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Safe Assets, Liquidity, and Monetary Policy†

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This paper studies monetary policy in models where multiple assets have different liquidity properties: safe and "pseudo-safe" assets coexist. A shock worsening the liquidity properties of the pseudo-safe assets raises interest rate spreads and can cause a deep recession-cum-deflation. Expanding the central bank's balance sheet fills the shortage of safe assets and counteracts the recession. Lowering the interest rate on reserves insulates market interest rates from the liquidity shock and improves risk sharing between borrowers and savers. (*JEL* E31, E32, E43, E44, E52)

This paper presents monetary models in which multiple assets have different liquidity properties, and studies the propagation mechanism of a shock that deteriorates the quality of some assets, in line with what has been observed at the onset of the US financial crisis.[1] In our framework there are some safe assets, such as money, which can be perfect stores of value and immediately resaleable, and other assets, labeled "pseudo-safe" or "pseudo-liquid" assets, which are also perfect stores of value but might instead have imperfect liquidity properties that can vary over time.[2]

Our framework shows that a liquidity shock can imply, if monetary policy is non-responsive, significant effects on the economy along two main dimensions. First, the overall shortage of liquidity implies a corresponding shortage of demand for goods since fewer assets remain available for goods purchasing.[3] The consequent

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[1] More recently, on April 2015, a shortage of high-quality bonds created disruption in the US repos market, creating bottlenecks for a key source of liquidity in the financial system.

[2] See Brunnermeier and Pedersen (2009) and Gorton, Lewellen, and Metrick (2012) for a discussion on how market liquidity can suddenly dry up.

[3] The mirror image of a disequilibrium in the financial market is a disequilibrium in the goods market, as often discussed in commentaries of the crisis, such as Lucas (2008) and DeLong (2010), bringing out ideas that go back to Mill (1844).

contraction in nominal spending can depress real activity in the presence of nominal rigidities, causing a deep recession-cum-deflation. Second, the liquidity shock raises the premium required to hold pseudo-safe assets. The funding costs for intermediaries, which borrow in the pseudo-safe assets, increase and at the same time force them to charge higher interest rates on loans. Due to rising borrowing costs, debtors need to cut their spending, amplifying the contraction in the real economy with important distributional effects between savers and borrowers.

Monetary policy has an important role in mitigating the adverse effects on the economy along the two transmission mechanisms underlined above. First, the central bank can heal the shortage of safe assets and prevent the contraction in nominal spending by issuing more money, which remains a perfectly safe asset in circulation. The expansion of the central bank's balance sheet is necessary to maintain price stability. Second, monetary policy can insulate the interest rates on the pseudo-safe assets from the liquidity shock, by lowering the interest rate on reserves and therefore improving risk sharing between borrowers and savers. For a large shock, the zero lower bound can be an important constraint along this dimension.

A policy in which the interest rate on reserves is lowered, while the balance sheet of the central bank is not expanded, does not prevent the contraction in nominal spending. Additionally, a policy in which more liquidity is injected into the system, but the policy rate is not lowered, can only partially contain the rise in the liquidity premia and the distributional costs of the liquidity shock. It is important to note that the two policy prescriptions coming out of our model do not depend on the degree of nominal rigidities. In particular nominal spending is the key variable to stabilize in the face of a liquidity shock, as frequently discussed in the recent public debate.[4]

Our analysis is complementary to a recent literature that has provided possible explanations of the macroeconomic adjustment following the recent crises. Compared to Cúrdia and Woodford (2010, 2011), Eggertsson and Krugman (2012), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Guerrieri and Lorenzoni (2011), we focus on different forces for understanding financial crises.[5]

To gauge the difference between our model and theirs, let i_t^b be the interest rate at which intermediaries lend, while i_t^d is their funding or deposit rate; i_t^m is the policy rate. In Cúrdia and Woodford (2010, 2011), Gertler and Karadi (2011), and Gertler and Kiyotaki (2010) the main shock diretly hits the credit spread $i_t^b - i_t^d$ by raising the interest rate on loans and lowering the deposit rate which, in their models, is equivalent to the risk-free rate i_t^m. The consequent reduction in credit flows causes the recession.

The key spread in our framework is instead between the deposit and the policy rate, $i_t^d - i_t^m$, which critically depends on the liquidity properties of deposits relative to money.[6] A liquidity shock worsening the quality of deposits raises this spread which is then passed through into higher lending rates. The deposit rate increases in our model rather than falling as in the literature. Most important, we emphasize

[4] See, among others, Woodford (2012).

[5] Justiniano, Primiceri, and Tambalotti (2013) analyze the quantitative implications of such models.

[6] In Gertler and Kiyotaki (2010) the deposit rate can also differ from the risk-free rate but in an idiosyncratic way across intermediaries participating in the interbank market.

an additional and new channel of transmission: the shortage of assets available to purchase goods creates a contraction in aggregate demand, driving the economy into a recession.

The different source of shock originating the crisis enables us to rationalize as optimal policies the main monetary policy actions adopted during the recent crisis such as balance sheet and zero lower bound policies focusing on different forces than those underlined by the literature above.

First, zero lower bound policies are also optimal in our context, but for different reasons. Lowering the policy rate is the way to insulate the market real rates from the liquidity shock and achieve a better risk sharing of consumption between borrowers and savers. In Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), instead, they are necessary to accommodate as much as possible the required fall in real rates and to avoid a deep contraction in aggregate output. In contrast, in our model, the recession is not averted at all by zero lower bound policies.

Second, we also emphasize the need for expansionary policies on central bank balance sheets, but again for different reasons. In the literature they act directly on credit spreads, as discussed in Cúrdia and Woodford (2010, 2011), Gertler and Karadi (2011), and Gertler and Kiyotaki (2010). Here, instead, unconventional policies should be primarily focused on healing the shortage of safe assets since this is the source of the fall in aggregate demand and of the ensuing recession. Without this intervention, the macroeconomic consequences of the crisis cannot be avoided. To this end, our analysis shows that it is not sufficient to intervene in the financial markets and reduce credit or liquidity spreads. Instead, safe assets should be supplied as needed to maintain stable nominal spending.[7]

Our approach to model liquidity is in line with that of Lagos (2010), where financial assets are valued for the degree to which they are useful in exchange for goods. In his model, agents are free to choose which assets to use as means of payment, between bonds and equity shares. However, he also restricts the analysis to cases in which bonds are assumed to be superior to equity shares for liquidity purposes.[8] The finance constraint in our model, through which goods and assets are exchanged, is of a simple form in line with the works of Lucas (1982), Svensson (1985), and Townsend (1987). The way we characterize a liquidity shock, as a change in the degree of resaleability of assets, is close to Kiyotaki and Moore (2012).[9] In their model, entrepreneurs face a borrowing constraint to finance investment and they need to use internal resources, among which are money and previous holdings of equity. Equity can be used only in part to finance investment, where the fraction available is known at the time when liquidity is needed. Instead, we model the

[7] Our analysis does not aim to explain all features of the financial crisis and the consequent slow recovery. But it might suggest that a delayed monetary policy reaction to a sudden deterioration in the liquidity properties of some assets could have produced real effects and additional financial consequences. Indeed the crisis started in July 2007 and the Fed began its balance sheet expansion and zero lower bound policies only at the end of 2008.

[8] In Aiyagari and Gertler (1991), instead, transaction costs in trading equities are responsible for a lower degree of liquidity of the latter with respect to bonds.

[9] Del Negro et al. (2016) estimate a quantitative version of Kiyotaki and Moore (2012) with nominal rigidities to describe the US financial crisis. Within a different framework, their result supports our policy prescriptions. Chiesa (2013) also presents a model in which liquidity holdings are an input to the investment process and assets have different degrees of pledgeability.

exchange of assets for goods at the level of consumption. The shortcut taken here has the benefit of producing a highly tractable model of the role of liquidity, which extends standard monetary models currently used for the analysis of monetary policy. Moreover, the partial resaleability of Lagos (2010) and Kiyotaki and Moore (2012) concerns a risky asset like equity and not risk-free assets as in our model. Finally, Trani (2012) is an example of an open-economy model in which multiple assets (equities) provide collateral services with time-varying properties specific to each asset, and therefore have different liquidity premia.

In monetary analysis, several works have introduced a transaction role for bonds, although an indirect one. In Canzoneri and Diba (2005), current income can also be used for liquidity purposes in a fraction that depends on the quantity of bonds held in the portfolio. In their model, bonds have indirect liquidity services since they enhance the fraction of income used to purchase goods. Woodford (1991) is an early example of a model in which current income has immediate liquidity value but bonds do not provide liquidity services. In our context, a liquidity constraint literally disciplines the exchange of assets for goods and the imperfect substitutability of pseudo-safe assets for money through a random factor. In Belongia and Ireland (2006, 2012), money and deposits are bundled together through a Dixit-Stiglitz aggregator, and can be used for liquidity purposes as in the work of Canzoneri et al. (2011) where bonds are instead imperfect substitutes for money. Canzoneri et al. (2008) consider instead a model in which bonds provide direct utility to the consumer. These latter works are not concerned with variation over time of the liquidity properties of assets. Finally Benk, Gillman, and Kejak (2010) present a model in which money and credit services can both provide liquidity services to study movements in the US velocity of money.

Another important feature of our model is that we allow the central bank to pay an interest rate on reserves. This is key to giving monetary policy two instruments, rather than a single one as in the literature, to counteract at the same time the fall in the supply of liquid assets and the distributional consequences of the rising spread in financial markets. Kashyap and Stein (2012) and Canzoneri, Cumby, and Diba (2015) are examples of other works in which reserves pay an interest rate. The dual monetary policy instruments can be used to achieve macroeconomic and financial stability objectives. However, in their case, the role of interest rate on reserves is for macroprudential purposes rather than as a useful tool to react to the surge of a liquidity crisis.

The paper is structured as follows. Section I presents the framework in which liquidity and pseudo-safe assets are introduced. It discusses a simple monetary model with flexible prices. In Section II the propagation of liquidity shocks is analyzed depending on alternative monetary policy regimes. Section III analyzes a monetary model with heterogenous agents (savers and borrowers) where an inside asset plays the role of a pseudo-safe security. Section IV uses the general model to study the macroeconomic implications of a liquidity shock and the role of monetary policy. Section V concludes.

I. A Simple Model with Pseudo-Safe Assets

We model liquidity as the resaleability of an asset in exchange for consumption goods. Several assets can be brought to buy goods, but they have different liquidity properties which can be discovered only at the time of purchasing. In the goods market, the portfolio of assets cannot be rebalanced; nor can new assets be traded. The following liquidity constraint applies:

$$\sum_{j=1}^{N} \gamma_t(j)\,(1 + i_{t-1}(j))\,B_{t-1}(j) \geq P_t C_t, \tag{1}$$

where N is the number of assets available and $B_{t-1}(j)$ is the value of asset j, in units of currency, held in the agent's portfolio. At the time of purchasing goods, each security already matures a predetermined nominal interest rate, specific to the asset and given by $(1 + i_{t-1}(j))$; P_t is the nominal price index while C_t is real consumption. Securities differ in their liquidity properties, which are only known when they are exchanged for goods: $\gamma_t(j)$ indicates literally the fraction of assets held from a previous period that can be used to purchase goods, with $0 \leq \gamma_t(j) \leq 1$. Assets can be ordered from the worst to the best in terms of liquidity properties assuming that $\gamma_t(j)$ is a non-decreasing function of j. In this set of assets, money may have the role of the best security for liquidity purposes, meaning $\gamma_t = 1$.[10]

Constraint (1) is in the spirit of Barnett's (1980) view that monetary aggregates should not only include money but also other assets weighted for their relative liquidity properties.

Liquidity in this model can have a dual interpretation. On the one hand, it can simply capture the degree of "acceptance" of an asset in exchange for goods. We could think of a consumer who goes to the goods market and discovers that, among all the securities that he has carried along, only a fraction are accepted to buy goods. On the other hand, it could simply refer to the fraction of securities which can be fully mobilized and exchanged for goods. In line with this interpretation, it can capture the intrinsic liquidity of the asset or a sort of delay in payment. To this end, we can think of these assets as the corresponding liabilities of some other agent, not modeled, that can be liquidated only in part at the exact time in which the creditor needs to purchase goods. There is a subtle difference between the two interpretations. In the first case, liquidity is a property that the "market" (seen from the perspective of who is offering goods) attributes to the asset. This property might have to do with the trust in the security as a medium of exchange. In the second case, it is an intrinsic property of the asset, although it can vary over time. Mixed interpretations could be given since the distinction is really subtle. Indeed illiquidity at the origin can also be correlated with a low degree of acceptance of the asset at the destination or vice versa.

In any case, all the securities traded in this model are perfect stores of value; γ_t captures just liquidity risk, and not credit risk. By this virtue, all securities are remunerated at their specific predetermined nominal interest rate. The remaining fraction $(1 - \gamma_t(j))$—which cannot be used as liquidity—remains in the financial account, becoming immediately available just after goods purchasing.

[10] This might not be the case in an imperfectly credible fiat-money system.

Money, the security with $\gamma_t = 1$, is the safe asset. Here safeness has a double meaning. First, it captures the property of an asset as a perfect store of value. In this model all assets share this property because each is remunerated at its specific risk-free nominal interest rate.[11] On top of this, the safe asset is fully liquid because it can always be accepted or mobilized to purchase goods. The other assets, with $\gamma_t(j) < 1$, are imperfect substitutes as means of exchange and can be labeled "pseudo"-safe or "pseudo"-liquid assets.

Following goods purchasing, the financial market opens and consumers reallocate their portfolio according to the following constraint:

$$\sum_{j=1}^{N} B_t(j) = \sum_{j=1}^{N} (1 - \gamma_t(j))(1 + i_{t-1}(j)) B_{t-1}(j) + P_t Y_t + T_t$$

$$+ \left[\sum_{j=1}^{N} \gamma_t(j)(1 + i_{t-1}(j)) B_{t-1}(j) - P_t C_t\right],$$

where Y_t is exogenous output and T_t are transfers from the central bank or government. In the above constraint it is clear that the assets which are not carried in the goods market or are unspent remain in the financial account.

Given the above general framework, we start our analysis from a simple model in which there are two outside assets, money and government bonds, which can provide liquidity services. Later we develop a model with inside assets.

Consider a closed economy with a representative agent maximizing the expected discounted value of utility,

$$E_{t_0} \sum_{t=t_0}^{+\infty} \beta^{t-t_0} U(C_t), \tag{2}$$

where E_t is the conditional expectation operator; β is the intertemporal discount factor with $0 < \beta < 1$; and $U(\cdot)$ is the utility flow which is a function of current consumption, C, and has standard properties.

At the end of a generic period $t - 1$, the representative agent invests M_{t-1} in money and B_{t-1} in bonds. At the beginning of the next period t, money and bonds mature their nominal interest rates, given respectively by $(1 + i_{t-1}^m)$ and $(1 + i_{t-1})$, which are both predetermined. At this time, both assets can be used to purchase goods, according to the following liquidity constraint:

$$(1 + i_{t-1}^m) M_{t-1} + \gamma_t (1 + i_{t-1}) B_{t-1} \geq P_t C_t, \tag{3}$$

[11] We could easily amend this assumption by allowing for default risk. However, the purpose of this paper is to analyze the effects of the change in the liquidity properties of assets which do not necessarily materialize in a credit event.

where P_t is the price level. Since γ_t lies in the interval $[0, 1]$, bonds are an imperfect substitute for money for purchasing purposes. As discussed above, γ_t is a measure of the degree of resaleability of bonds for goods when liquidity is needed.

After the goods market closes, the representative agent receives a stochastic endowment Y_t and transfers T_t from the government and, together with the unspent money and bonds, reallocates its overall wealth into new money and bonds to be carried over into the next period. The financial portfolio adjusts according to the following constraint:

$$M_t + B_t \leq (1 - \gamma_t)(1 + i_{t-1})B_{t-1} + P_t Y_t + T_t$$
$$+ [(1 + i_{t-1}^m)M_{t-1} + \gamma_t(1 + i_{t-1})B_{t-1} - P_t C_t],$$

where M_t and B_t denote the holdings of money and bonds to carry into the next-period goods market. Since the endowment and the transfer are given to the agent after the goods market closes, they both have to be turned into either money or bond holdings, to be used for transaction purposes in the next period. The term in the square bracket captures the residual holdings of assets after goods purchases. It should be noted that the fraction $(1 - \gamma_t)$ of bonds, which cannot be used for transaction purposes, still remains in the financial account and is available for asset trading when the financial markets open. The above constraint simplifies to

$$P_t C_t + M_t + B_t \leq (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^m)M_{t-1} + P_t Y_t + T_t. \tag{4}$$

The representative agent maximizes the expected utility (2) under the constraints (3) and (4), and subject to an appropriate borrowing-limit condition, by choosing consumption, C_t, and asset holdings (M_t, B_t). Given Lagrange multipliers ψ_t and λ_t attached to the constraints (3) and (4), the following first-order condition holds with respect to consumption:

$$\frac{U_c(C_t)}{P_t} = \psi_t + \lambda_t, \tag{5}$$

showing that the liquidity constraint creates a wedge between the marginal utility of nominal consumption and that of nominal wealth—the latter being captured by λ_t. This wedge depends on the multiplier ψ_t, measuring the marginal utility of liquidity. Optimality conditions with respect to money and bonds imply respectively

$$\lambda_t = \beta(1 + i_t^m)E_t(\psi_{t+1} + \lambda_{t+1}), \tag{6}$$

$$\lambda_t = \beta(1 + i_t)E_t(\gamma_{t+1}\psi_{t+1} + \lambda_{t+1}). \tag{7}$$

A unit of currency carried from period t and invested in money delivers a return $(1 + i_t^m)$ which can be used at time $t + 1$ to purchase goods or for the remaining part to contribute to next-period wealth. Instead, a unit of wealth invested in bonds is remunerated at $(1 + i_t)$ but provides liquidity services only for the fraction γ_{t+1}. Equations (6)–(7) show already that when $\gamma_{t+1} = 1$ interest rates on money and bonds are equalized because the two assets become perfect substitutes as a means of payment. This also happens when the liquidity constraint is never binding, i.e., when $\psi_t = 0$.

To see this formally, simplify the first-order conditions to

$$\frac{i_t - i_t^m}{1 + i_t} E_t\left\{\frac{U_c(C_{t+1})}{P_{t+1}}\right\} = E_t\{(1 - \gamma_{t+1})\psi_{t+1}\}, \tag{8}$$

$$\psi_t = \frac{U_c(C_t)}{P_t} - \beta(1 + i_t^m) E_t\left\{\frac{U_c(C_{t+1})}{P_{t+1}}\right\}. \tag{9}$$

In general, since ψ_{t+1} and γ_{t+1} are non-negative and γ_{t+1} is bounded above by 1, money has a lower return than bonds, $i_t^m \leq i_t$, which depends on ψ_{t+1} and γ_{t+1} and their covariance. For given ψ_{t+1}, when the liquidity properties of bonds improve, the interest rate on bonds falls closer to that of money. Moreover, the premium on bonds will be high when their liquidity properties, measured by γ_{t+1}, correlate inversely with the marginal utility of liquidity, represented by ψ_{t+1}.

Money and bonds are supplied by the central bank and government, respectively. Their integrated budget constraint can be written as

$$M_t^s + B_t^s = (1 + i_{t-1})B_{t-1}^s + (1 + i_{t-1}^m)M_{t-1}^s + T_t.$$

Equilibrium in asset markets implies

$$M_t = M_t^s,$$

$$B_t = B_t^s,$$

while in goods market

$$Y_t = C_t.$$

We solve for the equilibrium allocation of this model. The following set of equations:

$$(1 + i_{t-1}^m)M_{t-1}^s + \gamma_t(1 + i_{t-1})B_{t-1}^s \geq P_t Y_t, \tag{10}$$

$$\frac{i_t - i_t^m}{1 + i_t} E_t\left\{\frac{U_c(Y_{t+1})}{P_{t+1}}\right\} = E_t\{(1 - \gamma_{t+1})\psi_{t+1}\}, \tag{11}$$

$$\psi_t = \frac{U_c(Y_t)}{P_t} - \beta(1 + i_t^m) E_t\left\{\frac{U_c(Y_{t+1})}{P_{t+1}}\right\}, \tag{12}$$

characterizes the equilibrium of prices, interest rates, and the Lagrange multiplier ψ_t for given exogenous processes $\{Y_t, \gamma_t\}$ considering that $\psi_t \geq 0$. When $\psi_t > 0$, constraint (10) holds with equality. We further assume that there exists a technology through which the representative agent can store currency unaltered across periods so that the zero lower bound on the nominal interest on money (or reserves) applies.[12] The following inequalities hold: $i_t \geq i_t^m \geq 0$.

The equilibrium conditions have six unknowns $\{M_t^s, B_t^s, i_t, i_t^m, P_t, \psi_t\}$ which leave room for the choice of three policy instruments.[13] Considering an exogenous path of the supply of bonds, we are left with two dimensions along which to choose monetary policy.

II. Liquidity Shocks and Monetary Policy

We study the effects of a liquidity shock which worsens the quality of the pseudo-safe assets. At time 0 it is learnt that the liquidity properties of bonds temporarily deteriorate—meaning a fall in γ starting from period 1—and return back to normal levels in each period with a constant probability ξ. Ex post, the shock lasts T periods until period $T + 1$.[14]

There are clearly no real effects of the shock because in the simple model of the previous section prices are fully flexible. However, the way prices and interest rates react to the shock can be meaningful to intuit what will happen in more complicated models.

The specification of monetary policy is important for the results. First, we consider a benchmark policy in which the monetary policymaker is completely "passive." This policymaker keeps the interest rate on reserves unchanged and at the same time does not alter the path of money growth with respect to the previous trend. More broadly we can think of a policymaker that does not react at all to the shock either with conventional policy, through the policy rate, nor with unconventional policy, through the balance sheet.

In this context, the liquidity shock has two effects. The liquidity properties of bonds deteriorate and this is immediately reflected in a fall in their price and a rise in their yield, as shown in (11). To hold bonds, consumers ask for a higher return to compensate for the worsening in their quality. On the other side, there is a shortage of liquidity because the pseudo-safe assets now have a lower acceptance rate in exchange for goods, as shown in (10). The overall shortage of assets as means of payment implies a shortage of demand for consumption goods. Since prices are flexible, they fall to keep the goods market in equilibrium. These effects are shown in Figure 1 by the dotted line. The calibration implies that before the shock the interest rate on bonds is about 5 percent at annual rates and the interest rate on reserves is at 0.75 percent; money, prices, and the supply of bonds grow at 2 percent at annual rates. Under the benchmark "passive" policy, the interest rate on reserves remains at

[12] In our model M_t^s are the liabilities of the central bank and i_t^m is the interest rate paid by the central bank on such liabilities. We call it interest rate on money or reserves interchangeably. Note that a hypothetical corridor system is zero in our model so that the interest rate on reserves coincides at the same time with the policy rate and the interest rate on the marginal lending facility.

[13] We are implicitly assuming that lump-sum taxes T_t vary appropriately in a way that the transversality condition of households is satisfied at any equilibrium path of the stochastic processes $\{M_t^s, B_t^s, i_t, i_t^m, P_t, \psi_t\}$.

[14] We assume that the realization of the shock is known one period in advance by the monetary policymaker, to allow the latter to have the possibility to stabilize it completely. This is because in our model it is the money supply of the previous period that influences the current price level. In this way, a feasible policy is one in which there is complete stabilization of inflation—or the price level—around the target. Moreover, consumers cannot infer M_t^s from the transfer T_t since the latter also includes B_t^s.

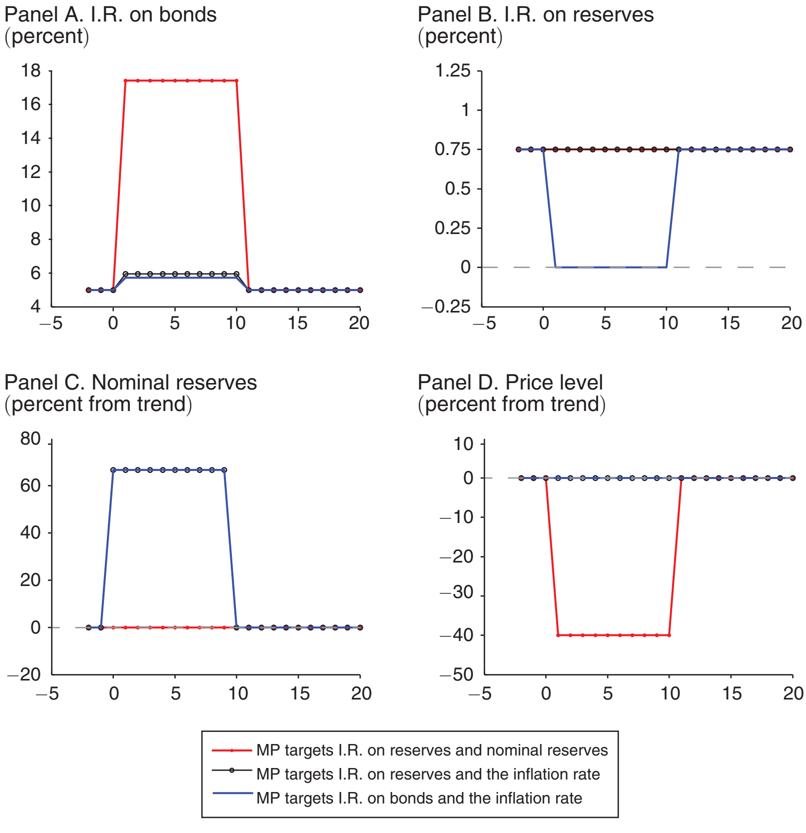

FIGURE 1. RESPONSE OF SELECTED VARIABLES TO A 20 PERCENT FALL IN THE LIQUIDITY PROPERTIES OF BONDS

Notes: Dotted line: passive monetary policy. Circled line: monetary policy targets interest rate on reserves and the inflation rate. Solid line: monetary policy targets interest rate on bonds and the inflation rate. The probability that in each period the shock returns back to steady state is $\xi = 10$ percent; the shock actually returns back after 10 quarters.

0.75 percent while money supply grows at 2 percent. We study the effects of a full deterioration in the quality of pseudo-safe assets which brings γ from 20 percent to 0 for 10 quarters. This shock leads to an increase in the spread between pseudo-safe and safe assets of about 13 percent. The price level falls substantially with respect to the previous trend through a deep deflation.[15]

We compare the benchmark policy with two other policies in which the policy-maker seeks to stabilize inflation rate at the 2 percent target. The two policies differ because in one (the circled line in Figure 1) the interest rate on reserves is kept

[15] The following quarterly calibration is used: $\beta = 0.99$. The initial ratio of $M/(PY)$ is set at $\bar{m} = 0.15 \cdot 4$ while that of $B/(PY)$ at $\bar{b} = 0.5 \cdot 4$, as implied by the US post-WWII average of the velocity of M1 and of the debt-to-GDP ratio. Such calibration implies that the steady state share of bonds providing liquidity services consistent with the constraint (10) is about 20 percent, and the steady state annualized interest rate on bonds is about 5 percent. In the initial equilibrium, the interest rate on money is set at 0.75 percent at annual rates while the growth rates of money, prices, and bonds are all 2 percent at annual rates.

unchanged at the initial level while in the other (the solid line in Figure 1) the policymaker tries to insulate the interest rate on bonds from the shock.

We have seen that the excess demand of liquidity and the corresponding excess supply of goods translate into a fall in the price level. To keep prices on their target, the excess demand of liquidity should be filled by assets with a high degree of acceptance in exchange for goods. To this end, the growth of money—the only safe asset in circulation since γ has fallen to zero—should increase substantially with respect to the previous target.[16] The effectiveness is shown in Figure 1 for both the circled-line and solid-line policies: the balance sheet is expanded by about 70 percent and prices are stabilized. The expansion should last until the liquidity properties of bonds return back to the initial level. However, this policy does not prevent the spillovers of the liquidity shock into a higher interest rate on bonds. The expectation of a stable inflation throughout the period, though, mutes the response of the interest rate on bonds, and the latter therefore rises less than in the case of passive policy, as shown by the circled line. To completely offset this surge, the monetary policymaker can in principle lower the interest rate on reserves, up to the point at which the zero lower bound becomes relevant. If the shock is large enough, as in Figure 1, the constraint is binding and the interest rate on bonds still rises, although by a lower amount, as shown by the solid line.[17]

At this point, it is useful to comment on the comparison between our analysis and that of Poole (1970). In Poole (1970), the driving shock is on money *demand*, and an interest rate targeting policy fully stabilizes the economy because money supply endogenously adjusts. In our framework, instead, we can interpret the shock to γ either as a shock to the *supply* of liquidity or to *demand*. In the case in which γ captures the intrinsic liquidity of the security in terms of the fraction that can be mobilized, a change in γ can be interpreted as a shock to the *supply* of liquidity. When instead γ captures the degree of acceptability of the securities in exchange for goods, it can be interpreted as a shock to the *demand* for liquidity. In both cases, given the overall demand for liquidity, a shock on γ endogenously shifts the demand for money/reserves. However, in our model, differently from Poole (1970), the monetary policymaker should specify two instruments of policy rather than just one to be able to determine the equilibrium. In the model discussed in this section, we are concerned about the determination of prices while Poole (1970) considers instead a sticky-price economy and therefore is concerned about the determination of output. Moreover our model economy is subject to a zero lower bound on the interest rate on reserves, which can prevent a complete insulation of the interest rate on bonds from the liquidity shock. Finally, it should be noted that the timing of openings of goods and financial markets implies that the nominal interest rate on bonds at time

[16] If γ falls to a positive number it would be possible to meet the liquidity needs by increasing the supply of bonds B_t^s, since this is an available policy instrument in this model. However, in the more general model of the next section in which the pseudo-safe asset is a deposit security, its supply is endogenously determined given some financial constraint at the level of the intermediary sector. We view the latter as the most relevant case to consider.

[17] In the experiment detailed in this section, we have focused on a monetary policy that acts directly through injection of liquidity into the system. However, since bonds and money are not perfect substitutes for liquidity purposes, the monetary policymaker can also operate by expanding money supply to buy pseudo-safe assets. In this case, the consumers' holdings of bonds, B_t, would fall during the experiment.

t is related negatively to expected output and prices at time $t+1$, through the Euler equations (11) and (12), rather than their current levels as in Poole (1970).

The simple model of this section does not have welfare implications because agents get utility from consumption, which is always equal to output in equilibrium. However, two important results already emerge from the analysis. First, to prevent prices from falling with respect to the target, the shortage of liquidity should be offset by issuing more safe assets. Second, a negative liquidity shock induces an upward pressure on the interest rate on bonds. The monetary policymaker can lean against it by cutting the interest rate on reserves. The extent to which it can be successful, however, depends on whether the liquidity shock is strong enough to drive the interest rate on reserves to the zero lower bound.

It is interesting to note that the two policy implications underlined in this section would be the result of an optimal policy problem in an extended version of the model with endogenous production and nominal rigidities. However, in the next section, we pursue a different avenue by proposing a model in which the pseudo-safe security is an inside asset. The aim is to capture some features of the recent financial crisis, in which the securities that lost their liquidity properties were indeed privately issued.

III. A Model with an Inside Pseudo-Safe Asset

Building on the insights of the previous simple model, we now present a more articulated framework in which money coexists with an inside security that plays the role of the pseudo-safe asset. In the model outlined in this section, financial intermediaries finance lending to the economy through deposits. Deposits and money are the only two assets available in the economy. However, deposits have imperfect liquidity properties which can suddenly deteriorate.[18] There are two groups of agents: Savers hold deposits and can use them to purchase goods together with money. Borrowers can use only money for their goods purchases and fund themselves directly from intermediaries. In this environment, we analyze the real effects of a liquidity shock and the ensuing implications for monetary policy.

A. *Households*

Consider a closed-economy model with two types of agents: borrowers, denoted with "b," and savers, with "s." There is a mass χ of savers and $(1-\chi)$ of borrowers. Utility is given by

$$E_t \sum_{T=t}^{\infty} \beta^{T-t}\left[U(C_T^j) - V(L_T^j)\right] \tag{13}$$

[18] Indeed, at the onset of the US financial crisis, intermediaries and other special vehicle purposes that were financing credit securities by issuing structured asset-backed securities experienced a drop in the liquidity properties of their financing means. In addition, short-term financing lines supporting holdings of asset-backed securities suddenly lost their liquidity value.

for $j = b, s$, where E_t denotes the standard conditional expectation operator and β is the discount factor, with $0 < \beta < 1$. Define C as a consumption bundle

$$C \equiv \left[\int_0^1 C(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where $C(i)$ is the consumption of a generic good i produced in the economy and θ is the intratemporal elasticity of substitution with $\theta > 1$; L^j is hours worked of quality of labor j.

At the beginning of period t the goods market opens and the following constraint limits the purchase of goods:

$$(1 + i_{t-1}^m) M_{t-1}^j + \gamma_t (1 + i_{t-1}^d) I_{t-1}^j B_{t-1}^j \geq P_t C_t^j, \tag{14}$$

for each $j = b, s$, where B_{t-1}^j represents the per-capita holdings of the inside security and I_t^j is an indicator function which takes the unit value only when B_{t-1}^j is positive—in which case it pays off $(1 + i_{t-1}^d)$—and zero otherwise. When in positive holdings, the inside asset takes the form of a deposit issued by the intermediary and it is a substitute for money to a certain degree, where γ_t is the quality value that the market attaches to it for its liquidity properties. All other variables have been previously defined.

When the goods market closes, the asset market opens and agents adjust their portfolios according to

$$\begin{aligned} M_t^j + B_t^j \leq {} & (1-\gamma_t)(1+i_{t-1}^d) I_{t-1}^j B_{t-1}^j + (1+i_{t-1}^b)(1-I_{t-1}^j) B_{t-1}^j + W_t^j L_t^j \\ & + \Psi_t^j + \Upsilon_t^j + T_t^j + [(1+i_{t-1}^m) M_{t-1}^j + \gamma_t (1+i_{t-1}^d) I_{t-1}^j B_{t-1}^j - P_t C_t^j], \end{aligned} \tag{15}$$

where $(1 + i_{t-1}^b)$ is the nominal interest on borrowing, i.e., when B_{t-1}^j is negative and therefore $I_{t-1}^j = 0$; W_t^j denotes the nominal wage which is specific to labor of agent $j = b, s$; Ψ_t^j are profits obtained from goods production while Υ_t^j are the profits of the intermediary sector.

"Savers" have positive holdings of the inside security, $B_t^s > 0$, while "borrowers" have negative, and specular, ones, $B_t^b < 0$. Since agents in the economy share the same intertemporal discount factor, we can choose an initial steady state in which a fraction, χ, of agents is savers while the remaining fraction is borrowers. In our experiment, we verify that borrowers and savers do not switch their portfolio positions during the time in which the liquidity shock hits the economy.[19]

Agents choose consumption and hours worked to maximize utility (13) under (14) and (15) taking into account standard borrowing-limit constraints. First-order

[19] Later we specify assumptions such that the initial distribution of wealth is locally determinate.

conditions of the two optimization problems are symmetric with respect to consumption, money, and labor:

$$U_c(C_t^j) = (\psi_t^j + \lambda_t^j) P_t, \tag{16}$$

$$\lambda_t^j = \beta(1 + i_t^m) E_t(\psi_{t+1}^j + \lambda_{t+1}^j), \tag{17}$$

$$V_l(L_t^j) = \lambda_t^j W_t^j, \tag{18}$$

for $j = b, s$, where ψ_t^j and λ_t^j are the respective Lagrange multipliers of constraints (14) and (15). The first-order condition of the savers with respect to deposit holdings implies

$$\lambda_t^s = \beta(1 + i_t^d) E_t(\gamma_{t+1} \psi_{t+1}^s + \lambda_{t+1}^s), \tag{19}$$

while that of the borrowers with respect to loans implies[20]

$$\lambda_t^b = \beta(1 + i_t^b) E_t \lambda_{t+1}^b. \tag{20}$$

We can combine more compactly the above first-order conditions to obtain the interest rate spread between deposits and money,

$$\frac{i_t^d - i_t^m}{1 + i_t^d} E_t\left\{\frac{U_c(C_{t+1}^s)}{P_{t+1}}\right\} = E_t\left\{(1 - \gamma_{t+1}) \varphi_{t+1}^s \frac{U_c(C_{t+1}^s)}{P_{t+1}}\right\}, \tag{21}$$

and between loans and money,

$$\frac{i_t^b - i_t^m}{1 + i_t^b} E_t\left\{\frac{U_c(C_{t+1}^b)}{P_{t+1}}\right\} = E_t\left\{\varphi_{t+1}^b \frac{U_c(C_{t+1}^b)}{P_{t+1}}\right\}. \tag{22}$$

Deposit and loan rates are in general higher than the interest rate on money (or reserves) insofar as the variables φ_{t+1}^s and φ_{t+1}^b are non-zero in some contingency, where

$$\varphi_t^j = 1 - \beta(1 + i_t^m) E_t\left\{\frac{U_c(C_{t+1}^j)}{U_c(C_t^j)} \frac{P_t}{P_{t+1}}\right\}, \tag{23}$$

and we have used the definitions $\varphi_t^j \equiv \psi_t^j P_t / U_c(C_t^j)$ for $j = b, s$. The liquidity shock γ_t affects the interest rate spread between deposits and money. When the liquidity properties of deposits worsen, the interest rate on deposits rises. This is one of the two new channels of transmission of financial crisis emphasized by our framework. Differently from the existing literature the "financial" shock does not directly raise the lending rate i_t^b but first it increases the deposit rate i_t^d. The other channel of

[20] In writing the intertemporal first-order conditions of savers and borrowers we have already accounted for the fact that in equilibrium $B_t^s > 0$ and $B_t^b < 0$.

transmission of the financial crisis is through equation (14) in which the liquidity shock reduces the nominal value of security available for purchasing goods.

Finally, we can write in a more compact way the marginal rate of substitution between labor and consumption through the following conditions:

$$\frac{V_l(L_t^j)}{U_c(C_t^j)} = (1 - \varphi_t^j)\frac{W_t^j}{P_t}, \tag{24}$$

for $j = b, s$. In this model, the liquidity constraint implies a financial friction which is captured by the variables φ_t^j. This friction creates a wedge between the real wage and the marginal rate of substitution between leisure and consumption, as shown in (24).[21]

B. *Financial Intermediaries*

The additional financial friction of the model, on top of the liquidity constraints, is that borrowing and lending should occur only through intermediaries which channel liquidity from savers to borrowers. The aggregate level of deposits is $D_t = \chi B_t^s$, and that of loans is $A_t = -(1-\chi)B_t^b$. The intermediaries' balance sheet in each period implies $A_t = D_t$. An interesting feature of the liquidity services provided by deposits is that, in the steady state, the interest rate on loans is above the one on deposits. This can be seen by using equations (21), (22), and (23) to get

$$\frac{\bar{\imath}^d - \bar{\imath}^m}{1 + \bar{\imath}^d} = (1 - \bar{\gamma})\frac{\bar{\imath}^b - \bar{\imath}^m}{1 + \bar{\imath}^b}, \tag{25}$$

where an upper bar denotes the steady state value of the variable. Unless $\bar{\gamma} = 0$, in which case deposits do not provide liquidity services, $\bar{\imath}^b > \bar{\imath}^d$.

Positive profits of intermediation naturally arise in our model because the liabilities of intermediaries are more liquid than their assets. Out of the steady state, the spread between lending and deposit rates $(i_t^b - i_t^d)$ is determined by the optimal choice of lending capacity on the part of intermediaries, because of the financial friction, which is the case in other models like Belongia and Ireland (2006, 2012) and Cúrdia and Woodford (2010). In these papers, however, the source of the financial crisis is a credit shock that hits this spread directly by increasing the lending rate while lowering the deposit rate. Instead, in our framework, this credit spread is not the key driver of the transmission mechanism. As shown in the Euler equation (21) a negative liquidity shock, worsening the liquidity properties of deposits, raises the spread between deposit and money rate $(i_t^d - i_t^m)$. Differently from the literature, intermediaries face higher funding costs while often in the literature the deposit rate falls following a credit shock. As we have already emphasized, this is only one of the two transmission mechanisms of the financial crisis in our model.

[21] This is consistent with the cash-in-advance constraint model of Cooley and Hansen (1989).

To close the model, we need to characterize the way credit spreads are set, and therefore the pass-through of the liquidity shock to the lending rate. In line with Belongia and Ireland (2006, 2012) and Cúrdia and Woodford (2010) we assume costs of intermediation. These might well capture agency costs in increasing the lending capacity on the side of intermediaries or other types of costs discussed in the literature.

Period t profits can be written in real terms as

$$\frac{\Upsilon_t}{P_{t-1}} = (1+i_{t-1}^b)\,a_{t-1} - (1+i_{t-1}^d)\,d_{t-1} - k_t\cdot\phi\Big(\frac{a_{t-1}}{\overline{a}}\Big),$$

which depends on the volume of lending and deposit supplied in the previous period, where $a_t \equiv A_t/P_t$ and $d_t \equiv D_t/P_t$. The cost function is given by $\phi(\cdot)$ with the properties $\phi(1) = 0$, $\phi'(1) = 1$, and $\phi''(1) > 0$ where $\phi'(\cdot)$ and $\phi''(\cdot)$ are respectively the first and second derivatives of $\phi(\cdot)$. The variable $\overline{a}$ defines the steady state level of lending and k_t is an appropriate scaling factor given by $k_t = (1+i_{t-1}^d)\overline{\delta}\,\overline{a}$ where $\overline{\delta}$ is the steady state spread between lending and deposit rates defined by $(1+\overline{\delta}) \equiv (1+\overline{\imath}^b)/(1+\overline{\imath}^d)$. We assume that the costs $k_t\cdot\phi(\cdot)$ are paid directly to the savers as are the profits of intermediation Υ_t, which are known in period $t-1$ and delivered in period t.[22]

An important implication of assuming a cost of intermediation penalizing lending with respect to the steady state value is that the steady state asset or liability positions of savers and borrowers become locally determinate in this case. This ensures that after a liquidity shock, the distribution of wealth converges to the initial steady state and savers and borrowers remain in their respective portfolio positions.

In a competitive market, the spread between borrowing and lending rates that maximizes profits is:

$$1+\delta_t \equiv \frac{(1+i_t^b)}{(1+i_t^d)} = \Big[1+\overline{\delta}\phi'\Big(\frac{a_t}{\overline{a}}\Big)\Big]. \tag{26}$$

It follows that the spread δ_t is increasing in the overall level of loans and is consistent with its steady state value since $\phi'(1) = 1$.[23] However, it is worth emphasizing again that this credit spread moves only endogenously in our model depending on how the overall level of lending reacts to the liquidity shock directly affecting the deposit rate i_t^d. Instead, in the literature, the main financial shock hits this spread exogenously and then propagates into the economy.

C. *Firms*

We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function $Y(i) = L(i)$ is linear in a bundle of labor which is a Cobb-Douglas index of the two types of labor:

[22] Quantitative results can be affected but not overturned by different assumptions on the distribution of intermediation profits, as we will discuss later.

[23] The scaling factor k_t appropriately normalizes the optimal spread around its steady state value.

$L(i) = (L^s(i))^{\chi}(L^b(i))^{1-\chi}$. Given this technology, labor compensation for each type of worker is equal to total compensation, $W_j L_j = WL$, where the aggregate wage index is appropriately given by $W = (W^s)^{\chi}(W^b)^{1-\chi}$. Each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta} Y$ where aggregate output is

$$Y_t = \chi C_t^s + (1-\chi) C_t^b. \tag{27}$$

Firms are subject to price rigidities as in Calvo's model: in each period a fraction of measure $(1-\alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price, while the remaining fraction α of firms index their previously adjusted price to the inflation target rate $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of the profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place:

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left[\bar{\Pi}^{T-t} \frac{P_t(i)}{P_T} Y_T(i) - (1-\tau) \frac{W_T}{P_T} Y_T(i) \right],$$

where Λ_T is the stochastic discount factor used to evaluate profits at a generic time T, which is a linear combination of the marginal utilities of consumption of the two agents, $\Lambda_T = \beta^{T-t}[\chi U_c(C_T^s) + (1-\chi) U_c(C_T^b)]$, and τ is an employment subsidy. The first-order condition of the optimal pricing problem implies

$$\frac{P_t^*}{P_t} = (1-\tau) \frac{\theta}{\theta-1} \frac{E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}}, \tag{28}$$

where we have set $P_t(i) = P_t^*$ since all firms adjusting their prices will fix it at the same price. Calvo's model further implies the following law of motion for the general price index:

$$P_t^{1-\theta} = (1-\alpha) P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \bar{\Pi}^{1-\theta}, \tag{29}$$

through which we can write the aggregate supply equation

$$\left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{1}{1-\theta}} = (1-\tau) \frac{\theta}{\theta-1} \frac{E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\theta} \frac{W_T}{P_T} Y_T \right\}}{E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t} \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{1-\theta} Y_T \right\}}. \tag{30}$$

We assume that the utility flow from consumption is exponential $U(C^j) = 1 - \exp(-\upsilon C^j)$ for some positive parameter υ while the disutility of working is isoelastic: $v(L^j) = (L^j)^{1+\eta}/(1+\eta)$. The above assumptions are convenient for aggregation purposes and to keep tractability. These features can be easily discovered by taking a weighted average of (24), for $j = s, b$, with weights χ and $1 - \chi$ respectively obtaining[24]

$$\frac{(Y_t \Delta_t)^{\eta}}{\upsilon \exp(-\upsilon Y_t)} = \frac{W_t}{P_t}(1 - \varphi_t^s)^{\chi}(1 - \varphi_t^b)^{1-\chi}, \tag{31}$$

where aggregate output and labor are related through $Y_t \Delta_t = L_t$ and Δ_t is an index of price dispersion defined by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di,$$

which evolves as

$$\Delta_t \equiv \alpha\big(\Pi_t^{\theta} \bar{\Pi}^{-\theta}\big)\Delta_{t-1} + (1 - \alpha)\left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha}\right)^{\frac{\theta}{\theta-1}}. \tag{32}$$

D. *Government Budget Constraint and Monetary Policy*

To complete the characterization of the model we specify the consolidated budget constraint of government and central bank. We assume that there are no government bonds or public spending. The consolidated budget constraint simply reads as

$$M_t = (1 + i_{t-1}^m) M_{t-1} + \chi T_t^s + (1 - \chi) T_t^b + \tau W_t L_t. \tag{33}$$

It is clear that with heterogenous agents the distribution of transfers matters for the equilibrium allocation. Here we assume that each agent receives transfers corresponding to its holdings of money after subtracting a proportional share of the employment subsidy:

$$T_t^s = M_t^s - (1 + i_{t-1}^m) M_{t-1}^s - \tau W_t L_t, \tag{34}$$

$$T_t^b = M_t^b - (1 + i_{t-1}^m) M_{t-1}^b - \tau W_t L_t. \tag{35}$$

Later, in Section IVE, we discuss an alternative transfer rule.

[24] In deriving (31), the assumption of a Cobb-Douglas production technology is also critical. It should be noted that another implication of our specification of preferences and production technology is that the steady state level of output implied by (31) does not depend on the distribution of wealth. Indeed, in a steady state in which $\Pi_t = \bar{\Pi}$ prices will be set as a markup over wages and moreover $\bar{\varphi}^s = \bar{\varphi}^b$ in a way that the steady state output implied in (31) is invariant to the distribution of wealth across agents.

E. *Equilibrium in Goods and Asset Markets*

In equilibrium, money supply is equal to money demand:

$$M_t = \chi M_t^s + (1-\chi) M_t^b, \tag{36}$$

while financial market equilibrium requires

$$(1-\chi) B_t^b + \chi B_t^s = 0. \tag{37}$$

Goods market equilibrium is given by (27).

Moreover, since only savers hold deposit for purchasing purposes, constraints (14) for $j = b, s$ imply

$$(1+i_{t-1}^m) M_{t-1}^b \geq P_t C_t^b, \tag{38}$$

$$(1+i_{t-1}^m) M_{t-1}^s + \gamma_t (1+i_{t-1}^d) B_{t-1}^s \geq P_t C_t^s. \tag{39}$$

F. *Equilibrium Conditions*

We will now collect the equations that characterize the equilibrium of the model. On the demand side, there are equations (21) and (22), and (23) for each $j = b, s$. Lending and borrowing interest rates are connected through equation (26). The two liquidity constraints (38) and (39) can be written in real terms as

$$(1+i_{t-1}^m) \frac{m_{t-1}^b}{\Pi_t} \geq C_t^b \tag{40}$$

and

$$(1+i_{t-1}^m) \frac{m_{t-1}^s}{\Pi_t} + \gamma_t (1+i_{t-1}^d) \frac{(1-\chi)}{\chi} \frac{b_{t-1}}{\Pi_t} \geq C_t^s, \tag{41}$$

where $m_t^b \equiv M_t^b / P_t$, $m_t^s \equiv M_t^s / P_t$ while b_t denotes the real debt of the borrowers given by $b_t \equiv -B_t^b / P_t$. To obtain (41), we have also used (37).

In real terms equation (36) implies

$$\chi m_t^s + (1-\chi) m_t^b = m_t, \tag{42}$$

where m_t denotes aggregate real money balances, defined as $m_t \equiv M_t / P_t$.

The flow budget constraint of the borrowers can be simplified, using (35), to

$$b_t = (1+i_{t-1}^b) \frac{b_{t-1}}{\Pi_t} + C_t^b - Y_t, \tag{43}$$

where we have used the fact that the Cobb-Douglas technology implies that $W_t^b L_t^b = W_t L_t$ together with the assumption $\Upsilon_t^b = 0$.[25]

On the aggregate supply side, there is equation (30) together with (31) and (32) and the relationship $Y_t \Delta_t = L_t$.

The set of equations (21), (22), and (23) for each $j = b, s$ together with (26), (27), (30), (31), (32), (40), (41), (42), and (43), describe the equilibrium conditions of the model together with the two Kuhn-Tucker conditions associated with the constraints (40), (41), and the inequalities $\varphi_t^s \geq 0$, $\varphi_t^b \geq 0$. There are 13 equations in the following 15 unknowns: $Y_t, C_t^b, C_t^s, i_t^b, i_t^d, i_t^m, \Delta_t, W_t/P_t, P_t, b_t, \varphi_t^s, \varphi_t^b, m_t^s, m_t^b, m_t$, leaving the possibility to specify two instruments of policy.

IV. Liquidity Shocks and Optimal Monetary Policy

We repeat the experiment of a shock that worsens the liquidity properties of the pseudo-safe asset. The model now has a richer transmission mechanism and there is also a propagation of the shock to the real economy for two reasons: first, there are redistributive effects between borrowers and savers because the inside asset is in nominal terms; second, the presence of nominal rigidities. We compare alternative monetary regimes with the Ramsey policy that maximizes the weighted sum of the utility of the consumers belonging to the economy:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t}\left[\tilde{\chi}(U(C_T^s) - V(L_T^s)) + (1-\tilde{\chi})(U(C_T^b) - V(L_T^b))\right], \tag{44}$$

given the equilibrium conditions of the model, in which $\tilde{\chi}$ and $(1-\tilde{\chi})$ are the relative weights, respectively, of savers and borrowers in the objective function.

To get intuition about the underlining trade-offs, we can derive a simple quadratic loss function corresponding to a second-order approximation of (44) under some relatively minor restrictions. In particular, we use assumptions such that the steady state resulting from the Ramsey problem coincides with the efficient steady state allocation of consumption and labor. This efficient allocation solves the maximization of (44) under the resource constraint

$$\chi C_t^s + (1-\chi) C_t^b = Y_t = (L_t^s)^{\chi} (L^b)^{1-\chi}.$$

In particular, as discussed in the Appendix, the first-order conditions of this problem imply

$$\frac{\tilde{\chi}}{1-\tilde{\chi}} \frac{U_c(\bar{C}^s)}{U_c(\bar{C}^b)} = \frac{\chi}{1-\chi}, \tag{45}$$

$$\frac{V_l(\bar{L}^j)}{U_c(\bar{C}^j)} = \frac{\bar{Y}}{\bar{L}^j}, \tag{46}$$

[25] If all the profits of intermediation were rebated to the borrowers, it would be easy to see that the relevant interest rate in (43) is the deposit rate, i_t^d, instead of the loan rate, i_t^b.

where the latter holds for each $j = b, s$. This is clearly the best allocation that can be achieved in this model, for given weights $\tilde{\chi}$ and $(1-\tilde{\chi})$. If the Ramsey policymaker could implement this allocation in the steady state, this would correspond to the Ramsey optimal policy when there are no stochastic disturbances.

We show that the combination of policies: $\Pi_t = \overline{\Pi}$ and $i_t^m = \overline{\imath}^m$ where $\overline{\Pi} \geq \beta$ and $0 \leq \overline{\imath}^m \leq \overline{\Pi}/\beta - 1$ can indeed implement the first best, under two minor restrictions.[26] The first restriction requires that the employment subsidy to firms is set at $\tau = (\overline{\mu} + \overline{\varphi})/(1+\overline{\mu})$ where $\overline{\mu} \equiv (\theta-1)^{-1}$ and $\overline{\varphi} = \overline{\varphi}^s = \overline{\varphi}^b$ are, respectively, the steady state net markup and level of the financial friction. Indeed, a policy in which $\Pi_t = \overline{\Pi}$ implies in equation (30) that

$$\frac{\overline{W}}{\overline{P}} = \frac{1}{(1-\tau)(1+\overline{\mu})},$$

which can be used in the steady state version of (24) to get

$$\frac{V_l(\overline{L}^j)}{U_c(\overline{C}^j)} = \frac{1-\overline{\varphi}}{(1-\tau)(1+\overline{\mu})}\frac{\overline{Y}}{\overline{L}^j}, \tag{47}$$

for each $j = b, s$. With the chosen subsidy $\tau = (\overline{\mu} + \overline{\varphi})/(1+\overline{\mu})$, equation (47) is equivalent to (46).

To implement (45), note that $\Pi_t = \overline{\Pi}$ and $i_t^m = \overline{\imath}^m$ imply that

$$\overline{C}^s = \frac{1-\chi}{\chi}\left(\frac{1-\beta}{\beta}\right)\overline{b} + \overline{Y}, \tag{48}$$

$$\overline{C}^b = -\left(\frac{1-\beta}{\beta}\right)\overline{b} + \overline{Y}, \tag{49}$$

where we have used the steady state version of (20), (27), and (43). Given that $\overline{b} > 0$, it follows that $\overline{C}^s > \overline{C}^b$. For given $\overline{C}^s$ and $\overline{C}^b$, in order to implement condition (45), we add the second restriction which involves an appropriately chosen weight $\tilde{\chi}$: in particular, $\tilde{\chi} > \chi$.

An interesting implication of our implementation exercise is that the Ramsey policymaker can achieve the first best for any choice of $\overline{\imath}^m$ in the interval $[0, \overline{\Pi}/\beta - 1]$ implying, from the steady state version of (23), that the Lagrange multiplier $\overline{\varphi}$ can fall anywhere in the interval $[0, 1-\beta/\overline{\Pi}]$. At the lower end of the range, the case $\overline{\varphi} = 0$ corresponds to the Friedman's rule which requires us to completely eliminate the financial friction. However, in our case positive values of $\overline{\varphi}$ are equivalent in terms of welfare. We can indeed choose a steady state value of $\overline{\varphi}$ different from zero, in the above interval, and get the same first-best allocation of consumption and labor and, at the same time, be consistent with the optimal choice of a

[26] It should be recalled that we can specify two policy instruments in our model.

Ramsey policymaker.[27] Notice that a positive steady state value of the multiplier $\overline{\varphi}$ is particularly convenient in our framework because it makes the financial friction non-negligible in a first-order approximation of our model.[28] To further justify this choice, it is worth noting that in heterogenous-agent stochastic models with incomplete markets the Friedman's rule is not achievable. In our model, indeed, it requires us to set φ_t^s and φ_t^b simultaneously to zero at all times, but this is not feasible unless $U_c(C_{t+1}^s)/U_c(C_t^s) = U_c(C_{t+1}^b)/U_c(C_t^b)$ in all contingencies as shown in (23).[29]

We show in the Appendix that under these two assumptions, a second-order approximation of (44) delivers the following simple quadratic loss function:

$$\frac{1}{2}E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}\left[\hat{Y}_T^2+\chi(1-\chi)\lambda_c(\hat{C}_T^s-\hat{C}_T^b)^2+\chi(1-\chi)\lambda_l(\hat{L}_T^s-\hat{L}_T^b)^2+\lambda_\pi(\pi_T-\overline{\pi})^2\right]\right\},$$

where in general variables with a "hat" are log deviations from their own steady state, while $\hat{C}_t^j \equiv (C_t^j-\overline{C}^j)/\overline{Y}$ for each $j = b, s$, $\pi_t = \ln P_t/P_{t-1}$, and $\overline{\pi} = \ln\overline{\Pi}$. The positive coefficients λ_c, λ_l, and λ_π are all defined in the Appendix.

The loss function contains some familiar terms to the literature. The only shock of the model is to liquidity, which is an inefficient shock, therefore deviations of output with respect to the efficient steady state are penalized appropriately. Inflation is also costly when it deviates from the trend to which price setters index prices, implying inefficient fluctuations of relative prices among goods produced according to the same technology. The other two terms in the loss function instead depend on the additional features that the heterogeneity of agents brings into the model. Since risk sharing of consumption and labor is efficient in the chosen steady state, departures from this allocation cause losses for aggregate welfare. In particular, the labor risk-sharing term can be further simplified noting that in a first-order approximation

$$\hat{L}_t^s-\hat{L}_t^b = -\frac{\rho}{1+\eta}(\hat{C}_t^s-\hat{C}_t^b)-\frac{1}{(1-\overline{\varphi})(1+\eta)}(\varphi_t^s-\varphi_t^b),$$

where $\rho = \upsilon\overline{Y}$. In standard models without financial frictions, the labor risk-sharing argument is proportional to the consumption risk-sharing term. Here, instead, it is also influenced by the difference between the financial distortions faced by savers and borrowers. Since in a first-order approximation of (23) we get

$$\varphi_t^j = \overline{\varphi}+(1-\overline{\varphi})E_t\left[(\pi_{t+1}-\overline{\pi})-\hat{\imath}_t^m+\rho\Delta\hat{C}_{t+1}^j\right],$$

for $j = b, s$, we can simplify the labor risk-sharing term to

$$\hat{L}_t^s-\hat{L}_t^b = -\frac{\rho}{1+\eta}E_t(\hat{C}_{t+1}^s-\hat{C}_{t+1}^b). \tag{50}$$

[27] Note that our framework nests standard results on the Friedman's rule. In the case in which $\overline{\mu} = 0$, $\tau = 0$, and prices are flexible, it is optimal only to set $\overline{\varphi} = 0$. However, this does not necessarily imply a rate of deflation equal to the time discount factor, since $\overline{\imath}^m$ is also a policy variable and therefore $\overline{\Pi} = \beta(1+\overline{\imath}^m)$. Only if $\overline{\imath}^m = 0$, as assumed in the literature, $\overline{\Pi} = \beta$.

[28] The two liquidity constraints, for borrowers and savers, will hold with equality in this steady state.

[29] See Woodford (1990) for a general discussion.

Because of the financial friction, labor effort at time t is producing income which is only liquid to purchase goods in the next period.[30] As shown by the first-order condition (24), using (23), the consumers are optimally choosing labor given current wages and future prices taking into account their expectations of future consumption. It follows that the cross-agent difference in labor effort is proportional to the difference in the one-period-ahead consumption expectations. Equivalently, we can write the loss function as

$$(51)\quad \frac{1}{2}E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}\left[\hat{Y}_T^2+\chi(1-\chi)\lambda_c\left(\hat{C}_T^R\right)^2+\chi(1-\chi)\,\tilde{\lambda}_l\left(E_T\hat{C}_{T+1}^R\right)^2+\lambda_\pi(\pi_T-\overline{\pi})^2\right]\right\}$$

for some $\tilde{\lambda}_l$, where $\hat{C}_T^R \equiv \hat{C}_T^s - \hat{C}_T^b$. We compute the optimal policy under commitment by minimizing this loss function with respect to the log-linear approximation of the equilibrium conditions.[31]

In order to provide a numerical illustration of the policy implications of our framework, we calibrate the model (quarterly) as follows. As in Cúrdia and Woodford (2010), we set the steady state real interest rate on deposits to 4 percent and the steady state credit spread $\overline{\delta}$ to 2 percent, both in annual terms. Moreover, we also set the steady state inflation rate $\overline{\pi}$ to 2 percent, annualized. Accordingly, the implied steady state annualized nominal interest rate faced by the borrowers $\overline{\imath}^b$ is equal to 8 percent, and the time discount factor is thereby $\beta = 0.985$. The steady state interest rate on reserves $\overline{\imath}^m$ is calibrated at 75 annual basis points.[32] We use the average velocity of M1 for the US economy during the Great Moderation (1984–2007) to calibrate the steady state money to GDP ratio: $M/PY = 0.125 \cdot 4$, and we follow Eggertsson and Krugman (2012) in setting the initial private debt to income ratio $\overline{b}/4\overline{Y}$ to 100 percent. The economy-wide liquidity constraint and equation (25) thereby imply that about 27 percent of assets provide liquidity services (i.e., $\overline{\gamma} = 0.27$) and that the share of savers in the economy is about 55 percent.[33] We also follow Eggertsson and Krugman (2012) in setting the elasticity of the credit spread to the stock of real debt, such that a 30 percent increase in debt doubles the spread.[34] A given value for the initial debt to income ratio, then, pins down a unique initial distribution of wealth: indeed, equations (48) and (49) yield the distribution of personal consumption, which in turn implies the distribution of money holdings, through equations (40) and (41). Finally, the relative risk-aversion coefficient is set to $\rho = 1$, the inverse of the Frisch-elasticity of labor supply to $\eta = 2$, the parameter α capturing the degree of nominal rigidity in the model implying an average duration of consumer prices of four quarters ($\alpha = 0.75$).

[30] It should be noted that (50) is also valid when $\overline{\varphi} = 0$ and that it captures the impossibility of achieving the Friedman's rule in the stochastic equilibrium ($\varphi_t^s = \varphi_t^b$) unless $\Delta\hat{C}_{t+1}^b = \Delta\hat{C}_{t+1}^s$ at all times.

[31] The solution of the above optimization problem with respect to all the endogenous variables involved corresponds to the first-order approximation of the solution of the non-linear Ramsey problem (see Benigno and Woodford 2012).

[32] This is the level at which the interest rate on reserves was set when it was introduced by the Fed.

[33] The implied fraction of pseudo-safe asset is consistent with the 30 percent estimated by Gorton, Lewellen, and Metrick (2012).

[34] This elasticity corresponds to the parameter ϕ defined in the Appendix.

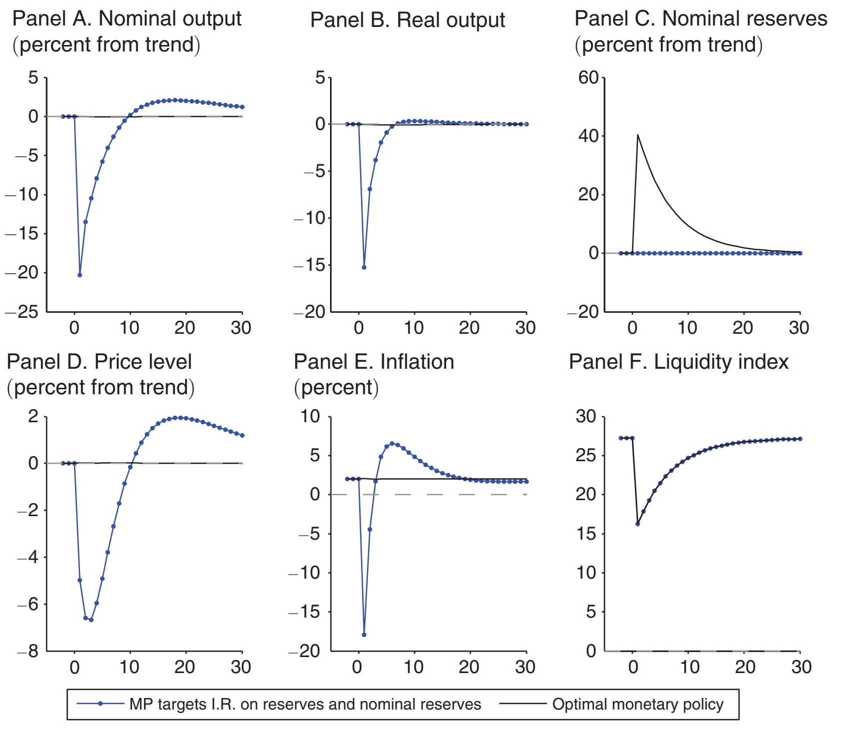

FIGURE 2. RESPONSES OF SELECTED VARIABLES TO AN 11 PERCENT FALL IN THE LIQUIDITY PROPERTIES OF PSEUDO-SAFE ASSETS: PASSIVE MONETARY REGIME VERSUS OPTIMAL POLICY

Notes: Dotted line: passive monetary policy keeps both the money growth and the interest rate on reserves i_t^m constant (passive regime). Solid line: optimal monetary policy. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

In the simulations below, we assume that the liquidity index $\hat{\gamma}_t \equiv \frac{\gamma_t - \bar{\gamma}}{1 - \bar{\gamma}}$ follows an autoregressive process of the kind $\hat{\gamma}_t = \rho_\gamma \hat{\gamma}_{t-1} + \varepsilon_\gamma$ and analyze the dynamic effect of an 11 percent negative shock which gradually reverts back to mean, with half-life of about four quarters ($\rho_\gamma = 0.85$). The size of the shock is chosen in order to imply a peak response of the credit spread of 4 percentage points above steady state, which corresponds to the exogenous credit spread shock simulated in Cúrdia and Woodford (2010).

A. *Optimal Unconventional Policies Following a Liquidity Shock*

In Figures 2 and 3 we compare the optimal policy (solid line) with a passive monetary policy in which the interest rate on reserves and the growth rate of the nominal money supply are kept constant at the levels before the shock hits (dotted line).

There are two important policy implications on what monetary policy should do when facing a liquidity shock: inject more liquidity in the form of reserves, as shown in Figure 2, and lower the interest rate on reserves up to the zero lower bound, as shown in Figure 3. Although it is in general hard to isolate the effects of the two channels in the general equilibrium of the model, we argue that the injection of

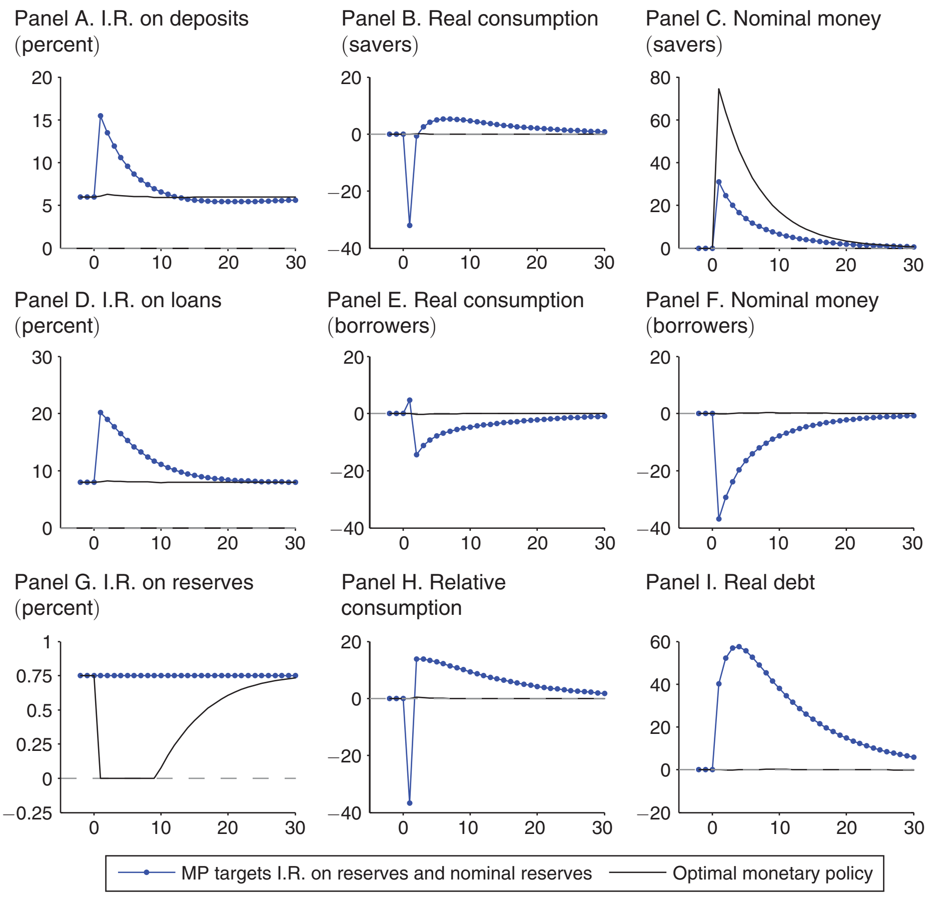

Figure 3. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets: Passive Monetary Regime versus Optimal Policy

Notes: Dotted line: monetary policy keeps both the money growth and the interest rate on reserves i_t^m constant (passive regime). Solid line: optimal monetary policy. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

liquidity avoids the deflation and the contraction in real activity, while lowering the interest rate on reserves helps to achieve a better risk sharing of the shock between savers and borrowers.[35]

The transmission of the liquidity shock can be understood in a simple way through two main mechanisms.

First, the liquidity shock creates, at an aggregate level, a shortage in the supply of the assets available for goods purchasing, because pseudo-safe assets have partially lost their qualities. An excess supply of goods is the corresponding disequilibrium in the goods market to that in the asset market due to the shortage of safe assets. Nominal spending falls, and the split between prices and real output depends on

[35] As will be discussed later, the second policy prescription is ineffective without the first.

the degree of price rigidities. This is what happens under the passive policy: real output drops widely, as shown in Figure 2 with a contraction of about 15 percent while prices fall by about 7 percent compared to their trend, through a deep deflation. Figure 2 clearly illustrates the dramatic effects of a liquidity shock when the monetary policymaker is completely helpless. Under optimal policy, instead, the contraction in real output is very mild as well as the response of inflation and prices. The key change in policy, which leads to a near stabilization of output and prices, is the increase in the growth rate of money, as shown in Figure 2, which goes up to a path about 40 percent above the previous trend. A substantial expansion in the central bank's balance sheet and an increase in the supply of safe assets are required to optimally absorb the shock. Interestingly, the expansion should last for as long as the liquidity conditions remain deteriorated, slowly returning back to the initial path as the liquidity properties of assets go back to normal.

The second mechanism of propagation works through asset prices. The liquidity shock requires a higher premium to hold pseudo-safe assets. This in turn increases the cost of funding for intermediaries which need to raise the interest rate on loans. Under the passive monetary policy, spreads and interest rates increase as shown in Figure 3. Under optimal policy, all market interest rates are instead insulated from the shock and stable around the previous steady state levels. The important change in policy that helps explain this result is the reduction in the interest rate on reserves up to the zero bound and for quite a long horizon. To intuit this result, Figure 3 displays the consumption of savers and borrowers, respectively, and their difference. As shown in the loss function (51), imperfect consumption risk sharing is costly in this economy. Under the passive monetary policy, the rise in the real interest rates has important wealth and redistributive effects between borrowers and savers. Borrowers are hit in a significant way by the increase in the real rate on loans, so that they have to cut their consumption.[36] Their real debt even rises, mostly because of the deflation. Savers instead benefit from the increase in the real return on their savings and can raise their consumption by holding more safe assets at the expense of borrowers.[37] Under optimal policy, instead, the interest rate on reserves falls and this stabilizes all other market interest rates.[38] The redistributive channel, which was strong under the passive policy, is now muted. Borrowers face approximately the same real interest rate as before the shock, and do not have to cut consumption. Savers can increase their money holdings to replace the deteriorated pseudo-safe assets without crowding out money holdings of borrowers. This is because the central bank injects more liquidity.

We can now appreciate some important differences between our model and those of the recent literature on financial crisis and unconventional policies. First, as we have already emphasized, the main shock on which the literature has focused directly hits the credit spread $i_t^b - i_t^d$ by raising the interest rate on lending and lowering the

[36] This is also the case if the profits of the financial intermediary are all distributed to the borrowers. In this case, the relevant interest rate is the deposit rate.

[37] On impact, savers' consumption falls, as the liquidity value of their accumulated pseudo-safe assets shrinks and they are unable to reallocate their portfolio.

[38] It is worth noting that the expansion of the central bank's balance sheet, by stabilizing output and inflation, also contributes to more stable nominal interest rates, as the latter also depend on the expected paths of the former.

deposit rate. Instead, in our model, the liquidity shock directly raises the deposit rates; this is then passed through into higher lending rates, at the same time creating a shortage of safe assets. Second, in models like Eggertsson and Krugman (2012), a deleveraging shock on borrowers' debt is responsible for the same rise in credit spreads emphasized by the literature but also for a drop in the natural rate of interest, which requires a parallel fall in the real interest rate. As a consequence, savers need to increase their consumption to compensate for the fall of the deleveragers. Instead, in our model, the different source of shock implies that the real rate does not need to move much under optimal policy. Consumption of savers and borrowers should remain at the same levels as before the shock hits. Third, in Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), the zero lower bound on nominal interest rate is a constraint to achieve the optimal stabilization of aggregate objectives like output and inflation. In our model, instead, the zero lower bound is only a constraint to achieve a better risk sharing of the shock between borrowers and savers.[39] Instead, balance sheet policies should take care of aggregate objectives: their primary role is to fulfill the shortage of safe assets and not necessarily to reduce credit spreads, as mostly emphasized by the literature.

B. *The Optimality of Nominal GDP Targeting at the Zero Bound*

We further investigate the features of the optimal policy and compare it with a targeting regime that can approximate it, as shown in Figures 4 and 5. The scale of these figures is rather different from the previous ones and enables us to better appreciate the variation of the variables of interest. We display the optimal policy in contrast with a simple policy in which nominal GDP is stabilized in each quarter and the interest rate on reserves is lowered in order to insulate the interest rate on deposits from the liquidity shock. Under the calibration considered, the shock is too large to successfully stabilize the interest rate on deposits in all periods because the zero lower bound becomes a relevant constraint for some quarters. This simple targeting regime can approximate quite well the optimal policy along the objectives of the loss function (51): the deviations are in general small and imply negligible losses in terms of welfare, unlike the case of passive monetary policy.[40]

We can further study the extent to which lowering the interest rates on reserves up to the zero lower bound is an important tool for sterilizing the shock. Consider a policy in which nominal GDP is stabilized in each period but where the interest rate on reserves is held constant. In Figure 6, we add this policy in comparison with the previous one and the optimal policy. When the interest rate on reserves is unchanged, a policy of targeting nominal GDP can completely stabilize inflation at the target but also real output. However, consumption dispersion across agents substantially rises.

[39] As will be shown later, in our model, in contrast to Eggertsson and Woodford (2003) and Eggertsson and Krugman (2012), an inflation targeting policy or a nominal GDP targeting policy can be implemented in all periods if the central bank is willing to commit to it, but this necessarily requires an expansion in the balance sheet exactly for the periods in which the liquidity properties have deteriorated. The zero lower bound is not necessarily a constraint for these policies.

[40] A result not shown in the graph is that a combined policy of strict inflation targeting with a stable interest rate on deposits gets very close to optimal policy, but performs slightly worse than nominal GDP targeting along all three dimensions relevant for welfare (relative consumption, real output, and inflation).

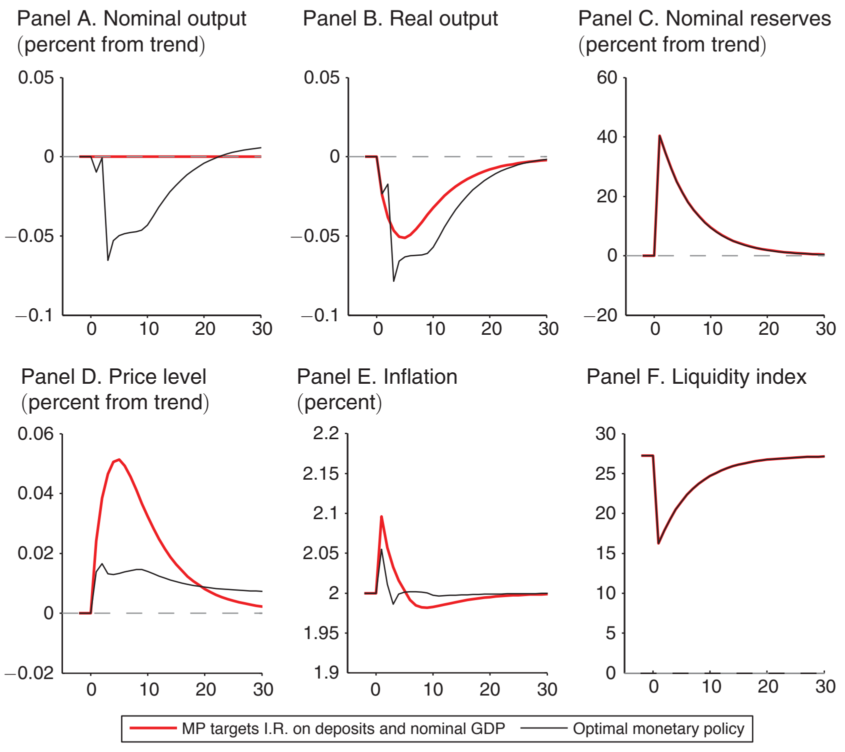

Figure 4. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets: Active Monetary Regime versus Optimal Policy

Notes: Bold solid line: monetary policy implements nominal GDP targeting and seeks stabilization of i_t^d. Thin solid line: optimal monetary policy. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

There is a trade-off between aggregate targets (output and inflation) on the one side, and cross-sectional ones (consumption dispersion) on the other. [41] As we conjectured, therefore, it is the cut in the interest rate on reserves that allows the central bank to improve the risk sharing of the liquidity shock, albeit at the cost of slightly more volatile output and inflation.[42]

Notice, however, that such a cut in the interest rate on reserves would be mostly ineffective unless it is associated with the expansion in the central bank's balance sheet. Indeed, consider a policy in which nominal reserves are kept on the initial path, and the central bank seeks to stabilize the interest rate on deposits in each period through adjusting the interest rate on reserves. Figure 7 displays the response of the economy under this regime (bold solid line) and contrasts it with the passive regime of Figures 2 and 3 and the nominal GDP targeting regime of Figures 4 and

[41] The same implication clearly follows if monetary policy targets inflation instead of nominal GDP.

[42] Note, however, that expansionary balance sheet policies, even when interest rates on reserves are kept constant, mitigate the rise in the deposit and loan rates, in contrast to the completely passive policy of Figures 3 and 4, thus substantially stabilizing the credit spread.

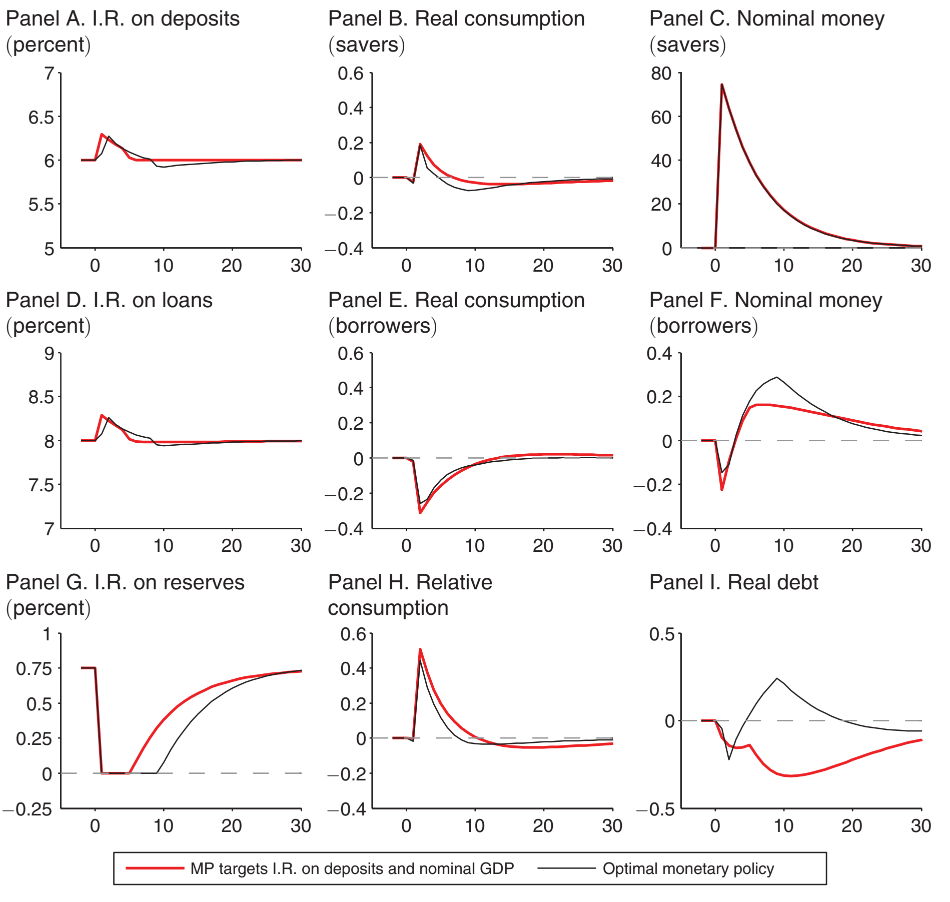

Figure 5. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets: Active Monetary Regime versus Optimal Policy

Notes: Bold solid line: monetary policy implements nominal GDP targeting and seeks stabilization of i_t^d. Thin solid line: optimal monetary policy. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

5. As shown by the bold solid line in Figure 7, under a regime that seeks to stabilize the interest rate on deposits without adjusting the rate of growth of money, the interest rate on reserves would have to stay at the zero lower bound about twice as long as in the case in which the central bank was also adjusting its balance sheet, and substantially overshoot its long-run equilibrium level afterwards. Overall, however, the gains from this policy regime are limited to a faster convergence of the interest rate on deposits (and the credit spread) to steady state, while inflation, real activity, and consumption dispersion evolve in the same way as under the completely passive one.

Finally, Figure 4 shows another interesting feature of optimal policy, i.e., that the long-run price level remains above the initial trend. A policy of nominal GDP targeting, instead, implies long-run convergence of the price level to the initial path. In this respect, a policy that commits to raising the nominal GDP target permanently

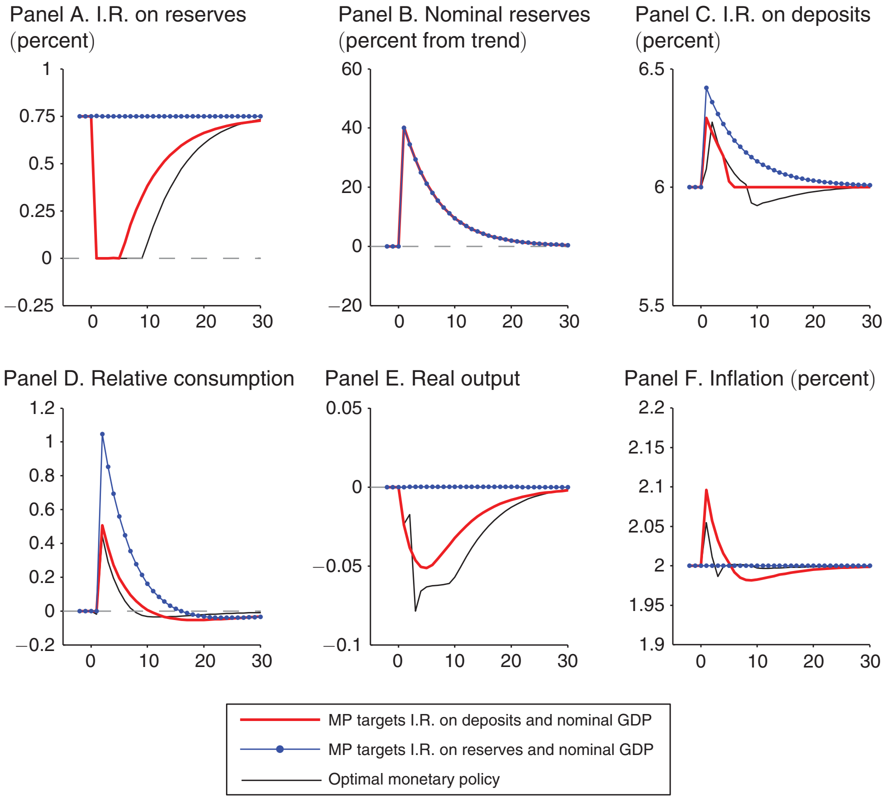

Figure 6. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets: Alternative Active Monetary Regimes versus Optimal Policy

Notes: Bold solid line: monetary policy implements nominal GDP targeting and seeks stabilization of i_t^d. Dotted line: same as before but now monetary policy keeps i_t^m constant. Solid line: optimal monetary policy. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

after the shock will perform better.[43] In the next section, we also investigate the nature of this long-run price divergence under optimal policy.

C. *Monetary Policy Tapering and Exit from the Zero Bound*

We now turn to analyze how the unconventional policy responses depend on the dynamic features of the liquidity shock. Figure 8 displays the responses of the interest rate on reserves, the detrended level of reserves, and the price level to an 11 percent negative liquidity shock with different dynamic features, displayed in panel D. In particular, the dash-dotted and solid lines show the cases of autoregressive processes with a half-life of one and four quarters, respectively; the dashed line shows the case of a Markov switching process for which, in every period, the liquidity properties go back to normal with probability $\xi = 10$ percent and stay at the lower

[43] We do not show this in the graph, but overall the improvements are marginal.

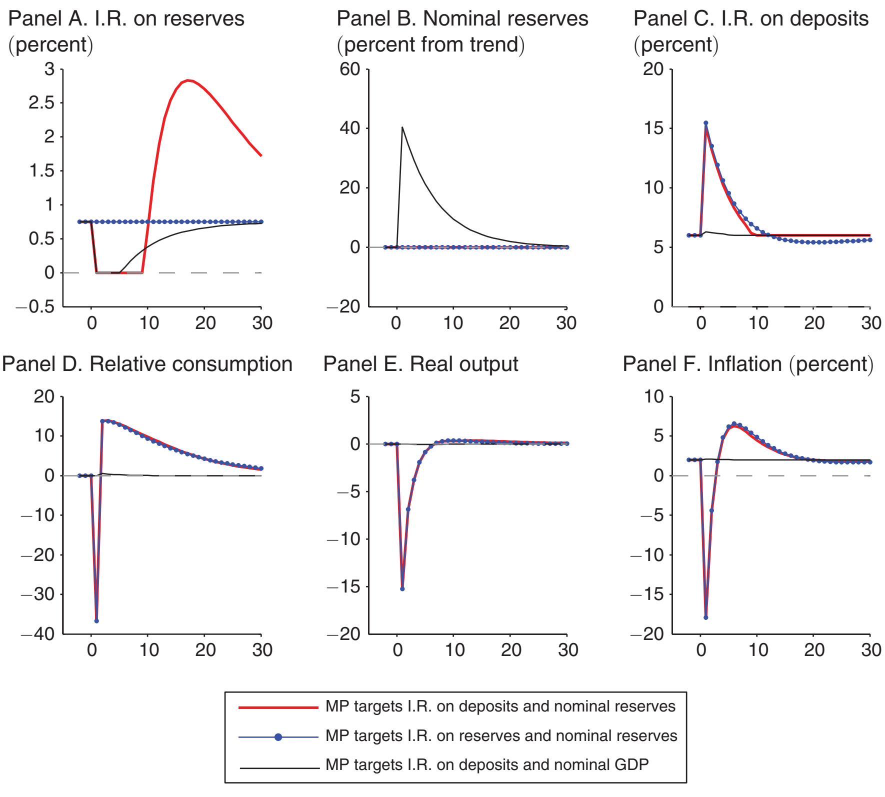

Figure 7. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets under Alternative Targeting Regimes

Notes: Bold solid line: monetary policy seeks stabilization of i_t^d but keeps money growth constant. Dotted line: monetary policy keeps both money growth and i_t^m constant. Solid line: monetary policy seeks stabilization of both i_t^d and nominal GDP. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

level with probability $1 - \xi$. In all cases, the liquidity properties fall initially by the same amount (-11 percent), but then return to normal conditions with different speed.

As Figure 8 shows, the expansion of the central bank's balance sheet is, on impact, the same across the three specifications, but the shape of the tapering is inherited from the properties of persistence of the shock. In the case of the Markov switching process, in particular, the monetary policymaker keeps nominal reserves on the new path and suddenly brings them back to the old one as soon as the liquidity properties are back to normal. Moreover, the more persistent the shock, the longer the stay at the zero lower bound. If the liquidity shock reverts back fast enough, the interest rate on deposits can be insulated from the shock without bringing the interest rate on reserves to zero. Analogously to Eggertsson and Woodford (2003), to enhance the effectiveness of the policy action during the shock, in the case of a Markov switching process the central bank is required to commit to keeping the interest rate on reserves at the zero bound longer than the duration of the shock, and to overshoot its long-run level afterwards.

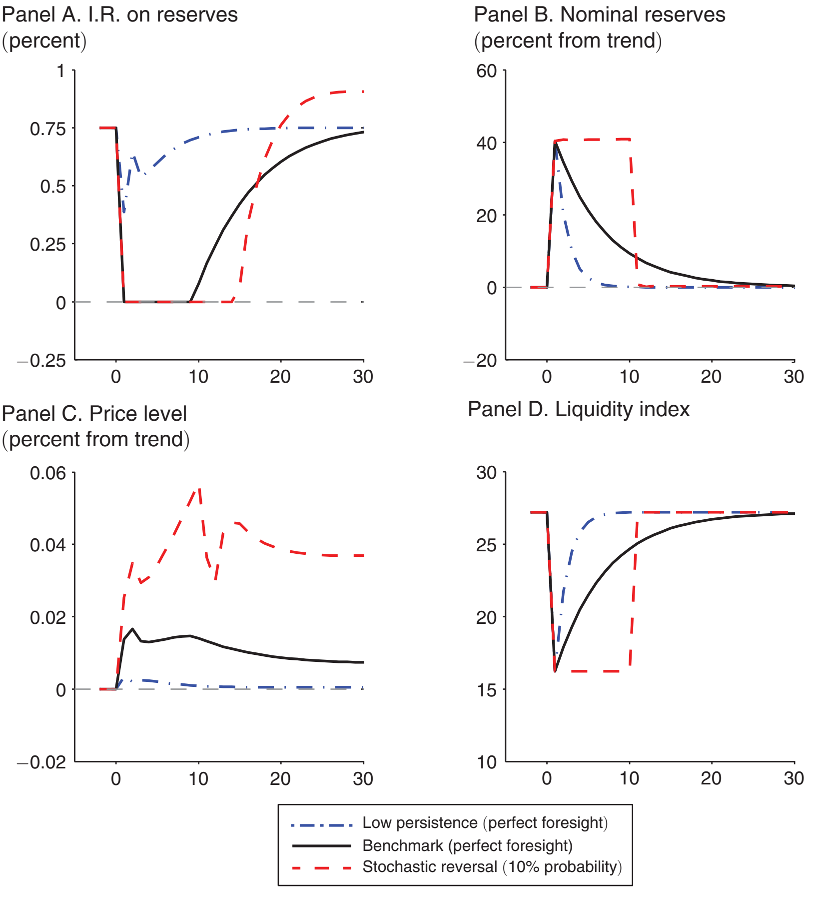

FIGURE 8. RESPONSES OF SELECTED VARIABLES TO NEGATIVE LIQUIDITY SHOCKS OF SAME SIZE BUT DIFFERENT PERSISTENCE, UNDER THE OPTIMAL MONETARY POLICY

Notes: Dash-dotted line: low persistence, half-life of one quarter ($\rho_\gamma = 0.5$). Solid line: benchmark case, half-life of about four quarters ($\rho_\gamma = 0.85$). Dashed line: 11 percent shock with 10 percent probability that in each period the shock returns back to steady state; the shock actually returns back after ten quarters.

Interestingly, the duration at the zero lower bound is linked to a commitment to keep the future price level permanently above the initial trend even after the shock has faded away.[44] Indeed, the longer the stay at the zero bound, the higher the long-run price-level path to which monetary policy should commit. This is in line with other models of optimal policy under the zero bound constraint, like Eggertsson and Woodford (2003).

D. *The Role of Price Rigidities*

Finally, Figure 9 studies whether the degree of price stickiness plays a role in driving the results. In particular, the figure displays the response of the same variables

[44] If nominal interest rates could turn negative, the price level would increase more, the more persistent the shock, but it would always converge back to the initial trend as the shock reverses to zero, regardless of its persistence.

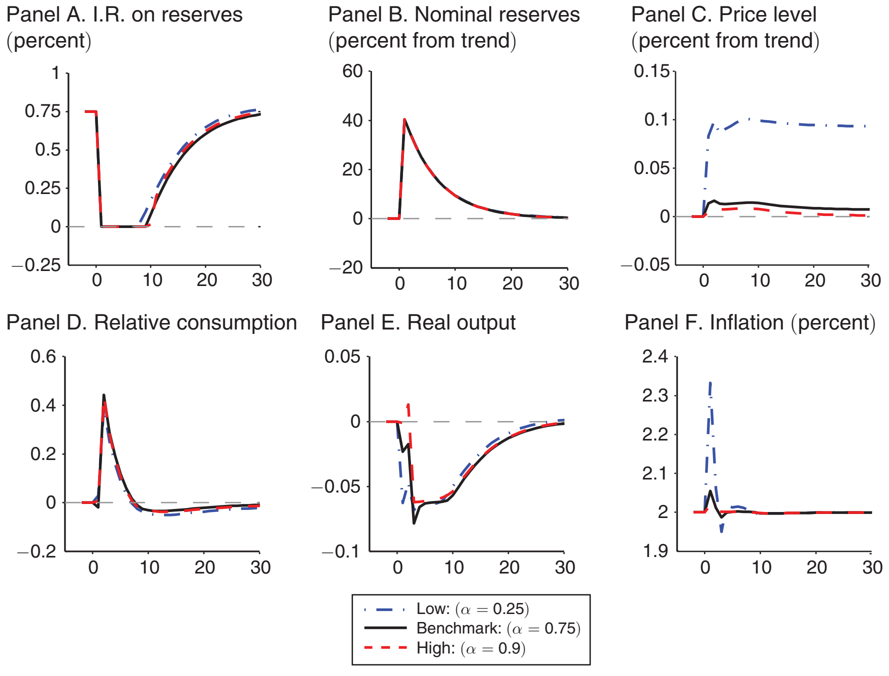

Figure 9. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets under the Optimal Monetary Policy: The Role of Price Stickiness

Notes: Dash-dotted line: prices are reset every 13 weeks. Solid line: prices are reset every four quarters. Dashed line: prices are reset every 10 quarters. The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

as in Figure 7 to an 11 percent deterioration of the liquidity properties of the pseudo-safe asset, for different degrees of price stickiness. Interestingly, Figure 9 shows that under optimal monetary policy, the economy is required to stay at the zero lower bound longer, the stickier the consumer prices. Differently from Figure 7, however, the stay at the zero lower bound in this case is inversely related to the increase in the price level path to which monetary policy has to commit in the distant future. On the one hand, indeed, more flexible consumer prices reduce the welfare costs of inflation, thereby allowing the central bank to focus more actively on the other stabilization objectives by committing strongly to increasing the long-run price level. On the other hand, the stronger commitment to an increase of the future price level requires a shorter stay at the zero lower bound, because more flexible consumer prices favor a quicker convergence to the new target path.

An additional important insight of Figure 9 is that the main policy implications of our model do not depend much on the degree of nominal rigidity. The size of the expansion in the balance sheet of the central bank, as well as the shape of the exit path, is independent from the degree of price stickiness. Monetary policy should offset the shortage of *nominal* safe assets by supplying more money in order to stabilize *nominal* GDP in the economy.

The need to lower the interest rate on reserves, hitting the zero lower bound at least on impact, is also independent from the degree of price stickiness. Indeed, even under high price flexibility (the dash-dotted line in Figure 9) a liquidity shock has substantial real redistributive effects between savers and borrowers, which require a policy intervention to counteract them.

E. *The Role of the Transfer Scheme*

Until now we have assumed that each agent receives transfers corresponding to its own holdings of money, net of a proportional share of the employment subsidy, as shown by equations (34)–(35). It is clear, however, that with heterogenous agents the distribution of transfers is not neutral for the equilibrium allocation.

Here we evaluate the implications of assuming an alternative transfer scheme, namely one in which each agent receives a proportional share of the overall increase in money supply:

$$T_t^s = T_t^b = M_t - (1 + i_{t-1}^m) M_{t-1} - \tau W_t L_t. \tag{52}$$

The first implication of this alternative transfer scheme is that the budget constraint of borrowers is no longer described by equation (43), but now reflects the wedge between their own holdings of money and the new transfers:

$$b_t = (1 + i_{t-1}^b) \frac{b_{t-1}}{\Pi_t} + C_t^b - Y_t + m_t^b - m_t - (1 + i_{t-1}^m) \frac{m_{t-1}^b - m_{t-1}}{\Pi_t}. \tag{53}$$

Figure 10 displays the response of selected variables to a liquidity shock under optimal monetary policy, and compares the two transfer schemes described by equations (34)–(35) on the one hand and (52) on the other. The main implication of Figure 10 is twofold.

On the one hand, it confirms that an interest rate cut to the zero lower bound and an increase in the size of the central bank's balance sheet are required to minimize the welfare costs of a liquidity deterioration of pseudo-safe assets. The balance sheet expansion is slightly larger than under idiosyncratic transfers, both on impact and along the transition. The stay at the zero lower bound, on the contrary, is considerably shorter than before, and the interest rate on reserves overshoots its steady state level immediately afterwards. Behind these dynamics there is a different response regarding the borrowers' money holdings (which now increase both on impact and along the transition) and a stronger effectiveness of the balance sheet channel in stabilizing market interest rate at short horizons. Indeed, a regime in which the central bank expands its balance sheet while leaving the interest rate on reserves constant would be very effective in stabilizing the interest rate on deposits, especially three quarters after the shock.[45] This leaves much less need for the interest rate on reserves to stay at the zero lower bound for longer than a couple of quarters.

[45] The complete set of simulations under all regimes is available from the authors upon request.

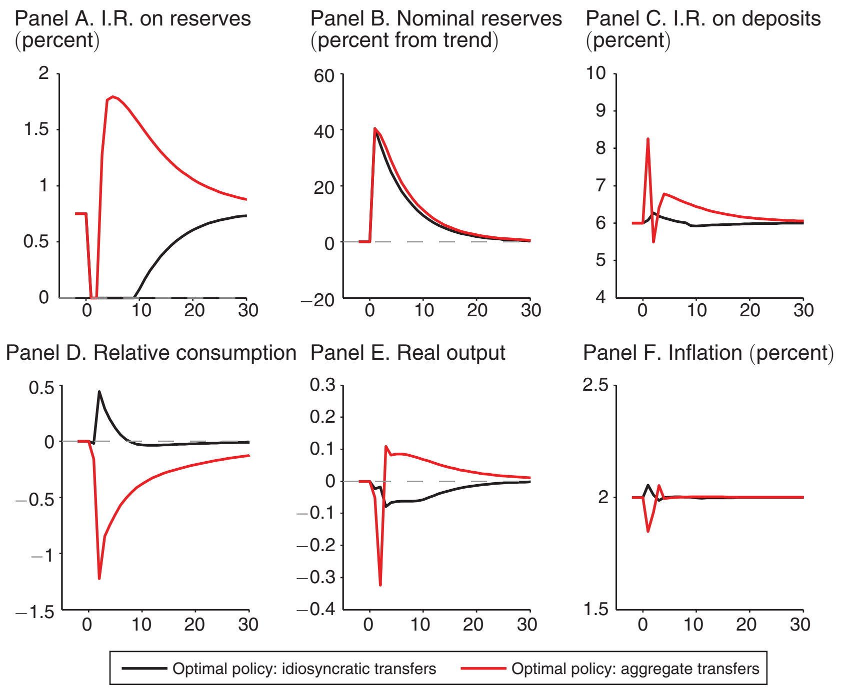

Figure 10. Responses of Selected Variables to an 11 Percent Fall in the Liquidity Properties of Pseudo-Safe Assets under Optimal Monetary Policy: The Role of Money Transfers

Notes: Thin solid line: benchmark specification with idiosyncratic transfers, as in equations (34)–(35). Bold solid line: alternative specification with aggregate transfers, as in equation (52). The half-life of the liquidity shock is about four quarters ($\rho_\gamma = 0.85$).

On the other hand, panels D, E, and F of Figure 10 show that the implied fluctuations of the welfare-relevant variables (hence the implied welfare loss) under the aggregate transfer scheme are substantially larger than those under the idiosyncratic scheme. The latter, indeed, allows the central bank to provide more safe assets specifically to those agents who need them the most, to substitute the pseudo-safe assets that have suddenly become illiquid.

V. Conclusions

We have presented monetary models in which the main novelty is that financial assets can have different liquidity properties. In this framework, we studied the effects on the economy of a change in these properties for some assets, which we labeled pseudo-safe or pseudo-liquid assets. The overall shortage of safe assets can produce significant effects on nominal spending, and thereby on aggregate prices and real activity, in a proportion that depends on the degree of nominal rigidities. A deep recession-cum-deflation can emerge for a reasonable parametrization. At the same time, in a model in which the pseudo-safe asset is a deposit security through which intermediaries finance their loans, a liquidity shock raises the funding costs of intermediaries, which is passed through into higher loan rates. This shock has important distributional effects between borrowers and savers, with borrowers adversely hit by the rise in the loan rates.[46]

The role of monetary policy is critical for the propagation of the shock. Two instruments can be used to minimize the welfare consequences of the shock both at the aggregate and distributional level. The monetary policymaker should offset the shortage of safe assets by issuing more liquidity in the form of money, which remains a safe asset in circulation in the model. This can be achieved by a policy of increasing the path of nominal reserves in the vein of quantitative easing, to stabilize the inflation rate around the target as well as nominal and real output. Moreover, the interest rate on reserves should be reduced in order to insulate the interest rate on the pseudo-safe assets and the interest rate spreads. This policy improves the risk sharing of the liquidity shock between savers and borrowers and avoids a consumption recession, in particular for borrowers. For large shocks, the zero lower bound becomes a constraint to this action.

Our work has contributed to an ongoing literature studying the cause and propagation of the financial crisis by analyzing liquidity shocks in monetary models which have been frequently used for policy analysis before the crisis. A small departure from the standard framework is sufficient to produce an interesting transmission mechanism of the shock and capture macroeconomic behavior and policy intervention consistent with what economies have experienced.

There are some limitations of our framework which can constitute ground for further work and analysis. We have abstracted from credit risk and credit events, which can be important channels of transmission mechanism of the recent crisis. However, the research objective of this work is to identify clearly a liquidity shock and liquidity risk as drivers of the macroeconomic adjustment, and to shed some light on the transmission mechanism of such shock. The sector of intermediaries is quite rudimentary and could be further elaborated to endogenize the creation of pseudo-safe assets. In relation to this point, the degree of acceptance of assets in exchange for goods is exogenous as in Lagos (2010), but the literature spurred from the latter work has been trying to endogenize it through differences in the information set on the quality of assets between borrowers and savers. This might be an important qualification to add to our analysis which could change some policy implications. In this vein, it could be interesting to model the exchange of assets for goods through a bargaining process instead of market equilibrium conditions. These are clearly important issues, which we leave for future research.

[46] In an extension of the model presented in Section I with endogenous production and nominal rigidities, it will still be optimal to lower the interest rate on reserves to offset any impact on consumption due to the change of the nominal interest rate on bonds.

Appendix A

In this Appendix we present the log linear approximation of the model of Section I which is used for the analysis of Section II. The first-order approximation is taken around a deterministic steady state where we can combine equations (11) and (12) to imply

$$\text{(A1)} \qquad \frac{\bar{\imath} - \bar{\imath}^m}{1 + \bar{\imath}} = (1 - \bar{\gamma})\left[1 - \frac{\beta(1 + \bar{\imath}^m)}{\bar{\Pi}}\right],$$

in which a bar denotes the steady state value. A first-order approximation of equations (11) and (12) around the above steady state delivers

$$\text{(A2)} \qquad \hat{\imath}_t - \hat{\imath}_t^m = -\vartheta_1 E_t \hat{\gamma}_{t+1} + \vartheta_2 E_t(\hat{r}_{t+1} + (\hat{\pi}_{t+2} - \bar{\pi}) - \hat{\imath}_{t+1}^m),$$

where we have defined $\hat{\imath}_t \equiv \ln(1 + i_t)/(1 + \bar{\imath})$, $\hat{\imath}_t^m \equiv \ln(1 + i_t^m)/(1 + \bar{\imath}^m)$, $\hat{\gamma}_{t+1} = (\gamma_{t+1} - \bar{\gamma})/(1 - \bar{\gamma})$, $\pi_t = \ln P_t/P_{t-1}$, and the coefficients ϑ_1 and ϑ_2 are $\vartheta_1 \equiv (\bar{\imath} - \bar{\imath}^m)/(1 + \bar{\imath}^m)$ and $\vartheta_2 \equiv 1 - \bar{\gamma}(1 + \bar{\imath})/(1 + \bar{\imath}^m)$. Notice that $\vartheta_1 \geq 0$ and in particular $\vartheta_1 = 0$ when the cash-in-advance constraint is not binding, while $0 \leq \vartheta_2 \leq 1$.[47]

In equation (A2), $\hat{r}_{t+1}$ captures the real interest rate that would apply in a model in which money and bonds are perfect substitutes and is defined as

$$\hat{r}_{t+1} = \rho E_{t+1}(\hat{Y}_{t+2} - \hat{Y}_{t+1}),$$

where $\rho \equiv -\bar{U}_{cc}\bar{Y}/\bar{U}_c$; $\hat{Y}_t \equiv \ln Y_t/\bar{Y}$. To complete the characterization of the equilibrium condition through a first-order approximation, we approximate the cash-in-advance constraint (10) to obtain

$$\text{(A3)} \qquad \hat{m}_t + s_b(\hat{b}_t + \vartheta_3 \hat{\gamma}_t) = 0,$$

where $\hat{m}_t$ represents the log deviations from the steady state of the ratio of money over nominal GDP defined as $m_t \equiv \tilde{M}_{t-1}^s/(P_t Y_t)$ where $\bar{m}$ is its steady state value; $\hat{b}_t$ is instead the log deviations from the steady state of the ratio of bonds over nominal GDP, defined as $b_t = \tilde{B}_{t-1}^s/(P_t Y_t)$ with steady state value $\bar{b}$, while $\vartheta_3 \equiv (1 - \bar{\gamma})/\bar{\gamma}$ and $s_b \equiv \bar{\gamma}\bar{b}/\bar{m}$. Moreover

$$\text{(A4)} \qquad \hat{m}_t = \hat{m}_{t-1} + \mu_{t-1}^m - \pi_t,$$

$$\text{(A5)} \qquad \hat{b}_t = \hat{b}_{t-1} + \mu_{t-1}^b - \pi_t,$$

where μ_t^m and μ_t^b are the rate of growth of money supply and bond supply from time $t - 1$ to time t. Given exogenous processes $\{\mu_t^b, \hat{\gamma}_t\}$, equations (A2), (A3), (A4),

[47] Indeed, equation (A1) implies $\bar{\gamma}(1 + \bar{\imath})/(1 + \bar{\imath}^m) = \bar{\gamma}/[\bar{\gamma} + (1 - \bar{\gamma})\bar{\Pi}^{-1}\beta(1 + \bar{\imath}^m)] \in [0, 1]$, which in turn implies $\vartheta_2 \in [0, 1]$.

and (A5) determine the path of $\left\{\hat{m}_t, \mu_t^m, \pi_t, \hat{b}_t, \hat{\imath}_t, \hat{\imath}_t^m\right\}$. Accordingly, monetary policy should specify the path of two instruments of policy.

We describe now some analytical results which are used to produce Figures 1 and 2.

Under the regime in which the monetary policymaker is passive and keeps the interest rate on reserves and the growth of money constant at the rate followed before the shock hits, we have that the growth of money is given by

$$\bar{\mu}^m = (1 + s_b)\bar{\pi} - s_b\bar{\mu}^b, \tag{A6}$$

while the inflation rates vary with the liquidity shock:

$$\pi_t = \bar{\pi} + \frac{s_b}{1 + s_b}\vartheta_3 \Delta\hat{\gamma}_t, \tag{A7}$$

and the path of the interest rate on bonds follows

$$\hat{\imath}_t = -\vartheta_1 E_t\hat{\gamma}_{t+1} + \vartheta_2\vartheta_3\frac{s_b}{1 + s_b}E_t\Delta\hat{\gamma}_{t+2}.$$

When the policymaker sets inflation rate at the steady state level of 2 percent at annual rate, as shown with the solid line in Figure 1, the path of money growth follows

$$\mu_{t-1}^m = (1 + s_b)\bar{\pi} - s_b(\bar{\mu}^b + \vartheta_3\Delta\hat{\gamma}_t). \tag{A8}$$

To keep inflation on target, the growth rate of money supply rises momentarily when the liquidity properties of bonds deteriorate, and falls to return to the previous path when liquidity conditions improve. A negative liquidity shock raises the interest rate on bonds which, under inflation targeting, follows

$$\hat{\imath}_t = -\vartheta_1 E_t\hat{\gamma}_{t+1}.$$

When, instead, the policymaker insulates the interest rate on bonds from the shock, the interest rate on money follows

$$\hat{\imath}_t^m = \max\left(-\ln(1 + \bar{\imath}^m), \vartheta_1\sum_{s=0}^{\infty}\vartheta_2^s E_t\hat{\gamma}_{t+1+s}\right),$$

which for the large shock discussed in the text can hit the zero lower bound.

Appendix B

We solve the model of Section III by taking a first-order approximation around the initial steady state. The Euler equations of the savers imply

$$\hat{\imath}_t^d - \hat{\imath}_t^m = -\vartheta_1^s E_t\hat{\gamma}_{t+1} + \vartheta_2^s E_t(\hat{r}_{t+1}^s + (\pi_{t+2} - \bar{\pi}) - \hat{\imath}_{t+1}^m), \tag{B1}$$

where we introduce the following additional notation with respect to previous sections: $\hat{\imath}_t^d \equiv \ln(1+i_t^d)/(1+\bar{\imath}^d)$, and the coefficients ϑ_1^s and ϑ_2^s are defined as $\vartheta_1^s \equiv (\bar{\imath}^d - \bar{\imath}^m)/(1+\bar{\imath}^m)$ and $\vartheta_2^s \equiv 1 - \bar{\gamma}(1+\bar{\imath}^d)/(1+\bar{\imath}^m)$. The Euler equation of the borrowers read in a first-order approximation as

$$\hat{\imath}_t^b - \hat{\imath}_t^m = E_t(\hat{r}_{t+1}^b + (\pi_{t+2} - \bar{\pi}) - \hat{\imath}_{t+1}^m). \tag{B2}$$

In both equations,

$$\hat{r}_{t+1}^j = \rho E_{t+1}(\hat{C}_{t+2}^j - \hat{C}_{t+1}^j)$$

for each $j = b, s$, where $\rho \equiv \upsilon \bar{Y}$, while $\bar{Y}$ is the steady state output, and we use the following definitions $\hat{C}_t^j \equiv (C_t^j - \bar{C}^j)/\bar{Y}$ for each $j = b, s$.

Appropriately, goods market equilibrium (27) implies in a first-order approximation that

$$\hat{Y}_t = \chi \hat{C}_t^s + (1-\chi)\hat{C}_t^b, \tag{B3}$$

where now $\hat{Y}_t = (Y_t - \bar{Y})/\bar{Y}$.

Finally in a first-order approximation the spread schedule (26) implies

$$\hat{\imath}_t^b = \hat{\imath}_t^d + \phi \hat{b}_t \tag{B4}$$

for some parameter ϕ, where $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$.

A first-order approximation of the flow budget constraint of the borrowers (43) implies that

$$\beta \hat{b}_t = \hat{b}_{t-1} + \tilde{b} \cdot \hat{\imath}_{t-1}^b - \tilde{b}(\pi_t - \bar{\pi}) + \beta \hat{C}_t^b - \beta \hat{Y}_t, \tag{B5}$$

where $\tilde{b} \equiv \bar{b}/\bar{Y}$.

Euler equations (B1) and (B2) together with (B3), (B4), and (B5) constitute the aggregate demand block of the model.

In a log linear approximation, the supply block comes from approximating (30), (31) taking into account the definitions of φ_t^j for $j = b, s$. The following modified New-Keynesian Phillips curve is obtained:

$$\pi_t - \bar{\pi} = \kappa(\eta + \rho)\hat{Y}_t + \kappa[\hat{r}_t + E_t(\pi_{t+1} - \bar{\pi}) - \hat{\imath}_t^m] + \beta E_t(\pi_{t+1} - \bar{\pi}), \tag{B6}$$

where we have defined $\kappa \equiv (1-\alpha)(1-\alpha\beta)/\alpha$ and now

$$\hat{r}_t = \rho E_t(\hat{Y}_{t+1} - \hat{Y}_t).$$

The New-Keynesian Phillips curve is augmented by a term reflecting the variations in the monetary frictions at the aggregate level.

Finally, using the definition $\hat{m}_t^j \equiv (m_t^j - \bar{m}^j)/\bar{m}$ for each $j = b, s$, we take a first-order approximation of the equilibrium conditions for the money market and obtain:

$$\text{(B7)} \quad \frac{1}{\tilde{m}^s}\hat{m}_{t-1}^s + \hat{\imath}_{t-1}^m + \vartheta_3\left(\hat{b}_{t-1} + \tilde{b}\cdot\hat{\imath}_{t-1}^d + \vartheta_4\hat{\gamma}_t\right) = \frac{1+\vartheta_3\tilde{b}}{\tilde{c}^s}\left(\hat{C}_t^s + \tilde{c}^s(\pi_t - \bar{\pi})\right),$$

$$\text{(B8)} \quad \frac{1}{\tilde{m}^b}\hat{m}_{t-1}^b + \hat{\imath}_{t-1}^m = \frac{1}{\tilde{c}^b}\hat{C}_t^b + (\pi_t - \bar{\pi}),$$

where we defined $\tilde{m}^j \equiv \bar{m}^j/\bar{m}$, $\vartheta_3 \equiv \bar{\gamma}\frac{1-\chi}{\chi}\frac{1+\bar{\imath}^d}{1+\bar{\imath}^m}(\tilde{m}\cdot\tilde{m}^s)^{-1}$, $\tilde{m} \equiv \bar{m}/\bar{Y}$, $\vartheta_4 \equiv \tilde{b}(1-\bar{\gamma})/\bar{\gamma}$, and $\tilde{c}^j = \bar{C}^j/\bar{Y}$ for each $j = s, b$. Real money balances follow

$$\text{(B9)} \quad \hat{m}_t \equiv \chi\hat{m}_t^s + (1-\chi)\hat{m}_t^b,$$

$$\text{(B10)} \quad \hat{m}_t = \hat{m}_{t-1} + \mu_t - \pi_t,$$

and μ_t is the nominal money supply growth.

Equations (B1), (B2), (B3), (B4), (B5) together with (B6), (B7), (B8), (B9), (B10), and the definitions of $\hat{r}_{t+1}^s$, $\hat{r}_{t+1}^b$, and $\hat{r}_{t+1}$ determine the equilibrium allocation for $\pi_t, \hat{C}_t^b, \hat{C}_t^s, \hat{Y}_t, \hat{\imath}_t^b, \hat{\imath}_t^d, \hat{\imath}_t^m$, $\hat{b}_t$, $\hat{m}_t^s$, $\hat{m}_t^b$, $\hat{m}_t$, μ_t, where two policy instruments should be specified.

Appendix C

In this Appendix, we show the derivations of the second-order approximation of the welfare (44). The approximation is taken with respect to an efficient steady state. This efficient steady state maximizes (44) under the resource constraint (27) considering that $L = (L^s)^\chi(L^b)^{1-\chi}$.

At the efficient steady state the following conditions hold:

$$\tilde{\chi}\,\bar{U}_c^s = \chi\bar{\lambda};$$

$$(1-\tilde{\chi})\,\bar{U}_c^b = (1-\chi)\bar{\lambda};$$

$$\tilde{\chi}\,\bar{V}_l^s = \chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s};$$

$$(1-\tilde{\chi})\,\bar{V}_l^b = (1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b},$$

where all upper bars denote steady state values and $\bar{\lambda}$ is the steady state value of the Lagrange multiplier associated with the constraint (27). Note that the above conditions imply $\bar{U}_c^s/\bar{U}_c^b = \chi(1-\tilde{\chi})/[(1-\chi)\tilde{\chi}]$ so that an appropriately chosen $\tilde{\chi}$ determines the efficient distribution of wealth in a consistent way with the steady state debt position of the borrowers in the model, given by $\bar{b}$. For the above efficient steady state to be consistent with the steady state of the model we need to offset the distortions of the model appropriately. Note that at the efficient steady state

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j}$$

for each $j = b, s$. On the other side, the steady state of the model, when inflation is at the target level, implies

$$\frac{\bar{V}_l^j}{\bar{U}_c^j} = \frac{\bar{Y}}{\bar{L}^j}\frac{\bar{W}}{\bar{P}}(1-\bar{\varphi})$$

for each $j = b, s$ and

$$\frac{\bar{W}}{\bar{P}} = \frac{1}{(1-\tau)(1+\bar{\mu})},$$

where $\bar{\mu} \equiv 1/(\theta - 1)$, while

$$\bar{\varphi} = 1 - \frac{\beta(1+\bar{\imath}^m)}{\bar{\Pi}}.$$

It is clear from the above equations that we just need to set the employment subsidy at the level

$$\tau = \frac{\bar{\mu}+\bar{\varphi}}{1+\bar{\mu}}$$

in order to make the steady state of the decentralized allocation efficient.

Having defined the efficient steady state, we take a second-order expansion of the utility flow around it to obtain

$$\begin{aligned} U_t = {} & \bar{U} + \tilde{\chi}\left[\bar{U}_c^s(C_t^s - \bar{C}^s) + \frac{1}{2}\bar{U}_{cc}^s(C_t^s - \bar{C}^s)^2\right] \\ & + (1-\tilde{\chi})\left[\bar{U}_c^b(C_t^b - \bar{C}^b) + \frac{1}{2}\bar{U}_{cc}^b(C_t^b - \bar{C}^b)^2\right] \\ & - \tilde{\chi}\left[\bar{V}_l^s(L_t^s - \bar{L}^s) + \frac{1}{2}\bar{V}_{ll}^s(L_t^s - \bar{L}^s)^2\right] \\ & - (1-\tilde{\chi})\left[\bar{V}_l^b(L_t^b - \bar{L}^b) + \frac{1}{2}\bar{V}_{ll}^b(L_t^b - \bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3), \end{aligned}$$

where an upper bar variable denotes the efficient steady state while $\mathcal{O}(||\xi||^3)$ collects terms in the expansion which are of an order higher than the second. We can use the steady state conditions to write the above equation as

$$\begin{aligned} U_t \;=\; & \bar{U} + \chi\bar{\lambda}\left[(C_t^s - \bar{C}^s) + \frac{1}{2}\frac{\bar{U}_{cc}^s}{\bar{U}_c^s}(C_t^s - \bar{C}^s)^2\right] \\ & + (1-\chi)\bar{\lambda}\left[(C_t^b - \bar{C}^b) + \frac{1}{2}\frac{\bar{U}_{cc}^b}{\bar{U}_c^b}(C_t^b - \bar{C}^b)^2\right] \\ & - \chi\bar{\lambda}\frac{\bar{Y}}{\bar{L}^s}\left[(L_t^s - \bar{L}^s) + \frac{1}{2}\frac{\bar{V}_{ll}^s}{\bar{V}_l^s}(L_t^s - \bar{L}^s)^2\right] \\ & - (1-\chi)\bar{\lambda}\frac{\bar{Y}}{\bar{L}^b}\left[(L_t^b - \bar{L}^b) + \frac{1}{2}\frac{\bar{V}_{ll}^b}{\bar{V}_l^b}(L_t^b - \bar{L}^b)^2\right] + \mathcal{O}(||\xi||^3). \end{aligned}$$

Note that for a generic variable X, we have

$$X_t \;=\; \bar{X}\left(1 + \hat{X}_t + \frac{1}{2}\hat{X}_t^2\right) + \mathcal{O}(||\xi||^3),$$

where $\hat{X}_t \;\equiv\; \ln X_t/\bar{X}$ and moreover recall that

$$Y_t \;=\; \chi\, C_t^s + (1-\chi)\, C_t^b,$$

implying that

$$\chi(C_t^s - \bar{C}^s) + (1-\chi)(C_t^b - \bar{C}^b) \;=\; \bar{Y}\left[\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right] + \mathcal{O}(||\xi||^3).$$

We can write the above approximation as

$$\text{(C1)} \quad \begin{aligned} U_t = {} & \bar{U} + \bar{\lambda}\bar{Y}\left[\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right] - \frac{1}{2}\bar{\lambda}\upsilon\left[\chi(C_t^s - \bar{C}^s)^2 + (1-\chi)(C_t^b - \bar{C}^b)^2\right] \\ & -\chi\bar{\lambda}\bar{Y}\left[\hat{L}_t^s + \frac{1}{2}(1+\eta)\,\hat{L}_t^s)^2\right] - (1-\chi)\bar{\lambda}\bar{Y}\left[\hat{L}_t^b + \frac{1}{2}(1+\eta)\,(\hat{L}_t^b)^2\right] + \mathcal{O}(||\xi||^3), \end{aligned}$$

where we have also used the fact that with the preference specification used $\bar{U}_{cc}^s/\bar{U}_c^s \;=\; \bar{U}_{cc}^b/\bar{U}_c^b \;=\; -\upsilon$ and $\bar{V}_{ll}^s\bar{L}^s/\bar{V}_l^s \;=\; \bar{V}_{ll}^b\bar{L}^b/\bar{V}_l^b \;=\; \eta$.

Note that in equilibrium $L_t \;=\; \Delta_t Y_t$ where $L_t \;=\; (L^s)^{\chi}(L^b)^{1-\chi}$. It follows that the following condition holds exactly:

$$\hat{Y}_t \;=\; \chi\,\hat{L}_t^s + (1-\chi)\,\hat{L}_t^b + \hat{\Delta}_t.$$

Using the above equation in (C1), the latter can be simplified to

$$\text{(C2)}\qquad \frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = \frac{1}{2}\hat{Y}_t^2 - \frac{1}{2}\rho\left[\chi\left(\hat{C}_t^s\right)^2 + (1-\chi)\left(\hat{C}_t^b\right)^2\right]$$
$$- \frac{1}{2}(1+\eta)\left[\chi\left(\hat{L}_t^s\right)^2 + (1-\chi)\left(\hat{L}_t^b\right)^2\right] - \hat{\Delta}_t + \mathcal{O}(||\xi||^3),$$

where $\rho \equiv \upsilon\bar{Y}$ and we have used the definitions of $\hat{C}_t^s$ and $\hat{C}_t^b$. Note that to a first-order approximation:

$$\hat{C}_t^s = \hat{Y}_t - (1-\chi)(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(||\xi||^2),$$
$$\hat{C}_t^b = \hat{Y}_t + \chi(\hat{C}_t^b - \hat{C}_t^s) + \mathcal{O}(||\xi||^2),$$
$$\hat{L}_t^s = \hat{Y}_t - (1-\chi)(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(||\xi||^2),$$
$$\hat{L}_t^b = \hat{Y}_t + \chi(\hat{L}_t^b - \hat{L}_t^s) + \mathcal{O}(||\xi||^2),$$

which can be used to simplify (C2) to

$$\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = -\frac{1}{2}(\rho+\eta)\hat{Y}_t^2 - \frac{1}{2}\chi(1-\chi)\rho\left(\hat{C}_t^s - \hat{C}_t^b\right)^2$$
$$- \frac{1}{2}\chi(1-\chi)(1+\eta)\left(\hat{L}_t^s - \hat{L}_t^b\right)^2 - \hat{\Delta}_t + \mathcal{O}(||\xi||^3).$$

Note that

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}.$$

By taking a second-order approximation of $\hat{\Delta}_t$, as it is standard in the literature and integrating appropriately across time, we obtain that

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{(\pi_t - \bar{\pi})^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).$$

We can then obtain a second-order approximation of the utility of the consumers as

$$W_t = -\bar{\lambda}(\eta+\rho)\bar{Y}\cdot\frac{1}{2}E_t\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}Loss_t\right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

where

$$Loss_t = \hat{Y}_t^2 + \chi(1-\chi)\lambda_c\left(\hat{C}_t^s - \hat{C}_t^b\right)^2 + \chi(1-\chi)\lambda_l\left(\hat{L}_t^s - \hat{L}_t^b\right)^2 + \lambda_\pi(\pi_t - \bar{\pi})^2,$$

where we have defined

$$\lambda_c \equiv \frac{\rho}{\rho+\eta},$$

$$\lambda_l \equiv \frac{1+\eta}{\rho+\eta},$$

$$\lambda_\pi \equiv \frac{\theta}{\kappa(\rho+\eta)}.$$

Note finally that

$$\frac{(L_t^s)^{1+\eta}}{\upsilon \exp(-\upsilon C_t^s)} = \frac{W_t}{P_t}\Delta_t Y_t(1-\varphi_t^s),$$

$$\frac{(L_t^b)^{1+\eta}}{\upsilon \exp(-\upsilon C_t^b)} = \frac{W_t}{P_t}\Delta_t Y_t(1-\varphi_t^b),$$

which implies in a log linear approximation that

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta}(\hat{C}_t^s - \hat{C}_t^b) - \frac{\overline{\varphi}}{(1-\overline{\varphi})(1+\eta)}(\hat{\varphi}_t^s - \hat{\varphi}_t^b).$$

Moreover, from log linear approximations of (23), we get

$$\hat{\varphi}_t^j = \frac{1-\overline{\varphi}}{\overline{\varphi}} E_t\big[(\pi_{t+1}-\overline{\pi}) - \hat{\imath}_t^m + \rho\Delta\hat{C}_{t+1}^j\big]$$

for $j = b, s$ and therefore

$$\hat{L}_t^s - \hat{L}_t^b = -\frac{\rho}{1+\eta}E_t(\hat{C}_{t+1}^s - \hat{C}_{t+1}^b),$$

which can also be used in the loss function to replace the term $\hat{L}_t^s - \hat{L}_t^b$.

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