Efficient Investment in Children and Implications for Policy*

Hakki Yazici†

University of Minnesota and Federal Reserve Bank of Minneapolis

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Abstract

Raising children is an important productive activity for a society since children's outcomes depend on their parents' investments. This paper develops an intergenerational framework in which adult outcomes are determined by parental investment, analyzes Pareto efficient allocations, and derives implications for policy. There are two key frictions: first, parental altruism type is private information; second, it is impossible for society to monitor parental investment. The main characterization result is that in any ex post Pareto efficient allocation, in any generation, society should transfer extra resources to all poor parents. This implies all agents, including the poor, should live above a certain welfare level, independent of whether or not society cares about them. Regarding implementation, the paper first shows that if initial generation parents can sign exclusive contracts ex ante that legally bind all their descendants, then the resulting equilibrium allocation is Pareto efficient. Second, the paper considers a more realistic market structure in which parents cannot sign contracts binding their descendants. Under such a market structure, implementing any Pareto efficient allocation requires government intervention. A feature shared by all Pareto efficient income tax schedules is that income taxes of agents with currently low income are negative.

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†Contact: Department of Economics, University of Minnesota, 4-101 Hanson Hall, 1925 Fourth Street South, Minneapolis, MN 55455. Email: hyazici@econ.umn.edu.

1 Introduction

It is mostly parents who choose the level of human capital investment their children receive. For instance, parents choose how many books to buy their children, how much time to spend reading to them, or whether to pay for private tutors for their children. It is empirically an established fact that these parental investments are significant in determining adult outcomes.[1] Hence, parental investments are important productive activities from society's perspective. However, it is hard to monitor the level of investment each child receives from her parents. This implies that society potentially faces an agency problem regarding investment in children: society has to provide parents with the right incentives so as to make them invest in their children.

This paper develops an intergenerational model in which parental investment in children is subject to such an agency problem in order to accomplish two specific goals. The first goal is to analyze the structure of the entire set of Pareto efficient allocations. The main characterization result is that, independent of social preferences, in all generations, poor parents should receive subsidies from the rest of the society. This implies all agents, including the poor, should live above a certain welfare level, independent of whether or not society cares about them. The second goal is to explore the types of market arrangements and policies that attain Pareto efficiency.

Specifically, the paper considers a dynastic model with heterogenous intergenerational altruism. In any generation, there are two types of parents: *altruistic* parents care about their children whereas *selfish* do not. Agents live for two periods. In the childhood period, they simply receive human capital investment from their parents which determines their output next period, when they become parents.[2] Investment in children is in terms of forgone consumption, and there is diminishing marginal returns to investment. In parenthood, agents first realize their altruism type and then choose how to divide family resources between consumption and

[1]Todd and Wolpin (2006) is a recent paper that provides evidence on the importance of parental investment in explaining children's cognitive test score gaps. This provides an indirect evidence on the importance of parental inputs in determining adult outcomes since, as it is documented by Leibowitz (1974), Neal and Johnson (1996), and Keane and Wolpin (1997), cognitive tests taken during adolescence are predictive of adult labor market outcomes. For more direct support of the effect of parental investments in determining future outcomes see Cunha and Heckman (2006).

[2]Becker and Tomes (1979), Loury (1981), and Becker and Tomes (1986) are seminal papers that model adult output as being determined by parental human capital investments undertaken during childhood.

investment in children.

I make two crucial informational assumptions. First, parents privately observe whether they are altruistic or not. Second, parental investment in a child is private information at the time of investment. Society learns the level of human capital investment a child receives from her parents by observing the child's adult outcome, and, thus, with a lag. The unobservability of parental investment implies that there is an agency problem regarding parental investment in children. Society cannot force parents to invest in their children; parents must be provided with incentives to do so. Since parents are already replaced by their children by the time investment becomes observable, the only way to provide parents with incentives is by rewarding or punishing their children in terms of consumption.

In the absence of informational frictions, Pareto efficient resource allocation involves two separate steps: (1) productive efficiency requires equating marginal returns to parental investment across children within any generation; (2) distributive efficiency then requires making transfers between agents so as to achieve the desired consumption distribution, which depends on the welfare criterion of society. However, I show that, under informational frictions, productive efficiency and distributive efficiency cannot be completely separated. Attaining productive efficiency implies certain properties that are common to all Pareto efficient distributions of consumption, independent of any social welfare criterion. The first major goal of this paper is to establish these properties.

I show in any Pareto efficient allocation, for all generations, all poor parents receive subsidies. That is, society should transfer additional resources to parents who have not received much human capital investment from their parents, and, hence, have low output. This then implies all agents at all periods live above a certain welfare level. The intuition is simple. Without a subsidy, the amount of investment children receive would be limited by parental output. Due to diminishing marginal returns to investment, this means children of poor parents would have relatively high marginal returns. By redistributing resources from rich parents to poor ones, society reallocates investment from low return children to high return children. As a result, average returns to childhood investment increase, and, hence, there are more resources next period to be distributed among those society cares about. However, since parental investment is unobservable, in order to make parents invest, society has to provide them with incentives in terms of their children's consumption, because otherwise parents would consume the subsidies intended for investment in children. Therefore, subsidies toward the poor implies welfare transfers toward them, which implies, all parents, including the poor, live above a certain welfare level.

Consequently, subsidizing the poor is required by Pareto efficiency because it enhances productive efficiency. Since there are no assumptions about any particular social welfare function, equality or insurance considerations play *no* role in the subsidy result. In this sense the subsidy result in the current paper is distinct from most of the redistribution results in public finance as these results are generally conditional on assumptions about social welfare functions that value equity. This distinction is a very important one because the result in this paper establishes that subsidizing the poor is a necessary condition to be efficient whereas previous results merely say redistributing to the poor is needed if we assume that society has a particular social welfare criterion.

The second major goal of the paper is to find actual market structures and tax systems that implement Pareto efficient allocations. The paper first considers a market in which initial generation parents can sign exclusive contracts ex ante that are legally binding for all their descendants. A version of the First Welfare Theorem holds: the market equilibrium is Pareto efficient.[3] As a result, if such a market is in operation, any government intervention would be based on redistributive motives rather than efficiency considerations. However, such a market structure is highly unrealistic as it crucially requires enforcement of contracts which, when signed by an agent, bind all future descendants.

The paper then considers implementing Pareto efficient allocations in a more realistic market setup in which contracts signed by parents do not bind their children. Agents are allowed to sign all other types of contracts that respect informational constraints. The market equilibrium is Pareto inefficient, which implies government intervention is essential under such a market structure. An interesting property of this market arrangement is that there is a subset of Pareto efficient allocations for which there is *no* tax implementation. Then, the paper focuses on

[3] The result that ex ante efficient allocations under private information can be decentralized as through ex ante contracts is originally due to Prescott and Townsend (1984) for static economies. Atkeson and Lucas (1992) and Golosov and Tsyvinski (2007) provide similar decentralizations for different dynamic private information economies.

Pareto efficient allocations that can be implemented with income taxes and derives an important property shared by all Pareto efficient income tax systems: in any generation, income taxes of agents with currently low income are negative. This result is essentially a translation of the subsidy result about Pareto efficient allocations into a result on Pareto efficient income taxation.

Becker and Tomes (1979), Loury (1981), and Becker and Tomes (1986) are seminal papers that develop dynastic models in which individuals' adult income is a function of the parental investment they receive during childhood. These papers consider equilibria under different incomplete market structures with the aim of establishing investments within family as an important source of income inequality. A recent paper in the same tradition is Aiyagari, Greenwood, and Seshadri (2002) which quantitatively compares the performance of different market structures in terms of welfare, aggregate output, and inequality in a model in which parental inputs are important determinants of adult outcomes. The main distinction of the current paper from all these papers is that, instead of assuming the existence of market incompleteness exogenously, I endogenize market incompleteness by assuming that there are key informational frictions regarding investment in children. Furthermore, all the aforementioned papers are essentially positive analyzes, whereas the current paper focuses on the properties of Pareto efficient allocations and implications for optimal policy.[4]

Banerjee and Newman (1991), Galor and Zeira (1993), and Aghion and Bolton (1997) are also related to the current paper. These papers study the link between the distribution of wealth and productive activity in dynastic economies in which there is individual level production with diminishing returns to scale. Even though they do not focus on policy analysis, an implicit conclusion common to all these papers is that redistribution towards the poor can enhance productive efficiency.[5] The main difference with the current paper is that I focus on Pareto efficient allocations and the resulting optimal public policies.

This paper is also related to a number of recent papers that explore constrained efficient allocations in intergenerational settings with private information frictions, such as Phelan (2006),

[4]An exception is Loury (1981) which provides some policy experiments. Furthermore, Seshadri and Yuki (2004) also investigates numerical policy experiments in an incomplete markets setup. However, unlike the current paper, neither of these papers analyze optimal policy.

[5]Indeed, Aghion and Bolton (1997) explicitly carries out such a normative analysis. The paper takes output maximization as the social objective and shows that when there are moral hazard problems due to unobservable effort, redistributing resources from the rich to the poor may increase total output, boosting economic growth.

Farhi and Werning (2007), and Farhi and Werning (2008). The most important difference between the papers above and the current one is that the papers above focus on social planning problems that maximize specific social welfare functions whereas I analyze the properties shared by the entire set of Pareto efficient allocations. This is a very important distinction because analyzing all Pareto efficient allocations allows me to, as Stiglitz (1987) puts it, "separate out efficiency considerations from the value judgements associated with choices among Pareto efficient points." As a result, I isolate and focus on the implications of productive efficiency while the efficiency concepts used by previous papers include social preference for equality.

The rest of the paper is structured as follows. In section 2, I introduce the model formally. Section 3 characterizes properties common to all ex post Pareto efficient allocations. Section 4 discusses two different implementations of Pareto efficient allocations. Finally, section 5 concludes.

2 Model

2.1 Environment

The economy is populated by a continuum of unit measure of dynasties who live for a countable infinity of periods, $t = 1, 2, \dots$. Each agent within a dynasty lives for two periods: childhood and parenthood. In each period t, each parent has a child. In period $t+1$, the child becomes a parent and replaces the period t parent.[6] The economy starts at the beginning of period one with the first members of each dynasty: initial parent generation and their children.

A period t parent has expected utility preferences, with utility function:

$$U_t = u(c_t) + \beta_t U_{t+1},$$

where c_t is period t parent's consumption, $u(\cdot)$ is instantaneous utility function, β_t is period t parent's altruism factor, and U_{t+1} is the utility of the offspring of the period t parent. Agents have to consume a non-negative amount, i.e. $c_t \geq 0$. The above specification of altruism is

[6] The terms period t parent and period t adult will be used interchangeably to refer to agents who have children in period t.

consistent with agents having a preference over the entire future consumption of their dynasty:

$$U_t = \sum_{\tau=t}^{\infty} \mathbb{E}_t[\prod_{s=t}^{\tau-1} \beta_s u(c_\tau)].$$

I make the following assumptions about the instantaneous utility function.

Assumption 1. *The instantaneous utility function* $u : \Re_+ \to \Re$ *satisfies:*

a. $u', -u'' > 0$.

b. $u(0) = \kappa > -\infty$.

The first part of the assumption is standard: instantaneous utility functions is strictly increasing and strictly concave in consumption. The second part of the assumption simply says that the lowest period utility that an agent can receive is finite.

A period t parent chooses how to allocate family resources between own consumption, c_t, and human capital investment in the child, n_t. So, investment in children is in terms of forgone consumption.[7] Investment cannot be negative, i.e., $n_t \geq 0$. When invested n_t units in period t, a child produces $y_{t+1} = f(n_t)$ units in period $t+1$ as an adult. During childhood, an agent is merely a production unit operated by the parent and does not make any decisions. I make the following somewhat standard assumptions on the production function f.

Assumption 2. *The production function* $f : \Re_+ \to \Re_+$ *satisfies:*

a. $f', -f'' > 0$.

b. $\lim_{n\to 0} f'(n) = \infty$.

c. $f(0) = 0$.

The first part of the assumption is standard and says children's outcomes are strictly increasing and strictly concave in the amount of investment they receive from their parents. The second part is simply the Inada condition. Finally, the last condition means if children do not receive any investment then their adult outcomes are zero.

Parents are heterogenous with respect to the level of their altruism towards their children. Some of them care about their children $\beta = \bar{\beta}$, and some do not, $\beta = \underline{\beta}$, where $0 \leq \underline{\beta} < \bar{\beta} < 1$.

[7] I follow Loury (1981), Becker and Tomes (1986), and Cunha and Heckman (2006), among others, in assuming that investment in children is in terms of forgone consumption.

Define $H = \{\underline{\beta}, \bar{\beta}\}$ to be the set of all altruism factors. Parents with altruism factor $\bar{\beta}$ are called altruistic, and those with altruism factor $\underline{\beta}$ are called selfish. Agents learn how altruistic they are only after becoming parents. Consequently, a period t child learns how altruistic she is in the beginning of period $t+1$, before making any decisions. The probability that a child in period t is going to be an altruistic parent in period $t+1$ is $\mu(\bar{\beta})$. With probability $\mu(\underline{\beta})$, the child becomes a selfish parent. The distribution of β is i.i.d. across dynasties and time.

In period t, dynasties are different from each other only with respect to the realizations of altruism factors up to that period. Therefore, in period t, each dynasty is identified with its realized history of altruism factors, $h^t = (\beta_1, \beta_2, ..., \beta_t)$. Define H^t to be the set of all histories at time t. Let the notation $h^{t+j} \succeq h^t$ mean that h^{t+j} follows h^t (i.e. its first t components are equal to h^t). With some abuse of notation, define $\mu(h^t)$ to be the ex ante probability of a period t parent having history h^t. By the Law of Large Numbers, $\mu(h^t)$ is also the fraction of dynasties with history h^t at time t.

To simplify notation, I define $\delta_t(h^\tau) = \prod_{s=t}^{\tau-1} \beta_s$. In words, $\delta_t(h^\tau)$ is the cumulative altruism towards agent with history h^τ from her period t ancestor. For initial parents, the cumulative altruism towards generation τ history h^τ descendant is denoted by $\delta(h^\tau) = \prod_{s=1}^{\tau-1} \beta_s$.

I make two key informational assumptions. First, each parent privately observes her altruism factor. I believe this is a reasonable assumption as it is hard, if not possible, for outsiders to know how much a parent cares about her child. Furthermore, how a parent allocates family resources between consumption and investment in the child is not publicly observable in the period in which the decision is made. Society learns the level of human capital investment in a child by observing the child's adult income next period, and thus, with a lag. When the child becomes an adult society observes her output and deduces the investment she received retrospectively.

An *allocation* in this economy is defined to be $x \equiv (c_t, n_t)_{t \geq 1}$, where

$$c_t, n_t : H^t \rightarrow \Re_+.$$

Here $c_t(h^t)$ and $n_t(h^t)$ are period t consumption and investment levels for a dynasty with an altruism history of h^t. An allocation x is *feasible* if for all t,

$$\sum_{h^t} \mu(h^t)[c_t(h^t) + n_t(h^t)] \leq \sum_{h^{t-1}} \mu(h^{t-1}) f(n_{t-1}(h^{t-1})), \tag{1}$$

$$c_t(h^t), n_t(h^t) \geq 0, \tag{2}$$

where $f(n_0) > 0$ is given. The first condition merely says that for any generation t the sum of aggregate consumption and aggregate investment should be no greater than aggregate output.[8] The second condition ensures that consumption and investment are non-negative.

2.2 Incentive-compatibility

There are two sources of private information in the model. First, there is hidden information: parents privately observe their altruism types. Second, parents are involved in hidden action: their consumption and human capital investment choices are unobservable. Hence, in this world, parents can deviate from an allocation recommended by the planner in three ways: they can lie about their altruism factor, they can choose an investment level that is different from what the planner recommended, or they can double deviate by doing both at the same time. A powerful Revelation Principle due to Myerson (1982) says that if an allocation is implemented as an equilibrium of an arbitrary mechanism, then it can also be implemented as a truth-telling and obedient equilibrium of a direct revelation mechanism. Therefore, without loss of generality, the rest of the paper restricts attention to allocations that can be implemented as truth-telling and obedient equilibria of direct revelation mechanisms.

In this subsection, I first define what it means for an allocation to be implemented as a truth-telling and obedient equilibrium of a direct revelation mechanism. Then, I define a subset of the set of all direct revelation mechanisms which I call the set of worst punishment direct revelation mechanisms. I prove that an allocation is implementable as a truth-telling and obedient equilibrium of a direct revelation mechanism if and only if it is implementable as a truth-telling and obedient equilibrium of a worst punishment direct revelation mechanism. This implies that one can further restrict attention to allocations that can be implemented as a truth-telling and

[8] Observe that one can write the feasibility condition as in (1), even though c_t and n_t are private information. The reason is that the sum $c_t + n_t$ is publicly observable.

obedient equilibrium of worst punishment direct revelation mechanisms. This is an important simplification because it is enough to check a single inequality to see whether an allocation can be implemented as a truth-telling equilibrium of a worst punishment direct revelation mechanism. This inequality is the incentive compatibility condition and the allocations that satisfy this condition are called incentive compatible.

In the last part of the subsection, I show that one can replace the incentive compatibility condition by a set of much simpler conditions, often referred to as temporary incentive compatibility constraints in the dynamic contracting literature.[9] Consequently, these inequalities completely represent the restrictions that informational problems place on the set of allocations achievable by the society.

A *direct revelation mechanism* is $(C,N) \equiv (C_t, N_t)_{t\geq 1}$, where $C_t, N_t : H^t \times \Re_+^{t-1} \rightarrow \Re_+$. Here, C_t and N_t are the outcome functions. The planner transfers a total of $C_t(h^t, \tilde{n}^{t-1}) + N_t(h^t, \tilde{n}^{t-1})$ to the dynasty with history $(h^t, \tilde{n}^{t-1})$ of reported altruism types and observed human capital investment levels, and recommends the parent to spend $N_t(h^t, \tilde{n}^{t-1})$ units on the child and the rest on her own consumption. Note that the outcome functions depend on the past observed history of investment levels as well as the history of reported types. This is due to the feature of the model that investment becomes publicly observable with one period lag.

A *dynastic strategy* is $(\hat{\sigma}, \hat{n}) \equiv (\hat{\sigma}_t, \hat{n}_t)_{t\geq 1}$, where $\hat{\sigma}_t : H^t \times \Re_+^{t-1} \rightarrow H$ and $\hat{n}_t : H^t \times \Re_+^{t-1} \rightarrow [C_t(h^t, \tilde{n}^{t-1}) + N_t(h^t, \tilde{n}^{t-1})]$. Here, $\hat{\sigma}_t(h^t, \tilde{n}^{t-1})$ is the report of the parent with history (h^t, n^{t-1}), and $\hat{n}_t(h^t, \tilde{n}^{t-1})$ is the investment of the same parent in her children. Define $\hat{\sigma}^t(h^t, \tilde{n}^{t-1})$ as the history of reports along $(h^t, \tilde{n}^{t-1})$ and $\hat{n}^t(h^t, \tilde{n}^{t-1})$ as the history of investment levels along the same path. Let Γ be the set of all dynastic strategies. Finally, define (σ, N) as the truth-telling and obedient dynastic strategy.

Define $W(\hat{\sigma}, \hat{n}|C, N)$ to be the expected lifetime payoff to a dynasty under the dynastic strategy $(\hat{\sigma}, \hat{n})$, given the direct revelation mechanism, (C, N) :

$$W(\hat{\sigma}, \hat{n}|C,N) = \sum_{t=1}^{\infty} \sum_{h^t} \mu(h^t)\delta(h^t) u\Big[C_t\Big(\hat{\sigma}^t(\cdot), \hat{n}^{t-1}(\cdot)\Big) + N_t\Big(\hat{\sigma}^t(\cdot), \hat{n}^{t-1}(\cdot)\Big) - \hat{n}_t\Big(h^t, \hat{n}^{t-1}(\cdot)\Big)\Big].$$

An allocation x is implemented as a truth-telling and obedient equilibrium of a direct revelation

[9] The term temporary incentive compatibility condition is due to Green (1987).

mechanism (C, N) if two conditions hold. First, the payoff to each dynasty of telling the truth and obeying recommendations should be at least as large as the payoff from any other strategy. Second, consumption and human capital investment allocations along the equilibrium path of the direct revelation game should correspond to the allocation x. The formal definition is given below.

Definition 1. *An allocation x is implemented as a truth-telling and obedient equilibrium of a direct revelation mechanism if and only if*

a. For all $(\hat{\sigma}, \hat{n}) \in \Gamma$,

$$W(\sigma, H|C, N) \geq W(\hat{\sigma}, \hat{n}|C, N).$$

b. For all h^t,

$$C_t(h^t, N^{t-1}(\cdot)) = c_t(h^t),$$
$$N_t(h^t, N^{t-1}(\cdot)) = n_t(h^t).$$

By the Revelation Principle, Definition 1, together with the feasibility constraints 1 and 1, characterizes the set of all allocations that are achievable by society.

I define a *worst punishment direct revelation mechanism* to be a direct revelation mechanism that sets consumption and investment levels to zero at all nodes following a deviation from recommended investment. For each direct revelation mechanism, (C, N), there exists a corresponding worst punishment direct revelation mechanism, (C', N'), defined as follows:

For any $t \geq 1$ and $(h^t, \tilde{n}^{t-1}) \in H^t \times \Re_+^{t-1}$,

$$Z'_t(h^t, \tilde{n}^{t-1}) = Z_t(h^t, \tilde{n}^{t-1}), \quad \text{if } \tilde{n}^{t-1} = N^{t-1}(h^{t-1}, N^{t-2}(\cdot));$$
$$Z'_\tau(h^\tau, \tilde{n}^{\tau-1}) = 0, \quad \text{if else},$$

for $Z = C, N$. Observe that for any period t, one can divide the public histories of the direct revelation game, $(h^t, \tilde{n}^{t-1})$, into two groups. First, there are paths along which there have been no deviations from the recommended investment levels up to t. Second, there are histories in which there has been at least one deviation in the first $t-1$ generations. I call the first group of histories the obedience path. So, (C', N') is identical to (C, N) on the path of obedience. The mechanisms differ in their reaction to dynasties that deviate from the recommended investment level, i.e., off the obedience path. The worst punishment direct revelation mechanisms punish disobeying dynasties in the harshest way possible. Note that the set of all worst punishment direct revelation mechanisms is a subset of the set of all direct revelation mechanisms.

Lemma 1. ***(Worst punishment)*** *An allocation x is implemented as a truth-telling and obedient equilibrium of a direct revelation mechanism (C, N) if and only if it is implemented as a truth-telling and obedient equilibrium of the corresponding worst punishment direct revelation mechanism, (C', N').*

Proof. Relegated to Appendix A. □

Consequently, the rest of the paper restricts attention to worst punishment direct revelation mechanisms. Observe that worst punishment mechanisms are much simpler objects compared to general direct revelation mechanisms. In fact, a worst punishment mechanism offers a dynasty an allocation x along the obedience path and identically zero transfers off the obedience path. To see this, let $C'_t(h^t, N'^{t-1}(h^{t-1}, N'^{t-2}(\cdot))) = c_t(h^t)$ and $N'_t(h^t, N'^{t-1}(h^{t-1}, N'^{t-2}(\cdot))) = n_t(h^t)$. In the rest of the paper, I refer to a worst punishment mechanism by the allocation it corresponds to on the obedience path since all worst punishment mechanisms are the same off the obedience path.

Restricting attention to worst punishment mechanisms is an important simplification because it allows one not to worry about the nodes that are reached after a deviation from an action recommended by the planner. At all such nodes, the planner will be transferring the dynasty zero units no matter what the dynasty reports. Consequently, reports following a deviation from recommended action are payoff irrelevant. Also, human capital investment has to be zero at any node following a deviation. These imply that one can redefine reporting and investment strategies independent of the history of investments.

Redefine a dynastic reporting strategy as a sequence of functions, $(\hat{\sigma}_t)_{t\geq 1}$, where $\hat{\sigma}_t : H^t \to H$. $\hat{\sigma}_t(h^t)$ gives a dynasty's report at node h^t, independent of the levels of investment made in the past. Now, I similarly redefine human capital investment as a function of the history of past types only. First, observe an implication of Lemma 1 : it is optimal from the point of view of a dynasty that deviates from the recommended investment level to consume all the resources given by the planner in the period of deviation. This is straightforward since under a worst punishment mechanism a dynasty's payoff following a deviation from recommended action is zero independent of the investment level the dynasty deviates to.

Therefore, without losing generality, we can restrict attention to investment strategies in which, at any period and node, the dynasty either invests at the level recommended by the planner to the reported type or invests zero. As a result, $\hat{\phi} = (\hat{\phi}_t)_{t\geq 1}$, where $\hat{\phi}_t : H^t \to \{0, 1\}$, is sufficient to represent a dynasty's investment strategy. Here, $\hat{\phi}_t(h^t) = 1$ means a dynasty that follows strategy $\hat{\phi}$ invests at the level recommended by the planner to the reported type at node h^t, $n_t(\hat{\sigma}^t(h^t))$, whereas $\hat{\phi}_t(h^t) = 0$ implies the dynasty chooses to invest zero.

Redefine $(\hat{\sigma}, \hat{\phi}) \equiv (\hat{\sigma}_t, \hat{\phi}_t)_{t\geq 1}$ as a dynastic strategy. Let (σ, ϕ) be the truth-telling and obedient strategy, where $\sigma_t(h^t) = \beta_t$ and $\phi_t(h^t) = 1$, for all t, h^t. Now, define the expected dynastic payoff under strategy $(\hat{\sigma}, \hat{\phi})$ given the worst punishment direct revelation mechanism x :

$$W(\hat{\sigma}, \hat{\phi}|x) = \sum_{t=1}^{\infty} \sum_{h^t} \mu(h^t)\delta(h^t)u\Big[\Big(\min_{h^\tau \prec h^t} \hat{\phi}_\tau(h^\tau)\Big)\Big(c_t(\hat{\sigma}^t(h^t)) + (1 - \hat{\phi}(h^t))n_t(\hat{\sigma}^t(h^t))\Big)\Big]. \quad (3)$$

Finally, I can define what it means for an allocation to be incentive compatible. An allocation x is incentive compatible if and only if there exists a worst punishment direct revelation mechanism x that implements it. The formal definition is given below.

Definition 2. ***(Incentive compatibility)*** *An allocation x is incentive compatible if and only if for all $(\hat{\sigma}, \hat{\phi})$,*

$$W(\sigma, \phi|x) \geq W(\hat{\sigma}, \hat{\phi}|x). \quad (4)$$

An allocation that is feasible and incentive compatible is called *incentive feasible.* These are the allocations society can achieve.

The remainder of this subsection shows that the incentive-compatibility conditions represented by (4) can be replaced with temporary incentive constraints. Given allocation x, define $V(h^t|x)$ to be the continuation utility to dynasty h^t from period $t + 1$ on when truth-telling and obedience is followed in all the nodes succeeding h^t and there has been no detection of disobedience in the past:

$$V(h^t|x) = \sum_{\tau=t+1}^{\infty} \sum_{h^\tau \succ h^t} \mu(h^\tau|h^t)\delta_{t+1}(h^\tau)u(c_\tau(h^\tau)).$$

If a dynasty member has already deviated in the past, then the continuation utility from period $t+1$ onwards is equal to the expected discounted utility of consuming zero in all nodes from period $t+1$ onwards, which is equal to $\frac{1}{1-\mathbb{E}\beta}\kappa$, where $\mathbb{E}$ denotes the expectation operator.

Lemma 2. ***(Temporary incentive compatibility)*** *An allocation x satisfies incentive constraints* (4) *if and only if it satisfies for all $t, h^{t-1}, \beta, \beta^o$*

$$(IC_L) \qquad u(c_t(h^{t-1},\beta)) + \beta V(h^{t-1},\beta|x) \geq u(c_t(h^{t-1},\beta^o)) + \beta V(h^{t-1},\beta^o),$$

$$(IC_D) \qquad u(c_t(h^{t-1},\beta)) + \beta V(h^{t-1},\beta|x) \geq u(c_t(h^{t-1},\beta) + n_t(h^{t-1},\beta)) + \beta \tfrac{1}{1-\mathbb{E}\beta}\kappa, \tag{5}$$

$$(IC_{LD}) \qquad u(c_t(h^{t-1},\beta)) + \beta V(h^{t-1},\beta|x) \geq u(c_t(h^{t-1},\beta^o) + n_t(h^{t-1},\beta^o)) + \beta \tfrac{1}{1-\mathbb{E}\beta}\kappa.$$

Proof. Relegated to Appendix A. □

The rest of the paper uses the three sets of inequalities in (5) to represent incentive compatibility of allocations. The first set of incentive-compatibility conditions, type L, is the usual one that says no parent should find it profitable to lie to be of the other type at any time and node. Type D incentive constraints say that parents should not find it profitable to disobey the investment level recommended by the planner. Finally, type LD constraints say that under an incentive-compatible allocation, parents should not be getting better off by double deviating: lying to be of the other type and investing zero in the child.

Observe that type D and type LD incentive constraints are novel in the dynamic contracting literature. These incentive constraints are very simple compared to the ones that usually arise in the presence of hidden action. This is due to the assumption that private information regarding actions (investment in children in the current context) is temporary.

2.3 Pareto Efficient Allocations

Now, I define Pareto efficient allocations for this economy.[10] In that regard, define $U(\beta^t|x) = u(c_1(\beta^t)) + \beta V(\beta^t|x)$ to be the dynastic welfare of agent with history h^t under allocation x.

Definition 3. *An allocation x^* is Pareto efficient if it is incentive feasible, delivers welfare $(U(\beta^t|x^*))_{t,\beta^t}$ and there is no other incentive feasible allocation x that delivers $(U(\beta^t|x))_{t,\beta^t}$ with $U(\beta^t|x) \geq U(\beta^t|x^*)$ for all β^t, t and at least one of these inequalities holding strictly.*

Pareto efficient allocations solve the following social planner's problem:

$$\max_x \sum_{t=1}^{\infty} \sum_{\beta^t} \mu(\beta^t) U(\bar{\beta}|x) \tag{6}$$

subject to for any t, β^t

$$U(\beta^t|x) \geq \bar{v}(\beta^t)$$

and subject to incentive-feasibility.

Observe that the social planning problem is indexed by $(\bar{v}(\beta^t))_{t,\beta^t \neq \bar{\beta}}$. Therefore, by changing the stochastic process $(\bar{v}(\beta^t))_{t,\beta^t \neq \bar{\beta}}$, we can trace the infinite-dimensional Pareto frontier of this economy.[11]

3 Characterization of Pareto Efficient Allocations

The aim of this section is to analyze the properties that are common to all Pareto efficient allocations. Put differently, I am interested in features that are shared by all allocations on the Pareto frontier of the economy. I assume $\underline{\beta} = 0$ in the main body of the paper. This assumption is mainly for expositional purposes and is not needed for any of the results that follow.[12]

[10] Throughout the paper, the term Pareto efficient allocation refers to Pareto efficient allocation under informational problems.

[11] Obviously, not all such processes define a Pareto efficient allocation. For a stochastic process $(\bar{v}(\beta^t))_{t,\beta^t \neq \bar{\beta}}$ to define a Pareto efficient allocation, there should be at least one incentive feasible allocation delivering each agent a welfare level that is weakly greater than the welfare level assigned to her by that stochastic process.

[12] I show in subsection 5.3 of the paper that the main result of the paper holds as long as $\underline{\beta}$ is below an upper bound $\tilde{\underline{\beta}}$.

Before analyzing the Pareto problem of the previous section, I first analyze efficiency under two different benchmark environments, which makes it easier to understand the role of informational assumptions and the resulting agency problem.

3.1 Two Benchmark Cases

Full Information Benchmark. First, consider an economy that is identical to the one described in section 2 except for all economic activity is public information. I want to characterize the whole set of Pareto efficient allocations. An allocation x is *full information Pareto efficient* if it is feasible, delivers welfare $(U(h^t|x^*))_{t,h^t}$, and there is no other feasible allocation x that delivers $(U(h^t|x))_{t,h^t}$ with $U(h^t|x) \geq U(h^t|x^*)$ for all h^t, t and at least one of these inequalities holding strictly.

Full information Pareto efficient allocations solve the following social planner's problem:

$$\max_x \sum_{t=1}^{\infty} \sum_{h^t} \mu(h^t) U(\bar{\beta}|x) \tag{7}$$

subject to for any t, h^t

$$U(h^t|x) \geq \bar{v}(h^t)$$

and subject to feasibility.

It follows simply from the first-order optimality conditions of the above problem that in any full information Pareto efficient allocation marginal returns to human capital investment in children is equated across children within any generation. This implies that in any generation all children receive same investment independent of which dynasty they belong to, i.e., $n_t(h^t)$ is independent of h^t. Since there are no ex ante productivity differences among children and there is diminishing marginal returns to investment, productive efficiency calls for an equal division of investment across children.

Under the full information assumption, productive efficiency is separated from distributive efficiency. Society invests in children in the productively efficient way and then distributes the output among agents in the way social norms call for.

Observable Investment Benchmark. Consider the same economy as before, but, now, the only source of private information is parental altruism types. In such a world, due to parental types being unobservable, only a subset of the set of all feasible allocation, those which are incentive compatible, are achievable by society. When the only source of private information is parental type, an allocation is incentive compatible if IC_L is satisfied in all generations and histories. Observe that the incentive constraints IC_D and IC_{LD} are not required for incentive compatibility when investment is observable. This is simply due to the fact that in this case the only way a parent can deviate is by lying to be of a different type; since investment is observable, parents have to obey the social planner's recommendation about how to allocate family resources.

Below I define a planner's problem, the solution of which gives the set of all observable investment Pareto efficient allocations as one varies $(\bar{v}(h^t))_{t,h^t \neq \bar{\beta}}$ appropriately. Observe that the only difference between the problem below and the social planning problem in (7) are the incentive constraints, IC_L.

$$\max_x \sum_{t=1}^{\infty} \sum_{h^t} \mu(h^t) U(\bar{\beta}|x) \tag{8}$$

subject to for any t, h^t

$$U(h^t|x) \geq \bar{v}(h^t)$$

and subject to feasibility and $IC_L(h^{t-1}, \beta)$, for all h^{t-1}, β, β^o.

Similar to the full information case, in any observable investment Pareto efficient allocation, human capital investment in children is independent of their dynastic history. Since investment is still observable, there is nothing preventing society from ensuring that marginal returns to investment are equated across children. To see how this follows from the social planner's problem above, observe that the only way investment enters the problem is through the resource constraints; it does not enter incentive constraints. Therefore, productive efficiency and distributive efficiency are separated in the model with observable investment as well. The main difference from the full information case is that, due to types being unobservable, society has to take into account incentive constraints IC_L when distributing consumption among agents.

Consequently, both in the full information and observable investment benchmark cases, investment in children is independent of history. This is due to the fact that the agency problem regarding investment in children disappears when investment is observable.

3.2 Characterization of Pareto Efficient Allocations

This subsection analyzes Pareto efficient allocations in the economy introduced in section 2 when investment in children, as well as altruism types of parents, is private information. The goal is to prove that in any Pareto efficient allocation, for any generation, poor parents are subsidized, and, hence, all agents, including the poor, live above a certain welfare level.

The first step in analyzing Pareto efficient allocations is simplifying the incentive compatibility constraints. In that regard, Lemma 3 below shows one can disregard some incentive-compatibility conditions from the outset. First, given an allocation, if an altruistic parent does not lie to be a selfish parent and the latter does not deviate by disobeying the recommended investment, then the altruistic parent does not deviate by disobeying either. Hence, incentive constraints regarding deviations in which a currently altruistic parent disobeys (type L and type LD constraints) are automatically satisfied. Second, observe that for any h^{t-1}, if x satisfies $IC_{LD}(h^{t-1}, \underline{\beta})$, then it also satisfies $IC_L(h^{t-1}, \underline{\beta})$. In words, if selfish parents do not deviate by lying and disobeying, then they do not deviate by only lying either. This follows straightforwardly from the facts that selfish parents do not care about their descendants and investment in children is non-negative.

Lemma 3. *For any h^{t-1}, incentive constraints $IC_D(h^{t-1}, \bar{\beta})$, $IC_{LD}(h^{t-1}, \bar{\beta})$, and $IC_L(h^{t-1}, \underline{\beta})$ are redundant.*

Proof. Relegated to Appendix B. □

As a result, in the rest of the paper, incentive compatibility conditions in which altruistic parents disobey and those in which selfish parents only lie are going to be omitted without loss of generality.

It is evident from the Pareto problems that social and parental objectives are not necessarily aligned. When parental investment is unobservable, this implies that society faces an agency problem regarding childhood investment. As a result, if society wants parents to invest in their children, it has to provide them with incentives to do so. This implies that resource cost is not the only social cost of investment in children; there is also an incentive cost. Since this incentive cost is potentially different for parents with different histories, it is no longer Pareto optimal to equate marginal returns to investment across children in any generation. To see the incentive cost of investment mechanically, observe that investment enters not only into the resource constraints but also incentive compatibility constraints in the social planner's problem. Consequently, productive efficiency and distributive efficiency are not separated when there is an agency problem in childhood investment.

Lemma 4 below which establishes two important properties of parental investment in children that are common to all Pareto efficient allocations. The subsidy result follows directly from these properties. The first property is that the Pareto efficient level of investment in children of selfish parents is zero in any generation. This follows directly from IC_D and $\underline{\beta} = 0$:

$$u(c_t^*(h^{t-1}, \beta)) \geq u(c_t^*(h^{t-1}, \beta) + n_t^*(h^{t-1}, \beta))$$

$$\Rightarrow n_t^*(h^{t-1}, \beta) = 0.$$

Since deviation from the recommended investment level is detected only after the children become adults, the only way society can make parents invest in their children is by punishing the children of deviating parents. However, since selfish parents do not care about their children at all, there is no reward/punishment mechanism that can make them invest in their children.

The second property of Pareto efficient investment in children is that, in all generations, children of all altruistic parents receive strictly positive investment. That is, even if society does not care at all about the welfare of an altruistic parent or any of her successors, it should still make sure that her child receives parental investment. Clearly, no notion of equality is responsible for the result. It follows solely from productive efficiency. Since there is diminishing marginal returns, society wants to exploit investment opportunities in all children. Smoothing investment across children in a period means higher average returns and, hence, higher output next generation, which the society can distribute among those it cares about.

Lemma 4. *For any allocation x^* in the set of Pareto efficient allocations,*

1. $n_t^*(h^{t-1}, \underline{\beta}) = 0$, *for all* h^{t-1}.

2. $n_t^*(h^{t-1}, \bar{\beta}) > 0$, *for all* h^{t-1}.

Proof. Relegated to Appendix B. □

Now, I show the main result of this section. First, define

$$\Delta_t(h^t) \equiv n_t(h^t) + c_t(h^t) - f(n_{t-1}(h^{t-1})).$$

In words, $\Delta_t(h^t)$ is the net transfer that an agent with history h^t receives from society under allocation x. I say that an agent h^t is *subsidized* under allocation x if $\Delta_t(h^t) > 0$. I make one more definition. I say that an agent h^t lives in misery under allocation x if $c_\tau(h^\tau) = 0$, for all $h^\tau \succeq h^t$. In words, an agent lives in misery if own and all future descendants' consumption is equal to zero.

The main result of this section is Proposition 1 below. It has three parts. First, children of selfish parents have zero output when they become adults. This follows from part one of Lemma 4 : children of selfish parents do not receive any human capital investment.

Second, in any generation, poor agents receive subsidy from the rest of society, independent of the social welfare criterion. The intuition is very simple. According to part two of Lemma 4, productive efficiency requires all altruistic parents to invest in their children. This then implies that altruistic parents with no output should be subsidized so that they can invest in their children. Selfish agents with no output are subsidized because parental altruism is private information.

Third, in any Pareto efficient allocation, all agents live strictly above misery. Exploiting investment opportunities in children requires society to reward people who operate them, their parents, due to the private information nature of the investment. Then, selfish parents should also receive consumption, since otherwise they would lie to be altruistic and consume the transfers intended for altruistic agents. Consequently, Pareto efficiency requires society to ensure that none of its members live in misery, even when some of these agents are not valued at all by society.

Proposition 1. *In any Pareto efficient allocation* x^*,

1. *If* $h^{t-1} = (h^{t-2}, \underline{\beta})$, *then* $y_t^*(h^{t-1}, \beta) = 0$.

2. *If* $y_t^*(h^t) = 0$, *then* $\Delta_t^*(h^t) > 0$.

3. *No agent lives in misery.*

Proof. Part 1. By Lemma 4 part one, $n_{t-1}^*(h^{t-2}, \underline{\beta}) = 0$, which implies $y_t^*(h^{t-2}, \bar{\beta}, \beta) = 0$.

Part 2. By Lemma 4 part two, $n_t^*(h^{t-1}, \bar{\beta}) > 0$. If $y_t^*(h^{t-1}, \beta) = 0$, then $\Delta_t^*(h^{t-1}, \bar{\beta}) > 0$. Then, from incentive-compatibility condition $IC_{LD}(h^{t-1}, \underline{\beta})$, it follows that $\Delta_t^*(h^{t-1}, \underline{\beta}) > 0$ too.

Part 3. $n_t^*(h^{t-1}, \bar{\beta}) > 0$ and $IC_{LD}(h^{t-1}, \underline{\beta})$ imply $c_t^*(h_{t-1}, \underline{\beta}) > 0$. Thus, no selfish agent is miserable. Since the altruism factor is privately observed, $IC_L(h^{t-1}, \bar{\beta})$ implies altruistic agents are not miserable either. □

Note that this subsidy result is a powerful one, as it does not depend on society's preferences. Even if a society does not care at all about its poor citizens, it still should subsidize them on productive efficiency grounds. Similarly, independent of the social welfare criterion, society should guarantee a minimum level of welfare for all its members. Equity or insurance considerations play no role in these results. The results depend crucially on two features of the model. First, all parents in the model "operate" individual level "production technologies," their children, with decreasing returns to scale. Obviously, there would be no issue of productive efficiency if the current model were an endowment economy or there were constant returns to scale to investment in children; in that case, there would be no need to subsidize any agent unless it is required by a particular social welfare criterion. Second, parental investment is unobservable, which implies that in order to exploit these investment opportunities, society has to provide parents with incentives, and, thus, welfare.

In order to understand the generality of Proposition 1, the next subsection analyzes a Pareto efficient allocation that arises from the social planning problem when society cares only about the ex ante welfare of initial generation parents directly. Since this welfare function is heavily analyzed in the existing literature, focusing on it also allows me to highlight the novel forces at work in the current model.

3.3 Ex ante Efficient Allocation

Consider the following family of social welfare functions parameterized by generational Pareto weight ρ :

$$\sum_{t=1}^{\infty}\sum_{h^t}\mu(h^t)\rho^{t-1}U(h^t|x).$$

I focus on the case where $\rho = 0$. This is the case in which society cares about only period one parents directly; all other generations are valued solely indirectly through their parents.[13]

I first analyze the full information benchmark. In the efficient allocation, all children receive the same level of investment. The distribution of consumption is as follows:

$$c_t(h^{t-1}, \beta) = 0, \text{for all } h^{t-1} \neq (\bar{\beta}, ..., \bar{\beta});$$
$$c_t(h^{t-1}, \bar{\beta}) = c_t(h^{t-1}, \underline{\beta}) > 0, \text{for all } h^{t-1} = (\bar{\beta}, ..., \bar{\beta}).$$

In words, only those agents with all altruistic ancestors consume in the full information efficient allocation. Since society directly values only the initial generation, those agents who have had selfish parents are not valued at all. This implies that in the full information efficient allocation those agents are miserable. Also, since agents with altruistic ancestors enter the social welfare function symmetrically, their consumption does not depend on their current type.

Now, consider the benchmark case in which altruism type is private information but investment in children is observable. I propose that the full information efficient allocation solves the planning problem in this case.

Proposition 2. *When the social welfare function is a weighted average of the initial generations' expected discounted utilities, the solution to the planning problems under full information and under observable investment coincide.*

Proof. To show this, it suffices to show that the full information efficient allocation is in the constraint set of the planning problem with observable investment. The feasibility condition is automatically satisfied since the full information planning problem has the same feasibility condition. The only thing that needs to be checked is IC_L, but this follows straightforwardly from (3.3). □

[13]Note that the social planning problem with the objective function in (3.3) corresponds to the Pareto problem in which $\bar{v}(h^t) = 0$, for all $t > 1$ and for all $h^t \neq \bar{\beta}$.

Consequently, in the ex ante efficient allocation any agent who has a selfish ancestor has zero welfare, i.e., $U(h^t|x^*) = 0$, for all $h^t \neq (\bar{\beta}, ..., \bar{\beta})$. This means, when investment is observable there exists a particular Pareto efficient allocation in which there are some agents with zero welfare, which further proves that the unobservability of parental investment is crucial for Proposition 1.

Consequently, under this particular social objective, I draw two conclusions about the impact of the agency problem caused by the unobservability of human capital investment in children. First, when investment is observable, in any generation, only those agents whom society cares about get to consume. When it is unobservable, no agent lives is misery. This implies some of those whom the society does not care about also consume. Therefore, in any generation, the distribution of consumption is less concentrated when investment is unobservable.

Second, when investment is observable, intergenerational persistence of consumption (welfare) is almost one in the long run: almost all the dynasties have stationary consumption equal to zero. On the other hand, with unobservability, persistence is necessarily different from one. Thus, unobservability of investment in children implies lower intergenerational persistence of welfare.

4 Implementation

This section is concerned with the following question. What kinds of markets and policies are sufficient to attain Pareto efficiency? I analyze the implementation of Pareto efficient allocations in two different market setups.

4.1 Market with Dynastic Contracts

The first market structure I consider is one in which generation one parents can sign contracts ex ante, before any uncertainty is realized, and, importantly, these contracts are legally binding for all their descendants. There is a continuum of intermediaries that are owned equally by all dynasties. In the beginning of generation one, before realization of any uncertainty, each intermediary signs a contract with a continuum of initial parents which binds all following generations. The contracts are offered competitively, and initial parents choose the contract with the highest promised ex ante expected utility.

A contract is $x = (c_t, n_t)_{t\geq 1}$, where $c_t, n_t : H^t \to \Re_+$. Here, $c_t(h^t) + n_t(h^t)$ is the total amount of resources transferred to a dynasty who have reported h^t and obeyed the intermediaries' recommendations up to period t. Any dynasty that deviates from the actions recommended by an intermediary receives zero units forever after the deviation is detected. Thus, without loss of generality, I confine intermediaries to offer worst punishment direct revelation mechanisms. Facing such a contract, each dynasty chooses a participation strategy, $(\hat{\sigma}_t, \hat{\phi}_t)_{t\geq 1}$, receives either $c_t(\hat{\sigma}_t(h^t)) + n_t(\hat{\sigma}_t(h^t))$ or zero depending on past deviation from recommendation, and chooses how to allocate the total amount received. Intermediaries take into account that each generation a parent can choose a different allocation of total resources handed to her than what is intended by the intermediary. The incentive compatibility conditions summarized by (5) in section 2 completely characterize the set of allocations intermediaries can offer to dynasties through period one contracts.

Intermediaries trade one period claims with each other. Denote by q_t the price of a claim a_t that pays one unit of consumption good in period $t+1$. I consider symmetric equilibria in which all intermediaries are price takers. Letting $\underline{\text{U}}$ be the equilibrium utility each dynasty receives, representative intermediary's problem is

$$\max_{x,a} \sum_{t=1}^{\infty} \sum_{h^t} \mu(h^t) \prod_{s=1}^{t-1} q_s \left[f(n_{t-1}(h^{t-1})) - c_t(h^t) - n_t(h^t) \right] \tag{9}$$

subject to (5) and $\sum_h \mu_h U(h|x) \geq \underline{\text{U}}$.

A formal definition of market equilibrium is as follows:

Definition 4. *A competitive equilibrium is prices* $(q_t)_{t\geq 1}$*, allocations* $(c_t, n_t)_{t\geq 1}$*, trades of one period claims* $(a_t)_{t\geq 1}$*, and a utility level* $\underline{U}$ *such that*

1. *Taking* $(q_t)_{t\geq 1}$ *and* $\underline{U}$ *as given,* $(c_t, n_t, a_t)_{t\geq 1}$ *solves the intermediaries' problem* (9);

2. *Initial generation parents choose contracts that offer the highest utility;*

3. *Claims market clears every period.*

I now prove a version of the First Welfare Theorem for this private information economy. This result was originally proven by Prescott and Townsend (1984) for a static private information economy. Atkeson and Lucas (1992) and Golosov and Tsyvinski (2007) prove the same result for different dynamic private information economies. The result follows from the observation that the intermediary's problem is the dual of the social planner's problem of maximizing ex ante dynastic utility. The proof uses a property of the equilibrium that is easy to show: in equilibrium, perfect competition drives intermediary's profits to zero.

Proposition 3. *The competitive equilibrium in which initial parents can sign contracts that bind all their descendants is Pareto efficient.*

Proof. It suffices to show that equilibrium x solves the social planner's problem of maximizing ex ante utility. Suppose this is not true. That the claims market clears every period implies that the equilibrium allocation satisfies aggregate feasibility. Incentive compatibility of the equilibrium allocation follows directly from the intermediary's problem. Hence, it must be that ex ante expected utility dynasties receive under the efficient allocation, call it U^*, is strictly greater than the utility under equilibrium allocation $\underline{U}$. Observe that the efficient allocation is available to the intermediary since $U^* > \underline{U}$. Now, consider a third allocation x' identical to the efficient one except for $c_1'(\bar{\beta}) = c_1^*(\bar{\beta}) - \epsilon$. For $\epsilon > 0$ small, the ex ante utility this allocation offers, U', is still strictly greater than $\underline{U}$, and x' is incentive compatible. Observe that under x' the intermediary makes strictly positive profits, which is a contradiction. □

The analysis above suggests private markets achieve Pareto efficiency without government intervention. As a result, any intervention would be based on redistributive motives rather than efficiency considerations. Put differently, government should intervene only if social norms call for a different Pareto efficient allocation than the one achieved under private markets. [14] However, observe that efficiency of private markets crucially depends on a very strong assumption about the set of contracts available: period one parents can write contracts that legally bind all their descendants. This assumption is highly unrealistic, as in most countries around the world contractual arrangements signed by individuals are not binding for their children. The next subsection analyzes a different, more realistic market structure in which there is no intergenerational trade.

[14] Under this market structure, markets only attain the Pareto efficient allocation that maximizes the ex ante welfare of initial generation parents. This result is in the same spirit as Bernheim (1989) which shows that many Pareto efficient allocations cannot be achieved in dynastic models with altruistically linked individuals.

4.2 Market without Intergenerational Trade

Consider a market setup in which contracts signed by parents do not bind their children. Agents are allowed to sign all other types of contracts that respect informational constraints. The aim is to provide an implementation of Pareto efficient allocations in this market setup. Towards this goal, let $T = (T_t)_{t\geq 1}$ be an income tax system where

$$T_t : \Re^t_+ \to \Re.$$

Here, $T_t(y^t)$ is the income tax levied on a period t parent with a dynastic history of incomes $y^t = (y_1, ..., y_t)$. Therefore, a dynasty's problem now reads:

$$\max_x \sum_\beta \mu(\beta) U(\beta|x) \tag{10}$$

subject to for all t and h^t,

$$c_t(h^t) + n_t(h^t) \leq y_t(h^t) - T_t(y^t(h^t)).$$

Definition 5. *An equilibrium with income taxes in a market without intergenerational borrowing and lending is income taxes T and an allocation x such that*

1. *Given T, x solves the dynastic problem;*

2. *Government budget balances every period.*

It is easy to see that the unique equilibrium allocation under laissez-faire is equivalent to the autarkic allocation. In that allocation, call it x^a, $n^a_t(h^t) = 0$, for all $t \geq 1$, $h^t \neq (\bar{\beta}, ..., \bar{\beta})$. By Proposition 4, this is clearly Pareto inefficient.

Proposition 4. *The laissez-faire market equilibrium without intergenerational borrowing and lending is Pareto inefficient.*

Under the equilibrium allocation, all agents who have at least one selfish ancestor have zero income. Therefore, altruistic parents who have at least one selfish ancestor cannot invest in their children, which means society cannot exploit very high return investment opportunities in children of altruistic parents. This breaks productive efficiency and creates Pareto inefficiency. As a result, as long as intergenerational borrowing and lending is absent, government intervention in the economy is essential since there is room for Pareto improvement over the existing market. A natural next step is to see what a government should do in order to ensure Pareto efficiency.

An interesting property of this market structure is that there is *no* tax implementation for some Pareto efficient allocations. To see this, remember that in the current model an agent's income is determined by parental investment during her childhood, i.e., $y_t = f(n_t(h^{t-1}))$. As a result, current income, y_t, does not reveal any information to the government about the agent's current type, β_t. But, then, the tax system cannot differentiate between agents with the same history of ancestors, h^{t-1}. Thus, period t parents with histories $(h^{t-1}, \bar{\beta})$ and $(h^{t-1}, \underline{\beta})$ receive the same transfers in any allocation that is implemented with income taxes. This implies that Pareto efficient allocations in which, in some period t, transfers depend on agents' period t types cannot be implemented through taxes on income under the market structure with no intergenerational trade.

If there were an intergenerational market of some kind, it could be possible for the government to deduce information about whether an agent is altruistic or not by observing her actions in the market. However, when such markets are absent, government has no way of screening altruistic and selfish parents with the same dynastic histories. As a result, Pareto efficient allocations in which selfish and altruistic parents with the same histories receive different transfers cannot be implemented through taxes. Fortunately, it is possible to completely characterize the set of Pareto efficient allocations that can be implemented by income taxes, as shown by Proposition 5.

Define $Y_t^* = \{y^t \in \Re_+^t | y_\tau = f(n_{\tau-1}^*(h^{\tau-1})), \text{for all } \tau \leq t \text{ and } h^{\tau-1} \preceq h^{t-1}, \text{ for some } h^{t-1}\}$. In words, y^t is in Y_t^* if there exists some type h^{t-1} that receives income history y^t in the Pareto efficient allocation.

Now set the tax system as follows:

$$\begin{aligned} T_t^*(y^t) &= -\Delta_t^*(h^{t-1}), \quad && \text{if } y^t \in Y_t^*; \\ T_t^*(y^t) &= y^t, && \text{if else.} \end{aligned} \tag{11}$$

Proposition 5. *A Pareto efficient allocation x^* can be implemented in a market with no intergenerational borrowing and lending through taxes on income if and only if $(\bar{v}(h^t))_{t\geq 1, h^t \neq \bar{\beta}}$ is such that $IC_{LD}(h^{t-1}, \underline{\beta})$ holds with equality for all h^{t-1}.*

Proof. $(\Rightarrow)$: Suppose a Pareto efficient allocation x^* can be implemented by an income tax system in a market with no intergenerational trade. This implies $c_t^*(h^{t-1}, \beta) + n_t^*(h^{t-1}, \beta)$ is independent of β, for all h^{t-1}, which implies $IC_{LD}(h^{t-1}, \underline{\beta})$ holds with equality for all h^{t-1}.

$(\Leftarrow)$: Now, suppose $IC_{LD}(h^{t-1}, \underline{\beta})$ holds with equality for all h^{t-1} in a Pareto efficient allocation x^*. Then, since $IC_D(h^{t-1}, \underline{\beta})$ binds for all h^{t-1} in any Pareto efficient allocation, $c_t^*(h^{t-1}, \beta) + n_t^*(h^{t-1}, \beta)$ is independent of β. Set the taxes as in (11).

Consider the dynastic problem with taxes. Define a dynasty's income strategy as $y = (y_t)_{t\geq 1}$, where $y_t : H^{t-1} \rightarrow \Re_+$. A dynastic income strategy gives a dynastic children investment strategy since income is a one-to-one function of childhood investment. The investment strategy then implies a consumption strategy through the budget constraint. Therefore, given taxes, the only object a dynasty is choosing is its income strategy. Let y^* be the income strategy in which $y_t^*(h^{t-1}) = f(n_{t-1}^*(h^{t-1}))$.

First, observe that if a dynasty chooses an income strategy y' such that for all t, h^{t-1}, $y_t'(h^{t-1}) = f(n_{t-1}^*(\sigma_{t-1}'(h^{t-1})))$, then by construction of the tax system, flow budget constraints imply that the dynasty receives allocation $(c_t^*(\sigma_t'(h^t)), n_t^*(\sigma_t'(h^t)))$, where $c_t^*(\sigma_t'(h^t))$ is the consumption of history h^t dynastic member. Hence, the expected discounted dynastic welfare under y' would be equal to $W(\sigma', \phi | x^*)$.

Second, suppose a dynasty chooses income strategy y. Suppose in some period t in some node h^t, y calls for setting human capital investment in the child to some level such that the dynastic income history next period is not going to be in Y_{t+1}^*. By the construction of income taxes, this implies, beginning from generation $t+1$ on, all output of the dynasty will be seized by the government. Hence, it is optimal for the period t member of the dynasty to consume all resources.

As a result, any potentially optimal income strategy y can be decomposed into two parts: an income strategy y' such that for all t, h^{t-1}, $y_t'(h^{t-1}) = f(n_{t-1}^*(\sigma_{t-1}'(h^{t-1})))$, and a decision made at each node about whether to deviate from the income path that occurs under y'. Denote the latter decision by $\nu' = (\nu_t')_{t\geq 1}$, where $\nu_t'(h^t) = 1$ means at node h^t the dynasty makes an amount of investment in children that makes the dynastic income equal to $y'_{t+1}(h^t)$. Thus, the expected discounted dynastic welfare under y is equal to $W(\sigma', \phi'|x^*)$, for $\phi' = \nu'$.

Then, under the tax system T^*, if a dynasty chooses y^*, then the dynasty receives (c^*, h^*) and the expected discounted utility, $W(\sigma, \phi|x^*)$. If a dynasty chooses any other y, then the dynastic utility is $W(\sigma', \phi'|x^*.)$ Then, by incentive compatibility of x^*, the dynasty chooses y^*. □

Section 3 already proved that $IC_D(h^{t-1}, \underline{\beta})$ binds for all h^{t-1} in any Pareto efficient allocation. If it is also the case that $IC_{LD}(h^{t-1}, \underline{\beta})$ holds with equality for all h^{t-1}, then $c_t^*(h^{t-1}, \beta) + n_t^*(h^{t-1}, \beta)$ is independent of β, which means $\Delta_t^*(h^{t-1}, \beta)$ is independent of β.

In the rest of the subsection, I confine analysis to Pareto efficient allocations that can be implemented with income taxes and derive an important property of Pareto efficient taxes. The corollary below essentially translates the subsidy result about Pareto efficient allocations derived in section 3 into a result on income taxes. Remember that by Proposition 4, in any Pareto efficient allocation, x^*, $\Delta_t^*(h^{t-2}, \underline{\beta}, h) > 0$, for all h^{t-2}. Remember also that by the same proposition, $n_t^*(h^{t-1}, \underline{\beta}) = 0$, for all h^{t-1}. Combining these two results, I get the following corollary.

Corollary 1. *Under the market structure with no intergenerational trade, in all Pareto efficient income tax systems, all agents with zero income receive strictly positive subsidies, i.e.,* $T_t^*(y^{t-1}, 0) < 0$, *for all* t *and* $y^{t-1} \in Y_{t-1}^*$.

5 Discussion

In the first two subsections of this section, I discuss whether the main characterization result of the paper survives generalizations of the model along two important dimensions. The last subsection provides a robustness check: the main result of the paper does not depend on the assumption that $\underline{\beta}$ is exactly zero.

5.1 Public Investment in Children

Throughout the paper I assume that government can only indirectly regulate investment in children through subsidizing their parents. In real life, on the other hand, governments also try to control childhood investment directly through in-kind subsidies.[15]

A natural question then is: What happens to the main result if we equip the government in the model with in-kind subsidies? One way to answer this question is to expand the "production function" of children by introducing an additional investment good, g, which is publicly observable.[16] The adult output of a child is now equal to $f(n, g)$, where n is the investment in children that is carried out privately by the parent as before. Now, the government has two ways to regulate investment in children. First, it can provide a parent with resources and incentives to invest in her child. Second, the government can directly invest in the child through publicly observable investment good.[17] The latter way represents in-kind subsidies in the model.

Whether or not the main result of the paper still holds depends on how good public investment is in terms of substituting parental investment. To see this, consider the extreme case in which parental and public investments in children are perfect substitutes and $f(n, g) = (n+g)^\alpha$, $\alpha \in (0, 1)$. In this case, the government can completely control the level of investment children receive through public investment. The set of Pareto efficient allocations is identical to the one in the observable benchmark case studies in subsection 3.1. All children receive the same level of investment which implies, in any period, all agents have the same level of output. Therefore, there is no poor to subsidize. Furthermore, since investment is completely controlled by the government, there is no need to provide parents with incentives to invest in their children. As a result, some people do live in misery depending on the social welfare function. Consequently, if public investments can completely substitute parental investments without any extra costs, then the main characterization result does not hold.

If, on the other hand, we assume that some components of parental investment are essential in raising children and cannot be substituted by public investment, then one can show that the main result of the paper still holds provided that we still have the assumption that there are

[15]Head Start is an example of an in-kind subsidy program in the U.S. It is a preschool program that aims to prepare disadvantaged children to school. See Currie (2001) for more information about Head Start and various other early childhood education programs.

[16]It is important to note that the only aspect that is public about this good is information. Each child receives her publicly observable investment separately and this investment affects only her adult output. Put differently, there is no public good aspect to g.

[17]Alternative to direct public provision, one can also think that the government is transferring consumption goods to the parent and the transfer is conditional on investing a specific amount of g in the child. The exact way in-kind subsidy is provided is not important for the question I am after.

arbitrarily high returns to parental investment as the level of this investment decreases to zero, i.e. $\lim_{n\to 0} \partial f(n,g)/\partial n = \infty$.

5.2 Persistent Ability

The main characterization result of the paper follows from the intermediate result that the children of all altruistic parents, rich or poor, should receive some level of investment in any Pareto efficient allocation. In this subsection I ask the question: How does this intermediate result depend on the simplifying assumption that there are equal returns to investing in poor and rich children?

One way to answer this question is to modify the model so that adult output is not only a function of parental investment but it is also a function of the child's ability, θ, and ability is persistent over generations. So, suppose parents know their children's θ before they make the investment decision. A child with ability θ and investment n produces $f(n,\theta)$ next period. Suppose also that the children of parents who are highly able are more likely to be highly able themselves. Also, for simplicity, suppose children's ability is public information. In such a world, marginal returns to investing in rich children is on average higher than marginal returns to investing in poor children at any investment level. Therefore, keeping everything else the same, one would expect that, on average, poor children should receive less investment compared to the level of investment they receive in the original model where poor and rich children have equal returns to investment. As a result, the amount subsidies poor receive is smaller than the amount they receive in the original model. However, as long as Inada condition holds at all ability levels, meaning $\lim_{n\to 0} \partial f(n,\theta)/\partial n = \infty$ for all θ, children of all altruistic parents should receive investment, which implies the main result of the paper still holds.

5.3 A Robustness Check: $\underline{\beta} > 0$

I show the main characterization result of the paper under the assumption that selfish parents do not care at all about their children, i.e., $\underline{\beta} = 0$. This subsection shows that this specific assumption is not required for the main result. The crucial assumption is that $\underline{\beta}$ is sufficiently close to zero. More precisely, I show that, in any finite horizon version of the model economy, there is an upper bound for $\underline{\beta}$, call it $\tilde{\underline{\beta}}$, such that, as long as $\underline{\beta}$ is weakly below $\tilde{\underline{\beta}}$, the main characterization result holds.

Proposition 6. *For a version of the model with arbitrary finite length, there exists $\tilde{\underline{\beta}}$ such that if $\underline{\beta} \leq \tilde{\underline{\beta}}$, then in any Pareto efficient allocation x^* :*

1. *For any t, there exists $\underline{y}_t$ such that if $h^{t-1} = (h^{t-2}, \underline{\beta})$, then $y_t^*(h^{t-1}, \beta) < \underline{y}_t$.*
2. *If $y_t^*(h^t) < \underline{y}_t$, then $\Delta_t^*(h^t) > 0$.*
3. *No agent lives in misery.*

Proof. Relegated to Appendix C. □

6 Conclusion

Parental investment is a key factor in determining children's adult outcomes, which implies it is an important productive activity for society. However, society cannot force parents to invest in their children since it is hard to monitor such investments. Furthermore, social and parental objectives are often not aligned. As a result, society faces an agency problem regarding childhood investment. This paper provides an intergenerational framework with such an agency problem and focuses on the entire set of ex post Pareto efficient allocations. The main characterization result is that, independent of social preferences, in all generations, poor parents receive subsidies. This implies that all parents, including the poor, live above misery.

The paper then provides two market structures which achieve Pareto efficiency. In the first one, parents can sign dynastic contracts that bind all their descendants. A version of the First Welfare Theorem holds for this economy: market equilibrium attains a Pareto efficient allocation. However, this market structure is not practical as in most parts of the world contracts signed by parents are not binding for their children. In the second one, I construct a more realistic market structure in which no intergenerational trade is enforced and show that a government can restore efficiency through taxes on income. A property common to all Pareto efficient income tax structures is that agents with low income pay negative taxes.

Empirical literature often interprets the dependence of children's outcomes on parental incomes as a sign of inefficiency.[18] When the only cost of investing in children is the resource cost, put differently, when there are no informational frictions, the current model confirms this interpretation: in all full information Pareto efficient allocations, investment in children is independent of dynastic history and, hence, parental income. Since children's outcomes are determined by parental investment in children, this means if there are no informational frictions, efficiency requires children's outcomes to be independent of parental incomes. On the other hand, when there is an agency problem regarding investment in children, there is an additional incentive cost of investment in children, which depends on the dynastic history of the children. In this case, I show that investment in children and, hence, their outcomes depend on parental incomes in Pareto efficient allocations. Consequently, it is not obvious a priori that intergenerational persistence in outcomes observed in data implies data is generated in an inefficient world. One can see whether the agency problem in childhood investment, not the inefficiency of the society in allocating resources, can account for the persistence in outcomes between generations by taking an appropriately modified version of the current model to data. I believe that this is an exciting area for future research.

[18]See Solon (1992), Zimmerman (1992), and Mazumder (2001) among others for empirical papers that establish intergenerational persistence in outcomes such as earnings and income. See Stokey (1996) for a paper that interprets intergenerational persistence as a sign of inefficiency.

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Appendix

A Incentive Compatibility

Proof of Lemma 1.

Proof.

$$W(\sigma, N|C', N') = W(\sigma, N|C, N)$$
$$\geq W(\hat{\sigma}, \hat{n}|C, N) \geq W(\hat{\sigma}, \hat{n}|C', N'),$$

where the first and last inequalities are true by construction and the second inequality is true since (C, N) induces truth-telling and obedience. □

Proof of Lemma 2.

Proof. Obviously, incentive constraints (4) imply temporary incentive constraints (5). Therefore, I only show the other direction.

Suppose for contradiction that there exists an allocation x satisfying (5) and violating (4). This implies there exists a strategy $(\bar{\sigma}, \bar{\phi})$ such that

$$W(\bar{\sigma}, \bar{\phi}|x) - W(\sigma, \phi|x) = \epsilon > 0.$$

Define a new strategy $(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau})$ that is identical to $(\bar{\sigma}, \bar{\phi})$ up to and including period τ and that, for $t > \tau$, prescribes telling the truth and obeying the recommendation at all nodes, i.e., $\bar{\sigma}_t^{\tau}(h^t) = \beta_t$ and $\bar{\phi}_t^{\tau}(h^t) = 1$.

Since $\underline{\beta}, \bar{\beta} < 1$, and assuming the appropriate boundary condition, it is obvious that as $\tau \to \infty$, $W(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau}|x)$ converges to $W(\bar{\sigma}, \bar{\phi}|x)$. Thus, there exists a τ such that

$$|W(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau}|x) - W(\bar{\sigma}, \bar{\phi}|x)| < \epsilon/2.$$

As a result, if one can prove that $W(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau}|x) \leq W(\sigma, \phi|x)$, one gets a contradiction.

Define a new strategy $(\bar{\sigma}^{\tau-1}, \bar{\phi}^{\tau-1})$ that is identical to $(\bar{\sigma}, \bar{\phi})$ up to and including period $\tau - 1$ and that, for $t > \tau - 1$, prescribes telling the truth and obeying the recommendation at all nodes, i.e., $\bar{\sigma}_t^{\tau-1}(h^t) = \beta_t$ and $\bar{\phi}_t^{\tau-1}(h^t) = 1$.

$$W(\bar{\sigma}^{\tau-1}, \bar{\phi}^{\tau-1}|x) - W(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau}|x)$$

$$= \sum_{h^\tau} \mu(h^t) \Big\{ \Big[u\Big(\Big(\min_{h^t \prec h^\tau} \bar{\phi}_t(h^t) \Big) c_\tau(\bar{\sigma}^{\tau-1}(h^{\tau-1}), \beta_\tau) \Big) + \beta_\tau V \Big(\Big(\min_{h^t \prec h^\tau} \bar{\phi}_t(h^t) \Big) (\bar{\sigma}^{\tau-1}(h^{\tau-1}), \beta_\tau |x) \Big) \Big]$$

$$- \Big[u\Big(\Big(\min_{h^t \prec h^\tau} \bar{\phi}_t(h^t) \Big) c_\tau(\bar{\sigma}^\tau(h^\tau)) + (1 - \bar{\phi}_\tau(h^\tau)) n_\tau(\bar{\sigma}^\tau(h^\tau)) \Big) + \beta_\tau V \Big(\Big(\min_{h^t \prec h^\tau} \bar{\phi}_t(h^t) \Big) (\bar{\phi}_\tau(h^\tau)) (\bar{\sigma}^\tau(h^\tau)|x) \Big) \Big] \Big\},$$

where $V(0)$ is defined as the expected discounted utility from consuming zero in all future nodes, i.e., $V(0) = \frac{1}{1-\mathbb{E}\beta}\kappa$.

For histories h^τ such that there exists a $h^t \prec h^\tau$ with $\bar{\phi}_t(h^t) = 0$, the value inside the brackets is zero, since both expressions inside the brackets is equal to the expected discounted utility of consuming zero in the current period and in all future nodes.

For histories h^τ such that there is no $h^t \prec h^\tau$ with $\bar{\phi}_t(h^t) = 0$, the value of the expression inside the brackets is greater than or equal to zero due to the temporary incentive constraints (5). Thus,

$$W(\bar{\sigma}^{\tau-1}, \bar{\phi}^{\tau-1}|x) \geq W(\bar{\sigma}^{\tau}, \bar{\phi}^{\tau}|x).$$

Define new strategies $(\bar{\sigma}^s, \bar{\phi}^s)$, for $s \in \{0, 1, ..., \tau - 2\}$, that are identical to $(\bar{\sigma}, \bar{\phi})$ up to and including period s and that prescribe telling the truth and obedience for periods $t > s$. By backwards induction, one can show that

$$W(\bar{\sigma}^0, \bar{\phi}^0|x) \geq W(\bar{\sigma}^\tau, \bar{\phi}^\tau|x).$$

Observing that the strategy $(\bar{\sigma}^0, \bar{\phi}^0|x)$ is by definition the truth telling and obedience strategy, (σ, ϕ), gives the desired contradiction. □

B Pareto Efficient Allocations

Proof of Lemma 3.

Proof. First, I prove $IC_D(h^{t-1}, \bar{\beta})$ and $IC_{LD}(h^{t-1}, \bar{\beta})$ are redundant.

$$\begin{aligned}
&u(c_t(h^{t-1}, \bar{\beta})) + \bar{\beta} V(h^{t-1}, \bar{\beta}|x) \\
&\geq u(c_t(h^{t-1}, \underline{\beta})) + \bar{\beta} V(h^{t-1}, \underline{\beta}|x) \\
&\geq u(c_t(h^{t-1}, \underline{\beta})) + \underline{\beta} V(h^{t-1}, \underline{\beta}|x) \\
&\geq u(c_t(h^{t-1}, \beta) + n_t(h^{t-1}, \beta)), \forall \beta \in H,
\end{aligned}$$

where the second inequality follows from $\bar{\beta} > \underline{\beta}$, the first inequality follows from $IC_L(h^{t-1}, \bar{\beta})$, and the third inequality follows from $IC_D(h^{t-1}, \underline{\beta})$ and $IC_{LD}(h^{t-1}, \underline{\beta})$.

Next, I show $IC_L(h^{t-1}, \underline{\beta})$ is redundant. $IC{-}C(h^{t-1}, \underline{\beta})$ being satisfied implies $u(c_t(h^{t-1}, \underline{\beta})) \geq u(c_t(h^{t-1}, \bar{\beta}) + n_t(h^{t-1}, \bar{\beta}))$, which implies $u(c_t(h^{t-1}, \underline{\beta})) \geq u(c_t(h^{t-1}, \bar{\beta}))$ since investment has to be non-negative. □

Proof of Lemma 4.

Proof. *Part 1.* This follows directly from $IC_D(h^{t-1}, \underline{\beta})$.

Part 2. Suppose for a contradiction that for some constrained Pareto efficient allocation x^*, $n_t^*(h'^{t-1}, \bar{\beta}) = 0$, for some h'^{t-1}.

First, I show that $n_t^*(h'^{t-1}, \bar{\beta}) = 0$ implies incentive-constraint $IC_{LD}(h'^{t-1}, \underline{\beta})$ binds. Suppose for a contradiction that it does not. Dropping this incentive-constraint and the non-negativity condition on $n_t(h'^{t-1}, \underline{\beta})$ from the planner's problem and taking the first order condition with respect to $n_t(h'^{t-1}, \underline{\beta})$, one gets $f'(n_t^*(h'^{t-1}, \underline{\beta})) = \lambda_t^*/\lambda_{t+1}^* > 0$. By Inada assumption on the production function, this implies $n_t^*(h'^{t-1}, \underline{\beta}) > 0$, which is a contradiction.

Now, I show that $IC_{LD}(h'^{t-1}, \underline{\beta})$ binding implies $c_t^*(h'^{t-1}, \bar{\beta}) > 0$. Suppose to the contrary that $c_t^*(h'^{t-1}, \bar{\beta}) = 0$. Consider the solution to the planner's problem without the incentive constraint $IC_{LD}(h'^{t-1}, \underline{\beta})$. In the solution, it has to be the case that $c_t^*(h'^{t-1}, \underline{\beta}) \geq 0$. Then, $IC_{LD}(h'^{t-1}, \underline{\beta})$ is automatically satisfied, meaning it is indeed slack, which is a contradiction.

Therefore, $c_t^*(h'^{t-1}, \bar{\beta}) > 0$. Now, consider the allocation $\tilde{x}$, which is the same as the constrained Pareto efficient allocation except for:

$$\tilde{c}_t(h'^{t-1}, \bar{\beta}) = c_t^*(h'^{t-1}, \bar{\beta}) - \epsilon,$$

$$\tilde{n}_t(h'^{t-1}, \bar{\beta}) = n_t^*(h'^{t-1}, \bar{\beta}) + \epsilon = \epsilon,$$

and

$$\tilde{c}_{t+1}(h'^{t-1}, \bar{\beta}, \beta) = c_t^*(h'^{t-1}, \bar{\beta}, \beta) + \epsilon',$$

where $\epsilon > 0$ is small and ϵ' is chosen to keep the welfare of parent with dynastic history $(h'^{t-1}, \bar{\beta})$ unchanged:

$$\begin{aligned} u(c_t^*(h'^{t-1}, \bar{\beta}) - \epsilon) + \bar{\beta} \textstyle\sum_\beta \mu(\beta)[u(c_{t+1}^*(h'^{t-1}, \bar{\beta}, \beta) + \epsilon') + \beta V(h'^{t-1}, \bar{\beta}, \beta | \tilde{x})] = \\ u(c_t^*(h'^{t-1}, \bar{\beta})) + \bar{\beta} \textstyle\sum_\beta \mu(\beta)[u(c_{t+1}^*(h'^{t-1}, \bar{\beta}, \beta)) + \beta V(h'^{t-1}, \bar{\beta}, \beta | x^*)]. \end{aligned} \tag{12}$$

By construction, $V(h'^{t-1}, \bar{\beta}, \beta | \tilde{x}) = V(h'^{t-1}, \bar{\beta}, \beta | x^*)$. Then, for small ϵ, (12) implies that the resources needed in period $t+1$ to keep the welfare of parent $(h'^{t-1}, \bar{\beta})$ unchanged are:

$$\epsilon' = \epsilon \frac{u'(c_t^*(h'^{t-1}, \bar{\beta}) - \epsilon)}{\bar{\beta} \sum_\beta \mu(\beta) u'(c_{t+1}^*(h'^{t-1}, \bar{\beta}, \beta) + \epsilon')}. \tag{13}$$

New resources created in $t+1$ due to human capital investment in the children of parents with dynastic history $(h'^{t-1}, \bar{\beta})$ is equal to $f(\epsilon)$. For ϵ small, this is equal to $\epsilon f'(\epsilon)$. As a result, in period $t+1$, total resource change is:

$$\epsilon \Big[f'(\epsilon) - \frac{u'(c_t^*(h'^{t-1}, \bar{\beta}) - \epsilon)}{\bar{\beta} \sum_\beta \mu(\beta) u'(c_{t+1}^*(h'^{t-1}, \bar{\beta}, \beta) + \epsilon')} \Big].$$

As ϵ approaches 0, by the Inada condition on f, the first term in the brackets tends to ∞, whereas the second term goes to $\frac{u'(c_t^*(h'^{t-1}, \bar{\beta}))}{\bar{\beta} \sum_\beta \mu(\beta) u'(c_{t+1}^*(h'^{t-1}, \bar{\beta}, \beta))}$), which is a finite number. Therefore, for small ϵ, the allocation $\tilde{x}$ raises some extra resources in period $t+1$. By construction, the welfare levels of all parents besides those with dynastic history $(h'^{t-1}, \bar{\beta}, \beta)$ are unchanged, and the welfare of the latter is strictly increased under $\tilde{x}$. Hence, showing that the new allocation $\tilde{x}$ is incentive compatible amounts to showing that it Pareto improves over x^*, which is a contradiction.

Now I show that $\tilde{x}$ is indeed incentive compatible. Since the only change from the original allocation is following the node $(h'^{t-1}, \bar{\beta})$, and by construction this agent's utility is left unchanged, all incentive-constraints preceding node $(h'^{t-1}, \bar{\beta})$ are satisfied in the new allocation. $IC_{LD}(h'^{t-1}, \underline{\beta})$ is satisfied since $c_t^*(h'^{t-1}, \bar{\beta}) + n_t^*(h'^{t-1}, \bar{\beta})$ is unchanged in the new allocation.

$IC_L(h'^{t-1}, \bar{\beta})$ is satisfied since the welfare of parent $(h'^{t-1}, \bar{\beta})$ is unchanged. Since the new allocation adds the same amount, ϵ', on top of $c^*_{t+1}(h'^{t-1}, \bar{\beta}, \beta)$ for both β, $IC_L(h'^{t-1}, \bar{\beta}, \bar{\beta})$ and $IC_{LD}(h'^{t-1}, \bar{\beta}, \underline{\beta})$ are also satisfied. There is no reason for the descendants of parents with history $(h'^{t-1}, \bar{\beta}, \beta)$ to deviate since their allocations remain unchanged in the new allocation. Finally, allocations in all other branches are also kept the same, completing the proof that the new allocation is incentive-compatible. □

C $\underline{\beta} > 0$

Proof of Proposition 6.

Proof. I need to find (1)$\underline{y}_t$ for every t and (2) $\tilde{\underline{\beta}}$ such that the proposition holds.

(1) Remember that I already proved that $n^*_t(h^{t-1}, \bar{\beta}) > 0$ for all t, h^{t-1}, when $\underline{\beta} = 0$. Similarly, one can show that $n^*_t(h^{t-1}, \bar{\beta}) > 0$ for all t, h^{t-1}, for all $\underline{\beta} \in [0, \bar{\beta}]$. Then, for any t, let $\underline{y}_t = \min_{\underline{\beta} \in [0,\bar{\beta}]} \min_{h^{t-1}} n^*_t(h^{t-1}, \bar{\beta})$. Clearly, $\underline{y}_t > 0$ for all t.

(2) I first prove the following claim:

Claim: $n^*_t(h^{t-1}, \underline{\beta}) \to 0$ for all t, h^{t-1} as $\underline{\beta} \to 0$.

To see this, suppose this is not true for some t, h^{t-1}. That means there exists $\epsilon > 0$ such that no matter how small $\underline{\beta}$ is $n^*_t(h^{t-1}, \underline{\beta}) \geq \epsilon$. Then, consider the incentive constraint $IC_D(h^{t-1}, \underline{\beta})$:

$$u(c^*_t(h^{t-1}, \underline{\beta})) + \underline{\beta} V(h^{t-1}, \underline{\beta}|x^*) \geq u(c^*_t(h^{t-1}, \underline{\beta}) + n^*_t(h^{t-1}, \underline{\beta})) + \underline{\beta} \frac{1}{1 - \mathbb{E}\beta} \kappa.$$

By rearranging this constraint we get:

$$u(c^*_t(h^{t-1}, \underline{\beta})) - u(c^*_t(h^{t-1}, \underline{\beta}) + n^*_t(h^{t-1}, \underline{\beta})) \geq \underline{\beta} \Big[\frac{1}{1 - \mathbb{E}\beta} \kappa - V(h^{t-1}, \underline{\beta}|x^*) \Big].$$

Since there is a finite amount of resources in this economy at any point in time, L.H.S. of the above expression is a strictly negative number bounded away from zero for any $\underline{\beta} \in [0, \bar{\beta}]$. R.H.S. of the above expression, on the other hand, converges to 0 as $\underline{\beta} \to 0$. But this is a contradiction.

Given the claim it is straightforward that, for every t, there exists $\tilde{\underline{\beta}}_t$ such that for all $\underline{\beta} \leq \tilde{\underline{\beta}}_t$, $f(n^*_t(h^{t-1}, \underline{\beta})) < \underline{n}_{t+1}$. I also choose $\tilde{\underline{\beta}}_t$ small enough so that $IC_{LD}(h^{t-1}, \underline{\beta})$ implies that $c_t^*(h^{t-1}, \underline{\beta}) - [c_t^*(h^{t-1}, \bar{\beta}) + n_t^*(h^{t-1}, \bar{\beta})] \geq -\delta_t$ where $\delta_t > 0$ is arbitrarily small. Now let $\tilde{\underline{\beta}} = \min_t \tilde{\underline{\beta}}_t$.

Now I am ready to prove the proposition.

Part 1. $y_t^*(h^{t-2}, \underline{\beta}, \beta) = f(n_{t-1}^*(h^{t-2}, \underline{\beta})) < \underline{y}_t$ where the inequality is true by construction.

Part 2. Suppose $y_t^*(h^{t-1}, \beta) < \underline{y}_t$. Then, $\Delta_t^*(h^{t-1}, \bar{\beta}) = c_t^*(h^{t-1}, \bar{\beta}) + n_t^*(h^{t-1}, \bar{\beta}) - y_t^*(h^{t-1}, \bar{\beta}) > 0$ since $n_t^*(h^{t-1}, \bar{\beta}) > \underline{y}_t$ by construction. Then, $\Delta_t^*(h^{t-1}, \underline{\beta}) = c_t^*(h^{t-1}, \underline{\beta}) + n_t^*(h^{t-1}, \underline{\beta}) - y_t^*(h^{t-1}, \bar{\beta}) > 0$ since, by construction, $c_t^*(h^{t-1}, \underline{\beta}) - [c_t^*(h^{t-1}, \bar{\beta}) + n_t^*(h^{t-1}, \bar{\beta})] \geq -\delta_t$ where $\delta_t > 0$ is arbitrarily small and $n_t^*(h^{t-1}, \bar{\beta}) > \underline{y}_t$.

Part 3. Similar to proof of Part 3 of Proposition 1.

□