Network Hazard and Bailouts *

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Abstract

This paper characterizes strongly stable networks under general threshold contagion. Among other applications, the theory is applied to interbank lending and financial contagion wherein a government can intervene to stop contagion. In the absence of intervention, banks form disjoined clusters to minimize contagion. In the presence of intervention, banks become less concerned with the counterparties of their counterparties, which we dub *network hazard*. Network hazard allows some banks to become systemically important and gives the network a core-periphery structure. The counterparty risk of a large part of the economy becomes correlated through the core banks' solvency. Core banks serve as a buffer against contagion when solvent and an amplifier of contagion when insolvent. As such, bailouts create welfare volatility and increase systemic risk via network hazard. It is shown that network hazard is a novel force distinct from moral hazard. Results are historically relevant to the pyramiding of reserves and the establishment of the Federal Reserve.

JEL classification: D85, G01, H81.

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1 Introduction

Contagion of certain attributes or actions over a network of bilateral relationships has been well studied in sociology, mathematics, physics, economics, and other fields. Applications include the spread of diseases, defaults by banks, the spread of rumors, the adoption of new products or platforms, innovation and patenting, the adoption of political or religious views, and more. Understanding what properties of a network facilitate contagion has been a key question in the literature. In certain scenarios, however, the network in question is strategically formed. A prime example is financial and economics networks. An immediate question is, if agents anticipate the possibility of contagion, what kinds of networks would they form? This question has drawn more attention in recent papers such as Blume et al. (2013). Once we understand how networks are strategically formed in the presence of the risk of contagion, another question follows: How would the anticipation that a principal might intervene with contagion in order to mitigate it alter the incentives regarding the formation of the network prior to contagion? For example, if a vaccine or quarantining were available to stop the spread of a disease, or bailouts were available to stop failures of banks, how would the anticipation of intervention impact the structure of the network? Would network effects lead to more or less contagion given that agents are less concerned with protecting themselves against contagion in response to the anticipation of intervention? These questions are the focus of this paper. We provide a general theory and a detailed application to interbank lending and bailouts.[1]

A widely used mechanics for modeling contagion is the threshold contagion model (see Jackson (2010), Easley and Kleinberg (2010), Centola and Macy (2007)). Under threshold contagion, once a certain fraction of counterparties of an agent adopt the behavior or the attribute, the agent in question also adopts the behavior or the attribute. As for network formation, we use the strong stability as our solution concept. Roughly speaking, strongly stable networks preclude deviations by any subset of agents that improve the deviating subset. We show the existence and uniqueness of strongly stable networks under threshold contagion. In the absence of intervention, strongly stable networks roughly consist of many disjoined cliques[2] of heterogenous sizes, and potentially another interconnected subnetwork. A disjoined clique structure serve to minimize risk of contagion while maintaining a satiation level in terms of the connectivity desired by each agent. In the presence of intervention, contagion is mitigated by intervention. As a by-product of intervention, the cliques dissolve into a large interconnected network that facilitates more potential for contagion. We introduce this theory in Section 2.

We then move in Section 3 to our main application. Threshold contagion models are useful for understanding financial contagion and systemic risk. Major examples are Elliott et al. (2014) and Acemoglu et al. (2015c). Our main application is financial networks, and interbank lending in particular, which is timely. The financial crisis of 2008 alerted many to the risk that the failure of a few individual financial institutions might, through the interconnectedness of the financial system, damage the economy as a whole. Such systemic risk can be ameliorated ex-ante using regulatory

[1] In Appendix D we provide another detailed application to real firms with joint projects.
[2] A clique is a subnetwork in which all nodes in the clique have links with each other.

tools, yet the inability of government to credibly commit to not intervening suggests that an ex-post response, in the form of bailouts, is unavoidable. Bailouts of failing institutions are criticized because they encourage excessive risk taking by individual institutions. Yet these negative individual implications could perhaps be offset if the net effect of ex-post intervention, given the ex-ante market response during network formation, is to mitigate contagion. Therefore, it is important to understand whether the anticipation of intervention leads to networks that are more or less prone to contagion. Could we expect some form of moral hazard with regards to network formation in response to the anticipation of bailouts?

In Section 3.1 we introduce and study a model of interbank lending. We borrow some features from Allen and Gale (2000), Moore (2011), and Erol and Ordonez (2017). Banks first form bilateral borrowing and lending partnerships.[3] After the strategic formation of these partnerships, some banks receive insolvency shocks and default. Once a certain number of counterparties of a bank default, it becomes too costly to raise liquidity for the bank, and the bank prefers to liquidate its dividend-yielding project to save management costs. Call these illiquid banks. Once many solvent banks become illiquid this way, a solvent bank with too many solvent-but-illiquid counterparties also becomes illiquid. Contagion of illiquidity spreads this way. Foreseeing the extent of contagion, government intervenes with capital injections to stop contagion in a time-consistent fashion. Government lacks the ability to commit itself to not intervene. This makes banks realize that contagion will be mitigated by the government, which reduces banks' own incentives to protect themselves against contagion. This has a subtle but powerful impact on the structure of the network and the resulting welfare, which we explain next.

In the absence of the anticipation of intervention, a bank prefers that its counterparties are counterparties only of each other. A bank that does not have a second-order counterparty eliminates exposure to second-order counterparty risk−that is, the risk that it incurs losses due to defaults by good counterparties that default because of their own defaulting counterparties. This force generates a market discipline that leads uniquely to the formation of dense clusters that are isolated from each other, and this network structure eliminates second-order counterparty risk.

In the presence of intervention all banks know that illiquid banks are going to be saved by bailouts. Therefore, a solvent bank knows that all of its solvent counterparties will continue their operations and maintain their dividend-yielding projects. In other words, a solvent bank does not need to worry about its solvent counterparties becoming illiquid due to their own counterparties. All solvent banks become immune to illiquidity. Second-order counterparty risk is eliminated as a by-product of optimal intervention, which we call *network hazard*. Network hazard loosens the market discipline because banks no longer concern themselves with the counterparties of their counterparties during network formation. This effect of bailouts on the network topology emerges because each bank anticipates that its counterparties can get bailed-out, and not because each bank anticipates that it will be bailed-out.

Network hazard changes the topology of the network in two ways. First, because banks no longer

[3]Prior work has shown that interbank lending mostly occurs over long-term partnerships. See Weber (2003), Calomiris and Carlson (2017), and Afonso et al. (2013).

concern themselves with second-order counterparty risk, the isolated clusters that form in the absence of intervention dissolve, and an interconnected network emerges in the presence of intervention. As the network becomes more interconnected, the extent of potential contagion increases. Second, large banks that need many counterparties would carry on risk across the separate parts of the network in the absence of intervention. In the presence of intervention, other banks no longer worry about large banks carrying excess second-order counterparty risk, and so large banks become highly connected to the rest of the network. This turns the network into a core-periphery network. Large banks make up the core of the network and small banks make up the periphery. Because of the banks at the core of the network, the counterparty risks faced by peripheral banks are correlated. In return, when a sufficient number of core banks get bad shocks, a large number of small banks become illiquid. An insolvent core serves as both a direct source of contagion and an amplifier of contagion across the periphery. When a sufficient number of core banks experience good shocks, peripheral banks overall become more resilient. Only a large number of bad shocks to the peripheral counterparties of a peripheral good bank can force the latter to default. Thus, the core serves as a buffer against contagion. Network hazard leads to a core-periphery arrangement, which makes very bad and very good outcomes more likely. This generates welfare volatility endogenously. We further argue that network hazard is distinct from moral hazard. In the model, banks that are bailed out are made indifferent between defaulting or not. The anticipation of intervention does not necessarily lead banks to shift more risk to depositors.

In Section 3.2 we dig deeper into understanding the separate impacts of second-order and first-order counterparty risks. In Section 3.3 we discuss the relevance of the theory to the establishment of Federal Reserve (FED henceforth) and the resulting change in the interbank network and volatility.

Literature. Our theory builds on threshold contagion models and strongly stable networks. Threshold contagion was first introduced by Granovetter (1978). Lim et al. (2015) study threshold contagion and include an instructive and brief overview of the large literature on cascades and the diffusion in networks. Strong stability was introduced in Dutta and Mutuswami (1997) and Jackson and Van den Nouweland (2005). Goyal and Vega-Redondo (2005) consider a network formation problem with additive payoffs across links. In case of a unilateral link formation game with no uncertainty in payoffs, they find that an empty network, a complete network, or a network that consists of two cliques is formed, depending on the cost of links. Erol and Vohra (2014) consider additive payoffs with uncertainty in the benefits that are derived from links, and they show that if the behavior is very contagious, the unique strongly stable network consists of disjoined cliques. Using our framework, we study strongly stable networks (neither unilaterally formed links nor pairwise stable networks) under any payoff function (not necessarily additive), any threshold rule (neither necessarily linear nor necessarily very contagious), and uncertainty regarding nodes (not edges).

Our application is related to a growing literature on systemic risk and networks that does not make particular emphasis on moral hazard. Early contributors include Allen and Gale (2000), Eisenberg and Noe (2001), Kiyotaki and Moore (1997), and in more recent years, Acemoglu, Ozdaglar and Tahbaz-Salehi (2015c), Elliott, Golub and Jackson (2014), Glasserman and Young (2015), and others. These papers examine contagion within fixed networks. Other scholars, among them Drakopoulos, Ozdaglar and Tsitsiklis (2015a), Freixas, Parigi and Rochet (2000), and Minca and Sulem (2014), examine the problem of how to most effectively stop contagion in exogenous networks. Acemoglu, Ozdaglar and Tahbaz-Salehi (2015b), Elliott and Hazell (2015), Erol and Vohra (2014), and others study the formation of networks by agents who take systemic risk into account. In contrast to our approach, these studies do not consider the possibility that the anticipation intervention with contagion might affect the network structure. To the best of our knowledge, this paper is the first to study how the anticipation of intervention affects endogenous networks and systemic risk, whether in banking or in other fields. Most closely related are two recent papers by Bernard et al. (2017) and Kanik (2017), who compare certain classes of networks under bailouts and bail-ins. More references are given in Section 5.

It is worth noting that most financial networks feature a core-periphery structure. Some authors, such as Farboodi (2015), Wang (2016), and Neklyudov and Sambalaibat (2017), show that core-periphery networks emerge endogenously without any government intervention. But these papers are not about systemic risk. Our results indicate that in a self-fulfilling process, the anticipation of government intervention makes some institutions systemically important and induces a core-periphery structure because government intervention mitigates systemic risk ex-post. Moreover, we show in Section 3.3.1 that even in the absence of intervention and under systemic risk, when ex-ante side payments are allowed, banks form core-periphery networks. In this case, the anticipation of intervention makes the network "more core-periphery." This, too, is a novel result.

Structure. Section 2 introduces the theory and provides the main theoretical results. In Section 3 the theory is applied to interbank lending. In Section 4 we briefly present other applications of the theory. We discuss the theory and the application to interbank lending in Section 5 and conclude in Section 6. Appendix A includes the proofs of Section 2. Appendix B continues with the theory and presents more results. Appendix C includes the proofs of Section 3 that concern the application to interbank lending. Appendix D details another application to real firms with joint projects.

2 Pareto Strong Stability under threshold contagion

2.1 Environment

Agents: Let $N = \{n_1, n_2, ..., n_k\}$ be a finite set of k agents. Each agent $n_i \in N$ has an ex-ante type $\gamma_i \in \Gamma$, where Γ is a finite set.

Network: Agents can form bilateral partnerships, called *links*. If n_i and n_j form a link, they are called *counterparties*, and the link is denoted $\{n_i, n_j\}$. The set of formed links is denoted $E \subset [N]^2$ and the network formed is denoted (N, E). N_i denotes the set of counterparties of n_i and $d_i = |N_i|$ the *degree* of n_i.

Agent n_i can form $d_i \in D(\gamma_i)$ many links where $D(\gamma)$ is the set of feasible number of links for type γ. Given the following contagion dynamics and the resulting expected payoffs for agents for any given network, agents are assumed to form Pareto Strongly Stable (PSS) networks in which they all have feasible degrees. The solution concept PSS, described in Section 2.2, roughly states that no coalition has a Pareto-improving deviation.

Contagion: After the network is formed, each agent n_i gets a *good shock* $\theta_i = G$ or a *bad shock* $\theta_i = B$. Shocks are i.i.d. and the probability of a good shocks is α.[4] Agents with good (bad) shocks are called the *good (bad) agents*, the set of which is denoted N_G (N_B). Bad agents are the initial set of *defaulting agents* during the first round of contagion. During any round along contagion if a non-defaulting agent n_i has strictly more than $R(d_i, \gamma_i)$ many defaulting counterparties, n_i defaults as well. $R(d_i, \gamma_i)$ is called the *resilience* (threshold) of n_i. The function R satisfies $R(d_i, \gamma_i) \in [0, d_i]$ for all d_i and γ_i. Contagion is irreversible. Contagion progresses round by round, and it stops when no new agent defaults during some round. Denote the default of n_i with $a_i = D$ and the final set of defaulting agents $N_D \supset N_B$. Call the remaining agents *continuing agents.* Denote the continuation of n_i with $a_i = C$ and denote the set of continuing agents $N_C \subset N_G$. Denote $b_i = |N_i \cap N_B|$ the number of bad counterparties of n_i and $f_i = |N_i \cap N_D|$ the number of defaulting counterparties of n_i.

Payoffs: At the end of contagion, a bad agent receives $P_B(d_i, \gamma_i)$ and a defaulting good agent receives $P_G(d_i, \gamma_i)$. A continuing agent n_i obtains payoff $P(f_i, d_i; \gamma_i)$. P is strictly decreasing in f_i.[5] Given the *payoff function* P and the resilience function R, agents can calculate the payoffs for any given network and form a feasible PSS network.

Interpretation: We provide detailed mappings of the theory to various financial, economic, and social networks in Sections 3 and 4. The essential two features are, first, that defaulting agents do not care about the number of defaulting counterparties and, second, continuing agents are strictly hurt by each defaulting counterparty. To fix ideas, think of a lead example wherein agents are banks that rely on each other to meet their short-term liquidity needs. Bad shocks mean large operational costs or large withdrawals of demand deposits (as in Allen and Gale (2000)) that force the bank with the bad shock into default. Then a good bank with too many defaulting counterparties may find that it is now too costly raise liquidity on demand. Such illiquidity can cause an otherwise healthy bank to be unable to meet the refinancing needs of projects it manages or be unlikely to meet the withdrawal demands of depositors. It can also create the opportunity costs that are caused by missing out on funding profitable projects. These costs can force even a good bank into default.

Remark: The outcome of the exogenous contagion dynamics can be micro-founded with an equilibrium of a network game with complementarities. Details are given in Section 5.

2.2 Solution Concept

Before shocks are realized, agents evaluate a network according to the expectation of their payoffs with respect to shocks given the contagion that follows. Agents form *Pareto Strongly Stable (PSS)*

[4] All results follow with conditionally independent shocks: for any given $\sigma \in \Delta[0,1]$ there is probability $\sigma(\alpha)$ that all shocks are i.i.d. with probability α. We use i.i.d. shocks for the sake of simplicity.

[5] In fact, it suffices to assume that for any $\gamma \in \Gamma$ and any $d \in D(\gamma)$, $P(f, d, \gamma)$ is strictly decreasing in f for $f \in [R(d, \gamma), d]$.

networks as defined in Jackson and Van den Nouweland (2005). Consider a candidate network (N, E) and a subset N' of banks. A *feasible deviation* by N' from E is one in which N' can simultaneously add any missing links within N', cut any existing links within N', and cut any of the links between N' and N/N'. A feasible deviation is illustrated in Figure 1.

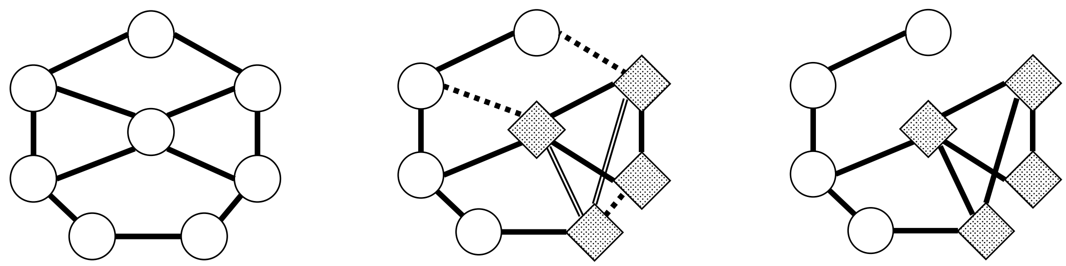


Figure 1: A feasible deviation

A *Pareto profitable deviation* by N' from E is a feasible deviation in which the resulting network yields weakly higher expected payoffs to every member of N' and a strictly higher payoff to at least one member of N'. A network (N, E) is *Pareto Strongly Stable (PSS)* if there are no subsets of N with a Pareto profitable deviation from E. In Appendix B we study weaker solution concepts.

2.3 Pareto Strongly Stable networks

Throughout the paper, we refer to losses due to bad counterparties as *first-order counterparty losses.* Losses due to defaulting good counterparties who default due to their bad counterparties are dubbed *second-order counterparty losses.* Higher order counterparty losses are defined analogously. For a given network, the expected counterparty losses of order t are called the *counterparty risk of order* t. Notice that if an agent n_i faces no counterparty risk of order t, then it faces no counterparty risk of order $t' > t$ either. This is because contagion that originates at distance t' from n_i has to go through agents at distance t in order to hurt n_i. If bad shocks to agents at distance t' from n_i cannot hurt n_i via contagion, then bad shocks to agents at distance t to n_i cannot hurt n_i via contagion either. It will turn out that first-order counterparty risk (FOCPR) and second-order counterparty risk (SOCPR) are the governing forces of network formation. FOCPR captures the propensity of agents to form links and determines the density of the network. SOCPR captures how concerned agents are with the counterparties of their counterparties, and it determines the structure of the network. FOCPR is illustrated in Figure $2{-}a$ and SOCPR is illustrated in Figure $2{-}b$.

A *star network* is useful to illustrate both contagion and some forces behind network formation. A star network, as shown in Figure 3, is one in which one agent, called the *center*, has links with all other agents. All other agents, which are called *leaf* agents, have links only to the center agent. Consider a disjoined star subnetwork with center n_i who has degree d_i.

If the center agent n_i gets a bad shock, then all leaf agents default (supposing that $R(1, \gamma_j) = 0$ for all leaves n_j). This is direct contagion from center to leaves. If the center agent n_i gets a good

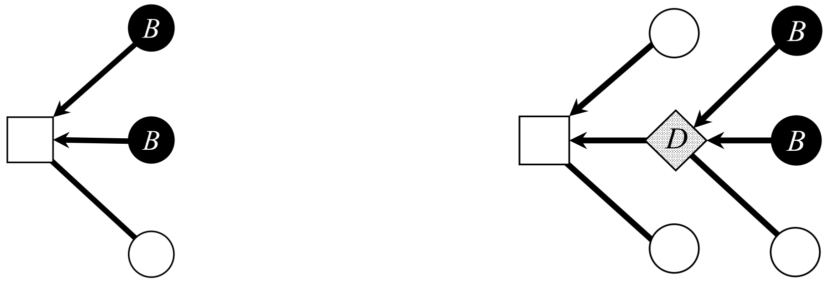
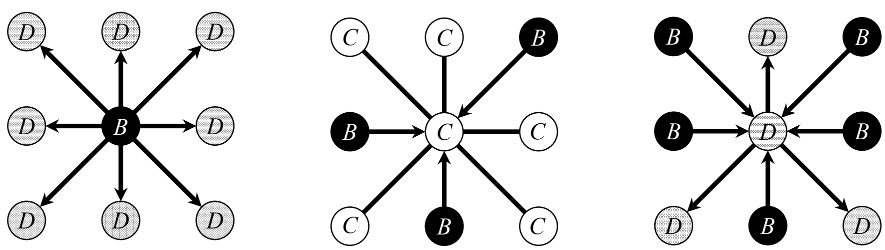


Figure 2−a : FOCPR faced by “square”

Figure 2−b : SOCPR faced by “square” through “diamond”

Figure 2: First-order and second-order counterparty risks

shock and $R(d_i, \gamma_i)$ many or less leaf agents get bad shocks, then n_i continues, and in turn the leaf agents that received good shocks also continue. In this case the center is sufficiently resilient and serves as a buffer. Contagion does not transmit through the center. If the center n_i gets a good shock and more than $R(d_i, \gamma_i)$ many leaf agents get bad shocks, then n_i defaults, which, in turn forces all leaf agents to default as well. This is the case in which peripheral shocks accumulate and force the center into default, creating cascading failures.



Downstream: Center to leaves

Resilient center: Leaves only

Up then downstream: Cascading failures

Figure 3: A star network and types of contagion

A particular feature of the star network is that the center is not exposed to any SOCPR because its counterparties, the leaves, have no counterparties other than the center. There is no way in which a good counterparty of n_i can hurt n_i. Moreover, because bad shocks are exogenous, n_i can not affect FOCPR conditional on degree d_i. This leads to a value for n_i that is a function of its degree, conditional on there being no SOCPR for n_i. Denote the payoff of n_i when it is at the center of a star network with $V(d_i, \gamma_i)$. Call V the *ideal value function*. V is given by

$$\begin{aligned} V(d_i,\gamma_i) = {} & \underbrace{(1-\alpha)\,P_B(d_i,\gamma_i)}_{if\, n_i\, is\, bad} + \underbrace{\alpha\mathbb{P}_{b_i}\left[b_i > R(d_i,\gamma_i)\right]\times P_G(d_i,\gamma_i)}_{if\, n_i\, is\, good\, but\, defaults} \\ & + \underbrace{\alpha\mathbb{P}_{b_i}\left[b_i \le R(d_i,\gamma_i)\right]\times \mathbb{E}_{b_i}\left[P(d_i,b_i,\gamma_i)\middle|\, b_i \le R(d_i,\gamma_i)\right]}_{if\, n_i\, is\, good\, and\, does\, not\, default}. \end{aligned} \tag{1}$$

Notice that in the expression of V, P is evaluated at $f_i = b_i$. The number of defaulting counterparties

f_i is equal the number of bad counterparties b_i because n_i is not exposed to any SOCPR in the star configuration.

Proposition 1. *For any network, n_i's expected payoff is at most $V(d_i, \gamma_i)$.*

Consequently, a disjoined star subnetwork is one ideal configuration for n_i, conditional on its degree d_i, in the sense that n_i cannot achieve a higher expected payoff in any other network in which it has the same degree d_i. This brings us to the next question: What other structures can give n_i its ideal value, conditional on its degree? Eliminating SOCPR is a necessary and sufficient condition for achieving the ideal value conditional on degree. The next proposition pins down how SOCPR for n_i can be eliminated. Denote $d_{ij} = |N_i \cap N_j|$.

Proposition 2. *n_i has $V(d_i, \gamma_i)$ expected payoff if and only if $\min\{R(d_i, \gamma_i), d_{ij}\} + (d_j - d_{ij} - 1) \leq R(d_j, \gamma_j)$ is satisfied for all $n_j \in N_i$.*

Proposition 2 is essential to our following results. An explanation is offered in Figure 4. Consider all events in which n_i is not forced into default by its bad counterparties. Can any such event cause a good counterparty $n_j \in N_i$ to default, which would be a second-order counterparty loss for n_i? This can happen if and only if at the event in which at most $R(d_i, \gamma_i)$ common counterparties of n_i and n_j get bad shocks and n_j's own counterparties get bad shocks, n_j is forced to default. This situation is illustrated in Figure 4.

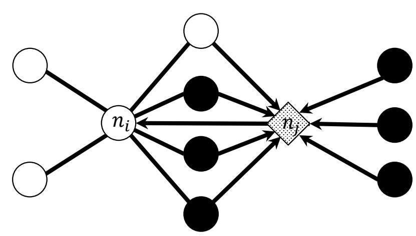


Figure 4: Eliminating second-order counterparty risk

Proposition 3. *Consider a disjoined clique with $d+1$ agents and an agent n_i in this clique. If all agents in the clique have the same or higher resilience than n_i (i.e., $R(d, \gamma_j) \geq R(d, \gamma_i)$ for all n_j in the clique), then n_i has the expected payoff $V(d, \gamma_i)$.*

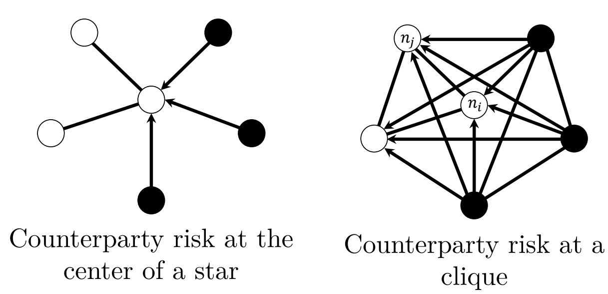


Figure 5: Counterparty risk at the center of a star and in a clique

The star network and those described by Proposition 2 are desirable configurations for n_i. One such configuration is a disjoined clique wherein all agents are at least as resilient as n_i. Nevertheless, such structures may not be desirable for counterparties of n_i; hence such a network may not be PSS. For example, in the star network the leaves are exposed to SOCPR through the center, and pairs of leaves could deviate and form links with each other. Or in the clique structure, counterparties of n_i who are more resilient than n_i may want to deviate and form more resilient cliques. Next we examine whether or how two counterparties can both eliminate SOCPR and achieve their ideal value conditional on their degrees. But first we need to introduce some definitions.

Define the set of *safe γ-counterparty degrees* $S(\gamma) := \{d \in D(\gamma) : R(d, \gamma) \geq d - 1\}$. This is the set of degrees such that having a γ-type counterparty of such a degree does not carry any SOCPR.

Proposition 4. *Consider two counterparties, n_i and n_j, that both achieve their ideal values conditional on their degrees. Then, either*

- *they both have unsafe counterparty degrees, their set of counterparties are identical except each for other, and they have the same resilience, or*
- *they both have safe counterparty degrees.*

The only way two counterparties with unsafe counterparty degrees get their ideal payoff conditional on their degrees is if none of them creates any SOCPR for the other. This is only possible if they have exactly the same counterparties and the same resilience. Also, an agent with a safe counterparty degree cannot achieve its ideal payoff conditional on its degree if it has any counterparty with an unsafe counterparty degree. The next and final step in finding the structure of the network formed is to consider whether or how all agents in a component[6] can achieve their ideal value. This is important to determine since our solution concept PSS allows for joint deviations by any number of agents.

Proposition 5. *Take any component. All agents in the component achieve their ideal payoffs given their degrees if and only if either*

- *they all have unsafe counterparty degrees, the component is a disjoined clique (hence all have the same degree), and they all have the same resilience, or*
- *they all have safe counterparty degrees.*

This results indicates that the perfectly resilient types can form an arbitrary structure because they naturally do not carry over any SOCPR to one another. Imperfectly resilient types, however, will form cliques in order to endogenously eliminate SOCPR. Consequently, all agents will end up eliminating SOCPR endogenously.

A next question follows: Conditional on there being no SOCPR and agents achieving their ideal values conditional on their degrees, what degree would agents want to have? The number of links

[6] A component is a connected subnetwork that is not connected to the rest of the network.

n_i can have is restricted, first, by its feasible set of degrees $D(\gamma_i)$ and, second, by the set of agents who are willing to form links. Define the *ideal degree for type* γ as[7]

$$d^*(\gamma) := argmax_{d<k,\, d\in D(\gamma)} V(d,\gamma). \quad (2)$$

Also call $V(d^*(\gamma),\gamma)$ the *ideal value for type* γ. According to Proposition 5, if $d^*(\gamma)+1$ agents of type γ come together and form a disjoined clique, they achieve their ideal value. However, this is not the only way agents can achieve their ideal values. For two counterparties to achieve their ideal values they need not have the same type but identical ideal degrees and identical resilience at their ideal degrees. Accordingly, call two types $\gamma,\gamma'\in\Gamma$ *similar* if their ideal degrees are equal, $d^*(\gamma)=d^*(\gamma')$, and the resulting resiliences are equal $R(d^*(\gamma),\gamma) = R(d^*(\gamma'),\gamma')$. Notice that similarity is an equivalence relation. Consider the equivalence classes induced by similarity. Index the equivalence classes by ι. Let k^ι be the number of agents in equivalence class ι. For an equivalence class ι, denote the ideal degree and induced resilience of the class with $d^{*\iota}=d^*(\gamma)$ and $R^{*\iota}=R(d^*(\gamma),\gamma)$, where γ is an element of the equivalence class ι. For an equivalence class ι, if the ideal degree is a safe counterparty degree, $R^{*\iota}\geq d^{*\iota}-1$, call this class a *safe class*; otherwise call it an *unsafe class*. According to Proposition 5, for unsafe classes, disjoined cliques that consist of similar types, wherein all agents have their ideal degree, give all of them their ideal value. This is the only way in which all agents achieve their ideal values.

Theorem 1. *Suppose that the number of agents from each class is larger than the ideal degree of the class:* $\forall\iota$, $k^\iota\geq d^{*\iota}+1$. *A network is PSS if and only if it consists of disjoined cliques of agents from the same unsafe class, each of which has their ideal degree, and a subnetwork of all safe classes in which each agent has its ideal degree.*[8]

The resulting network is illustrated in Figure $6-a$. High resilience agents prefer to form links with high resilience agents. Safe classes do not form links unsafe classes. So agent from the safest unsafe class forms links only with agents from the same safest unsafe class. Therefore there is sorting in terms of resilience. Agents from safe classes form links only with each other in a way that each agent achieves its ideal degree. Agents from the second most resilient class, which is an unsafe class, forms links with only with each others, making up disjoined cliques. The same argument holds iteratively for less resilient classes. By this argument, our result is a consequence of matching and sorting, but not symmetry across certain groups of agents. More results concerning PSS and weaker solution concepts are given in Appendix B.

2.4 Prologue to network hazard

In our general theory, P and R are both fundamentals of the model and do not need to be linked to each other. An example in which the two are not related is the spread of diseases. The social

[7] Generically, R, P, P_G, and P_B are such that V admits no indifferences over integers. We assume that V admits no such indifferences in order to rule out some cumbersome and unintuitive indifferences.

[8] In the "safe" part of the network, agents can also become counterparties with other classes because they all have safe counterparty degrees.

benefits of links do not need to be linked to the infectiousness of a disease. However, in many economic applications, it is possible that a good agent defaults once the payoff from continuing falls below the payoff from defaulting. For example, once a bank cannot repay its depositors, it goes bankrupt, but is protected by limited liability. Consequently, in many economics applications, R is going to be given by

$$R(d_i, \gamma_i) = max \left\{ f_i \,:\, P\left(d_i, f_i, \gamma_i\right) \geq P_G\left(d_i, \gamma_i\right) \right\}.$$

In these economic and financial applications, government intervention that stops ex-post contagion is going have an ex-ante consequence: agents are not going to be worried about contagion during network formation. This by-product of intervention will break this tie between payoff P and resilience R. Market discipline during network formation will be reduced, which is going to generate what we call *network hazard*.

To make things more concrete, imagine a principal that can intervene with contagion. The forms that intervention can take and the objective of the principal are context-specific. Each particular application of the theory is going to yield particular form and constraints for policy; it will yield a particular welfare criterion, and, hence, yield a particular optimal policy. We fix the timing of policy throughout the paper: intervention is going to be time-consistent in the sense that the principal can not commit to a policy before the realization of shocks. That is, the policy cannot be used directly to regulate the formation of the network. The principal observes the network and shocks, foresees the extent of contagion, and tries to alter contagion. In applications to financial and economics networks, such a principal could naturally be a government and the form of policy could be subsidizing the losses of agents using transfers from outside the system. These transfers are typically bailouts, such as capital injections or purchase of legacy assets. In Section 3 we present detailed applications of the theory to interbank lending and study in detail the resulting welfare criteria. Nonetheless, before moving on to detailed applications, it is useful to summarize what we mean by *network hazard* under a high-level welfare criteria.

Consider banks that rely on each other to meet their short-term liquidity needs. Suppose that bad shocks mean large operational costs that force bad banks into bankruptcy. These defaults can render good banks illiquid and create cascading defaults. In this situation, it is conceivably optimal to let insolvent bad banks fail and provide liquidity to illiquid good banks. This would ensure that all good banks and only good banks continue. As a result, all good banks always continue and the tie between P and R is broken. The resilience function R artificially becomes

$$\tilde{R}(d_i, \gamma_i) = d_i.$$

All degrees become safe counterparty degrees. Then as a corollary of Theorem 1, the cliques dissolve and an interconnected network emerges. This can be seen in Figure 6.

In general, time-consistent government intervention is going to alter resilience R and ideal value function V into some $\tilde{R}$ and $\tilde{V}$. Under any welfare criterion that requires that good agents are directly or indirectly assisted, the resilience function R is modified into $\tilde{R}(d_i, \gamma_i) = d_i$ for all d_i and

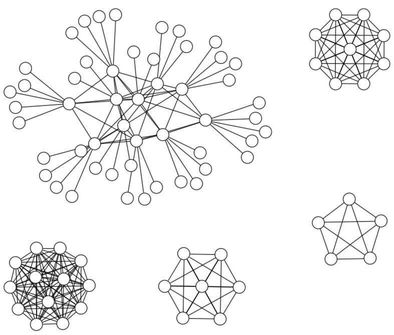


Figure 6$-a$: In the absence of intervention unsafe classes form cliques, safe classes form an interconnected component

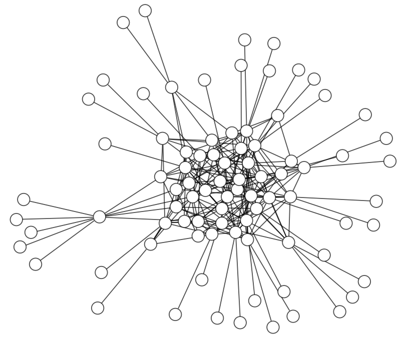


Figure 6$-b$: In the presence of intervention, all agents form an interconnected network

Figure 6: PSS network in the absence and presence of intervention

γ_i. That is, all degrees become safe counterparty degrees. SOCPR is eliminated as a by-product of intervention, which is the formal definition of *network hazard*.

3 Application to interbank lending and bailouts

3.1 Second-order counterparty risk

We first study the sole impact of the elimination of SOCPR. To do this, we introduce in Section 3.1.1 a simple framework wherein FOCPR is not mitigated by bailouts but SOCPR is eliminated. In Section 3.1.2 we show that network hazard creates welfare volatility in this framework. Finally, in Section 3.1.3 we contrast network hazard with moral hazard.

3.1.1 Model with solvency shocks

Absence of intervention. This application involves banks that form borrowing and lending partnerships that allow them to meet their short term liquidity needs. There are three stages, as shown in Figure 7.

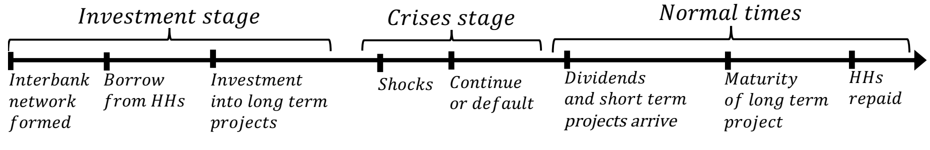


Figure 7: Timeline of events

Investment stage: Some banks are large banks located at central locations denoted $\gamma_i = L$. Some banks are small and rural banks, and they have $\gamma_i = S$. There are k_L many large banks and k_S many small banks. Each bank n_i has D_{γ_i} deposits and needs to pay depositors at the end. Deposit level is a proxy for the size of the bank, so we assume $D_L > D_S$. Banks can form bilateral partnerships for borrowing and lending in the future, which are represented by links. The utility cost of forming d_i links to n_i is $c_{\gamma_i} d_i^2$. We take $c_S > c_L = 0$ to capture the location advantage of large banks. After forming a PSS network, banks invest their deposits into some proprietary projects that pay a flow of dividends at rate λ and a safe rate of return $1 + r$ at maturity. We call this the long project.

Crises stage: In this stage, some banks receive shocks. A bad shock represents a very large operational cost for the bank and enforces its bankruptcy. Banks with bad shocks are called *insolvent bad banks*. A good shock represents small operational costs, and a bank that experiences such a shock is called a *good bank*. A bad shock to n_i is denoted $\theta_i = B$ and has probability $1 - \alpha$. A good shock to n_i is denoted $\theta_i = G$ and has probability α.

Insolvent bad banks cannot continue because of the high cost of their operations. They liquidate their long projects and repay their depositors as much as they can. Each good bank either continues its operations or defaults. Continuation is denoted $a_i = C$, and it has an operational cost κ_{γ_i} per unit of investment managed. Default, denoted $a_i = D$, does not entail any operational cost. In the event of default, the project is liquidated. If n_i liquidates its project, for some $\eta < 1$, n_i recovers $(1 - \eta) D_{\gamma_i}$ and pays it to the depositors. ηD_{γ_i} loss is covered by the deposit insurance. Accordingly, the payoffs of bad insolvent bad banks and the payoffs of solvent good banks that default are given by

$$P_B (d_i; \gamma_i) \equiv P_G (d_i; \gamma_i) = -c_{\gamma_i} d_i^2.$$

Normal times: All good banks that continue move on to normal times. Normal times consist of $\bar{t} \geq 1$ periods. During each period $t \leq \bar{t}$, n_i receives λD_{γ_i} dividend payments from the project. Moreover, during every period $t \leq \bar{t}$, one new outside borrower with a safe short-term project arrives at the economy. Call these short projects. Short projects have a $r' > 1$ rate of return that materializes at the end of the period. Each bank has probability $p < k^{-1}$ of receiving this borrower with the short project. The links help banks borrow and lend to each other and to channel their dividends to the bank with the short project. If n_i has a counterparty with the short project, say n_j, then n_i makes a take-it-or-leave-it offer of lending λD_{γ_i} to n_j at rate r'. n_j, in return, borrows from all counterparties, and lends these funds and its own dividends to the borrower with the short project. At the end of the period, the short project yields safe returns, the outside borrower pays n_j, and n_j pays all counterparties. Then all returns and dividends are consumed by banks.[9] The main friction interbank market is that funds cannot travel more than one link. Then, at the beginning of period t, n_i's expected return within period t is $p (d_i - f_i + 1) \lambda D_{\gamma_i} r' + \lambda D_{\gamma_i}$. At the end of period $\bar{t}$, n_i's project matures and returns $(1 + r) D_{\gamma_i}$. The original depositor is repaid D_{γ_i} and the remaining resources are consumed. The expected payoff of bank n_i at the beginning of normal times is

$$\bar{t} p \lambda r' D_{\gamma_i} (d_i - f_i + 1) + \bar{t} \lambda D_{\gamma_i} + r D_{\gamma_i} - \kappa_{\gamma_i} D_{\gamma_i}.$$

Corresponding R and V: Denote $\chi_1 = \bar{t} p \lambda r'$ and $\chi_2 = \chi_1 + \bar{t} \lambda + r$. If $a_i = C$, n_i's ex-post payoff P

[9] No dividend retention or perishable goods.

is given by

$$P\left(f_i, d_i; \gamma_i\right) = -c_{\gamma_i} d_i^2 + \left[\left(d_i - f_i\right)\chi_1 + \chi_2 - \kappa_{\gamma_i}\right] D_{\gamma_i}.$$

If $\left(d_i - f_i\right)\chi_1 + \chi_2 - \kappa_{\gamma_i} < 0$, then n_i defaults. That is, the contagion dynamics are governed by

$$R\left(d_i, \gamma_i\right) = d_i - \tau_{\gamma_i},\ \tau_{\gamma_i} = \left\lceil \frac{\kappa_{\gamma_i} - \chi_2}{\chi_1} \right\rceil. \tag{3}$$

In this case call n_i an *illiquid good bank*. Moreover, the ideal value function is

$$V\left(d_i, \gamma_i\right) = -c_{\gamma_i} d_i^2 + \alpha \mathbb{E}_{b_i}\left[\left[\left(d_i - b_i\right)\chi_1 + \chi_2 - \kappa_{\gamma_i}\right]^+\right] D_{\gamma_i}. \tag{4}$$

Define $d_S^* = argmax_{d_i \geq 0} V\left(d_i; S\right)$. Note that $d_S^* < \infty$ because $c_S > 0$.

Assumption 1. *Large banks have higher management costs and, thus, they are less resilient than small banks: $\tau_L > \tau_S$. Small banks are not entirely safe as counterparties: $\tau_S > 1$.*

Assumption 2. *There are few large banks and many small banks: $k_L < d_S^* < k_S$.*

Proposition 6. *In the absence of intervention, a network is PSS if and only if it consists of disjoined cliques of small banks, each of which has degree d_S^*, and one disjoint clique of large banks, each of which has degree $k_L - 1$.*

This result is not a direct corollary of Theorem 1; instead, it is a corollary of both Theorem 1 and Proposition 2 together. Large banks incur no cost of forming links, and so they want to form as many links as possible. Large banks, simply by forming links among themselves, cannot reach their ideal degree. They need to form links with small banks, too. But large banks are not sufficiently resilient, and so small banks are not willing to form links with large banks. Small banks instead form links with one another.[10]

Presence of intervention. Now suppose that a government can intervene with the market. Intervention is allowed only during the crises stage; it is not allowed in normal times.[11]

We assume that the government can intervene just before contagion during the crisis stage by implementing transfers from households−e.g., through bailouts, as in capital injections. Figure 8 show the timeline of events. A transfer can be executed to save troubled banks on the basis of their financial standing or to stop contagion. Formally, after the shocks and before the default decisions, government commits to a transfer scheme, $\left\{T_i\left(a_i \middle| \vec{\theta}, E\right)\right\}_{i\in N} \geq 0$. Here T_i describes the amount of transfer to bank n_i if n_i has $a_i \in \{C, D\}$.[12] The intervention is time-consistent in the sense that the government cannot commit to a transfer policy before the shocks.

[10] Note that this network is not efficient. What is efficient is large banks forming links with some small banks too. We study the efficient network in Section 3.3.1.

[11] Section 13(3) of the FED Act allows the FED to provide an uncapped amount of liquidity to the banking system *only under unusual and exigent circumstances* (https://www.federalreserve.gov/aboutthefed/section13.htm).

[12] In general, T_i depends on the entire action profile $\left(a_i\right)_{i\in N}$. The specification, wherein the transfer to n_i depends only on a_i but not a_{-i}, is without a loss of generality as far as optimal policy is concerned.

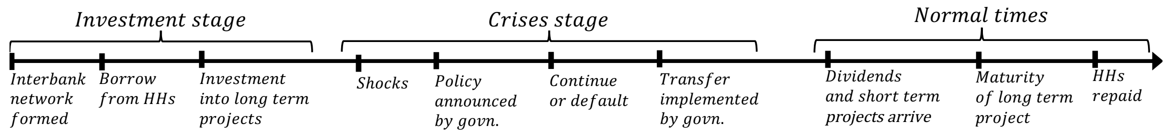


Figure 8: Timeline of events

T can induce an otherwise defaulting bank to continue. But for the receiving bank these transfers are not always free. For the sake of simplicity we assume that a bailed-out bank forfeits all future payoffs to the government, which is then redistributed to households. We assume that transfers from households do not entail any extra distortionary cost in terms of welfare. Accordingly, we define an *optimal policy* as an ex-post welfare maximizing transfer scheme that uses minimal transfers among all welfare maximizing transfer schemes.[13] Consequently, the optimal policy T^* satisfies $T_i^* (D|\cdot) \equiv 0$, meaning that defaulting banks do not receive any transfers. Then with a slight abuse of notation, denote $T_i^* = T_i^* \left(C|\vec{\theta}, E\right)$, which is the amount of transfer n_i gets if it continues. We say that an optimal policy T^* *bails-out* n_i if $T_i^* > 0$.

A good bank with less than τ_{γ_i} good counterparties faces default and is indirectly illiquid. If part of the bank's debt to depositors was covered it could continue, which by saving the liquidation costs and future dividends of the projects could improve welfare.

Assumption 3. *It is welfare improving that solvent good banks continue if they have at least one continuing counterparty:* $\chi_1 + \chi_2 + \eta > \kappa_L$.

Proposition 7. *The unique optimal policy bails out illiquid good banks that are facing default because of insolvent bad counterparties. Formally*

$$T_i^* = D_{\gamma_i} \left[\kappa_{\gamma_i} - \chi_2 - (d_i - b_i)\, \chi_1\right]^+$$

if $\theta_i = G$ *and* $T_i^* = 0$ *if* $\theta_i = B$.

Corresponding R and V: Under the implementation of the optimal policy T^*, good banks cannot be forced into default by their counterparties. In other words, $\tilde{R}(d_i; \gamma_i) = d_i$. Good banks become immune to contagion and all good banks always continue. Because of the anticipation of intervention, banks no longer worry about their illiquid good counterparties. SOCPR is eliminated. However, insolvent bad banks are not bailed out, and so all banks still concern themselves with their bad counterparties as much as they would in the absence of intervention. FOCPR is not altered and $\tilde{V} \equiv V$.

Proposition 8. *In the absence of intervention, a network is PSS if and only if all large banks have degree $k-1$ and small banks have degree d_S^*.*

[13] Because of the requirement that optimal policy must use minimal transfers, our results are robust to allowing for small costs of transfers. To make our point about network hazard as simple as possible, we do not introduce any distortionary costs of transfers.

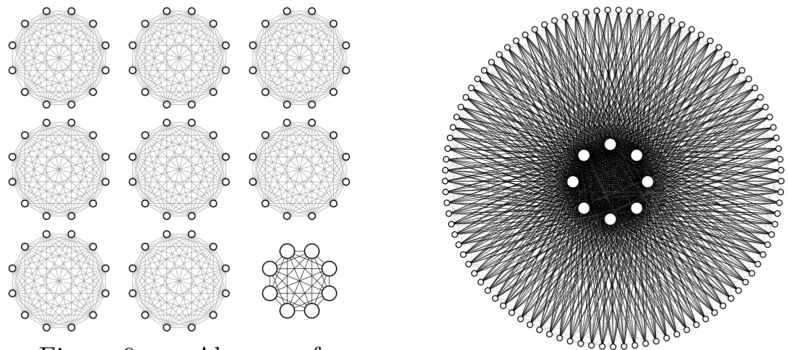


Figure 9−a : Absence of intervention, disjoined cliques.

Figure 9−b : Presence of intervention, core-periphery.

Figure 9: PSS network under solvency risk

As a by-product of intervention, SOCPR is eliminated. Banks do not concern themselves with the illiquidity risk of their counterparties, be it the direct liquidity risk from shocks or indirect illiquidity risk due to counterparty failures that make the bank illiquid at normal times. This makes all degrees safe counterparty degrees for both large and small banks during network formation. Then the cliques dissolve and the network acquires a core-periphery structure. The resulting change is illustrated in Figure 9. For two reasons the network acquires a core-periphery character.

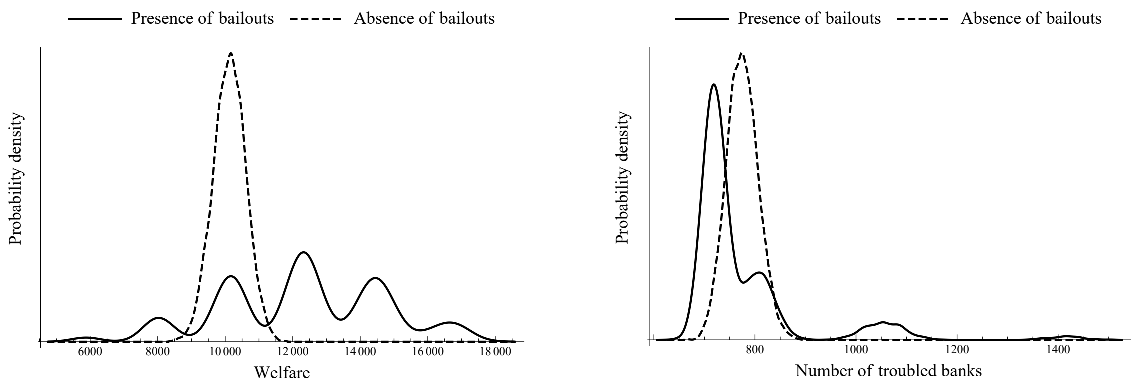
First, because bad banks are not bailed out, all bad banks still default. Then FOCPR is unchanged and $\tilde{V} \equiv V$. In other words, a bank's propensity to form links does not change. Accordingly, small banks form the same number of links: $\tilde{d}_S^* = d_S^*$. Because SOCPR is eliminated small banks no longer need to form cliques. This makes the periphery interconnected within itself.

Second, because SOCPR risk is eliminated, large banks are no longer susceptible to carrying over SOCPR across different parts of the network. Small banks start connecting with large banks. The propensity of a large bank to form links has not changed. Yet small banks no longer refuse to form links with large banks. This *allows* large banks to form their desired number of links, making them highly connected. Large banks become the core of the system, which renders them systemically important.

This change in the topology of the network is solely a result of the elimination of SOCPR (i.e. network hazard). FOCPR is not altered by intervention. We elaborate more on the contrast between FOCPR and SOCPR in Section 3.2.

3.1.2 Network hazard and volatility

The major impact of network hazard is to make large banks systemically important, which leads to volatility. Under the core-periphery structure, the risk of the whole economy is correlated through the solvency of the core. If a solvent core bank is illiquid because it has many insolvent bad counterparties, it gets bailed out and continues its business. Yet the insolvent core banks fail, as a consequence of which the potential benefits of the many links between the core and periphery are lost. If many core banks are insolvent the economy performs very poorly, and if many core banks are solvent the economy performs very well. Therefore, the anticipation of intervention creates volatility both in welfare and in the performance of the banking sector. The probability distribution of ex-post welfare and the number of troubled banks are illustrated Figure 10.



Absence: $\mu = 10214, \sigma = 483$
Presence: $\mu = 12300, \sigma = 2433$

Absence: $\mu = 775, \sigma = 31$
Presence: $\mu = 775, \sigma = 120$

Figure 10: Volatility as a consequence of intervention

It is important to note that mean ex-ante welfare increases as a consequence of intervention. The reason for this is as follows. Defaulting banks are made indifferent between defaulting or not; consequently FOCPR is not reduced. Therefore, banks' propensity for connections does not change. This and the fact that the cost of links are borne fully by banks, network hazard allowing the large banks to achieve the individually rational level of connections does not hurt welfare.

Proposition 9. *There exists $\bar{k}_L$ and $\underline{k}_S$ such that if $k_L < \bar{k}_L$ and $k_S > \underline{k}_S$, the mean and standard deviation of welfare is higher in the presence of intervention than in the absence of intervention.*[14]

3.1.3 Moral hazard

To the extent that volatility and increased likelihood of a tail event are hazardous, the elimination of SOCPR has hazardous consequences. Now we compare and contrast network hazard and moral hazard. We first introduce into our framework a form of individual moral hazard in operational risk choice that arises from limited liability and deposit insurance. We find that interconnectedness that arises from network hazard does not alter the extent of individual moral hazard by small banks. The systemic importance of large banks has ambiguous effects on the extent of individual moral hazard by large banks.

[14] That the welfare or volatility increases should not be seen as a general statement about the implications of network hazard for other applications of the theory. For example, in certain applications it is possible that the cost of links is not perfectly internalized by the agents who forming the links. Thus, the anticipation of intervention that allows certain agents to become more connected can lead a reduced welfare.

Suppose that banks can choose to incur some of their operational costs in advance. Smaller operational costs during the crises stage makes a bank more resilient. This partially reverses the operational risk shifted onto depositors. Formally each n_i can incur a cost $c_{op}(\kappa; \gamma_i)$ during the investment stage to set its operational cost of continuing during the crises stage to κ. c_{op} is decreasing in κ. For example, if $c_{op}(\kappa; \gamma_i) = \bar{\kappa}_{\gamma_i} - \kappa$ for some $\bar{\kappa_{\gamma_i}}$ so that incurring the cost earlier or incurring the cost later are perfect substitutes, then n_i always chooses $\kappa = \bar{\kappa}_{\gamma_i}$ to incur the operational cost during the crises stage and shifts all the operational risk onto the depositors. For general functional forms c_{op}, there is an optimal level $\kappa^*_{\gamma_i}$ in the absence of intervention. In the presence of intervention, this optimal choice becomes $\tilde{\kappa}^*_{\gamma_i}$. We are interested in whether $\tilde{\kappa}^*_{\gamma_i} > \kappa^*_{\gamma_i}$ or not. If $\tilde{\kappa}^*_{\gamma_i} > \kappa^*_{\gamma_i}$, the bank n_i is shifting more operational risk onto depositors in response to the anticipation of bailouts.

Proposition 10. *There exists κ^*_S, d^*_S, κ^*_L, and $\tilde{\kappa}^*_L$ such that*

- *In the absence of intervention, a network is PSS if and only if*
  - *all small banks choose operational cost κ^*_S and all large banks choose operational cost κ^*_L, and*
  - *the network consists of disjoined cliques of small banks, each of which has degree d^*_S, and one disjoined clique of large banks, each of which has degree $k_L - 1$.*
- *In the presence of intervention, a network is PSS if and only if*
  - *all small banks choose operational cost κ^*_S and all large banks choose operational cost $\tilde{\kappa}^*_L$, and*
  - *in the network, all small banks have degree d^*_S and all large banks have degree $k - 1$.*

Because FOCPR is not altered small banks have the same ideal value function in both the absence and presence of intervention. Accordingly, the realized degrees of small banks do not change, and so the optimal level of investment in their operations does not change. Large banks, on the other hand, are allowed form more connections because SOCPR is eliminated. Such a change in the degree does affect the level of investments in operations. The direction of the change, however, is ambiguous: it depends on the curvatures of the cost function c_{op} and the CDF of the binomial distribution. To infer a direction of change it is not sufficient to assume that c_{op} is just convex.

3.2 First-order counterparty risk

Now we dig deeper into understanding network hazard. Our main goal in Section 3.2 is to compare and contrast the impact of FOCPR and SOCPR. In Section 3.2.1, we tweak the model presented in 3.1.1 to incorporate liquidity shocks to banks rather than solvency shocks. Under liquidity shocks, optimal policy will be to bail out banks with bad shocks, which will eliminate FOCPR and SOCPR. Then in Section 3.2.2 we compare the two models and dissect the distinct effects of eliminating FOCPR and SOCPR.

3.2.1 Model with liquidity shocks

Consider the model in 3.1.1 and change the nature of a shock as follows. For some banks, their depositors need to withdraw all deposits early. These banks are called *illiquid bad banks.* Early withdrawal from bank n_i is denoted $\theta_i = B$ and has probability $1-\alpha$. In this case n_i has to liquidate the long project. Otherwise, with probability α, $\theta_i = G$ and there is no withdrawal. The bank is then called a *good bank.* After withdrawals, each good bank decides to continue its operations or default. Continuation is denoted $a_i = C$ and has an operational cost of κ_{γ_i} per unit of investment managed. Default, denoted $a_i = D$, entails no operational cost.

Proposition 6 holds identically with regards to the absence of intervention. As for the presence of intervention, the optimal policy is rather different.

Proposition 11. *The unique optimal policy bails out all illiquid bad banks. Formally $T_i^* = D_{\gamma_i}$ if $\theta_i = B$ and $T_i^* = 0$ if $\theta_i = G$.*

Now that all banks with bad shocks receive bailouts, all banks always continue. FOCPR is eliminated, as are all orders of counterparty risk, particularly SOCPR. Accordingly, $\tilde{R}(d_i;\gamma_i) = d_i$. Since FOCPR is eliminated

$$\tilde{V}(d_i,\gamma_i) = -c_{\gamma_i}d_i^2 + \alpha\left[d_i\chi_1 + \chi_2 - \kappa_{\gamma_i}\right]^+ D_{\gamma_i}.$$

$\tilde{R}(d_i;\gamma_i) = d_i$ turns all degrees into safe counterparty degrees and both L and S turn from unsafe types into safe types. The clique structure dissolves into an interconnected network. The ideal degree of large banks is $k-1$ since $c_L = 0$. The ideal degree of small banks increases because FOCPR is eliminated. The new ideal degree becomes $\tilde{d}_S^* = argmax_{d_i \geq 0}\tilde{V}(d_i;\gamma_i)$. Note that $\tilde{d}_S^* \geq d_S^*$.

Proposition 12. *In the absence of intervention, a network is PSS if and only if all large banks have degree $k-1$ and small banks have degree $\tilde{d}_S^*$.*

The change in the topology of the network from cliques to core-periphery is solely a result of the elimination of SOCPR (i.e., network hazard). Eliminating FOCPR increases the propensity of banks to form links, which increases the realized degree of small banks from d_S^* to $\tilde{d}_S^*$.

3.2.2 Liquidity vs. solvency shocks: dissecting the impact of bailouts on the network

The anticipation of intervention impacts the network and welfare through two main channels: FOCPR and SOCPR. Their effects and channels are summarized in Figure 11.

Under liquidity shocks, bailouts eliminate FOCPR, which alters the ideal value function V into $\tilde{V} \not\equiv V$. The individual propensity of banks to form links increases, which makes banks form more links compared to the absence of intervention. This is a consequence of bilateral agreements between banks that individually want to have more links. When FOCPR is eliminated, so is SOCPR, and when SOCPR is eliminated, the resilience function R becomes $\tilde{R}$, which is no longer tied down to payoff function P. Good banks become perfectly resilient. This relaxes market discipline and leads

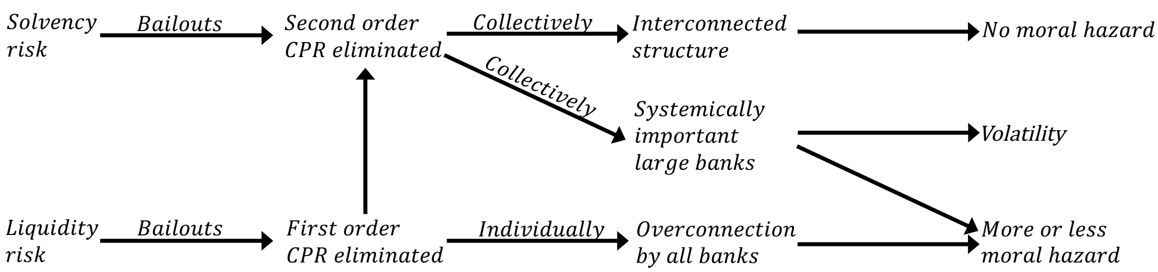


Figure 11: Intervention and its consequences

to the dissolution of cliques. Banks form interconnected networks. The interconnection, however, is a collective decision rather than a bilateral or an individual decision. Interconnection arises because groups of counterparties do not enforce discipline on each other with regards to each other's other counterparties.

The elimination of FOCPR is sufficient but not necessary for the elimination of SOCPR. Indeed, under solvency shocks rather than liquidity shocks, only good banks are bailed-out, which eliminates SOCPR, whereas FOCPR remains intact. Accordingly, the ideal value function V is not altered by bailouts, $\tilde{V} \equiv V$, and banks are not willing to form more links. Nonetheless, R becomes $\tilde{R} \not\equiv R$, and good banks become perfectly resilient. The dependence between resilience $\tilde{R}$ and payoff P again is broken. Network hazard exists for both liquidity shocks and insolvency shocks that lead to interconnectedness, whereas willingness to form more links exists only for liquidity shocks.

Under liquidity shocks, intervention eliminates FOCPR, which increases the willingness to form links. Since the elimination of SOCPR is necessary but not sufficient for the elimination of FOCPR, one can argue that network hazard is a *gateway for over-connectedness.* But there is another form of over-connection. Recall that under solvency shocks, intervention eliminates SOCPR but not FOCPR, which makes large banks more connected even though it does not increase the willingness of large banks to form links. This kind of over-connection occurs not because large banks' FOCPR is altered but because their SOCPR is eliminated, which makes small banks willing to connect with large banks. What is important here is that network hazard *allows* large banks to over-connect; it does *not make* them more willing to connect. That large banks are willing to form many links even in the absence of intervention is consistent with our "gateway" insight. Overall, network hazard allows for the emergence of over-connectedness; it does not directly cause over-connectedness. In particular, network hazard *allows* large banks to become systemically important.

3.3 The historical relevance of the theory

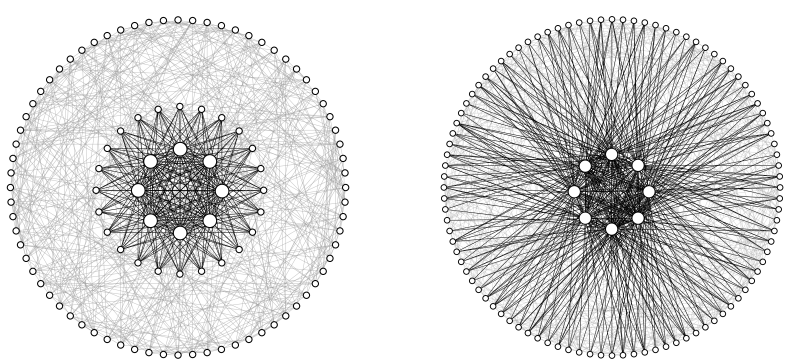
3.3.1 Side payments and the efficient network

The model predicts that in the absence of intervention small banks do not connect with large banks at all. This is so in part because large banks are not able to compensate small banks with risk premiums to convince small banks to connect with them. If side payments were allowed during network formation, then large banks could share with small banks (which are hurt by the SOCPR that large banks impose) the benefits that they enjoy from having extra links.

Suppose, for example, that during network formation banks can write arbitrary and enforceable contracts for how the cost of links is shared. In this case, banks form the utilitarian efficient network. In the absence of intervention, large core banks compensate their small counterparties with SOCPR premiums and, as shown in Figure 12$-a$, a hierarchical core-periphery emerges. In the presence of intervention, a large core-periphery emerges. The impact of network hazard on the network topology makes the network "more core-periphery" in character. To keep the insight general, we allow for both liquidity and solvency shocks for the following result. For technical reasons we alter Assumption 1 into $\tau_L > 1 \geq \tau_S$. We relax $c_L = 0$.

Theorem 2. *Suppose that $\tau_L > 1 \geq \tau_S$ and $c_S > c_L \geq 0$. The unique optimal policy bails out all illiquid banks, both illiquid bad banks with bad liquidity shocks and illiquid good banks that are illiquid due to counterparties. Insolvent bad banks are not bailed out. As for network formation, there exists $\tilde{d}_L^* \geq d_L^*$ and $\tilde{d}_S^* \geq d_S^*$ such that:*

- *In the absence of intervention a network is PSS if and only if there is a set S^* of $d_L^* - k_L + 1$ many small banks such that*
  - *all large banks are counterparties with S^* and all other large banks, but with no other bank,*
  - *all small banks have d_S^* counterparties (i.e., members of S^* have $d_S^* - k_L$ small counterparties, whereas other small banks have d_S^* small counterparties).*
- *In the presence of intervention, a network is PSS if and only if all large banks have degree $\tilde{d}_L^*$ and all small banks have degree $\tilde{d}_S^*$.*



12$-a$: Absence of intervention

12$-b$: Presence of intervention

Figure 12: Network with side payments

In the absence of intervention, large banks impose SOCPR on their small counterparties. Taking into account the imposed SOCPR, the efficient number of links that large banks have is smaller than what large banks would normally prefer if SOCPR were eliminated. In other words, large banks have to pay positive SOCPR premiums to their small counterparties, which causes them to connect less often than they would in the presence of intervention.

There are other ways to think about transfers. We can think of a situation in which large banks compensate their small counterparties via bilateral contracts that are contingent only on whether they have the link or not. This, however, seems to not change the qualitative result: large banks compensate their small counterparties with SOCPR premiums, and a smaller and limited core-periphery network emerges. At the level of connectedness that large banks would like to have, non-zero SOCPR premiums must be paid to small banks. This increases the effective cost incurred by large banks to connect with small banks. That is, large banks are limited in their ability to compensate small banks with SOCPR premiums, and, thus, large banks cannot convince all small banks to connect with them. In the presence of intervention, however, SOCPR is eliminated. Small banks are happy with arbitrarily small SOCPR premiums. Large banks are able to connect with more small banks and the periphery grows large as it spans a larger part of the economy.

Ex-post side payments during contagion, however, are different problem. This would correspond to bail-ins, wherein banks rescue each other from default. In recent papers this important problem has been studied by Bernard et al. (2017) and Kanik (2017).

3.3.2 The FED Act and volatility

Our model can be used to understand certain changes in the interbank networks that occurred after the FED was established as the lender of last resort. In our model, and in the absence of side payments and intervention, the clique structure resembles the clearinghouses that banks formed during the free banking era. During bank runs these clearinghouses would issue joint notes and sometimes suspend payments. This was the manner in which a pre-FED era banking panic typically would end (Gorton and Tallman (2016)). The FED Act of 1913, which established the FED as the lender of last resort, abolished the need for clearinghouses—an outcome that can be seen as the dissolution of the cliques that our model predicts.

Yet even in 19th century prior to the establishment of the FED, some banking networks featured a core-periphery structure. Our model with side payments predicts that banking networks are core-periphery even in the absence of intervention and feature a three-layered pyramiding of reserves. This is exactly the topology of the banking networks documented by Anderson et al. (2016). Our theory predicts that the network becomes more core-periphery in nature in the presence of intervention. Indeed Anderson et al. (2015) show evidence consistent with the view that after the FED was established, small state banks started to keep bigger parts of their reserves in large New York banks, which intensified the links between the core and the periphery.

The establishment of the FED, which would lend to member banks via discount facilities and serve as a lender of last resort, can be interpreted as switching from a regime without bailouts to a regime with bailouts. Starting with the free-banking era, small country banks used to hold part of their reserves at large banks. The goal was to accommodate out-of-city withdrawals and earn interest on these reserves. Large city banks would use these funds in other markets. The National Banking acts of the 1860s further incentivized small banks to keep deposits at large banks by allowing such reserves to count partially for the legal reserve requirements. This reinforced the pyramiding of reserves, from small country banks to reserves city banks, and from reserve city banks to central reserve banks in New York. This historical episode and its consequences with regards to the interbank network structure is discussed in detail by Anderson et al. (2016). The pyramiding system was inherently prone to instability. During certain seasons farmers would demand liquidity at large amounts. These seasonal liquidity shocks created credit crunches, and some years it caused panics. Some small banks started withdrawing their deposits from larger banks, and the distress in large New York banks propagated back to all the small banks in the pyramid. Many legislative attempts were made to address the pyramiding of reserves and frequent banking panics. Selgin (2016) provides an historical account.

Perhaps due to pyramiding, "The most important single factor to be considered in estimating the strength of the system as a whole," as Sprague (1910) puts it, had become the financial standing of the largest New York banks. While other options were being considered, such as an asset-backed (rather than government-bond backed) decentralized currency system to address instability, the FED was established in 1913 to issue a currency and lend to members at the discount rate in order to have an elastic money supply. However, small banks kept reserves at the reserve city banks to keep earning interest, while large banks borrowed from the FED. Thus, the establishment of the FED strengthened the pyramiding. In his review, Selgin (2016) notes: "...the Fed's discount facilities made it appear less likely that New York banks would ever have to suspend payments, and therefore less risky for other banks to send funds to them." Furthermore, a study by Anderson et al. (2015) indicates that small state banks kept their reserves in large New York banks to keep earning interest and maintain indirect access to FED discount facilities.

This led to an expansion of the large New York bank's balance sheets and their network position. Selgin (2016) notes: "Instead of declining, balances in the three reserve cities grew rapidly, with those in New York growing most rapidly of all. . . [T]he share of such balances belonging to the six-largest banks had risen from 65 percent to almost 78 percent."[15] In a recent paper Anderson et al. (2017) are studying the changes in the detailed structure of banking networks after the FED Act.

These changes had a variety of consequences. Especially after the 1917 amendment to the FED Act, the FED controlled the money supply elastically, which eliminated excess seasonal interest rate volatility. Bernstein et al. (2010) demonstrate the existence of these effects and Mankiw et al. (1987) show how quickly the adjustment occurred. Nonetheless, other forms of volatility remained, perhaps because of the pyramiding of reserves. Miron (1988) compares 25 years before and 25 years after the establishment of the FED, and even when the Great Depression period is excluded from the sample, he shows that "the variance of both the rate of growth of output and of the inflation rate increased significantly, while the average rate of growth of output fell, and real stock prices became

[15]Resources: Watkins (1929), Beckhart and Smith (1932), and "Annual Report of the Comptroller of the Currency, 1863–1980," Federal Reserve Bank of St. Louis archive, https://fraser.stlouisfed.org/title/56

substantially more volatile" . More recently, Selgin et al. (2012) have shown that the US economy has seen more volatility since the establishment of the FED. More on the historical period that followed the FED Act can be found in Goodfriend (2013).

Our theory is in line with increased volatility. Nonetheless, we take stances neither on the average and distributional welfare consequences of volatility nor the political debates surrounding the FED Act. Our results suggest that the network effects of the FED Act might be welfare enhancing, whereas volatility might have increased through the making of large banks more heavily connected to the system. Banks might have engaged in more or less individual moral hazard after the establishment of the FED, but documenting whether this happened is beyond the scope of this paper.

4 Other applications

In our theory, each link, in its most general form, represents a potentially beneficial relationship between the counterparties involved, such as credit lines, trade agreements, joint projects, or even friendships for the study of social networks. We assume that benefits realize in full only if neither party defects. Below are some applications of the general theory other than interbank lending. For simplicity we introduce these applications with identical agents in this section and drop the type γ from notation.

Real firms and joint investments. In this application agents are real firms that borrow from banks. After borrowing from banks, firms can undertake joint projects. The return from projects is safe but firms can fail due to exogenous events, such as high operational costs. If too many counterparties fail, it becomes too costly for a firm to complete its projects and the firm as a whole becomes unable to repay the bank. The details of the environment and a full analysis of network formation in the absence and presence of intervention are given in Appendix D. Sturm (2017) provides another insightful application regarding joint projects with endogenous size of projects.

Liquidity coinsurance and credit lines. Erol and Ordonez (2017) present another major application that involves credit lines that banks use to ride future refinancing shocks to their proprietary projects. If too many counterparties fail because of productivity shocks to their managed projects, then a bank with a healthy project can find it too unlikely to ride a potential future refinancing shock, and it may prefer to default on its project to reduce managements costs.

Loan syndication and risk diversification. Elliott et al. (2014) study diversification and integration via cross-ownership across banks. Our model can capture a similar tradeoff using our model. Consider banks that have illiquid assets A, liquid endowment ω, and liabilities L. Each bank has a proprietary project. Banks can invest 1 unit in each other's' project. A link represents two counterparties that each invest 1 unit into each other's' project. This helps them diversify and integrate. A link costs c to each counterparty−for example, costs incurred for monitoring counterparties. A bank that has d counterparties has invested $\omega - d$ into its own project. Then shocks are realized; $\theta_i \in \{G, B\}$ captures the management cost of continuation. If n_i continues, it incurs the utility cost $\kappa(\theta_i)$ to manage its project. Suppose that $\kappa(B)$ is very large so that all bad banks are forced into defaulting. All good banks that continue move on to the return and repayment stage. All projects that are still being managed realize a rate of return $r \sim \mathbb{F}$ realized. Denote $t \otimes \mathbb{F}$ for the distribution of the sum of t i.i.d. random variables distributed with respect to $\mathbb{F}$. A bank, n_i, for each good counterparty $n_j \in N_i$ that has continued, receives r_j. Then n_i's total liquidity is

$$X_i = (\omega - d)\, r_i + \sum_{n_j \in N_i, a_j = C} r_j.$$

If this liquidity suffices to pay L, n_i pays its debt and consumes the rest. Otherwise n_i has to liquidate its illiquid asset at a per unit cost η. Hence, n_i's realized payoff is $A + X_i - L - \eta A \times \mathbf{1}_{X_i < 0}$. Therefore,

$$P(f, d) = \mathbb{E}_{r, \tilde{r}} \left[\left[A + (\omega - d)\, r + \tilde{r} - L - \eta A \times 1_{r + \tilde{r} - L < 0} \right]^{+} \right] - cd - \kappa(G),$$

where $r \sim \mathbb{F}$ and $\tilde{r} \sim (d - f) \otimes \mathbb{F}$, and $P_G(d) \equiv P_B(d) = -cd$.

Social coordination and behavior adoption. Suppose that agents are individuals in the society who form social ties, such as in Morris (2000) and Goyal and Vega-Redondo (2005). It is beneficial to have friends who behave similarly, whereas it is costly to have conflict with friends. Shocks capture agents who start acting in their own interest. Because of peer pressure it can be costly to maintain friendships that involve such anti-social behavior, and other agents may start acting in their own interest. Such situations can be modeled with a payoff function

$$P(f, d) = (d - f)\, r - fc,$$

$$P_G(d) \equiv P_B(d) \equiv 0,$$

where r captures some homophylic benefit and c captures the cost of conflict. If the behavior adoption is a strategic choice, it can be modeled with

$$R(d) = \frac{d}{1 + \frac{c}{r}}.$$

On the other hand, behavior adoption could stem from psychological factors. Then R could be given by any other threshold rule irrespective of P.

Spread of diseases. One can also consider the spread of diseases, such as flu. Having social links provides some benefits, but active links can facilitate the transmission of disease. Each person has a resilience that captures the strength of their immune system against the contagion of the disease. If there are more than $R(d_i) \equiv \tau$ infected friends, the agent also gets infected, where τ is a constant. Infected friends drop out of the group, at least temporarily. So payoffs are given by

$$P(f, d) = u(d - f) - v(d) - c(d),$$

$$P_G(d) \equiv P_B(d) = v(d) - c(d),$$

where $u(d-f)$ is the benefit of social links with a temporary loss, $v(d)$ is the continuation value after all friends heal from the disease, and $c(d)$ is the cost of forming these links.

5 Discussions

5.1 Discussion of the theory

Other solution concepts for network formation. Strong stability is defined differently in Jackson and Van den Nouweland (2005) and Dutta and Mutuswami (1997). In Jackson and Van den Nouweland (2005), deviations that Pareto improve the coalition are precluded, whereas in Dutta and Mutuswami (1997) deviations that strictly improve every member of the coalition are precluded. To minimize confusion we use the name Pareto Strong Stability (PSS) for Jackson and Van den Nouweland (2005) and Strong Stability (SS) for Dutta and Mutuswami (1997). Appendix B provides results for SS.

In our framework, the advantage of PSS is that it yields a unique prediction, but to avoid unintuitive cycles of deviations its existence requires some divisibility assumptions regarding the number of agents. SS yields existence without divisibility assumptions regarding the number of agents, but it leaves some small room for multiplicity.[16]

It is natural to employ a solution concept that captures the fact that forming links requires mutual consent; it is less natural to employ a formation game in which a counterparty cannot reject a link. The simplest solution concept that captures this is Pairwise stability. Pairwise stability roughly precludes deviations by pairs of banks. SS is a demanding solution concept compared to Pairwise stability. The typical challenge is the existence of SS networks. We have been able to prove the existence of SS networks in our general contagion framework, but we do not see SS as a positive description of how banks become counterparties or how social links are formed. Indeed, SS should be seen as selections among many possible pairwise stable networks that can arise in our framework. In fact, one source of multiplicity among Pairwise stable networks in our framework is an unintuitive one that arises due to discreteness. The function $V(d,\gamma)$ typically features many non-monotonicities, such as $V(d+2,\gamma) > V(d,\gamma) > V(d+1,\gamma)$ for some d. In this case, $d-$regular networks are pairwise stable in the presence of intervention because the feasible deviations can feature only two agents and not three. Agents cannot increase their degree from d to $d+2$ with feasible deviations, and agents do not want to increase their degree from d to $d+1$. Thus, a $d-$regular network is Pairwise stable because of this unintuitive discrete fall in V at d. Even if coalitions of three agents were allowed to deviate, similar problems would occur at some d such that $V(d+3,\gamma) > V(d,\gamma) > V(d+2,\gamma) > V(d+1,\gamma)$. SS resolves all such discreteness problems by allowing all sizes of coalitions.[17]

[16] Other papers in the literatures, such as Farboodi (2015) and Erol and Vohra (2014), use the strong stability notion of Dutta and Mutuswami (1997). Strongly stable networks correspond to strong Nash equilibria of an underlying proposal game. See Dutta and Mutuswami (1997) for more on the relation between strong Nash equilibria and strongly stable networks.

[17] For more on various notions of network formation, see Bala and Goyal (2000), Bloch and Dutta (2011), Bloch and Jackson (2006), Dutta, Ghosal and Ray (2005), Fleiner, Janko, Tamura and Teytelboym (2015), Galeotti, Goyal

Divisibility assumptions. The existence of PSS networks is contingent on being a certain number of agents from each type. As shown in Theorem 3 in Appendix B, one can also use the SS solution concept, which does not require the divisibility assumptions for existence when agents are identical ex-ante. Such discreteness problems are common in endogenous network formation. Moreover, we show in Theorem 4 that if at least one type has a high propensity to form links, this type of node can absorb the residual demand, and we restore the existence.

Correlated shocks. All our results still hold if shocks are conditionally independent. Suppose that with some probability $\sigma_B < 1$ all shocks are bad, with probability $\sigma_G < 1$ all shocks are good, and with probability $1 - \sigma_B - \sigma_G$ all shocks are determined independently as in the model. All results concerning the topology of the network hold identically because the such correlation does not cause any contagion. The level of connectedness d^*, however, can be different in the case of $\sigma_B = \sigma_G = 0$.

Micro-foundation of contagion. Our theory takes the contagion dynamics to be exogenous. This allows us to have a general and arbitrary form for the resilience function R. This generality is useful in studies of situations in which contagion dynamics are not tied to the benefits that agents enjoy from links, such as in epidemics. However, contagion can be seen as the spread of a defective behavior that is taken strategically. Suppose that in stage three, agents play a simultaneous game and *choose* to play C or D. Given the realized network and shocks, this game is supermodular. Then, via Topkis' Theorem, the best responses are increasing functions of the actions of others, where C is the higher action. In return, Tarski's Theorem indicates that the set of Nash equilibria is a complete lattice. The action profile at the highest element of the lattice corresponds to the outcome of the exogenous contagion dynamics. Call this the *cooperating equilibrium.*[18]

This equilibrium can be obtained in two ways other than exogenous dynamics. In the first, the myopic best-response dynamics that starts with the 'everyone plays C for all projects' action profile is iterated, as in Morris (2000). The second, which is subtly different, applies the iterated elimination of strictly dominated strategies. In both cases, the constructed sequence of action profiles reaches and stops at the cooperating equilibrium. Following the second way to reach the cooperating equilibrium, an alternative definition of the cooperating equilibrium can be given via rationalizable strategies.[19] 'The rationalizable strategy profile in which all banks play the highest action they can rationalize' is identical to cooperating equilibrium. This natural contagion interpretation maps onto exogenous contagion dynamics.

Incomplete information. For the sake of simplicity, we introduced cooperating equilibrium for complete information games. Vives (1990) also shows that any Bayesian game with supermodular ex-post payoff functions has a maximal pure strategy Bayesian-Nash equilibrium. Thus, cooperating equilibrium can be identically defined for incomplete information.

and Kamphorst (2006), Goyal and Vega-Redondo (2005), Jackson and Watts (2002), Jackson and Wolinsky (1996), Ray and Vohra (2015), Shahrivar and Sundaram (2015), Tarbush and Teytelboym (2015) and Teytelboym (2013).

[18] The cooperating equilibrium is a standard selection used in the literature, such as in Vives (1990), Eisenberg and Noe (2001), Elliott et al. (2014), Morris (2000), Goyal and Vega-Redondo (2005), Erol and Vohra (2014) and others. See Vives (1990) for additional discussion of how complementarities generate a lattice structure on the set of Nash equilibria. See Milgrom and Shannon (1994) for more on supermodular games.

[19] This link between rationalizability and the extreme points of the lattice is introduced in Milgrom and Roberts (1990).

Now suppose that each agent observes the shocks to itself, its counterparties, and possibly some other agents. Formally, n_i observes the shocks to a subset $\mathcal{I}_i(E, \vec{\theta})$ of banks that includes n_i and all counterparties of n_i. $\{n_i\} \cup N_i \subset \mathcal{I}_i\left(E, \vec{\theta}\right) \subset N$. All results hold identically both for the absence and presence of intervention. The key observation is that in the absence of intervention, if agents form cliques, the shocks of every agent in the clique are common knowledge across all clique members. Therefore, by forming a clique with its own type, n_i has $V\left(d_i, \gamma_i\right)$ expected payoff. Clearly, this payoff cannot be exceeded in any configuration or any information structure because bad counterparties always default. As for presence of intervention, an agent does not need to know anything more than the shocks of its counterparties. This is so because intervention makes sure all good agents continue and there is no contagion.

Heterogenous link benefits. We have assumed that agents do not get different benefits or harm from links with different types. Moreover, the probability that an agent receives a good shock is the same across all types. This modeling choice is intentional. In this manner preferences over the composition of types of counterparties are endogenously determined via network formation, and the emphasis is on the network structure. One can relax these assumptions and assume that the payoff of n_i is given by $P\left(\left(f_i^\gamma, d_i^\gamma\right)_{\gamma\in\Gamma}, \gamma_i\right)$ where d_i^γ is the number of counterparties with type γ and f_i^γ are the number of defaulting counterparties with type γ. In other words, the composition of the types of counterparties of n_i could matter for n_i. One can also assume different α_γ for each type γ. Under appropriate regularity assumptions on P, R, and α's, such that there is a common linear to which types are more preferred, then similar results hold. Such arguments can be used to study also the formation of directed links.

Generalization of results. Our network formation results can be generalized to allow the ex-post payoff function P to depend on various other variables such as the shocks of counterparties and the number of defaults by agents at various distances to n_i.

Phase transitions. The network formed is prone to phase transitions in several ways. In an example provided by Erol and Ordonez (2017), the liquidity coinsurance network that banks form is influenced by the level of reserve requirement that the government sets ex-ante. They show that beyond a threshold level of reserve requirement, the network collapses and all liquidity coinsurance is lost. This leads to a discontinuous jump in systemic risk.

Behind this result is the theory that intervention effectively changes the parameters in the P function. The shape of V changes smoothly in these parameters, yet the argmax d^* of V features discontinuous changes. Although V has multiple local maxima that change smoothly, at a certain tipping point of the parameter in question, the global max d^* of V changes discontinuously from one local max to another local max. At this point, connectivity in the network changes discontinuously.

Generally speaking, in any application where certain parameters in the P function change, such phase transitions are possible. Moreover, a form of phase transition with respect to the number of agents is possible. It is possible that global max of V changes from one local max to another local max once there is one more agent in the economy. We explore these in future work.

5.2 Discussion of the application to interbank lending

Costly bailouts and welfare: One reason that welfare increases with time-consistent bailouts is that these transfers entail no extra cost. One can think of some costs per unit of transfer as a distortionary tax on households that funds the bailout budget. Another cost can be a fixed cost of executing each or all bailouts, such as passing a certain budget for bailouts or political costs. In such cases, the optimal policy is more complex than what our model proposes. We conjecture that with side payments and costly bailouts, there is a threshold for the cost of bailouts, above which welfare is smaller in the presence of intervention than in the absence of intervention. Because a detailed analysis of the optimal policy in the costly bailout case makes the analysis complicated, we do not go into details in this paper.

Undoing network hazard: constructive ambiguity. The goal of this paper is to identify network hazard. As we have shown, ex-ante welfare actually increases despite network hazard. Network hazard is the elimination of the SOCPR, which leads to volatility. Moreover, if bailouts entail distortionary costs, welfare, too, could decrease. That said, a partial commitment to not bailing out banks—if that were possible—would undo network hazard. Without side payments, even a small probability that there would not be any bailouts would be enough to restore the clique structure. However, with side payments, an arbitrarily small probability would not to suffice to return the core-periphery structure to the clique structure. Large banks are ready to pay high SOCPR premiums, including a premium for the risk of not getting bailed out. Large banks have a limited "budget" to distribute for these premiums. This limit can be used to determine the minimum probability of "no-bailouts" that is necessary to revert the core-periphery structure to the clique structure, or to induce an optimally sized core-periphery. Hence the model also can be used to find the optimal level of constructive ambiguity. We pursue this question in a follow-up paper.

Commitment to bailing out only systemically important institutions. If government could commit to bailing out only SIFIs, then the interconnectedness across the periphery would disappear and a core-periphery would emerge in which the periphery banks would form disjoined cliques besides their links with core banks. This would not undo the benefit or harm that would occur if some banks became systemically important, but it would restore the market discipline within the periphery.

A similar prediction can be made for the times when only large national banks were members of the FED. Indeed, after the establishment of the FED, not all state banks immediately joined the FED. The large national banks that became members of the FED enjoyed the discount lending. Our model predicts in this case a core-periphery network in which the peripheral small banks form disjoined cliques in addition to their links with large banks.

Optimal policy in the combined model. In the combined model, presented in Section 3.3.1, that allows for both liquidity and solvency shocks, the optimal policy is to bail out all illiquid banks, both exogenously illiquid banks due to bad liquidity shocks and endogenously illiquid banks due to defaulting counterparties. Closely related is the discussion by Rochet and Tirole (1996) of whether TBTF institutions should be bailed out to assist their troubled counterparties or troubled counterparties should be assisted directly. We argue that a mixture is optimal because all troubled illiquid banks, including those that are not TBTF, should receive bailouts, whereas TBTF institutions should fail if they are insolvent.

Other forms of individual risk and moral hazard. In Section 3.1.3 we introduce an endogenous choice of operational risk to study moral hazard. A similar analysis can be made for other forms of individual risk taking. These include the choice of idiosyncratic risk α and the choice of the level of deposits D_γ. Similar insights hold in that network hazard does not cause any moral hazard by itself.

Risk shifting. Consider the forms of individual risk taking we have mentioned. Interconnectedness caused by network hazard does not exacerbate risk shifting. This is clear in the case of solvency shocks with identical banks. Only good banks receive bailouts, and when they receive bailouts they are made indifferent between defaulting or not defaulting. As discussed above, small banks do not over-connect or under-connect in response to network hazard. Since the degree does not change, small banks undertake identical decisions regarding their individual risk in the absence and presence of intervention. However, the systemic importance consequence of network hazard can involve more or less risk shifting.

Other references. Papers that study systemic risk given exogenous networks include Allen and Gale (2000), Eisenberg and Noe (2001), Kiyotaki and Moore (1997), Acemoglu, Ozdaglar and Tahbaz-Salehi (2015a), Acemoglu, Ozdaglar and Tahbaz-Salehi (2010), Allen, Babus and Carletti (2012), Amini and Minca (2014), Blume et al. (2011), Bookstaber et al. (2015), Caballero and Simsek (2013), Eboli (2013), Elliott, Golub and Jackson (2014), Freixas, Parigi and Rochet (2000), Gai and Kapadia (2010), Gai et al. (2011), Gale and Kariv (2007), Gottardi, Gale and Cabrales (2015), Glover and Richards-Shubik (2014), Gofman (2011), Gofman (2014), Kiyotaki and Moore (2002), Lim, Ozdaglar and Teytelboym (2015), Vivier-Lirimonty (2006), Acemoglu, Ozdaglar and Tahbaz-Salehi (2015c), Elliott, Golub and Jackson (2014), and Glasserman and Young (2015). Some that study the efficient ways of stopping contagion for fixed networks are Drakopoulos, Ozdaglar and Tsitsiklis (2015a), Freixas, Parigi and Rochet (2000), Minca and Sulem (2014) Amin, Minca and Sulem (2014), Drakopoulos, Ozdaglar and Tsitsiklis (2015b), and Motter (2004). Moreover, Acemoglu, Ozdaglar and Tahbaz-Salehi (2015b), Elliott and Hazell (2015), Erol and Vohra (2014), Goldstein and Pauzner (2004), Moore (2011), Cabrales, Gottardi and Vega-Redondo (2017), Babus and Hu (2015), Blume et al. (2013), Chang and Zhang (2015), Condorelli and Galeotti (2015), Kiyotaki and Moore (1997), Lagunoff and Schreft (2001), and Zawadowski (2013) study the formation of networks.

6 Conclusion

This paper characterizes strongly stable networks in a general model of threshold contagion. The theory is used to document a novel force that we call network hazard. The risk of contagion across the network has a disciplining effect on the market during the network's formation. Ex-post intervention by a principal to mitigate contagion weakens such ex-ante market discipline. The anticipation of intervention affects the detailed structure of the network. The change in the structure of the network has adverse effects such as making the network more prone to contagion. Through these network effects, a large number of agents may become exposed to contagion, which may require a large-scale intervention with the market.

Various applications of the theory are presented. The main application involves interbank lending. In an environment in which banks form bilateral lending partnerships, the anticipation of bailouts reduces second-order counterparty risk—that is, the risk that a bank can incur losses due to solvent counterparties that default because of their own insolvent counterparties. In the presence of intervention, banks concern themselves less with the counterparties of their counterparties. As a result, "firebreaks" that would form in the absence of intervention dissolve, and banks form a core-periphery network. In return, the performance of a large part of the economy becomes correlated through the solvency of the core banks. This creates welfare volatility. Nonetheless, expected welfare increases, too. These predictions are consistent with some historical account of the pyramiding of reserves and the establishment of the FED.

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A Theory: Pareto Strongly Stable networks

This Appendix consists of proofs of results in Section 2.

Proof of Proposition 1. Consider any E and take any agent n_i. The distribution of f_i first-order-stochastically dominates the distribution of b_i due to potential spillovers. The latter equals the distribution of the total number of defaulting counterparties of n_i if n_i were at the center of a disjoined star with d_i leaves, because there is no SOCPR for n_i in the star configuration. $P(b_i, d_i, \gamma_i)$ is a strictly decreasing function of b_i . Since the expectation of a decreasing function decreases with respect to first order stochastic dominance, n_i gets at most $V(d_i, \gamma_i)$ in any network wherein n_i has degree d_i. □

Proof of Proposition 2. Suppose that there exists $n_j \in N_i$ such that

$$min\{R(d_i, \gamma_i), d_{ij}\} + (d_j - d_{ij} - 1) > R(d_j, \gamma_j).$$

Consider the event that $min\{R(d_i, \gamma_i), d_{ij}\}$ many agents in $N_i \cap N_j$ and all the $d_j - d_{ij} - 1$ many agents in $(N_j \backslash \{n_i\}) \backslash N_i$ get bad shocks, and all else get good shocks. n_j defaults while n_i does not default without n_j defaulting. This causes n_i to incur an extra loss on top of the direct costs from bad counterparties. That is, there is SOCPR for n_i through n_j. Due to the existence of such a positive probability event, conditional on the event that both n_i and n_j are good, and less than $R(d_i, \gamma_i)$ many counterparties of n_i are bad, the distribution of f_i in (N, E) first order stochastically dominates the distribution of b_i. Hence, n_i's expected payoff is strictly less than $V(d_i, \gamma_i)$.

Now suppose that

$$min\{R(d_i, \gamma_i), d_{ij}\} + (d_j - d_{ij} - 1) \leq R(d_j, \gamma_j)$$

is satisfied for all $n_j \in N_i$. By the same argument, there is no event in which n_i continues and n_j defaults without being bad, i.e. there is no SOCPR for n_i. Hence the only risk that n_j imposes on n_i is the first-order counterparty risk of n_j getting a bad shock. Then conditional on n_i having at least $R(d_i, \gamma_i)$ many good counterparties, the distribution of f_i of n_i in (N, E) is equivalent to the distribution of b_i. This means that n_i has $V(d_i, \gamma_i)$ payoff. □

Proof of Proposition 3. All other agents have the same or higher resilience: $R(d, \gamma_j) \geq R(d, \gamma_i)$ for all n_j in the clique. Then if $f \leq R(d, \gamma_i)$ many agents are bad in the clique, all the good agents in the clique continue. Thus from the viewpoint of n_i, if n_i gets a good shock, and $f \leq R(d, \gamma_i)$ many agents get bad shocks, n_i gets payoff $P(b_i, d_i, \gamma_i)$. If n_i gets a good shock, but $f > R(d, \gamma_i)$, then it defaults and gets $P_G(d_i, \gamma_i)$ which does not depend on f. If n_i gets a bad shock, it gets $P_B(d_i, \gamma_i)$. Thus, its n_i's payoff is $V(d, \gamma_i)$. □

Proof of Proposition 4. By Proposition 2 we have both

$$min\{R(d_i, \gamma_i), d_{ij}\} + d_j - d_{ij} - 1 \leq R(d_j, \gamma_j),$$

$$min\left\{R(d_j,\gamma_j),d_{ij}\right\}+d_i-d_{ij}-1\leq R(d_i,\gamma_i).$$

If one of n_i and n_j, say n_i, has a safe counterparty degree, $R(d_i,\gamma_i)\geq d_i-1\geq d_{ij}$, so that $min\left\{R(d_i,\gamma_i),d_{ij}\right\}=d_{ij}$. Then the latter inequality becomes $d_j-1\leq R(d_j)$. Thus, the other n_j must also have a safe counterparty degree.

Now consider the case in which both have unsafe counterparty degrees. By $d_j\notin S(\gamma_j)$ we have $R(d_j,\gamma_j)<d_j-1$. Then by the former inequality we have $min\left\{R(d_i,\gamma_i),d_{ij}\right\}< d_{ij}$. That implies $min\left\{R(d_i,\gamma_i),d_{ij}\right\}=R(d_i,\gamma_i)$. Then the former inequality becomes. $R(d_i,\gamma_i)+d_j-d_{ij}-1\leq R(d_j,\gamma_j)$. Similarly since $d_j\notin S(\gamma_j)$, we have $R(d_j,\gamma_j)+d_i-d_{ij}-1\leq R(d_i,\gamma_i)$. Add both up to get $d_i+d_j\leq 2(d_{ij}+1)$. That implies that $d_i=d_j=d_{ij}+1$, which in turn implies that $N_i\backslash\{n_j\}=N_j\backslash\{n_i\}$. Put that back into the inequalities to get $R(d_i,\gamma_i)=R(d_j,\gamma_j)$. □

Proof of Corollary 5. Either of the two conditions make sure that conditions of Proposition 4 are satisfied for all agents in the component, so all agents in the component have their ideal value.

Take any component. Suppose that there exists two agents with one safe and one unsafe counterparty degree. Then by the connectivity of the component, there are two counterparties with one safe and one unsafe counterparty degree in the component meaning that there is at least one agent which does not achieve its ideal payoff. This is a contradiction. If all agents in the component have safe counterparty degrees, the second condition is satisfied. If all agents in the component have unsafe counterparty degrees, by Proposition 4 the set of counterparties of any two counterparties in the component must be identical. Then by the connectivity of the component, the component must be a disjoined clique. All agents in the component are then counterparties of each other. Hence their resiliences also must be identical. □

Proof of Theorem 1. If there is any agent with type $\gamma\in\iota$ is not achieving its ideal value, then this agent, and $d^{*^{\iota}}$ other agents with similar types could deviate to forming a disjoined clique of order $d^{*\iota}+1$ and all get their ideal value. This would be a Pareto improvement. Hence, in any PSS network, all agents must achieve their ideal values. On the other hand, if all agents have their ideal value, then this network is clearly PSS. By Proposition 5, all agents have their ideal value if and only if the network consists of disjoined cliques of agents from the same unsafe class each with their ideal degree, and a subnetwork of all safe classes in which each agent has its ideal degree. □

B Theory: Strongly Stable networks

PSS networks may not exist due to integer problems in the number of agents from each type, as reflected in Theorem 1. However, SS networks always exist if $|\Gamma|=1$. This Appendix consists of results concerning strong stability and the proofs of these results.

Stating the set of SS networks requires some additional notation. Construct a sequence iteratively as follows. Let $\Gamma=\{\gamma\}$. Set $n_0=k$. For $t\geq 1$, as long as $d^*(k_t,\gamma)\notin S\left(\gamma\right)$, set $k_t=k_{t-1}-d^*\left(k_{t-1},\gamma\right)-1\geq 0$. Let k_κ be the last element of the sequence: $d^*\left(k_\kappa,\gamma\right)\in S\left(\gamma\right)$. That is, find the ideal degree among the remaining number of agents, and separate that many plus one agents aside. Iterate, and stop when ideal degree is a safe counterparty degree.

Theorem 3. *(Strongly stable networks) Suppose that* $\Gamma = \{\gamma\}$.

- *(Existence) The following is a strongly stable network: There are* κ *disjoined cliques with orders* $d^*(k_{t-1}, \gamma) + 1$, *for* $t = 1, 2, ..., \kappa$, *and another disjoined residual subnetwork which is almost-*$d^*(k_\kappa, \gamma)$*-regular*[20] *among the* k_κ *remaining agents.*
- *(Almost uniqueness) In any strongly stable network, there are* κ *disjoined cliques with orders* $d^*(k_{t-1}, \gamma) + 1$ *agents, for* $t = 1, 2, ..., \kappa$. *The remaining* k_κ *agents constitute an approximately-*$d^*(k_\kappa, \gamma)$*-regular*[21] *network.*[22]

Proof. (Existence) As stated before, by Corollary 5, being part of a clique with order $d^*(k_0, \gamma) + 1$ gives the highest payoff any configuration can achieve for a agent among a network of k_0 agents. Therefore, agents in the clique with order $d^*(k_0, \gamma) + 1$ have no incentive to deviate to any other network. The argument can be applied iteratively for the κ cliques. As for the remaining almost-$d^*(k_\kappa, \gamma)$-regular part, all agents have degree $d^*(k_\kappa, \gamma) \in S(\gamma)$ (except possibly one which is not connected to anyone). That is, all these remaining agents (except the singleton) have safe counterparty degrees. Then there is no SOCPR and two good counterparties are sufficient for each other to resist defaulting. Hence for any agent (except the singleton) has $V(d^*(k_\kappa, \gamma), \gamma)$ expected payoff, which is the highest any agent can achieve among k_κ people. If there is a singleton left-over agent with degree 0, it cannot convince anyone to deviate either, because everyone else is already getting their maximum possible payoff among people they could convince to deviate.

(Almost uniqueness) Take any strongly stable network. Let $d^* = d^*(k_0, \gamma)$. First consider $d^* \notin S(\gamma)$. If all agents have strictly less than $V(d^*, \gamma)$ expected payoff, $d^* + 1$ of them can deviate to a $(d^* + 1)$-clique and improve. Hence, there is at least one agent who gets $V(d^*, \gamma)$ payoff, say n_{i_0}. Then $d_{i_0} = d^* \notin S(\gamma)$.

For any counterparty of n_{i_0} which gets $V(d^*, \gamma)$, say n_j, it must be the case that $d_j = d^* \notin S$. By Proposition 4, $N_{i_0} \backslash \{n_j\} = N_j \backslash \{n_{i_0}\}$. Let $N_0 = N_{i_0} \cup \{n_i\}$. Thus all agents in N_0 which get $V(d^*, \gamma)$ are adjacent to all other agents in N_0, and none else.

Then consider agents in N_0 that get less than $V(d^*, \gamma)$, say N_1. Suppose that $N_1 \neq \emptyset$. Consider the deviation by N_1 in which they keep all existing edges with N_0, they connect all of the missing edges in N_1, and they cut all edges they have with N_0^C. After this deviation, N_0 becomes a $(d^* + 1)$-clique and all agents get $V(d^*, \gamma)$ so that all the deviators in N_1 get strictly better off. Therefore, $N_1 = \emptyset$, so that N_0 is already a $(d^* + 1)$-clique.

[20] A network is almost-d-regular if all agents, except at most one of them, have degree d and the possible residual agent has degree 0. An almost-d-regular network always exists among $d + 1$ or more agents.

[21] A network is approximately-d-regular if all agents, except at most d of them, have degree d.

[22] Concerning the remaining k_κ agents, more can be said on the structure of the subnetwork using Erdos-Gallai Theorem. If the degree sequence of the remaining k_κ agents is given by x_1,...,x_κ, then the sequence $d^*(k_\kappa) - x_k$,...,$d^*(k_\kappa) - x_1$ cannot be a graphic sequence.

A graphic sequence is sequence of integers such that there is a simple graph whose agent degrees are given by the sequence. Erdos-Gallai Theorem provides a necessary and sufficient condition for a sequence being graphic.

All in all, in any strongly stable network of k_0 agents, if $d^*(k_0,\gamma) \notin S$, there exists a disjoined clique of order $d^*(k_0,\gamma)+1$. Now the same arguments can be repeated for agents in the remaining $k_1 = k_0 - d^*(k_0,\gamma) - 1$ agents. Then among those, there must be a clique with $d^*(k_1,\gamma)+1$ agents, then $d^*(k_2,\gamma)+1$ agents.... as long as $d^*(k_t,\gamma) \notin S(\gamma)$.

When $d^*(k_\kappa,\gamma) \in S(\gamma)$ first time in the sequence, for the remaining k_κ people, among them there cannot be $d^*(k_\kappa,\gamma)+1$ or more people that have degree other than $d^*(k_\kappa,\gamma)$ because then $d^*(k_\kappa,\gamma)+1$ many would deviate and form a clique, and get $V(d^*(k_\kappa,\gamma),\gamma)$.

The tighter condition mentioned in the Footnote 22 is also necessary. If the sequence $d^*(k_\kappa,\gamma) - x_1$,...,$d^*(k_\kappa,\gamma) - x_\kappa$ is graphic, then an appropriate isomorphism of the graph with this particular degree sequence can be joined with the existing remainder, so that all deviators increase their degree to $d^*(k_\kappa,\gamma)$. This way, all agents achieve their ideal payoffs among k_κ agents, so that all deviators get strictly better off. $\square$

We were able to prove existence of SS networks without any divisibility assumptions. When there is heterogeneity, our proof strategy for the existence of SS networks does not work at the last small residuals. If there are remainder agents from each class ι after forming cliques of order $d^{*\iota}+1$, it is not clear how the remainders from different classes would mix with each other. For example, if remainders consist of 1 agent from each class, we essentially need to solve for a SS network for $|N| = |\Gamma|$. This is a hard task at this level of generality in the payoff and resilience functions P and R. The "trick" of using ideal payoffs do not work anymore when there is too much heterogeneity.

Nevertheless, by assuming there exists one type that is resilient and has high propensity to form links, we can restore the existence. Agents of this type would absorb the demand of residual agents. Formally, suppose that there exists $\gamma_0 \in \Gamma$ such that $R(d,\gamma_0) \geq d-1$ for all d and $P(f,d,\gamma_0) = \phi(d-f)$ for some increasing function ϕ.

Theorem 4. *Suppose that $k_0 > \sum_\iota d^{*\iota}$ for all ι such that $\gamma_0 \notin \iota$. Then SS networks exist. In particular, networks that consist of the following components are SS:*

- *disjoined cliques of ideal order of same unsafe equivalence class,*
- *A disjoined subnetwork of safe classes in which all agents have their ideal degree,*
- *A core-periphery component wherein agents with type γ_0 are the core and the periphery consists of*
  - *cliques of same unsafe class that are disjoined besides their links to the core,*
  - *a subnetwork of agents from safe classes which have their ideal degree including their links to the core.*

The proof is omitted since it is a simple variation of previous proofs. Without having such a type γ_0 that absorbs the residual demand of other types, and without having any divisibility assumptions on the number of agents from each type, we can show by an example that SS networks may not exist. Suppose that there are three agents n_1, n_2, n_3. n_1 and n_2 have type γ and n_3 has type γ'. $P_I \equiv P_S \equiv 0$ for both types. $R(d,\gamma) = R(d,\gamma') = d$ for all d. For some small $\varepsilon > 0$,

$$P(f,d,\gamma) = \begin{cases} 2-\varepsilon f & if\, d=2, \\ -\varepsilon f & if\, d=1, \\ 1-\varepsilon f & if\, d=0, \end{cases} \qquad P(f,d,\gamma') = \begin{cases} 1-\varepsilon f & if\, d=2, \\ 2-\varepsilon f & if\, d=1, \\ -\varepsilon f & if\, d=0. \end{cases}$$

n_1 and n_2 prefer having 2 links to 0 links to 1 link. n_3 prefers having 1 link to 2 links to 0 links. In this situation there are no strongly stable networks. The deviations from each candidate network are illustrated in Figure 13. All agents jointly deviate from G_1 to G_2. n_3 deviates from G_2 to G_3. n_2 deviates from G_3 to G_4. n_1 deviates from G_4 to G_1. n_1 and n_2 jointly deviate from G_5 and G_6 to G_1.

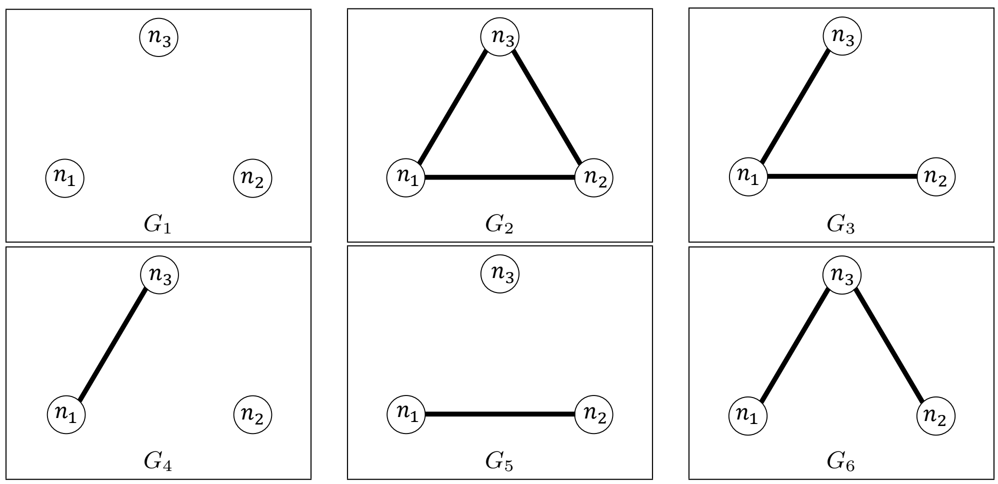


Figure 13: Labels of networks and deviations

Putting more structure on R and P can yields existence of SS networks in general without any particular assumption on the number of agents from each type. We leave this to future work.

C Application: Interbank lending

This Appendix consists of proofs of results in Section 3.

Proof of Proposition 6. The proof is a corollary of Theorem 1 for the most part and Proposition 2 for the remaining part. The only tweak to the proof of Theorem 1 concerns the large banks. Because $c_L = 0$ the ideal degree of large banks is $k-1$, which is larger then the number of large banks by Assumption 2.

By Assumption 2, the number of small banks is larger the ideal degree of small banks, in any PSS network all small banks must achieve $V(d_S^*, S)$. Otherwise, $d_S^* + 1$ small banks including the small bank that does not have $V\left(d_S^*,S\right)$ can deviate to a disjoined clique. This weakly improves all and strictly improves at least one deviating bank. By Proposition 2 and Assumption 1, a small bank can never achieve $V\left(d_S^*,S\right)$ if it has a large counterparty because large banks are not as resilient as small banks. Therefore, in any PSS network, there is no small bank that is connected to a large bank. By Assumption 1, small banks are not perfectly resilient, so they must form cliques among themselves in any PSS network. For the large banks, on the other hand, $V\left(d,L\right)$ is strictly increasing d since $c_L=0$. Then, in any PSS network, they must be doing the best thing among themselves, which is to be adjacent to as many large banks as possible, which leads to a clique among themselves of order k_L. Therefore, the only candidate PSS network is the candidate network wherein all small banks are formed to cliques of order d_S^*+1 and large banks into one clique of order k_L. Notice that this network is indeed PSS because if a small ever deviates and forms a link with a large bank, it can not achieve $V\left(d_S^*,S\right)$. So no large bank can connect to a small bank in a Pareto improving deviations. Then the only possible deviations are among large banks themselves, but they already achieve the best they can among k_L large banks. So the candidate network is PSS. □

Proof of Proposition 7. QED. □

Proof of Proposition 8. Now the resilience functions are $R\left(d,\gamma\right)=d$ for both $\gamma\in\{S,L\}$. Hence by Theorem 1 and Proposition 2, a network is PSS if and only if all banks have their ideal degrees. Since FOCPR is not changed, $V\equiv\tilde{V}$. Hence the ideal degree of small banks is still d_S^* and the ideal degree of large banks is $k-1$. □

Proof of Proposition 9. QED. □

Proof of Proposition 10. The payoff functions in the absence of intervention are given by

$$P\left(d_i,f_i\kappa_i;\gamma_i\right)=-c_{op}\left(\kappa_i;\gamma_i\right)-d_ic_{\gamma_i}+D_{\gamma_i}\left[\left(d_i-f_i\right)\chi_1+\chi_2-\kappa_i\right]^+,$$

$$P\left(d_i,f_i\kappa_i;\gamma_i\right)\equiv P\left(d_i,f_i\kappa_i,\gamma_i\right)=-c_{op}\left(\kappa_i;\gamma_i\right)-d_ic_{\gamma_i},$$
$$V\left(d_i,\kappa_i;\gamma_i\right)=-c_{op}\left(\kappa_i;\gamma_i\right)-d_ic_{\gamma_i}+\alpha D_{\gamma_i}\mathbb{E}\left[\left[\left(d_i-f_i\right)\chi_1+\chi_2-\kappa_i\right]^+\right].$$

In the presence of intervention, all illiquid banks are bailed out by being made indifferent between defaulting or not. $\tilde{V}\equiv V$ since FOCPR is intact. Therefore, $\tilde{d}_S^*=d_S^*$ and $\tilde{\kappa}_S^*=\kappa_S^*$. However, $\tilde{R}\left(d_i,\kappa_i\right)=d_i$ since SOCPR is eliminated. Therefore, large banks start connecting with small banks, and their realized degree becomes $k-1$. This in turn affects their choice of operational risk (which does not scare of other banks since there are bailout guarantees in case the operational cost ever becomes relevant) and they choose a different level $\tilde{\kappa}_L^*\neq\kappa_L^*$. □

Proof of Proposition 11. QED. □

Proof of Proposition 12. Since SOCPR is eliminated, the resilience functions are $R\left(d,\gamma\right)=d$ for both $\gamma\in\{S,L\}$. Hence by Theorem 1 and Proposition 2, a network is PSS if and only if all banks have their ideal degrees. In contrast with Proposition 8, FOCPR is also eliminated so, $V \not\equiv \tilde{V}$. Hence the ideal degree of small banks is now $\tilde{d}_S^* \neq d_S^*$. The ideal degree of large banks is still $k-1$ since $c_L = 0$. □

Proof of Theorem 2. Consider the absence of intervention. Since $\tau_S = 0$, the small banks' limited liability constraint never binds. Hence the payoff of a small bank n_i can be additively separated into ideal payoff and various orders of counterparty risk as

$$V\left(d_i, S\right) - \sum_{n_j \in N_i} \left[SOCPR\, imposed\, on\, n_i\, by\, n_j\right] - \left[higher\, CPR\, imposed\, on\, n_i\right].$$

Moreover, since $\tau_S = 0$, small banks do not impose any SOCPR on counterparties so that all SOCPR comes from large banks. For n_j, denote g_j the number of good counterparties, g_{jS} the number of small good counterparties, and d_{jS} the number of small counterparties of n_j. The SOCPR imposed on good n_i by large n_j is given by

$$\chi_1 \mathbb{P}\left[g_j < \tau_j, \theta_i = G\right].$$

Accordingly, the sum of payoffs of small banks is

$$\sum_{n_i:\gamma_i=S} V\left(d_i, S\right) - \chi_1 \sum_{n_i:\gamma_i=S} \left[\sum_{n_j:\gamma_j=L} \mathbb{P}\left[g_j < \tau_j, \theta_i = G\right]\right]$$

$$= \sum_{n_i:\gamma_i=S} V\left(d_i, S\right) - \chi_1 \sum_{n_j:\gamma_j=L} \mathbb{E}\left[g_{jS} | g_j < \tau_j\right] \mathbb{P}\left[g_j < \tau_j\right].$$

Denote

$$\Phi\left(d_{jS}, d_j\right) = \mathbb{E}\left[g_{jS} | g_j < \tau_j\right] \mathbb{P}\left[g_j < \tau_j\right].$$

Notice that $\Phi\left(d_{jS}, d_j\right) \leq \Phi\left(d_j - \left(k_L - 1\right), d_j\right)$ and the equality holds if and only if n_j has $k_L - 1$ large counterparties, i.e. n_j is counterparties with all other large banks.

Now consider the payoff of large banks. Large bank n_j has payoff at most $V\left(d_j, L\right)$. By Proposition 2, this payoff is attained if and only if all large counterparties of n_j have the exact same set of counterparties with n_j.

Then the sum of payoffs of banks is less than

$$\sum_{n_i:\gamma_i=S} V\left(d_i, S\right) - \chi_1 \sum_{n_j:\gamma_j=L} \Phi\left(d_j - \left(k_L - 1\right), d_j\right) + \sum_{n_j:\gamma_j=L} V\left(d_j, L\right).$$

where the equality holds if and only if all large banks are counterparties of each other and all large banks have exactly the same set of counterparties. Denote

$$d_L^* = argmax_{d_j} V\left(d_j, L\right) - \Phi\left(d_j - \left(k_L - 1\right), d_j\right),$$

$$d_S^* = argmax_{d_i} V\left(d_i, S\right).$$

Then the sum of payoffs of banks is less than

$$k_L \left[V\left(d_L^*, L\right) - \Phi\left(d_L^* - \left(k_L - 1\right), d_L^*\right)\right] + k_S V\left(d_S^*, S\right)$$

where the equality holds if and only if

- all large banks are counterparties of each other,
- all large banks have exactly the same set of counterparties,
- all large banks degree d_L^* and small banks have degree d_S^*.

Notice that these are consistent conditions, so the upper bound we have found is attained only at such networks:

- there is a set S^* of $d_L^* - k_L + 1$ small banks such that all large banks have S^* and all other large banks as their counterparties,
- all small banks have d_S^* counterparties (i.e. members of S^* has $d_S^* - k_L$ small counterparties whereas other small banks have d_S^* small counterparties).

For the presence of intervention, the there is no SOCPR through large banks due to bailouts so the efficient networks are those in which all banks have their ideal payoffs. □

D Application: Real firms with joint projects

Environment. In this economy firms first borrow from banks, then undertake some joint projects. Following investments, firms receive some productivity shocks and choose to continue their business or not. Then returns from projects realize and banks are repaid.

Formally, there are k *firms* denoted $N = \{n_1, n_2, ..., n_k\}$. Firm n_i has type $\gamma_i \in \Gamma$. n_i access to D_{γ_i} credit line from its bank. Denote $l_i \leq D_{\gamma_i}$ the total amount n_i borrows from its bank, and promises to pay $r_F^{\gamma_i} l_i$ after returns from investments where $r_F^{\gamma_i} > 1$. [23]

Firms then undertake joint investments by mutual consent. A mutual investment requires 1 unit of investment from both counterparties. The investment by n_i and n_j is called a *link*. Counterparties, network, degree are defined identically. The solution concept is PSS.[24]

[23] r_F^i is a constant for now. At the end of the section we generalize our results to endogenous interest rates and show that our results remain intact.

[24] We allow for only one link between a pair of firms. A link can be seen as the optimally chosen number of projects each of which have some fixed capacity.A link can also be seen as optimally sized project between two counterparties. One can engineer a cost function such that the optimal size of the investment is 2, and so each counterparty pays 1. Sturm (2017) studies a case in which the size of the projects are endogenously chosen. Our focus is the impact of bailouts on the network architecture.

Shocks are also defined similarly: $\theta_i \in \{G, B\}$ denotes the realized shock to bank n_i. A shock θ_i means that n_i has to incur a utility cost $\kappa_{\gamma_i}(\theta_i)$ per project to continue managing its projects. Call firms that receive bad shocks *bad firms* and firms that receive good shocks *good firms*.

After shocks are realized and observed, firms can -choose- to continue with their projects or not. The solution concept is the cooperating equilibrium.[25] A firm n_i chooses $(\tilde{a}_{ij})_{j \in N_i} \in \{C, D\}^{d_i}$. $\tilde{a}_{ij} = C$ means that n_i *continues with the project* with n_j and $\tilde{a}_{ij} = D$ means that n_i *defaults on the project* with n_j. We assume that once one counterparty quits managing a project the project fails.[26]

n_i incurs the management cost for each project that it continues with. If n_i and its counterparty continue their project, each get safe return r from the project. Otherwise both get 0. n_i had borrowed l_i and used d_i for investment, and owes $r_F^{\gamma_i} l_i$ to its bank. Accordingly, n_i's payoff is

$$\left[r \times |\{j \in N_i : \tilde{a}_{ij} = \tilde{a}_{ji} = C\}| + (l_i - d_i) - r_F^{\gamma_i} l_i\right]^+ - \kappa_{\gamma_i}(\theta_i) \times |\{j \in N_i : \tilde{a}_{ij} = C\}|.$$

Absence of intervention. *Best responses.* First note that n_i will never borrow more than d_i since banks are promised $r_F^{\gamma_i} > 1$. We assume that projects are ex-post profitable for good firms, or in other words management costs of good firms are small. Otherwise firms would never continue with projects ex-post, hence they would neither invest nor borrow in the first place.

Assumption 4. *Projects are profitable for good firms:* $r - r_F^{\gamma_i} > \kappa_{\gamma_i}(G)$ *for all* γ_i.

Then n_i's best response is given by

$$\tilde{a}^*_{ij} = \begin{cases} \tilde{a}_{ji} & if\ \left[r \times |\{j \in N_i : \tilde{a}_{ji} = C\}| - r_F^{\gamma_i} d_i\right]^+ - \kappa_{\gamma_i}(\theta_i) \times |\{j \in N_i : \tilde{a}_{ji} = C\}| \geq 0 \\ D & o.w. \end{cases}$$

In words, the best response is to

- reciprocate: continue with all projects in which the counterparty is continuing with the project, and try to repay banks, or to
- default: default on all projects and get 0,

depending on whichever gives higher payoff. Then without loss of generality we can actually restrict the strategy space of n_i. Represent the action of n_i with $a_i \in \{C, D\}$ where C is called *continuing,* representing the choice of reciprocating to each counterparty, and D is called *defaulting* representing the choice to default on all counterparties. Clearly, the representation overlaps when all counterparties are defaulting, so we represent this overlapping choice with $a_i = D$ which is more intuitive.

[25] See Section 5 for more on cooperating equilibrium.

[26] There are two ways to think about this. It becomes too costly for the other counterparty to continue managing the joint project on its own hence it is optimally dropped. Another way is to think that the project needs expertise or speciality of both counterparties.

Denote the number of counterparties of n_i that default with $f_i = |\{n_j \in N_i : a_j = D\}|$. Then n_i's best response is simply to play C if

$$(r - \kappa_{\gamma_i}(\theta_i)) \times (d_i - f_i) \geq r_F^{\gamma_i} d_i.$$

Contagion. Next we assume that a bad shock implies large management costs, enough to force a firm into default even if no counterparties are defaulting. Otherwise contagion never starts and all firms always play C.

Assumption 5. *Projects are not profitable for bad firms:* $\kappa_{\gamma_i}(B) > r - r_F^{\gamma_i}$ *for all* γ_i.

Accordingly bad firms default and get 0. Good firms also get 0 if they default. A good firm n_i that continues gets

$$P(f_i, d_i, \gamma_i) = A_{\gamma_i} d_i - B_{\gamma_i} f_i, \text{ where}$$

$$A_{\gamma_i} = r - r_F^{\gamma_i} - \kappa_{\gamma_i}(G), \;\; B_{\gamma_i} = r - \kappa_{\gamma_i}(G).$$

In the cooperating equilibrium, a good firm n_i plays C if only if

$$f_i \leq \tau_{\gamma_i} \times d_i, \text{ where } \tau_{\gamma_i} = \frac{A_{\gamma_i}}{B_{\gamma_i}}.$$

Network formation. The ex-ante characteristics of a firm n_i are captured by D_{γ_i} and τ_{γ_i}. For our purposes it suffices to consider two types of firms. Suppose that firms are either *large firms* or *small firms*: $\Gamma = \{S, L\}$. N_L is the set of large firms and N_S is the set of small firms. $N = N_L \cup N_S$. There are $k_L = |N_L|$ many large firms and $k_S = |N_S|$ many small firms.

Assumption 6. *There are few large firms. They have large credit lines:* $D_S > k_S$, $D_S > k_L$, $D_L > k_L$.

Assumption 7. *Firms are not too resilient:* $\frac{1}{1-\tau_S} < D_S$, $\frac{1}{1-\tau_L} < k_L$. *Large firms are less resilient than small firms:* $\tau_L + \frac{1}{D_S} < \tau_S$.[27]

Large firms want to have many counterparties and their challenge will be to convince the small firms to connecting with them. $\tau_L + \frac{1}{D_S} < \tau_S$ means that large firms are less resilient per project compared to smaller firms, potentially due to higher management costs. The role that this plays is that large firms are able to and willing to take on more projects than small firms, nonetheless, when they do, they become risky counterparties for small firms. $\frac{1}{1-\tau_S} < D_S$ and $\frac{1}{1-\tau_L} < k_L$ simply mean that a small firm that has D_S counterparties and a large firm that has k_L counterparties are not immune to contagion.

Proposition 13. *There exists* $\bar{\alpha} < 1$ *such that for* $\alpha > \bar{\alpha}$ *a network is PSS if and only if it consists of disjoined cliques of small firms each of which has degree* D_S *and one more disjoined clique of large firms with order* k_L.[28]

[27] Normally $\tau_L < \tau_S$ suffices but the additional $\frac{1}{D_S}$ resolves some integer problems.

[28] $\bar{\alpha}$ is roughly equal to τ_S. Due to discrete nature of the problem $\bar{\alpha} = \tau_S$ does not always work. The small gap between smallest such $\bar{\alpha}$ that Proposition 13 holds and τ_S can be approximated well by using Chernoff bounds.

Proof. If n_i's counterparties have no counterparties other than n_i, this situation would give n_i the highest payoff it can get conditional on degree d_i. This is by first order stochastic dominance. Call this payoff $V(d_i, \gamma_i)$. $V(d_i, \gamma_i)$ is given by

$$V\left(d_i, \gamma_i\right) = \alpha \mathbb{E}_{\theta_{-i}} \left[P\left(\left|N_i \cap N_B\right|, d_i, \gamma_i\right)^{+} \right].$$

The only way to achieve this payoff is to eliminate SOCPR. In particular, n_i has $V\left(d_i, \gamma_i\right)$ payoff if and only if there is no positive probability event in which at least one good counterparty of n_i, say n_j, defaults whereas n_i does not default without n_j defaulting. That is, n_i has $V\left(d_i, \gamma_i\right)$ payoff if and only if for all $n_j \in N_i$

$$min\left\{R\left(d_i, \gamma_i\right), d_{ij}\right\} + d_j - d_{ij} - 1 \leq R\left(d_j, \gamma_j\right). \tag{5}$$

Recall that $\tau_S D_S < D_S - 1$. Then if n_i and n_j are both small firms, then by Equation (5), n_i and n_j both get $V\left(D_S, S\right)$ if and only if $D_S = d_{ij} + 1$, i.e. the set of counterparties of n_i and n_j are identical except each other, i.e. $N_i \backslash \left\{n_j\right\} = N_j \backslash \left\{n_i\right\}$.

Now consider the case in which n_i is small and n_j is large. Suppose that n_i gets $V\left(D_S, S\right)$. Then

$$min\left\{\left\lfloor \tau_S D_S \right\rfloor, d_{ij}\right\} + d_j - d_{ij} - 1 \leq \left\lfloor \tau_L d_j \right\rfloor.$$

Consider $\left\lfloor \tau_S D_S \right\rfloor \leq d_{ij}$. Then

$$\left\lfloor \tau_L d_j \right\rfloor \geq \left\lfloor \tau_S D_S \right\rfloor + d_j - d_{ij} - 1 \geq \left\lfloor \tau_S D_S \right\rfloor \geq \left\lfloor \left(\tau_L + \frac{1}{D_S}\right) D_S \right\rfloor = \left\lfloor \tau_L D_S + 1 \right\rfloor$$

which implies that $d_j \geq D_S$. Then

$$D_S - \left\lfloor \tau_L D_S \right\rfloor \leq d_j - \left\lfloor \tau_L d_j \right\rfloor \leq d_{ij} + 1 - \left\lfloor \tau_S D_S \right\rfloor \leq D_S - \left\lfloor \tau_S D_S \right\rfloor$$

which implies that

$$\left\lfloor \tau_L D_S \right\rfloor \geq \left\lfloor \tau_S D_S \right\rfloor \geq \left\lfloor \tau_L D_S + 1 \right\rfloor.$$

A contradiction. Then it must be the case that $\left\lfloor \tau_S D_S \right\rfloor > d_{ij}$. Then Equation (5) becomes

$$d_j - 1 \leq \left\lfloor \tau_L d_j \right\rfloor.$$

In words, n_j must be immune to contagion conditional on n_i being a good bank. But note that $\tau_L k_L < k_L - 1$. So if a large firm has degree k_L, its small counterparties cannot achieve $V\left(D_S, S\right)$.

Now consider our candidate network. If there were a Pareto improving deviation from this network, the strict improvement has to be by a large firm since a small firm cannot go above $V\left(D_S, S\right)$. A large firm is getting $V\left(k_L - 1, L\right)$ in the candidate network. α is large enough that $V\left(\cdot, L\right)$ is increasing. So for a strict improvement, a large firm must connect with a small firm and have degree more than or equal to k_L in the deviation. But then its small counterparty is strictly hurt. So this network is PSS.

Now we show that there is no other PSS network. In any PSS network, since α is large enough, all large firms must be counterparties of each other. Otherwise large firms would deviate and form the links missing between each other. Then if a large firm has a link with a small firm in a PSS network, the small firm gets strictly worse than $V(D_S, S)$. Then D_S+1 small firms would deviate to forming a clique among each other which would be a Pareto improvement. Therefore, in a PSS network, small firms are counterparties of only each other. We have shown earlier that if two small firms n_i and n_j both get $V(D_S, S)$ then $N_i \backslash \{n_j\} = N_j \backslash \{n_i\}$. Since the component of n_i and n_j consists of small firms, by the connectivity of the component, the set of counterparties must be identical for every pair in the component, meaning that the component must be disjoined clique. $\square$

Presence of intervention. A policy is $\left\{T_i\left(\left(\tilde{a}_{ij}\right)_{j\in N_i}\middle|\vec{\theta}, E\right)\right\}_{i\in N} \geq 0$. Here T_i describes the amount of transfer to firm n_i if it continues with projects given by $\left(\tilde{a}_{ij}\right)_{j\in N_i}$. In order to illustrate what the optimal policy would look like, consider a firm n_i who has two counterparties n_j and $n_{j'}$. Suppose that n_i, n_j are good firms and $n_{j'}$ is a bad firm. Suppose that $\kappa_{\gamma_i}(G) + \kappa_{\gamma_j}(G) < 2r < \kappa_{\gamma_i}(G) + \kappa_{\gamma_{j'}}(B)$. Then the project $\{n_i, n_j\}$ is ex-post profitable and government would like both counterparties to continue with this project, whereas the opposite holds for the project $\{n_i, n_{j'}\}$. If $r - \kappa_{\gamma_i}(G) - 2r_F^{\gamma_i} < 0$, then n_i is facing default which implies that the ex-post profitable project $\{n_i, n_j\}$ fails too. If government promises to compensate n_i for the ex-post profitable project and part of n_i's debt to banks, but not for the inefficient project with $n_{j'}$, then n_i would continue with the project with n_j. Formally, government promises to pay $-\left[r - \kappa_{\gamma_i}(G) - 2r_F^{\gamma_i}\right]$ if $(\tilde{a}_{ij}, \tilde{a}_{ij'}) = (C, D)$ and 0 otherwise. Then n_i continues with $\{n_i, n_j\}$, conjecturing that n_j will also continue with $\{n_i, n_j\}$. In the induced cooperating equilibrium, n_j does continue with $\{n_i, n_j\}$ since it conjectures that n_i will continue with $\{n_i, n_j\}$.

In general, an ex-post welfare maximizing policy always makes sure that ex-post profitable projects are continued by both counterparties by sufficient transfers. An ex-post welfare maximizing policy clearly does not compensate banks for their projects that are not ex-post profitable. Hence with some abuse of notation we denote T_i to be the transfer promised to n_i if it continues with all ex-post profitable projects. In all other cases, optimal policy entails 0 transfer due to minimal transfer restriction. A strictly positive transfer to n_i, who would default on some ex-post profitable projects without the transfer, which makes sure that n_i continues with all ex-post profitable projects in the induced cooperating equilibrium can be interpreted as bailing-out firm n_i. Formally, an optimal policy T^* is said to *bailout* n_i if $T_i^* > 0$ and in the cooperating equilibrium induced by T^*, $\tilde{a}_{ij} = C$ for all $n_j \in N_i$ such that $\kappa_{\gamma_i}(\theta_i) + \kappa_{\gamma_{j'}}(\theta_j) < 2r$.

Proposition 14. *Suppose that $\kappa_S(B) = \kappa_L(B)$.*[29] *The optimal policy T^* is the following.*

(Case 1: Direct assistance policy) If $2r < \kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G)$ good firms which are facing default due to bad counterparties and have a good counterparty are bailed out. The transfers

[29]When $\kappa_S(B) \neq \kappa_L(B)$, the optimal policy has 18 different cases to consider, all of which yield the same network and same qualitative insights. We make this assumption only for clarity of the exposition.

are

$$T_i^* = \begin{cases} -\left[\left(r-\kappa_{\gamma_i}(G)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i\right]^- & \theta_i = G,\, N_i\cap N_G \neq 0 \\ 0 & \textit{otherwise.} \end{cases}$$

(Case 2: Mixed policy) If $\kappa(B)+\kappa_S(G) < 2r < \kappa(B)+\kappa_L(G)$ *all bad firms which have a good small counterparty are bailed out. Also all good large firms which are facing default due to bad counterparties and have a good counterparty are bailed out. The transfers are*

$$T_i^* = \begin{cases} -\left[\left(r-\kappa_{\gamma_i}(B)\right)\times|N_i\cap N_G\cap N_S| - r_F^{\gamma_i} d_i\right] & \theta_i = B,\, N_i\cap N_G\cap N_S \neq \emptyset \\ -\left[\left(r-\kappa_L(G)\right)\times|N_i\cap N_G| - r_F^{L} d_i\right]^- & \theta_i = G,\, n_i\in N_L,\, N_i\cap N_G \neq \emptyset \\ 0 & \textit{otherwise.} \end{cases}$$

(Case 3: Indirect assistance policy) If $\kappa(B)+\kappa_S(G) < \kappa(B)+\kappa_L(G) < 2r$ *and* $\kappa(B) > r$ *all bad firms with at least one good counterparty are bailed out. The transfers are*

$$T_i^* = \begin{cases} -\left[\left(r-\kappa_{\gamma_i}(B)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i\right] & \theta_i = B,\, N_i\cap N_G \neq 0 \\ 0 & \textit{otherwise.} \end{cases}$$

(Case 4: Indirect assistance policy) If $\kappa(B) < r$, *all bad firms with at least one counterparty are bailed out. The transfers are*

$$T_i^* = \begin{cases} -\left[\left(r-\kappa(B)\right)\times d_i - r_F^{\gamma_i} d_i\right] & \theta_i = B,\, N_i \neq 0 \\ 0 & \textit{otherwise.} \end{cases}$$

Proof. The proof strategy is to first show that the action profile in which all firms continue with their (ex-post) profitable projects and default on other projects is the cooperating equilibrium under the described transfer scheme. Second we show that this outcome can not be achieved with smaller transfers.

For the case of $2r < \kappa_B(B)+\kappa_S(G) < \kappa(B)+\kappa_L(G)$, if a good firm n_i conjectures that all of its good counterparties will continue with their projects with n_i, then then its best response to reciprocate and continue with all projects with good counterparties which yields

$$-\left[\left(r-\kappa_{\gamma_i}(G)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i\right]^- + \left(r-\kappa_{\gamma_i}(G)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i \geq 0.$$

Then by definition, the described action profile is rationalizable. Note that bad firms and good firms who do not have any good counterparties can not rationalize continuing with any projects. Hence the intended outcome is the cooperating equilibrium. This outcome cannot be achieved with smaller transfers. Consider any transfer scheme that implements this outcome. Bad firms default on all projects in this outcome. Then if a good firm n_i such that $N_i\cap N_G \neq 0$ and $\left(r-\kappa_{\gamma_i}(G)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i < 0$ does not receive at least $-\left[\left(r-\kappa_{\gamma_i}(G)\right)\times|N_i\cap N_G| - r_F^{\gamma_i} d_i\right]$ transfer, it defaults on at least one project with a good counterparty.

Consider $\kappa(B) + \kappa_S(G) < 2r < \kappa(B) + \kappa_L(G)$. If a bad firm n_i conjectures that all good and small counterparties continue with their projects with n_i and the rest default on n_i, then n_i's best response is to reciprocate which yields

$$-\left[(r - \kappa_{\gamma_i}(B)) \times |N_i \cap N_G \cap N_S| - r_F^{\gamma_i} d_i\right] + \left[(r - \kappa_{\gamma_i}(B)) \times |N_i \cap N_G \cap N_S| - r_F^{\gamma_i} d_i\right] = 0.$$

If a good small firm n_i conjectures that all of its counterparties continue with all projects, then its best response is clearly to reciprocate. If a good and large firm n_i conjectures that all of its good counterparties are continuing with their projects with n_i and its bad counterparties are defaulting on n_i, then n_i's best response is to reciprocate which yields

$$-\left[(r - \kappa_L(G)) \times |N_i \cap N_G| - r_F^{\gamma_i} d_i\right]^{-} + (r - \kappa_L(G)) \times |N_i \cap N_G| - r_F^{\gamma_i} d_i \geq 0.$$

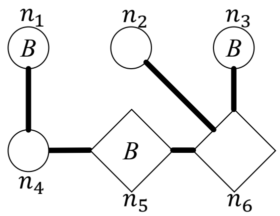
This outcome cannot be achieved with smaller transfers. Consider any transfer scheme that implements this outcome. Good large firms and bad firms default on bad firms. So bad firms can not continue with projects with good small firms without being compensated $(r - \kappa^{\gamma_i}(B)) \times |N_i \cap N_G \cap N_S| - r_F^{\gamma_i} d_i$. Bad firms default on good large firms in this outcome. So if a good and large firm n_i has $(r - \kappa_L(G)) \times |N_i \cap N_G| - r_F^{\gamma_i} d_i < 0$, then it must be compensated at least $-\left[(r - \kappa_L(G)) \times |N_i \cap N_G| - r_F^{\gamma_i} d_i\right]$ in order to continue with its projects with good firms.

The remaining cases are shown similarly. $\square$

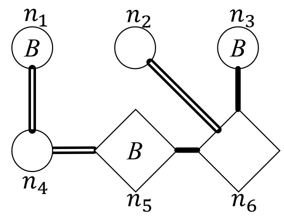
The optimal policy incorporates a form counter-contagion. First, bad firms that are worthy are bailed out. Then given that worthy bad firms are not defaulting anymore, good firms that still face default are bailed out as well. Being worthy of a bail out is determined by the characteristics of the firm in question and the set of counterparties of it.

If there is a project between the firm in question and another firm such that the sum of the management costs of the two is smaller than the total return $2r$ from the project, both members of the pair shall continue their business in order save the project. So there is a sense in which projects are being bailed out rather than firms, yet through firms. Notice that the policy does not have to compensate a firm at an amount that covers all potential losses. Policy covers firms that have ex-post efficient projects at an amount that makes sure only ex-post efficient projects are continued, not the other projects. Indeed, the policy implements the ex-post efficient outcome by saving all ex-post efficient projects and only such projects.

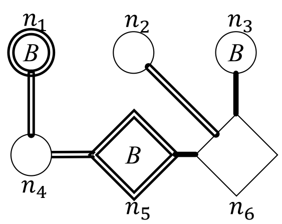
In Figure 14, bailouts under the mixed policy case are portrayed. This is richest case that most clearly illustrates the optimal policy. First, a bad firm which has a small and good counterparty is bailed out directly. Because bad firms will never continue business on their own without a bailout even if all of their counterparties are bailed out. It is just not possible to induce bad firms to continue via indirect support. This is the starting point of the counter-contagion. After such bad firms are bailed out, good bad firms are examined to see which ones are still troubled, i.e. facing default. Good and large firms who are surrounded by bad firms are not worthy of a bailout since they have no projects worthy of saving. Good and large firms that have a good counterparty are bailed out if they are facing default due to bad counterparties. This way these good firms will continue with their projects with other good firms. Now that good small firms are not hurt by bad counterparties and good large firms can not hurt other good firms either, good firms do not face the risk of default anymore. All profitable projects are saved via minimal injections. This concludes the counter-contagion.



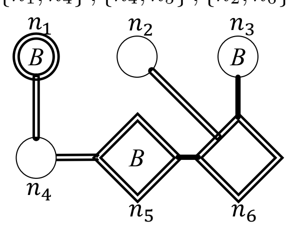
$\gamma_5 = \gamma_6 = L$, others S.
$\theta_1 = \theta_3 = \theta_5 = B$, others G.



Identify ex-post profitable projects: $\{n_1, n_4\}, \{n_4, n_5\}, \{n_2, n_6\}$.



Bad firms with good small counterparties bailed-out: n_1 and n_5.



Good firms who still face default and has a good counterparty bailed-out: n_6.

Figure 14: Illustration of optimal policy.

Proposition 15. *There exists $\bar{\alpha} < 1$ such that for all $\alpha > \bar{\alpha}$, a network is PSS if and only if large firms have min $\{k-1, D_L\}$ counterparties and small firms have D_S counterparties.*

Proof. For the case of $2r < \kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G)$, all good firms continue with good firms, and default on others. Bad firms default on everyone. Then in any network, a small firm n_i has $V(d_i, S)$ and a large firm n_i has $V(d_i, L)$ expected payoff. Since $\alpha > \bar{\alpha}$, $V(\cdot, S)$ and $V(\cdot, L)$ are both increasing. Then the core-periphery network that is described gives everyone their maximal payoff. Then clearly there is no other PSS network.

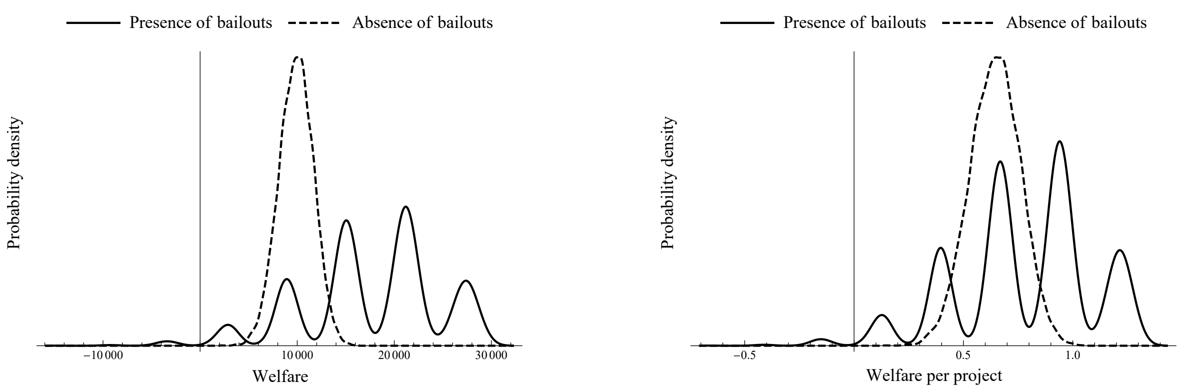
For $\kappa(B) + \kappa_S(G) < 2r < \kappa(B) + \kappa_L(G)$, a small firm n_i has expected payoff $\alpha\left(r - r_F^S\right) d_i$ regardless of the network. A large firm n_i has expected payoff $V(d_i, L)$ regardless of the network. Then again, all firms get their maximal payoff in the core-periphery and so core-periphery is the class of PSS networks.

For $\kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G) < 2r$, a small firm n_i has expected payoff $\alpha\left(r - r_F^S\right) d_i$ regardless of the network. A large firm n_i has expected payoff $\alpha\left(r - r_F^L\right) d_i$ regardless of the network. Same argument holds. $\square$

Proposition 16. *Government intervention strictly increases ex-ante welfare.*

Proof. All ex-post profitable projects reach maturity. The number of total projects does not decrease. The projects are ex-ante positive NPV since $\alpha > \bar{\alpha}$. Hence welfare increases. $\square$

It is not surprising that ex-ante mean of welfare increases. The number of projects increases, each project has positive NPV, and government can effectively bail out projects selectively through the amounts of transfers. However, welfare can be highly volatile due to network hazard. When the network becomes a core-periphery, the aggregate risk in the economy is correlated through idiosyncratic risks of the few large firms. This effect is shown in Figure 15 for direct assistance case, Figure 16 mixed assistance case, and Figure 17 indirect assistance case. The common feature is the endogenous volatility. The reason is that core-periphery structure makes the 'very good' and 'very bad' outcomes more likely. Very bad outcome is that the core gets bad shocks, and amplifies the contagion among the periphery, causing most peripheral firms into indirect defaults, which calls for a amount for bailouts. Very good outcome is that the core gets good shocks, and mitigates the contagion among the periphery by contributing to resilience of the periphery. The performance of the all firms and projects in the economy become highly correlated through the financial standings of large firms. That is, network hazard creates endogenous volatility in welfare through the shocks that large core firms receive.



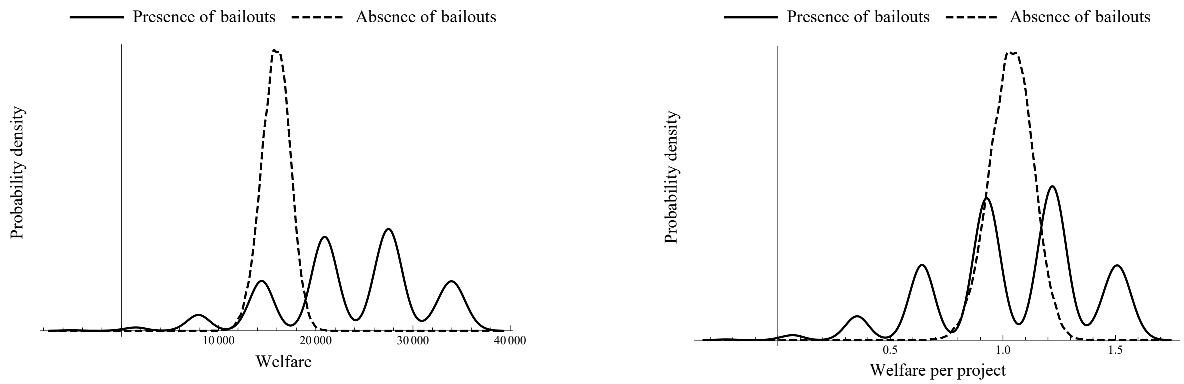
$\mu_{pre} = 17491, \sigma_{pre} = 7039.$
$\mu_{abs} = 9864, \sigma_{abs} = 1766.$

$\mu_{pre} = 0.776, \sigma_{pre} = 0.312.$
$\mu_{abs} = 0.643, \sigma_{abs} = 0.115.$

Figure 15: Distribution of welfare. Direct assistance case.

Since welfare follows a similar pattern for all cases, in the remainder we provide comparative statics regarding the mixed case to avoid repetitiveness. In the direct assistance case almost all good firms receive bailouts. In the indirect assistance case, almost all bad firms receive bailouts and all firms always continue. The mixed assistance case is the richest and the most intuitive case as far as bailouts are concerned.

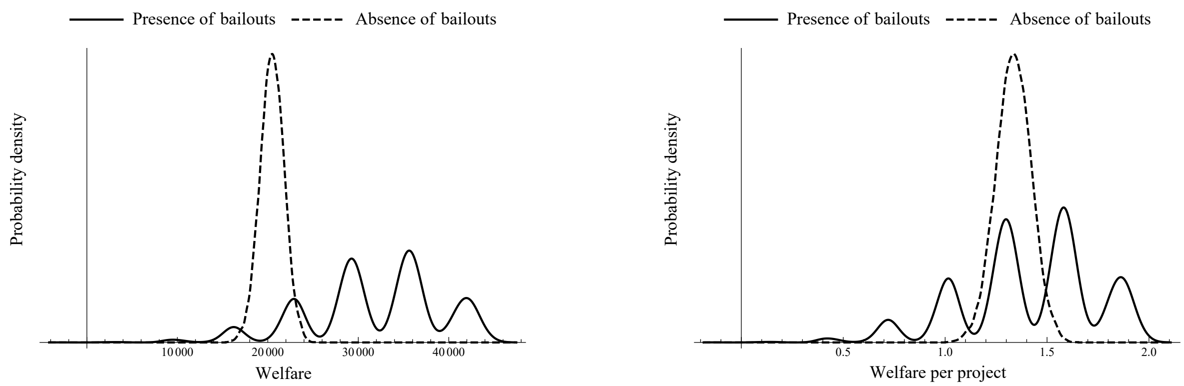
Figure 18 shows the number of ex-post efficient projects that are at the risk of failing. The probability that a project needs assistance in the presence of intervention is smaller than the probability that a project fails in the absence of intervention. Again, due to the correlation through the core, standard deviation is much larger, even when it is calculated for the fraction of projects that need assistance in order to account for the larger number projects undertaken in the presence of intervention.



$\mu_{pre} = 23538, \sigma_{pre} = 7411.$
$\mu_{abs} = 15894, \sigma_{abs} = 1455.$

$\mu_{pre} = 1.044, \sigma_{pre} = 0.328.$
$\mu_{abs} = 1.036, \sigma_{abs} = 0.094.$

Figure 16: Distribution of welfare. Mixed case.



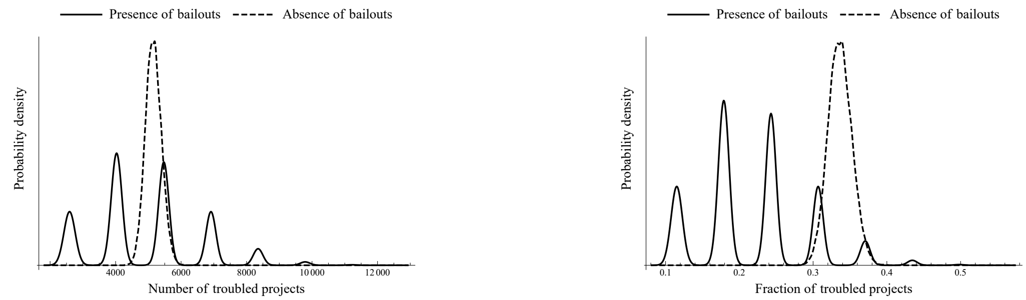
$\mu_{pre} = 31758, \sigma_{pre} = 7299.$
$\mu_{abs} = 20482, \sigma_{abs} = 1335.$

$\mu_{pre} = 1.409, \sigma_{pre} = 0.324.$
$\mu_{abs} = 1.336, \sigma_{abs} = 0.087.$

Figure 17: Distribution of welfare. Indirect assistance case.

Figure 19$-a$ plots the number of firms that default in the no intervention case against the number of troubled firms in the intervention case. Here troubled firm refers to defaulting firms and bailed-out firms. After shocks arrive, these firms are materially facing distress, and would all default without receiving direct assistance. Here intervention does not create volatility because large firms are bailed out and there is no need to provide direct assistance to good small firms. Big chunk of the number of bailouts is the small bad firms that have small good counterparties. Then the distribution of the total number of defaulting firms and bailed-out firms in the presence of intervention is roughly the distribution of the number of bad small firms. In the absence of intervention, contagion causes many small good firms to default as well. So in response to the anticipation of bailouts, the distribution roughly decreases in FOSD sense.[30] On the other hand, the number of firms at risk shown in Figure

[30]Note that for *Case* 1, $2R < \kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G)$, the optimal policy is to directly save small good firms instead of saving them with indirect assistance via the bad large firms. Then the volatility is reflected on the number of troubled banks as well. Bad large firms force many small good firms into default and government saves a large number small and good firms with direct assistance.

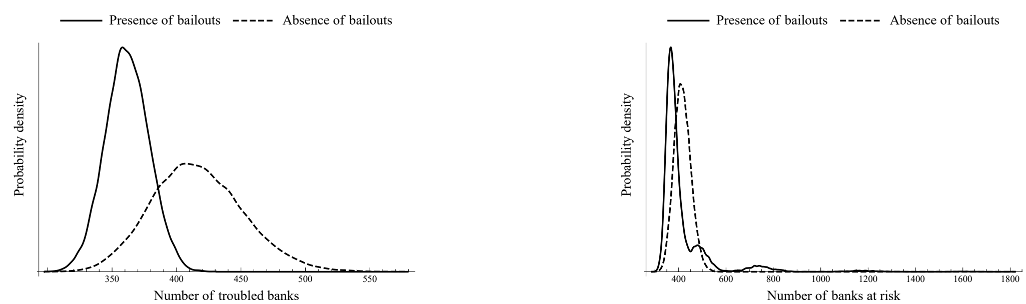


$\mu_{pre} = 4893, \sigma_{pre} = 1625,$
$\mu_{abs} = 5155, \sigma_{abs} = 241.$

$\mu_{pre} = 0.217, \sigma_{pre} = 0.072$
$\mu_{abs} = 0.336, \sigma_{abs} = 0.015.$

Figure 18: Distribution of the number of ex-post efficient projects that are at the risk of failing

19$-b$ becomes more volatile. A firm at risk refers to all firms that would default, hypothetically, if government did not intervene despite the anticipation of intervention. These firms rely on government's -direct or indirect- assistance. In the event that the core gets many bad shocks, all firms in the economy are at risk.



$\mu_{pre} = 361, \sigma_{pre} = 16$
$\mu_{abs} = 416, \sigma_{abs} = 35.$
19$-a$: Troubled firms

$\mu_{pre} = 415, \sigma_{pre} = 124$
$\mu_{abs} = 416, \sigma_{abs} = 35.$
19$-b$: Firms at risk

Figure 19: Distribution of the number of troubled firms and firms at risk.

Despite these, in fact, network hazard does not make the large firms inherently safer than smaller firms. Remember that the common impact of network hazard across all cases of the optimal policy is that network hazard makes good firms immune to contagion. For example under *Case* 1, $2r < \kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G)$, only good firms with good counterparties bailed out and being large or small does not factor into the ex-post decision of a firm being bailed-out or not. However, being large or small factors into the ex-ante likelihood of being bailed out. Network hazard does make large firms too big to fail in the sense that large firms become ex-ante more likely to continue business compared to small firms. This is more stark under *Case* 2, $\kappa(B) + \kappa_S(G) < 2r < \kappa(B) + \kappa_L(G)$ where bad firms with a good small counterparty are bailed out. Large firms have a large number of small counterparties and this makes it almost certain that large firms are bailed out even when they receive a bad shock. That is, large firms are almost certain to continue business. In this sense, network hazard makes large firms ex-ante TBTF.

Endogenous interest rates. The interest rates so far were taken to be exogenous for simplicity. Here we show how that endogenous interest rates would react to the anticipation of intervention and show that the previous results are robust. Given the anticipated network formation and endogenous risk of contagion that arises, banks are promised an interest rate that makes them indifferent between lending or not. By an *equilibrium* we mean one interest rate r_F^L for all large firms, one interest rate r_F^S for all small firms, and a PSS network given these interest rates. Consider members of a clique of order $d+1$ consisting of firms of type $\gamma \in \{S, L\}$. Let $\mathbb{F}$ denote the Binomial CDF. The interest rate r_F^γ is determined according to

$$1 = r_F^\gamma \alpha \mathbb{F}\left[\left(1 - \frac{r_F^\gamma}{r - \kappa_\gamma(G)}\right) d, d, 1-\alpha\right]. \tag{6}$$

Proposition 17. *(Existence) Suppose that $1 < \alpha^2 (r - \kappa_S(G))$. There exists $\underline{D}_S$ and $\bar{k}_L$ with $\underline{D}_S > \bar{k}_L$ such that for all $D_S > \underline{D}_S$ and $k_L < \bar{k}_L$ the following holds. In the absence of intervention, there is an equilibrium in which $r_F^S < r_F^L$ and the network formed consists of disjoined cliques of order $D_S + 1$ among small firms and one disjoined clique of order k_L among large firms.*

Proof. Conjecture that

$$\tau_L = 1 - \frac{r_F^L}{r - \kappa_L(G)} < 1 - \frac{r_F^S}{r - \kappa_S(G)} = \tau_S \text{ and}$$

$$r < \alpha (r - \kappa_S(G)).$$

By the first conjecture, network formed is in cliques of order D_S+1 among small firms and one clique of order k_L among large firms. In order for a bank to get a repayment, her corresponding firm must be a good firm and its counterparties suffer fewer bad shocks than what the firm can absorb. The probability of repayment is then $\alpha\mathbb{F}[\tau_S D_S, D_S, 1-\alpha]$ for small firms and $\alpha\mathbb{F}[\tau_L(k_L-1), k_L-1, 1-\alpha]$ for large firms, where $\mathbb{F}$ is the Binomial CDF. Accordingly the interest rates satisfy

$$r_F^S \alpha \mathbb{F}\left[\left(1 - \frac{r_F^S}{r - \kappa_S(G)}\right) D_S, D_S, 1-\alpha\right] = 1,$$

$$r_F^L \alpha \mathbb{F}\left[\left(1 - \frac{r_F^L}{r - \kappa_L(G)}\right)(k_L - 1), k_L - 1, 1-\alpha\right] = 1.$$

It is clear that $r_F^S > \alpha^{-1}$ and $r_F^L > \alpha^{-1}$. By the second conjecture $r_F^S < \alpha(r - \kappa_S(G))$. Then for all ε there exists $\underline{\mathrm{D}}_S$ such that for all $D_S \geq \underline{\mathrm{D}}_S$ we have

$$\mathbb{F}\left[\left(1 - \frac{r_F^S}{r - \kappa_S(G)}\right) D_S, D_S, 1-\alpha\right] \geq 1 - \varepsilon.$$

That is, if D_S is large enough, r_F^S is arbitrarily close α^{-1}. This verifies the second conjecture since $1 < \alpha^2 (r - \kappa_S(G))$.

Now consider r_F^L. Since $r_F^L > \alpha^{-1}$,

$$\mathbb{F}\left[\left(1-\frac{r_F^L}{r-\kappa_L(G)}\right)(k_L-1),k_L-1,1-\alpha\right] < \mathbb{F}\left[\left(1-\frac{1}{\alpha(r-\kappa_L(G))}\right)(k_L-1),k_L-1,1-\alpha\right].$$

Then there exists $\varepsilon' > 0$ and $\bar{k}_L$ such that for all $k_L \leq \bar{k}_L$, RHS is smaller than $1-\varepsilon'$. Therefore, r_F^L is bounded away from α^{-1} so that $r_F^L > 1-\varepsilon > 1-\varepsilon' > r_F^S$. Then

$$\frac{r_F^L}{r-\kappa_L(G)} > \frac{r_F^S}{r-\kappa_S(G)},$$

verifying the first conjecture. □

This does not rule out multiplicity yet. For example if $1 < \alpha^2(r-\kappa_L(G))$ then the following is a -potentially- consistent scenario. Banks charge a small interest rate around α^{-1} to large firms making sure that $\tau^L > \tau^S$, expecting that large firms will be highly connected and be very safe (due to law of large numbers and $1 < \alpha^2(r-\kappa_L(G))$). This way small firms are willing to connect with large firms so that large firms can indeed achieve very large degrees and indeed become worthy of small interest rates α^{-1}. It is still not clear whether there would exist a PSS network in this case since large firms are not entirely safe, but just safer.[31] Nonetheless it is a possible scenario. Regardless, $1 > \alpha^2(r-\kappa_L(G))$ rules this possibility out, as we show in the next proposition.

Proposition 18. *(Uniqueness) Suppose that $\alpha^2(r-\kappa_L(G)) < 1 < \alpha^2(r-\kappa_S(G))$. There exists $\underline{D}_S$ such that for all $D_S > \underline{D}_S$ the following holds. In the absence of intervention, there is no equilibrium with bounded prices*[32] *in which $\tau^L < \tau^S$. Accordingly there is no equilibrium network other than the one described in Proposition 17.*

Proof. Suppose that there exists an equilibrium with bounded prices in which $\tau_L < \tau_S$. Recall that a firm with degree d_i payoff gets payoff $V(d_i,\gamma_i)$ if and only if all of its counterparties n_j satisfy $min\{\lfloor\tau_{\gamma_i}d_i\rfloor, d_{ij}\} + d_j - d_{ij} + 1 \leq \tau_{\gamma_j}d_j$. Since small firms can always achieve the payoff $V(D_S,S)$ collectively for each of them, in any PSS network, each small firm must have this payoff. Hence a small firm accepts a configuration if and only if it has degree D_S and the counterparty risk not more than what it is in a clique of order D_S+1 consisting of small firms. Therefore, in any PSS network candidate, the probability of repayment by a small firms is identical to the probability of repayment in a clique of order D_S+1 consisting of small firms. That probability is $\alpha\mathbb{F}[\tau_S D_S, D_S, 1-\alpha]$. If $r_F^S > \alpha(r-\kappa_S(G))$, since D_S is large, $\alpha\mathbb{F}[\tau_S D_S, D_S, 1-\alpha] \approx 0$, so $r_F^S \approx \infty$. So $r_F^S < \alpha(r-\kappa_S(G))$. Since D_S is large, $\alpha\mathbb{F}[\tau_S D_S, D_S, 1-\alpha] \approx \alpha$ so $r_F^S \approx \alpha^{-1}$. Since $\tau_L > \tau_S$, we have $r_F^L < r_F^S$ which means $r_F^L \approx \alpha^{-1}$. That is, in the equilibrium network, for large firms n_i we have $\mathbb{F}\left[\theta^L d_i, d_i, 1-\alpha\right] \approx 1$. This necessitates d_i being large and $r_F^L < \alpha(r-\kappa_L(G))$. By the latter, we have $1 < \alpha^2(r-\kappa_L(G))$, a contradiction. Hence $\tau_L < \tau_S$ in any equilibrium.

[31] Recall the proof of Proposition 15 that it is not sufficient to have $\tau_L > \tau_S$ to convince small banks to connect with large banks.

[32] Another option for multiplicity is that banks charge infinite interest rates so any borrower firm is certain to default, and certain to not repay. We ignore this option.

Then by the previous arguments in the Proof of Proposition 13, the only candidate PSS network is the one described. $\square$

The only possible multiplicity that remains is due to the curvature of the Binomial CDF in Equation (6). Nevertheless this multiplicity does not have any impact on the network structure and the qualitative results.

When government intervention is anticipated, the analysis is simpler. The details of the network architecture does not play an ex-post role in optimal policy. All interest rates are determined by the type of a firm, the number of its counterparties, and the types of its counterparties.

Proposition 19. *Suppose that $\kappa_S(B) = \kappa_L(B)$. There exists $\bar{\alpha} < 1$ such that for all $\alpha > \bar{\alpha}$ the following holds. In the presence of intervention,*

If $2r < \kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G)$,

$$r_F^S = \left[\alpha\left(1 - (1-\alpha)^{D_S}\right)\right]^{-1},$$

$$r_F^L = \left[\alpha\left(1 - (1-\alpha)^{k-1}\right)\right]^{-1}.$$

If $\kappa(B) + \kappa_S(G) < 2r < \kappa(B) + \kappa_L(G)$,

$$r_F^S = \left[(1-\alpha)\left(1 - (1-\alpha)^{D_S - k_L}\right) + \alpha\right]^{-1},$$

$$r_F^L = \left[(1-\alpha)\left(1 - (1-\alpha)^{k-k_L}\right) + \alpha\left(1 - (1-\alpha)^{k-1}\right)\right]^{-1}.$$

If $\kappa(B) + \kappa_S(G) < \kappa(B) + \kappa_L(G) < 2r$ and $\kappa(B) > r$

$$r_F^S = \left[(1-\alpha)\left(1 - (1-\alpha)^{D_S}\right) + \alpha\right]^{-1},$$

$$r_F^L = \left[(1-\alpha)\left(1 - (1-\alpha)^{k-1}\right) + \alpha\right]^{-1}.$$

If $\kappa(B) < r$,

$$r_F^S = r_F^L = 1.$$

In all cases, a network is PSS if and only if large firms have degree $\min\{k-1, D_L\}$ and small firms degree D_S.

Proof. Q.E.D. by the optimal policy in Proposition 14. $\square$