

Italian Public Debt Valuation

Student: Alfonso Calvanese

Advisor: Prof. Facundo Piguillem

Einaudi Institute for Economics and Finance (EIEF) & LUISS

September 14, 2025

Abstract

This thesis investigates the valuation of public debt in Italy by adapting the asset pricing framework of Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2024a to a different macroeconomic and institutional setting. I construct a novel dataset on the market value of Italian marketable debt, combining information on bond prices, maturities, and cash flows to obtain consistent estimates over time. Using this dataset, the analysis examines whether the market value of government debt aligns with the present discounted value of future fiscal surpluses, as implied by a no-arbitrage condition. The empirical results provide a descriptive assessment of debt valuation and misalignment between prices and fundamentals.

I would like to thank my advisor, Professor Facundo Piguillem, for his availability and invaluable guidance throughout this journey. I am also sincerely thankful to the entire EIEF faculty for having been a constant source of inspiration and for having fostered my passion and curiosity during these two years.

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1 Introduction

Public finance theory implies that the market value of sovereign debt should equal the risk-adjusted present discounted value of future primary surpluses (from the government's budget constraint). However, recent research finds persistent and large deviations from this parity. In the United States Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2024a document a "public debt valuation puzzle", a growing gap between the market value of U.S. government debt and the present value of projected surpluses, the debt's fundamental value. Other work shows that dominant safe-asset issuers (e.g. the U.S. and U.K.) historically ran debt beyond what their future surpluses alone could justify Chen, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2023.

This thesis examines sovereign debt valuation in the context of Italy. Its debt-to-GDP ratio is among the highest in advanced economies, and its fiscal outlook under the euro regime poses challenges for proper valuation. Italy cannot unilaterally devalue its debt; moreover, it does not enjoy the same "exorbitant privilege" of dominant safe-asset status Chen, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2023. Thus a key question arises: **Does the market value of Italian government debt align with the present value of its expected future primary surpluses?** Is there evidence of a debt valuation gap in Italy similar to that documented for the U.S.?

To address this question, I build on recent advances in sovereign debt valuation. In particular, Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2024a, who use a vector autoregression to forecast fiscal variables and compute a risk-adjusted present discounted value (PDV) of surpluses. This study adapts the above methods to Italy: I estimate a similar VAR model incorporating past primary surplus components, GDP growth, and other macro-fiscal controls, then project Italy's future primary surpluses. I finally compute a model-free upper bound on the PDV of surpluses and compare this fundamental value to the current market value of Italian debt outstanding.

This thesis contributes new evidence on European debt. It complements the work on U.S. fiscal capacity by extending the analysis to a large euro-area economy, it also provides a benchmark for comparing Italy's debt risk to that of other countries. Debates persist over whether Italian government bonds are overpriced due to insufficient fiscal backing. IMF (2024) emphasize that sovereign risk in Italy remains high over the medium to long term and recommend for a faster-than-planned fiscal adjustment to strengthen the credibility of its debt commitments. Similarly, Canelli, Fontana, Realfonzo, and Passarella (2022) argue

that, particularly in light of recent economic shocks, long-run debt sustainability will likely require substantial future adjustments at either the national or European level.

The remainder of the thesis proceeds as follows: the next section describes the data and VAR forecasting methodology; Section 3 reconstruct the market value of debt, Section 4 present the PDV calculations and compare them to market debt; and Section 5 concludes.

2 Data

The purpose of this thesis will be to reconstruct separately the two sides of the valuation equation (2), namely the market value of government debt and the expected present discounted value of future surpluses, separately and then to compare them to check possible discrepancies. In this chapter I will first explain the data collection process and the construction of the dataset for the market value of debt, then I will move to the reconstruction of the present discounted value of the surpluses and the application of the methodology used in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2024a, (JLVNX) from now on.

Data on debt comes from two different sources, one is Pecchi, Valente, and Fiorito 2002 which reconstruct the market value of debt from 1970 to 1996, and after that information on the bonds at the ISIN code level comes from the Bank of Italy online database¹ which gives me the most relevant characteristics of every bond issued from 1990 to today, including quantity issued, coupon rate, date of issuance and date of maturity. Prices are obtained by looking at the last day of the year for which *Il Sole 24 Ore* online archive had them available (between 29th and 31st of December) and matched by ISIN code. A pool of 60-70 bonds is available on average in the newspaper so at this stage some bonds are still unpriced. Using these information, I computed the market value of debt from 1997-2024, more on this in LHS Reconstruction, the Market Value of Debt.

Fiscal data on tax revenues and government spending net of interest expenditures are taken from Golinelli and Monterastelli 1990 for the period 1975-1995 and then from ISTAT² annual data on Public Administration and National accounts. Other macrodata are obtained from the OCPI³ historical series, which pools data from ISTAT and Bank of Italy to give a simple dataset of macro time series, among these I pick Inflation, Nominal GDP and Real

¹Tables TDEE0120 to TDEE0123 in <https://infostat.bancaditalia.it/inquiry/home?spyglass/taxo:CUBESET=&ITEMSELEZ=&OPEN=/&ep:LC=EN&COMM=BANKITALIA&ENV=LIVE&CTX=DIFF&IDX=1&/view:CUBEIDS=&graphMode=>

²http://dati.istat.it/Index.aspx?DataSetCode=DCCN_FPA

³<https://osservatoriocpi.unicatt.it/ocpi-servizi-serie-storiche>

GDP Growth. Nominal yields computed at maturity have been reconstructed by Piselli and Verricelli 2023 from which I use the annual series RLPUBBT and TATIB12M, respectively for the BTP yields and the 1-year BOT yields. To proxy the price-dividend ratio of the Italian market I use $\frac{\text{Stock Market Capitalization}}{\text{Nominal GDP}}$, obtained from FRED⁴ (1975-1989) and the Bank of Italy⁵ (1990-2024).

The GDP risk premium is set to 4.35%, it is obtained by unlevering the risk premium on the stock market. According to Damodaran 2023, Italy has an equity risk premium of 7.26%, Corsetti, Del Gatto, and Salleo 2016 measures Italian firms' leverage from 2000 to 2015, as total debt over debt plus equity, which averages at 40%. Therefore, I obtain the output risk premium = equity risk premium $\cdot$ (1 – leverage).

3 Theoretical Framework

In this section, the key points of the methodology used in (JLVNX) are reported. In the absence of bubbles, the market value of outstanding government debt is pinned down by the government budget constraint, and should be equal to the present discounted value of primary surpluses.

$$G_t - T_t + Q_{t-1}^{\$}(1) = \sum_{h=1}^H (Q_t^{\$}(h) - Q_{t-1}^{\$}(h+1)) P_t^{\$}(h) \quad (1)$$

Equation (1) is the Budget Constraint where G_t is government spending at time t , T_t are tax revenues, $Q_t^{\$}(h)$ is the face (nominal) value of a bond that was issued at time t and has maturity h and $P_t^{\$}(h)$ its market price. Therefore the left-hand side represents the new financing needs of the government at time t , and the right-hand side the new money raised from debt issuance. To see that, consider that a bond issued at time $t - 1$ with maturity $h + 1$ is the same as a bond issued at time t with maturity h . Therefore the difference $Q_t^{\$}(h) - Q_{t-1}^{\$}(h+1)$ is the face value of bonds issued at time t . Following (JLVNX), I will now show two theoretical results that build on three assumptions (two of which are weak ones on the government cash flows):

1. Absence of arbitrage opportunities.

⁴<https://fred.stlouisfed.org/series/DDDM01ITA156NWDB>

⁵https://infostat.bancaditalia.it/inquiry/home?spyglass/taxo:CUBESET=&ITEMSELEZ=&OPEN=false&ep:LC=IT&COMM=BANKITALIA&ENV=LIVE&CTX=DIFF&IDX=1&/view:CUBEIDS=MFN_VALM.M.52000200.100096.101.MKV.SBI136.SO.WRDBI2.EUR

2. Tax revenues and spending are cointegrated with GDP.
3. Spending is counter-cyclical and revenues are pro-cyclical.

Under those and imposing a transversality condition on the market value of debt,

Proposition 1 is derived:

$$\sum_{h=1}^H P_t^{\$}(h) Q_t^{\$}(h) = \mathbb{E}_t \left[\sum_{j=1}^{\infty} M_{t,t+j}^{\$} (T_{t+j} - G_{t+j}) \right] \quad (2)$$

Equation (2) is the **Valuation Equation**, and is the core equation of this analysis, it links the market value of government debt to the expected present discounted value of future surpluses. The proof derived in (JLVNX) can be found in Appendix, Proposition 1. The no-arbitrage condition guarantees the existence of a Stochastic Discount Factor $M_{t,t+j}^{\$}$ for all t and j . Importantly, all possible aggregate risks, including default, are reflected in the bond prices, the valuation equation holds also in the case of positive default probability.

Tax revenues and spending have different cyclical properties, as highlighted by Marcellino 1995, in the short run spending compensate for reductions of the GDP, being counter-cyclical, and tax revenues follow the same path of GDP, being pro-cyclical.

Nonetheless, in the long run, since spending and revenues are cointegrated with GDP, they also have the same growth path of GDP, with spending becoming "more pro-cyclical" than revenues as a fraction of GDP. According to Marcellino 1995 this can be due to a generalization of the Wagner law, according to which when GDP grows, there is a more than proportional growth in government spending demand, reflecting higher needs of the citizens.

Because of these different behaviors, the discount rates associated with a tax and a revenue claim are expected to be different (as revenues are riskier than spending). Let's define the holding period returns:

$$R_{t+1}^D = \frac{\sum_{h=1}^H P_{t+1}^{\$}(h-1) Q_t^{\$}(h)}{\sum_{h=1}^H P_t^{\$}(h) Q_t^{\$}(h)}, \quad R_{t+1}^T = \frac{P_{t+1}^T + T_{t+1}}{P_t^T}, \quad R_{t+1}^G = \frac{P_{t+1}^G + G_{t+1}}{P_t^G}.$$

where $P_t^T = \mathbb{E}_t \left[\sum_{j=1}^{\infty} M_{t,t+j}^{\$} T_{t+j} \right]$ and $P_t^G = \mathbb{E}_t \left[\sum_{j=1}^{\infty} M_{t,t+j}^{\$} G_{t+j} \right]$. From Proposition 1, $D_t = P_t^T - P_t^G$, and under the same assumptions we get to

Proposition 2:

$$\mathbb{E}_t [R_{t+1}^D] = \frac{P_t^T}{D_t} \mathbb{E}_t [R_{t+1}^T] - \frac{P_t^G}{D_t} \mathbb{E}_t [R_{t+1}^G] \quad (3)$$

(Proof in (JLVNX) and Appendix, Proposition 2). This equivalence of the risk premia tells us that the average discount rate applied to government debt must equal the average discount rate applied to government assets — that is, to the stream of future primary surpluses. On top of that, because primary surpluses are simply the difference between revenues and spending, the discount rate on debt is going to be the difference between the rates of revenues and spending, properly weighted.

Now I can subtract the risk-free rate from both sides of this identity, and express the relationship in terms of expected excess returns, or risk premia. Given the cyclical properties I talked about before, we can conclude that a claim on tax revenues is riskier than a claim on government spending, therefore it earns a higher risk premium:

$$\mathbb{E}_t \left[R_{t+1}^D - R_t^f \right] > \mathbb{E}_t \left[R_{t+1}^T - R_t^f \right] > \mathbb{E}_t \left[R_{t+1}^G - R_t^f \right]$$

This implies that government debt carries a positive risk premium, and therefore cannot be priced using the risk-free rate. In the following chapters, debt will not be treated as risk-free, but instead its risk properties will be derived from the observed surplus process.

3.1 Discussion on the assumptions

The robustness of this thesis' findings are contingent on the validity of the key assumptions underpinning the theoretical framework, adapted from (JLVNX) .

Absence of Arbitrage is the first main assumption and it is also a foundational assumption in modern finance, ensuring the existence of a stochastic discount factor and thus providing the theoretical basis for the asset pricing approach.

Cointegration of Fiscal Variables with GDP is the assumption that tax revenues and government spending share a long-run equilibrium relationship with GDP. It is empirically plausible and supported by our own cointegration analysis for Italy and it imposes a reasonable long-run anchor on fiscal policy. However, this long-run relationship may not be stable. Relevant structural changes, such as the adoption of Euro, has happened and may happen again, major fiscal reforms could also alter these dynamics.

Cyclical Properties of Fiscal Policy completes the cointegration assumption, I assume counter-cyclical spending and pro-cyclical revenues. These are consistent with the operation of automatic fiscal stabilizers and have been empirically verified for Italy in other studies (Marcellino 1995). Nonetheless, governments, particularly those with high debt levels, may be forced

into pro-cyclical fiscal contractions during a downturn, violating this assumption. The political will to maintain fiscal discipline can also vary, leading to changes in the cyclicity of discretionary fiscal policy.

4 LHS Reconstruction, the Market Value of Debt

As mentioned above, the first 20 observations are coming from Pecchi, Valente, and Fiorito 2002 and the market value of marketable debt, namely the share of debt that can be traded once issued (all type of bonds), is obtained by summing over BTPs, BOTs and CCTs. From 1997 to 2024, I have used the same method as they use to get to the market value. Having stored the datasets on government bonds from BoI and after having extracted from the archive of *Il Sole 24 Ore* the market prices of the available bonds at the end of every year I merge the two datasets by ISIN code. From the merger I notice that many bonds are not priced for the whole period in which they were active (issue date $< t <$ expiring date) or not priced at all. To estimate the missing prices, I proceed in two main steps: (i) estimation of the yield curve, and (ii) imputation of prices based on the estimated yield curve, using this pricing equation:

$$P_t^{\$}(h) = \sum_{i=1}^h \frac{C}{(1 + Y_t(h))^i} + \frac{1}{(1 + Y_t(h))^n} \quad (4)$$

where C is the semiannual coupon payment and h is the number of remaining periods.

Yield Curve Estimation. For each year from 1997 to 2024, I estimate a smooth zero-coupon yield curve using the Nelson and Siegel 1987 functional form, which should be more precise than the log-function that Pecchi, Valente, and Fiorito 2002 have:

$$Y_t(h) = \beta_0 + \beta_1 \left(\frac{1 - e^{-h/\tau}}{h/\tau} \right) + \beta_2 \left(\frac{1 - e^{-h/\tau}}{h/\tau} - e^{-h/\tau} \right)$$

where m is the bond's time to maturity in years. To fit this model, I use only the subset of bonds for which both the market price and the coupon rate are observed. For each of these bonds, I compute its yield-to-maturity (YTM) by numerically solving the bond pricing equation. Then, I estimate the parameters $\beta_0, \beta_1, \beta_2, \tau$ by minimizing the squared distance between observed YTMs and those implied by the Nelson–Siegel specification via non-linear least squares. The resulting parameters are stored year-by-year as JSON files.

Market Value Estimation. With the estimated yield curve in hand, I compute the market value of the entire stock of marketable government debt at the end of each year. For each active bond, if the market price is available, I use it. Otherwise, I estimate its price by applying the Nelson–Siegel curve to obtain the appropriate YTM based on its residual maturity, and then discount its future cash flows accordingly using (4).

The price is then multiplied by the bond’s outstanding amount to compute its market value contribution. Summing across all bonds yields the total market value of government debt for each year which is plotted in Figure 1. Interestingly, before 2000, the market value was systematically below the nominal value. This reflects a high-interest-rate environment in which outstanding bonds with low coupons traded at a discount. After 2000, the opposite happens: the market value exceeds the nominal value. This corresponds to a structural drop in interest rates, probably also enhanced credibility of fiscal policy, and increased demand for safe assets following EMU accession and, later, ECB interventions (such as, Quantitative Easing).

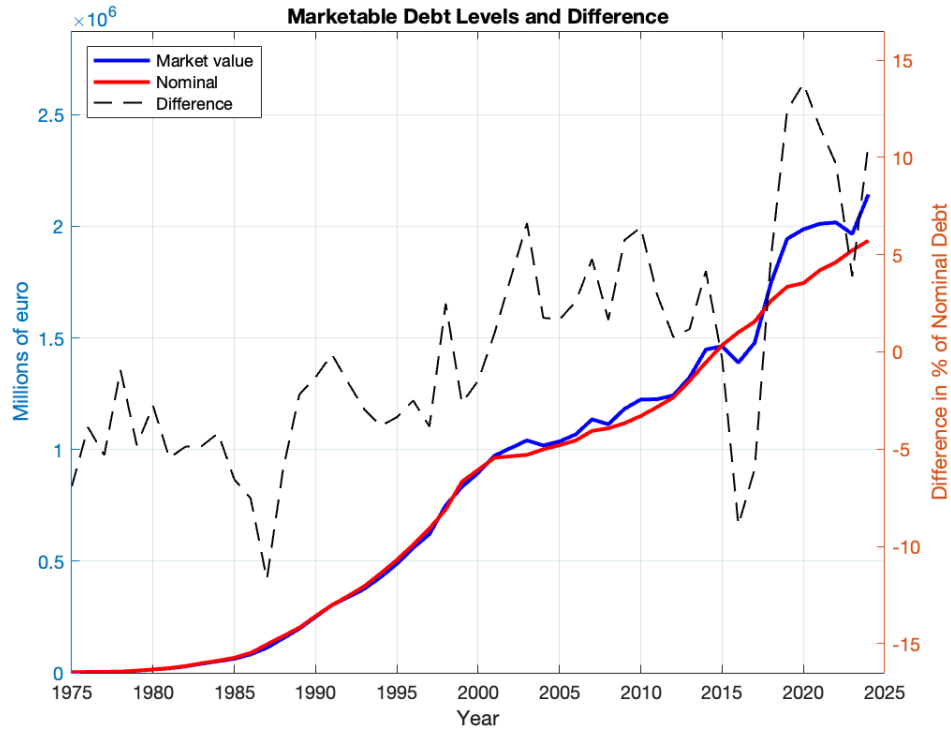


Figure 1: Market and Nominal Value of Debt

Note: Here are reported the Market Value (computed as said above) and the Nominal Value of the Italian government debt from 1975 to 2024. Their difference relative to each year’s Nominal debt is also plotted.

Breakdown by Maturity. Finally, for the period 1997-2024, I decompose the total market and nominal value of debt into three categories based on residual maturity: short-term (0–3 years), medium-term (3–10 years), and long-term (10+ years), Figure 2. This allows me to analyze whether pricing differences between market and nominal values are concentrated in specific segments of the maturity structure. Differences are expressed in terms of the Nominal Value and they are generally positive in low interest rate years such as the 2014–2021 period.

The long-term segment shows the largest deviations, as longer maturities are more sensitive to changes in interest rates. This aligns with standard bond duration theory: when yields drop, long-term bond prices rise more. During episodes of stress or monetary tightening (e.g., 2011–2012 sovereign debt crisis, post-2022 rate hikes), the differences narrow or even turn negative as market prices adjust downward.

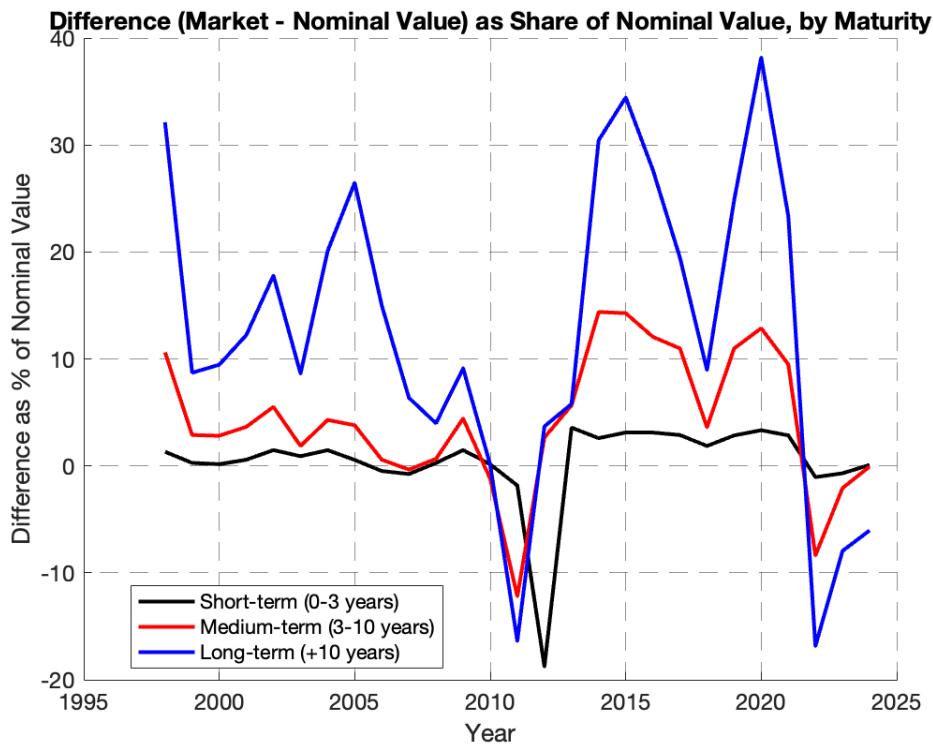


Figure 2: Differences of Debt Values by Maturity

Note: This picture shows the difference in behavior of different bonds, grouped by maturity, from 1995 to 2024. Within this period, for all classes of bonds, the market value was higher than the nominal value most of the times, the higher the maturity, the larger the difference.

5 RHS Reconstruction, Present Value of Future Surpluses

5.1 Estimating future Surpluses

As shown by Proposition 1, the market value of government debt reflects investors' expectations about future primary surpluses discounted at appropriate risk-adjusted rates. In this section, I reconstruct the RHS of the valuation equation — the fundamental component — following the approach of (JLVNX) and adapting it to the Italian context and data. I use a VAR-based macrofiscal model to forecast the expected path of revenues and expenditures and try to compute their discounted present value under the no-arbitrage assumption.

VAR esimation Assume that the $N \times 1$ vector of state variables $\mathbf{z}$ follows a Gaussian first-order VAR:

$$\mathbf{z}_t = \Psi \mathbf{z}_{t-1} + \mathbf{u}_t = \Psi \mathbf{z}_{t-1} + \Sigma^{\frac{1}{2}} \boldsymbol{\varepsilon}_t$$

with homoskedastic innovations $\mathbf{u}_t \sim i.i.d. \mathcal{N}(0, \Sigma)$ and using the Cholesky decomposition of the covariance matrix $\Sigma = \Sigma^{\frac{1}{2}} (\Sigma^{\frac{1}{2}})'$. This structure implies that the vector of reduced-form shocks $\mathbf{u}_t$ can be written as a linear combination of structural shocks $\boldsymbol{\varepsilon}_t \sim \mathcal{N}(0, I)$, with each shock affecting its own equation and those ordered later in the VAR. Specifically, shocks to each state variable are driven by its own innovation as well as the contemporaneous structural shocks to the variables that precede it in the VAR ordering. The vector of state variables $\mathbf{z}_t$ contains the demeaned macroeconomic and fiscal variables listed in Table 1, ordered as they appear in the VAR.

Table 1: State Variables

Position	Variable	Description	Sample Mean
1	π_t	Log Inflation	6.02%
2	$y_t^s(1)$	Log 1-Year Nominal Yield	6.67%
3	$yspr_t^s$	Log Long-term Minus Log 1-Year Nominal Yield Spread	0.66%
4	x_t	Log Real GDP Growth	1.34%
5	pd_t^M	Market Capitalization-to-GDP	0.25
6	$\Delta \log \tau_t$	Log Tax Revenue-to-GDP Growth	1.07%
7	$\log \tau_t$	Log Tax Revenue-to-GDP Level	0.41
8	$\Delta \log g_t$	Log Spending-to-GDP Growth	0.8%
9	$\log g_t$	Log Spending-to-GDP Level	0.42

Cointegration Analysis of Government Budget Components To investigate the long-run equilibrium relationship among Italian government spending, revenues, and nominal GDP, I conduct a cointegration analysis using annual data from 1951 to 2024, obtained from ISTAT and MEF. All variables are transformed into logarithmic form to facilitate interpretation as elasticities and to stabilize variance.

I begin by testing for unit roots using the Augmented Dickey–Fuller (ADF) test with an intercept ("ARD" model) and lag lengths from 0 to 4. The results indicate that the null hypothesis of a unit root cannot be rejected for any variable at lags greater than zero. This suggests that all three series — log spending, log revenues, and log nominal GDP — are non-stationary in levels, consistent with being integrated of order one, $I(1)$.

Given the non-stationarity of the series, I apply Johansen's cointegration test to the trivariate system $[\log(T_t), \log(G_t), \log(GDP_t)]$, the test includes one lag. Trace statistic results indicate rejection of the null hypothesis of no cointegration ($r = 0$), but fail to reject at ($r = 1$) suggesting the presence of exactly one cointegration vector. This is interpreted as evidence of a long-run equilibrium linking government spending, revenues, and nominal GDP.

This cointegration analysis motivates the inclusion in the state variables of both the log levels and the growth rate of spending and revenues, if there was no cointegration, all shocks to those variables would be permanent. Therefore, using this **Error Correction Specification** I am imposing mean reversion, being optimistic about future fiscal cash-flows, this in turn raises the expected PDV of future surpluses. There only is one empirical

issue, that (JLVNX) also had, and that is spending/GDP and revenues/GDP having trends, which would bias my coefficients. Therefore, when estimating – and only when estimating – the dynamics of the state variables I will do as them, remove the sample averages from the growth rates by reconstructing log revenues/GDP and log spending/GDP like this:

$$\log \tau_t = \log \tau_1 + \sum_{k=1}^t (\Delta \log \tau_k - \hat{\mu}^T), \quad \log g_t = \log g_1 + \sum_{k=1}^t (\Delta \log g_k - \hat{\mu}^G).$$

where $\hat{\mu}^i$ is the sample average of $\Delta \log i_t$ for $i \in \{G, T\}$.

The estimation proceeds equation-by-equation using OLS. The cointegrating terms for tax revenues and spending (in logs) are appended as error-correcting levels in the system, leading to the fiscal error-correction model that I mentioned above. To ensure stationarity, the eigenvalues of the coefficient matrix Ψ are computed, and the system is found to be stable (maximum modulus < 1). The estimated coefficients for Ψ and $\Sigma^{1/2}$ are in Table 2.

5.2 Computing the PDV

The Present Discounted Value of Surplus can be rewritten as follows:

$$\begin{aligned} PV(S)_t &= \mathbb{E}_t \left[\underbrace{\sum_{j=1}^{\infty} M_{t,t+j}^S T_{t+j}}_{P_t^T} \right] - \mathbb{E}_t \left[\underbrace{\sum_{j=1}^{\infty} M_{t,t+j}^S G_{t+j}}_{P_t^G} \right] \\ \frac{PV(S)_t}{Y_t} &= \frac{T_t}{Y_t} \cdot \frac{P_t^T}{T_t} - \frac{G_t}{Y_t} \cdot \frac{P_t^G}{G_t} \\ &= \tau_t PD_t^T - g_t PD_t^G \end{aligned}$$

and that is because the expected value of future cashflows is always equal to the price of a claim, this is true both for a tax revenue claim and for a spending claim. Then, when we divide the price by the current cash-flow we can call that a price-dividend ratio PD_t^T and PD_t^G , while τ_t and g_t are the ratios of revenues-to-GDP and spending-to-GDP. This new formulation is useful as we can rewrite price-dividend ratios using the Campbell and Shiller 1988 standard decomposition, which comes from a log-linearization and by forward

iterating. It reads as follows:

$$\begin{aligned} pd_t^T &= \frac{\kappa_0^T}{1 - \kappa_1^T} + \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^T)^{j-1} \Delta \log T_{t+j} \right] - \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^T)^{j-1} r_{t+j}^T \right], \\ pd_t^G &= \frac{\kappa_0^G}{1 - \kappa_1^G} + \underbrace{\mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^G)^{j-1} \Delta \log G_{t+j} \right]}_{CF_t^G} - \underbrace{\mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^G)^{j-1} r_{t+j}^G \right]}_{DR_t^G}, \end{aligned} \quad (5)$$

with κ_0^i and κ_1^i for $i \in \{G, T\}$ being linearization parameters and depending on pd_0^i which is the mean os log price-dividend ratios, and r_t^i being log returns. CF_t^i are the cash-flow components and DR_t^i the discount rate components. The expected log returns are given by: $\mathbb{E}_t[r_{t+1}^i] = y_t(1) + yspr_t + rp_t^i$ for $i \in \{G, T\}$ where rp_t^i are the risk-premium of the i claim and they are the unknowns to pin down.

Upper bound on Fiscal Backing To understand what could the RHS of Equation 2 is, without assuming any particular model but only that the surplus process can be estimated by the above macrofiscal VAR, I will construct the (JLVNX) **upperbound** on the present value of surpluses.

First of all, given the different cyclical properties of the spending and revenues claims, I can say that: (i) the *lower bound* on the expected return of a revenues claim is given by the expected return on a GDP claim, and (ii) the *upper bound* on the expected return of a spending claim is given again by the one on a GDP claim. Because of the way that expected returns were defined just above, this means that:

$$rp_t^T \geq rp_t^Y \geq rp_t^G$$

Now assume that risk premia on the surplus components are constant at their unconditional mean rp_0^i for $i \in \{T, G\}$, then the price-dividend ratios can be written as:

$$pd_t^T = pd_0^T + [(\mathbf{e}_\pi + \mathbf{e}_x + \mathbf{e}_{\Delta\tau})' \mathbf{\Psi} - (\mathbf{e}_{y1} + \mathbf{e}_{yspr})'(I - \kappa_1^T \mathbf{\Psi})^{-1} \mathbf{z}_t]$$

where $\mathbf{e}_i$ is a selector vector that has 1 in the position of variable i and 0 everywhere else.

The triplet $(pd_0^T, \kappa_0^T, \kappa_1^T)$ is determined as the solution of the following system of equations:

$$\begin{aligned}\kappa_1^T &= \frac{\exp(pd_0^T)}{\exp(pd_0^T) + 1}, \quad \kappa_0^T = \log(1 + \exp(pd_0^T)) - \kappa_1^T pd_0^T \\ pd_0^T &= -\frac{(y_0(1) + yspr_0 + rp_0^T) - (x_0 + \pi_0)}{1 - \kappa_1^T} + \frac{\kappa_0^T}{1 - \kappa_1^T}\end{aligned}$$

And similarly for the spending claim. Call $\widetilde{CF}_t^T = (\mathbf{e}_\pi + \mathbf{e}_x + \mathbf{e}_{\Delta\tau})' \Psi (I - \kappa_1^T \Psi)^{-1} \mathbf{z}_t$ and $\widetilde{DR}_t^T = (\mathbf{e}_{y1} + \mathbf{e}_{yspr})' (I - \kappa_1^T \Psi)^{-1} \mathbf{z}_t$, namely the time-varying cash flow and discount rate components. I can rewrite the present value of surplus-to-GDP as:

$$\frac{PV(S)_t}{Y_t} = \tau_t \exp(pd_0^T + \widetilde{CF}_t^T - \widetilde{DR}_t^T) - g_t \exp(pd_0^G + \widetilde{CF}_t^G - \widetilde{DR}_t^G)$$

From here, the steps to get the upper bound are easier. We have already assumed constant risk premia, but to maximize the PDV of surplus we have to impose them both equal to rp_0^Y so that the risk premium of spending (the safer claim) will be maximized, and the one on revenues will be minimized, so the overall risk of a surplus claim is minimized.

Imposing such a thing means that all the components not directly influenced by the time t cash flows are common between revenues and spending, $pd_0^Y = pd_0^T = pd_0^G$, $\kappa_0^Y = \kappa_0^T = \kappa_0^G$, $\kappa_1^Y = \kappa_1^T = \kappa_1^G$, and $\widetilde{DR}_t^Y = \widetilde{DR}_t^T = \widetilde{DR}_t^G$

The time varying upper bound on fiscal backing is:

$$\frac{\overline{PV(S)}_t}{Y_t} = \tau_t \exp(pd_0^Y + \widetilde{CF}_t^T - \widetilde{DR}_t^Y) - g_t \exp(pd_0^Y + \widetilde{CF}_t^G - \widetilde{DR}_t^Y) \quad (6)$$

Which, when evaluated at $\mathbf{z} = 0$ (unconditional mean) is the all time upper bound:

$$\frac{\overline{PV(S)}_t}{Y_t}(\mathbf{z} = 0) = \exp(pd_0^Y)(\tau_0 - g_0) \quad (7)$$

Equation 7 tells us that a country only has positive debt capacity if primary surpluses are positive on average ($\tau_0 > g_0$) or if $pd_0^T > pd_0^G$. Then, given that both taxes and spending grow at the same rate as GDP in the long run, this condition can only be satisfied if $rp_0^T < rp_0^G$, the tax revenue stream must be less risky than the spending stream. However, this implication contradicts the empirical characteristics of Italian fiscal data.

Considering the argument above, and since the average primary surpluses for the **long sample** (1975-2024) in Italy is negative, its debt capacity cannot be positive. Nonetheless, the analysis still makes sense as if we restrict the sample, only considering the period 1991-2024,

the average surplus becomes positive. In the following sections, this and other differences are explored. More steps of the above derivations, taken from (JLVNX) , can be found in Appendix: Derivations from (JLVNX) .

6 Results

6.1 Benchmark

Using the estimated discount factor and the VAR dynamics, I compute the cash flow components for tax revenues and spending as explained above, and the unique discount rate component, using them I construct the upper bound on the present value-to-GDP as shown in Equation 6, and then plot it with 90 and 95 percent confidence intervals obtained by bootstrap in Figure 3.

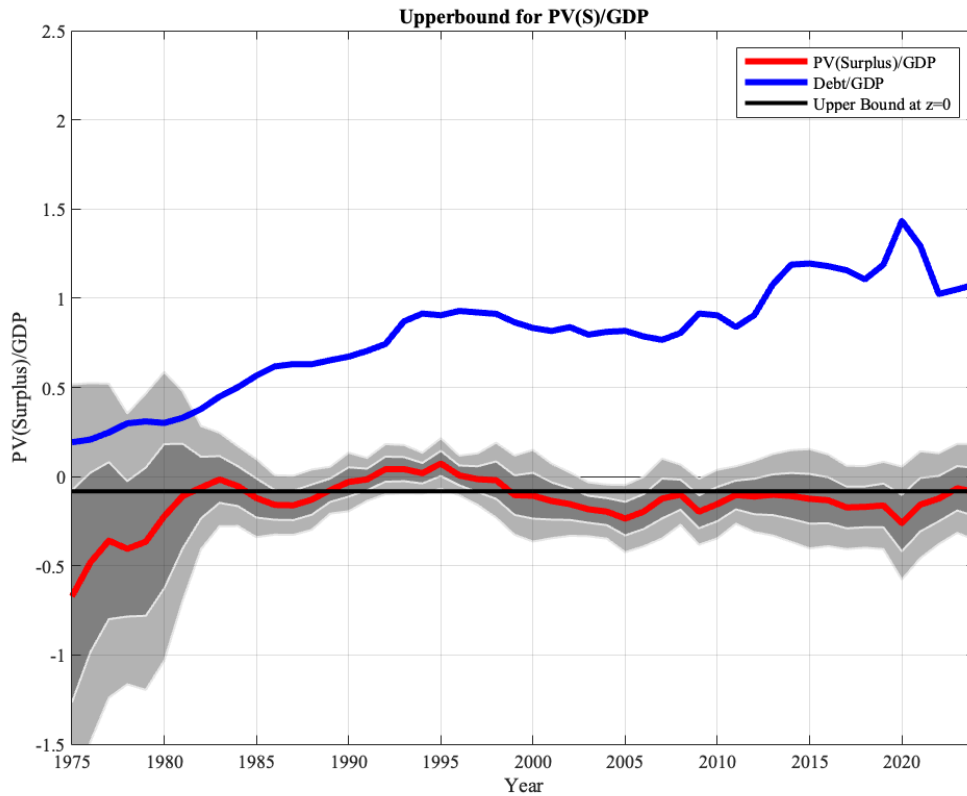


Figure 3: PDV of Surplus and Debt

Note: The blue line is the path of the observed Market value of debt divided by nominal GDP every year. The dynamic upperbound in red is computed as explained above and is enriched with its 95% C.I. (in gray). The fundamental value of the debt ($PV(S)/GDP$) is consistently below its market value.

In most years, especially after the 1990s, actual debt (blue) lies far above the valuation upper bound (red), this means that the market value of debt exceeds its fundamental value, computed as the present discount value of fiscal surpluses. Under current macro-financial conditions, part of the debt may not be backed by future fiscal revenues.

The average difference is 93.2% of GDP that year. This difference is huge, even when compared to (JLVNX) findings, that highlights a strong and increasing overvaluation of U.S. debt starting from 2010. From this analysis, it seems that the Italian fiscal capacity has never (since 1975 at least) been enough to back the country's level of debt.

Benchmark with Debt A version of the VAR that also includes debt/GDP and its growth rate is also estimated, allowing the surplus components to depend directly on the debt process, implying a stronger expected response of surpluses to an increase in the debt/GDP ratio than before. Figure 4 shows the new path obtained including the debt (green) compared to the previous valuation upper bound (red). Clearly, the difference is not big and far from being enough to explain the valuation gap.

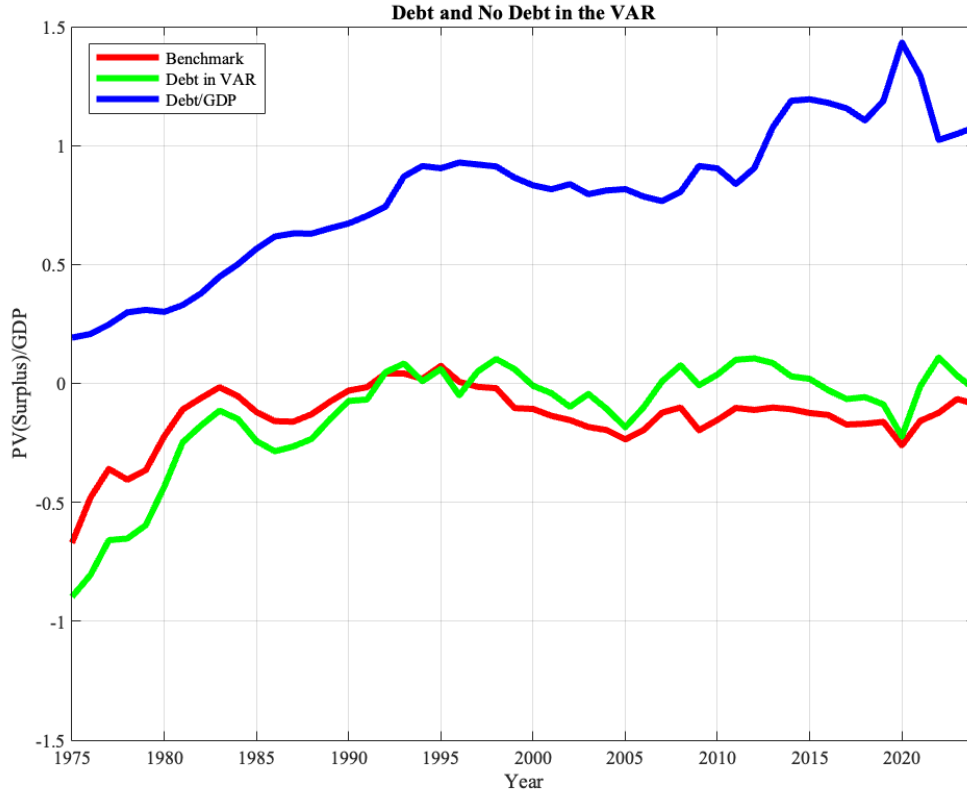


Figure 4: Upper bound for debt and no-debt in the VAR

Note: Not much difference between the red line (the previously computed $PV(S)/GDP$) and the green line, which reflects what is stated above.

6.2 Short sample, 1991-2024

Figure 5 shows the historical surplus-to-GDP path. Primary surpluses were systematically negative (but slowly turning positive) only in the first 17 years of the sample, that could be a problem since we are estimating the future behavior of the surplus components using their behavior in the past. In fact, when I restrict the sample and only consider the period 1991-2024, the average surplus becomes positive and over 2% of GDP. This restriction could only be problematic because of the reduced sample size, but it may be in principle even more correct to consider the country's history only from the 1990s as the national and international political frameworks had changed a lot before then and are more or less stable since then. Therefore, I repeat the whole analysis using the short sample. I re-estimate the VAR and re-compute the upper bound and show the results in Figure 6. The all time upper bound

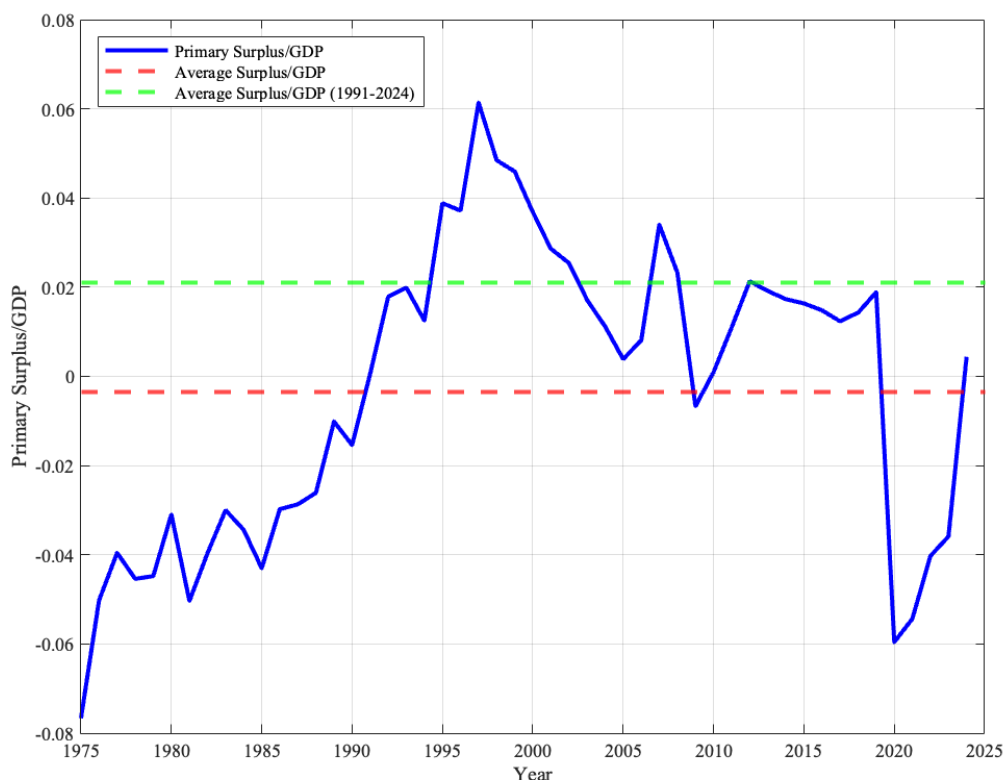


Figure 5: Surplus Path

Note: The green dashed line is the average over the "short sample", the period from 1991 to 2025.

(black) is now positive and the valuation upper bound (red) is always higher than before in the previous cases. However, even if the average difference-to-GDP has reduced, it is still very high and equal to 74.3% on average.

Using the reduced sample helps but doesn't close the gap.

6.3 Excessive Optimism

Another possibility is that the market consistently fails to estimate future surplus, either due to overly optimistic projections of GDP growth, which inflate expectations of tax revenues, or because of more general misperceptions about the government's ability to generate primary surpluses. To test this hypothesis, I simulate an artificial scenario in which I boost revenues relative to GDP, iteratively increasing the revenue share until the present-value computations for the upper bound on debt valuation align with observed market prices.

I had to increase revenues in every year by 10% to finally have an upper bound compatible with the market value of debt in the sample. This manipulation, however, increases average

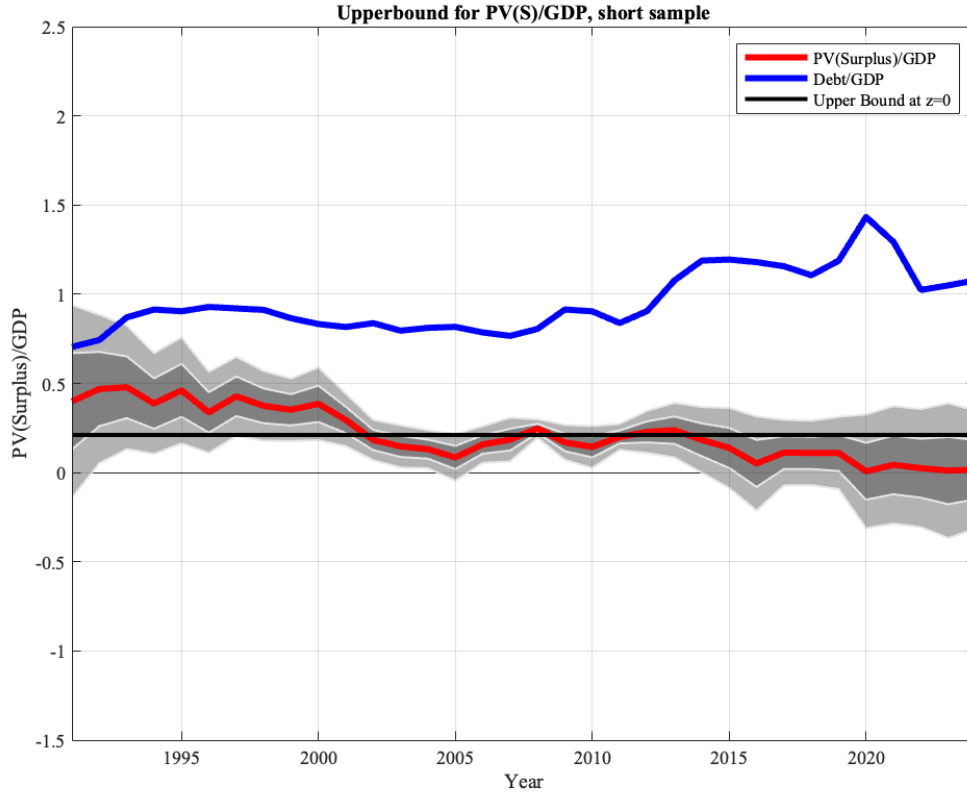


Figure 6: PDV of Surplus and Debt, Short Sample

Note: This is the same exercise as Figure 3 but done only considering the "short sample".

surplus from -0.34% of GDP to 3.8% , a change of more than **4 percentage points**, which raises the all time upper bound from -8% to 85% of GDP, which is exactly that 93% gap I found in Benchmark. However, such a large and systematic misjudgment of future surpluses is highly implausible, casting doubt on the idea that market over-optimism alone can explain the discrepancy.

6.4 Others

Even though the finding of a significant and persistent gap between the market value of Italian sovereign debt and its fundamental value, as determined by the present discounted value of future primary surpluses, aligns with the broader "debt valuation puzzle" documented by (JLVNX), the Italian case, situated within the unique institutional framework of the Eurozone, offers distinct insights and warrants a deeper discussion of the model's underlying assumptions and the potential drivers of this misalignment.

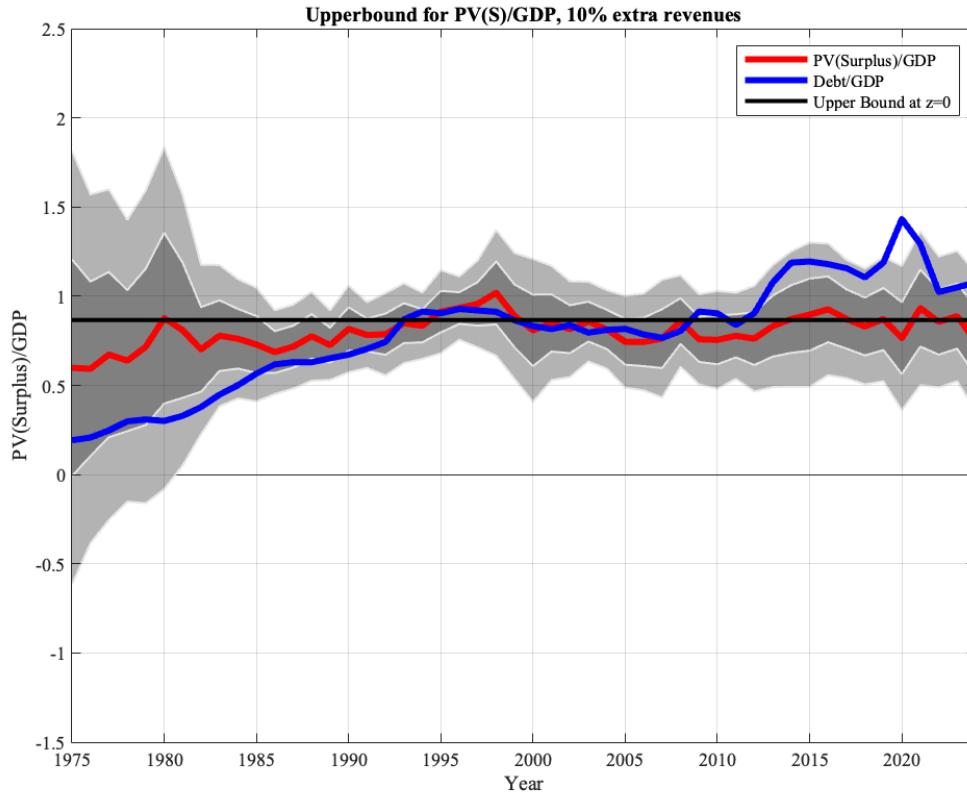


Figure 7: PDV of Surplus an Debt, higher revenues

Note: Same exercise as in Figure 3 with boosted revenues. Revenues are increased by 10% in every year.

As explained in the subsection above, "excessive optimism" is likely not to be the sole cause of the misalignment, but it could be one of the many factors contributing to it, together with several others, not explicitly captured by the model, which could partially explain why the market prices Italian debt at a premium relative to its fiscal fundamentals.

Among others, this premium may be attributable to a combination of the following:

- *The Role of the ECB*: Since the sovereign debt crisis, the ECB has implemented several non-standard monetary policies, including large-scale asset purchase programs. By acting as a large price-insensitive buyer of sovereign debt, the ECB has likely suppressed yields and inflated bond prices, partially enabling them to deviate from the fiscal outlook of individual member states. This institutional backstop may have provided a form of insurance to bondholders that is not reflected in the stream of primary surpluses alone.
- *Convenience Yield*: Italian government bonds, as a large and liquid asset class within

the Eurozone, may offer a "convenience yield." Financial institutions value these bonds for their use as high-quality collateral in repurchase agreements and for meeting regulatory requirements, such as Liquidity Coverage Ratios (LCR) under Basel III. This non-monetary return for holding the bonds leads investors to accept lower yields than the fiscal fundamentals would otherwise command.

On top of that, a segment of the market for government bonds is composed of captive domestic investors, such as banks and insurance companies, who may be implicitly or explicitly encouraged to hold domestic sovereign debt. This creates a stable demand base that is less sensitive to pure valuation metrics.

7 Conclusion

The analysis finds a substantial gap between Italy's public debt market value and the fundamental value implied by future primary surpluses. Under plausible macro-fiscal forecasts, the risk-adjusted PDV of Italy's projected surpluses falls well below the observed debt-to-GDP ratio. This suggests that Italian government bonds, like U.S. Treasuries, are priced at a premium relative to fiscal fundamentals. The market appears willing to finance debt at yields lower than those justified by expected future budgets.

Because this study is descriptive and empirical, I want to emphasize its evidence-based nature. I have not imposed a full general-equilibrium model, nor tested welfare-maximizing behavior, instead, by using historical data and VAR forecasts (a standard technique in the literature) to form expectations of Italy's fiscal path, and applying the no-arbitrage budget constraint, I compute a conservative measure of the PDV of surpluses. The conclusions thus rely on the accuracy of those forecasts and the chosen discount rates. In particular, the approach assumes no default (though it also holds in cases with default according to derivations), stable fiscal rules, and that past relationships remain valid; it also abstracts from political variables. Any model misspecification in the VAR (e.g., omitted shocks) or error in long-term trend assumptions will affect the PDV.

Looking forward, this work opens many doors for future extensions. One would be to adopt a flexible SDF model, similar to (JLVNX), that prices the entire term structure of government yields. Incorporating such a model could capture richer risk premia on Italian surpluses and may shrink or further explain the valuation gap. Another direction is cross-country analysis: applying the same PDV methodology to other euro-area countries (Spain, Portugal, etc.) or comparing Italy with a core economy (Germany) could isolate the effects of common shocks versus country-specific risk. Finally, the framework could be expanded by including alternative scenarios (for example, demographic change or euro-area financial shocks) to assess how sensitive Italy's debt valuation is to different macro paths.

In sum, my thesis shows that Italy's market value of debt does not respect the estimated PDV, thus failing to meet Equation 2, the Value Equivalence, this highlights the importance of risk considerations and market demand in sovereign debt pricing. Further research incorporating richer pricing models and broader international comparisons would shed more light on the dynamics of fiscal capacity in Italy and beyond.

A Appendix: VAR Results

Table 2: VAR Estimates:

	Ψ								
	π_{t-1}	$y_{t-1}(1)$	$yspr_{t-1}$	x_{t-1}	$p_{d,t-1}^M$	$\Delta \log \tau_{t-1}$	$\log \tau_{t-1}$	$\Delta \log g_{t-1}$	$\log g_{t-1}$
π_t	0.7489	0.1707	0.4632	0.6199	-0.0267	-0.1272	-0.0631	0.3791	-0.1359
$y_t(1)$	0.2434	0.6835	0.2148	0.2410	-0.0125	-0.0725	0.0191	0.0277	0.0703
$yspr_t$	-0.0406	0.0226	0.4825	-0.0081	-0.0021	-0.0007	-0.0050	0.0422	-0.0378
x_t	0.5125	-0.7560	-1.2078	0.2890	-0.0334	-0.2790	0.2049	0.2228	0.3261
$p_{d,t}^M$	1.4410	-2.3676	-1.1927	0.7251	0.6054	-0.0445	0.9100	0.6171	-0.1743
$\Delta \log \tau_t$	-0.4634	1.0092	0.4637	-0.2328	0.0042	-0.3304	-0.3046	-0.0456	-0.2388
$\log \tau_t$	-0.4634	1.0092	0.4637	-0.2328	0.0042	-0.3304	0.6954	-0.0456	-0.2388
$\Delta \log g_t$	-0.3943	0.8871	1.4299	0.0468	0.0112	0.1115	-0.1495	0.2094	-0.5896
$\log g_t$	-0.3943	0.8871	1.4299	0.0468	0.0112	0.1115	-0.1495	0.2094	0.4104

	$\Sigma^{1/2}$ Estimates								
	ϵ_t^π	$\epsilon_t^Y(1)$	ϵ_t^{Yspr}	ϵ_t^x	$\epsilon_{d,t}^{pd}$	$\epsilon_t^{\Delta \log \tau}$	$\epsilon_t^{\Delta \log g}$		
π_t	0.0130	0	0	0	0	0	0	0	0
$y_t^s(1)$	0.0052	0.0096	0	0	0	0	0	0	0
y_t^{spr}	-0.0004	-0.0016	0.0040	0	0	0	0	0	0
x_t	0.0021	0.0010	-0.0058	0.0173	0	0	0	0	0
$p_{d,t}^M$	-0.0031	0.0180	-0.0145	0.0086	0.0414	0	0	0	0
$\Delta \log \tau_t$	-0.0002	0.0072	0.0015	-0.0034	0.0031	0.0198	0	0	0
$\log \tau_t$	-0.0002	0.0072	0.0015	-0.0034	0.0031	0.0198	0	0	0
$\Delta \log g_t$	-0.0013	-0.0000	0.0039	-0.0195	-0.0024	0.0058	0	0.0231	0
$\log g_t$	-0.0013	-0.0000	0.0039	-0.0195	-0.0024	0.0058	0	0.0231	0

B Appendix: Proofs from (JLVNX)

Proofs and Derivations are taken from Jiang, Lustig, Van Nieuwerburgh, and Xiaolan 2024a.

B.1 Proposition 1

Proof: All objects in this proof are in nominal terms. The government faces the following one-period budget constraint:

$$G_t - T_t + Q_{t-1}^{\$}(1) = \sum_{h=1}^H \left(Q_t^{\$}(h) - Q_{t-1}^{\$}(h+1) \right) P_t^{\$}(h) \quad (8)$$

G_t represents the total nominal government spending and T_t is the total nominal government revenue. $Q_t^{\$}(h)$ is the quantity of nominal zero-coupon bonds with maturity h held in period t , each promising a \$1 payment at time $(t+h)$. $P_t^{\$}(h)$ is the current price of an h -period zero-coupon bond with a \$1 face value. An $(h+1)$ -period bond issued at $t-1$ transforms into an h -period bond in period t . Additionally, bonds can be issued or redeemed in period t , causing the stock of bonds for each maturity to evolve according to the equation:

$$Q_t^{\$}(h) = Q_{t-1}^{\$}(h+1) + \Delta Q_t^{\$}(h).$$

This notation also accommodates coupon-bearing bonds, as their price can be expressed as the sum of the present values of their coupons. We assume H is the maximum maturity issued in any period, meaning $Q_{t-1}^{\$}(H+1) = 0$ for all t .

The left side of the budget constraint indicates the current period's new financing requirements, equal to the sum of the primary deficit ($G - T$) and maturing one-period debt from the previous period. The right side illustrates that these funds are generated by issuing new bonds of various maturities.

We can now iterate the budget constraint forward. The period t constraint is given by

$$\begin{aligned} T_t - G_t = & Q_{t-1}^{\$}(1) - Q_t^{\$}(1)P_t^{\$}(1) + Q_{t-1}^{\$}(2)P_t^{\$}(1) - Q_t^{\$}(2)P_t^{\$}(2) \\ & + Q_{t-1}^{\$}(3)P_t^{\$}(2) - Q_t^{\$}(3)P_t^{\$}(3) + \dots - Q_t^{\$}(H)P_t^{\$}(H) + Q_{t-1}^{\$}(H+1)P_t^{\$}(H). \end{aligned}$$

Consider the period- $t+1$ constraint,

$$\begin{aligned} T_{t+1} - G_{t+1} = & Q_t^{\$}(1) - Q_{t+1}^{\$}(1)P_{t+1}^{\$}(1) + Q_t^{\$}(2)P_{t+1}^{\$}(1) - Q_{t+1}^{\$}(2)P_{t+1}^{\$}(2) \\ & + Q_t^{\$}(3)P_{t+1}^{\$}(2) - \dots - Q_{t+1}^{\$}(H)P_{t+1}^{\$}(H) + Q_t^{\$}(H+1)P_{t+1}^{\$}(H). \end{aligned}$$

Multiply both sides by $M_{t+1}^\$$ and take expectations conditional on time t :

$$\begin{aligned}\mathbb{E}_t \left[M_{t+1}^\$ (T_{t+1} - G_{t+1}) \right] &= Q_t^\$(1) P_t^\$(1) - \mathbb{E}_t \left[Q_{t+1}^\$(1) M_{t+1}^\$ P_{t+1}^\$(1) \right] + Q_t^\$(2) P_t^\$(2) \\ &\quad - \mathbb{E}_t \left[Q_{t+1}^\$(2) M_{t+1}^\$ P_{t+1}^\$(2) \right] + Q_t^\$(3) P_t^\$(3) \\ &\quad - \mathbb{E}_t \left[Q_{t+1}^\$(3) M_{t+1}^\$ P_{t+1}^\$(3) \right] + \cdots + Q_t^\$(H) P_t^\$(H) \\ &\quad - \mathbb{E}_t \left[Q_{t+1}^\$(H) M_{t+1}^\$ P_{t+1}^\$(H) \right] + Q_t^\$(H+1) P_t^\$(H+1),\end{aligned}$$

where $\mathbb{E}_t[M_{t+1}^\$] = P_t^\(1) , $\mathbb{E}_t[M_{t+1}^\$ P_{t+1}^\$(1)] = P_t^\$(2)$, $\dots$, $\mathbb{E}_t[M_{t+1}^\$ P_{t+1}^\$(H-1)] = P_t^\$(H)$, and $\mathbb{E}_t[M_{t+1}^\$ P_{t+1}^\$(H)] = P_t^\$(H+1)$.

Consider the period $t+2$ constraint, multiplied by $M_{t+1}^\$ M_{t+2}^\$$ and take time- t expectations:

$$\begin{aligned}\mathbb{E}_t \left[M_{t+1}^\$ M_{t+2}^\$ (T_{t+2} - G_{t+2}) \right] &= \mathbb{E}_t \left[Q_{t+1}^\$(1) M_{t+1}^\$ P_{t+1}^\$(1) \right] - \mathbb{E}_t \left[Q_{t+2}^\$(1) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(1) \right] \\ &\quad + \mathbb{E}_t \left[Q_{t+1}^\$(2) M_{t+1}^\$ P_{t+1}^\$(2) \right] - \mathbb{E}_t \left[Q_{t+2}^\$(2) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(2) \right] \\ &\quad + \mathbb{E}_t \left[Q_{t+1}^\$(3) M_{t+1}^\$ P_{t+1}^\$(3) \right] - \cdots \\ &\quad + \mathbb{E}_t \left[Q_{t+1}^\$(H) M_{t+1}^\$ P_{t+1}^\$(H) \right] - \mathbb{E}_t \left[Q_{t+2}^\$(H) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(H) \right] \\ &\quad + \mathbb{E}_t \left[Q_{t+1}^\$(H+1) M_{t+1}^\$ P_{t+1}^\$(H+1) \right].\end{aligned}$$

where by the law of iterated expectations and $\mathbb{E}_{t+1}[M_{t+2}^\$] = P_{t+1}^\$(1)$, $\mathbb{E}_{t+1}[M_{t+2}^\$ \times P_{t+2}^\$(1)] = P_{t+1}^\$(2)$, etc. Identical terms with opposite signs appear on the right-hand side of the last two equations. Adding up the expected discounted surpluses at t , $t+1$, and $t+2$:

$$\begin{aligned}&T_t - G_t + \mathbb{E}_t[M_{t+1}^\$ (T_{t+1} - G_{t+1})] + \mathbb{E}_t[M_{t+1}^\$ M_{t+2}^\$ (T_{t+2} - G_{t+2})] \\ &= \sum_{h=0}^H Q_{t-1}^\$(h+1) P_t^\$(h) - \mathbb{E}_t[Q_{t+1}^\$(1) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(1)] \\ &\quad - \mathbb{E}_t[Q_{t+2}^\$(2) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(2)] - \cdots - \mathbb{E}_t[Q_{t+2}^\$(H) M_{t+1}^\$ M_{t+2}^\$ P_{t+2}^\$(H)].\end{aligned}$$

Similarly, consider the one-period government budget constraints at times $t+3$, $t+4$, etc. Then add up all one-period budget constraints. Again, the identical terms appear with opposite signs in adjacent budget constraints. These terms cancel out upon adding up the budget constraints. Adding up all the one-period budget constraints until horizon $t+J$, I

get

$$\begin{aligned} \sum_{h=0}^H Q_{t-1}^{\$}(h+1)P_t^{\$}(h) &= \mathbb{E}_t \left[\sum_{j=0}^J M_{t,t+j}^{\$}(T_{t+j} - G_{t+j}) \right] \\ &+ \mathbb{E}_t \left[M_{t,t+J}^{\$} \sum_{h=1}^H Q_{t+J}^{\$}(h)P_{t+J}^{\$}(h) \right], \end{aligned}$$

where the cumulative SDF is $M_{t,t+j}^{\$} = \prod_{i=0}^j M_{t+i}^{\$}$ and by convention $M_{t,t}^{\$} = M_t^{\$} = 1$ and $P_t^{\$}(0) = 1$. The market value of the outstanding government bond portfolio equals the expected present discount value of the surpluses over the next J years plus the present value of the government bond portfolio that will be outstanding at time $t+J$. The latter is the cost the government will face at time $t+J$ to finance its debt, seen from today's vantage point. Now take the limit as $J \rightarrow \infty$:

$$\begin{aligned} \sum_{h=0}^H Q_{t-1}^{\$}(h+1)P_t^{\$}(h) &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}^{\$}(T_{t+j} - G_{t+j}) \right] \\ &+ \lim_{J \rightarrow \infty} \mathbb{E}_t \left[M_{t,t+J}^{\$} \sum_{h=1}^H Q_{t+J}^{\$}(h)P_{t+J}^{\$}(h) \right]. \end{aligned}$$

The market value of the outstanding debt inherited from the previous period equals the expected present-discounted value of the primary surplus stream $(T_{t+j} - G_{t+j})$ plus the discounted market value of the debt outstanding in the infinite future. According to the transversality condition:

$$\lim_{J \rightarrow \infty} \mathbb{E}_t \left[M_{t,t+J}^{\$} \sum_{h=1}^H Q_{t+J}^{\$}(h)P_{t+J}^{\$}(h) \right] = 0,$$

the market value of the outstanding debt cannot be growing faster than the stochastic discount factor. Otherwise, there is a government debt bubble. If the transversality condition is satisfied, the outstanding debt at the beginning of period t reflects the expected present-discounted value of the current and all future primary surpluses:

$$\sum_{h=0}^H Q_{t-1}^{\$}(h+1)P_t^{\$}(h) = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}^{\$}(T_{t+j} - G_{t+j}) \right].$$

Finally, using the one-period budget constraint (Equaion (8)) again, the end-of-period market

value of government debt is obtained, D_t , defined in Equation (2) in the main text:

$$D_t = \sum_{h=1}^H Q_t^{\$}(h) P_t^{\$}(h) = \mathbb{E}_t \left[\sum_{j=1}^{\infty} M_{t,t+j}^{\$} (T_{t+j} - G_{t+j}) \right].$$

Q.E.D.

B.2 Proposition 2

Proposition 2. Since

$$D_t = \sum_{h=1}^H Q_t^{\$}(h) P_t^{\$}(h),$$

It is true that

$$\begin{aligned} R_{t+1}^D D_t &= \sum_{h=0}^{\infty} Q_t^{\$}(h+1) P_{t+1}^{\$}(h) = (P_{t+1}^T + T_{t+1}) - (P_{t+1}^G + G_{t+1}) \\ &= P_t^T R_{t+1}^T - P_t^G R_{t+1}^G. \end{aligned}$$

Taking expectations, I get Equation (3) in the main text.

Q.E.D.

C Appendix: Derivations from (JLVNX)

C.1 Campbell-Shiller Decomposition of Tax and Spending Claims

Consider the return on a claim to the government's tax revenue:

$$r_{t+1}^T = \log \frac{P_{t+1}^T + T_{t+1}}{P_t^T} = \log \frac{T_{t+1}}{P_t^T} (1 + \exp(pd_{t+1}^T)).$$

We use pd_t^T to denote the log price-dividend ratio on the tax revenue claim: $pd_t^T = \log P_t^T - \log T_t$, where price is measured at the end of the period and the dividend flow is over the same period. Campbell and Shiller 1988 log-linearize the return equation around the mean log price/dividend ratio to derive the following expression for log returns on the tax claim:

$$r_{t+1}^T = \Delta \log T_{t+1} + \kappa_1^T pd_{t+1}^T - \kappa_0^T - pd_t^T,$$

with linearization coefficients as functions of the mean of the log price/dividend ratio pd_0^T :

$$\kappa_1^T = \frac{e^{pd_0^T}}{e^{pd_0^T} + 1} < 1, \quad \kappa_0^T = \log(1 + \exp(pd_0^T)) - \kappa_1^T pd_0^T.$$

By iterating forward on the linearized return equation, imposing a no-bubble condition: $\lim_{j \rightarrow \infty} (\kappa_1^T)^j pd_{t+j}^T = 0$, and taking expectations, we derive the following expression for the

log price/dividend ratio of the tax claim:

$$pd_t^T = \frac{\kappa_0^T}{1 - \kappa_1^T} + \mathbb{E}_t \left[\sum_{j=0}^{\infty} (\kappa_1^T)^j \Delta \log T_{t+j} \right] - \mathbb{E}_t \left[\sum_{j=0}^{\infty} (\kappa_1^T)^j r_{t+j}^T \right].$$

We use rp_t^T to denote the risk premium on tax claims relative to the long bond:

$$\mathbb{E}_t[r_{t+1}^T] = yspr_t + y_0(1 + rp_t^T).$$

We assume constant risk premia on the tax and spending claims, which we denote as rp_t^T and rp_t^G . We use e_{y1} and e_{yspr} to denote the column vectors that select the short rate and the yield spread. Because the state vector follows $\mathbb{VAR}(1)$ dynamics, we can compute the expected return as follows:

$$\mathbb{E}_t[r_{t+j}^T] = \psi_0^T(1) + yspr_0^R + rp_0^T + (e_{y1} + e_{yspr})' \Psi^j z_t$$

The DR (discount rate) term is given by the following expression:

$$DR_t^T \stackrel{\text{def}}{=} \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^T)^j r_{t+j}^T \right] = \frac{y_0(1) + yspr_0 + rp_0^T}{1 - \kappa_1^T} + (e_{y1} + e_{yspr})'(I - \kappa_1^T \Psi)^{-1} z_t.$$

The CF (cash flow) term is given by the following expression:

$$CF_t^T \stackrel{\text{def}}{=} \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^T)^{j-1} \Delta \log T_{t+j} \right] = \frac{x_0 + \pi_0^T}{1 - \kappa_1^T} + (e_{\pi} + e_x + e_{\Delta r})' \Psi (I - \kappa_1^T \Psi)^{-1} z_t,$$

We end up with the following expressions for the price/dividend ratio on the tax and spending claims:

$$pd_t^T = pd_0^T + ([e_{\pi} + e_x + e_{\Delta r}]' \Psi - (e_{y1} + e_{yspr})'(I - \kappa_1^T \Psi)^{-1}) z_t$$

where $e_{\Delta r}$ selects the tax-to-GDP growth rate in the state vector, and $(pd_0^T, \kappa_0^T, \kappa_1^T)$ solve

$$pd_0^T = \frac{x_0 + \pi_0 - y_0(1) - yspr_0 - rp_0^T}{(1 - \kappa_1^T)} + \frac{\kappa_0^T}{(1 - \kappa_1^T)},$$

$$\kappa_1^T = \frac{e^{pd_0^T}}{e^{pd_0^T} + 1}, \quad \kappa_0^T = \log(1 + \exp(pd_0^T)) - \kappa_1^T pd_0^T$$

Similarly, the log price/dividend ratio of the spending claim:

$$pd_t^G = \frac{\kappa_0^G}{1 - \kappa_1^G} + \mathbb{E}_t \left[\sum_{j=0}^{\infty} (\kappa_1^G)^j \Delta \log G_{t+j} \right] - \mathbb{E}_t \left[\sum_{j=0}^{\infty} (\kappa_1^G)^j r_{t+j}^G \right].$$

We can derive a similar expression for the spending claim. We use $\widetilde{CF}_t^T$ and $\widetilde{DR}_t^T$ to denote the mean-zero time-varying components of the cash flow and discount rate terms. The implied present value of surpluses/GDP ratio is given by

$$\frac{PV_t^S}{Y_t} = \tau_t \exp(pd_t^T + \widetilde{CF}_t^T - \widetilde{DR}_t^T) - g_t \exp(pd_t^G + \widetilde{CF}_t^G - \widetilde{DR}_t^G).$$

To derive some intuition, we can evaluate the expression at $z_t = 0$, that is, when all variables are at their unconditional mean. In this case, the present value of surpluses/GDP ratio is given by

$$\frac{PV_t^S}{Y_t}(z = 0) = \tau_0 \exp(pd_0^T) - g_0 \exp(pd_0^G).$$

C.2 Upper Bound on Debt Valuation.

To derive an upper bound, we equate the expected returns on taxes and spending to the expected return on GDP: $rp_t^S = rp_t^G = rp_t^Y$. This delivers an upper bound on the valuation of future surpluses, because it maximizes the value of the tax claim, and minimizes the value of the spending claim. Given these two assumptions, we derive the following expression for

the implied log price/dividend ratio on the tax claim and the spending claim:

$$\begin{aligned}
pd_t^T &= pd_t^Y + \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} (\Delta \log T_{t+j} - (x_0 + \pi_0)) \right] \\
&\quad - \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} (r_{t+j}^Y - (yspr_0 + y_0(1) + rp_0^Y)) \right], \\
pd_t^G &= pd_t^Y + \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} (\Delta \log G_{t+j} - (x_0 + \pi_0)) \right] \\
&\quad - \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} (r_{t+j}^Y - (yspr_0 + y_0(1) + rp_0^Y)) \right]
\end{aligned}$$

The long-run growth rate of tax and spending equals the long-run growth rate of output: $x_0 + \pi_0$. That follows directly from co-integration. We use a constant GDP risk premium rp_0^Y . We can back this number out of the unconditional equity risk premium by unlevering the equity premium. We use e_r to denote a column vector with zero with a 1 as the first element. The DR (discount rate) term is defined by

$$DR_t^Y = DR_t^G = DR_t^Y = \frac{y_0^S(1) + yspr_0 + rp_0^Y}{1 - \kappa_1^Y} + (e_{y1} + e_{yspr})'(I - \kappa_1^Y \Psi)^{-1} z_t.$$

The CF (cash flow) term for the tax claim is defined by

$$CF_t^T \equiv \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} \Delta \log T_{t+j} \right] = \frac{x_0 + \pi_0}{1 - \kappa_1^Y} + (e_\pi + e_x + e_{\Delta r})' \Psi (I - \kappa_1^Y \Psi)^{-1} z_t.$$

The CF (cash flow) term for the spending claim is defined by

$$CF_t^G = \mathbb{E}_t \left[\sum_{j=1}^{\infty} (\kappa_1^Y)^{j-1} \Delta \log G_{t+j} \right] = \frac{x_0 + \pi_0}{1 - \kappa_1^Y} + (e_\pi + e_x + e_{\Delta g})' \Psi (I - \kappa_1^Y \Psi)^{-1} z_t.$$

We use $\widetilde{CF}_t^I$ and $\widetilde{DR}_t^I$ to denote the time-varying components. Hence, we end up with the following expressions for the price/dividend ratio on the tax and spending claims:

$$\begin{aligned}
pd_t^T &= pd_0^Y + ([e_\pi + e_x + e_{\Delta r}]' \Psi - (e_{y1} + e_{yspr})'(I - \kappa_1^Y \Psi)^{-1}) z_t, \\
pd_t^G &= pd_0^Y + ([e_\pi + e_x + e_{\Delta g}]' \Psi - (e_{y1} + e_{yspr})'(I - \kappa_1^Y \Psi)^{-1}) z_t
\end{aligned}$$

A first-order Taylor expansion yields the following expression:

$$\frac{PV_t^S}{Y_t} \approx (\tau_t - g_t) \exp(pd_0^Y) + \tau_t(\widetilde{CF}_t^T - \widetilde{DR}_t^T) \exp(pd_0^Y) - g_t \exp(pd_0^G)(\widetilde{CF}_t^G - \widetilde{DR}_t^G).$$

This expression can be simplified. We obtain the following intuitive expression for an upper bound on the PDV of surpluses:

$$\frac{PV_t^S}{Y_t} \approx \exp(pd_0^Y)(\tau_t - g_t)(1 - \widetilde{DR}_t^Y) + \tau_t \widetilde{CF}_t^T - g_t \widetilde{CF}_t^G.$$

Suppose the country currently runs a primary surplus of zero. The discount rate effects cancel out, again to a first-order approximation. When the country runs a zero primary surplus, the upper bound on the value of debt/GDP is positive only if the expected tax revenue growth exceeds expected spending growth:

$$\frac{PV_t^S}{Y_t} \approx \exp(pd_0^Y) \tau_t (\widetilde{CF}_t^T - \widetilde{CF}_t^G) > 0 \quad \text{iff} \quad \widetilde{CF}_t^T > \widetilde{CF}_t^G.$$

This can be further simplified to yield the following expression:

$$\frac{PV_t^S}{Y_t} \approx \exp(pd_0^Y) \tau_t (e_{\Delta r} - e_{\Delta g})' \Psi (I - \kappa_1^Y \Psi)^{-1} z_t.$$

The discount rate dynamics and the dynamics of GDP growth are irrelevant (to a first-order approximation) for the upper bound. What matters is the dynamics in tax/GDP and spending/GDP. In other words, the expected cumulative effect of mean reversion in taxes has to outweigh the expected cumulative effect of mean-reversion in spending.

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