Topics In Empirical Industrial Organisation

Demand Systems for Highly Differentiated Products

Overview

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Outline I

Outline II

General focus of much of the innovation in E-IO has been in models of demand and models of entry.
In these lectures we focus on innovations in the modelling of systems of demand, with particular focus on industries characterised by imperfect competition and highly differentiated products

Demand Systems

1. the starting point for most applied analysis in I.O
2. the demand system is the only indispensable component of how prices are set in Nash equilibrium, and how prices and demand patterns might changes subsequent to a ...
    - merger
    - a new tariff
    - the entry of a new good
    - intervention by a regulator

Models of Firm Behaviour

Cournot: Assume firms choose quantity and the market determines price

Bertrand: Firms set price and the market determines the quantity sold (price setting oligopolists)

Homogenous products: small price differences can mean one firm captures all demand.

Heterogenous products: move away from a world of perfect substitutes.

Focus: price competition with imperfect substitutes

Deviation from Bertrand: Focus: price competition with imperfect substitutes and some other stuff

Bertrand-Nash Equilibrium

- F multiproduct firms competing in a Bertrand-Nash fashion; given their products and the prices and attributes of *competing* products, firms choose prices to maximise profits.
- Each firm f produces some subset J_f of the J total products.
- Firms have a MC function that is log-linear in a vector of observed ($\mathbf{w}_j$) and an unobserved ($\boldsymbol{\omega}_j$) cost characteristics.

Notation and Primitives

Characteristics of Products, characteristics of consumers (preferences) and an equilibrium notion.

Markets: $m = 1, ..., M$

Firms: N_m firms in market m.

Products: $j = 1, ..., J$. Choice set Ω_J.

Observable Product Attributes:

$\mathbf{b}_j : (L_d + L_c) \times 1$ vector

$\mathbf{b}_j : (\mathbf{v}_j, z_j)$

$\mathbf{v}_j : L_d \times 1$ vector of attributes that affect demand

$\mathbf{z}_j : L_c \times 1$ vector of attributes that affect marginal costs.

Unobservable Product Attributes:

(ξ_j, ω_j).

ξ_j : unobserved demand attribute.

ω_j : unobserved marginal cost variable.

Profits

We begin by assuming N firms are price setters.

Let

$C_j(q_j, \mathbf{z}_j, \boldsymbol{\tau}_j, \gamma)$ denote total costs for firm j.

$c_j(q_j, \mathbf{z}_j, \boldsymbol{\tau}_j, \gamma)$ denote marginal costs.

Profits:

$$\Pi_f = M \sum_{j \in J_f} (p_j - C_j(q_j, \mathbf{z}_j, \boldsymbol{\tau}_j, \gamma)) s_j(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d).$$

$s_j(.)$ is good j's predicted market share

$q_j(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) = M s_j(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d)$.

M is the number of households in the population

Given a pure-strategy interior equilibrium, the price vector satisfies the J FOCs for static price competition

$$s_j(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) + \sum_{j \in J_f} (p_j - mc_j) \frac{\partial s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d)}{\partial p_j} = 0. \tag{1}$$

The Industry Model

Marginal Costs and Mark-ups

This system of equations can be inverted to solve for the marginal costs

$$\begin{aligned} s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) &+ \boldsymbol{\Delta}(\mathbf{p}, \mathbf{v}; \boldsymbol{\theta})^{-1}(\mathbf{p} - \mathbf{c}) = 0 && (2) \\ \mathbf{c} &= \mathbf{p} - \boldsymbol{\Delta}(\mathbf{p}, \mathbf{v}; \boldsymbol{\theta})^{-1} s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) && (3) \\ \boldsymbol{\Delta}(\mathbf{p}, \mathbf{v}; \boldsymbol{\theta}) &= \boldsymbol{\Omega}(\partial s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) / \partial p_j) && (4) \end{aligned}$$

$\boldsymbol{\Delta}(\mathbf{p}, \mathbf{v}; \boldsymbol{\theta}) =$ is the appropriately defined matrix of own- and

cross-price share derivatives with elements $\partial s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d) / \partial p_j$

$\boldsymbol{\Omega} = \{\Omega_{jk}\} = 1$ if products j, k are produced by the same firm.

$\mathbf{mc}, \mathbf{p}$ and $s(\mathbf{v}, \xi, \mathbf{p}, \boldsymbol{\theta}_d)$ are respectively, the vectors of marginal costs, prices, and market shares.

Why Include the Supply Side?

Remark

Adding a supply side equation in the context of a demand model for differentiated products is analagous to adding a supply equation to the demand equation in a model of perfect competition.
As Pakes (2006) notes, the cost of adding a supply equation is that

1. *we need an equilibrium assumption*
2. *we need a specification for the cost function*

Market Power - much of the focus of antitrust analysis the extent to which a firm can exercise market power.

Are high profits and markups a result of product differentiation, market share, small number of substitutes, or some form of collusion.

Merger Analysis When firms merge the industry structure changes. Say two firms producing products j and l merge; the new firm is f'.

Firm f' will particularly care about the effect of a change in the price of product $j_{f'}$ on the demand for $l_{f'}$

The Market

Defining the Market We can define the market based on product technology but also from the standpoint of the consumer.

If a consumer is willing to switch from product A to B given an increase in the price of A, then both products are part of the same market.
Key issue in antitrust cases

In case of homogenous products MP is linearly related to the HHI)
In case of a Cournot oligopoly **homogenous** good industry, average price-cost margin is

$$\sum_j s_j \left(\frac{p - s_j}{p} \right) = \frac{\sum_j s_j^2}{\eta} = \frac{HHI}{\eta} \qquad (5)$$

η is industry demand elasticity

In an industry composed of firms selling differentiated products, market share is no longer good approximation of the ability to mark-up over cost.

Key issue: degree of substitutability. Firms with small market-share may command significant market power through a particular set of attributes and/or location.

Point of Departure: Logit Demand

- As we will see in the course of this course, a logit demand model implies that relative market share of two products depends on the difference in market prices
- large market share will always guarantee lower price elasticity, ceteris paribus.
- combined with uniform property on cross-elasticities this means that mergers between firms with high shares will always be more problematic in terms of reducing competition

The analysis of market concentration plays a key role in the selection stage of all notified mergers in the US and EU.

Remark

Following Article 2 of EU legislation, any merger that will significantly impedes effective competition in the common market or in a substantial part of it should be stopped.
US: mergers are prohibited if they are likely to result in a substantial lessening of competition

The screening rule is based on level and likely changes in the Herfindahl Index (HH). $= \sum_{i=1}^{N} s_i^2$.

s_i is the market share of i^{th} firm

$HH \in (\frac{1}{N}, 1)$

Within the framework of the **Industry Model** different types of strategic behaviour can be incorporated.

Different degrees of collusion or a merger occurs, we can alter the structure of Ψ accordingly.

When two firms merge they are especially interested in the effect of one firm's price on the demand of the others (+ vice versa).

Example

The ownership structure within an industry changes from period 1-2. : $\Psi_1 \rightarrow \Psi_2$.
Post merger prices: $\mathbf{p}_2 = (p_{1,2}, p_{2,2}, p_{3,2}, ..., p_{J,2})$ need to satisfy the revised F.O.C. over the different industry structure

$$\mathbf{p}_2 = \mathbf{c}_2 - (\Psi_2 \frac{\partial s(\mathbf{p}_2)}{\partial p})^{-1} s(\mathbf{p}_2)$$

$s(\mathbf{p}_2)$, $\frac{\partial s(\mathbf{p}_2)}{\partial p}$: inputs from logit, BLP, AIDS.

Different types of Ownership Structure

Example

single product firms $\rightarrow$ product differentiation

$$\Psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example

multiproduct firms $\rightarrow$ portfolio effect

$$\Psi = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Natural Experiments Can we estimate directly the effect of a change in price of product j on the demand for k.

Can we estimate the effect of one fewer firm on the equilibrium price

How does the effect of price and other dimensions of competition vary as a function of the number of firms in an industry?

Structural Estimation Product Space : AIDS modelling and multi-level budgeting

Characteristic Space : Logit

Characteristic Space : BLP

Automobile Industry: differentiated good oligopoly.

multiproduct firms - automobiles are produced by multiproduct firms selling differentiated products.

vertically differentiated - models differ across a number of attributes.

firms compete in prices to maximise profits from all models they sell.

Price are set by accounting for the competition from other firms as well as possible within fleet substitution for a given firm

Profits

$$\pi_j = \sum_{j \in F_f}^{\Omega_J} (p_j(1-t) - c_j) s_j(\mathbf{p}) M \qquad (6)$$

$p_j, c_j, t, M, \mathbf{p}$, respectively, price of car j, MC of j, tax, size of the market, and a vector of prices.

Competitive Pricing Behaviour in the Automobile Market

Example

Consider the impact of a tax? Will the introduction of a new tax change the market equilibrium, the prices of cars and possibly the equilibrium distribution of cars that each firm sells

Related Literature

- Berry, S. Levinsohn, J. and Pakes, A. (1995). "Automobile Prices in Market Equilibrium". *Econometrica* 63 (July), pp. 841-990.
- Ferraz, C., Fiuza , P. and da Motta, R. (1999). "Measuring the Effects of Environmental Regulation in Oligoplistic Markets with Differntiated Products". Working Paper No. 36, International Institute for Environment and Development, London.
- Fershtman, C., Gandal , N. and Markovich, S. (1999). "Estimating the Effect of Tax Reform in Differentiated Product Oligoplistic Markets". mimeo, Tel Aviv University.

See Sudhir paper
An Empirical Study of the Italian Automobile Market

Some of the key dimensions of a demand system that we will consider

single versus multi-products

representative vs heterogenous agents

product versus characteristic space

Dimensionality and Demand Systems

Modelling the demand for products $j = 1, .., J$ through a system of demand equations

$$\mathbf{q} = f(\mathbf{p}, \mathbf{z})$$

$\mathbf{p}$ a $J \times 1$ vector or prices. $\mathbf{z}$ - additional information
Three main issues that analyst needs to confront in specifying $f(.)$ and operationalising any of these approaches:

Curse of Dimensionality - what is this? Is this a real curse? Curse in product space/in characteristic space?

Consumer Heterogeneity - observed/unobserved heterogeneity.

Endogeneity of Prices - how to handle endogeneity in linear and non-linear demand systems.

Product Space versus Characteristic Space Models

product-space approach representative agent's utility defined on the product:

Data: expenditure shares, prices, and other drivers $\mathbf{z}$

$\{e_j, p_j, \mathbf{p_{-j}}, \mathbf{z}_j\}, j = 1, ..., J$

Example

J goods implies J^2 parameters. The curse of dimensionality results. Circumventing this problem requires assumptions in terms of separability of preferences and multi-level budgeting

Characteristic Space Models

characteristic-level approach representative agent's utility defined on the characteristics of the product.

Data: shares, prices, and attributes

$\{s_j, p_j, \mathbf{v}_j\}, j = 1, ..., J$

Remark

The number of parameters required to determine aggregate demand is independent of the number of products.

Example

4 characteristics. Distribution of characteristics joint normal, then 14 parameters would determine demand for all products.

MLB: a format of the underlying utility function that generates empirically convenient properties. (Pakes, Asker [Lecture Notes])

A "utility tree"

1. Create a mutually exclusive partition over Ω_J.
2. Allocate expenditures *across* groups - this is part of the estimation procedure.
3. Allocate expenditures *within* groups - this is also part of the estimation procedure.

Note: *within* segment expenditure: allocation of expenditure *within* segment s is independent of that within s'.

This implies a change in p_j^s is only felt via the change in expenditure at the group level.

See *An Almost Ideal Demand System* (Muellbauer and Deaton, (1980)) Three Stage Demand System: Multi-stage budgeting

1: Top Overall demand for product (i.e. beer)

2: Middle Demand across-segments i.e. premium beer. There is not always clear cut partition of Ω_J.

3: Bottom Demand within a segment. Competition among brands within a segment.

Estimation: begin at bottom level and use the theory of price indices to facilitate consistent estimation at aggregate levels of demand.

Buying behaviour within segments: imposes a set of zero restrictions on cross-elasticities *between* segments.

Competitive Analysis with Differentiated Products
Top Level Equation: Product

$$\log u_t = \beta_0 + \beta_1 \log y_t + \beta_2 \log \pi_t + Z_t\delta + \varepsilon_t \qquad (7)$$

Middle Level Equations: Segment

$$\begin{aligned} \log q_{mnt} &= \beta_m \log y_{Bnt} + \sum_{k=1}^{k} \delta_k \log \pi_{knt} + \alpha_{mn} + \varepsilon_{mnt} \\ m &= 1, ..., M, \quad n = 1, ..., N, \quad t = 1, ..., T \end{aligned}$$

Bottom Level Equation: Brand

$$\begin{aligned} s_{int} &= \alpha_{in} + \beta_i \log(y_{Gnt}/P_{nt}) + \sum_{j=1}^{J} \gamma_{ij} \log p_{jnt} + \varepsilon_{int}, \qquad (8) \\ i &= 1, ..., J, \quad n = 1, ..., N, \quad t = 1, ..., T \end{aligned}$$

see (Hausman et al (1994)

Within Segments: Econometric Specification

For each brand $j = 1, ..., J$ in segment G, city $n = 1, ..., N$, period $t = 1, ..., T$.

$$
\begin{aligned}
s_{int} &= \alpha_{in} + \beta_i \log(y_{Gnt}/P_{nt}) + \sum_{j=1}^{J} \gamma_{ij} \log p_{jnt} + \varepsilon_{int}, \quad (9) \\
i &= 1, ..., J, \quad n = 1, ..., N, \quad t = 1, ..., T
\end{aligned}
$$

s_{int} is the revenues share of total segment expenditure of the *ith* brand in city n in period t

y_{Gnt} is overall segment expenditure,

P_{nt} is a price index, p_{jnt} is the price of the *jth* brand in city n.

γ_{ij} permits a free pattern of cross-price elasticities and Slutsky symmetry can be imposed if desired $(\gamma_{ij} = \gamma_{ji})$.

Remark

An important econometric consideration is the use of segment expenditure y_{Gnt}, in the share specification of (9), rather than the use of overall expenditure.

Use of overall expenditure is inconsistent with the economic theory of multi-stage budgeting, and it can lead to decidedly inferior econometric results.

Middle Level Demand System

$$\begin{aligned} \log q_{mnt} &= \beta_m \log y_{Bnt} + \sum_{k=1}^{k} \delta_k \log \pi_{knt} + \alpha_{mn} + \varepsilon_{mnt} \\ m &= 1, \ldots, M, \quad n = 1, \ldots, N, \quad t = 1, \ldots, T \end{aligned}$$

q_{mnt} is log quantity of the m^{th} segment in city n in period t,

y_{Bnt} is total beer expenditure

π_{knt} are the segment price indices for city n.

π_{knt} can be estimated either by using the exact price index corresponding to equation (9), which is constructed from the expenditure function for each segment holding utility constant, or by using a weighted average price index of the Laspeyres type.

Top Level Equation
- estimate the overall price elasticity of beer

$$\log u_t = \beta_0 + \beta_1 \log y_t + \beta_2 \log \pi_t + Z_t \delta + \varepsilon_t \qquad (10)$$

u_t is overall consumption of beer

y_t is deflated disposable income

π_t is the deflated price index for beer

Z_t are variables which account for changes in demographics, monthly (seasonal) factors and minimum age for purchasing beer.

Substitution Patterns

The AIDS model makes some assumptions between products. This is obviously necessary in order to circumvent the estimation J^2 coefficients.

- Top level: Coors and another product (chips). If the price of Coors goes up, then the price index of beer π increases.
- Medium level: Coors and Old Style, two beers in separate segments. Increase in the price of Coors [Light beer segment] raises π_L, which is likely to decrease the quantity of light beer sold.
- Bottom level: Coors and Budweiser, two beers in the same segment. Increase in the price of Coors affects Budweiser through γ_{cb}.
- The AIDS model restricts substitution patterns to be the same between any two products in different segments. Is this a reasonable assumption?

TABLE 1

Beer Segment Conditional Demand Equations.

	Premium	Popular	Light
Constant	0.501	−4.021	−1.183
	(0.283)	(0.560)	(0.377)
log (Beer Exp)	0.978	0.943	1.067
	(0.011)	(0.022)	(0.015)
log ($P_{PREMIUM}$)	−2.671	2.704	0.424
	(0.123)	(0.244)	(0.166)
log ($P_{POPULAR}$)	0.510	−2.707	0.747
	(0.097)	(0.193)	(0.127)
log (P_{LIGHT})	0.701	0.518	−2.424
	(0.070)	(0.140)	(0.092)
Time	−0.001	−0.000	0.002
	(0.000)	(0.001)	(0.000)
log (# of Stores)	−0.035	0.253	−0.176
	(0.016)	(0.034)	(0.023)

Number of Observations = 101.

Figure 1: Demand Equations: Middle Level- Segment Choice

Brand Share Equations: Premium.

	1 Budweiser	2 Molson	3 Labatts	4 Miller	5 Coors
Constant	0.393	0.377	0.230	−0.104	–
	(0.062)	(0.078)	(0.056)	(0.031)	–
Time	0.001	−0.000	0.001	0.000	–
	(0.000)	(0.000)	(0.000)	(0.000)	–
log (Y/P)	−0.004	−0.011	−0.006	0.017	–
	(0.006)	(0.007)	(0.005)	(0.003)	–
log ($P_{Budweiser}$)	−0.936	0.372	0.243	0.150	–
	(0.041)	(0.231)	(0.034)	(0.018)	–
log (P_{Molson})	0.372	−0.804	0.183	0.130	–
	(0.231)	(0.031)	(0.022)	(0.012)	–
log ($P_{Labatts}$)	0.243	0.183	−0.588	0.028	–
	(0.034)	(0.022)	(0.044)	(0.019)	–
log (P_{Miller})	0.150	0.130	0.028	−0.377	–
	(0.018)	(0.012)	(0.019)	(0.017)	–
log (# of Stores)	−0.010	0.005	−0.036	0.022	–
	(0.009)	(0.012)	(0.008)	(0.005)	–
Conditional Own Price Elasticity	−3.527	−5.049	−4.277	−4.201	−4.641
	(0.113)	(0.152)	(0.245)	(0.147)	(0.203)

$$\Sigma = \begin{Bmatrix} 0.000359 & -1.436E-05 & -0.000158 & -2.402E-05 \\ - & 0.000109 & -6.246E-05 & -1.847E-05 \\ - & - & 0.005487 & -0.000392 \\ - & - & - & 0.000492 \end{Bmatrix}$$

Note: Symmetry imposed during estimation.

Figure 2: Demand Equations: Bottom-Level Brand Choice

Light Segment Own and Cross Elasticities.

	Genesee Light	Coors Light	Old Milwaukee Light	Lite	Molson Light
Genesee Light	−3.763	0.464	0.397	0.254	0.201
	(0.072)	(0.060)	(0.039)	(0.043)	(0.037)
Coors Light	0.569	−4.598	0.407	0.452	0.482
	(0.085)	(0.115)	(0.058)	(0.075)	(0.061)
Old Milwaukee Light	1.233	0.956	−6.097	0.841	0.565
	(0.121)	(0.132)	(0.140)	(0.112)	(0.087)
Lite	0.509	0.737	0.587	−5.039	0.577
	(0.095)	(0.122)	(0.079)	(0.141)	(0.083)
Molson Light	0.683	1.213	0.611	0.893	−5.841
	(0.124)	(0.149)	(0.093)	(0.125)	(0.148)

Figure 3: Segment Elasticities

	Elasticity	Standard Error
Budweiser	−4.196	0.127
Molson	−5.390	0.154
Labatts	−4.592	0.247
Miller	−4.446	0.149
Coors	−4.897	0.205
Old Milwaukee	−5.277	0.118
Genesee	−4.236	0.129
Milwaukee's Best	−6.205	0.170
Busch	−6.051	0.332
Piels	−4.117	0.469
Genesee Light	−3.763	0.072
Coors Light	−4.598	0.115
Old Milwaukee Light	−6.097	0.140
Lite	−5.039	0.141
Molson Light	−5.841	0.148

Figure 4: Overall Elasticities

1. Demand is cast in *characteristic space* whereby products are bundles of characteristics $\mathbf{v} = \{v_{jl}\}$
2. preferences are defined on these characteristics,
3. consumers exhibit differential marginal evaluation of characteristics.
4. that alternative generating the highest utility is preferred

$$U_{ij} = U(\mathbf{v}_j, p_j, t_i; \boldsymbol{\theta})$$

5. from the perspective of the analyst utility is an unobserved random variable.

   The perspective adopted here is that if all measures on both the product and decision maker could be observed, then preferences would not be random.

The Demand for New Goods

Remark

Demand systems cast in product space do not enable the analyst to analyse the demand for new goods prior to their introduction

See Petrin (2002), Nevo (2003)

Market Level Observe allocation of market demand across products.

Early studies focussed on market level data (market shares for a product), mostly theoretical.
Computer technology and simulation estimators have enabled us to specify demand systems that are conditional on a set of observed attributes, aggregate up from an *observed distribution* of consumer characteristics and a set of parameters determining the relationship between those characteristics and preferences over products (or product attributes)
see BLP (1995)

Product Level Observe individual choice across products.

Here we are able to directly match attributes of products with the characteristics of individuals making the choices
see Train and Weeks (2008)

Data: shares, prices, and attributes - $\{s_j, p_j, \mathbf{v}_j\}, j = 1, ..., J$

$$\begin{aligned} U_{ij} &= H(\mathbf{v}_j, p_j, t_i; \boldsymbol{\theta}) + \varepsilon_{ij} \\ &= \alpha_j + p_j \omega_p + \varepsilon_{ij} \end{aligned}$$

Parameters to estimate: $\theta = \{\boldsymbol{\alpha}, \omega_p\}$, $\boldsymbol{\alpha} = J \times 1$.

- Identifying information: single market, single period
- $J - 1$ shares; $s_J = 1 - \sum_{j=1}^{J-1} s_j$ used to identify BUT $J + 1$ parameters - not identified!
- Additional information? Characteristic model space.

Mean utility: additive in observed + unobserved attributes

$$\alpha_j = \sum_{l=1}^{L} \omega_l v_{jl} + \xi_j \tag{11}$$

(one element of attribute $\mathbf{v} = \{\mathbf{v}_{jl}\}$ is price)

Parameters: $\boldsymbol{\theta}' = \{\boldsymbol{\omega}\}$.

Compare to AIDS model in product space:

Characteristic-Space Additional identifying information with Market and Product Level data

i. - variation in shares over markets/time

ii. - product level data i.e we can match individuals to products

iii. - supply side

iv. - other stuff (next lecture).

Recent Trends in Demand Estimation

- a move away from "representative agent" models to models with heterogenous agents. This has been facilitated by increased power of computers and simulation-based inference. See Pakes and Pollard (1986), McFadden (1986).
- away from models cast in *product space* to those set in *characteristic space* (CS) (see Lancaster (1971)). Examples of recent CS studies include Berry, Levinsohln and Pakes (1995), Train and Winston (2001).
- away from models which used functional forms which restricted cross and own price (attribute) elasticities
- towards models which accommodated unobserved product attributes even in non-linear demand systems (see Berry (1994))

Homogenous Preferences

$$U_{ij} = \mathbf{v}_j \boldsymbol{\omega} + \varepsilon_{ij}$$

mean utility: $\mathbf{v}_j \boldsymbol{\omega}$

deviation from mean: ε_{ij}: independent across j and i for a given j

This specification of utility implies that the distribution of consumer preferences over products (other than that bought), does not depend on the product purchased.

Own and Cross Elasticities

$$\eta_{jk} = \frac{\partial s_j}{\partial p_k}\frac{p_k}{s_j} = \left\{ \begin{array}{ll} -\beta_p p_j(1-s_j) & j=k \\ \beta_p p_k s_k & \text{else} \end{array} \right\}$$

- two goods with the same shares (s_j) have the same cross-price elasticities with any other good
- there is no systematic differences in the price sensitivities of consumers attracted to different goods, own price derivatives ($\partial s/\partial p = s(1-s)$) only depends on shares.
- this then implies two goods with the same share must have the same markup in a single product firm "Nash in prices".

Oligopoly Pricing Under Logit

Remark

In general within oligopolistic markets products that face good substitutes will have low markups
And the converse applies: products with few substitutes will have higher markups, and higher prices relative to cost.

Remark

Specifically in the case of the standard vanilla logit model, two products with the same market share will have the same own-price demand derivatives.
In the context of oligopolistic pricing this implies that two products will have the same mark-up over marginal cost
We would expect mark-ups to depend on factors other than market shares

Ω_J is a very large set.

$$U_{ij} = \alpha_j + H_{ij} + \boldsymbol{\tau}_{ij} + \varepsilon_{ij} \ \forall j \in \Omega_J \tag{12}$$

j is a vehicle: Ford Mondeo, 1.8 L, Petrol, Automatic, Saloon

$\boldsymbol{\theta} = \{(\boldsymbol{\beta}, \sigma), \boldsymbol{\alpha}\}$
Given $\boldsymbol{\theta}$ there exists a unique $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_J)'$ such that the predicted shares equal the actual shares. (Berry (1994)).
Write $\boldsymbol{\alpha}$ as a function of $\boldsymbol{\theta}$, reducing the number of parameters that enter the LF.
$\boldsymbol{\theta}$ are estimated by Maximum Likelihood;
$\boldsymbol{\alpha}$ calculated st. predicted shares match observed shares given $\boldsymbol{\theta}$.