Default Cycles*

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July 2018

Abstract

Recessions are often accompanied by spikes of corporate default and prolonged declines of business credit. This paper shows that credit and default cycles can result from variations of self-fulfilling beliefs about credit market conditions. We develop a tractable macroeconomic model in which credit contracts reflect the expected default risk of borrowing firms. We calibrate the model to evaluate the macroeconomic impact of sunspots and fundamental shocks. Self-fulfilling changes in credit market expectations generate sizable reactions in default rates together with endogenously persistent credit and output cycles, accounting for most of the volatility of corporate default and over 20% of output volatility.

JEL classification: E22, E32, E44, G12

Keywords: Corporate default; Credit spreads; Sunspots; Financial Shocks; Risky Steady State

*We thank Mark Aguiar, Douglas Gale, Simon Gilchrist, Lars Hansen, Patrick Kehoe, Igor Livshits, Dominik Menno, Alexander Monge-Naranjo, Franck Portier, Morten Ravn, Guillaume Rocheteau, Vincent Sterk, Kathrin Schlafmann, and Yi Wen for helpful comments and discussions, and seminar and conference audiences at AFR Summer Institute of Economics and Finance, Bocconi University, CEMFI, CERGE, Chinese University of Hong Kong, Cologne, Durham, EEA 2017, Goethe University Frankfurt, Hong Kong Monetary Authority, Hong Kong University, Konstanz, Manchester, Oxford, SED 2017, SFU, St. Gallen, SUFE, Tsinghua (Economics), Tsinghua PBC, UBC, UCL, Vienna Macroeconomics Workshop, for their comments. Leo Kaas thanks the German Research Foundation (grant No. KA 442/15) for financial support. Wei Cui thanks the Centre for Macroeconomics (grant No. ES/K002112/1), the ADEMU European Horizon 2020 (grant No. 649396), and the HKIMR for financial support. The usual disclaimer applies.

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1 Introduction

Many recessions are accompanied by substantial increases of corporate default rates and credit spreads, together with declines of business credit. On the one hand, corporate defaults tend to be clustered over prolonged episodes which gives rise to persistent credit cycles.[1] Such clustering of default can only partly be explained by observable firm-specific or macroeconomic variables, but is driven by unobserved factors that are correlated across firms and over time.[2] On the other hand, credit spreads tend to lead the cycle and are not fully accounted for by expected default. Moreover, less than half of the volatility of credit spreads can be explained by expected default losses; instead, it is the "excess premium" on corporate bonds that has the strongest impact on investment and output (cf. Gilchrist and Zakrajšek (2012)).

This paper examines the joint dynamics of firm default, credit spreads, and output, using a tractable dynamic general equilibrium model in which firms issue defaultable debt. We argue that defaults in such economies are susceptible to self-fulfilling beliefs over credit conditions, and the size of the belief variation has important implications on spreads, leverage, and output. States of low default and good credit conditions can alternate between states of high default and bad credit conditions. Stochastic variation of self-fulfilling beliefs, which can be caused by both fundamental and non-fundamental shocks, play a key role in accounting for the persistent dynamics of default rates and their co-movement with macroeconomic variables.

To illustrate our main idea, we present in Section 2 a simple partial-equilibrium model of firm credit with limited commitment and equilibrium default. Leverage and the interest rate spread depend on the value that borrowing firms attach to future credit market conditions which critically impacts the firms' default decisions, and hence is taken into account in the optimal credit contract. This credit market value is a forward-looking variable which reacts to self-fulfilling expectations. A well-functioning credit market with a low interest rate and a low default rate is highly valuable for borrowing firms which makes credit contracts with few defaults self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by firms, and therefore it cannot sustain credit contracts that prevent high default rates.

After this illustrative example, we build in Section 3 a general-equilibrium model in order to analyze the role of self-fulfilling expectations and fundamental shocks for the dynamics of default rates, credit spreads, and their relationships with the aggregate economy. Credit constraints, credit spreads, default rates, and aggregate productivity all react endogenously to changes in credit market conditions. As in the simple model, leverage ratios and default rates depend on the value that borrowers attach to credit market participation which is susceptible

[1] See, e.g., Giesecke et al. (2011).

[2] See, e.g., Duffie et al. (2009).

to changes of self-fulfilling beliefs. Aggregate productivity is determined by the allocation of capital among firms which itself depends on current leverage ratios and on past default events. When credit is tightened or when more firms opt for default, less capital is operated by the most productive firms so that aggregate productivity and output fall.

Firms in our model differ in productivity and in their access to the credit market. High-productivity firms with a good credit standing borrow up to an endogenous credit limit at an interest rate which partly reflects the expected default loss (coming from both fundamentals and non-fundamentals) and which also includes an excess interest premium. This premium, subject to aggregate shocks, is a shortcut to account for the so-called "credit spread puzzle" according to which actual credit spreads could be far from expected default losses (explained by fundamentals).[3] Recovery rate can also fluctuate which affects the expected default loss and hence takes a direct impact on leverage and on the predicted component of credit spreads.

If a firm opts for default, a fraction of its assets can be recovered by creditors. After default, the firm's owners may continue to operate a business, but they lose the good credit standing and hence remain temporarily excluded from the credit market. Notice that the net worth of firms with credit market access is endogenous, and aggregate productivity and factor demand depend on the borrowing capacity (net worth multiplied by the leverage ratio) of firms. Then, periods of high default can have a long-lasting impact on credit and output.

In Section 4, we calibrate this model and show how it responds to sunspot shocks, as well as to (fundamental) shocks to the recovery rate, to the excess bond premium, or to aggregate productivity. A key feature of our model is that risky beliefs matter for leverage and credit spreads. For this reason, we log-linearize the model around the *risky steady state* (cf. Coeurdacier et al. (2011)) which describes a stationary model solution that takes aggregate risk (in particular, the impact of risky beliefs and recovery ability on the credit market) into account. With zero excess bond premium in the steady state, we show that average excess credit spreads in the data can be used to pin down the variance of belief shocks. This calibration strategy requires that parameters for the shock processes are jointly estimated together with parameters that affect the risky steady state.

Belief changes in the quantitative model can be induced by fundamental shocks, but they may also be completely unrelated to fundamentals (pure sunspot shocks). We show that variations in self-fulfilling beliefs are crucial for the dynamics of default rates. An adverse sunspot shock and a shock to the excess bond premium (via its impact on beliefs) raises the default rate and depresses leverage. We also show that without corporate default our economy would be more volatile due to a larger steady-state leverage ratio, which exaggerates the impact of belief variations on business cycles.

[3]See, e.g., Elton et al. (2001) and Huang and Huang (2012).

Although different shocks in our model can be called "financial shocks", the generated equilibrium responses are significantly different from each other, highlighting the complex dimensions of financial frictions. Our model links these dimensions in a tractable framework, and the model estimation suggests that all financial shocks together explain output dynamics since 1982 rather well and account for about two thirds of output growth volatility, of which about one third is induced by changes in credit market expectations.

Related Literature. Self-fulfilling beliefs matter in our model precisely because default is punished by the (permanent or temporary) exclusion from borrowing in future periods, which makes the value of credit market access a forward-looking variable. Early contributions on limited commitment, such as Eaton and Gersovitz (1981), do not consider the possibility of multiple equilibria by *assuming* that the borrowers' Bellman equation has a unique solution.[4] Our illustrative example of Section 2, which resembles the model of Eaton and Gersovitz,[5] shows that this assumption is not always valid.

This finding is not new (albeit often overlooked in the literature on limited commitment): Alvarez and Jermann (2000, 2001) show that the value function operator in a limited commitment economy is not a contraction so that multiple equilibria can arise. Building on Bulow and Rogoff (1989), Hellwig and Lorenzoni (2009) demonstrate that credit with limited commitment is equivalent to a bubble on an outside asset. Bethune et al. (2018) shows that multiple and periodic equilibria are possible in a matching model with credit subject to limited commitment constraints, and Krueger and Uhlig (2018) show that multiple equilibria can arise in a general-equilibrium model with one-sided commitment. Closely related to our paper is Azariadis et al. (2016) who consider the role of sunspot shocks in a model of unsecured firm credit with limited commitment.

However, there is no equilibrium default and credit spreads are zero in these previous articles.[6] An important contribution of our paper is to analyze the dynamics of corporate default and its impact on aggregate output within a tractable model. We show that the (risky) steady-state equilibrium is indexed by the variance of the aggregate belief shocks (sunspots) which also strongly affects the equilibrium dynamics. Building on this feature, we estimate the model using the standard perturbation method (around the risky steady state) and the Kalman filter. This new empirical strategy naturally constrains the size of the belief shocks from credit spread data. Our estimation exercise also highlights the importance of spill-over effects from the excess bond premium on belief variations.

[4] See the text after equation (5) in Eaton and Gersovitz (1981, p. 291).

[5] The major difference is that Eaton and Gersovitz consider stochastic income whereas our example has stochastic default costs on a two-point distribution.

[6] Further contributions on self-fulfilling expectations in macroeconomic models with financial frictions are Harrison and Weder (2013), Benhabib and Wang (2013), Liu and Wang (2014) and Gu et al. (2013). None of these papers addresses default and credit spreads.

In this regard, our approach is complementary to the literature of sentiments with imperfect information and "correlated equilibria" (e.g., Benhabib et al. (2013, 2015) and Angeletos and La'O (2013)), in which the variance of sentiments is uniquely determined in equilibrium. Our approach may be easier for a quantitative estimation exercise as (aggregate) belief shocks only have to satisfy a mild restriction.[7]

The co-existence of equilibria with high (low) interest rates and high (low) default rates relates to a literature on self-fulfilling sovereign debt crises. In a two-period model, Calvo (1988) shows how multiple equilibria emerge from a positive feedback between interest rates and debt levels. Lorenzoni and Werning (2013) extend this idea to a dynamic setting to study the role of fiscal policy rules and debt accumulation for the occurrence of debt crises. On the other hand, Cole and Kehoe (2000) find that self-fulfilling debt crises occur because governments cannot roll over their debt.[8] Our mechanism for multiplicity is different from these contributions by emphasizing the role of expectations about future credit conditions. We further focus on *strategic default* of borrowers in general equilibrium.

Finally, our work shows how shocks to the excess bond premium affect fundamentals and non-fundamentals through credit contracts and that they are indeed quantitatively important. This aspect relates to a number of recent contributions analyzing the macroeconomic implications of credit spreads and firm default. Building on Bernanke et al. (1999), Christiano et al. (2014) show that risk shocks in a quantitative business-cycle model not only generate countercyclical spreads but also account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include long-term defaultable debt in a macroeconomic model with financial shocks to the recovery rate. In line with empirical evidence, they find that credit spreads are countercyclical and lead output and stock returns. Gourio (2013) is motivated by the volatility of the excess bond premium and argues that time-varying risk of rare depressions (disaster risk) can generate plausible volatility of credit spreads and co-movement with macroeconomic variables. Self-fulfilling expectations do not matter in all these contributions which differ from our model in that default incentives do not depend on expected credit conditions. Furthermore, these papers do not allow for a link between the credit market and aggregate factor productivity.[9]

[7]Belief shocks are uncorrelated random variables with conditional mean zero.

[8]See also Aguiar et al. (2013), Bocola and Dovis (2016) and Conesa and Kehoe (2017)

[9]Gomes and Schmid (2012) develop a macroeconomic model with endogenous default of heterogeneous firms and analyze the dynamics of credit spreads. Khan et al. (2016) introduce firm dynamics and default risk and show that countercyclical default affect the capital allocation among firms, which amplifies and propagate real and financial shocks. Unlike our model, there is no role for self-fulfilling expectations. Benhabib et al. (2018) also features countercyclical credit spreads and pro-cyclical TFP with self-fulfilling expectations, but default disappears in equilibrium once reputation is introduced.

2 An Illustrative Example

We present a simple partial-equilibrium dynamic model to illustrate how default rates, credit spreads, and leverage can vary in response to changes in self-fulfilling expectations. We illustrate that a dynamic complementarity is the key to generate multiplicity.

2.1 The Setup

The model has a large number of firms who live through infinitely many discrete periods $t \geq 0$. Firm owners are risk-averse and maximize discounted expected utility

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t \left[(1-\beta)\log c_t - \mathbf{1}_{\{\text{defaulting}\}}\eta_t\right],$$

where c_t is consumption (dividend payout) in period t, $\beta < 1$ is the discount factor, and η_t is a default loss that materializes only when the firm defaults in period t. $\mathbf{1}_{\{\text{defaulting}\}}$ is the index function which is 1 if the firm chooses default, and 0 otherwise. Following Cui (2017), the utility cost with log utility ensures that there is closed form solution for binary choice (i.e., default or no default in the current model).[10] As an example, the default loss may reflect the additional labor effort of the firm owner in a default event.[11] The default loss is idiosyncratic and stochastic: with probability p it is zero, otherwise it is $\Delta > 0$. Hence in any given period, fraction p of the firms are more prone to default.

All firms are endowed with one unit of net worth in period zero and they have access to a linear technology that transforms one unit of the consumption good in period t into Π units of the good in period $t+1$. Firms may obtain one-period credit from perfectly competitive and risk-neutral investors who have an outside investment opportunity at rate of return $\bar{R} < \Pi$. Although firms cannot commit to repay their debt, there is a record-keeping technology that makes it possible to exclude defaulting firms from all future credit. That is, if a firm decides to default, it is subject to the default utility loss (if any) in the default period and it may not borrow in all future periods.

Investors offer standard debt contracts that specify the interest rate R and the volume of debt b. Competition between investors ensures that the offered contracts (R, b) maximize the

[10] Cui (2017) also shows that the utility cost is equivalent to a cost *proportional* to net worth after a proper adjustment. This is because the firm owners consume a $(1-\beta)$ fraction of the net worth given the log utility. Therefore, a cost that is a fraction of net worth implies a proportional reduction in consumption. With log utility, the proportional cost thus has the same effect as utility cost η.

[11] Alternatively, we may assume in this example, as well as in the full macro model of the next section, that a defaulting firm's net worth is subject to a real default cost shock proportional to the net worth. The reason behind is that product demand for a defaulting firm may or may not change. This alternative model has the same credit market equilibrium but slightly different aggregate dynamics. Details are available upon request.

borrower's utility subject to the investors' participation constraint. The latter requires that the expected return equals the outside return $\bar{R}$ per unit of debt.

In recursive notation, a firm owner's utility $V(\omega)$ depends on the firm's net worth ω and satisfies the Bellman equation

$$
\begin{aligned}
V(\omega) &= \max_{c,s,(R,b)} (1-\beta)\log(c) + \beta \mathbb{E} \max \left\{ V(\omega'), V^d(\omega'_d) - \eta' \right\} \text{ , s.t.} \qquad (1) \\
c &= \omega - s \ , \\
\omega' &= \Pi(s+b) - Rb \ , \\
\omega'_d &= \Pi(s+b) \ , \\
\mathbb{E}(R \cdot b) &= \bar{R} \cdot b \ .
\end{aligned}
$$

The firm owner chooses consumption c, savings s, and a particular credit contract (R, b), subject to the investors' participation constraint. Next period, she can choose to repay and obtain net worth ω'; she can also choose to default and obtain net worth ω'_d, in which case she has to bear the default cost η'. That is, default costs are realized ex-post to lending. The second maximization expresses the optimal ex-post default choice at the beginning of the next period, and the expectation operator $\mathbb{E}$ is over the firm's realization of the default cost $\eta' \in \{0, \Delta\}$. The defaulting firm is further punished by exclusion from future credit: $V^d(.)$ is the utility value of a firm with a default history, which satisfies the recursion

$$
\begin{aligned}
V^d(\omega) &= \max_{c,s}(1-\beta)\log(c) + \beta V^d(\omega') \text{ , s.t.} \qquad (2) \\
c &= \omega - s \ , \\
\omega' &= \Pi s \ .
\end{aligned}
$$

We show in the Appendix A (proof of Proposition 2) that all firms save $s = \beta\omega$ and that value functions take the simple forms

$$V(\omega) = \log(\omega) + V \text{ and } V^d(\omega) = \log(\omega) + V^d,$$

where V and V^d are independent of the firm's net worth. We write $v \equiv V - V^d$ to express the surplus value of access to credit; it is a forward-looking *endogenous* variable that reflects *expected credit conditions*.

2.2 The Optimal Credit Contract

Using the above notation, we can write the value function as

$$V(\omega) = \max_{s}(1-\beta)\log(\omega - s) + \beta[V^d + U(s)] \;, \tag{3}$$

where $U(s)$ is the surplus value of the optimal credit contract for a firm with savings s. It solves the problem

$$U(s) \equiv \max_{(R,b)} \mathbb{E} \max \Big\{ \log[\Pi(s+b) - Rb] + v, \log[\Pi(s+b)] - \eta' \Big\}, \quad \text{s.t.}$$

$$\bar{R}b = \mathbb{E}(Rb) = \begin{cases} Rb & \text{if } \log[\Pi(s+b) - Rb] + v \geq \log[\Pi(s+b)] \;, \\ (1-p)Rb & \text{if } \log[\Pi(s+b)] > \log[\Pi(s+b) - Rb] + v \geq \log[\Pi(s+b)] - \Delta \;, \\ 0 & \text{else.} \end{cases}$$

The participation constraint captures three possible outcomes. In the first case, the firm repays for any realization of the default loss in which case investors are fully repaid Rb. In the second case, the firm only repays when the default loss is positive, which is reflected in the expected payment $(1-p)Rb$. In the third case, the firm defaults with certainty.

It is straightforward to characterize the optimal contract.

Proposition 1. *Suppose that the parameter condition*

$$\frac{(e^{\Delta} - 1)(1-p)}{e^{\Delta} - 1 + p} < \frac{\bar{R}}{\Pi} < \frac{(e^{(1-p)\Delta} - e^{-p\Delta})(1-p)}{e^{(1-p)\Delta} - 1} \tag{4}$$

holds. Then, there exists a threshold value $\bar{v} \in (0, v^{\max})$ with $v^{\max} \equiv \log(\Pi/(\Pi - \bar{R}))$, such that

(i) *If $v \in [\bar{v}, v^{\max})$, the optimal contract is $(R, b) = (\bar{R}, b(s))$ with debt level and borrower utility*

$$b(s) = s\frac{\Pi(1-e^{-v})}{\bar{R} - \Pi(1-e^{-v})} \;,\; U(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R} - \Pi(1-e^{-v})}\right] .$$

(ii) *If $v \in [0, \bar{v})$, the optimal contract is $(R, b) = (\bar{R}/(1-p), b(s))$, with debt level and borrower utility*

$$b(s) = s\frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})} \;,\; U(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta \;.$$

Proof. See Appendix A. □

If expected credit conditions are good enough, $v \geq \bar{v}$, the threat of credit market exclusion is so severe that no firm defaults in the optimal contract. The corresponding debt level is the largest one that prevents default of firms with zero default loss whose binding enforcement constraint is $\log[\Pi(s+b)-Rb]+v = \log[\Pi(s+b)]$. A feasible solution to the optimal contracting problem further requires that debt is finite which necessitates $v < v^{\max}$.

Alternatively, if expected credit conditions are not so good, $v < \bar{v}$, the optimal contract allows for partial default since it is then relatively costly to prevent default of all firms. Instead, fraction p of firms default in the optimal contract, whereas firms with positive default cost are willing to repay which is ensured by $\log[\Pi(s+b) - Rb] + v = \log[\Pi(s+b)] - \Delta$.

The parameter conditions (4) imply that both outcomes are optimal for different values of expected credit conditions. If one of these inequalities fails, either no default (i) or partial default (ii) is the optimal contract for all feasible values of v.

2.3 Stationary Equilibria and Sunspot Cycles

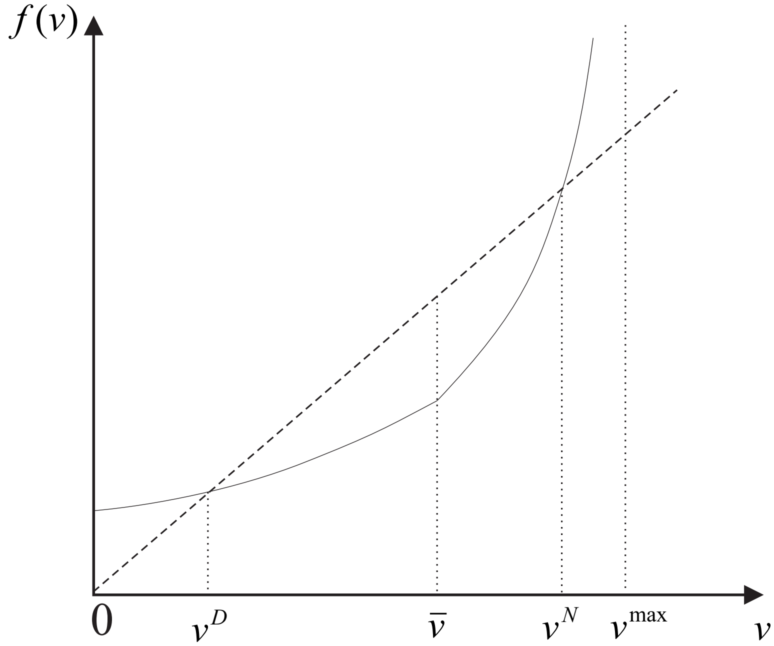
Expected credit conditions v depend themselves on the state of the credit market. In a stationary equilibrium, they are the solution to a fixed-point equation that maps next period's credit market condition into today's credit market condition. To derive this equation, we take the difference between Bellman equations (3) and (2). Utilizing the functional forms for $V(\omega)$, $V^d(\omega)$, and $s = \beta\omega$, as well as $U(s)$ from Proposition 1, the stationary value of expected credit conditions $v^* = V - V^d$ satisfies the fixed-point equation

$$v^* = f(v^*) \equiv \begin{cases} \beta \log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-e^{-v^*})}\right] & \text{if } v^* \geq \bar{v} \ , \\ \beta\Big\{ \log\left[\frac{\bar{R}}{\bar{R}-\Pi(1-p)(1-e^{-v^*-\Delta})}\right] - (1-p)\Delta \Big\} & \text{if } v^* < \bar{v} \ . \end{cases}$$

Any solution of this equation constitutes a stationary equilibrium of this economy. Under the conditions of Proposition 2, it can be verified that f is increasing and continuous, and it satisfies $f(0) > 0$ and $f(v) \to \infty$ for $v \to v^{\max}$. This shows that, generically, the fixed-point equation has either no solution, or two solutions. Moreover, if $f(\bar{v}) < \bar{v}$ holds, there is one equilibrium at $v^* = v^D < \bar{v}$ which involves default and a positive interest spread together with another equilibrium at $v^* = v^N > \bar{v}$ which has no default and a zero spread (see Figure 1, in which v on the horizontal axis measures the expected credit market conditions *tomorrow*). This result is summarized as follows.[12]

[12] If the parameter condition (5) (which is equivalent to $f(\bar{v}) < \bar{v}$) fails, there can exist at most two equilibria with default, or at most two equilibria without default. Since function f is convex and kinks upwards at $\bar{v}$, there cannot be more than two equilibria.

Figure 1: **Co-existence of Default and No-Default Equilibria**



Proposition 2. *Suppose that parameters satisfy*

$$\left(\frac{\bar{R}}{\bar{R}-\Pi(1-e^{-\bar{v}})}\right)^{\beta} < \frac{\Pi[1-(1-p)e^{-p\Delta}]}{\Pi-\bar{R}+e^{(1-p)\Delta}(\bar{R}-\Pi(1-p))}\ , \tag{5}$$

as well as condition (4). Then there are two stationary credit market equilibria $v^D < v^N$ such that default rates and interest spreads are positive at v^D and zero at v^N.

Proof. See Appendix A. □

The main insight of this proposition is that the state of the credit market is a matter of self-fulfilling expectations. A well-functioning credit market with a low interest rate and a low default rate is highly valuable for firms, and this high valuation makes credit contracts without default self-enforcing. Conversely, a weak credit market with a higher interest rate and more default is valued less by the firms, and therefore it cannot sustain credit contracts that prevent default.

Note that credit market expectations (and thus a fully dynamic model) are essential for this multiplicity result: conditional on credit market expectations v, the static credit market equilibrium as characterized in Proposition 1 is unique. This distinguishes our result from those where multiplicity of equilibria arises in static models (such as, e.g., Calvo (1988)). Key for multiplicity, as we emphasized in the introduction, is the observation that the fixed-point mapping $f(.)$ is not a contraction, so that the central Bellman equation does not have a unique fixed point.

Although the two equilibria are clearly ranked in terms of default rates, interest rates and utility, it is worth noticing that leverage, defined as the debt-to-equity ratio $b(s)/s$, can be higher or lower in the no-default state compared to the default state. On the one hand, the lower interest rate and the higher credit market valuation at the no-default equilibrium permit a greater leverage. On the other hand, preventing default of all firms requires a tighter borrowing constraint compared to the one that induces only firms with high default costs to repay.[13]

The additional parameter condition (5) of Proposition 2 is fulfilled whenever the discount factor β is low enough (because the fraction on the right-hand side is strictly greater than one). Conversely, the condition fails if β is sufficiently large.[14] In other words, a prerequisite for weak credit markets is that future consumption is discounted enough.

While the previous analysis describes stationary equilibria, this partial equilibrium model also gives rise to self-fulfilling sunspot cycles in which the economy fluctuates perpetually between states of positive spreads and default and states with zero spreads and no default:

Proposition 3. *Under the condition of Proposition 2, there exists a stochastic equilibrium in which the economy alternates between states with positive default $v_1 < \bar{v}$ and states without default $v_2 > \bar{v}$ with transition probability $\pi \in (0, 1)$.*

Proof. See Appendix A. □

Since the relationship between credit market expectations in periods t and $t+1$ is monotonic (see Figure 1), there are no deterministic cycles. Indeed, all deterministic non-stationary equilibria in this example converge to the steady state at v^N which is therefore locally indeterminate. The existence of sunspot cycles rests on a continuity argument (cf. Chiappori and Guesnerie (1991)) in the presence of multiple steady states; see the proof of Proposition 3 in the Appendix.

[13] To give a numeric example, set $\beta = 0.9$, $\Pi = 1$, $\bar{R} = 0.92$, $p = 0.1$, and two values of the default loss, $\Delta = 0.2$ and $\Delta = 0.4$. For both values of Δ, there is a no-default equilibrium at $v^N \approx 0.43$ with leverage $b/s \approx 0.61$. For $\Delta = 0.2$, the default equilibrium at $v^D \approx 0.11$ has lower leverage $b/s \approx 0.35$. For $\Delta = 0.4$, leverage at the default equilibrium $v^D \approx 0.2$ is $b/s \approx 0.79$. Hence, the default equilibrium can have *higher* leverage than the no-default equilibrium: the greater default loss relaxes the borrowing constraint which is imposed to preclude default of high-cost firms, while permitting default of the other firms.

[14] In this limiting case infinite debt levels would become sustainable, so that this partial model has no equilibrium at the given (low) interest rate $\bar{R} < \Pi$. In the general-equilibrium model, there always exists an equilibrium since the endogenous interest rate would rise when β becomes sufficiently large.

3 The Macroeconomic Model

We extend the insights of the previous section to a dynamic general equilibrium economy. The main departures from the partial model are as follows: (i) the safe interest rate is determined in credit market equilibrium; (ii) lenders can recover some of their exposure in default events; (iii) defaulters are *not* permanently excluded; (iv) due to idiosyncratic productivity shocks the credit market impacts aggregate factor productivity; (v) we introduce aggregate shocks to study business-cycle implications. These include fundamental shocks (technology and financial variables) as well as belief shocks (or sunspot shocks).

3.1 The Setup

Firms and Workers

The model has a unit mass of infinitely-lived firm owners with the same preferences as in the previous section: period utility is $(1-\beta)\log(c)-\eta$ where c is consumption and β is the discount factor. The idiosyncratic default loss η is distributed with cumulative function $G(.)$ with no mass points. Different from the stylized example of the previous section, a continuous distribution is necessary to capture continuous variation of default rates in response to aggregate shocks.

All firms operate a production technology which produces output (consumption and investment goods) $y=(zk)^\alpha(A_t\ell)^{1-\alpha}$ from inputs capital k and labor ℓ with capital share $\alpha\in(0,1)$. A_t is time-varying aggregate productivity that grows over time and is hit by exogenous productivity shocks,[15]

$$\log A_t=\mu_t^A+\log A_{t-1},$$

where μ_t^A follows a stationary process with mean μ^A.

Firms can have high or low idiosyncratic capital productivity z. Specifically, a firm has high productivity z^H with probability π and low productivity $z^L=\gamma z^H$ with probability $1-\pi$. To simplify algebra, we assume that the capital productivity shock affects the stock of capital (rather than the capital service), so that the firm's capital stock at the end of the period is $(1-\delta)zk$, where δ is the depreciation rate.

Next to firm owners, the economy includes a mass of hand-to-mouth workers who supply labor l and who consume their labor earnings $c=wl$. Their preferences are represented by a modified Greenwood-Hercowitz-Huffman utility function that allows for balanced growth

[15] To simplify notation, we use time index t to indicate time-varying *aggregate* variables. Idiosyncratic variables carry no index since we formulate them in recursive notation below.

paths, $u\left(c_t - \frac{A_t \kappa l_t^{1+\nu}}{1+\nu}\right)$, where u is increasing and concave, and κ, $\nu > 0$.[16] Workers supply labor according to

$$w_t / A_t = \kappa l_t^{\nu} \ . \tag{6}$$

That workers are hand-to-mouth consumers is not a strong restriction but follows from imposing a zero borrowing constraint on workers: If workers have the same discount factor β as firm owners, they do not save in the steady-state equilibrium in which the gross interest rate satisfies $\bar{R} < 1/\beta$ so that workers' consumption equals labor income in all periods.[17]

Consider a firm operating the capital stock k. In the labor market, the firm hires workers at the competitive wage rate w_t. This leads to labor demand which is proportional to the firm's effective capital input zk, so that the firm's net worth (before debt repayment) is $\Pi_t zk$, where the gross return per efficiency unit of capital is (see Appendix A for details)

$$\Pi_t = \alpha \left[\frac{(1-\alpha) A_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} + 1 - \delta \ . \tag{7}$$

Credit Market

The credit market channels funds from low-productivity firms (lenders) to high-productivity firms (borrowers). Competitive, risk-neutral banks pool the savings of lenders, taking the safe lending rate $\bar{R}_t$ as given, and offer credit contracts to borrowers.

The credit spread, i.e. the difference between borrowers' and lenders' interest rates, reflects the expected default cost and it also includes an "excess bond premium" term, denoted Φ_t per unit of debt, which is not directly related to the default risk and which may represent investor sentiment or risk appetite (cf. Gilchrist and Zakrajšek (2012)). Φ_t could also reflect intermediation costs or insurance premia against aggregate default risk.[18] We treat Φ_t as an exogenous parameter that may be subject to shocks. By focusing on the role of self-fulfilling beliefs, we choose to simplify here and do not model the reasons that generate endogenous fluctuations of this excess premium parameter. In our quantitative analysis, however, we take into account that variations of the excess premium can be correlated with credit market variables.

[16] The reason behind this utility function is that over time technological growth also increases the quality of leisure time (see Mertens and Ravn (2011)).

[17] This standard argument extends to a stochastic equilibrium around a steady-state equilibrium as long as shocks are not too large.

[18] Although banks insure lenders against *idiosyncratic* default risks, they cannot insure themselves against the *aggregate* component of default risk. The latter may be obtained from unmodeled (foreign) insurance companies selling credit default swaps (cf. Jeske et al. (2013)). In the absence of such insurance, banks could not offer a safe lending rate to depositors in combination with standard credit contracts, so they would need to offer risky securities to lenders to fund credit to risky borrowers.

Credit contracts take the form (R, b), where R is the gross borrowing rate, which reflects the firm's default risk, and b is the firm's debt. As in the previous section, the debt level in the optimal contract is proportional to the firm's internal funds (equity). Moreover, because all borrowing firms face the same ex-ante default incentives, the debt-to-equity ratio for all borrowing firms is the same and only depends on the aggregate state. This implies that we can write the equilibrium contract as (R_t, θ_t) where θ_t is the debt-to-equity ratio for any borrowing firm. We derive this credit contract below.

If a firm borrows in period t and defaults in period $t+1$, creditors can recover fraction λ_{t+1} of the borrower's gross return $\Pi_t zk$. The recovery parameter λ_{t+1} is the fraction of collateral assets that can be seized in the event of a default. It may be subject to "financial shocks" which can be understood as disturbances to the collateral value or to the cost of liquidation.[19] The owner of the defaulting firm keeps the share $(1-\lambda_{t+1})\zeta$ of the assets, where $\zeta < 1$ is a *real default* cost parameter. In subsequent periods, the firm carries a default flag which prevents access to credit. In any period following default, however, the default flag disappears with probability ψ in which case the firm regains full access credit.[20]

Timing

Timing within each period is as follows. First, the aggregate state $X_t = (A_t, \lambda_t, \Phi_t, \varepsilon_t^b)$ realizes. The first three components are the fundamental parameters described above which follow a Markov process. ε_t^b is a belief shock which is uncorrelated over time. Next to the aggregate state vector, idiosyncratic default costs η realize and indebted firms either repay their time $t-1$ debt or opt for default, after observing λ_t and ε_t^b. Firms with a default history lose the default flag with probability ψ. Second, firms learn their idiosyncratic productivity $z \in \{z^L, z^H\}$ and make savings and borrowing decisions, given A_t and Φ_t. Third, workers are hired and production takes place.[21]

[19] See e.g. Gertler and Karadi (2011) and Jermann and Quadrini (2012) for a similar modeling approach. See Chen (2010) for cyclical recovery rates.

[20] Such a default event can stand for a liquidation (such as Chapter 7 of the U.S. Bankruptcy Code) of the firm in which case the owner can start a new business with harmed access to credit. Alternatively, it can describe a reorganization (such as Chapter 11) in which case the same firm continues operation but may suffer from a prolonged deterioration of the credit rating which makes access to credit difficult. See Corbae and D'Erasmo (2016) for firm dynamics with an endogenous choice of the type of bankruptcy.

[21] Similar to the simple example, this timing implies that lenders observe he firm productivity ex ante, and default costs are realized ex post to lending.

3.2 Equilibrium Characterization

Credit Market

Write $V(\omega; X_t)$ for the value of a firm with a clean credit record and net worth ω in period t after default decisions have been made and before idiosyncratic productivity realizes. Similarly, $V^d(\omega; X_t)$ denotes the value of a firm with a default flag. After realization of idiosyncratic productivity, a borrowing firm ($z = z^H$) with net worth ω in period t chooses savings s and a credit contract (R, θ) to maximize

$$(1-\beta)\log(\omega-s)+\beta\mathbb{E}_t \max\left\{V\Big((z^H\Pi_t(1+\theta)-\theta R)s; X_{t+1}\Big), V^d\Big(z^H\Pi_t(1+\theta)(1-\lambda_{t+1})\zeta s; X_{t+1}\Big)-\eta'\right\},$$

where the expectation is over the realization of the aggregate state X_{t+1} and the idiosyncratic default cost η' in period $t+1$. A borrower who does not default earns the leveraged return $z^H\Pi_t(1+\theta)-\theta R$ and has continuation utility $V(.)$, whereas a defaulter earns $z^H\Pi_t(1+\theta)(1-\lambda_{t+1})\zeta$, incurs the default loss η' and has continuation utility $V^d(.)$.

In Appendix A, we show that these value functions take the form $V^{(d)}(\omega; X_t) = \log(\omega) + V^{(d)}(X_t)$, and we write $v_t \equiv V(X_t) - V^d(X_t)$ to denote the surplus value of a clean credit record ("*credit market expectations*"). Write

$$\rho \equiv R/(z^H\Pi_t)$$

for the interest rate relative to the borrower's capital return. Then the objective of a borrowing firm can be rewritten as [22]

$$(1-\beta)\log(\omega-s)+\beta\log(s)+\beta\mathbb{E}_t \max\Big\{\log[1+\theta(1-\rho)], \log[(1+\theta)(1-\lambda_{t+1})\zeta]-\eta'-v_{t+1}\Big\}.$$

It is immediate that every borrower saves $s = \beta\omega$. Moreover, there is an ex-post default threshold level

$$\tilde{\eta}' = \log\left[\frac{(1+\theta)(1-\lambda_{t+1})\zeta}{1+\theta(1-\rho)}\right] - v_{t+1}, \tag{8}$$

such that the borrower defaults if and only if $\eta' < \tilde{\eta}'$. The threshold $\tilde{\eta}'$ varies with next period's credit market value v_{t+1} and with the contract (ρ, θ).

Competitive banks offer contracts (ρ, θ). If a bank issues aggregate credit $B = \theta S$ (to borrowers with aggregate equity S), it needs to raise funds θS from lenders. In the next period $t+1$, the bank repays $\bar{R}_t\theta S$ to lenders, it pays the excess bond premium, and it earns risky revenue $[(1-G(\tilde{\eta}'))\rho\theta S+G(\tilde{\eta}')\lambda_{t+1}(1+\theta)S]z^H\Pi_t$ where $\tilde{\eta}'$ is the ex-post default threshold

[22] The terms $\log(z^H\Pi_t) + \mathbb{E}_t V(X_{t+1})$ are irrelevant for the maximization and hence cancel out.

for contract (ρ, θ). Competition drives expected bank profits to zero, which implies

$$\bar{\rho}_t(1+\Phi_t) = \mathbb{E}_t\Big\{(1-G(\tilde{\eta}'))\rho + G(\tilde{\eta}')\lambda_{t+1}\frac{1+\theta}{\theta}\Big\} \ , \tag{9}$$

where $\bar{\rho}_t \equiv \bar{R}_t/(z^H\Pi_t)$, similar to ρ, measures the safe interest rate in relation to the borrowers' capital return. The right-hand side of (9) is the expected revenue per unit of debt (relative to $z^H\Pi_t$). In default events $\eta' < \tilde{\eta}'$, banks recover $\lambda_{t+1}(1+\theta)/\theta$ per unit of debt.

Under perfect competition, the contracts offered in equilibrium maximize borrowers' expected utility,

$$\mathbb{E}_t\Big\{(1-G(\tilde{\eta}'))\log[1+\theta(1-\rho)] + \int_{-\infty}^{\tilde{\eta}'} \log[(1+\theta)(1-\lambda_{t+1})\zeta] - \eta' - v_{t+1}\, dG(\eta')\Big\} \ ,$$

subject to the ex-post default choice (8) and the zero-profit condition for banks (9).

We characterize the optimal contract as follows:

Proposition 4. *Given a safe interest rate $\bar{\rho}_t$, excess bond premium Φ_t, (stochastic) collateral parameter λ_{t+1}, and credit market expectations v_{t+1}, the optimal credit contract in period t, denoted (θ_t, ρ_t), together with the ex-post (stochastic) default threshold $\tilde{\eta}_{t+1}$ satisfy the following equations:*

$$\tilde{\eta}_{t+1} = \log\Big[\frac{(1-\lambda_{t+1})\zeta}{1-\xi_t}\Big] - v_{t+1} \ , \tag{10}$$

$$\theta_t = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t\left[\lambda_{t+1}G(\tilde{\eta}_{t+1}) + \xi_t(1-G(\tilde{\eta}_{t+1}))\right]} - 1 \ , \tag{11}$$

$$\mathbb{E}_t[G'(\tilde{\eta}_{t+1})(\xi_t - \lambda_{t+1})] = \mathbb{E}_t(1-G(\tilde{\eta}_{t+1}))\Big\{1 - \bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t[G(\tilde{\eta}_{t+1})(\xi_t - \lambda_{t+1})]\Big\} \ , \tag{12}$$

with $\xi_t \equiv \rho_t\theta_t/(1+\theta_t)$.

Proof. See Appendix A. ☐

Conditions (10) and (11) are the ex-post default choice and the zero-profit condition of banks, respectively. Condition (12) is the first-order condition of the contract value maximization problem.[23]

As in the partial model of the previous section, credit market expectations v_t depend themselves on the state of the credit market, satisfying the following recursive equation (see

[23] In our parameterizations with normally distributed default costs we verify that the second-order condition is also satisfied and that the solution is indeed a global maximum.

Appendix A for a derivation):

$$v_t = \beta\pi\mathbb{E}_t\Big\{\log(1+\theta_t) + \log(1-\lambda_{t+1}) + \log\zeta - \tilde{\eta}_{t+1}[1-G(\tilde{\eta}_{t+1})] - \int_{-\infty}^{\tilde{\eta}_{t+1}} \eta \; dG(\eta)\Big\} + \beta(1-\psi-\pi)\mathbb{E}_t v_{t+1} \; . \tag{13}$$

The value of access to the credit market in period t includes two terms. First, with probability π the firm becomes a borrower in which case it benefits from higher leverage θ_t, whereas a higher expected default threshold $\tilde{\eta}_{t+1}$ reduces the value of borrowing. Second, the term $\beta(1-\psi-\pi)\mathbb{E}_t v_{t+1}$ captures the discounted value of credit market access from period $t+1$ onward.

General Equilibrium

In the competitive equilibrium, firms and banks behave optimally as specified above, and the capital and labor market are in equilibrium.

Consider first the capital market. The gross lending rate $\bar{R}_t$ cannot fall below the capital return of unproductive firms $z^L\Pi_t$, which implies that $\bar{\rho}_t \geq \gamma = z^L/z^H$. When $\bar{\rho}_t > \gamma$, unproductive firms invest all their savings in the capital market; they only invest in their own inferior technology if $\bar{\rho}_t = \gamma$. Therefore, capital market equilibrium implies the following complementary slackness condition:

$$\gamma \leq \bar{\rho}_t \; , \quad f_t\pi\theta_t \leq (1-\pi) \; , \tag{14}$$

where $f_t \in [0,1]$ is the fraction of aggregate net worth owned by firms with access to credit. The left-hand side of the second inequality is total borrowing (as a share of capital): fraction $f_t\pi$ of capital is owned by borrowers and θ_t is borrowing per unit of equity. The right-hand side $(1-\pi)$ is the share of capital owned by unproductive firms, which is fully invested in the capital market if the safe interest rate $\bar{\rho}_t$ exceeds γ. Otherwise, if $\bar{\rho}_t = \gamma$, a fraction of the capital of unproductive firms is invested in their own businesses.

Since the labor market is frictionless, labor demand of any firm is proportional to the efficiency units of capital: $\ell = zk[(1-\alpha)A_t^{1-\alpha}/w_t]^{1/\alpha}$. Let Ω_t be the domestic aggregate net worth at the beginning of period t. Then, the capital stock operated by productive firms is $K_t^H = \beta\Omega_t\pi\Big[f_t(1+\theta_t)+1-f_t\Big]$. Savings of productive firms in period t are $\beta\Omega_t\pi$. Fraction f_t of this is owned by borrowing firms whose capital is $1+\theta_t$ per unit of internal funds. Fraction $1-f_t$ is owned by firms without access to credit whose capital is all internally funded. The capital stock operated by unproductive firms is $K_t^L = \beta\Omega_t\left[(1-\pi)-\pi f_t\theta_t\right]$. That is, these firms use the fraction of savings not invested in the capital market for production. Since labor supply l satisfies $\kappa l_t^\nu = w_t/A_t$ from (6), the real wage that clears the labor market is

$$w_t^{\frac{\nu+\alpha}{\nu}} (\kappa A_t)^{-\frac{\alpha}{\nu}} = (1-\alpha) A_t^{1-\alpha} (\beta \Omega_t)^\alpha \left(z^L \Big[(1-\pi) - \pi f_t \theta_t \Big] + z^H \pi \Big[f_t(1+\theta_t) + 1 - f_t \Big] \right)^\alpha . \tag{15}$$

It remains to describe the evolution of the aggregate net worth Ω_t and the share f_t of net worth owned by firms with credit market access. The aggregate net worth in period $t+1$ is

$$\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \left\{ (1-\pi)\bar{\rho}_t + \pi f_t \Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_{t+1})\zeta \Big] + \pi(1-f_t) \right\} . \tag{16}$$

In period t, all firms save fraction β of their net worth. Fraction $1-\pi$ are unproductive and earn return $z^H \Pi_t \bar{\rho}_t = \bar{R}_t$. Fraction πf_t of aggregate savings is invested by borrowing firms of which fraction $1 - G(\tilde{\eta}_{t+1})$ do not default and $G(\tilde{\eta}_{t+1})$ default in $t+1$. Fraction $\pi(1-f_t)$ of aggregate savings is invested by productive firms without credit market access who earn return $z^H \Pi_t$.

The net worth of firms with credit market access in period $t+1$ is

$$f_{t+1}\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \left\{ (1-\pi) f_t \bar{\rho}_t + \pi f_t (1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + (1-f_t)\psi[(1-\pi)\bar{\rho}_t + \pi] \right\} .$$

The right-hand side of this equation is explained as follows. Fraction f_t of net worth is owned by firms with access to the credit market in period t. Fraction $1-\pi$ of these firms earn $\bar{\rho}_t z^H \Pi_t$, and fraction $\pi(1-G(\tilde{\eta}_{t+1}))$ of firms borrow and do not default, earning return $[1+\theta_t(1-\rho_t)] z^H \Pi_t$. All these firms retain access to the credit market in the next period. Fraction $1-f_t$ of net worth is owned by firms without access to credit in period t. They earn $\bar{\rho}_t z^H \Pi_t$ with probability $1-\pi$, and $z^H \Pi_t$ with probability π, and they regain access to the credit market with probability ψ. Adding up the net worth of all these firms gives the net worth of firms with credit market access in period $t+1$, $f_{t+1}\Omega_{t+1}$. Division of this expression by (16) yields a dynamic equation for f_t.

$$f_{t+1} = \frac{f_t \Big[(1-\pi)\bar{\rho}_t + \pi(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) \Big] + (1-f_t)\psi[(1-\pi)\bar{\rho}_t + \pi]}{(1-\pi)\bar{\rho}_t + \pi f_t \Big[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_{t+1})\zeta \Big] + \pi(1-f_t)} . \tag{17}$$

A *competitive equilibrium* describes wages, credit contracts, aggregate net worth and capital, policy and value functions of firms such that: (i) firms make optimal savings and borrowing decisions, and borrowing firms decide optimally about default; (ii) banks make zero expected profits by offering standard debt contracts to borrowers and safe interest rates to lenders; (iii) the labor and the capital market are in equilibrium. The characterization of equilibrium described above is summarized as follows.

Definition 1. *Given an initial state* (f_{-1}, Ω_{-1}) *and an exogenous stochastic process for the state* $X_t = (A_t, \lambda_t, \Phi_t, \varepsilon_t^b)$*, a competitive equilibrium is a mapping* $(f_{t-1}, \Omega_{t-1}, X_t) \to (f_t, \Omega_t, X_{t+1})$*, together with a stochastic process for* $(\tilde{\eta}_t, \theta_t, \rho_t, \bar{\rho}_t, v_t, \Pi_t, w_t)$ *as a function of* $(f_{t-1}, \Omega_{t-1}, X_t)$*, satisfying the equations (7), (10), (11), (12), (13), (14), (15), (16), (17).*

In Appendix B we describe the steady-state (i.e. balanced growth where Ω_t/A_t is stationary) solutions of this model. We focus on dynamics around those steady states where $\bar{\rho} = \gamma$, which implies that some capital is used in low-productivity firms so that aggregate factor productivity responds endogenously to the state of the credit market.

As in the illustrative example of the previous section, this more general model typically generates two steady states, one of which is locally indeterminate and hence susceptible to sunspot shocks. Key for the possibility of self-fulfilling beliefs is the forward-looking dynamics of credit market expectations, described by equation (13), which entails a positive relationship (a dynamic complementarity) between future credit market values and today's value.

To illustrate this idea, rewrite equation (13) as $v_t = \mathbb{E}_t f(\tilde{X}_t, \tilde{X}_{t+1}, v_{t+1}) + \mathbb{E}_t \varepsilon_{t+1}^b$, where $\tilde{X}_t = (A_t, \lambda_t, \Phi_t)$ is the fundamental state vector, and ε_{t+1}^b is a random variable with mean zero and variance σ_b^2 ("belief shock"). If the function f is monotonically increasing in v_{t+1}, this equation can be solved for $v_{t+1} = \tilde{f}(\tilde{X}_t, \tilde{X}_{t+1}, v_t - \varepsilon_{t+1}^b)$, where $\tilde{f}(X_1, X_2, .)$ is the inverse of $f(X_1, X_2, .)$. If the steady state is locally indeterminate, this forward solution of equation (13) is a stationary process, implying that v_t can be treated as a *predetermined* variable which is subject to changes in self-fulfilling beliefs in period $t+1$. That is, the realization ε_{t+1}^b alters credit market expectations v_{t+1} which, in turn, impacts the default threshold in period $t+1$ via equation (10).

Note that, because $\tilde{f}$ is increasing in $v_t - \varepsilon_{t+1}^b$, *positive* realizations of the belief shock affect credit market expectations *negatively*, raising default rates. In the quantitative analysis of the next section, we allow the belief state ε_{t+1}^b to depend on fundamental shocks as well as on pure sunspot shocks that are unrelated to fundamentals. Such fundamental or non-fundamental shocks all correspond to self-fulfilling belief changes, as long as they satisfy the restriction $\mathbb{E}_t \varepsilon_{t+1}^b = 0$.

Different from the illustrative example in Section 2, we focus the quantitative analysis of the next section on local dynamics around the model's indeterminate steady state. As in Section 2, the indeterminate steady state features a higher v and therefore a larger volume of credit. The other (determinate) steady state does not permit local dynamics driven by with self-fulfilling beliefs.

4 Quantitative Analysis

In this section, we explore the quantitative implications of this model. We calibrate the model to suitable long-run targets, and we find that the model steady state is indeterminate. We employ in this paper the concept of a *risky steady state* (cf. Coeurdacier et al. (2011)). The risky steady state is a stationary equilibrium of our model given that all shock realizations are zero, while agents still take aggregate risk into account.

The risky steady state differs from the deterministic steady state due to the fact that the presence of aggregate risk alters the agents' decision rules. In our model, the presence of risky beliefs takes an impact on credit contracts (interest rates and leverage) via equations (11) and (12). In particular, risky beliefs affect the credit spread in our model, and this is why we use the credit spread to pin down the variance of belief shocks in our calibration. For this reason, several model parameters must be jointly calibrated with the exogenous parameters for the stochastic shock processes.

After describing our data in Section 4.1., we calibrate those parameters which can be set irrespective of the shock processes (Section 4.2). To analyze the dynamics around the risky steady state, we consider two scenarios. In Section 4.3, we study an economy with only sunspot shocks in order to highlight the role of self-fulfilling beliefs in isolation from other factors. In Section 4.4, we consider an economy with shocks to excess-bond premia, collateral, sunspots, and productivity, where we use credit spreads, recovery rates, default rates, and output growth to back out all the four shocks. Finally, in Section 4.5, we illustrate that modeling default is important quantitatively. Without default risks, the real effect of belief variations will be much larger.

4.1 Data

We obtain data for the recovery rate and the all-rated default rate for Moody's rated corporate bonds, covering the period 1982–2016, all in percentage terms, and we use the credit spread index developed by Gilchrist and Zakrajšek (2012) that is representative for the full corporate bond market. Moody's data are obtained from the 2016 annual report published by Moody's Investors Service. The recovery rate is measured by the post-default bond price for one dollar repayment. Regarding the spread series, we consider annual averages of the monthly series, updated until 2016.[24] Output is defined as the sum of private consumption and private investment in the U.S. national accounts, as we have a model of a closed economy without government. Output growth refers to the growth rate of real per capita output.

[24] See Simon Gilchrist's website http://people.bu.edu/sgilchri/Data/data.htm

Table 1: **Data vs. Model with Only Sunspot Shocks (Model Values in Brackets)**

Correlation	Spread	Recovery Rate	Default Rate	Output Growth
Spread	1	-0.40 (0.91)	0.64 (0.55)	-0.58 (-0.75)
Recovery Rate	-	1	-0.76 (0.16)	0.33 (-0.61)
Default Rate	-	-	1	-0.54 (-0.57)
Output Growth	-	-	-	1
Mean (%)	2.01 (2.01)	41.17 (41.17)	1.58 (1.58)	1.70 (1.70)
Std dev. (%)	0.86 (0.30)	8.97 (4.42)	1.05 (2.67)	1.90 (2.05)

Table 1 shows the basic statistics of these four variables. The sample means of credit spread, recovery rate, and default rate are 2.01%, 41.17%, and 1.58%, respectively. As expected, the spreads and default rate are highly positively correlated (0.64), and both of them are countercyclical (i.e., negatively correlated with output growth). The recovery rate is highly negatively correlated with the default rate (-0.76), but much less with the credit spread and it is mildly procyclical (i.e., the correlation with output growth is 0.33).

Time series of the three variables are shown in Figure 2. Evidently, the default rate spikes up in all three recessions since 1982, and most strongly during the Great Recession. The recovery rate reaches a trough during any recession. Interestingly, however, credit spreads did not increase during the 1991 recession; this further motivates the need to explore the distinct roles of credit spreads and corporate default for macroeconomic dynamics.

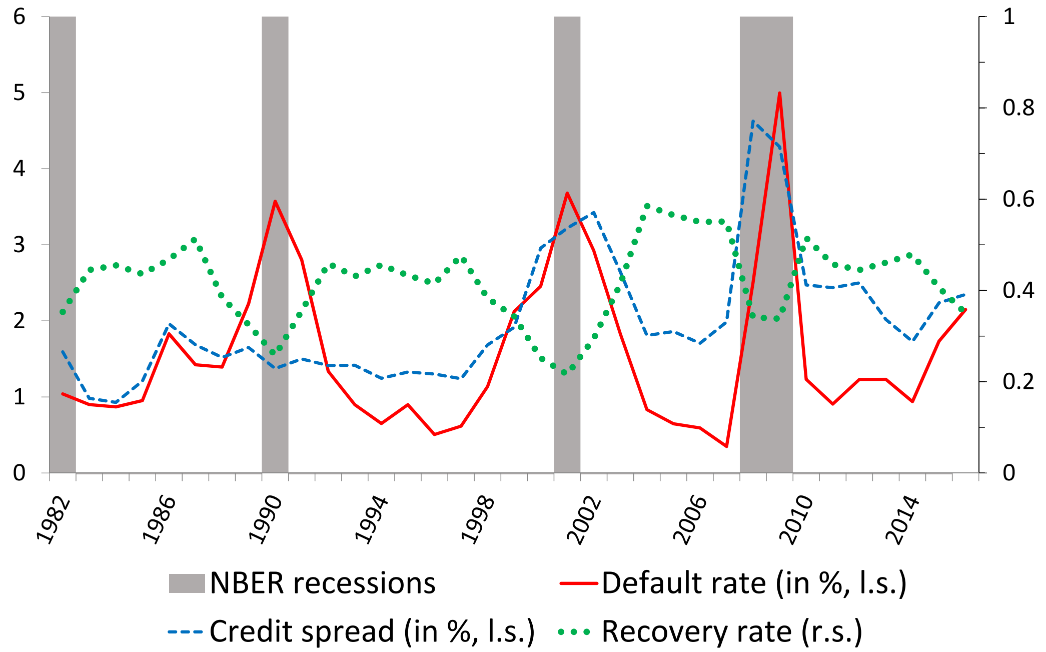
We will compare the same set of time series generated by the model to those in the data. In the model, default and output growth are straightforward. The credit spread in the model is $100(\rho_t/\bar{\rho}_t - 1)$ in percentage terms. For the (ex-post default) recovery rate, the consistent measure in the model is $\lambda_t/\xi_{t-1} = \lambda_t \Pi_{t-1} z^H \frac{1+\theta_{t-1}}{\theta_{t-1}} / R_{t-1}$ which is the ratio of the firm value recovered by the lenders per unit of debt to the lending rate R_{t-1}.

4.2 Basic Parameterizations

For analytical tractability, we assume that default costs η are normally distributed with mean μ and variance σ^2. Given that we consider annual time series for default rates and recovery rates, we calibrate the model at annual frequency. There are 15 model parameters that are not related to exogenous stochastic processes.

1. Preferences: β, κ, and ν.
2. Technology: α, δ, μ^A, z^H, z^L, and π.
3. Financial markets: ψ, λ, ζ, Φ, μ, and σ.

Figure 2: **Default Rates, Credit Spreads and Recovery Rates**



Directly calibrated are $1 - \alpha = 0.67$ (labor share), $\delta = 0.10$ (annual depreciation rate), $\mu^A = 0.017$ (growth rate of per capita output), $1-\zeta = 0.15$ (direct net-worth losses in default, see Davydenko et al. (2012)), $\psi = 0.1$ which implies a ten-year exclusion period.[25] We set the steady-state excess bond premium parameter to $\Phi = 0$, which implies that the credit spread in steady state is only accounted for by the default risk. Importantly, the default risk comes from both fundamentals (recovery ability and the distribution of default costs) and non-fundamentals (variation in beliefs). It is the risk of non-fundamental variations that allows the model to generate a large enough average credit spread as in the data given the average default rate (see more below).

According to Fiorito and Zanella (2012) and Keane and Rogerson (2012), the macro labor supply elasticity that allows for both intensive and extensive margin adjustments should be 1.5–2, so we set the elasticity to $1/\nu = 1.5$. We then set $\kappa = 2.38$ by arbitrarily normalizing steady-state labor supply at 0.25. We set probability $\pi = 0.20$ so that 20 percent of firms are financially constrained (see, e.g., Almeida et al. (2004) and Kaplan and Zingales (1997)).

[25]This corresponds to the bankruptcy flag for sole proprietors (or for partnerships with personal liabilities) filing for bankruptcy under Chapter 7 of the U.S. Bankruptcy Code.

Table 2: **Directly Calibrated Parameters**

Parameter	Value	Explanation/Target
α	0.33	Capital income share
δ	0.10	Depreciation rate
μ^A	1.72%	Trend growth
κ	2.38	Labor supply $\ell = 0.25$
ν	0.67	Macro labor supply elasticity $1/\nu = 1.5$
π	0.20	Constrained firms (Almeida et al. 2004)
ψ	0.10	10-year default flag
ζ	0.85	15% default loss (Davydenko et al. 2012)
Φ	0.00	Zero excess bond premium in steady state

See Table 2 for a summary of these parameter choices. Since we calibrate our model around a risky steady state, the values of all other parameters depend on which shocks are active. In the next section, we first consider the scenario where sunspot shocks are the only source of aggregate risk. Then, we turn to the case where also fundamental shocks are active.

4.3 Sunspot Shocks Only

In this section, we examine to what extent belief shocks alone can explain the data. To be specific, only belief shocks ε_t^b are random variables with mean zero and variance σ_b^2. The exogenous variables (λ_t, Φ_t) are at the steady state constant, and A_t grows at the constant rate μ^A (the economy is on the balanced growth path). Although this exercise is rather restrictive and gives too much power to sunspot shocks, it helps to understand the impact of belief variations on both financial and real variables.

Calibration with Risky Steady State

We log-linearize the system around the risky steady state. More precisely, we approximate those equations with expectation terms around the risky steady state to the second order by hand; doing this allows future risks to affect the steady state solution. We then solve the risky steady state around which we log-linearize (see Appendix B). Here, we explore the equilibrium dynamics in response to sunspot shocks alone. Using the zero-profit condition of lenders, equation (11), the variance of sunspot shocks is identified from the credit spread target. The rest of the parameters is then calibrated in a straightforward way (and is *independent* of aggregate dynamics).

Table 3 summarizes all the parameters from the calibration. We normalize average capital productivity at $\tilde{z} = \pi z^H + (1-\pi)z^L + f\pi\theta(z^H - z^L) = 1$.[26] The normalization pins down z^L, given parameters π, z^H, the debt-to-equity ratio θ, and the steady-state value of the fraction of firms with credit market access f.

Table 3: **Parameters (Risky Steady State with Only Sunspot Shocks)**

Parameter	Value	Explanation	Target
β	0.96	Discount factor	Capital-output ratio 200%
λ	0.20	Recovery parameter	Recovery rate 41.74%
σ_b	4.81%	Std. dev. of belief shocks	Credit spread 2%
z^H	1.13	High productivity	Debt-output ratio 82%
z^L	0.79	Low productivity	Average productivity $\tilde{z} = 1$
μ	-0.24	Mean of η	Default rate 1.58%
σ	7.24%	Std dev of η	Leverage $\theta = 2.1$

The remaining six parameters are calibrated jointly to match the following targets in the risky steady state (i.e., when the risk of sunspot shocks is taken into account): (i) the capital-output ratio $K/Y = 2$; (ii) the credit-output ratio $B/Y = 0.82$, based on all (non-financial) firm credit 1982–2016; (iii) the leverage ratio $\theta = 2.1$ in credit-constrained firms;[27] (iv) a recovery rate of 41.74%; (v) a 1.58% default rate; (vi) a 2% credit spread (see subsection 4.1). These targets identify the six parameters β, μ, σ, $\gamma = z^L/z^H$, λ, and the sunspot variance σ_b^2 uniquely (see Appendix B).

Intuitively, the discount factor β determines the investment rate and thus the capital-output ratio. The average default cost μ determines the default rate. The recovery parameter λ is identified from the recovery rate. The variance of sunspot shocks σ_b^2 is calibrated to match the fraction of the spread not accounted for by expected fundamental default losses.

Here the concept of a risky steady state is important: due to the non-linear distribution function $G(.)$ of default costs, the variance of sunspot shocks matters for steady-state values of the interest rate and of leverage. To illustrate the idea, suppose $\tilde{\eta}_t^e = \mathbb{E}_t[\tilde{\eta}_{t+1}]$ is the expected default threshold at time t. Then, the expected default rate that includes belief shocks ε_{t+1}^b is

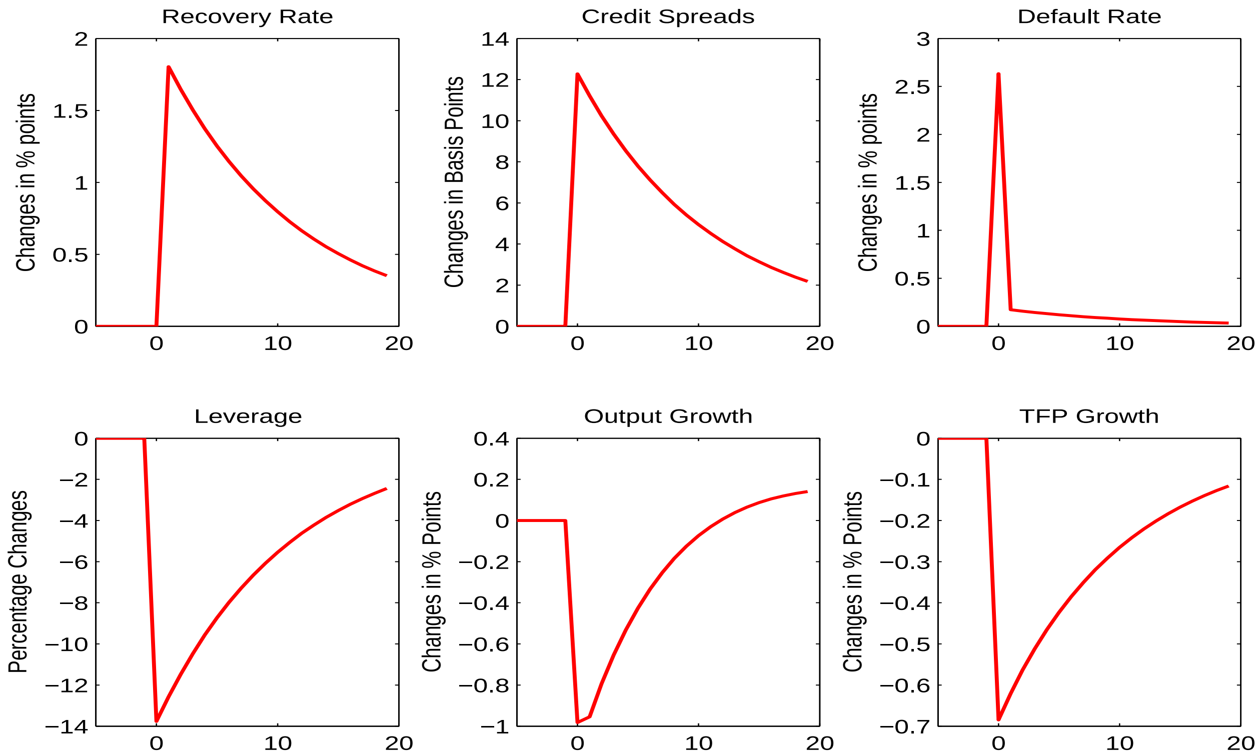
$$\mathbb{E}_t[G(\tilde{\eta}_{t+1})] = \mathbb{E}_t[G(\tilde{\eta}_t^e + \varepsilon_{t+1}^b)] \approx G(\tilde{\eta}_t^e) + 0.5G''(\tilde{\eta}_t^e)\sigma_b^2 \ ,$$

The steady-state expected default rate is thus larger than the fundamental default rate $G(\tilde{\eta}^e)$

[26] Without this normalization, δ would not be the depreciation rate of this economy: $(1-\delta)\tilde{z}K_t$ of the capital stock survives to the next period. Hence, depreciation is $K_t - (1-\delta)\tilde{z}K_t$, and the depreciation rate is $1-(1-\delta)\tilde{z}$ which equals δ if and only if $\tilde{z} = 1$.

[27] This corresponds to the 81st percentile of debt-equity ratios of firms in the Survey of Small Business Finances 2003 (Federal Reserve Board) and to the 83rd percentile in COMPUSTAT. Both are in line with the fact that about 20 percent of firms are constrained in our model.

Figure 3: **Impulse Responses to a Sunspot Shock**



at the calibrated steady-state value for $\tilde{\eta}$ where $G''(\tilde{\eta}^e) > 0$. This is because we target $G(\tilde{\eta}^e) = 1.58\%$ and $G(.)$ is convex at this point. At the risky steady state, the threshold $\tilde{\eta}$ is a constant as $\tilde{\eta}^e$, but the effect of ε^b_{t+1} on default risk is nonetheless taken into account in every period t.

The remaining two parameters, the variance of default costs σ and the productivity ratio γ, are determined from average credit and from the leverage ratio of constrained firms.

Impulse Responses and Model Fit

To gain intuition for the role of the sunspot shocks, we illustrate the transmission mechanisms by impulse response functions for various variables of interest. The economy starts from steady state and is hit by one standard deviation of a one-time sunspot shock that raises default in period zero (see Figure 3).

The shock raises the default rate on impact by 2.6 percentage points after which default falls back but remains persistently slightly above the steady-state level. An important consequence of sunspot shocks is that the leverage ratio falls significantly on impact and persistently, because credit market valuations (default incentives) remain persistently low (high) from time zero onward. Lenders, who take these incentives into account, tighten the credit constraints and charge (slightly) higher interest rates (about 12 basis points higher on impact). Since the leverage ratio is much lower (14% lower on impact), the recovery rate per unit of lending rises (1.8 percentage points on impact). Through tightened credit constraints, lenders are able to recover a greater share of their exposure after a default.

The persistent response of all variables (e.g., 10 years for output growth and more than 20 years for leverage ratio) is the key to sustain a self-fulfilling credit cycle. In fact, if the deterioration of credit conditions was rather short-lived, the value of credit market access would not fall much, which implies that default rates would go up only little at the time of the sunspot shock. That is, sizable responses of default rates require a persistent credit market response.

Productive firms as borrowers are hurt by this disturbance in the credit market. Because of the fall of leverage, these firms use a smaller share of the aggregate capital stock which dampens aggregate productivity. Therefore, we observe an endogenous fall of TFP which results in a 1 percentage-point reduction of the output growth rate, followed by a persistent reduction in subsequent years. After almost ten years, output growth is back to the steady state.

We next compare moments implied from the simulated model to those in the data (back to Table 1). The correlations among spreads, default, and output growth in the model are rather similar to the data, i.e., 0.55, -0.75, and -0.57 generated by the model compared to 0.64, -0.58, and -0.54 in the data. However, business cycles induced by sunspot shocks miss the co-movements of the aggregate recovery rate. Regarding volatility, the standard deviation of output growth is similar in the model and in the data. But credit spreads driven by sunspots have a standard deviation (0.30) that is about 35% of its data counterpart (0.86), while sunspots generate too volatile (about 1.54 times more volatile) default rates . These findings should not be too surprising, given that beliefs are the only source of aggregate risk in this exercise.

The reason why sunspot shocks miss the recovery co-movements is straightforward. When a sunspot shock that raises the default rate arrives, lenders tighten credit conditions mainly by reducing the amount of credit to protect default losses (while raising only slightly the lending rate). Ex-post recovery then actually goes up. This counterfactual result suggests that there should be exogenous components that move the recovery rate. Further, the too low volatility of credit spreads in the model suggests to introduce exogenous shocks to the excess bond premium. Thus, to bring our model closer to the data, we introduce these shocks in the next section.

4.4 Fundamental and Sunspot Shocks

In the previous exercise, sunspot shocks are the only exogenous shocks. We now include fundamental shocks to the collateral parameter, excess bond premium, and to output growth so as to quantify the respective contributions of fundamental and non-fundamental shocks to financial and macroeconomic volatility. We first describe the estimation procedure and show the smoothed shocks from the maximum likelihood estimation. Second, we illustrate the impulse responses after one standard deviation for each of the shocks. Third, we show the variance decomposition into the four independent shocks, and we examine the contribution of the credit market expectations channel for macroeconomic volatility.

Estimation Procedure and Estimated Shocks

We log-linearize the system around the risky steady state and express the system in a Kalman filter form. Now, the risks of the variations in recovery ability λ_{t+1} are also factored in, when we calculate the risky steady state (again, see Appendix B). We then explore the equilibrium dynamics in response to all shocks. That is, besides estimating the shocks to μ_t^A, we estimate shocks to the recovery parameter λ_t, to the excess bond premium Φ_t, and to sunspots.

We use the time series data for spreads, recovery rate, default rate, and output growth described above. We use the maximum likelihood method and estimate AR(1) processes for Φ_t and λ_t, and beliefs ε_t^b which satisfy

$$\begin{aligned}
\log(1+\Phi_t) - \log(1+\Phi) &= \rho_\Phi \left[\log(1+\Phi_{t-1}) - \log(1+\Phi)\right] + \varepsilon_t^\Phi \ , \\
\log(1-\lambda_t) - \log(1-\lambda) &= \rho_\lambda \left[\log\left(1-\lambda_{t-1}\right) - \log\left(1-\lambda\right)\right] + \varepsilon_t^\lambda \ + \chi_\lambda^\Phi \varepsilon_t^\phi \ , \\
\varepsilon_t^b &= \chi_b^\Phi \varepsilon_t^\Phi + \varepsilon_t^s \ ,
\end{aligned}$$

where ρ_Φ and ρ_λ are persistence parameters, and ε_t^Φ, ε_t^λ, and ε_t^s are i.i.d. normally distributed with mean zero and variances σ_Φ^2, σ_λ^2, and σ_s^2. These random variables are called below "EBP shocks" (where EBP stands for excess bond premium), "collateral shocks", and "sunspot shocks", respectively. Collateral shocks are essentially credit demand shocks, while EBP shocks affect the banks' willingness to supply credit. Sunspot shocks could reflect both credit demand and supply, because both of them depend on beliefs.

Further, we allow changes in the excess bond premium to take an impact on recovery and beliefs, which are reflected in the terms χ_λ^Φ and χ_b^Φ. This follows the empirical finding of Gilchrist and Zakrajšek (2012) in which excess bond premia can predict changes in real and financial variables. For example, high excess bond premia may reflect a situation where assets cannot be easily liquidated so that the recovery of defaulted corporate bonds is lower. Next

Table 4: **Parameters (Risky Steady State with All Shocks)**

Parameter	Value	Explanation	Target / T statistics (std errors)
β	0.96	Discount factor	Capital-output ratio 200%
λ	0.20	Recovery parameter	Recovery rate 41.74%
σ_b	3.42%	Std. dev. of belief shocks	Credit spread 2%
z^H	1.13	High productivity	Debt-output ratio 82%
z^L	0.79	Low productivity	Average productivity $\tilde{z} = 1$
μ	-0.23	Mean of η	Default rate 1.58%
σ	7.31%	Std. dev. of η	Leverage $\theta = 2.1$
σ_s	2.69%	Std. dev. of pure sunspots	Constrained $\sigma_b^2 = \sigma_s^2 + (\chi_b^\Phi)^2\sigma_\Phi^2$
ρ_Φ	0.73	Persistence of EBP shocks	Estimated: 6.22 (0.12)
ρ_A	0.25	Persistence of productivity shocks	Estimated: 1.23 (0.20)
ρ_λ	0.58	Persistence of collateral shocks	Estimated: 6.55 (0.09)
σ_Φ	0.0087	Std. dev. of EBP shocks	Estimated 10.27 (0.0009)
σ_A	0.0334	Std. dev. of productivity shocks	Estimated 7.81 (0.0043)
σ_λ	0.0313	Std. dev. of collateral shocks	Estimated 11.63(0.0027)
χ_b^Φ	2.4279	Spill over to beliefs variation	Estimated 3.54 (0.69)
χ_λ^Φ	0.0650	Spill over to collateral shocks	Estimated 5.80 (0.01)

to the impact of EBP shocks on credit market expectations, reflected in the parameter χ_b^Φ, there are also pure sunspot shocks ε_t^s that directly affect the belief state ε_t^b. Note that the requirement $\mathbb{E}_t\varepsilon_{t+1}^b = 0$ is satisfied which implies that belief shocks are self-fulfilling.

Finally, we also estimate an AR(1) process for productivity growth μ_t^A,

$$\log(1+\mu_t^A) - \log(1+\mu^A) = \rho_A\left[\log\left(1+\mu_{t-1}^A\right) - \log\left(1+\mu^A\right)\right] + \varepsilon_t^A \ ,$$

where ρ_A is another persistent parameter and ε_t^A is i.i.d. normally distributed with mean zero and variance σ_A^2.

Table 4 presents the estimation results and the targets used to compute the other parameters. Note that the calibration of the other model parameters and the estimation of shock parameters need to be *jointly* implemented. This is because the risky steady state depends on the dynamics (because the variance of belief shocks enters credit market variables), and the dynamics depends on the calibrated steady state (around which the model is log-linearized). For details how we calculate these parameters from the calibration targets, see Appendix B.

Compared to the case with sunspot shocks only, β, λ, z^H, and z^L are almost unchanged (some differences only in the 4th digit which are not reported). μ is slightly larger and σ is slightly smaller. As expected with the introduction of fundamental shocks, the standard deviation of the belief shocks σ_b^2 is 3.42%, a 29% reduction in the size of these shocks in the previous scenario (4.81%). The size of pure sunspot shocks is even smaller (2.69%). Therefore, the spill-over effect from EBP shocks accounts for 21% of the size of belief shocks, reflecting the importance of EBP shocks on the belief channel (to be discussed below). We interpret this result as follows. The EBP is partially related to lenders' risk appetite,[28] regardless of the actual default risk. This change in the view of risks can also lead to a change of expected credit market conditions.

EBP shocks seem to generate the most persistent responses ($\rho_\Phi = 0.73$ is the largest among persistence parameters). The estimates of standard deviations of all shocks are highly significant, implying that all fundamental shocks are indeed important to capture different aspects of the business cycle. Only the persistence of productivity shocks, ρ_A, is not statistically significant (and the persistence is rather small compared to the persistence of shocks to EBP and collateral). Finally, the spill-over effect from the EBP on the collateral value is much smaller than the spill-over effect from EBP on belief variation.

All four estimated shocks (at their mean levels) from Kalman smoother are plotted in Figure 4, normalized by their respective standard deviations. Because of the specification of the processes, positive innovations to EBP, collateral, and beliefs, imply a higher premium, a lower recovery rate, and a higher default rate. That is, positive shocks stand for adverse financial shocks.

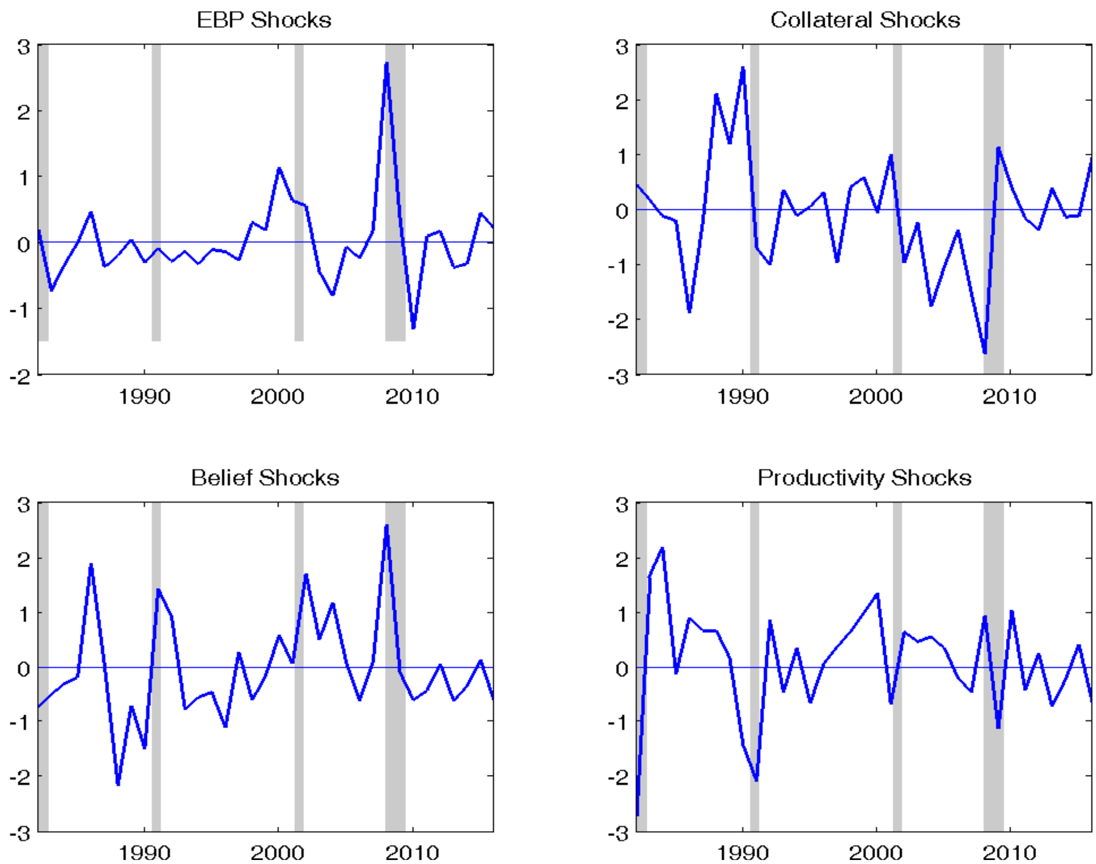
Through the lens of our model, the 2007-2009 recession is indeed special compared to the previous ones. It has a combination of a fall in recovery ability (about one standard deviation increase in $1 - \lambda$, corresponding to a 6% fall in λ), deteriorated credit market expectations (two standard-deviation increase of the belief state, corresponding to a 2.8 percentage-points increase in the default probability), and the recession is led by a larger-than-usual EBP (close to a three standard-deviation increase or 260 basis points increase). Aggregate productivity shocks do not show a clear pattern but are negative in 2007 and 2009.

The Great Recession featured a large liquidity and pledgeability drop of financial assets, which is captured in our model by the positive shocks to $1 - \lambda$ (i.e., a fall in recovery ability). Note also the negative shocks to $1 - \lambda$ in the years prior to the Great Recession which may reflect the real-estate boom and the surge of collateral assets in this period, leading to a higher-than-usual recovery ability. After the recession, recovery rises for some period, reflecting the asset-purchase programs implemented by the Federal Reserve in 2009-2010.

The positive EBP shocks in 2007 and 2008 generate the sharp increase of the excess bond premium induced by the banking crisis at the onset of the financial crisis. Notice also the sharp fall of innovations to Φ_t in 2009 and 2010. We can interpret this result as a consequence

[28] See a recent estimation of the measure of risk appetite by Pflueger et al. (2018) which is related to belief variations.

Figure 4: **Estimated Shocks at the Mean Levels**
Note: All shocks are normalized by their respective standard deviations. Shaded areas are NBER dated recessions.



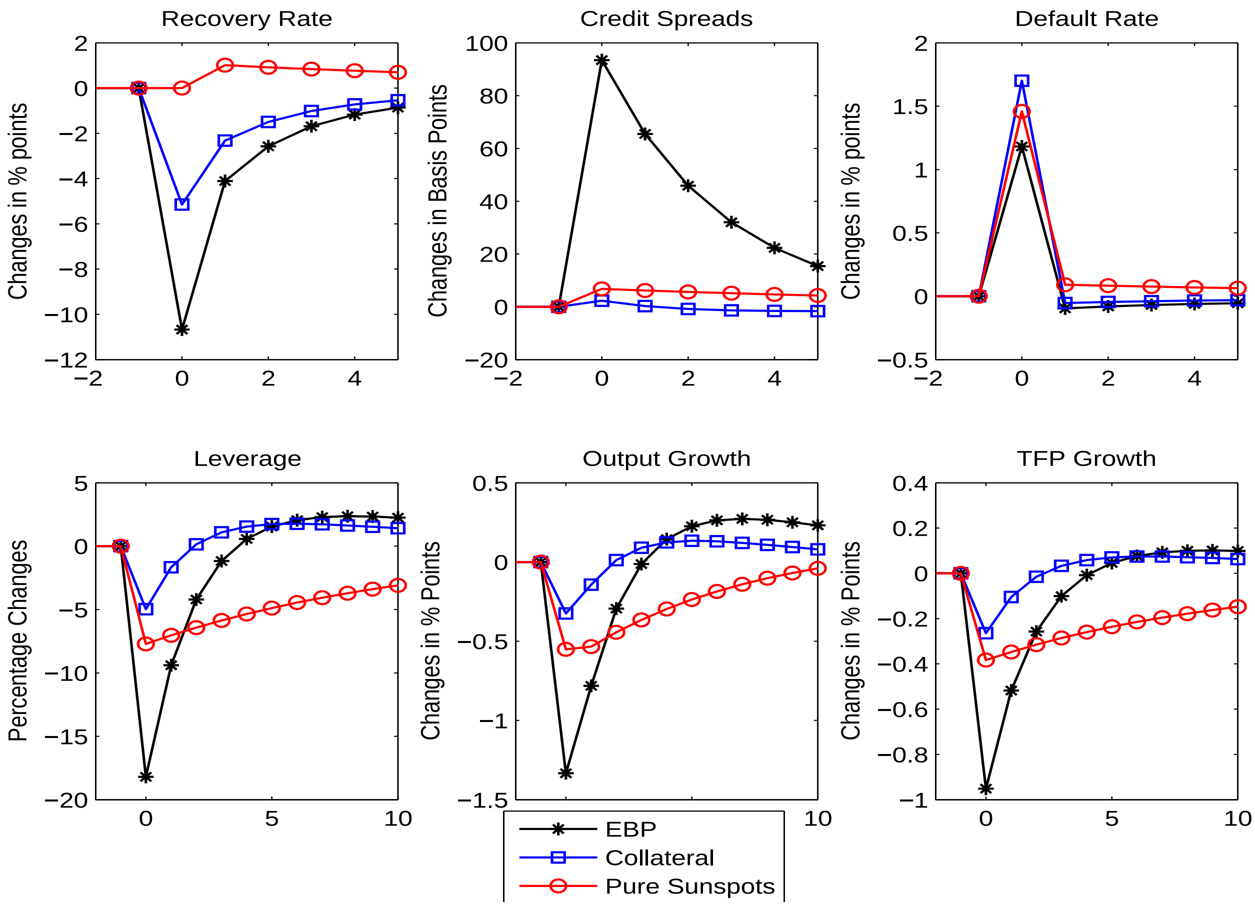
of government intervention in asset markets which may have significantly reduced risk aversion and other factors such as liquidity risks, contributing to the fall in bond premia. We observe a deterioration of credit market expectations (positive belief shocks) prior to all three recessions since 1982. As shown above, credit spreads did not increase during the 1991 recession, despite a significant increase of the default rate. This is mirrored in the absence of positive shocks to EBP in this period.

Impulse Responses

We plot impulse response functions after all three financial shocks in Figure 5.

A positive one standard-deviation innovation in $(1-\lambda)$ (collateral shocks) lowers the recovery value, so that lenders tighten credit on impact (5% fall in leverage) and charge a (slightly) higher interest rates (3 basis points more) to compensate for the losses. But the fall in leverage only lasts for 2 years and then overshoots, as the autocorrelation of λ is rather

Figure 5: **Impulse Responses after Adverse Financial Shocks**



small. Since the surprise fall in recovery value gives the borrowers more incentive to default on impact, we have a 1.7 percentage-point spike of the default rate. But then the default rate immediately falls below the steady-state level, as lenders fully take into account the changes of λ into the credit contract. One can also understand the fall of default from the borrowers' point of view. In expecting higher leverage in the future, firms have less incentives to default which is why the default rate falls (about -0.05 percentage points below the steady-state value) immediately after the initial rise.

In response to a rise of the excess bond premium, the credit spread increases significantly (92 basis points) and the default rate rises by 1.2 percentage points. Since the higher excess bond premium also reduces the recovery parameter, leverage and output dynamics are similar to those after a collateral shock. One should notice that the credit-market response of the excess bond premium is also driven by the spill-over impact of the shock on the recovery parameter λ_t and on beliefs ε_t^b. Consistent with Gilchrist and Zakrajšek (2012), the credit-spread shock induces a pronounced decline of output growth, while the default rate spikes on impact and immediately falls back to the steady-state level. This is because the fall of leverage offsets the negative impact on default incentives.

A pure sunspot shock generates much similar dynamics as in the exercise with only belief variations. Because of the presence of other shocks, the magnitudes are smaller, however. For example, the default rate increases by 1.5% on impact (compared with 2.6% in the previous section). The recovery rate again rises slightly which is explained by the fall of leverage. The crucial difference compared to shocks to the collateral value and to the excess bond premium is the persistence: default remains (slightly) above the steady-state level and leverage remains persistently below the steady-state level. Therefore, TFP growth and output growth are lower by 0.38 and 0.55 percentage points on impact and they are persistently (again, at least 10 years) below their respective steady-state levels. Notice that the dynamics of financial variables (excluding leverage) are much shorter lived than the dynamics of real variables and leverage. To visualize this, we plot the impulses of the financial variables in the first row up to year five (where year zero is the time when shocks hit), and the impulses of the remaining variables in the second row up to year ten. As a comparison, for the previous two fundamental financial shocks, the responses of all financial and real variables are rather short-lived.

Variance Decomposition

We now examine how much of the variation in the data can be separately explained by the three financial shocks and by aggregate productivity shocks. Then, we illustrate the channels through which these shocks operate: self-fulfilling credit market expectations, on the one hand, or fundamental variables, on the other hand. This is a non-trivial exercise because EBP shocks affect fundamental variables and beliefs at the same time.

Evidently from Table 5, the dynamics of recovery rates is mainly explained by collateral shocks and EBP shocks (due to the spill-over effect). Collateral shocks directly affect the recovery ability, while EBP shocks affect both spreads and the recovery ability. The variations of default rates are explained by all three financial shocks: direct sunspot shocks (33.38%), collateral shocks (44.56%), and EBP shocks (22.06%). Sunspot shocks and EBP shocks affect belief variations which then affect defaults. Collateral shocks also change the default incentives on impact. Credit spread fluctuations are predominately explained by EBP shocks which take a direct impact on spreads. Finally, by construction, shocks to productivity do not affect credit contract. This is because in the type of equilibria we focus on, z^L firms are indifferent between production and lending; $\bar{\rho} = \bar{R}_t/(z^H\Pi_t) = \gamma$ is a constant, and the credit contract shown in Proposition 4 is thus not affected by productivity.

Regarding output growth, sunspot shocks can explain 17.63% of the variation, while EBP

Table 5: **Variance Decomposition in Percents**

	Exogenous Shocks to				
	EBP (1)	Collateral (2)	Sunspot (3)	Productivity (4)	All financial shocks (1) + (2) + (3)
Credit Spreads	98.25	0.18	1.57	0	100
Recovery Rate	77.15	19.59	3.26	0	100
Default Rate	22.06	44.56	33.38	0	100
Output Growth	41.16	3.32	17.63	37.88	62.12
Debt-to-Output	37.73	5.77	54.25	2.26	97.74
TFP Growth	17.30	1.75	10.72	70.23	29.77

shocks and collateral shocks explain 41.16% and 3.32%, respectively. Notice that collateral shocks generate small impacts on output because the small reactions and overshoots in leverage following the shocks (recall Figure 5). Financial shocks together contribute to about two thirds (62%) of output variations because they affect the credit flow to productive firms. There are two ways how the credit flow impacts output dynamics. On the one hand, the credit flow affects the capital allocation among productive and unproductive firms. This is the *productivity effect* of credit. On the other hand, the credit flow also affects the firms' aggregate demand for capital and labor, and therefore aggregate production. This is the *factor effect* of the credit flow.

To shed light on these two effects, we show how the variation of debt growth and TFP growth in the model can be explained by each shock in the last two rows of Table 5. Endogenous fluctuation of productivity growth due to the credit allocation is about 30%, which is much less important than the exogenous fluctuations in productivity growth (70%). In other words, credit generates modest endogenous variation in TFP growth. Therefore, the main transmission mechanism of financial shocks is through the effect on the firms' factor demands.

While Table 5 reports the decompositions into exogenous shocks, we show now how these shocks impact credit market and macroeconomic variables through fundamental channels and the belief channel separately. This decomposition is important since we allow EBP shocks to affect beliefs as well.

We use a simple R-square statistics for this exercise. Let $\varepsilon_t = [\varepsilon_t^\Phi, \varepsilon_t^\lambda, \varepsilon_t^s, \varepsilon_t^A]'$ be the collection of structural shocks. Then, the belief variable can be expressed as $b_t = c\varepsilon_t$, where $c = [\chi_b^\Phi, 0, 1, 0]$. If we are interested in output growth g_t, for example, we run a population regression $g_t = \beta^b b_t + \nu_t$, where ν_t is orthogonal to b_t. Then, the variation in output growth explained by the belief channel can be expressed as the R-square of this regression.[29] For

[29]That is, $R^2 = \frac{\text{Var}(\beta^b b_t)}{\text{Var}(g_t)}$, where $\beta^b = \frac{\text{Cov}(g_t, b_t)}{\text{Var}(b_t)}$. Therefore, the variance of g_t explained by the belief channel is simply the square of the correlation between g_t and b_t, i.e., $R^2 = (\text{Corr}(g_t, b_t))^2$.

other variables of interests, we simply repeat this procedure.

Table 6: **Variance Decomposition in Percents: Fundamentals versus Beliefs**

	Shocks that change	
	Fundamentals	Beliefs
Credit Spreads	77.04	22.96
Recovery Rate	76.63	23.37
Default Rate	45.96	54.04
Output Growth	78.70	21.30
Debt-to-Output	78.48	21.52
TFP Growth	90.93	10.07

Table 6 shows that shocks that affect beliefs ε_t^b explain about 21.3% of output growth variations. The transmission mechanism can be seen again from a rather small effect on productivity growth: 10% of TFP growth variation is explained by the belief channel which shows that this channel operates mainly through factor demands. The belief channel is particularly important for the volatile default rates (accounting for 54% of default variations), and it also matters, although to a lesser extent, for fluctuations of recovery rates and spreads.

4.5 The Role of Default

In our model, the default rate moves with the business cycle, and it responds strongly to financial shocks. But a natural question arises: Do fluctuations of corporate default matter to understand the impact of financial shocks on macroeconomic aggregates?

To answer this question, we examine an economy without default, in order to contrast its steady-state and dynamic behavior to the one in our baseline scenario with endogenous default.

To be specific, we now assume that the default rate G is zero and that all firms have a fixed threshold default cost $\tilde{\eta} = -\infty$. We further assume that collateral shocks and sunspot shocks are observed when credit contracts are signed. Then firms do not default voluntarily when equation (10) holds (given that the default cost $\tilde{\eta}$ is known). The sunspot shock still affects expected credit market conditions v, and hence leverage in the optimal credit contract, but it does not change the default rate. Furthermore, the variance of belief shocks takes no impact on credit spreads, so that the equilibrium spread offered by the lender is zero and the leverage ratio (2.62) is about 25% higher than in the baseline model (where it is 2.1). In order to have a close comparison, we maintain all parameters and the estimated shock processes.

Figure 6: **Impulse Responses to a Sunspot Shock**

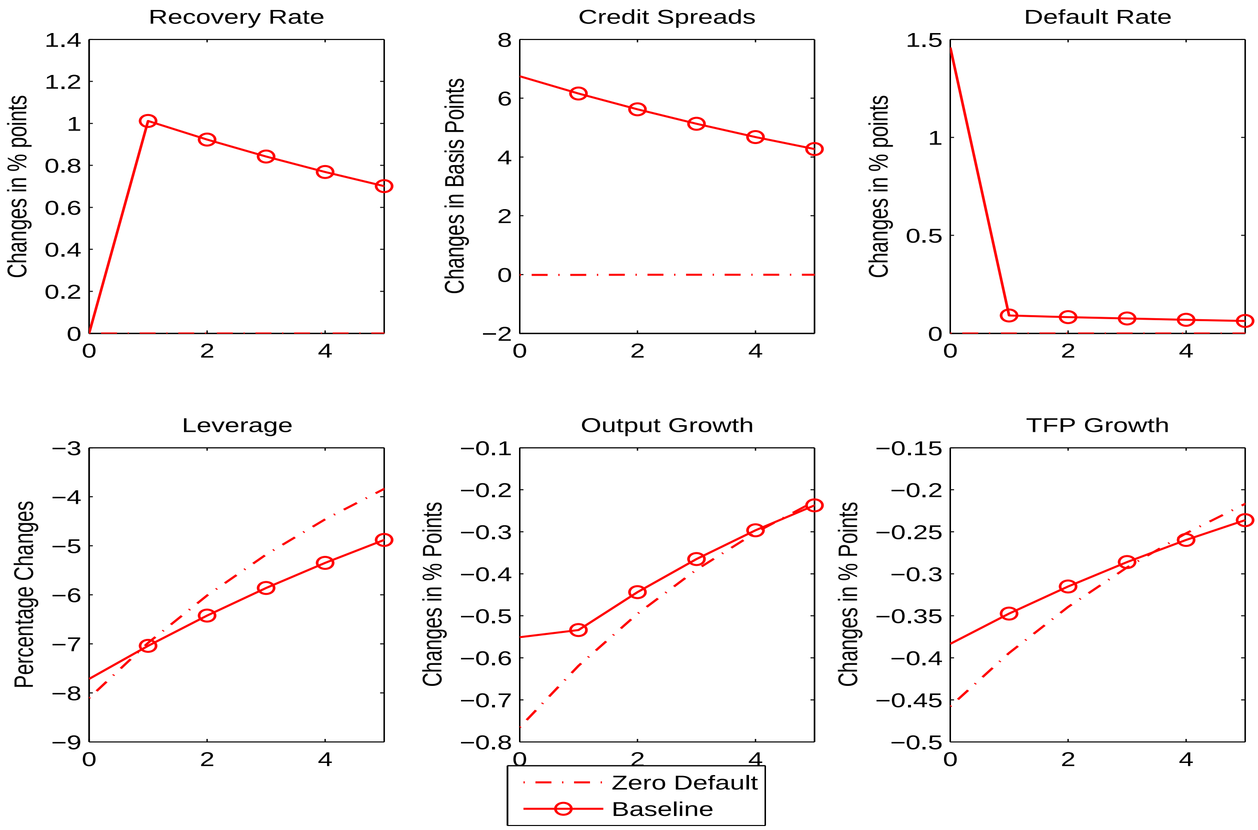


Figure 6 shows the model response to the same one-time sunspot shock (that raises the default rate in our baseline model) but in the counterfactual scenario with a zero default rate (the lines denoted by "Zero Default"). In the latter case, a positive sunspot implies that credit constraints tighten so as to induce firms not to default voluntarily. Since the steady-state leverage is higher in this economy, the leverage is more volatile in response to sunspot shocks compared to the baseline economy. The consequence is a deeper fall both in TFP growth and in output growth. The extra decline of TFP growth is almost 18.5% of that in the baseline on impact, and output growth falls by 0.77 percentage points, compared to 0.55 in the baseline model.

The message from this exercise is that modeling default is important. It certainly affects the financial variables to a large extent, but it also has a non-trivial impact on the real side. The impact of cyclical default affects both the steady state and the dynamics. In particular, the effect of similarly large belief variations could be much larger without modeling default. Lenders do not care about default losses when there are is no default. But default risk becomes quite important when it is taken into account in the dynamics around the risky steady state.

5 Conclusions

We develop a theory of firm default that is susceptible to changes in self-fulfilling beliefs. Variations in credit market expectations affect incentives to default and thereby take an impact on credit spreads and leverage. In turn, credit market conditions react to changes in default rates and interest rates, and this dynamic relationship generates multiple equilibria and the possibility of belief-driven cycles.

We use this idea in a tractable macroeconomic model which we calibrate so as to match selected long-run credit market features for the U.S. economy in order to explore the respective roles of shocks to credit market expectations (sunspots), recovery rates, excess bond premia, and aggregate productivity.

Our findings suggest that default cycles, driven in part by self-fulfilling beliefs, are an important source for output growth variations. Compared to direct financial shocks that affect the recovery ability or risk premia, sunspot shocks can generate a persistent credit cycle and prolonged reductions of credit and output growth. Besides non-fundamental shocks, our estimation shows that shocks to the excess bond premium also significantly affect credit and output. Some of this response is channeled through the impact of the excess bond premium on beliefs (and hence on default).

On the theoretical side, an interesting avenue for further research is the examination of long-term debt for the impact of self-fulfilling beliefs on default rates. One may conjecture that strategic default incentives are less sensitive to market expectations when borrowers hold long-term debt. Nevertheless, the ability of firms to role over long-term debt may react to investors' sentiments, as is known from the literature on sovereign debt cited in the introduction. Regarding policy implications, government policies that alter belief variations could strongly affect economic activity, both in the long run and over the business cycle.

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Appendices

A Proofs and Derivations

Proof of Proposition 1: To characterize the optimal contract (R, b), note first that, conditional on an interest rate and on a default regime, the firm's utility is increasing in b. Hence, b should be as large as possible within a default regime, so that only one of the following three contracts can be optimal:

1. No credit: $b = 0$ with utility $U^0(s) = \log(\Pi s)$.

2. Partial default: $R = \bar{R}/(1-p)$, debt is at the largest level that prevents default in state $\eta = \Delta$, which is $b^D(s) = \frac{\Pi(1-p)(1-e^{-v-\Delta})}{\bar{R}-\Pi(1-p)(1-e^{-v-\Delta})} \cdot s$. Utility is

$$U^D(s) = \log\Big(\frac{\Pi s \bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\Big) - (1-p)\Delta \; .$$

3. No default: $R = \bar{R}$, debt is at the largest level that prevents default for both states $\eta = 0, \Delta$, which is $b^N(s) = \frac{\Pi(1-e^{-v})}{\bar{R}-\Pi(1-e^{-v})} \cdot s$. Utility is

$$U^N(s) = \log\Big(\frac{\Pi s \bar{R}}{\bar{R} - \Pi(1-e^{-v})}\Big) \; .$$

Observe first that the level of savings s is irrelevant for the choice among these three contracts. Next, because of $U^N(s) \geq U^0(s)$ for all $v \geq 0$ (with strict inequality for $v > 0$), option 1 (no credit) can be ruled out (for any $v > 0$).

No default dominates partial default if $U^N(s) \geq U^D(s)$ which is equivalent to

$$v \geq \bar{v} = \log\Big(\frac{\Pi e^{-\Delta}(p + e^{p\Delta} - 1)}{(\Pi - \bar{R})e^{-(1-p)\Delta} + \bar{R} - \Pi(1-p)}\Big) \; .$$

$\bar{v}$ is well-defined because the expression in the log(.) is positive: the denominator is positive if $(\Pi - \bar{R})e^{-(1-p)\Delta} > \Pi(1-p) - \bar{R}$. The latter condition follows from the first inequality in (4). Moreover, the first inequality in (4) is equivalent to $\bar{v} < v^{\max} = \log(\Pi/(\Pi - \bar{R}))$. Hence, no default is the optimal contract for all $v \in [\bar{v}, v^{\max})$.

The second inequality in condition (4) is equivalent to $\bar{v} > 0$. Because $U^D(s) > U^N(s)$ is equivalent to $v < \bar{v}$, the partial default contract is optimal for all $v \in [0, \bar{v})$. □

Proof of Proposition 2: Substituting $V(\omega) = \log(\omega) + V$, $V^d(\omega) = \log(\omega) + V^d$, and $U(s)$

from Proposition 1 into Bellman equations (1) and (2) yields

$$\begin{aligned}\log(\omega) + V = & \max_s (1-\beta)\log(\omega - s) + \beta[\log(\Pi s) + V^d] \\ & + \beta \max\left\{ \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-e^{-v})}\right], \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta \right\} , \\ \log(\omega) + V^d = & \max_s (1-\beta)\log(\omega - s) + \beta[\log(\Pi s) + V^d] \ .\end{aligned}$$

This shows that the savings policy $s = \beta\omega$ is optimal for both types of firms and that the terms $\log(\omega)$ cancel out on both sides of these Bellman equations, leaving the constant terms V and V^d to be determined from

$$\begin{aligned}V = & (1-\beta)\log(1-\beta) + \beta[\log(\beta\Pi) + V^d] \\ & + \beta \max\left\{ \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-e^{-v})}\right], \log\left[\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-v-\Delta})}\right] - (1-p)\Delta \right\} , \\ V^d = & (1-\beta)\log(1-\beta) + \beta[\log(\beta\Pi) + V^d] \ .\end{aligned}$$

Differentiate the second from the first equation yields the fixed-point equation $v = f(v)$ for the value difference $v = V - V^d$, as specified in the main text.

It is immediate from the definition of f and parameter condition (4) that f is well-defined for $v \in [0, v^{\max})$, that $f(v) \to \infty$ for $v \to v^{\max}$, and that f is increasing and continuous. Furthermore $f(0) > 0$ if and only if

$$\frac{\bar{R}}{\bar{R} - \Pi(1-p)(1-e^{-\Delta})} > e^{(1-p)\Delta} \ ,$$

which is equivalent to the second inequality in (4) (which is in turn equivalent to $\bar{v} > 0$). Then the claim of the proposition follows if $f(\bar{v}) < \bar{v}$ holds. This inequality is equivalent to the one stated in (5). □

Proof of Proposition 3: We prove the existence of a sunspot cycle that alternates between two sunspot states $i = 1, 2$ with transition probability π. In this stochastic case we use the timing convention that the sunspot state is realized *after* borrowers repay their debt (or not) in the beginning of the period.[30] The Bellman equation of a firm in sunspot state i is

$$V_i(\omega) = \max_{s,(R,b)} (1-\beta)\log(\omega - s) + \beta \mathbb{E} \max\left\{ \hat{V}_i[\Pi(s+b) - Rb], \hat{V}_i^d[\Pi(s+b)] - \eta' \right\} ,$$

[30]This timing is different from the one that we use in the macroeconomic model, but considerably simpler to prove the existence of sunspot cycles in the partial model.

where $\hat{V}_i(.)$ and $\hat{V}_i^d(.)$ respectively, are the continuation values before the realization of next period's sunspot state. With (common) transition probability π we have

$$\hat{V}_i(\omega) = \pi V_{-i}(\omega) + (1-\pi)V_i(\omega) \ ,$$
$$\hat{V}_i^d(\omega) = \pi V_{-i}^d(\omega) + (1-\pi)V_i^d(\omega) \ .$$

The utility value of a firm with a default history satisfies the recursion

$$V_i^d(\omega) = \max_s (1-\beta)\log(\omega - s) + \beta \hat{V}_i^d(\Pi s) \ .$$

As in the deterministic case, all firms save $s = \beta\omega$ and value functions take the forms $V_i(\omega) = \log(\omega) + V_i$, $V_i^d(\omega) = \log(\omega) + V_i^d$, where V_i and V_i^d are independent of net worth. Write $v_i \equiv V_i - V_i^d$ for the surplus value of access to credit (*expected credit conditions*), and $\hat{v}_i = \pi v_{-i} + (1-\pi)v_i$ for expected credit conditions before realization of the sunspot state when i is last period's state. Rewrite the firm's value in state i as $V_i(\omega) = \max_s(1-\beta)\log(\omega - s) + \beta[\hat{V}_i^d + U_i(s)]$ where $U_i(s)$ is the surplus value of the optimal credit contract for a firm with savings s in state i:

$$U_i(s) \equiv \max_{(R,b)} \mathbb{E} \max \Big\{ \log[\Pi(s+b) - Rb] + \hat{v}_i, \log[\Pi(s+b)] - \eta' \Big\} \quad \text{s.t.}$$

$$\bar{R}b = \mathbb{E}(Rb) = \begin{cases} Rb & \text{if } \log[\Pi(s+b) - Rb] + \hat{v}_i \geq \log[\Pi(s+b)] \ , \\ (1-p)Rb & \text{if } \log[\Pi(s+b)] > \log[\Pi(s+b) - Rb] + \hat{v}_i \geq \log[\Pi(s+b)] - \Delta \ , \\ 0 & \text{else.} \end{cases}$$

It is immediate to see that this problem is the same as in the deterministic case, with $\hat{v}_i$ replacing the stationary value v. Hence Proposition 1 applies:

1. If $\hat{v}_i > \bar{v}$, the optimal credit contract has no default and surplus value $U_i(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R} - \Pi(1-e^{-\hat{v}_i})}\right]$.
2. If $\hat{v}_i < \bar{v}$, the optimal credit contract has positive default with surplus value $U_i(s) = \log\left[\frac{\bar{R}\Pi s}{\bar{R} - \Pi(1-p)(1-e^{-\hat{v}_i - \Delta})}\right] - (1-p)\Delta$.

From the Bellman equations for V_i and V_i^d, it follows that expected credit conditions $v_i = V_i - V_i^d$ satisfy the system of equations

$$v_i = f(\hat{v}_i) = f(\pi v_{-i} + (1-\pi)v_i) \ , \ i = 1, 2 \ ,$$

where $f(.)$ is defined as in the main text. We can write this system of equations in the form $\phi(v_1, v_2, \pi) = 0$, where $\phi : \mathbb{R}^3 \to \mathbb{R}^2$, and $\phi_i(v_1, v_2, \pi) \equiv f(\pi v_{-i} + (1-\pi)v_i) - v_i$ for

$i = 1, 2$. Under the requirement of Proposition 2, this equation system has the solution $\phi(v^D, v^N, 0) = 0$ since both v^D and v^N are stationary equilibria. Moreover, ϕ is differentiable at $(v^D, v^N, 0)$. Therefore, we can invoke the implicit function theorem to prove the existence of non-degenerate (i.e., stochastic) cycles for positive transition probabilities $\pi > 0$ such that v_1 is sufficiently close to v^D (so that the default rate is positive in state $i = 1$) and v_2 is close to v^N (so that the default rate is zero in state $i = 2$). The Jacobian matrix of ϕ with respect to (v_1, v_2) evaluated at $(v^D, v^N, 0)$ is

$$\begin{pmatrix} \frac{d\phi_1}{dv_1}(v^D, v^N, 0) & \frac{d\phi_1}{dv_2}(v^D, v^N, 0) \\ \frac{d\phi_2}{dv_1}(v^D, v^N, 0) & \frac{d\phi_2}{dv_2}(v^D, v^N, 0) \end{pmatrix} = \begin{pmatrix} f'(v^D) - 1 & 0 \\ 0 & f'(v^N) - 1 \end{pmatrix} .$$

Because of $f'(v^D) < 1 < f'(v^N)$ (see Figure 1), this matrix has full rank. By the implicit function theorem, there exists a solution $v_i(\pi)$, $i = 1, 2$, for $\pi > 0$ such that $v_1(0) = v^D$, $v_2(0) = v^N$. This proves the existence of two-state sunspot cycles. □

Derivation of the Capital Return Π_t

For a firm with capital k, the first-order condition for hiring labor is

$$(1 - \alpha)A \left(\frac{zk}{A_t \ell}\right)^{\alpha} = w_t \ .$$

Therefore, labor demand is

$$\ell = zk \left[\frac{(1 - \alpha)\, A_t^{1-\alpha}}{w_t}\right]^{1/\alpha} ,$$

and net worth before interest expense (or interest income) is

$$\left[\alpha \left[\frac{(1 - \alpha)A_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}} + 1 - \delta\right] zk \equiv \Pi_t zk \ .$$

Proof of Proposition 4: The contract (ρ, θ), together with state-specific default thresholds $(\tilde{\eta}')$, maximizes

$$\mathbb{E}_t \left\{ (1 - G(\tilde{\eta}')) \log[1 + \theta(1 - \rho)] + \int_{-\infty}^{\tilde{\eta}'} \log[(1 + \theta)(1 - \lambda_{t+1})\zeta] - \eta' - v_{t+1} \; dG(\eta') \right\} ,$$

subject to (8) and (9). Substitution of $1+\theta(1-\rho)$ via (8) gives the objective function

$$\mathbb{E}_t\left\{ \log\left((1+\theta)(1-\lambda_{t+1})\zeta\right) - \tilde{\eta}'(1-G(\tilde{\eta}')) - \int_{-\infty}^{\tilde{\eta}'} \eta \, dG(\eta) - v_{t+1} \right\} .$$

The additive terms $\log((1-\lambda_{t+1})\zeta)$ and $-\mathbb{E}_t v_{t+1}$ are irrelevant for the maximization. Solving (9) for $1+\theta$, using $\rho = \xi(1+\theta)/\theta$, gives

$$1+\theta = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \Psi(\xi)} ,$$

with

$$\Psi(\xi) \equiv \mathbb{E}_t\left\{ \lambda_{t+1} G(\tilde{\eta}(\xi)) + \xi(1-G(\tilde{\eta}(\xi))) \right\} ,$$

and

$$\tilde{\eta}(\xi) = \log\left[\frac{(1-\lambda_{t+1})\zeta}{1-\xi}\right] - v_{t+1} ,$$

which is the ex-post default threshold. Substitution into the objective function yields a maximization problem in ξ:

$$\max_{\xi} - \log(\bar{\rho}_t(1+\Phi_t) - \Psi(\xi)) - \mathbb{E}_t\left\{ \tilde{\eta}(\xi)(1-G(\tilde{\eta}(\xi))) + \int_{-\infty}^{\tilde{\eta}(\xi)} \eta' \, dG(\eta') \right\} .$$

The first-order condition for this problem is

$$\frac{\Psi'(\xi)}{\bar{\rho}_t(1+\Phi_t) - \Psi(\xi)} = \frac{1}{1-\xi}\mathbb{E}_t(1-G(\tilde{\eta}(\xi))) . \tag{18}$$

Then, using the derivative $\tilde{\eta}'(\xi) = 1/(1-\xi)$:

$$\Psi'(\xi) = \mathbb{E}_t(1-G(\tilde{\eta}(\xi))) + \frac{1}{1-\xi}\mathbb{E}_t[G'(\tilde{\eta}(\xi))(\lambda_{t+1}-\xi)] .$$

Substituting this expression into the first-order condition (18) yields (12) in the proposition. Furthermore, the default threshold (10) follows directly from (8), and $\theta_t = \bar{\rho}_t(1+\Phi_t)/(\bar{\rho}_t(1+\Phi_t) - \Psi(\xi_t)) - 1$, which leads to (11). □

Derivation of the Value Functions $V(\omega; X_t)$ and $V^d(\omega; X_t)$

Recall that $V(\omega; X_t)$ and $V^d(\omega; X_t)$ are values of firms with (without) a clean credit record whose net worth is ω in period t. Therefore

$$V(\omega;X_t) = \pi\hat{V}_b(\omega;X_t) + (1-\pi)\hat{V}_l(\omega;X_t)\ ,$$
$$V^d(\omega;X_t) = \pi\hat{V}_b^d(\omega;X_t) + (1-\pi)\hat{V}_l^d(\omega;X_t)\ ,$$

where $\hat{V}_\tau^{(d)}(\omega;X_t)$, $\tau = b,l$, are values of borrowing (or not lending) and lending firms after realization of idiosyncratic capital productivities. These satisfy the Bellman equations

$$\begin{aligned}
\hat{V}_b(\omega;X_t) =& \max_s (1-\beta)\log(\omega - s) \\
&+ \beta\mathbb{E}_t \max\Big\{V([1+\theta_t(1-\rho_t)]z^H\Pi_t s;X_{t+1}), V^d((1+\theta_t)(1-\lambda_{t+1})\zeta z^H\Pi_t s;X_{t+1}) - \eta'\Big\}\ , \\
\hat{V}_l(\omega;X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta\mathbb{E}_t V(\bar{R}_t s;X_{t+1})\ , \\
\hat{V}_b^d(\omega;X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta(1-\psi)\mathbb{E}_t V^d(z^H\Pi_t s;X_{t+1}) + \beta\psi\mathbb{E}_t V(z^H\Pi_t s;X_{t+1})\ , \\
\hat{V}_l^d(\omega;X_t) =& \max_s (1-\beta)\log(\omega - s) + \beta(1-\psi)\mathbb{E}_t V^d(\bar{R}_t s;X_{t+1}) + \beta\psi\mathbb{E}_t V(\bar{R}_t s;X_{t+1})\ .
\end{aligned}$$

Expectation operators are over the realizations of aggregate states and of the idiosyncratic default loss η' in period $t+1$.

It is straightforward to verify that all value functions take the form $\hat{V}_\tau^{(d)}(\omega;X_t) = \log(\omega) + \hat{V}_\tau^{(d)}(X_t)$ for $\tau = b,l$, $V^{(d)}(\omega';X_t) = \log(\omega') + V^{(d)}(X_t)$, and that savings are $s = \beta\omega$. With $B \equiv (1-\beta)\log(1-\beta) + \beta\log(\beta)$, it follows

$$\begin{aligned}
\hat{V}_b(X_t) =& B + \beta\mathbb{E}_t \max\Big\{\log([1+\theta_t(1-\rho_t)]z^H\Pi_t) + V(X_{t+1}), \qquad (19)\\
&\log((1+\theta_t)(1-\lambda_{t+1})\zeta z^H\Pi_t) + V^d(X_{t+1}) - \eta'\Big\}\ , \\
\hat{V}_l(X_t) =& B + \beta\log\bar{R}_t + \beta\mathbb{E}_t V(X_{t+1})\ , \qquad (20)\\
\hat{V}_b^d(X_t) =& B + \beta\log(z^H\Pi_t) + \beta(1-\psi)\mathbb{E}_t V^d(X_{t+1}) + \beta\psi\mathbb{E}_t V(X_{t+1})\ , \qquad (21)\\
\hat{V}_l^d(X_t) =& B + \beta\log\bar{R}_t + \beta(1-\psi)\mathbb{E}_t V^d(X_{t+1}) + \beta\psi\mathbb{E}_t V(X_{t+1})\ . \qquad (22)
\end{aligned}$$

Moreover,

$$V(X_t) = \pi\hat{V}_b(X_t) + (1-\pi)\hat{V}_l(X_t)\ , \qquad (23)$$
$$V^d(X_t) = \pi\hat{V}_b^d(X_t) + (1-\pi)\hat{V}_l^d(X_t)\ . \qquad (24)$$

Derivation of Equation (13)

Define $v_t = V(X_t) - V^d(X_t)$, take the difference between (23) and (24) and use (19)–(22)

to obtain

$$
\begin{aligned}
v_t =& \pi\Big\{\hat{V}_b(X_t) - \hat{V}_b^d(X_t)\Big\} + (1-\pi)\Big\{\hat{V}_l(X_t) - \hat{V}_l^d(X_t)\Big\} \\
=& \beta\pi\mathbb{E}_t\Bigg\{(1-\psi)v_{t+1} + \max\Big\{\log[1+\theta_t(1-\rho_t)], \log[(1+\theta_t)(1-\lambda_{t+1})\zeta] - \eta' - v_{t+1}\Big\}\Bigg\} \\
& + \beta(1-\pi)(1-\psi)\mathbb{E}_t v_{t+1} \\
=& \beta\pi\mathbb{E}_t \max\Big\{\log[1+\theta_t(1-\rho_t)], \log[(1+\theta_t)(1-\lambda_{t+1})\zeta] - \eta' - v_{t+1}\Big\} + \beta(1-\psi)\mathbb{E}_t v_{t+1} \ .
\end{aligned}
$$

Using the default threshold $\tilde{\eta}_{t+1}$, the max{.} term is equal to

$$
\log\big[(1+\theta_t)(1-\lambda_{t+1})\zeta\big] - v_{t+1} + \mathbb{E}_t \max\{-\tilde{\eta}_{t+1}, -\eta'\} \ .
$$

This proves equation (13).

B Miscellaneous

B.1 Collection of Equilibrium Conditions

We list all equilibrium conditions used for numerical exercises. There are 10 equations and we have 10 unknowns ($\tilde{\eta}_t$, θ_t, ρ_t, $\bar{\rho}_t$, v_t, Π_t, w_t, Ω_{t+1}, f_{t+1}, ξ_t).

$$
\tilde{\eta}_{t+1} = \log\left[\frac{1-\lambda_{t+1}}{1-\xi_t}\right] - v_{t+1} + \log\zeta \tag{25}
$$

$$
\theta_t = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t\left[\lambda_{t+1}G(\tilde{\eta}_{t+1}) + \xi_t(1-G(\tilde{\eta}_{t+1}))\right]} - 1 \tag{26}
$$

$$
\mathbb{E}_t\left[G'(\tilde{\eta}_{t+1})(\xi_t - \lambda_{t+1})\right] = \mathbb{E}_t\left[1 - G(\tilde{\eta}_{t+1})\right]\Bigg\{1 - \bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t\left[G(\tilde{\eta}_{t+1})(\xi_t - \lambda_{t+1})\right]\Bigg\} \tag{27}
$$

$$
\begin{aligned}
v_t =& \beta\pi\mathbb{E}_t\Big\{\log(1+\theta_t) + \log(1-\lambda_{t+1}) + \log\zeta - \mu - (\tilde{\eta}_{t+1} - \mu)(1 - G(\tilde{\eta}_{t+1})) + \sigma^2 G'(\tilde{\eta}_{t+1})\Big\} \\
& + \beta(1-\psi-\pi)\mathbb{E}_t\left[v_{t+1}\right]
\end{aligned} \tag{28}
$$

$$
\xi_t = \rho_t\theta_t/(1+\theta_t)
$$

$$
\left[\frac{\Pi_t - (1-\delta)}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} = \left(\frac{\kappa}{1-\alpha}\right)^{\frac{\alpha}{\alpha+\nu}}\left[\beta\frac{\Omega_t}{A_t}\Big(z^L\Big[(1-\pi) - \pi f_t\theta_t\Big] + z^H\pi\Big[f_t(1+\theta_t) + 1 - f_t\Big]\Big)\right]^{\frac{\alpha\nu}{\alpha+\nu}}
$$

$$w_t = (1-\alpha)A_t\left[\frac{\Pi_t + \delta - 1}{\alpha}\right]^{\frac{\alpha}{\alpha-1}}$$

$$\Omega_{t+1} = \beta z^H \Pi_t \Omega_t \left\{(1-\pi)\bar{\rho}_t + \pi f_t \left[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + \zeta G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_{t+1})\right] + \pi(1-f_t)\right\}$$

$$f_{t+1} = \frac{f_t\left[(1-\pi)\bar{\rho}_t + \pi(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t))\right] + (1-f_t)\psi[(1-\pi)\bar{\rho}_t + \pi]}{(1-\pi)\bar{\rho}_t + \pi f_t\left[(1-G(\tilde{\eta}_{t+1}))(1+\theta_t(1-\rho_t)) + \zeta G(\tilde{\eta}_{t+1})(1+\theta_t)(1-\lambda_{t+1})\right] + \pi(1-f_t)}$$

with $\gamma \leq \bar{\rho}_t$ and $f_t \pi \theta_t \leq (1-\pi)$ satisfied. Note that the fourth equation for v_t uses that the default cost distribution G is normal with mean zero and variance σ. On the balanced growth path, every variable is a constant except Ω_t. Ω_t grows at the rate of μ^A on the balanced growth path, and Ω_t/A_t becomes a constant.

In the full model, there are four exogenous processes. Specifically,

$$\log(1+\Phi_t) - \log(1+\Phi) = \rho_\Phi\left[\log(1+\Phi_{t-1}) - \log(1+\Phi)\right] + \varepsilon_t^\Phi \ ,$$

$$\log(1-\lambda_t) - \log(1-\lambda) = \rho_\lambda\left[\log(1-\lambda_{t-1}) - \log(1-\lambda)\right] + \varepsilon_t^\lambda + \chi_\lambda^\Phi \varepsilon_t^\Phi \ ,$$

$$\log(1+\mu_t^A) - \log(1+\mu^A) = \rho_A\left[\log(1+\mu_{t-1}^A) - \log(1+\mu^A)\right] + \varepsilon_t^A \ ,$$

$$\varepsilon_t^b = \chi_b^\Phi \varepsilon_t^\Phi + \varepsilon_t^s \ ,$$

where ε_t^Φ, ε_t^λ, ε_t^A, and ε_t^s are i.i.d. normal random variables with mean zero and variance σ_Φ^2, σ_λ^2, σ_A^2, and σ_s^2. Other parameters include ρ_Φ, ρ_λ, ρ_A which are persistence parameters, and χ_λ^Φ and χ_b^Φ, which are exposure parameters to Φ shocks.

Given an equilibrium, aggregate output is

$$Y_t = (A_t l_t)^{1-\alpha}(\tilde{z}_t K_t)^\alpha \ , \tag{29}$$

and average capital productivity

$$\tilde{z}_t = \pi z^H + (1-\pi)z^L + f_t \pi \theta_t [z^H - z^L] \ .$$

Define *total factor productivity* (TFP) as the residual of the aggregate production function, i.e.,

$$\tilde{A}_t = \frac{Y_t}{K_t^\alpha l_t^{1-\alpha}} = A_t^{1-\alpha}\tilde{z}_t^\alpha \ .$$

Two things affect capital efficiency $\tilde{z}_t$ of this economy. First, a greater share of firms with access to the credit market leads to a more efficient capital allocation. Second, the higher the ability to raise external capital θ_t, the more capital is employed by productive firms.

B.2 Calibration and Estimation

We look at the risky steady state instead of the deterministic steady state. This is the steady state in which risks are taken into account.

Approximating the Expectation Terms

In order to utilize the risky steady state, we approximate the equilibrium conditions with expectation terms by 2nd-order Taylor expansion. These conditions include (25)-(28). Notice that the impact of future risks show up. Other equilibrium conditions do not have expectation terms and there is no need to approximate them. Then, we solve for the risky steady state and we log-linearize around the risky steady state.

Recall that we impose the AR(1) structure

$$\log(1-\lambda_{t+1}) = \rho_\lambda \log(1-\lambda_t) + (1-\rho_\lambda)\log(1-\lambda) + \varepsilon^\lambda_{t+1} + \chi^\Phi_\lambda \varepsilon^\Phi_{t+1}$$

so that

$$\lambda_{t+1} = 1 - (1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda} e^{\varepsilon^\lambda_{t+1} + \chi^\Phi_\lambda \varepsilon^\Phi_{t+1}} \ .$$

The relationship between $\tilde{\eta}_{t+1}$ and v_{t+1} in (25) implies that

$$\tilde{\eta}^e_t = \mathbb{E}_t\left[\tilde{\eta}_{t+1}\right] = \rho_\lambda \log(1-\lambda_t) + (1-\rho_\lambda)\log(1-\lambda) - \log(1-\xi_t) - \mathbb{E}_t\left[v_{t+1}\right] + \log\zeta \ .$$

From the zero-profit condition (26)

$$\theta_t = \frac{\bar{\rho}_t(1+\Phi_t)}{\bar{\rho}_t(1+\Phi_t) - \mathbb{E}_t\left[\lambda_{t+1}G(\tilde{\eta}_{t+1}) + \xi_t(1-G(\tilde{\eta}_{t+1}))\right]} - 1$$

we are able to obtain an expression

$$\begin{aligned}
\frac{1+\Phi_t}{\Delta_t} &= \mathbb{E}_t\left[1 - G(\tilde{\eta}_{t+1}) + G(\tilde{\eta}_{t+1})\frac{\lambda_{t+1}}{\xi_t}\right] \\
&\approx 1 - \left[G(\tilde{\eta}^e_t) + \frac{G''(\tilde{\eta}^e_t)}{2}\left[\sigma_b^2 + \sigma_\lambda^2 + \left(\chi^\phi_\lambda\right)^2 \sigma_\Phi^2\right]\right]\left(1 - \frac{1-(1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda} e^{\frac{\sigma_\lambda^2 + \left(\chi^\phi_\lambda\right)^2 \sigma_\Phi^2}{2}}}{\xi_t}\right) \\
&\quad - \frac{G'(\tilde{\eta}^e_t)(1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda}}{\xi_t}\left(\sigma_\lambda^2 + \chi^\Phi_b \chi^\Phi_\lambda \sigma_\Phi^2\right) \ ,
\end{aligned}$$

with $\Delta_t = \rho_t/\bar{\rho}_t$ being the gross credit spread (ratio). From the contracting equation (27):

$$
\begin{aligned}
& -\left[1-\mathbb{E}_t G(\tilde{\eta}_{t+1})\right]\left\{1-\bar{\rho}_t(1+\Phi_t)-\mathbb{E}_t G(\tilde{\eta}_{t+1})(\xi_t-\lambda_{t+1})\right\} \\
=& -\left[1-\mathbb{E}_t G(\tilde{\eta}_{t+1})\right]\left\{1-\bar{\rho}_t(1+\Phi_t)+\frac{\theta_t\bar{\rho}(1+\Phi_t)}{\theta_t+1}-\xi_t\right\} \\
\approx& -\left[1-G(\tilde{\eta}_t^e)-\frac{G''(\tilde{\eta}_t^e)}{2}\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2\right]\right]\left[1-\xi_t-\frac{\bar{\rho}(1+\Phi_t)}{\theta_t+1}\right] \\
=& \mathbb{E}_t\left[G'(\tilde{\eta}_{t+1})(\lambda_{t+1}-\xi_t)\right] \\
\approx& \left[1-(1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda}e^{\frac{\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2}{2}}-\xi_t\right]\left[G'(\tilde{\eta}_t^e)+\frac{G'''(\tilde{\eta}_t^e)}{2}\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2\right]\right] \\
& -G''(\tilde{\eta}_t^e)(1-\lambda_t)^{\rho_\lambda}(1-\lambda)^{1-\rho_\lambda}\left(\sigma_\lambda^2+\chi_b^\Phi\chi_\lambda^\Phi\sigma_\Phi^2\right) \ .
\end{aligned}
$$

From the forward-looking equation (28) for v:

$$
\begin{aligned}
\frac{v_t}{\beta\pi}-\frac{1-\psi-\pi}{\pi}\mathbb{E}_t\left[v_{t+1}\right] \approx& \log(1+\theta_t)+\rho_\lambda\log(1-\lambda_t)+(1-\rho_\lambda)\log(1-\lambda)+\log\zeta-\mu \\
& +\sigma^2\left[G'(\tilde{\eta}_t^e)+\frac{G'''(\tilde{\eta}_t^e)}{2}\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2\right]\right] \\
& -\ (\tilde{\eta}_t^e-\mu)\left[1-G(\tilde{\eta}_t^e)-\frac{G''(\tilde{\eta}_t^e)}{2}\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2\right]\right] \\
& +G'(\tilde{\eta}_t^e)\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\phi\right)^2\sigma_\Phi^2\right] \ ,
\end{aligned}
$$

where we have used

$$
\begin{aligned}
\mathbb{E}_t\left[\tilde{\eta}_{t+1}G(\tilde{\eta}_{t+1})\right] =& \mathbb{E}_t\left[\tilde{\eta}_{t+1}\right]\mathbb{E}_t\left[G\left(\tilde{\eta}_{t+1}\right)\right]+\mathrm{Cov}(\tilde{\eta}_{t+1},G(\tilde{\eta}_{t+1})) \\
\approx& \tilde{\eta}_t^e\mathbb{E}_t\left[G\left(\tilde{\eta}_{t+1}\right)\right]+\mathrm{Cov}\left(\varepsilon_{t+1}^b+\varepsilon_{t+1}^\lambda+\chi_\lambda^\Phi\varepsilon_{t+1}^\Phi,G'(\tilde{\eta}_t^e)\left(\varepsilon_{t+1}^b+\varepsilon_{t+1}^\lambda+\chi_\lambda^\Phi\varepsilon_{t+1}^\Phi\right)\right) \\
=& \tilde{\eta}_t^e\mathbb{E}_t\left[G\left(\tilde{\eta}_{t+1}\right)\right]+G'(\tilde{\eta}_t^e)\left[\sigma_b^2+\sigma_\lambda^2+\left(\chi_\lambda^\Phi\right)^2\sigma_\Phi^2\right] \ .
\end{aligned}
$$

Parameterizations of the Risky Steady State

The capital share is exogenously set at $\alpha = 0.33$, the fraction of productive firms π is set at 0.2, and the depreciation rate is set $\delta = 0.1$. The annual growth rate of real per capita output is $\mu_A = 1.017$ in our sample period. Labor elasticity is set to $\nu = 1/1.5$, a conventional number in macroeconomics. $\psi = 0.10$ is used for a 10-year default flag. $\zeta = 0.85$ targets a 15% real default loss. The excess bond premium in steady state is set to $\Phi = 0$ so that the excess bond premium comes entirely from the belief channel in the steady state. The target for steady-state labor hours is $\ell = 0.25$. From the capital–output ratio $K/Y = 2$ (which will be used to identify β), we obtain $\Pi = 1 - \delta + \alpha \frac{Y}{\tilde{z}K} = 1 - \delta + \alpha \frac{Y}{K} = 1.07$. We then calibrate $\kappa = 2.38$ to target the steady-state labor share $\ell = 0.25$.

Given the debt-output ratio $B/Y = 0.82$, the capital-output ratio $K/Y = 2$, the leverage ratio in constrained firm $\theta = 2.1$, the sample average default rate 1.58%, the sample average recovery rate $r = 41.74\%$, together with the spread ratio $\Delta = \rho/\bar{\rho} = 1.0201$ and $\Phi = 0$, identify the seven parameters $\gamma = \frac{z^L}{z^H}$, β, ζ, $\tilde{\eta}^e$, σ, λ, and σ_b^2, as we show now. (Note: For z^L and z^H, we normalize the average productivity to be 1. More details below.)

First, we guess a pair of (μ, σ). Then we know the function G and we can back out $\tilde{\eta}^e$ which is the inverse of the average default rate. Second, the steady-state value of f (share of firms with credit market access) follows from $\pi\theta f = \frac{B}{K} = \frac{B}{Y} \cdot \frac{Y}{K} = 0.41$, hence $f = 0.41/(\pi\theta) = 0.9665$. From the steady-state equation for f, we have the quadratic equation

$$af^2 + bf + c = 0$$

where $a = \pi\theta(1-\rho) + \pi G[\theta\rho - (1+\theta)[1 - \zeta(1-\lambda)]] > 0$, $b = \pi - \pi(1-G)[1 + \theta(1-\rho)] + \psi[(1-\pi)\bar{\rho} + \pi]$, and $c = -\psi[(1-\pi)\bar{\rho} + \pi] < 0$. Use this quadratic equation, $\rho = \Delta\gamma$, $\lambda = r\xi = \frac{r\theta\rho}{1+\theta}$, and the numbers for f, θ, ψ, π, to solve uniquely for

$$\gamma = \bar{\rho} = \frac{[\pi\theta - \pi G(1+\theta)(1-\zeta)] f^2 + \pi [1 - (1-G)(1+\theta)] f - \psi\pi(1-f)}{\psi(1-\pi)(1-f) + \pi\theta\Delta(1-G)f(f-1) + \zeta\pi\theta G r \Delta f^2}$$

and therefore

$$\lambda = r\xi = \frac{r\theta}{1+\theta}\gamma\Delta \ .$$

From the normalization $\tilde{z} = 1$, we have

$$z^H = \frac{1}{\pi + (1-\pi)\gamma + f\pi\theta(1-\gamma)} \ .$$

From stationarity of Ω_t/A_t follows

$$e^{\mu_A} = \beta z^H \Pi \left\{ (1-\pi)\gamma + \pi f \Big[(1-G)[1+\theta(1-\rho)] + \zeta G(1+\theta)(1-\lambda) \Big] + \pi(1-f) \right\}$$

and hence $\beta = \frac{e^{\mu_A}}{z^H \Pi} \left\{ (1-\pi)\gamma + \pi f \Big[(1-G)[1+\theta(1-\rho)] + \zeta G(1+\theta)(1-\lambda) \Big] + \pi(1-f) \right\}^{-1}$ (i.e. β is identified from the K/Y ratio).

Third, from the approximated zero-profit condition of lenders, we have

$$\frac{1+\Phi}{\Delta} \approx 1 - \left[G(\tilde{\eta}^e) + \frac{G''(\tilde{\eta}^e)}{2}\left(\sigma_b^2 + \sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2\right)\right]\left(1 - \frac{1-(1-\lambda)e^{\frac{\sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2}{2}}}{\xi}\right)$$
$$- \frac{G'(\tilde{\eta}^e)(1-\lambda)}{\xi}\left(\sigma_\lambda^2 + \chi_b^\Phi\chi_\lambda^\Phi\sigma_\Phi^2\right). \tag{30}$$

Notice that λ is known at this stage, and $\xi = \lambda/r$. That is, the spread Δ (together with Φ and recovery) calibrates σ_b^2.

Finally, we use two equations to pin down the parameters (μ, σ). Since all other parameters above depend on (μ, σ), we use a nonlinear solver to achieve this goal. The first equation is from the contracting equation. In the steady state

$$\left[1 - G(\tilde{\eta}^e) - \frac{G''(\tilde{\eta}^e)}{2}\left(\sigma_b^2 + \sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2\right)\right]\cdot\left[\xi + \frac{\bar{\rho}(1+\Phi)}{\theta+1} - 1\right] = \left(1 - (1-\lambda)e^{\frac{\sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2}{2}} - \xi\right)\cdot$$
$$\left[G'(\tilde{\eta}^e) + \frac{G'''(\tilde{\eta}^e)}{2}\left(\sigma_b^2 + \sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2\right)\right] - G''(\tilde{\eta}^e)(1-\lambda)\left(\sigma_\lambda^2 + \chi_b^\Phi\chi_\lambda^\Phi\sigma_\Phi^2\right).$$

The second equation is the forward-looking equation for v. In analyzing this equation, we also express $\mathbb{E}_t\left[v_{t+1}\right]$ by using η^e according to the default threshold condition:

$$\left(\frac{1}{\beta\pi} - \frac{1-\psi-\pi}{\pi}\right)\left[\log(1-\lambda) - \log(1-\xi) + \log\zeta - \eta^e\right]$$
$$= \log(1+\theta_t) + \log(1-\lambda) + \log\zeta - \mu$$
$$+\sigma^2\left[G'(\tilde{\eta}^e) + \frac{G'''(\tilde{\eta}^e)}{2}\left(\sigma_b^2 + \sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2\right)\right] - (\tilde{\eta}^e - \mu)\left[1 - G(\tilde{\eta}_t^e) - \frac{G''(\tilde{\eta}^e)}{2}\left(\sigma_b^2 + \sigma_\lambda^2 + (\chi_\lambda^\Phi)^2\sigma_\Phi^2\right)\right]$$
$$+G'(\tilde{\eta}^e)\left[\sigma_b^2 + \sigma_\lambda^2 + \left(\chi_\lambda^\Phi\right)^2\sigma_\Phi^2\right].$$

Two Cases

In Section 4, we consider two cases. The first is an economy with only belief shocks, the second is an economy with all four shocks.

In the first case, one can simply set $\sigma_\lambda^2 = \sigma_\Phi^2 = \sigma_A^2 = 0$ in the above expressions. The exposure parameters χ_λ^Φ and χ_b^Φ do not matter, and so do those persistence parameters. The variance of beliefs σ_b^2 is uniquely pinned down from (30). There is no estimation procedure involved.

In the second case, we need to estimate variances, exposure parameters, and persistence parameters. Since σ_λ^2, σ_b^2, σ_Φ^2, χ_λ^Φ and χ_b^Φ affect the steady state, the steady state and the dynamics must be jointly determined. Specifically, we log-linearize the system and place the system as a Kalman filter form. Then, we estimate the system by using the maximum-likelihood estimator method. One should notice that this is a constrained MLE procedure since we directly compute many parameters illustrated before and also there is an important constraint on beliefs:

$$\sigma_b^2 = \sigma_s^2 + \left(\chi_b^{\Phi}\sigma_{\Phi}\right)^2 \ .$$