Incentives and Credit in a Quasi-Linear Environment

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1 The Physical Environment

- Consider an economy consisting of many people (a continuum).

- These people are identical *ex ante*; but may differ *ex post*.

- Denote an individual by $i \in [0, 1]$.

- Time is discrete and the horizon is infinite; $t = 0, 1, 2, ..., \infty$.

- Each time period is divided into two subperiods; labelled **day** and **night**.

1.1 Day Subperiod

- All agents are in a position to produce or consume goods (utility).

- Let $x > 0$ denote day consumption and $x < 0$ denote night production (transferable utlity).

- This day good is nonstorable; hence, there is an aggregate resource constraint:

$$\int x_t(i)di = 0;$$

where $x_t(i)$ is consumption at date t by individual i.

1.2 Night Subperiod

- At night, individuals discover **either** a desire to consume **or** an ability to produce.

- This desire/ability (their **type**) is determined randomly by an exogenous stochastic process (*i.i.d.* across people and time).

- Assume (for simplicity) a 50-50 chance of becoming either type.
  - If they have a desire to consume, then utility is $u(c)$.
  - If they have an ability to produce, then utility is $-g(y)$.

- Note: this stochastic preference/technology shock structure mimics what would happen with random pairwise meetings (and lack of double coincidence of wants) in a search environment.

- The night good is also nonstorable; hence there is another aggregate resource constraint:

$$\int c_t(i)di = \int y_t(i)di;$$

- where $\{c_t(i), y_t(i)\}$ denotes consumption and production at date t by individual i.

1.3 Preferences

- As people are *ex ante* identical, their preferences can be represented as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \left(\frac{1}{2}\right) \left[u(c_t(i)) - g(y_t(i))\right] \right\}.$$

- where $0 < \beta < 1$ (note that there is no discounting across subperiods).

2 The First-Best Allocation

- Let's think about this.

- Assume that we weight all agents equally (and why shouldn't we–they are *ex ante* identical).

- Then all agents will receive the same **expected** (*ex ante*) utility payoff.

- First question: How should $\{x_t(i)\}$ be allocated across people?

- Note that the resource constraint requires:

$$\int x_t(i)di = 0.$$

- One obvious solution is to have $x_t(i) = 0$ for all i and all t.

- In fact, because utility (for all people) is **linear** in x, there are many other solutions as well.

- For example:

$$x_t(i) = \begin{cases} +x & \text{w.p.} \quad 1/2; \\ -x & \text{w.p.} \quad 1/2; \end{cases}$$

  for any $x \in \mathbb{R}$ is also a solution.

- Because people are risk-neutral in $x_t(i)$, any lottery over $x_t(i)$ such that $E_t x_t(i) = 0$ is both feasible and in no way reduces **expected** utility (people will, of course, differ *ex post*).

- Second question: How should $\{c_t(i), y_t(i)\}$ be allocated across people?

- Efficiency dictates that $c_t(i) = 0$ if i is a producer (he does not value consumption).

- Feasibility dictates that $y_t(i) = 0$ if i is a consumer (he does not have an ability to produce).

- Given the curvature in u and g, it will make sense here to set $c_t(i) = c$ for i who are consumers and $y_t(i) = y$ for i who are producers.

- Since there is an equal number of consumers and producers, the resource constraint implies $c = y$.

- Hence, with $E_0 x_t(i) = 0$, ex ante utility for a given y can be expressed as:

$$W(y) = \left(\frac{1}{2}\right)(1-\beta)^{-1}\left[u(y) - g(y)\right].$$

- Clearly, the function W is strictly concave and achieves a unique maximum at y^* satisfying $u'(y^*) = g'(y^*)$.

- That is, the planner assigns $c_t^*(i) = y^*$ and $y_t^*(i) = 0$ if i is a consumer during the night; and $c_t^*(i) = 0$ and $y_t^*(i) = y^*$ if i is a producer during the night.

3 Discussion

- The first-best allocation described above can be thought of as the outcome of a **social credit** or **social insurance** arrangement.

- As an insurance arrangement, you can think of producers as paying a net premium y^* and consumers receiving a net benefit y^*.

- As a credit arrangement, you can think of consumers borrowing y^* from society in exchange for the promise to produce y^* at some time in the future (when they have an ability to produce).

- If there was a social planner (benevolent, of course) who was all-powerful in the sense of being able to observe types and enforce allocations, then the resource allocation problem is easily solved:

  From each according to his ability; To each according to his need.

- The same good outcome can be achieved if all people behaved **cooperatively**; i.e., in the sense of always honoring their promises (debt obligations) and always reporting their types truthfully.

- Moreover, note that **if** agents can commit to honor their promises (debt obligations) and **if** types are publicly observable, then the standard **welfare theorems** would appear to apply.

- That is, the first-best allocation is implementable as a competitive equilibrium (there are potentially many market structures that could achieve this).

- Consider, for example, a competitive insurance market. Prior to realization of personal shocks, agents can issue promises to deliver output y^* contingent on being a producer.

- They can then use the value of these promises to purchase consumption insurance (allowing them to consume y^* in the event that they have a desire to consume).

- In short, this is a boring world (reminds me of Canada)...so let's spice things up a bit.

4 Private Information

- Imagine that types are private information; but that agents can commit (not to telling the truth, of course).

- In this case, an allocation will have to be made contingent on **reports** of types.

- In fact, in a dynamic setting, it is generally desirable to make an allocation contingent on the agent's **entire personal history of reports and actions**.

- As it turns out, our quasi-linear environment simplifies matters greatly; i.e., we may without loss condition allocations on an agent's personal history going back **one period only**.

- **Key Point:** Some **memory** will be **essential** (although limited memory suffices in a quasi-linear environment).

  - **Memory:** a public-access database of personal histories (Kocherlakota, 1998);

  - requires that people are **identifiable** by their personal histories (i.e., they are not anonymous).

- We can then appeal to the **Revelation Principle**: Any constrained-efficient allocation can be implemented as the equilibrium of a direct revelation game in which people (strategically) report their types truthfully.

- Let's see how this might work...

- Let me focus on (stationary) allocations of the form (x, y), where:

$$x_t(i) = \begin{cases} +x & \text{if} \quad (c_{t-1}(i), y_{t-1}(i)) = (0, y); \\ -x & \text{if} \quad (c_{t-1}(i), y_{t-1}(i)) = (y, 0); \end{cases}$$

  with $x_0(i) = 0$ for all i.

- Observe that this allocation embeds within it a **punishment-reward system** (those who produced in the last period are rewarded today; and those who consumed in the last period are punished today)–similar to a **credit market** (producers are savers and consumers are borrowers).

- Note that, under truthtelling, any such allocation is feasible and generates the welfare function:

$$W(y) = \left(\frac{1}{2}\right)(1-\beta)^{-1}\left[u(y) - g(y)\right].$$

- To induce truth-telling behavior, the allocation must be **incentive-compatible** (people must have no incentive to misreport their type).

- In this setup, it is clear that consumers cannot pretend to be producers (i.e., they have no ability to produce).

- However, producers may have an incentive to pretend to be consumers (i.e., they can economize on the disutility of production).

- Hence, the relevant incentive-compatibility (IC) condition is given by:

$$-g(y) + \beta \left[x + W(y) \right] \geq 0 + \beta \left[-x + W(y) \right];$$

or

$$x \geq \left(\frac{1}{2\beta} \right) g(y).$$

- In words, the IC condition says that to get producers to reveal themselves as such, they need to be compensated with a sufficiently large future reward x.

Since there is no upper bound on x here, the first-best allocation is clearly implementable; i.e., simply choose any x such that:

$$x \geq \left(\frac{1}{2\beta}\right) g(y^*).$$

- Note that this x is supplied in the next day by those people who consumed (borrowed) the previous night.

- Our assumption of commitment here implies that this debt will be honored, regardless of the size of x.

5 Limited Commitment

- What if individuals cannot be expected to automatically honor their debt obligations?

- If individuals could renege on their debt with impunity, then the only implementable allocation here is **autarky**.

- To encourage **voluntary** debt repayment, debtors must be threatened with some credible **punishment** for noncompliance.

- Generally, the harshest punishment (or **penal code**) available is to be preferred.

- Absent overt coercion, the harshest punishment for noncompliance is the threat of **perpetual banishment to autarky**.

- Note that as each agent is of measure zero, the banishment of any countable number of agents will have no impact on aggregates (so that the threat of banishment is legitimately credible).

- Moreover, this punishment is feasible because agents are (by assumption) identifiable.

- Let $W(0) = 0$ denote the payoff associated with autarky.

- The lack of commitment implies that the allocation (x, y) must also satisfy a set of **sequential participation constraints** (also called sequential or *ex post* individual rationality constraints).

- The potentially binding SP constaint here is with respect to the person in the day who is obliged to repay the debt x (this person was a consumer in the previous night); i.e.,

$$-x + W(y) \geq 0;$$

or

$$x \leq W(y).$$

- In words, the maximum amount of debt that will be voluntarily repaid is bounded above by the value of continued participation in the social credit arrangement.

- Together, the IC and SP constraints restrict the set of implementable allocations (x, y) to the set:

$$\left(\frac{1}{2\beta}\right) g(y) \leq x \leq W(y).$$

Proposition 1 *Define* $\beta_0 \equiv g(y^*)/u(y^*)$. *Then the first-best allocation is implementable for any* $\beta \in [\beta_0, 1)$.

Proposition 2 *For any* $\beta \in [0, \beta_0)$, *the constrained-efficient (second-best) allocation is characterized by* $0 \leq y_0 < y^*$ *satisfying:*

$$\left(\frac{1}{2\beta}\right) g(y_0) = W(y_0);$$

or

$$g(y_0) = \beta u(y_0).$$