A State-Space Stochastic Frontier Panel Data Model

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Abstract

In this paper we introduce a state-space approach to the econometric modelling of cross-sectional specific trends (temporal variation in individual heterogeneity) and time varying slopes in the context of panel data regressions. We show that our state-space panel stochastic frontier model nests some of the popular models proposed in the literature on stochastic frontier to accommodate time varying inefficiency and its dynamic version (productivity). A detailed discussion of alternative model specifications is provided and estimation (along with testing procedures for model selection) is presented. The empirical application uses the EU-KLEMS dataset which provides data in the period 1977-2007 for 13 countries and 20 sectors of each economy. Our main empirical interest is centered on productivity analysis and thus we focus on the stochastic frontier interpretation of this cross-sectional specific temporal variation. A post-estimation growth accounting is introduced in order to provide a quantitative assessment of the main factors behind sectoral labour productivity growth for each country.

Keywords: sectoral productivity, state-space, Kalman filter, panel data, EU-KLEMS dataset, Malmquist

JEL Classification:

1 Introduction

In this paper we propose a general framework to deal with cross-sectional specific trends (temporal variation in individual heterogeneity) and time variation (trends) in the vector of slope coefficients in panel data models. The econometric framework is represented by a state-space which is particularly useful when dealing with "long" panels. Our model specification is compatible with a stochastic frontier interpretation if the post-estimation parameter transformation proposed by Schmidt and Sickles (1984) is applied. In fact, our specification accommodates time varying inefficiencies, a flexible trend specification for technical change, and it is useful to identify possible biases in technical change (time variation in the slope coefficients) in a flexible way. We first develop a general version of the panel stochastic frontier model and then show that the model is able to nest a number of commonly used panel models introduced in the literature which deal with technical change and time-varying inefficiency. Jin and Jorgenson (2009) recently introduced a state-space representation of a cost function, although their work is confined to a single time series and therefore unable to accommodate technical inefficiency deviations from the production frontier (individual time varying effects). Thus, this paper can also be viewed as a generalization of their approach to the panel stochastic frontier literature. We present our modelling strategy in a primal setting, i.e. a production function instead of a cost function. The production function is assumed to have time-varying parameters and time-varying technical inefficiency and we show that the model can produce a growth accounting decomposition able to identify the main drivers behind observed labour productivity growth. The state-space representation can be estimated by Kalman filter procedures. The framework is illustrated using the EU-KLEMS dataset to identify the main trends (and biases) in productivity growth in the period 1977-2007 for 13 OECD countries and 20 industrial sectors of each economy. This paper is also related to the work of Kneip et al (2011) who present an alternative strategy based on common factor modelling to accommodate cross-sectional time varying heterogeneity. Although different in spirit and in the econometric estimation approach, this work try to address the same issue raised by Kneip et al (2011).

The classical stochastic frontier model (SFM) introduced by Aigner et al (1997) and Meeusen and van den Broeck (1977) is well established in cross-sectional settings with the main purpose of allowing one-sided deviations from the production frontier (regression line). From a historical perspective, this attempt should be contrasted with classical regression analysis where firms or countries were assumed to be perfectly efficient and any deviation from the regression line was attributed to noise. The interest in SFM as a tool for efficiency analysis came later thanks to the contributions of Jondrow et al (1982) and Battese and Coelli (1988). A first discussion of panel data settings for SFM was provided by the seminal paper of Schmidt and Sickles (1984), although under the restrictive assumption of time invariant technical efficiency. In such a context technical efficiency can be interpreted as unobserved heterogeneity and panel data estimators are available to deal with it. Once the literature started to think in a panel data setting, the path was open to productivity measurement. As emphasized by Lovell (1996) in a review of the issue, nonparametric methods (such as DEA) accommodated productivity measurement in a more satisfactory way than SFM. In fact, in a dynamic context of productivity measurement, one has to model at least two different contributors to productivity change: technical change and technical efficiency change. While the latter has been addressed widely, the former has been basically left to ad hoc solutions and, in a sense, SFM analysis has been biased towards the static (efficiency measurement) and not the dynamics (productivity measurement). Kumbhakar (1990, 2004) proposed a model with deterministic time varying technical inefficiency and Battese and Coelli (1992) parameterized the deterministic function of time in a different fashion. At the same time Cornwell et al (1990) proposed to accommodate for time varying technical inefficiency using quadratic time varying firm specific intercepts. Ahn et al. (2000) proposed to model technical inefficiency as an AR(1) process (stationary) and the same route was followed, with some interesting novelties, by Desli et al. (2003) and Tsionas (2006). All these specifications place emphasis on technical efficiency change, with less attention to technical change. Particularly, stationary specifications (like the AR(1)) are unlikely to perform well when modeling a structurally non-stationary phenomena like technical change. Thus, it is not surprising that technical change has been accommodated by the applied researcher using ad hoc methods. A common way of addressing such an event is introducing time as an explanatory variable in the regressor vector of inputs, possibly having it interacting with the inputs to provide the second order approximation typical of translog functional specifications. This is indeed the way explicitly put forward by Orea (2002) to build up the generalized Malmquist productivity index. This strategy has been also proposed by Coelli et al (2003) and Ahn et al (2000).

The paper is organized as follow. Section 2 provides an explanation of the production model. Section 3 presents the panel state space SFMl and its nested models. Estimation is discussed in section 4. Section 5 presents the main findings of the empirical analysis and, finally, section 6 concludes.

2 Background: the production model

In this paper the technology is represented via a production function where a single output y_{it} (log of output) is produced by means of multiple inputs X_{it} where $i = 1, \dots, N$ indexes the number of countries, $t = 1, ..., T$ indexes the number of time periods. A translog specification is assumed and thus X_{it} is a $1 \times k$ vector containing the log of inputs, the squared log of inputs and interaction terms:

$$y_{it} = \mu_t + \gamma_{it} + X_{it}\beta_t + \epsilon_{it} \tag{1}$$

In this specification μ_t, β_t are time varying parameters common to all the countries,γ_{it} is a country specific time varying intercept (cross-sectional specific time trend) and ϵ_{it} is a normally distributed error term. The country specific intercepts are given by $\mu_t+\gamma_{it} = a_{it}$. In order to identify all the parameters we need to assume $\frac{1}{N}\sum_i a_{it} = \mu_t$, so that μ_t represents the time variation in the average intercept (it is an average function or common shock). This leads to the following possible reparameterization of the model:

$$y_{it} = a_{it} + X_{it}\beta_t + \epsilon_{it} \tag{2}$$

Since technical efficiency can be interpreted as a shift in the intercept, Schmidt and Sickles (1984) proposed also to reparameterize equation (2) using the following transformation: $\underset{i=1,\ldots,I}{max}\{a_{it}\} = a_t$, $u_{it} = a_{it} - a_t$ (this is a measure of technical inefficiency) which returns the following stochastic production frontier:

$$y_{it} = a_t + X_{it}\beta_t - u_{it} + \epsilon_{it} \tag{3}$$

The production frontier embedded in (3) is time varying (due to the time varying coefficients (a_t, β_t)) and technical inefficiency is time varying (as a notational convenience we use a_t for the maximum intercept and $a_{it} = a_t - u_{it}$ for the country specific intercept). If one is willing to assume no technical change then the production frontier becomes time invariant $y_{it} = a + X_{it}\beta - u_{it} + \epsilon_{it}$; going a step further and assuming also time invariant technical inefficiency one obtains the model discussed by Schmidt and Sickles (1984) in their seminal paper as a special case of specification (1):

$$y_{it} = a + X_{it}\beta - u_i + \epsilon_{it} \tag{4}$$

In equation (4) the technology is fixed (no technical change as the parameters are fixed) and the technical efficiency term is a fixed effect unobserved heterogeneity component. It is possible to estimate such a model using standard panel data estimators (for a detailed discussion of this point see Schmidt and Sickles, 1984). Of course, the validity of such a procedure is predicated on the assumption that technical efficiency is time invariant and this is a tolerable assumption for "short" panels. On the other hand, when T becomes larger the time invariant technical efficiency and no technical change model becomes less appealing. In this paper we propose a stochastic time-varying parameters specification which provides a general formulation of technical change nesting the standard practice of including deterministic time trends as a special case. Specification (3) emphasizes that there is always at least one country that lies onto the international production frontier and this country is the one with the maximum value of the intercept. This assumption leads to two fundamental advantages: *first*, technical change can be easily and elegantly modeled as a stochastic trend instead of being forced to be deterministic as in standard stochastic frontier models; *second*, the one sided technical inefficiency variable is free from any assumption about its statistical distribution and free to move according to the stochastic trend specification. Although reparameterizations (2) and (3) are useful in terms of interpretation, we will use model specification (1) for estimation purposes.

3 A State-Space Representation of the panel SFM

3.1 A general Model

Parameterization (3) provides the post-estimation interpretation of the coefficients of our proposed econometric model from which the main components of productivity change can be recovered (this is discussed in detail in Section 3.4). In this section we develop a general state-space representation of the econometric model from specification (1) which provides a more appealing econometric interpretation. Noise is accommodated with the standard normally distributed two-sided disturbance ($\epsilon_{it} \sim N\left(0, \sigma_\epsilon^2\right)$) and the stochastic production frontier can be written as for equation (1). The country specific intercept moves in time due to the time varying common average μ_t and the country specific trend γ_{it}. The common shock μ_t is represented by a stochastic double trend specification (random walk with time varying drift):

$$\begin{cases} \mu_t = c_\mu + \nu_{t-1} + \mu_{t-1} + \eta_{\mu t} \\ \nu_t = c_\nu + \nu_{t-1} + \eta_{\nu t} \end{cases} \tag{5}$$

where $\eta_{\mu t}$ and $\eta_{\nu t}$ are independent innovations assumed to be normally distributed $\eta_{\mu t} \sim N\left(0, \sigma_\mu^2\right)$ and $\eta_{\nu t} \sim N\left(0, \sigma_\nu^2\right)$. This stochastic double trend specification with drifts c_μ and c_ν are able to nest the common practice of using deterministic quadratic trends (this is shown explicitly in the appendix). The time varying country specific shock γ_{it} is assumed to follow a country specific stochastic double trend with drifts:

$$\begin{cases} \gamma_t = c_\gamma + \phi_{t-1} + \gamma_{t-1} + \eta_{\gamma t} \\ \phi_t = c_\phi + \phi_{t-1} + \eta_{\phi t} \end{cases} \tag{6}$$

where γ_t is a $N \times 1$ vector, ϕ_t is a $N \times 1$ vector and the other vectors dimensionality are defined by conformability. The innovation vectors are normally distributed $\eta_{\gamma t} \sim N\left(0, \sigma_\gamma^2 I_{N\times 1}\right)$ and $\eta_{\phi t} \sim N\left(0, \sigma_\phi^2 I_{N\times 1}\right)$. The last piece of our model is the vector of slope coefficients which models the bias in technical change. They are assumed to move according to a double stochastic trend with drifts:

$$\begin{cases} \beta_t = c_\beta + \tau_{t-1} + \beta_{t-1} + \eta_{\beta t} \\ \tau_t = c_\tau + \tau_{t-1} + \eta_{\tau t} \end{cases} \tag{7}$$

where, by conformability, $\tau = \begin{bmatrix} \tau_1 & \dots & \tau_K \end{bmatrix}'$ is a $K \times 1$ vector of drifts, $\eta_{\beta t} \sim N\left(0, \sigma_\beta^2 I_{K\times 1}\right)$ the vector of innovations and $\eta_{\tau t} \sim N\left(0, \sigma_\tau^2 I_{K\times 1}\right)$. Putting equations (5), (6) and (7) together, the full model ready for matrix state-space representation will be:

$$\begin{cases} y_t = \gamma_t + 1_N \mu_t + X_t \beta_t + \epsilon_t \\ \gamma_t = c_\gamma + \phi_{t-1} + \gamma_{t-1} + \eta_{\gamma t} \\ \phi_t = c_\phi + \phi_{t-1} + \eta_{\phi t} \\ \mu_t = c_\mu + \nu_{t-1} + \mu_{t-1} + \eta_{\mu t} \\ \nu_t = c_\nu + \nu_{t-1} + \eta_{\nu t} \\ \beta_t = c_\beta + \tau_{t-1} + \beta_{t-1} + \eta_{\beta t} \\ \tau_t = c_\tau + \tau_{t-1} + \eta_{\tau t} \end{cases} \tag{8}$$

In this model the variance of the country specific intercept comes from two different sources: first, a common shock to all the countries and, second, from a country specific shock. This is a quite general model, able to nest some quite common used models. Before showing this, it is useful to provide its state-space representation:

$$\begin{aligned} y_t &= Z_t \alpha_t + \epsilon_t \\ \alpha_t &= D\alpha_{t-1} + c_t + \eta_t \end{aligned} \tag{9}$$

where $\alpha_t = \begin{bmatrix} \gamma_t \\ \phi_t \\ \mu_t \\ \nu_t \\ \beta_t \\ \tau_t \end{bmatrix}$, $D = \begin{bmatrix} I_N & I_N & & & & \\ & I_N & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & I_K & I_K \\ & & & & & I_K \end{bmatrix}$, $Z_t = \begin{bmatrix} I_N & 0_{N\times N} & 1_N & 0_N & X & 0_{N\times K} \end{bmatrix}$, $c_t = \begin{bmatrix} c_\gamma \\ c_\phi \\ c_\mu \\ c_\nu \\ c_\beta \\ c_\tau \end{bmatrix}$. This means $size\,(\alpha_t) = N+N+1+1+K+K = 2\,(N+K+1) = B$ and $size\,(c) = 2\,(N+K+1) = B$ for a total of $4\,(N+K+1) = 2B$ parameters (plus all the hyperparameters). We apply the following transformation to the state vector α_t:

$$\alpha_t^* = \begin{bmatrix} \alpha_t - \Gamma_t c \\ c \end{bmatrix} \tag{10}$$

where the matrix Γ_t is defined as (see the appendix):

$$\Gamma_t = \textstyle\sum_{j=0}^{t-1} D^j = I + D + D^2 + ... + D^{t-1} =$$
$$= \begin{bmatrix} tI_N & \frac{t(t-1)}{2}I_N & & & & \\ & tI_N & & & & \\ & & t & \frac{t(t-1)}{2} & & \\ & & & t & & \\ & & & & tI_K & \frac{t(t-1)}{2}I_K \\ & & & & & tI_K \end{bmatrix} \tag{11}$$

After transformation the state space becomes:

$$y_t = Z_t^* \alpha_t^* + \epsilon_t \tag{12}$$

$$\alpha_t^* = D^* \alpha_{t-1}^* + \eta_t^* \tag{13}$$

where $Z_t^* = \begin{bmatrix} Z_t & (Z_t\Gamma_t) \end{bmatrix}$, $D^* = \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix}$, $\eta_t^* = \begin{bmatrix} \eta_t \\ 0 \end{bmatrix}$, $E(\eta_t^* \eta_t^{*'}) = Q_t$ and $E(\epsilon_t \epsilon_t') = H_t$. The system matrices are transformed accordingly. In this representation all hyper-parameters (i.e. those parameters in Q_t and H_t) are unknown and must be estimated. We discuss estimation in Section 4. Given estimates of these parameters, the Kalman filter and Kalman smoothing algorithms are then used to obtain estimates of the state vector, α_t^*, and its covariance matrix, P_t.

3.2 Nested models

Our model specification (8), although very general, is not very parsimonious since requires estimation of $4\,(N+K+1)$ parameters (plus all the hyperparameters) using $N \times T$ observations. Thus, unless T is very long, some restrictions on the model are needed in order to improve its finite sample estimation performance. This is also a useful exercise, since it gives us the possibility of discussing some commonly used models as special cases of our specification (8).

We introduce a generic set of J linear restrictions on the parameters of the model:

$$R_t \alpha_t^* = q_t \tag{14}$$

and focus our attention on the set of restrictions which fix $q_t = 0$. The following four restricted models will be discussed in detail in this section: 1) the fixed effects time invariant model; 2) fixed effects with common deterministic time trend model; 3) the Cornwell et al (1990) model (which includes the quadratic specification of Battese and Coelli, 1992); 4) a simple stochastic time-varying model. Since all these models are nested in our general model specification, it is possible to make a systematic statistical comparison among them using the AIC and BIC measures of fit. The two measures are:

$$AIC = log\left(\frac{e'e}{NT}\right) + \frac{2K}{NT}$$

$$BIC = log\left(\frac{e'e}{NT}\right) + K\frac{log\,NT}{NT}$$

We use these measures of statistical fit in order to penalize for the loss in degrees of freedom of models with a high number of parameters.

3.2.1 Fixed effects (FE) and fixed effects deterministic time trend model (FEDT)

One way of dealing with technical inefficiency in panel data frameworks is to treat it as unobserved heterogeneity (time invariant) as in specification (4). This approach has been discussed, for example, by Schmidt and Sickles (1984) and by Sickles (2005) in his review. By setting $Q = 0$ (see (13)), and defining R_t as follows:

$$R_t = \begin{bmatrix} 0_{(N+2)\times N} & I_{N+2} & 0_{(N+2)\times(2K+B)} \\ & 0_{(K+B)\times(2+2N+K)} & I_{K+B} \end{bmatrix}$$

one obtains the following restrictions:

$$\begin{bmatrix} \phi_t - tc_\phi \\ \tau_t - tc_\tau \\ \mu_t - tc_\mu - \frac{t(t-1)}{2}c_\nu \\ \nu_t - tc_\nu \\ c_\gamma \\ c_\phi \\ c_\mu \\ c_\nu \\ c_\beta \\ c_\tau \end{bmatrix} = 0$$

The last B restrictions impose $c = 0$. The other restrictions impose $\phi_t = 0$, $\mu = \nu = 0$, $\tau = 0$. Then the general model is restricted to a standard time invariant fixed effects model:

$$\begin{cases} y_t = \gamma_t + X_t\beta_t + \epsilon_t \\ \gamma_t = \gamma_{t-1} = \gamma \\ \beta_t = \beta_{t-1} = \beta \end{cases}$$

It is then easy to see that the general specification (8) collapses to (4), where technical inefficiency is unobserved heterogeneity and the parameters of the production function are time invariant. In such a framework, one can attempt to accommodate technical change by introducing deterministic time varying parameters. A common (and simple) procedure is to treat time as a standard variable in the translog specification (i.e., adding time and its interaction with inputs as additional regressors). This model implicitly assume the following deterministic time varying coefficients:

$$\gamma_{it} = \delta_0 + \delta_1 t + \delta_2 t^2 - u_i$$

$$\beta_{nt} = \tau_{n0} + \tau_n t, \quad n = 1, ..., N$$

This model can be obtained by setting $Q = 0$ (see (13)) and defining R_t as follows,

$$R_t = \begin{bmatrix} 0_{N\times N} & I_N & 0_{N\times(2+2K+B)} \\ 0_{(2N+K)\times(2+2N+K)} & I_{2N+K} & 0_{(2N+K)\times(2+2K)} \\ & 0_{K\times(B+2+2N+K)} & I_K \end{bmatrix}$$

obtaining the following model:

$$\begin{cases} y_t = \gamma_t + 1_N\mu_t + X_t\beta_t + \epsilon_t \\ \gamma_t = \gamma_{t-1} \\ \mu_t = c_\mu + \nu_{t-1} + \mu_{t-1} \\ \nu_t = c_\nu + \nu_{t-1} \\ \beta_t = c_\beta + \beta_{t-1} \end{cases}$$

Under these weaker restrictions the equation for the country specific intercept in (8) collapses to $\mu_t + \gamma_i, \ i = 1, ..., N$ and the common time trend follows a deterministic quadratic trend specification. If, additionally, $\nu = 0$ the intercept deterministic time trend becomes linear (which is a quite common way of dealing with trends, i.e. Hicks neutral linear deterministic time trend).

3.2.2 The Cornwell Schmidt and Sickles (1990) model (CSS)

Cornwell et al. (1990) proposed to model the country specific time-varying intercepts as country specific deterministic quadratic time trends. Their model is:

$$y_{it} = \gamma_{it} + X_{it}\beta + \epsilon_{it}$$

with $\gamma_{it} = \delta_{0i} + \delta_{1i}t + \delta_{2i}t^2$. This can be easily accommodated by our formulation (8) imposing the restrictions $Q = 0$ (see (13)) and defining R_t as,

$$R_t = \begin{bmatrix} 0_{2\times 2N} & I_2 & 0_{2\times(2K+B)} \\ 0_{K\times(2+2N+K)} & I_K & 0_{K\times B} \\ & 0_{(2+2K)\times(B+2N)} & I_{2+2K} \end{bmatrix}$$

These restrictions imply the following model:

$$\begin{cases} y_t = \gamma_t + X_t\beta_t + \epsilon_t \\ \gamma_t = c_\gamma + \phi_{t-1} + \gamma_{t-1} \\ \phi_t = c_\phi + \phi_{t-1} \\ \beta_t = \beta_{t-1} \end{cases}$$

and impose a country specific deterministic quadratic time trend on the time varying intercepts.

The Battese and Coelli (1992) model Battese and Coelli (1992) presented a very popular way of introducing time varying inefficiency. The technology is fixed (i.e., no technical change) and changes in the country specific intercepts will come from the time varying technical inefficiency. Technical inefficiency will vary as a deterministic function of time: $u_{it} = \delta(t)u_i$. They first propose an exponential function for $\delta(t)$ which, as the authors note, is very rigid. To remedy this problem, they propose a quadratic specification: $\delta(t) = \delta_0 + \delta_1 (t-T) + \delta_2 (t-T)^2$. This function can be expressed as $\delta(t) = \left(\delta_0 - \delta_1 T + \delta_2 T^2\right) + \left(\delta_1 - 2T\delta_2\right)t + \delta_2 t^2$, which is a quadratic deterministic time trend specification for the time varying intercepts. It is easy to see that this specification is a special case of the Cornwell et al (1990) model where the firm specific time varying parameters are different but proportional to each other, i.e.:

$$\delta_i \propto \delta_j$$

We note that the Battese and Coelli (1990) specification is restrictive in the sense that inefficiency follows the same trend for all the firms, while the Cornwell et al (1990) specification is able to accommodate different patterns (at the cost of increasing enormously the number of parameters to be estimated). Since Battese and Coelli (1990) is nested in Cornwell et al (1990), it follows that it is also nested in our general model specification.

3.2.3 A simple stochastic trend model (TV)

In order to reach a good balance between flexibility (statistical fitting) and parsimony (low number of parameters), we present another nested model. As a starting point it should be noted that the general formulation (8) requires the estimation of $4(N+K+1)$ parameters using $N \times T$ observations, therefore, unless T is very large, the estimation will be very imprecise as the model is flexible but not parsimonious. A similar problem arises with the Cornwell et al (1990) model, where the number of parameters is equal to $3N+K$ (with the time invariant slope parameter formulation). On the contrary the Battese and Coelli (1992) and Fixed effects models are very parsimonious but quite inflexible. The specification is given in (15) and it is obtained under the following restrictions to (8):

$$Q = \begin{bmatrix} \sigma_\gamma^2 I_N & & & \\ & 0_{(N+2)\times(N+2)} & & \\ & & \sigma_\beta^2 I_K & \\ & & & 0_{K\times K} \end{bmatrix}$$

$$R_t = \begin{bmatrix} 0_{(N+2)\times N} & I_{N+2} & 0_{(N+2)\times(2K+B)} \\ & 0_{(K+B)\times(2N+2+K)} & I_{K+B} \end{bmatrix}$$

$$\begin{cases} y_{it} = \gamma_{it} + X_{it}\beta_t + \epsilon_t \\ \gamma_{it} = \gamma_{it-1} + \eta_{\gamma it} \\ \beta_t = \beta_{t-1} + \eta_{\beta t} \end{cases}, \qquad i = 1, ..., N \tag{15}$$

Here the common shock is suppressed (although this saves only 4 parameters) and all the drifts are suppressed (there is no deterministic trend in this model). This basically means estimating $N+K$ parameters with $N+T$ observations, which is quite reasonable. Moreover the stochastic time trend formulation is flexible enough to accommodate many different types of trends in the data.

4 Estimation

4.1 Estimation of the parameters

The parameters of the stochastic frontier model are those in the state-space representation (12) and (13). This system can be augmented by incorporating the set of restrictions (14) and estimated by Kalman filter algorithms. Doran (1992), Doran and Rambaldi (1997), and Pizzanga et al. (2008) show the Kalman filter and smoothing algorithms estimates of the state vector, α_t^* and its mean square error matrix ($P_{t/T}$) obey the constraints exactly.

The Kalman filter algorithm (see for example Harvey (1989) Chapter 3 for the algorithm equations) provides estimates $\hat{\alpha}_{t|t}^*$, and $\hat{P}_{t|t}$ given estimates of the system matrices, H_t and Q_t, and an initial distribution (and values) of the state vector, α_0^* and P_0. These estimates are then smoothed (see for example Harvey (1989) Chapter 3 for a discussion of smoothing algorithms) to obtain $\hat{\alpha}_{t|T}^*$, and $\hat{P}_{t|T}$. Estimates of the hyperparameters in H_t and Q_t can be obtained by Bayesian or Likelihood approaches (see Durbin and Koopman (2001) for detail treatment). In this study we use maximum likelihood estimation. A suitable form of the log-likelihood function for the task is by writing the function using prediction errors (see Harvey (1989) or Durbin and Koopman (2001)). A brief sketch of the estimation algorithm used to estimate the TV model is:

1. Given an initial guess for $\psi = \{\sigma_\epsilon^2, \sigma_\gamma^2, \sigma_\beta^2\}$, ψ_0, obtain a value of the conditional likelihood function.

   The following definition of a conditional probability density function is used

   $$L(y;\psi) = \prod_{t=1}^{T} p(y_t|Y_{t-1}) \tag{16}$$

   where $p(y_t|Y_{t-1})$ denotes the distribution of y_t conditional on the information set at time $t-1$, that is $Y_{t-1} = \{y_{t-1}, y_{t-2}, ..., y_1\}$.

   Using the measurement equation (12), a prediction of the conditional distribution of y_t, $(N \times 1)$ is given by

   $$\tilde{\vec{y}}_{t|t-1} = Z_t \hat{\alpha}_{t|t-1}^*$$

   $\hat{\alpha}_{t|t-1}^*$ is the Kalman filter conditional estimate of the state vector.

   A prediction error is given by $u_t = \vec{y}_t - \tilde{\vec{y}}_{t|t-1}$ which has covariance matrix $cov(u_t) = F_t$

   Therefore for a Gaussian model, the log conditional likelihood function can be written as:

   $$lnL(\sigma_\epsilon^2, \sigma_\gamma^2, \sigma_\beta^2; y_t) = -\frac{NT}{2} ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln|F_t| - \frac{1}{2}\sum_{t=1}^{T} u_t' F_t^{-1} u_t$$

   Newton type numerical optimization methods are used to find the values of ψ. A given set of values for the parameters $\psi = \psi_*$ and starting distribution of the state vector, the Kalman filter algorithm provides a value of u_t, F_t and therefore a value for lnL. These steps are easily set as an iterative algorithm to find the maximum likelihood estimates of the hyperparameters, given by:

   $$\hat{\psi} = argmax_\psi \ln L(y_t|\psi)$$

   The distribution of the initial state vector, α_0 is assumed to be normal with a diffuse mean square error matrix, $P_0 = \kappa I$ (where κ is a very large number and I is the identity).

2. Given $\hat{\psi}$, estimates of the covariances Q_t and H_t , $\hat{Q}_t$ and $\hat{H}_t$, are now available. The estimates of α_t and its Mean Squared Error matrix are obtained by running the state-space model through the equations of the Kalman filter and smoother with initial state vector α_0.

4.2 Post-estimation growth accounting

The time-varying specification (1) has some consequences on the measure of productivity which have to be considered. It should be noted that time enters the production function via the time-varying parameters and not as a standard explanatory variable in the translog specification. This means that the functional specification we are dealing with is more general than the standard translog specification. In fact, it is a translog production function at any point in time, but it allows the parameters to move in time in a non-translog way. One of the consequences of this modelling strategy is that the quadratic identity lemma (Diewert, 1976) can be applied only at a given point in time, i.e. only to the input variables and not to the time variable. Since productivity is something that happens in time, the lemma cannot be used to build the translog productivity index in the spirit of Orea (2002). Therefore we have to re-build a measure of productivity growth. We assume that standard symmetry conditions of the translog specification holds at any point in time. In order to build a measure of productivity, let us consider the difference in output between two time periods:

$$y_{it+1} - y_{it} = a_{it+1} + X_{it+1}\beta_{t+1} - a_{it} - X_{it}\beta_t \tag{17}$$

This observed change can be imputed to three different effects: i) the shift in the country specific intercept, ii) the shift in the slope parameters and iii) the growth in the inputs. The first two effects are what one usually refers to as productivity change, while the last effect is an input growth effect. To isolate the productivity effect from the input growth effect we use a Malmquist logic, keeping some of the variables fixed while moving the others in order to separate the relative contribution of the different effects. Productivity growth can be measured keeping the level of inputs at the base period level, obtaining the equivalent of the base period Malmquist productivity index:

$$TFP^t = a_{it+1} - a_{it} + X_{it}\left(\beta_{t+1} - \beta_t\right) \tag{18}$$

Keeping the level of inputs at the comparison period value, we obtain the equivalent of the comparison period Malmquist productivity index:

$$TFP^{t+1} = a_{it+1} - a_{it} + X_{it+1}\left(\beta_{t+1} - \beta_t\right) \tag{19}$$

and to avoid the arbitrariness of choosing the base, we use the geometric mean of these two indexes in order to get an index of productivity growth:

$$\begin{aligned} TFP &= \tfrac{1}{2}\left(TFP^t + TFP^{t+1}\right) = \\ &= a_{it+1} - a_{it} + \frac{1}{2}\left(X_{it} + X_{it+1}\right)\left(\beta_{t+1} - \beta_t\right) \end{aligned} \tag{20}$$

The input growth effect or factor accumulation effect (FA) can be computed using the same logic. The base period index is:

$$FA^t = \left(X_{it+1} - X_{it}\right)\beta_t \tag{21}$$

The comparison period index is:

$$FA^{t+1} = \left(X_{it+1} - X_{it}\right)\beta_{t+1} \tag{22}$$

Finally, we use the geometric mean as a measure of the input growth effect:

$$FA = \tfrac{1}{2}\left(FA^t + FA^{t+1}\right) = \frac{1}{2}\left(\beta_t + \beta_{t+1}\right)\left(X_{it+1} - X_{it}\right) \tag{23}$$

It is easy to verify that the two effects are an exhaustive and mutually exclusive decomposition of the log change in output:

$$y_{it+1} - y_{it} = TFP + FA \tag{24}$$

Therefore TFP has the standard interpretation of being the difference between output growth and input growth between two time periods. TFP growth can be further attributed to two very different components: first the change in the country intercept $UTC = a_{it+1} - a_{it}$ and second the bias in technical change deriving form the time variation of the slope coefficients:

$$BTC = \frac{1}{2}\left(X_{it+1} + X_{it}\right)\left[\beta_{t+1} - \beta_t\right] \tag{25}$$

Summing up, the overall growth accounting decomposition will be:

$$y_{it+1} - y_{it} = TFP + FA = UTC + BTC + FA \tag{26}$$

where $UTC + BTC = TFP$. All these components are country and time specific. A positive (negative) BTC measures the extent to which technical change has been biased (for example, capital-using or capital saving). UTC incorporates the effect of the change in the country specific intercept (which is a measure of the productivity level). FA is the contribution of the increase (decrease) in the inputs endowment. We note that this decomposition is exhaustive (does not leave any residual) and the different components are independent from each other. In fact, UTC and BTC depends from different coefficients of the model and FA depends on how much a country is investing and accumulating factors of production.

5 Empirical Application

5.1 Data

The EU-KLEMS dataset is an official project that collects input and output data on prices and quantities for 26 industrialized countries in the time span 1970-2007 (see O'Mahony and Timmer (2009)). For each industry the database provides value data on gross output, capital compensation, intermediate inputs (materials and energy) along with fixed base price and quantity index numbers (1995=100). We used the amount of total hours worked by persons engaged as a proxy for the quantity of labour (the alternative of using the number of persons engaged is less satisfactory). Since gross output, intermediate inputs and capital services are measured in local currencies we used PPPs to adjust for cross-sectional differential in the general level of prices. PPPs indexes use US as benchmark (US=100, 1995=100), are sector specific (i.e., each sector has different PPPs) and different for sectoral output, intermediate inputs and capital services (i.e., there are three sets of PPPs). Due to lack of data (missing values) we limit our attention to a subset of data, specifically 13 countries and 20 industrial sectors in the time span 1977-2006. The list of countries and sectors is reported in Table 1.

[Table 1 here]

We build a balanced panel dataset in the following way. Be $j = 1, ...20$ the sector and $i = 1, ...13$ the country, then the index of sectoral output for country i at time t Y_{it}^{j} is:

$$Y_{it}^{j} = \frac{GO_{i1995}^{j}}{PPP_{i}^{j}} I_{it}^{j} \tag{27}$$

where GO_{i1995}^{j} is the value of gross output in 1995 for sector j in country i; I_{it}^{j} is the fixed base index of sectoral output quantity change between time t and the base period 1995; PPP_{i}^{j} is the purchasing power parity of country i in sector j. With a similar procedure quantity index numbers are build for intermediate output (materials) and capital services. With this procedure we obtain a "true" balanced panel data set where cross-sectional (cross-country) comparability is built using PPPs and time comparability is built using fixed base quantity index numbers. This procedure guarantee that in each time period countries can be compared. The result is a quantity index number that proxies sectoral output production, a quantity index number proxying capital services, a quantity index number that proxy the level of materials used and the number of hours worked for the labour input. All the variables obtained with this procedure have been normalized by the sample minimum. The usual procedure of normalizing variables by sample mean is very unfruitful in our modelling setting. In fact, consider that the sign of the capital bias component depends on: the sign of $[\beta_{kt+1} - \beta_{kt}]$ that is common to all countries and the sign of $(\log k_{it+1} + \log k_{it})$ that is country specific. Now, if we normalize by the sample mean for a positive (negative) sign of $[\beta_{kt+1} - \beta_{kt}]$ the countries below the sample mean will have a negative (positive) capital bias, while the countries above the mean will have a positive (negative) capital bias. This is unreasonable, since although the magnitude of the capital bias should be proportional to the quantity of capital used, it should be monotonic for all the countries, i.e. the same for all the countries. This is guaranteed by a transformation by the sample minimum.

5.2 Empirical Results

To illustrate the flexibility and parsimony of our proposed framework we first present a comparison of the simple TV model to CSS, FE and FEDT models using statistical fitting criteria such as BIC, AIC and adjusted R^2. We then present a selection of point- and interval- estimates of the time-varying parameters (local level intercept and slopes) as well as the computed growth accounting exercises. In our empirical model we assume constant returns to scale of the production function, expressing it in the intensive form: output per worker on the left hand side; capital per worker and materials per worker on the right hand side. We further assume for the TV model that second order parameters of the translog specification are time invariant. Consequently we have two time varying first order slope parameters and three time invariant second order slope parameters (in the TV specification):

$$y_{it} = a_{it} + \beta_{kt} k_{it} + \beta_{mt} m_{it} + \beta_{kk} k_{it}^{2} + \beta_{mm} m_{it}^{2} + \beta_{km} m_{it} k_{it} + \epsilon_{it}$$

where y is the log of output per worker, k is the log of capital per worker and m is the log of materials per worker. We estimate all the models separately for each of the 20 industrial sectors, thus applying them to each sectoral panel dataset individually.

5.2.1 Comparison of the models

In Table 2 we provide AIC and BIC values (along with R-squared and AdjR-squared) for each model in each sector. The first thing to note is that the fixed effects model provides a very good fit of the data. In all sectors analyzed the R-square is above 90% and the corresponding AIC value below 0.100. The fixed effects model is always the best performer in terms of both AIC and BIC criteria: the loss in statistical fitting (when compared to the other models) is always more than compensated by the lower number of parameters to be estimated. The second general pattern of these results is that our simple stochastic time trend model (TV) outperform the CSS model uniformly. The level of flexibility of the TV model is very close to the CSS model, but it is parsimonious to the extent that its performance is comparable to the fixed effects model. For example in sector Fuel (Coke, Refined Petroleum and Nuclear Fuel) the R-square of the TV model is equal to the R-square of the CSS model, although this fitting is reached with less parameters and this explains why the BIC and AIC statistics are so different. For this sector the AIC value of the CSS model is 0.228 against a 0.105 of the TV model; interestingly enough the fixed effects, with a value of 0.101, has a performance very similar to the TV model. In terms of the BIC the results are qualitatively the same, although (due to the stronger penalization for degree of freedom loss of the BIC) the differences are larger. The story in other sectors is very similar, leading us to conclude that the TV model represents a very good compromise between flexibility (statistical fitting) and parsimony (low number of parameters). In the following section we discuss the results on productivity change and growth accounting decomposition derived from the TV model.

[Table 2 here]

5.2.2 Patterns of growth

Applying the TV econometric model to the 20 panel datasets (sectors) returns 260 time varying intercepts, 40 time varying slope coefficients and 7540 growth accounting decompositions (one for each country, in each sector, in each time period). In order to make all this information manageable and interpretable, we decided to illustrate only the trends of 4 countries (US, Germany, Italy and Japan) and present the growth accounting exercise only for 3 sectors: Electrical and Optical Equipment, Post and Telecommunications, Chemicals. The first two sectors represent the most dynamic sectors in terms of productivity since they are the protagonists of the IT industrial revolution (basically computers and mobile phones). The last sector is interesting because it is a "classical" industrial sector and it illustrates well the notion of biased technical change. Results for all the other countries and all the other sectors are available on request.

Figure 1 reports intercept trends for the selected countries in the Electrical and Optical Equipment sector. From this figure emerges clearly the US boom in the IT sector, with a consistently uptrending intercept. On the contrary the other selected countries do not show any clear trend and indeed present a quite stable pattern. For this sector we report in Figure 2 the trends in the first order slope coefficients. Both the capital and materials coefficients are basically time invariant (the capital coefficient is not significantly different from zero, although the second order coefficients (not shown) are). This points out to a Hicks neutral productivity change in the Electrical and Optical Equipment sector.

[FIGURES 1 and 2 HERE]

Figures 3 and 4 report the same trends in the parameters for the Post and Telecommunications sector. Time varying intercepts follow different trends for the different countries. For example the US shows a declining intercept, Italy slightly growing and Germany very stable. The slope coefficients are very stable indicating that in this sector productivity change is basically Hicks neutral.

[FIGURE 3 and 4 HERE]

Figures 5 and 6 show the results on parameter trends for the Chemical sector. Here the picture is very different with country intercepts following very different trends and slope coefficient showing a bias in technical change. The capital coefficient is strongly upward trending, benefitting countries in which the capital per worker endowment is higher. This also means that for the chemical sector the bias in productivity change is a potentially important contributor to productivity change; increased capital deepening will increase labour productivity through the direct factor accumulation channel and indirectly through a larger benefit from the bias.

[FIGURE 5 and 6 HERE]

In Figure 7 we report the growth accounting of the US for different sectors (averaging across decades). From this picture emerges clearly the central role played by the Electrical and Optical Equipment sector in US productivity growth. Labour productivity growth (output per worker growth) in the Electrical and Optical Equipment sector is above 8% as an average on the 30 years period analyzed and almost a third of this growth is due to a TFP growth of around 2.5% per annum (the rest is due to capital and materials deepening, i.e. an increase in capital and materials per worker). Since the Electrical and Optical Equipment sector shows a Hicks neutral type productivity growth, the bias in technical change does not appear as a contributor to productivity change. In terms of labour productivity growth, the second best performing sector in the US is Post and Telecommunications with slightly above 4% average annual growth. TFP in this sector for the US has been slightly negative which means all the growth in output per worker is explained with an increase in the endowment of capital and materials per worker (i.e., factor accumulation). Finally, it is interesting to note the patterns of productivity for the Chemical sector in the US. Labour productivity growth has been around 2.5% on average. TFP explains a quarter of this growth, although the pattern in this sector is very different from the others. In fact, since the chemical sector faced a bias in technical change (with a particularly strong growth in the capital first order coefficient), the bias component is driving up productivity change. It should be noted that the US was not able to exploit this upward pattern in the bias because of a trending down intercept, resulting in an overall TFP growth lower than the bias component. This is especially evident in the average across the first decade (1977-1987) where a positive 2% bias was accompanied with a much lower growth in labour productivity (due to the downward trend in the intercept). Finally, it is interesting to point out that the US experimented a negative TFP growth (although more than compensated by capital per worker and materials per worker growth) in the Construction sector and the Mining and Quarrying sector. This results are very close to the finding of Jin and Jorgenson (2009); in fact their results too point out to a high TFP growth in the two IT sectors (Electrical and Optical Equipment, Post and Telecommunication) and a negative TFP growth in the resource base traditional sectors (Construction, Mining and Quarrying). These overall results indicate that there has been an IT-paradigm growth in the last 30 years in the US.

[FIGURE 7 HERE]

Figure 8 reports the growth accounting results for Germany. Here we see a very different pattern emerging: Germany TFP growth has been below 0.5% in all sectors (with some of them having no growth at all), while labour productivity growth has been well above 2% per annum in all sector. This points to a labour productivity growth sustained basically by accumulation of factors. Although the Electrical and Optical Equipment sector presents a growth of more than 5% per annum, this is not the best performing sector in Germany. Post and Telecommunication plays an important role (especially in the last decade), but all the more traditional industrial sectors have a very strong growth in Germany. For example, Textile, Transport and Storage, Electricity and Gas Supply, and Agriculture all presents labour productivity growth of around 4% per annum. This points to a different paradigm for Germany labour productivity growth based on a stable flow of investment aimed at increasing capital per worker and the ability to process materials into final products. The boom in the IT industry, although present in the data (they are amongst the best performing sectors) is not as strong as in the US and the IT sectors are not the only sectors in which strong growth is present.

[FIGURE 8 HERE]

Finally, it is interesting to address here explicitly the case of the Italian industrial crisis. In Figure 9 the growth accounting for Italy is reported. The figures for the overall average across all the 30 years period hide the big differences across decades. Labour productivity growth for most sectors during the first two decades (1977-1987, 1988-1997) has been very stable around 4% per annum (and in many sectors well above this figure). This pattern is very close to the type of growth observed for Germany, with high labour productivity growth in most sector driven by accumulation of factor. However, in the last decade (1998-2007) there was a collapse in the labour productivity growth of most sectors to around 2% per annum (this is a drop of more than 2 percentage points for most sectors). An interesting example which illustrates this is the Chemical industry. A growth in labour productivity of around 6% in the previous two decades (with a positive contribution of TFP) becomes a growth of 1.5% in the last decade. The only two exceptions to this trend are Post and Telecommunications and Electricity and Gas Supply. The possible reason for such a good performance in these two sectors is the liberalization of the mid 90's which produced a boom both in labour productivity and TFP. It is also interesting to stress the collapse in the Electrical and Optical Equipment sector. Labour productivity growth declined from above 5% in the first two decades to less than 2% in the last decade.

[FIGURE 9 HERE]

Figure 10,11 and 12 report a comparison of growth accounting for all the countries in the sample for the selected 3 sectors. From these figures it is evident the role played by the bias in technical change in the Chemical sector and the IT paradigm in the US. It is also interesting to see how each country follows a different pattern and thus the US IT paradigm cannot be generalized to all the OECD countries included in this study.

[FIGURE 10, 11, 12 HERE]

6 Conclusion

In this paper we introduced a state-space approach to deal with time variation of individual heterogeneity in a panel. The model is flexible enough to accommodate time variation also in the vector of slope parameters. Our preferred interpretation of this flexible specification is as a stochastic frontier model, with individual time varying heterogeneity being interpreted as time varying inefficiency (in line with Schmidt and Sickles, 1984, Cornwell, Schmidt and Sickles, 1990 and Kneip et al., 2011). We show how the model can nest some of the models introduced in the stochastic frontier literature and present a detailed discussion of a number of those models. Among these nested models, our preferred model is a pure stochastic trend model which preserves a high degree of flexibility (statistical fitting) with a low number of parameters (parsimony). The preferred model shows better performance according to the BIC and AIC criteria when compared to the other nested models. We present the state space representation and an estimation procedure based on the Kalman filter. A post estimation growth accounting is derived for the framework developed in this paper which allows the decomposition of observed labour productivity growth (output per worker) into TFP, bias in technical change and factor deepening effect (increase in capital per worker and materials per worker). The model is applied to the EU-KLEMS dataset, from which we selected 13 countries across 30 years (1977-2007) for 20 industrial sectors. In empirical terms the model is able to identify the IT productivity boom in the US, the stable labour productivity growth in Germany (based on a stable flow of investment) and the industrial crises of Italy observed in the past 15 years. These empirical results support our original expectation that the model retains a high level of flexibility while being very parsimonious on the number of estimated parameters.

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Table 1. Descriptive Statistics

	Descriptive Statistics by sector (1977-2007)															
	Output				Labour				Capital				Materials			
	Min	Mean	STD	Max	Min	Mean	STD	Max	Min	Mean	STD	Max	Min	Mean	STD	Max
Wood and products of wood and cork	828	13,440	20,080	99,378	21	285	349	1,426	64	1,233	2,013	8,693	689	8,372	12,996	71,547
Coke, refined petroleum and nuclear fuel	594	26,013	41,693	194,522	1	60	99	491	2	4,827	9,946	56,428	699	24,225	35,924	169,595
Chemicals and chemical	2,396	72,095	95,269	461,750	29	486	645	2,629	352	12,673	21,886	101,700	1,335	42,621	57,827	282,576
Rubber and plastics	960	29,991	36,060	170,767	21	421	559	2,103	115	3,475	5,760	29,137	564	16,357	21,961	109,391
Other non-metallic mineral	1,478	25,253	24,045	96,428	22	376	402	1,515	155	3,580	5,386	22,047	790	12,112	12,117	52,665
Machinery, nec	3,363	60,374	74,962	288,313	64	922	1,110	4,237	135	5,932	9,393	48,983	2,140	34,887	41,582	172,417
Post and telecommunications	840	56,011	108,844	734,119	66	748	1,175	5,383	211	15,496	31,465	194,701	276	19,173	43,284	314,477
Food , beverages and tobacco	5,235	91,169	115,655	550,211	62	1,037	1,161	4,218	394	10,913	17,076	71,771	3,900	65,003	85,400	436,668
Textiles, textile , leather and footwear	959	31,519	40,667	166,466	14	926	1,183	4,901	93	2,511	3,819	15,639	784	22,634	26,644	109,259
Pulp, paper, paper , printing and publishing	3,088	54,036	85,259	387,482	59	874	1,276	5,090	134	7,279	13,058	60,036	2,187	31,507	46,726	231,100
Basic metals and fabricated metal	2,735	87,388	105,092	419,053	68	1,282	1,532	6,872	187	10,834	16,935	63,705	1,875	54,146	62,838	255,769
Electrical and optical equipment	1,274	77,569	143,818	865,165	59	1,214	1,756	6,721	33	10,635	21,627	107,885	1,109	46,756	73,305	410,514
Transport equipment	1,004	86,388	141,471	669,023	19	899	1,261	5,095	5	6,538	11,182	51,337	1,132	64,643	97,235	488,287
Manufacturing nec; recycling	1,255	22,571	28,298	144,508	26	521	648	2,557	4	1,961	3,092	15,774	529	13,574	16,633	82,133
Transport and storage	5,281	79,752	106,201	578,825	211	2,060	2,357	9,000	601	11,208	16,467	95,496	2,661	47,121	55,018	282,599
Agriculture, hunting, forestry and fishing	3,912	40,804	61,179	302,858	111	2,482	2,853	16,180	33	10,390	17,425	70,888	3,169	27,188	37,504	166,401
Mining and quarrying	128	22,024	43,684	179,254	5	250	501	2,652	43	7,034	14,528	68,419	150	10,456	21,917	118,322
Electricity, gas and water supply	1,388	42,786	66,832	297,766	19	310	381	1,633	871	19,406	33,591	154,517	726	23,747	29,299	151,814
Construction	10,950	161,265	175,377	816,186	226	3,953	4,669	19,975	72	11,843	13,984	67,926	7,269	91,119	90,980	491,109
Hotels and restaurants	2,471	59,378	98,319	512,795	83	2,643	3,803	15,852	23	8,964	15,126	71,724	1,076	33,016	49,066	262,974

SOURCE: EU-KLEMS Dataset. List of countries included: Australia, Austria, Belgium, Denmark, Spain, Finland, France, Germany, Italy, Japan, Netherlands, UK, USA.

Table 2 - Comparison of Models

	WOOD				FUEL				CHEMICAL			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.989	0.989	0.309	0.105	0.991	0.991	0.308	0.105	0.999	0.999	0.306	0.103
CSS	0.997	0.997	0.674	0.226	0.991	0.99	0.676	0.228	0.998	0.998	0.673	0.226
FEDT	0.991	0.991	0.339	0.115	0.97	0.969	0.345	0.121	0.993	0.993	0.338	0.114
	PLASTICS				NON-METALLIC MINERAL				MACHINERY			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.996	0.996	0.307	0.103	0.994	0.993	0.307	0.103	0.995	0.995	0.307	0.103
CSS	0.997	0.997	0.674	0.226	0.997	0.996	0.674	0.226	0.997	0.996	0.674	0.226
FEDT	0.976	0.975	0.342	0.118	0.99	0.989	0.338	0.114	0.981	0.979	0.339	0.116
	TELECOMMUNICATIONS								FOOD AND BEVERAGES			
	R^2	R^2 Adj.	BIC	AIC					R^2	R^2 Adj.	BIC	AIC
TV	0.996	0.996	0.308	0.104					0.991	0.99	0.308	0.104
CSS	0.996	0.996	0.675	0.227					0.999	0.999	0.673	0.226
FEDT	0.986	0.985	0.343	0.119					0.995	0.995	0.337	0.114
TEXTILES, LEATHER AND FOOTWEAR					PULP, PAPER, PRINTING				METAL			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.985	0.984	0.309	0.106	0.999	0.999	0.306	0.103	1	1	0.306	0.103
CSS	0.999	0.999	0.673	0.226	0.998	0.998	0.673	0.226	0.998	0.998	0.673	0.226
FEDT	0.996	0.996	0.337	0.114	0.994	0.993	0.337	0.114	0.983	0.982	0.339	0.115
	ELECTRICAL AND OPTICAL				TRANSPORT EQUIPMENT				MANUFACTURING			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.958	0.956	0.32	0.117	0.964	0.962	0.316	0.113	0.999	0.999	0.306	0.103
CSS	0.999	0.998	0.674	0.226	0.998	0.998	0.674	0.226	0.997	0.997	0.674	0.226
FEDT	0.986	0.985	0.341	0.118	0.996	0.995	0.338	0.114	0.992	0.991	0.338	0.115
	TRANSPORT AND STORAGE				ELECTRICITY, GAS AND WATER				AGRICULTURE AND FISHING			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.985	0.984	0.308	0.105	0.973	0.972	0.311	0.108	0.902	0.897	0.372	0.169
CSS	0.998	0.997	0.673	0.226	0.996	0.995	0.674	0.227	0.999	0.998	0.674	0.227
FEDT	0.993	0.993	0.338	0.114	0.986	0.985	0.339	0.116	0.996	0.996	0.339	0.116
	MINING AND QUARRYING				HOTELS AND RESTAURANTS				CONSTRUCTION			
	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC	R^2	R^2 Adj.	BIC	AIC
TV	0.957	0.954	0.364	0.148	0.999	0.999	0.352	0.121	0.97	0.968	0.33	0.114
CSS	0.994	0.993	0.676	0.233	0.998	0.997	0.668	0.231	0.997	0.996	0.671	0.228
FEDT	0.979	0.978	0.361	0.134	0.989	0.988	0.353	0.123	0.991	0.99	0.344	0.118

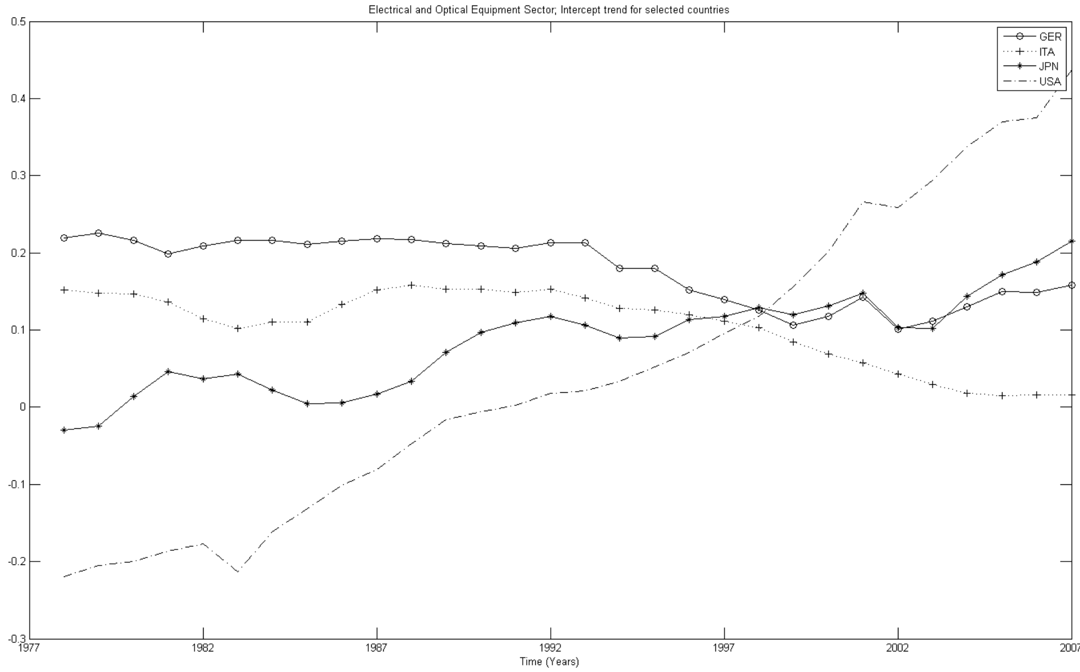

Figure 1: Intercept Trend (local level) for Selected Countries in the Electrical and Optical Equipment Sector

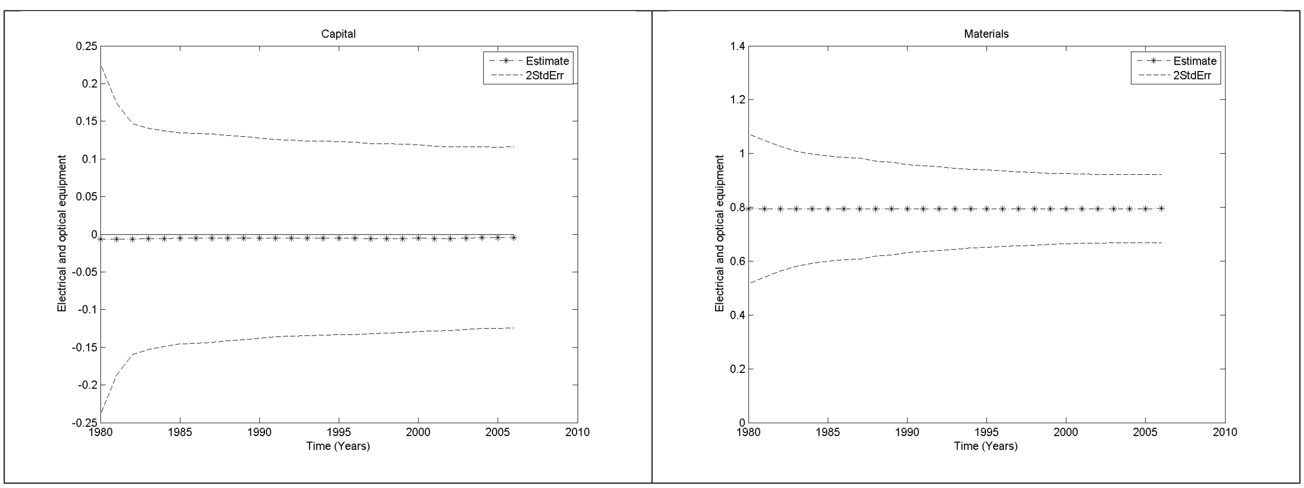

Figure 2: Estimates of the First Order Slope Coefficients for the Electrical and Optical Equipment Sector

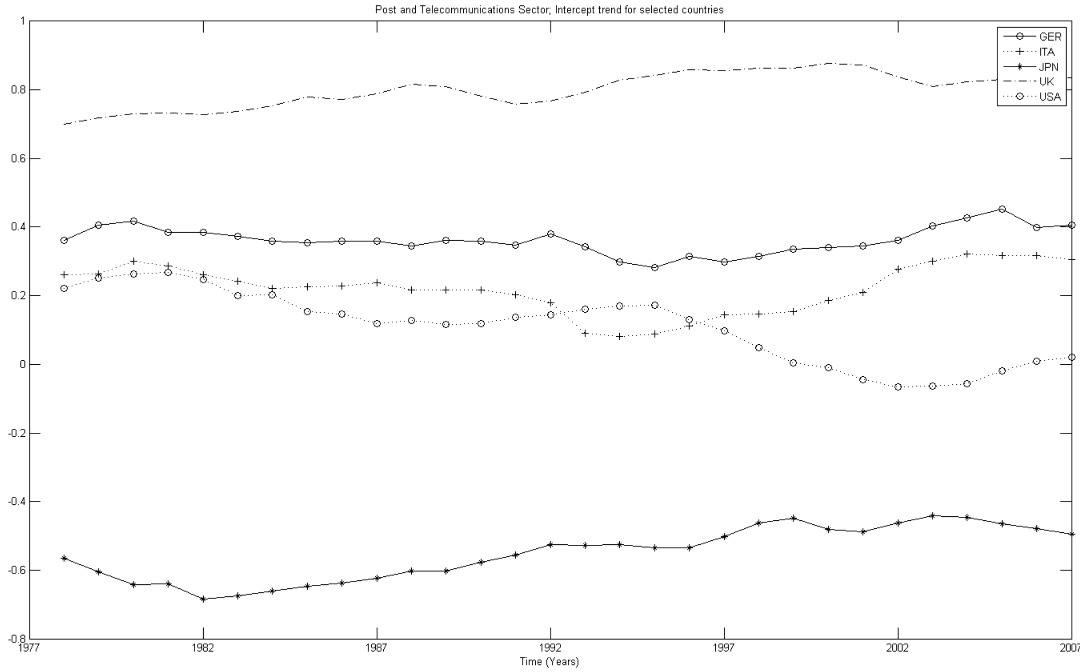

Figure 3: Intercept Trend (local level) for Selected Countries in the Postal and Telecomunications Sector

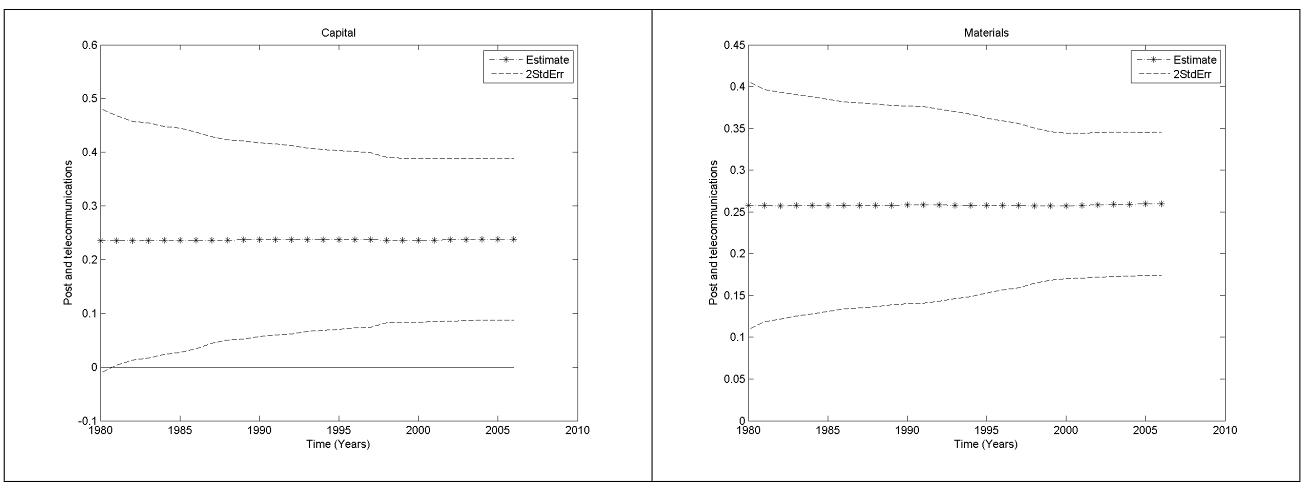

Figure 4: Estimates of the First Order Slope Coefficients for the Postal and Telecommunications Sector

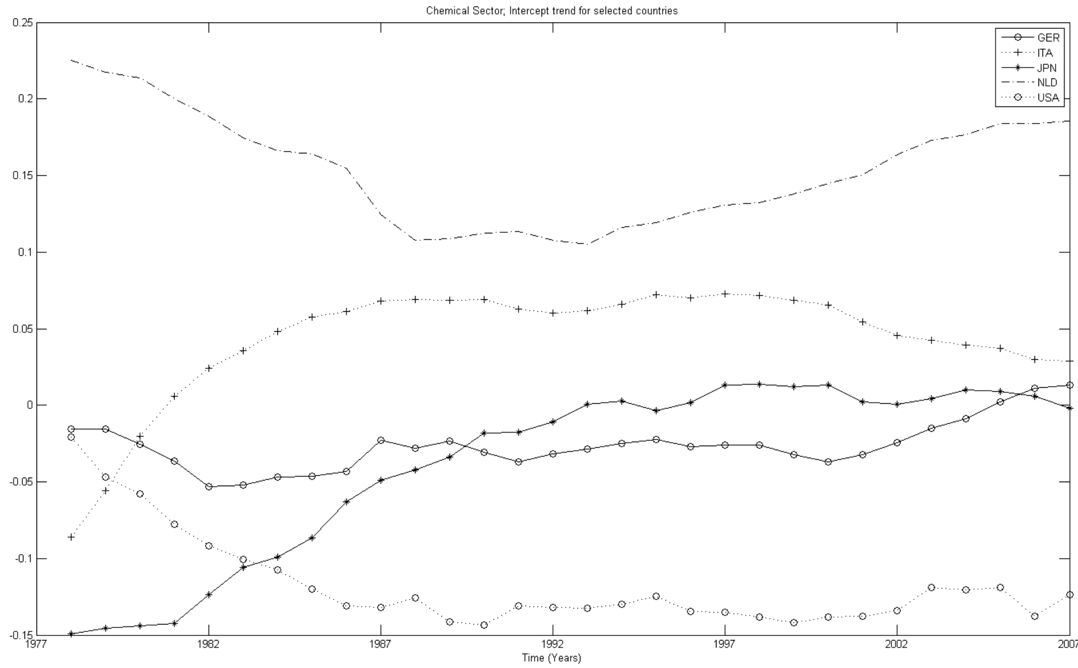

Figure 5: Intercept Trend (local level) for Selected Countries in the Chemical Sector

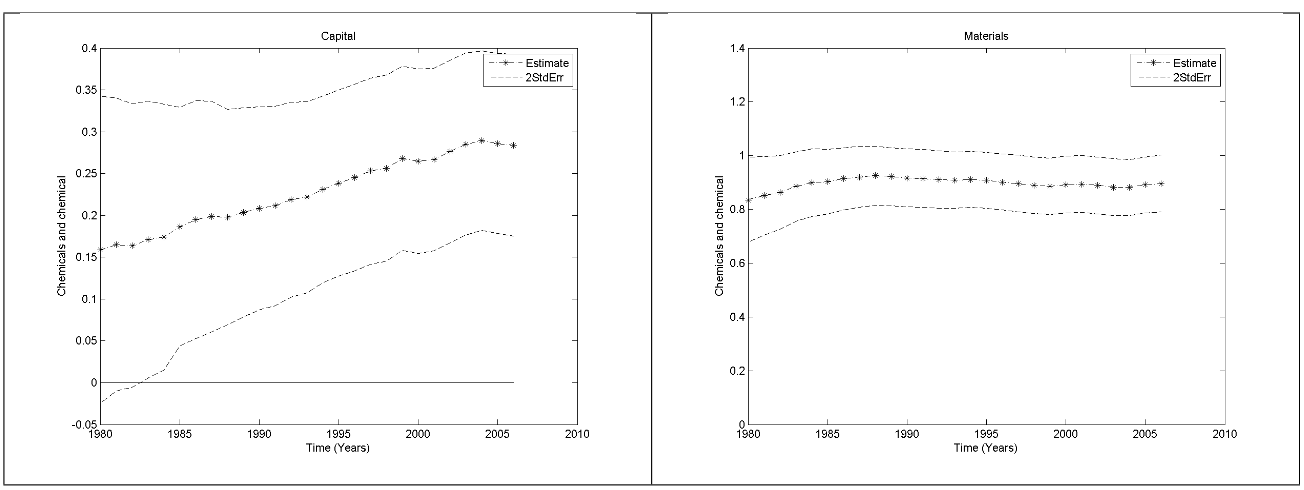

Figure 6: Estimates of the First Order Slope Coefficients for the Chemical Sector

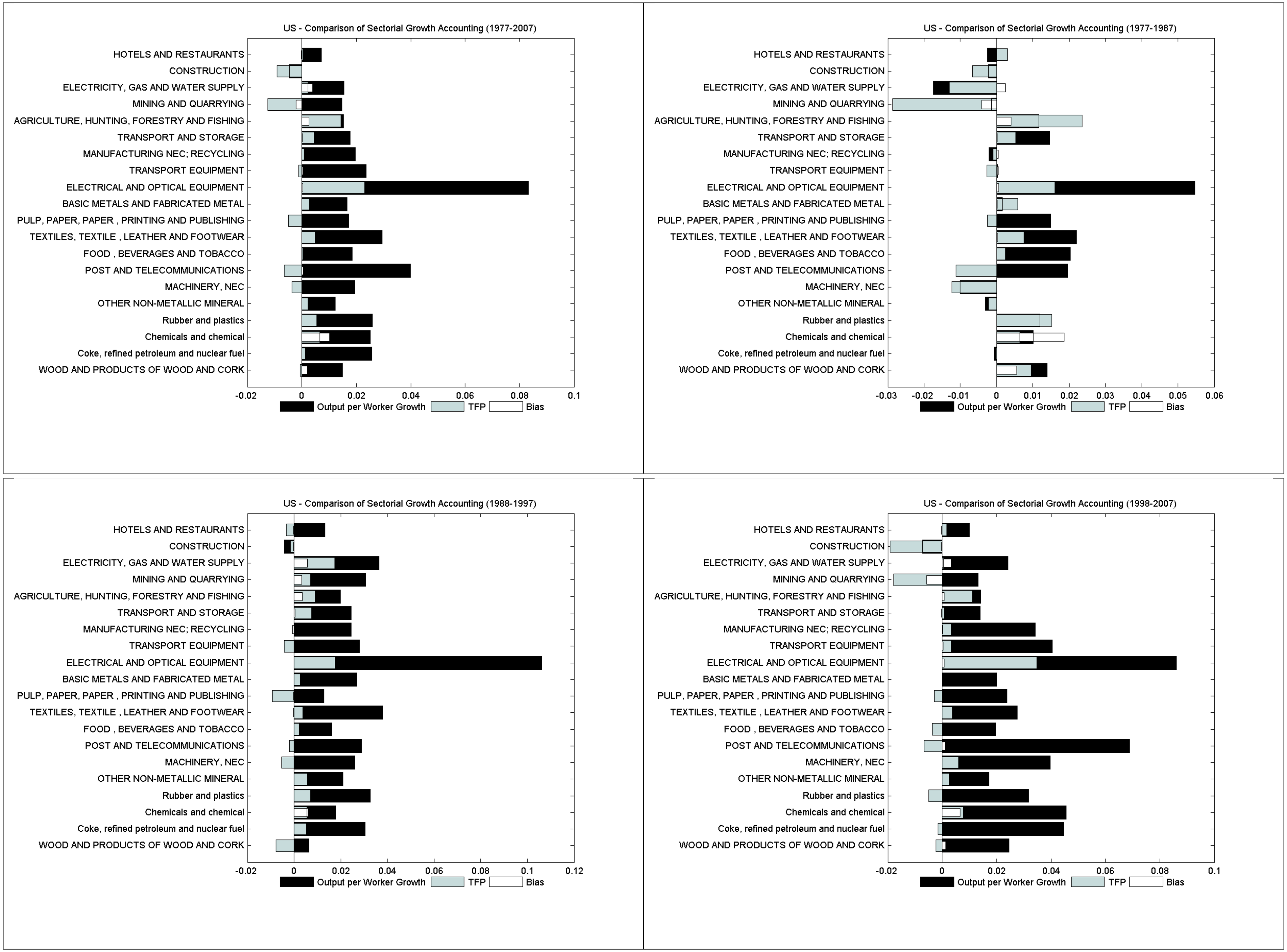

Figure 7: US Growth Accounting

Figure 8: Germany Growth Accounting

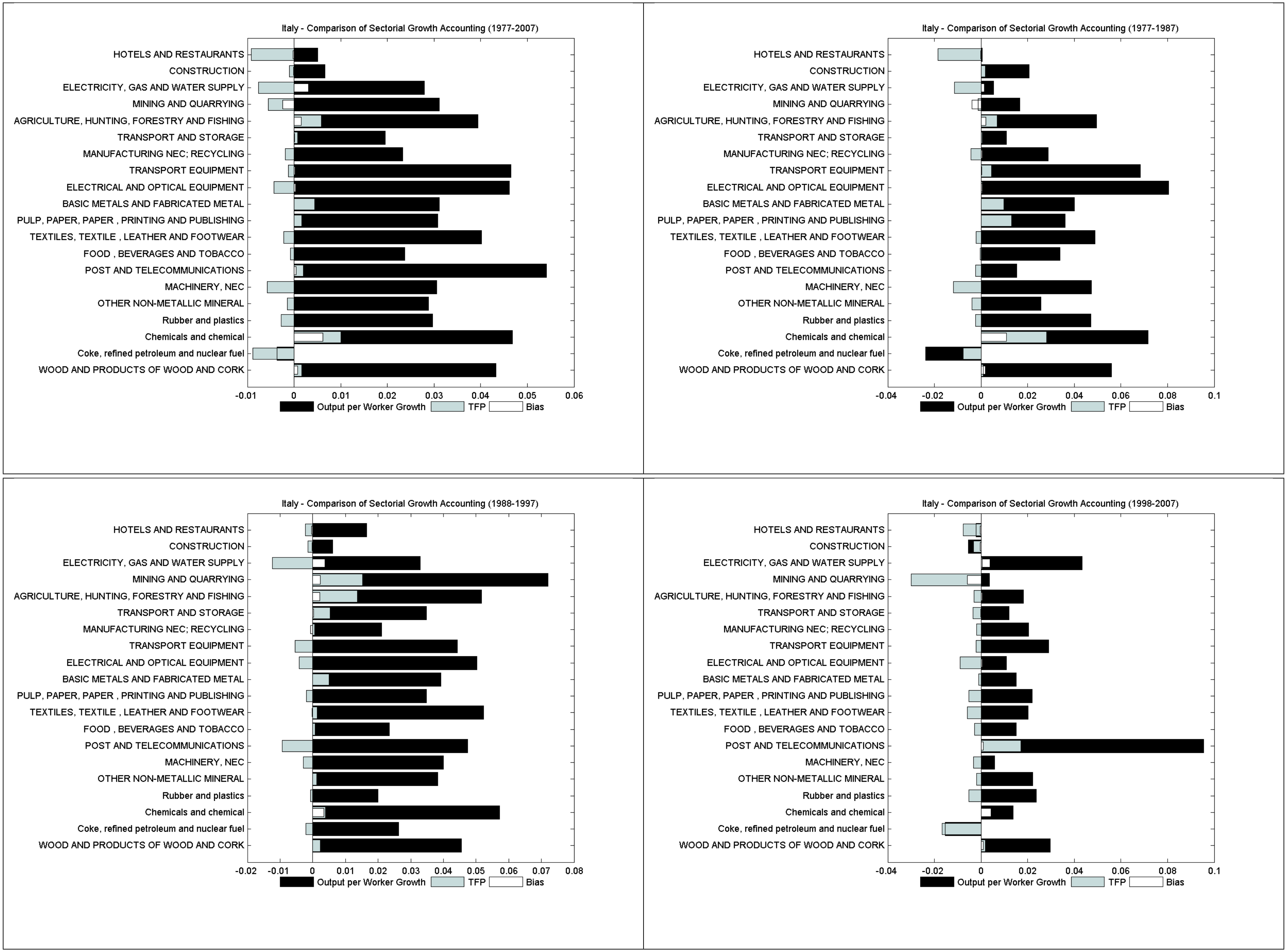

Figure 9: Italy Growth Accounting

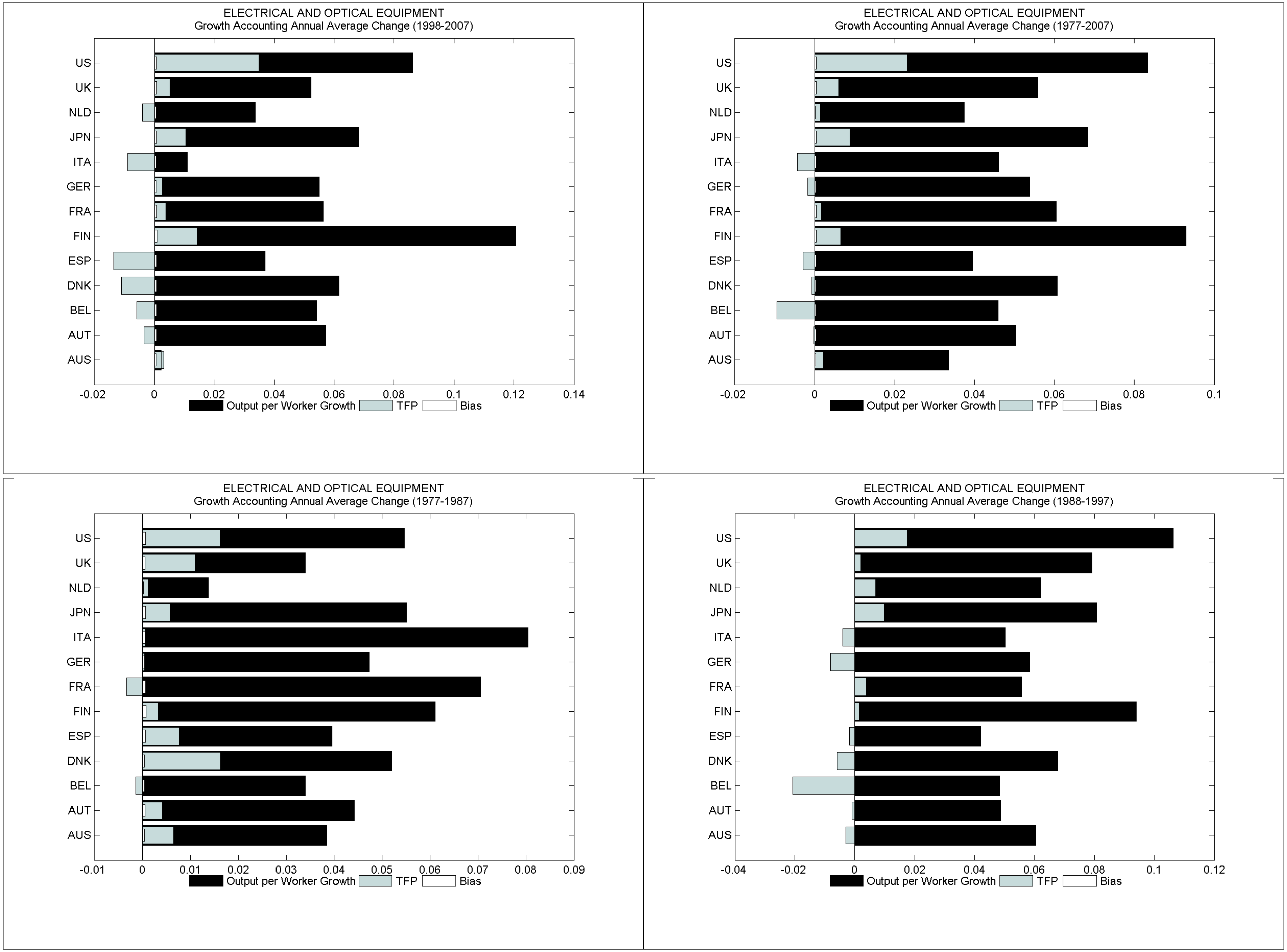

Figure 10: Electrical and Optical Equipment Growth Accounting

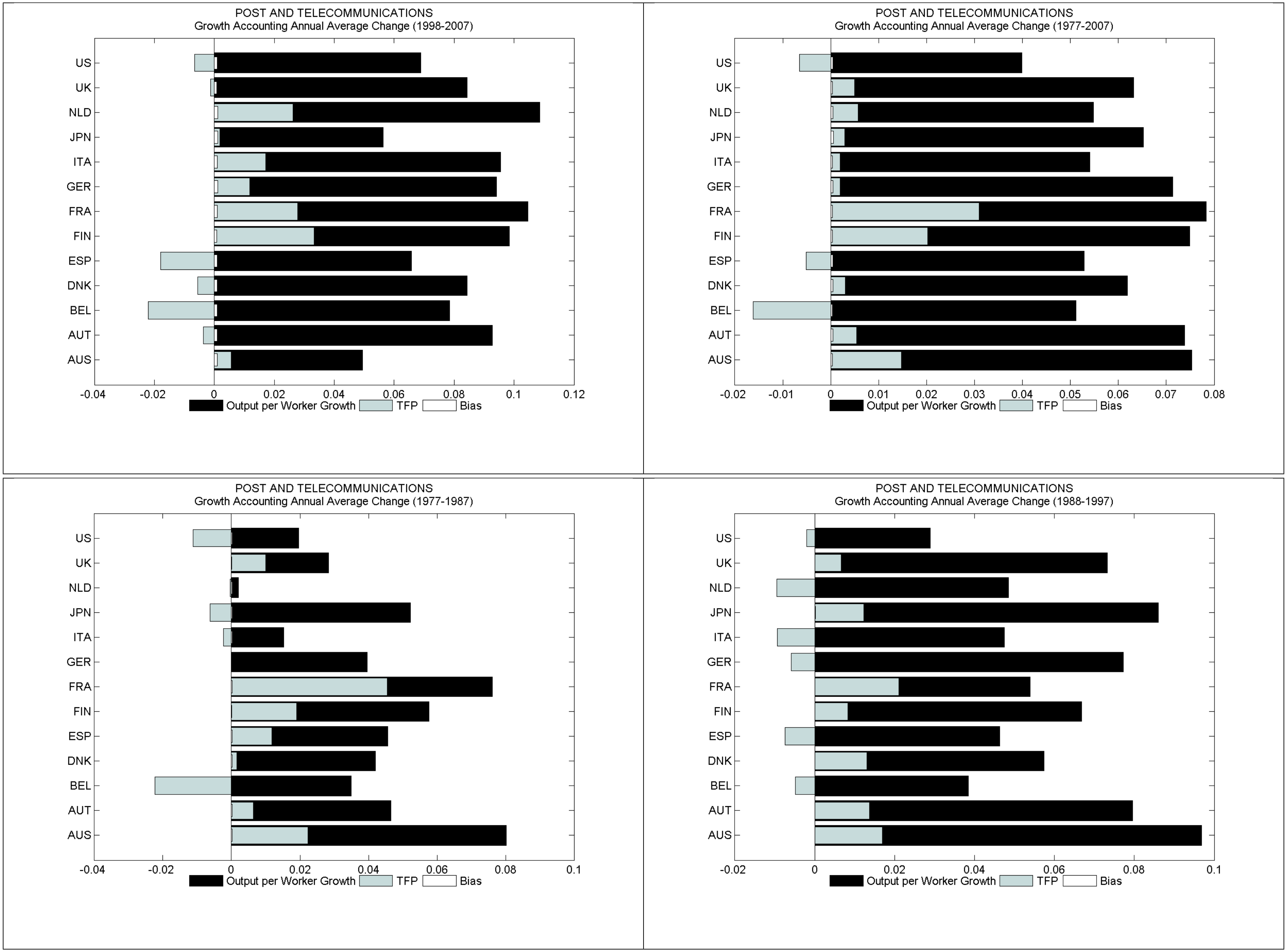

Figure 11: Post and Telecomunication Sector Growth Accounting

Figure 12: Chemicals and chemical Sector Growth Accounting

A Appendix

A.1 Quadratic Trend Specification

Consider a quadratic time trend specification for a parameter γ_t:

$$\gamma_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2$$

The double trend deterministic specification would be:

$$\gamma_t = c_{t-1} + \gamma_{t-1}$$

$$c_t = \mu + c_{t-1}$$

where a starting value as to be specified for the time varying parameters: c_0, γ_0. Inserting the second equation into the first, one obtains:

$$\gamma_t = c_{t-1} + \gamma_{t-1} = \mu + c_{t-2} + \gamma_{t-1} = 2\mu + c_{t-3} + \gamma_{t-1} = ... = (t-1)\,\mu + c_0 + \gamma_{t-1}$$

Further developing this expression:

$$\gamma_t = (t-1)\,\mu + c_0 + \gamma_{t-1} = (t-1)\,\mu + c_0 + (t-2)\,\mu + c_0 + \gamma_{t-2} = ... = \mu \sum_{\tau=1}^{t} (t-\tau) + (t-1)\,c_0 + \gamma_0$$

and re-arranging terms: $\mu \sum_{\tau=1}^{t} (t-\tau) = \mu \left(\sum_{\tau=1}^{t} t - \sum_{\tau=1}^{t} \tau \right) = \mu \left(t^2 - \frac{t(t+1)}{2} \right) = \frac{\mu}{2} \left(2t^2 - t^2 - t \right) = \frac{\mu}{2} \left(t^2 - t \right)$

$$\gamma_t = \frac{\mu}{2} \left(t^2 - t \right) + c_0 t + \gamma_0 - c_0 = \frac{\mu}{2} t^2 - \frac{\mu}{2} t + c_0 t + \gamma_0 - c_0 = (\gamma_0 - c_0) + \left(c_0 - \frac{\mu}{2} \right) t + \frac{\mu}{2} t^2$$

which implies the following:

$$\alpha_0 = \gamma_0 - c_0$$
$$\alpha_1 = c_0 - \frac{\mu}{2}$$
$$\alpha_2 = \frac{\mu}{2}$$

i.e. the quadratic time trend specification correspond to a deterministic double trend specification.

A.2 The Transformation Matrix

In order to compute the transformation matrix Γ_t, the following calculations are useful:

$$D = \begin{bmatrix} I_N & I_N & & & & \\ & I_N & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & I_K & I_K \\ & & & & & I_K \end{bmatrix} \tag{28}$$

$$D \cdot D = \begin{bmatrix} I_N & I_N & & & & \\ & I_N & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & I_K & I_K \\ & & & & & I_K \end{bmatrix} \begin{bmatrix} I_N & I_N & & & & \\ & I_N & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & I_K & I_K \\ & & & & & I_K \end{bmatrix} = \\ = \begin{bmatrix} I_N & 2I_N & & & & \\ & I_N & & & & \\ & & 1 & 2 & & \\ & & & 1 & & \\ & & & & I_K & 2I_K \\ & & & & & I_K \end{bmatrix} \tag{29}$$

$$D \cdot D \cdot \ldots \cdot D = \begin{bmatrix} I_N & tI_N & & & & \\ & I_N & & & & \\ & & 1 & t & & \\ & & & 1 & & \\ & & & & I_K & tI_K \\ & & & & & I_K \end{bmatrix} \tag{30}$$

Finally, the transformation matrix Γ_t will be:

$$\Gamma_t = \textstyle\sum_{j=0}^{t-1} D^j = I + D + D^2 + ... + D^{t-1} = \\ \begin{bmatrix} tI_N & \frac{t(t-1)}{2} I_N & & & & \\ & tI_N & & & & \\ & & t & \frac{t(t-1)}{2} & & \\ & & & t & & \\ & & & & tI_K & \frac{t(t-1)}{2} I_K \\ & & & & & tI_K \end{bmatrix} \tag{31}$$

$$\Gamma_t c = \begin{bmatrix} tI_N & \frac{t(t-1)}{2} I_N & & & & \\ & tI_N & & & & \\ & & t & \frac{t(t-1)}{2} & & \\ & & & t & & \\ & & & & tI_K & \frac{t(t-1)}{2} I_K \\ & & & & & tI_K \end{bmatrix} \begin{bmatrix} c_\gamma \\ c_\phi \\ c_\mu \\ c_\nu \\ c_\beta \\ c_\tau \end{bmatrix} = \begin{bmatrix} tc_\gamma + \frac{t(t-1)}{2} c_\phi \\ tc_\phi \\ tc_\mu + \frac{t(t-1)}{2} c_\nu \\ tc_\nu \\ tc_\beta + \frac{t(t-1)}{2} c_\tau \\ tc_\tau \end{bmatrix}$$

$$
\alpha_t^* = \begin{bmatrix} \alpha_t - \Gamma_t c \\ c \end{bmatrix} = \begin{bmatrix} \gamma_t - tc_\gamma - \frac{t(t-1)}{2} c_\phi \\ \phi_t - tc_\phi \\ \mu_t - tc_\mu - \frac{t(t-1)}{2} c_\nu \\ \nu_t - tc_\nu \\ \beta_t - tc_\beta - \frac{t(t-1)}{2} c_\tau \\ \tau_t - tc_\tau \\ c_\gamma \\ c_\phi \\ c_\mu \\ c_\nu \\ c_\beta \\ c_\tau \end{bmatrix} \tag{32}
$$