whether the model captures the pattern of predictability observed in actual data. Thus, I regress long-horizon excess returns on the log price-dividend ratio. The regression is performed on returns measured over horizons ranging from 1 to 7 years. In each simulation, the economy is simulated at the monthly frequency over a 60-year horizon. Table 6 shows that the model reproduces the negative and statistically significant intertemporal relationship between the current valuation ratios and future excess returns that we observe in actual data (Cochrane (2008), Lettau and van Nieuwerburgh (2008), van Binsbergen and Koijen (2010)). Moreover, the explanatory power and statistical significance increase with the horizon. Overall, income insurance, combined with business cycle uncertainty, helps to jointly understand the timing of dividend risk, the dynamic slope of the term-structure of equity, the main properties of the market asset and risk-free rate, as well as long-horizon predictability of equity excess returns.

**Table 6: Long-Horizon Predictability of Excess Equity Returns.**

The table reports the coefficient, t-statistics and adjusted-R$^2$ of the regressions of cumulative excess returns over the horizons of 1, 2, 3, 5 and 7 years on the current log price-dividend ratio. One thousand simulations are run at the monthly frequency over a 60-year horizon.

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient</td>
<td>-0.369</td>
<td>-0.413</td>
<td>-0.425</td>
<td>-0.494</td>
<td>-0.619</td>
</tr>
<tr>
<td>t-stat</td>
<td>-5.92</td>
<td>-9.12</td>
<td>-11.42</td>
<td>-12.49</td>
<td>-10.81</td>
</tr>
<tr>
<td>adj-R$^2$</td>
<td>0.37</td>
<td>0.59</td>
<td>0.69</td>
<td>0.73</td>
<td>0.67</td>
</tr>
</tbody>
</table>

5 Conclusion

This paper documents that the timing of risk of output, wages and dividends is heterogeneous. Income insurance from shareholders to workers, which only concerns transitory risk, empirically and theoretically explains those term-structures and provides a rationale to downward-sloping dividend risk. Once the resulting dividend dynamics is embedded in an otherwise standard general equilibrium asset pricing model, the negative slope of dividend risk transmits to equity returns under standard preferences. Thus, income insurance allows to reconcile in equilibrium traditional asset pricing facts—such as the risk-free rate and equity premium puzzles, the excess volatility of equity, the increasing real yields on bonds, the

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19The huge explanatory power is due to the fact that the model only accounts for priced shocks, whereas idiosyncratic noise and other sources of unpriced risk are not taken into account for the sake of simplicity and exposition.
long horizon equity predictability— with the new evidence about the term-structures of both fundamentals and equity and their dynamics.

The model can be extended in a number of directions, such as cross-sectional equity returns. Marfè (2015) empirically and theoretically supports the idea that the term-structure effect of income insurance explains the dynamics of the value premium.

Appendix

Income Insurance. The income insurance parameter maximizes the geometric average of the unconditional expectations of the transitory components of wages and dividends with bargaining parameter \( \bar{\vartheta} \) given in Eq. (10): \( \mathbb{E}_x \mathbb{E} [ \bar{W} (\phi) ]^\vartheta \mathbb{E}_x [ \bar{D} (\phi) ]^{1-\vartheta} \). We have

\[
\mathbb{E}_x [ \bar{W} (\phi) ] = \alpha^{-1} e^{\bar{z}(1-\varphi)+\sigma_\varphi^2 (1-\varphi)^2/(4\lambda_x)},
\]

\[
\mathbb{E}_x [ \bar{D} (\phi) ] = e^{\bar{z}+\sigma_\varphi^2/(4\lambda_x)} - \alpha^{-1} e^{\bar{z}(1-\varphi)+\sigma_\varphi^2 (1-\varphi)^2/(4\lambda_x)}.
\]

Taking the derivative with respect to \( \phi \),

\[
0 = \frac{\alpha^{-\varphi}}{2\lambda_x} \left( \alpha^{-1} e^{(4\lambda_x \bar{z}+\sigma_\varphi^2 (\varphi-1)) (\varphi-1)} - \alpha^{-1} e^{\varphi z+\sigma_\varphi^2 (\varphi-1)) (\varphi-1)} \right)^{\vartheta} \left( \varphi z+\sigma_\varphi^2 \vartheta - \alpha \lambda_x z - \sigma_\varphi^2 (\varphi-1) + 2\lambda_x \log \alpha \right),
\]

we get three solutions:

\[
\bar{\varphi} = 1 + \frac{2\lambda_x}{\sigma_\varphi^2} (\bar{z} + \log \alpha) + \frac{\vartheta}{2\sigma_\varphi^2} \sqrt{(-4\lambda_x \bar{z} - 2\sigma_\varphi^2 - 4\lambda_x \log \alpha)^2 + 16\lambda_x^2 \sigma_\varphi^2 (\varphi-1) + 2\lambda_x \log \alpha} \quad \text{for} \ \varPsi = \{ -1, 0, 1 \}.
\]

Since we are looking for a potential solution \( \varphi \in (0, 1) \), we can exclude the first two solutions for \( \bar{\varphi} \) given \( \bar{z} = 0 \) (which is used in the model calibration and which guarantees \( \Delta_t > 0 \) \( \forall t \) under the log-linearized dynamics of Eq. (14)). Note that under reasonable parameters the second term common to all solutions for \( \bar{\varphi} \) is by far lower than minus one. Therefore, we consider the third solution (\( \varPsi = 1 \)) and set \( \varphi = \max(0, \min(1, \bar{\varphi})) \).

Equilibrium State-Price Density. Under the infinite horizon, the utility process \( J \) satisfies the following Bellman equation: \( \mathcal{D} J (X, \mu, z) + f (C_s, J) = 0 \), where \( \mathcal{D} \) denotes the differential operator. Then we have

\[
0 = J X \mu X + \frac{1}{2} J X \sigma_x^2 X^2 + J \mu \lambda (\mu - \mu) + \frac{1}{2} J \mu \sigma_\mu^2 + J \lambda \varsigma (\varsigma - \mu) + \frac{1}{2} J \varsigma \sigma_\varsigma^2 + f (C_s, J).
\]

Guess a solution of the form \( J (X, \mu, z) = \frac{1}{1-\gamma} X^{1-\gamma} g (\mu, z) \). The Bellman equation reduces to

\[
0 = \mu - \frac{1}{2} \gamma \sigma_x^2 + \frac{g \mu}{g} \lambda (\mu - \mu) + \frac{1}{2} \frac{g \mu}{g} \sigma_\mu^2 + \frac{g \varsigma}{g} \lambda (\varsigma - \mu) + \frac{1}{2} \frac{g \varsigma}{g} \sigma_\varsigma^2 + \frac{\beta}{1-\gamma} \left( g^{-1} \chi e^{(1-1/\psi)} (d_0 + d_z z) - 1 \right).
\]

(A1)

Under limited market participation \( (C_{s,t} = D_t) \) and stochastic differential utility, the pricing kernel has dynamics given by

\[
d \xi_{0,t} = \xi_{0,t} \frac{dC}{C} + \xi_{0,t} \rho_{fJ} dt = -r (t) \xi_{0,t} - \theta (t) \xi_{0,t} dB_{t,x} - \theta \mu (t) \xi_{0,t} dB_{\mu,t} - \theta z (t) \xi_{0,t} dB_{z,t},
\]

(A2)
where, by use of Itô’s Lemma and Eq. (A1), we get

\[ r(t) = -\frac{\partial X_{fc}}{fc} \mu X - \frac{1}{2} \frac{\partial^2 X_{fc}}{fc^2} \sigma^2 X^2 - \frac{\partial \phi_{fc}}{fc} \lambda_{\mu} (\bar{\mu} - \mu) - \frac{1}{2} \frac{\partial \phi_{fc}}{fc} \sigma_\mu^2 - \frac{\partial \phi_{fc}}{fc} \lambda_z (\bar{z} - z) - \frac{1}{2} \frac{\partial \phi_{fc}}{fc} \sigma_z^2 - f, \]

\[ \theta_x(t) = -\frac{\partial X_{fc}}{fc} \sigma_x X, \quad \theta_{\mu}(t) = -\frac{\partial \phi_{fc}}{fc} \sigma_\mu, \quad \theta_z(t) = -\frac{\partial \phi_{fc}}{fc} \sigma_z, \]

An exact solution for \( g(\mu, z) \) satisfying Eq. (A1) does not exist for \( \psi \neq 1 \). Therefore, I look for a solution of \( g(\mu, z) \) around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by

\[ Q_{s,t} = E_t \left[ \int_t^\infty \xi_{t,u} C_{s,u} du \right], \]

and, applying Fubini’s Theorem and taking standard limits, the consumption-wealth ratio satisfies

\[ \frac{C_{s,t}}{Q_{s,t}} = r(t) - \frac{1}{\alpha} E_t \left[ \frac{dQ}{Q} \right] - \frac{1}{\alpha} E_t \left[ \frac{dE}{E} \right]. \quad (A3) \]

Guess

\[ Q_{s,t} = C_{s,t} \beta^{-1}(g(\mu_t, z_t) e^{(\gamma-1)(d_0 + d_z z_t)})^{1/\chi} \]

and apply Itô’s Lemma to get \( \frac{dQ}{Q} \). Then, plug in the wealth dynamics, the risk-free rate and the pricing kernel into Eq. (A3); after tedious calculus you can recognize that the guess solution is correct. Notice that the consumption-wealth ratio approaches \( \beta \) when \( \psi \to 1 \) as usual.

Denote \( cq = E[\log C_{s,t} - \log Q_{s,t}] \), hence, a first-order approximation of the consumption-wealth ratio around \( cq \) produces

\[ \frac{C_{s,t}}{Q_{s,t}} = \beta g(\mu_t, z_t)^{-1/\chi} e^{(1-1/\psi)(d_0 + d_z z_t)} \approx e^{cq} \left( 1 - cq + \log \beta - \frac{1}{\chi} \log g(\mu, z) + (1 - 1/\psi)(d_0 + d_z z) \right). \]

Using such approximation in the Bellman equation (A1) leads to

\[ 0 = \mu - \frac{1}{2} \sigma_x^2 + \frac{g_x}{g} \frac{\lambda_x(\bar{\mu} - \mu)}{1 - \gamma} + \frac{g_{\mu\mu}}{g} \sigma_\mu^2 + \frac{g_x}{g} \frac{\lambda_x(\bar{z} - z)}{1 - \gamma} + \frac{1}{2} \frac{g_{zz}}{g} \sigma_z^2 + \frac{1}{1 - \psi} e^{cq} \left( 1 - cq + \log \beta - \frac{1}{\chi} \log g(\mu, z) + (1 - 1/\psi)(d_0 + d_z z) \right) \]

which has exponentially affine solution \( g(\mu, z) = e^{u_0 + (1 - \gamma)d_0 + u_\mu \bar{\mu} + (u_z + (1 - \gamma)d_z)z} \), where \( u_0, u_\mu \) and \( u_z \) have explicit solutions and the endogenous constant \( cq \) satisfies \( cq = \log \beta - \chi^{-1}(u_0 + u_\mu \bar{\mu} + u_z \bar{z}) \) (recall \( \bar{z} = E[z_t] = 0 \)). The risk-free rate and the prices of risk take the form:

\[ r_0 = \frac{1}{2} \left( \frac{2(\beta \psi^2 (\gamma - 1)^2 + e^{cq}((1 - \psi)u_0 + \psi(cq - 1 - \log(\beta))(\gamma - 1))}{\psi(1 - \psi)/1 - \psi} + \frac{2\psi(u_0 - (1 - \psi)d_0)(\gamma - 1))\gamma \lambda_x}{\psi(1 - \psi)} \right), \]

\[ r_\mu = \frac{\psi(1 - \gamma) + u_\mu (\psi(1 - \gamma))(e^{cq} + \lambda_x)}{\psi(1 - \gamma)}, \]

\[ r_z = \frac{-\psi d_z (1 - \gamma) \lambda_x + u_z (\psi(1 - \gamma))(e^{cq} + \lambda_x)}{\psi(1 - \gamma)}, \]

\[ \theta_x(t) = \gamma \sigma_x, \quad \theta_\mu(t) = \frac{u_\mu(1 - \psi)}{1 - \gamma} \sigma_\mu, \quad \theta_z(t) = \left( d_z \gamma + \frac{u_z(1 - \psi)}{1 - \gamma} \right) \sigma_z, \]

and the results in the text easily follow.
Proposition A. The following conditional expectation has exponential affine solution:

\[ M_{t,\tau}(\bar{c}) = E_t\left[e^{c_0+c_1\log\xi_0+c_2X_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}\right] = \xi_0^{c_1} X_t^{c_2} e^{\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu+\ell_z(\tau,\bar{c})z}, \]  

(A4)

where \( \bar{c} = (c_0, c_1, c_2, c_3, c_4) \), model parameters are such that the expectation exists finite and \( \ell_0, \ell_\mu \) and \( \ell_z \) are deterministic functions of time.

Proof of Proposition A: Consider the following conditional expectation:

\[ M_{t,\tau}(\bar{c}) = E_t\left[e^{c_0+c_1\log\xi_0+c_2\log X_{t+\tau}+c_3\mu_{t+\tau}+c_4z_{t+\tau}}\right] \]

(A5)

where \( \bar{c} = (c_0, c_1, c_2, c_3, c_4) \) is a coefficient vector such that the expectation exists. Guess an exponential affine solution of the kind:

\[ M_{t,\tau}(\bar{c}) = e^{c_1\log\xi_0+c_2\log X_{t}+\ell_0(\tau,\bar{c})+\ell_\mu(\tau,\bar{c})\mu+\ell_z(\tau,\bar{c})z}, \]

(A6)

where \( \ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c}) \) and \( \ell_z(\tau,\bar{c}) \) are deterministic functions of time. Feynman-Kac gives that \( M \) has to meet the following partial differential equation

\[
0 = M_t - M_\xi(r_0 + r_\mu \mu + r_z z) + \frac{1}{2} M_\xi_\xi (\theta_\xi (t)^2 + \theta_\mu (t)^2 + \theta_z (t)^2) + M_X (\mu X) + \frac{1}{2} M_X X \sigma_x^2 X^2 \\
+ M_\mu \lambda_\mu (\bar{\mu} - \mu) + \frac{1}{2} M_\mu \mu \sigma_\mu^2 + M_z \lambda_z (\bar{z} - z) + \frac{1}{2} M_z z \sigma_z^2 - M_\xi \theta_x(t) \sigma_x X \\
- M_\xi \mu \theta_\mu(t) \sigma_\mu - M_\xi \mu \theta_z(t) \sigma_z,
\]

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives a linear function of the states \( \mu \) and \( z \). Hence, we get three ordinary differential equations for \( \ell_0(\tau,\bar{c}), \ell_\mu(\tau,\bar{c}) \) and \( \ell_z(\tau,\bar{c}) \):

\[
0 = \ell'_0(\tau,\bar{c}) - c_1 r_0 + \frac{1}{2} c_1 (c_1 - 1) (\theta_\xi (t)^2 + \theta_\mu (t)^2 + \theta_z (t)^2) + \frac{1}{2} c_2 (c_2 - 1) \sigma_x^2 + \ell_\mu(\tau,\bar{c}) \lambda_\mu \\
+ \frac{1}{2} \ell_\mu(\tau,\bar{c})^2 \sigma_\mu^2 + \ell_z(\tau,\bar{c}) \lambda_z \bar{z} + \frac{1}{2} \ell_z(\tau,\bar{c})^2 \sigma_z^2 - c_1 c_2 \theta_x(t) \sigma_x - c_1 \ell_\mu(\tau,\bar{c}) \theta_\mu(t) \sigma_\mu \\
- c_1 \ell_z(\tau,\bar{c}) \theta_z(t) \sigma_z \\
0 = \ell'_\mu(\tau,\bar{c}) - c_1 r_\mu + c_2 - \ell_\mu(\tau,\bar{c}) \lambda_\mu \\
0 = \ell'_z(\tau,\bar{c}) - c_1 r_z - \ell_z(\tau,\bar{c}) \lambda_z
\]

with initial conditions: \( \ell_0(0,\bar{c}) = c_0, \ell_\mu(0,\bar{c}) = c_3 \) and \( \ell_z(0,\bar{c}) = c_4 \). Explicit solutions are available.

Dividends. The conditional moment generating function \( \mathbb{D}_t(\tau, n) \) is a weighted sum of exponential affine functions. In particular, we have:

\[
\mathbb{E}_t [D_{t+\tau}] = \mathbb{E}_t \left[X_{t+\tau} (e^{z_{t+\tau}} - \alpha^{1-\phi} e^{(1-\phi)z_{t+\tau}})\right] = M_{t,\tau}(\bar{c}) - M_{t,\tau}(\bar{c}'),
\]

where \( \bar{c} = (0, 0, 1, 0, 1) \) and \( \bar{c}' = ((1 - \phi) \log \alpha, 0, 1, 0, 1 - \phi) \), and

\[
\mathbb{E}_t [D^2_{t+\tau}] = \mathbb{E}_t \left[X_{t+\tau}^2 (e^{z_{t+\tau}} - \alpha^{1-\phi} e^{(1-\phi)z_{t+\tau}})^2\right] = M_{t,\tau}(\bar{c}) - 2M_{t,\tau}(\bar{c}') + M_{t,\tau}(\bar{c}''),
\]

where \( \bar{c} = (0, 0, 2, 0, 2), \bar{c}' = ((1 - \phi) \log \alpha, 0, 2, 0, 2 - \phi) \) and \( \bar{c}'' = (2(1 - \phi) \log \alpha, 0, 2, 0, 2(1 - \phi)) \). With this results in hand, it is possible to compute the term-structures of the growth rates’ volatility \( \sigma^2_{D}(t, \tau) \).

Under the log-linearized dynamics in Eq. (14), \( \mathbb{D}_t(\tau, n) \) is exponential affine and obtains as a special
case of \( M_{t,\tau}(\vec{c}) \) with \( \vec{c} = (n_0, 0, n_0, n_0) \). Therefore,

\[
\sigma_D^2(t, \tau) = v_{D,\tau,x}^2 \sigma_x^2 + v_{D,\tau,\mu}^2 \sigma_\mu^2 + v_{D,\tau,z}^2 \sigma_z^2,
\]

where the coefficients are given by

\[
v_{D,\tau,x} = 1, \quad v_{D,\tau,\mu} = \frac{4e^{-\lambda_\mu \tau} - e^{-2\lambda_\mu \tau} + 2\lambda_\mu \tau - 3}{2\lambda_\mu^2}, \quad v_{D,\tau,z} = \frac{e^{-\lambda_z \tau} \sinh(\lambda_z \tau) d_0^2}{\lambda_z \tau}.
\]

The moment generating functions and the term-structures of volatility for wages and total consumption are computed in a similar way.

**Dividend Strips.** The equilibrium price of the market dividend strip with maturity \( \tau \) of Eq. (23) obtains as a special case of \( M_{t,\tau}(\vec{c}) \) with \( \vec{c} = (d_0, 1, 1, 0, d_z) \). Therefore, it is given by

\[
P_{t,\tau} = \xi_{0,t}^{-1} M_{t,\tau}(\vec{c}) = X_t e^{A_0(\tau) + A_\mu(\tau) \mu + A_z(\tau) z} \text{ with}
\]

\[
A_0(\tau) = \ell_0(\tau, \vec{c}) = \frac{1}{2} \left( -4e^{-\lambda_\mu \tau} \mu - (1 + r_\tau) \left( 1 + e^{\lambda_\mu \tau} \right) + e^{-2\lambda_\mu \tau} \right) \lambda_\mu^2
\]

\[
\times \left( -e^{2\lambda_\mu \tau} (r_\tau + d_z \lambda_\mu) \frac{4 \lambda_\mu^2 \sigma_x^2 + 4 e^{2\lambda_\mu \tau} \left( \lambda_\mu + \lambda_z \right)}{\lambda_\mu^2} \left( -\lambda_\mu^2 + \frac{2}{\lambda_\mu} \lambda_\mu (\theta) \lambda_z + r_z \lambda_z \right) \right)
\]

\[
-e^{2\lambda_\mu \tau} \left( \frac{1}{2} \lambda_\mu (1 + \lambda_\mu) \left( \lambda_\mu (\lambda_\mu + \lambda_z) \right) \right)
\]

\[
+ e^{2\lambda_\mu \tau} \left( \lambda_\mu (2 \lambda_\mu (\lambda_\mu + \lambda_z) \lambda_z + \sigma_z + \frac{d_0 - \tau \lambda_\mu}{1 + r_\tau} + \lambda_\mu \lambda_z \lambda_z \right)
\]

\[
+ (d_0 \lambda_\mu^2 - 2d_z r_z \lambda_z + \lambda_\mu^2 (2\tau \lambda_z - 3)) \lambda_z^2 \right)
\]

\[
+ 4 (r_\tau - 1) \theta \lambda_\mu^2 \lambda_z (\lambda_\mu - 1) \frac{\sigma_x^2}{\lambda_\mu} + (r_\tau - 1)^2 \lambda_z^2 (2\tau \lambda_\mu - 3) \frac{\sigma_z^2}{\lambda_\mu^2}
\)

\[
A_\mu(\tau) = \ell_\mu(\tau, \vec{c}) = \frac{1 + e^{-\lambda_\mu \tau} \mu - (1 + r_\tau) \mu - r_\tau}{\lambda_\mu},
\]

\[
A_z(\tau) = \ell_z(\tau, \vec{c}) = -r_\tau + e^{-\lambda_\mu \tau} \left( r_\tau + d_z \lambda_\mu \right)
\]

and \( A_0(0) = d_0 \), \( A_\mu(0) = 0 \) and \( A_z(0) = d_z \). Itô’s Lemma gives the dynamics of the market dividend strip price:

\[
dP_{t,\tau} = \left[ \frac{1}{2} \left( d_{0,\tau} \sigma_x dX_{t,\tau} + d_\mu P_{t,\tau} \sigma_\mu dB_{\mu,\tau} + d_z P_{t,\tau} \sigma_z dB_{z,\tau} \right) \right]
\]

Therefore the return volatility and premium are given by

\[
\sigma_P(t, \tau) = P_{t,\tau}^{-1} \sqrt{\left( \frac{d_{0,\tau} \sigma_x dX_{t,\tau} + d_\mu P_{t,\tau} \sigma_\mu dB_{\mu,\tau} + d_z P_{t,\tau} \sigma_z dB_{z,\tau} \right)^2 + (d_\mu P_{t,\tau} \sigma_\mu)^2 + (d_z P_{t,\tau} \sigma_z)^2} = \sqrt{\sigma_x^2 + (A_\mu(\tau) \sigma_\mu)^2 + (A_z(\tau) \sigma_z)^2},
\]

\[
(\mu_P - r)(t, \tau) = -\frac{1}{dt} \frac{d_{0,\tau} \sigma_x dX_{t,\tau} + d_\mu P_{t,\tau} \sigma_\mu dB_{\mu,\tau} + d_z P_{t,\tau} \sigma_z dB_{z,\tau}}{P_{t,\tau}} = \theta_x(t) \sigma_x + \theta_\mu(t) A_\mu(\tau) \sigma_\mu + \theta_z(t) A_z(\tau) \sigma_z.
\]

The slopes of the return volatility and premium for the market dividend strip obtain by standard calculus.

**Market Asset and Equity Premium.** Under the assumption of limited market participation, the shareholders act as a representative agent on the financial markets and, hence, the equilibrium price of the market asset is equal to the shareholders’ wealth. Therefore, using the previous results, the market asset price can be written as

\[
P_t = Q_{s,t} = C_{s,t} e^{-c_{st}} = X_t e^{\log (\beta + u_0 x^{-1} + d_0 + u_\mu x^{-1} \mu + (u x^{-1} + d_z) z)}.
\]

The dynamics of the market asset price obtains by applying Itô’s Lemma to \( P_t \):

\[
dP_t = \left[ \frac{1}{2} \left( d_{0,\tau} \sigma_x dX_{t,\tau} + d_\mu P_t \sigma_\mu dB_{\mu,t} + d_z P_t \sigma_z dB_{z,t} \right) \right]
\]
Therefore the return volatility and premium are given by

\[ \sigma_P(t) = P_t^{-1} \sqrt{(\partial_x P_t \sigma_x)^2 + (\partial_\mu P_t \sigma_\mu)^2 + (\partial_z P_t \sigma_z)^2} = \sqrt{\sigma_x^2 + (u_\chi^{-1} \sigma_\mu)^2 + ((u_z \chi^{-1} + d_z) \sigma_z)^2}, \]

\[ (\mu_P - r)(t) = -\frac{1}{\partial_t} \left( \int_0^\infty \frac{dp_t}{P_t} \right) = \theta_x(t)\sigma_x + \theta_\mu(t)u_\chi^{-1} \sigma_\mu + \theta_z(t)(u_z \chi^{-1} + d_z) \sigma_z. \]

Using the definition of the dividend strip price \( P_{t,\tau} = \mathbb{E}_t[\xi_{t,t+\tau} D_{t+\tau}] \) and market asset price \( P_t = \int_0^\infty P_{t,\tau} d\tau \), we can write the instantaneous premium of the market return as follows:

\[ (\mu_P - r)(t) = \theta_x(t)\frac{\partial_x P_t}{P_t} \sigma_x + \theta_\mu(t)\frac{\partial_\mu P_t}{P_t} \sigma_\mu + \theta_z(t)\frac{\partial_z P_t}{P_t} \sigma_z = \int_0^\infty P_t^{-1} P_{t,\tau} (\theta_x(t)\sigma_x + \theta_\mu(t)A_\mu(\tau)\sigma_\mu + \theta_z(t)A_z(\tau)\sigma_z) d\tau \]

and \( \Pi(t, \tau) \) easily follows using the previous results.

**Bond and Equity Yields.** The equilibrium price of the zero-coupon bond with maturity \( \tau \) obtains as a special case of \( \mathcal{M}_{t,\tau}(\bar{c}) \) with \( \bar{c} = (0, 1, 0, 0, 0) \). Then, it is given by \( B_{t,\tau} = \xi_{0,t}^{-1} \mathcal{M}_{t,\tau}(\bar{c}) \).

Given \( p(t, \tau) = \log(D_t/P_{t,\tau})/\tau \) and \( g_D(t, \tau) = \log(D_t(\tau, 1)/D_t)/\tau \), it is easy to verify that the premium on the equity yield, \( g(t, \tau) = p(t, \tau) - \varepsilon(t, \tau) + g_D(t, \tau) \) in Eq. (38), is state-independent.

**References**


Online Appendix A. Empirical Support and Robustness

Table OA.I: Johansen Test for Co-Integration

The table reports the Johansen’s trace statistics, the associated 5% critical values and the SBIC (Schwarz’s Bayesian) and HQIC (Hannan and Quinn) information criteria for the number of co-integrating equations in a VECM model for co-integration among the levels of the following variables. Panel A: GDP, wages, EBIT, dividends of the aggregate economy from NIPA tables on the sample 1929-2012 at yearly frequency. Panel B: gross value added, net value added, wages, EBIT and dividends of the non-financial corporate sector from the Flow of Funds on the sample 1946-2013 at yearly frequency. The Johansen test considers a VECM model with two lags and a constant trend. The symbol * denotes the number of co-integrating equations selected by the test at a 5% level of significance.

**Panel A – Aggregate Economy**

<table>
<thead>
<tr>
<th>rank</th>
<th>parms</th>
<th>log-likelihood</th>
<th>eigenvalue</th>
<th>trace stat</th>
<th>5% c.v.</th>
<th>SBIC</th>
<th>HQIC</th>
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<td>41.166</td>
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Panel B – Non-Financial Corporate Sector

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<th>eigenvalue</th>
<th>trace stat</th>
<th>5% c.v.</th>
<th>SBIC</th>
<th>HQIC</th>
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</table>

variables: GDP, wages, EBIT, dividends.

variables: gross value added, net value added, wages, EBIT, dividends.
Table OA.II: Determinants of the Dividend-Share: Granger Causality

The table reports the estimates of the model

\[ D/Y_t = b_0 + b_{01} D/Y_{t-1} + b_{01} D/Y_{t-2} + b_{11} \text{Lev}_{t-1} + b_{12} \text{Lev}_{t-2} + b_{21} I/Y_{t-1} + b_{22} I/Y_{t-2} + b_{31} W/Y_{t-1} + b_{32} W/Y_{t-2} + \epsilon_t, \]

where the dependent variable is the dividend-share \((D/Y)\) at time \(t\); the independent variables are the first and second lag of the financial leverage ratio of the corporate sector \((\text{Lev})\), the investment-share \((I/Y)\) and the labor-share \((W/Y)\). Panel A considers the shares of GDP in the aggregate economy from NIPA tables on the sample 1946-2012 at yearly frequency. Panel B considers the shares of gross value added in the non-financial corporate sector from the Flow of Funds on the sample 1946-2013 at yearly frequency. The \(t\)-statistics are reported in parenthesis. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. A set of Granger causality tests reports the \(F\) statistics and p-values of the hypothesis that all coefficients on the lags of each independent variable are jointly zero. The number of lags (2) is selected according to the final prediction error (FPE), Akaike’s information criterion (AIC) and the Hannan and Quinn information criterion (HQIC).

<table>
<thead>
<tr>
<th>Panel A – Aggregate Economy</th>
<th>Panel B – Non-Financial Corporate Sector</th>
</tr>
</thead>
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<tr>
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<td>(D/Y_t) Granger test</td>
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<td>coeff          F   prob</td>
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<tr>
<td>(W/Y)</td>
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<td>(t-1)</td>
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</table>
Table OA.III: The Term-Structure Effect of Income Insurance: Robusteness (I)

The table reports the estimates of the regressions where all variables have been de-trended:

\[
VR_W - VR_D|_{t,\tau} = a + b_{wd} \log W_t + b_y VR_Y|_{t,\tau} + \epsilon_t,
\]

\[
VR_W|_{t,\tau} = a + b_w \log W_t + b_y VR_Y|_{t,\tau} + \epsilon_t,
\]

\[
VR_D|_{t,\tau} = a + b_d \log W_t + b_y VR_Y|_{t,\tau} + \epsilon_t,
\]

where the dependent variables are either the variance ratios of wages, those of dividends or their difference computed at time \(t\) with horizon \(\tau\) ranging from 2 to 7 years; the independent variables are the time \(t\) logarithm of the wage-to-dividend ratios and the time \(t\) variance ratios of output with horizon \(\tau\). Panel A considers the aggregate economy: wages, dividends and GDP are from NIPA tables on the sample 1947:1-2013:3 at quarterly frequency. Panel B considers the non-financial corporate sector: wages, dividends and value added are from the Flow of Funds on the sample 1951:4-2015:1 at quarterly frequency. The time-series of variance ratios are computed through a rolling window of 10 years centered at \(t\). The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

### Panel A – Aggregate Economy

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</tr>
<tr>
<td>t-stat</td>
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<td>(9.14)</td>
<td>(10.50)</td>
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### Panel B – Non-Financial Corporate Sector

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</table>
The table reports the estimates of the regressions:

\[ VR_W - VR_D \bigg|_{t,\tau} = a + b_{wd} \log \frac{W}{D}_t + b_{lev} \text{Lev}_t + b_{inv} I/Y_t + b_{Y} VR_Y \bigg|_{t,\tau} + \epsilon_t, \]

where the dependent variable is the difference between the variance ratios of wages and those of computed at time \( t \) with horizon \( \tau \) ranging from 2 to 7 years; the independent variables are the time \( t \) wage-to-dividend ratios, financial leverage, investment-share and the time \( t \) variance ratios of output with horizon \( \tau \). Panel A considers the aggregate economy: wages, dividends, investments and GDP are from NIPA tables on the sample 1947:1-2013:3 at quarterly frequency. Panel B considers the non-financial corporate sector: wages, dividends, investments, financial leverage and value added are from the Flow of Funds on the sample 1951:4-2015:1 at quarterly frequency. The time-series of variance ratios are computed through a rolling window of 10 years centered at \( t \). The symbols \(^*\), \(^*\), and \(^*\) denote statistical significance at the 10%, 5%, and 1% levels.

### Panel A – Aggregate Economy

<table>
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<tr>
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<td>1.48***</td>
<td>1.72***</td>
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<td>1.54</td>
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<td>0.56</td>
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### Panel B – Non-Financial Corporate Sector

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Table OA.V: The Term-Structure Effect of Income Insurance: Robustness (III)

The table reports the estimates of the regressions:

$$VR_W|_{t,\tau} = a + b_{wd} \log \frac{W}{D}_t + b_{lev} \text{Lev}_t + b_{inv} \frac{I/Y}{t} + b_{y} VR_Y|_{t,\tau} + \epsilon_t,$$

where the dependent variable is the variance ratio of wages computed at time $t$ with horizon $\tau$ ranging from 2 to 7 years; the independent variables are the time $t$ wage-to-dividend ratios, financial leverage, investment-share and the time $t$ variance ratios of output with horizon $\tau$. Panel A considers the aggregate economy: wages, dividends, investments and GDP are from NIPA tables on the sample 1947:1-2013:3 at quarterly frequency. Panel B considers the non-financial corporate sector: wages, dividends, investments, financial leverage and value added are from the Flow of Funds on the sample 1951:4-2015:1 at quarterly frequency. The time-series of variance ratios are computed through a rolling window of 10 years centered at $t$. The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

### Panel A – Aggregate Economy

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Table OA.VI: The Term-Structure Effect of Income Insurance: Robustness (IV)

The table reports the estimates of the regressions:

\[ \text{VR}_D|_{t,\tau} = a + b_{wd}\log \frac{W}{D}_t + b_{lev}\text{Lev}_t + b_{inv}\text{I/Y}_t + b_{y}\text{VR}_Y|_{t,\tau} + \epsilon_t, \]

where the dependent variable is the variance ratio of dividends computed at time \( t \) with horizon \( \tau \) ranging from 2 to 7 years; the independent variables are the time \( t \) wage-to-dividend ratios, financial leverage, investment-share and the time \( t \) variance ratios of output with horizon \( \tau \). Panel A considers the aggregate economy: wages, dividends, investments and GDP are from NIPA tables on the sample 1947:1-2013:3 at quarterly frequency. Panel B considers the non-financial corporate sector: wages, dividends, investments, financial leverage and value added are from the Flow of Funds on the sample 1951:4-2015:1 at quarterly frequency. The time-series of variance ratios are computed through a rolling window of 10 years centered at \( t \). The symbols *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels.

### Panel A – Aggregate Economy

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<td>0.18</td>
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</table>
Table OA.VII: Long-Horizon Predictability of Dividend Growth: Robustness

The table reports the coefficients, Newey-West t-statistics and adjusted-$R^2$ of the regressions of cumulative dividend growth rates over the horizons of 3, 5, 10 and 15 years on the current logarithm of the wages-to-dividends ratio (log W/D), the financial leverage ratio (Lev) and the investment-share (I/Y). Panel A and B consider respectively the aggregate economy on the sample 1946-2012 and the non-financial corporate sector on the sample 1946-2013 at yearly frequency.

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<th>Panel B – Non-Financial Corporate Sector</th>
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<td>Horizon (years) 3 5 10 15</td>
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<td>log W/D 0.265** 0.339*** 0.666*** 1.121***</td>
<td>log W/D 0.358** 0.438*** 0.786*** 1.130***</td>
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<td>t-stat 2.59 5.45 5.34 8.06</td>
</tr>
<tr>
<td>Lev 0.490 0.923** 0.718* -0.149</td>
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<tr>
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<td>t-stat 0.34 1.85 1.90 -0.85</td>
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<tr>
<td>I/Y 1.027 1.855 5.080*** 6.116***</td>
<td>I/Y 0.351 0.606 2.373** 2.563***</td>
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<tr>
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<td>t-stat 0.45 0.47 2.06 2.94</td>
</tr>
<tr>
<td>adj-$R^2$ 0.23 0.41 0.61 0.66</td>
<td>adj-$R^2$ 0.20 0.39 0.60 0.63</td>
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</tbody>
</table>

Figure OA.I: The Timing of Macroeconomic Risk: Generalized Payout

Variance-ratios of the net payout from the non financial corporate sector as a function of the horizon. Data are yearly on the sample 1946:2004. The net payout is from Motohiro Yogo webpage and is defined as the sum of dividends, interest, and net repurchases of equity and debt. (Larrain and Yogo, 2008). The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations.
Figure OA.II: The Timing of Consumption Risk and Limited Market Participation

Variance-ratios of the aggregate consumption (solid line) and the sum of wages and dividends from the non-financial corporate sector (dashed line) as a function of the horizon. Data are yearly on the sample 1946:2004. The variance ratios of consumption are computed from the growth rates in Beeler and Campbell (2012). The variance ratio procedure uses the theoretical exposition of Campbell, Lo and MacKinlay (1997, pp. 48-55), which accounts for heteroscedasticity and overlapping observations.
Online Appendix B. Results Under Business Cycle Uncertainty

Recall that total resources are characterized by
\[ Y_t = X_t Z_t \]
with
\[ \log X_t = x_t, \log Z_t = z_t = \bar{z} - \hat{z}_t \]
and
\[ dx_t = (\mu_t - \sigma_x^2/2) dt + \sigma_x dB_{x,t}, \]
\[ d\mu_t = \lambda_\mu (\bar{\mu} - \mu_t) dt + \sigma_\mu dB_{\mu,t}, \]
\[ d\hat{z}_t = \lambda_z (\bar{z} - \hat{z}_t) dt + \hat{\sigma}_z \sqrt{\hat{\sigma}_z^2} dB_{\hat{z},t}. \]

Wages and dividends satisfy
\[ W_t = X_t \alpha^{1-\phi} e^{(1-\phi)(\bar{z} - \hat{z}_t)}, \]
\[ D_t = X_t (e^{\bar{z} - \hat{z}_t} - \alpha^{1-\phi} e^{(1-\phi)(\bar{z} - \hat{z}_t)}). \]

The linearization of the logarithm of the dividend process implies:
\[ \log D_t \approx \log X_t + \partial_z \log D \bigg|_{z=0} z_t = x_t + d_0 + d_z \hat{z}_t, \]
where \( d_0 = \log (1 - \alpha^{1-\phi}) - d_z \bar{z} \) with
\[ d_z = -\left(1 + \frac{\phi_0}{\alpha^\phi} \right). \]

**Proposition OA.1.** The shareholders’ utility process under recursive preferences and dynamics in Eq. (OA1)-(OA2)-(OA3)-(OA6) is given by
\[ J(X_t, \mu_t, \hat{z}_t) = \frac{1}{1-\gamma_s} X_t^{1-\gamma_s} \exp \left( u_0 + (1 - \gamma_s) d_0 + u_\mu \mu_t + (u_z + (1 - \gamma_s) d_z) \hat{z}_t \right), \]
where \( u_0, u_\mu = \frac{1 - \gamma_s}{e^{\phi_0} + \lambda_\mu}, u_z = \frac{\lambda_z (\gamma_s - 1)}{e^{\phi_0} + \lambda_z} \left(1 + \frac{\phi_0 \alpha}{e^{\phi_0} e^{\phi_0 \hat{z}}} - \alpha \right) \),
and \( cq = E[cq_t] \) are endogenous constants depending on the primitive parameters. The shareholders’ consumption-wealth ratio is equal to
\[ cq_t = \log C_{s,t}/Q_{s,t} = \log \beta - \chi^{-1}(u_0 + u_\mu \mu_t + u_z \hat{z}_t). \]

**Proposition OA.2.** The equilibrium state price density has dynamics given by
\[ d\xi_{0,t} = \xi_{0,t} \frac{df_t}{f_t} + \xi_{0,t} f_j dt = -r(t) \xi_{0,t} dt - \theta_x(t) \xi_{0,t} dB_{x,t} - \theta_\mu(t) \xi_{0,t} dB_{\mu,t} - \theta_z(t) \xi_{0,t} dB_{z,t}, \]
where the instantaneous risk-free rate satisfies
\[ r(t) = r_0 + r_\mu \mu_t + r_z \hat{z}_t. \]
and the instantaneous prices of risk are given by
\[ \theta_x(t) = \gamma_s \sigma_x, \]
\[ \theta_\mu(t) = \frac{\gamma_s - 1/\psi}{e^{\psi t} + \lambda_\mu}, \]
\[ \theta_z(t) = \sqrt{\tilde{z}_t} \left( d_s \gamma_s \tilde{z}_t - \frac{(\psi_{\gamma_s-1})(e^{\psi t} - \lambda_\mu + d_s(\gamma_s-1)\tilde{z}_t)}{\tilde{z}_t (\gamma_s-1)} \right). \]  

(14)

Proof of Proposition OA.1 and OA.2: Under the infinite horizon, the utility process \( J \) satisfies the following Bellman equation:
\[ \mathcal{D} J(X, \mu, \tilde{z}) + f(C_s, J) = 0, \]

(15)

where \( \mathcal{D} \) denotes the differential operator. Eq. (OA15) can be written as
\[ 0 = J_X X + \frac{1}{2} J_{XX} X^2 + J_{\mu} \lambda_\mu (\mu - \mu) + \frac{1}{2} J_{\mu\mu} \sigma_\mu^2 + J_z \lambda_z (\tilde{z} - \tilde{z}) + \frac{1}{2} J_{zz} \tilde{z}_t^2 + f(C_s, J). \]

Guess a solution of the form \( J(X, \mu, \tilde{z}) = \frac{1}{\gamma_s} X^{1-\gamma_s} g(\mu, \tilde{z}) \). The Bellman equation reduces to
\[ 0 = \mu - \frac{1}{2} \gamma_s \sigma_\mu^2 + \frac{2}{g} \lambda_\mu (\mu - \mu) + \frac{1}{2} \frac{2}{g} \lambda_z (\tilde{z} - \tilde{z}) + \frac{1}{2} \frac{2}{g} \tilde{z} \sigma_\mu^2 + \frac{\beta}{1 - \psi} \left( g^{-1/\psi } (1 - 1/\psi) (d_0 + d_\tilde{z}) - 1 \right). \]

(16)

Under the assumption of limited market participation and stochastic differential utility, the pricing kernel has dynamics given by
\[ d\xi_{0,t} = \xi_{0,t} \frac{d\xi}{C} + \xi_{0,t} f_d d\tau = -r(t) \xi_{0,t} - \theta_x(t) \xi_{0,t} dB_{x,t} - \theta_\mu(t) \xi_{0,t} dB_{\mu,t} - \theta_z(t) \xi_{0,t} dB_{\tilde{z},t}, \]

(17)

where, by use of Itô’s Lemma and Eq. (OA16), we get
\[ r(t) = - \frac{\partial X}{\partial \mu} \mu X - \frac{1}{2} \frac{\partial^2 X}{\partial \mu^2} \sigma_\mu^2 X^2 - \frac{\partial \mu}{\partial \mu} \lambda_\mu (\mu - \mu) - \frac{1}{2} \frac{\partial^2 \mu}{\partial \mu^2} \sigma_\mu^2 - \frac{\partial \mu}{\partial \mu} \lambda_z (\tilde{z} - \tilde{z}) - \frac{1}{2} \frac{\partial^2 \mu}{\partial \mu^2} \tilde{z}_t^2 - f_j, \]
\[ \theta_x(t) = - \frac{\partial X}{\partial \mu} \sigma_\mu, \]
\[ \theta_\mu(t) = - \frac{\partial \mu}{\partial \mu} \theta_x(t), \]
\[ \theta_z(t) = - \frac{\partial \mu}{\partial \mu} \theta_x(t). \]

An exact solution for \( g(\mu, \tilde{z}) \) satisfying Eq. (OA16) does not exist for \( \psi \neq 1 \). Therefore, I look for a solution of \( g(\mu, \tilde{z}) \) around the unconditional mean of the consumption-wealth ratio. Aggregate wealth is given by
\[ Q_{s,t} = \mathbb{E}_t \left[ \int_t^\infty \xi_{t,u} C_{s,u} \, du \right], \]

and, applying Fubini’s Theorem and taking standard limits, the consumption-wealth ratio satisfies
\[ \frac{C_{s,t}}{Q_{s,t}} = r(t) - \frac{1}{\beta} \mathbb{E}_t \left[ \frac{d\eta}{Q} \right] - \frac{1}{\beta} \mathbb{E}_t \left[ \frac{d\xi}{Q} \right]. \]

(18)

Guess
\[ Q_{s,t} = C_{s,t} \beta^{-1} (g(\mu_t, \tilde{z}_t)) e^{(\gamma_s-1)(d_0 + d_\tilde{z})} \]

and apply Itô’s Lemma to get \( \frac{d Q}{Q} \). Then, plug in the wealth dynamics, the risk-free rate and the pricing kernel into Eq. (OA18): after tedious calculus you can recognize that the guess solution is
correct. Notice that the consumption-wealth ratio approaches to $\beta$ when $\psi \to 1$ as usual.

Denote $cq = E[\log C_{s,t} - \log Q_{s,t}]$, hence, a first-order approximation of the consumption-wealth ratio around $cq$ produces
\[
\frac{C_{s,t}}{Q_{s,t}} = \beta g(\mu_t, \hat{z}_t)^{-1/(1-\psi)}(d_0 + d_z \hat{z}_t) \approx e^{cx} \left( 1 - cx + \log \beta - \frac{1}{\chi} \left( \log g(\mu_t, \hat{z}_t) + (\gamma_s - 1)(d_0 + d_z \hat{z}_t) \right) \right).
\]
Using such approximation in the Bellman equation (OA16) leads to
\[
0 = \mu - \frac{1}{2} \gamma_s \sigma^2 + \frac{g}{\psi} \lambda_s(\mu - \mu) + \frac{1}{2} \frac{g(\psi - 1)}{\gamma_s} \sigma^2 \lambda_s + \frac{1}{2} \frac{g(\psi - 1)}{\gamma_s} \sigma^2 \lambda_s^2 + \frac{1}{2} \frac{g(\psi - 1)}{\gamma_s} \sigma^2 \lambda_s^2 \\
+ \frac{1}{\gamma (1-\psi)} \left( e^{cx} \left( 1 - cx + \log \beta - \frac{1}{\chi} \log g(\mu_t, \hat{z}_t) + (1 - 1/\psi)(d_0 + d_z \hat{z}_t) \right) - \beta \right),
\]
which has exponentially affine solution $g(\mu, \hat{z}) = e^{u_0 + (1 - \gamma_s) d_0 + u_{\mu} \mu + (u_z + (1 - \gamma_s) d_z) \hat{z}}$ where
\[
u_0 = \frac{1}{2} e^{-cq} (1 - \gamma_s) \left( \frac{2 \beta \psi}{\psi} + \frac{2 e^{c_{\mu}} \psi (\gamma_s - 1)}{\psi - 1} - \sigma^2 \gamma_s + \frac{2 \mu \lambda_s e^{c_{\mu}} + \lambda_s}{\psi - 1} \right) - \frac{2 \bar{z} \lambda_s e^{c_q + \lambda_s - \psi \lambda_s^2 / \psi}}{(\gamma_s - 1) \sigma^2},
\]
\[u_{\mu} = \frac{1 - \gamma_s}{e^{c_{\mu}} + \lambda_s},
\]
\[u_z = e^{c_q + \lambda_s + d_z (\gamma_s - 1) \sigma^2 / \sigma^2} - \sqrt{e^{c_q + \lambda_s^2 + 2 e^{c_{\mu}} (\lambda_s + d_z (\gamma_s - 1) \sigma^2 / \sigma^2)}},
\]
and the endogenous constant $cq$ satisfies $cq = \log \beta - \chi^{-1}(u_0 + u_{\mu} \mu + u_z \hat{z})$. The risk-free rate and the prices of risk take the form:
\[r_0 = \frac{2 \beta \psi + 2 d_z \bar{z} \lambda_s - (1 + \psi) \gamma_s \sigma^2}{2 \psi} + \frac{(1 - \psi)(\psi \gamma_s - 1) \sigma^2}{2 \psi^2 (e^{c_q + \lambda_s})^2},
\]
\[r_{\mu} = \frac{1}{\psi},
\]
\[r_z = \frac{1}{2 \psi^2 (1 - \gamma_s)^2 \sigma^2} \left( 2 e^{c_q} (\psi - 1)(\psi \gamma_s - 1) + 2(\psi - 1)(\psi \gamma_s - 1) \lambda_s^2 - 2(\psi - 1) \lambda_s (\Omega \psi \gamma_s - 1) + d_z (\gamma_s - 1) \sigma^2 \right) \\
+ d_z (\gamma_s - 1) \sigma^2 \frac{2 \Omega (\psi \gamma_s - 1) + d_z (\gamma_s - 1) \sigma^2 + 2 e^{c_q} (\psi \gamma_s - 1)}{(\Omega - \psi \psi + 2(\psi - 1) \lambda_s + (\psi - 2) d_z (\gamma_s - 1) \sigma^2)},
\]
\[\theta_x(t) = \gamma_s \sigma^2,
\]
\[\theta_{\mu}(t) = \frac{(\gamma_s - 1)(1 + \psi) \sigma_{\mu}}{e^{c_q} + \lambda_s},
\]
\[\theta_z(t) = \sqrt{\frac{\sigma^2}{\sigma^2 + \psi (\psi - 1) \sigma^2 / \sigma^2}} \left( d_z \gamma_s \sigma^2 - \frac{(\psi \gamma_s - 1)(e^{c_q - \theta_x + \lambda_s + d_z (\gamma_s - 1) \sigma^2}}{(\sigma^2 \psi (\gamma_s - 1)} \right),
\]
where $\Omega = \sqrt{e^{c_q} + \lambda_s^2 + 2 e^{c_{\mu}} (\lambda_s + d_z (\gamma_s - 1) \sigma^2 / \sigma^2)}$. *q.e.d.*

**Proposition OA.3.** The following conditional expectation has exponential affine solution:
\[
\mathcal{M}_{t,\tau}(\bar{c}) = E_{t}[e^{c_0 + c_1 \log \xi_{t+\tau} + c_2 x_{t+\tau} + c_3 u_{t+\tau} + c_4 z_{t+\tau}}] = e^{c_{\theta_{0,t}}} X_t^{c_{\theta_{0,t}}} e^{\ell_{0}(\tau, c) + \ell_{\mu}(\tau, c) \mu + \ell_{z}(\tau, c) \hat{z}}, \quad (OA19)
\]
where $\bar{c} = (c_0, c_1, c_2, c_3, c_4)$, model parameters are such that the expectation exists finite and $\ell_0, \ell_\mu$ and $\ell_z$ are deterministic functions of time.

**Proposition OA.4.** The moment generating function of the logarithm of dividends has the following approximation:
\[
D_{t}(\tau, n) = E_{t}[D_{t+\tau}^n] \approx e^{n x_t + B_0(n, \tau) + B_\mu(n, \tau) \mu + B_z(n, \tau) \hat{z}_t}, \quad (OA20)
\]
where $n$ and model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA6) and $B_0, B_\mu$ and $B_z$ are deterministic functions of time.

Proof of Proposition OA.3: Consider the following conditional expectation:

$$M_{t,\tau}(\bar c) = \mathbb{E}[e^{c_0 + c_1 \log \xi_0 + c_2 \log X_t + c_3 \mu_t + c_4 \delta z_t}] \quad (OA21)$$

where $\bar c = (c_0, c_1, c_2, c_3, c_4)$ is a coefficient vector such that the expectation exists. Guess an exponential affine solution of the kind:

$$M_{t,\tau}(\bar c) = e^{c_1 \log \xi_0 + c_2 \log X_t + \ell_0(\tau, \bar c) + \ell_\mu(\tau, \bar c) \mu_t + \ell_z(\tau, \bar c) \delta z_t}, \quad (OA22)$$

where $\ell_0(\tau, \bar c), \ell_\mu(\tau, \bar c)$ and $\ell_z(\tau, \bar c)$ are deterministic functions of time. Feynman-Kac gives that $M$ has to meet the following partial differential equation

$$0 = M_t - M_\xi(\ell_0(\tau, \bar c) + \ell_\mu(\tau, \bar c) \mu + \ell_z(\tau, \bar c) \delta z - c_1 (r_0 + r_\mu + r_z \delta z) + \frac{1}{2} c_1 (c_1 - 1) (\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + M_\mu \lambda_x(\bar \mu - \mu) + \frac{1}{2} M_\mu \sigma_x^2 + M_z(\bar z - \bar z) + \frac{1}{2} M_{zz} \sigma_z^2 - \theta_x(t) \sigma_x$$

where the arguments have been omitted for ease of notation. Plugging the resulting partial derivatives from the guess solution into the pde and simplifying gives a linear function of the states $\mu$ and $\bar z$:

$$0 = \ell_0'(\tau, \bar c) + \ell_\mu'(\tau, \bar c) \mu + \ell_z'(\tau, \bar c) \delta z - c_1 (r_0 + r_\mu + r_z \delta z) + \frac{1}{2} c_1 (c_1 - 1) (\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2)$$

Hence, we get three ordinary differential equations for $\ell_0(\tau, \bar c), \ell_\mu(\tau, \bar c)$ and $\ell_z(\tau, \bar c)$:

$$0 = \ell_0'(\tau, \bar c) - c_1 r_0 + \frac{1}{2} c_1 (c_1 - 1) (\theta_x(t)^2 + \theta_\mu(t)^2 + \theta_z(t)^2) + \frac{1}{2} c_2 (c_2 - 1) \sigma_x^2 + \ell_\mu(\tau, \bar c) \lambda_z$$

with $\theta_z^0 = \theta_z(t)/\sqrt{z}$ and initial conditions: $\ell_0(0, \bar c) = c_0, \ell_\mu(0, \bar c) = c_3$ and $\ell_z(0, \bar c) = c_4$. Explicit solutions are available. $q.e.d.$

Proof of Proposition OA.4: The conditional moment generating function $D_t(\tau, n)$ of Eq. (OA20) obtains as a special case of $M_{t,\tau}(\bar c)$ with $\bar c = (n d_0, 0, n, 0, n d_z)$. Therefore, it is given by

$$D_t(\tau, n) = X_t e^{\ell_0(\tau, \bar c) + \ell_\mu(\tau, \bar c) \mu_t + \ell_z(\tau, \bar c) \delta z_t}$$

with $\ell_0(0, \bar c) = n d_0, \ell_\mu(0, \bar c) = 0$ and $\ell_z(0, \bar c) = n d_z$ and $B_0(n, \tau), B_\mu(n, \tau)$ and $B_z(n, \tau)$ are implicitly defined. $q.e.d.$

Proposition OA.5. The equilibrium price of the market dividend strip with maturity $\tau$ is given by

$$P_t,\tau = \mathbb{E}[\xi_{t,\tau} D_{t+\tau}] \approx X_t e^{A_0(\tau) + A_\mu(\tau) \mu_t + A_z(\tau) \delta z_t}, \quad (OA23)$$
where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA6) and \( A_0, A_\mu, \) and \( A_z \) are deterministic functions of time. The instantaneous volatility and premium on the dividend strip with maturity \( \tau \) are given by

\[
\sigma_p(t, \tau) = \left[ \sigma_z^2 + \frac{(1-e^{-\lambda_\mu \tau})(r_\mu-1)^2}{\lambda_\mu^2} \sigma_\mu^2 + \left( \frac{\lambda_\mu}{\sigma_z} + \theta'_z + \frac{\hat{\theta}}{\sigma_z} \tan \left( \tau \hat{\Omega}/2 - \arctan((\theta'_z \sigma_z + \lambda_z - d_z \sigma_z^2)/\hat{\Omega}) \right) \right)^2 \hat{z}_t, \right. 
\]

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA6) and \( A_0, A_\mu, \) and \( A_z \) are deterministic functions of time. The dynamics of the market dividend strip price obtains by applying Itô’s Lemma to

\[
\ell \equiv \frac{\mu}{\sigma_z} \sigma_z^2 = 0 \text{ and } \Theta = \frac{\lambda_\mu}{\sigma_z} \sigma_z^2 - (\theta'_z + 2r_z)\sigma_z^2. 
\]

**Proof of Proposition OA.5:** The equilibrium price of the market dividend strip with maturity \( \tau \) of Eq. (OA25) obtains as a special case of \( M_{t,\tau}(c) \) with \( c = (d_0, 1, 1, 0, d_z) \). Therefore, it is given by

\[
P_{t,\tau} = \xi_0^{-1} M_{t,\tau}(c) = X_t e^{\ell_0(\tau,c) + \ell_\mu(\tau,c)\mu_z + \ell_z(\tau,c)\hat{z}_t} 
\]

with \( \ell_0(0,c) = d_0, \ell_\mu(0,c) = 0 \) and \( \ell_z(0,c) = d_z \) and \( A_0(\tau), A_\mu(\tau) \) and \( A_z(\tau) \) are implicitly defined. The dynamics of the market dividend strip price obtains by applying Itô’s Lemma to \( P_{t,\tau} \):

\[
dP_{t,\tau} = \left[ \right] dt + \partial_x P_{t,\tau} \sigma_x dB_{x,t} + \partial_\mu P_{t,\tau} \sigma_\mu dB_{\mu,t} + \partial_z P_{t,\tau} \sigma_z \sqrt{\hat{z}_t} dB_{z,t}.
\]

Therefore the return volatility is given by

\[
\sigma_p(t, \tau) = P_{t,\tau}^{-1} \sqrt{(\partial_x P_{t,\tau} \sigma_x)^2 + (\partial_\mu P_{t,\tau} \sigma_\mu)^2 + (\partial_z P_{t,\tau} \sigma_z \sqrt{\hat{z}_t})^2} = \sqrt{\sigma_x^2 + (A_\mu(\tau)\sigma_\mu)^2 + (A_z(\tau)\sigma_z \sqrt{\hat{z}_t})^2},
\]

and the premium is given by

\[
(\mu_P - r)(t, \tau) = -\frac{1}{\hat{z}_t} \left( \frac{d_{0,t}}{\xi_0,t} \right. \frac{dP_{t,\tau}}{P_{t,\tau}} = \theta_x(t)\sigma_x + \theta_\mu(t)A_\mu(\tau)\sigma_\mu + \theta_z(t)A_z(\tau)\sigma_z \sqrt{\hat{z}_t}.
\]

q.e.d.

**Proposition OA.6:** The equilibrium price of the market asset is given by

\[
P_t = \mathbb{E}_t \left[ \int_t^{\infty} \xi_{t,u} D_u du \right] \approx X_t \beta^{-1} e^{u_0 x^{-1} + d_0 + u_\mu x^{-1} \mu_t + (u_z x^{-1} + d_z) \hat{z}_t} 
\]

where model parameters are such that the expectation exists finite, the approximation makes use of Eq. (OA6), and \( u_0, u_\mu, \) and \( u_z \) are endogenous constants depending on the primitive parameters. The instantaneous volatility and premium on the market asset are given by

\[
\sigma_p(t) = \sqrt{\sigma_x^2 + \frac{(1-1/\psi)^2 u_\mu^2 + u_z^2 \sigma_z^2}{(1-\gamma_\mu)^2} + \left( d_z \sigma_z + \frac{(1-1/\psi) u_\mu \sigma_z}{1-\gamma_\mu} \right)^2 \hat{z}_t}, \quad (O6)
\]

\[
(\mu_P - r)(t) = \theta_x(t)\sigma_x + \frac{(1-1/\psi) u_\mu \theta_\mu(t)\sigma_\mu}{1-\gamma_\mu} + \left( \theta'_z d_z \sigma_z + \theta_\mu(1-1/\psi) u_\mu \sigma_z \right) \hat{z}_t, \quad \hat{z}_t.
\]

where \( \theta'_z = \theta_z(t)/\sqrt{\hat{z}_t} \).

**Proof of Proposition OA.6:** Under the assumption of limited market participation, the shareholders act as a representative agent on the financial markets and, hence, the equilibrium price of
the market asset is equal to the shareholders’ wealth. Therefore, using the results of Proposition OA.1, the market asset price can be written as

\[ P_t = Q_{s,t} = C_{s,t} e^{-cqt} = X_t e^{-\log \beta + \mu_0 \chi^{-1} + d_0 + u_\mu \chi^{-1} \mu_\chi + (u_z \chi^{-1} + d_z) \xi_t}. \]

The dynamics of the market asset price obtains by applying Itô’s Lemma to \( P_t \):

\[ dP_t = \left[ \frac{\partial}{\partial x} P_t \sigma_x dB_{x,t} + \frac{\partial}{\partial \mu} P_t \sigma_\mu dB_{\mu,t} + \frac{\partial}{\partial z} P_t \hat{\sigma}_z \sqrt{\hat{z}_t} dB_{z,t} \right] dt. \]

Therefore the return volatility is given by

\[ \sigma_P(t) = P_t^{-1} \sqrt{\left( \frac{\partial}{\partial x} P_t \sigma_x \right)^2 + \left( \frac{\partial}{\partial \mu} P_t \sigma_\mu \right)^2 + \left( \frac{\partial}{\partial z} P_t \hat{\sigma}_z \sqrt{\hat{z}_t} \right)^2} = \sqrt{\sigma_2^2 + (u_\mu \chi^{-1} \sigma_\mu)^2 + (u_z \chi^{-1} + d_z) \hat{\sigma}_z \sqrt{\hat{z}_t}^2}, \]

and the premium is given by

\[ (\mu_P - r)(t) = -\frac{1}{dP_t} \left< \frac{d\xi_{0,t}}{\xi_{0,t}}, \frac{dP_t}{P_t} \right> = \theta_x(t) \sigma_x + \theta_\mu(t) u_\mu \chi^{-1} \sigma_\mu + \theta_z(t) (u_z \chi^{-1} + d_z) \hat{\sigma}_z \sqrt{\hat{z}_t}. \]

q.e.d.