# Trading Frictions in the Interbank Market, and the Central Bank[*]

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## Abstract

We present a core-periphery model of trading in the overnight interbank market. We model periods of crises as an increase in the number of periphery banks that lose access to core dealers, resulting in segmentation between core and periphery markets. Our model implies that such an increase in segmentation raises (i) the bargaining power of periphery banks connected to the core, (ii) the dispersion of prices in the interbank market, and (iii) inefficient resort to the central bank standing facilities. We argue that these implications are consistent with stylised facts and we use the model to study how optimal monetary implementation should respond to increased segmentation of interbank markets.

*Keywords*: Interbank markets, OTC markets, Bargaining Power, Monetary Policy Implementation, Corridor System.

# 1    Introduction

Overnight interbank markets play an important role in the economy. They serve as a mechanism to reallocate reserves from banks with an excess of reserve balances to banks with a deficit of reserve balances. The interest rates at which transactions take place in this market determine short term interest rates, and thus indirectly all interest rates in the economy. They also increasingly serve for indexing interest rates derivatives.[1] Hence, central banks often intervene in overnight interbank markets to implement their monetary policy (see, for instance, Bindseil (2005)). As a result, understanding how prices are formed in these markets is of considerable interest.

Virtually all interbank markets are organized as over the counter (OTC) markets. The terms of trade are arranged bilaterally between lenders and borrowers and reflect their respective bargaining power. The literature on OTC markets emphasizes the importance of search frictions to understand price formation in these markets (see, for instance, Duffie, Garleanu, and Pedersen (2005) or Afonso and Lagos (2015)). In this paper, we focus on another type of frictions: the extent to which smaller (periphery) traders (banks) are connected to larger (core) traders, and we study how the implementation of monetary policy should depend on the level of segmentation between the periphery and the core markets.

Our analysis is motivated by a series of stylised facts about interbank markets. First, there is increasing evidence that these markets have a tiered ("core-periphery") structure (see, for instance, Bech and Atalay (2010), Craig and von Peter (2014), or Finger, Fricke, and Lux (2013)). The core is populated by a few large dealer banks that trade very frequently among each other, while the periphery is composed of smaller banks with less frequent trading activity.[2]    As a

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[1]For instance, the periodic floating payments of overnight indexed swaps (OIS) swaps is computed using rates for overnight unsecured lending between banks, such as the FED fund rates for U.S. dollars or the Eonia for the Euros. The spread between the OIS and LIBOR rates is also considered as a measure of the fragility of the banking system and stress in money markets. Thus, understanding the way overnight rates are determined is key since it is a possible cause of fluctuation in the LIBOR-OIS spread.

[2]Existing studies (e.g. Cocco, Gomes, and Martins (2009), Brauning and Fecht (2012), Afonso, Kovner, and Schoar (2011)) show that the interbank network is very sparse, and that most banks rely on a limited set of

result, few banks (the core) trade with many counterparties (the periphery), who themselves have relatively few trading options. This structure closely resembles that of many other OTC markets, e.g., for municipal bonds (Li and Schuerhoff (2014)), European sovereign bonds (Dunne, Hau, and Moore (2015)), and Credit Default Swaps (Peltonen, Scheicher, and Vuillemey (2014)).

Importantly, the structure of the interbank market is subject to variations over time. In particular, there is evidence that peripheral banks tend to lose access to the core during crisis episodes.[3] Figure 1 illustrates this fact for the Euro area. It plots kernel density estimates for the distribution of the number of counterparties for across 271 Euro area banks (aggregated at the group level). The blue line refers to June 2008 (i.e., before the Lehman bankruptcy), while the red line corresponds to data for November 2011 (at the height of the sovereign debt crisis and before the ECB's 3-year long-term refinancing operation in December 2011). Two observations can be made. First, the market is generally very concentrated (in both periods), as most banks trade with relatively few counterparties. Second, the interbank network has thinned out throughout the sovereign debt crisis, so that banks have to rely on fewer counterparties for their trading needs (the distribution has shifted to the left over time).

A second observation is that this segmentation of the euro area interbank market during the crisis also coincided with an increase in the dispersion of interest rates at which transactions take place in the interbank market (Figure ??) and the likelihood that banks resort to the ECB lending and borrowing facilities (Figure 3). The ECB operates a so-called corridor system, which allows banks with excess reserve balances to deposit them at the central bank and earn interest on them, while banks with a deficit of reserves can borrow from the ECB at a penalty rate.[4]

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counterparties.

[3]There is considerable evidence that the global financial crisis and the European sovereign debt crisis profoundly affected the structure of interbank markets. For example, Afonso, Kovner, and Schoar (2011) show a decline in borrowing volumes and the number of counterparties directly following the Lehman bankruptcy. In turn, access to the Fed's discount window increased significantly. Similarly, Garcia de Andoain, Hoffmann, and Manganelli (2014) document that market access was limited for banks from peripheral countries throughout the sovereign debt crisis.

[4]Modern central banks implement their monetary policy through steering the overnight interest rate. Many rely on a corridor system (e.g., the Bank of England, the Bank of Canada, and to some extent the US Federal

Figure 3 shows that the volume of transactions at the ECB lending and borrowing facilities was close to zero until October 2008 and has then considerably increased.

A natural question is whether these facts are related and how monetary policy should respond to segmentation in interbank markets. To address this question, we develop a core-periphery model of trading in the overnight interbank market. In the periphery market, only a fraction $(1 - \lambda)$ of banks are connected (i.e., can trade with) to core banks (the "core"). Thus, when $\lambda$ increases, the periphery market becomes more segmented from the core market.

As is standard in the literature (e.g., Poole (1968), Bech and Monnet (2015) or Afonso and Lagos (2015)), core and periphery banks are hit by liquidity shocks (random shocks to their reserves balances). These shocks generate gains from trade between banks with a shortage of reserves ("borrowers") and banks with an excess of reserves ("lenders").

In our model, banks trade in three steps: (i) Periphery banks first trade together, (ii) periphery banks that do not find a trading partner in the periphery market trade with core banks if they are connected to them, and with the central bank otherwise, and (iii) core banks resort to the central bank's standing facilities if necessary (that is, if they end up with a shortage or an excess of reserve balances). Thus, core banks effectively act as dealers for periphery banks and the central bank acts as a liquidity provider of last resort for all banks. The central bank chooses the rates at which it lends or remunerates reserves (the difference between these rates defines the width of the corridor). In equilibrium, the rate in the core market is such that core banks' demand for reserves is just equal to the supply of reserves to core banks by periphery banks and the central bank. Thus, core banks' marginal valuations given their equilibrium reserves holdings

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Reserve since October 2008), whereby banks can borrow at a rate $R^P$ and deposit at a rate $R^D$, with $R^P > R^D$, thus giving a lower and upper bound on the rates at which banks trade with each other. The central bank steers their supply of reserves via open market operations in order to smoothen out aggregate liquidity shocks. Banks need to fulfil their reserve requirements with the central bank over a specific time period, and they trade with each other in the interbank market in order to neutralize idiosyncratic shocks. See Bindseil (2005) for a more detailed description.

is just equal to the equilibrium core rate, very much as in Poole (1968).[5]

As in other models of interbank markets (e.g., Bech and Monnet (2015) or Afonso and Lagos (2015)), the bargaining power of lenders relative to borrowers declines with aggregate excess reserves of periphery banks. Thus, equilibrium rates decrease with the aggregate reserves of periphery banks and the amount of liquidity injected by the central bank. The new effects in our model stem from (i) the distinction between core and periphery banks and (ii) the segmentation of the core and periphery markets.

In the benchmark case, all periphery banks are connected to the core ($\lambda = 0$). In this case the equilibrium in our model is identical to that in Poole (1968): all trades (in the core and the periphery market) take place at the same rate (no price dispersion). Moreover, the equilibrium outcome is efficient: all mutually beneficial trades take place and core banks need to resort to the central bank only when, in aggregate, there is an excess or a shortage of liquidity in the system.

Instead, when there is segmentation between the core and the periphery market ($\lambda > 0$), the equilibrium outcome is different because periphery banks connected to the core have a more attractive outside option than unconnected banks. Thus, in the periphery market, connected banks have more bargaining power than unconnected ones. This feature generates price dispersion: in equilibrium, trades in the periphery market take place at different rates across different trading partners. For instance, when, in aggregate, periphery banks are short of reserves, periphery banks with excess reserve balances lend their reserves at a markup relative to core dealers' rate and this markup is larger for connected banks.[6] Moreover, uncertainty on whether a trading partner

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[5]The determination of the core market equilibrium rate is also related to the determination of equilibrium prices in so called inventory models of dealership markets in market microstructure (see Foucault, Pagano, and Roell (2013), Ch.3 and 4).

[6]A significant share of the empirical literature examines the dispersion in borrowing rates across different types of banks and relates it to variations in bargaining power (see, for instance, Furfine (2001), Ashcraft and Duffie (2007), Bech and Klee (2011), Allen, Chapman, Echenique, and Shum (2012)). For instance, Ashcraft and Duffie (2007) document that lenders with a higher trading activity charge higher interest rates to borrowers with little trading activity, in line with an important role of traders' positioning inside the interbank network and the associated trading opportunities. Similarly, they show that bank's eagerness to trade (i.e. their relative level of reserves) is an important pricing factor.

is connected or not can lead periphery banks to ask "too much" in the negotiation (i.e., ask for rates such that connected banks prefer to trade with core dealers). When this happens, the equilibrium is inefficient in the sense that mutually profitable trades between periphery banks do not take place.[7] This has two related consequences: (i) the volume of trading in the core market increases, (ii) the frequency at which unconnected periphery banks resort to central bank facilities increases as well (relative to the benchmark case). Holding the parameters of monetary policy (e.g., the amount of liquidity injected by the central bank or the width of the corridor) constant, the equilibrium outcome is more likely to be inefficient when more periphery banks are lacking access to the core.[8]

Overall, these implications accord well with the stylised facts documented in Figures 1 to 3. Moreover, our model predicts that inefficient outcomes should be more frequent when fewer banks are connected to the core and/or injections of central bank reserves with core banks are larger. This is consistent with the empirical findings in Allen, Chapman, Echenique, and Shum (2012) for the Canadian overnight interbank market. Indeed, they find that inefficient outcomes in this market are more frequent after the onset of the financial crisis (which we associate with an increase in segmentation between the periphery and the core) and are positively correlated with injections of cash balances by the Bank of Canada.[9]

In the last part of the paper, we use our model to study the optimal implementation of monetary policy by the central bank. We show that when all banks are linked to the core, or in

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[7]Our definition of efficiency is related to that used by Allen, Chapman, Echenique, and Shum (2012) in their study of the canadian interbank market.

[8]Acharya, Gromb, and Yorulmazer (2012) develop a model of interbank loans that also generates a suboptimal equilibrium due to the market power of surplus banks. Their model features two banks bargaining over a loan with a long maturity. In contrast, we study a market for overnight loans with a continuum of banks, and the inefficiency is due to the combination of asymmetric information and OTC trading. Heider, Hoerova, and Holthausen (2015) show how counterparty risk, which is absent in our model, can also generate inefficient equilibria in interbank markets.

[9]Garcia de Andoain, Hoffmann, and Manganelli (2014) analyze the European interbank market throughout the sovereign debt crisis. They document that banks from peripheral countries are charged significantly higher borrowing rates, and increasingly fail to satisfy their liquidity needs in the private market, forcing them to resort to the ECB's liquidity operations.

"normal times", the central bank optimally uses liquidity injections (e.g., through open market operations) to offset expected liquidity imbalances, and sets the rates of its two standing facilities symmetrically around the target rate. This is a "corridor system", typical of monetary policy implementation of many central banks before the crisis.

When many banks lose access to the core, or in "crisis times", such a policy leads to deviations from the target rate because different types of banks face different rates. To reach the target, it is possible to inject large amounts of liquidity, and set the rate of the deposit facility equal to the target rate. Many central banks have actually responded to the crisis by moving to such a "floor system". However, our framework highlights two limitations. First, such a system can work only if the central bank can give more liquidity to both periphery banks and core banks. In normal times, this is not necessary because all periphery banks have access to the core and are thus indirectly affected by liquidity injections.[10] Second, if allowing the central bank to expand its balance sheet a lot has long-run costs, there is a trade-off between interest rate targeting and balance sheet expansion. Our model allows us to study this trade-off and can shed light on the optimal mix between injecting liquidity and manipulating the rates of the standing facilities.

The paper is organized as follows. In Section 2, we describe the model and derive its equilibrium. In Section 3, we discuss the testable implications of the model and in Section 4, we analyze its implications for the implementation of monetary policy. Section 5 concludes. Proofs of the main results are in the appendix.

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[10]In line with this result, we observe that several central banks eased access to liquidity by creating new liquidity-providing facilities, enlarged the set of eligible participants for regular and ad-hoc operations, and widened the pool of eligible collateral. Examples for new facilities include programmes for foreign currency swaps, and long-term refinancing operations. See, e.g., Ben Bernanke's speech "The Crisis and the Policy Response", January 13, 2009, Jean-Claude Trichet's speech "The ECB's response to the crisis", 20 February, 2009, and Paul Tucker's speech "The Repertoire of Official Sector Interventions in the Financial System: Last Resort Lending, Market-Making, and Capital", 28 May, 2009.

# 2 A core-periphery model of the overnight interbank market

In this section we describe the model of trading in the interbank market that we use to analyze monetary policy implementation by the central bank. As explained later, although the model is highly stylised, it delivers many of the empirical facts mentioned in the previous section.

## 2.1 Assumptions

### 2.1.1 Market participants and timing

We consider an interbank market with three types of participants: (i) a continuum of "periphery" banks, (ii) a continuum of "core" banks, and (iii) the central bank. All banks can borrow reserves from the central bank, using its *marginal lending facility*, at rate $R^P$ or deposit reserves at the central bank, using the *marginal deposit facility*, at rate $R^D$ (in the paper, rates are gross rates: one plus the rate of return). Thus, $R^P$ and $R^D$ define the corridor set by the central bank.

Gains from trade exist in the interbank market because $R^P > R^D$. Indeed, a bank with a deficit of reserves at the end of the game pays a cost of $R^P$ to the central bank, while a bank with a surplus of reserves earns $R^D$. Hence, a trade in which banks with a surplus of reserves lend reserves to banks with a deficit of reserves, at a rate within the corridor $[R^D, R^P]$, is mutually beneficial. Thus, a natural measure of the inefficiency of the interbank market is the fraction of trades that eventually take place with the central bank.

The timing of the model and the set of possible actions for market participants at each date, $t$ are as follows.

- **Date t = −1**: The central bank sets its corridor rates, $R^P$ and $R^D$ and chooses $m = m^{co} + m^{pe}$, the amount of liquidity provided ($m > 0$) or withdrawn ($m < 0$) in the interbank market, through open market operations. Parameters $m^{co}$ and $m^{pe}$ denote the resulting changes

in reserves for core and periphery banks.[11] We endogenize $m^{co}$, $m^{pe}$, $R^P$, $R^D$ in Section 4.

- **Date t = 0**: Periphery banks are hit by an aggregate liquidity shock $a^{pe} \hookrightarrow \mathcal{N}(\bar{a}_{pe}, \sigma_{pe}^2)$, observed by all banks. We denote $\Phi_{pe}(\cdot)$ the c.d.f. of $a^{pe}$, and $\phi_{pe}(\cdot)$ the pdf. Following this shock, the aggregate amount of excess reserves for periphery banks is equal to $\theta(a^{pe} + m^{pe}) = a^{pe} + m^{pe}$.[12] For brevity, we will often denote $\theta(a^{pe} + m^{pe})$ simply by $\theta$.

A mass $\pi_L(\theta)$ of periphery banks (lenders) has a net excess reserve equal to $l(|\theta|) > 0$ and a mass $\pi_B(\theta)$ of banks (borrowers) has a net excess reserve of $-l(|\theta|)$, with $\pi_L(\theta) + \pi_B(\theta) = 1$. As, by definition, the amount of excess reserves for periphery banks, $l(|\theta|)(\pi_L(\theta) - \pi_B(\theta))$, must be equal to $\theta$, we set:

$$\pi_L(\theta) = \frac{\theta}{2l(|\theta|)} + \frac{1}{2}. \tag{1}$$

We assume the function $l(.)$ to be such that the ratio $\frac{|\theta|}{l(|\theta|)}$ is increasing in $|\theta|$, is strictly lower than 1 and converges to 1 as $|\theta|$ goes to infinity.[13] These assumptions imply that both $\pi_L$ and $\pi_B$ are less than 1, that $\pi_L(\theta) = \pi_B(-\theta)$, and that $\pi_L$ increases with $\theta$. Moreover, $\pi_L$ converges to 1 when $\theta$ goes to $+\infty$, and $\pi_B$ converges to 1 when $\theta$ goes to $-\infty$. If $\theta > 0$ (i.e., periphery banks have strictly positive net excess reserves in aggregate) then the mass of lenders among periphery banks exceeds the mass of borrowers ($\pi_L(\theta) > \pi_B(\theta)$) and vice versa if $\theta < 0$. If $\theta = 0$, the mass of lenders is just equal to the mass of borrowers.

- **Date t = 1**: Trading takes place between periphery banks (see below).

- **Date t = 2**: Trading between core banks and high types periphery banks that have not found a trading partner at date $t = 1$ takes place at rate $R^{co}$.

- **Date t = 3**: Core banks receive a final shock on their excess reserves $a^{co} \hookrightarrow \mathcal{N}(\bar{a}_{co}, \sigma_{co}^2)$.

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[11]In practice, some central banks control the set of banks that are eligible for its monetary policy operations (see Kraenzlin and Nellen (2015)), which allows for differences in $m^{co}$ and $m^{pe}$. If the central bank cannot discriminate between core and periphery banks then $m^{co} = m^{pe} = m/2$.

[12]We normalize the required reserve balances for banks to zero. Thus, banks with negative reserves must borrow reserves to reach the required level of reserves and banks with positive reserves have excess reserves.

[13]For instance, one can assume $l(|\theta|) = 1 + |\theta|$.

We denote by $\Phi_{co}(.)$ the c.d.f of $a^{co}$, and $\phi_{co}(\cdot)$ the pdf.[14]

- **Date t = 4** : Banks with a surplus of reserves deposit them at rate $R^D$ at the central bank while banks with a deficit of reserves borrow at rate $R^P$ from the central bank.

This timing captures the fact that the core market acts as a market of "last resort" for periphery banks. This assumption is consistent with evidence on OTC markets with a core-periphery structure (see, for instance, Li and Schuerhoff (2014)). We assume that a fraction $\lambda \leq \frac{1}{2}$ of periphery banks are not connected to core dealers. Thus, if these banks do not find a trading partner at date 1, they have to trade with the central bank at date 4. We refer to periphery banks that are connected to the core as "*high type*" banks and to banks that are not as "*low type*" banks.

The main goal of the paper is to study how the equilibrium of the interbank market depends on $\lambda$. This parameter captures the fact that, in reality, some periphery banks cannot trade with large dealer banks and that, in crisis periods, more periphery banks are in this situation. Thus, as $\lambda$ increases, the market becomes more segmented: fewer periphery banks can trade with core banks. This segmentation is one source of friction in the model that prevents banks from efficiently realizing gains from trade in the interbank market. Another source of friction is the fact that trading in the periphery market is organized in a less efficient way than trading in the core market, as explained in the next subsection.

### 2.1.2   Market structure

**The periphery market.** The periphery market is a decentralized market where banks arrange trade bilaterally. Periphery banks arrive sequentially. Upon arrival, bank $j$ can either make a take-or-leave it offer (a rate) for lending or borrowing $l(|\theta|)$ units of reserves (depending on

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[14]The possibility of a last liquidity shock after trading in the interbank market is a standard asumption in the literature on interbank market (see Poole (1968) and Bindseil (2005)). If $\sigma_{co}^2 = 0$ (no shock), the equilibrium rate in the core market is necessarily $R^P$ or $R^D$ (see Section 2.3). This is a special case of the model.

whether it is a borrower or a lender) to its successor, $j + 1$, or accept the offer (if it exists) of its predecessor, $j - 1$.[15] A bank automatically rejects the offer of its predessor if it has the same trading need (e.g., both banks are borrowers). Otherwise, it rejects this offer if it expects to obtain a better rate with another trading partner (see below).

If bank $j$ is a lender, its successor is a borrower with probability $\beta(\theta)$ (or a lender with probability $(1 - \beta(\theta))$). If bank $j$ is a borrower then its successor is a lender with probability $\alpha(\theta)$. If two periphery banks trade together, they exit the market. If bank $j$ makes an offer and this offer is rejected then bank $j$ can trade in the core market at date $t = 2$ if it is connected to this market (its type is high). Otherwise, it trades at the central bank standing facilities at date $t = 4$.[16]

Thus, $\beta(\theta)$ and $\alpha(\theta)$ denote the transition probabilities from a lender to a borrower and a borrower to a lender in the "chain" of periphery banks. For consistency, these transition probabilities must be such that the stationary probability that bank $j$ is a lender (resp., borrower) is $\pi_L$ (resp., $\pi_B$). This imposes: $\frac{\alpha(\theta)}{\beta(\theta)} = \frac{\pi_L(\theta)}{\pi_B(\theta)}$. A simple specification for $\alpha(\theta)$ and $\beta(\theta)$ that satisfies this requirement is:

$$\alpha(\theta) = \min\left(\frac{1 - \pi_B(\theta)}{\pi_B(\theta)}, 1\right), \tag{2}$$

$$\beta(\theta) = \min\left(\frac{1 - \pi_L(\theta)}{\pi_L(\theta)}, 1\right). \tag{3}$$

Observe that $\alpha(\theta)$ weakly increases with $\theta$ while $\beta(\theta)$ weakly decreases with $\theta$. Moreover, when there are more lenders than borrowers (i.e., $\theta > 0$ so that $\pi_B(\theta) < 1/2$ and $\pi_L(\theta) > 1/2$)

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[15]For tractability, we assume that all trades between periphery banks are for exactly $|l(\theta)|$ units of reserves so that if banks trade together, their excess reserves after trading are just equal to zero. This assumption is natural since, by assumption, periphery banks have excess reserves of the same size ($|l(\theta)|$). Trading exactly this size removes exposure of periphery banks involved a trade to the risk of having to resort to the central bank at date $t = 3$.

[16]Trading with a core bank dominates, at least weakly, resorting to the central bank facility since $R^D \leq R^{co} \leq R^P$.

then $\alpha(\theta) = 1$ while $\beta(\theta) < 1$. In this case, borrowers are sure to be matched with a lender while lenders might not find a borrower in the periphery market since liquidity is plentiful. The opposite obtains when there are more borrowers than lenders. Thus, our model captures, in a simple way, the idea that finding a counterparty is more difficult for lenders (resp., borrowers) when, in aggregate, there is an excess (resp., deficit) of liquidity in the interbank market.

**The core market.** The core market operates like a centralized Walrasian market. Namely, each core dealer $i$ submits a demand function $q_i(R^{co})$ that specifies the amount of reserves that core dealer $i$ demands (i.e., borrows if $q_i(R^{co}) > 0$) at rate $R^{co}$. Periphery banks that trade in the core market just place an order to buy or sell $l(|\theta|)$ units of reserves depending on the direction of their excess reserve.[17] Thus, core banks effectively act as dealers for periphery banks since they absorb their net excess reserves. The equilibrium rate in the core market is such that the demand of reserves by core banks is equal to the supply of reserves (see Section 2.3).

## 2.2 Equilibrium in the periphery

In this section, we characterize the equilibrium actions of periphery banks in the periphery market. Periphery banks know their type but do not observe the types of other banks. Moreover, they form rational expectations on the rate in the core market. There is no uncertainty on the realization of this rate because, as we shall see in the next section, it is uniquely determined by $m^{co}$ and $\theta$, which are known to all banks at date 1.

Our first step is to formally define periphery banks' strategies and the equilibrium concept. Consider a high type borrower who has just received an offer $R_l$ from a lender. The borrower first decides whether to accept the offer or not. If he rejects the offer, he may either go to the core market, or make a new offer in the periphery market. In the latter case, he decides on the interest

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[17] As we shall see, $R^{co}$ is known once $\theta$ is known and is within the corridor in equilibrium. As periphery banks are price takers in the core market and face no additional reserve shocks, it is optimal for them to unload their reserves to core dealers.

rate he wants to offer. These decisions can be characterized by a triplet $S_b^H = \{d_b^H, q_b, \rho_b^H\}$, where: (i) $d_b^H(.)$ is a function of $R_l$ that takes a value 1 if the borrower accepts an offer at rate $R_l$, and 0 otherwise; (ii) $q_b$ is the probability that the borrower makes a new offer in the periphery market, rather than go to the core market, if he does not accept a previous offer; (iii) $\rho_b^H$ is a mixed strategy profile over the interest rates the borrower might offer to the next bank, with support $\mathbb{R}^+$. A particular case is of course a pure strategy, which simply specifies the rate offered by the borrower. The case of a low-type borrower is similar, except that a low-type bank does not have access to the core market. A low-type borrower's strategy is thus characterized by a pair $S_b^L = \{d_b^L, \rho_b^L\}$.

Symmetrically, a high type lender's strategy is characterized by $S_l^H = \{d_l^H, q_l, \rho_l^H\}$, where $d_l^H(R_b)$ takes a value 1 if the lender accepts an offer at rate $R_b$, $q_l$ is the probability that if the lender does not accept a previous offer he makes a new offer in the periphery market, and $\rho_l^H$ is a mixed strategy profile over the interest rates the lender can offer to the next bank. A low type lender's strategy is described by $S_b^L = \{d_l^L, \rho_l^L\}$.

We focus on Markov strategies (i.e., strategies chosen by banks in a given pair do not depend on the history of the trading game until they are matched) because trading in the interbank OTC market is opaque, which precludes the observation of the history of trades. A bank's strategy is not contingent on the type of the bank with which it is matched. This reflects the fact that banks do not know the type of other banks.[18]

Let $\varphi_l(R_l)$ be the likelihood that an offer $R_l$ made by a lender is accepted. The expected return of a high type lender who receives an offer $R_b$ from a borrower (i.e., an offer to borrow

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[18]Trading in OTC markets is usually not anonymous. Non anonymity however does not imply that a bank knows with certainty the current economic conditions of its trading partner, e.g., whether it has access or not to core dealers. Thus, $\lambda$ can be interpreted as a the likelihood that a given trading partner does not have access to the core market.

cash from the lender at $R_b$), follows strategy $S_l^H$ with a new offer at rate $R_l$ is:

$$V_l^H(R_b, S_l^H) = d_l^H(R_b)R_b + (1 - d_l^H(R_b))[q_l \varphi_l(R_l)R_l + (1 - q_l \varphi(R_l))R^{co}]. \quad (4)$$

That is, if the lender accepts the borrower's offer, he gets a return $R_b$ on his loan. If instead he turns down this offer, with probability $q_l$ he makes a new offer at $R_l$, which is accepted with probability $\varphi_l(R_l)$. With probability $1 - q_l \varphi_l(R_l)$, either he does not make a new offer in the periphery, or his offer is rejected, and in both cases the lender obtain a return $R^C$ by trading in the core market.

The situation for a low-type lender is similar, with the difference that he cannot choose to trade in the core, and has to lend at the deposit facility at rate $R^D$ if his offer is rejected:

$$V_l^L(R_b, S_l^L) = d_l^L(R_b)R_b + (1 - d_l^L(R_b))[\varphi_l(R_l)R_l + (1 - \varphi(R_l))R^D]. \quad (5)$$

Denoting $\varphi_b(R_b)$ the probability that an offer at rate $R_b$ is accepted by a lender, we obtain symmetric expressions for borrowers:

$$V_b^H(R_l, S_b^H) = -d_b^H(R_l)R_l - (1 - d_b^H(R_l))[q_b \varphi_b(R_b)R_b + (1 - q_b \varphi(R_b))R^{co}] \quad (6)$$

$$V_l^L(R_b, S_l^L) = -d_b^L(R_l)R_l - (1 - d_b^L(R_l))[\varphi_b(R_b)R_b + (1 - \varphi(R_b))R^P]. \quad (7)$$

Conditional on the offer it receives, a bank chooses the strategy that maximizes its expected payoff (hence, a borrower seeks to minimize his funding cost).

**Definition 1.** *A Markov-perfect equilibrium of the periphery market is a set of strategies $\{S_l^{H*}, S_l^{L*}, S_b^{H*}, S_b^{L*}\}$ such that (i) $S_l^{i*}$ maximizes the expected return of a lender of type $i \in \{H, L\}$ given that other banks behave according to strategies $\{S_l^{H*}, S_l^{L*}, S_b^{H*}, S_b^{L*}\}$ and (ii) $S_b^{i*}$ minimizes the expected*

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*funding cost of a borrower of type $i \in \{H, L\}$ given that other banks behave according to strategies*

$\{S_l^{H*}, S_l^{L*}, S_b^{H*}, S_b^{L*}\}.$

We denote by $V_l^{i*}$ and $V_b^{i*}$ the expected *equilibrium* payoffs of a lender and a borrower of type $i$, respectively, **after** rejecting an offer (i.e., in the continuation game that starts just after a bank rejects an offer). The following observations help to characterize basic properties of any equilibrium of the periphery market:

1. A bank accepts an offer if and only if it is as good as the average rate it could obtain by rejecting it. Hence, in equilibrium we have: $d_b^i(R_l) = 1$ if $R_l \leq -V_b^{i*}$, and 0 otherwise. Symmetrically, we have $d_l^i(R_b) = 1$ if $R_b \geq V_l^{i*}$, and 0 otherwise.

2. High-type banks can always play the same strategy as a low-type bank (the opposite is not true). Thus, $V_l^{H*} \geq V_l^{L*}$ and $V_b^{H*} \geq V_b^{L*}.$[19] Hence, any lender optimally chooses one of two rates when he makes an offer: (i) a relatively low rate $R_l = -V_b^{H*}$ that is accepted by all borrowers or (ii) a relatively high rate $R_l = -V_b^{L*}$ that is accepted by low-type borrowers only. Symmetrically, borrowers optimally announces one of two rates when they make an offer: $R_b = V_l^{H*}$, or $R_b = V_l^{L*}.$

3. High-type banks can borrow or lend at rate $R^{co}$ in the core market. Thus, in the periphery market, a high type lender cannot obtain a rate less than $R^{co}$ while a high type borrower cannot obtain a rate greater than $R^{co}$. As $V_l^{H*} \geq R^{co}$ and $V_b^{H*} \geq -R^{co}$, this means that a high type lender always makes offers at $R_l = -V_b^{L*}$, while a high-type borrower always makes offers at $R_b = V_l^{L*}$. Consequently, there is no transaction between high type banks in the periphery market.

From these observations, we deduce that all equilibria, including equilibria in mixed strategies,

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[19] Observe that $V_b^{H*}$ and $V_b^{L*}$ are negative because they represent the expected cost of funding for borrowers.

can be characterized as follows:

**Lemma 1.** *An equilibrium is fully characterized by the 4-tuple $(p_l, p_b, q_l, q_b)$, and the equilibrium behavior of banks is as follows:*

*- A high-type lender accepts any offer at rate $R_b \geq V_l^{H*}$. If he does not accept an offer, with probability $q_l$ he makes a new offer at rate $R_l = -V_b^{L*}$, and with probability $1 - q_l$ he directly goes to the core market.*

*- A low-type lender accepts any offer at rate $R_b \geq V_l^{L*}$. If he does not accept an offer, he makes an offer at rate $R_l = -V_b^{H*}$ with probability $p_l$, and an offer at rate $R_l = -V_b^{L*}$ with probability $1 - p_l$.*

*- A high-type borrower accepts any offer at rate $R_l \leq -V_b^{H*}$. If he does not accept an offer, with probability $q_b$ he makes a new offer at rate $R_b = V_l^{L*}$, and with probability $1 - q_b$ he directly goes to the core market.*

*- A low-type borrower accepts any offer at rate $R_l \leq -V_b^{L*}$. If he does not accept an offer, he makes an offer at rate $R_b = V_l^{H*}$ with probability $p_b$, and an offer at rate $R_b = V_l^{L*}$ with probability $1 - p_b$.*

To further characterize the equilibrium of the periphery market, we need to derive equilibrium rates and the probabilities $(p_l, p_b, q_l, q_b)$ with which these rates are offered by banks. It turns out that all equilibria belong to one four possible categories that we describe below:

- **Core-Periphery balanced:** $p_l = p_b = q_l = q_b = 1$. Low type lenders make offers that are accepted by all borrowers, and low type borrowers make offers that are accepted by all lenders. High-type banks' offers are only accepted by low-type banks on the other side. When they do not accept an offer, high-type banks always choose to make a new offer in the periphery.

- **Glut in the periphery:** $p_l = q_b = 1, p_b = q_l = 0$. Low type borrowers make offers that are only accepted by low type lenders ($p_b = q_l = 0$), while low type lenders make offers that are

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accepted by all borrowers ($q_b = p_l = 1$). High-type borrowers make offers that are accepted by low-type lenders only, while high-type lenders do not make an offer in the periphery market.

- **Shortage in the periphery:** $p_b = q_l = 1, p_l = q_b = 0$. Low type lenders make offers that are only accepted by low type borrowers ($q_b = p_l = 0$), while low type borrowers make offers that are accepted by all lenders ($p_b = q_l = 1$). High-type lenders make offers that are accepted by low-type borrowers only, while high-type borrowers do not make an offer in the periphery market.

- **Mixed:** In this equilibrium, either: (i) $p_l = q_b = 1$ and either $p_b$ or $q_l$ is not in $\{0, 1\}$ or (ii) $p_b = q_l = 1$ and either $p_l$ or $q_b$ is not in $\{0, 1\}$.

As we shall see (see Proposition 3), the first type of equilibrium obtains if the liquidity conditions are similar for core and periphery banks (e.g., both types of banks have an excess of liquidity). This is the reason why we refer to this equilibrium as "core-periphery balanced". In contrast, when periphery banks have a shortage of reserves while core banks enjoy an excess of reserves, one is more likely to obtain the third type of equilibrium. Hence we call this equilibrium "Shortage in the periphery." The "Glut in the periphery" equilibrium is symmetric (core banks have a deficit of reserves and periphery banks an excess). The mixed equilibrium is obtained for intermediate areas in which the differences in trading needs between core and periphery banks are large but not too large.

We now derive conditions under which a core-periphery balanced equilibrium obtains, taking $R^{co}$ as given. This is useful for two reasons. First, it shows how we solve for equilibrium rates. Second, these conditions are a necessary step to identify parameter values for which each type of equilibrium is obtained (see Proposition 3).

Denote $R_l^{i*}$ the interest rate offered by type $i$ lenders, and $R_b^{i*}$ the interest rate offered by type $i$ borrowers. A high-type lender's offer is accepted if and only if the lender is matched with

a low type borrower. This event has probability $\lambda\beta$, so that:

$$V_l^{H*} = \lambda\beta R_l^{H*} + (1 - \lambda\beta)R^{co}. \tag{8}$$

Symmetrically, a high-type borrower's offer being accepted with probability $\lambda\alpha$, we have:

$$V_b^{H*} = -\lambda\alpha R_b^{H*} - (1 - \lambda\alpha)R^{co}. \tag{9}$$

In a core-periphery balanced equilibrium, a low type lender's offer is accepted if and only if the lender is matched with a borrower, of any type, which gives:

$$V_l^{L*} = \beta R_l^{L*} + (1 - \beta)R^D. \tag{10}$$

And, symmetrically:

$$V_b^{L*} = -\alpha R_b^{L*} - (1 - \alpha)R^P. \tag{11}$$

Since low-type borrowers accept any offer with a rate $R_l \leq -V_b^{L*}$, it has to be the case that $R_l^{H*} = -V_b^{L*}$, otherwise the offers of high-type lenders would not be optimal. Symmetrically, $R_b^{H*} = V_l^{L*}$ in equilibrium. Conversely, since high-type borrowers accept offer $R_l$ as long as $R_l \leq -V_b^{H*}$, we necessarily have $R_l^{L*} = -V_b^{H*}$. Symmetrically, $R_b^{L*} = V_l^{H*}$. These four conditions combined with (8), (9), (10), and (11) form a linear system of 8 equations and 8 unknowns, the solution of which is:

$$R_b^{L*} = V_l^{H*} = \frac{(1 - \beta\lambda)R^{co} + \beta\lambda(1 - \alpha)R^P}{1 - \alpha\beta\lambda}, \quad R_b^{H*} = V_l^{L*} = \frac{(1 - \beta)R^D + \beta(1 - \alpha\lambda)R^{co}}{1 - \alpha\beta\lambda} \tag{12}$$

$$R_l^{L*} = -V_b^{H*} = \frac{(1 - \alpha\lambda)R^{co} + \alpha\lambda(1 - \beta)R^D}{1 - \alpha\beta\lambda}, \quad R_l^{H*} = -V_b^{L*} = \frac{(1 - \alpha)R^P + \alpha(1 - \beta\lambda)R^{co}}{1 - \alpha\beta\lambda}. \tag{13}$$

Equations (12) and (38) yield equilibrium rates in a core-periphery balanced equilibrium. It

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remains to derive the conditions under which banks have no incentive to deviate from their prescribed behavior in a core-periphery balance equilibrium. First, observe that we have $R_b^{H*} \leq R^{co}$ and $R_l^{H*} \geq R^{co}$. Hence, $q_l = q_b = 1$ is optimal for high type banks if they expect other banks to behave as in core-periphery balanced equilibrium. Second, we have $R_b^{L*} \leq R^P$ and $R_l^{L*} \geq R^D$: low-type banks are better off making an offer in the periphery than going to the central bank.

Last, we need to check that $p_l = p_b = 1$ is optimal for low type banks, that is, they must be better off making offers at $R_i^{L*}$, that are accepted by all types of banks, rather than offers at $R_i^{H*}$ that are accepted only by low type banks. Consider a low-type lender. In equilibrium, he obtains an expected profit of $V_l^{L*}$ when he makes an offer at $R_l^{L*}$ (the equilibrium offer). As explained previously, his most profitable deviation is to choose an offer at $R_l^{H*} = -V_b^{L*}$, which is accepted only by low-type borrowers, i.e., with probability $\beta\lambda$. Thus, this deviation is unprofitable if and only if:

$$V_l^{L*} \geq (1 - \beta\lambda)R^D - \beta\lambda V_b^{L*}. \tag{14}$$

Symmetrically, low-type borrowers do not deviate to targeting low-type lenders if and only if:

$$V_b^{L*} \geq -(1 - \alpha\lambda)R^P - \alpha\lambda V_l^{L*}. \tag{15}$$

Using (12) and (38), conditions (14) and (15) can be rewritten as:

$$\frac{R^P - R^{co}}{R^P - R^D} > \frac{\lambda(1 - \beta)}{1 - \beta\lambda(2 - \alpha\lambda)} \tag{16}$$

$$\frac{R^{co} - R^D}{R^P - R^D} > \frac{\lambda(1 - \alpha)}{1 - \alpha\lambda(2 - \beta\lambda)} \tag{17}$$

When $\theta = 0$, the core-periphery balanced equilibrium exists for all parameter values and is the unique equilibrium. Moreover, in this particular case, all transactions take place at $R^{co}$ (see Eq.(12) and (38)). For other realizations of $\theta$, at least one of these two conditions is satisfied

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because either $\alpha = 1$ (when $\theta > 0$) or $\beta = 1$ (when $\theta < 0$).

Now consider the case in which periphery banks have an excess of reserves, i.e., $\theta > 0$ (the case $\theta < 0$ is symmetric). In this case, Condition (16) is satisfied because $\alpha = 1$. Condition (17) will be satisfied as well if $R^{co}$ is close enough to $R^D$ (so that the L.H.S of (16) is close to one). In this case, low type borrowers find attractive to make offers that attract high type lenders because high type lenders are willing to accept low rates (since anyway the rate they can obtain in the core market is low). As $R^{co}$ increases however, it becomes increasingly costly for low type borrowers to make offers that attract both high and low type lenders and their incentive to switch to offers that only attract low type lenders increase. In the Appendix (Section B.1), we show that when $R^{co}$ is such that Condition (16) holds as an equality, they are just indifferent and a mixed equilibrium is obtained and that when $R^{co}$ is even higher, so that Condition (16) does not hold, then an equilibrium with glut in the periphery is obtained.

The next proposition summarizes these results and proves that for each set of parameter values, the equilibrium is unique.

**Proposition 1.** *The balanced equilibrium always obtains when $\theta = 0$. When $\theta > 0$, the balanced equilibrium obtains if and only if:*

$$\frac{R^P - R^{co}}{R^P - R^D} > \frac{\lambda(1 - \beta)}{1 - \beta\lambda(2 - \lambda)}. \tag{18}$$

*A mixed equilibrium with $p_l = q_b = 1$ and any $(p_b, q_l) \in [0, 1]^2$ obtains if there is equality in (18). Otherwise the equilibrium with glut in the periphery obtains.*

*When $\theta < 0$, the balanced equilibrium obtains if and only if:*

$$\frac{R^{co} - R^D}{R^P - R^D} > \frac{\lambda(1 - \alpha)}{1 - \alpha\lambda(2 - \alpha\lambda)}. \tag{19}$$

*A mixed equilibrium with $p_b = q_l = 1$ and any $(p_l, q_b) \in [0,1]^2$ obtains if there is equality in (19).*

*Otherwise the equilibrium with shortage in the periphery obtains.*

The existence conditions for the various possible equilibria and equilibrium rates depend, *inter alia*, on $R^{co}$, the rate at which high type periphery banks expect to trade in the core market. As this rate is endogenous and depends on other parameters of the model (e.g., $\lambda$), we postpone the complete description of the conditions under which the various equilibria are obtained and the expressions of equilibrium rates in terms of the exogenous parameters to Section 2.3 (see Propositions 3 and 4).

At the end of date 1, some periphery banks have not found a trading partner in the periphery market, either because they have been matched with a bank that has the same trading need or because their offer has been rejected. We denote by $\mu_{kl}^{u*}(\theta, \lambda)$ and $\mu_{kb}^{u*}(\theta, \lambda)$, the masses of lenders and borrowers of type $k \in \{L, H\}$ that remain unmatched in the periphery market. These masses are derived in Appendix B. They depend on which equilibrium is obtained but they do not depend on $R^{co}$.

Let $\Delta_1(\theta, \lambda) = l(\theta)[\mu_{Hl}^{u*} - \mu_{Hb}^{u*}]$ be the aggregate excess reserves of high type banks that have not been matched at the end of date 1. This variable is important as it determines the amount of reserves that core banks will have to lend to or borrow from high type periphery banks at date 2. Thus, it will play a role in the determination of the rate in the core market.

**Lemma 2.** *In equilibrium, the aggregate excess reserves $\Delta_1(\theta, \lambda)$ of high type periphery banks at the end of date 1 are positive if $\theta > 0$ and negative if $\theta < 0$. Their size $|\Delta_1(\theta, \lambda)|$ is equal to zero if $\theta = 0$ and increases with $|\theta|$.*

These properties are intuitive. If $\theta > 0$, there are more lenders than borrowers in the interbank market. Thus, high type borrowers are more likely to find a counterparty in the periphery market than high type lenders. As a result, the imbalance of reserves between periphery lenders and

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borrowers that trade in the core market has the same sign as the imbalance at the start of the periphery market.

## 2.3    Equilibrium in the Core Market

In this section, to close the model, we derive the equilibrium rate in the core market at date 2. Remember that $q_i(R^{co})$ denotes the demand of reserves of core bank $i$ at rate $R^{co}$ and that each core dealer can be hit by a shock $a^{co}$ at date 3. Thus, the expected profit of core dealer $i$ is:

$$E(\Pi_i) = R^D E(q_i + a^{co} | q_i + a^{co} > 0) + R^P E(q_i + a^{co} | q_i + a^{co} < 0) - R^{co} q_i. \tag{20}$$

Core dealers determine their demand for excess reserves to maximize their expected profit. Using Leibniz rule and the fact the c.d.f of shock $a^{co}$ is $\Phi_{co}$, the first order condition to this problem imposes:[20]

$$R^D(1 - \Phi_{co}(-q_i^*)) + R^P \Phi_{co}(-q_i^*) = R^{co}. \tag{21}$$

To understand this condition, observe that $\Phi_{co}(-q_i^*)$ is the probability that core dealer $i$ will be short of reserves at date $t = 4$ if its position in reserves at the end of date 2 is $q_i^*$. In this case, the bank will have to borrow at the central bank borrowing facility at $R^P$. Otherwise, core dealer $i$ will have a surplus of reserves that it will deposit at rate $R^D$. Thus, the L.H.S of Condition (21) is the rate at which core dealer $i$ expects to trade at date 4 if its position at the end of date $t = 2$ is $q_i^*$. That is, it gives the marginal valuation of dealer $i$ for reserves when it takes position $q_i^*$. Hence, Condition (21) states that each core bank determines its demand for reserves so as to equalize its marginal valuation for reserves to the rate at which trades take place in the core market. Condition (21) must hold for all core banks. Thus, in equilibrium, all core banks have the same demand function $q^*(R^{co})$.

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[20]The second order condition for a maximum is always satisfied because $R^D < R^P$.

Aggregate excess reserves owned by banks participating to the core market at date 2 have two components: (i) core banks excess reserves, $m^{co}$, and (ii) high type periphery banks who choose to trade in the core, $\Delta_1(\theta, \lambda)$. Thus, the aggregate supply of reserves in the core market is $S^{co}(m^{co}, \theta, \lambda) = m^{co} + \Delta_1(\theta, \lambda)$. The equilibrium rate in the core market, $R^{co*}$, is set such that the demand of core dealers for excess reserves is equal to the supply of excess reserves:

$$q^*(R^{co*}) = S^{co}(m^{co}, \theta, \lambda). \tag{22}$$

Hence, using eq. (21), we obtain the following result.

**Proposition 2.** *The equilibrium rate in the core market is:*

$$R^{co*} = (1 - \Phi_{co}(-S^{co}))R^D + \Phi_{co}(-S^{co})R^P, \tag{23}$$

*It decreases with the amount of excess reserves of banks trading in the core market ($S^{co}$) and is equal to the mid-point of the corridor ($\frac{R^P + R^D}{2}$) if and only $S^{co} = -\bar{a}_{co}$. Thus, the equilibrium rate in the core market decreases with the amount of monetary injection, $m^{co}$ or $m^{pe}$, and the aggregate liquidity shock of periphery banks, $a^{pe}$.*

As a benchmark, consider first the case in which $\lambda = 0$. In this case, all periphery banks have a high type and they all choose to trade in the core market. The amount of excess reserves owned by periphery banks is $\Delta_1(\theta, 0) = l(|\theta|)(2\pi_L - 1) = a^{pe} + m^{pe}$. Thus, $S^{co} = m^{co} + \Delta_1(\theta, 0) = m + a^{pe}$. In equilibrium, core dealers end up with a position in reserves equal to $S^{co}$: they "buy" (borrow) reserves if $S^{co} > 0$ and "sell" (lend) reserves if $S^{co} < 0$. The likelihood that after receiving their final liquidity shock, they end up with a long (resp., short) position is $(1 - \Phi_{co}(-S^{co}))$ (resp., $\Phi_{co}(-S^{co})$). Thus, as in Poole (1968), $R^{co*}$ is the expected rate at which core dealers expect to unwind their final inventories of reserves with the central bank given their position at the end of

date $t = 2$. It decreases with $S^{co}$ because when $S^{co}$ is higher, it becomes more likely that core dealers will have at date 4 a long position in reserves that they will deposit at rate $R^D$.

The same logic applies when $\lambda > 0$. In this case, however, the level of segmentation between core banks and periphery banks ($\lambda$) becomes an additional determinant of the equilibrium rate in the core market. The reason is that an increase in $\lambda$ reduces the size of excess reserves of high type periphery banks after trading at date 2 (Lemma 2). Thus, in absolute value, the supply of excess reserves in the core market becomes smaller when $\lambda$ becomes larger. As a result, $R^{co*}$ becomes closer to $\frac{R^P + R^D}{2}$. For instance, suppose that $\theta > 0$ and $m = 0$. In this case, $S^{co} = \Delta_1(\theta, \lambda) > 0$. Thus, $R^{co*} > \frac{R^P + R^D}{2}$. If $\lambda$ increases then $\Delta_1(\theta, \lambda)$ and therefore $S^{co}$ decreases. As a result, $R^{co*}$ gets closer to the mid point of the corridor. Symmetric effects are obtained if $\theta < 0$.

Using the expression (23) for the equilibrium rate in the core market, we can now rewrite the conditions of existence for the various type of equilibria (given in Proposition 1) in terms of the exogenous parameters of the model ($\theta$, $\lambda$, and $m^{co}$) only.

Suppose first that $\theta > 0$, so that a core-periphery balanced equilibrium obtains if and only f Condition (18) holds. As $R^P - R^{co} = (R^P - R^D)(1 - \Phi_{co}(-S^{co}))$, Condition (18) is equivalent to:

$$m^{co} > -\Phi_{co}^{-1}\left(\frac{(1-\lambda)(1-\lambda\beta)}{1-\lambda\beta(2-\lambda)}\right) - \Delta_1(\theta, \lambda). \tag{24}$$

Thus, when periphery banks have an excess of reserves ($\theta > 0$), a core-periphery balanced equilibrium obtains if and only if the reserves of core banks ($m^{co}$) are large enough. Intuitively, this condition guarantees that $R^{co*}$ is small enough so that low type borrowers find attractive to make offers large enough to attract high type lenders, even though low type lenders would be willing to accept lower offers.

Now, let $m_b^+$ be the smallest value of $m^{co}$ such that Condition (24) holds as an equality. In

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this case one can build a mixed strategy equilibrium. When $m^{co}$ passes below $m_b^+$, one can build mixed equilibria such that Condition (24) holds as an equality as long as $m^{co}$ is larger than a threshold $m_g^+$. Intuitively, the mixing probabilities $q_l$ and $p_b$ affect $\Delta_1(\theta, \lambda)$ and can be chosen so that Condition (24) holds as an equality for a range of value for $m^{co}$. These mixed equilibria are ranked in terms of aggregate welfare, and we select the most efficient one. Finally, when $m^{co} \leq m_g^+$, Condition (24) cannnot be satisfied (even as an equality) and one obtains the glut in the periphery equilibrium.

The case in which $\theta < 0$ is symmetric. In sum we obtain the following result.

**Proposition 3.** *For any $\theta > 0$, there exist three thresholds $m_g^+ < m_1^+ < m_b^+$, and two functions $q^*(.), \bar{p}_n(.)$ such that a balanced equilibrium obtains for $m^{co} \geq m_b^+$ and an equilibrium with glut in the periphery obtains for $m^{co} \leq m_g^+$. For $m^{co} \in [m_1^+, m_b^+)$ we have a mixed equilibrium with $p_l = 1, q_b = q^*(1) \in (0, 1)$, and for $m^{co} \in (m_g^+, m_1^+]$ a mixed equilibrium with $p_l = \bar{p}_n(m^{co}), q_b = 0$.*

*For any $\theta < 0$, there exist three thresholds $m_s^- < m_1^- < m_b^-$ such that a balanced equilibrium obtains for $m^{co} \leq m_b^-$ and an equilibrium with shortage in the periphery obtains for $m^{co} \geq m_s^-$. For $m^{co} \in (m_b^-, m_1^-]$ we have a mixed equilibrium with $p_b = 1, q_l = q^*(1) \in (0, 1)$, and for $m^{co} \in (m_1^-, m_s^-]$ a mixed equilibrium with $p_b = \bar{p}_n(m^{co}), q_l = 0$.*

*If $\mu^{co} = 0$, we have $m_s^- = -m_g^+, m_1^- = -m_1^+$, and $m_b^- = -m_b^+$.*

Figure 4 illustrates Proposition 3.[21] When both $\theta$ and $m^{co}$ have the same sign, we obtain a balanced equilibrium. If they have opposite signs and large enough in absolute values, we obtain either an equilibrium with glut in the periphery or an equilibrium with shortage in the periphery, depending on the sign of $\theta$. Mixed equilibria are obtained in the regions between the balanced equilibrium and the corresponding glut/shortage equilibrium, but these regions are too small to be visualized.

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[21]We set $\ell(|\theta|) = 1 + |\theta|, \lambda = 0.4, \sigma^{co} = 1$.

Proposition 3 implies that monetary policy (the choice of $m^{co}$) affects the efficiency of the outcome obtained in the periphery market. Massive injection of liquidity in the core market when periphery banks have a deficit of reserve balance increases the likelihood of obtaining a liquidity shortage equilibrium in the periphery market and therefore the likelihood that periphery banks will use the borrowing facility.

The next proposition provides the rate obtained in each possible equilibrium type in the periphery market. Let $\omega_l^L(\lambda, \theta, m^{co}) = \frac{(1-\lambda)\Phi_{co}(-S^{co})}{1-\beta(\theta)\lambda}$, $\omega_l^H(\lambda, \theta, m^{co}) = 1 - \frac{\alpha(\theta)(1-\lambda)(1-\Phi_{co}(-S^{co}))}{1-\alpha(\theta)\lambda}$, $\omega_b^H(\lambda, \theta, m^{co}) = 1 - \frac{\beta(\theta)(1-\lambda)\Phi_{co}(-S^{co})}{1-\beta(\theta)\lambda}$, and $\omega_b^L(\lambda, \theta, m^{co}) = \frac{(1-\lambda)(1-\Phi_{co}(-S^{co}))}{1-\alpha(\theta)\lambda}$.

**Proposition 4.** *In a core-periphery balanced equilibrium, low type lenders make offers at $R_l^{L*}(\theta, \lambda, m^{co}) = (1-\omega_l^L)R^D + \omega_l^L R^P$ and high type lenders make offers at $R_l^{H*}(\theta, \lambda, m^{co}) = (1-\omega_l^H)R^D + \omega_l^H R^P$. Moreover, low type borrowers make offers at $R_b^{L*}(\theta, \lambda, m^{co}) = \omega_b^L R^D + (1-\omega_b^L)R^P$ and high type borrowers make offers at $R_b^{H*}(\theta, \lambda, m^{co}) = \omega_b^H R^D + (1-\omega_b^H)R^P$. In a shortage in the periphery equilibrium, rates are identical except that low type lenders make offers at $R_l^{H*}(\theta, \lambda)$. In a glut in the periphery equilibrium, rates are identical except that low type borrowers make offers at $R_b^{H*}(\theta, \lambda)$*

Each transaction in the periphery market involves a borrower and a lender and gains from trade are equal to $R^P - R^D$. The weights $\omega_l^k(\lambda, \theta, m^{co})$ are equal to the fraction of gains from trade earned by a lender of type $k \in \{H, L\}$ when the lender makes an offer. Symmetrically, the weights $\omega_b^k(\lambda, \theta, m^{co})$ are the fraction of gains from trade earned by a borrower of type $k \in \{H, L\}$ when the borrower makes an offer. Thus, $\omega_l^k(\lambda, \theta, m^{co})$ measures the bargaining power of a lender of type $k$ and $\omega_b^k(\lambda, \theta, m^{co})$ measures the bargaining power of a borrower of type $k$.

# 3   Implications

In this section, we describe various testable implications of the model.

## 3.1 Equilibrium Rates and Bargaining Power

We first study how periphery banks' aggregate reserves and open market operations ($m^{co}$ and $m^{pe}$) affects banks' bargaining power and equilibrium rates. The next corollary describes how the rates offered by lenders and borrowers vary with $\theta$.

**Corollary 1.** *In equilibrium, periphery lenders' bargaining power decreases with $\theta$ and $m^{co}$ while periphery borrowers' bargaining power increases with $\theta$ and $m^{co}$. Thus, equilibrium rates in the periphery market decreases with $\theta$ and $m^{co}$. When $\theta = 0$ and $m^{co} = 0$, the bargaining power of lenders is equal to the bargaining power of borrowers. In this case all transactions in the periphery market and in the core market take place at the same rate: $R^{co*}$.*

Consider the effect of an increase in $\theta$ when $\theta > 0$, so that $\beta < 1$ and $\alpha = 1$. This increase reduces lenders' bargaining power for two reasons. First, periphery banks expect $R^{co*}$ to be smaller. Hence high type lenders have a less attractive outside option. Second, lenders are less likely to find a counterparty in the periphery market if they reject an offer ($\beta$ decreases with $\theta$ for $\theta > 0$). This allows borrowers to extract more surplus from each transaction. When $\theta < 0$, the first effect is not changed, but the second is. Indeed, $\beta = 1$ and $\alpha < 1$. Thus, an increase in $\theta$ does not affect lenders' chance of finding a counterparty. However, it increases borrowers' likelihood of finding a counterparty if they reject an offer ($\alpha$ increases with $\theta$). This effect forces lenders to make more attractive offers to borrowers.

**Corollary 2.** *In a core-periphery balanced equilibrium, low type banks' bargaining power is strictly less than high type banks'. Thus, low type lenders receive lower rates than high type lenders and low type borrowers pay higher rates than high type borrowers. In a glut in the periphery equilibrium, low type borrowers' bargaining power is equal to that of high type lenders' bargaining power, while in a shortage in the periphery equilibrium, low type lenders' bargaining power is equal to that of high type lenders' bargaining power.*

In Afonso and Lagos (2015), the bargaining power of lenders and borrowers is determined by whether the measure of banks with excess reserve balances exceeds the measure of banks with a shortage of reserve balances. The effects of $\theta$ and $m^{co}$ on lenders and borrowers' bargaining power in our model (Corollary 1) is reminiscent of this result. Corollary 2 however is specific to our model. Combined with Proposition 3, it implies that for low values of $m^{co}$, low type borrowers' bargaining power should be higher than for relatively high values of $m^{co}$ and vice versa for low type lenders. Moreover, the differences in bargaining power between high type and low type banks should be higher for intermediate values of $m^{co}$. To test these predictions, one could build implied estimates of high types and low types banks' bargaining power using the same method as in Bech and Klee (2011).

Figures 5 and 6 illustrate Corollary 1. Figure 5 shows the evolution of lenders' bargaining power (Panel A) and borrowers' bargaining power (Panel B) for two different levels of monetary injections in core banks: $m^{co} = 0$ (black lines) and $m^{co} = 3$ (red lines). Parameter values are such that a balanced equilibrium always obtain. The dashed lines depict high type banks' bargaining power while the plain lines depict low type banks' bargaining power. Figure 6 shows equilibrium rates. The black lines are the rates offered by lenders and the red lines are the rates offered by borrowers. Plain lines are the rates offered by high type banks while dashed lines are the rates offered by low type banks. The green line is the rate in the core market.

Observe that in the balanced equilibrium, there are always three possible rates observed in the periphery market. The reason is that when $\theta > 0$, high type and low type lenders offer the same rate equal to $R^{co*}$ while when $\theta < 0$, high type and low type borrowers offer the same rate equal to $R^{co*}$. Thus, in equilibrium, rates vary from transaction to another holding $\theta$ constant. That is, the model features rate dispersion (cross-sectional dispersion of rates across transactions in the same trading session) as observed empirically.

## 3.2 Distribution of rates in the interbank market

Figure ?? shows that the increase in segmentation between periphery and core banks in the euro area interbank market coincides with an increase in the dispersion of the rates in the interbank market. Moreover, the distribution of these rates move from being unimodal before the onset of the crisis to being bi-modal during the crisis (see Figure ??). In this section, we show that the model can explain these stylised facts.

When $\lambda = 0$, all transactions take place at $R^{co*}$ in the model. Variations in this rate then stems only from variations in shocks to periphery banks' reserves ($\theta = a^{pe} + m^{pe}$). In contrast when $\lambda > 0$, there is dispersion in rates at which trading takes place between banks for each realization of $\theta$, as explained in Section 3.1. In this case, the probability distribution of rates within the corridor is determined by (i) the frequency at which various rates are offered in the periphery market, (ii) the mass of periphery banks that choose to trade with core dealers, and (iii) $\theta$.

Figure 7 shows this distribution for specific parameter values of the model. It is obtained as follows. We draw randomly 100,000 realizations of the shock to periphery banks' aggregate reserves, $a^{pe}$, from a normal distribution with mean zero and standard deviation 0.5. Then for each draw, we compute equilibrium rates and record the *likelihood* that these rates are observed in a transaction in equilibrium (using the expressions for the stationary probability distributions derived in Appendix B.2). For the equilibrium rate in the core market, this likelihood is given by the mass of periphery banks that trade with core banks. We then divide the interval between $R^D$ and $R^P$ in 1,000 bins and report the sum of all likelihoods for rates observed in each bin. Thus, by construction, if all transactions were taking place at a single rate for all values of $\theta$, Figure 7 would show a vertical line peaking at 100,000 at this rate.

In all simulations, we assume that core banks have zero excess reserves on average (i.e.,

$\overline{a}^{co} = 0$). Panel A of Figure 7 shows the distribution of the interest rate in the benchmark case, i.e., when $\lambda = 0$ and $m^{pe} = m^{co} = 0$. In this benchmark, all transactions take place at rate $R^{co*}$. Variations in this rate arise only because periphery banks experience random shocks to their reserves, as in the Poole (1968)'s model. However, the distribution of rates is unimodal and centered on the mid-point of the corridor.

In panel B we still assume that $\lambda = 0$ but we set $m^{pe} = -2$ and $m^{co} = 3$. In this case, all transactions still take place at the same rate $R^{co*}$ but, in aggregate, banks have an excess of reserves ($m = m^{pe} + m^{co} = 1$). This excess of liquidity shifts the distribution of $R^{co*}$ to the left relative to the mid-point of the corridor, as one would expect. The distribution of the rates however remains unimodal (it peaks at the value of $R^{co*}$ obtained for $a^{pe} = 0$, i.e., the mean value of the shock to periphery banks' aggregate reserves).

In panel C, we introduce segmentation and assume that $\lambda = 0.4$ (that is 40% of all periphery banks lose access to the core market) and we assume that $m^{pe} = m^{co} = 0$. The latter assumption implies that the equilibrium is always of the core-periphery balance type. In contrast to the benchmark case, for a given realization of $\theta = a^{pe}$, there is now dispersion of rates in the periphery market. In particular, we show the distribution of each possible rates ($R_l^H$, $R_l^L$, $R_b^H$, $R_b^L$). Rates offered by lenders are skewed to the left while rates offered by borrowers are skewed to the right. The resulting distribution of *all* rates however remains similar to that in the benchmark case (Panel A).

Finally in panel D, we assume that $\lambda = 0.4$ and $m^{pe} = -2$ and $m^{co} = 3$. In this case, depending on the realization of $a^{pe}$, the equilibrium can be core-periphery balance, or mixed, or with a shortage in the periphery. In this case, the distribution of rates is significantly different than that obtained in the benchmark case. In particular, it becomes clearly bimodal, as observed empirically during the European sovereign debt market. The first mode is below the mid point of

the corridor. It simply reflects the fact that (i) many transactions take place in the core market (especially because some connected periphery banks fail to trade in the periphery) and (ii) the rate in the core market is skewed to the left due to the excess of reserves for core banks ($m^{co} = 3$). The second hump is above the mid point of the corridor. It stems from the fact that, in aggregate, periphery banks have a shortage of liquidity. This shortage tends to push up the rates at which transactions take place in the periphery market. In particular, relatively many transactions take place at rates offered by low type borrowers, $R_b^H$.

Overall, this numerical simulation shows that the combination of (i) the loss of access to the core market for some periphery banks and (ii) the imbalance in terms of liquidity needs between periphery and core banks is a possible explanation for the bimodal distribution of interbank rates observed in the data (Figure **??**).

## 3.3 Inefficiency

*To be written.*

# 4 Central bank policy

In this section, we define an objective function for the central bank, which trades off achieving interest rates close to its target with inflating its balance sheet. We then study the optimal policy of the central bank in normal times ($\lambda = 0$), and show that this policy becomes suboptimal in crisis times ($\lambda > 0$).

## 4.1 Objectives

**Interest rate targeting.** The first objective of the central bank is to achieve a target interest rate $R^T$. For given $a^{pe}, m^{pe}, m^{co}$, we know the type of equilibrium that obtains and the different

interest rates offered by market participants. Denote $\nu_i^k$ the frequency at which the interest rate $R_i^{k*}$ is obtained on the market, with $i \in \{b, l\}$ and $k \in \{H, L\}$. Denote $\nu^{co}$ the frequency at which banks trade at $R^{co*}$. The central bank wants to minimize the squared error $\mathcal{E}$ between the realized interest rates and its target:

$$
\begin{aligned}
\mathcal{E} &= \mathbb{E}[(R - R^T)^2] = \int_{-\infty}^{+\infty} \varepsilon(a)\phi_{pe}(a)da \qquad (25) \\
\text{with } \varepsilon(a) &= \sum_{i=b,l}\sum_{j=H,L} \nu_i^j(R_i^{j*} - R^T)^2 + \nu^{co}(R^{co*} - R^T)^2
\end{aligned}
$$

$\varepsilon(a)$ measures the dispersion of interest rates in the cross-sectional dimension, i.e., the rates offered by different types of banks for a given shock $a^{pe}$. $\mathcal{E}$ additionally takes into account that the same type of bank will face a different interest rate depending on the shock $a^{pe}$. It thus includes volatility in the time dimension. The central bank should try to minimize $\mathcal{E}$ and thus have rates close to its target. Due to the square, the central bank cares not only about the average interest rate, but also about the expected difference to the mean.

**Central bank's costs.** Many central banks have reacted to the financial crisis by injecting large amounts of liquidity and thereby increasing the size of their balance sheets. Such a policy has costs: for instance, a central bank that buys bonds to loosens monetary policy and sells them when it tightens monetary policy will typically sell at a lower price than it purchased the bonds. While the central bank's objective is not to make a profit per se, ultimately losses have to be covered either by fiscal transfers or by inflation.[22] To take this phenomenon into account, we assume that the central bank faces a cost $C(|m|)$ that depends on the magnitude $|m|$ of its liquidity injection/absorption, where $C(.)$ is increasing, convex, and $C(0) = 0$.

We also take into account that the central bank makes intermediation profits. If banks

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[22]For more on these issues, see for instance Cochrane (2011) and Berentsen, Marchesiani, and Waller (2014).

deposit a volume $T^D$ at rate $R^D$ and borrow a volume $T^P$ at rate $R^P$, the central bank earns $T^P R^P - T^D R^D$. Finally, the central bank earns an interest on the liquidity it lends to the system. We model this by assuming that the central bank earns $R^T \times m$, or loses this amount if $m < 0$. The central bank's costs minus revenues are thus:

$$\mathcal{C} = C(|m|) - R^T \times m - \mathbb{E}[T^P R^P - T^D R^D]. \tag{26}$$

**Comparative inefficiency of the central bank.** The central bank could in principle entirely replace interbank trading by setting $R^P = R^D$. It would take deposits from all lenders, and lend to all borrowers. There are several reasons why central banks seem reluctant to pursue such a policy. Two reasons in particular are frequently mentioned: (i) Information: an active interbank market gives the central bank important information about liquidity shocks, as well as about the health of the banking system; (ii) Efficiency: the central bank faces operating costs when trading with banks, such as monitoring or risk management. If banks already perform these activities for other reasons (e.g., extending long term interbank credit), this is a wasteful duplication of resources.[23] We parsimoniously model these costs by assuming that all trades with the central bank have a social cost $c$ and define:

$$\Gamma = \mathbb{E}[T^D + T^P] \times c. \tag{27}$$

**Objective function.** Putting the different pieces together, the central bank's objective can be written as follows:

$$\min_{R^P, R^D, m} \mathcal{E} + \psi_1 \mathcal{C} + \psi_2 \Gamma, \tag{28}$$

where $\psi_1$ and $\psi_2$ measure the importance relative to rate targeting of reducing costs and ineffi-

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[23]See Bindseil (2005) for a longer discussion.

ciencies, respectively.

## 4.2 Monetary policy in normal times

We first consider the case of "normal times", with $\lambda = 0$ and thus no market segmentation. A balanced equilibrium obtains in which all periphery banks trade in the core, so that all trades take place at $R^{co}$. There is no dispersion of interest rates for a given shock $a^{pe}$, but there is still variation over time. Using Proposition 2, (25) reduces to:

$$\mathcal{E} = \int_{-\infty}^{+\infty} [\Phi_{co}(-a - m)R^P + (1 - \Phi_{co}(-a - m))R^D - R^T]^2 \phi_{pe}(a) da. \qquad (29)$$

The number of trades with the central bank only depends on the aggregate liquidity in the system $a^{co} + a^{pe} + m^{co} + m^{pe}$. If this quantity is positive then $T^D > 0$ and $T^P = 0$, if it is negative we have the opposite. $a^{co} + a^{pe}$ follows a normal distribution $\mathcal{N}(\bar{a}_{pe} + \bar{a}_{co}, \sigma_{pe}^2 + \sigma_{co}^2)$, whose pdf we denote $\phi_{tot}$. In expectation, we have:

$$\mathbb{E}(T^P) = -\int_{-\infty}^{-m} (m + a)\phi_{tot}(a) da, \ \mathbb{E}(T^D) = \int_{-m}^{+\infty} (m + a)\phi_{tot}(a) da. \qquad (30)$$

Based on these equations, we have the following:

**Proposition 5.** *When $\lambda = 0$ and central bank's costs are negligible ($\psi_1 = 0$), an optimal policy is to: (i) Offset the average liquidity shock with a liquidity injection $m = -(\bar{a}_{co} + \bar{a}_{pe})$; (ii) Set a symmetric corridor around $R^T$, $\frac{R^P + R^D}{2} = R^T$.*

This is the typical policy pursued by many central banks before the crisis. The optimality, which is proven in the Appendix, comes from the symmetry of the problem: rates are too low when $a^{co} + a^{pe} > 0$, but it is impossible to lower them without having too high rates when $a^{co} + a^{pe} < 0$. Interestingly, only the total quantity $m$ matters, not the breakdown between

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$m^{co}$ and $m^{pe}$. This comes from the absence of fragmentation, which implies that only the total amount of liquidity matters for determining the interest rates.

The proposition assumes that the central bank neglects the costs of its policy, in line with the idea that these costs are negligible in normal times, when the required amount of liquidity injection is small. Although deriving a closed-form optimum in the case $\psi_1 > 0$ is difficult, the direction in which this changes the optimal policy compared to the case of Proposition 5 is clear. Assume for instance that $\bar{a}_{co} + \bar{a}_{pe} << 0$. Proposition 5 dictates to inject a large amount of liquidity. However, this is now costly for the central bank. It will thus choose $m > 0$ but lower than $-(\bar{a}_{co} + \bar{a}_{pe})$. As a result, when the average shocks $a^{pe}$ and $a^{co}$ realize, the interest rate $R^{co}$ is strictly above $\frac{R^P + R^D}{2}$. This justifies a so-called "asymmetric" corridor, with $\frac{R^P + R^D}{2} < R^T$. In other words, $R^T$ is closer to $R^D$ than to $R^P$.

### 4.3  Monetary policy in crisis times

**Interest rate targeting only.** Monetary policy becomes more complex in crisis times, when $\lambda > 0$ and the market is fragmented. As a result, for a given shock $a^{pe}$, banks trade at different rates in the core and in the periphery. Moreover, the equilibrium is not necessarily balanced, so that banks can trade too much with the central bank, which increases the inefficiency $\Gamma$. In particular, we can simultaneously have $T^D > 0$ and $T^P > 0$. Even with fragmentation, the central bank can still achieve its interest-rate targeting objective:

**Proposition 6.** *If the central bank focuses only on interest-rate targeting, $\psi_1 = \psi_2 = 0$, the following three policies achieve a perfect targeting: (i) Inject $m^{co} \to +\infty, m^{pe} \to +\infty$ and set $R^D = R^T$; (ii) Absorb $m^{co} \to -\infty, m^{pe} \to -\infty$, and set $R^P = R^T$; (iii) Set $R^P = R^D = R^T$.*

*If the central bank cannot control $m^{pe}$, injecting or absorbing an infinite amount of liquidity in the core only does not achieve perfect targeting.*

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Policy (i) is sometimes called a "floor policy": the banking system is awash with liquidity, all banks ultimately make a deposit at the central bank, and any interbank transactions have to take place at close to $R^D$. This can be seen as an extreme version of how the Fed and the ECB responded to the crisis. Clearly, the cost of such a policy is a rapid growth of the central bank's balance sheet. (ii) is the symmetric case of a "ceiling policy", which has historical precedents (see Bindseil (2005)). (iii) is always a solution if $\psi_2 = 0$ and the term $\Gamma$ can be neglected. Of course, such an extreme policy effectively suppresses the interbank market. Still, in line with the idea that a narrower corridor can help reduce the dispersion and volatility of market interest rates, the ECB for instance announced on 8 October 2008 a reduction of the corridor width from 200 to 100 bps.

An interesting difference with normal times is that the central bank needs to inject liquidity both in the core and in the periphery. To see why, assume that we have $\theta < 0$ and $m^{co} \to +\infty$. Then indeed the rate $R^{co}$ is equal to $R^D$. However, according to Proposition 3, we obtain an equilibrium with shortage in the periphery, and low type banks will trade at rates higher than $R^D$. In particular, if $\theta \to -\infty$, we can simultaneously have banks trading at $R^D$ in the core and at $R^P$ in the periphery. In the United States, open market operations are traditionally limited to primary dealers, so that liquidity cannot be transmitted through this channel to banks that are not linked to the dealers. Accordingly, the Federal Reserve put in place additional facilities to provide liquidity to the banking system. In the Euro area, all banks can access the ECB marginal refinancing operations, but banks hit by the crisis may lack collateral to participate in these operations. In line with the idea that the central bank needs to increase $m^{pe}$ when $\bar{a}_{pe} < 0$, the ECB repeatedly widened the set of assets it accepts as collateral in periods of market stress.

**General case.** *To be written.*

# 5    Conclusion

Our model offers a theory of OTC trading in interbank markets in crisis times, that also nests the traditional model of Poole (1968) as a special, "normal times" case. We show that when some banks lose access to the group of "core" banks, liquidity imbalances between core and periphery banks can give rise to inefficient equilibria and a significant dispersion of the rates at which banks trade with each other. For instance, if there is a liquidity shortage in the periphery and an excess in the core, the lenders in the periphery try to exploit the fact that some borrowers cannot benefit from the low interest rate offered by core banks. As a result, some periphery banks can trade at very high rates even if the market as a whole benefits from excess liquidity.

This segmentation between core and periphery banks poses significant challenges to central banks. The central bank should fight the dispersion of interest rates, as it implies that the average rate no longer properly reflects the borrowing conditions of banks. This typically requires to move to a "floor system", in which the central bank injects a lot of liquidity. However, this system can be efficient only if the central bank can allocate liquidity both to core and to periphery banks, which can be challenging. Moreover, large liquidity injections can be costly, in which case the optimal policy trades off interest rate targeting with costs and uses both excess liquidity and the rates of the two standing facilities.
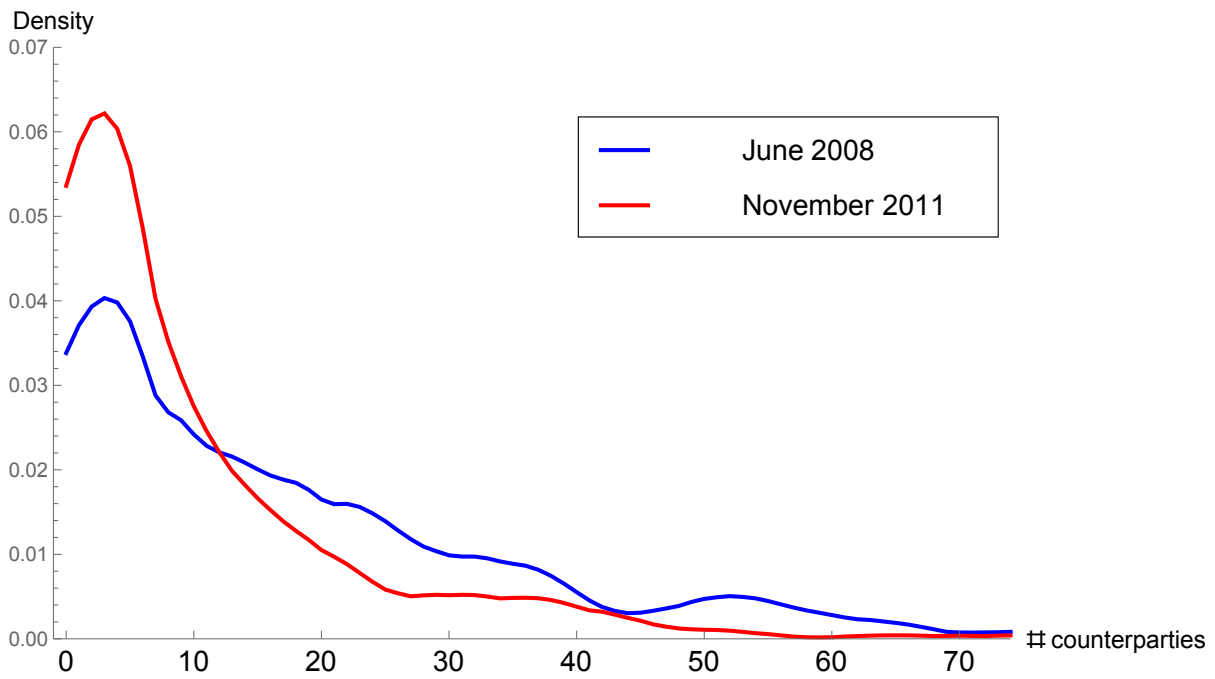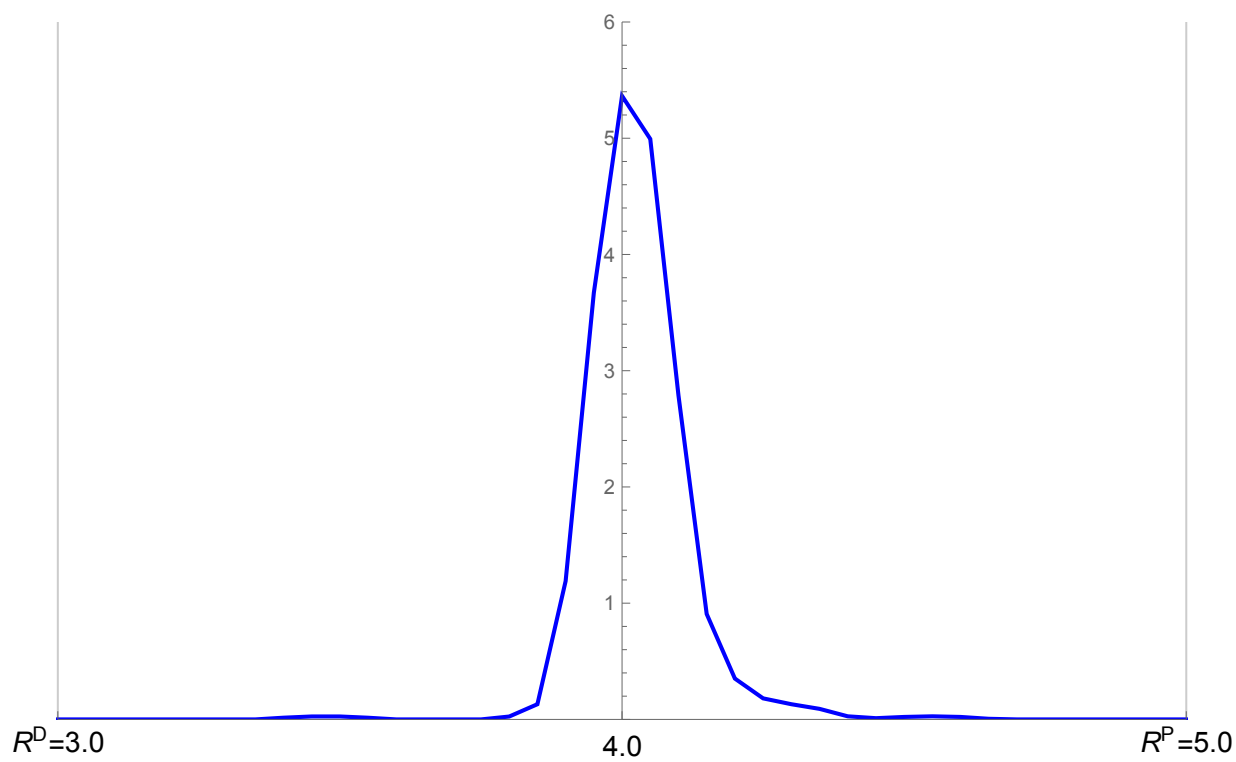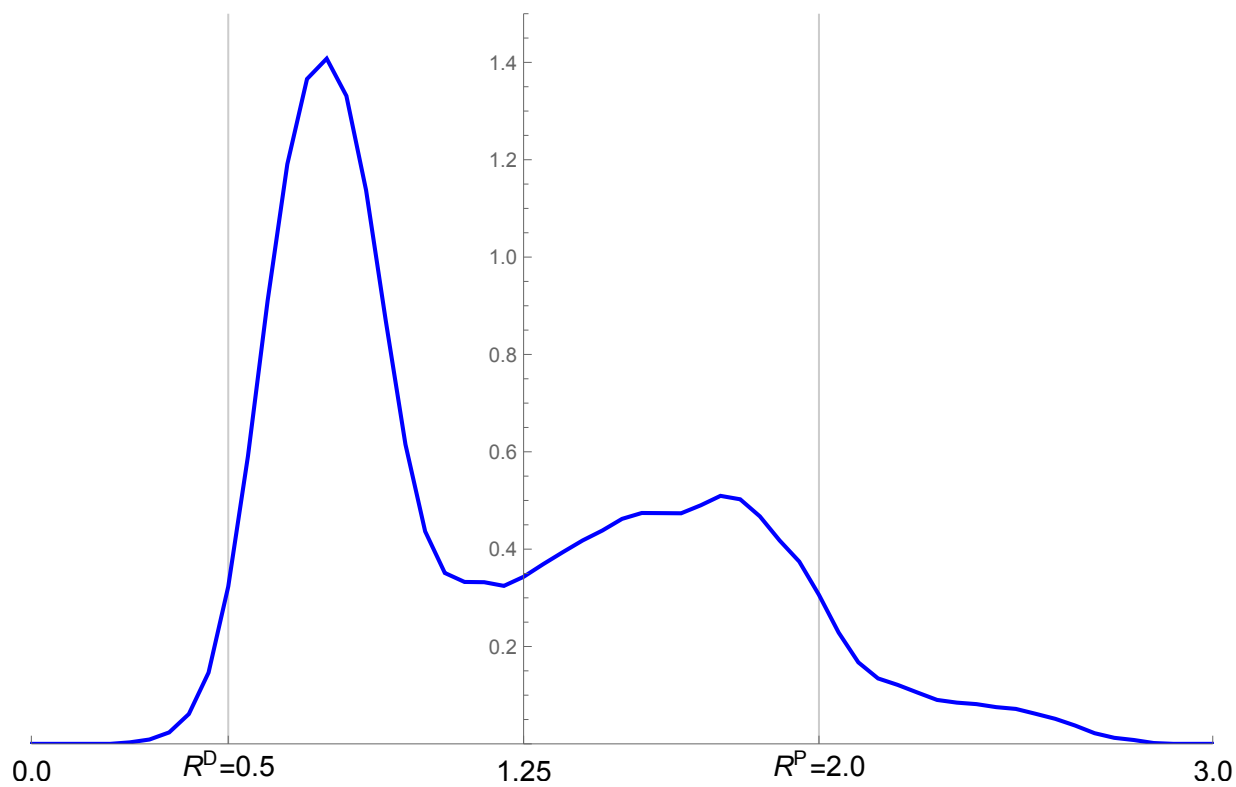
# A Figures



Figure 1: Distribution of the number of bank counterparties, both in normal times and crisis times.

Distribution of interbank interest rates - June 2008.



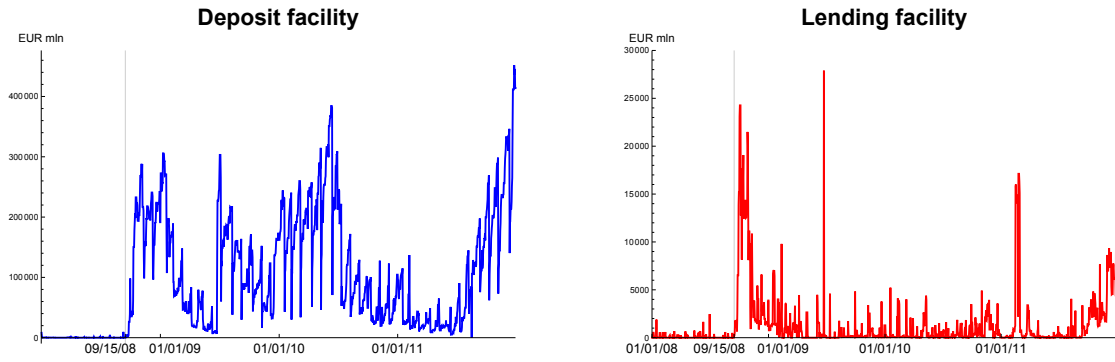Distribution of interbank interest rates - November 2011.

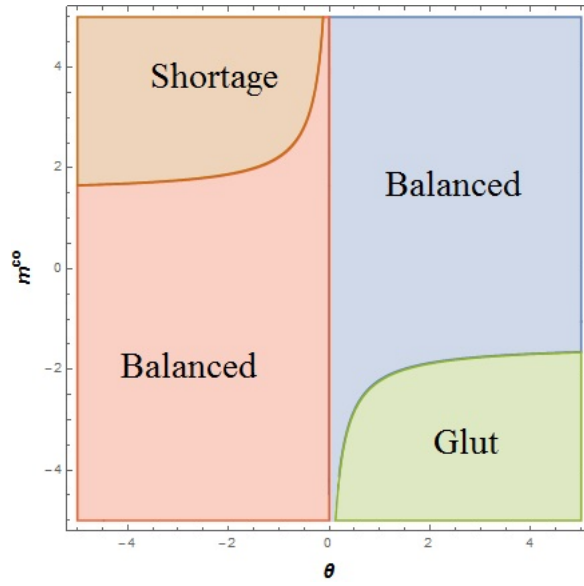Figure 3: Recourse to the central bank's standing facilities.
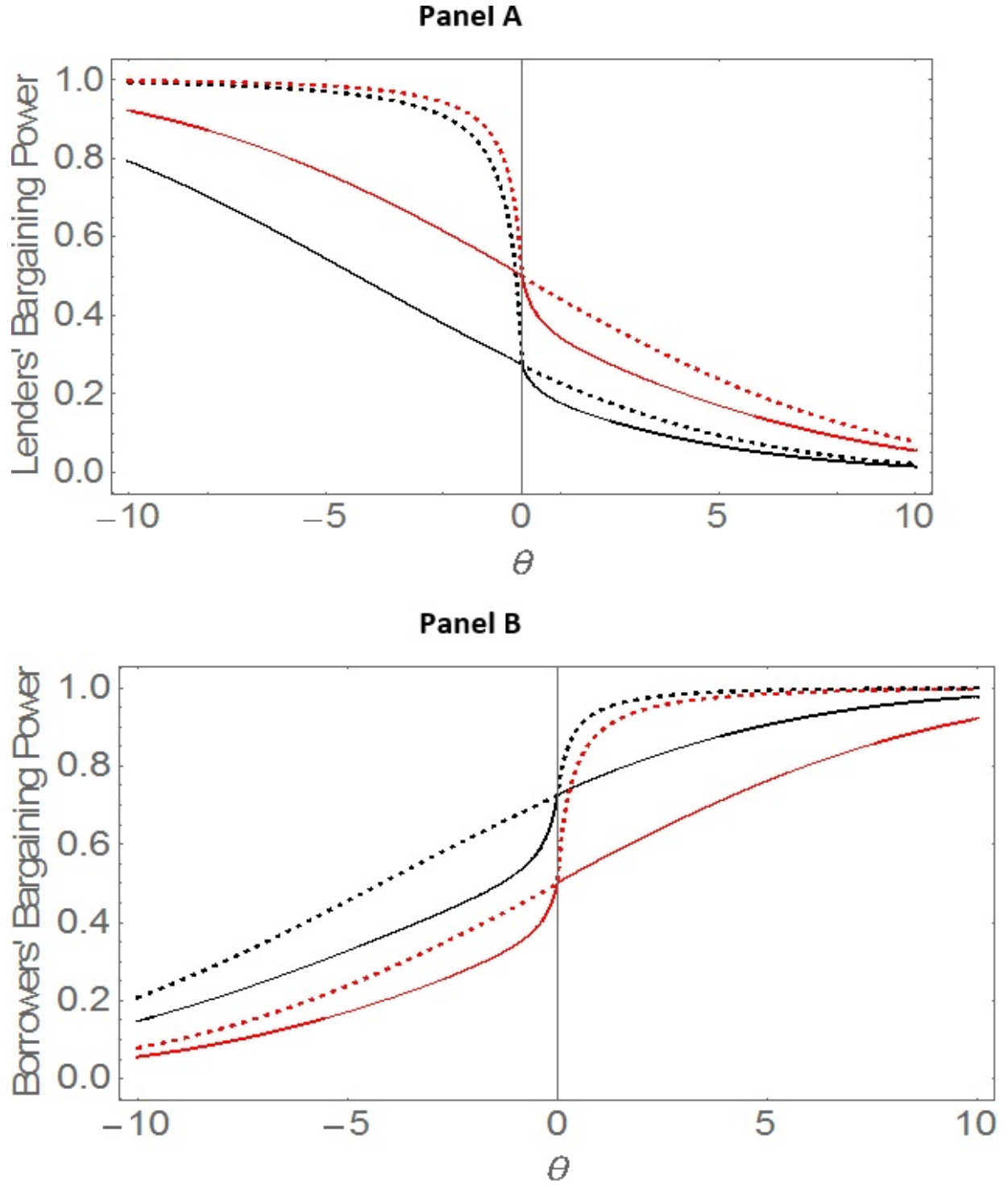


Figure 4: Equilibrium type as a function of $\theta$ and $m^{co}$.

## Panel A



## Panel B

Figure 5: Periphery banks' bargaining power.
Parameter values: $\lambda = 0.3$, $\sigma_{co} = 5$. Black lines: $m^{co} = 0$; Red line: $m^{co} = 3$. In all cases a balanced equilibrium obtains. The dashed lines represent low type banks' bargaining power while plain lines represent high type banks' bargaining power.
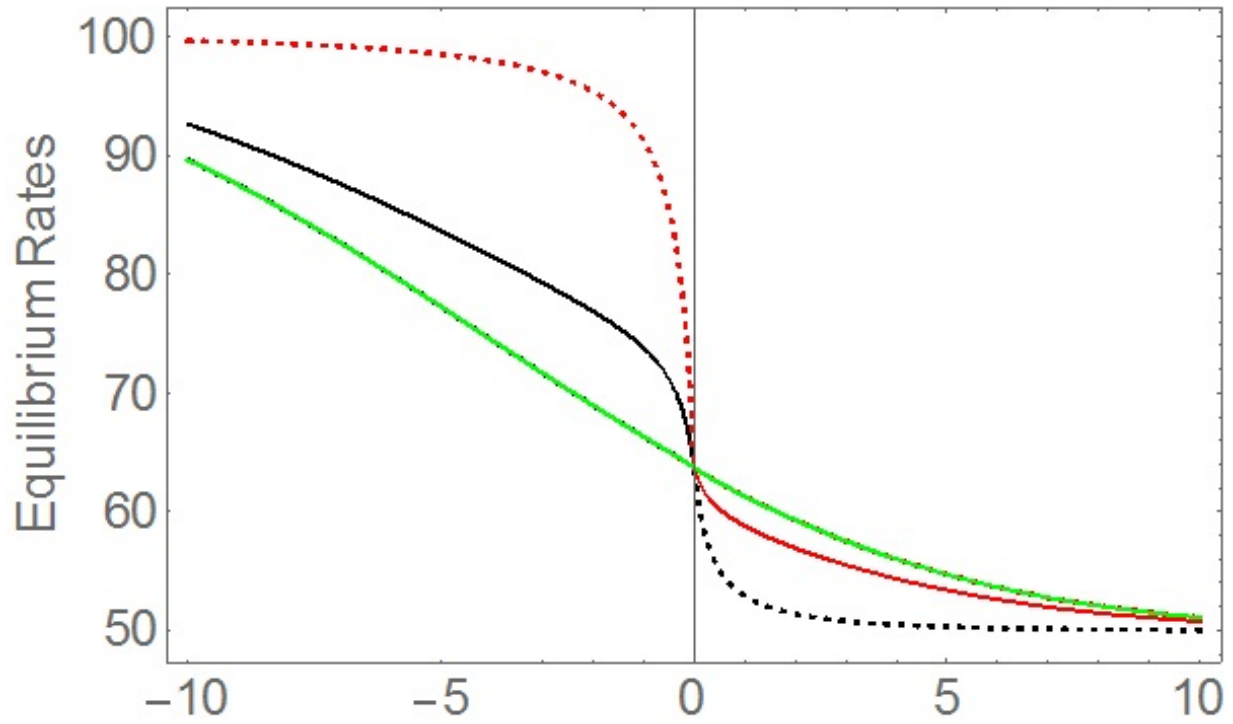
Figure 6: Equilibrium interest rates, as a function of $\theta$.
Parameter values: $\lambda = 0.3$, $\sigma_{co} = 5$. Black lines: $m^{co} = 0$; Red line: $m^{co} = 3$. $R^P = 100bps$ and $R^D = 50bps$. In all cases a balanced equilibrium obtains. The dashed curves represent rates offered by high type banks while the plain lines are rates offer by low type banks. The black lines are rates offered offered by borrowers while red lines are rates offered by lenders. The green line is the equilibrium rate in the core market.

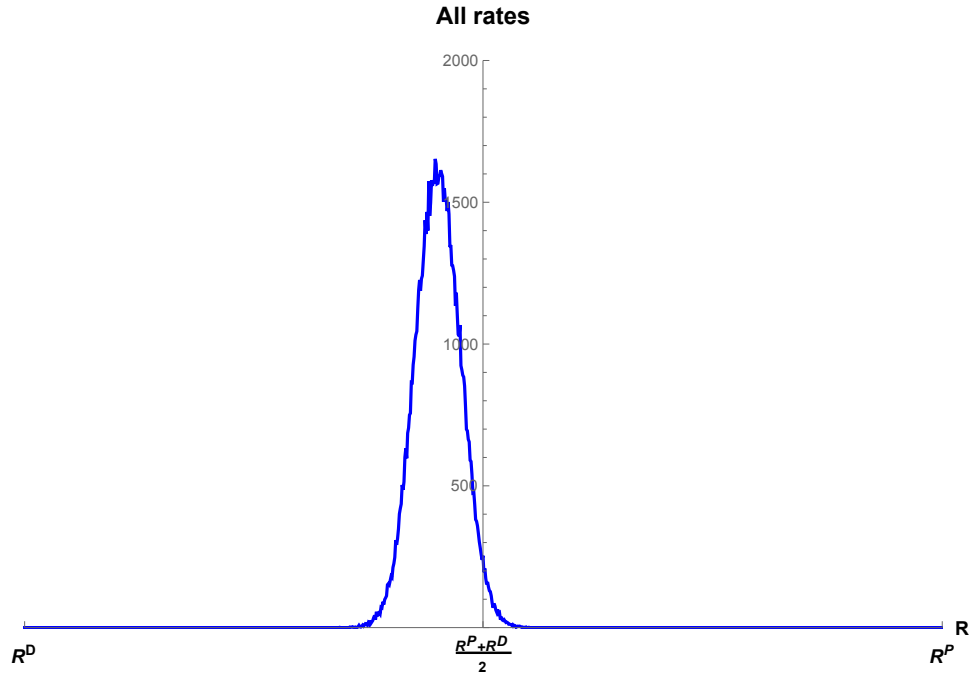## Figure 7: Distribution of equilibrium interest rates

### Panel A: No segmentation. Balanced liquidity conditions.

Parameters: $\lambda = 0$, $m^{co} = m^{pe} = 0$, $\bar{a}_{co} = \bar{a}_{pe} = 0$, $\sigma_{co} = 8$, $\sigma_{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. All banks trade at $R^{co}$.
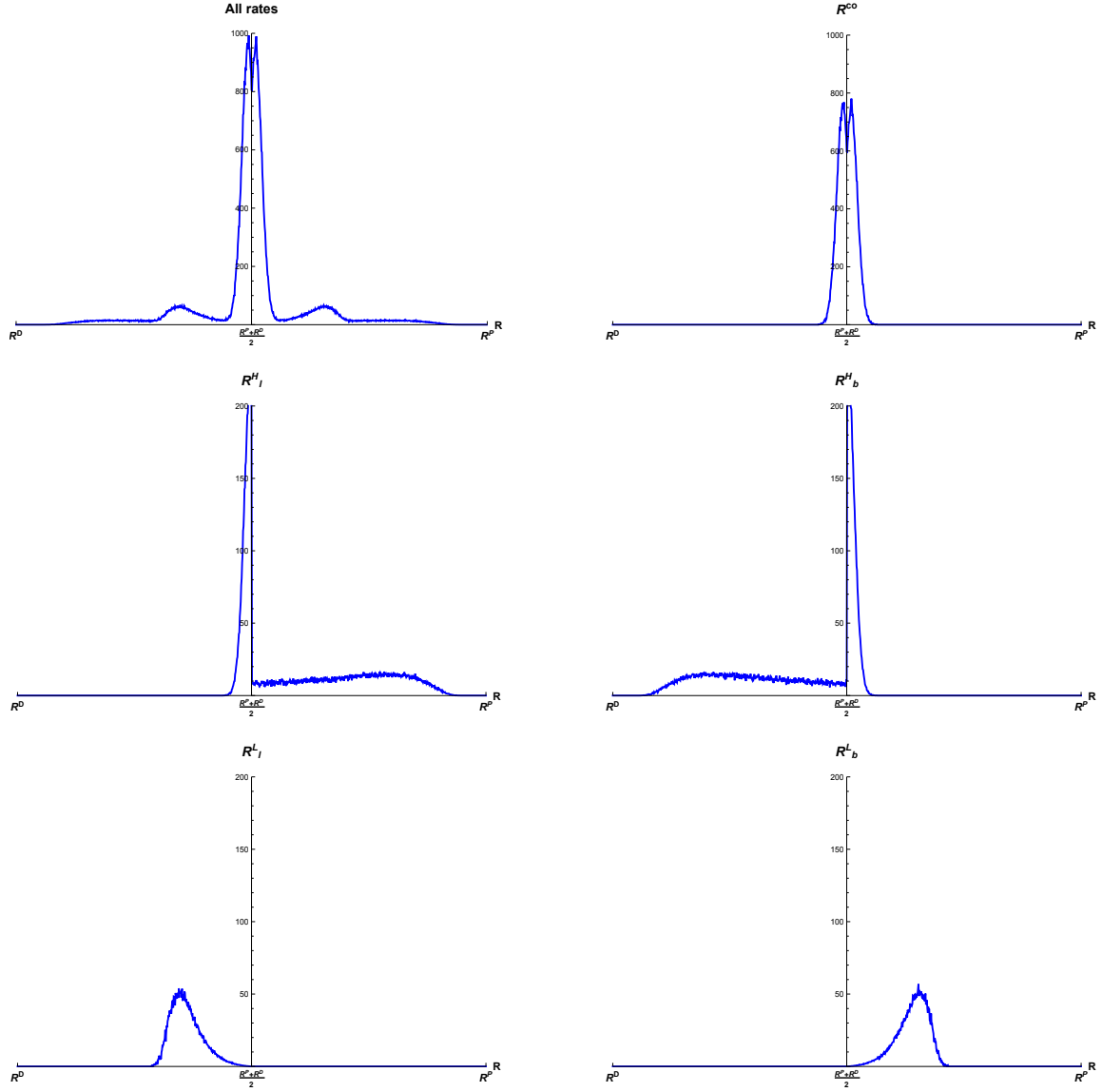


**All rates**
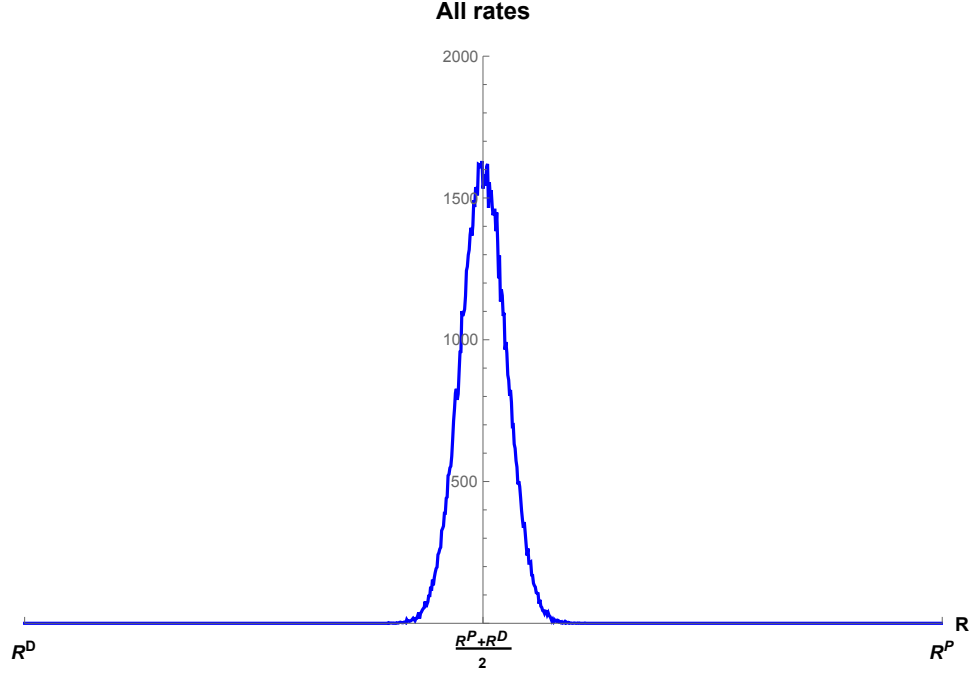
### Panel B: No segmentation. Excess liquidity in the core, shortage in the periphery.

Parameters: $\lambda = 0$, $m^{co} = 3$, $m^{pe} = -2$, $\bar{a}_{co} = \bar{a}_{pe} = 0$, $\sigma_{co} = 8$, $\sigma_{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. All banks trade at $R^{co}$.



**All rates**

Panel C: Segmentation. Balanced liquidity conditions.

Parameters: $\lambda = 0.4$, $m^{co} = m^{pe} = 0$, $\bar{a}_{co} = \bar{a}_{pe} = 0$, $\sigma_{co} = 8$, $\sigma_{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. We report the distribution of all interest rates pooled together, as well as the distribution of each type of rates.

Panel D: Segmentation. Excess liquidity in the core, shortage in the periphery.

Parameters: $\lambda = 0.4$, $m^{co} = 3, m^{pe} = -2$, $\bar{a}_{co} = \bar{a}_{pe} = 0$, $\sigma_{co} = 8, \sigma_{pe} = 0.5$, $\ell(|\theta|) = 1 + |\theta|$. We report the distribution of all interest rates pooled together, as well as the distribution of each type of rates.
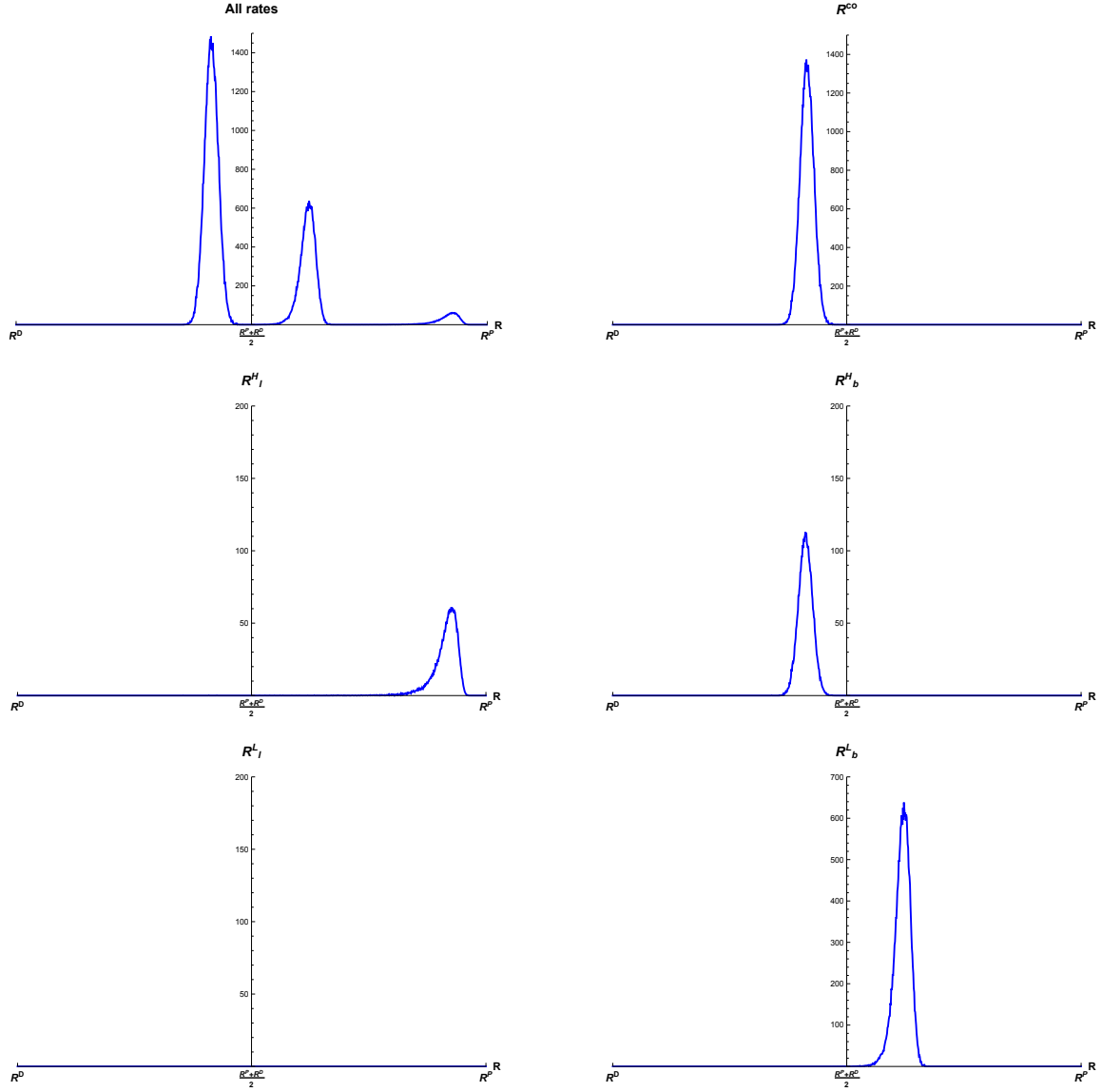
# B Proofs

## B.1 Proof of Proposition 1

We study the other types of equilibria.

**Equilibrium 2 - Glut in the periphery.** High-type borrowers still make offers that are accepted by low-type lenders only. However, high-type lenders now go directly to the core market, so that we have:

$$V_l^{H*} = R^{co} \tag{31}$$

$$V_b^{H*} = -\lambda\alpha R_b^{H*} - (1-\lambda\alpha)R^{co}. \tag{32}$$

$$\tag{33}$$

Low-type lenders still make offers that are accepted by all borrowers, whereas low-type borrowers now make offers that are accepted by low-type lenders only, hence with probability $\alpha\lambda$. This gives us:

$$V_l^{L*} = \beta R_l^{L*} + (1-\beta)R^D \tag{34}$$

$$V_b^{L*} = -(1-\alpha\lambda)R^P - \alpha\lambda R_b^{L*}. \tag{35}$$

$$\tag{36}$$

In such an equilibrium, we have $R_b^{H*} = V_l^{L*}$, $R_l^{L*} = -V_b^{H*}$, and $R_b^{L*} = V_l^{L*}$. This gives us a system of 7 equations and 7 unknowns, the solution of which is:

$$R_b^{L*} = R_b^{H*} = V_l^{L*} = \frac{(1-\alpha\lambda)\beta R^C + (1-\beta)R^D}{1-\lambda\alpha\beta}, \quad V_b^{L*} = -\frac{\lambda\alpha\beta(1-\alpha\lambda)R^C + \alpha\lambda(1-\beta)R^D}{1-\lambda\alpha\beta} - (1-\alpha\lambda)R^P \tag{37}$$

$$R_l^{L*} = -V_b^{H*} = \frac{(1-\alpha\lambda)R^C + \alpha\lambda(1-\beta)R^D}{1-\lambda\alpha\beta}, \qquad V_l^{H*} = R^C. \tag{38}$$

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We now need to check that no type of bank has an incentive to deviate from this equilibrium. First, we have $V_b^H \geq -R^C$, so that high-type borrowers do no want to go directly to the core market. Conversely, if high-type lenders would make an offer they would target low-type borrowers and obtain $-V_b^L$, which is lower than $R^C$. Hence, they make no offer and go to the core market directly. For low-type borrowers, we need to check that they do not prefer to target high-type lenders:

$$V_b^L \geq -(1-\alpha)R^P - \alpha V_l^H. \tag{39}$$

If $\beta = 1$, this inequality is equivalent to $R^C \geq R^P$, and is thus wrong. This type of equilibrium is thus only possible when $\theta > 0$ and $\alpha = 1, \beta < 1$. In that case, (39) can be rewritten as:

$$\frac{R^P - R^C}{R^P - R^D} \leq \frac{\lambda(1-\beta)}{1 - \lambda\beta(2-\lambda)}. \tag{40}$$

Finally, we need to check that low-type lenders do not prefer to target low-type borrowers:

$$
\begin{aligned}
V_l^L &\geq (1-\beta\lambda)R^D - \beta\lambda V_b^L \\
\frac{R^P - R^C}{R^P - R^D} &\leq \frac{1-\lambda}{1-\lambda^2\beta}.
\end{aligned}
\tag{41}
$$

When $\lambda < 1/2$, condition (41) is implied by (40), so that $\theta > 0$ and (40) are necessary and sufficient conditions for such an equilibrium to obtain.

**Equilibrium 3 - Shortage in the periphery.** This equilibrium is entirely symmetric to the previous one: the roles of borrowers and lenders are inverted, as well as $R^P$ and $R^D$. We obtain that this equilibrium obtains if and only if $\theta < 0$ and:

$$\frac{R^C - R^D}{R^P - R^D} \leq \frac{\lambda(1-\alpha)}{1 - \lambda\alpha(2-\lambda)}. \tag{42}$$

**Equilibrium 4 - Mixed equilibrium.** If $\theta > 0$ and there is equality in (40), then low-type borrowers are indifferent between targeting high-type or low-type lenders. Moreover, we also have $-V_b^L = R^C$, so that high-type lenders are exactly indifferent between making an offer and going to the core market. As a result, we have an equilibrium with $p_l = q_b = 1$ and any $(p_b, q_l) \in [0,1]^2$. Symmetrically, when $\theta < 0$ and there is equality in (42), we have an equilibrium with $p_b = q_l = 1$ and any $(p_l, q_b) \in [0,1]^2$.

## B.2  Stationary measures

In this section, we derive the stationary probabilities of the various types of events that can occur when a new bank arrives during the periphery market session. Each time a new periphery bank arrives, eight possible events can happen: (i) A high-type lender makes an offer; (ii) A high-type lender accepts an offer; (iii) A low-type lender makes an offer; (iv) A low-type lender accepts an offer; (v) A high-type borrower makes an offer; (vi) A high-type borrower accepts an offer; (vii) A low-type borrower makes an offer; (viii) A low-type borrower accepts an offer.

We denote by $\mu_i^*$ the stationary probabilities of event $i \in \{1,2,3,4,5,6,7,8\}$ in equilibrium and let $M^* = (\pi_{i,j}^*)$ be the transition matrix from event i to event j, where $\pi_{i,j}^*$ is the likelihood that event $i$ is followed by event $j$ given banks' equilibrium actions and the arrival process for borrowers and lenders. Let $\mu^*$ be the vector column vector of the $\mu s$. It solves

$$\mu^* = M^* \mu^* \text{ and } 1^t \mu^* = 1, \tag{43}$$

where $1^t = (1, 1, ..., 1)$.

Consider the case $\theta > 0$, so that we necessarily have $p_l = q_b = 1$ for any equilibrium type.

The equilibrium actions are such that:

$$M^* = \begin{pmatrix} A(\beta, \lambda) & C(\beta, \lambda, 1, q_l) \\ C(\alpha, \lambda, p_b, 1) & A(\alpha, \lambda) \end{pmatrix},$$

where $A(x, \lambda)$ and $C(x, \lambda)$ are two matrixes defined as:

$$A(x, \lambda) = (1 - x) \begin{pmatrix} (1 - \lambda) & 0 & \lambda & 0 \\ (1 - \lambda) & 0 & \lambda & 0 \\ (1 - \lambda) & 0 & \lambda & 0 \\ (1 - \lambda) & 0 & \lambda & 0 \end{pmatrix},$$

and

$$C(x, \lambda, p, q) = x \begin{pmatrix} (1 - \lambda) & 0 & \lambda(1 - q) & \lambda q \\ (1 - \lambda) & 0 & \lambda & 0 \\ (1 - \lambda)(1 - p) & (1 - \lambda)p & 0 & \lambda \\ (1 - \lambda) & 0 & \lambda & 0 \end{pmatrix}$$

Solving eq.(43), we obtain the stationary measures. Moreover, the reasoning is entirely symmetric

for the case $\theta < 0$. Define:

$$\mu_{1a}(x,p,q) = (1-\lambda)[1 - x\lambda(1 + (1-\lambda)p)]/M(x,p,q)$$

$$\mu_{2a}(x,p,q) = x(1-\lambda)^2\lambda p[1 - q(1 - x\lambda(1-\lambda))]/M(x,p,q)$$

$$\mu_{3a}(x,p,q) = \lambda[1 - x + x\lambda(1-\lambda)q(1 - p(1 - x(1-\lambda)))]/M(x,p,q)$$

$$\mu_{4a}(x,p,q) = x\lambda(1-\lambda)[1 - \lambda q(1 + xp(1-\lambda)^2)]/M(x,p,q)$$

$$\mu_{1b}(x,p,q) = x(1-\lambda)^2[1 - x\lambda q(p + (1-p)\lambda)]/M(x,p,q)$$

$$\mu_{2b}(x,p,q) = x\lambda(1-\lambda)[1 - x + x\lambda(1-\lambda)q(1 - p(1 - x(1-\lambda)))]/M(x,p,q)$$

$$\mu_{3b}(x,p,q) = x\lambda(1-\lambda)[1 - q(1 - x\lambda(1-\lambda))]/M(x,p,q)$$

$$\mu_{4b}(x,p,q) = x\lambda[\lambda - x\lambda + (1-\lambda)q(1 - x\lambda p)(1 - \lambda x(1-\lambda))]/M(x,p,q)$$

$$M(x,p,q) = (1+x)[1 + x\lambda(1 - pq(1-\lambda)(1 - \lambda x(1-\lambda)))]$$

When $\theta > 0$, we have $\mu_i^* = \mu_{ia}(\beta, p_b, q_l)$ for $i \leq 4$ and $\mu_i^* = \mu_{ib}(\beta, p_b, q_l)$ for $i > 4$. When $\theta < 0$, we have $\mu_i^* = \mu_{ib}(\alpha, p_l, q_b)$ for $i \leq 4$ and $\mu_i^* = \mu_{ia}(\alpha, p_l, q_b)$ for $i > 4$.

## B.3  Proof of Lemma 2

Using the results of B.2, when $\theta > 0$, for any type of equilibrium we have:

$$\Delta_1(\theta, \lambda) = l(|\theta|)[\mu_1^*(1 - \beta\lambda q_l) - \mu_5(1-\lambda)] = l(|\theta|)\frac{N(\beta(\theta), \lambda)}{D(\beta(\theta), \lambda)}$$

with $N(\beta, \lambda) = (1-\lambda)[1 - \beta + \beta\lambda[(1-\lambda)(1-p_b) - q_l] + q_l\lambda\beta^2[\lambda(1-p_b)(1-\lambda)^2 + \lambda + p_b(1-\lambda)]]$

$$D(\beta, \lambda) = (1+\beta)[1 + \beta\lambda - \beta\lambda p_b q_l(1-\lambda)(1 - \beta\lambda(1-\lambda))]$$

Simple computations show that $\Delta_1(0, \lambda) = 0$. Moreover, in $\theta = 0$ a balanced equilibrium obtains. Then, differentiating the right-hand side of (18) shows that it is decreasing in $\beta$, and hence

increasing in $\theta$. Thus, there exists a threshold $\theta_g$ such that a balanced equilibrium with $p_b = q_l = 1$ obtains for $\theta \in [0, \theta_g)$, a mixed equilibrium obtains for $\theta = \theta_g$, and an equilibrium with glut in the periphery and $p_b = q_l = 0$ obtains for $\theta > \theta_g$. For given values of $p_b$ and $q_l$, $N(\beta, \lambda)$ is decreasing in $\beta$ and $D(\beta, \lambda)$ is increasing in $\beta$. Moreover, $l(|\theta|)$ is increasing in $\theta$. $\Delta_1$ is thus increasing in $\theta$ for $\theta \in [0, \theta_g)$ and for $\theta > \theta_g$. Around $\theta_g$, $\Delta_1(\theta, \lambda)$ is larger for $p_b = q_l = 0$ than for $p_b = q_l = 1$. Hence, $\Delta_1$ increases discontinuously in $\theta = \theta_g$, regardless of the values of $p_b$ and $q_l$ selected in the mixed equilibrium. This reasoning also shows that $\Delta_1 > 0$ when $\theta > 0$. The reasoning for the case $\theta < 0$ is entirely symmetric.

## B.4 Proof of Proposition 2

Equation (23) follows directly from eq.(22) and (21). Moreover, using eq.(23), we obtain:

$$\frac{\partial R^{co*}}{\partial S^{co}} = -(R^P - R^D)\frac{\partial \Phi_{co}(-S^{co})}{\partial S^{co}} < 0,$$

since $\Phi_{co}(x)$ increases with $x$. As $S^{co} = m^{co} + \Delta_1(\theta, \lambda)$ and $\Delta_1(\theta, \lambda)$ increases with $\theta = m^{pe} + a^{pe}$ (see Lemma 2), we deduce the last part of the proposition. Moreover, $\Phi_{co}(\bar{a}_{co}) = 1/2$ since $\Phi_{co}(.)$ is the cumulative probability distribution of a normally distributed variable with mean $\bar{a}_{co}$. Thus, using eq.(23), if $S^{co} = -\bar{a}_{co}$, $R^{co*} = (R^P + R^D)/2$.

## B.5 Proof of Proposition 3

We focus on the case $\theta > 0$, the other case being symmetric. For a given $\theta$ positive, $\Delta_1$ depends on the equilibrium strategies $q_l$ and $p_b$. Write $\Delta_1(p_b, q_l) = [\mu_1(p_b, q_l)(1 - \beta\lambda q_l) - \mu_5(p_b, q_l)(1 - \lambda)]\ell$, with $\mu_1(p_b, q_l) = \mu_{ia}(\beta(\theta), p_b, q_l)$ and $\mu_5(p_b, q_l) = \mu_{ib}(\beta(\theta), p_b, q_l)$. It will be more convenient to work on the following monotonic transformation of $m^{co}$ (note that it does not depend on $p_b$ and

$q_l$):

$$\eta(m^{co}) = \frac{-1}{\ell} \times \left( m^{co} + \Phi_{co}^{-1} \left( \frac{(1-\lambda)(1-\lambda\beta)}{1 - \lambda\beta(2-\lambda)} \right) \right). \tag{44}$$

Using Condition (24) and Proposition 1, we see that a balanced equilibrium obtains for $\eta(m^{co}) \leq \eta_b = \Delta_1(1,1)/\ell(\theta)$, and an equilibrium with glut in the periphery obtains for $\eta(m^{co}) \geq \eta_g = \Delta_1(0,0)/\ell(\theta)$. This defines $m_b^+ = \eta^{-1}(\eta_b)$ and $m_g^+ = \eta^{-1}(\eta_g)$.

We now focus on $\eta \in [\eta_b, \eta_g]$, a region in which only mixed equilibria can be obtained. We know from Proposition 1 that such an equilibrium can be obtained only if there is equality in condition 18, which is equivalent to:

$$\Delta_1(p_b, q_l) = \ell(\theta) \times \eta. \tag{45}$$

Solving for a mixed equilibrium with an endogenous $R^{co}$ corresponds to finding $p_b$ and $q_l$ in $[0,1]$ that solve (45). As we have two unknowns and one equation, we actually need to characterize a continuum of possible equilibria. In order to do this, we first take $p$ as given and solve (45) in $q$. This gives us:

$$q^*(p) = \frac{N(p)}{D(p)} = \frac{(a_1 + b_1\eta) + c_1 p}{a_2 + (c_2 + d_2\eta)p} \tag{46}$$

With the following quantities:

$$a_1 = (1-\lambda)(1-\beta(1-\lambda(1-\lambda)))$$

$$b_1 = -(1+\beta)(1-\beta\lambda)$$

$$c_1 = -\beta\lambda(1-\lambda)^2$$

$$a_2 = \beta\lambda(1-\lambda)[1-\lambda\beta(2-\lambda(2-\lambda))]$$

$$c_2 = -(\beta(1-\lambda))^2\lambda[1-\lambda(1-\lambda)]$$

$$d_2 = -\beta\lambda(1-\lambda)(1+\beta)[1-\lambda\beta(1-\lambda)]$$

Our method is to first check that $q \in [0,1]$. This will typically determine a range of values for $p$. If the intersection of this range with $[0,1]$ is non-empty, then we have a mixed equilibrium, or more often a set of equilibria. The analysis is complicated by the fact that the numerator $N(p)$ and the denominator $D(p)$ of $q^*(p)$ need not be positive, depending on the value of $\eta$. Define:

$$\eta_1 = -\frac{a_1+c_1}{b_1}$$

$$\eta_2 = -\frac{a_2+c_2}{d_2}$$

$$\eta_3 = \frac{a_2-a_1}{b_1}$$

$$\eta_4 = \frac{c_1-c_2}{d_2}$$

$$\eta_b = \frac{a_2+c_2-c_1-a_1}{b_1-d_2}$$

$$\eta_e = \frac{-a_1}{b_1}$$

$$\bar{p}_n(\eta) = -\frac{a_1+b_1\eta}{c_1}$$

$$\bar{p}_d(\eta) = -\frac{a_2}{c_2+d_2\eta}$$

$$\bar{p}_q(\eta) = \frac{a_1+b_1\eta-a_2}{c_2-c_1+d_2\eta}$$

We obtain the following Lemma, proven at the end of this section:

**Lemma 3.** *For $\theta > 0$ and $\lambda < 1/2$, all the equilibria of the game are characterized as follows:*

- *For $\eta < \eta_b$, the balanced equilibrium obtains.*

- *For $\eta \in (\eta_b, \min(\eta_1, \eta_3)]$, a mixed equilibrium obtains with any $p \in [\bar{p}_q, 1]$ and $q = q^*(p)$.*

- *For $\eta \in [\eta_1, \eta_3]$, if this interval exists, a mixed equilibrium obtains with any $p \in [\bar{p}_q, \bar{p}_n]$ and $q = q^*(p)$.*

- *For $\eta \in [\eta_3, \eta_1]$, if this interval exists, a mixed equilibrium obtains with any $p \in [0, 1]$ and $q = q^*(p)$.*

- *For $\eta \in [\max(\eta_1, \eta_3), \eta_g)$, a mixed equilibrium obtains with any $p \in [0, \bar{p}_n]$ and $q = q^*(p)$.*

- *For $\eta > \eta_g$, the excess liquidity equilibrium obtains.*

In case of multiplicity, we then select the most efficient equilibrium. We use the following Lemma, proven at the end of this section:

**Lemma 4.** *For a given $\theta > 0$ and $\lambda < 1/2$, if there is a range of mixed equilibria with $p \in [\underline{p}, \bar{p}]$ and $q = q^*(p)$, then the equilibrium with $p = \bar{p}$ and $q = q^*(\bar{p})$ is the most efficient one.*

Combining Lemmas 3 and 4 gives us the following result:

**Proposition 7.** *For $\lambda < 1/2$ and $\theta > 0$, if the most efficient equilibrium is always selected, we have the following equilibrium:*

- *A balanced equilibrium for $\eta < \eta_b$.*

- *A mixed equilibrium with $p = 1$ and $q = q^*(p) \in (0, 1)$ for $\eta \in [\eta_b, \eta_1]$.*

- *A mixed equilibrium with $p = \bar{p}_n(\eta)$ and $q = 0$ for $\eta \in [\eta_1, \eta_g]$.*

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- *An excess equilibrium for $\eta > \eta_g$.*

Rewriting this proposition in terms of $m^{co}$ instead of $\eta$ proves Proposition 3.∎

**Proof of Lemma 3.** Let us first see where the different threshold values for $\eta$ come from. We first want to have $q^*(p) = N(p)/D(p) \geq 0$. If $\eta \leq \eta_1$ then $N(p)$ is positive for any $p$. If $\eta > \eta_1$, then $N(p)$ is positive if and only if $p \leq \bar{p}_n(\eta)$. Similarly, if $\eta \leq \eta_2$ then $D(p)$ is positive for any $p$. If $\eta > \eta_2$, then $D(p)$ is positive if and only if $p \leq \bar{p}_d(\eta)$.

Assume that both $N(p)$ and $D(p)$ are positive. We further want $q^*(p) \leq 1$, which we can rewrite as:

$$a_1 - a_2 + b_1\eta \leq (c_2 - c_1 + d_2\eta)p. \tag{47}$$

The left-hand side is positive if and only if $\eta \leq \eta_3$, and the right-hand side is positive if and only if $\eta \leq \eta_4$.

We will use several inequalities on the various thresholds: (i) When $\lambda < 1/2$, we have $\eta_1 \in (\eta_b, \eta_g)$ and $\eta_3 \in (\eta_b, \eta_g)$. Moreover, we have $\eta_1 < \eta_2$ and $\eta_4 > \max(\eta_1, \eta_3)$; (ii) When $\eta > \eta_1$, we have $\bar{p}_d(\eta) > \bar{p}_n(\eta)$; (iii) When $\eta > \eta_4$, we have $\bar{p}_q(\eta) > 1 > \bar{p}_n(\eta)$; (iv) When $\eta < \eta_3$, we have $\bar{p}_q(\eta) < \bar{p}_d(\eta)$.

Assume first that $\eta \leq \eta_1$. Since $\eta_1 \leq \eta_2$ we have $N(p)$ and $D(p)$ positive for any $p$, hence $q^*(p)$ is positive for any $p$. In order to have $q^*(p) \leq 1$, we also need to satisfy (47). If $\eta \leq \eta_3$, both sides of the inequality are positive and (47) is equivalent to $p \geq \bar{p}_q$. Hence we can select any $p \in [\bar{p}_q, 1]$ and the associated $q^*(p)$ to obtain an equilibrium. If $\eta > \eta_3$, then the left-hand side of (47) is negative and the right-hand side positive, so that the inequality is satisfied for any $p$. We can thus select any $p \in [0, 1]$ and the associated $q^*(p)$ to obtain an equilibrium.

Assume now that $\eta > \eta_1$. Since $p_d(\eta) > p_n(\eta)$, in order to have $q^*(p) \geq 0$ we need to have either $p \leq \bar{p}_n(\eta)$ so that $N(p)$ and $D(p)$ are positive, or $p \geq \bar{p}_d(\eta)$ so that both quantities are

negative. $p \geq \bar{p}_d(\eta)$ is possible only if $\eta > \eta_2$.

Case $p \leq \bar{p}_n(\eta)$: again we need to satisfy (47). If $\eta \leq \eta_3$ we must select $p \geq \bar{p}_q$, so that an equilibrium obtains for $p \in [\bar{p}_q, \bar{p}_n]$ and the associated $q^*(p)$. If $\eta > \eta_3$ then the left-hand side of (47) is negative. If $\eta < \eta_4$ then the right-hand side is positive and $p \leq \bar{p}_n(\eta)$ is thus enough to have an equilibrium. If $\eta \geq \eta_4$ then the right-hand side is also negative and we need $p \leq \bar{p}_q$. But since $p_q(\eta) > p_n(\eta)$, having $p \leq \bar{p}_n(\eta)$ is again sufficient to have an equilibrium.

Case $p \geq \bar{p}_d(\eta)$: We have both $N(p) \leq 0$ and $D(p) \leq 0$. In order to have $q^*(p) \leq 1$ we need to *contradict* (47). If $\eta < \eta_3$ both sides of the inequality are positive and we must have $p \leq \bar{p}_q$. We then need $\bar{p}_q \geq \bar{p}_d$, which is false. If $\eta \in [\eta_3, \eta_4]$ the left-hand side of (47 is always negative and the right-hand side positive, hence it is impossible to contradict. Finally, if $\eta > \eta_4$ both sides are negative and we need $p \geq \bar{p}_q$ but $\bar{p}_q > 1$, so this is impossible. To conclude, it is never possible to obtain an equilibrium with $p \geq \bar{p}_d(\eta)$.

This analysis covers all the possible cases for the position of $\eta$ relative to the different thresholds, the proof is thus concluded.∎

**Proof of Lemma 4.** We want to select the most efficient equilibrium. Denote $T^D$ the number of trades with the central bank at rate $R^D$, and $T^P$ the number of trades at rate $R^P$. In a centralized market, we would have $\min(T^P, T^D) = 0$, as all banks are matched together and there are trades with the central bank only in the presence of an aggregate surplus or shortage. This is not necessarily the case when the market is decentralized. The quantity $\min(T^P, T^D)$ is then positive and gives us a natural measure of inefficiency. Still considering the case $\theta > 0$, the

(expected number) of transactions with the central bank is:

$$T^P(p, q) = \ell\mu_3(p, q)(1 - \beta) + S^c$$

$$T^D(p, q) = \ell\mu_7(p, q)(1 - p)(1 - \lambda)$$

We want to study how both vary when we change $p$ and $q$ for a given $\theta$. Notice that for a given $\theta$ the quantity $S^c$ is pinned down independently of $p$ and $q$ in a mixed equilibrium, so that we do not need to take $S^c$ into account. We observe that $q^*(p)$ is decreasing in $p$ for $\eta \in (\eta_b, \eta_g)$. Moreover, $T^P$ is decreasing in $p$ and increasing in $q$, while $T^D$ is decreasing both in $p$ and in $q$.

If $T^P < T^D$, then $T^P$ is the relevant quantity to do welfare comparisons. Since $q^*(p)$ decreases in $p$, we have that $T^P(p, q^*(p))$ is necessarily decreasing in $p$. Hence a higher $p$ always increases welfare in this case. The situation is more complicated when $T^D < T^P$. First, remember that $p$ and $q$ need to be set such that $\Delta_1(p, q)$ is equal to a certain constant. Using implicit differentiation, we have:

$$q^{*\prime}(p) = \frac{-\partial\Delta_1/\partial p}{\partial\Delta_1/\partial q}.$$

The total derivative of $T^D(p, q^*(p))$ with respect to $p$ is thus:

$$\frac{dT^D(p, q^*(p))}{dp} = \frac{\partial T^D}{\partial p} + \frac{\partial T^D}{\partial q} \times q^{*\prime}(p).$$

And we have the following:

$$\frac{dT^D(p, q^*(p))}{dp} \leq 0 \Leftrightarrow \frac{\partial T^D/\partial p}{\partial T^D/\partial q} \geq \frac{\partial\Delta_1/\partial p}{\partial\Delta_1/\partial q}. \tag{48}$$

In other words, the impact of an increase in $p$ accompanied by a decrease in $q$ so as to keep a mixed equilibrium depends on how $p$ and $q$ relatively affect both $T^D$ and $\Delta_1$. As it turns out,

condition (48) is satisfied for any $p, q, \beta$ and $\lambda$. This shows that an increase of $p$ accompanied by a decrease in $q$ diminishes our measure of inefficiency. ∎

## B.6   Proof of Proposition 5

Denote $\mathcal{L} = \mathcal{E} + \psi_1 \mathcal{C} + \psi_2 \Gamma$ the function that the central bank wants to minimize, and assume that $\psi_1 = 0$. Differentiating with respect to $m, R^P, R^D$, we obtain:

$$
\frac{\partial \mathcal{L}}{\partial m} = -2 \int_{-\infty}^{+\infty} [R^{co}(a) - R^T] \phi_{co}(-a - m) \phi_{pe}(a) da + \psi_2 c \left[ \int_{-m}^{+\infty} \phi_{tot}(a) da - \int_{-\infty}^{-m} \phi_{tot}(a) da \right] (49)
$$

$$
\frac{\partial \mathcal{L}}{\partial R^P} = 2 \int_{-\infty}^{+\infty} [R^{co}(a) - R^T] \Phi_{co}(-a - m) \phi_{pe}(a) da \tag{50}
$$

$$
\frac{\partial \mathcal{L}}{\partial R^D} = 2 \int_{-\infty}^{+\infty} [R^{co}(a) - R^T](1 - \Phi_{co}(-a - m)) \phi_{pe}(a) da \tag{51}
$$

We check that these three derivatives are null at the proposed solution. In equation (49), when $a = \bar{a}_{pe}$ we have $R^{co} = \Phi_{co}(-\bar{a}_{pe} - m) R^P + (1 - \Phi_{co}(-\bar{a}_{pe} - m)) R^D$. Since $-\bar{a}_{pe} - m = \bar{a}_{co}$, we obtain $R^{co}(\bar{a}_{pe}) = R^T$. The first integral is symmetric around this point, and is thus equal to zero. The term in $\psi_2$ is also null, because $\phi_{tot}$ is symmetric around $\bar{a}_{pe} + \bar{a}_{co} = -m$. The derivatives with respect to $R^P$ and $R^D$ are null for the same reason that the integral in (49) null.

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