EIEF WORKING PAPER SERIES

EIEF

EINAUDI INSTITUTE FOR ECONOMICS AND FINANCE

EIEF Working Paper 16/09
December 2009

Monetary Policy and Price Responsiveness to Aggregate Shocks under Rational Inattention

by

Luigi Paciello

(EIEF)

Monetary Policy and Price Responsiveness to Aggregate Shocks under Rational Inattention*

Luigi Paciello

Einaudi Institute for Economics and Finance

September 25, 2011

Abstract

This paper studies a general equilibrium model that is consistent with recent empirical evidence showing that the U.S. price level and inflation are much more responsive to aggregate technology shocks than to monetary policy shocks. Specifically, we show that the fact that aggregate technology shocks are more volatile than monetary policy shocks induces firms to pay more attention to the former than to the latter. However, most important, this work adds to the literature by analytically showing how monetary policy feedback rules affect the incentives faced by firms in allocating attention. A policy rule responding more actively to inflation fluctuations induces firms to pay relatively more attention to more volatile shocks, helping to rationalize the observed behavior of prices in response to technology and monetary policy shocks.

JEL Codes: E31, E4, C11, C3

Keywords: Inattention, inflation, monetary policy

*Email: luigi.paciello@eief.it. Mailing address: EIEF, Via Sallustiana, 62, 00187 Roma.
I thank the Editor Paul Evans and two anonimous referees, as well as Pierpaolo Benigno, Martin Eichenbaum, Christian Hellwig, Francesco Lippi, Giorgio Primiceri, Daniele Terlizzese, and Mirko Wiederholt for their comments.

1 Introduction

Empirical work on nominal price adjustment has shown that the U.S. aggregate price level and inflation are much more responsive to aggregate technology shocks, such as innovation in total factor productivity, than to monetary policy shocks, such as unexpected innovations in the Federal Funds rate.[1] Moreover, the difference between inflation adjustment speed to the two shocks is more evident in the sub-sample coinciding with Volcker and Greenspan at the helm of the Federal Reserve.[2] Standard models of sticky prices have a hard time explaining the different behavior of the price level and inflation in response to these two aggregate shocks.[3] Indeed, one of the central issues in modern macroeconomics is understanding how firms set their prices in response to different aggregate shocks. This is an important task for monetary policy analysis and implementation. Understanding the transmission of technology and monetary policy shocks is particularly relevant as these shocks account together for a large fraction of business cycle fluctuations.[4]

We study a model that is consistent with the empirical evidence that prices respond much more quickly to aggregate technology shocks than to monetary policy shocks. We show that this response pattern arises naturally in a framework based on imperfect information with an endogenous choice of information structure similar to Sims (2003). In this model, firms will optimally choose to allocate more attention to those particular shocks that, in expectations, most reduce profits when prices are not adjusted properly. The more attention firms pay to a type of shock, the faster they respond to it.

This is a result that has been emphasized in the seminal paper by Maćkowiak and

[1]See Altig, Christiano, Eichenbaum and Linde (2011).
[2]See Paciello (2011).
[3]See Dupor, Han and Tsai (2009).
[4]See, for intance, Smets and Wouters (2007).

Wiederholt (2009), where these authors have shown that firms pay more attention to sector specific shocks than to aggregate nominal shocks roughly because the former are much more volatile than the latter. So, at first sight, this result would directly translate to a framework with aggregate technology and monetary policy shocks: since in the U.S. aggregate technology shocks are more volatile than monetary policy shocks, everything else being equal, firms allocate more attention to the former than to latter, inducing faster price responses to technology shocks.[5]

However, most important, we show that this is not the whole story. In a standard general equilibrium model, for given shock volatilities, the coefficients of the monetary policy rule may amplify or reduce differences in attention allocation across different types of shocks. This paper adds to the literature on two dimensions. First, the paper provides an analytical characterization of the solution to the general equilibrium model. This analytical solution allows to capture fully the interaction between monetary policy, real rigidities and complementarity in attention allocation. Second, most important, the paper shows that allowing for a feedback rule in monetary policy improves substantially the ability of the rational inattention model to match the relatively fast response of inflation to aggregate productivity shocks, and the relatively slow response to monetary policy shocks. In particular, we show that, when the systematic component of monetary policy responds more to inflation fluctuations, complementarity in attention allocation increases. This higher complementarity induce firms to pay more attention to the same variables that other firms pay more attention to, amplifying the difference in price responsiveness to technology and monetary policy shocks. We show that a high degree of monetary policy feedback to inflation helps to reconcile the rational inattention model with the empirical evidence about

[5]Figure 1 plots the growth rate in total factor productivity and the change in the Federal Funds rate from 1960 to 2007. Other authors have estimated the volatility of technology and monetary policy shocks within DSGE models. See, for instance, Smets and Wouters (2007).

the response of inflation to aggregate technology and monetary policy shocks.

Moreover, this novel mechanism of transmission introduces an asymmetry in the way changes in the systematic response of monetary policy to inflation influence price responsiveness to different shocks. When, for instance, policy responds more against inflation, the new equilibrium is characterized by a larger fraction of attention paid to the most volatile shocks, and a smaller fraction paid to the least volatile ones. As a consequence of the change in policy, everything else being equal, this channel of transmission causes price variability to reduce relatively less conditional on the most volatile shocks, and more conditional on the least volatile ones.

The results of this paper are obtained within a standard general equilibrium framework with a representative household, monopolistically competitive firms and a central bank that controls money supply so that the equilibrium dynamics of the nominal interest resemble the ones of a Taylor-type policy rule. In this model, prices respond more to the realizations of shocks about which firms are better informed. Technology shocks are aggregate innovations to labor productivity, while monetary policy shocks are shocks to money supply. The only friction introduced in this framework is that firms might not be well informed about the realizations of the shocks when changing their prices. The information structure of the economy is modeled along the lines of Maćkowiak and Wiederholt (2009). There is a limit on the total attention each firm can pay to the different shocks. This limit introduces a trade-off in the allocation of attention.

This paper relates to the large literature studying price setting decisions under incomplete information. Incomplete information theories have been popular in accounting for the sluggish price adjustment in response to monetary policy shocks. Behind these theories there is the assumption that firms only pay attention to a relatively small number of economic indicators. With imprecise information about aggregate conditions, prices respond with delay to changes in nominal spending. This simple idea was first proposed by Phelps (1970) and formalized by Lucas (1972). More recently Woodford (2002), Mankiw and Reis (2006), and Sims (2003), have renewed attention to imperfect information and limited information processing as sources of inertial prices.

Sims (2003) and Maćkowiak and Wiederholt (2009) study the endogenous optimal choice of the information structure. In particular, Maćkowiak and Wiederholt (2009) focus on the differential responses of prices to aggregate nominal shocks versus idiosyncratic shocks in a framework with limited information-processing capabilities, and with an exogenous process for nominal spending. In a recent and parallel work Maćkowiak and Wiederholt (2010) have extended their previous analysis to study business cycle dynamics under rational inattention in a DSGE model. Differently from this paper, Maćkowiak and Wiederholt (2010) consider an economy where both households and firms are imperfectly informed. This paper focuses instead on a simpler model where only firms are imperfectly informed so to study more in detail the interaction between monetary policy and allocation of attention in the price setting decision. This approach allows for a closed form solution to the model, which fully characterizes the channel of transmission from policy to the attention allocation decision.

Finally, within the imperfect information literature, Hellwig and Veldkamp (2009) have recently emphasized the interaction of strategic complementarity in price setting with endogenous information acquisition by firms. Relative to these authors, this paper further shows how the interaction of strategic complementarity in price setting and endogenous information acquisition depends on monetary policy activism.

The paper is organized as follows. Section 2 introduces the model. Section 3 describes the solution of the model in the case of i.i.d. shocks. Section 4 discusses the solution to persistent innovations and implications for inflation responses. Section 5 considers the case of a nominal interest rate rule. Section 6 considers the case of a cashless economy. Section 6 concludes.

2 The model

Apart from the information structure, this paper studies a standard general equilibrium model of incomplete nominal adjustment with monopolistic firms along the lines of Blanchard and Kiyotaki (1987). The information structure of firms is modeled along the lines of Maćkowiak and Wiederholt (2009). Time is discrete and infinite. Most of the analysis will focus on a model with *iid* productivity and monetary policy shocks, allowing for an analytical characterization of the equilibrium. In Section 4 we will consider the case of persistent shocks.

Household: The representative household's preferences over sequences of the final good consumption C_t and labor supply L_t are given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\psi}}{1+\psi} \right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, and $\psi > 0$ is the inverse of the Frisch elasticity. The final consumption good is obtained through a Dixit-Stiglitz aggregator over the different varieties $c_{i,t}$,

$$C_t = \left[\int_0^1 (c_{i,t})^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ determines the elasticity of substitution across varieties. The household's objective is to maximize (1) with respect to $\{C_t, L_t, c_{i,t},\ B_{t+1}, M_{t+1}\}_{\tau=0}^{\infty}$ subject to its sequence of flow budget constraints, for $\tau = 0, 1, ...$

$$M_t + B_t = R_{t-1}B_{t-1} + W_tL_t + D_t + (M_{t-1} - P_{t-1}C_{t-1}), \tag{3}$$

where P_t is the price of the final consumption good, W_t the nominal wage rate, and D_t the aggregate profits of the corporate sector rebated to the household. The representative household can transform pre-consumption wealth in period t into money balances, M_t, and bond holdings, B_t, paying an interest rate R_t in period $t+1$. The purpose of holding money is to purchase goods. We assume that the representative household faces the following cash-in-advance constraint

$$P_tC_t \leq M_t. \tag{4}$$

The representative household also faces a no-Ponzi-scheme condition. We assume for simplicity that $R_t > 1$ for all t, so that (4) is always binding.

We introduce the cash-in-advance constraint because it allows to obtain a mapping from the monetary policy instrument, i.e. the control of money supply, to the monetary policy target, i.e. the nominal interest rare. The advantage of explicitly deriving this mapping is that of avoiding multiplicity of equilibria. In Section 5 we show that our results can be extended to a cashless economy à la Woodford (2003). The formulation of the cash-in-advance constraint given above implies that there are no monetary transaction frictions because wage income can be transformed immediately into cash and cash can be spent immediately on goods. This specification of the cash-in-advance constraint is equivalent to the specification in Atkeson, Chari and Kehoe (2010).[6] We decided to adopt this formulation in the benchmark economy

[6] Atkeson, Chari and Kehoe (2010) show how this formulation is equivalent to an alternative formulation where cash has to be held for one period before it can be spent on consumption goods

for two reasons. First, this formulation of the cash-in-advance constraint allows for a more efficient and transparent solution of the model.[7] Second, this specification of our economy is observationally equivalent to the one adopted by Melosi (2011), so that we can use his empirical estimates to evaluate the predictions of our model.

The optimal demand of variety i is given by

$$c_{i,t} \equiv c\left(p_{i,t}\right) = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta} C_t. \tag{5}$$

where $p_{i,t}$ is the price of variety i. The first order conditions for labor supply, money and nominal bond demands imply

$$\frac{W_t}{P_t} = C_t L_t^{\psi}, \tag{6}$$

$$\frac{1}{R_t} = E_t\left[\frac{M_t}{M_{t+1}}\right], \tag{7}$$

$$M_t = P_t C_t. \tag{8}$$

Monetary Policy: The monetary authority controls money supply M_t, according to the following rule

$$\ln M_t = \phi_p \ln P_t + \phi_x \ln X_t + \ln Q_t, \tag{9}$$

where $X_t \equiv \frac{C_t}{C_t^*}$ is the ratio of aggregate consumption to efficient consumption C_t^*.[8] The variable Q_t is an exogenous disturbance to money supply and is given by

$$\ln Q_t = \mu_q + \rho_q \ln Q_{t-1} + \varepsilon_{q,t}, \tag{10}$$

and the government subsidizes labor with a subsidy rate equal to the risk-free nominal interest rate.

[7] This assumption avoids forward-looking variables in the problem of firms, so that the Kalman filter can be used to solve the signal extraction problem as shown by Woodford (2002).

[8] In this model output coincides with consumption. Efficient consumption corresspponds to the level of consumption in the model with flexible prices and perfect information, $C_t^* = \alpha^{\frac{\alpha}{1+\psi}} A_t$.

where $\varepsilon_{q,t}$ is an *iid* and normally distributed monetary policy disturbance, $\varepsilon_{q,t} \backsim N\left(0, \sigma_q^2\right)$, and $\rho_q \in [0,1]$. The money supply rule in (9)-(10) is the sum of a systematic component, $\phi_p \ln P_t + \phi_x \ln X_t$, that adjusts money supply in response to fluctuations in inflation and output-gap, and an exogenous stochastic process, $\ln Q_t$, capturing exogenous dynamics in money supply. The money supply rule in (9)-(10) is appealing for two reasons. First, the intertemporal Euler condition in (7) and the rule for money supply in (9)-(10) imply that the path for the nominal interest rate is, to a first order approximation, given by

$$\ln \frac{R_t}{\bar{R}} \approx \phi_p E_t \left[\ln \frac{\Pi_{t+1}}{\bar{\Pi}} \right] + \phi_x E_t \left[\ln \frac{X_{t+1}}{X_t} \right], \tag{11}$$

where $\bar{R}$ is the nominal interest rate in the non-stochastic steady state, $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. The expression for the nominal interest rate in (11) is appealing as it resembles policies of the type suggested by Taylor rules, where the nominal interest rate responds to expected inflation and output fluctuations.[9] Second, if $\phi_p = \phi_x = 0$, i.e. there is no feedback, the money rule in (9)-(10) is comparable to the rules studied by Woodford (2002), Maćkowiak and Wiederholt (2009) and Melosi (2011). This facilitates the comparison of results of this paper to those benchmark.

In Section 5, we will solve the model in the case of a cashless economy where monetary policy is specified directly in terms of the nominal interest rate.

Firms: Each variety i is produced by a single monopolistic firm using labor as the only input into production, according to

$$y_{i,t} = A_t l_{i,t}^{\alpha}, \tag{12}$$

[9] On the empirical side, a number of authors have emphasized that policy rules like (11) provide reasonable good descriptions of the way major central banks behave, at least in recent years. See, e.g., Orphanides (2003).

where A_t is an aggregate exogenous process evolving according to

$$\ln A_t = \mu_a + \rho_a \ln A_{t-1} + \varepsilon_{a,t}, \tag{13}$$

where $\varepsilon_{a,t}$ are normally distributed technology innovations to aggregate labor productivity, $\varepsilon_{a,t} \backsim N(0, \sigma_a^2)$, and $\rho_a \in [0,1]$. The parameter $\alpha \in [0,1]$ determines the returns to scale in production.[10] Firm i's nominal profits are given by

$$\pi_{i,t} = p_{i,t} y_{i,t} - W_t l_{i,t}. \tag{14}$$

Given (5), (12) and (14), profit-maximization implies

$$\log\left(p_{i,t}^*\right) = \alpha\xi \log\left(\frac{1}{\alpha}\frac{\theta}{\theta-1}\right) + \log(P_t) + \xi\left(\log(C_t) - A_t\right), \tag{15}$$

where $p_{i,t}^*$ denotes the desired price, and $\xi \equiv \frac{1+\psi}{\alpha+\theta(1-\alpha)}$ is the degree of real rigidity.

Information structure: We now specify the assumptions about the information structure. he information set of the price setter in firm i in period t is

$$\Omega_{i,t} = \Omega_{i,-1} \cup \{s_{i,0}, s_{i,1}, \ldots, s_{i,t}\}, \tag{16}$$

where $\Omega_{i,-1}$ is the initial information set of the price setter in firm i and $s_{i,t}$ is the signal that he or she receives in period t. The latter is a two-dimensional vector consisting of a noisy signal concerning aggregate technology and a noisy signal concerning money supply:

$$s_{i,t} = \begin{cases} s_{ai,t} = \ln(A_t) + \eta_{i,t} \\ s_{qi,t} = \ln(Q_t) + \zeta_{i,t} \end{cases}. \tag{17}$$

[10] This may correspond for instance to the presence of a firm-specific factor that is costly to adjust at short horizons.

We assume that the noise in the signal is due to limited attention by the decision-maker.[11] The noise in the signal has the following properties: (i) the processes $\{\eta_{i,t}\}_{t=0}^{\infty}$ and $\{\zeta_{i,t}\}_{t=0}^{\infty}$ are independent of the processes $\{A_t\}_{t=0}^{\infty}$ and $\{Q_t\}_{t=0}^{\infty}$, (ii) the processes $\{\eta_{i,t}\}_{t=0}^{\infty}$ and $\{\zeta_{i,t}\}_{t=0}^{\infty}$ are independent across firms and independent of each other, and (iii) $\eta_{i,t}$ and $\zeta_{i,t}$ follow Gaussian white noise processes with variances σ_{η}^2 and σ_{ζ}^2. The assumption that the noise in the signal is idiosyncratic accords well with the idea that the source of noise is limited attention by individual decision-makers rather than lack of publicly available information. The assumption that decision-makers in firms receive independent signals concerning aggregate technology and money supply is for tractability. This assumption is a simplification of reality that has the important advantage of introducing an endogenous information choice into an otherwise standard general equilibrium framework, while keeping the model tractable enough to allow for a closed form solution. This solution provides valuable information on the interaction between the different components of the model. Moreover, this structure is easily comparable to most of the new-Keynesian literature, where frictions in price setting received large attention.

Firms decide how to allocate their attention in period zero, before making the price setting decisions, by maximizing the discounted sum of profits from future activity, $E\sum_{t=0}^{\infty} Q_{0,t}\pi_{i,t}$.[12] In order to have an analytical solution to the attention allocation problem, this paper considers a second order Taylor expansion of the discounted sum of future profits around the non-stochastic steady state, in deviation from the discounted value of profits under the full-information profit-maximizing behavior.

[11] Think of the noise in the signal as the noise in the answers you get when you ask a sample of economists what the official CPI inflation rate for the United States was last year.

[12] In the static equilibrium of this model this assumption is irrelevant as the attention allocation choice is time-consistent.

The attention allocation problem of firm i reads[13]

$$\max_{\{\sigma_\eta,\,\sigma_\varsigma\}} -\lambda \sum_{t=0}^{\infty} \beta^t E\left[\left(\log\left(p_{i,t}\right) - \log\left(p^*_{i,t}\right)\right)^2\right], \tag{18}$$

subject the optimal price setting behavior conditional on the information available at each period,

$$\log\left(p_{i,t}\right) = E\left[\log\left(p^*_{i,t}\right) \mid \Omega_{i,t}\right], \tag{19}$$

and to to the information flow constraint

$$\kappa = \frac{1}{2}\log_2\left(\frac{\sigma^2_{A|t-1}}{\sigma^2_{A|t}}\right) + \frac{1}{2}\log_2\left(\frac{\sigma^2_{Q|t-1}}{\sigma^2_{Q|t}}\right), \tag{20}$$

where $\Omega_{i,t} = \Omega_{i,t-1} \cup \{s_{ai,t}, s_{qi,t}\}$ is the information set available to i, $\sigma^2_{A|t}$ and $\sigma^2_{A|t-1}$ denote the conditional variance of A_t given information of the price setter of firm i in period t and $t-1$ respectively, and $\sigma^2_{Q|t}$ and $\sigma^2_{Q|t-1}$ denote the conditional variance of Q_t given information of the price setter of firm i in period t and $t-1$ respectively. The constraint (20) measures the mutual information between the signal process $\{s_{it}\}$ and the structural shocks $\{A_t, Q_t\}$.[14] The more information about the structural shocks into signals, the larger the measure of mutual information. The parameter κ indexes firm's total attention, measuring the per-period information flow about the two shocks. In practice, if κ is finite, the information flow constraint prevents decision

[13] The parameter λ is given by $\lambda \equiv \frac{1}{2}\bar{L}^{1+\psi}\frac{\theta}{\theta-1}\frac{1}{\alpha}\left(1+\frac{\theta}{\theta-1}\left(\frac{1}{\alpha}-1\right)\right) > 0$ where $\bar{L}$ is the level of consumption in the non-stochastic steady state. See Maćkowiak and Wiederholt (2009) for a derivation. and a discussion of the reliability of the approximation.

[14] Formally, the mutual information is given by

$$\kappa = h\left(A_t, Q_t|\Omega_{i,t-1}\right) - h\left(A_t, Q_t|\Omega_{i,t}\right),$$

where h$(A_t, Q_t|\Omega_{i,t-1})$ denotes the conditional entropy of A_t and Q_t given $\Omega_{i,t-1}$ and $h\left(A_t, Q_t|\Omega_{i,t}\right)$ denotes the conditional entropy of A_t and Q_t given $\Omega_{i,t}$. Given that $\{A_t, s_{ai,t}\}$ and $\{Q_t, s_{qi,t}\}$ are independent and normally distributed the mutual information can be written as in (20).

makers from choosing $p_{i,t} = p^*_{i,t}$ in each period and state of the world. The units of measure of κ relates to the capacity of a channel of transmitting information, and are given by bits. For instance, one bit is the channel capacity needed to communicate the realization of a fair coin flip.[15] The signal structure (17), together with constraint (20), implies a trade-off in the attention allocation across the two types of shocks: if a firm pays more attention to one type of shock (i.e. chooses the corresponding signal process to be relatively more informative), it necessarily has to pay less attention to the other type of shock.

2.1 The equilibrium

The model is solved through a log-linearization of the first order conditions characterizing the equilibrium of the economy in a neighborhood of the non-stochastic steady state. The economy as two aggregate states, A_t and Q_t. Solving for the equilibrium of this economy requires solving for a fixed point. In fact, the attention allocation problem in $(18)-(19)$ depends on the stochastic process for the profit-maximizing price, $\hat{p}^*_{i,t}$, which in turn depends on the stochastic process for the price level, $\hat{P}_t$. The latter is an average over all intermediate good prices and therefore depends itself on the solution to the attention allocation problem of firms. In what follows $\hat{X}_t \equiv \log X_t - \log \bar{X}$ denotes the value of X_t in log-deviations from the non-stochastic steady state.

Definition 1 *A stationary equilibrium is a set of functions,* $C_t(\cdot)$, $L_t(\cdot)$, $P_t(\cdot)$, $W_t(\cdot)$, M_t, B_t, $R_t(\cdot)$, *of the two aggregate states,* A_t *and* Q_t, *a set of functions,* $p^*_{i,t}(\cdot)$, $p_{i,t}(\cdot)$, $c_{i,t}(\cdot)$, $l_{i,t}(\cdot)$, *of the two aggregate states,* A_t *and* Q_t, *as well as of the realizations of firm specific signals, and two scalars,* σ_η *and* σ_ς, *such that:*

[15] See Cover and Thomas (1991) for more details.

(i) $\{C_t(\cdot), L_t(\cdot), M_t, B_t\}$ *maximizes* (1) *subject to* (2) $-$ (4)

(ii) $P_t(\cdot)$ *satisfies* $P_t = \left[\int_0^1 (p_{i,t})^{1-\theta} di\right]^{\frac{1}{1-\theta}}$;

(iii) σ_η *and* σ_ς *maximize* (18) *subject to* (19) $-$ (20);

(iv) $p^*_{i,t}(\cdot)$ *satisfies* (15), *and* $p_{i,t}(\cdot)$ *satisfies* (19);

(v) $c_{i,t}(\cdot) = y_{i,t}(\cdot)$ *and* $L_t(\cdot) = \int_0^1 l_{i,t}(\cdot)\, di$;

(vi) all other markets clear.

3 The case of i.i.d. aggregate shocks

In this section we obtain an analytical solution to the model in a special case where A_t and Q_t are *iid* over time, i.e. $\ln Q_t = \mu_q + \varepsilon_{q,t}$ and $\ln A_t = \mu_a + \varepsilon_{a,t}$.

Proposition 1 *Let* $\rho_a = \rho_q = 0$. *Let* $\sigma \equiv \sigma_a/\sigma_q$ *and* $\phi \equiv (1-\phi_p)/(1-\phi_x)$. *There exists a stationary equilibrium in which the equilibrium dynamics of economic variables in period* t *are given by a set of linear functions of* $\varepsilon_{a,t}$ *and* $\varepsilon_{q,t}$. *In this equilibrium, the price level and consumption are given by*

$$\begin{aligned} \hat{P}_t &= \frac{\xi}{1-\phi_x}\left(\gamma_q \varepsilon_{q,t} - \gamma_a \varepsilon_{a,t}\right), & (21) \\ \hat{C}_t &= \hat{M}_t - \hat{P}_t, & (22) \end{aligned}$$

where γ_a *and* γ_q *are coefficients given by*

$$\left(\gamma_a;\ \gamma_q\right) = \begin{cases} (\bar{\gamma};\ 0) & if \quad \sigma > \bar{\sigma} \\ \left(\Gamma(\sigma);\ \Gamma\left(\frac{1}{\sigma}\right)\right) & if \quad \frac{1}{\bar{\sigma}} \le \sigma \le \bar{\sigma} \\ (0;\ \bar{\gamma}) & if \quad \sigma < \frac{1}{\bar{\sigma}} \end{cases}, \tag{23}$$

while $\bar{\gamma}$, $\bar{\sigma}$, *and the function* $\Gamma(\cdot)$ *are given by*

$$\bar{\gamma} = \frac{1-2^{-2\kappa}}{1-(1-\phi\xi)(1-2^{-2\kappa})}, \tag{24}$$

$$\Gamma(x) = \frac{\phi\xi + 2^{-2\kappa}(1-\phi\xi) - 2^{-\kappa}\frac{1}{x}}{(\phi\xi)^2 - 2^{-2\kappa}(1-\phi\xi)^2}, \tag{25}$$

$$\bar{\sigma} = Max\left\{1;\quad 2^{\kappa}\phi\xi + 2^{-\kappa}(1-\phi\xi)\right\}. \tag{26}$$

Proof. See Appendix A. ■

The equilibrium responses of prices to the two shocks depend on relative volatility, σ, on the degree of real rigidity, ξ, on the average quantity of information processed per period, κ, and on ϕ. The parameter ϕ has an important economic meaning, as it indexes relative monetary policy aggressiveness on expected inflation and output-gap.

The function $\Gamma(\cdot)$ determines the equilibrium price level responsiveness to a given shock as a function of relative volatility of that shock. The function $\Gamma(\cdot)$ is increasing in its argument for values of $\sigma \in (1/\bar{\sigma},\ \bar{\sigma})$. The latter follows from noticing that the denominator of (25) is strictly positive at an interior solution of the attention allocation problem.[16] Therefore, the equilibrium price level is more responsive to relatively more volatile shocks.

Moreover, the slope of $\Gamma(\cdot)$ with respect to its argument depends on $\phi\xi$ and κ : the smaller $\phi\xi$ or κ, the larger the impact of a change in relative volatility, σ, on price level responsiveness to the two shocks. The next proposition describes the relationship between equilibrium price responsiveness and structural parameters.

Proposition 2 *At an interior solution of the attention allocation problem in* (18) – (20)*,*

[16] From (26) an interior solution requires $2^{\kappa}\phi\xi + 2^{-\kappa}(1-\phi\xi) > 1$, which then implies that the denominator in (25) is positive.

1. *Relative price responsiveness, $\gamma \equiv \frac{\gamma_a}{\gamma_q}$, is strictly increasing in the relative standard deviation of technology shocks, σ, i.e. $\Gamma'(\cdot) > 0$.*

2. *If $\sigma > 1$ $(\sigma < 1)$, relative price responsiveness to technology shocks, γ, is strictly decreasing (increasing) in the degree of real rigidity, ξ, in the degree of relative monetary policy aggressiveness, ϕ, and in the upper bound on information flow, κ.*

3. *If $\sigma = 1$, relative price responsiveness is equal to $\gamma = 1$, for all values of $\phi\xi$ and κ.*

Proof. The results follow immediately from the definition of $\Gamma(\cdot)$ and $\bar{\sigma}$ in $(23) - (26)$. ■

The higher the volatility of technology shocks relatively to monetary policy shocks, the higher price responsiveness to technology shocks relative to policy shocks. The lower $\phi\xi$ or κ, the higher the difference in price responsiveness to the two shocks. When $\phi\xi$ is low enough, the model implies a corner solution where prices only respond to the most volatile shock. Finally notice that the level of the volatility of the two shocks does not matter for the allocation of attention, as long as their ratio is constant.[17]

3.1 Equilibrium attention allocation and monetary policy rule

Solving the attention allocation problem implies choosing the precision of signals (17) so to maximize (18) subject to $(20) - (19)$. This problem depends on the equilibrium dynamics of the desired price. These dynamics are obtained by substituting (22) into

[17] One could also think of a model where κ is endogenous. In that case it is possible that the allocation of attention responds to the level of volatility of the economy when κ responds to changes in the level of volatility.

(15),

$$\hat{p}^*_{it} = (1-\xi\phi)\,\hat{P}_t + \frac{\xi}{1+\phi_x}\,(\varepsilon_{q,t} - \varepsilon_{a,t}) \tag{27}$$

where the equilibrium dynamics of $\hat{P}_t$ are given by $(21)-(26)$. The coefficient $\xi\phi$ can be interpreted as the degree of strategic complementarity in price setting: the smaller $\xi\phi$, the larger the feedback from the price level to desired prices.

In equilibrium, the optimal attention allocation is such that signal precision to each type of shock is given by

$$\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_\eta^{2*}};\frac{\sigma_q^2}{\sigma_q^2+\sigma_\varsigma^{2*}}\right) = \begin{cases} (1-2^{-2\kappa};\ 0) & if\ \sigma > \bar{\sigma} \\ \left(1-\frac{2^{-\kappa}}{\omega\sigma};\ 1-2^{-\kappa}\omega\sigma\right) & if\ \frac{1}{\bar{\sigma}} \leq \sigma \leq \bar{\sigma} \\ (0;\ 1-2^{-2\kappa}) & if\ \sigma < \frac{1}{\bar{\sigma}} \end{cases} \tag{28}$$

where ω represents desired price responsiveness to technology shocks relative to monetary policy shocks,

$$\omega \equiv \frac{(1-\xi\phi)\,\gamma_a + 1}{(1-\xi\phi)\,\gamma_q + 1}. \tag{29}$$

firms allocate relatively more attention to technology shocks than to monetary policy shocks either because technology shocks are more volatile, i.e. $\sigma > 1$, or because they have a larger impact on the desired price than monetary policy shocks, i.e. $\omega > 1$.[18] However, while shock volatilities are exogenous to the model, desired price responsiveness is not. In particular, by substituting (23) into (29) it is possible to derive ω as a function only of the structural parameters of the model,

$$\omega = \frac{\phi\xi - \frac{1}{\sigma}2^{-\kappa}\,(1-\phi\xi)}{\phi\xi - \sigma 2^{-\kappa}\,(1-\phi\xi)}. \tag{30}$$

It follows from (28) and (30) that shock volatilities affect the attention allocation

[18] More details on the solution to the attention problem are available in Appendix A.

through two channels. First, as discussed above, for given desired price responsiveness to shocks, more attention is paid to more volatile shocks. Second, shock volatilities influence the attention allocation problem through relative desired responsiveness, ω: since more volatile shocks receive relatively more attention by all firms, they also have a higher associated price level responsiveness; the feedback effect from price level responsiveness to the desired price responsiveness affects the attention allocation decision.

Whether this feedback reinforces or reduces the impact of differences in volatilities of shocks on the attention allocation decision depends on the value of $\phi\xi$. It is at this stage that parameters of the interest rate feedback rule affect the attention allocation decision. If $\phi\xi < 1$, firm i pays more attention a type of shock whenever its competitors are more responsive to that type of shock, i.e. there are positive complementarity in attention allocation. In this case, the feedback effect reinforces the impact of different volatilities on attention allocation; in contrast, if $\phi\xi > 1$ the opposite is true and there are negative complementarity in attention allocation and the feedback effect reduces the impact of different volatilities.

Intuitively, an increase in ϕ_p, and therefore a lower ϕ, reduces the fluctuations in output-gap to all shocks. For given price level responsiveness, the smaller responsiveness of the output-gap to shocks induces the price level to be relatively more important for desired price dynamics. Hence, complementarities are higher.

While studying optimal policy is beyond the scope of this paper, the paper yields novel predictions on the impact of a change in the coefficients of the policy rule on the economy when compared to more standard models of sticky prices. When monetary authority changes the coefficients of the Taylor rule, it affects the economy through two channels. The first channel is a standard one, taking place also in models of nominal rigidities: for given information structure, a nominal interest rate responding more (less) to expected inflation accommodates technology shocks and offsets monetary policy shocks more (less); this reduces (increases) output-gap fluctuations, causing a smaller (larger) variability of prices to both types of shocks. The second channel is novel: by affecting the degree of complementarity in attention allocation, a more (less) active policy induces firms to pay more (less) attention to the most volatile shocks and less (more) to the least volatile ones.

4 The case of persistent innovations

In this section we solve for the impulse responses of inflation and output to permanent innovations in money supply and productivity, as of (10) and (13) respectively. In particular, we assume that both innovations in money supply and productivity follow a random walk, i.e. $\rho_a = \rho_q = 1$. We show that the feedback component of the monetary policy rule is important to reconcile prediction from the rational inattention model of this paper with empirical evidence about the impulse responses of inflation to productivity and monetary policy shocks.

Differently from the case of i.i.d. shocks, the model with persistent innovations cannot be solved analytically. However, for given allocation of attention, i.e. for given specification of the signal process in (17), the model dynamics in log-deviations from the non-stochastic steady state can be obtained by following the procedure in Woodford (2002): by applying the Kalman filter, one can then compute an exact linear rational expectations equilibrium of the log-linearized model by solving a Riccati equation. This is true despite in this paper, differently from Woodford (2002), the monetary policy rule can depend on endogenous variables as specified in (9). We solve the model in two steps. In the first step, for given guess of σ_η and σ_ζ, we solve for the equilibrium dynamics of the economy as just described. In the second step, we solve for the optimal attention allocation, i.e. for the optimal σ_η and σ_ζ, given the equilibrium dynamics derived in step 1. Then We update the guess for σ_η and σ_ζ, and go to step 1 until convergence.

The time unit is the quarter. We drew on the business cycle literature for the values of the preference parameter, θ, of output elasticity to labor, α, and discount factor, β. These parameters are the same used by Melosi (2011) in his estimation of the model. In particular, similar to Woodford (2003), the demand elasticity parameter θ is set equal to $\theta = 11$, while the parameter α is set equal to $2/3$, to match the average labor share of output in the U.S. The parameter ψ is set to $\psi = 1$, so to imply the Frisch labor elasticity equal to $1/2$. This implies a degree of real rigidity $\xi = 2/3$. The discount factor β is set to $\beta = 0.993$, so to have an annual nominal interest rate in steady state equal to 3 percent. We choose the standard deviation of innovations in money growth, i.e. σ_q, to match the standard deviation of the quarterly inflation in U.S. nominal GDP from 1980 to 2008, i.e. 0.6%. For simplicity we set the response of money growth to output-gap to zero, i.e. $\phi_x = 0$. This normalization is irrelevant for the results of this section: what matters for the allocation of attention, and therefore the speed of inflation adjustment, is the ratio $\phi \equiv \left(1 - \phi_p\right) / \left(1 - \phi_x\right)$.

As the analytical solution in the i.i.d. case showed, the remaining three parameters, $\sigma = \sigma_a/\sigma_q$, $\phi \equiv \left(1 - \phi_p\right) / \left(1 - \phi_x\right)$ and κ are particularly important in determining the equilibrium allocation of attention to productivity and monetary policy shocks, and therefore the speed of impulse responses of inflation and output to these two shocks. We proceed as follows. We choose a grid of values for the monetary policy feedback parameter, $\phi \in \{0.01, 0.5, 1\}$. A stationary solution of the model requires $\phi > 0$. For each value of ϕ, we choose σ and κ to match the speed of inflation adjustment to a technology and monetary policy shocks estimated by Paciello (2011) on U.S. data from 1980 to 2006. In particular, inflation persistence to shock i, 4 quarters after the

TABLE 1			
ϕ	0.01	0.50	1.00
σ	1.2	1.7	1.8
k	3.2	0.37	0.22
k_a/k_q	1.85	2.36	2.66

Table 1: k_a is allocation of attention to productivity shock; k_q is allocation of attention to monetary policy shock. $\phi = \left(1 - \phi_p\right) / \left(1 - \phi_x\right)$; $\sigma = \sigma_a/\sigma_q$.

shock, is measured as

$$r^i \equiv 1 - \frac{\sum_{s=0}^{4} \left(\hat{\pi}_s^i\right)^2}{\sum_{s=0}^{\infty} \left(\hat{\pi}_s^i\right)^2}, \tag{31}$$

where $\hat{\pi}_s^i$ is the response of the inflation rate to shock $i \in \{a,\ q\}$, evaluated s quarters after the shock. According to this measure, inflation is weakly persistent when the effects of shocks decay quickly, and it is strongly persistent when they decay slowly. When the effects of shock i die quickly, r^i is close to zero for shorter horizons. But when the effects of shock i decay slowly, r^i remains far from zero for longer horizon. According to Paciello (2011), inflation response to a productivity shock accomplishes 64% of its adjustment within the first year, i.e. $r^a = 0.34$, while it accomplishes only 37% of its adjustment to a monetary policy shock within the first year, i.e. $r^q = 0.63$. Therefore, for each value of ϕ, we choose σ and κ to match $r^a = 0.34$ and $r^q = 0.63$. Table 1 reports the values of σ and κ associated to each value of ϕ obtained from the procedure just described, as well as the implied allocation of attention between productivity and monetary policy shocks. Figure 2 plots inflation responses to a productivity and to a monetary policy shock predicted by the model in the case of $\phi = 0.01$. Two main results emerge from this exercise.

First, at $\phi = 1$, i.e. in absence of monetary policy feedback, the model needs the volatility of the productivity shock to be 80% larger than the volatility of the monetary policy innovation to match the empirical functions. This result is at odd with empirical evidence from U.S. data. For instance, the volatility of the growth rate in TFP estimated by Fernald (2007) is only about 30% larger than the volatility in the growth rate of nominal GDP. This is a result emphasized by Melosi (2011) who estimates the same model with $\phi = 1$ and argues that, given the relatively low empirical estimate of σ, the rational inattention model cannot generate enough asymmetry in the allocation of attention between the aggregate productivity and monetary policy shock to match the empirical functions. However, Table 1 shows that as the feedback in the monetary policy rule increases, i.e. ϕ goes towards zero, the ratio of volatilities between the two aggregate shocks, σ, needed to account for the empirical impulse responses decreases. In fact, at $\phi = 0.01$, we need the volatility of the aggregate productivity shock to be just 20% larger than the volatility of the monetary policy shock, i.e. $\sigma = 1.2$. This result helps to reconcile the model with the empirical evidence. In fact, direct estimates of on U.S. data suggest values of σ smaller than 1.3.[19]

Second, the estimated value of κ decreases with ϕ. This result is intuitive as the larger complementarity in price setting induced by the smaller value of ϕ, substitutes for the precision of the signals in generating inertia in inflation responses to productivity and monetary policy shock. A relatively higher value of κ is appealing as it implies that firms face, everything else being equal, smaller losses from being inattentive to the aggregate productivity and monetary policy shocks. In fact, the overall loss from inattention decreases from 8% of steady state profits in the case of

[19] This value of σ is within the 95% confidence interval estimated by Melosi (2011). It is also consistent with the ratio between the volatility of the quarterly U.S. TFP growth rate estimated by Fernald (2007) and the volatility of the growth rate of nominal GDP in the U.S. from 1980 to 2006. The latter implies a value of $\sigma = 1.3$.

$\phi = 1$, to 0.8% of steady state profits in the case of $\phi = 0.01$.[20]

Finally, this paper does not attempt to estimate directly the value of ϕ, but rather aims to show that the feedback component of monetary policy may help reconciling the rational inattention model with the empirical evidence. Given the empirical evidence on the volatility of innovations in productivity and monetary policy shocks, and given the estimated inertia in inflation responses to productivity and monetary policy shocks the model of this paper implies a relatively low value of ϕ, in the order of $\phi = 0.01$. The latter means that the feedback component of monetary policy is an important determinant of the variability in the money supply.[21] More generally, a low value of ϕ implies higher strategic complementarity in price setting. In fact, notice that the degree of complementarity in price setting in our model is given by the product $\xi\phi$. As argued by Woodford (2003), the New-Keynesian model needs relatively low degree of complementarity, i.e. low ξ and/or ϕ, to explain inflation inertia to nominal shocks. We find that the rational inattention model also need higher strategic complementarity in price setting to explain the relatively fast response of inflation to technology shocks and the relatively slow response to monetary policy shocks. In absence of feedback in monetary policy, i.e. $\phi = 1$, the value of ξ obtained from standard calibration, does not imply enough strategic complementarity in price setting to obtain this result. Allowing for a feedback component in the monetary policy rule can help bridging the gap and, therefore, help the model generating inflation responses to productivity and monetary policy shocks that are closer to the empirical estimates. However, notice that other sources of strategic complementarity in price setting and/or the presence of firm specific productivity shocks, may reduce the need of feedback in monetary policy to explain inflation responses to monetary policy and

[20] Losses are expressed relatively to profits evaluated at the profit-maximizing price under perfect information.

[21] See Clarida, Gali and Gertler (1999) for a review.

productivity shocks.[22] In this sense, we should interpret the value of $\phi = 0.01$ as a lower bound on the extent of feedback in monetary policy needed to match inflation responses.

5 Extension: a nominal interest rate rule

In this section we assume that monetary policy is not specified in terms of a money supply rule, but rather an interest rate rule. In particular, equations (9)-(11) are replaced by the following equation

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_r} \left[\left(E_t \frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}\right]^{1-\rho_r} e^{\varepsilon_{r,t}}, \tag{32}$$

where $\varepsilon_{r,t} \backsim N\left(0, \sigma_r^2\right)$ and i.i.d., and money supply M_t is adjusted residually to satisfy demand determined by nominal spending through (4). The rest of the economy is unchanged from previous sections. It is not possible to solve the model analytically so I use numerical methods.[23]

Given the importance of monetary policy and shocks volatilities in determining allocation of attention and shaping price responses in my model, we consider two different set of parameters governing the monetary policy rule in (32) and the productivity process in (13).[24] Monetary policy parameters are set equal to estimates of (32) obtained by Clarida et al. (1999) on U.S. data, for the pre-Volcker and Volcker-Greenspan periods. The volatility and persistence parameters of the productivity

[22] In Mackowiak and Wiederholt (2011) higher strategic complementarity in price setting may arise from households being imperfectly informed. See instead Melosi (2011) for a setup where firm specific productivity shocks imply higher attention allocation to aggregate productivity shocks.

[23] See Appendix B for more detalis.

[24] In fact, several studies (e.g. Clarida et al. (1999)) suggest that U.S. monetary policy has changed substantially since the election of Volcker at the helm of the Federal Reserve. In addition, Justiniano and Primiceri (2008) among others have shown that volatility of productivity and monetary policy shocks have declined substantially since the mid-80's.

TABLE 2						
	ϕ_π	ϕ_y	ρ_r	σ_r	ρ_a	σ_a
Pre-Volcker	0.83	0.27	0.68	0.004	0.61	0.008
Volcker-Greenspan	2.15	0.93	0.79	0.002	0.72	0.005

Table 2: k_a is allocation of attention to productivity shock; k_q is allocation of attention to monetary policy shock. $\phi = \left(1-\phi_p\right)/\left(1-\phi_x\right)$; $\sigma = \sigma_a/\sigma_q$.

process are set equal to estimates obtained from fitting (13) to the detrended logarithm of U.S. total factor productivity estimated by Fernald (2007) over the two subsamples.[25]

These parameters are displayed in Table 2: policy coefficients on inflation and output-gap more than double in the Volcker-Greenspan period, while shock volatilities reduce approximately by a half, although the ratio σ is approximately equal to two in both periods. We set $\kappa = 3.5$ so that the cost of processing information is similar to the value estimated by Zbaracki et al. (2004), i.e. 4.4% of steady state profits.[26] Notice that this parametrization of κ is conservative if compared to existing studies such as Maćkowiak and Wiederholt (2009), where firms devote less than one unit of κ to track aggregate shocks. Remaining parameters are chosen as in the previous section.

Figure 3 plots the impulse responses of inflation and output to one standard deviation technology and monetary policy shocks under both the pre-Volcker and the Volcker-Greenspan parametrizations. The model predicts that inflation responds more and faster to technology shocks in the Volcker-Greenspan period than in the pre-Volcker period. In contrast, the model predicts that inflation responds relatively more

[25] Fernald (2007) estimates TFP in the U.S. with a Solow residual approach, adjusting for labor hoarding and capital utilization.

[26] We measure the cost of processing information in my model as the product of κ and the Lagrangian multiplier associated to the constraint on information flow. The latter can be interpreted as the shadow price of κ.

to monetary policy shocks in the pre-Volcker period than in the Volcker-Greenspan period. These predictions are consistent, at least qualitatively, with empirical findings in Paciello (2011). In addition, the model predicts that the impact of a technology shock on output is higher in the Volcker-Greenspan period than in the pre-Volcker period, while monetary policy shocks have larger real effects in the pre-Volcker period than in the Volcker-Greenspan period. In fact, the endogenous fraction of κ allocated to technology shocks across the two subsamples goes from about 53 percent in the pre-Volcker period to 89 percent in the Volcker-Greenspan period, implying a relatively higher (lower) degree of price rigidity to technology (monetary policy) shocks in the pre-Volcker period than in Volcker-Greenspan period. This explains the different impact of the policy change on inflation or output responses conditional on the two shocks. But what is the main cause of the shift in allocation of attention towards technology shocks? We know from the results of the previous section that the allocation of attention is determined by the relative volatility of the two shocks and by the coefficients of the monetary policy rule. The relative volatility of the two shocks has not changed much across the two subsamples, going from $\frac{\sigma_a}{\sigma_r} = 2$ in the pre-Volcker period to $\frac{\sigma_a}{\sigma_r} = 2.5$ in the Volcker-Greenspan period. In fact, it turns out that the main cause of the shift in allocation of attention towards technology shocks is the change in monetary policy.[27] The latter has induced an increase in complementarity in allocation of attention across firms, amplifying the incentives to allocate relatively more attention to technology shocks, for given volatilities of shocks.

These predictions are consistent with empirical evidence by Galí, Lopez-Salido and Valles (2003) and Boivin and Giannoni (2006). However, differently from the

[27] I have solved the model in the counterfactual case where all parameters are identical across the two sub-samples, while policy changes. I have found that results in terms of allocation of attention and impulse responses in the two subsamples are very similar.

case of the previous section, the version of the model with the nominal interest rate rule cannot generate enough persistence in inflation response to both types of shocks. In fact, the basic framework of this paper allows for a closed form solution, but is too simple to fully account for business cycle dynamics. For instance, the model does not allow for physical capital accumulation, or for habit persistence in consumption, which typically increase inflation persistence in response to aggregate shocks. Extending the basic framework to incorporate these features is an important step to evaluate the ability of these models to account for business cycle dynamics. Maćkowiak and Wiederholt (2010) is an important contribution in this direction.

6 Concluding remarks

This paper has analytically derived the channel through which the feedback component of a monetary policy rule affect the attention allocation decision by firms. According to this channel, a monetary policy rule responding relatively more aggressively to inflation increases relative differences in price responsiveness to technology and monetary policy shocks by inducing firms to allocate more attention to the most volatile shock.

This paper shows that a simple model of price setting under rational inattention and attention allocation naturally generates inflation to adjust faster to aggregate technology shocks than to monetary policy shocks. Similarly to more standard Calvo-style models of price rigidities, inertia in inflation response arises from frictions in price setting. The less informed firms are on aggregate shocks, the larger inertia in inflation responses. However, differently from the Calvo model of price rigidity where firms adjust prices to all shocks at the same time, with endogenous attention decision the extent of the imperfect information friction may vary conditional on different types of aggregate shocks. In the model of this paper, firms have incentives to allocate more attention to technology shocks than to monetary policy shocks because the former are more volatile than the latter. However, most importantly, this paper shows that a combination of relatively high real rigidity and aggressive monetary policy is needed to magnify the impact of different volatilities on relative price responsiveness. In particular, a monetary policy rule responding to inflation and output amplifies the effects of exogenous shock volatility differential on the attention allocation decision and, therefore, on price responsiveness differential to the two shocks.

References

[1] Adam, Klaus. (2007) "Optimal Monetary Policy with Imperfect Common Knowledge." *Journal of Monetary Economics*. Vol. 54(2), 276-301, 2007.

[2] Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Linde. (2005) "Firm-Specific Capital, Nominal Rigidities and the Business Cycle", *National Bureau of Economic Research* working paper 11034.

[3] Angeletos, George M. and Jennifer La'O. (2008) "Dispersed Information over the Business Cycle: Optimal Fiscal and Monetary Policy". MIT working paper.

[4] Angeletos, George M. and Alessandro Pavan. (2009) "Policy with Dispersed Information." *Journal of the European Economic Association*, Vol. 7, No. 1, Pages 11-60.

[5] Atkeson, Andrew, V.V. Chari, and Patrick Kehoe. (2010) "Sophisticated Monetary Policies". *Quarterly Journal of Economics*, vol. 125, pages 47-89.

[6] Bernanke, B.S. and Mihov, I. (1998) "Measuring Monetary Policy", *Quarterly Journal of Economics*, vol. 113-3.

[7] Blanchard, Olivier Jean & Kiyotaki, Nobuhiro. (1987) "Monopolistic Competition and the Effects of Aggregate Demand," American Economic Review, American Economic Association, vol. 77(4), pages 647-66.

[8] Boivin, J. and Marc Giannoni. (2006) "Has Monetary Policy Become More Effective?" *The Review of Economics and Statistics*, 88(3).

[9] Clarida, R., Gali J. and Mark Gertler. (1999) "The science of monetary policy: a new Keynesian perspective." J*ournal of economic literature*, volume 37, number 4.

[10] Cogley, T. and Primiceri, G.E. and T.J Sargent. (2010) "Inflation-gap persistence in the US". *American Economic Journal: Macroeconomics*, vol. 2(1), pages 43-69, January.

[11] Cover, Thomas M., and Joy A. Thomas. (1991) "Elements of Information Theory." John Wiley and Sons, New York.

[12] Del Negro, M. and F. Schorfheide. (2008) "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)". *Journal of Monetary Economics*, vol 55-7.

[13] Dupor, Bill, Jing Han and Yi Chan Tsai. (2009) "What Do Technology Shocks Tells Us about the New Keynesian Paradigm?". *Journal of Monetary Economics*, volume 56-4, p 560–569.

[14] Fernald, John. (2007) "A Quarterly, Utilization-Corrected Series on Total Factor Productivity". mimeo.

[15] Hellwig, Christian and Laura Veldkamp. (2009) "Knowing What Others Know: Coordination Motives in Information Acquisition". *Review of Economic Studies*, vol. 76(1), pages 223-251, 01.

[16] Galí J., J. D. Lopez-Salido and J. Valles. (2003) "Technology shocks and monetary policy: assessing the Feds performance," *Journal of Monetary Economics* 50 (2003) 723743.

[17] Golosov, Mikhail and Robert Lucas. (2006) "Menu Costs and Phillips Curves." *Journal of Political Economy*, vol. 115(2), pages 171-199, April.

[18] Justiniano, A. and G.E. Primiceri. (2008) "The time-varying volatility of macroeconomic fluctuations". *American Economic Review*, vol 98-3.

[19] Lorenzoni, Guido. (2008) "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information". *Review of Economic Studies*, Vol 77.

[20] Lucas, Robert E. Jr. (1972) "Expectations and the Neutrality of Money". *Journal of Economic Theory*, 4, 103-124.

[21] Luo, Yulei and Eric R. Young. (2009) "Rational Inattention and Aggregate Fluctuations." *The B.E. Journal of Macroeconomics*, Vol. 9: Iss. 1 (Contributions), Article 14.

[22] Mankiw, N. Gregory, and Ricardo Reis. (2006) "Pervasive Stickiness". *The American Economic Review*, Volume 96, Number 2, May 2006 , pp. 164-169(6).

[23] Maćkowiak, Bartosz, and Mirko Wiederholt. (2009) "Optimal Sticky Prices under Rational Inattention". *American Economic Review*, vol. 99(3), pages 769-803, June.

[24] Maćkowiak, Bartosz, and Mirko Wiederholt. (2010) "Business Cycle Dynamics under Rational Inattention." Discussion paper Northwestern University.

[25] Matejka, Filip and Christopher A. Sims, (2010) "Discrete Actions in Information-Constrained Tracking Problems," manuscript, Princeton University.

[26] Melosi, Leonardo. (2011) "Estimating Models with Information Frictions". mimeo London Business School.

[27] Orphanides, Athanasios. (2003) "Historical monetary policy analysis and the Taylor rule," *Journal of Monetary Economics*, vol. 50(5), pages 983-1022, July.

[28] Paciello, Luigi. (2011) "Does Inflation Adjusts Faster to Technology Shocks than to Monetary Policy Shocks?". *Journal of Money, Credit and Banking*, Vol. 43, No. 8, pages 1663-1684.

[29] Paciello, Luigi, and Mirko Wiederholt. (2010) "Imperfect Information and Monetary Policy". mimeo Nortwestern Univesrity.

[30] Phelps, Edmund S. (1970) "Introduction: The New Microeconomics in Employment and Inflation Theory". In Microeconomic Foundations of Employment and Inflation Theory, edited by Edmund S. Phelps et al., Norton, New York.

[31] Sims, Christopher A. (1992) "Interpreting the Macroeconomc Time Series Facts: the Effects of Monetary Policy". *European Economic Review*, Elsevier, vol. 36(5), pages 975-1000.

[32] Sims, A. Christopher. (2003) "Implications of Rational Inattention". *Journal of Monetary Economics*, Volume 50, Number 3, April 2003 , pp. 665-690(26).

[33] Smets, F. and Rafael Wouters. (2007) "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, vol. 97(3), pages 586-606.

[34] Woodford, Michael. (2002) "Imperfect Common Knowledge and the Effects of Monetary Policy". In "Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps", Princeton University Press.

[35] Woodford, Michael. (2003) "Interest and Prices. Foundations of a Theory of Monetary Policy." Princeton University Press, Princeton and Oxford.

[36] Zbaracki, Mark J., Mark Ritson, Daniel Levy, Shantanu Dutta and Mark Bergen. (2004) "Managerial and Customer Costs of Price Adjustments: Direct Evidence from Industrial Markets." *Review of Economics and Statistics*, 86, 514-533.

A Appendix: Proof of Proposition 1

I use the method of undetermined coefficients to show that $(21) - (26)$ is an equilibrium.

(Step 1): Derivation of desired responses conditional on each shock

By substituting (22) into (15), it follows that

$$\hat{p}^*_{i,t} = (1 - \phi\xi)\,\hat{P}_t + \frac{\xi}{1+\phi_x}\left(\varepsilon_{q,t} - \varepsilon_{a,t}\right). \tag{33}$$

where $\phi \equiv \frac{1-\phi_p}{1-\phi_x}$. In addition, by substituting (21) into (33), $\hat{p}^*_{i,t}$ can be expressed as the sum of two independent components, each depending on one of the two types of shocks, $\hat{p}^*_{i,t} = \hat{p}^*_{ai,t} + \hat{p}^*_{qi,t}$, where $\hat{p}^*_{ai,t}$ and $\hat{p}^*_{qi,t}$ are defined as

$$\hat{p}^*_{ai,t} \equiv -\omega\left(\gamma_a\right)\frac{\xi}{1-\phi_x}\varepsilon_{a,t}, \tag{34}$$

$$\hat{p}^*_{qi,t} \equiv \omega\left(\gamma_q\right)\frac{\xi}{1-\phi_x}\varepsilon_{q,t}, \tag{35}$$

and where $\omega(\cdot)$ is a linear function of γ_a and γ_q,

$$\omega(x) = (1-\phi\xi)\,x + 1. \tag{36}$$

(Step 2): Solving the attention allocation problem

Given $(34) - (36)$, it is possible to solve the attention allocation problem in $(18) - (19)$ as a function of γ_a and γ_q. By substituting $(34) - (36)$ into (19), and using the independence assumption, $s_{ai,t} \perp s_{qi,t}$, it is possible to express $\hat{p}_{i,t}$ as $\hat{p}_{i,t} = \hat{p}_{ai,t} + \hat{p}_{qi,t}$, where $\hat{p}_{ai,t} = E\left[\hat{p}^*_{ai,t} \mid s^t_{ai}\right]$ and $\hat{p}_{qi,t} = E\left[\hat{p}^*_{qi,t} \mid s^t_{qi}\right]$. Notice that in the equilibrium of the model in Section 2 conditional expectations coincide with unconditional ones.

Therefore (18) can be expressed as

$$-\lambda\sum_{t=0}^{\infty}\beta^{t}E\left[\left(\log\left(p_{i,t}\right)-\log\left(p_{i,t}^{*}\right)\right)^{2}\right]=-\frac{1}{1-\beta}\lambda E\left[\left(\hat{p}_{ai,t}-\hat{p}_{ai,t}^{*}\right)^{2}\right]-\frac{1}{1-\beta}\lambda E\left[\left(\hat{p}_{qi,t}-\hat{p}_{qi,t}^{*}\right)^{2}\right].$$

From (17) and (34) – (36) it follows that

$$\hat{p}_{ai,t} = -\frac{\sigma_a^2}{\sigma_a^2+\sigma_\eta^2}\omega\left(\gamma_a\right)\frac{\xi}{1-\phi_x}s_{ai,t}, \tag{37}$$

$$\hat{p}_{qi,t} = \frac{\sigma_q^2}{\sigma_q^2+\sigma_\varsigma^2}\omega\left(\gamma_q\right)\frac{\xi}{1-\phi_x}s_{qi,t}. \tag{38}$$

By substituting the results above into the objective function, the attention allocation problem reads

$$\max_{\left(\sigma_\eta\geq 0,\ \sigma_\varsigma^2\geq 0\right)}-\frac{\lambda}{1-\beta}\left(\frac{1}{1+\frac{\sigma_a^2}{\sigma_\eta^2}}\left(\omega\left(\gamma_a\right)\sigma_a\right)^2+\frac{1}{1+\frac{\sigma_q^2}{\sigma_\varsigma^2}}\left(\omega\left(\gamma_q\right)\sigma_q^2\right)^2\right), \tag{39}$$

subject to the information flow constraint

$$\frac{1}{2}\log_2\left(1+\frac{\sigma_a^2}{\sigma_\eta^2}\right)+\frac{1}{2}\log_2\left(1+\frac{\sigma_q^2}{\sigma_\varsigma^2}\right)=\kappa.$$

The the first-order conditions to this problem imply

$$\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_\eta^{2*}};\frac{\sigma_q^2}{\sigma_q^2+\sigma_\varsigma^{2*}}\right)=\begin{cases}\left(1-2^{-2\kappa};\ 0\right) & if\ \omega\sigma>2^{\kappa}\\ \left(1-2^{-\kappa}\frac{1}{\omega\sigma};\ 1-2^{-\kappa}\omega\sigma\right) & if\ 2^{-\kappa}\leq\omega\sigma\leq 2^{\kappa}\\ \left(0;\ 1-2^{-2\kappa}\right) & if\ \omega\sigma<2^{-\kappa}\end{cases} \tag{40}$$

where $\omega\equiv\frac{\omega_a}{\omega_r}$ and $\sigma\equiv\frac{\sigma_a}{\sigma_q^2}$.

(Step 3): Solving for undetermined coefficients γ_a and γ_q

From the absence of ex-ante heterogeneity across firms, it follows that all firms make the same attention allocation decision: $\sigma_{\eta}^{2*} = \upsilon_{a}^{2*}$and $\sigma_{\varsigma}^{2*} = \upsilon_{r}^{2*}$ for all i. Using (21), (17) and $(37) - (38)$it follows

$$\begin{aligned}
\gamma_a \varepsilon_{a,t} &= \int_0^1 \frac{\sigma_a^2}{\sigma_a^2 + \sigma_\eta^{2*}} \omega\left(\gamma_a\right)\left[\varepsilon_{a,t} + u_{ai,t}\right] di = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_\eta^{2*}} \omega\left(\gamma_a\right) \varepsilon_{a,t}, \\
\gamma_q \varepsilon_{q,t} &= \int_0^1 \frac{\sigma_q^2}{\sigma_q^2 + \sigma_\varsigma^{2*}} \omega\left(\gamma_q\right)\left[\varepsilon_{q,t} + u_{qi,t}\right] di = \frac{\sigma_q^2}{\sigma_q^2 + \sigma_\varsigma^{2*}} \omega\left(\gamma_q\right) \varepsilon_{q,t},
\end{aligned}$$

where the second equality follows from the assumption that errors in information processing are independent across firms, $\int_0^1 \eta_{i,t} di = 0$, $\int_0^1 \varsigma_{i,t} di = 0$. By substituting (40) in the equations above it follows that at an interior solution of the attention problem:

$$\begin{aligned}
\gamma_a &= \left(\omega\left(\gamma_a\right) - 2^{-\kappa}\omega\left(\gamma_q\right)\frac{1}{\sigma}\right) = \Gamma\left(\sigma\right), \\
\gamma_q &= \left(\omega\left(\gamma_q\right) - 2^{-\kappa}\omega\left(\gamma_a\right)\sigma\right) = \Gamma\left(\frac{1}{\sigma}\right),
\end{aligned}$$

where the second equality follows from using (36) and function $\Gamma\left(\cdot\right)$ is given by

$$\Gamma\left(x\right) = \frac{2^{-2\kappa} + \xi\phi\left(1 - 2^{-2\kappa}\right) - 2^{-\kappa}\frac{1}{x}}{\left(\xi\phi\right)^2 - 2^{-2\kappa}\left(1 - \xi\phi\right)^2}.$$

At the corner solution the shock $j = a, q$ that receives all the attention as an associated $\gamma_j = \frac{1-2^{-2\kappa}}{1-(1-\xi\phi)(1-2^{-2\kappa})}$, while $\gamma_{-j} = 0$.

(Step 4): Derivation of $\bar{\sigma}$

An interior solution to the attention allocation problem requires that $0 < \frac{\sigma_a^2}{\sigma_a^2+\sigma_\eta^{2*}} < 1 - 2^{-2\kappa}$ and $0 < \frac{\sigma_q^2}{\sigma_q^2+\sigma_\varsigma^{2*}} < 1 - 2^{-2\kappa}$. The latter requires: $2^{-\kappa} \leq \omega\sigma \leq 2$. These two conditions are contemporaneously satisfied when $\frac{1}{\bar{\sigma}} \leq \sigma \leq \bar{\sigma}$, and $\bar{\sigma} = \max\left\{1; 2^{\kappa}\phi\xi + 2^{-\kappa}\left(1 - \phi\xi\right)\right\}$.

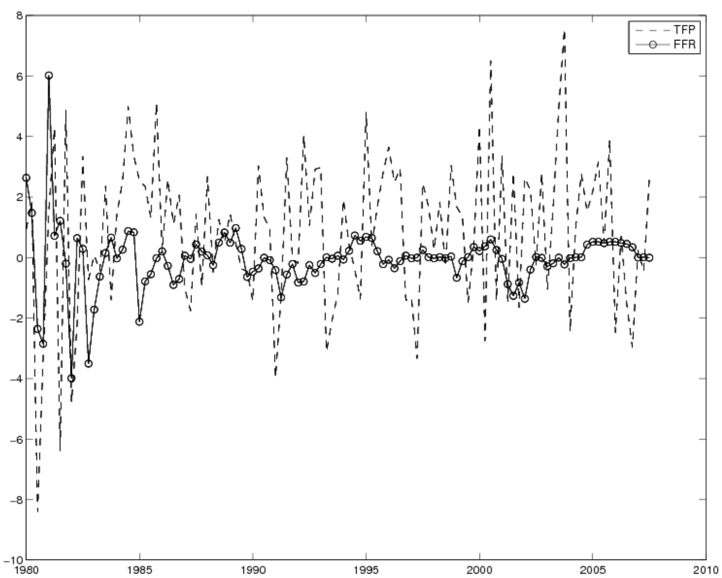

Figure 1: Growth rate in quarterly growth rate in U.S. TFP (annual basis) estimated by Fernald (2007) and change in the quarterly average of the FedFunds rate (annual basis).

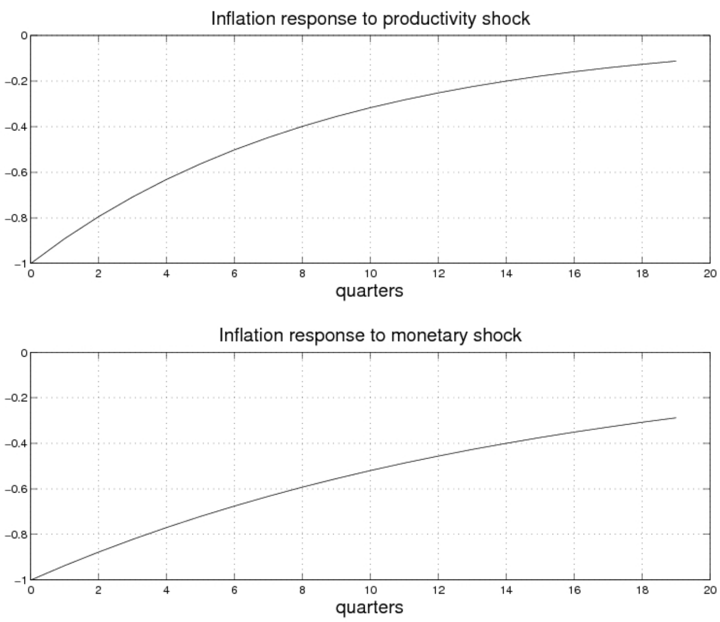

Figure 2: Inflation responses to a positive aggregate technology shock (top panel) and to a negative shock to money supply (bottom panel). Impulse responses are normalized so that the response of inflation on impact is equal to −1.

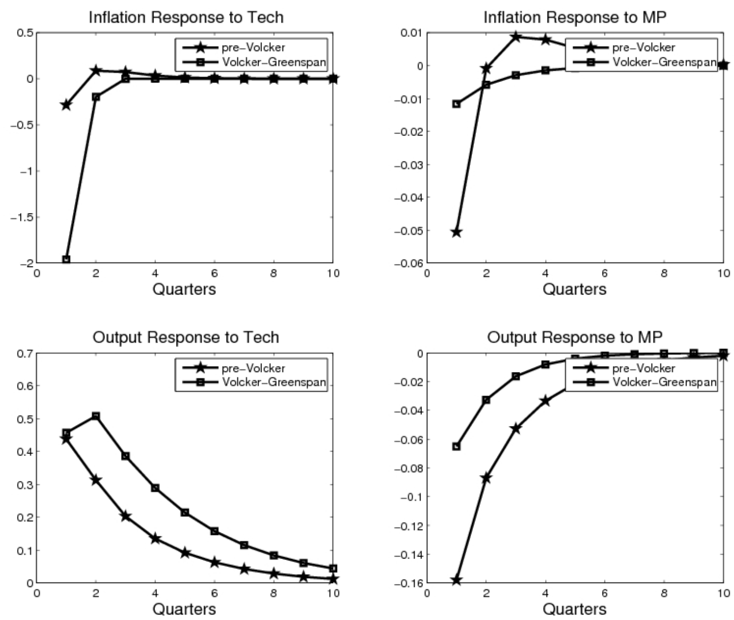

Figure 3: Inflation and output impulse responses to one standard deviation technology and monetary policy shocks, under the different parameterizations of the model described in Table 2. Responses are intended in % deviations from steady state.