PUBLIC PROTESTS AND POLICY MAKING*

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Technological advances and the development of social media have made petitions, public protests, and other form of spontaneous activism increasingly common tools for individuals to influence decision makers. To study these phenomena, in this article I present a theory of petitions and public protests that explores their limits as mechanisms to aggregate information. The key assumption is that valuable information is dispersed among citizens. Through petitions and protests, citizens can signal their private information to the policy maker, who can then choose to use it or not. I first show that if citizens’ individual signals are not sufficiently precise, information aggregation is impossible, no matter how large is the population of informed citizens, even if the conflict with the policy maker is small. I then characterize the conditions on conflict and the signal structure that guarantee information aggregation. When these conditions are satisfied, I show that full information aggregation is possible as the population grows to infinity. When they are not satisfied, I show that information aggregation may still be possible if social media are available. JEL Codes: D72, D78, D83.

I. INTRODUCTION

Petitions and public protests are a common feature of the U.S. political system. From the civil rights movement, to the war in Iraq, to the more recent debates on tax and health care reforms, petitioners and protesters have often attempted (and sometimes succeeded) to influence policy making. Petitions and public protests play similarly important roles in other established democracies, in the private sector, and even in nondemocratic regimes. Recent examples illustrate the range and magnitude of these phenomena: in 2006 over 28,000 Facebook users signed an online petition on petitionsonline.com against a change in its privacy settings with the introduction of News Feed; in 2007, over

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1.8 million U.K. citizens signed an online petition against “road pricing and car tracking,” prompting a reconsideration by the government; in 2012, United Airlines revised its PetSafe policy restricting dogs from flying in the main cabin on the basis of their breed after over 46,000 customers protested using the online portal Change.org.\(^1\) In other examples, fewer citizens are involved, but they represent an influential elite of the population: in 2003 over 400 economists, including 10 Nobel Prize winners, signed a petition published in the *New York Times* against the tax cuts of the Bush administration; in 2007, over 1,500 active-duty military personnel and reservists signed an appeal for troop withdrawal from Iraq; in 2016, 180 former Israeli top security officials signed a petition to urge their prime minister to accept a nuclear deal with Iran.\(^2\) In all these environments, it is rarely the case that active citizens and policy makers share the same preferences; still, citizens believe that the “power of numbers” allows them to overcome conflicts, change the policy maker’s mind, and affect public decisions.\(^3\)

In this article I present a theory of petitions and public protests to study the extent to which they can serve as a tool to aggregate information. The key assumption is that valuable information is dispersed among citizens. With petitions and protests, citizens can signal their private information to the policy maker, who can then choose to use it to select better policies. Two features make petitions and public protests a distinctive form of communication. First, they typically involve a large (or even very large) number of people; second, they are associated with very simple

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1. For the first example, see Arrington (2006) and Schmidt (2006). For the second example, see BBC News (2007). For the third example, see Karp (2012).

2. The petition of the first example was published in a paid page of the *New York Times* on February 11, 2003. For the second example, see Alvarez (2007) and Schorn (2007). For the third example, see the (Times of Israel 2015) and Eglash and Booth (2015).

3. This belief appears to be shared by policy makers, too. Exploiting recent technological advances, a number of governments are attempting to “institutionalize” public protests by providing online tools to channel them. In 2011 the Obama administration created the web portal We the People, a platform that gives U.S. citizens a tool to propose and/or endorse petitions (see https://petitions.whitehouse.gov/); similar portals have been opened by the U.K. government in 2010 (see http://epetitions.direct.gov.uk/) and the German Bundestag in 2005 (see https://epetitionen.bundestag.de/). Private companies such as Change.org, Avaaz.org, and 38degrees.org.uk are also providing tools for online campaigns and petitions.
“message spaces” that limit the ability of each individual to communicate independently from the others: sign a petition or not, participate in a rally or not, and so on. These features suggest a number of questions: Can we have effective aggregation of information even if conflict is large, just because of the “power of numbers?” That is, just because there is a large enough number of people involved? What are the limits of petitions and public protests as mechanisms for information aggregation?

These questions parallel corresponding questions asked in the literature on the Condorcet jury theorem. This literature—started by Condorcet (1785)—aims at studying conditions under which elections allow aggregation of information dispersed within a population. The Condorcet jury theorem shows that elections can be remarkably efficient in this respect: it features conditions under which the outcome of an elections converges in probability to the outcome that would be chosen with complete information, even if a biased voting rule is used.\(^4\) Petitions and public protests are similar to elections in the sense that they allow citizens to “vote” in favor or against a policy with their “voice.” The fundamental difference between voting and the protests studied herein is that in protests the policy maker does not commit to a decision rule (as opposed to elections, where the outcome is mechanically determined by the ballots and the voting rule). Can the ideas behind the Condorcet jury theorem be extended to petitions and protests? If they cannot be extended, what features of the environment determine if petitions and protests can be an effective tool for citizens to influence a policy maker?

To study these questions, we present a simple model in which citizens and a policy maker are faced with a decision between two policies, that we call \(A\) and \(B\). The citizens and the policy maker find policy \(A\) optimal in state \(a\), and policy \(B\) optimal in the complementary state \(b\). They have the same prior on these two states, but they disagree on the benefit of choosing policy \(A\) in state \(a\) and the cost of choosing policy \(A\) in state \(b\). Given the prior, the policy maker would choose \(B\) while the citizens have preferences that are more favorable to \(A\) (although they might also choose \(B\) in the absence of additional information). A fraction of the

\(^4\) The most general version of this result is proven by Feddersen and Pesendorfer (1997). A more formal presentation of the Condorcet jury theorem relevant for the environment of this article is presented in Section II.C. See also Feddersen and Pesendorfer (1999a) for a survey of this literature.
citizens receives an informative signal on the state of the world and all citizens—informed and uninformed alike—can signal their dissent by signing a petition or participating in a rally. The policy maker chooses the policy after observing the citizens’ activities, but he or she is otherwise unconstrained by the protesters.

I first characterize the conditions under which public protests can serve as a mechanism to aggregate information and when they will fail to do so. I show that the key factor determining whether public protests can be effective is the precision of the individual signals received by the informed citizens. If citizens’ individual signals are not sufficiently precise, information aggregation is impossible, even if the conflict with the policy maker is small, no matter how large is the population of citizens. In contrast, if individual signals are sufficiently precise, then information aggregation is possible even if conflict between the citizens and the policy maker is large. Indeed, we characterize a necessary and sufficient condition under which the Condorcet jury theorem can be extended to environments in which the policy maker cannot commit to a voting rule. We show that there are situations in which the probability of choosing the wrong policy is lower when the policy maker does not precommit to a decision rule than when he or she commits (if the policy rule is not appropriately chosen). This result implies that under certain conditions, political protests can be more effective than voting. Taken together, these results suggest that public protests can function as effective mechanisms to aggregate dispersed information. Large masses of poorly informed citizens, however, will not be able to aggregate information, even if they do not have a significant conflict of interest with policy makers.

When the condition for the effectiveness of petitions and public protests is not satisfied, what can citizens do to influence a policy maker? It is often argued that social media is empowering masses that were previously ignored by policy makers. To understand the effect of social media, I enrich the model by assuming that each citizen is affiliated with a smaller social circle (a group of friends, a YouTube or Facebook page, a Twitter hashtag, a union, or a political party). In the absence of social media, citizens act independently (or can coordinate only in very small social circles).

5. See Casciani (2010), Kirkpatrick (2010) for examples of the effect of Facebook and other social media on public protests, and Section V for a more extensive list of references.
Social media allows citizens to pool their information within their social circle and thus become better informed; but in the presence of conflict vis-à-vis the policy maker, social media potentially reduces citizens’ credibility because it allows social circles to coordinate the actions of its members and act as one. I show that improvements in social media expand the conditions under which protests can be effective.

The reason signaling may be impossible in the face of large numbers of informed citizens and the reason social media may improve communication can be explained intuitively. First, note that, as in voting models, a citizen’s decision to protest matters only when it is pivotal, that is, when a marginal increase in the number of protesters induces a change in the policy maker’s decision. Informative protests, therefore, are possible only if citizens are willing to act according to their signal, conditioning on being pivotal. In particular, it must be that at least the citizens who receive the signals that are most supportive of $B$ (and hence most in line with the default action of the policy maker) are willing not to protest conditioning on being pivotal, otherwise all citizens choose to protest and no information is conveyed. This puts an upper bound on the citizens’ posterior on the state in which $A$ is optimal. Second, note that, in public protests, the number of protesters that makes a citizen pivotal (the pivotal event) is endogenous, since it is determined by the number of protesters at which the policy maker is just indifferent between $A$ and $B$. To be indifferent, the policy maker cannot assign too low a probability that $A$ is optimal in states in which citizens are pivotal. This, in turn, puts a lower bound on the citizens’ posterior on $a$. When the conflict of interest is small and/or citizens’ signals are sufficiently precise, these two constraints are compatible; for sufficiently high conflict and/or insufficiently precise signals, they are not, even with an arbitrarily large number of citizens. In these cases no information can be conveyed by protesters. Social networks are useful because they relax the tension between the precision of individual signals and the conflict vis-à-vis the policy maker.

The organization of the remainder of the article is as follows. Section II outlines the model and present two relevant

6. The fact that the policy maker needs to be indifferent when changing decision from $B$ to $A$ is a consequence of the fact that the policy maker cannot ex ante commit to a decision rule so the change must be optimal after observing the protesters.
benchmarks. Section III characterizes the necessary and sufficient condition under which public protests aggregate information. Section IV studies information aggregation as the number of protesters becomes arbitrarily large, compares public protests with elections, and provides a new interpretation of the Condorcet jury theorem in light of this comparison. Section V studies the effects of social media on public protests. Section VI presents a number of extensions of the basic model. A discussion of the related literature is presented in the reminder of this section.

I.A. Related Literature

This work is related to two strands of literature. First, there is the literature on information aggregation in elections (Feddersen and Pesendorfer 1996, 1997; Austen-Smith and Banks 1996; Myerson 1998b). This was the first literature to pick up Condorcet’s idea that information is dispersed among citizens and to study how it can be aggregated by a political mechanism. The Condorcet jury theorem shows that under plausible assumptions, full information aggregation in large elections is possible, even if the voting rule is biased toward an alternative. The key difference between elections and the petitions and protests that we study is that in elections, the decision maker commits to an exogenous voting rule, whereas in the cases we study, the decision must be ex post optimal for the decision maker. To our knowledge, the only paper in this literature that has studied whether a voting rule is ex post optimal is Yildirim (2013). Yildirim (2013) characterizes a condition for a voting rule to be ex post optimal when the policy maker maximizes the citizens’ welfare and shows that, even without conflict between the policy maker and the citizens, not all voting rules are ex post optimal. In this work, I extend this literature to study conditions under which information aggregation,
or even full information aggregation, is feasible without formal elections in the presence of conflict between citizens and the decision maker, relying only on informal mechanisms like petitions or protests.

The second body of literature to which this work is connected is that on communication with multiple senders.\(^\text{10}\) This literature typically dispenses of the assumption that the decision maker has commitment power, but it focuses on environments with few senders (typically two). Thus, it does not study whether the presence of a large enough number of informed people facilitates effective aggregation of information even if conflict is large. Nor does it consider whether full information aggregation as in the Condorcet jury theorem is possible.\(^\text{11}\) Austen-Smith (1990, 1993) was the first to study a model of information aggregation with imperfectly informed citizens in which, as in our model, the decision maker does not commit to a voting rule. Because he focused on environments with few informed actors, he did not study the extent to which information can be aggregated as the number of protesters becomes large. In the same spirit, Battaglini and Benabou (2003) presented a signaling model of political activism in which two classes of equilibria coexist (equilibria with high participation and poor information aggregation, and equilibria with more selective participation and superior information aggregation) and study their welfare properties; (Battaglini 2004) presents a model of aggregation of noisy signals in a multidimensional policy space.\(^\text{11}\)

The literatures on communication with verifiable information and on Bayesian persuasion also study models of information aggregation with multiple informed agents. The first group

\(^{10}\) In between the literature on voting and the literature on communication there is the work by Coughlan (2000) and Austen-Smith and Feddersen (2006), who present models in which there is a communication stage before an election. In these models, however, the electoral rule is given. The focus is on how different voting rules affect the incentives to reveal information in the predeliberation communication stage.

\(^{11}\) These works should be distinguished from the rest of the literature on cheap talk with multiple senders that assumes all senders observe the same realization of the signal and that therefore does not consider aggregation of dispersed signals (see Gilligan and Krebski 1989; Krishna and Morgan 2001; Battaglini 2002 among others). Another related but different literature is the literature on public polls, which studies information aggregation when the policy maker can choose an unbiased sample of citizens to poll (see McKelvey and Ordeshook 1985; Cukierman 1991; Morgan and Stocken 2008). This literature shows that with polls full information aggregation is typically achieved.
studies information acquisition by an uninformed policy maker when the senders can conceal information, but cannot lie (Milgrom and Roberts 1986; Shin 1994; Lipman and Seppi 1995; Dewatripont and Tirole 1999; Bhattacharya and Mukherjee 2013). In this context, Wolinsky (2002) presents a model with verifiable information in which senders can only send binary signals, as in my model. Taking a mechanism design approach, he shows that the optimal mechanism for information aggregation with commitment is not monotonic in the messages. The model of Bayesian persuasion, first proposed by Kamenica and Gentzkow (2011) and extended to multiple senders by Gentzkow and Kamenica (2015), studies an environment in which senders have no ex ante private information, but can design public experiments to influence a policy maker.

Two noteworthy alternative models of public protests to ours mine Lohmann (1993, 1994) and Banerjee and Somanathan (2001). Lohmann (1993, 1994) proposes models in which the preferences of the policy maker coincide with the preferences of the median citizen and suggests that public protests can always improve policy making. The main focus of her models is to argue that when participation costs are sufficiently small, citizens with preferences close to the policy maker choose to overcome their free rider problem. The opposite conclusion is reached by Banerjee and Somanathan (2001), who show that when protesters and the policy maker have different priors, information transmission fails when the number of protesters is large. They assume that only one protester receives valuable information and focus on the ability of the informed citizen to communicate in spite of uninformed citizens, rather than on aggregation of dispersed information. In this work I reach very different conclusions from these two papers: I show that participation and information aggregation can fail even with no costs of participation; I characterize conditions for information aggregation and even full information aggregation. By studying information aggregation with conflict, my theory provides a unified framework to understand when petitions and protests work and when they do not.

12. She, however, does not provide a proof of the existence of an informative equilibrium and does not study information aggregation as the number of informed citizens becomes large.

13. The consequences of costs of participation in a generic collective decision-making process are also the focus of Osborne, Rosenthal, and Turner (2000) and Osborne and Turner (2010).
To focus on the signaling role of public protests, we deliberately ignore many important aspects of the problem. First, we assume that public protests affect the policy maker only through the informative channel. There is a significant literature in economics and political science assuming that protests have an exogenous direct effect on policy makers if participation is large enough. Second, as in the literature on the Condorcet jury theorem, I adopt an individualistic approach, ignoring ethical concerns or other behavioral factors that may affect participation. Finally, I do not model how preferences determining political conflict are formed. A significant literature in political science is dedicated to the study of what kind of conflict results in public protests; I assume preferences and conflict are exogenous variables. While the exclusive attention to the informative role of protests and an individualistic approach may limit the scope of the theory, these restrictions allow me to focus on and clarify an important channel. I leave to future research the task of integrating the informative theory developed here in a richer model of citizens’ behavior.

II. Model

II.A. Setup

Consider a model in which a policy maker has to choose between two policies, $A$ and $B$. The policy maker believes that policy $A$ is optimal in state $a$ and policy $B$ in the complementary state $b$. Formally, the policy maker’s preference is $V(p, \theta)$, where $p = A, B$ is the policy and $\theta = a, b$ is the state of the world. The prior probability that the state is $\theta$ is $\mu(\theta)$ with $\mu(a) = \mu$. If I define $V(\theta) = V(A, \theta) - V(B, \theta)$ to be the net expected benefit of $A$ in

14. The literature on private politics (Baron 2003; Baron and Diermeier 2007) studies how activists can change production practices of private companies by threatening actions that directly effect a company’s profit like a boycott. The literature on regime change assumes that a regime change occurs if participation in a mass protest is higher than a given exogenous threshold. The focus of this literature is on the possibility of multiple equilibria due to coordination problems (Weingast 1997) and on the political factors that can serve as coordination devices for political action such as a violent revolutionary vanguard (Bueno de Mesquita 2010) and elections (Little, Tucker, and LaGatta 2014).

state $\theta$, then $V(a) > 0$ and $V(b) < 0$. The policy maker is willing to choose $A$ if

$$\mu \geq -\frac{V(b)}{V(a) - V(b)} = \frac{1}{1 + V},$$

where $V = -\frac{V(a)}{V(b)} > 0$. We define $\mu^* = \frac{1}{1 + V} \in (0, 1)$ and assume that $\mu < \mu^*$. This implies that with no additional information, the policy maker chooses $B$.

There is a population of informed citizens. The number of citizens is a Poisson random variable with mean $n$. Citizens’ utilities are described by $v(p, \theta)$, where $p$ is the policy and $\theta$ is the state of the world. Citizens agree that $A$ is the best policy in state $a$ and $B$ is the best policy in state $b$. If we define $v(\theta) = v(A, \theta) - v(B, \theta)$, we have $v(a) > 0$ and $v(b) < 0$. A citizen is willing to choose $A$ if

$$\mu \geq -\frac{v(b)}{v(a) - v(b)} = \frac{1}{1 + v},$$

where $v = -\frac{v(a)}{v(b)}$. The policy maker and the citizens have different willingness to choose $A$. We assume $v > V$, so $\mu^{**} = \frac{1}{1 + v} < \mu^*$. Citizens are therefore more partial to choosing $A$ than the policy maker is. The difference between $v$ and $V$ (or equivalently $\mu^*$ and $\mu^{**}$) provides a natural way to measure the conflict of interest between the policy maker and the citizens.

Citizens observe a private informative signal $t$ with distribution $r(t; \theta)$, support $T = \{1, \ldots, T\}$ with $T \geq 2$ and $r(t, \theta) > 0$ for any $t, \theta$. For any $t' \geq t$, I assume a standard monotone likelihood ratio property for $r(t; \theta)$: for any $t' > t$, $\frac{r(t'; a)}{r(t'; b)} \geq \frac{r(t; a)}{r(t; b)}$ with strict inequality for some $t'$ and $t$. This implies that the posterior $\mu(a; t)$ of a citizen with signal $t$ is nondecreasing in $t$. After observing the private signal, each citizen chooses whether to protest against the policy maker’s default policy $B$ or to stay home. The policy maker observes the number of protesters and then chooses a policy that maximizes her utility.

16. The use of Poisson games to study large games with anonymous players has been pioneered by Myerson (1998b, 2000) and it has become quite common since then. See Myerson (1998a) for a discussion of the advantages of this approach.

17. Citizens, however, are not necessarily assumed to prefer $A$ to $B$ at the ex ante level, it may still be that $\mu < \mu^{**}$.

18. In some applications, it may be natural to assume that only a fraction of citizens receive informative signals, or that some citizens are better informed
In Section V I discuss how we model social networks in this environment. Until then, I assume that citizens act independently. In this case, a strategy for the policy maker is a function from the observed number of protesters to a probability of choosing A, that is, \( \rho: N \rightarrow [0, 1] \). A strategy for a citizen is a function from the signal to a probability of protesting, that is, \( \sigma: T \rightarrow [0, 1] \).

The model is best suited to describe public protests in established democracies in which the overthrow of the political regime is out of question and the purpose of the protests is to convince the policy maker to change policy. A key assumption of the model is that both the citizens and the policy maker agree on the optimal policy ex post. This ex post alignment of interests does not require that the players really agree on the ex post benefits of the policy outcome: the alignment may arise from political economy forces. Assume, for example, that the policy maker always prefers B to A: she receives a utility \( U > 0 \) if B is chosen and 0 otherwise in all states \( \theta = a, b \). However, the policy maker knows that at the end of the game the true state is revealed (when players observe the payoffs) and she is concerned about the opinion of the majority of the citizens. If the policy maker is an elected official, the concern is about her reelection probability; if the policy maker is a private company, the concern is about the possibility of losing market share. The literature on political agency (see Barro 1973; Ferejohn 1986; Besley 2006 among others) has shown that these concerns may provide potent incentives for a policy maker’s behavior. If we assume that the disutility of going against the majority is \( D > U \), then the policy maker’s state contingent preferences are described by \( V(a) = D - U \) and \( V(b) = -(D + U) \) with \( V = V = \frac{(D-U)}{(D+U)} < 1 \). Under the assumption that \( V = \frac{(D-U)}{(D+U)} < v \), the

19. The literature on political agency explains why it is optimal for citizens to discipline politicians on the basis of their past performance and the extent to which this can serve as a disciplining device. In Section I of the Online Appendix I describe a simple electoral game inspired by this literature in which the parameter \( D \) is interpreted as a penalty imposed on the policy maker if she chooses a policy that is disliked by the majority. Empirical evidence that voters punish ex post nonperforming policy maker is presented by Fiorina (1991) and Key (1966), among others.
analysis of this variant of the model is the same as the baseline model described above.

A case to which the model can be directly applied is that of a prime minister or president who decides whether to wage war on a foreign country (or if the war has already started, whether to continue the military presence abroad). In this scenario, it is reasonable to assume that both the president and the citizens would agree a dangerous country should be attacked and diplomacy should be used otherwise; still, in the presence of uncertainty, they may disagree on the expected cost/benefits of attacking.\(^20\) While it is hardly the case that the average citizen receives informative signal on these matters, often a significant number of citizens do, and they may decide to speak up. Recent examples in this sense include two of the cases cited in the introduction: the petition signed by the 180 former Israeli security chiefs in 2016 urging their prime minister to accept a nuclear deal with Iran; and the petition signed by 1,500 active-duty military personnel and reservists urging the president of the United States to withdrawal from Iraq in 2006. These petitions have a clear informative impact on policy maker’s beliefs. An article published in the *Washington Post* on the position taken by the Israeli security chiefs was titled: “How an Iran Deal Can Be Good for Israel, According to Some Israelis Who Know What They Are Talking About” (Tharoor 2015).\(^21\)

Other cases to which our model applies are petitions against private companies that introduce controversial product changes. It is reasonable to assume that in many of these cases the company and the consumers have different evaluations of the cost/benefit trade-off induced by the changes (i.e., \(v\) versus \(V\)); they might, moreover, be uncertain about the nature of the trade-off (i.e.,

\(^{20}\) In the model I assume only two states of the world for simplicity (in the example above, “dangerous” versus “nondangerous” country). Naturally in real life there could be (though not necessarily) more than two relevant states of the world. In addition, the conflict may not just be about the payoffs as in the model represented above, but also on prior beliefs. As discussed more extensively in Section VI, the model can be extended to environments in which the players do not have a common prior.

\(^{21}\) As a testimony that the petition had a significant informative impact on policy makers, the news was used by Secretary of State John Kerry as one of his main arguments in the U.S. Senate hearing on foreign affairs of July 2016 (see Saletan 2015). The Obama administration’s official Twitter account for the Iran deal, moreover, tweeted no less than 13 times the news (Gross 2015).
the state of the world). A pertinent example here is another case cited in the introduction: the opposition faced by Facebook when it changed its privacy setting, introducing news feed. News feed was controversial because it implied decreased privacy, but made it easier for users to “know what’s going on in your friends’ lives,” as stated by Facebook CEO in an open letter following the change (Zuckerberg 2006b). The relevant policy question was: are privacy costs higher than the convenience benefits of news feed? Given that in its current form news feed is a quite popular feature of Facebook, it is reasonable to assume that not even the protesters could be sure about the answer to that question when news feed was introduced. This is especially true because the protests started immediately after the change, before most users could have experienced a real privacy costs. It is also reasonable to assume that the users’ preferences were correlated in this respect, so the issue had a common value component, just as in the model described above. As in the previous example, Facebook explicitly acknowledged that the protests made them better learn about their customers’ preferences. Responding the day after the petitions started in an open letter to the users, Facebook CEO Mark Zuckerberg wrote: “Calm down. Breathe. We hear you. […] We are listening to all your suggestions about how to improve the product; its brand new and still evolving” (Zuckerberg 2006a).

A similar situation is found in the case of United Airlines and its PetSafe policy on animal transportation. From the beginning, the key issue of contention was the impact on animals’ health of denying travel in the aircraft cabin (Huffington Post 2013; Isenbeck 2015). Simplifying, the policy question was: are the health costs for the animals tolerable? Here too it is reasonable to assume a certain degree of uncertainty for the passengers and even for United Airlines. United Airlines initially believed it could maintain the animals in temperature-controlled

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22. The Facebook page “Students Against Facebook News Feed” closed in on 100,000 members as of 9:30 p.m. of the same day of the change (Arrington 2006). The petition, sponsored by the administrator of this page, reached 28,000 signatures by the following day (Schmidt 2006).

23. The type of potential privacy problems that were highlighted by protesters appear quite common among college students (at that time the prevalent users of Facebook). As Erik Ornitz, 18, a Brown student who started his own anti-news feed group declared to Time: “everyone will know that at 10 o’clock I updated my Facebook profile and I wasn’t in class” (Schmidt 2006).
environments (Huffington Post 2013), but it later appeared this was not the case (Isenbeck 2015). Customers’ reactions started as attempts to post evidence on Facebook and Twitter proving that the policy had detrimental effects on animals health (Huffington Post 2013; Isenbeck 2015).24 I should also note that the relevant policy maker was not just United Airlines, but also the Department of Transportation (DOT), which regulates United Airlines. United Airlines reported in 2012 that only 2% of animals were injured and 1% lost in transportation. The protests prompted a verification by the DOT of United Airlines’s reports on pet injuries. The verification resulted in a $350,000 fine for neglecting to accurately report statistics to the DOT. It is therefore the case that the protests provided information to the DOT that prompted a policy change.25

The examples do not mean that our model applies to all instances in which public protests may matter. A key feature of all examples presented above is that there is a significant common value component in the examples, that is, something to be learned aggregating the dispersed signals. The examples illustrate cases in which this common value component is relevant. Pure common value environments are clearly a simplification; as are pure private value models. In Section VI, I discuss extensions of the model in which citizens have heterogeneous preferences and present a simple pure private value environment (i.e., an environment in which the preferences of the citizens are fully private) to which the model can be applied.

II.B. Informative Equilibria

Given the strategies described in the previous section, the probability that a citizen protests in state \( \theta \) when the strategy is \( \sigma \) is \( \sum_t r(t, \theta) \sigma(t) \). The posterior probability that the state is \( a \) if \( Q \)

24. The Facebook page “United Airlines Almost Killed my Dog” was started by Janet Sinclair after her dog was allegedly mistreated by United Airlines. Sinclair used the Facebook page to post images of her dog left on the tarmac for over an hour in the summer heat. Many pet owners followed her posting similar stories (Huffington Post 2013). Passenger Barbara Galletly posted on Twitter a photo of her pet dog as it sat in a rain-soaked tarmac in Houston, Texas (Kitching 2014).

25. While PetSafe became controversial in 2012 when it was adopted by United Airlines, the program started earlier when it was introduced by Continental. It was extended to United Airlines in 2012 after the merger of the two airlines.
citizens protest is then: \(26\)

\[
\Gamma_n(a; Q, \sigma) = \frac{1}{1 + \frac{1 - \mu}{\mu} e^{-n\phi(b; \sigma) / n\phi(a; \sigma)} Q}. 
\]

The public protest game always has an equilibrium in which the policy maker ignores the protesters and chooses \(B\) with probability one: in such an equilibrium citizens use uninformative, state-uncontingent strategies. \(27\) In the following we study the conditions under which the policy maker’s decision is influenced by the “wisdom of the crowd,” that is, the citizens’ actions. Naturally, citizens’ protests can affect the policy maker’s action only if they are informative on the state of the world. We say that \(\sigma, \rho\) is an informative equilibrium if citizens use informative strategies and so the probability of protesting is higher in state \(a\), the state in which the policy maker’s default policy is incorrect: \(\phi(a; \sigma) > \phi(b; \sigma)\). In this case the probability of \(a\) is increasing in \(Q\) and there is a \(Q^*\) such that the policy maker is willing to choose \(A\) if and only if \(Q > Q^*\). \(28\) I am interested in studying the existence and the properties of informative equilibria in environments with an arbitrarily large number of citizens. To formalize this point, we say that an informative equilibrium exists in a large society if there is a \(n^*\) such that an informative equilibrium exists for all \(n > n^*\).

Informativeness of an equilibrium is only a minimal requirement for public protests to be useful: even if public protests are informative, information transmission can be minimal and the policy maker’s mistake can be significant; even when the population is arbitrarily large, informativeness may converge to 0 as \(n \to \infty\). The probability of a mistake in an

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26. In the model described above, the probability that \(Q\) citizens decide to protest in state \(\theta\) is a Poisson random variable with mean \(n\phi(\theta; \sigma)\). Given this, equation (1) follows from Bayes’s rule.

27. For example, \(\sigma(t) = \frac{1}{2}\) for all \(t\) and \(\rho(Q) = 0\) for all \(Q\) is an equilibrium: the strategy is such that \(\phi(a; \sigma) = \phi(b; \sigma) = \frac{1}{2}\) and so \(\Gamma_n(a; Q, \sigma)\) is independent from \(Q\), implying that \(\rho(Q) = 0\) is optimal; since the policy maker is unresponsive to \(Q\), \(\sigma(t) = \frac{1}{2}\) is optimal for the citizens as well.

28. We can also have informative equilibria in which citizens “protest” to show support to the policy maker and stay home to signal their disagreement: in this case \(\phi(a; \sigma) < \phi(b; \sigma)\). For the purpose of this article, there is no loss of generality to focus on the most natural case in which a protest is interpreted as a sign that citizens protests to induce a change in the policy maker’s action.
informative equilibrium \( \sigma, \rho \), is
\[
M(\sigma, \rho) = (1 - \mu) \Pr(A, b; \sigma, \rho) + \mu \Pr(B, a; \sigma, \rho),
\]
where \( \Pr(p, \theta; \sigma, \rho) \) is the probability that policy \( p \)
is chosen in state \( \theta \). We say that full information aggregation is
achievable if there is a sequence of informative equilibria \( \sigma_n, \rho_n \)
for environments with expected population \( n \) such that \( M(\sigma_n, \rho_n) \)
converges to 0 as \( n \to \infty \).

II.C. Two Benchmarks

To appreciate the peculiarities of the public protest game, it is
useful to introduce two natural benchmarks. The first is the case
in which citizens decide the policy through an election in which
citizens can vote for \( A \) or \( B \). The second is the case in which there
is no election, the citizens can sign a petition against \( B \) but the
policy maker can commit to a decision rule, that is to choose \( A \) if
at least a given share of citizens sign the petition.

Let us first consider the case in which an election is possible.
We say that an election is decided by a \( q \)-rule if policy \( A \) is chosen if
and only if the share of votes for \( A \) is larger or equal than \( q \). In the
same informational environment as the environment described
above, Myerson (1998b) has proven:

**Proposition 1.** Full information aggregation is achieved in an
election as \( n \to \infty \) if the outcome is decided by a \( q \)-rule with \( q \in (0, 1) \).

Proposition 1 is a version of the Condorcet jury theorem ap-
plied to our environment. What is remarkable in this result is
that full information aggregation can be achieved for any \( q \in (0, 1) \), even for very biased voting rules: the voters strategies ad-
just to the rule.\(^{29}\)

The petitions and public protests that are the focus of this
work differ in two ways from elections: first, the policy maker
does not commit to a voting rule as in an election; second, citizens
can only protest or abstain, they cannot directly vote for the two
alternatives. In our second benchmark, citizens can only protest
against \( B \), but the policy maker can commit to a response function.
With the policy maker’s commitment, the “protest” game still looks
similar to a voting rule: citizens can “vote” against the policy

\(^{29}\) As discussed in the previous section, many versions of this result have been
presented in the literature for alternative environments, showing its robustness,
for example, to more general signal structures (Feddersen and Pesendorfer 1997)
or the possibility of abstention (Feddersen and Pesendorfer 1999b).
maker’s default policy $B$ by protesting, or “vote” for the policy maker’s ex ante optimal policy by staying silent. This yields a result similar to Proposition 1. We say that a policy rule is a cut-off rule if there is a threshold $\hat{Q}$ such that $A$ is chosen if and only if the fraction of protesters over the expected population, $\frac{Q}{n}$, is larger than or equal to $\hat{Q}$. In the Appendix we prove:

**Proposition 2.** Full information aggregation is achieved in a petition as $n \to \infty$ if the policy maker can commit to a cut-off rule $\hat{Q} \in (0, 1)$.

The key feature of the public protest game, when compared with the two benchmarks presented above, is that the policy maker is unable to precommit to a policy rule. The two benchmarks clearly show that when commitment is possible, information aggregation is not a problem in our environment, whether a formal election is available or not. The inability of the policy maker to commit imposes an additional equilibrium condition requiring that, given the citizens’ strategies, the policy maker is willing to follow the policy rule after observing the citizens’ actions. In the following, we study the implications of the inability to commit on the effectiveness of public protests in aggregating information in large societies.

**III. Public Protests and Information Aggregation**

In this section, I study the conditions under which an informative equilibrium exists in the game described in the previous section. To this end, I first characterize an equilibrium in terms of simple cut off strategies (Proposition 3). I use this characterization to explain why information aggregation may fail even if the number of informed citizens is arbitrarily large and present the first of the main results of the article (Proposition 4).

**III.A. Characterization in Cut-Off Strategies**

The policy maker’s optimal choice naturally depends on his posterior belief $\Gamma_n(a; Q, \sigma)$ given the citizens’ strategy $\sigma$. If the citizens use informative strategies, $\Gamma_n(a; Q, \sigma)$ is increasing in $Q$ and the policy maker always finds it optimal to follow a cut-off rule. Let $Q_n(\sigma, \rho)$ be the minimal $Q$ such that:

$$\Gamma_n(a; Q, \sigma) \geq \mu^*.$$
The policy maker strictly prefers $B$ if $Q < Q_n(\sigma, \rho)$ and $A$ if $Q > Q_n(\sigma, \rho)$; if $Q = Q_n(\sigma, \rho)$ the policy maker is indifferent if $Q_n(\sigma, \rho)$ satisfies equation (2) with equality and strictly prefers $A$ otherwise. To account for the possibility of the policy maker using mixed strategies, it is convenient to represent the policy maker’s strategy $\rho_n(Q)$ as a function of a threshold $q_n$ on the real line:

$$\rho_n(Q) = \begin{cases} 0 & Q < \lfloor q_n \rfloor \\ \lfloor q_n \rfloor - q_n & Q = \lfloor q_n \rfloor \\ 1 & Q > \lfloor q_n \rfloor \end{cases}$$

where $\lfloor x \rfloor$ and $\lceil x \rceil$ are, respectively, the largest integer less than or equal to $x$ and the smallest integer greater than $x$. When $q_n$ is an integer, equation (3) describes a simple cut-off rule for action in pure strategies: type $q_n$ is the smallest number of protesters that induces the policy maker to choose $A$ with probability 1; $B$ is chosen if and only if less than $q_n$ citizens protest. When $q_n$ is not an integer, then $\lceil q_n \rceil$ is the smallest number of protesters that induces the policy maker to choose $A$ with probability 1.

A policy maker who observes $\lfloor q_n \rfloor$ chooses $A$ with probability $\lfloor q_n \rfloor - q_n$; a policy maker that observes less than $\lfloor q_n \rfloor$ chooses $B$ with probability 1. Following a strategy described by equation (3) is optimal for a policy maker if and only if $q_n \in [Q_n(\sigma, \rho), Q_n(\sigma, \rho) + 1]$, with $q_n = Q_n(\sigma, \rho)$ if $\Gamma_n(a; Q_n(\sigma, \rho), \sigma) > \mu^*$. In this case we say that $q_n$ is optimal given the citizens’ strategy.

The citizens’ strategies depend on their posterior belief, conditioning on being pivotal, that is, conditioning on being able to affect the policy maker’s decision. To evaluate the citizens’ decision, define $\varphi_n(\theta; \sigma, \rho)$ to be the pivot probability in state $\theta$ given an expected population size $n$ and the strategies $\sigma, \rho$. The pivot probability is the increase in the probability that $A$ is chosen, as induced by a citizen’s decision to protest. The pivot probability in state $\theta$ is:

$$\varphi_n(\theta; \sigma, \rho) = \beta_n \cdot P(Q_n(\sigma, \rho) - 1, n\phi(\theta; \sigma))$$

$$+ (1 - \beta_n) \cdot P(Q_n(\sigma, \rho), n\phi(\theta; \sigma))$$

$$= P(Q_n(\sigma, \rho), n\phi(\theta; \sigma)) \cdot \left[ \beta_n \frac{Q_n(\sigma, \rho)}{n\phi(\theta; \sigma)} + (1 - \beta_n) \right],$$

where $P(\cdot, n\phi(\theta; \sigma))$ is a Poisson with mean $n\phi(\theta; \sigma)$ and $\beta_n$ is the probability that $A$ is chosen if $Q_n(\sigma, \rho)$ citizens are protesting. To
interpret equation (4), note that a citizen is pivotal in only two events, when \( Q_n(\sigma, \rho) - 1 \) or \( Q_n(\sigma, \rho) \) other citizens are protesting (corresponding, respectively, to the first and second term in equation [4]). In the first event, a citizen’s protest increases the probability of \( A \) from 0 to \( \beta_n \); in the second event, a citizen’s protest increases the probability of \( A \) from \( \beta_n \) to 1.

A citizen chooses to protest if the expected benefit of the protest is nonnegative:

\[
\mu(a; t)v(a, \sigma, \rho) + \mu(b; t)v(b, \sigma, \rho) \geq 0. \tag{5}
\]

We can rewrite this condition as:

\[
\frac{\mu(a; t)}{\mu(b; t)} \geq \frac{v(b, \sigma, \rho)}{v(a, \sigma, \rho)} = \frac{\varphi_n(b; \sigma, \rho)}{\varphi_n(a; \sigma, \rho)}. \tag{6}
\]

The monotone likelihood assumption on citizens’ signals implies that there is a \( t_n(\sigma, \rho) \in [1, T] \) such that only citizens with \( t \geq t_n(\sigma, \rho) \) find it optimal to protest and citizens with \( t < t_n(\sigma, \rho) \) find it strictly optimal not to protest; if \( t_n(\sigma, \rho) \) satisfies equation (6) with equality, then citizens with \( t = t_n(\sigma, \rho) \) are indifferent and are willing to randomize their action. As with the policy maker, a citizen’s equilibrium strategy \( \sigma_n \) can be conveniently represented as a continuous function of a threshold \( \tau_n \in [1, T + 1] \) as follows:

\[
\sigma_n(t) = \begin{cases} 
0 & t < [\tau_n] \\
[\tau_n] - \tau_n & t = [\tau_n] \\
1 & t > [\tau_n]
\end{cases}
\tag{7}
\]

Following a strategy described by equation (7) is optimal for a citizen if and only if \( \tau_n \in [t_n(\sigma, \rho), t_n(\sigma, \rho) + 1] \), with \( \tau_n = t_n(\sigma, \rho) \) if equation (6) is strict at \( \tau = t_n(\sigma, \rho) \). In this case, we say that \( \tau_n \) is optimal given \( q_n \).

The representations of the strategies in equations (3) and (7) allow us to characterize an equilibrium in terms of two real numbers and simple cut-off strategies:

**Proposition 3.** An informative equilibrium is characterized by a pair of thresholds \( \tau_n^*, q_n^* \) such that \( q_n^* \) is optimal given \( \tau_n^* \), and \( \tau_n^* \) is optimal given \( q_n^* \).
As mentioned in Section II, existence of an equilibrium is easily established. The real question is whether information transmission is possible in equilibrium.

III.B. Information Aggregation

To appreciate the problems that may arise for the existence of an informative equilibrium, consider a simple example in which the citizens receive a binary signal \( T = \{1, 2\} \) with \( r(1,b) = r(2,a) = r > \frac{1}{2} \) and \( r(1,a) = r(2,b) = 1 - r \). If an informative equilibrium exists then there must be a threshold \( Q_n \) such that the policy maker is willing to choose \( A \) if and only if the number of protesting citizens \( Q \) is at least \( Q_n \). At this threshold the policy maker's posterior probability must be sufficiently large: \( \Gamma_n(a; Q_n, \sigma) \geq \mu^* \). This inequality can be rewritten as:

\[
\frac{P(Q_n, n\phi(a; \sigma_n))}{P(Q_n, n\phi(b; \sigma_n))} \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right).
\]

The equilibrium, however, is informative only if there is separation of the citizens' types. This is possible only if, at the very minimum, the citizens with the lowest signal are willing to be inactive. By condition (6), we must have:

\[
\frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \leq \frac{1}{V} \left( \frac{1}{\mu(a; 1)} - 1 \right).
\]

An informative equilibrium exists only if equations (8) and (9) are both satisfied. I now show that when conflict is sufficiently high and/or the precision of the individual signals is sufficiently low, these conditions are incompatible.

To this goal, first note that the left-hand sides of equations (8) and (9) are intimately connected. The left-hand side of equation (8) is the ratio between the probabilities of having \( Q_n \) protesters in, respectively, state \( a \) and in state \( b \). The left-hand side of equation (9) is the ratio of the pivot probabilities in, respectively, state \( a \) and \( b \). As can be seen from equation (4), the pivot probability in state \( \theta \) is a convex combination of the probabilities that \( Q_n \) and \( Q_n - 1 \) citizens are active in state \( \theta \) (since a citizen is pivotal only in these two events).\(^{30}\) There is therefore, a well-defined relationship between the right-hand side of equations (8) and (9). As

\(^{30}\) The weights in the convex combination depend on the policy maker's strategy (the probability of choosing \( A \) with \( Q_n \) protesters).
formally shown in the proof of Lemma 1, the relationship between them can be bounded as follows:

\[
\frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \geq \frac{P(Q_n, n\phi(a; \sigma_n))}{P(Q_n, n\phi(b; \sigma_n))} \left(\frac{1}{r} - 1\right).
\]

Using equation (10) we can now connect equations (8) and (9) and obtain the following necessary condition for information aggregation:

\[
\frac{1}{v} \left(\frac{1}{\mu(a; 1)} - 1\right) \geq \frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \geq \frac{P(Q_n, n\phi(a; \sigma_n))}{P(Q_n, n\phi(b; \sigma_n))} \left(\frac{1}{r} - 1\right) \geq \frac{1}{V} \left(\frac{1}{\mu} - 1\right) \left(\frac{1}{r} - 1\right).
\]

The first and last inequality follow from equations (8) and (9), the second inequality follows from equation (10). I conclude that an informative equilibrium exists in our example only if:

\[
V \geq \frac{\left(\frac{1}{\mu} - 1\right) \left(\frac{1}{r} - 1\right)}{\left(\frac{1}{\mu(a; 1)} - 1\right)} \cdot v.
\]

Remarkably, when equation (11) is not satisfied, an informative equilibrium does not exist even if the number of informed citizens is arbitrarily large.

Recall that the policy maker is less inclined to choose A than the citizens, so \(V < v\) and conflict is larger when \(V\) is smaller. Condition (11) therefore defines an upper bound on the conflict between the citizens’ and the policy maker’s preferences: the right-hand side is clearly smaller than \(v\), so when \(V\) is not sufficiently close to \(v\), no informative equilibrium exists.\(^{31}\) The upper bound on conflict, however, depends only on the precision of the individual signal (as measured, in this example, by \(r\)): for any conflict, even if arbitrarily small, information aggregation becomes impossible as \(r \to \frac{1}{2}\). If we reinterpret equations (8) and (9) in terms of beliefs we see why it is the precision of the individual signal and not the number of signals that matters for information

\(^{31}\) If, for example, we assume \(r = \frac{2}{3}\) and \(v\) is such that a citizen is indifferent when the posterior is \(\mu^{**} = \frac{1}{1 + v} = 0.5\), then no informative equilibrium exists if \(V\) is such that the indifference threshold for the policy maker is \(\mu^{*} = \frac{1}{1 + V} \geq 0.8\).
aggregation. Recall that condition (8) requires the policy maker’s posterior probability to be sufficiently large conditioning on \( Q_n \): \( \Gamma_n(a; Q_n, \sigma) \geq \mu^* \). Similarly, condition (9) can be written as requiring \( \Gamma_n(a; Piv_n, t = 1, \sigma) \leq \mu^{**} \), where \( \Gamma_n(a; Piv_n, t = 1, \sigma) \) is the posterior, conditioning on being pivotal, of a citizen with signal \( t = 1 \). Unsurprisingly, the citizens with the smallest signals are willing to abstain from protesting only if their posterior conditioning on being pivotal is below their indifference threshold \( \mu^{**} \).

Subtracting these two conditions, we have:

\[
\Gamma_n(a; Q_n, \sigma) - \Gamma_n(a; Piv_n, t = 1, \sigma) \geq \mu^* - \mu^{**}.
\]

The key observation is that the difference in the posteriors between the citizens with the lowest signals and the policy maker depend at most on two signals: the policy maker conditions her belief on the presence of \( Q_n \) protesters (i.e., the equilibrium threshold); the citizen conditions on the presence of \( Q_n \) or \( Q_n - 1 \) other citizens protesting and on his own signal. When the precision of the individual signals is not sufficiently high, these two signals alone are insufficient for equation (12) to be satisfied, even if the total number of signals is very large. Indeed, as \( r \to \frac{1}{2} \) the left-hand side of equation (12) converges to 0 and it is impossible to satisfy equation (12) even if conflict, as measured by \( \mu^* - \mu^{**} > 0 \), is small.

For the general case, we have the following impossibility result:

**Lemma 1.** No informative equilibrium exists if \( V < V_1(v) \) where:

\[
V_1(v) = \frac{1}{\mu(a; T)} - 1 \cdot v.
\]

Lemma 1 highlights a key difference between our public protests game and the voting games studied in the Condorcet jury theorem literature in which the policy maker can commit to a response function: with commitment, as shown in Proposition 1 and 2, not only does an informative equilibrium exist, but full information aggregation is achieved as the size of the
population increases, independently from the cut-off rules that are used. Conversely, when the policy maker cannot commit to a response plan, the fact that citizens receive informative signals is not sufficient for information transmission. Indeed when the condition of Lemma 1 is satisfied, no information is transferred at all, no matter how large the number of informed citizens is.

In what situations will public protests be useful and allow the policy maker to improve her choice when conflict is sufficiently small? The following result characterizes a simple sufficient condition for the existence of an informative equilibrium.

**Lemma 2.** An informative equilibrium exists if $V \geq V_2(v)$, where:

\[
V_2(v) = \frac{1}{\mu(a;1)} v.
\]

It is easy to verify that $V_2(v)$ is positive, larger than $V_1(v)$ and smaller than $v$. This condition implies that if conflict is sufficiently small, information transmission is possible for any population size. As the precision of the individual signals increases, moreover, \(\frac{(\frac{1}{2} - 1)}{\frac{1}{\mu(a;1)} - 1}\) converges to 0, so equation (14) is satisfied for any $V$.

The exact condition for the existence of an informative equilibrium naturally depends on the details of the environment, like the shape of the entire signal structure (as described by $r(t, \theta)$ for $t = 1, \ldots, T$), the expected number of agents, and so on. The next result completes the analysis by showing that a simple threshold characterizes when information transmission is possible in a large society. Recall that we say a property holds in a large society if there is a $n^*$ such that it holds for all $n > n^*$.

**Proposition 4.** There is a threshold $V^*(v) \in [V_1(v), V_2(v)]$ such that an informative equilibrium exists in a large society if $V > V^*(v)$ and it does not exist if $V < V^*(v)$.

In summary, two lessons on the effectiveness of public protests should be highlighted from the preceding discussion and Proposition 4. First, the size of conflict between the citizens and the policy maker is important. It is not surprising that the larger the conflict, the less effective public protests are. Much more surprising is the fact that if the conflict is sufficiently large, then protests are completely ineffective even if the number of informed
The citizens is arbitrarily large and each of them receives a strictly informative independent signal. The number of informed citizens is not a substitute for a small conflict of interest between the citizens and the policy maker.

The second lesson has to do with the importance of the precision of the signals received by the citizens. As the precision of the citizens’ private information converges to 0, we have that:

\[
\frac{r(1; b)}{r(1; a)} \to \frac{r(T; b)}{r(T; a)}.
\]

As we can see from equation (13), this implies that \( V_1(v) \) converges to \( v \). Thus, even if conflict is small, public protests may be ineffective if the citizens are not receiving sufficiently informative private signals.

Table I illustrates what informative equilibria look like as we change \( n, T, \) and \( \mu^* \) in an example in which the signal distribution is:

\[
r(t; \theta) = \frac{e^{-\alpha_at}}{\sum_{j=1}^{T} e^{-\alpha_b j}},
\]

with \( \alpha_a = 1 \), \( \alpha_b = 2.5 \), \( \mu = \frac{1}{2} \), and \( \mu^{**} = 0.4 \). It is interesting to note that many equilibria typically exist; perhaps more important, they can be very different from each other. When \( n = 1,000, T = 3, \) and \( \mu^* = 0.8 \), for example, the policy maker

<table>
<thead>
<tr>
<th></th>
<th>( \mu^* = 0.65 )</th>
<th>( \mu^* = 0.8 )</th>
<th>( \mu^* = 0.9 )</th>
</tr>
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<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>( n = 500 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T = 2 )</td>
<td>0.154</td>
<td>0.234</td>
<td>n/a</td>
</tr>
<tr>
<td>( T = 3 )</td>
<td>0.0321</td>
<td>0.338</td>
<td>0.186</td>
</tr>
<tr>
<td>( T = 5 )</td>
<td>0.0121</td>
<td>0.3742</td>
<td>0.218</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T = 2 )</td>
<td>0.154</td>
<td>0.252</td>
<td>n/a</td>
</tr>
<tr>
<td>( T = 3 )</td>
<td>0.0321</td>
<td>0.338</td>
<td>0.181</td>
</tr>
<tr>
<td>( T = 5 )</td>
<td>0.0031</td>
<td>0.43</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Notes. The values of the table represent the minimal and maximal equilibrium values of \( \frac{Q}{n} \). The value n/a is reported when an informative equilibrium could not be found.
is “convinced” by protesters if a little more than 18% of the expected population chooses to protest in the smallest equilibrium, or a little more than 21% in the largest equilibrium. But when conflict is smaller and/or $T$ is larger, the range of equilibria is significantly larger. When $n = 1,000$, $T = 3$, and $\mu^* = 0.65$ the policy maker can be convinced if just 3.2% of the expected population protests in the smallest equilibrium, and 33% in the largest equilibrium.

IV. LARGE PUBLIC PROTESTS AND INFORMATION AGGREGATION

Since Condorcet (1785) an important literature has been dedicated to the study of the conditions under which elections provide a mechanism for the full aggregation of citizens’ private information. An analogous question can be asked in the public protest game presented in the previous sections. Is there an equilibrium in which citizens’ actions are so informative that the policy maker’s decision reflects all the information available to the citizens? As a consequence, is there a sequence of equilibria along which the probability of a policy mistake converges to 0 as $n \to \infty$? This question pushes the line of inquiry of the Condorcet jury theorem one step further by asking not only whether it is optimal for the citizens to vote informatively but also whether it is ex post optimal for the policy maker to commit to a rule consistent with the citizens’ actions. We refer to the conjecture that the policy maker chooses the optimal action with probability one in a large society as the “Weak Condorcet Jury Theorem”.

Proposition 4 immediately informs us that the answer to the question stated above can not be unequivocally positive: when $\| v - V \|$ is too large, no informative equilibrium exists. Proposition 4, however, does not address this question when an informative equilibrium exists: even if we have an informative equilibrium for every $n$, it is still possible that the level of informativeness converges to 0 as population grows to infinity. The next result completes the analysis of Proposition 4 by proving the weak Condorcet jury theorem for our game of petitions and public protests. Recall from Section II that we say that full information aggregation is achievable if there is a sequence of informative equilibria along which the probability of mistake converges to 0.

**Proposition 5.** Full information aggregation is achievable as $n \to \infty$ if $V > V^*(v)$. 
Probability of Mistake When $\mu=\frac{1}{2}$, $\mu^*=0.6$, $\mu^{**}=0.5$, $T=3$, and $r(t; \theta)$ is described in the text with $\alpha_a=1$ and $\alpha_b=1.5$.

Figure I illustrates the equilibrium probability of committing an error as a function of $n$ in a specific example. The probability of error is computed as the probability of choosing $B$ in state $a$ plus the probability of choosing $A$ in state $b$:

$$M_n = \mu \Pr(B, a; \sigma_n, \rho_n) + (1 - \mu) \Pr(A, b; \sigma_n, \rho_n).$$

The lower curve (in black online) corresponds to the probability of committing an error in the best equilibrium, that is, in the equilibrium with the lowest error. The intermediate curve (in red online) corresponds to the probability of committing an error in the worst equilibrium. The higher curve (in blue online) is a benchmark: it represents the expected error that we would have if the policy maker could commit to choose $A$ if and only if more than 50% of the expected number of citizens protest. As the example shows, the probability of mistake converges to 0 relatively quickly; still, multiplicity of equilibria creates significant uncertainty when the number of citizens is not very large.

33. As an additional reference, we should recall that the policy maker with no information would commit a mistake with 50% probability when $\mu = \frac{1}{2}$, since with no information the policy maker chooses $B$ with probability 1.
The intuition for Proposition 5 is as follows. As we know from Proposition 4, when $V > V^\ast(v)$ we have a sequence of informative equilibria $\tau_n, q_n$ as $n \to \infty$; let $\tau_\infty$ be the limit of $\tau_n$. If $\tau_\infty$ is interior, that is, $\tau_\infty \in (1, T + 1)$, then the result is immediate. In this case, the monotone likelihood ratio assumption implies that $\phi(a; \tau_\infty) > \phi(b; \tau_\infty)$. This implies that the policy maker is drawing large samples from one of two distributions of independent random variables with different means: the result follows from a straightforward application of Chebyshev’s inequality. Problems arise when $\tau_\infty$ is not interior, but luckily this case can be ruled out.

Proposition 5 has significant implications on how to interpret the Condorcet jury theorem. A standard interpretation of the theorem is that it provides a positive theory of elections: elections are a good way to make public decisions because they have good informational properties (see for example Feddersen and Pesendorfer 1997). An implied assumption in this interpretation is that other political processes would not have the same informative properties. Proposition 5 shows that elections are not unique as mechanisms thorough which information can be aggregated: a policy maker—even one endowed with dictatorial powers and no commitment—can achieve full information relying just on the signals voluntarily provided by activist citizens as long as the ex ante conflict is not too large. Of course, this does not imply that autocratic regimes are better or even as good as liberal democracies in aggregating dispersed information. Both the possibility of

34. As $n \to \infty$, the distribution of protesters looks approximately like a normal concentrated around $n\phi(a; \tau^\ast)$ in state $a$, and around $n\phi(b; \tau^\ast)$ in state $b$. Observing a fraction of protesters $\tilde{Q}^n$ lower or equal than $\phi(b; \tau^\ast)$ is arbitrarily more likely in state $b$ than in state $a$; and observing a fraction of protesters $\tilde{Q}^n$ larger or equal than $\phi(a; \tau^\ast)$ is arbitrarily more likely in state $a$. This implies that the equilibrium threshold $q^\ast$ that makes the policy maker indifferent must be such that: $\phi(b; \tau^\ast) < \frac{q^\ast}{n} < \phi(a; \tau^\ast)$. Given this, the result follows from Chebyshev’s inequality: the probability that the fraction of protesters is larger than $\frac{q^\ast}{n}$ in state $b$ or lower than $\frac{q^\ast}{n}$ in state $a$ converges to 0 as $n \to \infty$.

35. Indeed, even a policy maker with a significant conflict of interest with the citizens can do better than a voting system. For example, Figure 1 suggests that the probability of a mistake in the worst equilibrium is significantly lower than the probability of mistake in the case in which the decision is taken by committing to a cut-off rule $\tilde{Q} = 0.5$. Commitment to a cut off rule guarantees that the mistake converges to 0 as population increases; if the cut-off rule is not appropriately chosen, however, it does not guarantee that the mistake converges faster than the equilibrium mistake in an equilibrium of the protest game.
holding elections and of staging petitions or public protests presuppose the existence of a certain degree of civil liberties that is typically not present in autocratic regimes. Formal mechanisms to aggregate information (like elections and referenda) and less formal ways (like petitions and public protests) should be seen as complementary ways for policy makers to acquire information and, indeed, as manifestations of the same phenomenon. Proposition 5 highlights the fact that if we are looking for a positive theory of why elections are good, we need to look beyond the mere fact that they allow for information aggregation.

So are elections especially good for information aggregation? And why? Proposition 5 provides a positive argument for the optimality of formal elections if we take a “behind the veil of ignorance” point of view. It is natural to assume that citizens choose institutions at an ex ante stage, before the issue to be decided on is defined. At this stage, it is plausible to assume that there is uncertainty regarding the conflict that may arise between the policy maker and the citizens. Assume $V$ is a random variable with density in $[0, v]$ and generic distribution $F$ with full support. This represents an environment in which citizens and the policy maker do not know what kind of issue they will have to face, so they are uncertain about the conflict of interest. Citizens have to choose between an election in which a nonunanimous $q$-rule is used as in Proposition 1 and a system in which the policy maker has full authority but citizens can protest. In this case we have:

**Proposition 6.** Assume we select the most informative equilibria in the public protest and voting games. There is a $n^*$ such that for $n > n^*$ both citizens and the policy maker prefer to commit to an election rather than to leave the policy to the policy maker’s discretion before they know the realization of $V$.

Proposition 6 puts very few constraints on the expected conflict between the policy maker and the citizens. Still the result claims that the policy maker would not find it optimal to retain discretion in policy making. The idea behind Proposition 6 is simple. When conflict is sufficiently small, public protests can perform as well or even better than elections. When the conflict is large

36. Even if we see elections and petitions/public protests as complementary mechanisms, when designing a “political system” we are left with the decision of what should be left to the discretion of the executive and what should instead be the object of an election or referendum.
public protests are ineffective but, by committing ex ante to a voting rule, the policy maker can achieve the first best in this case, too. As \( n \) increases, any possible advantage of the policy maker with discretionary power fades away, since in both systems the probability of mistake converges to 0 when \( V > V^*(v) \). In states in which \( V < V^*(v) \), on the contrary, the election does strictly better. The key observation following Proposition 6 is that what makes elections valuable for information aggregation is not the specific voting rule, but the commitment implicit in the decision rule. We should expect citizens to commit to electoral systems when expected conflict is high and when population is high. When population is not particularly high and or expected conflict is not too high, a more informal system in which an individual is entrusted with decision power and the other citizens can stage public protests may be more efficient.

V. PROTESTS AND SOCIAL MEDIA

Many scholars have recently emphasized the importance of social media in fostering collective action.\(^{37}\) From a conceptual point of view, the effect of social media on the ability of citizens to signal their information is ambiguous. Social media allows groups to share information and coordinate the activity of their members, but it reduces the number of independent actors. By sharing information, protesters become better informed, but they can also coordinate on strategies that are more effective at influencing the policy maker: this may reduce the policy maker’s willingness to “trust them.”

To study the impact of social media on the ability of public protests to aggregate information, consider a slightly simplified version of the model presented above in which the individual signal \( t \) has binary support \( T = \{0, 1\} \), with \( r(1; a) = r = r(0; b) \) and \( r > \frac{1}{2} \). I model the effect of social media assuming that each

\(^{37}\) Recent works include Wasserman (2010) and Manacorda and Tesei (2016) who discuss the importance of mobile phones and texting for collective action in Africa; Valenzuela, Arrigada, and Scherman (2012) who document the correlation between activity on Facebook and protests in Chile; Bennett and Segerberg (2011) who document the role of social media by activists during the 2009 G20 London Summit; Enikolopov, Makarin, and Petrova (2016) who study the diffusion of the social media platform VK on a wave of protests in Russia in 2011; and Acemoglu, Hassan, and Tahoun (2014) who document the use of Twitter during Egypt’s Arab spring.
citizen is affiliated to a group of size \( g \). The number of groups is a Poisson random variable with mean \( m \geq 2 \), so expected population is now \( n = m \cdot g \). The key assumption in this version is that groups’ members can communicate and share their information within their groups.\(^{38}\) Examples of relevant social groups are blogs, Facebook or YouTube pages, Twitter hashtags; or more old fashioned groups, like unions or parties. I say that citizens are in autarky when \( g = 1 \), as in the previous sections. I say that social groups are available if \( g \geq 2 \). In this model technological progress in social media corresponds to an increase in \( g \).

Consider the problem faced by the citizen in a social group. When citizens share their information, each citizen in a group receives an informative signal corresponding to the number of citizens with a \( t = 1 \) (instead of \( t = 0 \)) realization. This aggregate signal \( \tilde{t} \) has support \( \tilde{T} = \{0, \ldots, g\} \) and distribution \( r_g(t; \theta) = \mathcal{B}_g(t; \theta) \), where \( \mathcal{B}_g(t, \theta) \) is a binomial with mean \( rg \) when the state is \( a \), and \( (1 - r)g \) when the state is \( b \). If we fix the strategies of the policy maker, it is indeed easy to see that citizens in a social circle find it optimal to truthfully share their information, coordinate their actions, and act as a block. Similarly, if all groups act in a coordinated way, the policy maker will find it optimal to treat each group as an individual agent.\(^{39}\) This implies that the extended game with \( m \) groups of size \( g \) can be treated as a game with \( m \) individual with signal \( r_g(t; \theta) \). The trade-off mentioned at the beginning of this section is now clear: on the one hand each group receives a more precise signal than any individual protesters; on the other hand, however, we have only \( m \) independent groups, rather than \( n = mg \) independent individuals.

To see why groups may help protesters signal their information, note that the likelihood ratio of the signal received by a group is now \( \frac{r(t; a)}{r(t; b)} = \left( \frac{r}{1 - r} \right)^{2t - g} \). As \( g \) increases, the posterior probability that the state is \( a \) after signals \( \tilde{t} = 0 \) converges to 0: these groups are going to be willing to abstain from protesting even when there is a very large conflict. The implications of this can be seen on

\(^{38}\) This idea reflects the evidence on the working of activist groups. For example, Bennett and Segerberg (2011) report that during the 2009 G20 London Summit 160 distinct civil society groups were active; Valenzuela, Arrigada, and Scherman (2012) report the case of a successful protest in which 118 independent Facebook pages were active. There is abundant evidence that social media allows these groups superior information sharing and coordination.

\(^{39}\) A formal proof of these claims is presented in the Appendix in the proof of Proposition 7.
$V_2(v)$ that now becomes:

$$V_2(v) = \left[ \frac{(1 - r)}{r} \right]^{\frac{g}{2}} \cdot v.$$  

As $g$ increases, $V_2(v)$ converges to 0 for any $v$, guaranteeing existence of an informative equilibrium no matter how large the conflict is. We have:

**Proposition 7.** For any $r$ and $m$, information aggregation is possible with social media if $g > g^*(r)$, where 

$$g^*(r) = \frac{\log (\frac{V}{v})}{\log (\frac{1 - r}{r})}.$$ 

Proposition 7 clarifies the importance of the information sharing and coordination services provided by social media in our model. Two familiar forces emerge from the threshold $g^*(r)$. First, the size of conflict. The ratio $\frac{V}{v}$ is a measure of the size of the conflict: it is minimal when $V = v$ (a case with no conflict) and it increases as the distance between $v$ and $V$ increases. For any $r$, as the conflict converges to 0, $g^*(r)$ converges to a value that is lower or equal than 1: so for sufficiently small conflict, communication in groups is irrelevant. For a given level of conflict, however, the minimal group size compatible with information aggregation decreases with $r$: as signals become uninformative, the size of a required group grows to infinity. The size of social media required for information aggregation is determined by these two forces.

The analysis becomes interesting when we assume that no information can be aggregated in autarky. This always occurs when the individual signals are not too informative. From equation (13), we see that no information is aggregated if $r < r^*$, where $r^* = \frac{1}{1 + \sqrt{\frac{V}{v}}}$. Define:

$$r(g) = \frac{1}{1 + \left( \frac{V}{v} \right)^{\frac{1}{2g}}}.$$  

Note that $r(g) < r^*$ for any $g \geq 2$ and $r(g) \to \frac{1}{2}$ and $g \to \infty$. An immediate implication of Proposition 7 is the following result:

**Corollary 1.** When $r \in [r(g), r^*]$, information aggregation is feasible with public protests if and only if social media is available.

To understand why social groups help public protests, it is useful to go back to why information aggregation fails in autarky.
As explained in Section III, a necessary condition for an informative equilibrium to exist is that citizens that receive a low signal choose to stay home even if the policy maker is indifferent and they condition on being pivotal. However, when the signal received by each agent is not very informative (i.e., $r < r^*$), the difference in the posterior beliefs of the policy maker and the citizens with the lowest realization of $\tilde{t}$ is not sufficient to compensate for the conflict of interest: in this case information aggregation is impossible. When citizens share information in groups, however, they improve the precision of their aggregate signal. The worst possible signal now is not receiving $t = 0$ rather than $t = 1$, but receiving zero positive signals out of $g$. If $g$ is sufficiently large, precision in a group is sufficient to compensate for the conflict, and information aggregation becomes possible.

The intuition described above suggests that a potential problem for Proposition 7 may arise when, as plausible to assume, signals are correlated at the level of the social group.\textsuperscript{40} To see this point let us assume that $g = 2$ and let the joint distribution of a group’s signals in state $\theta$ be described by Table II, where $r_a = 1 - r_b = r > \frac{1}{2}$.\textsuperscript{41} In this distribution, signals are imperfectly positively correlated and correlation is increasing in the parameter $\alpha$: when $\alpha = 0$ types are conditionally independent and we are in the case analyzed above (with $r(1; a) = r = r(0; b)$ for each agent as above); when $\alpha = 1$ types are perfectly correlated and all the mass is on the leading diagonal. We refer to this as the model with correlated types.

It is easy to verify the monotone likelihood ratio is satisfied and that the likelihood ratio of the signal received by a group at

\begin{table}
\centering
\small
\begin{tabular}{ll}
\hline
& $t = 0$ & $t = 1$ \\
\hline
$t = 0$ & $(1 - r_\theta)^2 + a r_\theta(1 - r_\theta)$ & $(1 - a) r_\theta(1 - r_\theta)$ \\
$t = 1$ & $(1 - a) r_\theta(1 - r_\theta)$ & $r_\theta^2 + a r_\theta(1 - r_\theta)$ \\
\hline
\end{tabular}
\caption{Distribution of Signals in the Case with Correlation and $g = 2$}
\end{table}

\textsuperscript{40} It is plausible to assume that signals in social groups may be correlated because, among other possible reasons, the members of a social groups are likely to share at least some of their sources of information.

\textsuperscript{41} This distribution was first introduced by Battaglini, Benabou, and Tirole (2005) to study correlation in a signaling model with multiple senders.
\( \tilde{t} = 0 \) is now:

\[
R(0, r, \alpha) = \frac{\Pr(0; a, \alpha)}{\Pr(0; b, \alpha)} = \frac{(1 - r)^2 + ar(1 - r)}{r^2 + ar(1 - r)}.
\]

This ratio is decreasing in \( \alpha \) and, for any \( \alpha \in (0, 1) \), it lies in between the level we have with conditionally independent signals and the case with perfectly correlated signals: \( \frac{(1-r)^2}{r^2} < R(0, \alpha) < \frac{(1-r)}{r} \). In autarchy the signals have the same distribution as in the case with independent signals so, as before, no information aggregation is possible if \( r < r^* \). Using equation (16), we can now see that with correlated signals, instead, we have information aggregation if \( r > r(\alpha) \), where \( r(\alpha) \) is uniquely defined by this equation: \( \tilde{R}(0, r(\alpha), \alpha) v = V \). As it can be verified \( r(\alpha) = r(g) \) as defined by equation (15) when \( \alpha = 0 \) and \( g = 2 \); \( r(\alpha) = r^* \) when \( \alpha = 1 \); \( r(\alpha) \), moreover, is increasing in \( \alpha \). We now have:

Corollary 2. In the model with correlated signals, when \( r \in [r(g, \alpha), r^*] \) information aggregation is feasible with public protests if only if social media is available.

Corollary 2 shows that with correlated signals, social groups may still make petitions and public protests informative when they are not informative in autarchy. However, correlation reduces the benefit of social groups: as \( \alpha \to 1 \) the group becomes just as informative as an individual.

VI. DISCUSSION AND EXTENSIONS

In the model analyzed in the previous sections, I made many simplifying assumptions. In this section I relax these assumptions, extending the basic framework in a number of directions.

VI.A. Different Priors

So far we have assumed that citizens and the policy maker assign the same prior probabilities on the states of the world. The origin of conflict between players lies on the fact that citizens and the policy maker face different trade-offs between \( A \) and \( B \), as measured by \( V = V = -\frac{V(a)}{V(b)} \) and \( v = -\frac{v(a)}{v(b)} \). In their model of voice, Banerjee and Somanathan (2001) propose an alternative

42. The condition corresponding to \( V > V_2(v) \) in the model with correlated signals is \( V \leq R(0, r(\alpha), \alpha) v \). This implies \( r > r(\alpha) \), with \( r(\alpha) \) as defined above.
model in which citizens and the policy maker assign the same payoffs to the effect of policies in the two states; the citizens and the policy maker, however, assign different priors probabilities to the states of the world. To see the relationship of our model to Banerjee and Somanathan’s model, consider a modified version of the model presented in the previous sections in which \( V = \hat{V} \); the policy maker assigns a prior probability \( \mu \in (0, 1) \) to state \( a \); citizens assign a probability \( \hat{\mu} \in (0, 1) \) with \( \hat{\mu} > \mu \). In this version, therefore, conflict arises because citizens assign a higher prior probability on \( a \) than the policy maker.\(^{43}\)

To see the implications of this change, consider equation (6), the condition characterizing when it is optimal for a citizen to protest. With the new prior, it can be written as:

\[
\frac{\hat{\mu}(a; t)}{\hat{\mu}(b; t)} = \frac{\hat{\mu} \cdot r(t; a)}{(1 - \hat{\mu}) \cdot r(t; b)} \geq \frac{\varphi_n(b; \sigma, \rho)}{\hat{\mu} \cdot \varphi_n(a; \sigma, \rho)}.
\]

where \( \hat{\mu}(\theta; t) \) is the posterior corresponding the different prior \( \hat{\mu} \). This condition can be easily rewritten as:

\[
\frac{\mu(a; t)}{\mu(b; t)} \geq \frac{\varphi_n(b; \sigma, \rho)}{\hat{\mu} \cdot \varphi_n(a; \sigma, \rho)},
\]

where \( \mu(\theta; t) \) is the posterior when the prior is \( \mu \) and \( \hat{\mu} = \frac{\hat{\mu}(1 - \mu)}{\mu(1 - \hat{\mu})} \).

Equation (17) says that citizens with prior \( \hat{\mu} \) and payoff parameter \( \hat{V} \) are willing to protest if and only if citizens with prior \( \mu \) and payoff parameter \( \hat{\mu} > \hat{V} \) are willing to protest. This implies that the modified game in which citizens have different prior probabilities than the policy maker is equivalent to the game studied above in which priors are the same but citizens have different payoffs than the policy maker. The model presented in Section II, therefore, can be reinterpreted as a model in which conflict originates from a disagreement in prior beliefs.

VI.B. Conflict of Interest between Citizens

Another assumption made in the previous sections is that all citizens have the same preferences, as described by the single parameter \( v \). Naturally, in real life citizens have heterogeneous preferences.

\(^{43}\) Of course I assume here that \( v = \hat{V} \) just to make a cleaner comparison with the model of Section II; we could assume both \( \hat{\mu} > \mu \) and \( v > \hat{V} \) and have a model with different preferences and different prior probabilities.
preferences. Extending the model to incorporate these conflicts is, at least conceptually, not difficult. Assume that there are $K$ types of citizens, each characterized by a preference parameter $v_i \geq 0$ for $i = 1, \ldots, K$; and that these types are independently distributed with probability $(\xi_i)_{i=1}^K$. In this case, the number of citizens of type $i$ is Poisson distributed with mean $\xi_i n$. The analysis of this version of the model is similar to the analysis presented above: voters decide to protest based both on their signal and their preference parameter; the equilibrium is described by a series of cut-points $(\tau_i^*)_{i=1}^K$ and a policy maker’s decision rule $q^*$.

A particularly convenient but still insightful version of the model with conflict is a model with partisans. Assume there are three types: type 1 with $v_1 = v \in (0, \infty)$, who wants $A$ in state $a$ and $B$ in state $b$; type 2 with large $v_2$, who finds $A$ optimal in both states; and type 3 with $v_3 = 0$, who finds $B$ optimal in both states. Types 2 and 3 are “partisans” who have a dominant strategy and so their actions would never provide information to the policy maker. Since type 3 agents would not find it optimal to protest, this model is equivalent to a model in which type 3 does not exist and $n = (1 - \xi_3)n$. Since type 2 agents are always active, the policy maker corrects his expectation for the fact that a share of agents $\xi_2$ is always active. Assume that $\tau^*, Q^*$ is the limit of an sequence of equilibria as $n \to \infty$ with no partisans ($\xi_1 = 1$). Then $\tau^*, Q^{**}$ is a limit of equilibria of a model with partisans, where $Q^{**} = Q^* \xi_1 + \xi_2$.

It is useful to consider this simple model because although it leads to similar positive predictions, it may have different

44. Note that for $v_2$ to be arbitrarily large we do not need to have arbitrarily large payoffs. Since $v_2 = -\frac{v(a)}{v(b)}$, it is sufficient to have $v(b)$ close to 0.

45. The assumption of “partisan” citizens who prefer a policy no matter what the state is quite common in the literature on elections, see for example Feddersen and Pesendorfer (1996).

46. A similar phenomenon is observed by Chiang and Knight (2011) who study the effect of newspaper endorsements on voters. They present evidence showing that endorsements are effective, but voters rationally account for biases in the newspapers’ ideological positions.

47. A related phenomenon occurs with heterogeneous preferences, even if preferences are not extreme. If a subset of citizens finds it optimal to act uninformatively, the policy maker will adjust her reaction function correcting for the uninformative behavior. Similarly as in Lemmas 1 and 2 and in Proposition 4, a key condition for the existence of an informative equilibrium is that there is at least a subset of citizens receiving sufficiently informative signals and/or having sufficiently low conflict with the policy maker.
implications for welfare. Assume for example that $\xi_3 = 0$ and $\xi_2 > 0.5$. In this case, an election would lead to an uninformative outcome, since $A$ would always win. The result is different in the public protest game. In this case, as $n \to \infty$, public protests may lead the policy maker to take the policy that maximizes her payoff in all states: that is, we may have information aggregation.

VI.C. Private Values

Although I allow for heterogeneity in the extension above, I maintain the assumption that all players share a common valuation for the policy ex post: they disagree on the relative costs of possible mistakes that can be committed at the interim stage, when the state is unknown; but they agree on the best policy given the state.\(^{48}\) This is the standard assumption adopted in the literature on the Condorcet jury theorem, and it is the appropriate assumption in many applications. There are, however, cases for which there is no obvious “best policy”: citizens have private values and the policy maker cares only about being with the majority. We now show that the model can be extended and reinterpreted to describe environments with these features.

Assume that citizens have private values and that they may either be for policy $A$ or for policy $B$: a citizen obtains a payoff of 1 when the preferred policy is chosen and 0 otherwise. Given the state, citizens’ types are independent realizations. In state $a$, the probability that a citizen is for $A$ is $p_a > \frac{1}{2}$; similarly, in state $b$, the probability that a citizen is for $B$ is $p_b > \frac{1}{2}$: so the majority is expected to be for $A$ in state $a$ and for $B$ in state $b$. The policy maker is “office motivated” as in the interpretation presented in Section II: he receives a benefit $U > 0$ for policy $B$ and 0 for $A$ in all states; in addition he suffers a disutility $D > U$ when he selects a policy that is in disagreement with the preferences of the majority of citizens.\(^{49}\)

The key assumption now is that at stage one, when the citizens decide whether to participate in a petition for $A$, they do not know with certainty if in the second stage they are going to be for $A$ or $B$, they just receive an informative signal $t = 1, \ldots, T$. The idea is that citizens may have received only partial information

\(^{48}\) As discussed in Section II.A, the reason the policy maker wants the same policy as the agent ex post may depend more on his desire for reelection or for not losing market shares than on real agreement on the policies.

\(^{49}\) See Section II for a discussion of these preferences.
on the pros and cons of the policy. Returning to the example of United Airlines’ PetSafe policy, for example, a citizen may decide to sign a petition against the policy on the basis of the feedback heard from a neighbor or a friend who had direct experience with it, without knowing all the details. As in Section II I assume that the signal distribution is \( r(s; \theta) \) in state \( \theta \) with the same monotone likelihood ratio properties as in the baseline model.50

In this environment, the policy maker’s problem is very similar to the problem of the previous model: he chooses \( A \) if the posterior after observing \( Q \) protesters is high enough, \( \Gamma_n(a; Q, \sigma) \geq \mu^* \). Using the terminology of the previous section we have \( V(a) = D - U \) and \( V(b) = -(D + U) \). This leads a condition similar to equation (8):

\[
\frac{P(Q_n, n \phi(a; \sigma_n))}{P(Q_n, n \phi(b; \sigma_n))} \geq \frac{D + U}{D - U} \left( \frac{1}{\mu} - 1 \right) = \frac{1}{\tilde{V}} \left( \frac{1}{\mu} - 1 \right),
\]

where \( \tilde{V} = \frac{(D-U)}{(D+U)} \) plays the same role as \( V \) in the previous analysis.

The citizens’ problem is now a little different. A citizen with a signal \( t \) finds it optimal to be active if:

\[
(18) \quad \sum_{\theta} \mu(\theta; t) \left[ p_{b} \cdot \phi_{n}(\theta; \sigma, \rho) - (1 - p_{b}) \phi_{n}(\theta; \sigma, \rho) \right] \geq 0.
\]

To interpret equation (18), note that the term in the square brackets is the net benefit of being active in state \( \theta \): with probability \( p_b \) the agent is for \( A \), and so by being active he or she increases the probability of the preferred policy by \( \phi_{n}(\theta; \sigma, \rho) \); with probability \( 1 - p_b \), instead, the agent is for \( B \) and he or she reduces the probability of the preferred policy by \( \phi_{n}(\theta; \sigma, \rho) \). Condition (18) can be rewritten as:

\[
\frac{\mu(a; t)}{\mu(b; t)} \geq \frac{2p_{b} - 1}{2p_{a} - 1} \cdot \frac{\phi_{n}(b; \sigma, \rho)}{\phi_{n}(a; \sigma, \rho)} = \frac{\phi_{n}(b; \sigma, \rho)}{\tilde{V} \cdot \phi_{n}(a; \sigma, \rho)},
\]

50. As in the previous section, I do not model the details of where these signals originate, but it is not difficult to describe environments with the features described above. Consider the case in which, in addition to a Poisson number of activists, there is a continuum of citizens who never engage in public activism, the “silent majority.” As for the activists, a member of the silent majority is more likely to be for \( A \) in state \( a \) than in state \( b \). A finite subset of these citizens have direct experience with the policy and know their preferences for certain. Each activist can see the experiences of a subset of these citizens (their neighborhood, their friends, family, etc.) and use these observations as signals on the state of the world.
where $\tilde{v} = \frac{(2p_a-1)}{(2p_b-1)}$ plays the same role as $v$ in the previous analysis. In the model of the previous sections with common values, the citizens’ preferences depend directly on the “true state”: the citizens condition on their signal and on being pivotal to assess the likelihood of this state and to choose the best action. Now each citizen is just trying to induce a policy equal to his or her expected private value. By doing so, however, the active citizens signal to the policy maker the position of the majority. Despite these differences, a comparison of equations (17) and (18) with (8) and (6) shows that the analysis of this model with private values is similar to the common values model if we assume $\tilde{V} < \tilde{v}$. This condition is satisfied if $U$ is sufficiently large or, for any $U$, if $p_a \geq p_b$.

VI.D. Participation Costs and Benefits

In all the examples presented in the introduction (the petition on Change.org by the customers of United Airlines, the petition of the 400 economists against President’s Bush tax cuts in 2003, for example), the costs of participating in the public protest are not particularly significant. There are, however, environments in which some citizens experience positive costs of participation, due to the effort of protesting; other citizens, on the contrary, experience a negative cost, if they enjoy voicing their concerns; others, finally, experience both costs and benefits, with a net that can be positive, negative, or zero. To model all these possibilities, I follow the approach of Palfrey and Rosenthal (1985) who presented the first rigorous analysis of participation costs in elections. I assume each active citizen pays a token $c$ that has finite support with distribution $G(c)$. I assume that the minimal value in the support is $c < 0$, the maximal value is $c > 0$ (sufficiently large that citizens with this cost will never protest), and 0 is in the support. I do not impose other restrictions on the support or the relative probabilities assigned to the possible realizations.

With participation costs, the analysis is considerably more complicated because now the choice to protest depends on both the signal $t$ and the cost $c$. In the previous analysis, citizens’ strategies could be described by a simple cut-point $\tau^*$. Now citizens’ strategies can be described by a family of cut-points $\tau^*(c)$, one for each possible realization cost $c$. We generally have three classes of citizens: there is a threshold $c_*$ such that for $c \geq c_*$ citizens will chose to be inactive no matter what their signal is; there is a threshold $c^*$, such that for $c \leq c^*$ citizens will chose to protest
no matter what their signal is; finally, for $c \in (c_*, c^*)$, we have citizens who use informative strategies.\footnote{Note that in general $c_*$ is strictly negative and $c^*$ strictly positive since even if a protester likes (respectively, dislikes) the act of protesting, he or she may be reluctant to do it after a bad (good) signal.} The results presented in the previous sections, however, can be extended to this more general version of the model. We have:

**Proposition 8.** There is a threshold $V^G(v) \in (V_1(v), V_2(v))$ such that an informative equilibrium exists in a large society if $V > V^G(v)$ and it does not exist if $V < V^G(v)$. When $V > V^G(v)$ full information aggregation is achieved as $n \to \infty$.

Although the cut point for the existence of an informative equilibrium may depend on the distribution of participation costs $G$, what matters is that $V^G(v) \in (V_1(v), V_2(v))$, so the range of environments in which information aggregation is impossible because of the size of the conflict and the range in which full information aggregation is feasible are nonempty for any distribution of costs $G$. What is remarkable is that the distribution of costs $G$ requires little restriction.

The intuition for this result is relatively straightforward. As in Palfrey and Rosenthal (1985), only the citizens with a sufficiently small cost/benefit of participation (i.e., a small $|c|$) will act according to their signal when population is large. This happens because the probability of being pivotal converges to 0 as $n \to \infty$, so $c$ dominates the citizen's decision. When $V < V^G(v)$, however, not even the citizens with a very small $|c|$ find it optimal to follow their signals: in this case therefore, no citizen behaves informatively. When $V > V^G(v)$ the logic is similar, with the only difference being that in this case the citizens with a very small cost/benefit of participation find it optimal to act according to their signal. Naturally the fact that full information aggregation is theoretically feasible in this case doesn't mean that the error will be zero or even small with finite population: the expected number of citizens who are willing to be informative may be drastically reduced, implying a reduction in the quality of information aggregation if $n$ is finite.

An important assumption in this analysis is that although $c$ may be heterogeneous, there is a fraction of population for which it is small in absolute value. This is a natural assumption in liberal democracies, but less so in autocracies where dissent of any
form may be severely punished. How would the analysis change in these cases? Let us assume that in addition to the costs $c$ described above, citizens always pay a strictly positive cost $\kappa > -c$ for activism in an autocracy. Unsurprisingly, this makes information aggregation even harder. The result that if the precision of the individual signals is insufficiently high, then an informative equilibrium cannot exist even for large populations remains valid: in addition, however, now the minimal precision required for information aggregation increases with $\kappa$; for any level of precision, there is a $\kappa^*$ such that an informative equilibrium is impossible if $\kappa \geq \kappa^*$. To see this, note that as precision of signals is reduced, the policy maker requires an increasingly larger number of active citizens to be convinced; but when the number of required active citizens is large, the probability that each of them is pivotal is necessarily small. When precision of signals is sufficiently small, the resulting pivot probabilities can not be sufficiently high to compensate for $\kappa$. The result that we have full information aggregation, moreover, also does not extend if $\kappa + c > 0$. In this case the expected number of protesters must be finite even if $n \to \infty$, to keep pivot probabilities above $\kappa + c$ and so ensure participation: this limits the number of informative signals that can be acquired by the policy maker. These observations confirm that while public protests and petitions can be effective tools to guide public policy, they require minimal level of civil liberties to work (in the form of sufficiently low costs of expressing dissent). When these costs are high, information transmission is either limited or impossible.

VI.E. Voters’ Motivation and Pivotality

Throughout the analysis, I have assumed that citizens have instrumental preferences: they care about their participation in petitions or protests only to the extent that it affects the policy outcome. This is a simplification since there is evidence that some citizens enjoy participating to protests or petitions: they may like the idea of expressing their opinion (Brennan and Lomasky 1993), they may be motivated by self-image considerations (Della Vigna, List, and Malmendier 2015) or ethical concerns (Coate and Conlin 2004; Feddersen and Sandroni 2006). The presence of voters with these types of noninstrumental preferences, however, is not in conflict with the basic result of the model: first, the results are robust to modification of the assumptions regarding the citizens’ preferences; second, the fact that citizens directly care about being
politically engaged does not imply that they are uninterested in policy outcomes and in the likelihood of affecting them.

Regarding the first point, note that, as I did in the extension presented above in this section the baseline model can be enriched by adding agents who like (or dislike) the act of protesting. Importantly, the logic behind the results of the baseline model generalizes to these alternative environments even if the fraction of citizens who care mostly about the outcome is small (as shown in Proposition 8). Intuitively, citizens that are not instrumentally motivated and act uninformatively do not affect the policy maker’s beliefs: the policy maker discounts the actions of these players.\footnote{In this respect, the issue of whether citizens have instrumental preferences and/or noninstrumental preferences is less relevant in my model than in standard voting models. In standard voting models the decision is mechanically determined by the voting rule, so if informative voters are a small group, they may not affect the outcome.}

Regarding the second point, there is a large body of evidence, from both field and experimental data, showing that instrumental preferences are an important factor in collective actions. Field data have been used to test for the comparative statics predictions of pivotal voter predictions, especially to test whether pivotal voter models can account for the variation in turnout in a cross section of districts with heterogeneous sizes. Supporting evidence of these comparative statics predictions have been found both in work relying on reduced-form estimations and aggregate data (Riker and Ordeshook 1968; Wolfinger and Rosenstone 1980; Powell 1986), and in structural estimations of models based on detailed micro data from local elections (Hansen, Palfrey, and Rosenthal 1987; Coate, Conlin, and Moro 2008).\footnote{Hansen, Palfrey, and Rosenthal (1987) structurally estimate a model of elections using data on school district referenda in Oregon. Coate, Conlin, and Moro (2008) perform a similar exercise with liquor referenda in Texas. Both papers show that pivotal voter models do a very good job in explaining cross-sectional data as they predict how turnout changes with the characteristics of the environment. Coate, Conlin, and Moro (2008) moreover show that a simple alternative model in which voters ignore the probability of being pivotal (the intensity model), is unable to explain the cross sectional data. Coate, Conlin, and Moro (2008) find that pivotal voter models find it difficult to explain the margin of victories observed in their data. This may be because they estimate a model with complete information and homogeneous citizens. As shown theoretically by Feddersen and Pesendorfer (1996) and experimentally by Battaglini, Morton, and Palfrey (2010), the prediction that the margin of victory should be small is not to be expected in a model with incomplete information and citizens with heterogeneous information.}

More recently, Kawai and Watanabe (2013) structurally estimated a model of

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strategic voting based on Myerson (2002), using aggregate municipality data from Japanese general elections. They find that strategic voters (in their definition, voters who make a voting decision conditioning on the event that their vote is pivotal) constitute a large proportion of the electorate: from 63% to 85% of the total, in their estimate. A similar conclusion is reached by Fujiwara (2011), who exploits a regression discontinuity design in the assignment of single-ballot and dual-ballot plurality systems in Brazilian mayoral races.

A recent wave of experimental work has provided a closer look at the performance of pivotal voter models allowing direct observation of individual strategies. Experimental evidence on the importance of pivotality for participation in elections has been presented, among others, by Schram and Sonnemans (1996), Goeree and Holt (2005), Grosser, Kugler, and Schram (2005), Herrera, Morelli, and Palfrey (2014), Kartal (2014), and especially Levine and Palfrey (2007) who present the most comprehensive laboratory study of voters’ participation. Although they find that there is undervoting in small electorates and overvoting in large electorates, they also find strong evidence of the main comparative statics predictions of pivotal voter models.54 Perhaps more important for this work, laboratory experiments have tested the extent to which citizens condition on pivotal events when forming beliefs in models with incomplete information. Evidence supporting the prediction of the pivotal models of information aggregation have been presented by Guarnaschelli, McKelvey, and Palfrey (2000), Battaglini, Morton, and Palfrey (2007, 2008, 2010), Bhattacharya, Duffy, and Kim (2014), Bouton, Castanheira, and Llorente-Saguer (2016), among others.55

Two additional features of the theory should be stressed here. First, for the theory to work we do not need extremely large num-

54. The analysis has been confirmed and extended by Duffy and Tavits (2008) who have documented a positive correlation between voting participation and the perceived probability of being pivotal in an experiment in which the authors elicited truthful beliefs with monetary incentives.

55. The first paper to experimentally study information aggregation in elections with incomplete information is Guarnaschelli, McKelvey, and Palfrey (2000). In subsequent work, the analysis has been extended to environments with heterogeneous preferences and heterogeneous signal precision (Battaglini, Morton, and Palfrey 2008, 2010), the possibility of sequential elections (Battaglini, Morton, and Palfrey 2007), the possibility of abstention (Bhattacharya, Duffy, and Kim 2014), and when alternative voting rules are used Bouton, Castanheira, and Llorente-Saguer (2016).
bers of protesters; indeed, some of the examples presented show that large numbers are not necessary for petitions and protests to be relevant. Consider the petition by the Israeli security chiefs: it comprised only 180 chiefs in total, but only dozens of them were really prominent (Times of Israel 2015). In this case, the probability of being pivotal for each petitioner is definitely not negligible. As discussed in Section VI, moreover, when social media is available and citizens can coordinate in groups, the groups are the basic players of the public protest game. The assumption that groups expect to be pivotal is perhaps more intuitively plausible than the corresponding assumption for individuals, at least to the extent that groups act in a coordinated way (see Morton 1991; Schram and van Winden 1991, for example). Evidence of the coordinated actions of these nonatomistic players is presented, among others, by Bennett and Segerberg (2011) for the London G20 protests in 2009 and by Valenzuela, Arrigada, and Scherman (2012) for youth protests in Chile. Second, the qualitative theoretical results do not rely on the assumption that citizens are particularly accurate in computing their beliefs. For example, consider the impossibility result stating that if individual signals are not sufficiently precise, then an informative equilibrium does not exist even if the population is arbitrarily large. As argued in Section IV, this result follows if the left-hand side of equation (12) converges to 0 as the individual precision converges to 0. For this to happen, it is not necessary for the posterior beliefs to be exactly correct, just that the citizens and the policy maker understand that they are conditioning on very similar events.56

VI.F. Multiple Policies, Multiple Actions

In the model, we have assumed that both the policy maker and the citizens have only two options: policy A or B for the policy maker; “action” or “no action” for the citizens. There are, however, environments in which the policy maker can graduate her response by choosing more than two policies, and citizens can choose among multiple levels of activism. Once more, extending the analysis to these environments presents no conceptual problem, but it would make the analysis considerably more complicated. It is interesting to discuss here the complications that it would entail.

56. Indeed, the logic of this result is not very dissimilar to Groucho Marx’s famous telegram to the Friar’s Club of Beverly Hills: “Please accept my resignation. I don’t care to belong to any club that will have me as a member.”
Assume first the case in which the policy takes more than two values, but the citizens’ actions are binary. Citizens still want to condition their beliefs on being pivotal: because more than two actions can be taken, however, there are now multiple ways of affecting the outcome. For example, with three policies citizens may be interested in the event in which policy $A$ is chosen instead of the default policy $B$, or they may be interested in the event in which policy $C$ is chosen instead of policy $A$. The optimal decision depends on the relative probabilities of all these events. Additional complications arise when the agents can choose multiple actions, perhaps at different costs, because different actions may allow the agents to be pivotal in different events. These issues are very similar to the complications that we encounter in extending voting theories to allow voters to cast more than one ballot (as it happens in approval voting or Borda voting) or if we allow for multiple candidates. In a series of seminal papers Myerson (2000, 2002) has shown how to extend the standard tools to deal with these complications in voting models, though at the price of a significantly more complicated analysis. The analysis would be even more complicated in our case since we are not assuming the policy maker commits to a decision rule as in voting models with an exogenous voting rule. I leave this extension for future research.

The presence of multiple actions (i.e., multiple forms of participation to the petitions/protests), each associated to a different cost of participation, should make it easier for citizens to screen themselves and therefore to be informative. An argument similar to the one presented in the previous section however, makes it clear that the key qualitative prediction that a sufficiently high precision of individual signals is necessary for information aggregation remains valid. If the individual precision of signals is low, then the policy maker is convinced only if many protesters are active; in this case, however, the probability that each of them is pivotal is low. When precision is sufficiently low, the pivot probability is not sufficiently high to make the costly forms of participation incentive compatible. This implies that the presence of different forms of participations with heterogeneous costs can make a difference only if the individual precision of the
citizens’ signals is sufficiently high, which is one of the main point of this article.\textsuperscript{57}

VI.G. Public Signals and Other Public Events

I have assumed that agents do not observe correlated events, but only conditionally independent signals. In real life, however, it is plausible to assume that agents can also receive a public signal, say $S = \{\bar{s}, \bar{s}\}$: $S$ could be a set of public informative signals (for example, it could represent the speech of a foreign minister), or it could be a set of uninformative events observed by all agents (for example, weather conditions). Extending the analysis to these cases is technically straightforward: because the signal is public, the equilibrium in the subgame after the signal is observed can be characterized as in the analysis presented in the previous sections. An equilibrium can now be described by a pair of functions $\tau^*(s)$, $q^*(s)$ for $s = \bar{s}, \bar{s}$. The public signal has now two main effects on the equilibrium outcomes. First, even if the signal is completely uninformative, it enlarges the set of equilibrium payoffs since it allows the players to correlate their strategies. Second, if the signal is correlated with the state, it also affects the players’ priors.

VII. Conclusion

This article has presented a theory of petitions and public protests to study when they may serve as mechanisms to aggregate information dispersed among citizens. I have shown that when citizens receive sufficiently precise signals and/or the conflict with the policy maker is sufficiently small, public protests aggregate dispersed information and improve the policy maker’s decisions. But when these conditions are not satisfied, no information aggregation is possible, even if the number of informed citizens is arbitrarily high. I have characterized the conditions for information aggregation and studied their properties as the number of citizens grows to infinity. For the case in which they are satisfied, we have shown that full information aggregation is possible. This means that as the population grows to infinity, there is a sequence of equilibria in which the probability of a policy

\textsuperscript{57} An analysis of a related signaling model with actions with heterogeneous costs (but only with three senders) is presented by Battaglini and Benabou (2003). A key assumption of this article is that the precision of individual signals is sufficiently high.
mistake converges to 0. For the case in which these conditions are not satisfied, I have shown that information aggregation may still be possible if social media are available. Our theory, in particular, provides new insights on why social media may enhance the effectiveness of public activism.

There are many different directions in which the ideas presented here might usefully be developed. Perhaps the most important extension would be to fully develop a version of the model with heterogeneous preferences among citizens. As mentioned in Section VI, there are no conceptual issues in pursuing this extension, except for a more complicated model. This extension would allow the model to study how the informative role of public protests is affected by the distribution of preferences among citizens and would make it easier to bring the theoretical predictions to the data. Numerical simulations based on the simple model with partisans presented in Section VI suggest that heterogeneity in preferences reduces information aggregation. Addressing heterogeneity would also allow the model to study environments with social groups of different sizes. Another interesting extension would be to allow for the behavioral factors highlighted by Coate and Conlin (2004) and Feddersen and Sandroni (2006), including altruism and ethical motives. These factors may play an important role in determining political participation and may improve the effectiveness of public protests.

APPENDIX

The proofs of the results omitted in this section are in the Online Appendix.

A. Proof of Proposition 2

I prove here a result that is more general than the result stated in Proposition 2. I say that a policy rule is a weak cut-off rule if there is a threshold $Q$ such that $A$ is chosen if the fraction of protesters over the expected population $Q_n$ is larger than $Q$; $B$ is chosen if the fraction of protesters of over the expected population $Q_n$ is smaller than $Q$; and $A$ is chosen with some probability $\beta_n \in [0, 1]$, if the fraction of protesters over the expected population $Q_n$ is equal to $Q$. A cut-off rule is a special case of a weak cut-off rule in which $A$ is chosen with probability 1 if the fraction of protesters over the expected population $Q_n$ is greater than or equal
to $Q$. We now proceed in two steps. I first prove the existence of a sequence of equilibria in correspondence of which citizens’ strategies are characterized by a cut-point $\tau_n^*(Q) \in (\tau_b(Q), \tau_a(Q))$; then I prove that in correspondence to this sequence we have full information aggregation. The proof presented below uses some definitions introduced in Section III, the reader may find it useful to read that section first.

**Step 1.** Consider the pivot probability in state $\theta$, $\varphi_n(\theta; \sigma_n, \rho_n)$, as defined in equation (4); and the probability that a citizen is protesting in state $\theta$, $\phi(\theta; \tau_n, q_n)$ as defined in Section II. By equations (3) and (7) we can represent them without loss of generality as continuous functions of the thresholds $\tau_n$ and $q_n$ as, respectively, $\varphi_n(\theta; \tau_n, q_n)$ and $\phi(\theta; \tau_n)$. Fix $Q \in (0, 1)$ and define $q_n = Q \cdot n$ and $\tau_\theta(Q)$ as the solution:

$$\phi(\theta; \tau_\theta(Q)) = Q.$$ 

It is easy to verify that $\tau_\theta(Q)$ is uniquely defined and $\tau_\theta(Q) \in (0, 1)$; the monotone likelihood ratio property, moreover, implies that $\tau_a(Q) > \tau_b(Q)$. I start from the following useful lemmas.

**Lemma A1.** If the decision is taken by a weak cut-off rule $Q$, then there is a $n_1$ such that for $n > n_1$, $\frac{\log(\varphi_b(\theta; \tau_n))}{\varphi_b(\theta; q_n)}$ is a strictly decreasing function of $\tau$ in $\tau \in [\tau_b(Q), \tau_a(Q)]$.

**Proof.** See Online Appendix.

**Lemma A2.** There is a $n_2$ such that a $\hat{\tau}_n(Q)$ satisfying $\log(\frac{\varphi_b(\theta; \hat{\tau}_n(Q))}{\varphi_a(\theta; \hat{\tau}_n(Q))}) = 0$ exists for any $n > n_2$. Moreover, $\hat{\tau}(Q) = \lim_{n \to \infty} \hat{\tau}_n(Q) \in (\tau_b(Q), \tau_a(Q))$ and

$$\frac{\log(\frac{\varphi_b(\theta; \tau_n)}{\varphi_a(\theta; \tau_n)})}{n} > 0,$$

(resp. $< 0$) if $\tau \in [\tau_b(Q), \hat{\tau}_n(Q))$ (resp. $\tau \in (\hat{\tau}_n(Q), \tau_a(Q)]$).

**Proof.** See Online Appendix.

Define $r(t) = \frac{r(t; a)}{r(t; b)}$ for $t = 1, \ldots, T$ and $r(0) = 0$, $r(T + 1) = r(T) + 1$. Define the following correspondence:

$$\Xi(\tau) = \begin{cases} [r(\tau - 1), r(\tau)] & \text{if } \tau \text{ is an integer} \\ r(\lfloor \tau \rfloor) & \text{else} \end{cases}.$$
I now show that for any $Q \in (0, 1)$, there is a unique $\tau_n^*(Q) \in (\tau_b(Q), \tau_a(Q))$ such that:

$$\frac{1}{v} \left( 1 - \frac{\mu}{\mu} \frac{\varphi_n(b; \tau_n^*(Q), q_n)}{\varphi_n(a; \tau_n^*(Q), q_n)} \right) \in \Xi(\tau_n^*(Q)),$$

for sufficiently large $n$. First, note that $\Xi(\tau)$ is a convex, compact-valued, upper-hemicontinuous correspondence with $\xi(\tau) = \min\{x \text{ s.t. } x \in \Xi(\tau)\} > 0$ $\forall \tau$ and $\overline{\xi}(\tau) = \max\{x \text{ s.t. } x \in \Xi(\tau)\}$ bounded above $\forall \tau$. Second, note that Lemma A2 implies that as $n \to \infty$, $\frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)} \to \infty$ for $\tau \in [\tau_b(Q), \hat{\tau}(Q))$ and $\frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)} \to 0$ for $\tau \in (\hat{\tau}(Q), \tau_a(Q))$. It follows that there is a $n^*$ such that for $n > n^*$,

$$\frac{1}{v} \left( 1 - \frac{\mu}{\mu} \frac{\varphi_n(b; \tau_b(Q), q)}{\varphi_n(a; \tau_b(Q), q)} \right) > \overline{\xi}(\tau_b(Q)) \quad \text{and} \quad \frac{1}{v} \left( 1 - \frac{\mu}{\mu} \frac{\varphi_n(b; \tau_a(Q), q)}{\varphi_n(a; \tau_a(Q), q)} \right) < \overline{\xi}(\tau_a(Q)).$$

This implies that there is a $\tau_n^*(Q) \in (\tau_b(Q), \tau_a(Q))$ satisfying equation (20) for $n > n^*$. Because $\frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)}$ is strictly decreasing, there is a unique point with this property.

We conclude this step by proving that there is a $n^*$ such that for all $n > n^*$, $\tau_n^*(Q)$ is an equilibrium of the public protest game in which the policy maker commits to a response $\rho_n = Q \cdot n$. Assume first that $\tau_n^*(Q)$ is an integer. If $t < \tau_n^*(Q)$, then we must have:

$$\frac{r(t; a)}{r(t; b)} \leq \frac{r(\tau_n^*(Q) - 1; a)}{r(\tau_n^*(Q) - 1; b)} \leq \frac{1}{v} \left( 1 - \frac{\mu}{\mu} \frac{\varphi_n(b; \tau_n^*(Q), q_n)}{\varphi_n(a; \tau_n^*(Q), q_n)} \right) \leq \frac{1}{v} \left( 1 - \frac{\mu}{\mu} \frac{\varphi_n(b; t, q_n)}{\varphi_n(a; t, q_n)} \right).$$

(21)

where the second follows from equation (20), and the third from Lemma A1. Condition (21) implies that

$$\frac{\mu(a; t)}{\mu(b; t)} \leq \frac{\varphi_n(b; t, q_n)}{\varphi_n(a; t, q_n)},$$

and so it is optimal to have $\sigma(t) = 0$. Similarly if $t \geq \tau_n^*(Q)$, then equation (20) implies it is optimal to have $\sigma(t) = 1$. Assume now that $\tau_n^*(Q)$ is not an integer, then

$$\frac{r(\lfloor \tau_n^*(Q) \rfloor a)}{r(\lfloor \tau_n^*(Q) \rfloor b)} = \frac{v - 1}{\mu} \frac{\varphi_n(b; \tau_n^*(Q), q_n)}{\varphi_n(a; \tau_n^*(Q), q_n)},$$

so type $[\tau_n^*(Q)]$ is indifferent. If follow that $\sigma(t) = 0$ for
\[ t < \lfloor \tau_n^*(Q) \rfloor, \sigma(t) = 1 \text{ for } t > \tau_n^*(Q) \text{ and } \sigma(t) = (\lfloor \tau_n^*(Q) \rfloor + 1 - \tau_n^*(Q)) \text{ for } t = \lfloor \tau_n^*(Q) \rfloor \] is an optimal reaction function. We conclude that the strategy described by equation (7) for \( \tau = \tau_n^*(Q) \) is an equilibrium.

**Step 2.** We now prove that we have full information aggregation in correspondence to the sequence of equilibria \( \tau_n^*(Q) \). Let \( \tau^*(Q) = \lim_{n \to \infty} \tau_n^*(Q) \) and \( \hat{\tau}(Q) = \lim_{n \to \infty} \hat{\tau}_n(Q) \). From the argument above we must have that \( \lim_{n \to \infty} \tau_n^*(Q) = \hat{\tau}(Q) \in (\tau_b(Q), \tau_a(Q)) \). It follows that \( \tau^*(Q) \in (\tau_b(Q), \tau_a(Q)) \) and that for \( \varepsilon > 0 \) sufficiently small we can find an \( n^* \) such that \( \tau_n^*(Q) \in (\tau_b(Q) + \varepsilon, \tau_a(Q) - \varepsilon) \) for all \( n > n^* \). This implies that there is a \( \bar{c} > 0 \) such that \( Q \in (\phi(b; \tau_n^*(Q)) + \bar{c}, \phi(a; \tau_n^*(Q)) - \bar{c}) \) for \( n > n^* \).

Let \( \eta_n = \min_{\theta = a, b} |Q - \phi(\theta; \tau_n^*(Q))| \) note that \( \eta = \lim_{n \to \infty} \eta_n > 0 \). The probability of a mistake on the sequence of equilibria can be bounded above as follows:

\[ (22) \quad M(\tau_n^*(Q)) \leq \sum_{\theta} \Pr \left( \left| \frac{\tilde{Q}_n}{n} - \phi(\theta; \tau_n^*(Q)) \right| > \eta_n \right) \leq \frac{\left( \sum_{\theta} \frac{\phi(\theta; \tau_n^*(Q))}{\eta_n^2} \right)}{n}, \]

where \( \tilde{Q}_n \) is the realized fraction of protesting citizens. The last inequality in equation (22) follows from the Chebyshev's inequality recognizing that the fraction of protesting citizens in state \( \theta \) is a Poisson random variable with mean \( \phi(\theta; \tau_n^*(Q)) \) and standard deviation \( \sqrt{\frac{\phi(\theta; \tau_n^*(Q))}{n}} \). Condition (22) implies that \( M(\tau_n^*(Q)) \to 0 \) as \( n \to \infty \).

**B. Proof of Lemma 1**

Assume by way of contradiction that an informative equilibrium exists and \( V < V_1(\nu) \). Define \( Q^* = \min_{Q \geq 0} \{Q \text{ s.t. } \Gamma_n(a; Q, \sigma^*) \geq \mu^* \} \). In correspondence to an informative equilibrium, assuming its existence, it must be that \( Q^* \) is finite for any (finite) \( n \). By definition of \( \mu^* \), we must have:

\[ (23) \quad \Gamma_n(a; Q^*, \sigma^*)V(a) + (1 - \Gamma_n(a; Q^*, \sigma^*))V(b) \geq 0. \]

By Bayes’s rule, we can rewrite equation (23) as: \( \mu \cdot P(Q^*, n\phi(a; \sigma^*))V(a) + (1 - \mu)P(Q^*, n\phi(b; \sigma^*))V(b) \geq 0 \) or:

\[ (24) \quad \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(b; \sigma^*))} \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right). \]
For any informative equilibrium, moreover, we need that type $t = 1$ is willing to stay inactive, otherwise all types would be active and no information would be revealed by the citizens’ actions. This requires:

\[
(25) \quad \frac{1}{\nu} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{\varphi_n(\alpha; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)},
\]

for any $n$. Observe that we can write:

\[
\frac{\varphi_n(\alpha; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} = \frac{\rho(Q^*) \cdot P(Q^* - 1, n\phi(a; \sigma^*)) + (1 - \rho(Q^*)) \cdot P(Q^*, n\phi(a; \sigma^*))}{\rho(Q^*) \cdot P(Q^* - 1, n\phi(b; \sigma^*)) + (1 - \rho(Q^*)) \cdot P(Q^*, n\phi(b; \sigma^*))}.
\]

\[
(26) \geq \frac{e^{-n\phi(a; \sigma^*)} \frac{Q^*}{(Q^*)!} \phi(b; \sigma^*)}{e^{-n\phi(b; \sigma^*)} \frac{Q^*}{(Q^*)!} \phi(a; \sigma^*)},
\]

The last inequality follows from the fact that \(\frac{(1 - \rho(Q^*) + \rho(Q^*) Q_{\phi(a; \sigma^*)}^r)}{(1 - \rho(Q^*) + \rho(Q^*) Q_{\phi(b; \sigma^*)}^r)}\) is nonincreasing in $\rho(Q^*)$. The following lemma is useful to complete the argument:

**Lemma A3.** For any pair of strategies $\sigma, \rho$, we have: $\frac{\phi(b; \sigma, \rho)}{\phi(a; \sigma, \rho)} \geq \frac{r(T; b)}{r(T; \mu)}$.

**Proof.** See Online Appendix.

From Lemma A3 and equation (26) we have:

\[
(27) \quad \frac{\varphi_n(\alpha; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} \geq \frac{e^{-n\phi(a; \sigma^*)} \frac{Q^*}{(Q^*)!} \mu(b; T)}{e^{-n\phi(b; \sigma^*)} \frac{Q^*}{(Q^*)!} \mu(a; T)}.
\]

\[
(28) = \frac{e^{-n\phi(a; \sigma^*)} \frac{Q^*}{(Q^*)!} \mu(b; T)\mu}{e^{-n\phi(b; \sigma^*)} \frac{Q^*}{(Q^*)!} \mu(a; T)(1 - \mu)} \geq \frac{\frac{1}{\mu(a; T)} - 1}{\frac{1}{\mu} - 1} \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(b; \sigma^*))}.
\]
I conclude from equation (28) that: \[ \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(a; \sigma^*))^2} \leq \frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} \leq \frac{1}{\mu(a; \tau)} - 1 \left( \frac{1}{\mu} - 1 \right)^2 P(Q^*, n\phi(a; \sigma^*)) \]

\[ \geq \frac{1}{\mu(a; \tau)} - 1 \left( \frac{1}{\mu} - 1 \right)^2 = \frac{1}{V} \left( \frac{1}{\mu(a; T)} - 1 \right). \]

This implies that \( V \geq V_1(v) \), a contradiction.

C. Proof of Lemma 2

I proceed in two steps. In step 1, I consider a modified game in which I force the lowest type (i.e., a citizen with a signal \( t = 1 \)) to be inactive and the highest type (i.e., a citizen with a signal \( t = T \)) to be active. I prove that an informative equilibrium exists in this modified game. In step 2, I prove that if \( V \geq V_2(v) \), then any equilibrium of the modified game is also an equilibrium of the original game. Recall that strategies \( \sigma, \rho \) can be represented by two thresholds \( \tau, q \) with \( \tau \in [1, T + 1] \) and \( q \in [0, \infty) \). In the rest of this section, I represent the policy maker’s posterior \( \Gamma_n(\theta; Q, \sigma) \) and the pivot probabilities \( \varphi_n(\theta; \sigma, \rho) \) as, respectively, \( \Gamma_n(\theta; Q, \tau) \) and \( \varphi_n(\theta; \tau, q) \).

Step 1. Restrict the strategy space imposing \( \tau \in [2, T] \). Let \( Q(\tau) = \max Q \{ Q \) s.t. \( \Gamma_n(a; Q, \tau) \leq \mu^* \} \) and \( \bar{Q} = \max_{\tau \in [2, T]} Q(\tau) \). It is easy to see \( \bar{Q} < \infty \). Restrict the set of strategies for the policy maker to \( q \in [0, \bar{Q} + 2] \). We now have a modified game in which \( \tau \in [1, T] \) and \( q \in [0, \bar{Q} + 2] \).

Given a strategy \( \tau, q \), define \( t(\tau, q) \) as follows: \( t(\tau, q) = 1 \) if \( \frac{\mu(a; \tau)}{\mu(b; \tau)} > \frac{1}{v \varphi_n(a; \tau, q)} \) for all \( t \geq 2 \) and

\[ t(\tau, q) = \max \left\{ t \in \{2, \ldots, T\} \text{ s.t. } \frac{\mu(a; t)}{\mu(b; t)} \leq \frac{1}{v \varphi_n(a; \tau, q)} \right\}. \]

otherwise. Using the notation introduced in Section II, a citizen’s strategy described by \( \tau \) is optimal, given the other players’ strategies \( \tau, q \), if and only if \( \tau \in R_1(\tau, q) \) where \( R_1(\tau, q) \) is defined as:

\[ R_1(\tau, q) = \left\{ \begin{array}{ll} \lfloor t(\tau, q) + 1 \rfloor & \text{for } \frac{\mu(a; t(\tau, q))}{\mu(b; t(\tau, q))} = \frac{1}{v \varphi_n(a; \tau, q)} \\lfloor t(\tau, q) + 1 \rfloor & \text{else} \end{array} \right. \]
Similarly, define $Q(\tau)$ as $Q(\tau) = -1$ if $\Gamma_n(a; Q, \tau) > \mu^*$ for all $Q \in \{0, \ldots, \hat{Q} + 2\}$ and

$$Q(\tau) = \max\{Q \in \{0, \ldots, \hat{Q} + 2\} \text{ s.t. } \Gamma_n(a; Q, \tau) \leq \mu^*\}$$

otherwise. The policy maker’s strategy described by $q$ is optimal, given the other players’ strategies $\tau, q$, if and only if $q \in R_2(\tau)$ where $R_2(\tau)$ is defined as:

$$R_2(\tau) = \{[Q(\tau), Q(\tau) + 1] \text{ if } \Gamma_n(a; Q(\tau), \tau) = \mu^* \}
\{Q(\tau) + 1 \text{ else}\}.$$

Let $X = [2, T] \times [0, \hat{Q} + 2]$. Define $R : X \mapsto X$ as $R = R_1(\tau, q) \times R_2(\tau)$. We have:

**Lemma A4.** $R$ has a closed graph.

**Proof.** See Online Appendix.

It is easy to verify that in addition to being closed valued, $R$ is nonempty and convex valued. The Kakutani fixed point theorem implies that there is a fixed point $(\tau^*, q^*) \in R(\tau^*, q^*)$: this fixed point is an equilibrium of the modified game.

**Step 2.** I now prove that the equilibrium of the restricted game $(\tau^*, q^*)$ is an equilibrium of the full game if $V \geq V_2(v)$. By definition of $Q, q^* < \hat{Q} + 1$. Given this, the strategy described by $q^*$ is optimal for the planner given $(\tau^*, q^*)$. To show that the strategy described by $\tau^*$ is optimal, we proceed in three steps. Assume first that $\tau^* \in (1, T)$. In this case, by construction types $t < t(\tau^*, q^*)$ and $t = t(\tau^*, q^*)$ if $\frac{\mu(a(t(\tau^*, q^*)))}{v(\mu(a; t(\tau^*, q^*)))} < \frac{1}{v(\phi_n(b; \tau^*, q^*))}$ find it optimal to abstain; type $t = t(\tau^*, q^*)$ if $\frac{\mu(a(t(\tau^*, q^*)))}{v(\mu(a; t(\tau^*, q^*)))} = \frac{1}{v(\phi_n(b; \tau^*, q^*)}$ is indifferent; and types $t > t(\tau^*, q^*)$ find it optimal to be active: this is exactly the action prescribed by $\tau^*$. It follows that $\tau^*$ is an optimal reaction function given $(\tau^*, q^*)$. We conclude that $(\tau^*, q^*)$ is a Nash equilibrium of the full game.

Assume now that $\tau^* = 0$, to prove that $(\tau^*, q^*)$ is an equilibrium we only need to prove that the lowest type finds it optimal to abstain, that is, $\sigma(1) = 0$ is optimal. For type 1 voters it is optimal to stay inactive for type 1 if:

$$\frac{1}{v} \left(\frac{1}{\mu(a; 1)} - 1\right) \geq \frac{\phi_n(a; \tau^*, q^*)}{\phi_n(b; \tau^*, q^*)}.$$
To verify this inequality, note that:

\[
\frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} = \frac{\rho(Q(\tau^*)) \Pr(Q(\tau^*) - 1; a) + (1 - \rho(Q(\tau^*))) \Pr(Q(\tau^*) - 1; b)}{\rho(Q(\tau^*)) \Pr(Q(\tau^*) - 1; b) + (1 - \rho(Q(\tau^*))) \Pr(Q(\tau^*) - 1; a)}
\]

\[
e^{-n\phi(a; \tau^*, q^*) / (Q(\tau^*)^q_a() / (Q(\tau^*))!)} \cdot (1 - \rho(Q(\tau^*))) + \rho(Q(\tau^*)) \frac{Q(\tau^*)}{n\phi(a; \tau^*)}) = P(Q(\tau^*), n\phi(a; \tau^*)) \frac{P(Q(\tau^*), n\phi(b; \tau^*))}{P(Q(\tau^*), n\phi(b; \tau^*))}.
\]

The last inequality in equation (30) follows from the fact that by construction of the strategies and the monotone likelihood ratio property assumption, \(\phi(a; \tau^*) \geq \phi(b; \tau^*)\). By definition of \(Q(\tau^*)\), we must have:

\[
P(Q(\tau^*), n\phi(a; \tau^*)) \leq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right).
\]

Conditions (30) and (31), together with \(V \geq V_2(v)\), then imply:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right) \geq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)},
\]

and so equation (29) is satisfied.

Assume now that \(\tau^* = T\), to prove that \((\tau^*, q^*)\) is an equilibrium we only need to prove that the highest type finds it optimal to be active, that is, \(\sigma(T) = 1\) is optimal. Define \(\tilde{Q}(\tau^*) = \min\{Q \in [0, \ldots, \tilde{Q} + 2]\} a.t. \Gamma_n(a; Q, \tau^*) \geq \mu^*\). Naturally \(\tilde{Q}(\tau^*) \leq \tilde{Q} + 2\) and, since \(\mu < \mu^*\), \(\tilde{Q}(\tau^*) > 0\) Consider the problem faced by a voter of type \(T\). It is optimal to stay active for type \(T\) if:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; T)} - 1 \right) \leq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)}.
\]

Using similar steps as in equation (28) we can show that:

\[
\frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} \geq \left[ \frac{(1 - \frac{1}{\mu(a; T)} - 1)}{(1 - \frac{1}{\mu}} - 1) \right], P(\tilde{Q}(\tau^*), n\phi(a; \tau^*)) \frac{P(\tilde{Q}(\tau^*), n\phi(b; \tau^*))}{P(\tilde{Q}(\tau^*), n\phi(b; \tau^*))}.
\]
We conclude that:

\[
\frac{1}{v} \leq \frac{1}{V} \leq \frac{P(\hat{Q}(\tau^*), n\phi(a; \tau^*))}{P(Q(\tau^*), n\phi(b; \tau^*))} \leq \frac{\phi_n(a; \tau^*, q^*)}{\phi_n(b; \tau^*, q^*)} \left( \frac{1}{\mu - 1} \right)
\]

This implies equation (32). I conclude that the equilibrium of the modified game is an equilibrium of the original game.

**D. Proof of Proposition 4**

I proceed in two steps. In step 1 I prove that if information aggregation is possible in a large society when the policy maker’s preference parameter is \( V \), then information aggregation is possible in a large society if the preference parameter is \( V' \geq V \) as well. In step 2 I prove that step 1 plus Lemmas 1 and 2 implies the result.

**Step 1.** Assume that information aggregation is possible in a large society when the policy maker’s preference parameter is \( V \). Then there is a \( n_1 \) such that for any \( n > n_1 \) there is an informative equilibrium \( \tau^*_n, q^*_n \) with \( \phi(a; \tau^*_n) > \phi(b; \tau^*_n) \), where \( \phi(\theta; \tau^*_n) \) is the expected probability that a random citizen chooses to protest given strategy \( \tau^*_n \) in state \( \theta \). Let \( Q^* = \lim_{n \to \infty} q^*_n \). We now show that there is a \( n_2 \geq n_1 \) such that there is an informative equilibrium \( \tau^{**}_n, q^{**}_n \) with \( \phi(a; \tau^{**}_n) > \phi(b; \tau^{**}_n) \) for \( n > n_2 \) when the policy maker’s preference parameter is \( V' \geq V \). To this goal, consider a modified game in which \( q_n \) is forced to be in \([0, q^*_n]\). Following the proof of Lemma 2 it is easy to show that this modified game has an equilibrium \( \hat{\tau}_n, \hat{q}_n \) for any \( n \). Moreover it can be easily verified that in no equilibrium of the modified game we can have \( \hat{q}_n = 0 \). We now prove that any equilibrium of the modified game is also an informative equilibrium of the original game when the policy maker’s preference parameter is \( V' \geq V \).

Assume first that there is a \( n_2 \) such that for any \( n > n_2 \), \( \hat{q}_n < q^*_n \). This implies that \( \hat{\tau}_n, \hat{q}_n \) is an equilibrium of the original game since \( \hat{q}_n < q^*_n \) implies that it is optimal for the planner to choose \( A \) with probability \( (\lfloor q^*_n \rfloor - q^*_n) \) if \( Q = \lfloor q^*_n \rfloor \) and with probability 1 if \( Q > \lfloor q^*_n \rfloor \). So the restriction on the modified game is irrelevant and an equilibrium of the modified game is an equilibrium of the original game as well.

Assume now that for any \( n_2 \) there is a \( n > n_2 \) such that \( \hat{\tau}_n, q^*_n \) is an equilibrium of the modified game. We can therefore find a sequence of equilibria \( \hat{\tau}_n, q^*_n \) for \( n \to \infty \). We have:
Lemma A5. If $\hat{\tau}_n, q^*_n$ is a sequence of equilibria of the modified game, then \( \lim_{n \rightarrow \infty} \hat{\tau}_n = \lim_{n \rightarrow \infty} \tau^*_n \).

Proof. See Online Appendix.

Since $\tau^*_n, q^*_n$ is an equilibrium of the original game, we must have:

\[
\frac{P(|q^*_n|, n\phi(a; \sigma^*_n))}{P(|q^*_n|, n\phi(b; \sigma^*_n))} = \frac{1}{V} \left( \frac{1}{\mu} - 1 \right) \geq \frac{P(|q^*_n| - 1, n\phi(a; \sigma^*_n))}{P(|q^*_n| - 1, n\phi(b; \sigma^*_n))}.
\]

This implies that there must be an $n'$ such that for $n > n'$:

\[
\frac{P(|q^*_n|, n\phi(a; \hat{\sigma}_n))}{P(|q^*_n|, n\phi(b; \hat{\sigma}_n))} > \frac{1}{V'} \left( \frac{1}{\mu} - 1 \right).
\]

The first inequality follows from the fact that $\frac{P(Q, n\phi(a; \hat{\sigma}_n))}{P(Q, n\phi(b; \hat{\sigma}_n))} \to 0$, equation (8) and $V' > V$. Equation (9) implies $Q = |q^*_n|$ the policy maker strictly prefers $A$, so it must be that $\hat{q}_n < q^*_n$, a contradiction. We conclude that for any sequence of equilibria $\hat{\tau}_n, \hat{q}_n$ of the modified game is a sequence of equilibria of the original game for $n$ sufficiently large. Since $\hat{q}_n \in (0, q^*_n)$ we must have $\hat{\tau}_n \in (1, T + 1)$ for any $n$, implying that $\phi(a; \hat{\tau}_n) > \phi(b; \hat{\tau}_n)$ for any $n$ sufficiently large.

Step 2. Define $V^*$ as the infimum of the set of $V$'s such that an informative equilibrium exists in a large society. By Lemma A1 and A2 we must have that $V^* \in [V_1(v), V_2(v)]$. By definition of $V^*$ if $V < V^*$, then public protests cannot be informative in a large society. Assume $V > V^*$, then by the definition of $V^*$ there is a $V' \in (V^*, V)$ such that public protests are informative in a large society when the policy maker’s preference parameter is $V'$. Since $V > V'$, step 1 implies that we have an informative equilibrium in a large election when the policy maker’s preference parameter is $V$.

E. Proof of Proposition 5

We consider two cases.

Case 1. Assume first that $V \geq V_2(v)$. We first prove that we must have a sequence of equilibria $\hat{\tau}_n, \hat{q}_n$ such that $\hat{Q} \in (\phi(b; \hat{\tau}), \phi(a; \hat{\tau}))$, where $\hat{\tau} = \lim_{n \rightarrow \infty} \hat{\tau}_n$ and $\hat{Q} = \lim_{n \rightarrow \infty} \hat{q}_n$. In Lemma 2 we have constructed a sequence of equilibria $\hat{\tau}_n, \hat{q}_n$ such that $\hat{\tau}_n \in [2, T]$. It follows that $\hat{\tau} = \lim_{n \rightarrow \infty} \hat{\tau}_n \in [2, T]$. This fact
and the monotone likelihood ratio imply that \( \phi(a; \tau) > \phi(b; \tau) > 0 \). Note that I can write:

\[
\varphi_n(\theta; \tau_n, q_n) = \frac{e^{-n\phi(\theta; \tau_n)}(n\phi(\theta; \tau_n))^{\hat{Q}_n}}{\hat{Q}_n!} \cdot \left[ \beta_n \frac{\hat{Q}_n}{n\phi(\theta; \tau_n)} + (1 - \beta_n) \right]
\]

\[
= \frac{e^{-n\phi(\theta; \tau_n)}(n\phi(\theta; \tau_n))^{\hat{Q}_n}}{\mathcal{t}(\hat{Q}_n)(\frac{Q_n}{e}) \sqrt{2\pi \hat{Q}_n + \pi}} \cdot \left[ \beta_n \frac{\hat{Q}_n}{n\phi(\theta; \tau_n)} + (1 - \beta_n) \right]
\]

\[
= \frac{e^{n\phi(\theta; \tau_n)}\psi(\frac{\hat{Q}_n}{n\phi(\theta; \tau_n) Q_n})}{\mathcal{t}(\hat{Q}_n)\sqrt{2\pi \hat{Q}_n + \pi}} \cdot \left[ \beta_n \frac{\hat{Q}_n}{n\phi(\theta; \tau_n)} + (1 - \beta_n) \right],
\]

where \( \hat{Q}_n = [\hat{Q}_n] \), \( \psi(x) = x(1 - \log x) - 1 \), and \( \mathcal{t}(Q) = \frac{Q}{\sqrt{2\pi Q + \pi}} \).

Taking the log of the ratio of the pivot probabilities in the two states and the limit as \( n \to \infty \), we have:

\[
(36) \quad \lim_{n \to \infty} \frac{\log \left( \frac{\varphi_n(a; \tau, n \cdot Q)}{\varphi_n(b; \tau, n \cdot Q)} \right)}{n} = \Omega(Q),
\]

where \( \Omega(Q) = \phi(a; \tau)\psi(\frac{Q_n}{\phi(a; \tau)}) - \phi(b; \tau)\psi(\frac{Q_n}{\phi(b; \tau)}) \).

Note that \( \Omega(Q) \) is increasing in \( Q \) and:

\[
\Omega(\phi(a; \tau)) = -\phi(b; \tau)\psi\left(\frac{\phi(a; \tau)}{\phi(b; \tau)}\right) > 0
\]

\[
\Omega(\phi(b; \tau)) = \phi(a; \tau)\psi\left(\frac{\phi(b; \tau)}{\phi(a; \tau)}\right) < 0.
\]

This implies that if \( \hat{Q} = \lim_{n \to \infty} \hat{Q}_n \leq \phi(b; \tau) \), then \( \frac{\varphi_n(a; \tau, \hat{Q}_n)}{\varphi_n(b; \tau, \hat{Q}_n)} \) converges to 0, and so \( \Gamma_n(a; [\hat{Q}_n], \sigma^*) < \mu^* \), a contradiction: we conclude that we must have \( \hat{Q} > \phi(b; \tau) \). Similarly, I can prove that we must have \( \hat{Q} < \phi(a; \tau) \). I conclude that \( \hat{Q} \in (\phi(b; \tau), \phi(a; \tau)) \). Given this, the proof that the probability of a mistake converges to 0 on the sequence of equilibria \( \hat{\tau}_n, \hat{q}_n \) as \( n \to \infty \) follows the same argument as in step 2 of Proposition 2.

Case 2. Assume now that \( V \in (V^*(v), V_2(v)) \). If \( \hat{\tau}_n \to \hat{\tau}_\infty \in (1, T + 1) \), then again we have \( \hat{Q} \in (\phi(b; \tau), \phi(a; \tau)) \); and the

58. Details of this derivation are presented in Lemma A1 in the Online Appendix.
result is proven as in step 2 of Proposition 2. To complete the proof, I therefore only need to prove that we can neither have $\hat{\tau}_n \to 1$ nor $\hat{\tau}_n \to T + 1$.

**Step 2.1.** Assume first that $\hat{\tau}_n \to 0$, so the citizens’ strategy becomes arbitrarily uninformative as $n \to \infty$. From the fact that $V < V_2(v)$ we have that there must be an $\varepsilon > 0$ such that

\[
\frac{V}{v} < \frac{\left(\frac{1}{\mu} - 1\right)}{\left(\frac{1}{\mu(a;1)} - 1\right)} - \varepsilon.
\]

(37)

For $n$ sufficiently large, moreover, it must be that $\hat{\tau}_n < 2$, implying that citizens of type 1 must be indifferent between $B$ and $A$. We must indeed have:

\[
\frac{1}{v} \left(\frac{1}{\mu(a;1)} - 1\right) = \frac{\varphi_n(a;\hat{\tau}_n, \hat{q}_n)}{\varphi_n(b;\hat{\tau}_n, \hat{q}_n)} > \frac{P(\lfloor \hat{q}_n \rfloor, n\phi(a;\hat{\tau}_n))}{P(\lfloor \hat{q}_n \rfloor, n\phi(b;\hat{\tau}_n))} - \frac{\varepsilon}{2}
\]

(38)

where the first inequality follows from the fact that a citizen with a signal $t = 1$ is indifferent between $A$ and $B$, the second follows from the fact that as $\hat{\tau}_n \to 0$, then $\phi(b;\hat{\tau}_n) \to 1$, $\phi(a;\hat{\tau}_n) \to 1$, implying $\varphi_n(a;\hat{\tau}_n, \hat{q}_n) = \frac{P(\lfloor \hat{q}_n \rfloor, n\phi(a;\hat{\tau}_n))}{P(\lfloor \hat{q}_n \rfloor, n\phi(b;\hat{\tau}_n))} \to 0$ as $n \to \infty$, the last inequality follows from the definition of $\hat{q}_n$. Conditions (37) and (38), however, are impossible to satisfy, implying that it cannot that $\hat{\tau}_n \to 0$.

**Step 2.2.** Assume now that $\hat{\tau}_n \to T + 1$. For $n$ sufficiently large it must be that $\hat{\tau}_n > T$, implying that citizens of type $T$ must be indifferent between $B$ and $A$. Let $\tilde{n}$ be such that $\hat{\tau}_{\tilde{n}} > T$. I can now construct a sequence of equilibria $(\tilde{\tau}_n, \tilde{q}_n)$ that is equal to $(\hat{\tau}_n, \hat{q}_n)$ for $n < \tilde{n}$ and that is such that $\tilde{\tau}_n = \hat{\tau}_{\tilde{n}}$, $\tilde{q}_n = \hat{q}_{\tilde{n}}$. To see that this remains a sequence of equilibria note that, as it can be easily verified from Bayes’s rule, strategies with $\tau_n > T$ are equally informative, that is, $\Gamma_n(a; Q, \tau_n)$ is constant for any $\tau_n > T$: this implies that if $\hat{q}_n$ is an optimal threshold for $\hat{\tau}_n > T$, then it continues to be optimal for $\tilde{\tau}_n > T$. Similarly, if $\hat{q}_n$ is the threshold and $\hat{\tau}_n > T$, then type $T$ is indifferent between $A$ and $B$ and $\hat{\tau}_{\tilde{n}}$ is an optimal threshold. I conclude that there is a sequence with $\hat{\tau}_n \to \hat{\tau}_{\tilde{n}} \in (1, T + 1)$ and the result follows from step 2 of Proposition 2.
F. Proof of Proposition 6

From Proposition 1, first proven by Myerson (1998b), we know that for any $q$-rule there is a sequence of equilibria as $n \to \infty$, in correspondence of which the probability of a policy mistake converges to 0. Given this, we know that if $V > V^*(v)$, the expected benefit of choosing the protest game (in terms of higher payoffs for the players) must be either negative or, if positive, arbitrarily small as $n \to \infty$. If $V < V^*(v)$, however, we know from Proposition 5 that no equilibrium of the protest game is informative. It follows that in this case, the expected payoffs of both citizens and the policy maker are strictly higher in the most informative equilibrium of the election than in the most informative equilibrium of the protest game for $n$ sufficiently large. Given that the distribution $F$ of $V$ has full support (and so puts positive mass on events with $V > V^*(v)$), it must be that, both for citizens and the policy maker, the expected utility of the election is higher for $n$ sufficiently large.

G. Proof of Proposition 7

Consider a game with $m$ citizens who receive a signal with distribution $r_g(t; \theta)$ and support $T = \{0, \ldots, g\}$. By Lemma 1, this game has an equilibrium $(\tau_g, q_g)$ if $V \geq V_2(v)$. Given $r_g(t; \theta)$, we have $\frac{r_g(0; a)}{r_g(0; b)} = \frac{1-r}{r}^g$, implying that an informative equilibrium exists if: $V \geq \left[\frac{(1-r)}{r}\right]^g v$. That is, if $g \geq g^*(r)$. I now use this equilibrium to construct an informative equilibrium for the original game in which there are $m$ groups with $g$ members each. Assume without loss of generality that citizens in each group are identified by a number and let us call the citizen with the lowest number the “leader.” 59 The strategies for the citizens are defined as follows. After observing the signals each citizen reports his signal to all the remaining members of the group. The leader recommends actions to the others according to strategy $\tau_g$. All citizens in the group follow the recommended action. The policy maker’s strategies and beliefs are as follows. In equilibrium the number of protesting citizens is a multiple of $g$. If the number of protesting citizens is a

59. There may be situations in which citizens are not identified by a name or number. The strategies presented above can be easily modified for these situations. For example, assume that at the communication stage citizens report the signal and a random number with uniform distribution in $[0,1]$. With probability one no agent will report the same number, so with probability 1 each agents is identified by a number.
multiple of \( g \), then the policy maker will form beliefs according to Bayes’s rule. If the number of protesters \( k \) is not a multiple, the policy maker will form the belief according to Bayes’s rule in the case in which there are \( gl \) protesters, where \( l \) is an integer such that \( k \in [gl, gl + 1] \). The policy maker chooses the action that is optimal given these posterior beliefs. It is straightforward to prove that these strategies form an equilibrium.

H. Proof of Proposition 8

I first prove that an informative equilibrium does not exist with large population if \( V < V_1(v) \). As said in the main text, with participation costs the strategies can be described by a set of thresholds \( \tau^*_n(c) \) with \( \tau^*_n(c) \in [1, T + 1] \), for the citizens and a threshold \( q^*_n \in [0, \infty] \) for the policy maker. In this environment, the probability that citizens protest is

\[
\phi(\theta; \tau^*_n) = \sum_c \left[ \left( \left\lceil \tau^*_n(c) \right\rceil - \tau^*_n(c) \right) \mu\left( \left\lceil \tau^*_n(c) \right\rceil; \theta \right) + \sum_{t > \left\lfloor \tau^*_n(c) \right\rfloor} \mu(t; \theta) \right] p(c).
\]

Assume by contradiction that an informative equilibrium exists with large population. This implies that there is a sequence of equilibria with \( \phi(a; \tau^*_n) > \phi(b; \tau^*_n) \). To prove that this is impossible, note first that given that \( V < V_1(v) \), all citizens with \( c \leq 0 \) find it strictly optimal to protest, that is, \( \tau^*(c) = T + 1 \) for \( c \leq 0 \). This follows from the fact that if \( V < V_1(v) \), then all citizens, conditioning on the pivotal event, find it optimal to protest with \( c = 0 \): a fortiori they find it optimal is \( c < 0 \). Because agents with \( c = \bar{c} \) never find it optimal to protest (i.e., \( \tau^*(\bar{c}) = T + 1 \)), we have \( \sum_{c \leq 0} p(c) \leq \phi(\theta; \tau^*_n) \leq 1 - p(\bar{c}) \) in any sequence of informative equilibria. This implies that the probability of being pivotal in state \( \theta \), converges to 0 in all sequences of informative equilibria. Let \( c_+ \) be the minimal strictly positive cost in \([c, \bar{c}]\). An agent with cost \( c_+ \) finds it optimal to protest if and only if:

\[
(39) \quad c_+ \leq \varphi_n(a; \sigma, \rho) \left[ \mu(a; t)\psi(a) + \mu(b; t)\psi(r) \frac{\varphi_n(b; \sigma, \rho)}{\varphi_n(a; \sigma, \rho)} \right].
\]

Given that the term in parentheses is bounded, there must be a \( n^* \) such that for \( n > n^* \), equation (39) is not satisfied. For \( n > n^* \), we have \( \tau^*(c) = 1 \) for \( c > 0 \). It follows that along the sequence of informative equilibria we have \( \phi(a; \tau^*) = \sum_{c \leq 0} p(c) = \phi(b; \tau^*) \), a
contradiction. We now prove that we have a sequence of informative equilibria for any \( n \) sufficiently large if \( V > V_2(v) \). Consider a modified game in which \( \tau^*(c) = T + 1 \) if \( c < 0 \) and \( \tau^*(c) = 1 \) if \( c > 0 \). In this game the strategies can be described as in Section III using only two thresholds: \( \tau^* \in [1, T + 1] \) for the citizens with \( c = 0 \); and \( q^* \in [0, \infty] \) for the policy maker. Following the same steps as in the proof of Lemma 2, we can prove that an informative equilibrium exists in this modified game. To prove that this is an informative equilibrium for the original game, note that for any informative equilibrium of the modified game, \( \tau^*(c) = T + 1 \) is optimal for citizens with \( c < 0 \); by the same argument as above, there is a \( n^* \) such that for \( n > n^* \), \( \tau^*(c) = 1 \) is optimal for citizens with \( c > 0 \). Following the same argument as in Proposition 4, we have that there is a \( V^G(v) \in (V_1(v), V_2(v)) \) such that no informative equilibrium exists if \( V < V^G(v) \); an informative equilibrium exists for any \( n \) sufficiently large if \( V > V^G(v) \). We can prove that full information aggregation is achievable if \( V > V^G(v) \) following the same steps as in Proposition 5.

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Supplementary Material

An Online Appendix for this article can be found at The Quarterly Journal of Economics online.

References


Miller, Nicholas, “Information, Electorates, and Democracy; Some Extensions and Interpretations of the Condorcet Jury Theorem,” in Information Pooling and
