Asset Pricing vs Asset Expected Returning in Factor Models

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Abstract

This paper proposes a new approach to factor modeling based on the long-run equilibrium relation between prices and related drivers of risk (integrated factors). We show that such relationship reveals an omitted variable in standard factor models for returns that we label as Equilibrium Correction Term (ECT). Omission of this term implies misspecification of every factor model for which the equilibrium ()cointegrating) relation holds. The existence of this term implies short-run mispricing that disappears in the long-run. Such evidence of persistent, but stationary, idiosyncratic risk in prices is consistent with deviations from rational expectations. Its inclusion in a traditional factor model improves remarkably the performance of the model along several dimensions. Furthermore, the ECT–being predictive–has strong implications for risk measurement and portfolio allocation.

Keywords: Asset Pricing, Asset Returns, Equilibrium Correction Term, Dynamic Factor Structure
JEL codes: G11, G17

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1 Introduction

In his AFA presidential address, John Cochrane (2011) states "... *We have to answer the central question, what is the source of price variation? When did our field stop being "asset pricing" and become "asset expected returning"?...*".

This paper shows that understanding price fluctuations leads to a remarkable emprical improvement in factor-based asset pricing models.

We propose a novel approach to factor modeling of prices and returns that naturally leads to the identification of an "Equilibrium Correction Term". Our approach allows to capture the long-run equilibrium relation between prices and related drivers of risk (integrated factors) and learn from that relation. This feature is relevant to explain the time-series dynamics of returns. The newly identified Equilibrium Correction Term (henceforth, ECT) enables to specify an improved dynamic factor structure for asset returns. Furthermore, the ECT–being predictive–has strong implications for risk measurement and portfolio allocation.

Factor models are vastly used to reduce the number of parameters to be estimated for asset allocation and risk measurement of portfolios including many assets (see, for example, Kolanovic and Wei (2013), Ang (2014)). The literature on the relationship between the choice of factors and the investment horizon has been much less developed. In particular, factor-based approach to asset allocation and risk measurement has concentrated almost exclusively on modeling returns with factors while devoting much less attention on the relationship between prices (value of buy-and-hold portfolios in any test assets) and risk drivers (value of buy-and-hold portfolios in factors). Also the literature on factor models has traditionally concentrated on factor representation of stationary variables and only very recently the factor framework has been extended to non-stationary cointegrated factors (e.g., Barigozzi et al. (2015), Banerjee et al. (2017)).

Long-run risk drivers are related to prices just as factors are related to asset returns. Risk drivers are variables that explain the long-run (buy-and-hold) performance of any given portfolio. As risk drivers and prices of a given portfolio are non-stationary variables, the validity of a given set of risk drivers to explain portfolio prices is investigated by assessing if there exists a stationary linear combination of them (i.e., if they are cointegrated).

We show that the analysis of the potential cointegration between risk drivers and prices is more informative for the validation of a factor model than the analysis between factors and returns. The existence of cointegration between risk drivers and prices delivers a stationary equilibrium correction term that captures the gap between asset prices and their long-run risk drivers. Importantly, the new term reverts to the mean, but slowly: it is stationary, but persistent. Therefore it is predictable and it is also statistically significant in explaining return dynamics. This term is typically omitted from traditional factor models in asset pricing, and its inclusion in factor models for returns improves remarkably their empirical performance.

The relevance of the ECT is grounded in the literature on the relationship between cointegrated variables and equilibrium correction models (see, for example, Engle and Granger (1987), Hendry (1986), Johansen (1995), Pesaran and Shin (1998)). Since (log) prices and risk drivers are cointegrated, the ECT represents the extent of disequilibrium in their equilibrium relation. Therefore, when cointegration holds, any factor model for asset returns that does not include the ECT is misspecified.

The existence of a persistent but stationary deviation of prices from an equilibrium that depend on their risk drivers can only be explained by a persistent but stationary idiosyncratic risk component. The literature on "diagnostic expectations" in which agents' over-reaction to news on prices is related to subsequent adjustments in the dynamics of returns (e.g., Bordalo et al. (2017), Gennaioli et al. (2015), Gennaioli and Shleifer (2018)) offers a natural framework to understand the dynamics of such a component.

The ECT is predictive by its nature. This characteristic has relevant consequences for portfolio allocation and risk measurement. When prices and risk drivers diverge from their long-run equilibrium, we expect an adjustment in the next-period relationship between asset returns and factors. We provide strong evidence of such equilibrium correction mechanism in our analysis.

This paper illustrates the importance of cointegration analysis between risk drivers and prices, the significance of the ECT for the specification of appropriate dynamic factor models, and the implications of this evidence for the derivation of the predictive distribution of returns. Furthermore, we show how to implement a zero-cost investment strategy that consistently exploits information embedded in the ECT. Such a strategy substantially outperforms alternative strategies in the long-run, earning a statistically significant average annual excess return of 6.21%.

We conduct our empirical analysis on annual database for the 25 Fama-French portfolios formed on Size and Book-to-Market and the Fama-French five-factor model (Fama and French (2015)) augmented with the momentum factor proposed by Jegadeesh and Titman (1993) and Carhart (1997).[1]

2 From Factors for Asset Returns to Risk Drivers for Asset Prices

Factor models are extensively used to characterize parsimoniously the predictive distribution of asset returns. To this end multifactor models, in which k factors characterize in a lower parametric dimension the distribution of n asset returns, have the following general form:

$$
\begin{aligned}
r^{i}_{t+1} &= \alpha^{i} + \beta'_{i}\mathbf{f}_{t+1} + v^{i}_{t+1} && (1)\\
\mathbf{f}_{t+1} &= E\left(\mathbf{f}_{t+1} \mid I_{t}\right) + \epsilon_{t+1} && (2)\\
\epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0}, \mathbf{\Sigma}\right)\\
Cov\left(v^{i}_{t+1}, v^{j}_{t+1}\right) &= 0, \quad for \quad each \quad i \neq j
\end{aligned}
$$

in which $\mathbf{f}_{t+1}$ is a k-dimensional vector of factors at time $t+1$, r^{i}_{t+1} is the return on the i-th of the n assets at time $t+1$ and the vector β'_{i} contains the loadings for asset i on the k factors. (1) specifies the conditional distribution of returns upon factors, while (2) specifies the predictive distribution for factors at time $t+1$ conditioning on information available at time t. The traditional baseline specification for this system assumes away factors predictability and it implies that the conditional expectations has no variance: $E\left(\mathbf{f}_{t+1} \mid I_{t}\right) = \mu$. Returns and factors are stationary variables.[2]

For a factor model to effectively characterize the distribution of n assets it is crucial that v^{i}_{t+1} represent idiosyncratic risks and therefore they are orthogonal to the factors, serially uncorrelated, and contemporaneously uncorrelated across assets. The diagonality of the variance-covariance matrix of the residuals coming from projecting asset returns on factors is a necessary–and testable–requirement for the validity of any factor model. In fact, only in this case the factor structure allows to reduce the problem of modeling the n-variate distribution of returns (in which n can be very large) to model the k-variate distribution of factors (in which k is typically small).[3] Testing for the validity of these assumptions is crucial to guarantee that any given factor model produces the

[1] Jegadeesh and Titman (1993) were the first to document the momentum anomaly that motivated the Fama-French-Carhart model (1997).

[2] The unconditional moments of returns are constant, although the conditional ones might be time varying.

[3] Using the joint distribution of returns requires the estimation of $n(n+1)/2+n$ parameters while using a factor structure requires the estimation of $n(k+1)+(k(k+1))/2+k$ parameters.

valid reduction of dimensionality which is particularly useful for asset allocation, risk measurement, and risk management.

Further validation of factor models is traditionally based on testing restrictions on their coefficients. Let's consider the time-series and the expected return-beta representation of a factor model (e.g., Cochrane (2005)). Factor loadings can be calculated via n time-series regressions of the n returns on the k factors

$$r_{t+1}^{i} = \alpha_1 + \beta_{i,f^1} f_{t+1}^{1} + \beta_{i,f^2}\ f_{t+1}^{2} + \cdots + \beta_{i,f^k} f_{t+1}^{k} + v_{t+1} \tag{3}$$

and the affine expected return-beta cross-sectional regression is:

$$E(\mathbf{r}) = \gamma_0 + \gamma_1 \hat{\beta}_{f1} + \gamma_2 \hat{\beta}_{f2} + \cdots + \gamma_k \hat{\beta}_{fk} \tag{4}$$

In the case in which all factors are excess returns, equation (4) holds also for the factors, and we have $\gamma_i = E(f^i)$ where $E\left(f^i\right)$ is the mean over time of the factor i.

A cross-sectional test for the validity of the factor model can be run by regressing the n-variate averaged over time vector of returns on the cross-section of the estimates of the exposures with length n obtained from the first step

$$E(\mathbf{r}) = \gamma_0 + \gamma_1 \hat{\beta}_{f1} + \gamma_2 \hat{\beta}_{f2} ... + \gamma_k \hat{\beta}_{fk}$$

and by testing the following null hypothesis:

$$\hat{\gamma}_0 = \bar{r}^f,\ \hat{\gamma}_i = E\left(f^i\right)$$

If both test assets and factors are excess returns, there is no need to run the cross-sectional test, as the validity of the model can be simply tested by evaluating the null that all intercepts in the time-series model are zero (see Gibbons et al. (1989)). Recently, Barillas and Shanken (2017) have proposed an alternative procedure based on examining the extent to which each factor models price the factors in alternative models.

In the case factors are not returns, and the factor risk premia might be different from the mean value of factors, the time-series tests cannot be run without the aid of the cross-sectional regression. In fact, by imposing the restriction $E(r^i) = \beta_i' \gamma$, we can rewrite the time-series regression as:

$$r_{t+1}^{i} = \beta_i' \gamma + \beta_i' (\mathbf{f}_{t+1}^{i} - E(\mathbf{f})) + v_{t+1} \tag{5}$$

Therefore the intercept restriction in the time-series specification becomes:

$$\alpha_i = \beta_i'(\gamma - E(\mathbf{f})) \tag{6}$$

Interestingly, the focus of the literature on these aspects has somewhat underscored the fact that the standard framework relating asset returns to stationary factors leaves (log) asset prices undetermined. Our proposal to overcome this limitation is to extend the framework defining the mapping of asset returns into factors to include the relationship between asset prices and risk drivers.

Meucci (2011) introduces the concept of risk drivers of any given security as a set of random variables that completely specifies the security price and that follow a stochastic process homogeneous across time. We define as factor-risk drivers the cumulative log returns on a portfolio investing in standard factors that have a period (log) return of f_t. The generic risk driver associated to a factor with a (log) period return of f_t evolves according to the following process:

$$F_t = F_{t-1} + f_t \tag{7}$$

Consider now a test asset i that has a (log) period return of r_t^i. The log price of this asset is defined as:

$$lnP_t^i = lnP_{t-1}^i + r_t^i \tag{8}$$

The determination of (log) prices is naturally obtained by considering the relation between asset prices and factor-risk drivers as the long-run equivalent of the relationship between returns and factors. Factor-risk drivers are the non-stationary variables that drive the non-stationary dynamics of prices. In other words, in a valid empirical model prices and risk drivers should be cointegrated and the exposure of any given portfolio P_t^i to risk drivers is determined by estimating parameters in the following model:

$$\ln P_t^i = \alpha_0^i + \alpha_1^i t + \beta_i' \mathbf{F}_t + u_t^i \tag{9}$$

For a correct specification of risk drivers, the estimation of (9) should deliver stationary error terms u_t^i. The residuals u_t^i capture "disequilibria" in the long-run relationship between prices and factor-risk drivers. The coefficient α_1 measures the systematic long-run component in the relative performance of portfolios and factor-risk drivers; a positive α_1 in the long-run generates "alpha" in returns.

Stationarity of disequilibrium is a minimal condition to identify a valid factor model for asset returns. Indeed, in this case the factor model is capable to replicate fluctuations in prices of any given portfolio up to stationary residuals. Non-stationary mispricing is a strong argument to discard any factor model. Note that the omission of a factor whose associated risk driver is relevant to determine the price dynamics of a given portfolio rules out cointegration between portfolio prices and any set of risk drivers that omits the relevant one.

A cointegrated relation between risk drivers and portfolio prices that delivers stationary deviations from the long-run equilibrium relation deserves some attention. Cointegration between portfolios and risk drivers has important implications for the specification of a model for asset returns and their associated factors. In fact, when risk drivers explain portfolio values, cointegration implies that portfolio returns respond to disequilibrium in the long-run cointegrating relationship identifying a variable so far omitted in the literature in empirical factor models that we label as "Equilibrium Correction Term".

Disequilibrium in the relation between prices and risk drivers is naturally related to the idiosyncratic component of risk of a given portfolio. Cointegration between prices and risk drivers allows for persistence in the risk component and–as we shall see in the next section–a statistically significant degree of persistence is inevitably found in the data when applying the proposed framework to the 25 Fama-French portfolios. We have already seen that omitting a relevant factor from the specification for returns is not compatible with cointegration between (log) portfolio prices and risk drivers. So persistence is most naturally interpreted as a property of the idiosyncratic risk component. A possible interpretation of this evidence is offered by models of belief formation based on representativeness heuristic. These models, which imply over-reaction to news caused by an exaggeration of probability of states that are objectively more likely, do deliver this feature of the data (e.g., Bordalo et al. (2017)).

The existence of an ECT implies that a positive deviation of prices from their equilibrium at time t predicts a decline in returns at time $t+1$. Such evidence would closely resemble the findings by La Porta (1996) that returns on stocks with the most optimistic analyst long-term earning growth forecasts are substantially lower than those for stocks with the most pessimistic forecast. Bordalo et al. (2017) rationalize these findings with the hypothesis of "diagnostic expectations". According to the expectations formation mechanism, agents are forward looking but deviate from rational expectations by over-reacting to news. However, deviations are temporary and over-reaction subsequently generates an equilibrium correction adjustment.

The joint distribution of factors, portfolios and risk drivers is described as follows:

$$
\begin{aligned}
\ln P_{t+1}^{i} &= \alpha_0^i + \alpha_1^i t + \beta_i' \mathbf{F}_{t+1}^{i} + u_{t+1}^{i} \\
u_{t+1}^{i} &= \rho_i u_t^i + v_{t+1}^{i} \\
\mathbf{f}_{t+1} &= \mu_0 + \mathbf{\Pi} \mathbf{F}_t + \epsilon_{t+1} \\
lnP_t^i &= lnP_{t-1}^i + r_t^i \\
\mathbf{F}_t &= \mathbf{F}_{t-1} + \mathbf{f}_t \\
\epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0},\ \mathbf{\Sigma}\right) \\
Cov\left(v_{t+1}^{i}, v_{t+1}^{j}\right) &= 0
\end{aligned}
\tag{10}
$$

The vector ϵ_{t+1} is the vector of invariants that drives, together with the idiosyncratic term v_{t+1}^{i}, the statistical distribution of returns. In the representative heuristic view, the idiosyncratic term v_{t+1}^{i} may capture the news orthogonal to risk drivers and factors that ignites the "deviation from equilibrium", followed by the mean-reversion driven by ρ_i.

When u_t^i is stationary, pricing errors disappear in the long-run, and they are only relevant to determine high-frequency dynamics. Stationarity of the u_t^i is achieved when risk drivers and (log) prices are cointegrated (i.e., $\mid \rho_i \mid < 1$). In this case, when $0 < \mid \rho_i \mid < 1$ we have positive persistence in the idiosyncratic term v_{t+1}^{i} and the effect of the "news" on this term lasts more than one-period, while when $\mid \rho_i \mid < 0$ the "news" ignites an oscillatory, but convergent, effect of the news. The effect of news behaves like a proper unpredictable shock only when $\mid \rho_i \mid = 0$. Note that this evidence, which has important implications for predictions, does not provide any information on the hypothesis of under-reaction versus over-reaction to news. A precise test of this hypothesis would require identification of news, which is beyond the scope of this paper.[4]

Bordalo et al. (2018) show that for an autoregressive process for the idiosyncratic risk like ours the diagnostic distribution has the following mean:

$$
E_t^{\theta}\left(u_{t+1}\right) = E_t\left(u_{t+1}\right) + \theta\left[E_t\left(u_{t+1}\right) - E_{t-1}\left(u_{t+1}\right)\right] \tag{11}
$$

Diagnostic Expectations are then a linear combination of the rational expectations held at time t and t-1, and they differ form rational expectations by a shift in the direction of the information received at time t. In the case of our process:

[4]In a different context, Bordalo et al. (2017) consider the case of stationary price-to-earnings that reacts to revisions on a stationary process for earning growth to find evidence of over-reaction.

$$E_t^\theta\left(u_{t+1}\right)=\rho\left(1+\theta\right)u_t \tag{12}$$

and the estimation of a single parameter for the residuals in our coitegrating specification does not allow to identify ρ and θ. Note, however, that the evidence of cointegration is consistent with a very low ρ and a high θ that causes the current shock to idiosyncratic risk to be extrapolated into the future.

We derive the relationship between returns and factors implied by our model by taking first differences of equation (10):

$$r_{t+1}^i=\alpha_1^i+\beta_i'\mathbf{f}_{t+1}^i+(\rho_i-1)u_t^i+v_{t+1}^i \tag{13}$$

where u_t^i is the Equilibrium Correction Term associated with asset i. If there is cointegration and the time horizon at which asset returns are defined is sufficiently long to allow for a reaction of returns at time $t+1$ to disequilibrium in the relationship between risk drivers and prices at time t, then the traditional factor specification for returns is augmented by an Equilibrium Correction Term. The inclusion of this term ensures that the specification for returns is consistent with the long-run relationship between risk drivers and portfolio prices. The omission of the ECT leads to a mis-specification of the factor model that might lead to statistical rejection of its validity along several dimensions.

In the case of cointegration, the long-run response of prices to the idiosyncratic term v_{t+1}^i is $\frac{1}{1-\rho_i}$. When $\mid \rho_i \mid< 0$, we have overreaction with respect to the long-run response, while when $0 <\mid \rho_i \mid< 1$, we have under-reaction with respect to the long-run response.

Interestingly, a traditional factor model would feature non autocorrelated residuals whenever risk drivers and (log) prices are not cointegrated (i.e., when $\mid \rho_i \mid= 1$). Therefore, the incapability of a given factor structure of pricing buy-and-hold portfolios might be missed when only the relationship between returns and factors is specified.

The predictive model for factors is also specified as a VECM allowing for the existence of cointegration among risk drivers.[5] This predictive model can be interpreted as the reduced-form of a forward-looking relation. Nevertheless, a backward-looking specification is needed to generate the predictive distribution of factors and returns at time $t+1$ given the information available at time t. Note that the predictive relation linking factors to linear combination of risk drivers identifies useful and potentially "strong" (in the sense of Stock et al. (2002)) instruments

[5] When there is cointegration among risk drivers, $\mathbf{\Pi}$ is reduced rank and we have $\mathbf{\Pi}=\gamma\delta'$ where γ is the matrix of adjustment coefficients and δ' is the matrix of cointegrating parameters.

that could be used to estimate the relevant parameters in the projection of returns on factors.

Note that our specification is designed for low-frequency asset returns in which a constant volatility specification is adopted and the dimension of predictability mostly exploited is that of the mean. Indeed, observations at a lower frequency are needed to better identify the correction of returns with respect to short-term over-reactions to price shocks. If the focus is instead at an higher frequency, the interest would shift from predicting means via VECM specifications to predicting volatilities using GARCH or stochastic volatility models.

2.1 On the Relevance of the Equilibrium Correction for Risk Measurement and Asset Allocation

The Equilibrium Correction Term is asset specific and it has an average of zero over time by construction. It cannot be considered as a "factor" in itself because its ability to price the cross-section of returns is zero. We interpret the ECT as a measure of mispricing with respect to a long-run equilibrium that is specific to any chosen factor structure. Importantly, every factor structure that generates cointegration between risk drivers and prices naturally leads to an Equilibrium Correction Term. The ECT would not be significant to explain returns only in the case of no-cointegration between risk drivers and (log) prices.

Whenever the ECT is significant, the empirical model of asset prices and returns (10) has a distinctive feature with respect to standard factor asset pricing models: the Equilibrium Correction Term is predictive and therefore the predicted distribution of returns at time $t + 1$ is centered on the (observed) ECT. The existence of this predictive factor is in line with the evidence reported in La Porta (1996) and Bordalo et al. (2017) according to which returns on stocks with the most optimistic analysts long-term earning growth forecasts are substantially lower than those for stocks with the most pessimistic forecasts. The ECT may capture agents' over-reaction to news in prices that are subsequently corrected in the dynamics of returns.

The presence of the Equilibrium Correction Term detects shifts in the predictive distribution of returns with relevant implications for risk measurement and asset allocation. For example, think of a situation in which a market crash causes a change in the relative position of prices and risk drivers so that the sign of u_t becomes very negative. The movement in u_t will shift the distribution of future asset returns to the right with crucial implications for portfolio allocation and risk management.

The existence of the ECT in our approach for asset allocation and risk measurement enables to settle the old-fashioned dispute between risk managers and portfolio managers after a crisis. In fact, in this situation typically the portfolio manager complains with the risk manager who does not allow to take positions geared to exploit the opportunity of a market crash because they are deemed “too risky”. The presence of an equilibrium correction mechanism in the model of asset prices and returns makes the distribution of predicted returns at time $t+1$ after a market crash very different from that of standard factor models for returns. In particular, after the crash the ECT shifts the predicted returns distribution to the right decreasing the associated conditional VaR.

Finally, let's suppose that there exists a market for hedging the factors. In this case, asset allocation can be based on the predictive distribution of $r_{t+1}^i - \gamma_i' \mathbf{f}_{t+1}^i$ which is centered around the estimated exposure of returns to the observable Equilibrium Correction Term. The possibility of hedging factors would then allow to neutralize the known exposure of a portfolio to factors within a portable (time-varying) alpha framework.

3 The Empirical Evidence

In our empirical analysis we focus on U.S. annual data made available from the Kenneth R. French's Data Library for the sample 1963–2018.[6] We consider data on returns for the 25 Portfolios formed on Size and Book-to-Market (5x5), the Fama-French five factors (EXC MKT, SMB, HML, RMW, CMA), and momentum (Mom).

In Figure 1 we show the yearly dynamics for the (log) prices and the risk drivers associated with the Fama-French six factors (FF6, Fama-French five factors plus Mom). We compute risk drivers and (log) prices as described in equations (7) and (8). It is noteworthy that the time-series behavior of risk drivers and prices portends a possibility to model a common stochastic trend between the two variables for the 25 Fama-French portfolios.

INSERT FIGURE 1

[6] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

3.1 The Statistical Evidence on the Equilibrium Correction Term

Our statistical investigation on the relevance of the Equilibrium Correction Term begins from the identification of a potential long-run relationship between (log) prices and risk drivers. We estimate the following regression for the long-run relationship between the log prices of the 25 Fama-French portfolios and the six-risk drivers:[7]

$$\begin{aligned} \ln P_t^i &= \alpha_0^i + \alpha_1^i t + \beta_i' \mathbf{F}_t + u_t^i \qquad i = 1, ..., 25 \\ \mathbf{F}_t' &= \left[F_t^{EXCMKT} \; F_t^{SMB} \; F_t^{HML} \; F_t^{RMW} \; F_t^{CMA} \; F_t^{Mom} \right] \end{aligned} \tag{14}$$

Figure 2 reports the evidence of tests for stationarity on the 25 residuals from equation (14) based on the augmented Dickey-Fuller (ADF) and the KPSS tests. The ADF shows rejection of the null hypothesis of non-stationary of residuals for 17 of the 25 Fama-French portfolios, and only in two cases the observed statistics is clearly below the critical value. The KPSS test does not reject the null of (level) stationarity of residuals for all 25 portfolios.

INSERT FIGURE 2

We proceed to specify a system of 25 equations for the annual returns of the 25 Fama-French portfolios that includes–beside the standard six-factors–the Equilibrium Correction Terms derived from the estimation of the long-run cointegrating relationships:

$$\begin{aligned} r_{t+1}^i &= \alpha_1^i + \beta_i' \mathbf{f}_{t+1} + \delta_i \hat{u}_t^i + v_{t+1}^i \\ lnP_t^i &= lnP_{t-1}^i + r_t^i \\ F_t^i &= F_{t-1}^i + f_t \\ \ln P_t^i &= \alpha_0^i + \alpha_1^i t + \beta_i' \; \mathbf{F}_t + u_t^i \\ \mathbf{F}_t' &= \left[F_t^{EXCMKT} \; F_t^{HML} \; F_t^{SMB} \; F_t^{RMW} \; F_t^{CMA} \; F_t^{MOM} \right] \end{aligned} \tag{15}$$

where r_{t+1}^i is the excess return of test asset i at time $t+1$ and $\hat{u}_t^i$ is the estimated ECT for test asset i.

[7] The evidence of cointegration and the estimates of the parameters in the cointegrating relationships are substantially unaltered when a dynamic model specifying simultaneously long-run and short-run dynamics is considered.

Figures 4 and 5 report all estimated coefficients with their associated 95% confidence intervals showing clear evidence for a uniform rejection of the non-significance of each Equilibrium Correction Term.[8]

INSERT FIGURES 4 and 5

The null hypothesis of joint non-significance of all ECT factors is strongly rejected by the data. When the null that all the 25 coefficients on the ECTs is tested after estimating the system of our 25 equations for the test assets using a Wald test, distributed as a Chi^2 with 25 degrees of freedom, a value of 293 is obtained with an associated probability well below one per cent. If this null is imposed and a traditional factor model is considered, then the null that all the 25 intercepts are zero is strongly rejected with a value for the relevant Wald test, again distributed as a Chi^2 with 25 degrees of freedom, of 73.

When the traditional factor model is augmented allowing for the ECT, the equivalent of the test that the intercept is zero becomes a test that $\hat{\alpha}_1^i + \hat{\delta}_i \hat{u}_t^i = 0$. We report in Figure 6 the time-series of the 25 terms with their associated 95% confidence intervals. Note that for each portfolio, zero is always included in the confidence interval. This is a strong supportive evidence for the ability of the ECT to improve the standard factor pricing model.

INSERT FIGURE 6

Further statistical evidence on the importance of the ECT is provided by analyzing the properties of the correlation matrix of the residuals of different factor models taking as a benchmark the correlation matrix of the returns on the 25 Fama-French portfolios. The heatmaps reported in Figure 7 show a progressive success of single-factor (CAPM), traditional six-factor (FF6), and the six-factor models augmented with the ECT (FECM) in modeling the strong correlation of returns for the 25 Fama-French portfolios.

INSERT FIGURE 7

[8]We also consider heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as proposed by Newey and West (1994). Using these standard errors do not change our conclusions. In fact, when considering robust standard errors only Portfolios 42 and 53 are not significant at 5% level of significance.

Finally, we gain further insights on the improvement generated by the FECM specifications by evaluating the evidence on the multivariate normality of residuals of the progressively more articulated specifications. Table 1 shows a test for multivariate normality of residuals that highlights the sizable relative contributions of the ECT in delivering normally distributed residuals.

INSERT TABLE 1

To sum up, the evidence in favor of the inclusion of the 25 ECTs in the system relating factors and returns is strong and uniform.

3.2 Long-Run Evidence and the Detection of Mis-Specification in Factor Models

Our integrated approach to modeling asset prices and returns is crucially relevant to detect mis-specification in factor models. In fact, even a gross misspecification in modeling buy-and-hold returns with risk drivers is not easily detected in a model relating returns to factors. When we consider only the projection of returns on factors–as in the traditional approach to factor asset pricing models, the unit root in the residuals from the projection of prices on risk drivers is removed.

Note that our framework cannot be introduced by considering a standard factor model and by showing that residuals are correlated. In fact, the joint analysis of prices, returns, factors and risk drivers, shows that a traditional factor model that is not supported by cointegration between risk drivers and (log) prices will show no-autocorrelation in its residuals.

To illustrate this phenomenon consider, as an example, a CAPM specification for the returns of the first Fama-French Portfolio–Small and Growth–along with the associated relationship between log prices and risk drivers:

$$\left(r_{t+1}^{11} - r_{t+1}^{rf}\right) = \alpha_1 + \beta_1 \left(r_{t+1}^{m} - r_{t+1}^{rf}\right) + u_{2,t+1}^{11} \tag{16}$$
$$\ln P_{t+1}^{11} = \alpha_0 + \alpha_1 t + \beta_1 F_{t+1}^{m} + \left(1 - \beta_1\right) F_{t+1}^{rf} + u_{1,t+1}^{11}$$

The ADF statistics in Table 2 is of particular interest. Table 2 illustrates for the single-factor model that the null hypothesis of a unit root in the residuals from the model for buy-and-hold portfolio (no-cointegration) cannot be rejected though the unit root in the residuals is removed when the standard CAPM is obtained by differencing. As a consequence, misspecification is apparent in the model in levels but much less evident when the standard CAPM specification for asset returns is considered. Figure 3, which shows the time-series of actual and fitted values for the two specifications, highlights this point.

INSERT TABLE 2 and FIGURE 3

3.3 On the Empirical Relevance of the Equilibrium Correction Term

In this section we motivate and illustrate the relevance of the Equilibrium Correction Term for risk management and portfolio allocation. We show that explicitly modeling the relationship between risk drivers and asset prices has important consequences for the predictive distributions of returns, which is relevant both for risk measurement and asset allocation. This happens for two reasons. Firstly, the ECT is a predictive variable that is observed at time t and is related to the distribution of returns at time $t+1$. Secondly, the equilibrium relationship(s) among risk drivers have predictive power for factors at time $t+1$ in a VECM specification.

3.3.1 ECT and Risk Measurement

To show the relevance of the ECT specification for risk measurement, we consider the properties of a traditional six-factor and of the FECM specifications for the returns of the Fama-French Portfolio Small and Growth (Port 11). The traditional factor model takes the following specification in which returns are projected on factors and the unconditional distribution of factors is used for simulations:

$$\begin{aligned} r_{t+1}^{11} &= \alpha_1^{11} + \beta_{11}' \mathbf{f}_{t+1}^{11} + v_{t+1}^{11} \\ \mathbf{f}_{t+1} &= \mu_0 + \epsilon_{t+1} \\ \epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0},\ \mathbf{\Sigma}\right) \\ \mathbf{f}_t' &= \left[f_t^{EXCMKT}\ f_t^{HML}\ f_t^{SMB}\ f_t^{RMW}\ f_t^{CMA}\ f_t^{MOM} \right] \end{aligned} \tag{17}$$

Instead, the FECM approach includes both the predictive Equilibrium Correction Term in the equation for returns and the levels of the risk drivers in the VECM specification for factors:

$$
\begin{aligned}
r_{t+1}^{11} &= \alpha_1^{11} + \beta_{11}' \mathbf{f}_{t+1}^{11} + \delta_{11} \hat{u}_t^{11} + v_{t+1}^{11} \\
\mathbf{f}_{t+1} &= \mu_0 + \mathbf{\Pi} \mathbf{F}_t + \epsilon_{t+1} \\
lnP_t^{11} &= lnP_{t-1}^{11} + r_t^{11} \\
\mathbf{F}_t &= \mathbf{F}_{t-1} + \mathbf{f}_t \\
\ln P_t^{11} &= \alpha_0^{11} + \alpha_1^{11} t + \beta_{11}' \ \mathbf{F}_t + u_t^{11} \\
\epsilon_{t+1} &\sim \mathcal{D}\left(\mathbf{0}, \ \mathbf{\Sigma}\right) \\
\mathbf{f}_t' &= \left[f_t^{EXCMKT} \ f_t^{HML} \ f_t^{SMB} \ f_t^{RMW} \ f_t^{CMA} \ f_t^{MOM} \right]
\end{aligned}
\tag{18}
$$

The results of the estimation of the FECM specification and of the VECM for factors are reported in Tables 3 and 4.

INSERT TABLES 3 and 4

Table 3 shows the estimation results for the specifications of returns for both the long-run cointegrating equation and the short-run factor error correction model. Interestingly, the specification of a full model for factors allows to estimate the FECM by Generalized Method of Moments (GMM), using as instruments for the returns the lagged levels of the risk drivers. The GMM estimation strongly confirms the significance of the ECT. Table 4 illustrates the relevance of risk drivers for predicting one-year ahead factors.

Further supporting evidence for the importance of the ECT for understanding the time-series dynamics of returns is provided by the analysis of the semi-partial R^2. We show the results for the semi-partial R^2 associated to each factor in the last column of Table 3. Importantly, the ECT is the fourth most relevant factor in explaining the total variance of returns.

Finally, we use the standard and the FECM specifications to predict the distributions for one-year ahead returns for 2008 and 2009 for Portfolio Small and Growth. The two models are fitted on the sample up to 2007, then the out-of-sample predicted return distributions are obtained by bootstrapping the correlated residuals ϵ_{t+1} and the idiosyncratic error v_{t+1}^{11}.[9]

The predictive distributions resulting from the traditional factor model are reported in Figure 8 which highlights the immutability of the distributions that do not change despite the crash of 2008. Differently,

[9] A remarkable stability of the parameters emerges for the FECM specification estimated on the full sample and estimated only up to 2007.

the predictive distributions from the FECM specification, reported in Figure 9, shows an evident shift after the crisis. This is due to the fact that the crash of 2008 brought the prices from above the long-run equilibrium to below the long-run equilibrium, causing a shift in the mean of the predictive distribution. Importantly, the one-year ahead 10% VaR from the FECM goes from -0.367 for 2008 to -0.067 for 2009, as a consequence of the crash, while the VaR from the standard model–reflecting the unconditional distribution of assets returns and factors–remains mostly unchanged.

INSERT FIGURES 8 and 9

3.3.2 ECT and Asset Allocation

Investors want to maximize risk-adjusted expected returns. In our framework, expected returns are time-varying and predictable, thus we can use implied conditional information to implement a tradable investment strategy. Let's consider equation (15). We can rewrite the equation as

$$r^i_{t+1} = \alpha^i_{t+1} + \beta'_i \mathbf{f}_{t+1} + v^i_{t+1} \tag{19}$$

where $\alpha^i_{t+1} = \alpha^i_1 + \delta_i u^i_t$ is a time-varying intercept. The fact that the intercept in our framework is time-varying allows to construct a long-short portfolio based on the alpha. In particular, we exploit the information conveyed by the ECT at time t to implement a zero-cost strategy at time $t+1$.

The ECT measures the temporary deviation of a specific asset price from the long-run equilibrium relation between the price and related priced risks. Our approach allows for a strategy based on the conditional alpha that consistently takes advantage of short-run idiosyncratic mispricing.

We illustrate how the strategy works for the 25 Fama-French portfolios. For every year t, we sort the 25 portfolios based on $\hat{\alpha}^i_1 + \hat{\delta}_i u^i_{t-1}$, we go long on the 5 portfolios with the highest $\hat{\alpha}_t$ and short the 5 portfolios with the lowest $\hat{\alpha}_t$. Th long-short portfolio is held for one year. We label this strategy as High-Minus-Low alpha (HMLα).

Figure 10 shows the performances of the HMLα strategy in the sample period from 1965 to 2018. The cumulative returns for the strategy over the whole period are in Panel (b). The strategy has an average excess return of 6.21% per year, with a robust t-stat of 5.05. Annual average percentage returns for portfolios sorted by time-varying alpha with respective robust standard errors are in Table 5.

Interestingly, average returns for each portfolio sorted by conditional alpha are economically and statistically significant. Most importantly, from Port 1 (portfolio associated with the lowest alpha) to Port 5 (portfolio associated with the highest alpha) returns are monotonically increasing. This evidence strongly supports the ability of our approach to consistently exploit temporary mispricing for asset allocation.

INSERT TABLE 5 and FIGURE 10

4 Robustness

In this section we address three main points. Firstly, we test for the significance of the new Equilibrium Correction Term to different specifications. Then, we provide a better understanding of the HMLα performance in terms of other factors. Finally, we implement a Principal Component Analysis on time-varying alphas.

4.1 Significance of ECT

According to standard asset pricing theory, two assets exposed to the same sort of systematic risk should have the same expected return. When it is not the case, the difference between the two expected returns is called "anomaly". The proliferation of anomalies is evident: Harvey et al. (2016) document more than 200 stocks cross-sectional anomalies. Furthermore, Asness et al. (2013) report that value and momentum anomalies have pervasive features across different asset classes.

In a recent paper, Stambaugh and Yuan (2016) propose two new measures of mispricing constructed by averaging on stocks' anomaly rankings. The main intuition motivating their approach is that anomalies partially reflect common mispricing components across stocks. The two mispricing factors considered are management (MGMT) and performance (PERF), resulting from clustering anomalies potentially related to firms' management and performance.[10]

We want to test whether our ECT is related to commonalities in stocks anomalies. We add the two mispricing factors MGMT and PERF to regression (15), and look at the significance of the ECTs across the 25

[10] Monthly time series for the two factors are available on Stambaugh's website.

Fama-French portfolios. Figure 11 shows the results of our test.[11] Even when we add MGMT and PERF to the regression, each ECT coefficient is statistically significant at 5% level of significance, providing supportive evidence that the Equilibrium Correction Term does not contain systematic mispricing elements.

Most interestingly, results are not surprising. In fact, the ECT is asset specific, as it measures the deviation of any asset from his long-run equilibrium relation with factor-risk drivers. This finding is also consistent with Daniel and Titman (1997) who points out that when expected returns reflect both compensation for systematic risks and mispricing, some of the mispricing is asset specific. The ECT conveys such information.

INSERT FIGURE 11

Ang et al. (2006) show that aggregate volatility is a priced factor in the cross-section of stock returns. In particular, aggregate volatility represents a systematic factor because leads to changes in the investment opportunity set of the marginal investor. Therefore, stocks that are more exposed to innovations in volatility earns a premium for risk. We include an idiosyncratic volatility (IVOL) factor in the short-run regression (15) to address the issue whether the ECT loses his significance when an asset-specific factor is included.

We compute IVOL for the 25 Fama-French portfolios as the standard deviation of the residual ϵ^i_{t+1} in the regression

$$r^i_{t+1} = \alpha^i + \beta_i' \mathbf{f}_{t+1} + \epsilon^i_{t+1} \tag{20}$$

where $\mathbf{f}$ are the FF6 factors. Then, we sort the 25 portfolios at time t based on IVOL at time $t-1$. Excess returns for the zero-investment strategy long on the portfolio associated with the lowest IVOL and short on the portfolio associated with the highest IVOL is the IVOL factor.

Figure 12 illustrates the estimated coefficients for IVOL factor and ECT obtained by running regression (15) when we add also the IVOL factor. Notice that the ECT does not lose any significance, i.e., the term remains always statistically significant at 5% level of significance. Systematic risk associated with idiosyncratic exposure to innovations in aggregate volatility cannot explain the Equilibrium Correction Term.

[11] When we consider a factor model with only market and size combined with the two mispricing factors–as in the original Stambaugh and Yuan (2016) paper–plus the ECT, results are the same.

INSERT FIGURE 12

4.2 Factor Spanning Tests

We compare the performance of HMLα to the standard FF6 factors, the mispricing factors, and the IVOL factor. For this purpose we perform a factor spanning test. We use excess returns from the HMLα strategy as dependent variable in equation (20) and look at the loadings associated with the other factors.

Table 6 illustrates the results from factor spanning tests. In Column (1) the considered regressors are only the traditional FF6 factors. Among the factors, HMLα is mostly related to SMB, with a negative sign. The alpha is economically and statistically strongly significant. We get similar results when we add the mispricing and the IVOL factors to the regression. Results in Column (2) show that the unique significant factor exposure is the one associated with SMB. Most importantly, also using the full sample of regressors, the constant is the variable most economically and statistically significant.

Finally, we ask what is the marginal contribution to the overall R^2 due to each regressor in Column (2). For this purpose, we perform a semi-partial R^2 analysis. It is noteworthy that only for the alpha the semi-partial R^2 is different from zero.

The analysis of the relation between the HMLα performances and other factors suggests that the returns associated with the strategy are poorly explained by existing factors. In fact, the performances of the time-varying alpha investment strategy are to great extent related to short-term mispricing.

INSERT TABLE 6

4.3 Principal Component Analysis

We analyze the principal components across the asset-specific time-varying alphas. The main goal is to better understand how many relevant orthogonal variables contribute to the total variation of the set of observed conditional alphas.

The analysis is interesting because the ECT is asset-specific thus the information embedded in the time-varying alphas are mostly specific. It means that we expect a large number of relevant components resulting from the Principal Component Analysis (PCA).

We perform the PCA on the sample composed by the estimated time-varying alphas for each portfolio in the 25 Fama-French portfolios for the period 1964–2018. We find that at least 11 principal components are needed to explain about 95% of total variation of the set of conditional alphas. This result strongly supports the view that ECT conveys information specific to each asset.

5 Conclusions

This paper has proposed a novel integrated approach to modeling asset prices, returns, factors and risk drivers. We have shown that focusing on the long-run (cointegrating) relation between asset prices and risk drivers naturally leads to the identification of a new "Equilibrium Correction Term". The ECT conveys important information about long-run disequilibria that strongly affect short-run dynamics of returns. Furthermore, the existence of the long-run relationship between prices and associated drivers of risk generate some relevant predictability.

We have mainly focused on the importance of the Equilibrium Correction Term for determining the predictive distribution of returns, and its consequences for risk measurement and portfolio allocation. Our empirical analysis has been based on modeling returns and log prices of the 25 Fama-French portfolios and on capturing the additional effects of the "Equilibrium Correction Term" on the Fama-French five-factor model augmented with a momentum factor along several dimensions. Our framework, based on constructing an Equilibrium Correction Term from the relation between prices and risk drivers, can be extended to any set of factors.

We have given less emphasis to the importance of the long-run relations among risk drivers to predict factors. This is an issue on our agenda for further research that could be particularly interesting in models based on the simultaneous utilization of local and global factors to model asset returns (Griffin (2002)). Cointegration among local and global risk drivers has an obvious potential for explaining the dynamics of local factors as determined by the response to an Equilibrium Correction Term in which global risk drivers determine local risk drivers.

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Tables and Figures

Table 1: E-Test for Multivariate Normality (Székely and Rizzo (2005))

CAPM	Test Statistic	2.684
	p-value	0
FF6	Test Statistic	2.618
	p-value	0.08
FECM	Test Statistic	2.611
	p-value	0.22

Notes: This table reports the E-test for multivariate normality proposed by Székely and Rizzo (2005) on CAPM, FF6, and FECM residuals. The null hypothesis is multivariate normality.

Table 2: Augmented Dickey-Fuller Tests

Single-Risk Driver (EXC MKT) Model	Test Statistic	−2.783
	p-value	0.259
Single-Factor (CAPM) Model	Test Statistic	−3.873
	p-value	0.022

Notes: This table reports the augmented Dickey-Fuller (ADF) tests for the single-factor specifications in equation (16) for Portfolio Small and Growth in the 25 Fama-French portfolios. The null hypothesis is non-stationarity.

Table 3: FECM for Portfolio 11

	Long-Run	FECM (OLS)	FECM (GMM)	FECM SpR^2
Trend	-0.059^{***}			
	(0.005)			
EXC MKT	1.050^{***}	1.179^{***}	1.263^{***}	0.262
	(0.038)	(0.041)	(0.245)	
SMB	1.370^{***}	1.404^{***}	1.673^{***}	0.259
	(0.050)	(0.087)	(0.207)	
HML	-0.353^{***}	-0.634^{***}	-1.004^{***}	0.025
	(0.092)	(0.071)	(0.363)	
RMW	-0.544^{***}	-0.273^{***}	-0.038	0.005
	(0.060)	(0.080)	(0.315)	
CMA	-0.396^{***}	-0.049	0.582	0
	(0.120)	(0.092)	(0.721)	
Mom	0.094^{***}	0.070^{**}	0.197^{*}	0.003
	(0.024)	(0.012)	(0.113)	
ECT		-0.724^{***}	-0.959^{***}	0.018
		(0.140)	(0.286)	
Constant	0.008	-0.075^{***}	-0.106^{***}	
	(0.028)	(0.007)	(0.036)	
Observations	55	54	54	
Adjusted R^2	0.980	0.967	0.940	

Notes: This table reports the estimated coefficients for the FECM specifications (18) for Portfolio Small and Growth in the 25 Fama-French portfolios. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as proposed by Newey and West (1994). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

Table 4: VECM for Factors as Functions of Risk Drivers

	Factor					
	EXC MKT	SMB	HML	RMW	CMA	MOM
EXC MKT RD(-1)	-0.283^{***}	-0.222^{**}	-0.077	0.211^{***}	-0.097	0.249^{**}
	(0.084)	(0.098)	(0.102)	(0.067)	(0.067)	(0.095)
SMB RD(-1)	0.011	-0.250^{***}	0.116	0.040	0.015	-0.023
	(0.093)	(0.089)	(0.093)	(0.061)	(0.061)	(0.086)
HML RD(-1)	0.196	-0.202	-0.542^{**}	-0.222	-0.032	0.444^{**}
	(0.209)	(0.197)	(0.204)	(0.135)	(0.134)	(0.190)
RMW RD(-1)	0.435^{***}	-0.183	-0.033	-0.584^{***}	0.037	-0.193
	(0.147)	(0.197)	(0.204)	(0.135)	(0.134)	(0.189)
CMA RD(-1)	-0.062	0.242	0.208	0.540^{***}	-0.418^{**}	-0.302
	(0.334)	(0.271)	(0.281)	(0.186)	(0.184)	(0.261)
Mom RD(-1)	-0.063	0.205^{**}	0.219^{**}	-0.029	0.241^{***}	-0.160^{*}
	(0.078)	(0.086)	(0.089)	(0.059)	(0.058)	(0.083)
Constant	0.015	0.115	-0.019	-0.025	-0.072	0.095
	(0.068)	(0.071)	(0.073)	(0.048)	(0.048)	(0.068)
Observations	42	42	42	42	42	42
Adjusted R^2	0.137	0.242	0.191	0.322	0.282	0.172

Notes: This table reports the estimated coefficients for the VECM specification in equation (18) for factors as functions of the risk drivers for Portfolio Small and Growth in the 25 Fama-French portfolios. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as proposed by Newey and West (1994). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

Table 5: Portfolios Sorted by Time-Varying Alpha

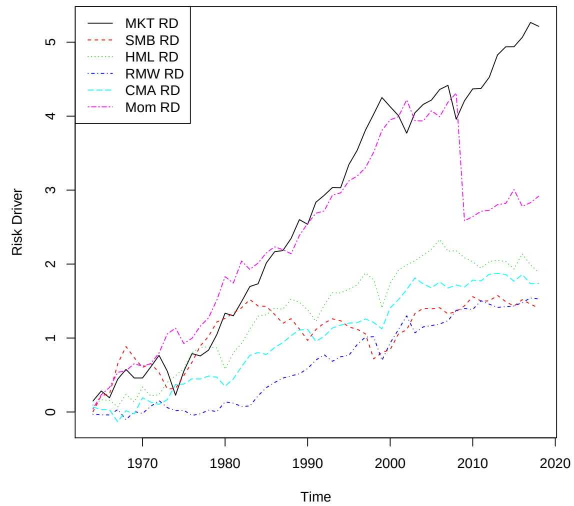
	Port 1	Port 2	Port 3	Port 4	Port 5	Port 1-5
Return (%)	9.176***	7.495***	6.758***	6.007***	2.964*	6.212***
	(1.408)	(1.411)	(1.493)	(1.360)	(1.553)	(1.230)

Notes: This table reports percentage average annual returns for the 25 Fama-French portfolios sorted by time-varying alpha. Port 1 is the portfolio associated with the highest alpha, Port 5 is the portfolio associated with the lowest alpha. Port 1-5 represents the HMLα portfolio. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as proposed by Newey and West (1994). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1964 to 2018.

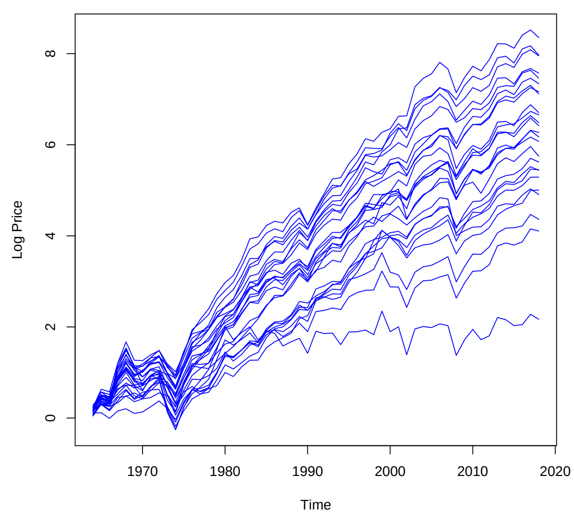
Table 6: Factor Spanning Tests

	HMLα	
	(1)	(2)
EXC MKT	-0.175^{*}	-0.148
	(0.104)	(0.094)
SMB	-0.287^{***}	-0.229^{**}
	(0.086)	(0.095)
HML	0.318^{*}	0.240
	(0.186)	(0.187)
RMW	0.196	0.199
	(0.240)	(0.131)
CMA	0.054	-0.056
	(0.258)	(0.248)
Mom	0.015	0.006
	(0.029)	(0.041)
MGMT		0.293
		(0.191)
PERF		0.021
		(0.146)
IVOL		0.113
		(0.147)
Alpha	6.119^{***}	4.690^{***}
	(2.023)	(1.711)
Observations	52	52
Adjusted R^2	0.549	0.554

Notes: This table reports the estimated coefficients for the factor spanning tests of HMLα on FF6, mispricing, and IVOL factors. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as proposed by Newey and West (1994). ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. The sample period is 1965 to 2016.



(a) This figure shows the yearly dynamics for the six risk drivers associated with FF6 factors.



(b) This figure shows the yearly dynamics for the log prices for the 25 Fama-French portfolios formed on Size and Book-to-Market.

Figure 1: Risk Drivers and Log Prices Dynamics

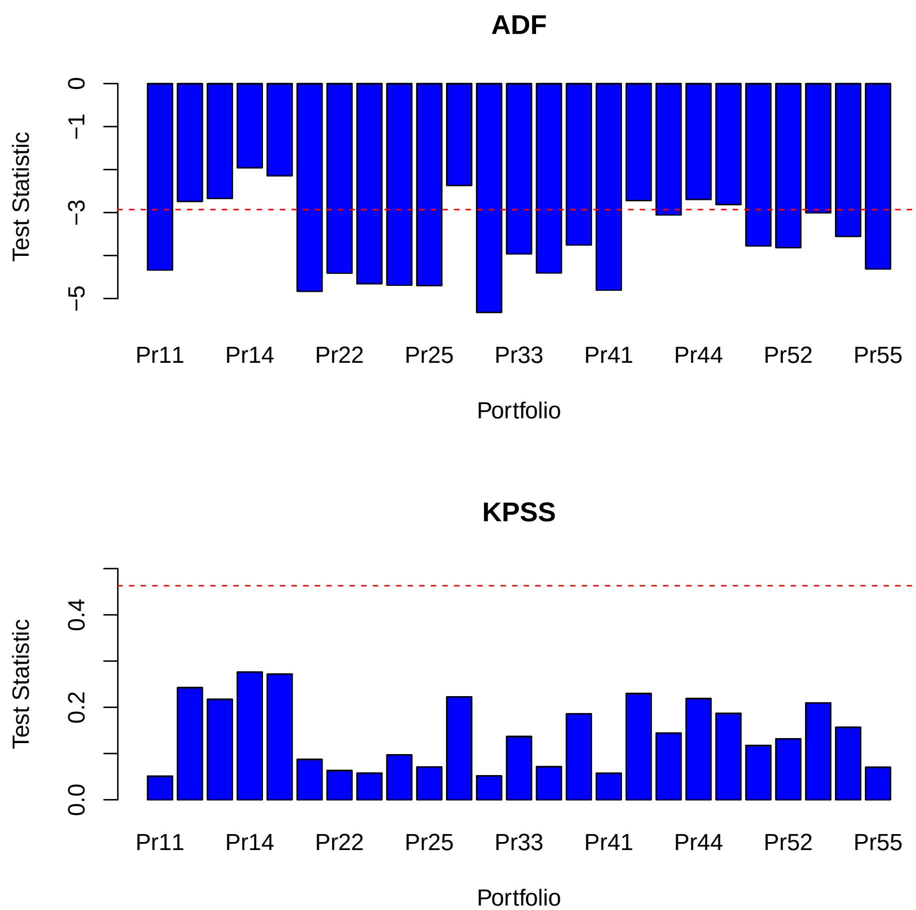


Figure 2: ADF and KPSS Tests for the Six-Risk Driver Model Residuals

This figure shows the test statistics for ADF and KPSS tests on the six-risk driver associated with FF6 model residuals. The (dashed) red line is the critical value at 5% level of significance. The null hypothesis for the ADF test is non-stationarity, while for the KPSS test is (level) stationarity.

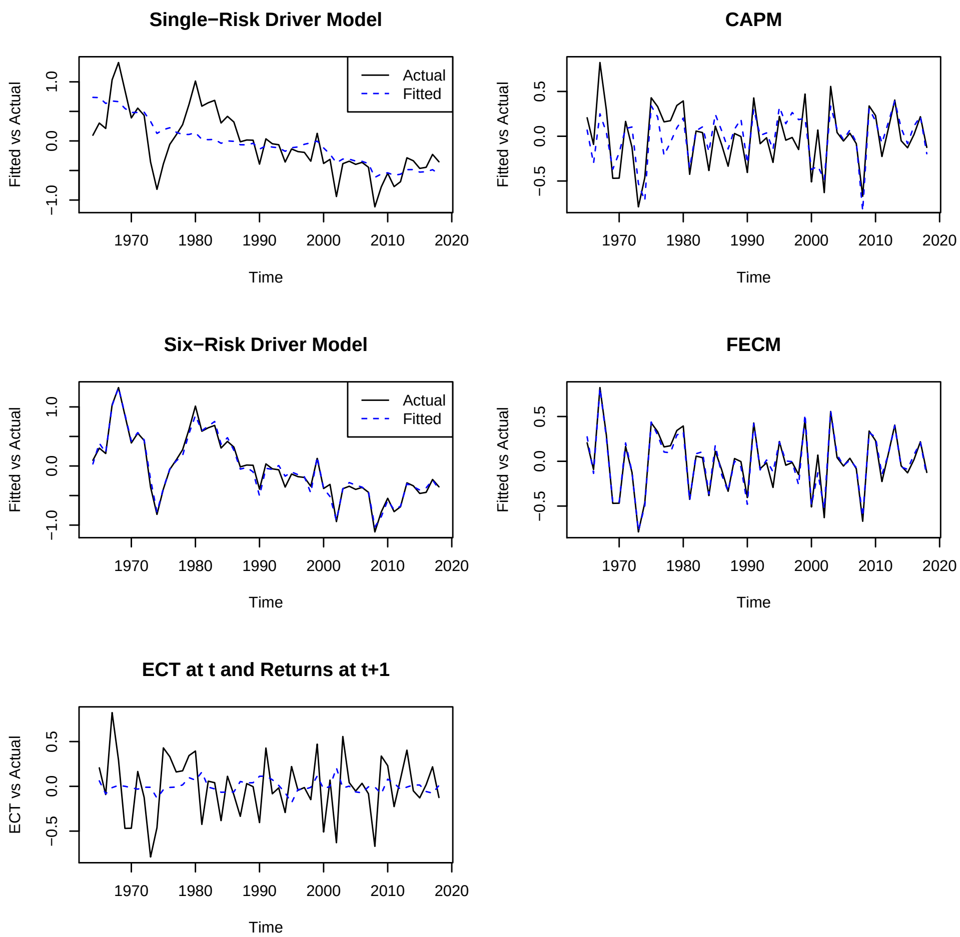


Figure 3: Single- and Six-Factor Specifications for Portfolio Small and Growth

This figure shows single-risk driver (EXC MKT) model, six-risk driver (FF6) model, CAPM and FECM for Portfolio Small and Growth in the 25 Fama-French portfolios. The black lines are actual values, the dashed blue lines are fitted values. The last figure illustrates the dynamics of the ECT compared to the portfolio Small and Growth observed returns.

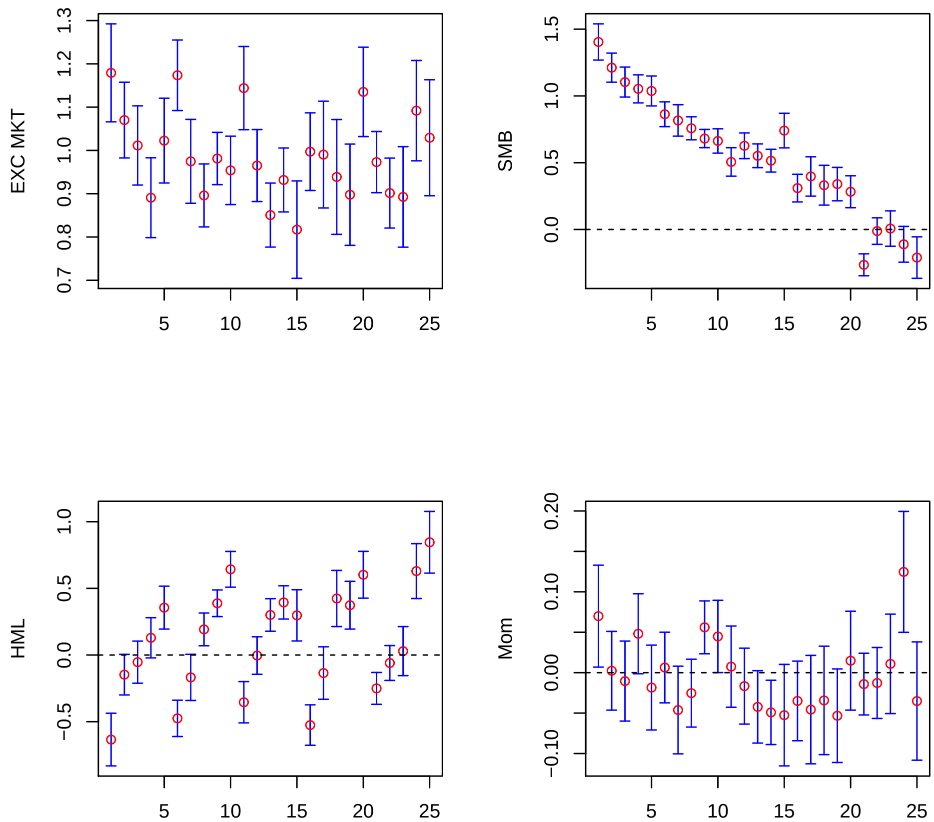


Figure 4: Factors Significance for the 25 Fama-French Portfolios (1)

This figure shows the estimated coefficients for EXC MKT, SMB, HML and Mom obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (15) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.

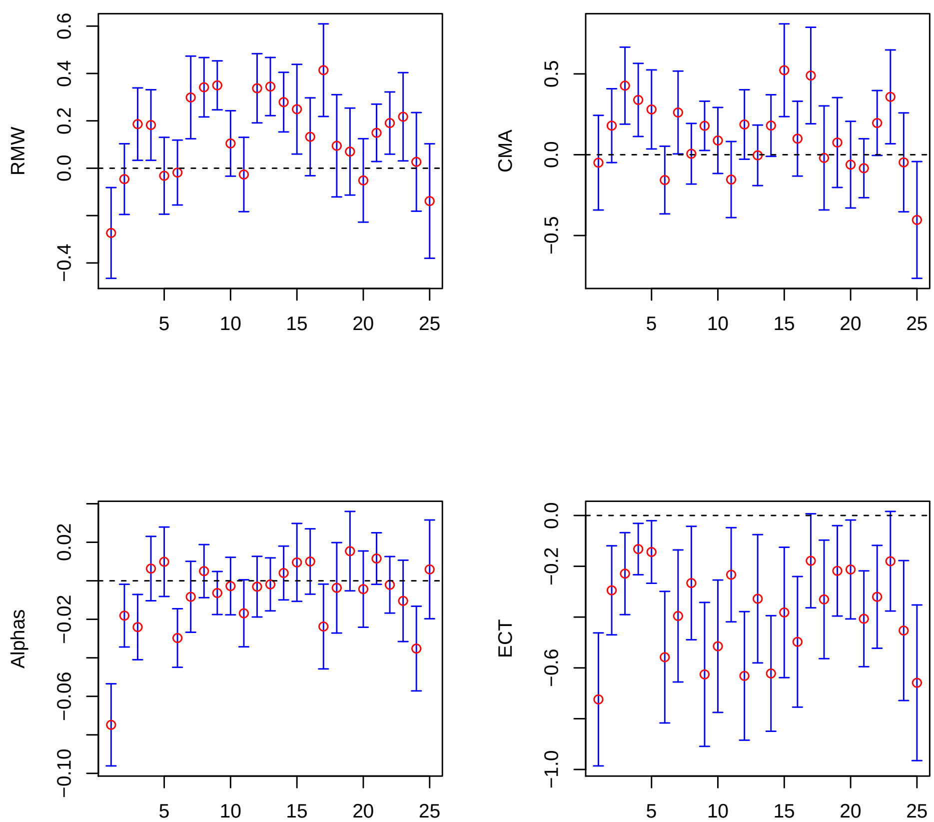


Figure 5: Factors Significance for the 25 Fama-French Portfolios (2)

This figure shows the estimated coefficients for RMW, CMA, alphas and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (15) with respective confidence intervals at 5% level of significance. The sample period is 1964 to 2018.

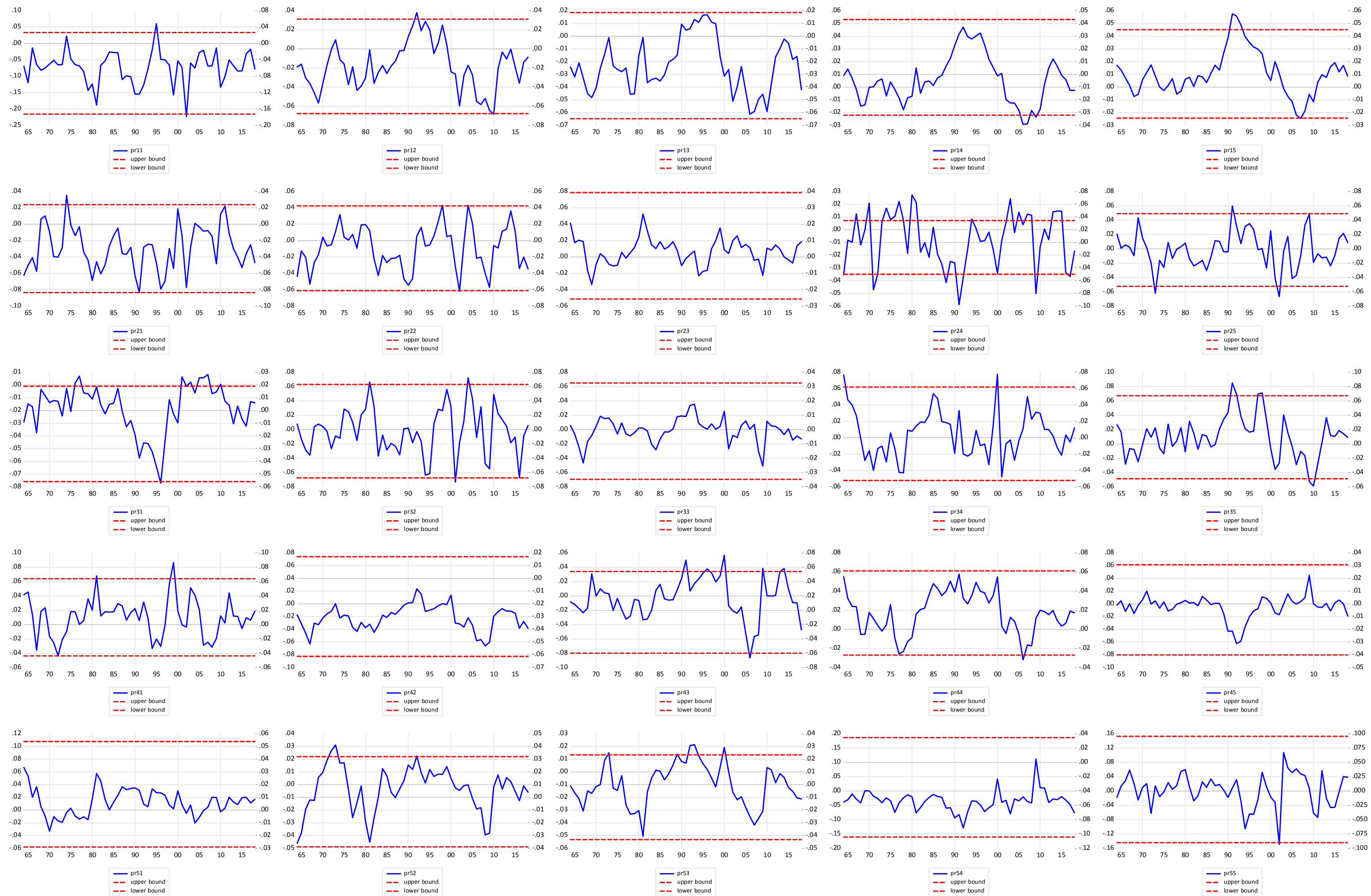
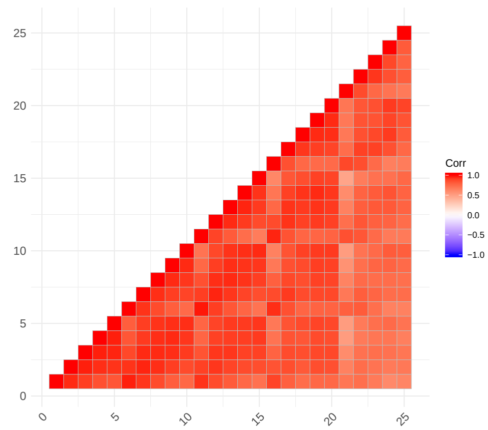
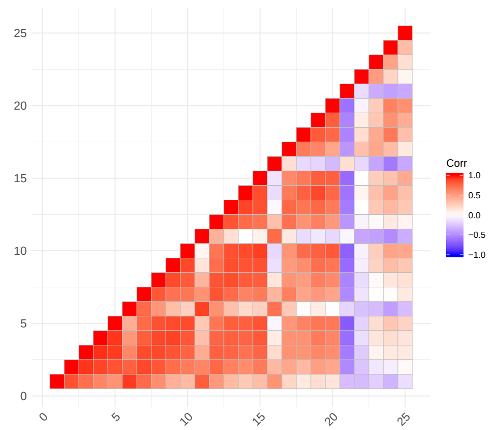


Figure 6: Intercept and ECT

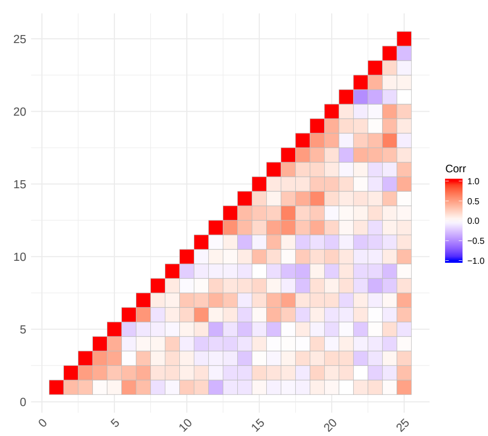
This figure shows the estimated values for the time-varying intercept from regression (19) for the 25 Fama-French portfolios with respective confidence intervals at 5% level of significance.



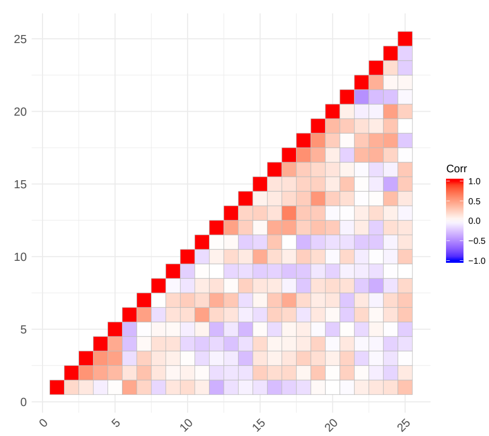
(a)



(b)



(c)



(d)

Figure 7: Correlation Matrix Plots

This figure shows the heatmap for the correlation matrix of the 25 Fama-French portfolios returns (a) and the residual of the 25 portfolios returns for the CAPM (b), FF6 (c), and FECM (d). The sample period is 1964 to 2018.

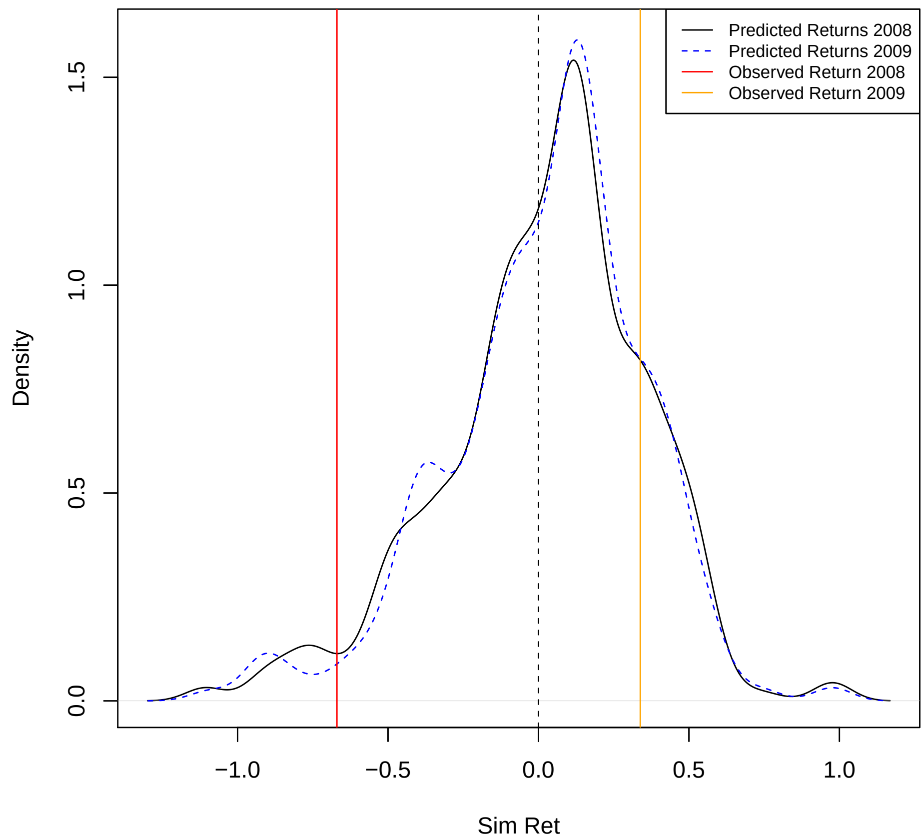


Figure 8: Predicted Returns: the Traditional Approach

This figure shows the predicted and observed returns for portfolio Small and Growth in the 25 Fama-French portfolios during the crash 2007–2009 using the traditional factor model. We estimate the model in the sample period 1964 to 2007 and we predict the distribution of returns by bootstrapping. Bootstrapped 10% VaR takes the value of -0.440 for 2008 and of -0.404 for 2009. The unconditional 10% VaR is -0.469.

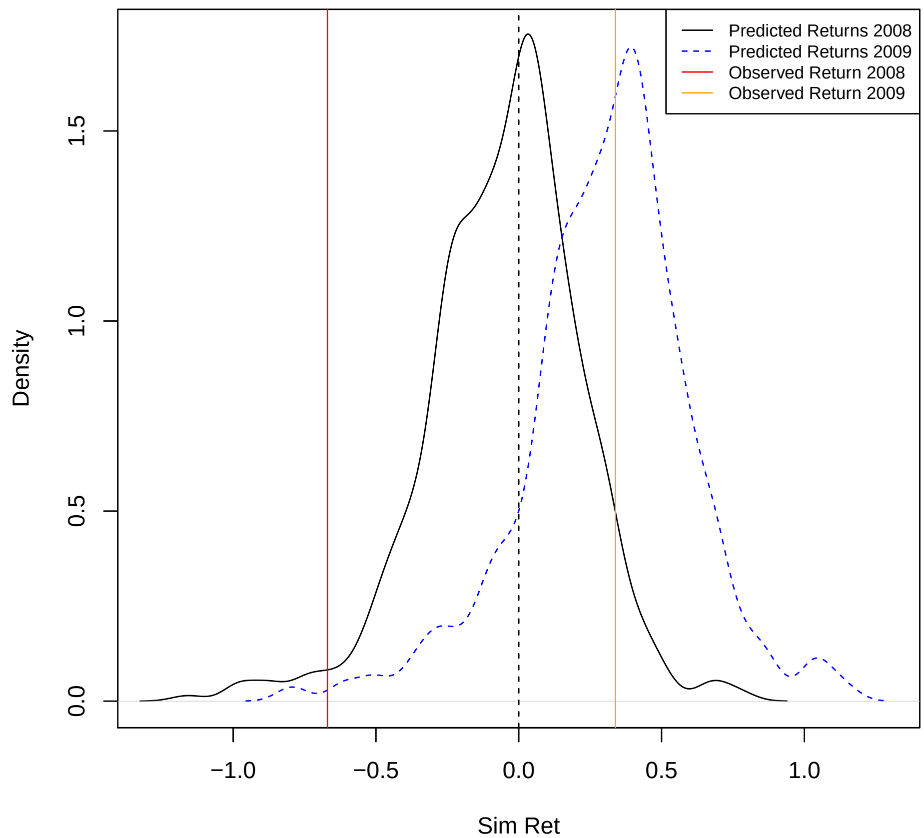
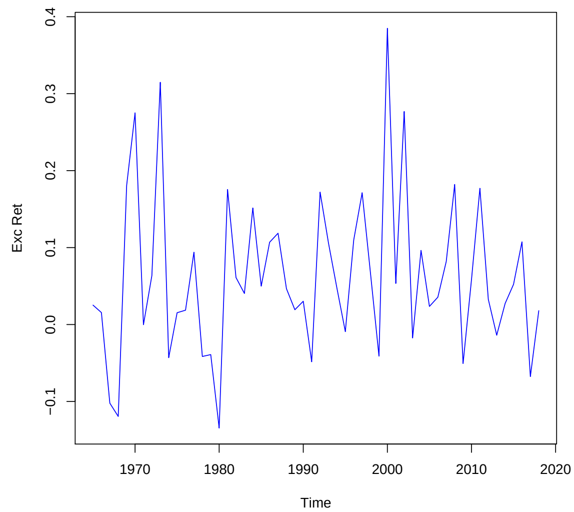
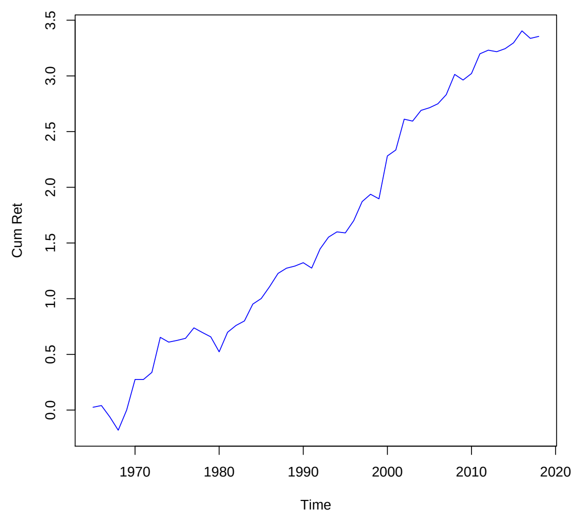


Figure 9: Predicted Returns: the EC Approach

This figure shows the predicted and observed returns for portfolio Small and Growth in the 25 Fama-French portfolios during the crash 2007–2009 using the EC model. We estimate the model in the sample period 1964 to 2007 and we predict the distribution of returns by bootstrapping. Bootstrapped 10% VaR takes the value of -0.367 for 2008 and of -0.067 for 2009.



(a) This figure shows the yearly excess returns for the HMLα strategy implemented on the 25 Fama-French portfolios.



(b) This figure shows the cumulative returns for the HMLα strategy implemented on the 25 Fama-French portfolios.

Figure 10: Returns and Cumulative Returns for the HMLα Strategy

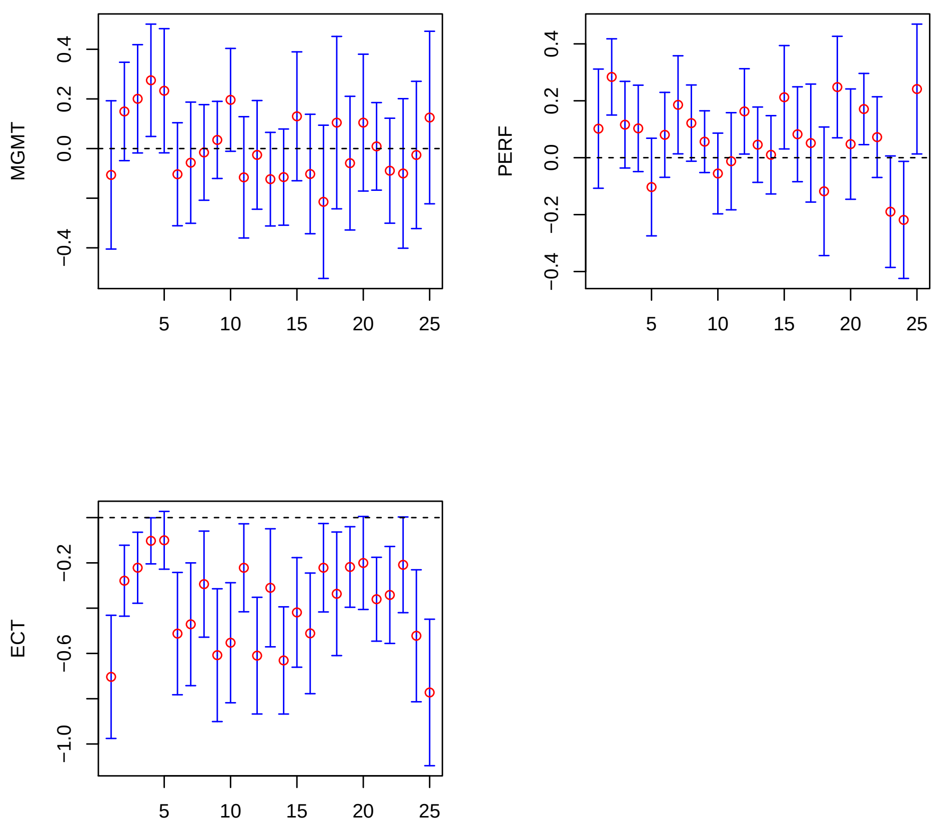


Figure 11: Mispricing Factors and ECT

This figure shows the estimated coefficients for MGMT, PERF, and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (15) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2016.

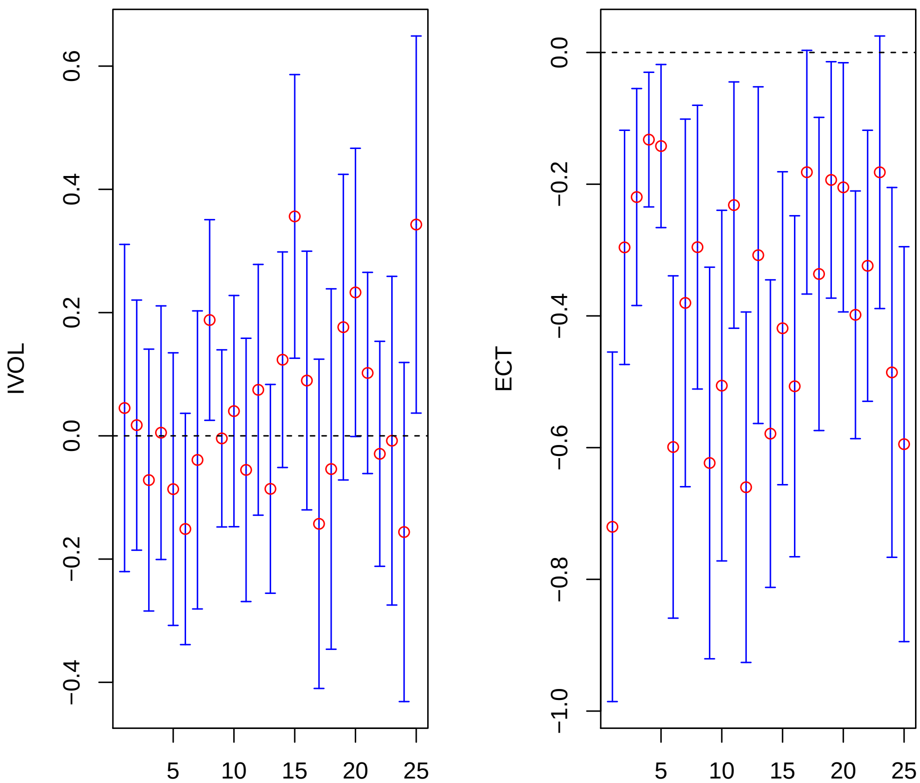


Figure 12: IVOL Factor and ECT

This figure shows the estimated coefficients for IVOL and ECT obtained by regressing the 25 Fama-French portfolios excess returns on factors as in equation (15) with respective confidence intervals at 5% level of significance. The sample period is 1965 to 2016.