

Pitfalls in Estimating Asymmetric Effects of Energy Price Shocks

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Abstract: A common view in the literature on the transmission of energy price shocks is that the effect of energy price shocks on macroeconomic aggregates such as output or employment is asymmetric in energy price increases and decreases. This perception has been bolstered by regression evidence that energy price increases (obtained by censoring percent changes in the price of energy to exclude energy price decreases) appear to have disproportionately larger effects on macroeconomic aggregates than decreases. We first show that commonly used asymmetric models of the transmission of energy price shocks are misspecified, resulting in inconsistent parameter estimates, and that the implied impulse responses have been routinely computed incorrectly. As a result, the quantitative importance of energy price increases for the U.S. economy has been exaggerated in the literature. Second, we develop alternative regression models, estimation methods and methods of computing responses to energy price shocks that yield consistent estimates regardless of the degree of asymmetry. Third, we develop improved tests of the null hypothesis of symmetry in the responses to energy price increases and decreases. We also develop symmetry tests for models involving net changes in energy prices. Fourth, an empirical study reveals little evidence against the null hypothesis of symmetry. Our analysis has important implications for the theoretical literature on the transmission of energy price shocks and for the debate about policy responses to energy price shocks. The practical importance of our analysis is illustrated by presenting alternative estimates of the effects of falling oil prices on U.S. real GDP since mid-2008.

Key Words: Asymmetry; oil price; energy prices; net increase; shocks; propagation; transmission.

JEL Classification: C32, E37, Q43

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1. Introduction

In the literature of how energy price shocks affect macroeconomic aggregates such as output or employment, a common view is that the responses are asymmetric. In particular, energy price increases are perceived to have larger effects than energy price decreases. This perception has been bolstered by empirical evidence that there are seemingly large effects of energy price increases (obtained by censoring energy price changes to exclude all energy price decreases) on the macro economy, whereas uncensored percent changes in energy prices tend to have much smaller effects. Vector autoregressive models relating energy price increases to macroeconomic aggregates in particular have shaped the discussion of the effects of energy price shocks in recent decades. In this paper, we demonstrate that the methods used in estimating these VAR models generate inconsistent estimates of the true effects of unanticipated energy price increases. We show that existing econometric methods are likely to have exaggerated the effects of energy price increases and that fundamental changes are needed in how these effects are estimated in practice. In addition to addressing the problem of how to estimate responses to energy price shocks, we develop improved tests of the null hypothesis of symmetric responses to energy price increases and decreases. Our empirical evidence suggests that there is no compelling evidence against the symmetry null. Our analysis has important implications for the theoretical literature on the transmission of energy price shocks and for the debate about policy responses to energy price shocks, especially in the current environment of sharply falling energy prices.

1.1. Review of the Literature

In the view of many economists, oil price shocks are perhaps the leading alternative to monetary policy as the determinant of postwar U.S. recessions. Increases in oil prices, for example, preceded the recessions of 1973-75, 1980-82, and 1990-91. Given the striking coincidence of deteriorating macroeconomic outcomes and rising oil prices in the 1970s and early 1980s, it was natural at the time to suspect a strong link from oil price increases to recessions. Nevertheless, it has proved surprisingly difficult to find an indicator of oil price shocks that produces the expected responses of macroeconomic and policy variables in a VAR setting. As discussed in Bernanke, Gertler and Watson (1997), finding a measure of oil price

shocks that "works" in a VAR context in practice is not straightforward. Simple measures of energy price shocks (such as linearly unpredictable changes in energy prices) in particular sometimes imply "anomalous" effects on macroeconomic outcomes, relative to the conventional wisdom about the effects of oil price shocks on the economy. They also tend to have an unstable relationship with macroeconomic outcomes.¹

Far from undermining the view that energy price shocks are important, these difficulties have led researchers to employ increasingly complicated specifications of the "true" relationship between oil prices and the economy. Today it is widely believed that the most appropriate specification of oil price shocks involves some measure of oil price increases, obtained by censoring oil price changes to exclude all oil price decreases. This consensus dates back to the work of Mork (1989). After the sharp oil price declines of 1985-86 failed to lead to an economic boom in oil importing economies, Mork (1989) pointed out that the effects of positive and negative oil price shocks on the economy need not be symmetric. He provided empirical evidence that positive changes in the real price of oil had far more important effects on U.S. real GDP than negative changes. This was widely interpreted as evidence that only oil price increases matter for the U.S. economy (see, e.g., Bernanke, Gertler and Watson 1997, p. 103). Given the a priori belief that oil price shocks have quantitatively important effects on macroeconomic aggregates and given researchers' inability to generate such responses from linear and symmetric models, regressions of macroeconomic aggregates on measures of oil price increases became accepted on the grounds that they produced "better looking" impulse responses (Bernanke, Gertler and Watson 1997, p. 104).²

The initial proposal to focus on oil price increases only has subsequently been refined by

¹ Kilian (2008a) recently has discussed some of the reasons for the apparent instability of such regressions in small samples and for the seemingly counterintuitive responses estimates occasionally obtained from such regressions.

² This finding reinforced results based on measures of oil supply disruptions such as the dummy variable constructed by Hoover and Perez (1994) and the quantitative dummy variable of Hamilton (1996, 2003). It also seemed consistent with evidence using an alternative VAR methodology provided in Davis and Haltiwanger (2001, p. 509), who considered "the evidence for asymmetric responses to oil price ups and downs as well established". Related evidence was provided by Mory (1993), Mork, Olsen and Mysen (1994); Ferderer (1996), Hooker (1996a,b; 2002), Hamilton (1996, 2003), Raymond and Rich (1997), Huntington (1998), and Balke, Brown and Yücel (2002), among others. For a critical review of this literature see Edelstein and Kilian (2007a,b).

Hamilton (1996, 2003) who introduced the “net oil price increase”. This measure distinguishes between oil price increases that establish new highs relative to recent experience and increases that simply reverse recent decreases. Specifically, in the context of monthly data, Hamilton's measure equals the maximum of (a) zero and (b) the difference between the log-level of the crude oil price for the current month and the maximum value of the logged crude oil price achieved in the previous 12 (or alternatively 36) months. Hooker (2002), for example, finds the net increase measure to perform “well”, in the sense of having a relatively stable relationship with macroeconomic variables, and Hamilton (1996, 2003, 2009) makes the case that this measure generates large effects on U.S. real GDP.

The net increase measure in particular has been the foundation of numerous VAR papers, the results of which have become accepted in academic and policy discussions of the transmission of energy price shocks (see, e.g. Dotsey and Reid 1992; Davis and Haltiwanger 2002; Lee and Ni 2002; Jones, Leiby, and Paik 2004; Jiménez-Rodríguez and Sánchez 2005; Herrera 2008). This model also plays a central role in VAR analyses of the role of monetary policy in propagating energy price shocks (see, e.g., Bernanke, Gertler and Watson 1997, 2004; Hamilton and Herrera 2004; Herrera and Pesavento 2007).³

1.2. Outline of the Paper

In this paper, we demonstrate that the estimation methods used in these VAR papers generate inconsistent estimates of the true effects of unanticipated energy price increases. We show, for example, that VAR models of the effects of energy price increases on macroeconomic outcomes will tend to overestimate the true response to energy price increases asymptotically, when the underlying data generating process is symmetric. Since these models were adopted precisely based on their ability to generate larger responses to energy price shocks than symmetric VAR models, as discussed above, our finding casts doubt on the results reported in this literature and indeed on the empirical relevance of these models. Our results apply with equal force to measures of oil price increases and net oil price increases. More generally, we

³ Similar econometric issues also arise in studying the transmission of crude oil price shocks to retail energy prices (see, e.g., Borenstein, Cameron, and Gilbert 1997; Bachmeier and Griffin 2003).

show that asymptotic biases in the VAR response estimates may arise whether the underlying data generating process is linear and symmetric or not.

In section 2, we illustrate this point in the context of a stylized static model. We establish the inconsistency of estimators of conventional asymmetric models analytically, and we study the determinants of the asymptotic bias of the estimator by simulation. We show that static censored regressor models will be consistent only in very special and empirically and theoretically implausible cases. In section 3, we strengthen this result by showing that estimates of dynamic censored VAR models of the type frequently employed in the literature are fundamentally misspecified and will be inconsistent regardless of the data generating process. We investigate both data generating processes that are symmetric in energy price increases and decreases and asymmetric data generating processes. We demonstrate that censored VAR models will distort the quantitative importance of energy price increases asymptotically whether the true model is symmetric or asymmetric. We show in particular that asymmetric data generating processes cannot be represented as censored VAR models. We discuss how alternative, correctly specified asymmetric models may be constructed and estimated consistently using restricted maximum likelihood estimators.

An important problem in practice is that we may not know whether the data generating process is symmetric or not, and whether energy price decreases should be included in the regression, if the data generating process is asymmetric. In section 4, we propose a model that can be estimated consistently whether the true model is symmetric or not, regardless of the precise form of the asymmetry. Moreover, this model may be estimated by standard methods such as unrestricted least-squares.

In section 5, we show that in addition to the misspecification of the regression models used in the literature and the resulting inconsistency of the parameter estimates, the dynamic responses of macroeconomic aggregates to unanticipated energy price increases have routinely been computed incorrectly in a way that further exaggerates the quantitative importance of these shocks. We adapt existing methods of constructing nonlinear impulse responses to the structural model proposed in section 4 and demonstrate how asymmetric impulse responses may be estimated consistently. Unlike existing methods of computing nonlinear impulse

responses in the econometric literature, our approach is fully structural and avoids the ambiguities of defining a shock in nonlinear reduced form models.

Both the regression model proposed in section 4 and the method of computing responses to energy price shocks developed in section 5 play a crucial role in designing tests of the symmetry of response functions in energy price increases and decreases. In section 6, we discuss the problem of testing the null hypothesis that the U.S. economy responds symmetrically to energy price increases and decreases. First, we show that traditional tests for asymmetries in the regression slopes of predictive regressions in the spirit of Mork (1989) are incompletely specified. We propose a modified test of symmetry in the slopes based on the structural model of section 4 with more accurate size and higher power. Second, we observe that statistically insignificant departures from symmetry in the slopes may cause statistically significant asymmetries in the implied impulse response functions, given the nonlinearity of these functions, while significant departures from symmetry in the slopes need not imply large asymmetries in the impulse response functions. Moreover, by construction the extent to which responses from the linear symmetric model provide a good approximation will depend on the magnitude of the energy price shock. We therefore propose a direct statistical test of the symmetry of the economy's response to unanticipated energy price increases and decreases. That test is shown to have reasonably accurate finite-sample size and nontrivial power.

In section 7 we apply these tools in examining the evidence against the symmetry null in three prominent empirical examples. Specifically, we model the relationship between quarterly U.S. real GDP and the real price of oil, between monthly U.S. unemployment and the real price of oil, and between monthly U.S. gasoline consumption and the U.S. real retail price of gasoline. We find no compelling evidence of asymmetric responses to positive and negative energy price shocks.

In section 8 we extend the analysis to models of net energy price increases motivated by the analysis in Hamilton (1996, 2003). Despite the widespread use of the net oil price increase measure in VAR models, none of the results in the literature provides a justification for the use of these VAR models. Notwithstanding the evidence for asymmetries in the predictive relationship between real GDP growth and oil price changes provided by Hamilton (2003), with

the partial exception of Balke, Brown, and Yücel (2002), no paper has adequately addressed the implications of the net increase model for impulse response analysis. In this paper, we address this question. We present several tests of symmetric impulse responses based on the net increase model, some based on the slope parameters of a modified version of the model proposed in section 4 and others based on the impulse response functions implied by that model. Our analysis clarifies and extends the earlier analysis in Balke et al., which recognized many of the problems discussed here, but had no apparent impact on the empirical practice in this literature.

Unlike Hamilton (2003) or Balke et al. (2002), we find little, if any, evidence against the null hypothesis of symmetric responses in our three empirical examples. Our results suggest that linear symmetric models will be adequate for many applications. This finding also has important implications for the theoretical literature on the transmission of energy price shocks and for the debate about policy responses to oil price shocks. Finally, to the extent that there is evidence of asymmetries, our analysis suggests that important changes are needed in the way these asymmetries are modeled in the literature. In section 9, we illustrate the importance of correctly specifying nonlinear models and of correctly forming expectations from nonlinear models in the context of multi-step ahead predictions of real GDP since mid-2008, when energy prices entered their steepest decline ever. Section 10 contains the concluding remarks.

2. A stylized model

In this section, we illustrate the estimation biases induced by censoring regressors in the simplest possible setting. It is well known that censoring dependent variables causes OLS estimates of the coefficients of linear models to be biased (see, e.g., Amemiya (1984) for a review). Notwithstanding the existence of a large literature on the effects of censoring the dependent variable, the problems arising from censoring the explanatory variable that are the focus of our paper have rarely been analyzed. A notable recent exception is Rigobon and Stoker (2007, 2008) who make the case that researchers encounter censored regressors as often or perhaps more often than censored dependent variables. Rigobon and Stoker focus on the problem of censoring regressors in static single equation models used to analyze cross-sections. Their primary interest is in double censoring where a regressor is top and bottom

coded, as might be the case in classifying household income by range, for example.⁴

In contrast, our focus in this paper is on multivariate time series models with censored endogenous regressors. The type of censoring of interest in our paper occurs below zero (or alternatively above zero). Moreover, motivated by the literature on the transmission of energy price increases, our main interest is the effect of censoring on the impulse response estimates rather than slope parameters. Finally, whereas Rigobon and Stoker study cases where the econometrician cannot observe the uncensored data, we study situations where the uncensored data is observed, but the researcher has chosen to estimate a regression using the censored data in order to estimate an asymmetric effect.

2.1. Asymptotic Biases from Using Censored Regressors

For expository purposes consider the following static linear two-equation data generating process (DGP):

$$\begin{aligned}x_t &= \alpha_1 + \varepsilon_{1,t} \\y_t &= \alpha_2 + x_t\beta + \varepsilon_{2,t}\end{aligned}\tag{1}$$

where $\alpha_1, \alpha_2, \beta$, are constants, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are mean zero i.i.d Gaussian random variables with variances σ_1^2 and σ_2^2 , and $t = 1, \dots, T$. It is straightforward to show that the OLS estimators of a and b in the regression model

$$y_t = a + x_t b + u_t\tag{2}$$

will be unbiased and consistent estimators of α and β . To illustrate the effect of replacing negative values of x_t with zero in this regression, define the variable x_t^+ as

$$x_t^+ = \begin{cases} x_t & \text{if } x_t > 0 \\ 0 & \text{if } x_t \leq 0 \end{cases}\tag{3}$$

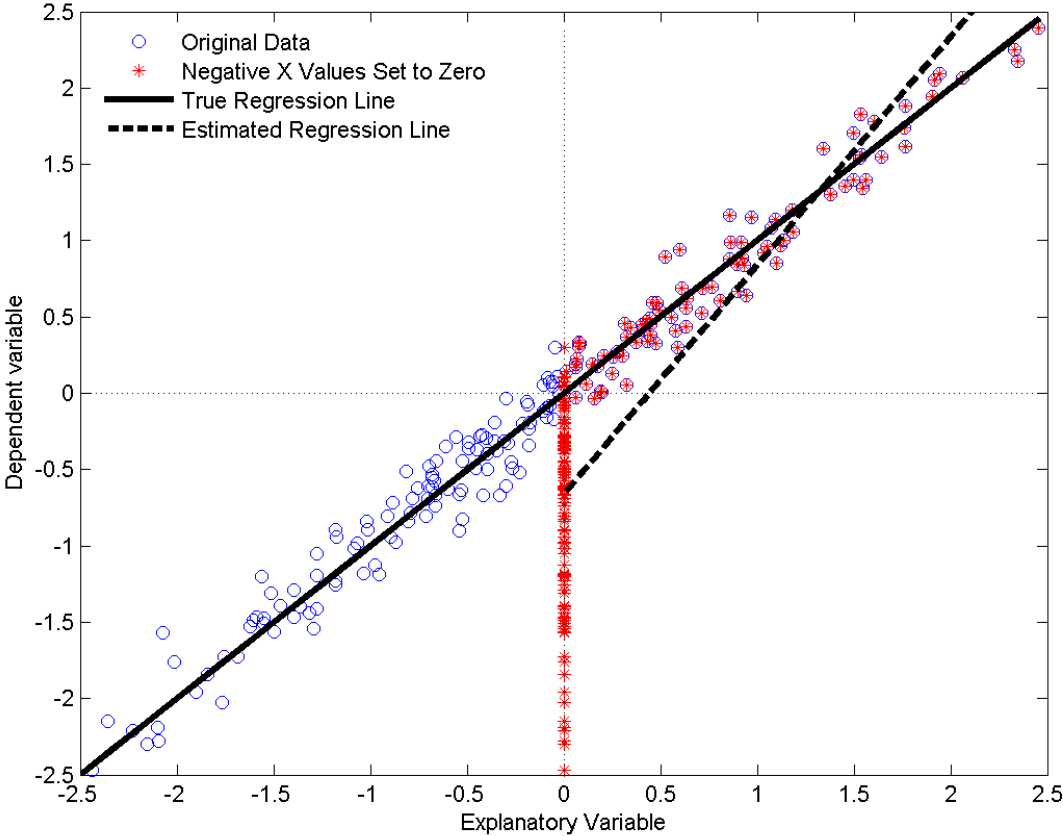
and consider estimating the censored regression model:

$$y_t = a + x_t^+ b + u_t\tag{4}$$

⁴ Related work also includes Manski and Tamer's (2002) work on interval data and the statistical literature on data that are missing at random (see, e.g., Little 1992).

rather than the unconstrained model (2). Censoring the explanatory variable renders the estimator of b inconsistent for β . Figure 1 illustrates the problem. The values of the dependent variable that are associated with negative values of x_t are moved to the y -axis. Given these points, the estimated intercept is negative and far below its true value of zero. As a result of the negative intercept, the value of the slope coefficient must be greater than the true value. In this example, the estimated slope is 1.5 when the true value is 1. This problem is equivalent to the expansion bias discussed by Rigobon and Stoker (2008) in the context of top-coding of cross-sectional data.

Figure 1: The Effect of Censoring Negative Values of the Explanatory Variable



The upward bias in the estimated effect of x_t on y_t is not a small-sample problem. For the simple case where α equals zero and where x_t has a symmetric distribution with mean zero

and variance 1 and is uncorrelated with $\varepsilon_{2,t}$, we can derive the limits for \hat{a} and \hat{b} . Observe that $E(x_t^+) = 0.5\mu$ where $\mu \equiv E(x_t|x_t > 0)$. As shown in the appendix, \hat{a} and \hat{b} converge in probability to the following limits:

$$\hat{a} \xrightarrow{p} -\beta \frac{0.5\mu}{1 - 0.5\mu^2} \quad [5]$$

and

$$\hat{b} \xrightarrow{p} \beta \frac{1}{1 - 0.5\mu^2} \quad [6]$$

The proof is a standard application of the Law of Large Numbers to the OLS estimators and is described in the appendix. If the variable x_t has a standard normal distribution, then \hat{a} converges to about minus 0.58β and \hat{b} converges to roughly 1.47β . Therefore, in this simple example, the effect of x_t on y_t is over-estimated by almost 50 percent.⁵

2.2. Further Illustrations of the Asymptotic Bias From Censoring

To illustrate further the effects of censoring and its determinants, we report some simulation evidence. The first simulation illustrates that the effect of censoring negative values depends on the relative frequency with which negative values occur. Table 2 reports the results of simulating data from model (1) but for different values of α_1 . As a consequence of varying α_1 , the fraction of x_t 's that are positive varies. When 90 percent of the observations are positive, the slope coefficient is still biased upwards, although by less than 10 percent. These simulation results approximate asymptotic results, as the number of observations per simulated dataset is 100,000 and results are averaged across 2000 simulated datasets.

The analysis so far has focused on data generating processes that are symmetric in x_t , but the same problem may arise even when the true response of y_t to x_t is asymmetric in positive and negative values. Consider the following data generating process:

⁵ Analogous results would hold if the researcher retained only negative values of x_t and censored all positive values. The only change in the analysis would be to replace μ with negative μ . In that case, the limit of the estimated intercept is positive and equals 0.58β in magnitude, but the limit of the slope estimator is the same as in the baseline case – roughly 50 percent larger than the true slope.

Table 2: Asymptotic Bias from Censoring in the Static Symmetric Model

Varying the Fraction of Positive x_t Observations			
$P(x_t > 0)$	$E[x_t]$	\hat{a} $\alpha_2 = 0$	\hat{b} $\beta = 1$
0.10	-1.28	-1.41	2.70
0.20	-0.84	-1.08	2.14
0.30	-0.52	-0.87	1.83
0.40	-0.25	-0.72	1.62
0.50	0.00	-0.59	1.47
0.60	0.25	-0.47	1.34
0.70	0.52	-0.36	1.24
0.80	0.84	-0.26	1.15
0.90	1.28	-0.15	1.08

Notes: Symmetric DGP: $\beta = 1$, $\alpha_1 = \alpha_2 = 0$, $\sigma_1 = \sigma_2 = 1$. Average results for 2000 samples of 100,000 observations each.

$$\begin{aligned}
 x_t &= \alpha_1 + \varepsilon_{1,t} \\
 y_t &= \alpha_2 + x_t\beta + x_t^+\gamma + \varepsilon_{2,t}
 \end{aligned}
 \tag{7}$$

where γ captures the asymmetric response that is of interest to many economic researchers. This process allows for both positive and negative values of x_t to affect y_t , but with different coefficients. Given this data generating process, if we estimated the equation

$$y_t = a + x_t^+b + u,
 \tag{8}$$

we would want the value of b to equal $\beta + \gamma$ in the limit. However, as shown in Table 3, this is not the case unless β equals zero, implying that only positive x_t have an effect on y_t . For all other values of β , the value of the estimated slope coefficient is biased upwards. Furthermore, in this example, if the slope for negative x_t is at least half of the slope for positive x_t , then the estimate of an increase in x_t on y_t is less biased when using the full sample than when using the censored sample.⁶

⁶ Additional simulation exercises (not shown to conserve space) confirm that the asymptotic biases reported in Tables 2 and 3 that arise from the misspecification of the regression model carry over to small samples. Relative to models with Gaussian errors, small sample biases may increase substantially when the errors are fat-tailed.

Table 3: Asymptotic Bias from Censoring in the Asymmetric Static Model

Population Slope Parameters			Average Estimated Slope \hat{b}	
β	γ	$\beta+\gamma$	Standard Model	Censored Model
0.00	1	1	0.50	1.00
0.25	0.75	1	0.62	1.12
0.50	0.50	1	0.75	1.23
0.60	0.40	1	0.80	1.28
0.70	0.30	1	0.85	1.32
0.80	0.20	1	0.90	1.37
1.00	0.00	1	1.00	1.47
1.25	-0.25	1	1.12	1.58
1.50	-0.50	1	1.25	1.70

Notes: Asymmetric DGP: $\alpha_1 = \alpha_2 = 0$, $\sigma_1 = \sigma_2 = 1$. Average results for 2000 samples of 100,000 observations each.

Table 3 illustrates that only when β is *known* to be zero, the censored regression model will consistently estimate the effect of an energy price increase in the static model. This point is important since Mork (1989) merely failed to reject the null hypothesis that energy price decreases have no effect on real GDP growth. He did not establish that $\beta=0$, and indeed was careful only to suggest that these coefficients are “perhaps zero”. Nor does economic theory predict that $\beta=0$ (see, e.g., Edelstein and Kilian 2007a, Kilian 2008b). If, in fact, both energy price increases and decreases matter for real GDP, but to a different extent, as suggested by economic theory, then the censored regressor model is bound to overestimate the effect of an energy price increase even in this simplest possible model.

Below we will show that in a dynamic setting the censored VAR models routinely employed in the literature will be inconsistent not only in the empirically plausible case of nonzero responses to both energy price increases and decreases, but even when all coefficients on current and lagged x_t are zero in population. In other words, even if oil price decreases had no effect on real GDP growth in population, as postulated in the literature, censored VAR models would yield inconsistent parameter estimates.

2.3. Problems Associated with Estimating Impulse Responses from Censored VAR Models

The static model of the preceding section is useful for building intuition, but, in practice, researchers are inevitably interested in the response of the economy to energy price increases

over time. Such impulse responses in general are defined as the

$$I_y(h, \varepsilon_{1,t}, \Omega_{t-1}) = E(y_{t+h} | \varepsilon_{1,t}, \Omega_{t-1}) - E(y_{t+h} | \Omega_{t-1})$$

where Ω_{t-1} is the information set at the time of the shock. Impulse responses can be computed from VAR models. For example, Bernanke, Gertler and Watson (1997, p. 103) observe that “Knut Mork provided evidence that only positive changes in the relative price of oil have important effects on output. Accordingly, in our VARs we employ an indicator that equals the log-difference of the relative price of oil when that change is positive and otherwise is zero.” Similarly, Leduc and Sill (2004, p. 790) state that “to get an empirical estimate of the output response to positive oil-price shocks, we run a VAR using ... oil-price increases [...] constructed by taking the first difference of the log of oil prices, then setting negative values to zero. Thus, only oil-price increases affect the other variables in the system.”

The standard dynamic model used in the literature is a recursively identified VAR model, in which the energy price increase variable is ordered above the macroeconomic aggregate of interest. The prototypical example is a linear bivariate autoregression for $(x_t^+ \ y_t)'$. Adding additional macroeconomic variables does not affect the econometric points of interest in this paper, and indeed is not required for consistently estimating the response of y_t to an unanticipated increase in energy prices under the maintained assumption of predetermined (or contemporaneously exogenous) energy prices (see Kilian 2008b). The identifying assumption of predetermined energy prices with respect to the macroeconomic aggregate of interest is not only standard in the literature, but is consistent with empirical evidence presented in Kilian and Vega (2008).

Estimates of this model have been used in constructing responses to a one-standard deviation shock in x_t^+ . The construction of impulse responses from nonlinear models such as the asymmetric models of interest in this paper is not straightforward. Whereas impulse responses in linear models are independent of the history of the observations, impulse responses in nonlinear models are dependent on the history of the observations and on the magnitude of the shock (see, e.g., Gallant Rossi and Tauchan 1993; Koop, Pesaran and Potter 1996). Thus, the construction of impulse responses requires Monte Carlo integration over all possible paths of the data. This point has been routinely ignored in the literature. In practice,

researchers have presented impulse response estimates computed exactly as in linear VAR models. For now we follow that convention since we wish to illustrate the asymptotic biases in the results reported in the literature. Moreover, discussing one problem at a time will facilitate the exposition. We will return to this point in section 5, however, and show how impulse responses can be computed correctly.

3. How empirically relevant is the asymptotic bias of VAR models of energy price increases?

3.1. Linear and symmetric VAR data generating processes

The points we made in section 2 directly extend to fully dynamic VAR models. Consider a linear symmetric bivariate VAR(p) data generating process of the form

$$\begin{aligned} x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1t} \\ y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \varepsilon_{2,t} \end{aligned} \quad [9]$$

where x_t denotes the percent change in energy prices, y_t denotes the percent change in the macroeconomic aggregate of interest, and $\varepsilon_t \sim (0, \Sigma)$ is uncorrelated white noise. We focus on three illustrative examples that are representative of models employed in the empirical literature:

- (1) A quarterly VAR in the percent changes in real crude oil prices and the growth rate of U.S. real GDP. The sample period is 1973.II-2007.IV. The oil price series is based on an index of U.S. refiners' acquisition cost, extrapolated as in Kilian (2008a), and deflated by the U.S. CPI. The real GDP data are from the BEA.
- (2) A monthly VAR in the percent change in crude oil prices and the change in the U.S. unemployment rate. The sample period is 1973.2-2007.12. The unemployment rate data are from the BLS.
- (3) A monthly VAR in the percent change in real gasoline prices and the percent change in real U.S. real gasoline consumption, as constructed by the BEA. The sample period is 1973.2-2007.12.

The lag order p is set to 6 for expository purposes. For each data set, we construct a data generating process by replacing the model parameters by their least-squares estimates obtained from fitting this model to the data set in question and by treating the structural errors as Gaussian white noise. For each data generating process, we generate data of length T , fit a VAR(6) model for $(x_t^+ y_t)'$ and construct the cumulative response of y_{t+h} , $h = 0, 1, \dots$ to a unit increase in energy prices. We compare that response to the response of y_{t+h} to a unit increase in energy prices in the data generating process. Since we are interested in evaluating the asymptotic bias of the responses implied by the censored VAR model, all results in Figure 2 are based on $T = 1,000,000$.

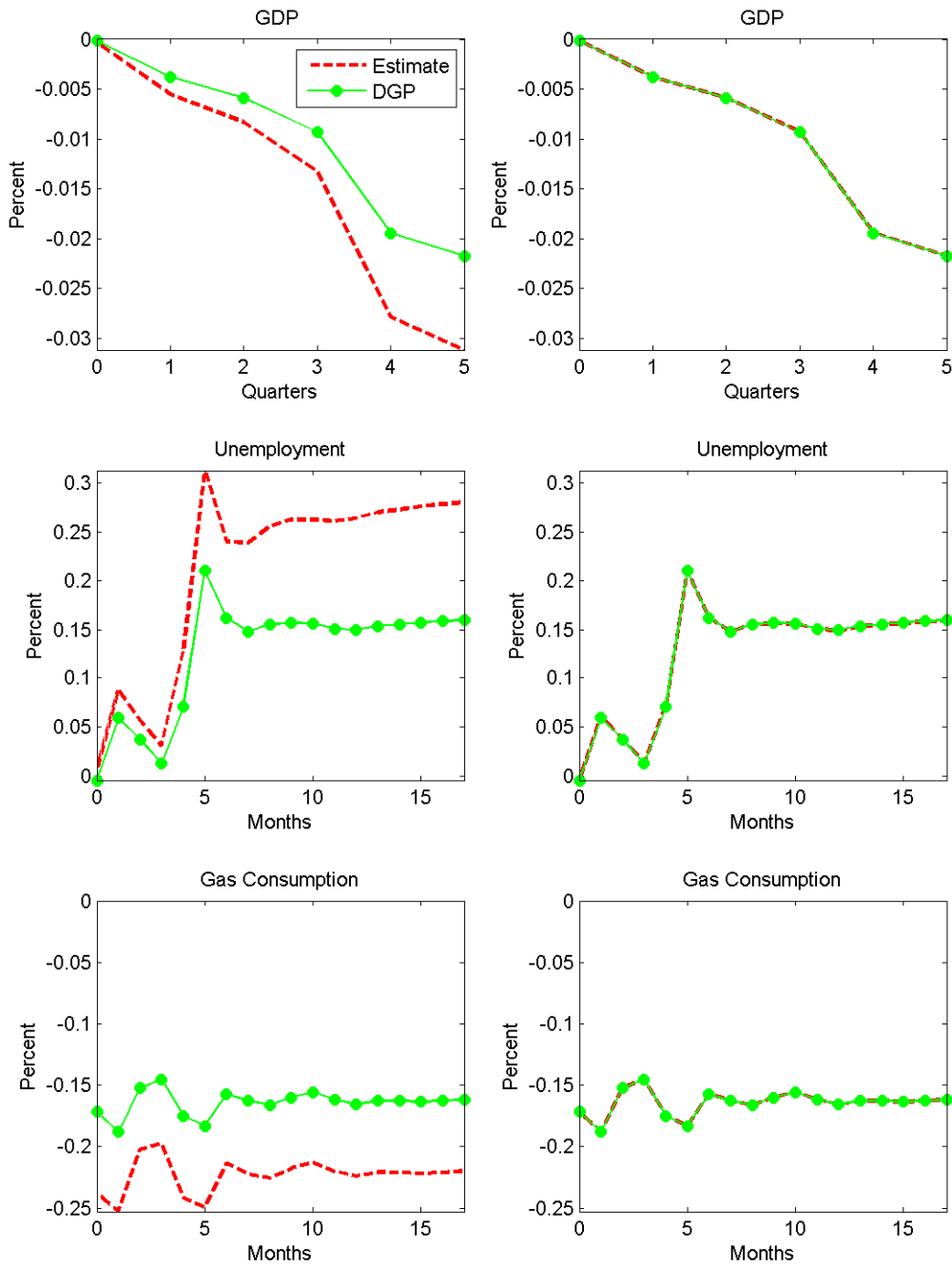
The left column of Figure 2 quantifies the asymptotic bias induced by censoring energy price decreases. The impulse response implied by the data generating process is shown as the solid line. The estimated impulse response from the censored VAR model is shown as the dashed line. As shown in section 2, the censored VAR response tends to overestimate the true response in each case. For example, the response of real GDP after 6 quarters is overestimated by about one third. The right column of Figure 2 illustrates the point that our results are not driven by sampling uncertainty. It shows that the true and the estimated response lie exactly on top of one another if we fit a linear symmetric VAR model to the same data. The results in Figure 2 suggest strong caution in interpreting the results of censored VAR models.

3.2. Asymmetric data generating processes

Figure 2 showed that censored VAR models will be misleading when the data generating process is linear and symmetric. It is equally interesting to investigate the consistency of the censored VAR model when the data generating process involves asymmetric effects of energy price increases on macroeconomic aggregates. Here we follow the bulk of the empirical literature on energy price shocks and focus on the leading example of models in which only energy price increases matter for macroeconomic aggregates

For expository purposes, first consider the simplest possible dynamic model, in which energy price decreases have no effect:

Figure 2: Inconsistency of the Estimated Effect of Energy Price Increases
Symmetric VAR DGP



Notes: Simulations from symmetric VAR DGP based on U.S. data. T=1,000,000.

$$\begin{aligned}
x_t &= \alpha_1 + \rho x_{t-1} + \varepsilon_{1,t} \\
y_t &= \alpha_2 + x_t^+ \gamma_2 + \varepsilon_{2,t}
\end{aligned}
\tag{10}$$

where x_t^+ is defined as above. Setting the initial conditions to zero, in this system the impact response of y_t to a positive shock to x_t would be γ . In the next period, the response would be $\rho\gamma$, provided that ρ were positive.⁷ If this system is estimated without censoring, then, as expected, estimates of γ and ρ will be unbiased. If instead a researcher estimated the censored system:

$$\begin{aligned}
x_t^+ &= \alpha_1 + \rho x_{t-1}^+ + \varepsilon_{1,t} \\
y_t &= \alpha_2 + x_{t-1}^+ \gamma + \varepsilon_{2,t}
\end{aligned}
\tag{11}$$

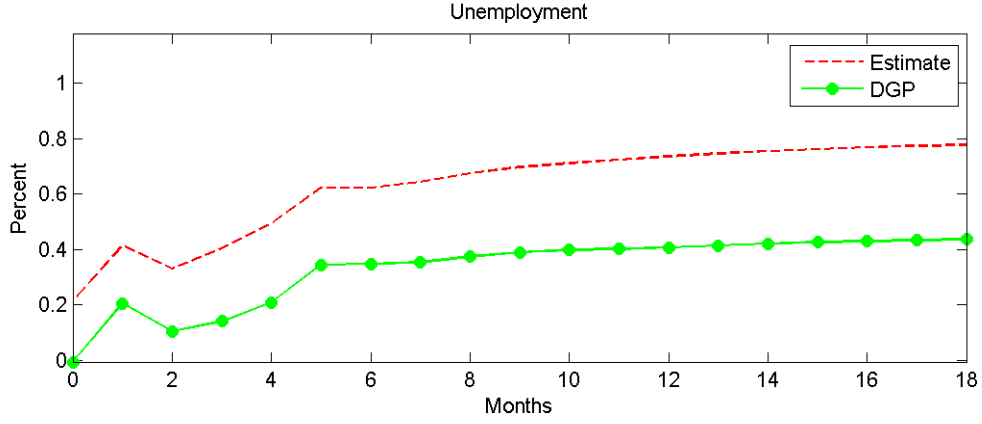
however, although the estimate of γ would be unbiased, as discussed in the context of the static model, the estimate of ρ would be asymptotically biased and so would be the impulse response estimate.

This example illustrates that the data generating process cannot be represented as a bivariate VAR for $(x_t^+ \ y_t)'$. In fact, a censored VAR data generating process with positive probability would generate realizations for x_t^+ that may be negative. It may seem that this contradiction could be avoided by censoring the realizations much like researchers have censored percent changes in actual energy prices, but in that case the same asymptotic biases would arise that we already documented for the linear symmetric model. This point is illustrated in Figure 3. Based on a censored VAR data generating process, the censored VAR run on censored realizations of x_t^+ generates responses to energy price increases that are systematically higher than the pseudo-true response even in the limit.

The source of the problem in Figure 3 is that the censored VAR regression model is an incomplete description of the data generating process. This problem can be avoided only by fully specifying the underlying structural model:

⁷ If ρ were sufficiently negative that x_t being positive implies that x_{t+1} would be negative, then the response in the second period would be zero. This result is discussed more in section 5.

Figure 3: Inconsistency of the Estimated Effect of Energy Price Increases
Censored VAR DGP with Censored VAR Variable



Notes: Simulations from censored VAR DGP based on U.S. data. T=1,000,000.

$$\begin{aligned}
 x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \dots + \varepsilon_{1t} \\
 y_t &= \beta_1 x_t^+ + \beta_2 x_{t-1}^+ + \beta_3 y_{t-1} + \dots + \varepsilon_{2t}
 \end{aligned}
 \tag{12}$$

where the structural shocks ε_{1t} and ε_{2t} are uncorrelated and where for expository purposes we have omitted the definition of x_t^+ as a function of x_t . Although the slope parameters of Model (12) can be estimated consistently by OLS, the resulting residuals will not be uncorrelated. To impose the latter restriction requires the use of a restricted maximum likelihood estimator.

The data generating process in Model (12) postulates that percent changes in energy prices evolve in an unconstrained fashion; only the feedback from energy prices to the macroeconomic aggregates is constrained. This model is easily recognizable as a generalization of Model (7) with $\beta = 0$ to the VAR context. Note that in this model a negative shock to x_t may have a non-zero effect on y_{t+h} if the negative shock over time results in positive values of x_{t+h} . Also note that Model (12) is not equivalent to the following model:

$$\begin{aligned}
 x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \dots + \varepsilon_{1t} \\
 y_t &= \beta_2 x_{t-1}^+ + \beta_3 y_{t-1} + \dots + \beta_1 \varepsilon_{1t} + \varepsilon_{2t}
 \end{aligned}
 \tag{13}$$

The key difference between Models (12) and (13) is that the impact effect of a negative value of ε_{1t} is zero in Model (12) and is β_1 in Model 13. Furthermore, Model (12) is not equivalent to estimating the model:

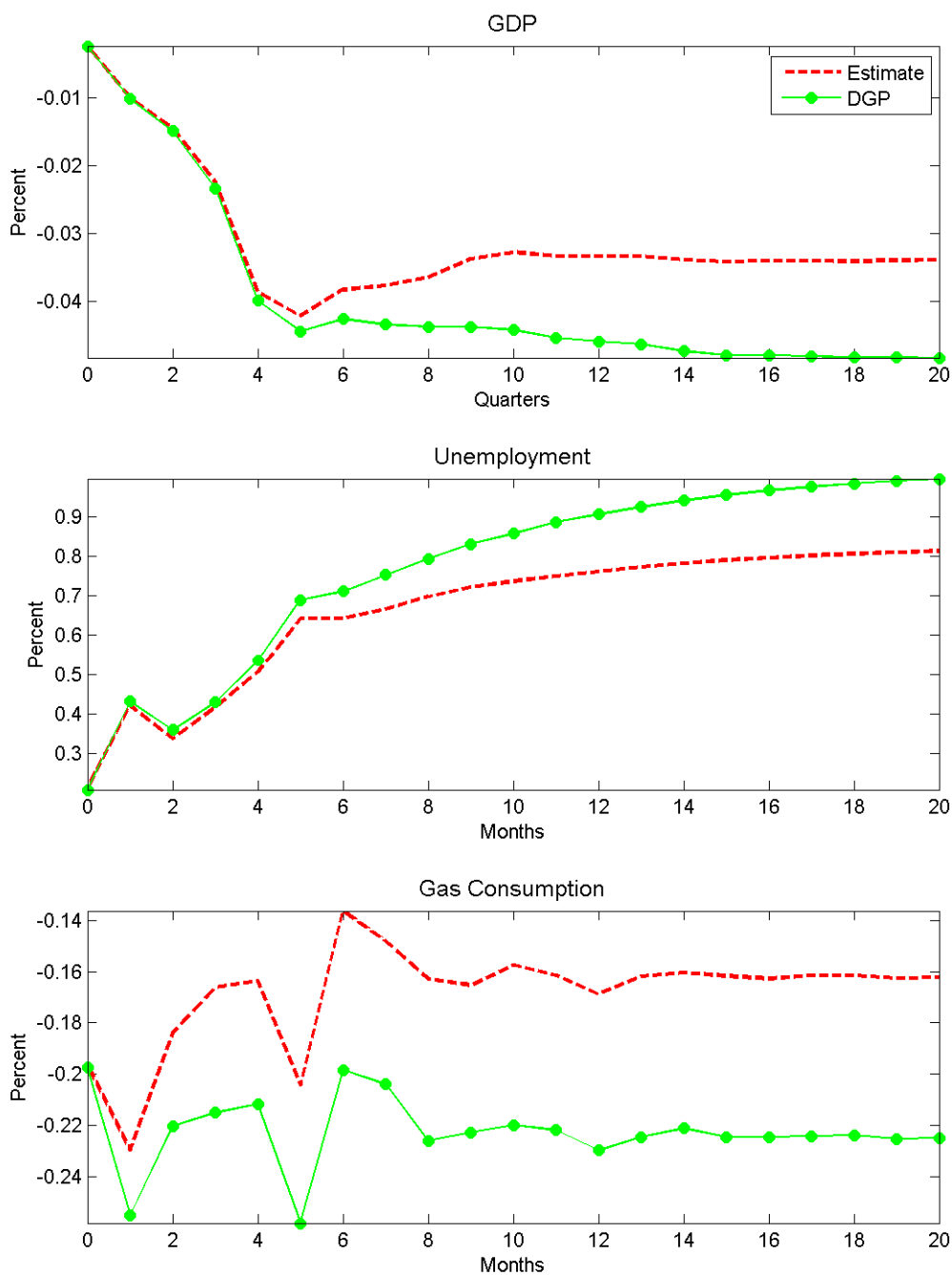
$$\begin{aligned}
x_t &= \alpha_1 x_{t-1} + \alpha_2 y_{t-1} + \dots + \varepsilon_{1t} \\
y_t &= \beta_2 x_{t-1} + \beta_3 y_{t-1} + \dots + u_{2t}
\end{aligned}
\tag{14}$$

where $u_{2t} = \beta_1 \varepsilon_{1t} + \varepsilon_{2t}$, and applying a Cholesky decomposition to the variance covariance matrix of the two error terms ε_{1t} and u_{2t} . The key difference is that the Cholesky decomposition does not discriminate between positive and negative shocks.

Below we confirm that even when the data are generated from Model (12), asymptotic biases may arise when estimating the response to energy price increases from a censored VAR model. We focus on the same illustrative examples as in Figure 2, except that we now construct the data generating processes under the working hypothesis that the data are generated by the asymmetric Model (12) in which there is no effect from current or lagged x_t on y_t . For each DGP example, we treat the least-squares estimates of the slope parameters and innovation variances obtained on the actual data as the population parameters in the simulation and impose the zero correlation of the innovation variances. All results are based on $T = 1,000,000$. Figure 4 shows that even in this case the responses implied by the censored VAR model will be asymptotically biased. The direction of the bias is ambiguous. Especially for the unemployment rate the bias is substantial. After 18 months, the estimated response to an energy price increase is only about 80 percent of the true response. By the same reasoning as in Section 2.3, this result is a consequence of inconsistently estimating the response of the price of energy to the energy price shock. Figure 5 illustrates this problem by plotting the corresponding responses of the price of energy. Although there is no problem in consistently estimating the second equation of the system that includes the censored regressors (and indeed the impact response of y_t is correctly estimated in Figure 4), the fact that the censored VAR model misspecifies the first equation of the system causes both response estimates to be inconsistent as the energy price shock is propagated over time.

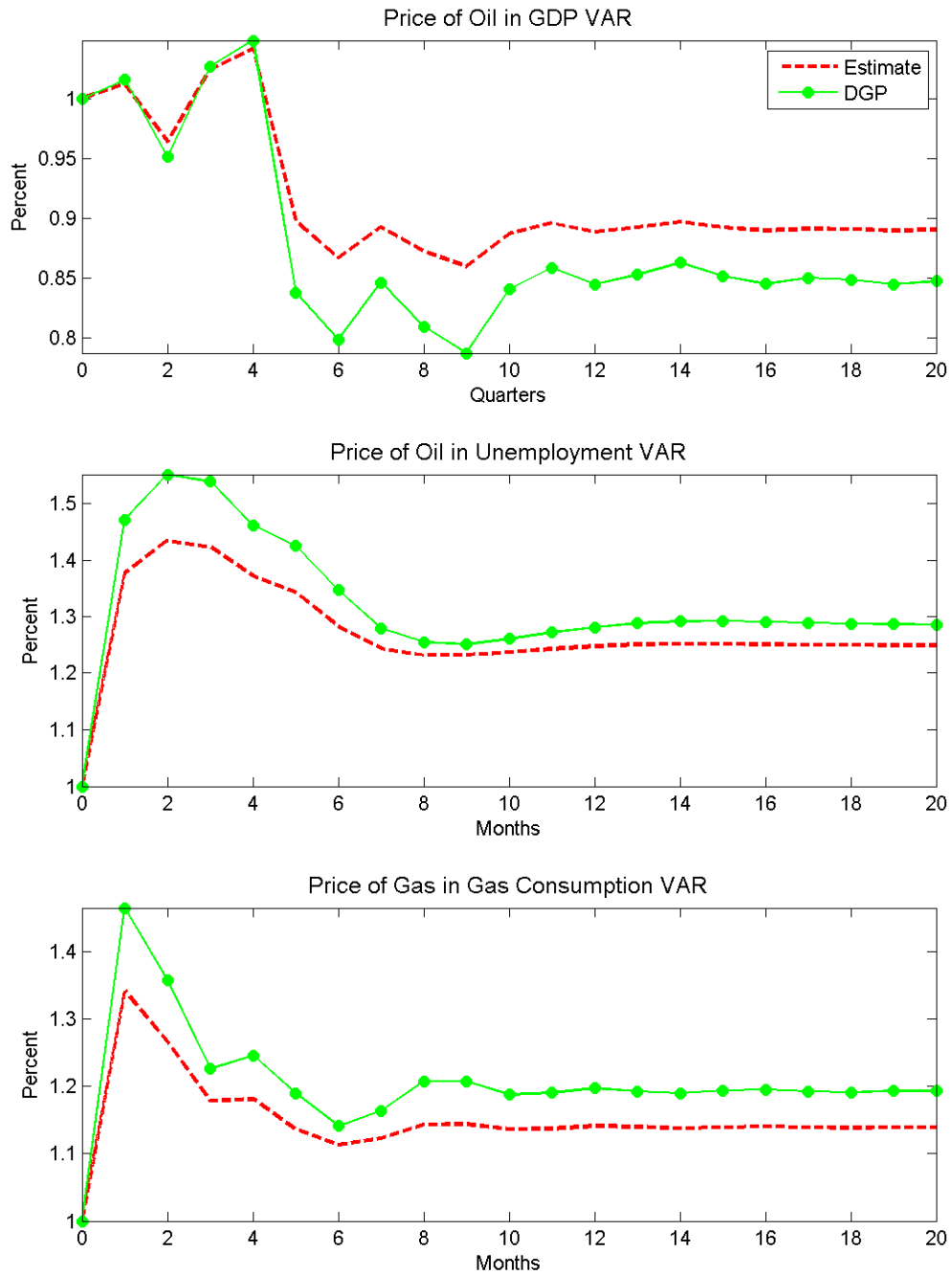
The results in Figures 4 and 5 represent the best possible scenario in that we postulated that only energy price increases matter in the DGP. Additional asymptotic biases would arise if the asymmetric DGP allowed for nonzero effects from energy price decreases, and those biases would affect even the impact responses.

Figure 4: Inconsistency of the Estimated Effect of Energy Price Increases on Outcome Variable
 Asymmetric Structural Model DGP
 Fitting Censored VAR Model



Notes: Simulations based on Model (12) estimated on U.S. data. T=1,000,000.

Figure 5: Inconsistency of the Estimate of Energy Price Increases on Energy Price
 Asymmetric Structural Model DGP
 Fitting Censored VAR Model



Notes: The responses are for the real price of oil and the real price of gasoline. Simulations based on Model (12) estimated on U.S. data. T=1,000,000.

4. Eliminating the Asymptotic Bias: A General Model of the Oil Price-Economy Link

Up until now, we have imposed a strong form of asymmetry in which energy price declines have no effect on the macro aggregate of interest. In the interest of full generality, we relax this assumption by allowing for both energy price increases and decreases to have an effect but to a different extent.⁸ The first equation of the resulting model is identical to the first equation of a standard linear VAR in x_t and y_t , but the second equation now includes both x_t and x_t^+ and, as such, both energy price increases and decreases affect y_t .

$$\begin{aligned}
 x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1t} \\
 y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^+ + \varepsilon_{2,t}
 \end{aligned}
 \tag{15}$$

Given estimates of these coefficients, one can calculate the dynamic responses to unanticipated positive and negative energy price changes.⁹ Note that the OLS residuals of Model (15) are uncorrelated, whereas the OLS residuals of Model (12) may be correlated. This means that model [15] may be estimated by standard regression methods.

As illustrated in Figure 6, the key advantage of Model (15) is that the dynamic responses are consistently estimated regardless of whether the true DGP is symmetric or asymmetric. We focus on the real GDP data to conserve space. Similar results hold for the other data sets. The response estimates of Model (15) are consistent when the DGP is the standard linear VAR (see Figure 6a). In contrast, as was shown earlier, the response estimates of both Model (12) and the censored VAR model do not converge to the population response in that case. Likewise, when the DGP is the asymmetric Model (12), although response estimates based on the standard VAR model are asymptotically biased, the response estimates based on Model (15) are consistent (see Figure 6b). Finally, Figure 6c illustrates that Model (15) allows consistent estimation of the dynamic response to a negative shock in Model (12). In short, the advantage

⁸ Theoretical models of asymmetry do not imply the strong form of asymmetry but allow for nontrivial effects of both energy price increases and decreases (see Edelstein and Kilian 2007a,b; Kilian 2008b).

⁹ If energy prices never declined, this model would suffer from collinearity. But in the data, we observe both energy price increases and declines.

Figure 6a: Estimating The Effect of an Energy Price Increase Using the General Model
Symmetric VAR DGP

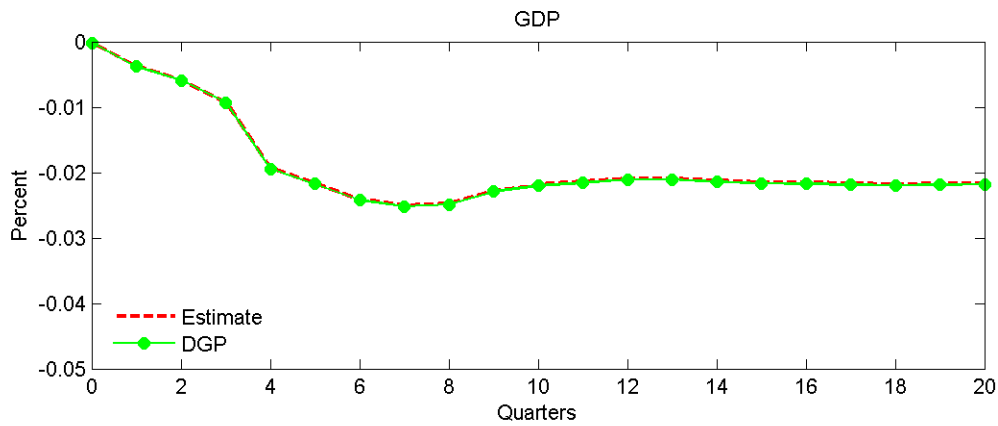


Figure 6b: Estimating The Effect of an Energy Price Increase Using the General Model
Asymmetric Structural Model DGP

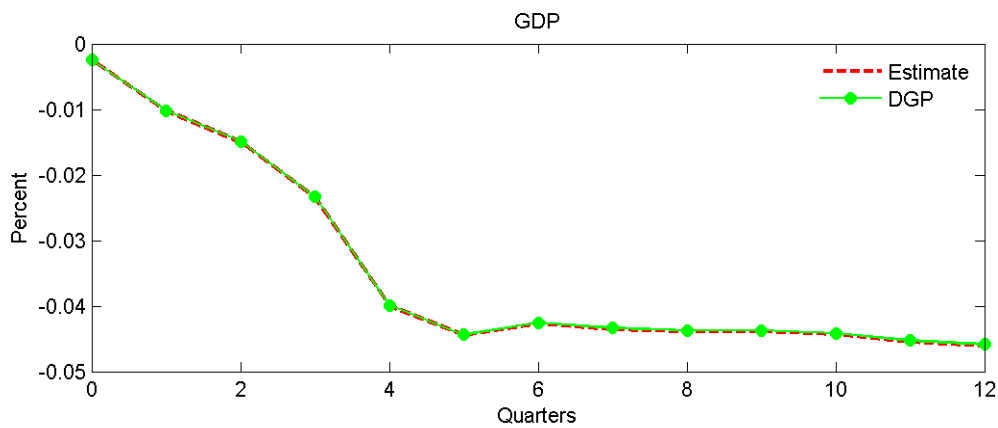
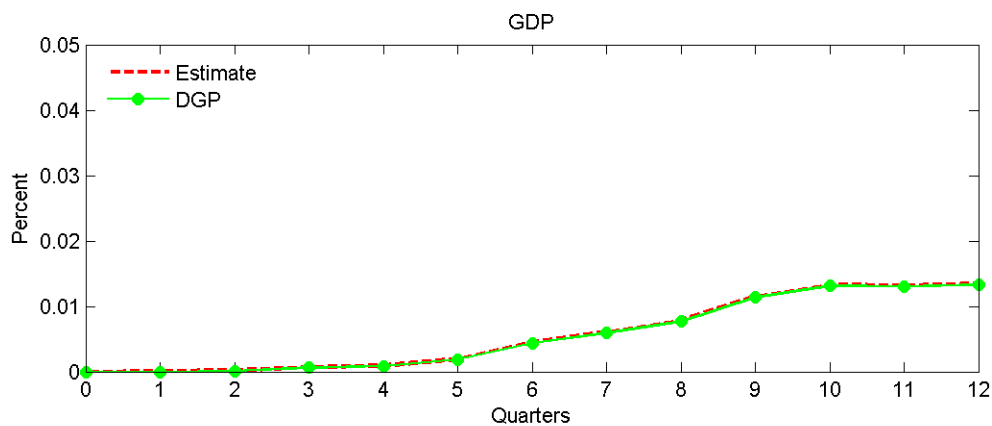


Figure 6c: Estimating The Effect of an Energy Price Decline Using the General Model
Asymmetric Structural Model DGP



Notes: Simulations based on models estimated on U.S. data. T=1,000,000.

of Model (15) is that it may be used without knowing the nature of the data generating process. Its only limitation is that the response estimates may be slightly biased in finite samples due to the nonlinearity of the response function and that they are not efficient asymptotically.

5. Computing Responses to Energy Price Shocks in Nonlinear Models

So far we have followed the convention in the empirical literature on energy price shocks of computing impulse responses as one would for linear and symmetric VAR models. While this approach simplifies the computation of the responses from asymmetric models, it can be misleading in that the effect of a given shock in asymmetric models depends on the recent history of the series in question and on the magnitude of the shock. For example, the net effect of a negative innovation to energy prices on macroeconomic aggregates will depend on the extent to which the effect of this shock on energy prices is dampened or amplified by the cumulative effect of previous shocks. Thus, nonlinear impulse responses must be computed for a given shock as the average of impulse response draws obtained using alternative initial conditions. This point is well known (see, e.g., Gallant et al. 1993; Koop et al. 1996), but has been routinely ignored in the literature on estimating the effects of energy price increases.

In this section we propose an adaption of methods for computing impulse responses from nonlinear models specifically designed for model (15).¹⁰ Having estimated model (15), we proceed as follows:

Step 1 Take a block of p consecutive values of x_t and y_t . This defines a history Ω^i . Note that the choice of history does not affect the coefficients of the model. For all histories, the model coefficients are fixed at their estimated values.

Step 2 Given Ω^i , simulate two time paths for x_{t+i} and y_{t+i} for $i = 0, 1, \dots, H$. In generating the first time path, the value of ε_{1t} is equal to a pre-specified value δ . In generating the other time path, the value of ε_{1t} is drawn from the marginal empirical distribution of ε_{1t} . The value of ε_{2t} and the values of all subsequent shocks ε_{1t+i} and ε_{2t+i} , $i = 1, \dots, H$, are drawn from their respective marginal distributions. Since the structural errors ε_{1t} and ε_{2t} are by definition uncorrelated, we treat the draws as independent in practice.

¹⁰ The same algorithm could be applied to model (12) as well, if that model were considered appropriate for applied work.

Step 3. Calculate the difference between the time paths for y_{t+i} $i = 1, \dots, H$.

Step 4. Average this difference across $m=10,000$ repetitions of Steps 2 and 3.

This average is the response of y_{t+i} at horizon $i = 0, \dots, H$, to a shock of size δ conditional on Ω^i :

$$I_y(h, \delta, \Omega^i). \quad [16]$$

The unconditional response $I_y(h, \delta)$ is defined as the value of $I_y(h, \delta, \Omega^i)$ averaged across all histories:

$$I_y(h, \delta) = \int I_y(h, \delta, \Omega^i) d\Omega^i \quad [17]$$

Depending on the application, both the conditional response $I_y(h, \delta, \Omega^i)$ and the unconditional response could be of potential interest. To determine the general importance of oil shocks as a source of fluctuations, the unconditional response is of most interest. Likewise, for the evaluation of DSGE models the unconditional response is the most appropriate object to match. In contrast, for forecasting and policy purposes, the response conditional on current history is the more relevant statistic. In the remainder of the paper, we will focus on the unconditional response (17).

It is important to highlight the difference between the response in (17) and the impulse response functions that have been discussed in the literature on the transmission of oil price shocks. For this purpose, it is useful to introduce several alternative definitions of impulse responses to energy prices. For example,

$$I_y(h, \delta, \underline{0}) \quad [18]$$

refers to the response obtained conditioning on a hypothetical historical path involving $x_{t-i} = y_{t-i} = 0$ for all i . Another possible definition would be the response

$$\begin{aligned} & I_y^*(h, \delta, \Omega^i) \\ &= E \left(y_{t+h} \mid \varepsilon_{1,t} = \delta, \Omega^i, \{\varepsilon_{1t+j} = 0\}_{j=1}^h, \{\varepsilon_{2t+j} = 0\}_{j=0}^h \right) \\ &- E \left(y_{t+h} \mid \Omega^i, \{\varepsilon_{1t+j} = 0\}_{j=0}^h, \{\varepsilon_{2t+j} = 0\}_{j=0}^h \right) \end{aligned} \quad [19]$$

Upon integrating out across alternative histories, this expression simplifies to:

$$I_y^*(h, \delta) = \int I_y^*(h, \delta, \Omega^i) d\Omega^i \quad [20]$$

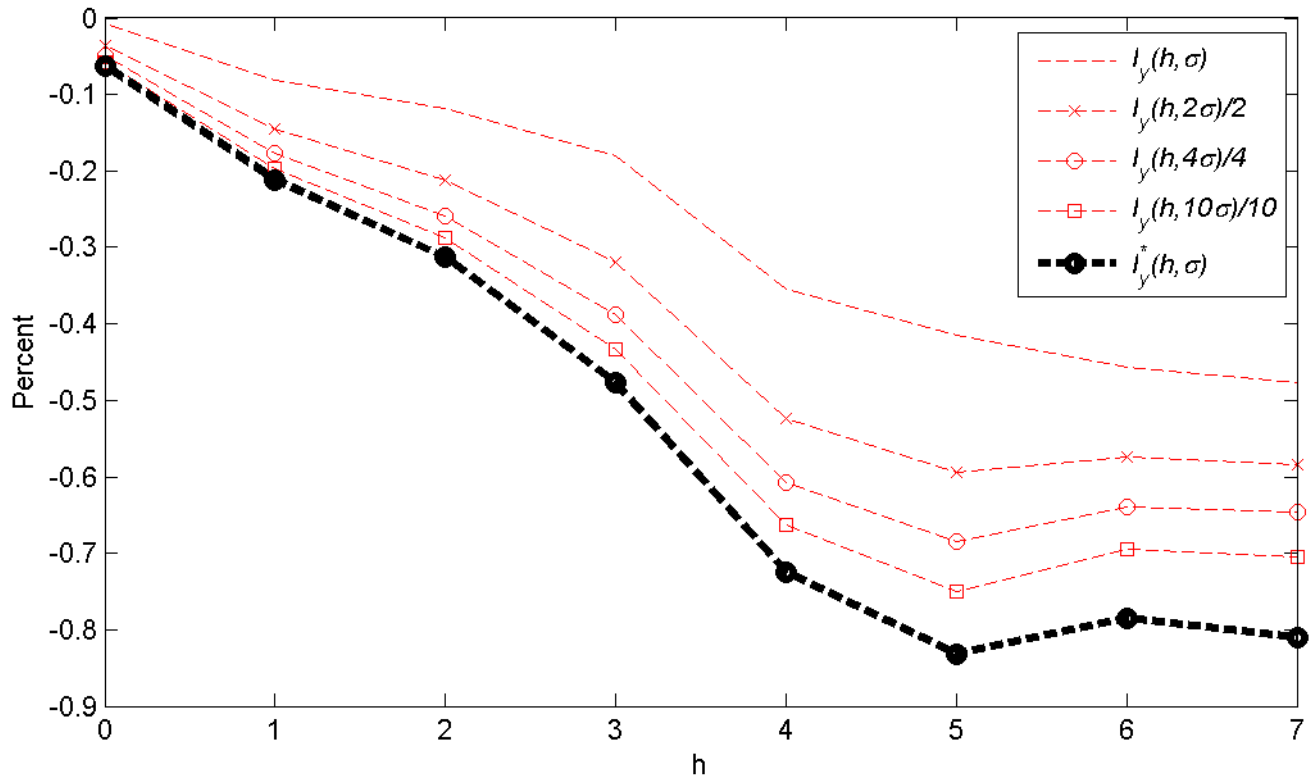
The response most often reported in the empirical literature on energy price shocks is $I_y^*(h, \delta, \underline{0})$ and may be viewed as a combination of (18) and (20). That response conditions neither on history nor does it allow for non-zero future shocks.

The time series literature on non-linear impulse responses has favored $I_y(h, \delta, \Omega^i)$ over $I_y^*(h, \delta, \underline{0})$ for at least three reasons. First, the process of $I_y^*(h, \delta, \underline{0})$ may not accurately characterize the true dynamics of a non-linear system. For example, Koop, Pesaran, and Potter (1996, p. 125) observe that $I_y^*(h, \delta, \underline{0})$ may not converge to its mean, even when the nonlinear stochastic data generating process is stationary. Second, Potter (2000) observes that the theoretical treatment of nonlinear impulse responses is facilitated by treating future shocks as random variables rather than fixed values. Third, there is no intrinsic economic interest in any impulse response that conditions on $\underline{0}$.

One of the chief drawbacks of nonlinear impulse response analysis of the type considered by Koop et al. (1996) is that shocks cannot be defined unambiguously in the context of reduced-form models. The reason is that reduced-form regression errors are inevitably mutually correlated. In practice, researchers have resorted to presenting responses for representative draws from the joint distribution of the reduced form errors. Such responses may be useful in characterizing the persistence of the data, but they are devoid of any economic interpretation. An attractive feature of the nonlinear impulse responses (16) and (17) that we defined above is that in the context of the structural models (12) and (15) an energy price shock is orthogonal to other shocks and uniquely defined (up to scale). This fact addresses the chief limitation of nonlinear impulse responses as discussed in Koop et al.

How the impulse response is computed directly affects the magnitude of the estimated response. As shown in Figure 7, in the context of our example, $I_y(h, \delta)$ is much smaller in absolute value than $I_y^*(h, \delta, \underline{0})$. The value of $I_y^*(h, \delta, \underline{0})$ is more comparable to the response to a very large shock in excess of ten standard deviations. In other words, even if the regression model were correctly specified and hence the parameter estimates consistent, the use of

Figure 7: The Response of GDP to a Positive Energy Price Shock by Shock Size



Notes: $I_y(h, \sigma)$ is calculated by Monte Carlo integration over 300 histories with 10,000 paths each. All impulse responses have been scaled (as denoted in the legend) to ensure compatibility. h denotes the horizon.

traditional impulse response functions would greatly exaggerate the effect of a positive oil price shock. This finding reinforces our earlier concern with the methods underlying the existing literature. To help us understand these results, below we provide a more in-depth analysis of the impact responses.

Impact Response of x_t

Because of the linear nature of the first equation in Model (15), the impact effect on x_t of a shock of size δ will be a constant:

$$I_x(0, \delta, \Omega^i) = \delta. \quad [21]$$

We minimize the simulation error involved in computing

$$I_x(\widehat{0}, \delta, \Omega^i) = \delta - \frac{1}{m} \sum_{i=1}^m \varepsilon_{1t,i} \quad [22]$$

by setting m to a sufficiently large number. If $\delta^2 = E\varepsilon_{1,t}^2$ then $\text{var}(I_x(0, \delta, \Omega^i)) = \frac{\delta^2}{m}$. In practice, we use $m = 10,000$ to ensure reasonably stable results.

Impact Response of y_t

The impact effect of ε_{1t} on y_t is

$$I_y(0, \delta, \Omega^i) = b_{21,0} \delta + g_{21,0} (E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)). \quad [23]$$

The term $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$ plays a central role in the construction of nonlinear impulse responses for y_{t+h} $h = 1, \dots, H$. Absent uncertainty about the value of $\varepsilon_{1,t}$, the value of $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$ would be easy to calculate. In particular, consider the value of $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i)$ where, for ease of notation, we define

$$\tilde{x}_t \equiv E(x_t | \varepsilon_{1,t} = 0, \Omega^i) \quad [24]$$

For $\delta > 0$, then

$$\begin{aligned} E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i) &= \delta && \text{if } \tilde{x} > 0 \\ E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i) &= \tilde{x} + \delta && \text{if } -\delta < \tilde{x} < 0 \\ E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i) &= 0 && \text{if } \tilde{x} < -\delta \end{aligned} \quad [25]$$

Computing the value of $E(x_t^+ | \delta, \Omega^i) - E(x_t^+ | \Omega^i)$ is more of a challenge because we need to account for uncertainty about $\varepsilon_{1,t}$. With uncertainty, we have that

$$\begin{aligned} E(x_t^+ | \delta, \Omega^i) &= E(\max(\tilde{x}_t + \delta, 0) | \delta, \Omega^i) = \max(\tilde{x}_t + \delta, 0) \\ E(x_t^+ | \Omega^i) &= E(\max(\tilde{x}_t + \varepsilon_{1t}, 0) | \Omega^i) \end{aligned} \quad [26]$$

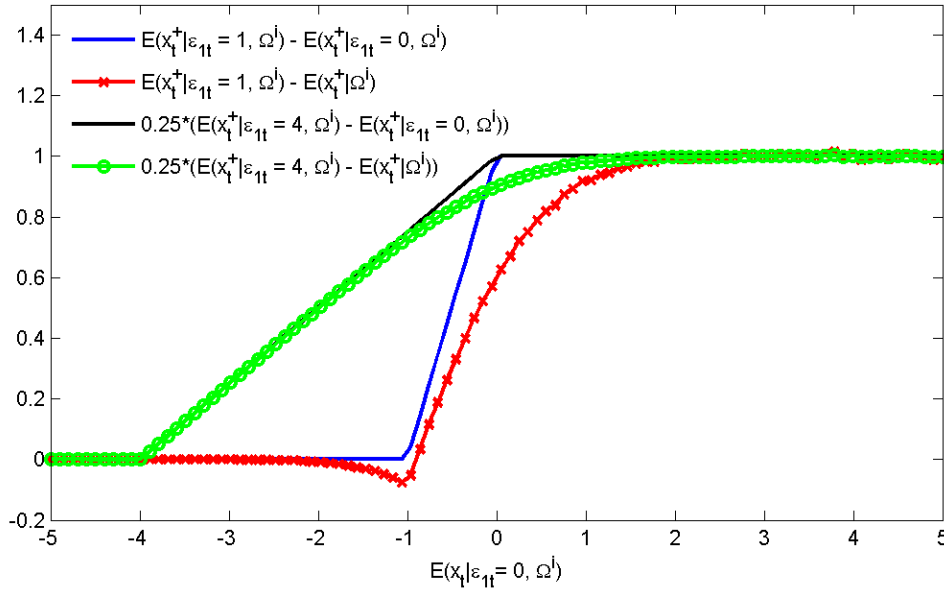
The value of $E(x_t^+ | \Omega^i)$ depends on the variance of the shocks. Note that $E(x_t^+ | \Omega^i)$ can be positive even if \tilde{x}_t is negative. In fact, by Jensen's inequality, $E(x_t^+ | \Omega^i) \geq \tilde{x}_t$ for all values of \tilde{x}_t .

In particular, if \tilde{x}_t equals zero and ε_{1t} has a standard normal distribution then $E(x_t^+|\Omega^i)$ has a value of 0.4. Hence, when $\tilde{x}_t = 0$ and ε_{1t} has a standard normal distribution, we have that

$$\begin{aligned} E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i) &= E(x_t^+|\delta, \tilde{x}_t = 0) - E(x_t^+|\tilde{x}_t = 0) \\ &= \delta - 0.4. \end{aligned} \quad [27]$$

This result implies that the larger δ , the smaller the effect of incorrectly treating ε_{1t} as equal to zero under the counterfactual path, relative to magnitude of the impulse response (see Figure 7). In other words, all else equal, the larger δ , the more similar the traditional incorrectly computed impulse response and the correctly computed unconditional response will be. This point is important because most energy price shocks measured at the monthly or quarterly frequency tend to be quite small (e.g. Edelstein and Kilian 2007a). We conclude that traditional, incorrectly computed impulse responses will tend exaggerate the effect of an unanticipated energy price increase.

Figure 8: $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)$



More generally, $E(x_t^+|\delta, \Omega^i) - E(x_t^+|\Omega^i)$ is a function of \tilde{x}_t . Figure 8 illustrates this relationship under the assumption that

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} = \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \right) \quad [28]$$

As δ increases, the importance of both histories \tilde{x}_t and the Jensen's Inequality term induced by

σ_1^2 declines. The reduced importance of the histories is illustrated by the following limit argument:

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\int E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \Omega^i) d\Omega^i \right) = \delta$$

As a first step, towards that proof, consider the simpler case of $\tilde{x}_t \equiv E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i)$. In that case, it can be shown that

$$\frac{1}{n} \left(\int E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \varepsilon_{1,t} = 0, \Omega^i) d\Omega^i \right) = \delta \left(P(\tilde{x}_t > -n\delta | \Omega^i) + \frac{1}{n\delta} \int_{-n\delta}^0 \tilde{x}_t d\Omega^i \right).$$

As long as \tilde{x}_t does not have too much mass in the left tail, it follows that

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \left(\int E(x_t^+ | n\delta, \Omega^i) - E(x_t^+ | \Omega^i) d\Omega^i \right) &= \delta \left(\lim_{n \rightarrow \infty} P(\tilde{x}_t > -n\delta | \Omega^i) + \lim_{n \rightarrow \infty} \frac{1}{n\delta} \int_{-n\delta}^0 \tilde{x}_t d\Omega^i \right) \\ &= \delta(1 + 0), \end{aligned}$$

where the value of $P(\tilde{x}_t > -n\delta | \Omega^i)$ converges toward one, as n increases, and $E(\tilde{x}_t | \tilde{x}_t < 0)$ is finite, and we exploit the fact that

$$\frac{1}{n\delta} E(\tilde{x}_t | \tilde{x}_t < 0) < \frac{1}{n\delta} \int_{-n\delta}^0 \tilde{x}_t d\Omega^i < 0.$$

As illustrated in Figure 7, the impulse response $I_y^*(h, \delta, \underline{0})$ can be thought of as the scaled version of the response to a very large shock of the form $I_y(h, n\delta, \Omega^i)/n$. Hence, for sufficiently large price energy shocks, one would expect that the importance of Ω^i diminishes such that the impulse response used in the existing VAR literature becomes a good approximation to the correctly constructed estimate. For smaller (and more typical) energy price shocks, however, such as a one-standard deviation increase in $\varepsilon_{1,t}$, the interaction of the innovation in the price of energy with the history Ω^i will be quantitatively important. That is why traditional VAR impulse responses may be quite different from the correctly computed nonlinear responses in practice.

6. Testing Symmetry in Energy Price Increases and Decreases

The preceding sections have shown that the presence of asymmetries considerably complicates the econometric analysis of the transmission of positive energy price shocks. While model (15)

may be used to sidestep the question of whether the model in question is asymmetric or not, this involves a cost in terms of asymptotic efficiency. Clearly, a linear symmetric VAR model would be preferable if we could convince ourselves that the data generating process is symmetric. This raises the question of how to test the symmetry null hypothesis. We first discuss the case of testing for symmetry between energy price increases and decreases, before adapting these tests to the problem of testing models of net energy price increases.

6.1. Slope-Based Tests

If energy price increases and decreases received exactly the same weight in regressions of y_t on lagged y_t and current and lagged x_t^- and x_t^+ , it would follow immediately that the dynamic responses to energy price shocks must be symmetric in positive and negative shocks. This line of reasoning has motivated the development of slope-based tests of symmetry. Such tests are attractive in that they not require the complete specification of the system to be estimated nor do they require the computation of impulse responses.

The traditional approach to testing for symmetry in the transmission of energy price shocks involves tests on the symmetry of the slope coefficients in regressions of y_t on lagged x_t^- and x_t^+ (see, e.g. Mork 1989). This is equivalent to testing

$$H_0: g_{21,1} = \dots = g_{21,p} = 0$$

in model (15). Closer inspection of model (15) reveals that this test fails to impose all restrictions implied by the null hypothesis of symmetry. Mork's test omits the contemporaneous regressors because he works with the reduced form. A complete test of all symmetry restrictions on the slopes would involve the null hypothesis:

$$H_0: g_{21,0} = \dots = g_{21,p} = 0$$

In other words, the proposal is that we estimate Model (15) and use a Wald Test to determine whether including $\{x_{t-i}^+\}_{i=0}^p$ improves the fit of the model. This test has an asymptotic χ_{p+1}^2 -distribution.

It is useful to compare the size and power properties of Mork's (1989) test with those of the slope-based test implied by the structural model (15). Table 5 summarizes the results from some Monte Carlo experiments. The size results are based on a DGP obtained by fitting the three empirical models under the assumption of symmetric responses. We simulate data under

the null assumption of symmetry and report the relative frequency at which the Wald Test rejects the null hypothesis of symmetry. In assessing the power, we follow a similar strategy. We first estimate the equations

$$\begin{aligned}
 x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{x,t} \\
 y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^+ + \varepsilon_{y,t}
 \end{aligned} \tag{29}$$

by OLS to obtain the parameter values for the DGP. The artificial data are then generated from

$$\begin{aligned}
 x_t &= b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{x,t} \\
 y_t &= b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \kappa \sum_{i=0}^p g_{21,i} x_{t-i}^+ + \varepsilon_{y,t}
 \end{aligned} \tag{30}$$

where $\kappa \in \{0.25, 0.5, 1, 2, 4\}$ controls the degree of asymmetry in the population response. The larger is κ , the more asymmetric is the DGP response. All simulations are based on the assumption of Gaussian innovations.

Table 5 shows that, for the monthly applications, the Wald tests are quite accurate under the null. For the quarterly application, there is a modest size distortion, reflecting the smaller quarterly sample size. It is likely that one could mitigate these size distortions by using bootstrap methods.

Table 6 shows the corresponding results for Mork's slope-based test. A comparison of Tables 5 and 6 shows that the correctly specified symmetry test proposed above has more accurate size and higher power than Mork's test. Under the null, Mork's test has a tendency to reject symmetry too often. The power gains of the correctly specified test may be substantial. In one case the power more than doubles.

There are two possible outcomes when conducting slope-based tests. If the test rejects symmetry, that is sufficient for concluding that the impulse responses are asymmetric, but it

Table 5: Proposed Test: Rejection Rates of the 5 Percent Test for Asymmetric Coefficients

	Size	Power				
κ		0.25	0.5	1	2	4
Unemployment	0.07	0.08	0.14	0.43	0.97	1.00
Gas Consumption	0.07	0.09	0.18	0.62	1.00	1.00
Real GDP	0.11	0.14	0.19	0.45	0.96	1.00

Notes: In all cases, the data sets are generated to have the same number of observations as the original data. Critical Values are from the χ_7^2 -distribution. The value of κ scales $g_{21,i}, i = 0, \dots, p$, by a fixed factor and determines the degree of asymmetry of the population response.

Table 6: Mork's (1989) Test: Rejection Rates of the 5 Percent Test for Asymmetric Coefficients

	Size	Power				
κ		0.25	0.5	1	2	4
Unemployment	0.10	0.07	0.09	0.21	0.66	0.99
Gas Consumption	0.11	0.09	0.17	0.57	1.00	1.00
Real GDP	0.16	0.12	0.18	0.42	0.94	1.00

Notes: In all cases, the data sets are generated to have the same number of observations as the original data. Critical Values are from the χ_6^2 -distribution. The value of κ scales $g_{21,i}, i = 0, \dots, p$, by a fixed factor and determines the degree of asymmetry of the population response.

does not tell us whether the departures from symmetry are economically or statistically significant. Given that impulse response functions are highly nonlinear functions of the slope parameters and innovation variances it is quite conceivable that the degree of asymmetry in the impulse responses to positive and negative energy price shocks could be quite small, making the linear model a good approximation, despite the statistical rejection. Moreover, the quality of the linear approximation will differ depending on the magnitude of the shock. For that reason, the applied user will want to plot the point estimates of the impulse response functions and inspect them.

If the test fails to reject symmetry, on the other hand, we again learn little because

statistically insignificant departures from symmetry in the slopes may cause statistically significant asymmetries in the implied impulse response functions, given the nonlinearity of these functions. This observation suggests that a more useful approach would be to test directly the symmetry of the economy's dynamic responses to unanticipated energy price increases and decreases directly based on the impulse response functions. The latter approach is in the spirit of recent work by Edelstein and Kilian (2007a,b). This alternative approach to testing symmetry is discussed next.

6.2. Impulse-Response Based Tests

The proposal is that we estimate the unrestricted model (15) and calculate the unconditional impulse responses to both positive and negative energy price shocks. Then we construct a Wald test of the joint null hypothesis of symmetric responses to positive and negative energy price shocks up to a pre-specified horizon H . Symmetry means that

$$I_y(h, \delta) = -I_y(h, -\delta) \text{ for } h = 0, 1, 2, \dots, H$$

or equivalently

$$I_y(h, \delta) + I_y(h, -\delta) = 0 \text{ for } h = 0, 1, 2, \dots, H$$

The variance covariance matrix of the vector sum of response coefficients can be estimated by bootstrap simulation. Given the asymptotic normality of the parameter estimators of model (15), the test has an asymptotic χ^2_{H+1} -distribution.

Unlike the slope-based test, this test depends on the magnitude of δ , so the evidence against symmetry will depend on the magnitude of the shock considered. For small shocks, a symmetric model will provide a better approximation than for large shocks. How accurate and powerful the impulse-response based test is relative to the slope-based test is an empirical question. Table 7 reports the size properties for tests of the symmetry of the impulse responses. Table 7 shows that, for our examples, these tests have acceptable size properties, despite a slight tendency for the test to overreject, as the horizon increases.

7. Empirical Tests of Symmetry in Energy Price Increases and Decreases

In this section, we use the two statistical tests discussed in section 6 to check for asymmetry in the empirical responses to energy price shocks. Table 8 shows that the evidence for

Table 7: Size of the 5 Percent Test of $H_0: I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$

H	Gas Consumption		GDP		Unemployment	
	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock
1	0.06	0.05	0.06	0.05	0.06	0.05
3	0.07	0.05	0.07	0.05	0.07	0.05
5	0.08	0.05	0.08	0.06	0.07	0.05
7	0.09	0.07	0.10	0.07	0.09	0.07

Notes: The data sets are generated to have the same number of observations as the original data. All results based on 20,000 simulations from the linear VAR DGP. Impulse responses based on an average of 90 histories. Significance levels are based on the χ_{H+1}^2 distribution.

Table 8: Empirical Symmetry Tests: Baseline Model

Variable	The Proposed Test of Symmetric Slope Coefficients	Marginal Significance Level	Mork's Test of Symmetric Slope Coefficients	Marginal Significance Level
Unemployment	7.7224	0.358	3.1317	0.792
Gas Consumption	11.3755	0.123	9.2366	0.161
Real GDP	10.4722	0.163	9.7565	0.135

asymmetry in the slope coefficients is weak. The p -values suggest no evidence against symmetry at the 10% significance level in monthly U.S. unemployment rates, in quarterly U.S. real GDP, and for U.S. gasoline. This result would be obtained even using Mork's test, although there are substantial differences in the p -values in some cases. An important question is whether these empirical results are sensitive to the choice of lag order. Our baseline results rely on six lags. Mork's (1989) test for real GDP relied on four lags. Table 9 shows that only for gasoline consumption the p -values are sensitive to the lag order. As the lag order of the model exceeds six lags, we find rejections of the symmetry null hypothesis at the 5% level.

Table 9: Marginal Significance Levels for the Proposed Slope-Based Symmetry Test

<i>Number of lags</i>	2	4	6	8	10	12
Unemployment	0.188	0.349	0.534	0.681	0.657	0.359
Gas Consumption	0.191	0.172	0.156	0.008	0.022	0.014
Real GDP	0.523	0.631	0.412	0.258	0.233	0.239

Notes: p -values are from the χ_{p+1}^2 distribution. Sample size chosen so that the estimation period is the same, regardless of the number of lags. The results may differ slightly from Table 8 given the difference in sample periods.

In general, there is a tendency for the p -values to decline with the addition of more lags. This reflects the fact that the χ_{p+1}^2 distribution becomes a less accurate approximation under the null, as illustrated in Table 10. Table 10 shows that, in the presence of excess lags, the size systematically increases from 6% to almost 10% for the monthly models and from near 9% to 28% for the quarterly model. These size distortions from overfitting help explain the general decline of the p -values with increasing numbers of lags in Table 9, but do not explain the rapid drop in the p -values for gasoline consumption beyond lag 6.

Even strong rejections of symmetry based on the slope-based test, however, need not imply large degrees of asymmetry in the impulse responses of ultimate interest, as discussed earlier. This point is illustrated in Figure 9. The difference between $I_y(h, \sigma)$ and $-I_y(h, -\sigma)$ tends to be quite small when 12 lags are included in the regression model. This example suggests that we can never rely on slope-based symmetry tests alone. This point holds with equal force if the slope-based symmetry test fails to reject. Even statistically insignificant departures from symmetry in the slope parameters may imply highly economically and statistically significant departures from symmetry in the impulse response functions. Thus, it makes sense to focus on direct tests of the symmetry of these responses.

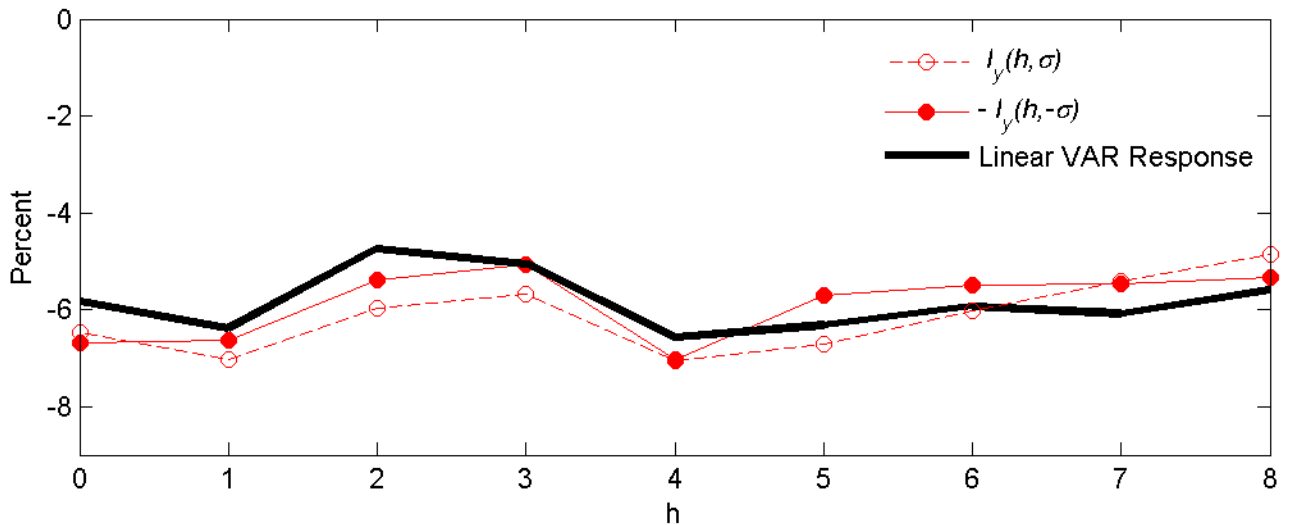
Table 11 reports the corresponding tests of the symmetry of the impulse response functions for the baseline model with six lags. Neither for U.S. real GDP nor for unemployment

Table 10: : Size of the 5% Test of $H_0: I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$: DGP with 4 Lags

Number of lags in regression model	4	6	8	10	12
Unemployment	0.060	0.065	0.074	0.084	0.095
Gas Consumption	0.060	0.065	0.072	0.081	0.094
Real GDP	0.085	0.117	0.157	0.214	0.282

Notes: p -values are from the χ^2_{p+1} distribution. Sample size chosen so that the estimation period is the same, regardless of the number of lags.

Figure 9: The Response of Gas Consumption to a One-Standard Deviation Energy Price Shock Model with 12 Lags



is there statistically significant evidence against the symmetry of the impulse response functions.¹¹ For gasoline consumption, the results are mixed. Whereas there is no evidence against symmetry based on two-standard deviation shocks, based on one standard-deviation shocks the test

¹¹ These results are qualitatively consistent with the findings in Edelstein and Kilian (2007a,b) based on a somewhat different methodology.

Table 11: Testing the Symmetry of the Response $I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$

H	Gas Consumption		GDP		Unemployment	
	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock	1 Std. Deviation Shock	2 Std. Deviation Shock
0	0.45	0.47	0.40	0.47	0.36	0.43
1	0.13	0.28	0.44	0.54	0.65	0.73
2	0.05	0.15	0.59	0.69	0.83	0.88
3	0.09	0.25	0.56	0.68	0.92	0.96
4	0.07	0.21	0.66	0.78	0.97	0.98
5	0.04	0.15	0.78	0.87	0.99	1.00
6	0.06	0.18	0.48	0.59	1.00	1.00
7	0.09	0.26	0.58	0.69	1.00	1.00

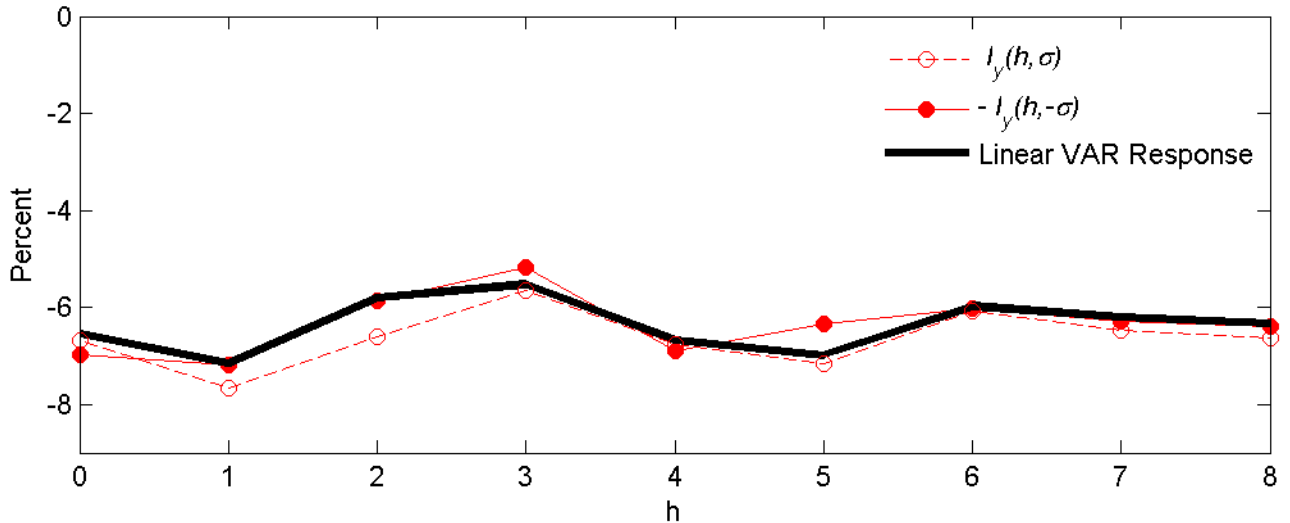
Notes: Based on 20,000 simulations of Model (15). p -values are based on the χ^2_{H+1} distribution.

rejects the null hypothesis of symmetry at the 5% level at one horizon and at the 10% level at several additional horizons. The evidence against symmetry appears stronger than that based on the slope-based test. However, as shown in Figure 10 which reports $I_y(h, \sigma)$ and $-I_y(h, -\sigma)$, the actual difference between these two responses seems fairly small, and one would be hard pressed to make the case for using the asymmetric model on economic grounds.

8. Testing Models of Net Energy Price Increases

As noted in the introduction much of the recent empirical work in the transmission of oil price shocks has focused on the net increase in the price of oil as defined in Hamilton (1996, 2003). For example, Lee and Ni (2002, p. 834) note that the “oil price variable [in their VAR] is Hamilton’s (1996) ‘net oil price increase’, defined as the percentage change of oil price over the maximum value of the preceding year if the price of the current month exceeds the previous year’s maximum, and zero otherwise.” Likewise, Bernanke et al.’s (1997, p. 104) VAR analysis relies as the main measure of oil price shocks on Hamilton’s measure which “equals the maximum of (a) zero and (b) the difference between the log-level of the crude oil price for the current month and the maximum value of the logged crude oil price achieved in the previous twelve months”. Similar net oil price increase measures have also been used by Davis and Haltiwanger (2001), Lee and Ni (2002), and Hamilton and Herrera (2004), among others.

Figure 10: The Response of Gas Consumption to a One-Standard Deviation Energy Price Shock
Baseline Model with 6 Lags



Given the widespread use of the net oil price increase measure in applied VAR work, it is important to assess the empirical support of that model. Hamilton (2003) proposed a formal test of the linear symmetric model against the alternative of a model in which only net increases matter based on the conditional expectation function. Rather than specifying the structural model of interest, Hamilton focused on predictive relationships in the data. His statistical evidence against the null hypothesis of linearity in the one-step ahead predictive relationship between real GDP growth and changes in oil prices by construction implies that the underlying data generating process must be asymmetric as well. Hamilton compared several nonlinear predictive models and concluded that a conditional expectations model based on the three-year net increase measure had the most predictive power one quarter ahead.

It is important to keep in mind, first, that the model Hamilton investigated differs from the VAR models estimated in the literature on estimating responses to energy price shocks such as Bernanke et al. (1997). Even if we take Hamilton's results about the existence of a nonlinear reduced-form relationship involving net oil price increases at face value, these results do not justify the use of censored VAR models of the form

$$x_t^{+,net} = b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i}^{+,net} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1t}$$

$$y_t = b_{20} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=1}^p g_{21,i} x_{t-i}^{+,net} + a \varepsilon_{1t} + \varepsilon_{2,t}$$

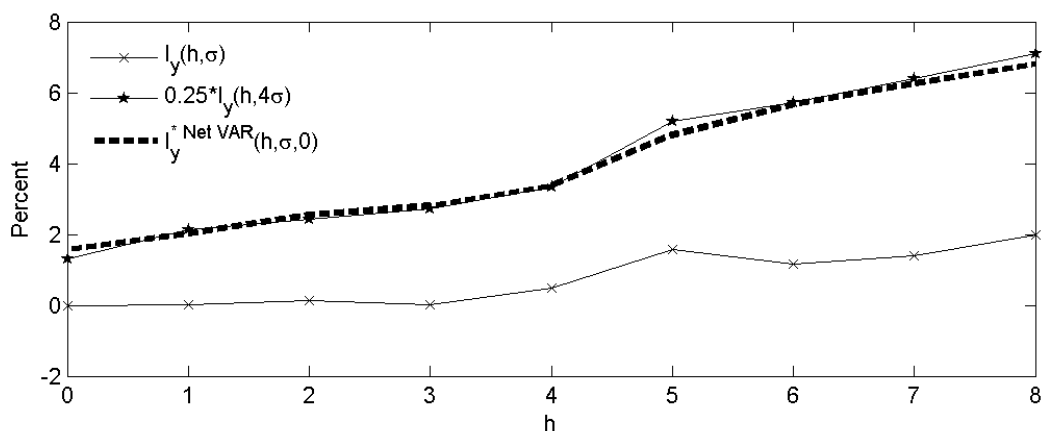
where a is estimated by the Cholesky factorization of the variance covariance matrix of the reduced-form residuals. For the reasons discussed earlier, the structural models that would give rise to such asymmetries in the reduced form cannot be represented as VAR models and must be modified prior to estimation. The implications of this point are illustrated in Figure 11. Using the data of the unemployment rate example, Figure 11 shows that the response to one-standard deviation shock implied by the incorrectly specified net increase VAR model, as estimated in the literature, is effectively equivalent to a four-standard deviation shock in the correctly specified net increase model. Similar results hold for the other two empirical examples, but are not shown to conserve space.

Second, Hamilton's work leaves unanswered the question of how much the response of real GDP to an exogenous oil price innovation is affected by the nonlinearity of the DGP relative to the linear case. Even if there is nonlinearity in the slope parameters of the reduced form, that nonlinearity need not have large effects on the implied impulse response function. Moreover, the extent to which responses from a linear symmetric VAR model provide a good approximation will be a function of the size of the energy price shock. Answering that question requires a fully specified multivariate structural model.¹²

In closely related work, Balke, Brown and Yücel (2002) conducted an alternative test of the net increase model based on the slope parameters of regressions of real GDP growth on lagged real GDP growth, lagged oil price changes and lagged net oil price increase changes. Like Hamilton, they concluded that the inclusion of net oil price increases improves the predictive power of this model. Unlike Hamilton, they explicitly recognized the need for a multivariate structural model in constructing estimates of the impulse response functions and they observed,

¹² Hamilton (2003, Figure 14) provides an estimate of the impulse response function eight quarters ahead based on a single-equation nonlinear reduced-form model designed for one-step-ahead prediction (see p. 392). That estimate suffers from three problems that make it unsuitable for our purposes. First, in constructing the impulse responses Hamilton imposes that the net energy price increases are strictly exogenous. That assumption is unnecessarily restrictive and has been shown to be economically and empirically implausible (see Kilian 2008b,c). Second, Hamilton's model omits the contemporaneous regressor in constructing the impulse response function. Third, Hamilton ignores the nonlinear nature of the model in computing the impulse response function.

Figure 11: Alternative Empirical Estimates of the Response of the Unemployment Rate to an Oil Price Increase: One-year Net Increase



consistent with our analysis, that such models do not have a VAR representation. Balke et al. also recognized the importance of accounting for the nonlinear nature of the model in constructing impulse responses. While the description of their algorithm is terse and incomplete, their approach appears substantively identical with the procedure that we propose below. However, no paper in the large empirical VAR literature on the nonlinear transmission of energy price shocks has adopted their proposal. Applied users have continued to employ incorrectly specified VAR models and have computed the model impulse responses incorrectly as though the model were linear. A likely reason is that Balke et al. do not elaborate on the limitations of commonly used asymmetric VAR models beyond observing that this “specification is not completely suitable for an examination of asymmetry”. For that reason, we believe that it is important to restate the basic points made in Balke et al. and to do so more explicitly. In addition, we will take their analysis a step further by proposing an explicit test of the symmetry of the impulse response functions.

As discussed below, our empirical conclusions are somewhat different from Balke et al. (2002). Whereas Balke et al. found some evidence in favor of asymmetries in output, we do not. There are several differences between their analysis and ours. First, we focus on the more relevant measure of net increases in the real price of oil (rather than the nominal price). Second, we have the advantage of a longer time span of data from the empirically relevant post-1972 period. The key difference, however, is in the econometric analysis. Like us, Balke et al. found

little evidence of asymmetries based on one-standard deviations shocks. In the published version of their paper they concentrated exclusively on two-standard deviations shocks which are much rarer in practice (and hence less precisely estimated). While increasing the shock size indeed increases the apparent degree of asymmetry in the response functions, for the reasons we discussed earlier, it also increases the uncertainty surrounding those estimates because there are fewer episodes of large oil price changes in the data. Below we propose and implement a formal joint test of the symmetry of the impulse responses for a shock of given magnitude. We find very little evidence against the linear symmetric model for shocks of typical magnitude.

8.1. A Slope-Based Symmetry Test for the Net Increase Model

In this section, we outline two tests of the net increase model, building on the analysis in section 4. Rather than testing the null hypothesis of symmetry between net oil price decreases and net oil price increases, as in Edelstein and Kilian (2007a), we nest the net increase model in the standard linear symmetric VAR model. In essence, we ask whether there is incremental explanatory power in including net oil price increases in the baseline model. This results in a model structure similar to model (15) with x_t^+ replaced by $x_t^{+,net}$, where $x_t^{+,net} = \max [0, x_t^*]$ and x_t^* is the maximum of x_t over the preceding year (or three years, alternatively), following Hamilton (1996, 2003). We follow Kilian (2008c) in specifying the net increase in the real price of oil rather than the nominal price as in Hamilton (1996, 2003) since the real price is the economically relevant measure of the price of oil. This does affect the time path of the net increase variable, as documented in Kilian (2008c), but is more consistent with the recent empirical literature.

The problems with the use of net oil price increase measures in VAR models are fundamentally the same as with the use of oil price increase measures and can be addressed along similar lines. By analogy to the discussion in section 4 the specified structural model

$$x_t = b_{10} + \sum_{i=1}^p b_{11,i} x_{t-i} + \sum_{i=1}^p b_{12,i} y_{t-i} + \varepsilon_{1t} \quad [31]$$

$$y_t = b_{20} + \sum_{i=0}^p b_{21,i} x_{t-i} + \sum_{i=1}^p b_{22,i} y_{t-i} + \sum_{i=0}^p g_{21,i} x_{t-i}^{+,net} + \varepsilon_{2,t}$$

may be estimated consistently by least squares. Note that model (31) also allows us to compute impulse response functions taking account of the magnitude and direction of the innovation ε_{1t} as well as the history of observations, whereas the shock in the commonly used censored VAR model is not well defined.

In assessing the evidence for this structural net increase model, a natural starting point is the slope-based test:

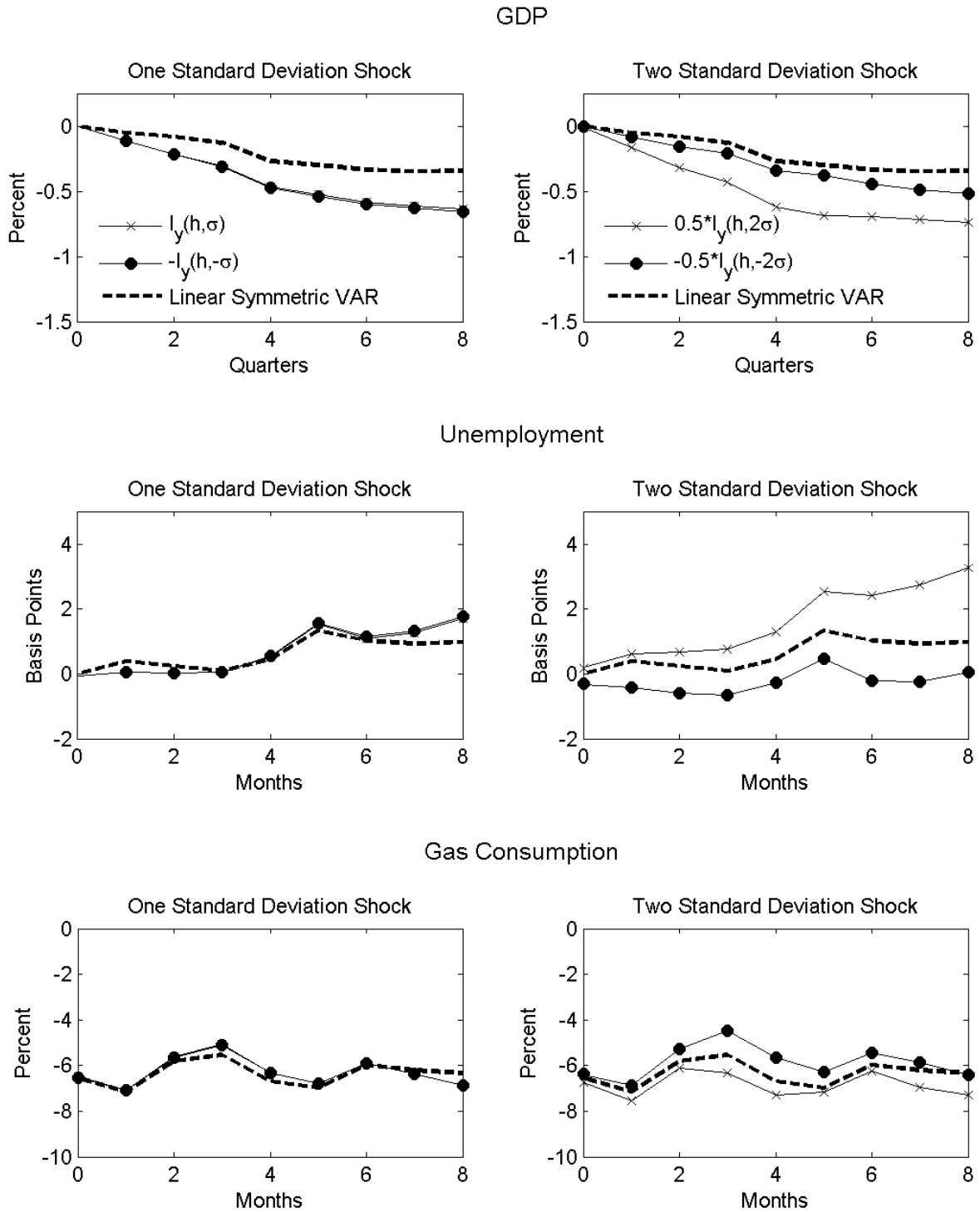
$$H_0: g_{21,0} = \dots = g_{21,p} = 0$$

based on [31]. This test relates to the test conducted in Balke et al. (2002) as the slope-based test in section 6.1 relates to Mork's tests of symmetry. The only difference to the analysis in Balke et al. is the additional inclusion of contemporaneous regressors. Table 12 suggests that there is no evidence of asymmetries using the one-year net increase measure, but using the three-year net increase measure the symmetry test rejects at the 5% level for gasoline consumption and real GDP.

Table 12: Slope-Based Test of the Linear Symmetric VAR Model against the Net Increase VAR Model: Baseline Model with 6 Lags

Variable	1-Year Net Increase		3-Year Net Increase	
	Test of Linear Symmetric Model	Marginal Significance Level	Test of Linear Symmetric Model	Marginal Significance Level
Unemployment	10.5099	0.162	9.6332	0.210
Gas Consumption	9.8879	0.195	14.5307	0.043
Real GDP	7.2617	0.402	14.2965	0.046

Figure 12: Empirical Responses to One- and Two-Standard Deviation Positive and Negative Energy Price Shocks in Baseline Model with 6 Lags: 3-Year Net Increase



Notes: The responses to negative shocks are shown as mirror images to facilitate the comparison. Some of the responses in the left panel are nearly invisible since the responses are almost perfectly symmetric.

Figure 12 once again illustrates that slope-based tests are of limited use in that they are not informative about the degree of asymmetry in the impulse response functions. We focus on the three-year net increase. Results for the one-year net increase are very similar. Figure 12 shows that correctly computed impulse responses from model (31) for one-standard deviation shocks are generally similar to the response implied by a linear symmetric model, especially for the unemployment model and gasoline consumption model.¹³ Moreover, responses to positive and negative one-standard deviation shocks are almost perfectly symmetric in all three models. Thus, despite the partial rejection of the hypothesis of symmetric slopes, there is no compelling reason to depart from the linear symmetric model when dealing with shocks of typical magnitude.

Broadly similar results hold for two standard deviation shocks with the glaring exception of the unemployment rate model. Ironically, the unemployment rate model was the one model that passed all slope-based tests of symmetry, highlighting the importance of actually computing the impulse response functions. In contrast, the other point estimates look fairly symmetric. Although the response of real GDP to a positive two standard deviations shock is somewhat larger in absolute terms than the response to a negative shock of this magnitude, both responses are clearly negative and have a similar pattern. In the gasoline consumption model, the symmetry of the two response functions is even more pronounced.

8.2. An Impulse-Response Based Symmetry Test for the Net Increase Model

It may be tempting to decide the question of symmetry based on the estimates of the impulse response functions in Figure 12. Figure 12 underscores that there is no reason to question the symmetry assumption for shocks of typical magnitude. For two-standard deviation shocks the evidence is less clear, however, especially in the unemployment example. Since the point estimates in Figure 12 are subject to considerable sampling uncertainty, especially when considering large energy price shocks, it is useful to conduct a formal test of the linear symmetric model based on the impulse response functions implied by model (31). As in section 7.1, the test is based on $I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$. Table 14 shows that, as

¹³ By construction, a one-standard deviation shock is a typical shock in that about two thirds of energy price shocks in historical data are no larger than one standard deviation.

Table 14: p -Values of Test of $H_0: I_y(h, \delta) = -I_y(h, -\delta)$ for $h = 0, 1, 2, \dots, H$

H	Gasoline Consumption		GDP		Unemployment	
	1 Std.	2 Std.	1 Std.	2 Std.	1 Std.	2 Std.
	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock	Deviation Shock
a. 1-Year Net Increase						
0	0.95	0.82	0.96	0.50	0.99	0.01
1	1.00	0.91	0.96	0.72	0.83	0.03
2	1.00	0.34	0.98	0.58	0.94	0.07
3	0.99	0.11	0.99	0.65	0.98	0.12
4	0.98	0.18	0.99	0.64	0.99	0.20
5	0.98	0.26	0.99	0.74	1.00	0.21
6	0.99	0.34	0.99	0.60	1.00	0.19
7	1.00	0.43	1.00	0.66	1.00	0.24
b. 3-Year Net Increase						
0	0.94	0.43	0.98	0.79	0.95	0.13
1	0.98	0.56	0.99	0.30	0.99	0.24
2	1.00	0.66	1.00	0.26	1.00	0.40
3	1.00	0.23	1.00	0.38	1.00	0.57
4	1.00	0.32	1.00	0.41	1.00	0.71
5	1.00	0.44	1.00	0.53	1.00	0.69
6	1.00	0.55	1.00	0.57	1.00	0.69
7	1.00	0.59	1.00	0.67	1.00	0.77

Notes: Based on 20,000 simulations of Model (31). p -values are based on the χ_{H+1}^2 distribution.

expected, the p-values decline with the magnitude of the shock. At conventional significance levels, there is no evidence against the symmetry null hypothesis in any of the three empirical examples in response to a one-standard deviation shock, whether we focus on the one-year or the three-year net changes. The same results hold in response to a two-standard deviation shock when using the three-year net changes. Similar results also hold for the one-year net changes with the partial exception of the unemployment rate at very short horizons. We conclude that there is very little, if any, evidence of asymmetric responses to energy price increases and decreases. In particular, there is no such evidence for U.S. real GDP.

The evidence in Table 14 is based on a model that rules out responses to net decreases, but there is no compelling a priori reason that the economy is not responding to net decreases in energy prices as well. We explore this possibility by augmenting model (31) with lags of net decreases in energy prices. The corresponding symmetry test results are in Table 15 [To be added]

These empirical results are important in light of the quote by Davis and Haltiwanger (2001, p. 509), who considered “the evidence for asymmetric responses to oil price ups and downs as well established”. Our analysis suggests that this evidence against the symmetry hypothesis has been overstated. This point is important, as a large literature has developed aiming to explain the perceived asymmetry of responses to energy price shocks from a theoretical point of view (see, e.g., Bernanke 1983; Hamilton 1988; Pindyck 1991). Our evidence casts doubt on the empirical relevance of these theoretical models. It also casts doubt on the results of many empirical studies of the transmission of energy price shocks.

9. An Illustration of Alternative Estimates of the Effects of the Recent Oil Price Declines

It is certainly possible that impulse-response-based tests lack the power to detect asymmetries in the data, especially if those asymmetries are relatively weak for shocks of typical size. As Table 12 illustrates, there certainly is room to differ on the practical importance of asymmetries in the transmission of energy price shocks, even if the impulse response based tests do not detect statistically significant asymmetries. None of the evidence presented in the literature, however, justifies the use of VAR models of net energy price increases or the way impulse responses have been routinely computed from these nonlinear VAR models. One of the central

objectives of this paper has been to clarify the importance of correcting these mistakes. The same point applies to multi-step ahead prediction. We conclude with an illustration of the importance of correctly specifying models of the transmission of energy price shocks. We focus on four models. Model (1) is the linear symmetric VAR model. Model (2) is the agnostic structural model that allows for asymmetries of the type envisioned by Hamilton (1996,2003). Model (3) imposes the strict form of asymmetry identified in Hamilton (2003) and in Balke et al. (2002), but ensures the correct computation of the prediction. Finally, model (4) corresponds to the single-equation predictive model proposed in Hamilton (2003). Predictions are computed without accounting for the nonlinear nature of the model. Based on model estimates obtained using data until June of 2008, we are interested in the path of real GDP predicted by these models in response to the decline in oil price since July of 2008 (for a similar exercise see Hamilton 2009). Figure 13 shows that ... [To be completed]

10. Conclusions

A common view in the literature on the transmission of energy price shocks is that the effect of energy price shocks on macroeconomic aggregates such as output or employment is asymmetric in energy price increases and decreases. This perception has been bolstered by empirical evidence that unanticipated percent changes in energy prices tend to have comparatively small effects on the U.S. economy, whereas similar regressions on energy price increases produce much larger effects. Measures of energy price increases in practice are obtained by censoring energy price changes to exclude all energy price decreases.

The censoring of explanatory variables is known to undermine the validity of regression estimates. This point has recently been illustrated in the context of cross-sectional models by Rigobon and Stoker (2007, 2008). As discussed in Rigobon and Stoker (2007), microeconomic data often are released in censored form, leaving the econometrician with no choice but to use these data. In contrast, in studying the effects of energy price shocks on the economy, censoring is optional. The econometrician observes both the censored and the uncensored time series data. This paper discussed various pitfalls that arise when working with censored energy price changes as regressors in dynamic time series models. The dynamic nature of the model was shown to reinforce the conclusions obtained in the static model in some cases and to

overtake them in others. Allowing for dynamics also introduces econometric complications that do not arise in cross-sectional models.

We showed that standard methods for estimating the response of macroeconomic aggregates to energy price increases produce inconsistent estimates that tend to exaggerate the quantitative importance of these shocks for the U.S. economy. Although it is common to rely on censored VAR models to analyze the effects of energy price increases on macroeconomic aggregates, we established that structural models of asymmetric effects of energy price increases and decreases do not permit a VAR representation. Estimates of these models are inconsistent. We proposed an alternative structural regression model that allows consistent estimation of the responses in question whether the underlying data generating process is symmetric or not. We also observed that conventional estimates of asymmetric responses to energy price shocks are computed as though the model were linear and symmetric. These response estimates are misleading in that they implicitly condition on initial conditions of zero rather than integrating over alternative paths and in that they ignore the dependence of the response on the magnitude of the energy price shock. In practice, they will overstate the importance of energy price increases, even if the underlying regression model is correctly specified and the parameter estimates are consistent. We proposed a suitable method for computing consistent impulse response estimates to positive and negative energy price shocks. We concluded that fundamental changes are required in the models and methods used by empirical researchers to quantify the asymmetric transmission of energy price shocks.

The literature has typically interpreted empirical evidence of quantitatively larger response estimates from censored VAR models of energy price shocks than from linear symmetric VAR models as persuasive evidence of asymmetries. Our analysis suggests that this interpretation is invalid, since the responses in question are inconsistent and biased upward by construction. Formal statistical evidence against the linear symmetric VAR model has been limited to tests of the symmetry of slope parameters in dynamic reduced-form regressions (see, e.g., Mork 1989, Balke et al. 2002, Hamilton 2003). We proposed a modified slope-based test with more accurate size and higher power than Mork's test. We also introduced a direct test of the symmetry of the impulse response functions, motivated by the practical limitations of

slope-based tests. Finally, we considered versions of these tests designed for models involving net energy price increases.

We applied these tests to representative regression models based on actual U.S. data. We found very little, if any, evidence of asymmetries in the response of U.S. real GDP, unemployment, and gasoline consumption to energy price shocks. Our empirical results have important implications for studies of the transmission of energy price shocks. First, one reason that researchers had been eager to accept the apparent finding of asymmetry in the 1990s was that it seemed consistent with theoretical models of the transmission of energy price shocks that emphasized asymmetries through shifts in uncertainty or frictions to the reallocation of factors of production within and across sectors (see, e.g., Bernanke 1983; Hamilton 1988; Pindyck 1991). The latter models were required to rationalize large effects from oil price shocks that are difficult to obtain in conventional models based on cost shocks or aggregate demand shocks. Our evidence provides no support for theoretical models with built-in asymmetries.

Second, in the absence of asymmetries, the responses of the U.S. economy to energy price shocks appear modest at best, which is fully consistent with conventional macroeconomic models of the transmission of energy price shocks that do not predict large fluctuations in U.S. output in response to energy price shocks (see, e.g., Kilian 2008b). Thus, the absence of larger effects is not a puzzle. We conclude that oil price shocks are only one of many factors contributing to recessions, not a key determinant. Our findings also lend credence to recent linear models of how the oil demand and oil supply shocks that drive oil price shocks affect the U.S. economy (see, e.g., Kilian 2008a).

Third, our analysis calls into question several empirical findings reported in the literature. To the extent that these studies used censored VAR models and/or computed impulse responses to energy price shocks incorrectly, they are invalid. For example, much of the consensus on how monetary policy responds to oil price shocks is based on the censored VAR model introduced by Bernanke, Gertler and Watson (1997). That study and subsequent papers using the same type of model will have to be reexamined in light of our findings. Similarly, influential studies of sectoral responses to oil price shocks such as Lee and Ni (2002) or employment responses at the plant level as in Davis and Haltiwanger (2001) will have to

reexamined. These studies are important in that they have been used to support or reject specific channels of the transmission of energy price shocks and have shaped our thinking about how these shocks are transmitted.

Appendix: Proof of Inconsistency

Suppose that an i.i.d. series y_t can be expressed as a linear combination of x_t and ε_t :

$$y_t = x_t \beta + \varepsilon_t \quad [32]$$

where β is a constant and x_t and ε_t are i.i.d. symmetrically distributed variables with mean zero and finite fourth moments. Consider the regression model:

$$y_t = a + x_t^+ b + u_t \quad [33]$$

where

$$x_t^+ = \begin{cases} x_t & \text{if } x_t > 0 \\ 0 & \text{if } x_t \leq 0 \end{cases}$$

The objective is to show that censoring of the explanatory variable renders \hat{b} , the OLS estimator of b , inconsistent for β . Recall that

$$\hat{a} = S_y - \hat{b} S_{x^+}$$

$$\hat{b} = \frac{S_{x^+ y} - S_{x^+} S_y}{S_{x^+ x^+} - S_{x^+}^2}$$

where $S_y \equiv \frac{1}{T} \sum_{t=1}^T y_t$, $S_{x^+} \equiv \frac{1}{T} \sum_{t=1}^T x_t^+$, $S_{x^+ x^+} \equiv \frac{1}{T} \sum_{t=1}^T x_t^{+2}$, $S_{y x^+} \equiv \frac{1}{T} \sum_{t=1}^T y_t x_t^+$, and $S_{\varepsilon x^+} \equiv \frac{1}{T} \sum_{t=1}^T \varepsilon_t x_t^+$.

Recall that $E(x_t^+) = 0.5\mu$ where $\mu \equiv E(x_t | x_t > 0)$. Observe that $E(x_t^{+2}) = 0.5E(x_t^2)$ if x_t is symmetrically distributed around zero. Because y_t , x_t^+ , and x_t^{+2} all are i.i.d with finite variances, a standard application of the weak law of large numbers implies that the sample averages of these series will converge in probability to their population means. As such we have that

$$S_y \xrightarrow{p} E y_t = 0$$

$$S_{x^+ x^+} \xrightarrow{p} E(x_t^{+2}) = \frac{1}{2} E x_t^2$$

$$S_{x^+} \xrightarrow{p} \frac{1}{2} \mu.$$

Finally, we have that

$$S_{yx^+} = \alpha S_{x^+} + \beta S_{x^+x^+} + S_{\varepsilon x^+}$$

where

$$\frac{1}{T} \sum_{t=1}^T x_t^+ x_t = \frac{1}{T} \sum_{t=1}^T x_t^{+2}$$

and

$$S_{\varepsilon x^+} \xrightarrow{p} E \varepsilon_t x_t^+ = E \varepsilon_t E x_t^+.$$

Since ε_t and x_t^+ are independent and ε_t is mean zero, we have that $E \varepsilon_t E x_t^+ = 0$. Since α equals zero, we have that

$$S_{yx^+} \xrightarrow{p} \frac{1}{2} \beta E x_t^2.$$

Combining these results, as T goes to infinity,

$$\hat{b} \xrightarrow{p} \beta \frac{E x_t^2}{E x_t^2 - 0.5 \mu^2}$$

and

$$\hat{a} \xrightarrow{p} -\beta \frac{0.5 \mu E x_t^2}{E x_t^2 - 0.5 \mu^2}.$$

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