



**EIEF Working Paper 26/13**  
**October 2013**

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Portfolio Choice over the Life-Cycle**

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First draft: September 2011  
This version: September 2013

## Abstract

We study the life cycle of portfolio allocation following for 15 years a large random sample of Norwegian households using error-free data on all components of households' investments drawn from the Tax Registry. Both, participation in the stock market and the portfolio share in stocks, have important life cycle patterns. Participation is limited at all ages but follows a hump-shaped profile which peaks around retirement; the share invested in stocks among the participants is high and flat for the young but investors start reducing it as retirement comes into sight. Our data suggest a double adjustment as people age: a rebalancing of the portfolio away from stocks as they approach retirement, and stock market exit after retirement. Existing calibrated life cycle models can account for the first behavior but not the second. We show that incorporating in these models a reasonable per period participation cost can generate limited participation among the young but not enough exit from the stock market among the elderly. Adding also a small probability of a large loss when investing in stocks, produces a joint pattern of participation and of the risky asset share that is similar to the one observed in the data. A structural estimation of the relevant parameters of the model reveals that the parameter combination that fits the data best is one with a relatively large risk aversion, small participation cost and a yearly large loss probability of around 1.3 percent.

**JEL Classification:** G11, D14.

**Keywords:** portfolio choice, portfolio rebalancing, asset allocation, asset market participation, life cycle model.

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\*We would like to thank Árpád Ábrahám, Jérôme Adda, Juan-Carlos Conesa, Francisco Cocco, Maria Gustafsson, Elin Halvorsen, Jonathan Heathcote, Dirk Krueger and Marno Verbeek for helpful comments and suggestions. We are grateful to Francisco Cocco for making the code of his life cycle model available to us. We thank seminar participants at the University of Cambridge, EIEF, University of Oxford, European University Institute, Statistics Norway, Norges Bank, NHH Bergen, ESEM 2011, SED 2012 and IIPF 2013. We are grateful to NETSPAR for financial support.

# 1 Introduction

Over the past decade a number of contributions have re-examined the life cycle behaviour of investors' portfolio. Inspired by empirical findings from novel microeconomic data on households portfolios, several papers have provided new models of the life cycle portfolio of individual investors that go beyond the seminal models of Mossin (1968), Samuelson (1969) and Merton (1969).

These earlier contributions have two sharp predictions: first, even in a dynamic setting, individuals should, at all points in their life-cycle invest a share of their wealth in risky assets. That is, independently of age, all investors should participate in the stock market - an extension of the participation principle in a static setting to a dynamic context. Second, assuming complete markets and in the absence of labor income, the share invested in the risky asset should be age-invariant. Thus, the portfolio - either described by the ownership of risky assets or by their share in total wealth - exhibits no life cycle pattern. However, the absence of rebalancing over the life cycle predicted by these earlier models is not robust to the (realistic) presence of human capital. As shown by Merton (1971), the presence of tradeable human capital in a complete market setting implies that since human capital is riskless and tradeable, it plays the same role as a large endowment of riskless bonds. It therefore creates a strong incentive to invest in risky securities when abundant, that is early in the life cycle, and to rebalance away from them as people get older and their human wealth shrinks. Importantly, this basic implication carries over to more complex environments that feature non-insurability of labor income and incomplete markets, as shown by several computational models of life cycle portfolio investments reviewed in Section 2 that amend the Samuelson-Merton model in one or more dimensions to add doses of realism. All these models uniformly predict that individuals should rebalance toward a safer portfolio as they approach retirement and the driving force is the life cycle pattern of human capital. On the other hand, without additional assumptions, they still imply that people should participate in the stock market.

In contrast, microeconomic data on household portfolios seem to show two remarkable features: first, not only participation in the stock market is limited at all ages but it tends to follow a life cycle pattern - in many instances a hump-shaped one (see Haliassos, Guiso, and Jappelli 2001). Second, the share invested in stocks tends to vary little over age, though in this case the specific empirical pattern is more controversial. Summarizing evidence for several countries, Haliassos, Guiso, and Jappelli (2001) argue that the age profile of the share of risky assets conditional on participation is relatively flat, though in some instances "there does seem to be some moderate rebalancing of the portfolio away from risky securities" as people age. Thus, a reasonable characterization of the empirical findings is that participation in risky assets follows a hump-shaped profile while the share invested varies little, if at all, with age. But how solid is the evidence on which this characterization rests? The empirical finding that people do not rebalance their risky portfolio share over the life cycle sounds particularly puzzling because rebalancing is implied by an indisputable fact of life - the decrease in the stock of human capital as people age.

While the lack of participation is a robust feature of the data, there are at least three reasons to doubt the empirical patterns over age in both participation and the portfolio share. First, most of the available evidence is obtained from cross sectional data. Hence, inferences on the age pattern of the portfolio must be drawn from comparisons of portfolio holdings of individuals of different age, rather than of the same individual as his age varies.

If individuals of different age differ in un-observables that are correlated with age, the estimated age pattern may reflect the former not the latter, in particular, cohort effects. Panel data may help sorting this out: though adding an extra source of variation to the data (time) also adds the need to model it, if one can impose reasonable restrictions on time effects one can then distinguish the effect of age from that of year of birth. Second, most studies ignore the fact that the risky portfolio share is only defined for the participants in the risky assets markets and that participation in assets markets is an endogenous choice. Thus, uncontrolled selection, if correlated with age, may be responsible for the failure to find evidence of rebalancing in the risky share. Third, evidence so far is based primarily on household surveys which are notoriously subject to measurement problems. Most importantly, measurement and reporting errors are likely to be correlated with age, hiding age patterns when present in the true data. For instance, this could arise because wealth is correlated with age and the wealthy may have a stronger motive to under-report or not-report specific assets (such as stocks). Hence, age profiles of the risky portfolio share (and participation) may appear flatter than they actually are.

One important exception is Ameriks and Zeldes (2002) who try to circumvent these problems by using a panel of TIAA-CREF contributors covering 13 years of data.<sup>1</sup> Thus, they can in principle distinguish between age, time and cohort effects. Because they use administrative data, non-reporting and under-reporting of assets in the program is not a major issue. Using a variety of identifying assumptions to separate age, time and cohort effects and distinguishing between ownership of stocks and conditional shares, they conclude that a good characterization of the portfolio life cycle is one where the life-cycle of stock market participation is hump shaped and the conditional share in stocks shows little action over the life cycle. Thus, in their view, most of the life cycle portfolio changes take place on the extensive rather than on the intensive margin.

While their results mark a clear progress in the literature, a number of open issues related in part to the data remain. First, TIAA-CREF reports only assets contributed to the program, not the complete portfolios of these individuals. Furthermore, the part left out is not negligible - retirement assets amount to less than 30% of total household financial assets in the 1998 SCF - and there is no obvious reason why the portfolio allocation in pension savings should be the same as the allocation in other financial assets or follow the same age profile (indeed it does not, see Guiso and Sodini 2011). Second, the data refer to individuals and not to households. If the asset allocation is a joint family decision, this may result in biased estimates. Third, participants at TIAA-CREF belong to a selected group of the population - typically employees at institutions of higher education - which have marked different characteristics compared to a representative population sample. Since the estimated portfolio life-cycle reflects the age pattern of portfolio-relevant household (or individual) variables, such as the age profile of human capital and that of its riskiness, if these profiles differ across groups also the profiles of their portfolios will be different. Hence, they may not be a good characterization of the average investor in a population. Finally, dynamic portfolio patterns of pension assets from a defined contribution plan such

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<sup>1</sup>Agnew, Balduzzi, and Sundén (2003) also use a four year panel dataset of about 7,000 people in a 401k retirement accounts and can thus distinguish age and time effects. They find that the risky portfolio share is decreasing in age. However, this result is obtained restricting cohort effects to zero; in addition, since they fit a Tobit model, no distinction is made between the optimal share and the participation decision. Thus it is unclear whether the age pattern stems from people exiting the market or lowering their share. Since they look at allocations in a 401k plan alone, all the issues raised about the Ameriks and Zeldes (2002) data extend to their data too.

as TIAA-CREF may be constrained by the rules of the plan, potentially resulting in less pronounced age patterns than in overall portfolios which reflect allocations of constraint-free financial wealth.

In this paper, we try to overcome these problems. We have assembled a new database drawing on administrative records from the Norwegian Tax Registry (NTR). Because Norwegian households are subject to a wealth tax, they have to report to the tax authority all their asset holdings, real and financial, item by item at the level of the single instrument as of the end of year. We have drawn a random sample of 20% (about 164,000) of the 1995 population of Norwegian households and then followed these households for 15 years up until 2009 - the latest year for which we could obtain the data. This dataset reports the complete portfolio of Norwegian people and is similar in structure and content to the one used by Calvet, Campbell, and Sodini (2007) but spans many more years - an essential feature when studying the life cycle profile of portfolio allocation. Being of administrative source, measurement error is minimized. The main cause of non-reporting or under-reporting should stem from incentives to evade the wealth tax, but the way the wealth tax is collected in Norway, as we argue in Section 3, suggests that tax evasion is unlikely to be an issue. Finally, since the whole population of Norwegian taxpayers has to report to the NTR, there is little attrition in the panel - apart from that due to death, migration to another country or divorce.

Taking into account the endogeneity of the participation decision and modelling cohort effects directly, we find that both participation in the stock market and the portfolio share in stocks show important life cycle patterns. As in other studies, we also find a hump-shaped life cycle profile in participation (besides limited stock market participation at all ages). But we also find that conditional shares decline significantly with investors' age. Specifically, the portfolio share in risky assets is high and fairly constant in the earlier and mid phases of the life cycle at a level just below 50%. As retirement comes into sight, households start rebalancing their risky asset share gradually but continuously at a pace of little less than one percentage point per year until they retire (around age 65). In retirement investors who remain in the stock market keep the share fairly flat at around 30%. On the other hand, participation in the stock market rises rapidly with age when young, reaching a value of around 60% at age 45 and stays roughly constant or slightly increasing until retirement. As soon as investors leave the labor market and retire, they start exiting the stock market as well.

Our data suggest a double adjustment as people age with a very specific timing: a rebalancing of the portfolio away from stocks *before* households reach retirement; exiting the stock market *after* retirement. Existing calibrated life cycle models can account for the first behaviour but not the second. We show that extending the models by Gomes, Kotlikoff, and Viceira (2008) and Gomes and Michaelides (2003) to incorporate a (relatively large) per period participation cost generates substantial limited participation among the young but not enough exit after retirement. However, adding also a small subjective probability of a large loss when investing in stocks (a "disaster" event), the model predicts a joint pattern and level of participation and the risky asset share over the life cycle similar to the one observed in the data, with early rebalancing of the share and pronounced exit from the risky asset market after retirement.

Numerical simulations reveal that a combination of small participation costs, small probability of a large loss and a relatively large risk aversion can explain well the shape and location of the life cycle profile of stock market participation and the risky asset share

of the average household.

The rest of the paper is organized as follows. Section 2 reviews the life cycle portfolio literature highlighting its core implications for the life cycle pattern of the participation and risky portfolio share. Section 3 discusses the Norwegian Registry data and presents descriptive evidence of the portfolio life cycle pattern. Section 4 lays down the methodology for estimating the life cycle portfolio profile and presents the estimation results. Section 5 shows how an extended calibrated life cycle model can account for the pattern of the portfolio that we observe in the data. Section 6 presents the results of the calibration and of estimates of the parameters of the models. Section 7 summarizes our contribution and draws implications for future research.

## 2 An overview of the literature

Over the past decade several papers have provided new models of optimal portfolio rebalancing over the life cycle that go beyond the seminal dynamic framework of Merton (1969), Mossin (1968) and Samuelson (1969). The Merton-Mossin-Samuelson (MMS) models generate two sharp predictions. First, individuals should participate in risky asset markets at all ages. Second, the share invested in the risky assets should not vary over the life cycle. The implications of the MMS models are in contrast both with the limited participation that we observe in the data at all ages and with the widespread advice of the financial industry practitioners to invest substantially in stocks when young and reduce the exposure to the stock market when older. Yet, these earlier contributions were not meant to provide sharp predictions about realistic features of the data but rather to establish the benchmark conditions under which a long term investor would choose “myopically”- i.e. show no life cycle pattern in his investments. As Samuelson (1969) points out, “A lifetime model reveals that investing for many periods does not itself introduce extra tolerance for riskiness at early, or any, stages of life”. One needs the MMS assumptions of no labor income, unpredictable stock returns, constant relative risk aversion and time-separable preferences to obtain an optimal portfolio risky share that does not vary with age (and wealth).

In fact, as shown by Merton (1971), adding to the model tradeable human capital in a complete market setting generates a strong rebalancing motive in the financial risky share. Since human capital is riskless and tradeable, it plays the same role as a large endowment in riskless bonds. Hence, it creates a strong incentive to invest in risky securities when human capital is abundant, that is early in the life cycle, and to rebalance away from stocks as people get older and their human wealth diminishes. The simple presence of human capital - an indisputable feature of any realistic model of household portfolio decisions - seems to be enough to provide a rationale for the practitioners’ advice to rebalance the financial portfolio away from stocks as people age and is consistent with recent evidence that human capital drives financial risk-taking positively (Calvet, Campbell, and Sodini 2007).

Merton (1971)’s result is obtained in a complete market setting with tradeable human capital; this allows him to obtain neat closed-form solutions. A new recent wave of papers has reconsidered the Merton (1971) model relaxing the assumption of complete markets and tradeability of human capital (see Gomes and Michaelides 2003, Gomes and Michaelides 2005, Heaton and Lucas 1997, Gakidis 1998, Michaelides and Haliassos 2002, Storesletten, Telmer, and Yaron 2007, Campbell and Viceira 2001, Viceira 2001, Cocco, Gomes, and Maenhout 2005, Davis, Kubler, and Willen 2006, Benzoni, Collin-Dufresne,

and Goldstein 2007, Gomes, Kotlikoff, and Viceira 2008). Because markets are incomplete and labor income is uncertain and non-tradeable, these models do not have closed form solutions and have to be solved numerically. A representative example of this literature is Cocco, Gomes, and Maenhout (2005). They develop, solve numerically and simulate a life cycle model of consumption and portfolio choice which allows for non-tradeable and uncertain labor income as well as many other features that characterize a typical household environment such as bequest motives, mortality risk, non-standard preferences, uncertain retirement income and catastrophic labor income shocks. They calibrate the labor income process with data from the PSID and compute average consumption and assets allocation by simulating the model over 10,000 households. A robust prediction of this and all the other models in this literature is that the portfolio share invested in stocks has a strong life cycle profile. Thus, Merton (1971)'s rebalancing implication holds true not only when labor income is tradeable and certain but also when it is non-tradeable and subject to uninsurable risk.<sup>2</sup>

Although the prediction that households should rebalance their portfolio as they approach retirement rests on an uncontroversial fact, namely the decline in human capital as people age, it has been hard to find it in the data, as we have argued. This is likely to be the reflection of limitations in the data that we are able to overcome using the Norwegian dataset; indeed, we find evidence that is consistent with the prediction that households should rebalance their financial risky portfolio as they approach retirement.

While the shape of the age profile of the portfolio share in stocks predicted by these models resembles the one we find in the data, there are two important differences between these models' predictions and our findings. First, the new models generate much higher shares in stocks, particularly at the beginning of the life cycle and in the middle ages, than those seen in the data among the stockholders. Second, they often do not give rise to limited participation and to exit from the stock market as people age. In particular, our evidence suggests a double adjustment as people age: as they approach retirement, they rebalance their portfolio share away from stocks but continue to stay in the market. After retirement they stop rebalancing but start exiting the market. In other words, before retirement households adjust along the intensive margin; after retirement they reduce exposure to financial risk along the extensive margin. Some models have addressed the issue of limited participation among the young by allowing for a once and for all fixed cost of participation (Cocco, Gomes, and Maenhout 2005), or for long run co-integration between labor income and stock market returns (Benzoni, Collin-Dufresne, and Goldstein 2007) or for costly access to the loans market (Davis, Kubler, and Willen 2006). None of these models, however, can deal with exit from the stock market as people retire. Hence they cannot explain the hump shape in participation over the life cycle and the timing of rebalancing in the optimal share and in participation that we observe in the data. In addition, these models tend to predict a far too high share in stocks among the stockholders at some point over the life cycle. To better mirror the data, we propose a

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<sup>2</sup>A declining life cycle portfolio profile may be generated also by other features than just the life cycle of human capital. For instance, Bodie, Merton, and Samuelson (1992) show that accounting for endogenous labor supply decisions can induce the young to invest more in stocks because greater labor market flexibility offers insurance against financial risks. A downward sloping age-portfolio profile can be generated by departure from CRRA utility (Gollier and Zeckhauser 2002), by life cycle patterns of risk aversion and background risk, as well as predictability of stock returns (Kandel and Stambaugh 1995, Campbell and Viceira 1999, Campbell and Viceira 2002). These factors may certainly contribute to induce a rebalancing motive over the life cycle but none is uncontroversial as the life cycle of human capital.

simple extension of the Cocco, Gomes, and Maenhout (2005) model enriched with two ingredients: a per period participation cost and a small probability of a left-tail event in stock performance, which can be interpreted either as a perceived probability of being frauded; a "trust" friction as in Guiso, Sapienza, and Zingales (2008) or as rare disaster as in Barro (2006) and Rietz (1988).<sup>3</sup> This model is able to generate a hump shaped pattern of stock market participation that peaks at retirement and declines thereafter. This pattern is the consequence of the hump-shaped wealth age profile: when young, wealth is typically increasing and thus gradually more and more consumers will cross the wealth threshold that makes it worthwhile to incur the per period cost, triggering entry into the stock market. After retirement, people begin to decumulate assets and at some age the level of assets left is too low for it to be worth paying the per period cost and remaining in the market, hence they exit.<sup>4</sup> At the same time, the portfolio share invested in stocks among the stockholders is relatively high and flat at young age, but as people foresee retirement, they start rebalancing the portfolio. The disaster probability helps not only to lower the portfolio share in stocks, bringing it close to the one observed in the data, but, as we will show, to account for the exit from the stock market following retirement. When we simulate the model, we find that a combination of low participation cost, low disaster probability and relatively high risk aversion explains quite well the joint behaviour of life cycle profile of stock market participation and the conditional share of the average household.

### 3 Data

The empirical study of household portfolio allocations over the life cycle has formidable data requirements. Ideally, one needs data on households' complete portfolio holdings over a long time span, free of measurement and reporting errors. The NTR data that we use in our empirical analysis come very close to meet these requirements. Because households in Norway are subject to a wealth tax, they are required to report every year their complete wealth holdings to the tax authority. We merge this information with administrative records of individual demographic characteristics and information on earnings from the same source and obtain a unique panel data set spanning the years from 1995 to 2009.

#### 3.1 The Norwegian administrative data

Each year, before taxes are filed in April, employers, banks, brokers, insurance companies and any other financial intermediary send both to the individual and to the tax authority, information on the value of the asset owned by the individual and administered by the

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<sup>3</sup>Alan (2012) also studies the life cycle portfolio allocation allowing for participation costs and a disaster probability and estimates it structurally on US data, using pseudo panel data constructed using various cross sections of the US Survey of Consumer Finances. She finds that the model fits the data poorly. As she notices when compares the simulated profile and that in the data, "What is particularly disturbing in this figure is that the model persistently generates a hump shape for shares and participation that does not exist in the data." However, it may be that it is not the model that needs to be amended but rather the data that are unable, due to the shortcomings of survey data discussed in the introduction, to reveal the true life cycle portfolio pattern.

<sup>4</sup>Some exit from the stock market after retirement may occur even without a per period participation cost if households liquidate stocks in bulk to finance durable consumption purchases or to face unusual lumpy expenses e.g. for health care (Alan 2006). In general, however, absent participation costs, one should see a decumulation of both stocks and bonds and very little exit.



employer or the intermediary, as well as information on the income earned on these assets. In case an individual holds no stocks, the tax authority pre-fills a tax form and sends it to the individual for approval; if the individual does not respond, the tax authority considers the information it has gathered as approved. In 2009, as many as 2 million individuals in Norway (60% of the tax payers) belonged to this category.<sup>5</sup> If the individual owns stocks then he has to fill in the tax statement - including calculations of capital gains/losses and deduction claims. The statement is sent back to the tax authority which, as in the previous case receives all the basic information from employers and intermediaries and can thus check its truthfulness and correctness.<sup>6</sup> Stockholders are treated differently because the government wants to save on the time necessary to fill in more complex tax statements and to reduce the risk of litigation due to miscalculated deductions on capital losses and taxes on capital gains.<sup>7</sup> This procedure, particularly the fact that financial institutions supply information on their customer's financial assets directly to the tax authority, makes tax evasion very difficult, and thus non-reporting or under-reporting of assets holdings very likely to be negligible.<sup>8</sup>

Tax statements on both labor income in the previous year and asset holdings, as of December 31 of the previous year, are filed separately by each taxpayer in the population even for married couples. Besides information on assets, the administrative data contains information on demographic characteristics of all individuals as well as an identifier for the family they belong to. Thus, we can aggregate assets at the household level. For our purposes, we define a household as a married couple (or a cohabiting couple possibly with children) and identify its age (and other characteristics such as education) with that of the husband. The term "cohort" refers to the year of birth of the husband. In order to extract a large but still computationally manageable sample, we first retain all households defined as above with both spouses alive as of 1995 and with at least 3,000 NOK of financial assets (480 USD at 1995 prices). We then randomly sample 20 percent of them obtaining an initial reference sample of 164,015 households which we follow over the subsequent 15 years until 2009. Households who exit the sample because individuals die, or migrate or divorce are not replaced. Overall, the sample contains 916,823 household-year observations.<sup>9</sup>

We focus on the financial portfolio and distinguish between bank deposits, bonds, stocks (of listed and non-listed companies), mutual funds and money market funds.<sup>10</sup> Following the literature, we consider a two asset-portfolio and define risky financial assets as the sum of mutual funds with a stock component and directly held stocks; the rest - the sum of bank deposits, money market funds and bonds - is classified as risk-free assets.<sup>11</sup>

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<sup>5</sup>See Norwegian Tax Administration annual report:  
<http://www.skatteetaten.no/Upload/annual-report-2009.pdf>

<sup>6</sup>Internet brokers tend to offer to their costumers calculations of realized returns over the previous year for free.

<sup>7</sup>Since year 2000 all this is done electronically; prior to 2000 tax reports were done on paper forms.

<sup>8</sup>The only exception is if households own and do not report foreign investments. Calvet, Campbell, and Sodini (2007) discuss this issue for Sweden and conclude that unreported foreign investments represent a modest fraction of households' assets - except perhaps for the very wealthy.

<sup>9</sup>The quality of this data is similar to that in the Swedish data studied by Calvet, Campbell, and Sodini (2007). Until 2007, Sweden like Norway collected taxes on both individual income and wealth. In 2007, however, Sweden abandoned the wealth tax, leaving Norway as the only Scandinavian country with this arrangement.

<sup>10</sup>Very few households (67 observations in the whole sample) hold more sophisticated instruments such as futures and options. We exclude them from the sample.

<sup>11</sup>Private old age pensions were not widespread in Norway during our observation window. All Norwegian citizens are entitled to a state pension from the age of 67 in accordance with the National Insurance Act.

Table 3.1: Descriptive Statistics - 1995

	Full Sample				Balanced Panel Sample			
	Obs	Mean	Std Dev	Median	Obs	Mean	Std Dev	Median
<b>Demographics:</b>								
Age Husband	164,015	50.88	14.14	49	106,369	47.67	11.64	47
Age Wife	164,015	48.12	14.01	47	106,369	45.00	11.40	45
Share Less High School Education	164,015	0.22			106,369	0.18		
Share High School Education	164,015	0.53			106,369	0.55		
Share College Education	164,015	0.24			106,369	0.27		
Household Size	164,015	3.24	1.19	3	106,369	3.44	1.17	3
<b>Asset Holdings in USD:</b>								
Financial Wealth	164,015	38,270	106,975	11,884	106,369	38,169	111,865	11,348
Stocks	164,015	12,797	91,438	0	106,369	14,386	97,230	0
Mutual Funds	164,015	1,173	3,895	0	106,369	1,245	3,989	0
Safe Assets	164,015	24,297	37,678	9,734	106,369	22,536	35,575	9,139
Net worth	164,015	120,354	143,051	97,543	106,369	116,213	142,199	93,318
<b>Participant share:</b>								
Risky Assets	164,015	0.33	0.47	0	106,369	0.35	0.48	0
Stocks	164,015	0.23	0.42	0	106,369	0.25	0.43	0
Mutual Funds	164,015	0.22	0.41	0	106,369	0.23	0.42	0
<b>Mean share participants:</b>								
Risky Assets	54,519	0.32	0.30	0.20	37,770	0.33	0.31	0.22
Stocks	54,519	0.23	0.31	0.05	37,770	0.24	0.32	0.06
Mutual Funds	54,519	0.09	0.15	0.03	37,770	0.09	0.15	0.04
<b>Attrition:</b>								
	58,863							
Share Death		0.62						
Share Migration		0.13						
Share Divorce/Separation		0.25						
Mean yearly attrition rate:		0.030	0.000					
Age at Exit		62.63	16.83					

*Note:* This table displays summary statistics for the main sample of married households in the first year of observation, 1995. In addition, the table provides summary statistics for the sample of households that remain in the panel throughout, until 2009. Where applicable, values are reported in 1995 USD. Education is missing for less than one percent of the sample.

Table 3.1 provides summary statistics for the whole household sample in 1995. Household average age is 51 years. High school diploma is the most common educational level, which is attained by 53% of the sample, while 26% hold a college degree. The average Norwegian household holds around 38,000 USD (1995 prices) in financial assets. Net worth, the sum of financial assets and real estate net of debt, amounts to 120,000 USD, of which about 2/3 is real estate.<sup>12</sup> The financial portfolio of the average household is mostly composed of safe assets which account for 63% of average financial assets. We define a participant in the risky financial assets market to be a household with at least 160 USD (1995 prices) of risky assets. The participation rate in risky asset markets amounts to 33% (37% if we include all those with positive risky assets), reflecting 23% of the population holding stocks directly and 22% percent participating via mutual funds. Thus, back in 1995 mutual funds were not as widespread as direct stock-holding among Norwegian households. Among participants, the average portfolio share in risky assets is 32% while mutual funds account for 9%; a similar figure for the total share prevails in other European countries, as documented in Haliassos, Guiso, and Jappelli (2001). Needless to say, over our sample period, asset markets worldwide and in Norway experienced both booms and busts and the mutual fund industry expanded significantly making it easier for many households to participate in the risky asset market, e.g. by lowering participation costs, offering more diversified investments and spreading information about mutual fund investments.

Although there is attrition in the sample at an average annual rate of 3%, we can track 2/3 of the households sampled in 1995 all the way until 2009. The main reason for exiting the sample is death of one spouse (62%), which is consistent with the high average age at exit (65 years, see bottom of Table 3.1). To get a sense of the importance of attrition for the composition of the sample, the right part of Table 3.1 displays summary statistics in 1995 for the balanced sample - households that are present continuously from 1995 to 2009. Balanced panel households are not surprisingly younger in 1995 and slightly better educated. However, the value of asset holdings, portfolio allocation and risky asset market participation are similar across the two groups suggesting that attrition is fairly random.

### 3.2 Portfolio life cycle patterns by cohort: descriptive evidence

Figure 3.1 plots the age participation profile in the risky assets market for selected cohorts spaced by 5-year intervals, beginning with the cohort born in 1970, aged 25 in 1995, the first sample year. Since we are able to follow each cohort for 15 years, just plotting the raw data provides a good picture of the life cycle portfolio pattern.

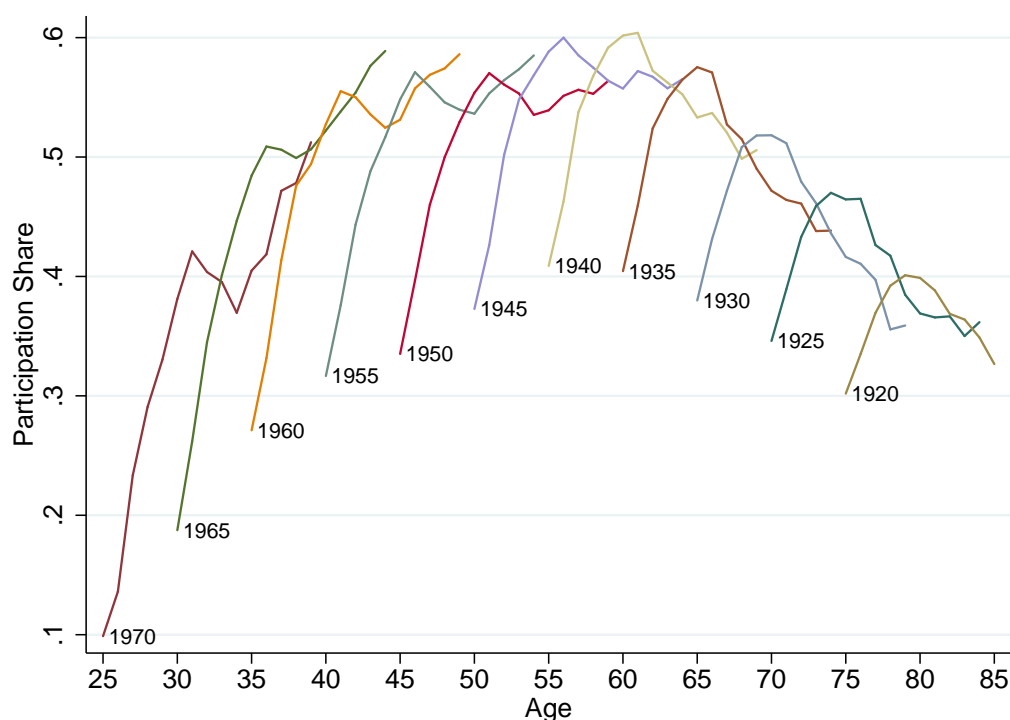
Consider the first cohort born in 1970 whose members are 25 years old in 1995; only slightly more than 10% of them were participating in risky asset markets in 1995. However, subsequently the share of participants in this cohort increases substantially, and five years later when this cohort ages 30, almost 50% of the households own risky financial assets. Clearly, this pattern is consistent with a marked age effect (an increase in participation

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The financing of this system is through a Pay-as-you-go system, but it is currently undergoing reforms and evolving towards a defined contribution system to be fully implemented by 2025. Furthermore, early retirement schemes are widespread in Norway and workers may be eligible for these from the age of 62, see e.g. Vestad (2013). Pension benefits are indexed according to the average wage growth of the population.

<sup>12</sup>The value of real estate is a proxy based on the reported tax values of Norwegian households, and is not updated every year. To obtain our estimate, we divide the reported tax value of real estate by 0.25. This follows the guidelines of the Norwegian Tax Authorities, which state that the tax value of real estate shall not exceed 30% of its market value.

Figure 3.1: **Participation shares in Risky Asset markets, selected cohorts**



*Note:* This Figure plots the mean participation rates in Risky Asset markets at observed age for selected cohorts in the period 1995-2009.

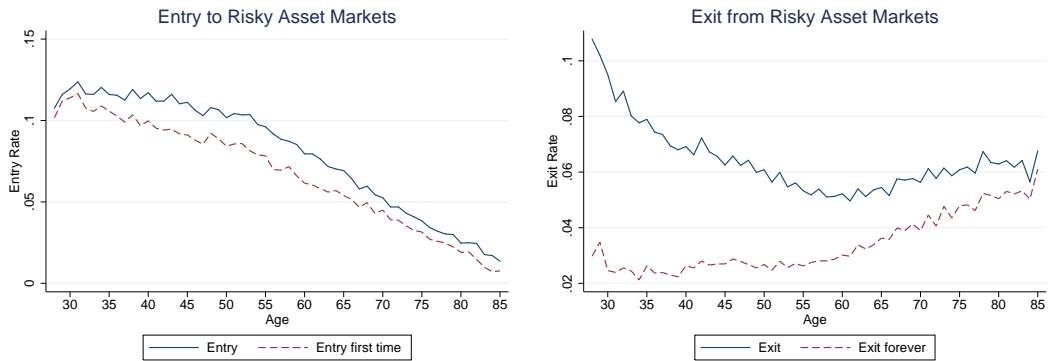
with age), with strong time effects (an increase in participation due to favourable improvements in market conditions, e.g. the boom of the mutual funds industry), as well as with a cohort-specific pattern. If this were the only cohort observed, these effects would be hard to disentangle as time and age evolve in parallel and we only observe one cohort. We could not make any claim on whether the increase in participation rate is cohort-specific, a pure age effect, or if it reflects a common time trend that affects all cohorts in the years 1995-2009.

The next plotted cohort - households born in 1965 - reveals a steep increase in average participation during the first years of our sample also for these households. This suggests that the increase in participation over age/time is unlikely to be cohort specific. But it is still unclear whether it is due to an age-effect, or to a common time trend. Comparing the evolution of participation across cohorts suggests that time effects are likely to be important; for instance, all cohorts experience a marked increase in participation during the first years of our sample, even those born in 1920 - who are 75 in 1995 - and thus typically exit risky asset markets. And a drop during the 2001 recession even among those born in the 1960's and 1970's who are typically entering the stock market. This graphical evidence also suggests that cohort effects are likely to play an important role. In fact, compared to younger cohorts, older cohorts at the same age, have lower participation

Table 3.2: **Entry and Exit Definitions**

<b>Measure 1:</b>	
Entry:	The fraction of households who do not hold stocks at age $a$ that enter the risky asset markets at $a+1$ .
Exit:	The fraction of those who are stockholders at age $a$ who exit the market at age $a+1$ .
<b>Measure 2:</b>	
Entry:	The fraction of households who <b>has never held any stocks</b> up until the age $a$ that enter the risky asset markets at $a+1$ .
Exit:	The fraction of those who are stockholders at age $a$ who exit the market at age $a+1$ and <b>never re-enters</b> the stock market.

Figure 3.2: **Entry and Exit rates from Risky Asset Markets**



*Note:* This Figure plots entry and exit rates into the risky asset markets. The left graph depicts entry and exit frequencies, allowing for re-entry/exits. In the right graph, frequencies of first time entry and once and for all exits are plotted.

rates. In Section 4, we describe our empirical strategy to separate age and time effects and test for the presence of cohort effects.

As a next step in the descriptive analysis of the life cycle patterns of participation, we consider two measures of entry into and exit from the stock market, as defined in Table 3.2. These two measures are plotted in Figure 3.2 for the same selected cohorts. The first measure refers to entry (exit) in a given year, regardless of the household’s past (future) participation pattern. The second, reports entry (exit) that was not preceded (followed) by a previous entry (a subsequent exit). The second measure captures first-time entry and permanent exit.

First-time entry is very high at the beginning of the life cycle, with a peak at 13%, and drops steadily thereafter. It is lower than total entry particularly for middle aged households. Instead permanent exit is low at the beginning of the life cycle and increases sharply after retirement.<sup>13</sup> By comparing the two measures, Figure 3.2 highlights that

<sup>13</sup>Because of the limited time span of our data, the second measure of entry and exit may be affected

early in life temporary entry and exit are very common phenomena. Among households in their early 30's, 13% enter the stock market and most of them enter for the first time. On the other hand, the fraction of young households that sells all risky financial assets to return to the stock market later in life is almost five times the fraction of households that exit permanently. The existence of intermittent participation suggests a role for per period participation cost and it will be interesting to see whether our extended model that allows for this type of costs can reproduce the observed pattern of entry and exit.<sup>14</sup>

Figure 3.3 plots the risky financial share among households who participate in the stock market for the same cohorts as in Figure 3.1. We refer to it as the conditional share. Looking at the overall age pattern the picture suggests that once people enter, they invest a relatively large share in risky assets, hold it fairly constant over the early part of the life cycle and reduce it as they age. A comparison across cohorts suggests less pronounced cohort effects than those that seem to characterize the participation profile. On the other hand, the pattern of the conditional share over time across cohorts reveals strong time effects, reflecting movements in stock prices that are only partially undone by active rebalancing, as suggested by Calvet, Campbell, and Sodini (2009). Instead, the raw data for the risky share suggest that there is substantial rebalancing over the life cycle, in particular as households approach retirement.

## 4 Estimation

The descriptive evidence suggests the existence of marked life cycle patterns for both the participation decision and the risky share of household's portfolio conditional on participation. However, it does not deal with two key issues: the endogeneity of participation in risky assets and the separation of time, age and cohort effects. In this section, we discuss how we address these issues in order to pin down the age profile of participation in the market for risky asset markets and the portfolio share for the participants.

### 4.1 Methodology: limited asset market participation

It is well established that not all households participate in risky asset markets. Empirical studies of the life cycle profile of household portfolios has, so far, neglected the endogeneity of participation when estimating the life cycle profile of the portfolio share (Ameriks and Zeldes 2002). This is unfortunate because unaccounted selection can bias the relation between the optimal share and age, for instance hiding it. Also calibrated life cycle models have, until recently, ignored limited participation in risky asset markets by abstracting from participation costs. Later, we will remedy this deficiency by introducing a per period participation cost in a standard life cycle portfolio model that already allows for several other realistic features.

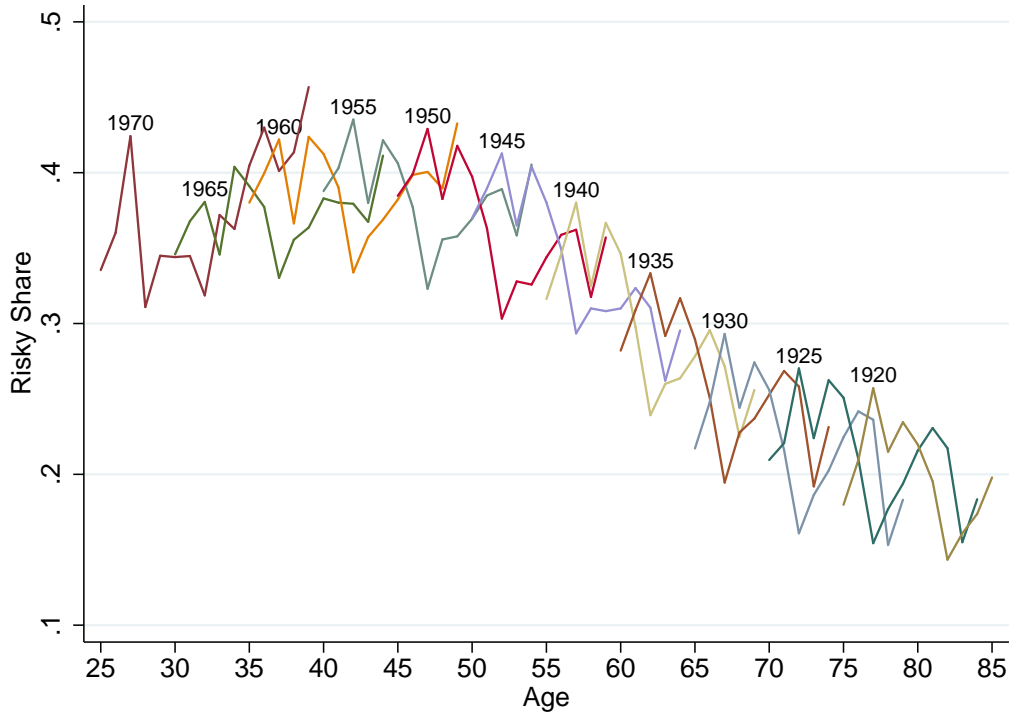
Empirically, we deal with the joint decision of whether to participate and how much financial wealth to allocate in risky assets using a Heckman selection model. To do so, we

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by censoring. Censoring should bias upward both the first time entry rate measure at young age, and the permanent exit rate measure at old age.

<sup>14</sup>Note that at low and high ages, the number of observations is limited because of fewer households and because of more limited stock market participation at extreme ages. This explains the higher variability at the two ends of the age range both in this Figure 3.2 and in Figure 3.3.

Figure 3.3: Risky Share of Financial Wealth by Cohort



Note: This Figure plots the average risky shares of households' financial portfolios conditional on participation, for selected cohorts at each age they are observed.

estimate a probit model for the household participation in the risky assets market and a share equation for the participants accounting for selection. To achieve identification, we let the decision to participate be affected by (the lagged value of) the overall lifetime wealth of the individual, obtained summing accumulated assets and an estimate of the individual stock of human wealth (see the Appendix in Section A.2 for details about the estimation of human wealth), and impose that lifetime wealth does not affect the financial portfolio share conditional on participation. This exclusion restriction is inspired by Merton (1971) whose model implies that in the presence of labor income, risky assets holdings as a share of total lifetime wealth is constant over the life cycle and thus independent of lifetime wealth. The financial portfolio share in risky assets depends on the *ratio* of human to financial wealth which evolves over the life cycle but not on the *level* of lifetime wealth. Hence, controlling for age to account for the life cycle of human to financial wealth, the

share should be unaffected by total lifetime resources.<sup>15</sup> We impose this restriction.<sup>16</sup> Of course, with a fixed participation cost the decision to participate will depend on the level of individual wealth (Vissing-Jorgensen 2002).

## 4.2 Methodology: treatment of cohort effects

Even though we observe households' investments over a substantial portion of their life span, it is well known that it is not possible, without additional restrictions, to identify cohort, time and age effects. This issue is extensively discussed in Ameriks and Zeldes (2002) in the context of estimates of the life cycle profile of portfolio choice. In fact, calendar time, age and year of birth are linearly related. Hence, if we observe that older people hold fewer stocks than younger ones, it may be because as they age they choose a safer portfolio (an age effect) or it may be because the older grew up in different times than the younger and this has led to develop different preferences towards risk or different beliefs about stock market returns (a cohort effect). Alternatively, it could be that over the years people are exposed to different types of shocks (a time effect). Panel data partially addresses this issue, but as much as they help identifying one extra dimension, they also add one more dimension to identify.

Since at the heart of the identification problem is the linear relationship "calendar year" = "age" + "year of birth", most solutions have proceeded by making assumptions or using prior information so as to break this multicollinearity, allowing the use of standard regression techniques. One strategy that has been followed is to re-specify the model to make it non-linear or to estimate it in first differences; another is to impose parametric restrictions; a third to replace the dummies that capture one of the effects with variables meant to capture a causal mechanism for that effect.<sup>17</sup> Here we rely on both the second

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<sup>15</sup>Let  $s(a)$  denote the share of financial wealth ( $W(a)$ ) invested in risky assets by an individual aged  $a$  and  $H(a)$  his stock of human capital. In Merton (1971) the share of risky assets as a fraction of lifetime wealth  $W(a) + H(a)$  is

$$\frac{s(a)W(a)}{W(a)+H(a)} = \frac{r_p}{\gamma\sigma_r^2}$$

where  $r_p$  denotes the equity premium,  $\sigma_r^2$  the variance of stock returns and  $\gamma$  the investor relative risk aversion. This share is constant over age; on the other hand,  $s(a) = \frac{r_p}{\gamma\sigma_r^2} (1 + \frac{H(a)}{W(a)})$  and varies over the life cycle because the ration of human capital to financial wealth  $\frac{H(a)}{W(a)}$  varies with age. Thus, capturing  $\frac{H(a)}{W(a)}$  with a set of age dummies,  $s(a)$  is unaffected by the level of human wealth or that of financial wealth.

<sup>16</sup>The restriction holds true in the context of the Merton (1971) model; whether it is still valid once one relaxes the assumption on which it builds, in particular the complete market markets assumption, is hard to tell because there is no closed form solution to the model. One may think that with uninsurable income risk, presumably investors with more cash on hand can stand background risk more easily, suggesting that cash on hand can reduce the effect of background risk on the risky portfolio share, which would invalidate the exclusion restriction. To check how important this may be, we have used the simulated data generated by the model in Section 5 and run regressions of the portfolio share on a full set of time dummies and cash on hand accounting for endogenous participation. We find that cash on hand has positive and strong effect on participation (one standard deviation increase in cash on hand increases the probability of participation in the stock market by 10 percentage points - about 25% the mean participation rate. On the other hand it has a negative but very small effect on the share. A one standard deviation increase in cash on hand lowers the risky share by 1.6 percentage points. Since its mean is around 40%, this is a tiny effect. Thus, though the exclusion restriction does not hold literally, it seems to hold approximately. An alternative exclusion restriction that is implied by the theory would be to use information on per period participation costs in the probit regression. Finding valid measures of individual participation costs is very difficult and we have not, so far, been able to come up with a convincing one. Thus, rather than trying unconvincing proxies for participation costs, we have preferred to impose an identifying restriction that simulations suggest is, economically speaking, not far from what literal validity requires.

<sup>17</sup>The use of one or another strategy is context specific and the choice depends on what assumption



(impose parametric restrictions) and the third strategy (model cohort effects explicitly).

As for the parametric restrictions, we rely on Deaton and Paxson (1994) and impose that time effects sum to zero once the variables have been detrended. Since our data cover several years, we should be able to separate trend and cycle, and thus be reasonably confident about the decomposition of age, time and cohort effect based on this restriction (Deaton 1997).

To implement the other strategy, we build on recent research by Giuliano and Spilimbergo (2009), which indicates that generations who grew up in recessions have systematically different socio-economic beliefs compared to generations who grew up in booms - suggesting important year of birth effects on beliefs preferences. Even closer to the spirit of our approach is the study by Malmendier and Nagel (2011), who show that households with experience of higher stock market returns early in life are more likely to participate in the stock market and, conditional on participation, invest a higher fraction of their wealth in risky assets. Furthermore, when asked, they report a higher willingness to bear risk, possibly because early experiences have enduring effects on risk preferences. This evidence suggests that one can rely on variations in experienced stock market returns among members of our sample to model cohort effects. Accordingly, we will use stock market returns (a weighted average of the Norwegian Stock Exchange (NSE) and the S&P 500 index) experienced during the household heads' youth (between age 18 and age 25, as in Giuliano and Spilimbergo 2009) as our proxy for cohort effects. As we will show, these returns significantly affect the decision to enter the risky assets market and to a lesser extent the conditional risky share. Thereby we can identify unrestricted time and age effects. However, we try to validate this second strategy by imposing a restriction on the age profile and estimate unrestricted cohort effects; then, we test whether the unrestricted cohort effects correlate with early age stock market experiences. In doing so, we follow Berndt and Griliches (1995) who solve the identification problem by restricting some of the age coefficients identical.

To assess the robustness of our results, we also follow a third approach imposing some restrictions on the age profiles and thereby allowing for unrestricted time and cohort effects. As we show in Section 5, our simulated model generates a hump shaped profile for the risky portfolio share and participation. Around the peak, the profile changes smoothly and tends to be relatively flat. Hence, a theory-based, reasonable restriction is to impose that the age effects are constant around the peak. We use the simulations in Figure 6.3 to identify the peak in the participation profile (age 60) and in the conditional share (age 38) and then we impose that age effects be the same two years before and two years after the peak.

### 4.3 Model specification

We specify the following two equation model for the share of financial wealth invested in stocks conditional on participation,  $s_{iact}$ , and for the decision to participate  $P_{iact}$  by

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appears reasonable in the specific context. Some recent papers propose generic, contest-independent solutions. One is suggested by Yang, Fu, and Land (2004) and Yang, SchulhoferWohl, Fu, and Land (2008) who propose what they call the intrinsic estimator. Another introduced by Browning, Crawford, and Knoef (2012) who show that when the range of the variable(s) of interest is bounded, the time, age and cohort effects are partially identified in the sense they are confined to a closed convex set. They then propose using a maximum entropy estimator to achieve point identification within that set. Because our variables are bounded (the decisions to participate being zero/one, and the share between 0 and 1) the later methodology could be applied in our context.

household  $i$ , aged  $a$ , belonging to cohort  $c$  in year  $t$ :

$$s_{iact} = \beta_a A_a + \beta_c C_c + \beta_t D_t + \beta_0 Trend + \theta_1 Z_{iact} + \theta_2 \lambda_{iact} + \varepsilon_{iact} \quad (4.1)$$

$$\begin{aligned} prob(P_{iact} = 1|x) &= prob(P_{iact}^* > 0|x) \\ &= prob(\delta_a A_a + \delta_c C_c + \delta_t D_t + \delta_0 Trend + \vartheta_1 Z_{iact} + \vartheta_2 L_{iact} + \eta_{iact} > 0) \end{aligned}$$

where  $P_{iact}$  is a dummy variable taking value 1 for households with positive risky assets and zero otherwise,  $P_{iact}^*$  is the unobserved latent variable triggering participation when positive,  $A_a, C_c$  and  $D_t$  denote dummies for age, cohort and time,  $Trend$  is a time trend,  $Z_{iact}$  a vector of individual controls,  $\lambda_{iact}$  the inverse Mills ratio computed from the participation equation and  $L_{iact}$  an estimate of lifetime wealth;  $\varepsilon_{iact}$  and  $\eta_{iact}$  are error terms.

When we use the Deaton and Paxson (1994) method to tell age, time and cohort effects apart, we also impose the restriction  $\sum \beta_t = \sum \delta_t = 0$ ; when we model cohort effects as a function of experienced stock market returns ( $R_{iact}$ ), we set  $C_c = R_{iact}$  and  $\beta_0 = \delta_0 = 0$ . Finally, identification through peak restrictions are obtained imposing  $\beta_{36} = \beta_{37} = \beta_{38} = \beta_{39} = \beta_{40}$  in the share equation and  $\delta_{50} = \delta_{59} = \delta_{60} = \delta_{61} = \delta_{62}$  in the participation equation and  $\beta_0 = \delta_0 = 0$ . Assuming  $\eta_{iact}$  is normally distributed, we estimate the above model using a two stage Heckman estimator.

#### 4.4 Results from estimating life cycle patterns

Table 4.1 reports the estimates of the Heckman selection model. Age and time effects as well as the coefficients of the other controls are, for brevity, not reported. The first two columns shows the estimates using the Deaton and Paxson (1994) restriction. In the first column, the time trend is positive, significant and economically important in the participation equation; it implies that in the final year of the sample the average participation rate is 18 percentage point higher than at the beginning of the sample. The trend is negative and statistically significant but economically small in the conditional share estimate. Unrestricted cohort effects are significant both for the participation decision and for the risky asset share, but particularly for the former (see the  $\chi^2$  test at the bottom of the Table). Interestingly, the probability that the household participates in the market for risky assets is strongly affected by the level of lifetime wealth, which suggests that the identifying strategy is, as expected, both consistent with the presence of fixed participation costs and powerful. In addition, the significance of the Mills ratio suggests the importance of adjusting for selection to obtain consistent estimates of the age profile of the conditional share.

Column 3 and 4 show the estimates obtained modelling cohort effects explicitly. Cohort effects captured by stock market returns experienced in youth have a positive and significant effect on the participation decision but not on the share of financial wealth invested in risky assets among the participants. Economically, investors who grow up in years of low stock market returns (5th percentile of the historical return distribution) are 6.12 percentage points less likely to own risky assets compared to investors exposed in youth to high stock market returns (95th percentile of the historical return distribution). The effect of lifetime wealth on participation and of the Mills ratio on the conditional share is essentially the same as when imposing the Deaton and Paxson (1994) restriction.

Finally, the last two columns report the results imposing peak restrictions. Again, the effects of initial total wealth in the participation regression and the Mills ratio is similar to the estimates obtained using the other two methods. Most importantly, if we retrieve the cohort effects from these estimates and correlate them with the youth stock returns proxy, we find that they are positively correlated, in particular, cohort effects in the participation regression which supports the identification strategy used in Columns 3 and 4.

The age profile for participation and the portfolio share obtained from the estimated Heckman model using the first two strategies are plotted in the two panels of Figure 4.1.<sup>18</sup> Independently of the method used to separate age from time and cohort effects, the figures document a distinct hump-shaped age pattern of asset market participation over the life cycle. Among younger households, the participation rate (right scale) increases steadily until the age of approximately 40, and then much more gradually, peaking when households are in their 60's, just prior to retirement. At peak the participation rate is around 60%. From then on participation in the risky asset market drops almost linearly until the age of 80. The age pattern of the conditional risky share is remarkably different. The share starts high at very young age and remains relatively constant for about a decade; from then on individuals rebalance the share in risky assets first gradually and then somewhat faster until retirement (around age 65), when the risky share stabilizes. During the transition, the share is reduced at a speed of around half of a percentage point a year (if the cohort proxy is used or 2/3 of a percentage point using the Deaton and Paxson (1994) restriction), half of the speed of adjustment that is typically recommended by practitioners.

The most interesting feature of the two profiles is the timing of the portfolio adjustment along the two margins - the intensive margin of the share invested in risky financial assets and the extensive margin of participation in risky assets. Our estimates show that consistent with life cycle portfolio models with labor income, households do limit exposure to the stock market by rebalancing their financial portfolio as they approach retirement and the stock of human capital falls. But they adjust also along the other margin, by leaving the stock market altogether as they age. However, this adjustment starts to take place only after the household retires, exactly when the adjustment along the intensive margin stops. The pattern and the timing of this double adjustment that we document empirically is the focus of the life cycle portfolio model developed in Section 5.

Figure 4.2 contrasts the estimated life cycle profiles of the share and participation considering also the peak restriction method. The three methods deliver very similar participation profiles. Instead, the Deaton and Paxson (1994) method predicts a significantly higher conditional particularly among the young.

Since the age profiles of human capital differ in level and shape according to education (see Appendix A.2), these differences may result in different portfolio share and participation profiles though their main qualitative features should be preserved since human wealth declines with age independently of education. As a robustness check, we have estimated the model presented in Section 4.3 separately for three education groups imposing the Deaton and Paxson (1994) restriction (results are similar using the other two methods). More educated households tend to participate more and to invest larger shares in risky assets conditional on participation. However, the age profile of the share and participation preserve the dual adjustment pattern that we have documented for the whole sample, with the conditional share being relatively flat in the middle ages and then

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<sup>18</sup>Obviously, since the value of lifetime wealth depends on age it contributes to confer a lifetime profile to the participation rate, in addition to the effect that the age dummies have on it. Figure 4.1 reflects this.

declining until retirement and the participation profile being hump shaped with exit from the stock market beginning only after households have already adjusted the share and are close to retire or just retired.

Finally, we apply the same methodology to separate age from year and cohort effects in the entry and exit patterns shown in Figure 3.2. We regress the two different measures of entry and exit of the risky asset market on age dummies, cohort dummies and calendar year fixed effects imposing the Deaton-Paxson restriction discussed in 4.2. The estimated profiles are reported in Figure 4.3. Interestingly, once we account for cohort and time effects, the entry age profiles are hump shaped with a peak around age 40 while the exit age profiles are somewhat U-shaped.

Table 4.1: **Heckman Selection Model**

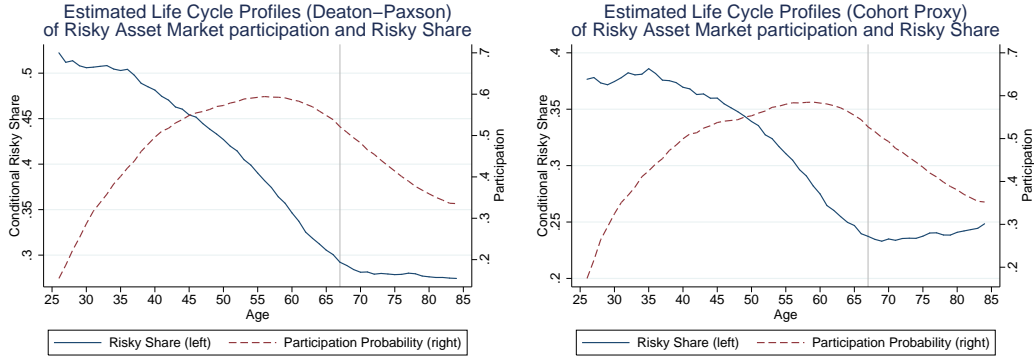
	Deaton-Paxson		Cohort Proxy		Peak Restriction	
	Part Eq	RS Eq	Part Eq	RS Eq	Part Eq	RS Eq
Trend	0.012*** (0.001)	-0.003*** (0.001)				
Youth Stock Return			0.361*** (0.017)	-0.070 (0.080)		
Lag Total Wealth	4.107*** (0.148)		4.186*** (0.030)		3.597*** (0.010)	
lambda		-0.186*** (0.001)		-0.186*** (0.001)		-0.185*** (0.001)
Observations	1,804,115	886,189	1,804,115	886,189	1,804,115	886,189
Joint sign. tests						
Year $\chi^2$ (12)	1575.79***	882.70***				
Cohort $\chi^2$ (59)	7644.51***	19.17***			1641.10***	9.99***

*Note:* This table displays the three estimated Heckman selection models (discussed in Section 4.2) for asset market participation and the conditional risky share. Lagged Total Wealth is the sum of Financial and Human Wealth and is in 100.000 USD (1995), and "lambda" is the inverse Mills ratio/nonselection hazard. Coefficients in the Selection Equation are calculated marginal effects of the underlying probit regression. For spacial reasons calendar year fixed effects and family size coefficients are not reported here, age coefficients and marginal effects are displayed in Figure 4.1. Standard errors are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

## 5 Model

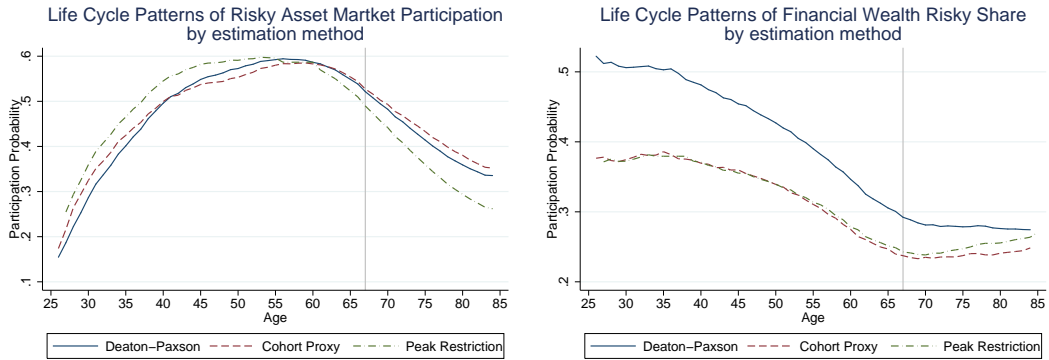
The previous sections have established novel stylized facts on the life cycle profile of Norwegian households' asset market participation and portfolio composition. Existing calibrated life cycle models can account for the rebalancing of the risky share away from stocks over the life cycle, but not for the joint patterns of adjustment of the share and the participation in the risky assets market. In this section, we present a life cycle model that can account for the life cycle profile of portfolio allocations along both margins. To facilitate comparisons with the literature, we use the workhorse portfolio choice model of Cocco, Gomes, and Maenhout (2005) but add two features. First, we allow for a fixed per-period stock market participation cost; this provides a motive for exiting the stock market as people age besides inducing limited participation in the stock market at early ages. Second, we allow for a small probability of a negative tail event when investing in stocks. There are two well established ways in the literature to interpret this tail event. One

Figure 4.1: **Estimation: Risky Asset Market Participation & Risky Shares**



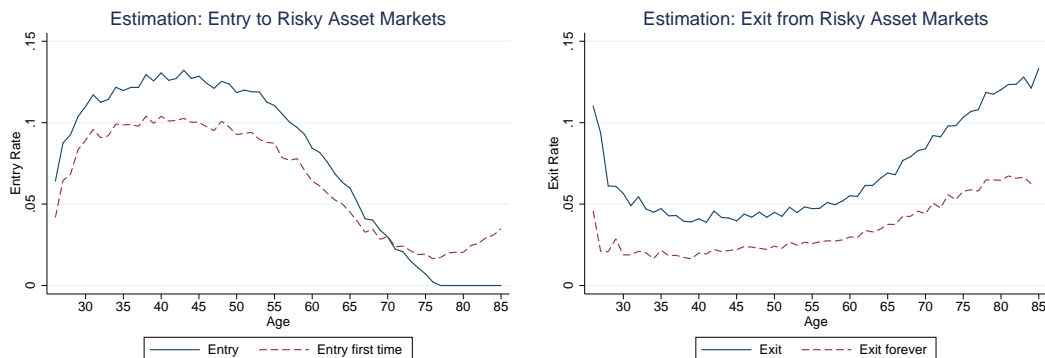
*Note:* The left panel of the Figure plots the life cycle patterns for both the risky asset market participation and the conditional risky share of financial wealth coming from the Heckman selection equation applying the Deaton and Paxson (1994) methodology reported in columns 1)-2) in Table 4.1. The right panel applying the cohort-proxy methodology reported in columns 3)-4) in Table 4.1. For the Selection/Participation Equation, we plot the marginal values of the estimated underlying probit equation, and for the risky share, the age coefficients of the Outcome equation in the Heckman model.

Figure 4.2: **Estimation: Comparing methodologies**



*Note:* The left panel of the Figure plots the life cycle patterns of **Risky Asset Market Participation** coming from the Heckman selection equations reported in Table 4.1 applying the three different methodologies, Deaton-Paxson, cohort-proxy and peak restriction. The participation graphs plot the marginal values of the estimated underlying probit equations. The right panel plots the life cycle patterns of **Conditional Risky Share** of financial wealth coming from the Heckman selection equations reported in Table 4.1 applying the three different methodologies: Deaton-Paxson, Cohort-Proxy and Peak Restriction.

Figure 4.3: Life Cycle Patterns of Entry and Exit from Risky Asset Markets



*Note:* These figures plot the estimated life cycle patterns of entry to and exit from risky asset markets, defined by two different measures in Table 3.2 imposing the Deaton-Paxson restriction of zero-sum time effects (see Section 4.2).

interpretation is that stocks are (more than other financial instruments) subject to frauds and investors receive less than full legal protection giving rise to limited trust as in Guiso, Sapienza, and Zingales (2008).<sup>19</sup> An alternative interpretation, pursued by Barro (2006) and Rietz (1988) argues that households anticipate rare disasters of the type considered. As we will see, this second feature is key for the model to generate enough exit from the stock market after retirement as well as to help addressing the “too” high conditional shares in risky assets at young and middle ages that a workhorse portfolio choice model typically generates. We proceed with the illustration of the model.

## 5.1 Households

In the model economy, households work from age  $T^b$  until age  $T^w$ , after which households retire. Households face uncertainty in the number of years they live ( $T$ ). We model this component as in Hubbard, Skinner, and Zeldes (1995) and denote  $p_a$  the probability that the household is still alive at age  $a + 1$  conditional on being alive at  $a$ . We assume that households leave no intentional bequests and their objective function is the sum of discounted life time utility:

$$E \sum_{a=1}^T \delta^a \underbrace{\left( \prod_{j=0}^{a-2} p_j \right)}_{=\beta_a} U(c_{i,a}) \quad (5.1)$$

where  $c_{i,a}$  is the consumption of household  $i$  at age  $a$  and  $\delta$  the discount factor, and  $\beta_a$  the age-dependent effective discount factor that takes into account the probability of

<sup>19</sup>In recent years, several claims of accused fraud against the biggest financial adviser firms in Norway for selling dubious products with high leverage are ongoing. In March 2013 the Norwegian Supreme Court ruled that DNB (Norway’s biggest commercial bank) would have to compensate a small investor who had suffered large losses on guaranteed savings products the bank had sold him in 2000. See <http://www.morningstar.com/invest/articles/727984-norways-dnb-compensate-investment-clients.html> or <http://www.reuters.com/article/2009/04/24/dnbnor-lawsuit-idUSL013864720090424>

death. We assume the utility function is of the CRRA type; the degree of risk aversion is denoted by  $\gamma$ .

## 5.2 Market structure

In the model economy, markets are incomplete. Households smooth consumption over the life cycle by holding a riskless asset and possibly a risky asset. The riskless asset can be thought of as a real bond and has a time-invariant return  $r_f$ . We denote the amount of bonds a household  $i$  holds at age  $a$  with  $b_{i,a}$ . Whereas the riskless asset can be purchased and sold at no cost, we impose a fixed per-period participation cost  $q$  to hold risky assets. The amount of risky asset held by household  $i$  at age  $a$  is denoted  $s_{i,a}$ . The risky asset has a time-dependent real return  $\tilde{r}_t$ , and a risk premium denoted  $r_p$ .

$$\tilde{r}_t = r_f + r_p + \nu_t, \quad \nu \sim \mathcal{N}(0, \sigma_r^2). \quad (5.2)$$

where  $\nu_t$  is the period  $t$  innovation to stock market returns drawn from a normal distribution. Finally, we assume that households can't borrow against future labor income and that the quantities of the two assets held are non-negative.

$$s_{i,a} \geq 0, \quad b_{i,a} \geq 0. \quad (5.3)$$

These constraints ensure that the share  $\alpha_{i,a}$  of financial wealth invested in risky assets at age  $a$ , is non-negative and  $\alpha_{i,a} \in [0, 1]$ . Finally, we incorporate a tail risk into the model reflecting either a subjective belief people have that corporate or mutual fund managers abscond with their money (as in Guiso, Sapienza, and Zingales 2008) or a probability of a "disaster" as in Barro (2006), Barro (2009) and Rietz (1988) - a rare event that damages their stock investment. In contrast to Barro (2009) and Rietz (1988), the disaster is individual specific. Thus, the tail risk can either reflect the probability a household attributes to being cheated or/and the probability of some extreme loss in investment value and is taken into account by the household when deciding its portfolio allocation. We denote with  $p_{tail}$  this low probability event that the household faces every period. The complementary probability  $(1 - p_{tail})$  measures the degree of confidence an investor has in recovering his investment with the accrued return.

## 5.3 Household problem

### 5.3.1 Household budget constraints

Households start a period with a certain amount of cash-on-hand which is the sum of their labor income ( $w_a z_{i,a}$ ) and financial wealth ( $x_{i,a}$ ). Then households decide how much to consume ( $c_{i,a}$ ) and to save in bonds ( $b_{i,a+1}$ ) if they don't participate in the stock market, and how much to consume and save in bonds and equity ( $s_{i,a+1}$ ), if they choose to participate in the stock market. Finally, they compare their utility in both scenarios (participation vs. non-participation) and decide whether to enter (stay) or exit the stock market.

The budget constraint of a working age household reads as follows:

$$c_{i,a} + \mathbf{1}_{i,a+1}(s_{i,a+1} + q) + b_{i,a+1} = w_a z_{i,a} + (1 + \tilde{r}_a) \mathbf{1}_{i,a} s_{i,a} + (1 + r_f) b_{i,a}, \quad a = 1, \dots, T^w \quad (5.4)$$

where  $\mathbf{1}_{i,a+1}$  is an indicator function taking value 1 if household  $i$  participates in the stock market at age  $a + 1$  and 0 if not;  $w_a z_{i,a}$  stands for the age-dependent labor income which is composed of an age-dependent deterministic component  $w_a$  and a random walk component  $z_{i,a}$ , as shown in equation (A.1). The labor income components are estimated from our dataset.<sup>20</sup>

The retired households' budget constraint reads as follows:

$$c_{i,a} + \mathbf{1}_{i,a+1}(s_{i,a+1} + q) + b_{i,a+1} = \phi_{ret} w T^w + (1 + \tilde{r}_a) \mathbf{1}_{i,a} s_{i,a} + (1 + r_f) b_{i,a}, \quad a = T^w + 1, \dots, T \quad (5.5)$$

Equation (5.5) is isomorphic to (5.4) with the difference that labor income is now time-invariant: Retirement income is a fixed share  $\phi_{ret}$  of the last working-age labor income of the household.

The household problem is to maximize its objective function given by equation (5.1) subject to the above constraints (5.2)-(5.5). In the following subsection, we formulate this problem recursively.

### 5.3.2 Recursive formulation

The household problem has a set of control variables  $\{c_{i,a}, s_{i,a+1}, b_{i,a+1}, \mathbf{1}_{i,a+1}\}_{a=1}^T$  and a set of state variables  $\{x_{i,a}, z_{i,a}\}_{a=1}^T$ . We denote  $V_a^{in}(x, z)$  the indirect utility of a  $a$ -year old household who participates in the stock market, has labor productivity  $z$  and financial wealth amounting to  $x$ ,<sup>21</sup> and is the solution to the following maximization problem:

$$V_a^{in}(x, z) = \max_{c, s', b'} U(c) + \beta_{a+1} E_{z', \tilde{r}'} [(1 - p_{tail}) V_{a+1}(x', z') + p_{tail} V_{a+1}((1 + r_f) b', z')] \quad (5.6)$$

where

$$x' = (1 + r_f) b' + (1 + \tilde{r}) s'. \quad (5.7)$$

Households participating in the stock market have with probability  $(1 - p_{tail})$  the law of motion given by equation (5.7). With probability  $p_{tail}$ , the household loses its stock investment ( $s' = 0$ ), and the law of motion of the household financial wealth is given by  $x' = (1 + r_f) b'$ .

The Bellman equation  $V_a^{out}(x, z)$  is the indirect utility of a  $a$ -year old household who does not invest in risky assets, has labor productivity  $z$  and financial wealth  $x$ . It is computed by solving the following maximization problem:

$$V_a^{out}(x, z) = \max_{c, b'} U(c) + \beta_{a+1} E_{z'} V_{a+1}(x', z') \quad (5.8)$$

where

$$x' = (1 + r_f) b'. \quad (5.9)$$

The budget constraint of the household problem reads as follows:

$$c + \mathbf{1}'(s' + q) + b' = wz + x, \quad (5.10)$$

---

<sup>20</sup>Details on the estimation of the age-dependent component of labor income and the variances of the transitory and permanent shocks to labor income are in the Appendix A.1.

<sup>21</sup>We drop the indices  $i$  and  $a$ , to lighten the notation, and index one period ahead variable as  $x'$  instead of  $x_{a+1}$ .



The Bellman equation  $V_a(x, z)$  for the household problem pins down the participation decision of the household problem.

$$V_a(x, z) = \max_{\mathbf{1}'=\{1,0\}} (V_a^{in}(x, z); V_a^{out}(x, z)) \quad (5.11)$$

The optimal policy correspondence  $\mathbf{1}$  for the participation decision is obtained as follows:

$$\mathbf{1}' = \begin{cases} 1, & \text{if } \bar{x} \in X_p, \\ 0, & \text{otherwise.} \end{cases} \quad \text{where } X_p \equiv \{\bar{x} : V_a^{in}(x, z) > V_a^{out}(x, z)\}. \quad (5.12)$$

The maximization problem of the retired households is analogous to the above formulation with the only difference that the uncertainty with regard to the realization of the income shocks is shut down and replaced with a deterministic income that is equal to a fraction  $\phi_{ret}$  of the last working year labor income.

## 5.4 Solution method

The problem is solved by backward induction. Given the terminal condition, the policy functions and the value function in the final period  $T$  are trivial: households consume all their wealth, and the value function equals to the utility function. We substitute this value function in the Bellman equation and compute the policy functions one period backward. We do this for 75 periods, from  $T = 100$  to age  $T_b = 25$ . We discretize the state space for cash-on-hand state variable and iterate on the value function. The density function for the labor income process as well as for the risky return is approximated using Gaussian quadrature. Finally, we simulate 10,000 agents, compute averages and confront the model averages with the age profiles estimated in Section 4.

## 5.5 Parametrization

The parameters that we have estimated outside our model or imposed are reported in Table 5.1. The first four parameters reported are fixed. In line with Norwegian law, we set the retirement age at 67 for all households. The risk free rate ( $r_f$ ) is set at 1.8% and the equity premium ( $r_p$ ) at 3% as documented by Dimson, Marsh, and Staunton (2008) for Norway.<sup>22</sup> The standard deviation of the return on the risky assets is set to 0.231, the standard deviation of returns on the Oslo Stock Exchange. The conditional survival probabilities ( $p_j$  in equation 5.1) are obtained from the Population Tables of Statistics Norway.<sup>23</sup>

The last five parameters in Table 5.1 are estimated from our dataset. The age profile and the variances of permanent ( $\sigma_p^2$ ) and transitory shocks ( $\sigma_t^2$ ) to labor income are obtained by applying to our measure of disposable household labor income the decomposition used by Carroll (1997) and Cocco, Gomes, and Maenhout (2005) (see Appendix A.1 for details).<sup>24</sup> Our estimates of the variances of labor income shocks are very close to those obtained by Blundell, Graber, and Mogstad (2013) who use the same data source, but a different methodology.

<sup>22</sup>These are historical values for the (average) risk free rate and risk premium

<sup>23</sup>Table 5 Life tables, 2010 Statistics Norway

<sup>24</sup>In Table 5.1, we report only the variances of permanent and transitory shocks to labor income for the whole population; in Appendix A.1 a similar exercise has been done by education group.

The replacement rate  $\phi_{ret}$  is pinned down by computing the ratio of mean pension income five years after retirement and mean labor income five years prior to retirement. The age profile of labor income is obtained from the fitted age polynomial for all the population as documented in Table A.2.

For the purpose of our simulations, we require an estimate of the wealth of the households at age 25. For this, we fit a Pareto distribution to the wealth distribution of household aged 25 in our sample, and obtain an estimate of shape  $\mu_{x_0}$  and scale  $\sigma_{x_0}$  parameters, which we then use to randomly assign initial wealth to each household.

Table 5.1: **Parameter choice**

Variable Name	Variable	Value	Source
Retirement age	$T^r$	67	Norwegian Law
Risk free return	$r_f$	0.018	Dimson, Marsh, and Staunton (2008)
Risk premium	$r_p$	0.03	Dimson, Marsh, and Staunton (2008)
Std deviation stock return	$\sigma_r$	0.231	Data - Oslo Stock Exchange
Variance of transitory shocks	$\sigma_t^2$	0.023	Table A.1
Variance of persistent shocks	$\sigma_p^2$	0.012	Table A.1
Income share of retired HH	$\phi_{ret}$	0.842	Table A.2
Shape of Pareto Distribution for $x_0$	$\mu_{x_0}$	0.4521	Data - Wealth at age 25
Scale of Pareto Distribution for $x_0$	$\sigma_{x_0}$	5711.7	Data - Wealth at age 25

Finally, we parametrize the discount factor  $\beta$ , the risk aversion parameter  $\gamma$ , the participation cost  $q$  and the disaster probability  $p_{tail}$ . In order to illustrate the role played by the per period cost and the tail-event probability in the model, we set the values of these parameters to standard values in the literature. In Section 6.3 we estimate them.

## 6 Results

This Section provides the solution to the model, and delivers the economic intuition behind the decision rules of the households. To facilitate comparisons with the literature, our analysis builds on Cocco, Gomes, and Maenhout (2005) and shows how the introduction of the per period participation cost and the idiosyncratic tail-event probability contribute to explaining the stylized facts obtained in Section 4.

### 6.1 Policy Functions

The four panels of Figure 6.1 plot the optimal portfolio share invested in risky assets conditional on participating in the stock market as a function of cash-on-hand at a given age. Each panel plots the optimal portfolio share for 3 models: first, the Cocco, Gomes, and Maenhout (2005) model parametrized to Norwegian data, second the same model augmented by a fixed cost (and a participation decision), and finally the fully fledged model as described in Section 5, with participation decision and idiosyncratic disaster probability.<sup>25</sup>

<sup>25</sup>The policy functions are obtained by using parameters taken from Table 5.1 and  $\beta = .96$ ,  $\gamma = 10$  (as in Cocco, Gomes, and Maenhout (2005)),  $q = 0.25$ , which amounts to 250 US\$ in 1995 prices, and  $p_{tail} = 1\%$ .

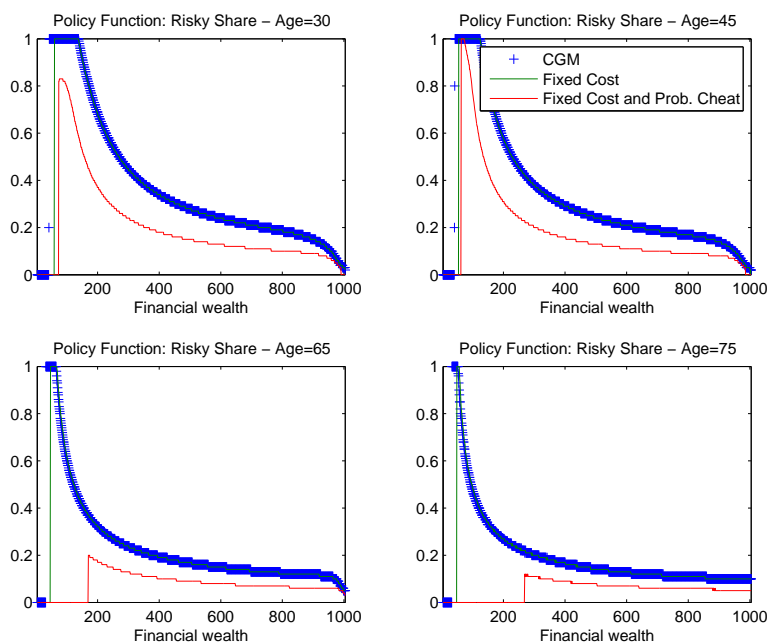


Figure 6.1: Policy Functions - Conditional Risky Share

The optimal portfolio rule is decreasing with both cash-on-hand, and age, a pattern that is consistent with the literature (Merton (1971) amongst others). The key driver is the importance of human capital (discounted stream of future labor income) relative to accumulated wealth. During working age, since shocks to labor income are uncorrelated with stock market returns, the deterministic component of labor income mimics the payoff of a risk free asset. Therefore, for a given level of human capital, households with low levels of financial wealth have a relatively large amount of future income from risk free assets (relative to their financial wealth) and thus invest more aggressively in stocks than wealthier households. A higher level of financial wealth reduces the relative importance of the safe human capital and leads households to rebalance their portfolio by investing less in stocks relative to their financial wealth.

As for the negative correlation with age, this follows from the same logic. The portfolio rule is less aggressive when agents grow older because the capitalized value of labor income drops with age, and households compensate for this drop in bond-like wealth by reducing their relative holding of risky assets.

The inclusion of a per-period participation cost introduces a wealth-participation threshold (Figure 6.2).<sup>26</sup> Overall, the wealth threshold of participation is mildly U-shaped with respect to age. The main drivers behind the U-shaped pattern of the participation threshold are jointly the hump-shaped labor income process, the age dependent discounting factor, and the diversification channel (young households seeking to hold equity more aggressively than older households). At early working age, labor income is low making it relatively expensive for households to participate. A higher labor income and a higher optimal risky share make stock market participation more worthwhile, which leads to a

<sup>26</sup>The participation thresholds can also be visualized in Figure 6.1 where they are the vertical cut-off line of the conditional risky share policy functions.

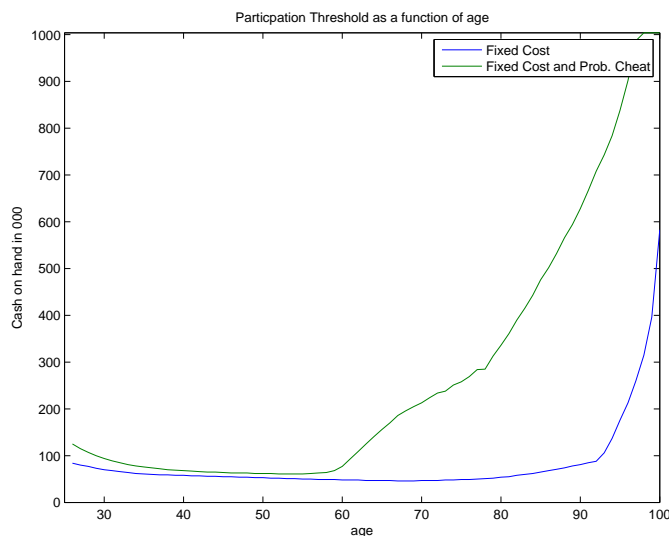


Figure 6.2: Wealth threshold of participation as a function of age

drop in the age-dependent wealth threshold. With increased age, households discount the future more (because of higher mortality risk), have lower optimal risky share, and lower labor incomes. These three facts jointly make participation less worthwhile for old households and consequently increases the wealth threshold of participation.

The introduction of individual disaster probabilities has three distinct effects. First, from Figure 6.1 one can notice that the idiosyncratic disaster probability has a stronger effect on the optimal conditional risky share of young households relative to old households. This asymmetry arises because the high level of human wealth relative to financial wealth among younger household increases their optimal risky shares above that of older households, meaning that they have more to lose from a disaster event, and therefore reduce their optimal risky share more strongly than older households. Second, the default probability makes stock market participation a less attractive choice, by reducing the expected return from holding stocks, which explains why at all ages the wealth threshold of participation is higher than the thresholds from the model with participation cost only (see Figure 6.2). Third, it is striking from both Figure 6.1 and 6.2 that the interplay between the participation cost and the default probability is significantly stronger for older households than young ones. The interplay between the default friction and the participation cost on the conditional risky share is well understood in the stylized model presented in Guiso, Sapienza, and Zingales (2008). However, its age dependent aspect is novel. This asymmetry is driven by two facts. First, because retired households rely heavily on accumulated assets to finance their consumption, a disaster event would hurt them substantially more than young households. Second, retired households have a substantially lower optimal risky share, implying that they need to invest substantially more than young households to benefit from the equity premium and cover the participation cost.

Overall, the policy functions show that the fixed per period participation cost can induce stock market entry and exit over the life-cycle, leaving the conditional risky share unaffected, whereas the idiosyncratic default probability has both an impact on the average

behavior of the participation margin and the conditional risky share. These differential responses of the share and participation will ease the identification of the disaster probability and the participation cost when we estimate them in Section 6.3.

## 6.2 Simulations

To highlight the role of per period participation costs and the disaster probability for the age profiles of participation and the conditional share, Figure 6.3 plots the average stock market participation rate (upper panel) and the average conditional risky share (lower panel) of simulated panels of 10,000 households from three models: the Cocco, Gomes, and Maenhout (2005) model calibrated to Norwegian data, the same model augmented with a per period fixed stock market participation cost and for the model presented in Section 5 which incorporates both the participation cost and the tail event.

There are a number of interesting features that emerge from the upper panel of Figure 6.3. First and obviously, in the absence of participation costs, the participation rate is 100% at all ages. Second, introducing a fixed per period participation cost generates limited participation. This effect is marked among the young because of their low levels of cash on hand. The per period cost generates also exit among the elderly, giving rise to a hump-shaped participation profile. However, for the assumed level of participation cost (250 US dollars, as estimated for the US by Vissing-Jorgensen (2002)) the hump in participation is much less pronounced and exit from the market takes place at a later age than observed in the data. This property does not change even if we double the participation cost suggesting that a reasonable participation cost is not sufficient to produce exit at the time and rate that we observe. Third, when we also add the small idiosyncratic disaster probability the simulated profile shows rapid exit initiating around retirement - a feature that is consistent with the data - while leaving the pattern of participation among the young barely affected. This is because the idiosyncratic disaster probability affects the participation rate of old households substantially more than younger households, a reflection of the age dependence of participation threshold depicted in Figure 6.2 and discussed in the previous subsection.

The lower panel of Figure 6.3 plots the average conditional risky share of the simulated panel by age. There are three noteworthy features. First, adding participation costs to the Cocco, Gomes, and Maenhout (2005) model leaves the level of the conditional share and its age profile little affected. The share is very (and unrealistically) high - hitting 100% - at relatively young age but households start rebalancing gradually until retirement. Second, introducing the small tail event probability lowers the conditional share at all ages towards levels that come closer to those observed in the data while the pattern of rebalancing as households approach and foresee retirement is unchanged. Third, on average, rebalancing of the risky share starts around the late thirties, much before households start exiting the stock market.

Looking jointly at the simulated life cycle profile of participation and of the conditional share reveals that the participation cost and the small probability tail event can together reproduce qualitatively, with one exception, the pattern and timing of portfolio adjustment along the intensive and the extensive margins documented in Section 4. The exception is that while the conditional share at the beginning of the life cycle is flat in the data (see Figure 4.1), it is increasing in the model. However, the model replicates well the joint pattern of life cycle rebalancing and exit. A key finding in the Norwegian data is that households *first* start reducing the conditional share before retirement, and *later* they

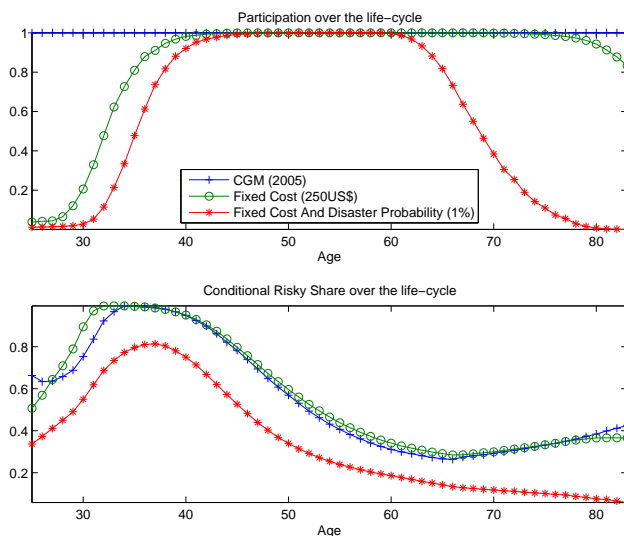


Figure 6.3: Model Simulation

begin exiting the stock market after retirement. This qualitative ability of the model to reproduce the empirical timing of the double adjustment is the main contribution of this section of the paper. We now move to assessing the ability of our model to quantitatively reproduce the empirical patterns identified in Section 4.

### 6.3 Estimation

In this subsection, we use indirect inference to estimate a set of unobserved parameters  $\kappa = [\gamma, \beta, q, p_{tail}]$ , namely risk aversion, discount factor, fixed participation cost and idiosyncratic tail event probability, such that the model matches the targeted moments best.

These set of target moments, denoted  $\Theta_D$ , are the stylized facts obtained in Section 3. From the Heckman selection model, and based on our identification strategy, we recover the age profile of the risky share and the participation decision. We then construct the model counterpart of our target moments: based on a draw of  $\kappa$  we solve and simulate the model presented in Section 5. From the model simulation, we obtain an average participation rate and an average conditional share at each age  $\Theta_M(\kappa)$ .

The estimation pins down  $\kappa^*$  so as to minimize the distance between moments from a simulated model  $\Theta_M(\kappa)$  and moments from the data  $\Theta_D$ :

$$\kappa^* = \arg \min_{\kappa} (\Theta_D - \Theta_M(\kappa))' W (\Theta_D - \Theta_M(\kappa))$$

where  $W$  is a weighting matrix, which is the identity matrix. The minimization procedure is performed using the Nelder-Mead simplex algorithm.

Table 6.1 summarizes the findings of our structural estimations. In the first three estimations, we target the age profiles estimates under the Deaton and Paxson (1994) restriction (DP); in Estimation 4 those generated using the cohort proxy (CP). For the first two estimations, we impose a value for the disaster probability of 1.75% and zero; the first corresponds to the value computed by Barro (2006) by pooling historical data for 35

Table 6.1: Structural estimation of parameters

Variable	Data	CGM	Est. 1	Est. 2	Est. 3	Est. 4	Est. 5
Risk aversion ( $\gamma$ )	-	10	7.5	17.3	10.4	12.5	13.3
Participation cost ( $q$ ) in US\$	-	n/a	17.1	119.8	24.5	14.1	29.1
Discount factor ( $\beta$ )	-	0.89	0.91	0.86	0.89	0.85	0.86
Probability of tail event ( $p_{tail}$ ) in%	-	n/a	<u>1.75</u>	<u>0</u>	1.3	1.4	1.34
Fraction of stock investment lost ( $\theta$ ) in%	-	n/a	<u>100</u>	n/a	<u>100</u>	<u>100</u>	0.78
Value of Objective Function	-	-	2.93	4.91	2.86	2.26	2.97
Target Data	-	-	DP	DP	DP	CP	DP
Mean wealth to income ratio (age 65)	1.62	2.23	1.74	3.07	1.75	1.71	1.81
Std. dev. of log wealth (age 65)	1.77	0.78	0.79	1.01	0.795	0.789	0.783

*Note:* Parameters with  $\underline{x}$  are fixed parameters. 'Target Data' indicates whether the target moments are taken from Deaton-Paxson methodology (DP) or from our Cohort Proxy (CP) identification strategies developed in Section 4.

countries and defining a macroeconomic "disaster" as a drop in GDP of at least 15% in a year respectively; the second to no disaster. A comparison of the two sets of estimates shows clearly that a small tail event probability allows to obtain more moderate estimates of the degree of risk aversion and of the participation cost. Forcing  $p_{tail} = 0$  (instead of 1.75%) results in an increase in the estimate risk aversion from 7.5 to 17.3 and of the participation cost from \$17.1 to \$119.8 per year (1995 prices). The other two columns estimate the four parameters jointly. In our context, the low probability tail event does not necessarily originate from a very adverse macroeconomic shock but rather from extreme idiosyncratic investment experiences, including frauds. Because direct estimates of the probability of these tail events are difficult, we let the structural estimates speak.<sup>27</sup> The two sets of estimates yield very similar results. The low-tail event probability is estimated between 1.3 and 1.4 percent and the participation cost is comprised between \$14 and \$25, while the risk aversion parameter is around 10 and the discount factor between 0.85 and 0.89. Table 6.1 instructs us that a relatively low idiosyncratic disaster probability enhances the quantitative performance substantially. Comparing Estimation 2 with Estimation 3 shows that a 1.3% probability of a large loss reduces the objective function by a factor close to 2, and more importantly allows to obtain estimates for the preference parameters that are in line with the literature (for instance Cocco, Gomes, and Maenhout (2005) use a  $\beta$  of 0.93 and a  $\gamma$  of 10).

Finally, we jointly estimate the fraction of stocks lost when the rare event occurs and its probability. Whereas in Estimations 1, 3 and 4, we posited that when the tail-event occurs, the whole investment in stocks is lost, and that household's next period's financial wealth is only composed of accrued interest on bonds, in Estimation 5, we relax this

<sup>27</sup>While deception and fraud is likely to be pervasive in financial markets, systematic evidence is hard to collect. Survey data from the European Social Survey show that about 1/3 of the people interviewed report, that over the past five years, they experienced that a bank or insurance company failed to offer the best deal they were entitled to. This corresponds to an annual probability of around 6%, but refers to a much broader notion of the type of frauds that we want to capture. The Financial Fraud Research Center at Stanford University reports estimates of 7.1% of Americans incurring at least one financial fraud over their lifetime and 13.5% a generic fraud every year (<http://fraudresearchcenter.org/2011/08/state-of-research-2/>). Estimates of the size of fraud losses are as difficult. However, they are unlikely to be negligible. The Bernie Madoff case alone resulted in \$50 billion losses, around 0.8% of the value of households investments in stocks.

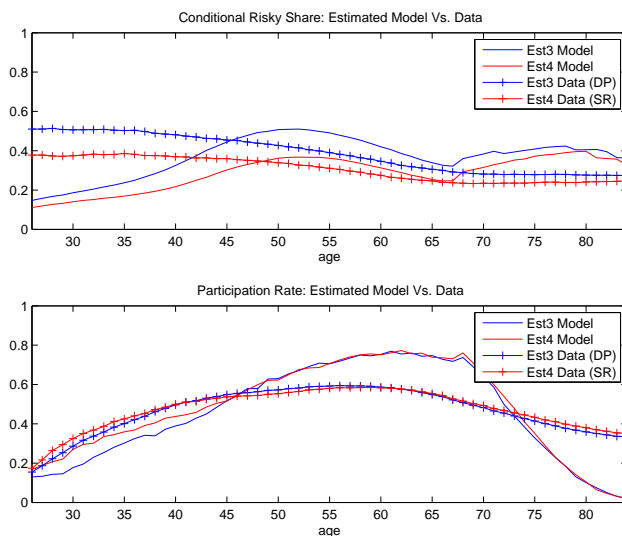


Figure 6.4: Model simulations and data using the parameters from Estimation 3 and 4

assumption, and indirectly infer the fraction of stocks lost that fits our stylized facts best.<sup>28</sup> We estimate a fraction lost of 78% and a probability of this tail event of around 1.34%. All our estimates remain close to those from Estimation 3 apart from the risk aversion parameter. This nuances the point made above which emphasized the importance of the idiosyncratic probability in aligning preference parameters to those of the literature. In fact, our estimates suggest that a small probability of a large loss when investing in stocks, a small participation cost, a relatively large degree of risk aversion and a discount factor in line with standard estimates, is the parameter combination that gets the life cycle profiles of stock market participation and the conditional share generated by the model closest to the empirically estimated life cycle profile of the risky share and stock market participation.

Figure 6.4 contrasts the model-generated age profiles using the set of estimated parameters and the data-estimated profiles for the two estimations set (Estimation 3 and 4). The model simulated profiles approximate our stylized facts well. In particular, they reproduce closely the hump-shaped pattern of the participation rate and capture the differential timing when people start rebalancing the share and exiting the stock market quite well. Yet, the simulated behaviour of the conditional risky fits poorly the observed one. Compared to the data, the model predicts a too low share in stocks when young and a too high one for people in the middle ages.

The last two lines of Table 6.1 show two moments that we did not target: the wealth to income ratio at age 65 and the standard deviation of wealth at the same age. The model generates a value for the first moment (1.75, Estimation 3) that is quite close to the observed value (1.62) but predicts considerably less wealth inequality (0.795, Estimation 3) than observed in the data (1.77). Yet, it is interesting to notice that allowing for a small disaster probability considerably improves the ability of the model to reproduce the observed wealth to income ratio.

<sup>28</sup>In this modelling environment  $x' = (1 + r_f)b' + (1 - \theta)s'$ , where  $\theta$  is the fraction of stock investment lost when the tail event occurs.



## 7 Conclusion

Over the past decade, many scholars have used calibrated models to study life cycle portfolio allocations, departing from the simplifying assumptions of early generations models and adding realistic features of households environments. Among them, uninsurable income risk, non-tradeable human capital and borrowing constraints. Despite these (and other) complications, these models uniformly predict that households should at a certain point before retirement start lowering exposure to the stock market in order to compensate for the decline in the stock of human wealth as people age, which in this models acts mostly as a risk-free asset. Finding empirical evidence in support of this rebalancing, however, has been hard. We have argued that this is likely to be due to data limitations, both because a proper treatment of the issue requires long longitudinal data and because the information on assets needs to be exhaustive and free of measurement error. Combining administrative and tax registry data from Norway, we are fulfilling these requirements and find that households do indeed manage their portfolio over the life cycle in a way that is consistent with model predictions. We find that they adjust their financial portfolios along two margins: the share invested if they participate in the stock market and the decision whether to stay or leave the market altogether. They tend to enter the stock market early in life as they accumulate assets and tend to invest a relatively large share of financial wealth in stocks. As they start foreseeing retirement, they rebalance their portfolio share, reducing it gradually. Around retirement, they start adjusting on the other margin, exiting the stock market. This double adjustment pattern along the intensive and extensive margin with its clear timing cannot be explained by any of the available life-cycle portfolio models; however as we show, an extension of these models that incorporates a small per period participation cost and a small probability of a large loss when investing in stocks is able not only to generate the double pattern of adjustment but also to replicate the profiles of stock market participation and portfolio shares observed in the data.

## A Appendix: earnings variances and human wealth

To estimate the variance of permanent and transitory shocks to labor income and the value of a households human wealth, we rely on a broad measure of household labor income obtained from tax records by summing the labor income of the two spouses for all households in our portfolio sample. The income data cover the same time span we observe the households portfolio. As in Carroll (1997), we define labor income as the sum of after-tax earnings at the household level. Besides earnings, it includes capital income and transfers (including sickness money, compensation for maternity leave, benefits paid during unemployment spells and pensions). Values are in 1995 USD - converted using the 1995 NOK/USD exchange rate. The statutory retirement age in Norway is 67, in practice however, a number of arrangements allow workers to retire earlier.<sup>29</sup> Our measure is then deflated using the growth in the National Insurance Scheme basic amount, which is used to adjust payments of unemployment insurance and pensions.<sup>30</sup>

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<sup>29</sup>In Norway, the actual average retirement age is around 64. See e.g.

[http://ec.europa.eu/economy\\_finance/publications/publication14992\\_en.pdf](http://ec.europa.eu/economy_finance/publications/publication14992_en.pdf). Early retirement schemes are widespread in Norway and workers may be eligible for these from the age of 62, see e.g. Vestad (2013)

<sup>30</sup>See The Norwegian Labour and Welfare Administration, <http://www.nav.no/English/Membership+in+The+National+Insurance+Scheme> for more information on

## A.1 Variance of permanent and transitory shocks to labor income

Following Carroll (1997) and Cocco, Gomes, and Maenhout (2005), we estimate the following model for (log) labor income,  $Y_{i,a,t}$  for household  $i$ , aged  $a$  at time  $t$  :

$$\log(Y_{i,a,t}) = \alpha + \beta(X_{i,a,t}) + \theta_a + \gamma_t + \varepsilon_{i,a,t} \quad (\text{A.1})$$

where  $\alpha$  is a constant,  $X_{i,a,t}$  a set of demographic controls (such as household size),  $\theta_a$  a full set of age dummies,  $\gamma_t$  the calendar year fixed effects and  $\varepsilon_{i,a,t}$  the error term capturing shocks to labor income. We estimate the model separately for three different levels of educational attainment of the household (using husband education), as well as for the whole sample of non-retired households.<sup>31</sup>

To estimate the variance components of the income process, we follow the procedure in Carroll and Samwick (1997) and Cocco, Gomes, and Maenhout (2005) and assume that labor income innovations can be decomposed as the sum of a permanent and a transitory shock, with variances  $\sigma_u^2$  and  $\sigma_\eta^2$  respectively. Using the estimated model we compute for each observation prediction errors  $d$ -years ahead -denoted  $r_{id}$  - for various  $d$  starting from the base year (1995); then, noticing that  $\text{var}(r_{id}) = d\sigma_u^2 + 2\sigma_\eta^2$ , we retrieve the two variances from an OLS regression of  $\text{var}(r_{id})$  on  $d$  and a constant term. The estimates are shown in Table A.1. The variance of transitory shocks is larger than that of permanent shocks for all education groups as well as the total sample, with some differences in its extent. We do not find large differences in the size of the variances across education groups with a tendency of households with High-School education to face lower labor income uncertainty than either households with less than high school or households with a college degree. Compared to Cocco, Gomes, and Maenhout (2005), we find much lower values of the transitory components.<sup>32</sup> There are two possible explanations. The first is that workers in Norway are covered by a generous social insurance scheme which dampens the labor income effects of temporary shocks. The second that, differently from Cocco, Gomes, and Maenhout (2005) who use PSID surveys, we use administrative records. Hence there is much less scope for measurement errors which otherwise inflates the estimated variance of transitory shocks.

Our estimates of the variance of permanent and transitory shocks are instead very close to those of Blundell, Graber, and Mogstad (2013) based on the same Norwegian data that we use, but obtained using a different methodology and excluding income from self employment. Unlike us, they allow for age-varying variances, finding that the earnings variances follow a U-shaped profile. Variances are higher at the very beginning of the (working) life cycle (particularly for high education workers) and essentially age-invariant for many years before retirement when they increase again. Our estimates are similar to their for middle aged households.

Finally, we also computed the correlation between labor income and stock market returns on the Norwegian stock market. A negative correlation would represent a hedging

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the basic amount, 'grunnbeløpet'.

<sup>31</sup>In fact, we cut the sample at the age of 65, to avoid variability in income coming from early retirement from influencing our results

<sup>32</sup>The estimates of the income variance are highly dependent on very low income realizations of few households. In previous contributions this has been taken care of by excluding households with realized incomes below some threshold, justifying the choice with the need to limit the influence of measurement error. Since we use highly reliable administrative records we retain the whole sample, including households with very low income realizations.

Table A.1: **Income variance decomposition and correlation with stock return**

	<High School	High School	College	All
Transitory	0.026 (9.97)	0.015 (6.76)	0.029 (11.89)	0.023 (16.5)
Permanent	0.013 (17.44)	0.011 (19.12)	0.011 (17.73)	0.012 (31.11)
Stock Market	0.017 (0.48)	0.045 (0.34)	0.005 (0.75)	0.008 (0.50)

*Note:* The table reports estimates of the variance of permanent and transitory labor income shocks. The estimation is based on the error terms from estimating the labor income process in Figure A.1. The procedure is based on the method in Carroll and Samwick (1997), which is also used in Cocco, Gomes, and Maenhout (2005). T-values in parentheses.

Table A.2: **Age polynomials for labor income process**

	Less High School	High School	College	All
Age	0.0418	0.0375	0.112	0.0572
Age <sup>2</sup> /10	-0.0231	-0.0176	-0.0689	-0.0308
Age <sup>3</sup> /100	0.00587	0.00454	0.0212	0.00888
Age <sup>4</sup> /1000	-0.101	-0.0906	-0.337	-0.161
Age <sup>5</sup> /10000	0.0749	0.0735	0.208	0.117
Constant	3.576	3.657	3.415	3.566
Observations	61	61	61	61
r2	0.995	0.996	0.988	0.995

*Note:* The table shows the coefficients of a 5th order polynomial describing labor income as a function of age.

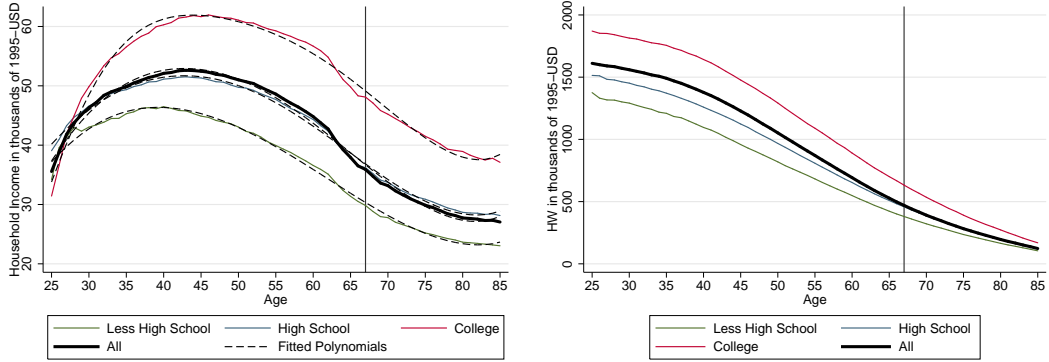
opportunity for the households, as argued in Bodie, Merton, and Samuelson (1992). Table A.1 shows that the correlation tends to be positive but very small and never statistically significant. This confirms the results in Cocco, Gomes, and Maenhout (2005) for the United States.<sup>33</sup>

## A.2 Human wealth

To obtain an estimate of the human wealth of a household of age  $a$  we estimate Equation 5.7 on the whole sample of households aged between 25 and 80 and separately for the three education groups. We then retrieve the age dummies and regress them on a 5th order polynomial. The age effects (solid lines) and the fitted polynomials are plotted in Figure A.1. Table A.2 shows the estimated 5th order polynomial.

<sup>33</sup>The same holds for a combined measure of returns from the S& P 500 and the Oslo stock exchange.

Figure A.1: Life Cycle Profiles for Income and Human Wealth



*Note:* The left panel of the Figure plots the estimated labor income processes by educational level and for the full sample, coming from Equation A.1 estimated on the different sub-samples. The right panel displays the life cycle profiles of human wealth, calculated as described in Guiso and Sodini (2011) (Equations A.2 & A.3) based on the income polynomials in the left panel.

The income profiles by educational attainment are consistent with the evidence in the literature showing much steeper profiles for high education workers. To compute lifetime wealth we proceed as follows. Let  $G_e(a)$  denote the estimated 5th order polynomial in age for log income for a households with education level  $e$ . Assuming that all the household characteristics apart from age will not change in the future the labor income (or pension benefits) at age  $a + \tau$  of a household with education level  $e$  and age  $a$  can be calculated using the function  $G_e(a)$  as follows:

$$L_e(a + \tau) = L_e(a) \frac{\exp(G_e(a + \tau))}{\exp(G_e(a))} \quad (\text{A.2})$$

The human wealth for a households of age  $a$  is then computed as:

$$H_{e,a} = L_e(a) + \sum_{\tau=1}^{T-a} p(a + \tau|a) \frac{L_e(a + \tau)}{(1 + r)} \quad (\text{A.3})$$

where  $p(a + \tau|a)$  is the probability of surviving to age  $a + \tau$  conditional on survival to  $a$ , from the population tables of Statistics Norway<sup>34</sup> and  $r$  is a risk free rate, which we set at 0.02. For each household we obtain an estimate of  $H_{e,a}$  for each age of household in the sample.

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<sup>34</sup>See <http://ssb.no/en/dode/>

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