# Money in a Quasi-Linear Environment

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# 1 Review

- **Memory:** a public access database containing the personal trading histories of all people in the economy; i.e., a record-keeping technology.

- Think of this memory as existing in an **intangible** form; i.e., as a book-entry object (like your credit history, or as an account at your bank).

- Under limited commitment and/or private information over types, I showed earlier that **memory** is **essential**; i.e.,

  - constrained-efficient allocation must be made contingent on personal trading histories to properly align incentives (recall that history-dependence is **not** essential absent these frictions).

- **Money:** a **tangible** object that circulates widely as a means of payment.

- If people are identifiable by their personal trading histories (i.e., if memory is available), then there is no need for money (money is not essential).

  – In such an economy, all transactions can be made via book-entry adjustments in personal accounts held at a centralized data bank.

- But what if this centralized data bank is not available? (Assume, for example, that it is prohibitively costly).

- Alternatively (and I think equivalently), assume that people can costlessly falsify their identity (trading histories); in either case, agents are rendered **anonymous**.

- In this case, the only feasible trades must involve *quid-pro-quo* exchanges of goods for some other object that does not correspond to a private liability.

- One such object that might serve this role is a **tangible token** created by society (and not by any individual).

- But why might people willingly sacrifice goods for an intrinsically useless token?

- Hint: in the model studied earlier, people were willing to sacrifice goods for an intrinsically useless memory object!

# 2 Monetary Exchange

- Let $M$ denote current money supply and let $M^+$ denote next period's money supply.

- Society expands (or contracts) the money supply so that $M^+ = \mu M$.

- Expansion (contraction) via a lump-sum transfer (tax) at the beginning of each period;

$$\tau = (\mu - 1)\, M.$$

- Assume, for the moment, that when $\tau < 0$, the tax is paid voluntarily.

- Trade is conducted on a sequence of competitive spot markets (where goods are exchanged for money).

- Let $v_d$ and $v_n$ denote the value of money (inverse of price-level) in the day and night markets, respectively.

## 2.1 The Day Market

- A person begins the day with $z$ dollars plus transfer $\tau$. He will want to save $m$ dollars for the night market (in case he has a desire to consume). He will use the rest of his money to purchase the day good $x$.

$$x = v_d(z + \tau - m).$$

- Let $D(z)$ denote the value of beginning the day with $z$ dollars; and let $N(m)$ denote the value of entering the night-market with $m$ dollars; then

$$D(z) = \max_{m \geq 0} \{v_d(z + \tau - m) + N(m)\}$$

- Here is what the quasi-linearity buys us:

$$v_d = N'(m^D) \text{ and } D'(z) = v_d$$

- The demand for money $m^D$ does not depend on a person's initial money holdings $z + \tau$

  $\Rightarrow$ *everyone will leave the day market with identical money balances.*

## 2.2 The Night Market

- Let $C(m)$ and $P(m)$ denote the value of being a consumer and producer in the night-market, respectively. Then,

$$N(m) = \frac{1}{2}C(m) + \frac{1}{2}P(m).$$

### 2.2.1 Consumers

$$C(m) = \max_{y, z^+ \geq 0} \left\{ u(y) + \beta D(z^+) : z^+ = m - v_n^{-1} y \right\}$$

- For $\mu > \beta$, solution is $z^+ = 0$ (or $y = v_n m$); that is, they spend all their money.

- Note that since consumers spend all their money at night, they will want to accumulate money the next day (by selling $x$);
  - remember that all agents leave the day market with the same money balances.

### 2.2.2 Producers

$$P(m) = \max_{y, z^+ \geq 0} \left\{ -g(y) + \beta D(z^+) : z^+ = m + v_n^{-1} y \right\}$$

- Recall that $D'(z) = v_d$; hence

$$v_n g'(y) = \beta v_d^+.$$

- Notice that $v_d^+$ here plays the role of the "promised utility" in our earlier analysis;
  - a producer willingly reveals himself here (even though nobody asks him) because it is in his interest to accumulate money at night, and then spend it the next day for his "future reward" $x$ (**relate to IC**)
  - remember that this future reward is supplied by those consumers who spent all their money and wish to rebalance their money holdings the next day (**relate to SP**)

- So in the day, consumers purchase money (sell $x$) and producers sell money (purchase $x$); and all agents leave the day market with identical money holdings.

## 2.3 Equilibrium

- Producer IC is automatically satisfied here; but consumer SP may not be (assume for the moment that it is).

- If both conditions are satisfied, then for a given $\mu$, the stationary equilibrium of night-output is characterized by:

$$u'(\hat{y}) = \left[2\left(\frac{\beta}{\mu}\right) - 1\right] g'(\hat{y})$$

- The amount of day output produced is given by

$$\hat{x} = \beta^{-1}\hat{y}g'(\hat{y}).$$

- Compare first-best to monetary equilibrium:

$$u'(y^*) = g'(y^*)$$

$$u'(\hat{y}) = \left[2\left(\frac{\beta}{\mu}\right) - 1\right] g'(\hat{y})$$

- Clearly, we have $\hat{y} < y^*$ when $\mu > \beta$

- First-best is implemented by setting $\mu = \beta$ (deflation); this is the so-called **Friedman rule**.

- Key Insight: money serves as a substitute (in this case, perfect) for a missing record-keeping technology (i.e., **money is memory**).

# 3 Incentives and the Limits to Deflation

- When agents are identifiable, they can be feasibly (and credibly) punished with perpetual autarky; this gave rise to a SP constraint:

$$x \leq W(y)$$

- But now, agents are not identifiable in the sense that their **past** records (histories) are hidden; i.e., they are anonymous in this sense.

- Moreover, the Friedman rule requires contraction of the money supply via the taxation of money balances.

- Question: What is to prevent anonymous agents from escaping their tax obligations?

- Answer: If agents can default on their tax obligations with impunity, then the Friedman rule is infeasible (the second-best allocation is achieved by setting $\mu = 1$; i.e., a constant money supply).

- However, things might be improved if agents could somehow be induced to pay their taxes "voluntarily."

## 3.1 Scenario 1: Observable Money is Taxable

- Assume that money holdings are observable in the day (this does not violate the assumption of anonymity).

- Assume that observable money is taxable.

- Then agents can avoid paying taxes by refusing to accumulate money balances in the day (and in the night).

- This strategy, however, means that money is never bought or sold–the resulting payoff is autarky.

## 3.2 Scenario 2: Society as a Gatekeeper

- Assume that society can control access to the night-market (a gatekeeper–like a bouncer at a bar); again note that this does not violate the assumption of anonymity.

- Now interpret the tax as an "entrance fee" (which is paid voluntarily, of course).

- Failure to pay the entrance fee essentially sends the agent to autarky, as above.

- Hence, either scenario motivates an identical SPC:

$$x \leq W(y)$$