The double bind of asymmetric information in over-the-counter markets

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October 10, 2017

Abstract

In over-the-counter markets with heterogeneous asset qualities and individual valuations, private information about both of these value components amplifies the adverse selection problem attributable only to privately known asset quality. Specifically, when gains from trade are low, asymmetric information creates a double bind: either the market breaks down due to a classic lemons problem or low-quality assets are traded excessively, generating a congestion externality. A market designer may improve efficiency without incurring losses by acquiring all assets, issuing asset-backed securities of publicly known quality and capturing at least a part of the surplus from the future trades of these securities.

JEL classification: D82, D83, G14.

Keywords: asymmetric information, over-the-counter markets, search frictions, market design.

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1 Introduction*

The liquidity of secondary markets is a key determinant of the success and the terms of trade of the initial issuance of securities traded in over-the-counter (OTC) markets.[1] This paper builds a model of a secondary OTC market to investigate how search frictions interact with asymmetric information on both *asset qualities* and agents' *private values* of holding an asset. These two dimensions of private information have been identified as salient features of OTC markets for various financial instruments.[2]

We find that bi-dimensional private information adds an additional layer of complexity to the endogenous market composition of sellers relative to a situation in which private information only pertains to asset quality. Specifically, all the holders of low quality assets, even those with no fundamental reason to trade, may be encouraged to mimic the trading strategy of sellers with high quality assets. This incentive creates two novel inefficiencies. First, it amplifies the standard lemons problem: the additional supply of lemons dilutes the average quality of the assets on sale, thus it further discourages buyers from offering a high, pooling price. Second, even when all asset qualities trade, a share of transactions take place among high-valuation lemon holders and buyers, who both enjoy an identical private benefit from holding assets. The latter trades never improve social welfare, but they are actually detrimental when bilateral trading opportunities arise according to a matching technology which exhibits congestions effects.[3] Indeed, for this class of matching technologies the individual time to sell—a measure of liquidity in markets with search frictions—becomes longer as the mass of agents who want to sell their assets increases. Even if buyers match more frequently with sellers, the additional delay endured by asset holders with a real need to sell leads to an overall social welfare loss. The results uncover that bi-dimensional asymmetric information creates a double bind in OTC markets: either there is too little trade as high quality assets are not exchanged, or there is an excessive amount of trades and sellers suffer from a deterioration in their market liquidity conditions. In light of these inefficiencies, we analyze to which extent a market designer may improve the decentralized outcome through budget-balanced interventions that aim at enlarging the set of economies[4] in which a first-best allocation is implementable.

Our model builds on the classic Duffie et al. (2005) model of OTC markets with search frictions and repeated trade, and it naturally extends their model along two dimensions. First, we consider a more general matching technology that includes as special cases the ones in Duffie et al. (2005) and Kiyotaki and Wright (1993). The former matching technology is non-competitive in the sense that for a fixed measure of buyers (sellers), the matching rate of sellers (buyers) with buyers (sellers) is independent of the measure of sellers (buyers) in the market. In contrast, Kiyotaki and Wright (1993) make use of a competitive matching technology as the rate at which buyers (sellers) match with sellers (buyers) is decreasing in the measure of buyers (sellers) in the market, i.e. there are congestion effects

*We are grateful to Max Bruche and Vincent Maurin for helpful comments and suggestions. The views expressed in this paper are those of the authors and do not involve the responsibility of the Bank of Italy. All errors remain ours.

[1] Aiyar et al. (2015) and the ECB (2016) explicitly refer to asymmetric information and the lack of a secondary market among the determinants of the very limited issuance of securities backed by non-performing loans (NPLs) in Europe whereas IOSCO (2016) presents evidence suggesting that liquidity concerns in the secondary market affect pricing and issuance in primary corporate bond markets.

[2] A number of scholars stress that asymmetric information about the quality of the assets underlying these securities have certainly played a role in the liquidity dry-up observed during the financial crisis. A non-exhaustive list includes Bigio (2015), Chari et al. (2014), Chang (2017), Friewald et al. (2016), Guerrieri and Shimer (2014) and Kurlat (2013).The impossibility to observe agents' trading motives (e.g. liquidity vs. informed trading) is a common assumption in the market microstructure literature (see Biais et al. (2005) for a review). More recently, Acharya and Bisin (2014) also point out the uncertainty over the trading motives of other counter parties in OTC markets.

[3] We later use the term 'competitive' to refer to this class of matching technologies.

[4] We refer to an economy as a set of admissible parameters.

as one side of the market grows in size.[5] Second, we introduce bi-dimensional private information on the individual cost of holding an asset (private value) and on asset quality (common value), hence introducing uncertainty on the motives for trade. Specifically, we consider a model with two asset qualities $i = b, g$, with a share λ $(1-\lambda)$ of peaches (lemons), both providing a strictly positive flow payoff δ_i such that $\delta_g > \delta_b$; agents holding an asset can either enjoy the full flow δ_i or incur a holding cost equal to $x \in (0, \delta_b]$, reducing their flow payoff to $\delta_i - x$. We can interpret agents with holding costs x as distressed, and the magnitude of x provides a measure of the gains from trade for each asset quality. Similarly to Plantin (2009), Guerrieri and Shimer (2014) and Chiu and Koeppl (2016), each asset holder has superior information on the quality of his asset relative to all other agents. Plantin (2009) refers to this assumption as 'learning by holding', and discusses why holders of ABS, MBS and corporate bonds obtain superior information on the underlying quality of their securities.[6] As in Duffie et al.'s model each agent exogenously transitions from strictly positive ($x > 0$) to zero asset holding costs and vice versa, independently from agents' trading decisions.[7] In this framework the expected utility of each agent type, and in turn his reservation value when deciding whether to trade or not, may depend on the expected utilities of all the other agent types.

If asymmetric information only concerned asset quality, in equilibrium only distressed agents could trade with buyers. This is no longer the case when private information is bi-dimensional: non-distressed lemon holders may find it convenient to offer their assets for sale as buyers cannot detect whether a seller trades for liquidity or informational reasons. When the information rent—as measured by the difference $\delta_g - \delta_b$—is low relative to the asset holding cost x, only distressed asset holders sell, and some equilibria achieve the first-best outcome in terms of utilitarian social welfare. In the opposite case—i.e. when $\phi := \frac{\delta_g - \delta_b}{x}$ is high—also non-distressed lemon holders participate in the market, mimicking peach sellers.[8] The fact that non-distressed lemon holders now offer their assets for sale has two negative effects. First, it worsens the classic adverse selection problem because it increases the endogenous share of lemons in the market; this quality dilution effect makes buyers more reluctant to offer a high price, in turn making distressed peach holders less willing to sell. Second, even when all asset qualities trade, the excessive amount of trade leads to an inefficient delay when the matching function is competitive. In particular, trades between non-distressed lemon holders and buyers do not entail any allocative gain but they impose a negative externality on all distressed sellers. Too many lemons for sale slow down the expected time to sell—a measure of market liquidity in search markets—for all sellers due to a market congestion effect on the asset supply side. In this excessive trade equilibrium lemons trade more frequently than peaches because each lemon is continuously offered for sale. Although the overall trade volume is higher than in the first-best outcome, the corresponding utilitarian social welfare is lower.

[5] More specifically, in Duffie et al. (2005) the matching rate of buyers and sellers depend on the total measure of agents in the economy and displays constant returns to scale in this argument but not necessarily in the measures of buyers and sellers. In Kiyotaki and Wright (1993), on the other hand, the matching technology exhibits constant returns to scale with respect to the measure of agents participating in the market. That is, the difference between the matching technologies pertains to which set of agents the uniform random matching technology applies: in Duffie et al. (2005) all agents in the economy, in Kiyotaki and Wright (1993) only agents choosing to participate in the market. This implies that they are identical only if all agents in the economy participate as buyers or sellers.

[6] 'Learning by holding' is a particularly suitable assumption in the market for syndicated loans, in which the managing agent and the syndicate participants may have access to material non-public information about the borrower while such information is usually unavailable to other participants in the secondary market (Wittenberg Moerman, 2010).

[7] Therefore, we keep the methodological distinction between exogenous preference shocks and endogenously determined trading patterns.

[8] Importantly, the minimum threshold ϕ is endogenous and it depends on the specific equilibrium. Moreover, this threshold determines not only whether non-distressed lemon holders participate in the market but also when buyers make losses from acquiring lemons at a pooling price.

In other words, in OTC markets where the adverse selection problem is sufficiently severe (i.e. ϕ high enough) bi-dimensional private information creates a double bind: all decentralized equilibria entail a welfare loss because either peaches do not trade (for λ small) or there is an excessive supply of lemons, which forces distressed sellers to keep their assets longer. The results also imply that a small decrease in λ—the share of peaches in the economy—may cause a substantial reduction in the volume of trade as the economy transitions from an equilibrium with excessive trade to a market in which only distressed lemon holders sell.

The existence of inefficiencies for a large set of economies motivates our interest in studying optimal market design interventions that restore a first-best outcome. Specifically, the first-best outcome is achieved if the set of incentive compatible and individually rational transfers implements an equilibrium in which all distressed agents, and only them, offer their assets for sale and trade once they match with a buyer. The designer is not able to ascertain asset qualities and reallocation is subject to the same search frictions of the purely decentralized OTC market. We restrict attention to budget-balanced mechanisms.[9]

In our analysis we consider two classes of mechanisms: (i) a *transfer scheme* to reallocate existing assets among agents; (ii) a *packaging scheme* in which the designer acquires the existing assets and pools them together to create a new asset whose quality is common knowledge.[10] Packaging is analogous to a securitization process that bundles together a large pool of loans belonging to a specific category. In the first class of mechanisms it is implicit that the market designer can set the terms of trade at which assets are exchanged. In the second class of mechanisms, we consider two cases separately: either the designer can set the terms of trade for the newly issued certificates, or this option is precluded.

If the designer can set the terms of trade in the secondary market, it turns out that imposing a wedge between the price at which agents buy and sell implements the first-best outcome. In the transfer scheme, in which asymmetric information is still present, the wedge discourages non-distressed lemon holders from trying to exchange their assets for peaches. In the packaging scheme, the price differential is a way to effectively raise revenues from the trades of the newly issued assets, recouping the initial losses incurred when acquiring all assets at the price of a peach. By removing asymmetric information on the common value component, a packaging scheme has a wider scope to implement a first-best outcome than a transfer scheme. In particular, when ϕ is high—i.e. adverse selection is severe—the only feasible mechanism requires to buy all assets at a premium such that also high valuation peach holders sell their assets. Although this rapid removal entails higher initial losses in the acquisition phase, it ensures that the quality of the acquired assets correspond to the quality of the average asset in the economy.

Our results provide insights into the functioning of markets for securitized loans. Namely, it is important to ensure that such markets operate at sufficiently high levels of aggregation, at which investors know the average quality of the underlying assets. Otherwise, the securitized loans may be traded at a discount due to remaining asymmetric information concerns. A second policy implication relates to the possibilities to offset the initial losses by intermediation profits generated in the secondary market. Indeed, one way to ensure that the intervention is budget-balanced is to earn profits by setting the terms of trade in the secondary market, thereby reducing the initial cost burden. To act as a monopolist in price setting a designer should have control of all trading venues. However, this solution is often not feasible

[9] Obviously, a loss-making scheme would further improve the possibility to implement a first-best outcome.

[10] Stated differently, the designer buys individual assets from their holders at a cost, and then issues certificates backed by the acquired assets. Since the market designer cannot observe individual asset qualities, the dividend flow of the certificates issued depends on the average quality of the purchased assets.

in financial markets, and it is likely to contrast with the current legislative framework. Nonetheless, a price differential could be analogously introduced through a financial transaction tax (FTT). Although this policy is often criticized for its potential negative effects on market liquidity, FTTs are already in place in several jurisdictions and ten European countries are currently discussing a proposal to adopt an harmonized FTT.[11] We provide a novel theoretical argument in favour of a FTT based on a beneficial change in the composition of market participants, i.e. the market exclusion of non-distressed lemon holders.

If the terms of trade in the secondary markets are beyond the control of the designer, it turns out that a packaging scheme may still be budget-balanced if sellers have sufficiently strong bargaining power.[12] This allows the designer to sell the newly created assets at a high price, which is a consequence of the sellers capturing a large share of the surplus from trade.

The paper is structured as follows. The next part discusses the related literature. Section 2 presents the model and the equilibrium definition. Sections 3 and 4 discuss the equilibria with complete and unidimensional private information. Section 5 characterize the set of equilibria with bi-dimensional private information. Section 6 discusses the equilibrium properties in terms of social welfare, volume, prices and time to sale. Section 7 studies the budget-balanced market intervention mechanisms. Section 8 concludes. All proofs of the results in the main text are in Appendix A. Some additional results are presented in Appendix B.

Related literature

Our paper contributes to the literature on OTC markets in the presence of search frictions and asymmetric information between asset holders and other investors. Duffie et al. (2005) is the seminal paper in the literature on the functioning of OTC markets. In their model a fixed set of infinitely lived agents have time-varying asset valuations, and they can exchange assets bilaterally after matching with a counterparty, who could be either a dealer (dealership market) or another agent (pure decentralized market). For analytical tractability, Duffie et al. make a number of assumptions: (i) only one type of asset is traded; (ii) its dividend flow is common knowledge; (iii) agents can hold either zero or one unit of the asset; (iv) agents have only two different asset valuations (e.g. distressed or not); (v) risk-neutrality; (vi) the uniform random matching applies to all agents in the economy.[13] Subsequent papers extend this framework in several directions, and they restrict attention either to a dealership market[14] or to a pure decentralized market.[15] In our model we depart from assumptions (i) and (ii) and consider a market in which two asset qualities (peaches and lemons) coexist and their dividend flows are only observable to asset holders. We also provide a more general characterization in terms of assumption (vi), thanks to a general matching technology which allows for endogenous market participation and encompasses Duffie

[11] See Section 7.1 for a discussion.

[12] Specifically, if sellers have all the bargaining power then the revenues generated by the intervention are identical irrespective of whether the designer can set the terms of trade or not.

[13] Formally, the matching technology is such that, for any two disjoint sets of agents, the total measure of matches is proportional to the product of their masses.

[14] Lagos and Rocheteau (2007, 2009), Gârleanu (2009) relax the unit holding restriction and consider search models in which trade can only take place through dealers.

[15] For pure decentralized markets Vayanos and Wang (2007), Vayanos and Weill (2008), Weill (2008) extend the model to multiple assets; Duffie et al. (2009) and Duffie et al. (2014) study information diffusion; Afonso and Lagos (2015) consider the possibility to hold an integer number of units; Hugonnier et al. (2016) extend the model to an arbitrary distribution of asset valuations; risk aversion is considered in Duffie et al. (2007); Afonso (2011) and Atkeson et al. (2015) consider the welfare implications of decentralized market participation decisions; a micro-foundation of Duffie et al. (2005) matching function is presented in Duffie and Sun (2007).

et al. (2005) and Kiyotaki and Wright (1993) as special cases. With the exception of non-competitive matching technologies à la Duffie et al. (2005), the endogenous market participation decision is going to be crucial because it affects prices, allocations and welfare through congestion effects on the supply side of the market.

Within the OTC literature Chiu and Koeppl (2016) is the closest paper to ours. They study how to restart a lemons market in an OTC model with search frictions, resale and asymmetric information. To reduce the complexity of the non-stationary dynamics in their model, Chiu and Koeppl adopt a series of simplifying assumptions relative to the original Duffie et al. (2005) model. These assumptions jointly create a cyclical trading pattern and underlie some of their key findings.[16] In this paper we introduce asymmetric information while preserving the structure of the Duffie et al. (2005) model with random valuation shocks, i.e. distress statuses are independent of asset holdings and the pattern of trade. We provide analytical results on stationary equilibria, but the complexity of the model requires to solve it numerically when we discuss non-stationary dynamics.[17]

Our results also contribute to the growing literature on dynamic adverse selection. In this strand of literature the typical model setup considers assets of two different qualities that low-valuation holders want to sell to high-valuation buyers. All assets qualities yield a strictly positive payoff, and in the first-best outcome all assets would be traded immediately. However, buyers cannot observe asset quality and adverse selection may lead to a classic Akerlof (1970) lemons market. Differently from both Duffie et al. (2005) and our setup, buyers and sellers exit the market forever after trade.[18] The possibility to trade at different points in time allows sellers of different qualities to separate over time (intertemporal separation): peach sellers trade at a high price by delaying trade, while on average lemons trade earlier. This result holds true even when the initial share of peaches is close to zero, although the resulting delay creates inefficiencies. The stationary equilibrium characterized in Moreno and Wooders (2010) offers the closest comparison to our results.[19] They consider a discrete time model in which sellers have a fixed per period probability to match with a buyer, and show the existence of a mixed strategy equilibrium in which all assets trade over time. In equilibrium, trade delays mitigate the mimicking incentives of lemon holders. In contrast, we show that in our model a lemons market is inevitable when the initial share of peaches is low.[20]

[16]In particular, lemons provide a non-positive dividend flow and all lemon holders offer their assets for sale; only peach holders experience a negative valuation shock, leading to an absorbing state until trade; all agents without assets participate as buyers, even sellers of peaches who had a low valuation just before trade. As a result, their model features: an increase in the relative market share of peaches in the absence of trade; continuous market participation of all lemon holders as they own a worthless asset and have no asset valuation shocks; a fixed measure of buyers participating in the market.

[17]Maurin (2016) shows the existence of cyclical equilibria in a discrete time model that shares with our setup bi-dimensionality of private information. However, his model is different from ours in several dimensions. First, the economy in his model is not closed in that a free entry condition determines the measure of buyers in the market. Second, attention is restricted to a subset of parameter values for which high-valuation lemon holders are always willing to sell at the pooling price. Lastly, he employs a Leontief-type matching function which implies that the short-side of the market matches for sure.

[18]In turn, these features imply that agents' reservation values are exogenously fixed, while in our model they are endogenously determined, and may depend on the expected utility of all other agent types.

[19]The dynamic adverse selection literature mainly considers non-stationary models. A non-exhaustive list includes Janssen and Roy (2002), Blouin (2003), Camargo and Lester (2014), Fuchs and Skrzypacz (2015) and Moreno and Wooders (2016). A recent extension by Kaya and Kim (2015) consider buyers that receive private informative signals on asset quality, in the spirit of Taylor (1999). In Kaya and Kim's model different trade dynamics arise depending on the initial share of high quality assets, but inter-temporal separation continues to be the driving economic mechanism when the initial share of high quality assets is low.

[20]In Moreno and Wooders (2010) buyers get a zero expected payoff and mix among a high price, say p_H, accepted by all sellers, a lower price, say p_L, accepted only by lemons, and a set of prices rejected with probability one by all sellers. Delaying trade with price offers rejected with probability one reinforces the incentives for lemon sellers to accept p_L rather than waiting until a buyer offers p_H. As lemons trade faster than peaches, the endogenous market quality can be increased to a point in which buyers are willing to offer p_H. Importantly, this strategy is possible only if lemon sellers have a (endogenous) reservation utility equal to the (exogenous) outside value of a lemon buyer. In contrast, in our model the expected utility difference between

Our paper is also related to Chang (2017) and Guerrieri and Shimer (2015) on competitive search markets with bi-dimensional private information. In this class of models, buyers first observe the prices posted by sellers—the market is not opaque—and then request to buy the asset from one seller. If there are more buyers (sellers) than sellers (buyers) the former are rationed. When private information only concerns asset quality, Guerrieri and Shimer (2014) show that a fully separating equilibrium always exists: a downward distortion in sellers' trade probabilities—interpreted as market illiquidity—separates higher quality assets from lower quality ones. If private information is bi-dimensional multiple semi-pooling equilibria exist. Chang (2017) characterizes the conditions on the joint distribution of asset types and holding costs that support the existence of a fire-sales equilibrium in which a set of agent types pools on the same low price and sell their assets quickly. We obtain a similar result in Section 6, where we discuss the dynamics of an increase in x—i.e. distressed agents have greater urgency to trade—such that the equilibrium switches from one with excessive trade to one in which only distressed asset holders trade. Although our model differs in many respects from that of Chang, we also find that distressed agents trade faster at a lower price, but total volume decreases moving from an excessive (all lemons are offered for sale) to an efficient (only distressed asset holders sell) level of trade.

Our normative analysis is related to the literature on public interventions in frozen markets. Within this collection of works, Tirole (2012) and Philippon and Skreta (2012) are closest in spirit to our study. In particular, their problem, as ours, gives rise to mechanism participation constraints which depend on the mechanism itself. Both papers establish that the market designer incurs a loss as he acquires the lowest quality assets at a premium. Although the presence of a private market affects the pattern of the intervention, there would be no welfare gain from shutting it down. In our model, if the government has to compete with the private asset market, offering the going market price does not always guarantee a separation of types because of bi-dimensional private information.[21] The inefficiency arising from type-dependent reservation values implies that interventions targeted at lemons, such as those considered in Tirole (2012) and Philippon and Skreta (2012), are only feasible in our setting when adverse selection is moderate, but they would not improve efficiency relative to our packaging scheme. Lastly, to the best of our knowledge, we provide a novel economic justification for the introduction of a financial transaction tax (FTT) for the purpose of mitigating the bi-dimensional private information problem. Two recent theoretical papers also argue in favour of a FTT based on social welfare considerations: in Berentsen et al. (2016) a FTT mitigates a pecuniary externality on liquid assets; in Davila (2016) a designer may find it optimal to introduce a FTT to reduce non-fundamental trading due to investors' differences in beliefs over asset returns.

2 Model

2.1 Economic environment

Time is continuous and the economy infinitely lived. There are two consumption goods, perishable fruit and a numéraire good. There is a measure A of durable assets. Each asset is one of two types: a peach

non-distressed (buyers) and distressed (sellers) lemon holders is always strictly positive; as a result, in equilibrium buyers never find it profitable to offer a price rejected with probability one. If this were the case, they would find it profitable to deviate and trade immediately by offering a price slightly higher than the reservation price of distressed lemon holders.

[21] Suppose types could be separated, leaving only peaches to be traded in the private market. This would encourage non-distressed lemon holders to participate in the private market, which would now offer a higher surplus due to lower information rents.

(g) or a lemon (b). A peach yields δ_g units of fruit per period, while a lemon yields δ_b, with $\delta_g > \delta_b > 0$. A proportion $\lambda \in [0,1]$ of assets are peaches.

Agents are infinitely lived, risk-neutral, and discount future payoffs at a rate $r > 0$. A measure A of agents are initially endowed with one unit of the asset, while a measure 1 (normalized) of agents initially holds no asset. At any point in time, an agent can hold either 0 or 1 unit of the asset. This restriction on asset holdings keeps the distribution of assets among agents tractable. The type of an asset is private information to its current owner. At any point in time, each agent is in one of two states: either he has a low (l) or a high (h) valuation of the asset. The instantaneous utility of an agent with high valuation from a type-i asset is δ_i , whilst that of an agent with low valuation is $\delta_i - x$, where $i \in \{b,g\}$. The parameter x satisfies $x < \delta_b$, ensuring that agents never want to dispose of their asset. The ratio $(\delta_g - \delta_b)/x$, a key object in what follows, is denoted by ϕ. All agents with high (low) valuation, with or without an asset, transit to the state of low (high) valuation with intensity κ (intensity ν). That is, a high-valuation (low-valuation) agent receives a valuation shock with Poisson arrival rate κ (rate ν). We assume $\nu \geq \kappa A$ as it guarantees that the measure of high-valuation agents without assets is always greater or equal to the measure of low-valuation asset holders. In other words, in a frictionless market assets could always be held by high-valuation agents. An agent's valuation state is his private information. Each agent can produce any amount of the numéraire good, and receives an instantaneous utility which is the sum of the numéraire good consumed/produced in that instant and the utility obtained from the asset, if any, he holds.

It is useful to label the agents according to their status, namely whether they are in the state of high (h) or low (l) valuation, and whether they hold a lemon (b), a peach (g), or no asset (n). Thus, the set of agent statuses is $\{hb, lb, hg, lg, hn, ln\}$. Agents with valuation $j \in \{h,l\}$ holding an asset $i \in \{b,g,n\}$ are denoted by ji, and their mass by γ_{ji}. In general, the mass of any group C of agents is γ_C. For notational convenience, we partition agents with different statutes into a set of potential sellers $\tilde{S} = \{hb, lb, hg, lg\}$ and a set of potential buyers $\tilde{B} = \{hn, ln\}$ with elements $\tilde{s}$ and $\tilde{b}$, respectively.

Trade is decentralized and takes place between an agent without an asset, but wishing to buy one, and an agent wishing to sell his asset. Buyers and sellers are bilaterally and randomly matched. We denote by ξ the set of agents with and without an asset to which the matching technology applies. For example, if matching always involves all agents in the economy then $\xi = \tilde{B} \cup \tilde{S}$; alternatively, if the matching technology requires a deliberate decision to participate in a trading platform, then ξ is the set of agents with and without assets, respectively, that participate in the platform. For any $\tilde{s} \in \tilde{S}$ ($\tilde{b} \in \tilde{B}$), we indicate with $q^S_{\tilde{s}}$ ($q^B_{\tilde{b}}$) the share of agents of type $\tilde{s}$ ($\tilde{b}$) to which the matching technology applies. Obviously, if all agents in the economy are always matched then $q^S_{\tilde{s}} = 1$ and $q^B_{\tilde{b}} = 1$ for all $\tilde{s} \in \tilde{S}$ and $\tilde{b} \in \tilde{B}$.

We derive our results using a general function $\mu M(\gamma_C, \gamma_D; \gamma_\xi)$ for the total meeting rate between two sets of agents C and D. In other words, the quantity $\mu M(\gamma_C, \gamma_D; \gamma_\xi)$ is the instantaneous measure of total matches between agents in C and D, and all agents belonging to C (to D) have an identical intensity to match with an agent in D (in C).[22] Therefore, an agent in C meets agents in D at intensity $\mu M(\gamma_C, \gamma_D, \gamma_\xi)/\gamma_C$ and an agent in D meets agents in C at intensity $\mu M(\gamma_C, \gamma_D, \gamma_\xi)/\gamma_D$. The parameter μ determines the

[22] If at times determined by a Poisson process with intensity parameter $\tilde{\mu}$ each agent contacts another agent from the set of agents participating in the platform 'at random', the rate at which a group C of agents contacts a disjoint group D of agents is $\gamma_C \tilde{\mu} \gamma_D / \gamma_\xi$. Similarly, the rate at which the group D contacts the group C is $\gamma_D \tilde{\mu} \gamma_C / \gamma_\xi$. Thus, the total meeting rate between the groups C and D is $2\tilde{\mu}\gamma_C\gamma_D/\gamma_\xi$ (see Duffie et al., 2005). Thus, the parameter μ in our expression for the total meeting rate can be equated to $2\tilde{\mu}$, double the intensity at which an agent contacts another agent in ξ. Also, it follows that the meeting rate between agents in C is given by $(\mu/2)\gamma_C^2/\gamma_\xi$.

degree of search frictions in the market. If $\mu \to \infty$ trade becomes frictionless. The general matching function $M(\gamma_C, \gamma_D, \gamma_\xi)$ is required to satisfy: (i) strict monotonicity in each of its first two arguments, i.e. $M_1, M_2 > 0$; (ii) a non-increasing ratio between actual matches $M(\gamma_C, \gamma_D, \gamma_\xi)$ and the total measure of possible matches $\gamma_C\gamma_D$, i.e. $\frac{M(\gamma_C,\gamma_D,\gamma_\xi)}{\gamma_C\gamma_D}$ being non-increasing in γ_C and γ_D;[23] (iii) non-decreasing returns to scale, i.e. $M_1(\gamma_C, \gamma_D, \gamma_\xi)\gamma_C + M_2(\gamma_C, \gamma_D, \gamma_\xi)\gamma_D \geq M(\gamma_C, \gamma_D, \gamma_\xi)$.[24] The second condition ensures that the intensity at which agents in set C (D) meet agents in D (C) does not increase when the measure of agent in set C (D) increases.[25] This assumption simply rules out matching functions that reverse the natural assumption that search among agents has a competitive component, at least weakly, and it is broadly satisfied by a large class of matching functions. We distinguish between matching functions for which the ratio $\frac{M(\gamma_C,\gamma_D,\gamma_\xi)}{\gamma_C\gamma_D}$ is constant in γ_C and γ_D and those for which the ratio is strictly decreasing in these two arguments, calling the former non-competitive and the latter competitive matching functions. In each of these two classes, we identify a noticeable matching technology:

1. *Duffie-Gârleanu-Pedersen (DGP) matching:* each agent has an identical probability to match with every other agent in the economy as there is no explicit participation decision to a matching platform. Hence, $\xi = \tilde{B} \cup \tilde{S}$ and $\gamma_\xi = 1 + A$. This matching technology is adopted in the seminal paper by Duffie et al. (2005). In the context of our model, the total meeting rate becomes $\mu M(\gamma_C, \gamma_D, \gamma_\xi) = \frac{\mu\gamma_C\gamma_D}{1+A}$. This matching technology is non-competitive since the ratio $\frac{M(\gamma_C,\gamma_D,\gamma_\xi)}{\gamma_C\gamma_D}$ is equal to a constant.[26] Thus, the intensity at which an agent in group C (D) meets agents in group D (C) is independent of the mass of agents in C (D). As a result, an increase in the number of sellers does not have an effect on the rate at which an individual seller meets buyers. A real world example of this matching technology is an over-the-counter market in which participants phone each other randomly and without knowing whether the agent they contact is willing or able to buy/sell an asset.

2. *Kiyotaki-Wright (KW) matching:* each participant in the platform is matched uniformly at random with another participant. This matching function, adopted in Kiyotaki and Wright (1993), has been microfounded in Stevens (2007), and it implies that matching takes place only among agents that actively participate in the market through the platform. Using the previous notation, $\gamma_\xi = \sum_{\tilde{s}\in\tilde{S}} q^S_{\tilde{s}}\gamma_{\tilde{s}} + \sum_{\tilde{b}\in\tilde{B}} q^B_{\tilde{b}}\gamma_{\tilde{b}} \leq 1 + A$. Formally, the matching function for groups C and D is $M(\gamma_C, \gamma_D, \gamma_\xi) = \gamma_C\gamma_D/\gamma_\xi$ when $C, D \subset \xi$, whereas the matching function is equal to zero if C or $D \subset \xi^c$. This matching technology is competitive as the ratio $\frac{M(\gamma_C,\gamma_D,\gamma_\xi)}{\gamma_C\gamma_D}$ is strictly decreasing in γ_C and γ_D. Thus, the intensity at which an agent in group C meets agents in group D is strictly decreasing in the mass of agents in C, and analogously for an agent in group D. Therefore, for example, when the number of sellers increases the rate at which an individual seller meets buyers falls. This matching technology can be thought of as a physical market place in which only parties interested in trading participate, but where it is not possible to direct search only towards buyers or sellers.

For notational convenience, in the remainder of the paper we suppress the γ_ξ argument of the matching function and simply write $M(\gamma_C, \gamma_D)$.

[23] The condition is equivalent to requiring $M(\gamma_C, \gamma_D, \gamma_\xi) \geq \max\{M_1(\gamma_C, \gamma_D, \gamma_\xi)\gamma_C, M_2(\gamma_C, \gamma_D, \gamma_\xi)\gamma_D\}$.

[24] Note that if $M(\cdot, \cdot)$ is a homogeneous function, this condition requires it to be homogeneous of at least order one.

[25] For example, if this condition were violated then, for a constant measure of buyers, sellers would meet buyers more frequently when the measure of sellers in the market *increases*.

[26] By the definition of a non-competitive matching technology, any such technology is characterized, up to a constant, by the DGP matching function. For this reason, we use the terms non-competitive and DGP matching technologies interchangeably.

2.2 Strategies and equilibrium definition

In a matched buyer-seller pair, the terms of trade depend on the bargaining protocol. Under complete information we consider the generalized Nash bargaining solution; under asymmetric information, we assume buyers make take-it-or-leave-it offers to sellers. This is a common assumption in the dynamic adverse selection literature[27] because it ensures that the price offer does not contain information about the type of the seller's asset. In section B.2 in the Appendix we consider an alternative bargaining protocol, namely asset holders propose prices and buyers can either accept or reject the offer. Once a seller trades, he becomes an agent without an asset.

For analytical tractability, we restrict attention to stationary equilibria, namely steady state solutions that do not depend on time t. Let $\sigma_{\tilde{s}}(p)$ be the probability that a potential seller of type $\tilde{s} \in \tilde{S}$ accepts price p. Similarly, $\sigma_{\tilde{b}}(p)$ is the probability that a potential buyer of type $\tilde{b} \in \tilde{B}$ offers price p.

Fix a strategy profile $\sigma = \{\sigma_{\tilde{s}}, \sigma_{\tilde{b}}\}$ and a set ξ of agents participating in the market. We define as *actual* sellers and buyers, and denote them by S and B respectively, the sets of agents with and without assets belonging to ξ and whose probability to trade is non-zero. For a formal definition, let $\tilde{s}(\xi) = \tilde{s} \cap \xi$ and $\tilde{b}(\xi) = \tilde{b} \cap \xi$ be the set of agents in $\tilde{s} \in \tilde{S}$ and $\tilde{b} \in \tilde{B}$ to which the matching technology applies. Moreover, denote by $S^T(\sigma) \subset \tilde{S}$ and $B^T(\sigma) \subset \tilde{B}$ the subsets of $\tilde{S}$ and $\tilde{B}$ that trade with positive probability, i.e.:

$$\begin{aligned} S^T(\sigma) &= \left\{ \bigcup_{\tilde{s} \in \tilde{S}} \tilde{s} : \exists p \text{ s.t. } \sigma_{\tilde{s}}(p) \left(\sum_{\tilde{b} \in \tilde{B}} \sigma_{\tilde{b}}(p) \right) > 0 \right\} \\ B^T(\sigma) &= \left\{ \bigcup_{\tilde{b} \in \tilde{B}} \tilde{b} : \exists p \text{ s.t. } \sigma_{\tilde{b}}(p) \left(\sum_{\tilde{s} \in \tilde{S}} \sigma_{\tilde{s}}(p) \right) > 0 \right\} \end{aligned} \tag{1}$$

The sets of active buyers and sellers are $B = B^T(\sigma) \bigcap \left(\bigcup_{\tilde{b} \in \tilde{B}} \tilde{b}(\xi) \right)$ and $S = S^T(\sigma) \bigcap \left(\bigcup_{\tilde{s} \in \tilde{S}} \tilde{s}(\xi) \right)$.

Agents hold beliefs on the probability to match with a possible trading counter party: m_B and m_S denote sellers and buyers' beliefs to meet a buyer and a seller in B and S, respectively. A potential buyer $\tilde{b} \in \tilde{B}$ holds a belief $\pi_{\tilde{b}}(i, p)$ to get asset quality i when he offers a price p to a seller $i \in S$. Analogously, a potential seller $\tilde{s} \in \tilde{S}$ holds a belief $\pi_{\tilde{s}}(p)$ to receive a price p from a buyer in B. Finally, let π be the collection of beliefs held by all agents. We proceed to define an equilibrium of the game.

Definition 2.1 *A stationary assessment (σ, π) is a Perfect Bayesian equilibrium of the game if and only if:*

1. *For every ji, σ_{ji} is a best response taking as given π and σ.*

2. $\xi = B^T(\sigma) \cup S^T(\sigma)$, *except for the DGP technology for which* $\xi = \tilde{B} \cup \tilde{S}$.

[27] See Moreno and Wooders (2010, 2016), Camargo and Lester (2014), Kaya and Kim (2015).

3. *Agents hold correct beliefs on the equilibrium path, i.e.:*

$$
\begin{aligned}
\pi_{\tilde{b}}(i,p) &= \sum_{j=h,l} \frac{q^S_{ji}\gamma_{ji}}{\gamma_S}\sigma_{ji}(p) & \textit{if} \quad & \sum_{\tilde{s}\in\tilde{S}} \sigma_{\tilde{s}}(p) > 0 \\
\pi_{\tilde{s}}(p) &= \sum_{\tilde{b}\in\tilde{B}} \frac{q^B_{\tilde{b}}\gamma_{\tilde{b}}}{\gamma_B}\sigma_{\tilde{b}}(p) & \textit{if} \quad & \sum_{\tilde{b}\in\tilde{B}} \sigma_{\tilde{b}}(p) > 0 \\
m_S &= \mu\frac{M(\gamma_B,\gamma_S)}{\gamma_S} & m_B &= \mu\frac{M(\gamma_B,\gamma_S)}{\gamma_B} \\
q^S_{\tilde{s}} &= \frac{\gamma_{\tilde{s}(\xi)}}{\gamma_{\tilde{s}}} & q^B_{\tilde{b}} &= \frac{\gamma_{\tilde{b}(\xi)}}{\gamma_{\tilde{b}}}
\end{aligned}
\tag{2}
$$

The first condition is standard as it requires each agent in ji to play a best response strategy. The second equilibrium requirement is not binding for the DGP technology (as by definition $\xi = \tilde{B}\cup\tilde{S}$), while for all other technologies it selects the equilibrium in which only agents that expect to trade with positive probability participate in the trade platform, implying that $\xi = B^T(\sigma)\cup S^T(\sigma)$. This equilibrium restriction can be seen as the limit outcome of a model setup with platform participation costs which tend to zero. Lastly, the third requirement is also standard as it imposes beliefs, matching rates and the share of agents participating in the platform to be correct on the equilibrium path.

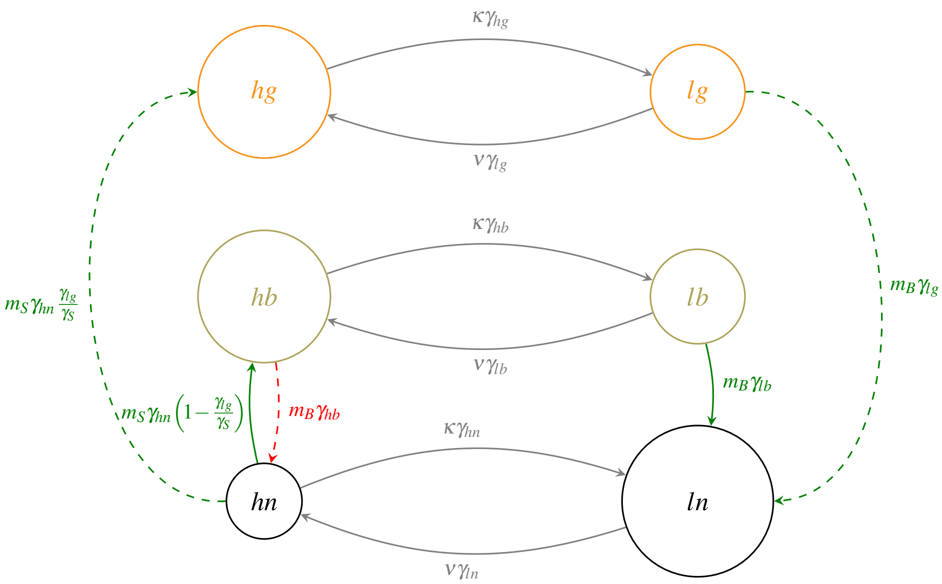

Figure 1: Transitions due to valuation shocks and trade.

Figure 1 illustrates how agents transition between different statuses due to valuation shocks and trade. Valuation shocks, giving rise to the flows in grey, apply to all agents, irrespective of their asset holdings and the equilibrium patterns of trade. The remaining flows are due to trade, thus depending on agents' equilibrium strategies. The flows in green denote transitions due to assets passing from low-valuation to high-valuation agents, between whom there are gains from trade. The flow in red, on the other hand, arises from low-quality assets changing hands between high-valuation agents. As we will see in what follows the flows indicated with dashed lines arise only in some of the possible equilibria.

Consider an equilibrium assessment. We denote by V_{ji} the value function of an agent with valuation $j\in\{h,l\}$ and holding an asset $i\in\{b,g,n\}$. A high-valuation owner of a type-i asset derives instantaneous utility δ_i until he either transits to the state of low valuation or meets a buyer and sells his asset. Denoting the random time at which a high-valuation owner transits to the state of low valuation with τ_d and that at which he meets a buyer in B with τ_B, his value function V_{hi} satisfies

$$V_{hi}(t) = \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} \delta_i \, \mathrm{d}s + e^{-r(\tau_d - t)} V_{li}(\tau_d) \mathbb{1}_{\{\tau_d = \tau\}} + e^{-r(\tau_B - t)} W_{hi}(\tau_B) \mathbb{1}_{\{\tau_B = \tau\}} \right] \tag{3}$$

where $\tau = \min\{\tau_d, \tau_B\}$ and W_{hi} denotes the value function of a high-valuation owner of type-i asset matched with a buyer. Note that if the matching technology matches only actively participating agents, then $\tau = \tau_d$ for agents who do not participate in the market.

Analogously, a low-valuation owner of a type-i asset obtains a utility flow $\delta_i - x$ until he either transits to the state of high valuation or sells his asset. Denoting the time at which a low-valuation asset holder changes valuation with τ_u, his value function V_{li} becomes

$$V_{li}(t) = \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} (\delta_i - x) \, \mathrm{d}s + e^{-r(\tau_u - t)} V_{hi}(\tau_u) \mathbb{1}_{\{\tau_u = \tau\}} + e^{-r(\tau_B - t)} W_{li}(\tau_B) \mathbb{1}_{\{\tau_B = \tau\}} \right] \tag{4}$$

where $\tau = \min\{\tau_u, \tau_B\}$.

Given an equilibrium assessment (σ, π), denote the distribution function of prices offered by buyers at time τ by $F_\tau(\cdot)$. The value function of an owner of a type-i asset matched with a buyer W_{ji} satisfies:

$$W_{ji}(\tau) = \int \max\{p + V_{jn}(\tau), V_{ji}(\tau)\} \, \mathrm{d}F_\tau(p) \tag{5}$$

Agents without an asset derive an instantaneous utility of zero, and they experience a change in their expected future utility when either their valuation state changes, or they meet a seller and buy an asset. Hence, denoting the time at which an agent without an asset experiences a change in his valuation with τ_c, and the next time at which he meets a seller in S with τ_S, his value function satisfies

$$V_{jn}(t) = \mathbb{E}_t \left[e^{-r(\tau_c - t)} V_{\tilde{j}n}(\tau_c) \mathbb{1}_{\{\tau_c < \tau_S\}} + e^{-r(\tau_S - t)} W_{jn}(\tau_S) \mathbb{1}_{\{\tau_S < \tau_c\}} \right] \tag{6}$$

where $j, \tilde{j} \in \{h, l\}$ and $j \neq \tilde{j}$.

A buyer who is matched with a seller in S decides on a price to offer depending on the probabilities of obtaining an asset of either type conditional on the offered price. The value function W_{jn} of a buyer matched with a seller in S satisfies

$$\begin{aligned} W_{jn}(\tau) = \max_p \big\{ & \pi_{jn}(g, p, \tau) [V_{jg}(\tau) - p] + \pi_{jn}(b, p, \tau) [V_{jb}(\tau) - p] \\ & + [1 - \pi_{jn}(g, p, \tau) - \pi_{jn}(b, p, \tau)] V_{jn}(\tau) \big\} \end{aligned} \tag{7}$$

Differentiating (3), (4) and (6) with respect to t and rearranging, one obtains the following Hamilton-Jabobi-Bellman equations

$$rV_{hi}(t) = \delta_i + \kappa[V_{li}(t) - V_{hi}(t)] + m_B(t)[W_{hi}(t) - V_{hi}(t)] + \dot{V}_{hi}(t) \tag{8}$$

$$rV_{li}(t) = \delta_i - x + \nu[V_{hi}(t) - V_{li}(t)] + m_B(t)[W_{li}(t) - V_{li}(t)] + \dot{V}_{li}(t) \tag{9}$$

$$rV_{hn}(t) = \kappa[V_{ln}(t) - V_{hn}(t)] + m_S(t)[W_{hn}(t) - V_{hn}(t)] + \dot{V}_{hn}(t) \tag{10}$$

$$rV_{ln}(t) = \nu[V_{hn}(t) - V_{ln}(t)] + m_S(t)[W_{ln}(t) - V_{ln}(t)] + \dot{V}_{ln}(t), \tag{11}$$

These general expressions encompass different possible equilibria. In particular, in our subsequent analysis it will be of great importance to consider if peaches trade in the market, and whether high-valuation lemon holders prefer to stay out of the market. As we restrict attention to stationary equilibria, value functions do not change over time, i.e. $\dot{V}_{ji} = 0$ and $V_{ji}(t) = V_{ji}$ for every ji.

3 Complete information

Before analysing equilibria under asymmetric information, we consider the environment described in the previous section when agents have complete information about both asset quality and each other's valuation states. This allows us to distinguish between features of the equilibrium outcomes which are attributable to asymmetric information as opposed to arising from search frictions.

It is straightforward to characterize the equilibrium under complete information. The next proposition summarizes the main equilibrium properties:

Proposition 3.1 *With complete information, a unique equilibrium exists for the whole parameter space, and satisfies:*

1. $S = \{lb, lg\}$ *and* $B = \{hn\}$.
2. *The matching rate* m_B^C *is the unique solution to*

$$m_B^C \gamma_S^C = m_S^C \gamma_B^C = \mu M\left(\gamma_B^C, \gamma_S^C\right) \quad \Rightarrow \quad \frac{\kappa A m_B^C}{\kappa + \nu + m_B^C} = \mu M\left(\frac{\kappa A}{\kappa + \nu + m_B^C}, \frac{\nu - \frac{\kappa A m_B^C}{\kappa + \nu + m_B^C}}{\kappa + \nu}\right) \tag{12}$$

3. *During any time interval* Δt *the trade volume is* $\frac{\kappa A m_B^C}{\kappa + \nu + m_B^C} \Delta t$.
4. *Let* β *and* $1 - \beta$ *be the Nash bargaining weights of sellers and buyers, respectively. For an asset of quality* i *the equilibrium price is:*

$$p_i = \frac{1}{r}\left[\delta_i - \frac{\kappa + (1-\beta)(r + m_S^C)}{\kappa + \nu + r + (1-\beta)m_S^C + \beta m_B^C} x\right] \tag{13}$$

where $m_S^C = \frac{\kappa A(\kappa + \nu) m_B^C}{(\kappa + \nu)\nu + m_B^C(\nu - \kappa A)}$.

With complete information assets of both qualities are traded. Buyers offer different prices for peaches and lemons, the difference being equal to $(\delta_g - \delta_b)/r$. Prices are equal to the discounted value of future dividends (δ_i/r) less a discount which depends on the loss x due to low asset valuation and, importantly, on the rates m_S^C and m_B^C at which buyers meet sellers and vice versa. From the equation linking the two rates, it is easy to see that the two quantities are positively related. Intuitively, markets with higher matching intensities may be interpreted as more liquid markets as buyers and sellers have to wait less until they encounter another agent willing to trade. The price equation (13) is analogous to the one presented in Duffie et al. (2005) for the case without market makers.[28]

The effect of liquidity (i.e. of the matching intensities m_B^C and m_B^S) on prices depends on the relative bargaining power between buyers and sellers. In particular, when buyers (sellers) have relatively more bargaining power prices decrease (increase) with more market liquidity. To understand the economic

[28] In Duffie et al. (2005) notation, their model excludes market makers when $\rho = 0$.

intuition underlying this result, it is useful to consider the two extreme cases of full bargaining power for buyers or sellers. If buyers hold all bargaining power ($\beta = 0$), the resulting price discount $\frac{\kappa+r+m_S^C}{\kappa+r+\nu+m_S^C}x$ is increasing in m_S^C, i.e. the higher m_S^C the lower the price buyers pay to sellers. Indeed, the equilibrium price is equal to the reservation price $\bar{p}_{li} = V_{li} - V_{ln}$, namely it is equal to the difference between the expected value of holding an asset when in the state of low valuation $\left(V_{li} = \frac{1}{r}\left[\delta_i - \frac{\kappa+r}{\kappa+\nu+r}x\right]\right)$ and the post-trading continuation value of being without asset $\left(V_{ln} = \frac{1}{r}\frac{m_S^C}{\kappa+\nu+r+m_S^C}\frac{\nu}{\kappa+\nu+r}x\right)$. The latter term is increasing in the matching intensity m_S^C because it becomes less time-consuming to match with an agent willing to sell. In other words, an improvement in the matching intensity of buyers m_S^C decreases the price that asset holders require to exchange their assets, because they expect to find a seller more easily once their valuation will be high again. This counter-intuitive effect of liquidity on prices captures in reality a common sense notion: if agents know it is going to be hard to buy a similar asset in the market because of illiquidity, they require a greater compensation to part with an asset which they are likely to desire in the future. On the contrary, when sellers have all the bargaining power ($\beta = 1$) they demand a price equal to V_{hi} (the expected value of an asset holder with high valuation) and the price discount is equal to $\frac{\kappa}{\kappa+\nu+r+m_B^C}x$. In this case, a higher matching intensity m_B^C for sellers improves V_{hi} because it makes it easier to sell the asset when the holder will switch to a low valuation. As a result, higher liquidity leads to higher market prices. This intuition is more familiar in the financial markets literature, and this positive effect of liquidity on securities is commonly referred as liquidity premium.

This salient relationship between market liquidity and bargaining power also applies to markets with asymmetric information between buyers and sellers. When buyers make take-it-or-leave-it offers, they implicitly have full bargaining power and there is a negative relationship between liquidity and transaction prices. In a similar vein, we show in Appendix B that flipping around the bargaining protocol—sellers make take-it-or-leave-it offers—reverses this relationship, i.e. a more liquid market displays higher transaction prices (see Section B.2.2). Nonetheless, despite the similarities, the setup with asymmetric information leads to additional effects because the average quality of the assets on sale affects market liquidity. We refer to Appendix B for a detailed analysis of asymmetric information equilibria with sellers making offers.

In Section 6.2 we are going to show that the decentralized equilibrium outcome of Proposition 3.1 belongs to the set of allocations maximizing utilitarian social welfare, subject to the search frictions due to the matching process. In other words, the decentralized nature of trade and the bargaining outcome do not adversely affect allocative efficiency.

4 Unidimensional private information

We proceed by introducing private information only in one dimension: concerning either asset quality or agents' valuation states. This allows us to understand whether the equilibrium outcomes in our setting with bi-dimensional private information are also shaped by an interaction between the two sources of asymmetric information. We first consider the case in which asset quality is publicly observable but agents' valuation states are not; then we consider the alternative case in which agents can distinguish between high- and low- valuation agents but the quality of an asset is private information of its holder.

4.1 Asset quality publicly observable

When the quality of each asset is public information, the results obtained with complete information continue to apply. This can be seen by applying the mechanism for bilateral bargaining in Myerson and Satterthwaite (1983). Suppose each pair of matched agents, one of which holds an asset, is proposed a mechanism in which the asset changes hands at the price that would arise from Nash bargaining between a low-valuation asset holder and a high-valuation agent with no asset. If either of the two agents is not willing to trade, the mechanism prescribes trade not to take place. With such a mechanism, no encounter between two agents in the same valuation state results in trade. Therefore, we can conclude that the market composition and equilibrium quantities are identical to those with complete information.

4.2 Individual valuation state publicly observable

If there is common knowledge about agents' valuation states, trade can only take place between a low-valuation asset holder and high-valuation agent with no asset. This result follows immediately from the no trade theorem in Milgrom and Stokey (1982). Whereas no allocative inefficiency arises with asymmetric information on the agents' valuations of the assets, this is no longer the case when asymmetric information concerns the underlying asset quality, i.e. the common value component, even if individual private states are publicly observable.

Under this informational setup the equilibrium characterization follows analogous steps to the one provided in Section 5 in which only low-valuation asset holders sell their good; see Proposition 5.2. In the interest of space we skip any formal statement here and we provide some additional details in Appendix B. A crucial difference relative to the analysis in Section 5 is the absence of any constraint to ensure *hb* agents do not sell their assets, as common knowledge on private valuation states coupled with asymmetric information on asset quality leads to the exclusion of trades when there are no gains from trade, i.e. between *hb* and *hn* agents as they both have a high private valuation. Nonetheless, in equilibrium high-quality assets may not be traded if their share λ is too low and buyers make take-it-or-leave-it offers: the relevant constraint to ensure trade in both goods is identical to the one presented in Section 5 for equilibrium E. However, the adverse selection problem is less severe relative to the equilibria in the bi-dimensional private information case, as in the latter the presence of an additional constraint on *hb* agents' market participation will limit the possibilities for the existence of an allocatively efficient outcome. We turn to address this point in detail.

5 Asymmetric information on asset quality and private valuations

5.1 Lemons market equilibrium

Since the seminal Akerlof (1970) paper, it is a well known result that the presence of asymmetric information between buyers and sellers may lead to an extreme form of adverse selection, with only the lowest quality goods exchanged. We start our analysis by considering when this market breakdown may materialize in our dynamic model with bilateral matches and resale.

A first important observation is that the existence of the lemons market equilibrium, say equilibrium L, depends on the matching technology, because it determines whether a buyer who deviates by offering a higher price can match with a peach owner. The next proposition provides a necessary and sufficient condition for the existence of a lemons market equilibrium.

Proposition 5.1 *Let $m_{\tilde{s}}$ denote the matching rate of buyers with asset owners of type $\tilde{s} \in \tilde{S}$ when $S = \{lb\}$. A lemons market equilibrium exists if and only if:*

$$\phi \geq \min\left\{ \frac{m_{lg}}{m_{lb}} \frac{r}{r+\kappa+\nu+m_{lb}}, \frac{m_{lg}}{m_{lb}} \frac{r}{r+\kappa+\nu+m_{lb}} + \frac{m_{hb}}{m_{lb}} \left(\frac{r}{r+\kappa+\nu+m_{lb}} - \phi \right) \right\} \tag{14}$$

Proposition 5.1 holds for all admissible matching functions, and in particular it implies the following corollary for different matching technologies:

Corollary 5.1 *With the DGP technology there exist $\lambda^* < 1$ such that for every $\lambda \geq \lambda^*$ a lemons market equilibrium does not exist. With any competitive matching technology a lemons market equilibrium always exists.*

In a lemons market equilibrium, buyers never find it optimal to offer a higher price if the matching technology only applies to agents participating in the market as buyers only encounter lemon holders. In this case, the lemons market equilibrium is self-fulfilling and it exists over the whole parameter space. On the contrary, with the DGP technology all agents are continuously matched, implying that buyers always have a non-zero probability of matching with a peach holder. As a consequence, if the share of peaches is sufficiently high, each buyer finds it convenient to offer a price accepted by peach holders, being likely to match with such an agent, whereas offering a price accepted only by lemon holders delays trade because there are few lemons in the economy.

The lemons market equilibrium is clearly suboptimal because not all mutually convenient trades take place, and it is a common feature of static adverse selection models. In the remainder of the section, we consider the existence of semi-pooling equilibria in which both peaches and lemons are traded.

5.2 Semi-pooling equilibria

In this section we discuss the existence of the stationary equilibria in which also peaches are traded. It is crucial to distinguish between two possible types of equilibria:

1. Equilibrium **E**: *lg* and *lb* agents are active sellers.
2. Equilibrium **H**: *lg*, *lb* and *hb* agents are active sellers.

Throughout the paper we denote by $\bar{p}_{ji}$ the price that makes *ji*-agents indifferent between selling and keeping their asset; i.e. $\bar{p}_{ji} = V_{ji} - V_{jn}$. Differently from Chiu and Koeppl (2016), in our model the quantity of lemons on the market is endogenous, and it depends on whether high-valuation lemon holders prefer to sell their assets at the price $\bar{p}_{lg}$ that low-valuation holders of peaches are willing to accept. Thus, we distinguish between two types of semi-pooling equilibria: in equilibrium E only low-valuation asset holders participate in the market, as in the complete information benchmark; in equilibrium H also high-valuation lemon holders participate as they find it convenient to sell their assets at price $\bar{p}_{lg}$. In this section we characterize the conditions for the existence of equilibrium E and H.

By Lemma A.3 in Appendix A, only *hn* agents are willing to participate as buyers since there are gains from trade only between low- and high-valuation agents. Moreover, Lemma A.4 shows that, in any equilibrium, buyers offer $\bar{p}_{lb}$ and/or $\bar{p}_{lg}$, lb agents accept $\bar{p}_{lb}$ and $\bar{p}_{lg}$ with probability one while lg agents only accept $\bar{p}_{lg}$. Recall that $\sigma_{hn}(\bar{p}_{lg})$ denotes the probability of buyers offering $\bar{p}_{lg}$. Then, given that lg agents reject the lower offer $\bar{p}_{lb}$, the measures of agents obey the following laws of motion:

$$\begin{aligned}
\dot{\gamma}_{hb} &= \nu\gamma_{lb} - \kappa[(1-\lambda)A - \gamma_{lb}] + m_B\gamma_{lb} \\
\dot{\gamma}_{hg} &= \nu\gamma_{lg} - \kappa(\lambda A - \gamma_{lg}) + m_B\sigma_{hn}(\bar{p}_{lg})\gamma_{lg} \\
\dot{\gamma}_{hn} &= \nu(1-\gamma_{hn}) - \kappa\gamma_{hn} - m_S\gamma_{hn}\tfrac{\gamma_{lb}+\sigma_{hn}(\bar{p}_{lg})\gamma_{lg}}{\gamma_S}
\end{aligned} \tag{15}$$

Note that whether hb agents participate or not affects the equilibrium masses only through the matching rates as an hb agent selling an asset to an hn agent leaves the masses of hb and hn agents unchanged. Using the equilibrium condition $m_S\gamma_B = m_B\gamma_S$, it is immediate to obtain the next lemma.

Lemma 5.1 *In equilibria E and H the following properties hold:*

1. *The measures of agents satisfy:*

$$\gamma_{lb} = \frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B} \qquad \gamma_{lg} = \frac{\kappa\lambda A}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B} \qquad \gamma_{hn} = \frac{\nu-\kappa A}{\kappa+\nu} + \gamma_{lb} + \gamma_{lg} \tag{16}$$

2. *The total measure of sellers is:*

$$\begin{aligned}
\gamma_S^E = &\quad \gamma_{lb} + \gamma_{lg} &&= \tfrac{\kappa(1-\lambda)A}{\kappa+\nu+m_B^E} + \tfrac{\kappa\lambda A}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B^E} \\
\gamma_S^H = &\quad \gamma_{hb} + \gamma_{lb} + \gamma_{lg} &&= (1-\lambda)A + \tfrac{\kappa\lambda A}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B^H}
\end{aligned} \tag{17}$$

It is important to notice that all the endogenous equilibrium masses depend on the matching rate m_B and in general it is different in equilibrium E and H. A higher matching rate decreases the equilibrium masses of low-valuation asset holders (lg and lb) and buyers (hn) because as they trade assets move from low-valuation to high-valuation agents. Hence, a higher intensity of trade decreases the measures agents on the two sides of the market. This occurs also in equilibrium H, albeit to a smaller extent as some trades occur between hb and hn agents, inducing no change in the masses of these two types of agents.

We now move to analyse the equilibrium value functions. In order to have expressions which cover both the E and H equilibria we introduce the indicator function $\mathbb{1}_{\{hb\in S\}}$ which equals one when hb agents sell their assets (equilibrium H) and zero otherwise. By Lemma A.4, the value function of a seller matched with a buyer is $W_{ji} = \sigma_{hn}(\bar{p}_{lg})(\bar{p}_{lg} + V_{jn})$. Hence, the value functions of asset holders are:

$$\begin{aligned}
rV_{hg} &= \delta_g + \kappa(V_{lg} - V_{hg}) \\
rV_{lg} &= \delta_g - x + \nu(V_{hg} - V_{lg}) + m_B(W_{lg} - V_{lg}) = \delta_g - x + \nu(V_{hg} - V_{lg}) \\
rV_{hb} &= \delta_b + \kappa(V_{lb} - V_{hb}) + m_B(W_{hb} - V_{hb})\mathbb{1}_{\{hb\in S\}} = \delta_b + \kappa(V_{lb} - V_{hb}) + m_B\sigma_{hn}(\bar{p}_{lg})(V_{lg} - V_{ln} + V_{hn} - V_{hb})\mathbb{1}_{\{hb\in S\}} \\
rV_{lb} &= \delta_b - x + \nu(V_{hb} - V_{lb}) + m_B(W_{lb} - V_{lb}) = \delta_b - x + \nu(V_{hb} - V_{lb}) + m_B\sigma_{hn}(\bar{p}_{lg})(V_{lg} - V_{lb})
\end{aligned} \tag{18}$$

The value functions of agents without assets are:

$$\begin{aligned}
rV_{hn} &= \kappa(V_{ln} - V_{hn}) + m_S(W_{hn} - V_{hn}) \\
rV_{ln} &= \nu(V_{hn} - V_{ln})
\end{aligned} \tag{19}$$

In the next Lemma we provide some useful expression that show what are the underlying economic forces that determine agents' values.

Lemma 5.2 *In equilibrium E and H equilibrium values satisfy these properties:*

1. *Holders of peaches have the following values in both equilibria:*

$$V_{hg}-V_{lg}=\frac{x}{\kappa+\nu+r} \qquad V_{lg}=\frac{\delta_g}{r}-\frac{x}{r}\frac{\kappa+r}{\kappa+\nu+r} \tag{20}$$

2. *Buyers' value is:*

$$V_{hn}=\frac{(\nu+r)m_S}{r(\kappa+\nu+r+m_S)}\left[\frac{\gamma_{lg}}{\gamma_S}V_{hg}+\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)V_{hb}-V_{lg}\right] \tag{21}$$

The first point in Lemma 5.2 reveals that peach holders have identical values in equilibrium E and H. This result follows from the bargaining protocol: being offered $\bar{p}_{lg}=V_{lg}-V_{ln}$ leaves lg agents indifferent between accepting the offer and keeping their asset; consequently, lg agents' value is equal to that under autarky, taking into account that they transition between high and low valuation over time. This result greatly improves the possibilities to compare outcomes between equilibria E and H.

The second point in Lemma 5.2 sheds light on the determinants of buyers' value. Their value depends on two main components: first, the expected value of acquiring an asset for a high valuation agent $\frac{\gamma_{lg}}{\gamma_S}V_{hg}+\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)V_{hb}$, which depends on the probability of obtaining a peach ($\frac{\gamma_{lg}}{\gamma_S}$), minus the price paid $\bar{p}_{lg}=V_{lg}-V_{ln}=V_{lg}-\frac{\nu}{\nu+r}V_{hn}$; second; a discounting term $\frac{(\nu+r)m_S}{r(\kappa+\nu+r+m_S)}$ which decreases with the discount rate r, the intensity at which a high-valuation buyer returns to the state of low valuation (κ), whilst it increases with the intensity at which an ln agent transits to the state of high valuation (ν) and, most importantly, with the rate at which buyers meet active sellers m_S. The value of being a buyer varies between equilibrium E and H because three endogenous variables take different values: (i) the share of active sellers with peaches $\frac{\gamma_{lg}}{\gamma_S}$; (ii) the value V_{hb} of being an hb agent; (iii) the matching rate m_S. In Section 6 we are going to prove that, whenever both equilibrium E and H exist, V_{hn} is higher in equilibrium E. Note that there is no *prima facie* reason to conclude this to be the case because the effect of the lower share of peaches offered in equilibrium H is (partially) counterbalanced by a higher matching rate for buyers m_S. This comparison is further complicated by the absence of an explicit expression for m_S and the fact that γ_{lg}/γ_S and V_{hb} also depend on m_S. A large part of our effort in solving the model is devoted to overcoming these difficulties.

The next lemma introduces the two relevant constraints that jointly characterize a semi-pooling equilibrium.

Lemma 5.3 *If peaches are traded, the equilibrium satisfies:*

1. *Buyers offer $\bar{p}_{lg}$, which is the case if and only if:*

$$\frac{\gamma_{lg}}{\gamma_S}(V_{hg}-V_{lg})+\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb}-V_{lg})\geq\frac{\gamma_{lb}}{\gamma_S}\frac{\kappa+\nu+r+m_S}{\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S}(V_{hb}-V_{lb}) \tag{22}$$

2. *In equilibrium hb agents do not participate in the market if and only if:*

$$V_{hb}-V_{lg}\geq\frac{\frac{\gamma_{lg}}{\gamma_S}m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}\frac{x}{\kappa+\nu+r} \tag{23}$$

The first condition is standard in the adverse selection literature: it requires the share of peaches to be high enough to provide buyers with incentives to offer a price which is accepted by both lemon and peach holders. Note that the term multiplying $(V_{hb} - V_{lb})$ captures the fact that the outside options of buyers and asset holders depend on their valuation states. If they were type-independent, as in the static benchmark model presented in Section B.3 of the Appendix, this term would be equal to 1. The second condition is novel and it determines whether *hb* agents participate in the market as active sellers.[29] In dynamic adverse selection models every asset holder is assumed to have a lower valuation than buyers. In other words, the maximum amount of trade possible is exogenous and coincides with the first-best volume of trade under complete information. In our setup this is no longer the case because *hb* agents would not trade under complete information but they might find it convenient to do so under asymmetric information. This endogenous market participation decision creates novel effects in terms of trade volume, prices and welfare. We are going to analyse all these implications in Section 6.

5.2.1 Pure strategy pooling equilibria

For analytical tractability, we continue by focusing on pure strategy equilibria, i.e. $\sigma_{hn}(\bar{p}_{lg}) = 1$ and $\sigma_{hb}(\bar{p}_{lg}) \in \{0,1\}$. From here on, we use equilibrium E and H to refer exclusively to pure strategy equilibria. In this subset of equilibria we are able to provide analytical results for the main variables of interest—price, volume of trade and matching intensities—to better highlight the main underlying economic forces at work in the model. We discuss mixed strategy equilibria in Section 5.2.2. The findings therein demonstrate that the main results of our analysis continue to hold when also taking into consideration the possibility of mixed strategy equilibria.

We begin by investigating the rate at which assets are traded. The following lemma demonstrates the properties of the matching intensities in equilibria E and H.

Lemma 5.4 *If peaches are traded the matching rate m_B^K, $K = E, H$, is unique and solves:*

$$m_B^K \gamma_S^K = \mu M \left(\frac{\nu - \frac{\kappa A m_B}{(\kappa+\nu+m_B)}}{\kappa+\nu}, \gamma_S^K \right) \tag{24}$$

Moreover, the matching rates have the following properties:

- *In equilibrium E, the matching rates m_B^E and m_S^E do not depend on λ for all matching technologies.*
- *In equilibrium H, $\frac{\mathrm{d}m_B}{\mathrm{d}\lambda} \geq 0$ and $\frac{\mathrm{d}m_S}{\mathrm{d}\lambda} < 0$. In particular:*
  - *For all competitive matching technologies, m_B^H is strictly increasing in λ. Furthermore, $m_B^H < m_B^E$, $m_S^H > m_S^E$ for every $\lambda \in (0,1)$ with $\lim_{\lambda\to 1} m_B^H = m_B^E$ and $\lim_{\lambda\to 1} m_S^H = m_S^E$.*
  - *For all non-competitive matching technologies, $m_B^H = m_B^E$. Moreover, $m_S^H > m_S^E$ for every $\lambda \in (0,1)$ and $\lim_{\lambda\to 1} m_S^H = m_S^E$.*

To understand why the matching intensities in equilibrium E are independent of λ, note that the measure of active sellers in equilibrium E is $\gamma_S^E = \frac{\kappa A}{\kappa+\nu+m_B^E}$, implying that equation (24) does not depend on λ. Intuitively, only low-valuation holders of both peaches and lemons participate in the market as

[29] Notice that the right-hand side of (23) reflects the difference in being without an asset for high- and low-valuation agents. If it were the case that $V_{hn} = V_{ln}$, this term would be equal to zero.

sellers, while high-valuation agents without assets participate as buyers. As a result, the share λ of peaches does not affect the total masses of buyers and sellers. In equilibrium H, on the contrary, all lemon holders participate as active sellers. Hence, the mass of sellers is $\gamma_S^H = (1-\lambda)A + \frac{\kappa\lambda A}{\kappa+\nu+m_B^H}$ and only hg agents do not wish to sell their assets. As a consequence, in this equilibrium a higher λ decreases the mass of active sellers because the share of hg agents increases. In turn, this change in the total mass of sellers may affect the matching intensities of buyers (m_S^H) and sellers (m_B^H) relative to equilibrium E. For any matching technology, buyers match more frequently with sellers, i.e. $m_S^H > m_S^E$, because the measure of sellers γ_S^H is higher than γ_S^E.[30] With a competitive matching technology, the larger measure of sellers leads to an individual seller waiting on average a longer time before trading ($m_B^H < m_B^E$). With a non-competitive matching technology sellers match at the same rate with buyers ($m_B^H = m_B^E$) because matching occurs among all agents, and not just by agents participating in platform. In other words, there is no congestion on sellers' side because there is no decision on whether to be matched, and no resulting impact on the sellers' matching rate.

The results in Lemmata 5.2–5.4 are crucial to characterize the conditions that allow equilibrium E and H to exist. The next two propositions formally state these conditions in terms of the primitives of the model, and we proceed by discussing their implications.

Proposition 5.2 *(Existence of equilibrium E)*
A stationary equilibrium in which only lb and lg asset holders are sellers exists if and only if:

1. $\phi \leq \phi^{*E}$ *where* $\phi^{*E} = \dfrac{r(\kappa+\nu+r+m_B^E)+\kappa m_B^E}{(\kappa+\nu+r+\lambda m_S^E)(\kappa+\nu+r+m_B^E)}$
2. $\lambda \geq \lambda^{*E}$ *where* $\lambda^{*E} \in (0,1)$ *is the unique value of* λ *that satisfies equation (22) with equality for the equilibrium values* V_{ji}^E *with* $j = h,l$ *and* $i = b,g,n$.

Proposition 5.3 *(Existence of equilibrium H)*
A stationary equilibrium in which lb, lg and hb asset holders are sellers exists if and only if:

1. $\phi \geq \phi^{*H}$ *where* $\phi^{*H} = \dfrac{r(\kappa+\nu+r+m_B^H)+\kappa m_B^H}{(\kappa+\nu+r+\frac{\gamma_{lg}^H}{\gamma_S^H} m_S^H)(\kappa+\nu+r+m_B^H)}$
2. $\lambda \geq \lambda^{*H}$ *where* $\lambda^{*H} \in (0,1)$ *is the unique value of* λ *that satisfies equation (22) with equality for the equilibrium values* V_{ji}^H *with* $j = h,l$ *and* $i = b,g,n$.

Both propositions introduce two conditions: the first is the hb agents participation condition while the second ensures that buyers prefer to offer $\bar{p}_{lg}$ rather than $\bar{p}_{lb}$. Given that the matching rates do not depend on ϕ, the first conditions show that hb agents are willing to sell their assets when gains from trade are low, i.e. x is small and by implication ϕ large, while they prefer to keep their assets when gains from trade are large. Intuitively, when x is small buyers offer a relatively high price and hb agents are willing to sell their assets as well. Differently from the static model presented in the Appendix, both the threshold values ϕ^{*E} and ϕ^{*H} are smaller than one. This stems from two features: (i) V_{lg} exceeds $\frac{\delta_g - x}{r}$, i.e. the value of being distressed forever; and (ii) being a buyer entails a positive expected payoff, i.e. $V_{hn} > V_{ln} > 0$. The former leads to a higher $\bar{p}_{lg}$ whereas the latter improves the attractiveness of becoming a buyer for hb agents. Lower threshold values for ϕ imply a more severe adverse selection problem than in the

[30] For any competitive matching technology, also the measure of buyers is higher in equilibrium H than E, which exerts a negative effect on m_S. However, the direct, positive effect of a larger number of sellers dominates.

static benchmark because *hb* agents have more incentives to participate in the market. Interestingly, this problem is more severe in economies with more pronounced search frictions.

The second conditions, requiring λ to be sufficiently high, are intuitive. As in the static benchmark, buyers are willing to offer the higher price $\bar{p}_{lg}$ if and only if the average quality of assets in the economy is sufficiently high. Otherwise, there are relatively many lemons on the market and, as a result, buyers prefer to offer the lower price $\bar{p}_{lb}$.

Before illustrating when both equilibria exist, we state how the threshold values defining the region of parameters in which each equilibrium exists vary with the parameters.

Lemma 5.5 *The threshold values λ^* and ϕ^* satisfy.*

1. *ϕ^{*E} is decreasing in λ.*

2. *λ^{*E} and λ^{*H} are increasing in ϕ.*

The fact that ϕ^{*E} is decreasing in λ reveals an important feature of our model, which constrains the parameter space in which equilibrium E exists. Namely, high-valuation lemon holders have a stronger incentive to participate in the market when the average quality of assets in the economy is higher. Indeed, the expected value of buying an asset is higher when there are fewer lemons for sale, as the latter enjoy an information rent. As a result, the value of being a buyer V_{hn}, as well as the difference $V_{hn} - V_{ln}$, is increasing in λ. For this reason, *hb* agents find it more attractive to become buyers when the average quality of assets increases. These two forces shape the relationship between λ and ϕ^{*E} When *hb* agents do not participate in the market; if *hb* agents offer their assets for sale and the matching technology is competitive, an additional force comes into play: sellers meet buyers at a higher intensity for larger values of λ. Due to this effect, the value of a high-valuation lemon holder is increasing in the average quality of assets, making them more willing to keep their asset. On the other hand, also in this case the value of being a buyer is increasing in λ. These two opposite effects determine whether ϕ^{*H} is decreasing or increasing in λ, and in turn this depends on the parameter values considered. On the other hand, for a non-competitive matching technology m_B^H is independent of λ and equal to m_B^E, and $\frac{\gamma_{lg}^H}{\gamma_S^H} m_S^H = \lambda m_S^E$ implies $\phi^{*H} = \phi^{*E}$. In words, in the absence of congestion externalities, the decision of a high-valuation lemon holder to participate in the market is independent of whether other *hb* agents participate or not.

The reason why λ^{*E} and λ^{*H} are increasing in ϕ is the following. A buyer offering $\bar{p}_{lg}$ captures all the gains from trade, which depend positively on x, when matched with a peach holder but has to concede information rents, which are increasing in $\delta_g - \delta_b$, to lemon holders. Therefore, when ϕ is higher, the value of λ at which a buyer is indifferent between offering $\bar{p}_{lg}$ and $\bar{p}_{lb}$ is higher.

In order to shed further light on the set of parameters for which equilibria E and H exist, we will illustrate three cases. In the first case, illustrated in Figure 2, matching takes place according to the DGP technology. As already pointed out, for this matching technology $\phi^{*E} = \phi^{*H}$. Thus, equilibrium E can exist below this curve and equilibrium H above it. When *hb* agents have the incentive to offer theirs assets for sale, the minimum λ^{*H} above which peach holders trade is higher than what would be if private valuation state were observable (λ^{*E}), i.e. when only low valuation asset holders could trade.

In the second and third case, illustrated in Figures 3 and 4, agents are matched according to the KW technology. In the economy considered in Figure 3, the positive effect of a higher matching rate m_B on V_{hb} is stronger than that of a higher λ on the value of being a buyer. As a consequence, ϕ^{*H} is increasing in λ and there is a set of (λ, ϕ) pairs for which both equilibrium E and H exist.

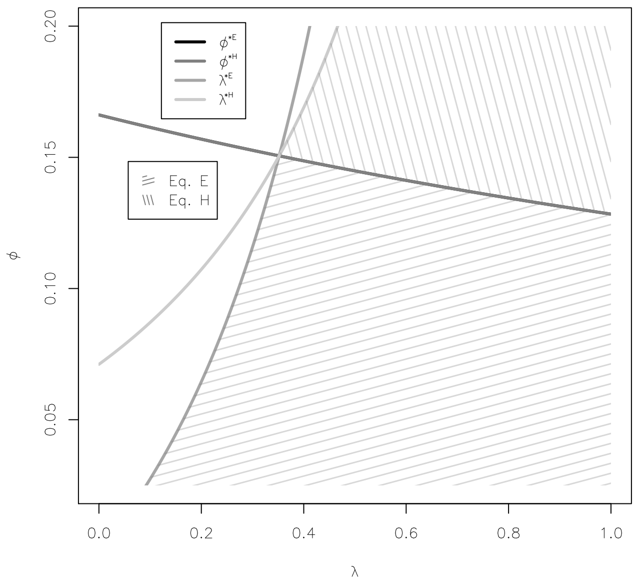

Figure 2: DGP technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 1$, $\mu = 2.5$ and $r = 0.05$.

In the economy of Figure 4, the positive effect of a higher λ on $V_{hn} - V_{ln}$ dominates that of the higher matching rate m_B on V_{hb}. For this reason, the ϕ^{*H} curve is downward sloping. In this case there exists a set of tuples (λ, ϕ) for which neither equilibria E nor H exists. However, as shown in the next section, in this region mixed strategy equilibria in which a fraction of *hb* agents participate exist.

5.2.2 Mixed strategy semi-pooling equilibria

In this section we discuss the existence and the properties of equilibria in mixed strategies. In this way we complete the picture of the possible equilibrium outcomes in our model. As we provide a more detailed treatment in Appendix B, here we focus on the main insights from that analysis.

By Lemma A.4, three possible types of mixed strategy equilibria exist: one in which buyers randomize between $\bar{p}_{lb}$ and $\bar{p}_{lg}$, one in which *hb* agents randomize between accepting or not an offer of $\bar{p}_{lg}$ and one in which both buyers and *hb* agents play mixed strategies. Table 1 lists the different types of mixed strategy equilibria and shows how we denote them.

Mixed strategy equilibrium	Strategy profiles	
M1	$\sigma_{hn}(\bar{p}_{lg}) \in (0,1)$	$\sigma_{hb}(\bar{p}_{lg}) = 0$
M2	$\sigma_{hn}(\bar{p}_{lg}) \in (0,1)$	$\sigma_{hb}(\bar{p}_{lg}) = 1$
M3	$\sigma_{hn}(\bar{p}_{lg}) = 1$	$\sigma_{hb}(\bar{p}_{lg}) \in (0,1)$
M4	$\sigma_{hn}(\bar{p}_{lg}) \in (0,1)$	$\sigma_{hb}(\bar{p}_{lg}) \in (0,1)$

Table 1: The strategy profiles of the agents employing mixed strategies in the mixed strategy equilibria.

Intuitively, for a given ϕ, the values of λ for which buyers are willing to randomize between the two prices is determined by the interplay of two forces. On the one hand, a decrease in $\sigma_{hn}(\bar{p}_{lg})$ entails a lower effective intensity $\sigma_{hn}(\bar{p}_{lg})m_B$ at which peaches are traded. On the other hand, the average quality of assets increases when $\sigma_{hn}(\bar{p}_{lg})$ falls as it implies that lemons trade faster than peaches. When agents

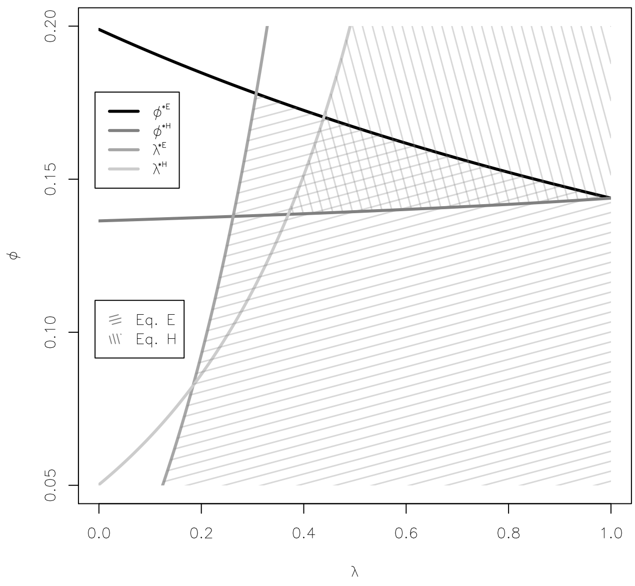

Figure 3: KW technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 5$ and $r = 0.05$.

discount less future payoffs, a buyer attaches relatively more weight to the former effect, as it has to do with the probability to trade a peach in the future. Therefore, for a given λ, buyers are less willing to offer the high price $\bar{p}_{lg}$ in a mixed than in a pure strategy equilibrium when r is sufficiently low. In such economies, mixed strategy equilibria exist for higher values of λ than the thresholds λ^{*E} and λ^{*H}.

Figures 5 and 6 illustrates the regions of the parameter space in which mixed strategy equilibria exist for the DGP and KW technologies, respectively. They show that for r low relative to κ and ν and when adverse selection is severe, mixed strategy equilibria do not enlarge the set of (λ, ϕ) pairs for which peaches can be traded. That is, above the ϕ^{*E} curve, where equilibrium E does not exist, mixed strategy equilibria exist for higher values of λ than equilibria H. This motivates our focus on the pure strategy equilibria L and H when due to severe adverse selection equilibrium E does not exist.[31]

5.3 Salient features of equilibria under asymmetric information

We close this section by discussing features of the equilibrium characterization which are unique to our setting. Let us first make a few additional remarks about the threshold ϕ^*, which determines whether or not high-valuation lemon holders are willing to participate as sellers. When ϕ is below the threshold, adverse selection is moderate as only low-valuation lemons holders are willing to sell their assets and for this reason the average quality of the assets on the market corresponds to that in the whole economy. When, on the contrary, $\phi \geq \phi^*$, the market suffers from severe adverse selection as all lemon holders, irrespective of their valuation state, are willing to sell their asset, worsening the average quality of assets on the market. In the case of severe adverse selection, the minimum average quality of assets in the economy to support a pooling equilibrium is higher than when adverse selection is moderate. The

[31] In Appendix B we show that for reasonable values of r, the necessary values of κ and ν which allow the existence of mixed strategy equilibria for lower values of λ than equilibria E and H are unreasonably low. Specifically, they would imply that agents' valuations change on average less frequently than every 50 years.

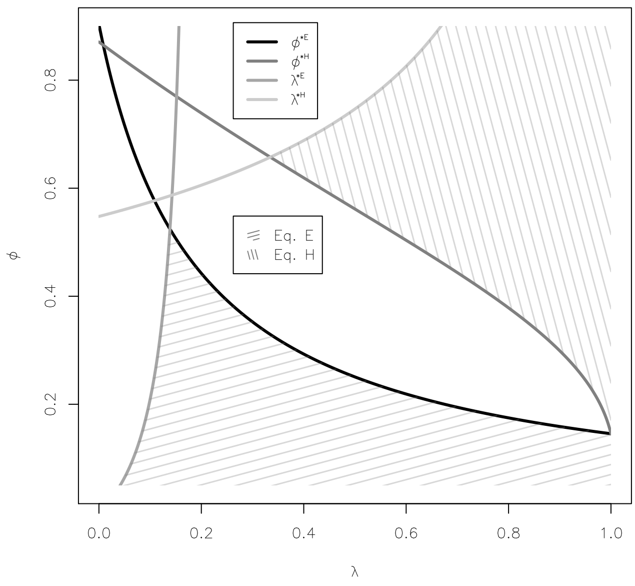

Figure 4: KW technology, parameter values: $A = 1$, $\kappa = 0.02$, $\nu = 0.02$, $\mu = 2.5$ and $r = 0.2$.

reason for this is that the participation constraint of *hb* agents $V_{hb} - (V_{lg} - V_{ln}) - V_{hn} \leq 0$ also determines when the surplus of a buyer from acquiring a lemon is negative. In other words, whenever adverse selection is severe, the pooling price $\bar{p}_{lg}$ exceeds the value of a lemon to a buyer. It is also worth pointing out once more the implications of the fact that ϕ^{*E} is decreasing in λ. This means that severe adverse selection arises for a larger set of parameter values in economies in which the average quality of assets in the economy is high. When adverse selection is severe, asymmetric information creates a double bind: either only lemons are traded or all lemons, irrespective of their holder's valuation state, are continuously offered for sale. As we will show in the next section, both outcomes are generically inefficient.

Another feature of equilibria worth pointing out is that decentralized trade does not mitigate the lemons problem as in a dynamic adverse selection model à la Moreno and Wooders (2010). Namely, in our model there is only limited scope to support equilibria in which assets of both qualities are traded by endogenously delaying the trade of peaches. In particular, when λ is low enough, no mixed strategy equilibrium with $\sigma_{hn}(\bar{p}_{lg}) < 1$—implying that the share of peaches on the market is above λ—exists. This is stated formally in the following proposition.

Proposition 5.4 *For λ sufficiently small, no equilibrium in which peaches are traded exists.*

In our model, V_{hb} always strictly exceeds V_{lb}. In particular, if V_{lb} increases due to buyers offering a higher price so does V_{hb} as agents transit between high and low valuation. For this reason, offering $\bar{p}_{lb}$ always yields a strictly positive surplus, making it impossible to construct, for any λ, an equilibrium in which buyers are indifferent between offering $\bar{p}_{lg}$ or $\bar{p}_{lg}$, and obtain with both a zero surplus. Hence, it is not possible to have trade delays for lemon holders, as they always trade once matched with a buyer.

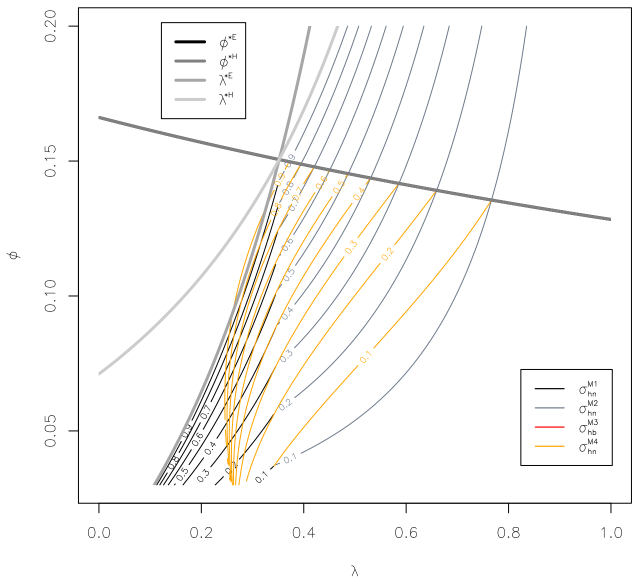

Figure 5: DGP technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 2.5$ and $r = 0.05$.

6 Equilibrium properties

6.1 Price, volume, time to sell and average quality of traded assets

We continue by investigating the properties of equilibria in terms of price, volume, time to sell and average quality of traded assets. First, we compare analytically these quantities across stationary equilibria. Then, we illustrate graphically the transitional dynamics of the four quantities as well as how they respond to changes in gains from trade and the average quality of assets in the economy.

Comparing the four quantities of interest in equilibria in which at least some *hb* agents participate, i.e. $\sigma_{hb}(\bar{p}_{lg}) > 0$, to those in equilibrium E, yields the following proposition.

Proposition 6.1 *(Prices, volume of trade and average quality of assets across equilibria)* *Consider parameter values such that equilibrium E and at least one equilibrium with $\sigma_{hb}(\bar{p}_{lg}) > 0$ and $\sigma_{hn}(\bar{p}_{lg}) = 1$ exist. Relative to equilibrium E, each of the latter features:*

1. *A higher price at which assets are traded, $\bar{p}_{lg}$.*

2. *A higher volume of trade, $M(\gamma_B, \gamma_S)$.*

3. *A longer expected time to sell, $1/m_B$.*

4. *A lower average quality of assets, γ_{lg}/γ_S.*

For all non-competitive matching technologies, $\bar{p}_{lg}$ and $1/m_B$ are equal to their values in equilibrium E.

To understand why the equilibrium price is higher when at least some high-valuation lemon holders participate in the market, it is useful to report the expression derived in the proof of the proposition for

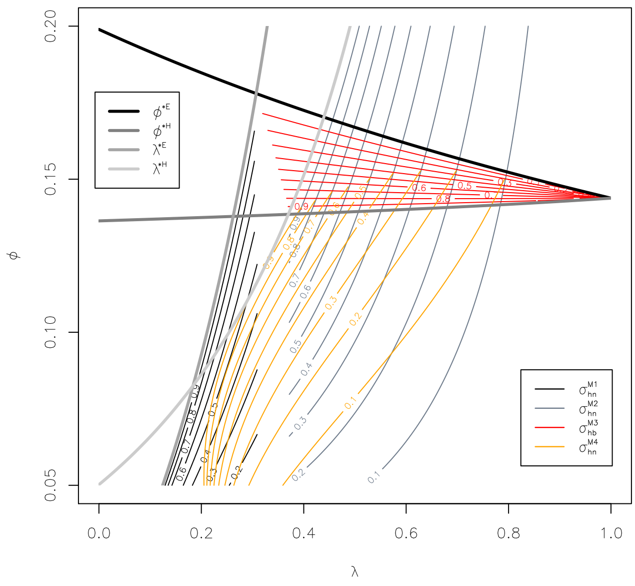

Figure 6: KW technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 5$ and $r = 0.05$.

the value of being a buyer:

$$V_{hn} = \frac{\nu + r}{r}\left[\frac{\frac{\gamma_{lg}}{\gamma_S}m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}(V_{hg}-V_{lg}) + \frac{\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}(\bar{p}_{hb}-\bar{p}_{lg})\right] \tag{25}$$

The second expression inside the square brackets shows that when *hb* agents are willing to sell their asset, i.e. $\bar{p}_{hb} \leq \bar{p}_{lg}$, a buyer obtains a negative surplus from acquiring a lemon at price $\bar{p}_{lg}$. In the opposite case, when *hb* agents prefer to keep their asset, a buyer obtains a positive surplus from acquiring both peaches and lemons at the pooling price. Due to the negative surplus from buying lemons, the value of being a buyer is lower when *hb* agents participate in the market. Consequently, low-valuation peach holders require a higher price as a compensation for the lower continuation value after selling their asset. For this reason, the equilibrium price is higher when *hb* agents participate in the market.

The ranking of the three other quantities is intuitive. The volume of trade is higher due to additional sellers in the market. For the same reason, an individual seller meets buyers at a lower intensity, rendering the time to sell longer. This leads to higher measures of agents in the economy, which further increases the volume of trade. Similarly, the lower average quality of traded assets is due to the presence of additional lemon holders in the pool of sellers.

Figures 7 and 8 show how the four quantities of our interest vary with λ in the three pure strategy equilibria, when they exist, for the DGP and KW technologies, respectively. Three observations are worth making. First, both in equilibrium E and H, the equilibrium price is decreasing in λ, while in equilibrium L it is increasing in λ. This is due to the fact the expected surplus captured by a buyer is increasing in the share of peaches when both assets are traded whereas it as decreasing in λ when buyers offer prices rejected by peach holders. Second, the average quality of traded assets is strictly lower and the volume of trade strictly higher in equilibrium H than in equilibrium E for both matching technologies. This stems from the higher measure of lemon holders participating in the market. Third,

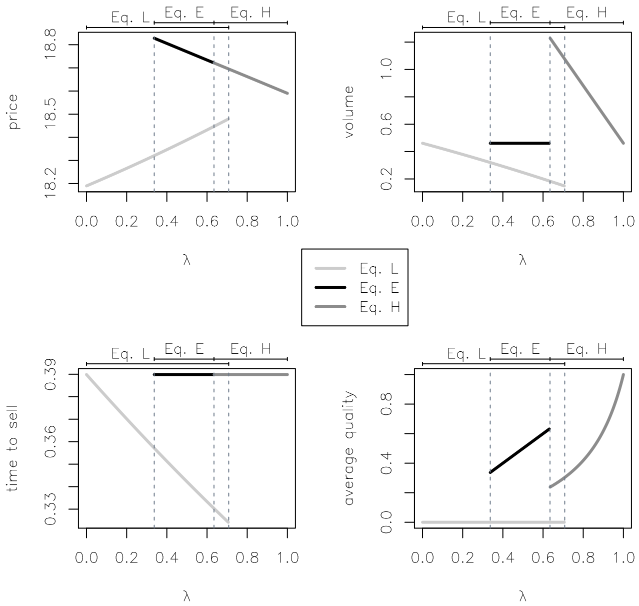

Figure 7: DGP technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 2.5$, $r = 0.05$, $\phi = 0.14$, $\delta_g = 1$ and $\delta_b = 0.98$.

for the KW technology, the expected time to sell is strictly higher in equilibrium H than in equilibrium E when both exist. The reason for this is the congestion externality that *hb* agents impose on all sellers, slowing the rate at which they are matched with buyers.

To gain further insight into how equilibrium quantities vary with the key parameters of the model and across equilibria, we carry out the following exercises. First, we illustrate how the equilibrium quantities respond to unanticipated changes in x and λ in equilibrium E and H, taking into consideration any transitional dynamics that such changes may induce. Second, we show the behaviour of the equilibrium quantities when the economy embarks on a transition from equilibrium E to H, with no change in the parameters. That is, we consider a transition that is due to a change in agents' beliefs. The exercises are conducted for the KW technology.

In Figure 9, we illustrate the effect of a gradually higher x, a measure of gains from trade, on equilibrium quantities. Until $t = 1$ the economy is in equilibrium H with $x = 3/30$. At $t = 1$ there is an unanticipated increase in x to $4/30$ but the economy remains in equilibrium H. Given that the matching rates do not depend on x, this change induces no transitional dynamics. The discrete decrease in the equilibrium price reflects the higher value of a buyer, capturing a part of the higher gains from trade. At $t = 2$, the economy embarks on a transition to equilibrium E due to a change in agents' beliefs. Note that this is possible as both equilibrium E and H exist for the parameter values under consideration (see Figure 3).[32] Given that the ratio of γ_{lg} to γ_{lb} in equilibrium H is equal to that in equilibrium E, the average quality of traded assets jumps immediately to its value in the limiting stationary equilibrium when *hb* agents exit the market. The volume of trade and the time to sell, on the other hand, are higher along the transition path than in the stationary equilibrium E. This stems from the fact that initially there are more low-valuation sellers in the market as previously their matching intensity was below that in equilibrium

[32] Naturally, we have checked that *hb* agents are not willing to participate and buyers prefer to offer $\bar{p}_{lg}$ rather than $\bar{p}_{lb}$ also along the transition path.

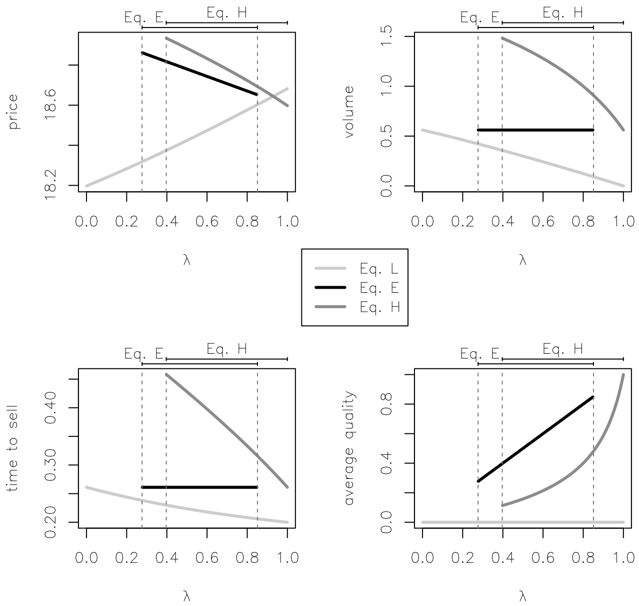

Figure 8: KW technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 5$, $r = 0.05$, $\phi = 0.15$, $\delta_g = 1$ and $\delta_b = 0.98$.

E. Due to the initially higher measure of sellers, the rate at which an individual buyer meets sellers is decreasing along the transition path. For this reason, the value of being a buyer is decreasing and the equilibrium price increasing along the transition path.[33] At $t = 3$, there is a further unanticipated increase in x to $5/30$. This induces a further decrease in the equilibrium price, attributable to an increase in the surplus captured by a buyer.

Figure 10 shows the results of an exercise in which the average quality of assets is gradually decreased.[34] At $t = 1$, there is an unanticipated decrease in λ from 0.9 to 0.6. Although the economy remains in equilibrium H, this induces transitional dynamics as the matching rates depend on λ. In particular, the initial measure of low-valuation sellers is lower than that in the limiting stationary equilibrium due to the previously higher m_B. As a result, the initial volume of trade is lower. The initially higher time to sell, on the other hand, is due to the initial measure of buyers being lower than in the limiting stationary equilibrium. Along the transition path, the average quality of traded assets improves as more low-valuation peach holders accumulate in the market. At $t = 2$, the economy embarks on a transition path from equilibrium H to E. The transitional dynamics share the same characteristics as in the previous exercise. At $t = 3$, λ further decreases to 0.3. In this case, equilibrium quantities do not display any transitional dynamics as in equilibrium E m_B does not depend on the average quality of assets in the economy.

The results of the two exercises can be summed up as follows. An increase in distress, as measured by x, leads to a decrease in the equilibrium price as distressed sellers are willing to sell their assets at a lower price. The decrease is amplified if the increase in x is associated with the economy moving from equilibrium H to E. However, in this case, time to sell goes down and the average quality of assets

[33] The increase in the equilibrium price along the transition path is relatively small and for this reason cannot be seen in the figure.

[34] More specifically, a randomly chosen set of peaches turns into lemons.

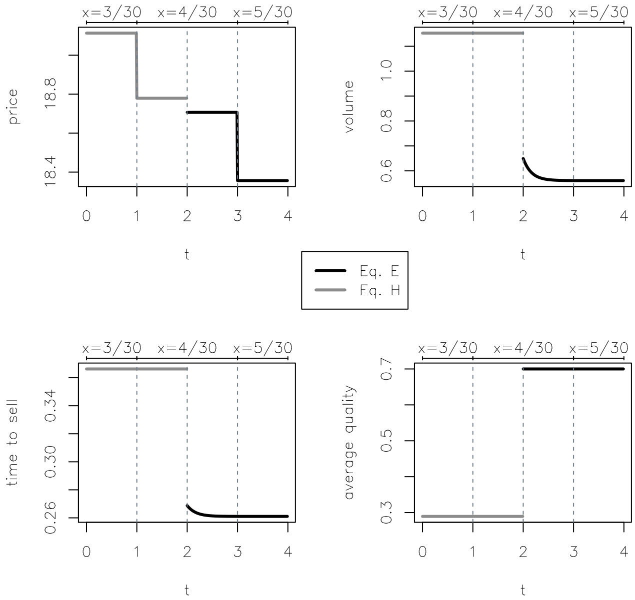

Figure 9: KW technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 5$, $r = 0.05$, $\lambda = 0.7$, $\delta_g = 1$ and $\delta_b = 0.98$.

on the market improves as high-valuation owners of lemons no longer find it convenient to participate as sellers. A decrease in the average quality of assets in the economy, on the other hand, affects the equilibrium quantities in a non-monotonic fashion. Namely, if a decrease in λ is associated with the economy moving to equilibrium E, the direct, positive effect of a lower λ on the equilibrium price can be at least partly offset by the lower price in equilibrium E. Similarly, the average quality of traded assets behaves non-monotonically when λ decreases, decreasing due to the direct effect and increasing due to the transition to equilibrium E.

6.2 Utilitarian welfare across equilibria

We turn to investigate how equilibria compare in terms of utilitarian social welfare. For any strategy profile $\sigma = \{\sigma_{ji}\}_{j=\{h,l\}}^{i=\{b,g,n\}}$, the utilitarian welfare value $W(\sigma)$ is the weighted sum of the corresponding value functions, say $V_{ji}(\sigma)$, using as weights the agents' masses $\gamma_{ji}(\sigma)$:

$$W(\sigma) = \sum_{ji} \gamma_{ji}(\sigma) V_{ji}(\sigma) \tag{26}$$

The next Lemma provides an explicit expression for the welfare values.

Lemma 6.1 *Consider an admissible strategy profile σ and the corresponding masses $\gamma_{ij}(\sigma)$. The utilitarian social welfare value is equal to:*

$$W(\sigma) = \frac{A}{r}\left[\lambda \delta_g + (1-\lambda)\delta_b\right] - \frac{x}{r}\left[\gamma_{lg}(\sigma) + \gamma_{lb}(\sigma)\right] \tag{27}$$

The utilitarian welfare expression point out that strategies profiles σ can be evaluated from a utilitarian welfare perspective by comparing the masses of low-valuation asset holders in the economy. The

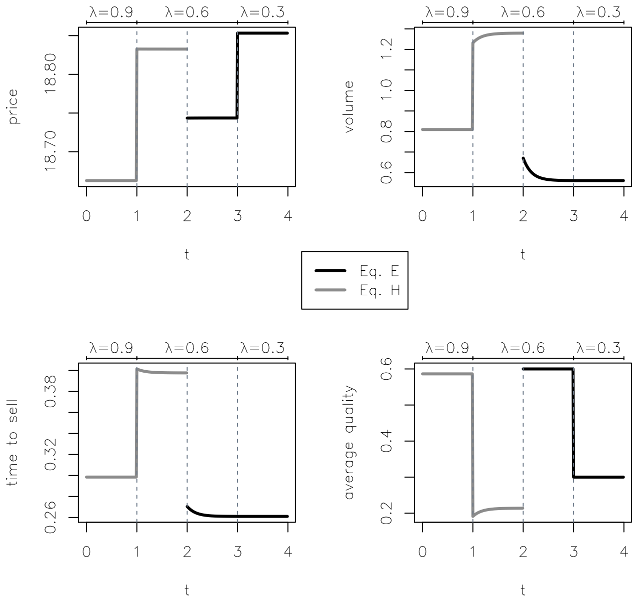

Figure 10: KW technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 5$, $r = 0.05$, $x = 4/30$, $\delta_g = 1$ and $\delta_b = 0.98$.

lower the masses, the higher the welfare because fewer assets are in the hands of agents that value them the least. In turn, the equilibrium masses depend on the strategy profile σ both directly and through the potential impact on the matching rate m_B. The effect on m_B substantially complicates the analysis, and it is not even straightforward to determine whether the complete information benchmark maximizes utilitarian welfare. In principle, it may be optimal to have buyers offer prices rejected with positive probability by peach holders, or even to restrict market participation of low-valuation asset holders. In the next proposition we show that this is never the case because a strategy profile maximizes utilitarian social welfare only if all low-valuation asset holders participate in the market and trade as soon as they match with a buyer. For this purpose, we introduce the following notation for the probability that asset holder ji trades when matched with a buyer:

$$q^T_{ji,z}(\sigma) = \int \sigma_{zn}(p|\mathscr{I})\sigma_{ji}(p)\mathrm{d}p \qquad j,z = \{h,l\} \quad i = \{b,g\} \tag{28}$$

where $\sigma_{zn}(p|\mathscr{I})$ is the probability that an agent zn offers price p given his information set $\mathscr{I}$ on the asset quality of his matched agent: with complete information $\mathscr{I}$ is equal to the true asset quality, while he has no information under incomplete information, hence his strategies are identical irrespective of the matched asset holder.

Proposition 6.2 *The first-best level of utilitarian social welfare is equal to the one in the complete information equilibrium:*

$$W(\sigma^C) = \frac{A}{r}[\lambda\delta_g + (1-\lambda)\delta_b] - \frac{x}{r}\frac{\kappa A}{\kappa + \nu + m_B^C} \tag{29}$$

where m_B^C solves equation (12).
A strategy profile σ satisfies $W(\sigma) = W(\sigma^C)$ if and only if the following conditions hold:

- $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ *constant in γ_B, γ_S (e.g. DGP technology):*

$$q^B_{hn} \sum_{i=b,g} q^S_{li} q^T_{li,h} = 1 \qquad q^B_{ln} \sum_{i=b,g} q^S_{hi} q^T_{hi,l} = 0 \tag{30}$$

- $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ *strictly decreasing in* γ_B, γ_S *(e.g. KW technology):*

$$q^B_{hn} \sum_{i=b,g} q^S_{li} q^T_{li,h} = 1 \qquad q^B_{ln} + \sum_{i=b,g} q^S_{hi} = 0 \tag{31}$$

Proposition 6.2 has straightforward implications about the welfare properties for the equilibria analysed in the previous sections. The next corollary provides an overview of the results.

Corollary 6.1 *The following statements hold:*

1. *For all admissible matching technologies, the complete information equilibrium (Proposition 3.1) and equilibrium E (Proposition 5.2) attain the first-best welfare level* $W(\sigma^C)$.
2. *Equilibrium H in Proposition 5.3 entails a welfare loss equal to:*

$$W(\sigma^C) - W(\sigma^H) = \frac{\kappa A x}{r}\left[\frac{1}{\kappa+\nu+m^H_B} - \frac{1}{\kappa+\nu+m^C_B}\right] \tag{32}$$

   where $W(\sigma^C) - W(\sigma^H)$ *is strictly positive and decreasing in* λ *if and only if* $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ *is strictly decreasing in* γ_B, γ_S.

3. *For all admissible matching technologies, the lemons market equilibrium in Proposition 5.1 entails a welfare loss equal to:*

$$W(\sigma^C) - W(\sigma^L) = \frac{\kappa A x}{r}\left[\frac{\lambda}{\kappa+\nu} + \frac{(1-\lambda)}{\kappa+\nu+m^L_B} - \frac{1}{\kappa+\nu+m^C_B}\right] \tag{33}$$

   where $W(\sigma^C) - W(\sigma^L)$ *is strictly positive and increasing in* λ.

The corollary points out an important feature of equilibrium H, in which also high-valuation lemon holders participate in the market. Namely, for any competitive matching technology, such an equilibrium is inefficient. This is due to the congestion externality that *hb* agents impose on all sellers: by slowing down trade, a larger number of assets are held by low-valuation agents, leading to a welfare loss relative to the first-best outcome.

The welfare loss in the lemons market equilibrium, on the other hand, simply stems from the fact that peaches are not traded at all and for that reason end up being held by low-valuation agents until they transit back to the state of high-valuation.

7 Market design interventions

The characterization of the equilibria in Section 5 and the results in 6.2 point out that under asymmetric information the first-best outcome cannot be implemented for a substantial set of parameters. In this section we explore whether and to which extent a market designer can implement the first-best outcome relative to the decentralized equilibria. In Section 7.1 we discuss mechanisms in which the designer can set the terms of bilateral trades; in Section 7.2 this possibility is precluded.

7.1 Designer sets the bilateral terms of trade

We first consider the possibility that the designer sets, directly or indirectly, the terms of trade at which agents can exchange assets. We further distinguish between two possible classes of mechanisms. In the first class, the designer implements a stationary equilibrium such that the first-best outcome is obtained by reallocating the existing assets among agents—subject to the same search frictions of the decentralized economy—through a budget-balanced system of transfers. In the second class, the designer—still subject to the search frictions—first acquires the existing assets from their holders, and then sells a newly created asset that combines the existing assets in a 'package' which is not affected by an asymmetric information problem about its asset quality. Then, we compare the two mechanisms in terms of their effectiveness in enlarging the region of parameters for which the first-best outcome is implementable. Lastly, we discuss the possible practical implementation of this policy and some related issues.

7.1.1 Transfer scheme for existing assets

In the first class of mechanisms the designer implements a stationary equilibrium through a mechanism $\mathscr{M}$ offered to all agents. At every point in time an agent can report a message to the designer: for each message the mechanism associates a flow transfer, an intensity at which agents sell or buy the asset—if the agent holds an asset or not, respectively—and a transfer conditional on trade. Importantly, we assume the designer cannot record what agents reported in their past but only their last message.[35]

Recall that in our notation agents hold a type $\tilde{b} \in \tilde{B} = \{hn, ln\}$ if buyers or $\tilde{s} \in \tilde{S} = \{hb, lb, hg, lg\}$ if sellers; in general, let $\theta \in \tilde{B} \cup \tilde{S}$ be a generic type. Types are bi-dimensional as they include both a private information on the asset quality (b, g or n) and whether the agent holds a high (h) or low (l) valuation for assets. By the direct revelation principle of Myerson (1981), we can restrict attention to direct mechanisms in which all agents report directly their type, and we denote by θ' the reported type. The designer promises a mechanism $\mathscr{M} := \{\alpha(\theta), t_T(\theta), t_N(\theta)\}$. If the agent reports type θ' then the quantity $\alpha(\theta')$ denotes the intensity rate at which the agent is going to trade: if $\theta' \in \tilde{S}$ an agent sells his asset to the designer, while if $\theta' \in \tilde{B}$ the agent receives an asset. We assume the designer observes whether an agent holds or not an asset, i.e. $\theta' \in \tilde{S}$ for $\theta \in \tilde{S}$ and $\theta' \in \tilde{B}$ for $\theta \in \tilde{B}$. Moreover, transfer $t_T(\theta')$ applies upon trade while $t_N(\theta')$ is a flow transfer received until the agent trades. Transfers are positive (negative) if the designer pays (receives) the consumption good to the agent. Therefore, we can express the value functions corresponding to a mechanism $\mathscr{M} = \{\alpha(\theta), t_T(\theta), t_N(\theta)\}$ as follows:

$$\begin{aligned} rV(\theta', \theta = ji, \mathscr{M}) &= \delta_i - x\mathbb{1}_{\{j=l\}} + \nu(V_{hi} - V_{li})\mathbb{1}_{\{j=l\}} + \kappa(V_{li} - V_{hi})\mathbb{1}_{\{j=h\}} + \alpha(\theta')\left[t_T(\theta') - V_{ji} + V_{jn}\right] + t_N(\theta') \\ rV(\theta', \theta = jn, \mathscr{M}) &= \nu(V_{hn} - V_{ln})\mathbb{1}_{\{j=l\}} + \kappa(V_{ln} - V_{hn})\mathbb{1}_{\{j=h\}} + \alpha(\theta')\left[\mathbb{E}[V_{ji}] + t_T(\theta') - V_{jn}\right] + t_N(\theta') \end{aligned} \tag{34}$$

where $\mathbb{E}[V_{ji}]$ is the expected value of holding an asset for a $jn \in \tilde{B}$ agent, given his expectation over the average quality of assets received by the designer from sellers in $\tilde{S}$. We assume the designer does not observe the quality of the assets, but simply assigns to buyers one asset at random from the ones received from sellers. Without loss of generality we set $t_T(\theta') = 0$ if $\alpha(\theta') = 0$.

The mechanism designer is subject to the same matching frictions of the decentralized equilibrium. To formally take into account these constraints, we introduce some notation similar to the one in Section

[35] If this opportunity would be feasible, it might be possible to implement the first-best outcome for a larger set of parameters, but we would crucially depart from the anonymity assumption imposed in the decentralized equilibrium with bilateral trades.

2.2. Let:

$$S(\mathscr{M}) = \left\{ \bigcup_{\tilde{s}\in\tilde{S}} \tilde{s} : \alpha(\tilde{s}) > 0 \right\} \qquad B(\mathscr{M}) = \left\{ \bigcup_{\tilde{b}\in\tilde{B}} \tilde{b} : \alpha(\tilde{b}) > 0 \right\} \tag{35}$$

Denote by $\gamma_S(\mathscr{M})$ and $\gamma_B(\mathscr{M})$ the measure of agents in $S(\mathscr{M})$ and $B(\mathscr{M})$, respectively. Given the matching technology available, the measure of matches between agents in $S(\mathscr{M})$ and $B(\mathscr{M})$ is equal to $M(\gamma_B(\mathscr{M}), \gamma_S(\mathscr{M}))$, and we can define the matching rates $m_B(\mathscr{M})$ and $m_S(\mathscr{M})$ as the solution to $m_S(\mathscr{M})\gamma_B(\mathscr{M}) = m_B(\mathscr{M})\gamma_S(\mathscr{M}) = \mu M(\gamma_B(\mathscr{M}), \gamma_S(\mathscr{M}))$. Therefore, the designer is subject to the constraints:

$$\alpha(\theta) \leq m_S(\mathscr{M}) \quad \text{for} \quad \theta \in \tilde{B} \qquad \alpha(\theta) \leq m_B(\mathscr{M}) \quad \text{for} \quad \theta \in \tilde{S} \tag{36}$$

In order to be feasible, we require the mechanism to be budget-balanced on the equilibrium path. In turn, in a stationary equilibrium this requirement implies the following constraint on the instantaneous flow of total transfers:

$$\sum_{\theta\in\tilde{S}\cup\tilde{B}} \gamma_\theta [t_N(\theta) + \alpha(\theta) t_T(\theta)] \leq 0 \tag{37}$$

Finally, the mechanism must satisfy incentive compatibility (IC) and individual rationality (IR) for each type θ, i.e.:

$$\begin{aligned} &\text{IC:} \quad V(\theta,\theta,\mathscr{M}) \geq V(\theta',\theta,\mathscr{M}) && \forall\theta \in \tilde{S}\cup\tilde{B} \\ &\text{IR:} \quad V(\theta,\theta,\mathscr{M}) \geq \frac{1}{r}\left(\delta_i - \frac{\kappa + r\mathbb{1}_{\{j=l\}}}{\kappa+\nu+r}x\right) && \forall\theta = ji \in \tilde{S} \quad V(\theta,\theta,\mathscr{M}) \geq 0 \ \forall\theta \in \tilde{B} \end{aligned} \tag{38}$$

where the reservation value for sellers is equal to the expected utility from holding the asset forever.

Our goal is to characterize for which set of parameters it is possible to implement the first-best outcome subject to the constraints in equations (36)–(38). In the first-best allocation the rate at which low-valuation asset holders trade is equal to the matching rate m_B^C under no asymmetric information on asset quality (Proposition 6.2), i.e. $\alpha(lg) = \alpha(lb) = m_B^C$, and similarly hn agent trade at rate $\alpha(hn) = m_S^C$; for all other θ we have $\alpha(\theta) = 0$.

The next proposition characterizes the main result of this section.

Proposition 7.1 *A mechanism $\mathscr{M} = \{\alpha(\theta), t_T(\theta), t_N(\theta)\}$ implements a first-best outcome under constraints (36)–(38) if and only if:*

$$\phi \leq \frac{r(\kappa+\nu+r+m_B^C) + \kappa m_B^C}{(\kappa+\nu+r)(\kappa+\nu+r+m_B^C)} \tag{39}$$

If inequality (39) holds, the first-best outcome can always be implemented if the transfer scheme satisfies:

$$t_N(lg) + m_B^C t_T(lg) = t_N(lb) + m_B^C t_T(lb) \tag{40}$$

$$t_N(h) = t_N(hi) = \tfrac{r}{r+m_B^C}\left\{t_N(li) + m_B^C\left[t_T(li) - \tfrac{1}{r}\left(\delta_g + t_N(ln) - \tfrac{\kappa+r}{\kappa+\nu+r}x\right)\right]\right\} \geq 0 \tag{41}$$

$$t_N(ln) = \tfrac{r}{r+m_S^C}\left\{t_N(hn) + m_S^C\left[t_T(hn) + \tfrac{1}{r}\left(\delta_g + t_N(h) - \tfrac{\kappa}{\kappa+\nu+r}x - (1-\lambda)\tfrac{r(\kappa+\nu+r+m_B^C)}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g - \delta_b)\right)\right]\right\} \geq 0 \tag{42}$$

$$\mu M(\gamma_B^C, \gamma_S^C)\left[\tfrac{x}{\kappa+\nu+r} - (1-\lambda)\tfrac{\kappa+\nu+r+m_B^C}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g - \delta_b)\right] \geq A t_N(h) + t_N(ln) \tag{43}$$

The conditions in (41)–(43) are necessary if equation (39) is satisfied with equality.

It is worth making two observations about the characterization. First, given that $m_B^C = m_B^E$, the threshold value of ϕ in (39) strictly exceeds the largest value of ϕ for which equilibrium E exists, i.e. ϕ^{*E} in Proposition 5.2. Thus, the mechanism under consideration enlarges the set of (λ, ϕ) pairs for which the first-best outcome can be implemented. Second, the reason for which the mechanism can implement the first-best outcome in regions of the parameter space in which *hb* agents are willing to participate in the decentralized economy is that the mechanism compresses the difference between V_{hn} and V_{ln}. As a result, *hb* agents are less willing to participate in the market than in the decentralized economy.

The next corollary follows immediately from Proposition 7.1.

Corollary 7.1 *If inequality (39) holds, the mechanism $\mathscr{M}^*$:*

$$\begin{aligned}
t_N^*(\theta) &= 0 \quad \forall\theta \qquad \alpha(lg) = \alpha(lb) = m_B^C \qquad \alpha(hn) = m_S^C \qquad \alpha(\theta) = 0 \quad o.w. \\
t_T^*(li) &= \frac{1}{r}\left(\delta_g - \frac{\kappa+r}{\kappa+\nu+r}x\right) = V(lg, lg, \mathscr{M}^*) \qquad i = b, g \\
t_T^*(hn) &= -t_T^*(li) - \frac{x}{\kappa+\nu+r} + \frac{\kappa+\nu+r+m_B^C}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(1-\lambda)(\delta_g - \delta_b) \\
&= -[\lambda V(hg, hg, \mathscr{M}^*) + (1-\lambda)V(hb, hb, \mathscr{M}^*)] = -\mathbb{E}[V_{hi}]
\end{aligned} \tag{44}$$

maximizes the revenues for the market designer conditional on implementing the first-best outcome.

In the revenue-maximizing reallocation scheme the market designer captures the whole surplus from trade. That is, in addition to low-valuation peach holders being indifferent between selling and keeping their asset, buyers obtain a surplus of zero. Consequently, $V_{ln} = V_{hn} = 0$, and *hb* agents have weaker incentives to sell their assets than in the decentralized economy.

7.1.2 Packaging schemes

We proceed by considering an alternative mechanism which consists of the designer gradually acquiring all the assets in the economy, and for each asset acquired issuing a certificate yielding a flow dividend equal to that of an average asset held by the designer. From the moment in which the mechanism is put in place, buyers can no longer buy the original assets but only the certificates issued by the designer. Therefore, original assets can either be sold to the designer or kept forever (autarky).[36] For newly created certificates, the only source of private information in bilateral negotiations pertains to agents' valuation states, and certificates can be exchanged subject to the same search frictions of the market for the original assets, but at terms of trade dictated by the designer.

We consider two possible schemes to acquire all the assets in the economy. In the first one, say slow packaging (SP), the designer offers to buy original assets at a price which is only accepted by low-valuation asset holders, while all high-valuation asset holders prefer to keep their assets until they transit to the low-valuation state. In the second one, say fast packaging (FP), the designer acquires assets at a price accepted by all asset holders. In both schemes, the total measure of certificates issued is equal to A and the average quality of the assets held by the designer at all times is equal to the one in the economy.[37]

We assume the designer can commit to make a single take-it-or-leave-it offer to each asset holder. That is, if an asset holder rejects the designer's offer, the former cannot sell his asset to the latter in the

[36] Alternatively, we could assume that the the terms of trade for the original assets are set to be such that all gains from trade accrue to the designer. This would ensure that agents continuing to trade existing assets would obtain their autarky values.

[37] If the designer offered a price accepted only by low-valuation lemon holders or by all low-valuation asset holders and high-valuation lemon holders, the share of peaches held by the designer would, at least temporarily, differ from λ. For simplicity we do not consider such non-stationary schemes, and restrict attention to the two stationary schemes outlined above.

future. It turns out that the designer commits to such a single take-it-or-leave-it offer in a FP mechanism, while he prefers not to do so in a SP one. In the former scheme committing to a single offer ensures the outside option of a peach holder is equal to his autarky value.[38] In the latter scheme instead the designer does not commit to a single offer because he wishes to acquire assets only from low-valuation asset holders, irrespective of whether agents previously received an offer while they were in the high-valuation state.[39]

A certificate changes hands only when there are gains from trade, meaning that trade takes place between low-valuation certificate holders and *hn* agents. As in the previous section, we characterize stationary schemes, in which the matching intensities are constant over time. Since the certificates are traded between low-valuation certificate holders and *hn* agents, sellers' and buyers' matching intensities are equal to m_B^C and m_S^C, respectively. In the asset acquisition phase, the search frictions determine the rate at which the designer acquires the original assets and issues certificates. However, we do not model this rate because our results only depend on the rate at which certificates trade. Another requirement underlying our mechanisms is that the designer encounters fewer asset holders than buyers. We justify this assumption on the grounds that when $\nu \geq \kappa A$ the measure of buyers exceeds that of low-valuation asset holders. This ensures that the designer meets fewer sellers than buyers in a SP scheme. Similarly, in a FP scheme the measure of buyers exceeds that of sellers as the designer can simultaneously buy an asset from and sell a certificate to the same agent when encountering a high-valuation asset holder.

We solve for mechanisms which maximize the designer's profits, conditional on the resulting allocation attaining the first-best welfare level. In this way we obtain the largest set of parameter values for which the mechanisms under consideration are feasible. Given that the rate at which trade takes place is constant, we can solve the problem of the designer asset by asset. To implement a first-best outcome, the terms of trade set by the designer have to be such that only low-valuation certificate holders are willing to sell their certificate and only *hn* agents are willing to buy one. Denoting the prices at which certificates are sold and bought by $p_T(hn)$ and $p_T(ln)$, respectively, the present value of profits Π from each certificate accruing to the designer solves:[40]

$$
\begin{aligned}
r\Pi_h &= \kappa(\Pi_l - \Pi_h) \\
r\Pi_l &= \nu(\Pi_h - \Pi_l) + m_B(p_T(hn) - p_T(lc) + \Pi_h - \Pi_l),
\end{aligned}
\tag{45}
$$

where lc refers to a low-valuation agent with a certificate while Π_l and Π_h denote the value of the designer's profits when the certificate is in the hands of a low-valuation and a high-valuation agent, respectively. Solving for Π_h, we obtain:

$$
\Pi_h = \frac{\kappa m_B}{r(\kappa + \nu + r + m_B)}[p_T(hn) - p_T(lc)] \tag{46}
$$

[38] Recall that in a fast packaging scheme the designer acquires all assets at the reservation price of a high-valuation peach holder, so every other agent type enjoys a strictly positive payoff from accepting this offer. Suppose on the contrary that an asset holder, even after rejecting an offer in the high-valuation state, could receive the same designer's offer in the future. Since low-valuation peach holders are strictly better off accepting than rejecting the offer, this surplus from trade would also increase the expected utility of high-valuation asset holders. As a consequence, the reservation price of high-valuation peach holders would be above their autarky value, and the designer would end up paying a higher price to acquire the assets. If instead the designer can commit to make a single take-it-or-leave-it offer, the value of a high-valuation peach holder matched with the designer offering p_A is $W_{hn} = \max\{p_A + V_{hn}, V_{hg}^a\}$, where V_{hg}^a denotes the autarky value of an hg agent.

[39] In contrast to a fast packaging scheme, in a slow one the designer offers the reservation price of low-valuation peach holders. As a result, the previous argument in favour of a single take-it-or-leave-it offer does not hold.

[40] Note that differently from the previous section we are expressing the designer's profits in terms of prices rather than transfers.

When matched with an asset holder and a buyer, the problem of the designer is:

$$\begin{aligned}
&\max_{p_A, p_T}\{-p_A + p_T(hn) + \Pi_h\} \\
&\text{subject to} \quad p_A + \hat{V}_{jn} \geq \frac{1}{r}\left(\delta_g - \frac{\kappa + r\mathbb{1}_{\{j=l\}}}{\kappa+\nu+r}x\right) \\
&\qquad\qquad\quad \hat{V}_{hc} - p_T(hn) \geq 0 \\
&\qquad\qquad\quad \hat{V}_{ln} + p_T(lc) \geq \frac{1}{r}\left(\hat{\delta} - \frac{\kappa+r}{\kappa+\nu+r}x\right)
\end{aligned} \tag{47}$$

where $j = l$ in a SP scheme, $j = h$ in a FP scheme and $\hat{\delta}$ denotes the flow dividend of a certificate. $\hat{V}_{ji}$ denote agents' values when certificates are traded. In words, the designer acquires the asset at price p_A, issues a certificate, sells it at price $p_T(hn)$ and the value of the future profits from the certificate is equal to Π_h. The first constraint is the IR constraint of an asset holder, the second the IR constraint of a buyer and the third the IR constraint of a certificate holder. Note that we are supposing that the IC constraints do not bind, which will be shown to be true in a revenue-maximizing mechanism. Solving the designer's problem yields the following proposition.

Proposition 7.2 *The following statements hold for packaging schemes:*

1. *A revenue-maximizing packaging scheme is characterized by:*

$$p_A = \frac{1}{r}\left(\delta_g - \frac{\kappa + r\mathbb{1}_{\{j=l\}}}{\kappa+\nu+r}x\right) \qquad p_T(hn) = \frac{1}{r}\left(\hat{\delta} - \frac{\kappa}{\kappa+\nu+r}x\right) \qquad p_T(lc) = \frac{1}{r}\left(\hat{\delta} - \frac{\kappa+r}{\kappa+\nu+r}x\right) \tag{48}$$

2. *Slow packaging is feasible if and only if:*

$$\phi \leq \frac{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}{(\kappa+\nu+r)(\kappa+\nu+r+m_B^C)} \tag{49}$$

3. *Fast packaging is feasible if and only if:*

$$(1-\lambda)\phi \leq \frac{\kappa m_B^C}{(\kappa+\nu+r)(\kappa+\nu+r+m_B^C)} \tag{50}$$

4. *When both slow and fast packaging are feasible, slow packaging yields higher profits than fast packaging.*

The first part of the proposition reveals that in a revenue-maximizing packaging scheme both buyers and certificate holders are kept at their outside option values. In this way, the designer maximizes revenue from the trade in the certificates. The designer offers a price equal to the autarky value of holding a peach to convince asset holders of both qualities to sell. Alternatively, the designer could set the terms of trade for certificates to be such that certificate buyers would obtain a positive surplus, and pay a lower price for the original assets. However, this is not optimal for the following reason: selling a certificate at a price below the outside option of a buyer entails an immediate cost for the designer in the form of a lower revenue; at the same time, the reservation price of an asset holder decreases by less than one-to-one as the seller benefits from the lower price of certificates only in the future. A similar reasoning applies to the price offered to a seller of certificates. As a consequence, it is optimal to maximize the revenues from the trades in certificates, although this implies that the designer has to acquire all assets at a higher price.

As regards the feasibility condition of a SP scheme, this coincides with the feasibility condition found in the Section 7.1.1, applying to mechanisms in which the original assets continue to circulate. This is due to the fact that both revenue-maximizing mechanisms implement the first-best outcome by setting $V_{hn} = V_{ln}$ while ensuring that *hb* agents prefer to keep their assets.

The condition for a FP scheme to yield a non-negative profit is intuitive. Since in a FP scheme the designer acquires all assets at a 'premium', such a scheme yields positive profits only when the average quality of assets in the economy is sufficiently high. Otherwise, the initial loss incurred due to acquiring lemons at the high price is larger than the present value of future profits from the certificate trades. Finally, the fact that a SP scheme yields higher profits than a FP scheme is due to the lower price at which the assets are acquired.

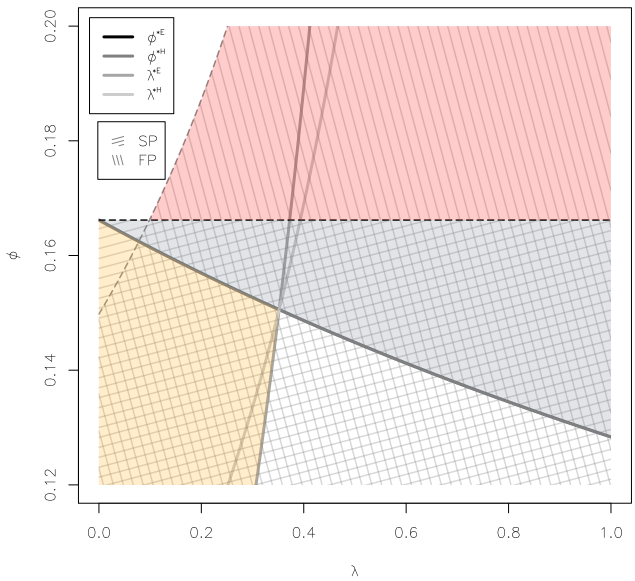

Figure 11: DGP technology, parameter values: $A = 1$, $\kappa = 1$, $\nu = 2$, $\mu = 2.5$, $r = 0.05$.

Figure 11 illustrates the feasibility of the two types of packaging schemes. In the yellow area, a packaging scheme improves efficiency as equilibrium E does not exist even though adverse selection is moderate. Either a SP or a FP scheme can restore a first-best outcome even in regions of the parameter space where the outcome in the decentralized economy is always inefficient due to severe adverse selection (the grey and the red areas). This is because for the parameter values considered certificates change hands relatively frequently, and a high volume of trade implies higher revenues for the designer. As a result, the market designer is able to recoup any initial losses from acquiring the original assets at a 'premium' by controlling the secondary market for certificates.

To conclude our discussion of market interventions in which the designer can dictate the terms of trade, we point out that an intervention involving packaging existing assets has a wider scope to implement the first-best outcome than a reallocation scheme for existing assets. Issuing certificates all of which yield the same dividend flow removes a source of asymmetric information—heterogeneous asset quality—from the economy. Given that bi-dimensional private information amplifies adverse selection in the decentralized economy, the first-best outcome is easier to implement in an economy in which only homogeneous assets circulate.

7.1.3 Implementation issues

The previous mechanisms rely on the possibility for the designer to alter the bilateral terms of trade among agents. Both when a mechanism applies to existing assets and to certificates, it is crucial to impose a wedge between the price at which agents buy and sell. In a market with asymmetric information on asset quality, this wedge mitigates the mimicking incentives of non-distressed lemon holders. In the secondary market for certificates asymmetric information is no longer an issue; nonetheless, imposing a wedge is the most effective way to raise revenues from trades. Indeed, unless sellers have all the bargaining power in bilateral negotiations, an initial fee when issuing the certificates raises a lower amount of revenues (in present value terms). In this sense, imposing a fee for each transaction alleviates an inefficiency due to bargaining. This will become clearer from the results in Section 7.2. To practically implement a price differential, two main types of intervention are possible: (i) trade intermediation; (ii) a financial transaction tax.

Market intermediation of all trades requires to directly set the terms of trade—by charging a bid-ask spread—for all the transactions taking place in the secondary markets. This implementation strategy presupposes that the designer controls all the market venues through which market participants can trade. For the case of the packaging schemes, this implies control of both the primary issuance market for certificates and their secondary market. This strategy has been implemented by several peer-to-peer (P2P) platforms that control both the primary and the secondary markets for their originated loans.[41] Selling on the secondary market platforms for P2P loans often requires paying a fee or getting a discount relative to the price paid by new buyers. Although the way P2P secondary markets work may provide practical insights for the design of other markets, such as the one for NPLs, it is disputable whether this option is actually feasible and cost-efficient. Indeed, it may be legally impracticable to restrict trade to one platform, in turn hindering the possibilities to recoup costs. Moreover, the fixed cost for the creation of a new trading platform may be substantial, further reducing the viability of such an intervention.

A financial transaction tax (FTT) can help to overcome the main hurdles previously pointed out for setting up a single market venue to trade all certificates. In Europe FTTs are already in place in some countries (Belgium, France, Greece and Italy), and there is an ongoing discussion on a proposal to harmonize FTT legislation among ten European countries.[42] The main challenge for an effective tax collection from a FTT comes from the lack of an uniform application of the tax among countries, asset classes and investors. The need for exemptions has been justified based on the potential negative liquidity effects on market participants,[43] while FTT proponents point out a beneficial composition effect

[41] FinTech lending companies originate (directly or through a third-party) loans that are bought by an initial set of investors. To improve the attractiveness of loans to investors, P2P lending companies have been developing internal secondary markets for their loans that allow investors to sell their portfolio before maturity; see for example 'Banking without banks', *The Economist*, March 1^{st}, 2014. Interestingly, some platforms allow individual loans to be traded, while others only allow to invest and divest in a portfolio of all platform loans outstanding; for a detailed overview of P2P lending platforms see the blog piece "Where can you buy or sell existing loans?" on 4thway.co.uk.

[42] European Commission, 2013."Proposal for a Council Directive Implementing Enhanced Cooperation in the Area of Financial Transaction Tax."

[43] The empirical evidence find mixed evidence on the liquidity effects; see Becchetti et al. (2014), Capelle-Blancard and Havrylchyk (2015), Coelho (2016), Cappelletti et al. (2016) and Colliard and Hoffmann (2017) for evidence on the recent implementation of FTTs in France and Italy.

arising from a more limited participation by non-fundamental traders.[44] In the same spirit, our results advocate introducing a FTT based on the fact that it limits market participation of non-distressed lemon holders who trade just to take advantage of their information rent and not because of asset valuation shocks. In the secondary market for certificates there is no asymmetric information, but the FTT is still optimal from an ex ante perspective as it increases the present value of future tax revenues. In turn, higher tax revenues better allow to cover the initial losses incurred during the initial asset acquisition, when all lemons and peaches are acquired at the price of peaches.

7.2 Designer cannot affect the bilateral terms of trade

The interventions characterized so far presuppose that the marker designer can impose a wedge between the price paid by buyers and the one received by sellers—e.g. a bid-ask spread or a tax—for both the original assets and the issued certificates. In practice this may not always be possible, and we now examine under which conditions a packaging scheme can be feasible when terms of trade are dictated by the market. This alternative assumption has two implications: first, the market designer may no longer be able to earn a profit from each secondary market transaction of certificates; second, when acquiring the original assets, the designer has to take into account the possibility that existing assets can continue to be traded in the private market.

To model this market constraint we consider the following protocol. Once a buyer and a seller meet, both the designer and the buyer make take-it-or-leave-it offers to the seller to obtain his asset. As in the previous packaging schemes, the designer can commit to make a single take-it-or-leave-it offer to each asset holder. In other words, if a seller rejects the designer's offer, the former will not receive any further offer from the designer for his asset. If the buyer's offer is rejected by the seller, he can still buy a certificate issued by the designer. As previously, the certificate offers a known dividend flow. The price of the issued certificate, on the contrary, is not set by the designer. Instead, the designer is constrained to sell certificates at the same price at which they are traded in the secondary market, in which private buyers and sellers engage in Nash bargaining to determine the terms of trade. This constraint can be motivated by the fact that now the designer has to compete with private sellers of certificates.[45] In this way, we also avoid the possible non-stationarities that would arise from private buyers finding it more convenient to reject the designer's offer when the measure of private certificate sellers increases over time.

We still focus on stationary schemes which implement the first-best outcome and maximize the designer's profits. Moreover, we restrict attention to fast packaging schemes for the following reasons. Suppose that the designer introduces a slow packaging scheme when terms of trade for both the original assets and the certificates are dictated by the market. If certificate buyers have at least some bargaining power, the price at which certificates are sold is such that buyers obtain a positive surplus. As a consequence, also the difference $\hat{V}_{hn} - \hat{V}_{ln}$ is strictly positive. As demonstrated in Section 7.1, this implies that high-valuation lemon holders have a stronger incentive to sell their assets than when the designer

[44] See Tobin (1978) and Stiglitz (1989) were early proponents of FTT to curb excessive volatility. The recent theoretical contribution by Berentsen et al. (2016) and Davila (2016) argue that social welfare would improve in the presence of a positive FTT.

[45] It should be pointed out that, due to search frictions, buyers may be willing to acquire a certificate from the designer at a higher price than that resulting from Nash bargaining with a private seller. Buyers' willingness to do so is due to an analogous economic mechanism as in Diamond (1971). However, to better highlight the limitations arising from not being able to freely set the terms of trade, we assume that the designer acts as a price-taker in the market for certificates.

captures all the gains from trade, in which case $\hat{V}_{hn} = \hat{V}_{ln} = 0$. Thus, a slow packaging scheme with market-determined terms of trade would be feasible for a smaller set of parameter values than that characterized in Proposition 7.2. Furthermore, in a slow packaging scheme, the designer, offering a price accepted only by low-valuation asset sellers, would face stronger competition from private sellers than in a fast packaging scheme, in which the designer offers a higher price accepted also by high-valuation asset holders. Therefore, the designer would have to offer a price above the reservation price of low-valuation peach holders, which would further strengthen the incentives of high-valuation lemon holders to sell their assets to the designer.

Formally, the designer's problem is:

$$\begin{aligned}
&\max_{p_A}\{-p_A+p_C\} \\
&\text{subject to} \quad p_A+\hat{V}_{hn} \geq \max\left\{p+\hat{V}_{hn}, \frac{1}{r}\left(\delta_g - \frac{\kappa}{\kappa+\nu+r}x\right)\right\} \\
&\qquad\qquad p = \arg\max\left\{\pi_{hn}(g,p)(V_{hg}-p)+\pi_{hn}(b,p)(V_{hb}-p)+\left[1-\pi_{hn}(g,p)-\pi_{hn}(b,p)\right](\hat{V}_{hc}-p_C)\right\}
\end{aligned} \tag{51}$$

where p denotes the price offered by a private buyer, p_A that offered by the designer and p_C the certificate price determined by Nash bargaining in the secondary market. A few comments are in order. First, sellers' outside options are no longer equal to their values under autarky. Rather they are determined by the terms of trade in the private asset market. Second, the price of the certificate issued by the designer is no longer a choice variable because he acts as a price-taker in the market for certificates. Finally, a private buyer, taking the strategy of the designer as given, offers a price that maximizes his expected utility. The strategy of the designer influences the probability that a buyer offering p obtains an asset of type-i, $\pi_{hn}(i,p)$. Given that we are interested in the feasibility of a packaging scheme, we focus on mechanisms in which all sellers accept the designer's offer. In this class of schemes, we obtain the following result.

Proposition 7.3 *With market-determined outside options and certificate prices set by Nash bargaining, a fast packaging scheme is feasible if and only if*

$$\begin{aligned}
(1-\lambda)\phi \leq\, & \frac{\beta\kappa m_B^C}{(\kappa+\nu+r)\left[\kappa+\nu+r+(1-\beta)m_S^C+\beta m_B^C\right]} \\
& - \frac{(1-\beta)r\left[\kappa+\nu+r+(1-\beta)m_S^C\right]\left[\kappa+\nu+r+\beta m_B^C-(1-\beta)m_S^C\right]}{(\kappa+\nu+r)(\kappa+\nu+r+\beta m_B^C)\left[\kappa+\nu+r+(1-\beta)m_S^C+\beta m_B^C\right]}
\end{aligned} \tag{52}$$

The feasibility of a packaging scheme in which the designer no longer sets the terms of trade in the secondary market for the certificates depends crucially on the relative bargaining power of sellers. When a seller of a certificate has all the bargaining power ($\beta = 1$), the designer essentially sells not only a certificate but also all the surplus from its future trade to the buyer. For this reason, the feasibility condition in this case coincides with that when the designer earns the maximum profit from all future trades of the certificate. On the other hand, when a buyer of a certificate has all the bargaining power ($\beta = 0$), the designer captures no surplus and has to sell the certificates at a lower price than that paid to asset holders, always making a loss.

To sum up, a packaging scheme can be budget-balanced even when the market designer does not control the secondary market for the issued certificates. However, feasibility requires that the bargaining power of a certificate seller, be it the designer or a private agent, is sufficiently high. This ensures that the designer, when issuing the certificates, captures a large share of the surplus from trade which private sellers obtain when certificates change hands in the future.

8 Conclusions

We have shown that, in a model of over-the-counter markets, private information about both the private (holding costs) and common value (asset quality) components amplifies the adverse selection problem due to privately known asset quality. Namely, private information about asset holding costs further constrains the parameter region in which an efficient market outcome can emerge. This is because of endogenous market participation; high-valuation holders of low-quality assets have an incentive to sell when either gains from trade are small, implying a high equilibrium price, or the average quality of assets in the economy is high, entailing that buyers capture a large surplus. The participation of high-valuation asset holders as sellers creates an inefficiency because they have the same asset valuation as buyers—hence no gains from trade exist—and their presence in the market potentially slows down trade due a congestion effect. High-valuation holders of lemons have a stronger incentives to offer their assets for sale when the share of peaches is high, as this improves the chances of exchanging their lemons for peaches.

When the economy suffer from severe adverse selection problem—i.e. when also high-valuation lemon holders choose to participate in the market—the resulting inefficiency can manifest itself in two starkly different ways. Either high-quality assets are not traded at all because of a classic market breakdown or the equilibrium exhibits excessive trade. In the first case, due to all lemons being on the market, a pooling equilibrium cannot exist as buyers do not find it convenient to offer the reservation price of peach holders. Instead, buyers offer low prices at which only low-valuation lemon holders are willing to sell. The equilibrium is inefficient as not all mutually beneficial trades take place. In the second case, although all lemon holders participate as sellers, the average quality of the assets offered for sale is sufficiently high, and buyers find it convenient to offer a high price accepted by all sellers. Such an equilibrium, featuring lemons being traded excessively, is inefficient whenever the matching technology exhibits congestion effects. It is worth mentioning that the two possible outcomes are markedly different in terms of observables. In the first case, the volume of trade and the market price are low, but time to sell is short; in the second case, the volume of trade and the market price are high, but the market is less liquid as it takes longer for sellers to find a buyer.

We have characterized two budget-balanced interventions which can implement the first-best outcome subject to the search frictions of the decentralized economy. The first intervention entails setting the terms of trade of the existing assets, and it can restore the first-best outcome in a region of parameters where the decentralized economy suffers from severe adverse selection. By reducing the surplus accruing to buyers, the intervention discourages high-valuation lemon holders from participating in the market. The second intervention consists of gradually buying the existing assets in the economy, and simultaneously issuing certificates backed by the whole pool of assets acquired. Acquiring assets at a premium to market price and earning a profit from trades in the secondary market for certificates, such an intervention can restore the first-best outcome for a set of economies for which the first type of intervention fails. The packaging scheme eliminates asymmetric information on asset quality—the certificates issued by the market designer are of homogeneous quality—and at the same time renders irrelevant the presence of private information on agents' valuations states. Such a scheme can be budget-balanced even when the designer cannot influence the terms of trade in the secondary market for the certificates as long as the designer's bargaining power in the primary market is strong.

Appendix A

A.1 Auxiliary results

We first make use of the Hamilton-Jabobi-Bellman equations to establish a few auxiliary results that hold in every stationary equilibrium.

Lemma A.1 *For $i \in \{b, g\}$, $V_{hi} > V_{li}$ and $V_{hn} \geq V_{ln}$.*

Proof of Lemma A.1.
On the contrary, suppose $V_{li} \geq V_{hi}$. Combining (8) and (9) yields

$$(\kappa + \nu + r)(V_{hi} - V_{li}) = x + m_B \left[(W_{hi} - V_{hi}) - (W_{li} - V_{li}) \right]. \tag{53}$$

if $V_{li} \geq V_{hi}$ then equation (53) implies $W_{hi} - V_{hi} < W_{li} - V_{li}$ as x and m_B are strictly positive. Since

$$W_{ji} - V_{ji} = \int \max\{p + V_{jn} - V_{ji}, 0\} \, \mathrm{d}F(p), \tag{54}$$

it must be the case that $V_{hn} - V_{hi} < V_{ln} - V_{li}$. Hence the following must hold:

$$V_{ln} - V_{hn} > V_{li} - V_{hi} \geq 0 \tag{55}$$

From equations (10) and (11) we have:

$$(\kappa + \nu + r)(V_{hn} - V_{ln}) = m_S \left[(W_{hn} - V_{hn}) - (W_{ln} - V_{ln}) \right], \tag{56}$$

where

$$W_{jn} - V_{jn} = \max_p \left\{ \pi_{jh}(g,p)(V_{jg} - p - V_{jn}) + \pi_{jh}(b,p)(V_{jb} - p - V_{jn}) \right\}. \tag{57}$$

By (55) it holds that $V_{ln} - V_{hn} > 0$, hence equation (56) implies

$$W_{hn} - V_{hn} < W_{ln} - V_{ln} \quad \Rightarrow \quad W_{ln} - W_{hn} > V_{ln} - V_{hn} > 0 \tag{58}$$

Let p^*_{ln} an optimal price offered by ln agents. Then, equations (55) and (58) together with the equilibrium condition $\pi_{hn} = \pi_{ln}$ lead to the following implications:

$$\begin{aligned}
0 &\leq V_{li} - V_{hi} < V_{ln} - V_{hn} < W_{ln} - W_{hn} \\
&= \max_p \left\{ \pi_{ln}(g,p)[V_{lg} - p] + \pi_{ln}(b,p)[V_{lb} - p] \right\} - \max_p \left\{ \pi_{hn}(g,p)[V_{hg} - p] + \pi_{hn}(b,p)[V_{hb} - p] \right\} \\
&\leq \pi_{hn}(g, p^*_{ln})[V_{lg} - V_{hg}] + \pi_{hn}(b, p^*_{ln})[V_{lb} - V_{hb}] \leq \max\left\{ V_{lg} - V_{hg}, V_{lb} - V_{hb} \right\}
\end{aligned} \tag{59}$$

This is a contradiction as the inequality cannot hold for the asset i with the highest value $V_{li} - V_{hi}$.

Now, suppose that $V_{ln} > V_{hn}$. Given that $V_{hi} > V_{li}$ for $i \in \{b, g\}$, it follows from (57) that $W_{ln} - V_{ln} \leq W_{hn} - V_{hn}$. Thus, by (56), $V_{hn} \geq V_{ln}$, a contradiction. Hence, it has been proved that $V_{hn} \geq V_{ln}$. ■

Lemma A.2 *For $j \in \{h,l\}$, $V_{jg} > V_{jb}$.*

Proof of Lemma A.2.
Consider a high-valuation agent holding an asset. From (8), it follows that

$$(\kappa + r)(V_{hg} - V_{hb}) = \delta_g - \delta_b + \kappa(V_{lg} - V_{lb}) + m_B\left[(W_{hg} - V_{hg}) - (W_{hb} - V_{hb})\right], \tag{60}$$

where

$$W_{hi} - V_{hi} = \int \max\{p + V_{hn} - V_{hi}, 0\}\, \mathrm{d}F(p). \tag{61}$$

Now, suppose that $V_{hb} \geq V_{hg}$. Then, from (61), $W_{hg} - V_{hg} \geq W_{hb} - V_{hb}$. Thus, for (60) to hold, it has to be that $V_{lg} < V_{lb}$. From (9), one obtains

$$(\nu + r)(V_{lg} - V_{lb}) = \delta_g - \delta_b + \nu(V_{hg} - V_{hb}) + m_B\left[(W_{lg} - V_{lg}) - (W_{lb} - V_{lb})\right], \tag{62}$$

where the term multiplied by m_B is positive by equation (54) and $V_{lg} < V_{lb}$. Thus, for both (60) and (62) to hold, the following inequalities have to be satisfied

$$r(V_{hg} - V_{hb}) + \kappa\left[(V_{hg} - V_{hb}) - (V_{lg} - V_{lb})\right] > 0 \tag{63}$$

$$r(V_{lg} - V_{lb}) + \nu\left[(V_{lg} - V_{lb}) - (V_{hg} - V_{hb})\right] > 0. \tag{64}$$

Given that $V_{hg} - V_{hb} \leq 0$ and $V_{lg} - V_{lb} < 0$, this would require

$$V_{hg} - V_{hb} > V_{lg} - V_{lb} > V_{hg} - V_{hb}, \tag{65}$$

a contradiction. Hence, it follows that $V_{hg} > V_{hb}$. A symmetric argument can be employed to establish that $V_{lg} > V_{lb}$. ■

Lemma A.3 *It is always a best response for ln agents not to trade. It is a strict best response if lemons and peaches are traded in the market.*

Proof of Lemma A.3.
From (5) and Lemma A.1, the lowest price at which a peach can be bought, $\bar{p}_{lg}$ satisfies

$$\bar{p}_{lg} + V_{ln} = V_{lg}. \tag{66}$$

Substituting this price into (7) yields

$$W_{ln} = \pi_{ln}(g, \bar{p}_{lg})V_{ln} + \pi_{ln}(b, \bar{p}_{lg})\left[V_{ln} - (V_{lg} - V_{lb})\right] + \left[1 - \pi_{ln}(g, \bar{p}_{lg}) - \pi_{ln}(b, \bar{p}_{lg})\right]V_{ln} \tag{67}$$

By Lemma A.2, $V_{lg} > V_{lb}$. Thus, it follows that $W_{ln} < V_{ln}$ as $\pi_{ln}(b, \bar{p}_{lg)} > 0$ because lemons trade at this price ($\bar{p}_{lg} > \bar{p}_{lb}$). That is, a low-valuation agent without an asset is worse off trading at the lowest price at which a peach can be acquired. The lowest price at which a lemon can be bought, on the other hand, is equal to:

$$\bar{p}_{lb} = V_{lb} - V_{ln}. \tag{68}$$

Given that $V_{lg} > V_{lb}$, this price is rejected by sellers of peaches, implying that $\pi_{ln}(g, \bar{p}_{lg}) = 0$. Thus, one obtains

$$W_{ln} = \pi_{ln}(b, \bar{p}_{lb})V_{ln} + [1 - \pi_{ln}(b, \bar{p}_{lb})]\, V_{ln}. \tag{69}$$

Consequently, a low-valuation agent without an asset is at least as well off not trading as trading. ■

Lemma A.4 *Buyers offer $\bar{p}_{lg} := V_{lg} - V_{ln}$ and/or $\bar{p}_{lb} := V_{lb} - V_{ln}$. In equilibrium, hg agents never sell their assets while active sellers accept prices greater or equal to their reservation price with probability one.*

Proof of Lemma A.4.
By Lemma A.3 equations (10) and (11) become:

$$\begin{aligned} V_{ln} =& \frac{\nu}{\nu + r} V_{hn} \\ V_{hn} =& \frac{m_S}{r + \frac{\kappa r}{\nu + r} + m_S} \max_p \{\pi_{hn}(g, p)[V_{hg} - p] + \pi_{hn}(b, p)[V_{hb} - p] + [1 - \pi_{hn}(g, p) - \pi_{hn}(b, p)]V_{hn}\} \end{aligned} \tag{70}$$

Step 1. In equilibrium buyers do not offer prices rejected with probability one.
Suppose per contra buyers offer a price rejected with probability one by all sellers. Then, equation (70) implies $V_{hn} = 0$. However, consider a deviation:

$$\tilde{p}_{lb} = V_{lb} - V_{ln} + \varepsilon = \bar{p}_{lb} + \varepsilon \tag{71}$$

As lb agents always participate in the market and accept $\tilde{p}_{lb}$, then $\pi_{hn}(b, \tilde{p}_{lb}) > 0$; hence buyers' expected payoff would be:

$$\pi_{hn}(b, \tilde{p}_{lb})\, [V_{hb} - V_{lb} + V_{ln} - \varepsilon] = \pi_{hn}(b, \tilde{p}_{lb})\, [V_{hb} - V_{lb} - \varepsilon] \tag{72}$$

since $V_{hn} = 0$. The deviation is profitable for ε sufficiently small because, by Lemma A.1, $V_{hb} - V_{lb} > 0$. Hence, buyers offer no price rejected with probability one.

Step 2. No buyer offers $p \geq \bar{p}_{hg}$.
First, notice that Lemma A.2 implies $\bar{p}_{jg} = V_{jg} - V_{jn} > V_{jb} - V_{jn} = \bar{p}_{jb}$. Moreover, it also holds that $\bar{p}_{hg} = V_{hg} - V_{hn} > V_{lg} - V_{ln} = \bar{p}_{lg}$, or equivalently $V_{hg} - V_{lg} > V_{hn} - V_{ln}$. Suppose per contra the last inequality does not hold. In equilibrium, if peaches are traded then buyers find it optimal to offer $p \geq \bar{p}_{lg}$. Let W_{hn} be the expected payoff from offering p. Then,

$$W_{hn} = \pi_{hn}(g, p)[V_{hg} - p] + \pi_{hn}(b, p)[V_{hb} - p] + [1 - \pi_{hn}(g, p) - \pi_{hn}(g, p)]V_{hn} \leq V_{hg} - p \leq V_{hg} - \bar{p}_{hg} = V_{hn} \tag{73}$$

However, if $W_{hn} \leq V_{hn}$ then $V_{hn} = 0$ and, in turn, by equation (70) it follows that $V_{ln} = 0$. As a result, by Lemma A.1, the following contradiction obtains: $0 = V_{hn} - V_{ln} \geq V_{hg} - V_{lg} > 0$. Hence, it must be $\bar{p}_{hg} > \bar{p}_{lg}$. Therefore, all asset holders are willing to sell at $\bar{p}_{hg}$. The expected gain for a buyer from offering $\bar{p}_{hg}$ is zero only if he receives a peach for sure. Thus, a buyer incurs a strictly negative expected payoff because he always receives a lemon with positive probability ($\pi_{hn}(b, \bar{p}_{hg}) > 0$). Hence, $\bar{p}_{hg}$ is never offered and high-valuation holders of peaches never trade. If offering $\bar{p}_{hg}$ is not convenient it is also suboptimal to offer any price above it.

Step 3. No buyer offers $\bar{p}_{hb}$.
Let's distinguish two cases:

Case 1. Let $\bar{p}_{lg} > \bar{p}_{hb}$ and suppose per contra a buyer offers $\bar{p}_{hb}$. Such an offer is only accepted by lemon holders. If the buyer is an *ln* agents any price above $\bar{p}_{lb} < \bar{p}_{hb}$[46] leads to a negative expected gain from the trade. An *hn* buyer makes an expected net gain of zero if he trades at $\bar{p}_{hb}$. Consider a deviation $p \in [\bar{p}_{lb}, \bar{p}_{hb})$. If an *hn* agent matches with a *hb* seller, the latter does not accept so the net gain is zero; if he matches with a *lb*-seller, the latter accepts because $p \geq \bar{p}_{lb}$ and the *hn* agent makes a strictly positive net gain from the trade, hence the deviation is profitable.

Case 2. Let $\bar{p}_{hb} > \bar{p}_{lg}$ and suppose per contra a buyer offers $\bar{p}_{hb}$. Consider a deviation $p \in [\bar{p}_{lg}, \bar{p}_{hb})$. If a buyer matches with an *hb* agent he does not trade as $p < \bar{p}_{hb}$, but the former's expected payoff is at most zero as there is no expected gain in acquiring a lemon at $\bar{p}_{hb}$. If a buyer matches with a *lg* or *lb* seller, both accept $p < \bar{p}_{hb}$ and the buyer's net gain from the trade is positive.

Step 4. Buyers offer either $\bar{p}_{lb}$, $\bar{p}_{lg}$ or both. A low-valuation asset holder li, $i = b, g$, accepts any price greater or equal to his reservation price $\bar{p}_{li}$ with probability one.
By definition of reservation price $\bar{p}_{li}$, the statement is obvious for any price $p > \bar{p}_{li}$. Hence, we can restrict attention to the following two cases:

Case 1. In equilibrium $\bar{p}_{lb}$ is accepted with probability one by lb agents. Suppose per contra this is not the case and in equilibrium $\bar{p}_{lb}$ is rejected with positive probability by *lb* agents. By Lemma A.2 only *lb*-agents may accept this price. For every $\varepsilon \in (0, \bar{p}_{lg} - \bar{p}_{lb})$ offering a price $\bar{p}_{lb} + \varepsilon$ makes all *lb*-agents willing to accept, and all *lg*-agents reject. Agents *hn* expected payoff from offering this price would be $\pi_{hn}(b, \bar{p}_{lb} + \varepsilon)(V_{hb} - p) + [1 - \pi_{hn}(b, \bar{p}_{lb} + \varepsilon)]V_{hn}$. Notice that for every $\varepsilon \in (0, \bar{p}_{lg} - \bar{p}_{lb})$ the probability of acceptance jumps discontinuously because all *lb*-sellers accept with probability one. Therefore, it holds $\pi_{hn}(b, \bar{p}_{lb} + \varepsilon) > \pi_{hn}(b, \bar{p}_{lb})$ for every $\varepsilon > 0$. It is a profitable deviation to offer $\bar{p}_{lb} + \varepsilon$ if:

$$\pi_{hn}(b, \bar{p}_{lb} + \varepsilon)(V_{hb} - \bar{p}_{lb} - \varepsilon)) + [1 - \pi_{hn}(b, \bar{p}_{lb} + \varepsilon)]V_{hn} > \pi_{hn}(b, \bar{p}_{lb})(V_{hb} - \bar{p}_{lb}) + [1 - \pi_{hn}(b, \bar{p}_{lb})]V_{hn} \tag{74}$$

which holds for $\varepsilon < \frac{[\pi_{hn}(b, \bar{p}_{lb} + \varepsilon) - \pi_{hn}(b, \bar{p}_{lb})](V_{hb} - \bar{p}_{lb} - V_{hn})}{\pi_{hn}(b, \bar{p}_{lb} + \varepsilon)}$. Therefore, if $\bar{p}_{lb}$ were not accepted with probability one then there would be a profitable deviation for buyers. However, if this were the case, no price $\bar{p}_{lb} + \varepsilon$ would be a best response as buyers would always have an incentive to slightly undercut the price offered by others. Therefore, the only possible equilibrium is that $\bar{p}_{lb}$ is accepted with probability one in order to be played.

Case 2. In equilibrium $\bar{p}_{lg}$ is accepted with probability one by lg and lb agents. First notice that *lb* agents always accept with probability one as $\bar{p}_{lg} > \bar{p}_{lb}$. For *lg* agents an analogous argument to the one in Case 1 shows that *lg* agents accept $\bar{p}_{lg}$ with probability one both when $\bar{p}_{lg} > \bar{p}_{hb}$ and $\bar{p}_{lg} < \bar{p}_{hb}$. Indeed, in this case, for a sufficiently small ε, offering a price $\bar{p}_{lg} + \varepsilon$ does not affect the decision of *hb* agents to accept or reject, respectively, the out-of-equilibrium deviation price. The argument has to be established when $\bar{p}_{lg} = \bar{p}_{hb}$ and both *lg* and *hb* accept this price with probability less than one; in this case offering a slightly higher price $\bar{p}_{lg} + \varepsilon$ leads both *lg* and *hb* agents to accept the deviation with probability one. A deviation $\bar{p}_{lg} + \varepsilon$ is profitable if and only if:

$$\begin{aligned} &\pi_{hn}(g, \bar{p}_{lg} + \varepsilon)(V_{hg} - \bar{p}_{lg} - \varepsilon) + \pi_{hn}(b, \bar{p}_{lg} + \varepsilon)](V_{hb} - \bar{p}_{lg} - \varepsilon) + [1 - \pi_{hn}(g, \bar{p}_{lg} + \varepsilon) - \pi_{hn}(b, \bar{p}_{lg} + \varepsilon)]V_{hn} \\ \geq &\pi_{hn}(g, \bar{p}_{lg})(V_{hg} - \bar{p}_{lg}) + \pi_{hn}(b, \bar{p}_{lg})](V_{hb} - \bar{p}_{lg}) + [1 - \pi_{hn}(g, \bar{p}_{lg}) - \pi_{hn}(b, \bar{p}_{lg})]V_{hn} \end{aligned} \tag{75}$$

[46] This inequality holds by an argument analogous to the one in Step 2 for peaches.

Since $\bar{p}_{lg} = \bar{p}_{hb}$, the expression can be rearranged as:

$$\begin{aligned}\varepsilon \leq & \frac{[\pi_{hn}(g,\bar{p}_{lg}+\varepsilon)-\pi_{hn}(g,\bar{p}_{lg})](V_{hg}-V_{hn}-\bar{p}_{lg})+[\pi_{hn}(b,\bar{p}_{lg}+\varepsilon)-\pi_{hn}(b,\bar{p}_{lg})](V_{hb}-V_{hn}-\bar{p}_{hb})}{\pi_{hn}(g,\bar{p}_{lg}+\varepsilon)+\pi_{hn}(b,\bar{p}_{lg}+\varepsilon)} \\ = & \frac{[\pi_{hn}(g,\bar{p}_{lg}+\varepsilon)-\pi_{hn}(g,\bar{p}_{lg})](\bar{p}_{hg}-\bar{p}_{lg})}{\pi_{hn}(g,\bar{p}_{lg}+\varepsilon)+\pi_{hn}(b,\bar{p}_{lg}+\varepsilon)}\end{aligned} \tag{76}$$

Since all lg agents accept $\bar{p}_{lg}+\varepsilon$ and, by hypothesis, while they mix between accepting and rejecting when offered $\bar{p}_{lg}$, it holds $[\pi_{hn}(g,\bar{p}_{lg}+\varepsilon)-\pi_{hn}(g,\bar{p}_{lg})] > 0$. Moreover, by Step 2 it holds $\bar{p}_{hg}-\bar{p}_{lg} > 0$. Therefore, an optimal deviation would exist for ε sufficiently small, contradicting the hypothesis that offering $\bar{p}_{lg}$ is a best response. As a result, it must be the case that all lg agents accept $\bar{p}_{lg}$ with probability one.

Finally, notice the previous arguments together with Steps 1–3 imply that it is never optimal to offer any price different from $\bar{p}_{lg}$ or $\bar{p}_{lb}$. ■

A.2 Proofs

Proof of Proposition 3.1.

1. Given that there are gains from trade only between low-valuation asset holders and high-valuation agents without assets, these two groups constitute the sets of sellers and buyers.

2. In equilibrium $m_S\gamma_B = m_B\gamma_S = \mu M(\gamma_B,\gamma_S)$. The equilibrium expressions for $\gamma_S = \gamma_{lg}+\gamma_{lb}$ and $\gamma_B = \gamma_{hn}$ are the steady state solution of the following system of differential equations for the evolution of masses:

$$\begin{aligned}\dot{\gamma}_{hg} =& \nu\gamma_{lg}-\kappa(\lambda A-\gamma_{lg})+\gamma_{lg}m_B = 0 \\ \dot{\gamma}_{hb} =& \nu\gamma_{lb}-\kappa[(1-\lambda)A-\gamma_{lb}]+\gamma_{lb}m_B = 0 \\ \dot{\gamma}_{hn} =& \nu(1-\gamma_{hn})-\kappa\gamma_{hn}-\gamma_{hn}m_S = \nu(1-\gamma_{hn})-\kappa\gamma_{hn}-\gamma_S m_B = 0\end{aligned} \tag{77}$$

   Solving the equations we get $\gamma_{lg} = \frac{\kappa\lambda A}{\kappa+\nu+m_B}$, $\gamma_{lb} = \frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B}$ and $\gamma_{hn} = \frac{\nu-\frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu}$. Substituting these expression in $m_B\gamma_S = \mu M(\gamma_B,\gamma_S)$ we get:

$$\frac{\kappa A m_B}{\kappa+\nu+m_B} = \mu M\left(\frac{\kappa A}{\kappa+\nu+m_B}, \frac{\nu-\frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu}\right) \tag{78}$$

   The LHS is strictly increasing in m_B and it is equal to zero for $m_B = 0$; the RHS is decreasing in m_B as $\frac{\partial\gamma_B}{\partial m_B} < 0$, $\frac{\partial\gamma_S}{\partial m_B} < 0$ and $M(\cdot,\cdot)$ is increasing in both its arguments. Moreover, for $m_B = 0$ the matching function $M\left(\frac{\kappa A}{\kappa+\nu},\frac{\nu}{\kappa+\nu}\right) > 0$. Therefore, there exists a unique value m_B that solves the equation. The value of m_S can be easily obtained once we consider the equilibrium condition $m_S\gamma_B = m_B\gamma_S$ and we substitute for the equilibrium masses $\gamma_B = \gamma_{hn}$ and $\gamma_S = \gamma_{lg}+\gamma_{lb}$ obtained before. It is straightforward to obtain that m_S is strictly increasing in m_B.

3. Notice that the total volume at every point in time is equal to $m_B\gamma_S$ (or alternatively $m_S\gamma_B$), and the results in point 2. lead immediately to the expression in the proposition.

4. Under the generalized Nash bargaining solution, the equilibrium price solves:

$$\max_p\,(p-V_{li}+V_{ln})^{\beta}\,(V_{hi}-p-V_{hn})^{1-\beta} \tag{79}$$

Hence, the solution is equal to $p = \beta\left(V_{hi} - V_{hn}\right) + (1-\beta)\left(V_{li} - V_{ln}\right)$. Substituting this price expression in the value functions and using point 1. we get:

$$rV_{hi} = \delta_i + \kappa(V_{li} - V_{hi}) \tag{80}$$

$$rV_{li} = \delta_i - x + \nu(V_{hi} - V_{li}) + \beta m_B(V_{hi} - V_{hn} - V_{li} + V_{ln}) \tag{81}$$

$$rV_{hn} = \kappa(V_{ln} - V_{hn}) + (1-\beta)m_S(V_{hi} - V_{li} + V_{ln} - V_{hn}) \tag{82}$$

$$rV_{ln} = \nu(V_{hn} - V_{ln}) \tag{83}$$

Using the last two equations we get:

$$V_{hn} - V_{ln} = \frac{(1-\beta)m_S}{\kappa + \nu + r + (1-\beta)m_S}(V_{hi} - V_{li}) \tag{84}$$

Subtracting the value functions of li agents from that of hi agents and using the last equation we get:

$$V_{hi} - V_{li} = \frac{\kappa + \nu + r + (1-\beta)m_S}{(\kappa + \nu + r)[\kappa + \nu + r + (1-\beta)m_S + \beta m_B]}x \tag{85}$$

Substituting $V_{hn} - V_{ln}$ and $V_{hi} - V_{li}$ into the value functions we get:

$$\begin{aligned} V_{hi} - V_{hn} &= \frac{1}{r}\left[\delta_i - \frac{(\kappa + \nu + r)[\kappa + (1-\beta)m_S]}{\kappa + \nu + r + (1-\beta)m_S}(V_{hi} - V_{li})\right] \\ V_{li} - V_{ln} &= \frac{1}{r}\left[\delta_i - \frac{(\kappa + \nu + r)[\kappa + r + (1-\beta)m_S]}{\kappa + \nu + r + (1-\beta)m_S}(V_{hi} - V_{li})\right] \end{aligned} \tag{86}$$

Plugging these values in the expression for the equilibrium price and using equation (85) we get:

$$p_i = \frac{1}{r}\left[\delta_i - \frac{\kappa + (1-\beta)(r + m_S)}{\kappa + \nu + r + (1-\beta)m_S + \beta m_B}x\right] \tag{87}$$

■

Proof of Proposition 5.1.

If only lemons are traded then $\gamma_S = \gamma_{lb}$. Let $m_{\tilde{s}}$ denote the matching rate of buyers with asset owners of type $\tilde{s} \in \tilde{S}$ when $\gamma_S = \gamma_{lb}$. For the DGP technology $m_{\tilde{s}} = \mu\frac{\gamma_{\tilde{s}}}{1+A}$ for every $\tilde{s} \in \tilde{S}$; for any competitive matching technology $m_{\tilde{s}} = 0$ if $\tilde{s} \neq lb$ and $m_{lb} = \mu\frac{\gamma_{lb}}{\gamma_{lb} + \gamma_{hn}}$. By Lemma A.4, in a lemons market buyers only offer $\bar{p}_{lb} = V_{lb} - V_{ln}$, while peaches do not trade. As a consequence, the equilibrium value functions are equal to:

$$\begin{aligned} rV_{hg} &= \delta_g + \kappa\left(V_{lg} - V_{hg}\right) \\ rV_{lg} &= \delta_g - x + \nu\left(V_{hg} - V_{lg}\right) \\ rV_{hb} &= \delta_b + \kappa\left(V_{lb} - V_{hb}\right) \\ rV_{lb} &= \delta_b - x + \nu\left(V_{hb} - V_{lb}\right) + m_B\left(W_{lb} - V_{lb}\right) = \delta_b - x + \nu\left(V_{hb} - V_{lb}\right) \\ rV_{hn} &= \kappa(V_{ln} - V_{hn}) + m_{lb}(W_{hn} - V_{hn}) = \kappa(V_{ln} - V_{hn}) + m_{lb}(V_{hb} - V_{lb} + V_{ln} - V_{hn}) \\ rV_{ln} &= \nu(V_{hn} - V_{ln}) \end{aligned} \tag{88}$$

From the set of equations in (88) it is straightforward to derive:

$$V_{hg} - V_{lg} = V_{hb} - V_{lb} = \frac{x}{\kappa + \nu + r} \qquad V_{hn} - V_{ln} = \frac{m_{lb}}{\kappa + \nu + r + m_{lb}}\frac{x}{\kappa + \nu + r} \tag{89}$$

It is sufficient to show that buyers do not find convenient to offer a price different from $\bar{p}_{lb}$. A similar argument to the one in Lemma A.4 establishes that the best possible deviation is to offer $\bar{p}_{lg} = V_{lg} - V_{ln}$. As a result, the no deviation condition is:

$$m_{lb}\left(V_{hb} - V_{lb} - V_{hn} + V_{ln}\right) \geq m_{lg}V_{hg} + \left(m_{lb} + m_{hb}\mathbb{1}_{\{\bar{p}_{lg} > \bar{p}_{hb}\}}\right)V_{hb} - \left(m_{lg} + m_{lb} + m_{hb}\mathbb{1}_{\{\bar{p}_{lg} > \bar{p}_{hb}\}}\right)\left(V_{lg} + V_{hn} - V_{ln}\right) \tag{90}$$

Rearranging we get:

$$m_{lb}\left(V_{lg} - V_{lb}\right) \geq m_{lg}\left(V_{hg} - V_{lg} - V_{hn} + V_{ln}\right) + m_{hb}\mathbb{1}_{\{\bar{p}_{lg} > \bar{p}_{hb}\}}\left(V_{hb} - V_{lg} - V_{hn} + V_{ln}\right) \tag{91}$$

Notice that $\bar{p}_{lg} > \bar{p}_{hb}$ is equivalent to $V_{hb} - V_{lg} - V_{hn} + V_{ln} < 0$, hence the last term on the RHS is non-positive. From (88) it is immediate to get $V_{lg} - V_{lb} = \frac{\delta_g - \delta_b}{r}$ and $V_{hb} - V_{lg} = \frac{x}{\kappa + \nu + r} - \frac{\delta_g - \delta_b}{x}$. Substituting the values in equation (89) in (91) and rearranging it is immediate to get the inequality in (14). To determine the equilibrium γ_{lg}, γ_{lb} and γ_{hn} notice that in a stationary lemons market equilibrium:

$$\begin{aligned}
\dot{\gamma}_{hg} &= \nu\gamma_{lg} - \kappa(\lambda A - \gamma_{lg}) = 0 \\
\dot{\gamma}_{hb} &= \nu\gamma_{lb} - \kappa[(1-\lambda)A - \gamma_{lb}] + \gamma_{lb}m_B = 0 \\
\dot{\gamma}_{hn} &= \nu(1 - \gamma_{hn}) - \kappa\gamma_{hn} - \gamma_{hn}m_{lb} = 0
\end{aligned} \tag{92}$$

Finally, the equilibrium condition $m_{lb}\gamma_{hn} = m_{hn}\gamma_{lb}$ ensures that the total number of matched buyers and sellers is equal. ■

Proof of Corollary 5.1.
For the DGP technology $m_{\tilde{s}} = \mu\frac{\gamma_{\tilde{s}}}{1+A}$ for every $\tilde{s} \in \tilde{S}$. From the system of equations in (92) we get:

$$\begin{aligned}
m_{lg} &= \mu\frac{\kappa\lambda A}{(1+A)(\kappa+\nu)} & m_{lb} &= \mu\frac{\kappa(1-\lambda)A}{(1+A)(\kappa+\nu+m_{hn})} \\
m_{hb} &= \mu\frac{(1-\lambda)A}{1+A} - m_{lb} & m_{hn} &= \mu\frac{\nu}{(1+A)(\kappa+\nu+m_{lb})}
\end{aligned} \tag{93}$$

Substituting the above quantities in equation (14) we get:

$$\phi \geq \min\left\{\frac{\lambda}{1-\lambda}\frac{(\kappa+\nu+m_{hn})r}{(\kappa+\nu)(\kappa+\nu+r+m_{lb})}, \frac{\lambda}{1-\lambda}\frac{(\kappa+\nu+m_{hn})r}{(\kappa+\nu)(\kappa+\nu+r+m_{lb})} + \frac{\nu+m_{hn}}{\kappa}\left(\frac{r}{\kappa+\nu+r+m_{lb}} - \phi\right)\right\} \tag{94}$$

The RHS of equation (94) is decreasing in m_{lb} and increasing in λ and m_{hn}. To prove the statement it is sufficient to show that m_{lb} is decreasing and m_{hn} is increasing in λ. If this is the case, the RHS is monotonically increasing in λ, and it tends to infinity for $\lambda \to 1$.

First, notice that it is possible to express m_{hn} as:

$$m_{hn} = \frac{\mu}{1+A}\gamma_{hn} = \frac{\mu}{1+A}\frac{\nu - \kappa(1-\lambda)A\frac{m_{hn}}{\kappa+\nu+m_{hn}}}{\kappa+\nu} \tag{95}$$

By implicit differentiation of this expression we get:

$$\frac{\mathrm{d}m_{hn}}{\mathrm{d}\lambda} = \frac{\frac{\mu\kappa A m_{hn}}{(1+A)(\kappa+\nu)(\kappa+\nu+m_{hn})}}{1 + \frac{\mu\kappa(1-\lambda)A}{(1+A)(\kappa+\nu+m_{hn})^2}} > 0 \tag{96}$$

By the system of equations in (92), we can express m_{lb} as:

$$m_{lb} = \frac{\gamma_{lb}}{\gamma_{hn}} m_{hn} = \frac{\gamma_{lb} m_{hn}}{\nu - \gamma_{lb} m_{hn}} (\kappa + \nu) \tag{97}$$

Hence, it is sufficient to show that $\gamma_{lb} m_{hn}$ is decreasing in λ. Notice that:

$$\gamma_{lb} m_{hn} = \kappa(1-\lambda) A \frac{m_{hn}}{\kappa + \nu + m_{hn}} \tag{98}$$

Differentiating this quantity with respect to λ and using the expression in equation (96) we have:

$$\frac{\mathrm{d}(\gamma_{lb} m_{hn})}{\mathrm{d}\lambda} = \kappa A \frac{m_{hn}}{\kappa + \nu + m_{hn}} \left[\frac{\mu \kappa (1-\lambda) A}{\mu \kappa (1-\lambda) A + (1+A)(\kappa + \nu + m_{hn})^2} - 1 \right] < 0 \tag{99}$$

Hence, we proved that the RHS of equation (94) is monotonically increasing in λ.

For any competitive technology it is sufficient to realize that in a lemons market equilibrium a buyer is only matched with lemon sellers as no peach participates to the market as the price is too low, i.e. $m_{lg} = 0$ and inequality (14) holds for every admissible parameter constellation. ■

Proof of Lemma 5.1.

1. Follows immediately from imposing stationarity, i.e. $\dot{\gamma}_{ji} = 0$ for $j = h, l$ and $i = b, g$.

2. Implied by the fact that the total measure of lemons on the market is $(1-\lambda)A$.

■

Proof of Lemma 5.2.

1. It follows immediately from the first two equations in (18).

2. From the value function for V_{ln} in (19), we get $V_{ln} = \frac{\nu}{\nu + r} V_{hn}$. To obtain an expression for V_{hn}, we first need to compute W_{hn}, i.e. the expected value from acquiring an asset on the market at price $\bar{p}_{lg} = V_{lg} - V_{ln}$:

$$W_{hn} = \frac{\gamma_{lg}}{\gamma_S} V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right) V_{hb} - (V_{lg} - V_{ln}) \tag{100}$$

Notice that whenever $\sigma_{hn}(\bar{p}_{lg}) < 1$ buyers are indifferent between offering $\bar{p}_{lg}$ and $\bar{p}_{lb}$ so this expression continues to hold. Substituting W_{hn} and V_{ln} into the expression for rV_{hn} in equation (19) and rearranging we get the expression in equation (21).

■

Proof of Lemma 5.3.

1. Buyers' expected payoff from offering $\bar{p}_{lg}$ is:

$$\Gamma(\bar{p}_{lg}) = \frac{\gamma_{lg}}{\gamma_S} V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right) V_{hb} - V_{lg} - (V_{hn} - V_{ln}) \tag{101}$$

Since $V_{hn} - V_{ln} = \frac{r}{\nu + r} V_{hn}$, and equation (19) provides V_{hn}, it is immediate to get:

$$\Gamma(\bar{p}_{lg}) = \frac{\kappa + \nu + r}{\kappa + \nu + r + m_S} \left[\frac{\gamma_{lg}}{\gamma_S} V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right) V_{hb} - V_{lg} \right] \tag{102}$$

Offering $\bar{p}_{lb} = V_{lb} - V_{ln}$ instead yields:

$$\Gamma(\bar{p}_{lb}) = \frac{\gamma_{lb}}{\gamma_S}(V_{hb} - V_{lb} - V_{hn} + V_{ln}) = \frac{\gamma_{lb}}{\gamma_S}\left[V_{hb} - V_{lb} - \frac{m_S}{\kappa+\nu+r}\Gamma(\bar{p}_{lg})\right] \tag{103}$$

The no-deviation condition requires $\Gamma(\bar{p}_{lg}) \geq \Gamma(\bar{p}_{lb})$. Rearranging this inequality we get (22).

To prove that $V_{hn} > 0$, ensuring that buyers are willing to participate in the market, equation (21) shows it is sufficient to show that $\Gamma(\bar{p}_{lg}) \geq \Gamma(\bar{p}_{lb})$ implies $\Gamma(\bar{p}_{lg}) > 0$. Suppose per contra $\Gamma(\bar{p}_{lg}) \leq 0$. Then, by Lemma A.2:

$$0 \geq \Gamma(\bar{p}_{lg}) \geq \Gamma(\bar{p}_{lb}) \geq \frac{\gamma_{lb}}{\gamma_S}(V_{hb} - V_{lb}) > 0 \tag{104}$$

a contradiction.

2. Agents of type hb do not find it convenient to participate in the market when $\bar{p}_{hb} \geq \bar{p}_{lg}$, i.e. $V_{hb} - V_{lg} \geq V_{hn} - V_{ln}$. Since $V_{hn} - V_{ln} = \frac{r}{\nu+r}V_{hn} = \frac{m_S}{\kappa+\nu+r+m_S}\left[V_{hb} - V_{lg} + \frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{hb})\right]$, then $V_{hb} - V_{lg} \geq V_{hn} - V_{ln}$ is equivalent to:

$$V_{hb} - V_{lg} \geq \frac{m_S}{\kappa+\nu+r+m_S}\left[V_{hb} - V_{lg} + \frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{hb})\right] \tag{105}$$

Hence,

$$V_{hb} - V_{lg} \geq \frac{m_S}{\kappa+\nu+r}\frac{\gamma_{lg}}{\gamma_S}\left[V_{hg} - V_{lg} - (V_{hb} - V_{lg})\right] \tag{106}$$

Rearranging and using $V_{hg} - V_{lg} = \frac{x}{\kappa+\nu+r}$ we get the expression in (23).

■

Proof of Lemma 5.4.
In equilibrium an equal mass of buyers and sellers trade at every instant, i.e.:

$$m_B\gamma_S = \mu M\left(\frac{\nu - \frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu}, \gamma_S\right) \tag{107}$$

From Lemma 5.1 we can substitute for γ_S to get:

$$\begin{aligned} &\text{Equilibrium E:} \quad \frac{\kappa A m_B}{\kappa+\nu+m_B} = \mu M\left(\frac{\nu - \frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu}, \frac{\kappa A}{\kappa+\nu+m_B}\right) \\ &\text{Equilibrium H:} \quad \left[(1-\lambda)A + \frac{\kappa\lambda A}{\kappa+\nu+m_B}\right]m_B = \mu M\left(\frac{\nu - \frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu}, (1-\lambda)A + \frac{\kappa A\lambda}{\kappa+\nu+m_B}\right) \end{aligned} \tag{108}$$

For both equations the LHS is increasing in m_B and it is equal to zero for $m_B = 0$; the RHS is strictly positive for $m_B = 0$ and it is decreasing in m_B because both γ_B and γ_S are decreasing in m_B and $M(\cdot,\cdot)$ is increasing in both arguments. Therefore, there is a unique value that equalizes both sides of the equation.

Notice that equation (108) for equilibrium E does not depend on λ, thus also m_B^E is independent of its value. Similarly, it is immediate to get that m_S^E does not depend on λ once we adopt the equivalent expressions $\gamma_B^E = \frac{\nu}{\kappa+\nu+m_S^E}$ and $\gamma_S^E = \frac{\kappa A - \frac{\nu m_S}{\kappa+\nu+m_S}}{\kappa+\nu}$, and we plug them into the equilibrium condition $m_S\gamma_B^E = \mu M(\gamma_B^E, \gamma_S^E)$.

We turn to study how m_B changes as a function of λ. Consider the function $g(\lambda, m_B) = m_B\gamma_S -$

$\mu M\left(\frac{\nu-\frac{\kappa A m_B}{\kappa+\nu+m_B}}{\kappa+\nu},\gamma_S\right)$. By implicit differentiation we get:

$$
\begin{aligned}
\frac{\mathrm{d}m_B}{\mathrm{d}\lambda} &= -\frac{\partial g/\partial\lambda}{\partial g/\partial m_B} \\
\frac{\partial g}{\partial\lambda} &= \frac{\partial\gamma_S}{\partial\lambda}\left[m_B-\mu M_2(\gamma_B,\gamma_S)\right] \\
\frac{\partial g}{\partial m_B} &= \gamma_S+\frac{\partial\gamma_S}{\partial m_B}\left[m_B-\mu M_2(\gamma_B,\gamma_S)\right]+\mu M_1(\gamma_B,\gamma_S)\frac{\kappa A}{(\kappa+\nu+m_B)^2}
\end{aligned}
\tag{109}
$$

Notice that by Lemma 5.1 $\frac{\partial\gamma_S^E}{\partial\lambda}=0$ and $\frac{\partial\gamma_S^H}{\partial\lambda}=-A\frac{\nu+m_B}{\kappa+\nu+m_B}<0$, hence $\frac{\partial g}{\partial\lambda}\leq 0$ as $m_B-\mu M_2(\gamma_B,\gamma_S)$ has, by definition of m_B, the same sign of $M(\gamma_B,\gamma_S)-M_2(\gamma_B,\gamma_S)\gamma_S$, i.e. it is by assumption greater or equal to zero. To determine the sign of $\frac{\partial g}{\partial m_B}$, first notice that $\frac{\partial\gamma_S^E}{\partial m_B^E}=-\gamma_S^E\frac{1}{\kappa+\nu+m_B^E}$ and $\frac{\partial\gamma_S^H}{\partial m_B^H}=-\lambda\frac{\kappa A}{(\kappa+\nu+m_B^H)^2}$. Substituting these expressions in $\frac{\partial g}{\partial m_B}$ and simplifying we get:

$$
\begin{aligned}
\frac{\partial g}{\partial m_B^E} &= \gamma_S^E\frac{\kappa+\nu+\mu M_1(\gamma_B^E,\gamma_S^E)+\mu M_2(\gamma_B^E,\gamma_S^E)}{\kappa+\nu+m_B^E}>0 \\
\frac{\partial g}{\partial m_B^H} &= (1-\lambda)A+\frac{\kappa\lambda A}{\kappa+\nu+m_B^H}\,\frac{\kappa+\nu+\mu M_1(\gamma_B^H,\gamma_S^H)+\mu M_2(\gamma_B^H,\gamma_S^H)/\lambda}{\kappa+\nu+m_B^H}>0
\end{aligned}
\tag{110}
$$

Therefore, it is possible to conclude that $\frac{\mathrm{d}m_B^E}{\mathrm{d}\lambda}=0$ and $\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda}\geq 0$. In particular, $\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda}>0$ if and only if the matching function satisfies $M(\gamma_B,\gamma_S)-M_2(\gamma_B,\gamma_S)\gamma_S>0$.

To show that m_S^H is strictly decreasing in λ we consider non-competitive and competitive matching technologies separately.

Case 1. Non-competitive matching technologies. By total differentiation of the equilibrium condition $m_S\gamma_B=\mu M(\gamma_B,\gamma_S)$ we have:

$$
\frac{\mathrm{d}m_S^H}{\mathrm{d}\lambda}\gamma_B^H+m_S^H\frac{\mathrm{d}\gamma_B^H}{\mathrm{d}\lambda}=\mu\left(M_1\frac{\mathrm{d}\gamma_B^H}{\mathrm{d}\lambda}+M_2\frac{\mathrm{d}\gamma_S^H}{\mathrm{d}\lambda}\right)
\tag{111}
$$

Given that for non-competitive matching technologies $m_S-\mu M_1=\frac{\mu}{\gamma_B}[M(\gamma_B,\gamma_S)-M_1\gamma_B]=0$, we obtain

$$
\frac{\mathrm{d}m_S^H}{\mathrm{d}\lambda}\gamma_B^H=\mu M_2\frac{\mathrm{d}\gamma_S^H}{\mathrm{d}\lambda}
\tag{112}
$$

From above, it is straightforward to show that $\mathrm{d}\gamma_S^H/\mathrm{d}\lambda<0$. Thus, for any non-competitive matching technology $\mathrm{d}m_S^H/\mathrm{d}\lambda<0$.

Case 2. Competitive matching technologies. From $m_B\gamma_S=\mu M(\gamma_B,\gamma_S)$ we obtain:

$$
\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda}\gamma_S^H+m_B^H\frac{\mathrm{d}\gamma_S^H}{\mathrm{d}\lambda}=\mu\left(M_1\frac{\mathrm{d}\gamma_B^H}{\mathrm{d}\lambda}+M_2\frac{\mathrm{d}\gamma_S^H}{\mathrm{d}\lambda}\right)
\tag{113}
$$

Rearranging yields:

$$
\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda}\gamma_S^H=\mu\frac{\mathrm{d}\gamma_S^H}{\mathrm{d}\lambda}\frac{1}{\gamma_S^H}\left[M_1\gamma_B^H\left(\frac{\gamma_S^H}{\gamma_B^H}\frac{\mathrm{d}\gamma_B^H/\mathrm{d}\lambda}{\mathrm{d}\gamma_S^H/\mathrm{d}\lambda}\right)+M_2\gamma_S^H-M(\gamma_B^H,\gamma_S^H)\right]
\tag{114}
$$

Given that $\mathrm{d}\gamma_S^H/\mathrm{d}\lambda<0$ and $M_1\gamma_B+M_2\gamma_S\geq M(\gamma_B,\gamma_S)$ by $M(\cdot,\cdot)$ satisfying non-decreasing returns to scale, it must be the case that $\frac{\gamma_S^H}{\gamma_B^H}\frac{\mathrm{d}\gamma_B^H/\mathrm{d}\lambda}{\mathrm{d}\gamma_S^H/\mathrm{d}\lambda}<1$ in order to have $\mathrm{d}m_B^H/\mathrm{d}\lambda>0$, which we have established for all competitive matching technologies. Then, since from $m_S\gamma_B = \mu M(\gamma_B, \gamma_S)$:

$$\frac{\mathrm{d}m_S^H}{\mathrm{d}\lambda}\gamma_B^H = \mu\frac{\mathrm{d}\gamma_B^H}{\mathrm{d}\lambda}\frac{1}{\gamma_B^H}\left[M_1\gamma_B^H + M_2\gamma_S^H\left(\frac{\gamma_B^H}{\gamma_S^H}\frac{\mathrm{d}\gamma_S^H/\mathrm{d}\lambda}{\mathrm{d}\gamma_B^H/\mathrm{d}\lambda}\right) - M(\gamma_B^H, \gamma_S^H)\right], \tag{115}$$

where $\frac{\gamma_B^H}{\gamma_S^H}\frac{\mathrm{d}\gamma_S^H/\mathrm{d}\lambda}{\mathrm{d}\gamma_B^H/\mathrm{d}\lambda} > 1$, it follows that $\mathrm{d}m_S^H/\mathrm{d}\lambda < 0$ as $\mathrm{d}\gamma_B^H/\mathrm{d}\lambda < 0$ for all competitive matching technologies.

For competitive technologies we have $M(\gamma_B, \gamma_S) - M_2(\gamma_B, \gamma_S)\gamma_S > 0$, hence $\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda} > 0$. Since $m_B^E = m_B^H$ for $\lambda = 1$ (as it implies $\gamma_S^E = \gamma_S^H$) and $\frac{\mathrm{d}m_B^H}{\mathrm{d}\lambda} > 0$, then it follows that $m_B^E > m_B^H$ for every $\lambda \in (0,1)$.

For non-competitive technologies $m_B - \mu M_2(\gamma_B, \gamma_S) = 0$ so $\frac{\mathrm{d}m_B}{\mathrm{d}\lambda} = 0$ both in equilibria E and H. Lastly, to show that $m_B^E = m_B^H$ for every $\lambda \in (0,1)$ notice that $m_B^E = m_B^H$ for $\lambda = 1$ as for competitive technologies.
■

Proof of Proposition 5.2.
To get an expression for the holders of lemons, first notice that $(\kappa + r)V_{hb} = \delta_b + \kappa V_{lb}$. Substituting this expression in the fourth equation in (18) and multiplying both sides by $\kappa + r$, we get:

$$(\kappa + r)rV_{lb} = (\kappa + r)(\delta_b - x) + \nu(\delta_b - rV_{lb}) + (\kappa + r)m_B(V_{lg} - V_{lb}) \tag{116}$$

Substituting V_{lg} from equation (20) and simplifying:

$$[r(\kappa+\nu+r) + (\kappa+r)m_B]V_{lb} = (\kappa+\nu+r)\delta_b - (\kappa+r)x + (\kappa+r)m_B\left[\frac{\delta_g}{r} - \frac{x}{r}\frac{\kappa+r}{\kappa+\nu+r}\right] \tag{117}$$

Hence:

$$V_{lb} = \frac{r(\kappa+\nu+r)}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_b}{r} + \frac{(\kappa+r)m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_g}{r} - \frac{\kappa+r}{\kappa+\nu+r}\frac{x}{r} \tag{118}$$

and

$$V_{hb} = \frac{r(\kappa+\nu+r+m_B)}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_b}{r} + \frac{\kappa m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_g}{r} - \frac{\kappa}{\kappa+\nu+r}\frac{x}{r} \tag{119}$$

The equilibrium exists if *hb*-agents do not find it convenient to participate in the market. In turn, this requires $\bar{p}_{hb} \geq \bar{p}_{lg}$, i.e. $V_{hb} - V_{lg} \geq V_{hn} - V_{ln}$.

From equations (20) and (119):

$$V_{hb} - V_{lg} = \frac{x}{\kappa+\nu+r} - \frac{\kappa+\nu+r+m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}(\delta_g - \delta_b) \tag{120}$$

Substituting (120) in (23) and noting that $\frac{\gamma_{lg}}{\gamma_S} = \lambda$ we can immediately rearrange to get:

$$\frac{\delta_g - \delta_b}{x} \leq \frac{r(\kappa+\nu+r+m_B)+\kappa m_B}{(\kappa+\nu+r+\lambda m_S)(\kappa+\nu+r+m_B)} \tag{121}$$

The optimality condition of inequality (22) becomes:

$$\lambda V_{hg} + (1-\lambda)V_{hb} - V_{lg} \geq \frac{\kappa+\nu+r+m_S}{\frac{\kappa+\nu+r}{1-\lambda}+m_S}(V_{hb} - V_{lb}) \tag{122}$$

Notice that:

$$\begin{aligned} V_{hg} - V_{hb} &= \frac{r(\kappa+\nu+r+m_B)}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_g-\delta_b}{r} > 0 \\ V_{hb} - V_{lb} &= \frac{r}{\kappa+\nu+r}\frac{x}{r} - \frac{rm_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}\frac{\delta_g-\delta_b}{r} > 0 \end{aligned} \tag{123}$$

By Lemmata A.1 and A.2 both quantities are positive. Lemma 5.4 guarantees m_B is independent of λ. Therefore, it is straightforward to notice that the LHS of equation (122) is increasing in λ, while its RHS is decreasing in λ. For $\lambda = 0$ equation (122) is not satisfied as it requires:

$$V_{hb} - V_{lg} \geq V_{hb} - V_{lb} \quad \Rightarrow \quad V_{lg} \leq V_{lb} \tag{124}$$

but the latter inequality contradicts Lemma A.2. If $\lambda = 1$ equation (122) holds strictly as $V_{hg} - V_{lg} > 0$ by Lemma A.1. Therefore, there exists a unique λ^* such that for every $\lambda \geq \lambda^*$ the condition in (22) is satisfied. Moreover, if equation (122) holds, then $\lambda V_{hg} + (1-\lambda)V_{hb} - V_{lg} > 0$ as it is greater than a positive quantity. As a result, equation (21) implies $V_{hn} > 0$. ■

Proof of Proposition 5.3.
In this equilibrium $\mathbb{1}_{\{hb \subset S\}} = 1$. To evaluate condition (23) in Lemma 5.3, let us solve for $V_{hb} - V_{lg}$. The second and third equation in (18) imply:

$$V_{hb} - V_{lg} = \frac{r}{r+m_B}\left(\frac{\kappa+r}{\kappa+\nu+r}\frac{x}{r} - \frac{\delta_g - \delta_b}{r} - \frac{\kappa}{\kappa+\nu+r+m_B}\frac{x}{r}\right) + \frac{\nu+r+m_B}{\kappa+\nu+r+m_B}\frac{m_B}{r+m_B}(V_{hn} - V_{ln}), \tag{125}$$

where, by (19) and (21):

$$V_{hn} - V_{ln} = \frac{m_S}{\kappa+\nu+r+m_S}\left[\frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{lg}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb} - V_{lg})\right]. \tag{126}$$

Combining these two equations yields:

$$\begin{aligned} &\frac{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B] + (\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}{(\kappa+\nu+r+m_B)(\kappa+\nu+r+m_S)}(V_{hb} - V_{lg}) \\ =&\frac{r(\kappa+\nu+r+m_S)+\kappa m_B}{(\kappa+\nu+r)(\kappa+\nu+r+m_B)}x - (\delta_g - \delta_b) + \frac{(\nu+r+m_B)m_B}{\kappa+\nu+r+m_B}\frac{\frac{\gamma_{lg}}{\gamma_S}m_S}{\kappa+\nu+r+m_S}\frac{x}{\kappa+\nu+r}, \end{aligned} \tag{127}$$

which simplifies to:

$$\begin{aligned} &V_{hb} - V_{lg} \\ =&\frac{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B] + (\nu+r+m_B)\frac{\gamma_{lg}}{\gamma_S}m_S m_B}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B] + (\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}\frac{x}{\kappa+\nu+r} \\ &- \frac{(\kappa+\nu+r+m_S)(\kappa+\nu+r+m_B)}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B] + (\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}(\delta_g - \delta_b) \end{aligned} \tag{128}$$

Substituting this expression in (23) and combining terms, one finds that hb agents are willing to sell their assets when:

$$\begin{aligned} &\frac{(\kappa+\nu+r)(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}\frac{x}{\kappa+\nu+r} \\ &\leq (\kappa+\nu+r+m_S)(\kappa+\nu+r+m_B)(\delta_g - \delta_b). \end{aligned} \tag{129}$$

Simplifying yields:

$$\frac{\delta_g - \delta_b}{x} \geq \frac{r(r+\kappa+\nu+m_B^H)+\kappa m_B^H}{(r+\kappa+\nu+\frac{\gamma_{lg}^H}{\gamma_S^H}m_S^H)(r+\kappa+\nu+m_B^H)}, \tag{130}$$

the condition 1. in Proposition 5.3.

As a first step in proving that (22) is satisfied for λ sufficiently high, note that the last two equations in (18) imply:

$$V_{hb} - V_{lb} = \frac{x}{\kappa + \nu + r + m_B} + \frac{m_B}{\kappa + \nu + r + m_B}(V_{hn} - V_{ln}). \tag{131}$$

Substituting for $V_{hn} - V_{ln}$ from (126) above yields:

$$\begin{aligned} V_{hb} - V_{lb} = & \frac{x}{\kappa + \nu + r + m_B} \\ & + \frac{m_B}{\kappa + \nu + r + m_B} \frac{m_S}{\kappa + \nu + r + m_S} \left[\frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{lg}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb} - V_{lg}) \right]. \end{aligned} \tag{132}$$

Plugging this expression into the right-hand side of (22) and combining terms, one gets:

$$\frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{lg}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb} - V_{lg}) \geq \frac{\kappa + \nu + r + m_S}{\frac{\gamma_S}{\gamma_{lb}}(\kappa + \nu + r + m_B) + m_S} \frac{x}{\kappa + \nu + r}. \tag{133}$$

Let us first prove that the right-hand side of (133) is strictly decreasing in λ. Given that the RHS is strictly increasing m_S and strictly decreasing in m_B, which are decreasing and increasing in λ, respectively, it suffices to show that γ_S/γ_{lb} is strictly increasing in λ. Lemma 5.1 implies that:

$$\frac{\gamma_S}{\gamma_{lb}} = \frac{1}{1-\lambda} + \frac{\nu + m_B}{\kappa}. \tag{134}$$

Given that m_B is increasing in λ, this quantity is indeed strictly increasing in λ. Thus, we have established that the RHS of (133) is strictly decreasing in λ. To prove that the left-hand side of (133) is strictly increasing in λ, note that:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left[\frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{lg}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb} - V_{lg}) \right] = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\gamma_{lg}}{\gamma_S}\right)(V_{hg} - V_{hb}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)\frac{\mathrm{d}V_{hb}}{\mathrm{d}\lambda}, \tag{135}$$

as V_{lg} and V_{hg} are independent of λ. Given that, by Lemma A.2, $V_{hg} > V_{hb}$, it suffices to show that γ_{lg}/γ_S and V_{hb} are strictly increasing in λ. Let us begin by considering γ_{lg}/γ_S. From Lemma 5.1 and the equilibrium condition $m_S\gamma_B = m_B\gamma_S$, one obtains:

$$\frac{\gamma_{lg}}{\gamma_S} m_S = \frac{\gamma_{lg}}{\gamma_S} m_B \gamma_S \frac{\kappa + \nu}{\nu - \frac{\kappa A m_B}{\kappa + \nu + m_B}}. \tag{136}$$

Substituting for γ_{lg} on the right-hand side from Lemma 5.1, this simplifies to:

$$\frac{\gamma_{lg}}{\gamma_S} = \frac{\kappa\lambda A}{m_S}(\kappa + \nu)\frac{m_B}{\nu(\kappa + \nu + m_B) - \kappa A m_B}. \tag{137}$$

Given that m_S is strictly decreasing in λ and

$$\frac{\mathrm{d}}{\mathrm{d}m_B}\left(\frac{m_B}{\nu(\kappa + \nu + m_B) - \kappa A m_B}\right) = \frac{\nu(\kappa + \nu)}{[\nu(\kappa + \nu + m_B) - \kappa A m_B]^2} > 0, \tag{138}$$

it follows that γ_{lg}/γ_S is strictly increasing in λ. Next, to be able to evaluate $\mathrm{d}V_{hb}/\mathrm{d}\lambda$, let us solve for V_{hb}. Combining (128) above with the expression for V_{lg} in Lemma 5.2 yields

$$
\begin{aligned}
V_{hb} =& \frac{(\kappa+\nu+r+m_S)\kappa m_B+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}\frac{\delta_g}{r} \\
&+\frac{(\kappa+\nu+r+m_S)r(\kappa+\nu+r+m_B)}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}\frac{\delta_b}{r} \\
&-\frac{\kappa}{\kappa+\nu+r}\frac{x}{r} \\
&-\frac{(\nu+r+m_B)rm_B}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}\frac{x}{r}
\end{aligned} \tag{139}
$$

Differentiating with respect to λ, one obtains

$$
\frac{\mathrm{d}V_{hb}}{\mathrm{d}\lambda}=\frac{\mathrm{d}m_B}{\mathrm{d}\lambda}\frac{\partial V_{hb}}{\partial m_B}+\frac{\mathrm{d}m_S}{\mathrm{d}\lambda}\frac{\partial V_{hb}}{\partial m_S}+\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\gamma_{lg}}{\gamma_S}m_S\right)\frac{\partial V_{hb}}{\partial\left(\frac{\gamma_{lg}}{\gamma_S}m_S\right)}, \tag{140}
$$

where by Lemma 5.4, m_B is increasing and m_S strictly decreasing in λ. To show that (140) is strictly positive, let us begin by considering the derivative of $(\gamma_{lg}/\gamma_S)m_S$ with respect to λ. Employing Lemma 5.1, one obtains:

$$
\begin{aligned}
\frac{\gamma_{lg}}{\gamma_S}m_S =& \frac{\gamma_{lg}}{\gamma_S}m_B\gamma_S\frac{\kappa+\nu}{\nu-\frac{\kappa A m_B}{\kappa+\nu+m_B}} \\
=& \kappa\lambda A(\kappa+\nu)\frac{m_B}{\nu(\kappa+\nu+m_B)-\kappa A m_B},
\end{aligned} \tag{141}
$$

where the equality is obtained by substituting in for γ_{lg}. Given that m_B is increasing in λ and:

$$
\frac{\mathrm{d}}{\mathrm{d}m_B}\left(\frac{m_B}{\nu(\kappa+\nu+m_B)-\kappa A m_B}\right)=\frac{\nu(\kappa+\nu)}{[\nu(\kappa+\nu+m_B)-\kappa A m_B]^2}>0, \tag{142}
$$

it follows that $(\gamma_{lg}/\gamma_S)m_S$ is strictly increasing in λ. What remains to sign $\mathrm{d}V_{hb}/\mathrm{d}\lambda$ is to evaluate the partial derivatives of V_{hb} with respect to m_B, m_S and $(\gamma_{lg}/\gamma_S)m_S$. From (139):

$$
\begin{aligned}
\frac{\partial V_{hb}}{\partial\left(\frac{\gamma_{lg}}{\gamma_S}m_S\right)} =& \frac{(\nu+r+m_B)m_B(\kappa+\nu+r+m_S)(\kappa+\nu+r+m_B)(\delta_g-\delta_b)}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2} \\
&+\frac{(\nu+r+m_B)^2m_B^2x}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2} \\
&>0.
\end{aligned} \tag{143}
$$

On the other hand,

$$
\begin{aligned}
\frac{\partial V_{hb}}{\partial m_S} =& \frac{(\nu+r+m_B)m_B[r(\kappa+\nu+r+m_B)+\kappa m_B]x}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2} \\
&-\frac{(\nu+r+m_B)(\kappa+\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B(\delta_g-\delta_b)}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2}.
\end{aligned} \tag{144}
$$

Thus, $\partial V_{hb}/\partial m_S \le 0$ if and only if:

$$
\frac{\delta_g-\delta_b}{x}\ge\frac{r(r+\kappa+\nu+m_B)+\kappa m_B}{(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)(r+\kappa+\nu+m_B)}, \tag{145}
$$

which is the condition for *hb* agents to be willing to sell their asset. Finally,

$$
\begin{aligned}
\frac{\partial V_{hb}}{\partial m_B} = & -\frac{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)m_B+\kappa m_B^2+(\nu+r+m_B)r(\kappa+\nu+r)]x}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2} \\
& +\frac{(\kappa+\nu+r+m_S)\{(\kappa+\nu+r+m_S)(\kappa+\nu+r)\kappa+(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)[(\nu+r+m_B)(\kappa+\nu+r+m_B)+\kappa m_B]\}(\delta_g-\delta_b)}{\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\}^2}
\end{aligned}
\tag{146}
$$

It follows that $\partial V_{hb}/\partial m_B \geq 0$ if and only if:

$$
\frac{\delta_g-\delta_b}{x} \geq \frac{r[(\nu+r+m_B)(r+\kappa+\nu+m_B)+\kappa m_B]+\kappa m_B^2}{(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)[(\nu+r+m_B)(\kappa+\nu+r+m_B)+\kappa m_B]+(\kappa+\nu+r)\kappa(\kappa+\nu+r+m_S)} \tag{147}
$$

Note that the right-hand side satisfies:

$$
\begin{aligned}
& \frac{r[(\nu+r+m_B)(r+\kappa+\nu+m_B)+\kappa m_B]+\kappa m_B^2}{(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)[(\nu+r+m_B)(\kappa+\nu+r+m_B)+\kappa m_B]+(\kappa+\nu+r)\kappa(\kappa+\nu+r+m_S)} \\
< & \frac{r}{r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S}+\frac{\kappa m_B}{(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)(\kappa+\nu+r+m_B)(1+\frac{\nu+r}{m_B})} \\
< & \frac{r(r+\kappa+\nu+m_B)+\kappa m_B}{(r+\kappa+\nu+\frac{\gamma_{lg}}{\gamma_S}m_S)(r+\kappa+\nu+m_B)}.
\end{aligned}
\tag{148}
$$

Employing the inequality ensuring that *hb* agents are willing to sell their asset, it follows that (147) is satisfied. Thus, we have have that:

$$
\frac{\mathrm{d}m_B}{\mathrm{d}\lambda} \geq 0, \qquad \frac{\partial V_{hb}}{\partial m_B} \geq 0, \qquad \frac{\mathrm{d}m_S}{\mathrm{d}\lambda} < 0, \qquad \frac{\partial V_{hb}}{\partial m_S} \leq 0, \qquad \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{\gamma_{lg}}{\gamma_S}m_S\right) > 0, \qquad \frac{\partial V_{hb}}{\partial\left(\frac{\gamma_{lg}}{\gamma_S}m_S\right)} > 0, \tag{149}
$$

implying that V_{hb} is strictly increasing in λ. Therefore, the left-hand side of (133) is increasing λ. ■

Proof of Lemma 5.5.

1. In equilibrium E, the threshold value ϕ^* is determined by

$$
\phi^{*E} = \frac{r(\kappa+\nu+r+m_B^E)+\kappa m_B^E}{(\kappa+\nu+r+\lambda m_S^E)(\kappa+\nu+r+m_B^E)} \tag{150}
$$

   By Lemma 5.4, m_B^E and m_S^E do not depend on λ. Hence, by inspection of (150), the threshold value of ϕ is decreasing in λ.

2. Let us begin by showing that λ^{*E} is increasing in ϕ. By Lemma 5.3, λ^{*E} satisfies:

$$
\lambda^{*E}(V_{hg}-V_{lg})+(1-\lambda^{*E})(V_{hb}-V_{lg}) = \frac{(1-\lambda^{*E})(\kappa+\nu+r+m_S)}{\kappa+\nu+r+(1-\lambda^{*E})m_S}(V_{hb}-V_{lb}). \tag{151}
$$

   Substituting the expressions for $V_{hg}-V_{lg}$ (from Lemma 5.2), $V_{hb}-V_{lg}$ and $V_{hb}-V_{lb}$ (from the proof of Proposition 5.2) yields:

$$
\begin{aligned}
& \lambda^{*E}\frac{x}{\kappa+\nu+r}+(1-\lambda^{*E})\left[\frac{x}{\kappa+\nu+r}-\frac{\kappa+\nu+r+m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}(\delta_g-\delta_b)\right] \\
= & \frac{(1-\lambda^{*E})(\kappa+\nu+r+m_S)}{\kappa+\nu+r+(1-\lambda^{*E})m_S}\left[\frac{x}{\kappa+\nu+r}-\frac{m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}(\delta_g-\delta_b)\right].
\end{aligned}
\tag{152}
$$

   Combining terms, we obtain:

$$\frac{\lambda^{*E}(\kappa+\nu+r)}{\kappa+\nu+r+(1-\lambda^{*E})m_S}\frac{1}{\kappa+\nu+r}$$
$$=\left[\frac{(1-\lambda^{*E})(\kappa+\nu+r+m_B)}{r(\kappa+\nu+r+m_B)+\kappa m_B}-\frac{(1-\lambda^{*E})(\kappa+\nu+r+m_S)}{\kappa+\nu+r+(1-\lambda^{*E})m_S}\frac{m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}\right]\phi. \tag{153}$$

Multiplying through by $\kappa+\nu+r+(1-\lambda^{*E})m_S$ gives:

$$\lambda^{*E}=\frac{(1-\lambda^{*E})(\kappa+\nu+r+m_B)(\kappa+\nu+r+(1-\lambda^{*E})m_S)-(1-\lambda^{*E})(\kappa+\nu+r+m_S)m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}\phi. \tag{154}$$

Simplifying, we obtain

$$\frac{\lambda^{*E}}{1-\lambda^{*E}}=\phi\frac{(\kappa+\nu+r)(\kappa+\nu+r+m_S)-\lambda^{*E}m_S(\kappa+\nu+r+m_B)}{r(\kappa+\nu+r+m_B)+\kappa m_B} \tag{155}$$

Given that m_B and m_S are independent of ϕ, it follows by inspection that λ^{*E} is increasing in ϕ.

Turning to λ^{*H} , Lemma 5.3 implies that

$$\frac{\gamma_{lg}}{\gamma_S}(V_{hg}-V_{lg})+\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb}-V_{lg})=\frac{\frac{\gamma_{lb}}{\gamma_S}(\kappa+\nu+r+m_S)}{\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S}(V_{hb}-V_{lb}). \tag{156}$$

By substituting the expressions for $V_{hg}-V_{lg}$ (from Lemma 5.2) and $V_{hb}-V_{lg}$ (from the proof of Proposition 5.3) we obtain

$$\begin{aligned}
&\frac{\gamma_{lg}}{\gamma_S}\frac{x}{\kappa+\nu+r}\\
&+\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)\Bigg[\left(1-\frac{(\nu+r+m_B)(\kappa+\nu+r)m_B}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}\right)\frac{x}{\kappa+\nu+r}\\
&\qquad-\frac{(\kappa+\nu+r+m_S)(\kappa+\nu+r+m_B)}{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B}(\delta_g-\delta_b)\Bigg]\\
&=\frac{\frac{\gamma_{lb}}{\gamma_S}(\kappa+\nu+r+m_S)}{\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S}(V_{hb}-V_{lb})
\end{aligned} \tag{157}$$

Substituting the expression for $V_{hb}-V_{lb}$ (from the proof of Proposition 5.3) and simplifying yields

$$\begin{aligned}
&\frac{1-\frac{\gamma_{lb}}{\gamma_S}}{\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S}x\\
&+\frac{\frac{\gamma_{lb}}{\gamma_S}(\kappa+\nu+r+m_S)(r+m_B)m_B-\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)(\nu+r+m_B)m_B}{\left\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\right\}(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)}x\\
&=\frac{\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)\left[(\kappa+\nu+r+m_S)(\kappa+\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)-\frac{\gamma_{lb}}{\gamma_S}(\kappa+\nu+r+m_S)m_Sm_B\right](\delta_g-\delta_b)}{\left\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\right\}(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)}
\end{aligned} \tag{158}$$

Simplifying further, we get

$$\frac{(\kappa+\nu+r+m_S)\left\{\frac{\gamma_{lg}}{\gamma_S}(r+m_B)(\kappa+\nu+r+m_B)+\frac{\gamma_{hb}}{\gamma_S}(\kappa+\nu+r)(r+m_B)-\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)\nu m_B\right\}}{\left\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\right\}(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)}$$
$$-\frac{\left(1-\frac{\gamma_{lg}}{\gamma_S}\right)(\kappa+\nu+r+m_S)(\kappa+\nu+r)(\kappa+\nu+r+m_B+\frac{\gamma_{lb}}{\gamma_S}m_S)\phi}{\left\{(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B]+(\nu+r+m_B)(\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S)m_B\right\}(\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S)}=0 \tag{159}$$

Given that the left-hand side is decreasing in ϕ and, by the proof of Proposition 5.3, increasing in λ, it follows that λ^{*H} is increasing in ϕ.

■

Proof of Proposition 5.4.
In proving the proposition, we consider separately equilibria in which *hb* agents participate in the market and equilibria in which only low-valuation agents are willing to sell their asset. Let us begin from the former.

Rewriting the expressions for $\Gamma(\bar{p}_{lg})$ and $\Gamma(\bar{p}_{lb})$ in the proof of Lemma 5.3 using (19) and the fact that *hb* agents participate in the market yields:

$$\Gamma(\bar{p}_{lg})=\frac{\kappa+\nu+r}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}\left[\frac{\gamma_{lg}}{\gamma_S}(V_{hg}-V_{lg})+\left(\sigma_{hb}(\bar{p}_{lg})\frac{\gamma_{hb}}{\gamma_S}+\frac{\gamma_{lb}}{\gamma_S}\right)(V_{hb}-V_{lg}-V_{hn}+V_{ln})\right] \tag{160}$$

$$\Gamma(\bar{p}_{lb})=\frac{\gamma_{lb}}{\gamma_S}\left(V_{hb}-V_{lb}-\frac{m_S}{\kappa+\nu+r}\Gamma(\bar{p}_{lg})\right) \tag{161}$$

Note that we have used the fact that by Lemma A.4 *lb* and *lg* agents accept offers equal to or greater than their reservation prices with probability one. Simplifying, we obtain

$$\Gamma(\bar{p}_{lg})\geq\Gamma(\bar{p}_{lb})\Leftrightarrow\frac{\kappa+\nu+r+\frac{\gamma_{lb}}{\gamma_S}m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}[\gamma_{lg}(V_{hg}-V_{lg})+(\sigma_{hb}(\bar{p}_{lg})\gamma_{hb}+\gamma_{lb})(V_{hb}-V_{lg}-V_{hn}+V_{ln})]\geq\gamma_{lb}(V_{hb}-V_{lb}) \tag{162}$$

First note that $m_B=\mu M(\gamma_B,\gamma_S)/\gamma_S$ and $m_S=\mu M(\gamma_B,\gamma_S)/\gamma_B$ are bounded between strictly positive lower and upper bounds as the two arguments are strictly positive and bounded from above. Thus, given that:

$$\gamma_{lg}=\frac{\kappa\lambda A}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\qquad \gamma_{lb}=\frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B}\qquad V_{hg}-V_{lg}=\frac{x}{\kappa+\nu+r} \tag{163}$$

and $V_{hb}-V_{lg}-V_{hn}+V_{ln}\leq 0$ as *hb* agents participate in the market, we can conclude that the left-hand side of (162) tends to a non-positive limit with $\lambda\to 0$. The right-hand side of (162), on the contrary, remains strictly positive as $\lambda\to 0$ since $V_{hb}>V_{lb}$ by Lemma A.1. Thus, by continuity, (162) cannot be satisfied for λ sufficiently small, implying that peaches cannot be traded as buyers do not find it convenient to offer the high price $\bar{p}_{lg}$.

To prove the result in the latter case when only low-valuation owners participate in the market, we make use of the explicit expression for the buyers' indifference condition derived in the proof of Proposition B.2:

$$\frac{\lambda}{1-\lambda}\frac{\kappa+\nu+m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\geq\phi\frac{(\kappa+\nu+r)\left(\kappa+\nu+r+\frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B}\frac{m_S}{\gamma_S}\right)-\kappa\lambda A\frac{m_S}{\gamma_S}\frac{\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \tag{164}$$

Note that as $\lambda \to 0$ the left-hand side of (164) tends to zero whereas its right-hand side tends to a strictly positive limit. Thus, also in this case, for λ sufficiently small, the buyers do not find it convenient to offer the high price $\bar{p}_{lg}$ and peaches cannot be traded. ■

Proof of Proposition 6.1.

Let us first compare m_B across the two types of equilibria. Given that we are comparing equilibria in which $\sigma_{hb}(\bar{p}_{lg}) > 0$ and $\sigma_{hn}(\bar{p}_{lg}) = 1$ to equilibrium E, characterized by $\sigma_{hb}(\bar{p}_{lg}) = 0$ and $\sigma_{hn}(\bar{p}_{lg}) = 1$, it is sufficient to show how m_B varies with $\sigma_{hb}(\bar{p}_{lg})$. First note that from the equilibrium condition $m_B\gamma_S = \mu M(\gamma_B, \gamma_S)$, we obtain

$$\frac{\mathrm{d}m_B}{\mathrm{d}\gamma_S}\left(\gamma_S - \frac{\mathrm{d}\gamma_B}{\mathrm{d}m_B}\mu M_1\right) = \mu M_2 - m_B \leq 0, \tag{165}$$

since

$$\frac{\mathrm{d}\gamma_B}{\mathrm{d}m_B} = -\frac{1}{\kappa+\nu}\frac{\kappa A}{(\kappa+\nu+m_B)^2} < 0 \tag{166}$$

Hence, given that γ_B does not depend directly on $\sigma_{hb}(\bar{p}_{lg})$, if $\mathrm{d}\gamma_S/\sigma_{hb}(\bar{p}_{lg}) \leq 0$, then $\mathrm{d}m_B/\sigma_{hb}(\bar{p}_{lg}) \geq 0$. However, this would constitute a contradiction as the equilibrium condition $m_B\gamma_S = \mu M(\gamma_B, \gamma_S)$ implies that

$$\frac{\mathrm{d}m_B}{\sigma_{hb}(\bar{p}_{lg})}\left\{\frac{(\kappa+\nu)\kappa A}{(\kappa+\nu+m_B)^2} + \sigma_{hb}(\bar{p}_{lg})\frac{[(\nu+2m_B)(\kappa+\nu)+m_B^2](1-\lambda)A}{(\kappa+\nu+m_B)^2}\right\} + \frac{(\nu+m_B)(1-\lambda)A}{\kappa+\nu+m_B} \tag{167}$$

$$=\mu\frac{\mathrm{d}\gamma_B}{\mathrm{d}\sigma_{hb}(\bar{p}_{lg})}M_1 + \mu\frac{\mathrm{d}\gamma_S}{\mathrm{d}\sigma_{hb}(\bar{p}_{lg})}M_2, \tag{168}$$

where

$$\frac{\mathrm{d}\gamma_B}{\mathrm{d}\sigma_{hb}(\bar{p}_{lg})} = \frac{\mathrm{d}m_B}{\sigma_{hb}(\bar{p}_{lg})}\frac{\mathrm{d}\gamma_B}{\mathrm{d}m_B} \leq 0. \tag{169}$$

Thus, we can conclude that $\mathrm{d}\gamma_S/\sigma_{hb}(\bar{p}_{lg}) > 0$, implying that $\mathrm{d}m_B/\sigma_{hb}(\bar{p}_{lg}) \leq 0$. Note that the inequality is strict whenever $\mu M_2 < m_B$, i.e. for all competitive matching technologies.

Having established that m_B is lower in equilibria in which $\sigma_{hb}(\bar{p}_{lg}) > 0$ and $\sigma_{hn}(\bar{p}_{lg}) = 1$ than in equilibrium E, we can prove the four statements in the proposition.

1. Given that $\bar{p}_{lg} = V_{lg} - V_{ln}$ and V_{lg} does not vary across equilibria, it is sufficient to prove that $V_{ln}^E \geq V_{ln}^A$, where A denotes any of the alternative equilibria under consideration. Given that $V_{ln} = \frac{\nu}{r}(V_{hn} - V_{ln})$, it is equivalent to show that $V_{hn}^E - V_{ln}^E \geq V_{hn}^A - V_{ln}^A$.

Let us begin by noting that the value functions in (19) imply that

$$V_{hn} - V_{ln} = \frac{m_S}{\kappa+\nu+r}\left[\frac{\gamma_{lg}}{\gamma_S}(V_{hg} - V_{lg} - V_{hn} + V_{ln}) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(V_{hb} - V_{lg} - V_{hn} + V_{ln})\right] \tag{170}$$

Rearranging and rewriting in terms of the reservation prices yields

$$V_{hn} - V_{ln} = \frac{\frac{\gamma_{lg}}{\gamma_S}m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}(V_{hg} - V_{lg}) + \frac{\left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)m_S}{\kappa+\nu+r+\frac{\gamma_{lg}}{\gamma_S}m_S}(\bar{p}_{hb} - \bar{p}_{lg}) \tag{171}$$

Note that the second term is positive in equilibrium E and negative in the alternative equilibria as they are characterized by $\sigma_{hb}(\bar{p}_{lg}) > 0$. Thus, to prove that $V_{hn}^E - V_{ln}^E \geq V_{hn}^A - V_{ln}^A$, it is sufficient to establish that $\frac{\gamma_{lg}}{\gamma_S}m_S$ is lower in the alternative equilibria than in E. Using the expressions for γ_{lg} and γ_B from Lemma

5.1, which continue to hold also in equilibria A, we obtain

$$\frac{\gamma_{lg}}{\gamma_S}m_S = \gamma_{lg}\frac{m_B}{\gamma_B} \tag{172}$$

$$= \kappa\lambda A(\kappa+\nu)\frac{m_B}{(\nu-\kappa A)(\kappa+\nu+m_B)+(\kappa+\nu)\kappa A} \tag{173}$$

Given that $m_B^A \leq m_B^E$, it follows by inspection that $(\frac{\gamma_{lg}}{\gamma_S}m_S)^A \leq (\frac{\gamma_{lg}}{\gamma_S}m_S)^E$, with the inequalities being strict for all competitive matching technologies. Thus, we have shown that $\bar{p}_{lg}^A \geq \bar{p}_{lg}^E$.

2. By Lemma 5.1, the measure of buyers in both equilibria E and A is given by

$$\gamma_B = \frac{1}{\kappa+\nu}\left(\nu - \frac{m_B}{\kappa+\nu+m_B}\kappa A\right). \tag{174}$$

Thus, given that $m_B^E \geq m_B^A$, $\gamma_B^A \geq \gamma_B^E$. As to the measure of sellers, we have:

$$\gamma_S^E = \frac{\kappa A}{\kappa+\nu+m_B^E}, \qquad \gamma_S^A = \frac{\kappa A}{\kappa+\nu+m_B^A} + \sigma_{hb}(\bar{p}_{lg})\frac{(\nu+m_B^A)(1-\lambda)A}{\kappa+\nu+m_B^A} \tag{175}$$

Making use of the fact that $m_B^E \geq m_B^A$, allows us to conclude that $\gamma_S^A \geq \gamma_S^E$. Given that both arguments of $M(\cdot,\cdot)$ assume higher values in any equilibrium A than in equilibrium E, we have established that $M(\gamma_B^A,\gamma_S^A) \geq M(\gamma_B^E,\gamma_S^E)$. Note that, by the same argument as above, the inequality is strict for all competitive matching technologies.

3. Follows directly from $m_B^E \geq m_B^A$.

4. Lemma 5.1 implies that:

$$\frac{\gamma_{lg}^E}{\gamma_S^E} = \lambda, \qquad \frac{\gamma_{lg}^A}{\gamma_S^A} = \lambda\frac{\kappa}{\kappa+(1-\lambda)(\nu+m_B)} \tag{176}$$

Thus, $\frac{\gamma_{lg}^H}{\gamma_S^H} \leq \frac{\gamma_{lg}^E}{\gamma_S^E}$ and the inequality is strict for $\lambda < 1$. ■

The statement in the proposition about all non-competitive matching technologies is true due to $m_B^A = m_B^E$ and $\phi^{*E} = \phi^{*H}$. The former feature ensures that expected time to sell is the same across the two equilibria. The latter property, on the other hand, implies that equilibrium E and an equilibrium in which $\sigma_{hb}(\bar{p}_{lg}) > 0$ and $\sigma_{hn}(\bar{p}_{lg}) = 1$ can only co-exist when $\bar{p}_{lg} = \bar{p}_{lb}$. This along with $m_B^A = m_B^E$ allows us to conclude that $\bar{p}_{lg}$ takes the same value in the two equilibria.

Proof of Lemma 6.1.

Consider a strategy profile σ and let q_{ji}^S and q_{jn}^B be the probability that an agent ji belongs to the set of actively participating sellers S or buyers B. In turn, the equilibrium masses satisfy:

$$\dot{\gamma}_{hi} = \nu\gamma_{li} - \kappa\gamma_{hi} + m_B q_{li}^S\gamma_{li}\frac{q_{hn}^B\gamma_{hn}}{\gamma_B}\int\sigma_{hn}(p|\mathscr{I})\sigma_{li}(p)\mathrm{d}p - m_B q_{hi}^S\gamma_{hi}\frac{q_{ln}^B\gamma_{ln}}{\gamma_B}\int\sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p)\mathrm{d}p = 0 \quad i=b,g \tag{177}$$

$$\dot{\gamma}_{hn} = \nu\gamma_{ln} - \kappa\gamma_{hn} - m_S q_{hn}^B\gamma_{hn}\left[\sum_{i=b,g}\frac{q_{li}^S\gamma_{li}}{\gamma_S}\int\sigma_{hn}(p|\mathscr{I})\sigma_{li}(p)\mathrm{d}p\right] + m_S q_{ln}^B\gamma_{ln}\left[\sum_{i=b,g}\frac{q_{hi}^S\gamma_{hi}}{\gamma_S}\int\sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p)\mathrm{d}p\right] = 0 \tag{178}$$

The corresponding values for γ_{lg}, γ_{lb} and γ_{ln} follow immediately from $\gamma_{hg}+\gamma_{lg} = \lambda A$, $\gamma_{hb}+\gamma_{lb} = (1-\lambda)A$ and $\gamma_{hn}+\gamma_{ln} = 1$.

For a generic strategy profile σ the corresponding value functions are:

$$rV_{hi} = \delta_i + \kappa(V_{li} - V_{hi}) + q^S_{hi} m_B \int \left[\frac{q^B_{hn}\gamma_{hn}}{\gamma_B} \sigma_{hn}(p|\mathscr{I})\sigma_{hi}(p) + \frac{q^B_{ln}\gamma_{ln}}{\gamma_B} \sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p) \right] (p - V_{hi} + V_{hn})\mathrm{d}p \quad i = b, g \tag{179}$$

$$rV_{li} = \delta_i - x + \nu(V_{hi} - V_{li}) + q^S_{li} m_B \int \left[\frac{q^B_{hn}\gamma_{hn}}{\gamma_B} \sigma_{hn}(p|\mathscr{I})\sigma_{li}(p) + \frac{q^B_{ln}\gamma_{ln}}{\gamma_B} \sigma_{ln}(p|\mathscr{I})\sigma_{li}(p) \right] (p - V_{li} + V_{ln})\mathrm{d}p \quad i = b, g \tag{180}$$

$$rV_{hn} = \kappa(V_{ln} - V_{hn}) + q^B_{hn} m_S \sum_{i=b,g} \int \left[\frac{q^S_{hi}\gamma_{hi}}{\gamma_S} \sigma_{hn}(p|\mathscr{I})\sigma_{hi}(p) + \frac{q^S_{li}\gamma_{li}}{\gamma_S} \sigma_{hn}(p|\mathscr{I})\sigma_{li}(p) \right] (V_{hi} - p - V_{hn})\mathrm{d}p \tag{181}$$

$$rV_{ln} = \nu(V_{hn} - V_{ln}) + q^B_{ln} m_S \sum_{i=b,g} \int \left[\frac{q^S_{hi}\gamma_{hi}}{\gamma_S} \sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p) + \frac{q^S_{li}\gamma_{li}}{\gamma_S} \sigma_{ln}(p|\mathscr{I})\sigma_{li}(p) \right] (V_{li} - p - V_{ln})\mathrm{d}p \tag{182}$$

Substituting the equilibrium expression $\frac{m_S}{\gamma_S} = \frac{m_B}{\gamma_B}$ and using the value functions above, it is straightforward to obtain the utilitarian welfare:

$$\begin{aligned}
W(\sigma) =& \sum_{ji} \gamma_{ji} V_{ji} \\
=& \frac{1}{r}(\gamma_{lg} + \gamma_{hg})\delta_g + (\gamma_{lb} + \gamma_{hb})\delta_b - (\gamma_{lg} + \gamma_{lb})x \\
&+ (V_{hi} - V_{li}) \left[m_B q^S_{li}\gamma_{li} \frac{q^B_{hn}\gamma_{hn}}{\gamma_B} \int \sigma_{hn}(p|\mathscr{I})\sigma_{li}(p)\mathrm{d}p - m_B q^S_{hi}\gamma_{hi} \frac{q^B_{ln}\gamma_{ln}}{\gamma_B} \int \sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p)\mathrm{d}p - \kappa\gamma_{hi} + \nu\gamma_{li} \right] \\
&+ (V_{hn} - V_{ln}) \left\{ \nu\gamma_{ln} - \kappa\gamma_{hn} - m_S q^B_{hn}\gamma_{hn} \left[\sum_{i=b,g} \frac{q^S_{li}\gamma_{li}}{\gamma_S} \int \sigma_{hn}(p|\mathscr{I})\sigma_{li}(p)\mathrm{d}p \right] + m_S q^B_{ln}\gamma_{ln} \left[\sum_{i=b,g} \frac{q^S_{hi}\gamma_{hi}}{\gamma_S} \int \sigma_{ln}(p|\mathscr{I})\sigma_{hi}(p)\mathrm{d}p \right] \right\} \\
=& \frac{1}{r}[(\gamma_{lg} + \gamma_{hg})\delta_g + (\gamma_{lb} + \gamma_{hb})\delta_b - (\gamma_{lg} + \gamma_{lb})x] + \sum_{i=b,g,n} (V_{hi} - V_{li})\dot{\gamma}_{hi}
\end{aligned} \tag{183}$$

Substituting $\gamma_{lg} + \gamma_{hg} = \lambda A, \gamma_{lb} + \gamma_{hb} = (1-\lambda)A$ and noting that $\dot{\gamma}_{ji} = 0$ for every ji we get the expression for utilitarian welfare:

$$W(\sigma) = \frac{A}{r}[\lambda\delta_g + (1-\lambda)\delta_b] - \frac{x}{r}[\gamma_{lg}(\sigma) + \gamma_{lb}(\sigma)] \tag{184}$$

where we write $\gamma_{li}(\sigma)$ to better stress the dependence of the mass of low-valuation asset holders on the strategy profile σ. ■

Proof of Proposition 6.2.

Let σ^C the strategy profile corresponding to the complete information equilibrium in Proposition 3.1. In this equilibrium we have $q^B_{hn} = q^S_{lg} = q^S_{lb} = 1$, $q^B_{ln} = q^S_{hg} = q^S_{hb} = 0$ and $q^T_{lg,h} = q^T_{lb,h} = 1$. The corresponding matching rate m^C_B solves $m_B\gamma^C_S = m^C_S\gamma^C_B = \mu M(\gamma^C_B, \gamma^C_S)$ where $\gamma^C_B = \gamma_{hn}(\sigma^C)$ and $\gamma_S = \gamma_{lg}(\sigma^C) + \gamma_{lb}(\sigma^C)$.

To show that σ^C achieves the maximum welfare value among the set of admissible strategy profiles, suppose per contra there exists a $\sigma' \neq \sigma^C$ such that $W(\sigma') \geq W(\sigma^C)$. By equation (177) for a generic strategy profile σ we have:

$$(\kappa + \nu) \sum_{i=b,g} \gamma_{li}(\sigma) = \kappa A - \frac{m_B(\sigma)}{\gamma_B(\sigma)} \left[q^B_{hn}(\sigma)\gamma_{hn}(\sigma) \sum_{i=b,g} q^S_{li}(\sigma)q^T_{li,h}(\sigma)\gamma_{li}(\sigma) - q^B_{ln}(\sigma)\gamma_{ln}(\sigma) \sum_{i=b,g} q^S_{hi}(\sigma)q^T_{hi,l}(\sigma)\gamma_{hi}(\sigma) \right] \tag{185}$$

Hence, it is sufficient to study under which conditions, if any, it is possible to have:

$$\frac{m^C_B}{\gamma^C_B} \gamma^C_{hn} \sum_{i=b,g} \gamma^C_{li} \leq \frac{m_B}{\gamma_B} \left[q^B_{hn}\gamma_{hn} \sum_{i=b,g} q^S_{li}q^T_{li,h}\gamma_{li} - q^B_{ln}\gamma_{ln} \sum_{i=b,g} q^S_{hi}q^T_{hi,l}\gamma_{hi} \right] \tag{186}$$

From the equilibrium condition $m_B\gamma_S = \mu M(\gamma_B, \gamma_S)$ it follows that $\frac{m_B}{\gamma_B} = \frac{\mu M(\gamma_B, \gamma_S)}{\gamma_B\gamma_S}$. Therefore, a (weak) upper bound for the RHS of equation (186) is:

$$\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S} q^B_{hn}\gamma_{hn}\sum_{i=b,g} q^S_{li}q^T_{li,h}\gamma_{li} = \frac{M\left(\sum\limits_{j=l,h} q^B_{jn}\gamma_{hn},\ \sum\limits_{j=l,h}\sum\limits_{i=b,g} q^S_{ji}\gamma_{ji}\right)}{\left(\sum\limits_{j=l,h} q^B_{jn}\gamma_{hn}\right)\cdot\left(\sum\limits_{j=l,h}\sum\limits_{i=b,g} q^S_{ji}\gamma_{ji}\right)} q^B_{hn}\gamma_{hn}\sum_{i=b,g} q^S_{li}q^T_{li,h}\gamma_{li} \tag{187}$$

It is easy to show that the RHS expression in equation (187) is increasing in $q^B_{hn}\gamma_{hn}$ and $q^S_{li}\gamma_{li}$, $i=b,g$. By Lemma 6.1 $W(\sigma') \geq W(\sigma^C)$ holds if and only if $\sum\limits_{i=b,g}\gamma_{li}(\sigma') \leq \sum\limits_{i=b,g}\gamma_{li}(\sigma^C)$. Moreover, by equations (177)–(178), for every strategy profile σ it holds $\gamma_{hn}(\sigma) - \sum\limits_{i=b,g}\gamma_{li}(\sigma) = \frac{\nu-\kappa A}{\kappa+\nu}$, hence it also holds $\gamma_{hn}(\sigma') \leq \gamma_{hn}(\sigma^C)$. Therefore, a (weak) upper bound of equation (187) for equilibrium σ' is:

$$\frac{M\left(\gamma^C_{hn} + q^B_{ln}\gamma_{ln},\ \sum\limits_{i=b,g}\gamma^C_{li} + \sum\limits_{i=b,g} q^S_{hi}\gamma_{hi}\right)}{\left(\gamma^C_{hn} + q^B_{ln}\gamma_{ln}\right)\cdot\left(\sum\limits_{i=b,g}\gamma^C_{li} + \sum\limits_{i=b,g} q^S_{hi}\gamma_{hi}\right)}\gamma^C_{hn}\sum_{i=b,g}\gamma^C_{li} \tag{188}$$

In turn, by assumption $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ is decreasing in both arguments γ_B and γ_S. Therefore, an upper bound for equation (188) is obtained by setting $q^B_{ln}\gamma_{ln}$ and $\sum\limits_{i=b,g} q^S_{hi}\gamma_{hi}$ equal to zero, i.e. their minimum possible value. But then the upper bound in equation (188) coincides with $M(\gamma^C_B,\gamma^C_S)$ as $\gamma_B = \gamma^C_B$ and $\gamma_S = \gamma^C_S$. Therefore, $W(\sigma^C)$ is the maximum possible value among the set of admissible strategy profiles. It is a strict maximum whenever one of the upper bounds in equation (187)–(188) is strict. Hence, utilitarian welfare is equal to $W(\sigma^C)$ only if:

$$\left[q^B_{hn}\gamma_{hn}\sum_{i=b,g} q^S_{li}q^T_{li,h}\gamma_{li} - q^B_{ln}\gamma_{ln}\sum_{i=b,g} q^S_{hi}q^T_{hi,l}\gamma_{hi}\right] = \gamma^C_B\gamma^C_S \tag{189}$$

If $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ is constant, as in the DGP technology, equation (189) is a necessary and sufficient condition. If $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ is strictly decreasing in its arguments, as in the KW technology, then the condition in equation (189) is only necessary, and utilitarian welfare is equal to $W(\sigma^C)$ only if the additional conditions $q^B_{ln}=0$ and $q^S_{hi}=0$, $i=b,g$, hold. ■

Proof of Corollary 6.1.

We prove each statement:

1. Under equilibrium E we have $q^B_{hn} = q^S_{lg} = q^S_{lb} = 1$, $q^B_{ln} = q^S_{hg} = q^S_{hb} = 0$ and $q^T_{lg,h} = q^T_{lb,h} = 1$. Hence, by Proposition 6.2, utilitarian welfare is equal to $W(\sigma^C)$.

2. In equilibrium H we have $q^B_{ln}=0$, $q^B_{hn}\sum\limits_{i=b,g} q^S_{li} = 1$ and $q^S_{hb}=1$. The fact that $q^S_{hb}=1$ may create a difference from the first best only if $m^H_B \neq m^C_B$ as m_B is the only variable that may be different in the expressions for $\sum\limits_{i=b,g}\gamma_{li}(\sigma^H)$ and $\sum\limits_{i=b,g}\gamma_{li}(\sigma^C)$. By Lemma 5.4 it holds $m^H_B = m^E_B = m^C_B$ if $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ is constant (e.g. DGP technology), and $m^H_B < m^E_B = m^C_B$ if $\frac{M(\gamma_B,\gamma_S)}{\gamma_B\gamma_S}$ strictly decreasing (e.g. KW technology). Therefore, there is no welfare loss in the former case, whereas in the latter there is a social welfare loss equal to equation (32). The welfare loss is decreasing in λ because, by Lemma 5.4, m^H_B is increasing in λ whereas m^C_B is independent of it.

3. In a lemons market $q^T_{lg,h}=0$ and the social welfare loss is equal to

$$W(\sigma^C)-W(\sigma^L)=\frac{\kappa Ax}{r}\left[\frac{\lambda}{\kappa+\nu}+\frac{(1-\lambda)}{\kappa+\nu+m_B^L}-\frac{1}{\kappa+\nu+m_B^C}\right] \tag{190}$$

where m_B^L solves

$$\frac{\kappa A(1-\lambda)m_B}{\kappa+\nu+m_B}=\mu M\left(\frac{\nu-\frac{\kappa(1-\lambda)Am_B}{\kappa+\nu+m_B}}{\kappa+\nu},\frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B}\right) \tag{191}$$

Notice that for $\lambda=0$ we have $m_B^L=m_B^C$ and the welfare loss is equal to zero. By implicit differentiation of equation (191), we get

$$\frac{\partial m_B^L}{\partial\lambda}=\frac{\left[m_B^L-\mu M_2+\mu M_1\frac{m_B^L}{\kappa+\nu}\right](\kappa+\nu+m_B^L)}{(1-\lambda)(\kappa+\nu+\mu M_1+\mu M_2)}>0 \tag{192}$$

hence $m_B^L>m_B^C$ for every $\lambda\in(0,1)$. To prove the social welfare loss is positive we show that total derivative of equation (190) is strictly increasing with respect to λ, i.e.

$$\frac{\mathrm{d}[W(\sigma^C)-W(\sigma^L)]}{\mathrm{d}\lambda}=\frac{\kappa A}{r}\left[\frac{1}{\kappa+\nu}-\frac{1}{\kappa+\nu+m_B^L}-(1-\lambda)\frac{1}{(\kappa+\nu+m_B^L)^2}\frac{\partial m_B^L}{\partial\lambda}\right]>0 \tag{193}$$

Substituting equation (192) and simplifying we get:

$$\frac{\mathrm{d}[W(\sigma^C)-W(\sigma^L)]}{\mathrm{d}\lambda}=\frac{\kappa A}{r(\kappa+\nu)}\left[1-\frac{\kappa+\nu+\mu M_1}{\kappa+\nu+\mu M_1+\mu M_2}\right]>0 \tag{194}$$

as $M_1,M_2>0$.

■

Proof of Proposition 7.1.

To reduce notation we simply denote by V_{ji} the equilibrium value $V(ji,ji,\mathscr{M})$ for a ji agent under mechanism $\mathscr{M}$.

From the set of incentive compatibility constraints, the following constraint apply:

$$m_S^C[\lambda V_{hg}+(1-\lambda)V_{hb}-V_{hn}+t_T(hn)]\geq t_N(ln)-t_N(hn)\geq m_S^C[\lambda V_{lg}+(1-\lambda)V_{lb}-V_{ln}+t_T(hn)] \tag{195}$$

$$t_N(hi)-t_N(\theta')\geq\alpha(\theta')[t_T(\theta')-V_{hi}+V_{hn}]\qquad \theta'\neq hi,\qquad i=b,g \tag{196}$$

$$t_N(li)+m_B^C[t_T(li)-V_{li}+V_{ln}]\geq t_N(\theta')+\alpha(\theta')[t_T(\theta')-V_{li}+V_{ln}]\qquad \theta'\neq li,\qquad i=b,g \tag{197}$$

From condition (196) and the fact that $\alpha(hi)=0$ for $i=b,g$ it follows that $t_N(hg)=t_N(hb):=t_N(h)$. From condition (197) and $\alpha(li)=m_B^C$, $i=b,g$, it follows that $t_N(lg)+m_B^Ct_T(lg)=t_N(lb)+m_B^Ct_T(lb):=t(l)$, i.e. condition (40). Hence, the conditions (195)–(197) can be expressed as:

$$m_S^C[\lambda V_{hg}+(1-\lambda)V_{hb}-V_{hn}+t_T(hn)]\geq t_N(ln)-t_N(hn)\geq m_S^C[\lambda V_{lg}+(1-\lambda)V_{lb}-V_{ln}+t_T(hn)] \tag{198}$$

$$t(l)-m_B^C(V_{li}-V_{ln})\geq t_N(h)\geq t(l)-m_B^C(V_{hi}-V_{hn})\qquad i=b,g \tag{199}$$

In particular, constraint (199) requires $V_{hb}-V_{hn}\geq V_{lg}-V_{ln}$. We next verify when this condition holds. From the equations in (34) it follows immediately that:

$$V_{hi}-V_{li}=\frac{x+t_N(h)-t(l)+m_B^C(V_{li}-V_{ln})}{\kappa+\nu+r}\qquad i=b,g \tag{200}$$

Substituting equation (200) into (34) we get:

$$V_{li} - V_{ln} = \frac{(\kappa+\nu+r)\delta_i - (\kappa+r)[x-t(l)] + \nu t_N(h) - r(\kappa+\nu+r)V_{ln}}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} \qquad i=b,g \tag{201}$$

To get an expression for $V_{hb}-V_{hn}$, we use (34) to express V_{hn} as a function of V_{ln}, i.e. $V_{hn} = \frac{(\nu+r)V_{ln}-t_N(ln)}{\nu}$. Using equations (200) and (201) into (34) and rearranging we get:

$$V_{hi} - V_{hn} = \frac{(\kappa+\nu+r+m_B^C)\delta_i - \kappa[x-t(l)] - \frac{\nu+r+m_B^C}{\nu} r(\kappa+\nu+r)V_{ln} + (\nu+r+m_B^C)t_N(h)}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} + \frac{t_N(ln)}{\nu} \qquad i=b,g \tag{202}$$

By equations (202) and (201), the condition $V_{hb}-V_{hn} \geq V_{lg}-V_{ln}$ is equivalent to:

$$\frac{r(x-t(l)) - \frac{r+m_B^C}{\nu} r(\kappa+\nu+r)V_{ln} + (r+m_B^C)t_N(h)}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} + \frac{t_N(ln)}{\nu} \geq \frac{(\kappa+\nu+r)\delta_g - (\kappa+\nu+r+m_B^C)\delta_b}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} \tag{203}$$

Next we substitute for $t(l)$, $t_N(h)$ and $t_N(ln)$ in order to characterize the largest set of parameters for which inequality (203) can hold. For this purpose, as the RHS is a constant, transfers have to maximize the LHS of equation (203). First, set $t_N(h)$ as high as possible consistently with constraint (199), i.e. $t_N(h) = t(l) - m_B^C(V_{lg}-V_{ln})$. Substituting in equation (201) and plugging back into equation (203):

$$\frac{\frac{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}{\kappa+\nu+r} x - m_B^C\delta_g - rV_{ln}\frac{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}{\nu}}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} + \frac{t_N(ln)}{\nu} \geq \frac{(\kappa+\nu+r)\delta_g - (\kappa+\nu+r+m_B^C)\delta_b}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} \tag{204}$$

Substituting the expression $rV_{ln} = \nu(V_{hn}-V_{ln}) + t_N(ln)$ into equation (204) the $t_N(ln)$ terms cancel out:

$$\frac{\frac{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}{\kappa+\nu+r} x - m_B^C\delta_g}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} - (V_{hn}-V_{ln}) \geq \frac{(\kappa+\nu+r)\delta_g - (\kappa+\nu+r+m_B^C)\delta_b}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C} \tag{205}$$

Therefore, the condition is going to be satisfied for the largest set of parameters when $V_{hn}-V_{ln}$ is as low as possible; by equation (34) and (196) the lowest incentive compatible value is $V_{hn}-V_{ln}=0$. Therefore, rearranging inequality (205) with $V_{hn}-V_{ln}=0$ we get condition (39).

We now characterize the set of transfers satisfying and the IC, IR and budget balance constraints. When condition (39) is slack, imposing $V_{hn}-V_{ln}=0$ is not necessary to implement the first-best outcome, but it becomes so when equation (39) is binding, as shown in the previous paragraph.

Imposing $V_{hn}-V_{ln}=0$ and using (34), (201) and (202) we get:

$$\begin{aligned}
V_{lg} =& \frac{\delta_g + t_N(h)}{r} - \frac{\kappa+r}{\kappa+\nu+r}\frac{x}{r} \\
V_{hg} - V_{lg} =& \frac{x}{\kappa+\nu+r} \\
V_{lg} - V_{lb} =& \frac{\kappa+\nu+r}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g-\delta_b) \\
V_{hb} - V_{lg} =& \frac{x}{\kappa+\nu+r} - \frac{\kappa+\nu+r+m_B^C}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g-\delta_b) \\
V_{hn} = V_{ln} =& \frac{m_S^C[\lambda V_{hg} + (1-\lambda)V_{hb} + t_T(hn)] + t_N(hn)}{r+m_S^C} = \frac{t_N(ln)}{r}
\end{aligned} \tag{206}$$

To get the corresponding transfers, first consider:

$$
\begin{aligned}
0 = r(V_{hn} - V_{ln}) =& m_S^C \left[\lambda V_{hg} + (1-\lambda)V_{hb} + t_T(hn) - V_{hn}\right] + t_N(hn) - t_N(ln) \\
=& m_S^C \left[\lambda V_{hg} + (1-\lambda)V_{hb} + t_T(hn) - \frac{m_S^C\left[\lambda V_{hg} + (1-\lambda)V_{hb} + t_T(hn)\right] + t_N(hn)}{r + m_S^C}\right] + t_N(hn) - t_N(ln)
\end{aligned}
\tag{207}
$$

Rearranging and using equation (206) we get:

$$
\begin{aligned}
t_N(ln) =& \frac{r}{r+m_S^C}\left\{t_N(hn) + m_S^C\left[\lambda V_{hg} + (1-\lambda)V_{hb} + t_T(hn)\right]\right\} \\
=& \frac{r}{r+m_S^C}\left\{t_N(hn) + m_S^C\left[\lambda(V_{hg} - V_{lg}) + (1-\lambda)(V_{hb} - V_{lg}) + V_{lg} + t_T(hn)\right]\right\} \\
=& \frac{r}{r+m_S^C}\left\{t_N(hn) + m_S^C\left[t_T(hn) + \frac{1}{r}\left(\delta_g + t_N(h) - \frac{\kappa}{\kappa+\nu+r}x - (1-\lambda)\frac{r(\kappa+\nu+r+m_B^C)}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g - \delta_b)\right)\right]\right\} \geq 0
\end{aligned}
\tag{208}
$$

As $V_{hn} = V_{ln} = \frac{t_N(ln)}{r}$, the IR constraint is always satisfied when this lower bound is positive, guaranteed by condition (42). Similarly, using (206) and imposing the IR constraint for the lg agents we get:

$$
t_N(h) = t(l) - m_B^C(V_{lg} - V_{ln}) = t(l) - m_B^C\left(\frac{\delta_g + t_N(h)}{r} - \frac{\kappa+r}{\kappa+\nu+r}\frac{x}{r} - \frac{t_N(ln)}{r}\right) \geq 0 \tag{209}
$$

Rearranging and substituting $t(l) = t_N(li) + m_B^C t_T(li)$, $i = b, g$, we get:

$$
t_N(h) = t_N(hi) = \frac{r}{r+m_B^C}\left\{t_N(li) + m_B^C\left[t_T(li) - \frac{1}{r}\left(\delta_g - \frac{\kappa+r}{\kappa+\nu+r}x\right) - \frac{t_N(ln)}{r}\right]\right\} \geq 0 \tag{210}
$$

Finally, the designer's flow of net revenues from the mechanism is:

$$
\begin{aligned}
\sum_{\theta\in\tilde{S}\cup\tilde{B}} \gamma_\theta\left[t_N(\theta) + \alpha(\theta)t_T(\theta)\right] =& (\lambda A - \gamma_{lg})t_N(hg) + \gamma_{lg}\left[t_N(lg) + m_B^C t_T(lg)\right] + \left[(1-\lambda)A - \gamma_{lb}\right]t_N(hb) \\
&+ \gamma_{lb}\left[t_N(lb) + m_B^C t_T(lb)\right] + \gamma_{hn}\left[t_N(hn) + m_S^C t_T(hn)\right] + (1-\gamma_{hn})t_N(ln)
\end{aligned}
\tag{211}
$$

Substituting the transfer restrictions $t_N(hg) = t_N(hb)$ and $t_N(lg) + m_B^C t_T(lg) = t_N(lb) + m_B^C t_T(lb)$, recalling that $m_B^C(\gamma_{lg}^C + \gamma_{lb}^C) = m_S^C\gamma_{hn}^C = \mu M(\gamma_B^C, \gamma_S^C)$, and using the results in (206), the budget balance condition simplifies to:

$$
\mu M(\gamma_B^C, \gamma_S^C)\left[\frac{x}{\kappa+\nu+r} - (1-\lambda)\frac{\kappa+\nu+r+m_B^C}{r(\kappa+\nu+r+m_B^C)+\kappa m_B^C}(\delta_g - \delta_b)\right] \geq At_N(h) + t_N(ln) \tag{212}
$$

i.e. condition (43). ■

Proof of Corollary 7.1.

From the proof of Proposition 7.1, within the class of mechanism such that $V_{hn} - V_{ln} = 0$ net revenues are maximized when $t_N(hi)$ and $t_N(ln)$ are as low as possible, i.e. both are equal to zero to satisfy (208) and (210). In this case $V_{hn} = V_{ln} = 0$ so the IR constraints are both binding, and no other mechanism with $V_{hn} - V_{ln} > 0$ could improve total revenues. The transfers in (44) follow immediately from conditions (40)–(42). ■

Proof of Proposition 7.2.

1. Let us begin by solving for $\hat{V}_{jn}$. To do so, note that agents' values when certificates are traded satisfy:

$$r\hat{V}_{hc} = \hat{\delta} + \kappa(\hat{V}_{lc} - \hat{V}_{hc}) \tag{213}$$

$$r\hat{V}_{lc} = \hat{\delta} - x + \nu(\hat{V}_{hc} - \hat{V}_{lc}) + m_B[p_T(lc) + \hat{V}_{ln} - \hat{V}_{lc}] \tag{214}$$

$$r\hat{V}_{hn} = \kappa(\hat{V}_{ln} - \hat{V}_{hn}) + m_S[\hat{V}_{hc} - \hat{V}_{hn} - p_T(hn)] \tag{215}$$

$$r\hat{V}_{ln} = \nu(\hat{V}_{hn} - \hat{V}_{ln}), \tag{216}$$

From this system of equations, we immediately obtain:

$$\hat{V}_{hn} = \frac{m_S(\nu+r)\{(\kappa m_B p_T(lc) - [r(\kappa+\nu+r+m_B)+\kappa m_B]p_T(hn) + (\kappa+\nu+r+m_B)\hat{\delta} - \kappa x\}}{r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(\nu+r+m_B)]} \tag{217}$$

$$\hat{V}_{ln} = \frac{m_S\nu\{\kappa m_B p_T(lc) - [r(\kappa+\nu+r+m_B)+\kappa m_B]p_T(hn) + (\kappa+\nu+r+m_B)\hat{\delta} - \kappa x\}}{r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(\nu+r+m_B)]} \tag{218}$$

Given that these values do not depend on p_A, it must be the case that:

$$p_A - \hat{V}_{jn} = \frac{1}{r}\left(\delta_g - \frac{\kappa + r\mathbb{1}_{\{j=l\}}}{\kappa+\nu+r}x\right) \tag{219}$$

Otherwise, the designer could increase p_A marginally and make strictly higher profits while still satisfying all the constraints. Thus, given that the autarky value of a jg agent does not depend on the choice variables, the designer's problem becomes:

$$\max_{p_A, p_T}\{\hat{V}_{jn} + p_T(hn) + \Pi_h\} \tag{220}$$

$$\text{subject to} \quad \hat{V}_{hc} - p_T(hn) \geq 0 \tag{221}$$

$$\hat{V}_{ln} + p_T(lc) \geq \frac{1}{r}\left(\hat{\delta} - \frac{\kappa+r}{\kappa+\nu+r}x\right) \tag{222}$$

Let us consider slow and fast packaging in turn. The designer's objective function in a SP scheme up to a constant is:

$$\frac{m_S\nu\{(\kappa m_B p_T(lc) - [r(\kappa+\nu+r+m_B)+\kappa m_B]p_T(hn)\}}{r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(\nu+r+m_B)]} + p_T(hn) + \frac{\kappa m_B[p_T(hn) - p_T(lc)]}{r(\kappa+\nu+r+m_B)} \tag{223}$$

Note that this objective function is increasing in $p_T(hn)$ and decreasing in $p_T(lc)$ if and only if:

$$r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(\nu+r+m_B)] \geq m_S\nu r(\kappa+\nu+r+m_B) \tag{224}$$

$$\Leftrightarrow \quad r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(r+m_B)] \geq m_S\nu r m_B \tag{225}$$

$$\Leftrightarrow \quad r(\kappa+\nu+r+m_S)[r(\kappa+\nu+r+m_B)+\kappa m_B] \geq 0 \tag{226}$$

Clearly, this inequality is always satisfied. In a FP scheme, the designer's objective function up to a constant is:

$$\frac{m_S(\nu+r)\{(\kappa m_B p_T(lc) - [r(\kappa+\nu+r+m_B)+\kappa m_B]p_T(hn)\}}{r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B + m_S(\nu+r+m_B)]} + p_T(hn) + \frac{\kappa m_B[p_T(hn) - p_T(lc)]}{r(\kappa+\nu+r+m_B)} \tag{227}$$

In this case, the objective is increasing in $p_T(hn)$ and decreasing in $p_T(lc)$ if and only if:

$$r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B+m_S(\nu+r+m_B)] \geq m_S(\nu+r)r(\kappa+\nu+r+m_B) \tag{228}$$

$$\Leftrightarrow \quad r(\kappa+\nu+r)[r(\kappa+\nu+r+m_B)+\kappa m_B+m_S m_B] \geq m_S(\nu+r)rm_B \tag{229}$$

$$\Leftrightarrow \quad r(\kappa+\nu+r)r(\kappa+\nu+r+m_B)+r(\kappa+\nu+r+m_S)\kappa m_B \geq 0 \tag{230}$$

Also this inequality is always satisfied. Thus, we have established that both in SP and in FP schemes the designer's objective function is increasing in $p_T(hn)$ and decreasing in $p_T(lc)$. Consequently, in a revenue-maximizing scheme $p_T(hn) = \hat{V}_{hc}$. From this, it follows that $\hat{V}_{ln} = \hat{V}_{hn} = 0$. Thus, the revenue-maximizing prices for a certificate are given by:

$$p_T(lc) = \frac{1}{r}\left(\hat{\delta} - \frac{\kappa+r}{\kappa+\nu+r}x\right) \qquad p_T(hn) = \frac{1}{r}\left(\hat{\delta} - \frac{\kappa}{\kappa+\nu+r}x\right) \tag{231}$$

2. The feasibility of a SP scheme is determined by the highest price p_A at which hb agents prefer to keep their asset. This is due to the fact that we are requiring the average quality of the assets held by the designer to be equal to the average quality of the assets in the economy. If, in addition to low-valuation agents, also hb agents would be willing to sell their asset, the share of peaches held by the designer would be initially lower than λ. Given that in a revenue-maximizing SP scheme $\hat{V}_{ln} = \hat{V}_{hn} = 0$, hb agents prefer to keep their assets when:

$$\frac{1}{r}\left[\delta_b + \frac{\kappa m_B}{r(\kappa+\nu+r+m_B)+\kappa m_B}(\delta_g-\delta_b) - \frac{\kappa}{\kappa+\nu+r}x\right] \geq \frac{1}{r}\left(\delta_g - \frac{\kappa+r}{\kappa+\nu+r}x\right) = p_A \tag{232}$$

The left-hand side represents the value of keeping a lemon until transiting to the state of low-valuation and is equal to V_{hb} in equilibrium E. Rearranging yields the expression in the proposition. Note that a SP scheme, when feasible, yields a positive profit as:

$$-p_A + p_T(hn) + \Pi_h = \frac{x}{r}\left[\frac{r(\kappa+\nu+r+m_B)+\kappa m_B}{(\kappa+\nu+r)(\kappa+\nu+r+m_B)} - (1-\lambda)\phi\right] \tag{233}$$

This expression is positive as, for the scheme to be feasible, ϕ is required to be smaller than the first term inside the square brackets.

3. The feasibility of a FP scheme, on the other hand, is determined by a non-negative profits condition. In a FP scheme:

$$-p_A + p_T(hn) + \Pi_h = \frac{x}{r}\left[\frac{\kappa m_B}{(\kappa+\nu+r)(\kappa+\nu+r+m_B)} - (1-\lambda)\phi\right] \tag{234}$$

Requiring this to be positive yields the expression in the proposition.

4. This follows immediately by comparing the expressions in (233) and (234).

■

Proof of Proposition 7.3.

Given that we are considering fast packaging schemes, the price offered by the designer has to satisfy $p_A \geq V_{hg} - \hat{V}_{hn}$. Let us suppose that the designer offers $V_{hg} - \hat{V}_{hn}$ and check whether a private buyer would have an incentive to match this offer. Offering $V_{hg} - \hat{V}_{hn}$, a private buyer obtains at most (for $\pi_{hn}(g,p) = 1$)

$\hat{V}_{hn}$. Any offer below that of the designer is rejected, in which case the buyer purchases a certificate at price p_C and his value is $\hat{V}_{hc} - p_C$. Using the expression in the proof of Proposition 3.1, we obtain:

$$p_C = \beta(\hat{V}_{hc} - \hat{V}_{hn}) + (1-\beta)(\hat{V}_{lc} - \hat{V}_{ln}) \tag{235}$$

A buyer prefers not to match the designer's offer when

$$\begin{aligned} \hat{V}_{hc} - p_C &\geq \hat{V}_{hn} \\ \Leftrightarrow \quad \beta\hat{V}_{hn} + (1-\beta)(\hat{V}_{hc} - \hat{V}_{lc} + \hat{V}_{ln}) &\geq \hat{V}_{hn} \\ \Leftrightarrow \quad (1-\beta)(\hat{V}_{hc} - \hat{V}_{lc} + \hat{V}_{ln} - \hat{V}_{hn}) &\geq 0 \end{aligned} \tag{236}$$

Using the results in the proof of Proposition 3.1, we get:

$$\hat{V}_{hc} - \hat{V}_{lc} + \hat{V}_{ln} - \hat{V}_{hn} = \frac{r(\kappa+\nu+r)}{\kappa+\nu+r+(1-\beta)m_S}(\hat{V}_{hc} - \hat{V}_{lc}) > 0 \tag{237}$$

Thus, (236) is satisfied, implying that a private buyer prefers not to match the designer's offer. What remains is to derive a condition for the feasibility of the scheme. Given that the designer can commit to making each buyer a single take-it-or-leave-it offer, the outside option of a peach holder is equal to his value under autarky. Therefore, the feasibility condition becomes:

$$-p_A + p_C = -\left[\frac{1}{r}\left(\delta_g - \frac{\kappa}{\kappa+\nu+r}x\right) - \hat{V}_{hn}\right] + \beta(\hat{V}_{hc} - \hat{V}_{hn}) + (1-\beta)(\hat{V}_{lc} - \hat{V}_{ln}) \geq 0 \tag{238}$$

Using the results in the proof of Proposition 3.1 yields

$$\begin{aligned} \hat{V}_{hc} - \frac{1}{r}\left(\delta_g - \frac{\kappa}{\kappa+\nu+r}x\right) &= \frac{1}{r}\left[\frac{\kappa}{\kappa+\nu+r}\frac{\beta m_B}{\kappa+\nu+r+(1-\beta)m_S+\beta m_B}x - (1-\lambda)(\delta_g-\delta_b)\right] \\ \hat{V}_{lc} + \hat{V}_{hn} - \hat{V}_{ln} - \frac{1}{r}\left(\delta_g - \frac{\kappa}{\kappa+\nu+r}x\right) &= \frac{1}{r}\left[\frac{\kappa}{\kappa+\nu+r}\frac{\beta m_B}{\kappa+\nu+r+(1-\beta)m_S+\beta m_B}x - (1-\lambda)(\delta_g-\delta_b)\right. \\ &\quad \left. - \frac{r}{\kappa+\nu+r}\frac{[\kappa+\nu+r+(1-\beta)m_S][\kappa+\nu+r+\beta m_B-(1-\beta)m_S]}{[\kappa+\nu+r+\beta m_B][\kappa+\nu+r+(1-\beta)m_S+\beta m_B]}\right] \end{aligned} \tag{239}$$

Substituting these expressions into (238) and simplifying yields the feasibility condition in the proposition. ■

Appendix B

B.1 Mixed strategy equilibria

First, we characterize analytically some properties of mixed strategy equilibria when trading opportunities arise according to the DGP technology. Then, we illustrate graphically mixed strategy equilibria for the KW technology.

Note that in general agents can randomize on two margins: whether to participate or not in the market and whether to play a mixed strategy either when offering a price or when accepting an offer. However, for the DGP technology the first margin is redundant as matching takes place among all agents in the economy and no participation decisions are taken. Hence, for the DGP technology, we can focus on the latter margin.

We begin by considering equilibria M3, described in Table 1, in which only *hb* agents play mixed strategies. For the existence of such equilibria, the following characterization is obtained.

Proposition B.1 *For the DGP technology, mixed strategy equilibria M3, in which only hb agents play mixed strategies exist for a set of pairs of* (λ, ϕ) *with measure zero.*

Proof. The proposition is proven in two steps. First, it is shown that the condition which needs to be satisfied for *hb* agents to be indifferent between accepting and not an offer of $\bar{p}_{lg}$ is invariant to $\sigma_{hb}(\bar{p}_{lg})$. Then, it is proven that this condition is satisfied for a set of pairs of (λ, ϕ) with measure zero.

The relevant indifference condition of *hb* agents is given by $V_{hb} - V_{lg} = V_{hn} - V_{ln}$. We slightly abuse notation and set $\gamma_S = \gamma_{lg} + \gamma_{lb} + \sigma_{hb}(\bar{p}_{lg})\gamma_{hb}$, which represents the measure of sellers accepting an offer of $\bar{p}_{lg}$ with probability one. Using this definition and noting that for the DGP technology $m_S = \mu\gamma_S/(1+A)$, we can find an explicit expression for the indifference condition following the same steps as in the proof of Proposition 5.3. We obtain the following:

$$V_{hb} - V_{lg} = V_{hn} - V_{ln} \quad \Leftrightarrow \quad \frac{\delta_g - \delta_b}{x} = \frac{r(\kappa + \nu + r + m_B) + \kappa m_B}{(\kappa + \nu + r + \frac{\gamma_{lg}}{\gamma_S} m_S)(\kappa + \nu + r + m_B)} \tag{240}$$

Given that for the DGP technology

$$m_B = \mu \frac{\gamma_B}{1+A}, \tag{241}$$

$$m_S \frac{\gamma_{lg}}{\gamma_S} = \mu \frac{\gamma_{lg}}{1+A} \tag{242}$$

are independent of $\sigma_{hb}(\bar{p}_{lg})$, it follows that the indifference condition of *hb* agents is invariant to $\sigma_{hb}(\bar{p}_{lg})$ and coincides with that in equilibrium H.

Finally, note that for the DGP technology the indifference condition of *hb* agents is identical in equilibria E and H as m_B and $m_S \frac{\gamma_{lg}}{\gamma_S}$ assume the same values in these two equilibria. Thus, by Lemma 5.5, the indifference condition $V_{hb} - V_{lg} = V_{hn} - V_{ln}$ defines a curve in the (λ, ϕ) space, a set with measure zero. ■

Intuitively, when trading opportunities arise according to the DGP technology, the indifference condition of *hb* agents does not depend on whether they actively trade or not. For this reason, the indifference condition is identical in equilibria E, H and M3. Consequently, such mixed strategy equilibria exist only on the curve along which *hb* agents are indifferent between participating and not.

A second aspect of mixed strategy equilibria that we wish to discuss is whether such equilibria exist for λ smaller λ^* above which pure strategy equilibrium in which both lemons and peaches are traded exist. In this case, mixed strategy equilibria would enlarge the parameter space in which both lemons and peaches can be traded.

We seek to find a sufficient condition for mixed strategy equilibria of any type to exist in a region of the (λ, ϕ) space where neither equilibrium E nor equilibrium H exists. For this reason, we can focus on M1, the most analytically tractable mixed strategy equilibria, in which only *lg* agents employ mixed strategies and *hb* agents do not trade. We begin by characterizing on how m_B, the rate at which a seller meets buyers, varies with $\sigma_{hn}(\bar{p}_{lg})$.

Lemma B.1 *For the DGP technology, the rate at which a seller meets buyers m_B is decreasing in $\sigma_{hn}(\bar{p}_{lg})$.*

Proof. For the DGP technology, the rate at which a seller meets buyers is given by $m_B = \mu\gamma_B/(1+A)$. Differentiating this with respect to $\sigma_{hn}(\bar{p}_{lg})$ yields

$$\frac{\mathrm{d}m_B}{\mathrm{d}\sigma_{hn}(\bar{p}_{lg})}\left(1-\frac{\mu}{1+A}\frac{\partial\gamma_B}{\partial m_B}\right) = -\frac{\mu}{1+A}\frac{\kappa\lambda A m_B}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}, \tag{243}$$

where

$$\frac{\partial\gamma_B}{\partial m_B} = -\frac{\kappa\lambda A\sigma_{hn}(\bar{p}_{lg})}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2} - \frac{\kappa(1-\lambda)A}{(\kappa+\nu+m_B)^2} < 0 \tag{244}$$

Thus, we have shown that m_B is decreasing in $\sigma_{hn}(\bar{p}_{lg})$. ■

As $\sigma_{hn}(\bar{p}_{lg})$ increases, the measure of buyers decreases, which for the DGP technology leads to a decrease in m_B. Equipped with this property we can prove the following result about the values of λ for which mixed strategy equilibria exist.

Proposition B.2 *For the DGP technology, if $r^2 \geq \kappa\left(\kappa+\nu+\phi\mu\frac{A}{1+A}\right)$, then mixed strategy equilibria exist for lower values of λ than equilibria in pure strategies in which peaches are traded.*

Proof. The result is proven by considering how the indifference condition of buyers in equation (22) varies with $\sigma_{hn}(\bar{p}_{lg})$. This is sufficient as when the condition for hb agents not to participate holds with strict inequality for $\sigma_{hn}(\bar{p}_{lg}) = 1$, by continuity, it also holds for $\sigma_{hn}(\bar{p}_{lg}) < 1$ in a neighbourhood of $\sigma_{hn}(\bar{p}_{lg}) = 1$. Given that the effective rate at which a seller receives an offer of $\bar{p}_{lg}$ in a mixed strategy equilibrium is $\sigma_{hn}(\bar{p}_{lg})m_B$ and the ratio of sellers of peaches to those of lemons is given by

$$\frac{\gamma_{lg}}{\gamma_{lb}} = \frac{\lambda}{1-\lambda}\frac{\kappa+\nu+m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}, \tag{245}$$

the indifference condition for buyers becomes (following the same steps as in the proof of Lemma 5.5):

$$\frac{\lambda}{1-\lambda}\frac{\kappa+\nu+m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B} = \phi\frac{(\kappa+\nu+r)(\kappa+\nu+r+m_S)-\frac{\gamma_{lg}}{\gamma_S}m_S(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \tag{246}$$

Given that by Lemma 5.1 $\gamma_{lb} = \kappa(1-\lambda)A/(\kappa+\nu+m_B)$ and $\gamma_{lg} = \kappa\lambda A/(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)$, we can rewrite the condition as follows:

$$\frac{\lambda}{1-\lambda}\frac{\kappa+\nu+m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B} = \phi\frac{(\kappa+\nu+r)\left(\kappa+\nu+r+\frac{\kappa(1-\lambda)A}{\kappa+\nu+m_B}\frac{m_S}{\gamma_S}\right)-\kappa\lambda A\frac{m_S}{\gamma_S}\frac{\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \tag{247}$$

Note that $m_S = \mu\gamma_S/(1+A)$. Thus, we obtain the following equivalent expression:

$$\begin{aligned}&-\phi\frac{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)\left[(\kappa+\nu+r)\left(\kappa+\nu+r+\frac{\kappa(1-\lambda)\mu}{\kappa+\nu+m_B}\frac{A}{1+A}\right)-\kappa\lambda\mu\frac{A}{1+A}\frac{\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\right]}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B}\\&+\frac{\lambda}{1-\lambda}(\kappa+\nu+m_B)=0\end{aligned} \tag{248}$$

Let us denote the left-hand side of (248) by g. Differentiating with respect to $\sigma_{hn}(\bar{p}_{lg})$ yields:

$$
\begin{aligned}
\frac{\partial g}{\partial \sigma_{hn}(\bar{p}_{lg})} =& \frac{\lambda}{1-\lambda} \frac{\partial m_B}{\partial \sigma_{hn}(\bar{p}_{lg})} \\
&+ \phi \frac{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)(\kappa+\nu+r)}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \frac{\kappa(1-\lambda)\mu}{(\kappa+\nu+m_B)^2} \frac{A}{1+A} \frac{\partial m_B}{\partial \sigma_{hn}(\bar{p}_{lg})} \\
&- \phi \frac{[r^2-\kappa(\kappa+\nu)]\left[(\kappa+\nu+r)\left(\kappa+\nu+r+\frac{\kappa(1-\lambda)\mu}{\kappa+\nu+m_B}\frac{A}{1+A}\right) - \kappa\lambda\mu \frac{A}{1+A}\frac{\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\right]}{[r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B]^2} \frac{\partial(\sigma_{hn}(\bar{p}_{lg})m_B)}{\partial \sigma_{hn}(\bar{p}_{lg})} \\
&+ \phi \frac{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)\kappa\lambda\mu\frac{A}{1+A}\frac{\kappa+\nu}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \frac{\partial(\sigma_{hn}(\bar{p}_{lg})m_B)}{\partial \sigma_{hn}(\bar{p}_{lg})}
\end{aligned} \tag{249}
$$

Note that the first two terms are negative as $\partial m_B/\partial\sigma_{hn}(\bar{p}_{lg}) \leq 0$. The last two terms, on the other hand, can be combined, using the expression for g, as follows

$$
-\frac{\lambda\left\{\frac{[r^2-\kappa(\kappa+\nu)](\kappa+\nu+m_B)}{1-\lambda} - \phi\kappa(\kappa+\nu)\mu\frac{A}{1+A}\right\}}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)[r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B]} \frac{\partial(\sigma_{hn}(\bar{p}_{lg})m_B)}{\partial \sigma_{hn}(\bar{p}_{lg})} \tag{250}
$$

For $r^2 \geq \kappa\left(\kappa+\nu+\phi\mu\frac{A}{1+A}\right)$ this expression is negative as

$$
\frac{\partial(\sigma_{hn}(\bar{p}_{lg})m_B)}{\partial \sigma_{hn}(\bar{p}_{lg})}\left(1+\frac{\kappa\lambda A\sigma_{hn}(\bar{p}_{lg})}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}\right) = m_B - \frac{\kappa(1-\lambda)A\sigma_{hn}(\bar{p}_{lg})}{(\kappa+\nu+m_B)^2}\frac{\partial m_B}{\partial \sigma_{hn}(\bar{p}_{lg})} > 0 \tag{251}
$$

Differentiating with respect to λ, on the other hand, we obtain:

$$
\begin{aligned}
\frac{\partial g}{\partial \lambda} =& \frac{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B} \left\{ \frac{1}{(1-\lambda)^2}(\kappa+\nu+m_B)\frac{r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B} \right. \\
&+ \phi\left[(\kappa+\nu+r)\frac{\kappa\mu}{\kappa+\nu+m_B}\frac{A}{1+A} + \kappa\mu\frac{A}{1+A}\frac{\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\right] \\
&+ \frac{\lambda}{1-\lambda}\left[\frac{(1-\sigma_{hn}(\bar{p}_{lg}))(\kappa+\nu)}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}[r(\kappa+\nu+r+\sigma_{hn}(\bar{p}_{lg})m_B)+\kappa\sigma_{hn}(\bar{p}_{lg})m_B] + \sigma_{hn}(\bar{p}_{lg})(\kappa+r)\frac{\kappa+\nu+m_B}{\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B}\right]\frac{\partial m_B}{\partial\lambda} \\
&\left. + \phi\left[(\kappa+\nu+r)\frac{\kappa(1-\lambda)\mu}{(\kappa+\nu+m_B)^2}\frac{A}{1+A} + \sigma_{hn}(\bar{p}_{lg})(\kappa+\nu)\frac{\kappa\lambda\mu}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}\frac{A}{1+A}\right]\frac{\partial m_B}{\partial\lambda} \right\}
\end{aligned} \tag{252}
$$

Given that

$$
\frac{\partial m_B}{\partial \lambda}\left(1+\frac{\sigma_{hn}(\bar{p}_{lg})\kappa\lambda A}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)^2}+\frac{\kappa(1-\lambda)A}{(\kappa+\nu+m_B)^2}\right) = \mu\frac{(1-\sigma_{hn}(\bar{p}_{lg}))m_B}{(\kappa+\nu+\sigma_{hn}(\bar{p}_{lg})m_B)(\kappa+\nu+m_B)} \geq 0 \tag{253}
$$

it holds that $\partial g/\partial\lambda > 0$. Thus, we have established that $\mathrm{d}\lambda/\mathrm{d}\sigma_{hn}(\bar{p}_{lg}) \geq 0$ for $r^2 \geq \kappa\left(\kappa+\nu+\phi\mu\frac{A}{1+A}\right)$. Thus, for any $\sigma_{hn}(\bar{p}_{lg}) < 1$, the value of λ for which the indifference condition of buyers is satisfied is smaller than the threshold value λ^{*E} above which equilibria in pure strategies exist. Given that for the DGP technology the indifference condition of *hb* agents is identical in equilibria E and H, for $\lambda < \lambda^{*E}$ and $\phi < \phi^{*E}$, no pure strategy equilibria in which peaches are traded exists. Thus, we can conclude that mixed strategy equilibria exists for lower values of λ than pure strategy equilibria in which $\sigma_{hn}(\bar{p}_{lg}) = 1$ when $r^2 \geq \kappa\left(\kappa+\nu+\phi\mu\frac{A}{1+A}\right)$. ■

Figure 12 illustrates a case in which equilibria M1 exist for lower values of λ than pure strategy equilibria E and H. Note that, for the parameters to satisfy the sufficient condition in Proposition B.2, the values of κ and ν have to be relatively low. For the chosen parameter values, an agent's valuation state changes in expectation once every 50 years. Figure 12 reveals that also mixed strategy equilibria M2, in which $\sigma_{hb}(\bar{p}_{lg}) = 1$ can exist for lower values of λ than pure strategy equilibria H. A difference between equilibria M1 and M2 is that in the latter the positive effect of a lower $\sigma_{hn}(\bar{p}_{lg})$ on the average quality of assets is asymmetric around $\lambda = 0.5$, being stronger for λ equal to $0.5+\varepsilon$ than for λ equal to $0.5-\varepsilon$.[47] For this reason, M2 can exist for higher values of λ than pure strategy equilibria H when the opposite holds for the equilibria E and M1. Finally, for the parameter values considered in Figure 12, also mixed strategy equilibria M4, in which both lg and hb agents employ mixed strategies exist for pairs of (λ, ϕ) for which no pure strategy equilibria in which peaches are traded exists. The existence of equilibria M4 is determined by two considerations. First, to render hb agents indifferent between trading and not, ϕ has to be lower than in equilibria in which $\sigma_{hn}(\bar{p}_{lg}) = 1$. This is due to the fact that, for a given λ, $\sigma_{hn}(\bar{p}_{lg})m_B$ is lower and $(\gamma_{lg}/\gamma_S)m_S$ is higher than when $\sigma_{hn}(\bar{p}_{lg}) = 1$. Second, to render buyers indifferent between offering $\bar{p}_{lg}$ and $\bar{p}_{lb}$, λ has to be lower than in the pure strategy equilibria, given that we are considering a case in which the discount rate is high.

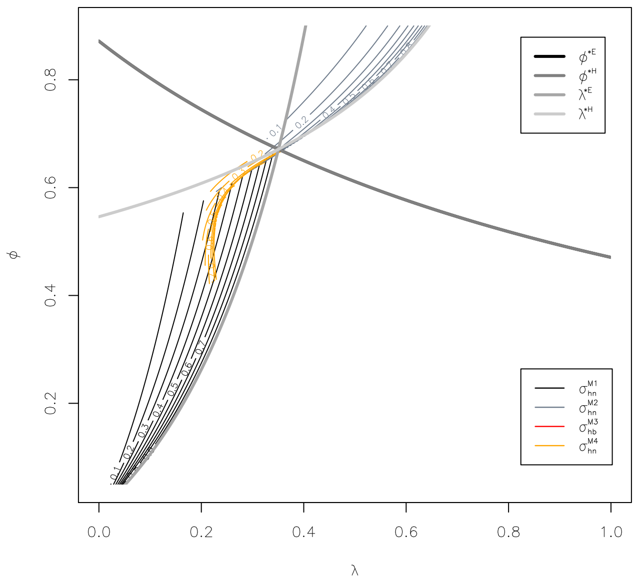

Figure 12: DGP technology, parameter values: $A = 1$, $\kappa = 0.02$, $\nu = 0.02$, $\mu = 1.25$, $r = 0.2$.

Let us conclude our discussion of mixed strategy equilibria when considering the DGP technology by further commenting Figure 5, illustrating where they exist for values of κ and ν which are larger relative to r, implying more reasonable intensities for the changes in agents' valuation states. For the chosen parameter values, mixed strategy equilibria M1 and M2 exist in the interior of the parameter space in which the pure strategy equilibria E and H exist. It is worth pointing out that the contour lines of M1 and M2 meet for each $\sigma_{hn}(\bar{p}_{lg})$. This is due to the fact that, for any $\sigma_{hn}(\bar{p}_{lg})$, m_B assumes the same value in M1 and M2 and when the hb agents' participation constraint is satisfied with equality the value functions across the two mixed strategy equilibria are identical. Mixed strategy equilibria of type M4, on the other hand, exist also in the parameter region in which neither equilibria E nor equilibria H exist. This is due to the fact that hb agents indifference condition is satisfied below the ϕ^{*E} curve for $\sigma_{hn}(\bar{p}_{lg}) < 1$ and buyers are indifferent between offering $\bar{p}_{lb}$ and $\bar{p}_{lg}$ on the right of the λ^{*H} curve when some hb agents trade.

[47] This can be seen by analysing how $\frac{\partial(\gamma_{lg}/\gamma_S)}{\partial(\sigma_{hn}(\bar{p}_{lg})m_B)} = -\frac{\kappa\lambda(1-\lambda)}{[\kappa+(\nu+\sigma_{hn}(\bar{p}_{lg})m_B)(1-\lambda)]^2}$ varies with λ.

When agents are matched according to the KW technology, the two margins on which agents can randomize are both potentially relevant. This is because the matching rates of buyers and sellers depend on who participates in the market. In particular, the fraction of *hb* agents participating in the market influences the matching rates. However, the probability of *hb* agents accepting an offer of $\bar{p}_{lg}$ when they are indifferent between accepting and rejecting has no effect on the matching rates and the value functions. This is because this acceptance probability does not affect the measures of agents with different statuses and buyers obtain zero surplus from trading with an *hb* agent. Consequently, when considering the KW technology, the only relevant margin for *hb* agents is whether they participate in the market or not. For this reason, we slightly depart from our equilibrium definition and assume that only a fraction of *hb* agents, denoted by σ_{hb}, participate in the market. This allows us to discover whether peaches can be traded in a larger region of the parameter space when *hb* agents randomize on the relevant margin.

We begin by illustrating, in Figure 13, a parameter configuration in which r is relatively large. The two effects of a decrease in $\sigma_{hn}(\bar{p}_{lg})$ discussed above are also at play when agents are matched according to the KW technology. However, in this case, the intensity at which peaches are traded varies more strongly with $\sigma_{hn}(\bar{p}_{lg})$ as m_B is increasing in $\sigma_{hn}(\bar{p}_{lg})$. This renders the effect of the effective trading intensity stronger, constraining the parameter space for which mixed strategy equilibria exist for lower values of λ than pure strategy equilibria E and H. For the chosen parameter values, the two effects of a lower $\sigma_{hn}(\bar{p}_{lg})$ almost cancel each other out, as evidenced by the fact the the mixed strategy equilibria M1 exist only for values of λ marginally smaller than λ^{*E}. Mixed strategy equilibria M2, on the other hand, exist for smaller values of λ than λ^{*H} for λ slightly larger than 0.5 due to the same reason as for the DGP technology: the positive effect of a decrease in $\sigma_{hn}(\bar{p}_{lg})$ on the average quality is strongest for λ in this range. As regards mixed strategy equilibria M3, in which only a fraction of *hb* agents participate in the market, such equilibria exist between the two indifference curves for *hb* agents in the pure strategy equilibria E and H, ϕ^{*E} and ϕ^{*H}. The reason for this is that when a fraction of *hb* agents participate in the market, $(\gamma_{lg}/\gamma_S)m_S$ is lower than in the pure strategy equilibrium E. Finally, mixed strategy equilibria M4 exist in a region between the equilibria M1 and M2 as in this part of the (λ,ϕ) space *hb* agents are indifferent between participating and not in the market and at the same time λ is sufficiently high such that buyers are indifferent between offering the low and the high price.

When r assumes a lower value relative to κ and ν, as illustrated in Figure 6, mixed strategy equilibria exist in the interior of the (λ,ϕ) space in which the pure strategy equilibria E and H exist. Note that differently from when considering the DGP technology, also equilibria M4 exist for higher values of λ than pure strategy equilibria in which peaches are traded. The reason for this, in this case, higher values of λ are needed to support equilibria where *hb* agents participate than for the DGP technology given that sellers meet buyers less frequently than in equilibrium E. Mixed strategy equilibria M3, in which only *hb* agents employ mixed strategies, again exist in between the two indifference curves for *hb* agents, ϕ^{*E} and ϕ^{*H}.

B.2 Pure strategy equilibria when sellers offer prices

We consider the same model setup of section 2.1 but we assume a different trading protocol: sellers make take-it-or-leave-it offers to buyers who can either accept or reject. Being a signalling game, there are many possible Perfect Bayesian equilibria and their complete characterization is a challenging task that goes beyond the scope of this paper. In this appendix we restrict attention to stationary pure strategy

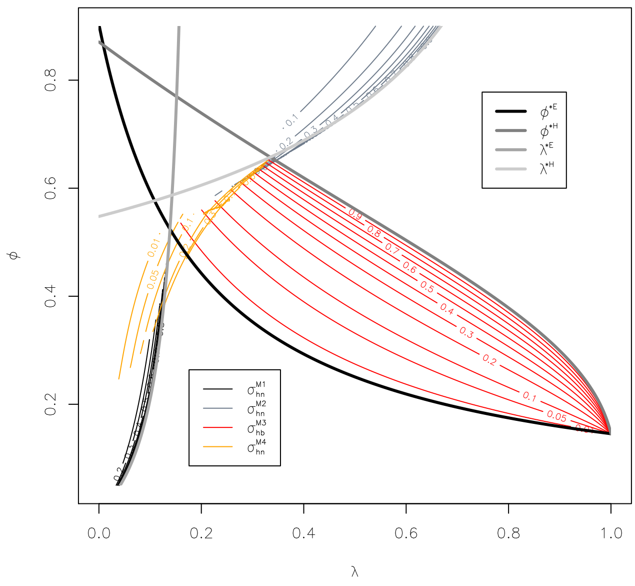

Figure 13: KW technology, parameter values: $A = 1$, $\kappa = 0.02$, $\nu = 0.02$, $\mu = 2.5$ and $r = 0.2$.

equilibria in which peaches and lemons trade. Our main results in terms of welfare inefficiencies, trade volume and liquidity continue to hold with this bargaining protocol. Similarly to the complete information benchmark, transaction prices and market liquidity are positively related when sellers fully exploit their bargaining power *vis-á-vis* buyers (Section B.2.2).

If peaches are traded, the restriction to pure strategy equilibria implies that all sellers offer the same price and buyers accept this price with probability one. Indeed, when peaches are traded and buyers only play pure strategies, the price offered by peach holders has to be accepted with probability one, thus lemon holders do not find convenient to offer a lower price because it would not allow them to trade more rapidly. As common in signalling models, infinitely many pooling prices may support an equilibrium; it is sufficient that buyers out of equilibrium beliefs assign a sufficiently high probability to the event of receiving a lemon. As in the previous analysis, we first characterize the equilibrium when individual private states are observable, and later we consider the case of private information also along this dimension. For this latter case we distinguish whether in equilibrium *hb* agents participate in the market as active sellers or not. For each of the two cases, we restrict attention to equilibria in which sellers demand the highest possible price that buyers accept. This restriction allows to characterize the largest set of parameters in which *lg* agents sell their assets.

B.2.1 Individual private states publicly observable

By Milgrom and Stokey (1982) no trade theorem, in equilibrium only low valuation asset holders trade their assets. If both peaches and lemons trade, then matching are equal to m_B^C and m_S^C and sellers offer the highest possible price that buyers accept:

$$p = \frac{\gamma_{lg}}{\gamma_S} V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S} V_{hb}\right) - V_{hn} = \lambda V_{hg} + (1-\lambda) V_{hb} - V_{hn} \tag{254}$$

In turn, at this price it follows immediately that $V_{hn} = V_{ln} = 0$. Hence, the relevant value functions are:

$$rV_{hi} = \delta_i + \kappa(V_{li} - V_{hi}) \qquad i = b, g \tag{255}$$

$$rV_{li} = \delta_i - x + \nu(V_{hi} - V_{li}) + m_B^C[\lambda V_{hg} + (1-\lambda)V_{hb} - V_{li}] \qquad i = b, g \tag{256}$$

It is immediate to realize that this equilibrium exists if and only if:

$$\lambda(V_{hg} - V_{lg}) + (1-\lambda)(V_{hb} - V_{lg}) \geq 0 \tag{257}$$

From equations (255) and (256) we get:

$$V_{hg} - V_{lg} = \frac{x - (1-\lambda)m_B^C(V_{hb} - V_{lg})}{\kappa + \nu + r + \lambda m_B} \tag{258}$$

$$V_{hb} - V_{lg} = \frac{x - (\delta_g - \delta_b) - \left(\nu + \lambda m_B^C\right)(V_{hg} - V_{lg}) - \kappa(V_{hb} - V_{lb})}{r + (1-\lambda)m_B^C} \tag{259}$$

$$V_{hg} - V_{hb} = \frac{\delta_g - \delta_b + \kappa(V_{lg} - V_{lb})}{\kappa + r} \tag{260}$$

$$V_{lg} - V_{lb} = \frac{\delta_g - \delta_b + \nu(V_{hg} - V_{hb})}{\nu + r + m_B^C} \tag{261}$$

From the above equations we get:

$$V_{hb} - V_{lg} = \frac{x}{\kappa + \nu + r + m_B^C} - \frac{\kappa + \nu + r + \lambda m_B^C}{r(\kappa + \nu + r + m_B^C) + \kappa m_B^C}(\delta_g - \delta_b) \tag{262}$$

Substituting (262) first in equation (258), and then in (257) we get that the equilibrium exists if and only if:

$$(1-\lambda)\phi \leq \frac{r(\kappa + \nu + r + m_B^C) + \kappa m_B^C}{(\kappa + \nu + r)(\kappa + \nu + r + m_B^C)} \tag{263}$$

B.2.2 Bi-dimensional private information: *hb* agents sell their assets

Consider sellers offering the highest possible price that buyers would accept:

$$p = \frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}V_{hb}\right) - V_{hn} \tag{264}$$

This price offer makes *hb* agents willing to sell their assets because $p + V_{hn} > V_{hb}$ as $V_{hg} > V_{hb}$.

If sellers trade at this price then agents with no assets have the following values:

$$rV_{hn} = \kappa(V_{ln} - V_{hn}) + m_S\left[\frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)V_{hb} - p - V_{hn}\right] = \kappa(V_{ln} - V_{hn}) \tag{265}$$

$$rV_{ln} = \nu(V_{hn} - V_{ln}) \tag{266}$$

The only solution to these two equations is $V_{hn} = V_{ln} = 0$, i.e. agents without asset do not get any surplus. Note that also in this case only *hn* agents participate as buyers, while *ln* agents wait until their valuation is high again. The remaining value functions are:

$$rV_{hg} = \delta_g + \kappa(V_{lg} - V_{hg}) \tag{267}$$

$$rV_{lg} = \delta_g - x + \nu\left(V_{hg} - V_{lg}\right) + m_B\left[\frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)V_{hb} - V_{lg}\right] \tag{268}$$

$$rV_{hb} = \delta_b + \kappa\left(V_{lb} - V_{hb}\right) + m_B\left[\frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)V_{hb} - V_{hb}\right] \tag{269}$$

$$rV_{lb} = \delta_b - x + \nu\left(V_{hb} - V_{lb}\right) + m_B\left[\frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)V_{hb} - V_{lb}\right] \tag{270}$$

This equilibrium exists if and only if lg agents are willing to offer this price, i.e.

$$\frac{\gamma_{lg}}{\gamma_S}V_{hg} + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)V_{hb} - V_{lg} \geq 0 \tag{271}$$

To compute when this condition holds, let's consider $V_{hg} - V_{lg}$ and $V_{hb} - V_{lg}$. From equations (267)–(270) we get:

$$V_{hg} - V_{lg} = \frac{x - \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)m_B\left(V_{hb} - V_{lg}\right)}{\kappa + \nu + r + \frac{\gamma_{lg}}{\gamma_S}m_B} \qquad V_{hb} - V_{lb} = \frac{x}{\kappa + \nu + r + m_B} \tag{272}$$

Subtracting equation (268) from (269) and using the results in (272) we have:

$$\begin{aligned} r(V_{hb} - V_{lg}) &= \delta_b - \delta_g + x - \kappa(V_{hb} - V_{lb}) - \nu(V_{hg} - V_{lg}) - m_B(V_{hb} - V_{lg}) \\ &= \delta_b - \delta_g + x - \kappa\frac{x}{\kappa + \nu + r + m_B} - \nu\frac{x - \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)m_B\left(V_{hb} - V_{lg}\right)}{\kappa + \nu + r + \frac{\gamma_{lg}}{\gamma_S}m_B} - m_B(V_{hb} - V_{lg}) \end{aligned} \tag{273}$$

Rearranging and simplifying we get:

$$V_{hb} - V_{lg} = \frac{x}{\kappa + \nu + r + m_B} - \frac{(\delta_g - \delta_b)\left(\kappa + \nu + r + \frac{\gamma_{lg}}{\gamma_S}m_B\right)}{r(\kappa + \nu + r + m_B) + \kappa m_B + \frac{\gamma_{lg}}{\gamma_S}m_B(\nu + r + m_B)} \tag{274}$$

Substituting the results in (272) and (274) in (271) and rearranging:

$$\frac{\gamma_{lg}}{\gamma_S}\left(V_{hg} - V_{lg}\right) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)\left(V_{hb} - V_{lg}\right) = \frac{x}{\kappa + \nu + r + m_B} - \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)\frac{(\kappa + \nu + r)(\delta_g - \delta_b)}{r(\kappa + \nu + r + m_B) + \kappa m_B + \frac{\gamma_{lg}}{\gamma_S}m_B(\nu + r + m_B)} \tag{275}$$

Therefore, this equilibrium exists only if:

$$\frac{\delta_g - \delta_b}{x} \leq \frac{r(\kappa + \nu + r + m_B) + \kappa m_B + \frac{\gamma_{lg}}{\gamma_S}m_B(\nu + r + m_B)}{\left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)(\kappa + \nu + r)(\kappa + \nu + r + m_B)} \tag{276}$$

Lastly, we derive an expression for the equilibrium price, and we show it is in increasing in the matching rate m_B. The price offered by sellers is equal to:

$$p = \frac{\gamma_{lg}}{\gamma_S}\left(V_{hg} - V_{lg}\right) + \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)\left(V_{hb} - V_{lg}\right) + V_{lg} \tag{277}$$

From the previous results, after some algebraic manipulations, we get the following expression in terms of the fundamentals:

$$p = \frac{1}{r}\left\{\delta_g - \frac{\kappa}{\kappa + \nu + r + m_B}x - \left(1 - \frac{\gamma_{lg}}{\gamma_S}\right)\frac{(\delta_g - \delta_b)[r(\kappa + \nu + r + m_B) + \kappa m_B]}{r(\kappa + \nu + r + m_B) + \kappa m_B + \frac{\gamma_{lg}}{\gamma_S}m_B(\kappa + r + m_B)}\right\} \tag{278}$$

In general, this expression can be increasing or decreasing in m_B because the term multiplying x is decreasing in m_B, while the last term can be increasing in m_B. However, if the inequality in (276) holds then the expression is unambiguously increasing in m_B. Indeed, consider a tuple (δ_g, δ_b, x) such that (276) holds with equality. Substituting in (278) the price expression becomes:

$$p = \frac{1}{r}\left(\delta_g - \frac{\kappa + r}{\kappa + \nu + r} x\right) \tag{279}$$

which is independent of m_B. Therefore, a higher x (or equivalently a lower $\delta_g - \delta_b$) satisfies the existence condition in (276), and it simultaneously increases the absolute value of the derivative with respect to m_B of $\frac{\kappa}{\kappa+\nu+r+m_B}x$ relative to the one for the last term in equation (278).

B.2.3 Bi-dimensional private information: *hb* agents do not sell their assets

Consider an equilibrium in which lg and lb agents propose a price that would never be offered by hb agents. The highest possible such price is equal to $p = V_{hb} - V_{hn}$.

Since hb agents do not trade while lg and lb agents trade at $p = V_{hb} - V_{hn}$, the value functions are:

$$rV_{hg} = \delta_g + \kappa(V_{lg} - V_{hg}) \tag{280}$$

$$rV_{lg} = \delta_g - x + \nu(V_{hg} - V_{lg}) + m_B(V_{hb} - V_{hn} - V_{lg} + V_{ln}) \tag{281}$$

$$rV_{hb} = \delta_b + \kappa(V_{lb} - V_{hb}) \tag{282}$$

$$rV_{lb} = \delta_b - x + \nu(V_{hb} - V_{lb}) + m_B(V_{hb} - V_{hn} - V_{lb} + V_{ln}) \tag{283}$$

$$rV_{hn} = \kappa(V_{ln} - V_{hn}) + \frac{\gamma_{lg}}{\gamma_S} m_S(V_{hg} - V_{hb}) \tag{284}$$

$$rV_{ln} = \nu(V_{hn} - V_{ln}) \tag{285}$$

After some simple algebraic manipulations we get:

$$V_{lg} - V_{lb} = \frac{\kappa + \nu + r}{r(\kappa + \nu + r + m_B) + \kappa m_B}(\delta_g - \delta_b) \tag{286}$$

$$V_{hg} - V_{hb} = \frac{\kappa + \nu + r + m_B}{r(\kappa + \nu + r + m_B) + \kappa m_B}(\delta_g - \delta_b) \tag{287}$$

$$V_{hn} - V_{ln} = \frac{\frac{\gamma_{lg}}{\gamma_S} m_S}{\kappa + \nu + r} \frac{\kappa + \nu + r + m_B}{r(\kappa + \nu + r + m_B) + \kappa m_B}(\delta_g - \delta_b) \tag{288}$$

$$(V_{hb} - V_{lg})\left(r + m_B \frac{\kappa + r}{\kappa + \nu + r}\right) = \delta_b - \delta_g + \left(1 - \frac{\kappa}{\kappa + \nu + r + m_B} - \frac{\nu}{\kappa + \nu + r}\right)[x + m_B(V_{hn} - V_{ln})] \tag{289}$$

Agents of type lg participate in the market only if the price satisfies $p + V_{ln} \geq V_{lg}$. In this equilibrium $p = V_{hb} - V_{hn}$, hence it must hold $V_{hb} - V_{lg} - V_{hn} + V_{ln} \geq 0$. To get a condition on the underlying parameters that satisfy this inequality we consider the equivalent condition $(V_{hb} - V_{lg} - V_{hn} + V_{ln})\left(r + m_B \frac{\kappa+r}{\kappa+\nu+r}\right) \geq 0$. Deducting $\left(r + m_B \frac{\kappa+r}{\kappa+\nu+r}\right)(V_{hn} - V_{ln})$ from both sides of (289) and substituting the expressions in equation (288) in the resulting expression we get (after some simplifications):

$$(V_{hb} - V_{lg} - V_{hn} + V_{ln})\left(r + m_B \frac{\kappa + r}{\kappa + \nu + r}\right) = \frac{r(\kappa + \nu + r + m_B) + \kappa m_B}{(\kappa + \nu + r + m_B)(\kappa + \nu + r)} x - \frac{\kappa + \nu + r + \frac{\gamma_{lg}}{\gamma_S} m_S}{\kappa + \nu + r}(\delta_g - \delta_b) \tag{290}$$

Since in equilibrium only lg and lb agents trade, we have $\frac{\gamma_{lg}}{\gamma_S} = \lambda$ (as in Proposition 5.2). Therefore, by equation (290), this equilibrium exists if and only if:

$$\frac{\delta_g - \delta_b}{x} \leq \frac{r(\kappa + \nu + r + m_B) + \kappa m_B}{(\kappa + \nu + r + \lambda m_S)(\kappa + \nu + r + m_B)} \tag{291}$$

B.3 Static benchmark

Consider an economy in which there two types of non-durable goods, peaches and lemons. The value in utils of a good of either type to an agent depends on his valuation state, which can be either high or low. An agent in the high valuation state derives a utility of δ_g from a peach and δ_b from a lemon, with $\delta_g > \delta_b$. An agent in the low valuation state, on the other hand, obtains a utility of $\delta_g - x$ from a peach and $\delta_b - x$ from a lemon. A proportion λ of all the goods are peaches. The fractions of peaches and lemons in the hands of low-valuation agents are equal. Thus, the probability of a randomly-chosen low-valuation agent with a good holding a peach is equal to λ.

Consider an agent in the high valuation state but without a good, called a buyer, making a take-it-or-leave-it offer to an agent with a good, without knowing the type of the good he holds and his valuation state. The buyer will offer either $\delta_g - x$ or $\delta_b - x$ as the latter dominates any price above it, whereas the former any price between the two. Note that the higher price $\delta_g - x$ is accepted also by high-valuation agents holding a lemon when $\delta_g - x \geq \delta_b$. Thus, there are two cases to consider: $\frac{\delta_g - \delta_b}{x} < 1$ and $\frac{\delta_g - \delta_b}{x} \geq 1$. In the former case, both of the two possibly prices are rejected by all high-valuation agents. Thus, offering the higher price is optimal in this case when:

$$\lambda \delta_g + (1-\lambda)\delta_b - (\delta_g - x) \geq (1-\lambda)[\delta_b - (\delta_b - x)] \quad \Leftrightarrow \quad \frac{\lambda}{1-\lambda} \geq \frac{\delta_g - \delta_b}{x} \tag{292}$$

In the latter case, in which $\frac{\delta_g - \delta_b}{x} \geq 1$, on the other hand, the higher price is accepted also by high-valuation agents holding lemons. Denoting the fraction of low-valuation agents with α, offering the higher price is optimal when:

$$\alpha\lambda[\delta_g - (\delta_g - x)] + (1-\lambda)[\delta_b - (\delta_g - x)] \geq \alpha(1-\lambda)[\delta_b - (\delta_b - x)] \quad \Leftrightarrow \quad \alpha\frac{\lambda}{1-\lambda} + (1-\alpha) \geq \frac{\delta_g - \delta_b}{x} \tag{293}$$

Figure 14, where ϕ denotes $\frac{\delta_g - \delta_b}{x}$, illustrates the different equilibria in the static benchmark. Equilibrium E refers to the equilibrium in which the higher price is offered and only low-valuation owners are willing to sell their asset. Equilibrium H, on the other hand, denotes the equilibrium in which also high-valuation owners of lemons are willing to sell their asset. Note that in the unshaded area, the unique equilibrium is one in which the lower price $\delta_b - x$ is offered and only low-valuation owners of lemons are willing to sell their asset.

It is worth making two observations, that also apply to the dynamic model considered in the main text. First, Figure 14 shows that when gains from trade are low, i.e. x is small and by implication ϕ large, the only equilibrium in which both types of assets are traded is H. This is because the price $\delta_g - x$ is sufficiently high such that also high-valuation owners of lemons are willing to sell their asset. Given that there are no gains from trade between such an owner and a buyer, the equilibrium exhibits excessive trade. Second, the fact that λ^{*H} lies to the right of λ^{*E} for $\phi > 1$ reflects buyers' lower willingness to offer the higher price when all the lemons are on the market. For this reason, for assets of both types to be traded, a higher share of peaches is required. Taken together, these two observations allow us to

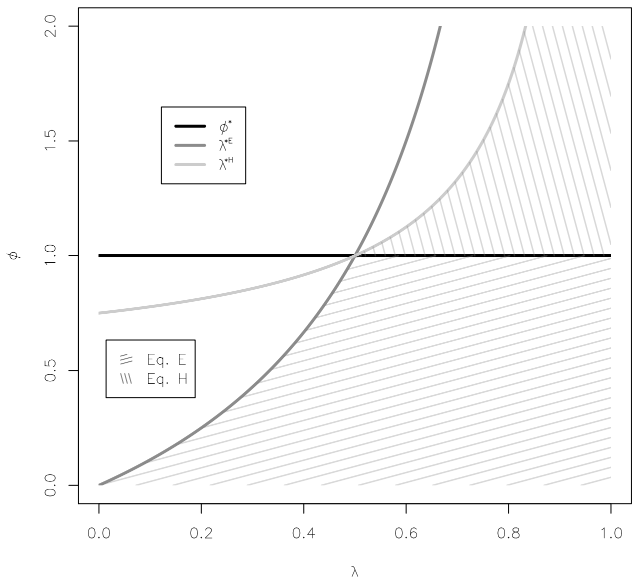

Figure 14: Equilibria in the static benchmark. The two curves λ^{*E} and λ^{*H} show the threshold values for which equations (292) and (293), respectively, hold with equality, while ϕ^* denotes the line $\phi = 1$.

conclude that when gains from trade are small there is a double bind of asymmetric information: either high-quality goods are not traded at all or too many low-quality goods are traded. In our dynamic model, with type-dependent, endogenous outside options of asset holders, the double bind is more pronounced as high-valuation owners of lemons are more willing to participate in the market when the average quality of goods in the economy is higher.

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