Asset Bubbles and Product Market Competition

Francisco Queirós*

Università di Napoli Federico II and CSEF

September 12, 2022

Abstract

This paper studies the interactions between asset bubbles and product market competition. It offers two main insights. The first is that imperfect competition creates a wedge between interest rates and the marginal product of capital. This makes rational bubbles possible even when there is no overaccumulation of capital. The second is that, when providing a production subsidy, bubbles stimulate competition and reduce monopoly rents. I show that bubbles can destroy efficient investment and have ambiguous welfare consequences. However, when stimulating competition, they can result in higher investment and output.

JEL Classification: E44, L13, L16

Key words: rational bubbles, competition, market power, dotcom bubble

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*Address: CSEF – Università di Napoli Federico II, Via Cintia – Monte Sant'Angelo, 80126 Napoli, Italia. e-mail: queiros.francisco@gmail.com. I am greatly indebted to my advisors, Fernando Broner and Jaume Ventura, for their guidance and motivation. I also thank Árpád Ábrahám, Vasco Carvalho, Russell Cooper, Matt Delventhal, Alessandro Ferrari, Luca Fornaro, Ramon Marimon, Alberto Martin, Xavier Mateos-Planas, Alessandro Spiganti, Haozhou Tang, Tomás Williams and seminar participants at CREI, EUI, CSEF, the Bank of Spain, the 2018 Transatlantic Doctoral Conference (LBS), the 1st QMUL Economics and Finance Workshop for PhD Students (Queen Mary University), the SED 2018 Annual Meeting (Mexico City), the EEA 2018 Congress (Cologne), and the SMYE 2019 (Brussels) for helpful discussions and suggestions. I acknowledge financial support from the Portuguese Science Foundation (grant SFRH/BD/87426/2012). All errors are my own.

1 Introduction

"With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the cost of low or even negative gross margins."

('Dotcom history is not yet repeating itself, but it is starting to rhyme', *Financial Times, 12/March/2015)*

Stock markets often experience fluctuations that seem too large to be driven entirely by fundamentals. Major historical events include the Mississippi and the South Sea bubbles of 1720 or the British railway mania of the 1840s. A more recent example is that of the US stock market during the dotcom bubble: between October 1995 and March 2000, the NASDAQ Composite index increased by almost sixfold to then collapse by 77% in the following two years. One common aspect among these episodes is that they appear to be concentrated on a particular industry, and to bring about a surge in competition.[1] The dotcom bubble constitutes a good example in this regard. In a period characterized by soaring prices of technology stocks, many internet firms had an IPO and entered the stock market. Furthermore, as the valuation of firms is typically based on metrics of size (revenues or market shares) and not on earnings, some of these firms sought rapid growth and engaged in aggressive commercial practices, such as unusually low penetration prices. For example, some online companies offered their services for free (e.g. *Kozmo.com* or *UrbanFetch*) or made money payments to consumers (e.g. *AllAdvantage.com*).[2]

The idea that the dotcom bubble was associated with a more competitive market structure is corroborated by indicators of market power. Figure 1 shows average price-cost markups for four high-tech industries that were at the center of the bubble. These are shown against the Shiller CAPE ratio, which is a popular measure for stock market overvaluation. A common pattern can be detected in these four industries — average markups decline from 1995 until the peak of the bubble in 2000/2001, and start increasing after the stock market crash. Note that at the peak of the bubble, the average firm in the four industries charged a price below its variable cost,

[1] The Mississippi and the South Sea bubbles involved two trading companies (the *Compagnie d'Occident* in France and the *South Sea Company* in Great Britain) that engaged in innovative financial schemes; the *railway mania* involved the British railway industry; the dotcom bubble was concentrated on a group of internet and high-tech industries.

[2] Even if following unsustainable business models, the new *dotcoms* often posed a threat to incumbents, which were in many cases forced to react. Some well-known examples involve GE or Microsoft (Queirós (2021)).

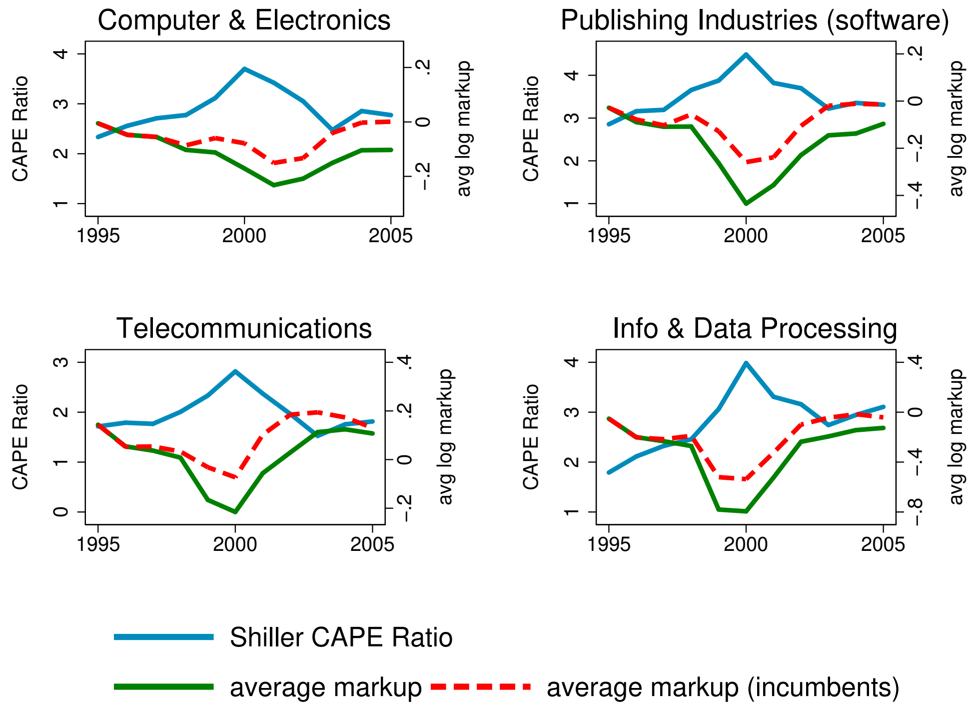


Figure 1: Average markups in the dotcom bubble

This figure shows the Shiller CAPE ratio and average (unweighted) markups for four industries during 1995-2005: 'Computer & Electronic Product Manufacturing' (NAICS 334), 'Publishing Industries (software)' (NAICS 511), 'Telecommunications' (NAICS 517), and 'Information & Data Processing' (NAICS 518-519). The CAPE ratio is the ratio of total stock market capitalization to a 10–year moving average of past earnings (EBITDA); the ratio is in logs and is measured at the beginning of the year. Markups are the ratio of sales to variable operating costs ('Cost of the Goods Sold' + 'General, Selling and Administrative Expenses'); the ratio is in logs.

implying that it exhibited negative earnings. These patterns could be observed for both the full sample of firms (green line) and for the set of firms already active in 1995 (dashed red line).

Motivated by these observations, I investigate the interactions between asset bubbles and product market competition. I present a model featuring imperfect competition and rational bubbles, which builds upon the classical OLG economy of Diamond (1965). Individuals live for two periods. They work when young and have to decide how much to consume and save for retirement. They can save by investing in capital, or by purchasing bubbles. The production side of the model consists of a multi-industry economy. In every industry, there is a productive firm that faces competition from a fringe of unproductive competitors. This firm can charge a (limit) price above its marginal cost, hence enjoying market power and making some monopoly rents.

Individuals can issue and trade two types of bubbly assets. One is an asset that is issued outside the corporate sector, which I label *government debt*. This asset will sustain a set of an intergenerational transfers, and will not have an impact on the industry market structure. The second is an asset that is issued by firms, which I label *bubbly stocks*. Importantly, the rents that firms can obtain when issuing bubbly stocks depend on their size, so that larger firms are also able to issue a larger amount of bubbles.[3] These assets will thus provide firms with the incentives to expand, thereby reducing prices, markups and monopoly rents. Insofar as they reduce monopoly rents in a given industry, bubbles will have a *pro-competitive effect* and correct a market failure. However, I show that if they are sufficiently large, these bubbles can generate situations of excessive production, with firms charging prices below their marginal cost. The model can thus explain the prevalence of low markups exhibited by high-tech firms in the dotcom bubble (as suggested by Figure 1), or examples of overinvestment in the British railway mania (Campbell and Turner (2015)).

Considering the general equilibrium properties of the model, the existence of a price-cost markup will create a wedge between factor prices and marginal products. In particular, interest rates will be below the (aggregate) marginal product of capital. This has two main consequences. First, rational bubbles will be possible even when there is no overaccumulation of capital and the equilibrium is Pareto efficient. Under certain conditions, bubbles may crowd-out efficient investment, and be detrimental for welfare. Thus, the classical equivalence result of Tirole (1985) — between the possibility of rational bubbles, overaccumulation of capital and Pareto inefficiency — is not necessarily satisfied in this economy. Second, the economy can be characterized by underinvestment. Since interest rates do not reflect the efficiency of investment, individuals may opt to save too little — and consume too much — when young. In such a case, the resulting equilibrium can be shown to be Pareto inefficient, with excessive first-period consumption and insufficient investment. However, if issued by the corporate sector and being *pro-competitive*, bubbles may increase the aggregate demand for investment; they may reduce first-period consumption and can lead to an efficient increase in the capital stock.

[3] This assumption is meant to capture one aspect of valuation techniques, namely the fact that they are often based on metrics of size (such as market shares) and not on profits. See section 2 for a discussion.

Related Literature This paper is mostly related to the literature that forms the theory of rational bubbles. Different models have explored different aspects of asset bubbles. The seminal contributions of Samuelson (1958) and Tirole (1985) explore the role of bubbles as a store of value, and show that they can be Pareto improving. Being a store of value, bubbles can also be a liquidity instrument as in Farhi and Tirole (2012), Miao and Wang (2012) and Xavier (2022). A different strand of the literature has put an emphasis on the appearance of new bubbles: the formation of a new pyramid scheme provides a rent or subsidy that can have economic consequences. In this category, Olivier (2000) shows that, if appearing attached to R&D firms, bubbles can stimulate growth. Martin and Ventura (2012, 2016) argue that the creation of new bubbles allows credit-constrained entrepreneurs to expand investment. Tang and Zhang (2022) study how bubbles affects the firm productivity distribution.[4] In this paper, I provide a theory of how asset bubbles can be expansionary. My theory is thus related to the class of models emphasizing how bubbles can alleviate credit market frictions and be associated with larger investment (Farhi and Tirole (2012), Martin and Ventura (2012, 2016), Hirano and Yanagawa (2017), Ikeda and Phan (2019)). Yet, there are important differences. First, my focus is on frictions in product markets, not in financial markets. Second, previous models fail to explain how overvaluation can generate overinvestment and negative earnings. These have been important aspects of the dotcom bubble and other episodes (Haacke (2004)).

Finally, this paper is related to the literature studying the aggregate consequences of market power (Rotemberg and Saloner (1986), Chatterjee et al. (1993), Jaimovich and Floetotto (2008), Ferrari and Queirós (2022)). One insight of my model is that markups create a wedge between interest rates and the marginal product of capital, allowing rational bubbles to emerge even when there is no overaccumulation of capital. A similar result has also been contemporaneously shown by Eggertsson et al. (2019) and by Ball and Mankiw (2021). In addition to this, a contribution of my paper is to show that bubbles can stimulate competition and reduce monopoly rents.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes the equilibrium of a single industry. Section 3 discusses the general equilibrium and the conditions for the existence of bubbles. Section 4 concludes.

[4] Recent contributions include the quantitative models of Larin (2019) and Guerron-Quintana et al. (2022). Galí (2014), Biswas et al. (2020) and Asriyan et al. (2021) study the interactions between bubbles and monetary policy.

2 The Model

2.1 Demographics and Preferences

Time is discrete and runs forever: $t = 0, 1, 2, \ldots$. The economy is populated by two overlapping generations, each of which has measure one. Each individual $j \in [0, 1]$ born at t has utility

$$u_{jt} = c_{jt}^{y} + \beta \, c_{j,t+1}^{o} \tag{1}$$

where c_{jt}^{y} and $c_{j,t+1}^{o}$ represent young-age and old-age consumption, and β is the discount factor. The choice of a linear utility function is made for analytical convenience. In Appendix B, I show that the central results of this paper hold under general CRRA preferences. Throughout, I will also assume that $\beta > 1$. This assumption implies that individuals have a preference for old-age consumption and that rational bubbles will be possible whenever $c_{jt}^{y} > 0$ (as shown below).

Assumption. $\beta > 1$

Individuals supply one unit of labor when young, and earn a wage W_t. They can save by purchasing financial assets (such as government debt) which deliver a gross return R_{t+1}. When old, they run a firm in the corporate sector and can receive lump sum transfers $\chi_{j,t+1}^{o}$ (e.g. the profits that they make as entrepreneurs). They thus face the budget constraint

$$c_{j,t+1}^{o} = R_{t+1} \left(W_t - c_{jt}^{y} \right) + \chi_{j,t+1}^{o} \tag{2}$$

The problem of each young individual i is to maximize (1) subject to (2). Denoting her savings level by $s_{jt} := W_t - c_{jt}^{y}$, the solution to this problem yields

$$s_{jt} \begin{cases} = W_t & \text{if } \; R_{t+1} > \dfrac{1}{\beta} \\[2ex] \in [0, \; W_t] & \text{if } \; R_{t+1} = \dfrac{1}{\beta} \end{cases} \tag{3}$$

If the interest rate is greater than the inverse of the discount factor, young individuals save all their income. When the two are identical, the young are indifferent between saving and consuming in their first period of life. As it will be clear below, the equilibrium interest rate cannot be lower than the inverse of the discount factor.

2.2 Technology

There is a final good Y_t, which is a CES composite of different varieties

$$Y_t = \left(\int_0^1 y_{it}^{\rho} \, di \right)^{1/\rho} \tag{4}$$

where y_{it} is the quantity of variety $i \in [0,1]$, $0 < \rho < 1$ and $\sigma := 1/\left(1-\rho\right) > 1$ is the elasticity of substitution. The final good is produced in a competitive sector and is chosen as the *numeraire*. The demand for each variety i is given by

$$p_{it} = \left(\frac{Y_t}{y_{it}} \right)^{1-\rho} \tag{5}$$

Entrepreneur $j \in [0,1]$ can produce variety $i \in [0,1]$ by means of a Cobb-Douglas technology $f_{ij}\left(k,l\right) = z_{ij}\, k^{\alpha}\, l^{1-\alpha}$. The term z_{ij} represents the productivity of individual j in the production of variety i. Labor is hired at the competitive wage W_t. Capital needs to be invested one period ahead and fully depreciates in production. Each unit of capital used at t therefore costs R_t. Given these assumptions, an entrepreneur with productivity z_{ij} can produce variety i with unit cost θ_t / z_{ij}, where

$$\theta_t := \left(\frac{R_t}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}$$

is the factor price frontier for a Cobb-Douglas technology with unit productivity.

Imperfect competition will arise because firms feature an unequal distribution of productivities. In particular, I assume that

$$z_{ij} = \begin{cases} 1 & \text{if} \quad j = i \qquad \text{(leader)} \\ \\ \gamma \leq 1 & \text{if} \quad j \neq i \qquad \text{(followers)} \end{cases} \tag{6}$$

Therefore, each variety $i \in [0,1]$ can be produced either with productivity $z_{ij} = 1$ by entrepreneur $j = i$ or with productivity $\gamma \leq 1$ by all the other entrepreneurs. I refer to entrepreneur j as the *leader* of industry $i = j$ and to all other entrepreneurs $j \neq i$ as the *followers*. The important aspect of (6) is that for every variety i there is only one individual with access to the best technology.

2.3 Bubbleless Industry Equilibrium

I assume that firms compete à la Bertrand. Assuming that $\gamma \geq \rho$, the leader must set a limit price equal to the followers' marginal cost[5]

$$p_{it} = \frac{\theta_t}{\gamma} \tag{7}$$

Assumption. $\gamma \geq \rho$

The parameter $\gamma \in [\rho, 1]$ is equal to the inverse of the markup, and can thus be taken as a measure of competition. In the particular case of $\gamma = 1$, the model features perfect competition in product markets. The price in (7) is the equilibrium price of good i when firms are not overvalued. As I show below, when firms have the possibility of issuing overvalued stocks, they may find it optimal to charge a price below (7). To conclude the discussion of the bubbleless industry equilibrium, I shall briefly characterize optimal policy interventions.

Optimal Policy A regulatory authority intervening in industry i would like to ensure that the leader produces a quantity consistent with marginal cost pricing, i.e. $p_{it} = \theta_t$. There are different ways of implementing this outcome. One possibility is to grant the followers with an *ad valorem* subsidy equal to $\phi^F := 1/\gamma - 1$, so that the followers effectively obtain $p_{it}/\gamma \geq p_{it}$ per each unit they sell. Given this subsidy, the followers will produce at any price $p_{it} > \theta_t$ and so the leaders will be forced to charge $p_{it} = \theta_t$.

A second possibility is to grant the leaders with an *ad valorem* subsidy equal to $\phi^L := 1/\rho - 1$. This reduces their optimal monopoly price to $p_{it} = \theta_t < \theta_t/\gamma$. Such a subsidy could be financed by a lump sum tax on the leaders equal to $\tau_t^L = \phi^L \, p_{it} \, y_{it}$.

2.4 Asset Bubbles

I next characterize the equilibrium with trade in bubbles. I will consider two types of rational bubbles. The first is an asset that is issued outside the corporate sector. As an illustration, I will consider a government debt scheme that is rolled over forever. The second is an asset that is issued by the corporate sector. The assumptions we make about how this asset is issued and distributed across firms can change the equilibrium in goods markets.

[5] When $\gamma < \rho$, the leaders can simply charge their desired monopoly price $p_{it}^M = \theta_t/\rho$. In this case, the monopoly price is below the followers' marginal cost θ_t/γ.

Government Debt Suppose that there is a government that can issue one-period debt, to be rolled over forever. Let D_t be the funds raised by the government in period t. I assume that

$$D_t = R_t D_{t-1} + d_t \quad \text{with} \quad d_t \geq 0 \tag{8}$$

According to this formulation, the government is capable of issuing an amount of debt D_t that is sufficient to cover previous debt repayments $R_t D_{t-1}$. I assume that $d_t \geq 0$ (i.e. the funds raised in excess of debt repayments) are distributed to the old generation as a lump sum transfer.

Bubbly Stocks Bubbles can also be initiated in the corporate sector. As an example of a bubble issued by firms, one can think of a stock that never pays any dividend or cash-flow (but which is still traded at a positive price). Let B_{it} be the value of bubbly stocks issued by firms in industry i and up to time t. I assume that it evolves according to

$$B_{it} = \underbrace{R_t B_{i,t-1}}_{\substack{\text{return on} \\ \text{past bubbles}}} + \underbrace{b_{it}}_{\substack{\text{new} \\ \text{bubbles}}} \quad \text{with} \quad b_{it} \geq 0 \tag{9}$$

According to the previous equation, the time t value of all bubbly stocks issued in industry i has two components. The first is the return on bubbly stocks that were issued in the past $(R_t B_{i,t-1})$. The second (b_{it}) represents the value of new stocks issued by firms in industry i at time t. An important assumption to make concerns how these new bubbles are distributed across firms. One could assume, for example, that b_{it} is equally split across firms (leader and followers), independently of whether they produce or not. In such a case, the industry equilibrium would be unchanged.[6] This assumption seems however unsatisfactory, as it would imply that the followers can issue stocks, even if producing nothing. I will be therefore making two assumptions

(i) the total amount of new stocks that industry i issues is exogenously determined by financial markets and equal to $b_{it} \geq 0$[7]

(ii) this value is split across firms according to market shares.

According to this formulation, investors' total demand for new stocks in industry i exceeds the industry's fundamental value by an exogenous amount b_{it}. Furthermore, larger firms can issue a greater amount of bubbles. Note that this process captures one important aspect of

[6]This is no longer the case when firms are subject to fixed costs, as in the extension considered in Appendix C.

[7]Following the literature, I assume free disposal of bubbles. This rules out the possibility of negative bubbles.

financial markets — namely the fact that valuation models are often based on multiples of revenues or market shares, and not on profits (Damodaran (2006)).[8] For instance, Hong et al. (2007) provide evidence that equity analysts offering valuations for Amazon in the 1997-1999 period tended to emphasize its growth path (in terms of sales) and highly disregarded operating margins. A well-known consequence of such valuation methods is that they induce firms to boost revenues and market shares, at the expense of profits (Aghion and Stein (2008)).

These assumptions have consequences on the industry equilibrium. Since firms get $p_{it} + b_{it}/y_{it}$ per each unit that they sell, the leader must charge a limit price such that

$$p_{it} + \frac{b_{it}}{y_{it}} = \frac{\theta_t}{\gamma} \tag{10}$$

This limit price can be shown to be decreasing in b_{it}, as stated in Proposition 1.

Proposition 1. *(Limit Price with Bubbles) The limit price in* (10) *is decreasing in* b_{it} *(for fixed aggregate variables* Y_t *and* θ_t*).*

Proof. From (5), y_{it} is decreasing in p_{it}. Using (10), it follows that p_{it} decreases in b_{it}. ∎

Figure 2 shows the industry price, quantity and production profits as a function of b_i (time subscripts are omitted for simplicity). There is a value of b_i such that the leader optimally sets a price equal to her marginal cost ($p_i = \theta$), in which case she produces the perfect competition quantity. This is an example in which the market can provide a substitute for the optimal production subsidy discussed above. However, as b_i gets large enough, the price p_i will fall short of the leader's marginal cost, making (production) profits negative. Therefore, sufficiently large bubbles can lead to a situation of excessive production and profit losses, as it was documented in the British railway mania, the dotcom and other bubble episodes. For example, as noted in the context of the recent Silicon Valley boom: "*With valuations based on multiples of revenue, there's ample incentive to race for growth, even at the cost of low or even negative gross margins. The many taxi apps and instant delivery services (...) are facing huge pressure to cut prices*".[9] Note that a regulatory authority can avoid a situation of overinvestment by imposing a minimum price $p_i \geq \underline{p} = \theta$.

[8] These valuation techniques are especially used for young firms: typically they start with low or even negative earnings, which makes it difficult to project future cash flows from current earnings.

[9] "Dotcom history is not yet repeating itself but it is starting to rhyme" (03/12/2015), Financial Times

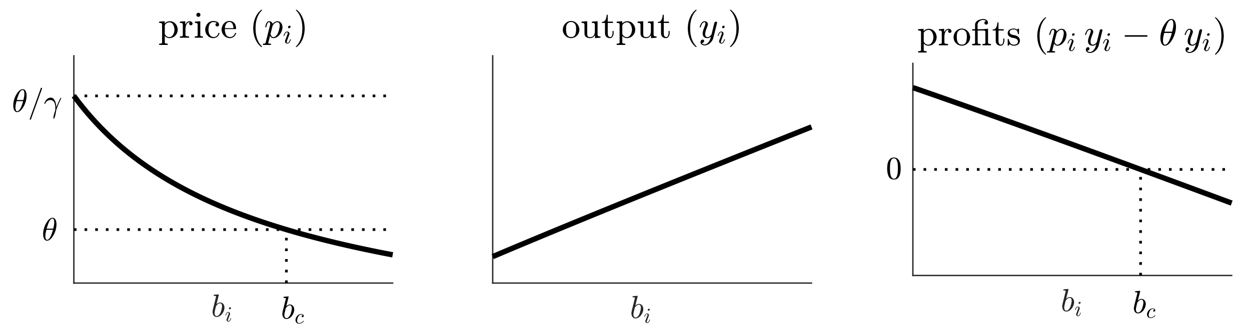


Figure 2: Equilibrium with an Industry Bubble

In the present model, bubbles can have an impact on production variables because their distribution across firms is linked to firms' size. This forces the leaders to set a lower price, in order to keep a high market share. Appendix C considers an extension where firms also have fixed costs. In this case, bubbles can provide firms with an entry subsidy, and thus have a consequence on the market equilibrium even when they are independent of size/market shares.

3 General Equilibrium

In this section, I characterize the general equilibrium of the model. I will focus on symmetric equilibria in which all industries are characterized by identical bubbles $b_{it} = b$. This ensures that industries will be identical and characterized by the same prices $p_{it} = p_t$ and quantities $y_{it} = y_t$. Denoting by L_t and K_t the aggregate stocks of labor and capital, we further have $L_t := \int_0^1 l_{it}\, di = l_{it}$ and $K_t := \int_0^1 k_{it}\, di = k_{it}$. Similarly, denoting by B_t the aggregate value of bubbly stocks, we have $B_t := \int_0^1 B_{it}\, di = B_{it}$.

Definition. An aggregate equilibrium consists of a non-negative sequence for aggregate bubbles (government debt and stocks), capital, labor and consumption $\{ d_t, b_t, D_t, B_t, K_t, L_t, C_t^y, C_t^o \}_{t=0}^{\infty}$ and prices $\{ W_t, R_t, p_t \}_{t=0}^{\infty}$ such that (i) individuals optimize, (ii) the leaders set prices given by (10), (iii) government debt and bubbly stocks evolve according to (8) and (9) (where d_t and b_t are exogenous), and (iv) labor and capital markets clear, i.e.

$$L_t = 1 \tag{11}$$

and

$$K_{t+1} = W_t - \left(D_t + B_t + C_t^y \right) \tag{12}$$

To facilitate the exposition, in subsection 3.1 I start by characterizing the general equilibrium without bubbles. I provide conditions for rational bubbles to be possible, and relate these conditions to the possibility of overaccumulation of capital. Then, in subsection 3.2 I characterize the aggregate equilibrium with government debt and with bubbly stocks.

3.1 General Equilibrium without Bubbles

Aggregate Output Using the fact that all industries are symmetric and that $L_t = 1$, aggregate output can be written as

$$Y_t \;=\; A\,K_t^{\alpha} \tag{13}$$

Equilibrium Factor Prices When no bubbles are traded, equilibrium factor prices are given by

$$\begin{aligned} W_t &= \;\gamma\,(1-\alpha)\;A\,K_t^{\alpha} \\ R_t &= \;\gamma\,\alpha\,A\,K_t^{\alpha-1} \end{aligned} \tag{14}$$

The parameter γ is the inverse of the markup, and corresponds to the aggregate factor share (i.e. the share of total labor and interest payments to aggregate output).

Capital Dynamics and Steady-State When no bubbles are traded, equilibrium in the capital market requires that aggregate investment is equal to aggregate savings

$$K_{t+1} = W_t - C_t^y$$

Combining the previous equation with equations (3) and (14), we find an expression for the dynamics of capital

$$K_{t+1} = \min\left\{\gamma\,(1-\alpha)\,AK_t^{\alpha},\;(\beta\gamma\alpha A)^{1/(1-\alpha)}\right\} \tag{15}$$

To understand (15), note that the equilibrium interest rate cannot fall short of $1/\beta$. In the first region, K_t is low enough so that the young save all their wage, convert it into capital and obtain a return $R_{t+1} \geq 1/\beta$. In the second region, K_t is sufficiently high so that one would observe $R_{t+1} < 1/\beta$ if the young were to convert all their labor income into capital. Therefore, when no bubbles are traded, the economy converges to a steady-state

$$K^* = (\gamma A \min\{\beta\alpha, 1-\alpha\})^{1/(1-\alpha)} \tag{16}$$

with an associated interest rate

$$R^* = \max\left\{\frac{\alpha}{1-\alpha}, \frac{1}{\beta}\right\} \tag{17}$$

When $\alpha/(1-\alpha) > 1/\beta$, the steady-state features $R^* > 1/\beta$ and the young save all their wage. When $\alpha/(1-\alpha) < 1/\beta$, the economy converges to a steady-state with $R^* = 1/\beta$ where the young only save part of their wage. There are two aspects that are worth highlighting about equations (16) and (17). First, K^* increases in the degree of competition γ. To understand this result, note that γ represents the aggregate factor share (as highlighted in (14)). When $R^* > 1/\beta$, the young save all their wage; a higher γ implies that a higher fraction of output is distributed to the young as wages. When $R^* = 1/\beta$, a higher degree of competition γ allows the economy to keep the same interest rate with a higher level of capital K^*. Second, the steady-state interest rate is independent of γ. This happens because γ has a dual role on interest rates, as equation (14) highlights. On the one hand, a higher γ implies a higher capital share and hence a higher R_t for the same K_t. On the other hand, it also means a higher K^*, which implies lower R^* (because of decreasing returns). Given the linear utility specification chosen, these two effects exactly cancel out. However, as shown in Appendix B, under general CRRA utility, the steady-state interest rate will be a function of γ.

Rational Asset Bubbles Rational bubbles are possible when $R^* < 1$. The next proposition states that this will happen if and only if the capital elasticity α is low enough.

Proposition 2. *(Possibility of Rational Bubbles) Rational asset bubbles are possible if and only if*

$$\alpha < \frac{1}{2}$$

Proof. Using (17) and given that $\beta > 1$, it follows that $R^* < 1$ if and only if $\alpha < 1/2$. ■

Capital Overaccumulation and Pareto Efficiency In the seminal paper of Tirole (1985), rational bubbles are possible if and only if the economy overaccumulates capital, i.e. if it converges to a steady-state above the *golden rule* capital stock (i.e. the level of capital that maximizes steady-state welfare). When this happens, the bubbleless equilibrium is also shown to be Pareto inefficient. In Tirole (1985), there is thus an equivalence between the conditions for the emergence of rational bubbles, overaccumulation of capital and Pareto inefficiency. As I show below, such an equivalence only holds in this paper in the particular case of perfect competition ($\gamma = 1$). However, when markets are characterized by imperfect competition ($\gamma < 1$), the equivalence result of Tirole (1985) may not hold. For example, the equilibrium may be Pareto inefficient even when there is no overaccumulation of capital. Additionally, rational bubbles can be possible when there is no overaccumulation of capital and the equilibrium is Pareto efficient.

To clarify these points I start by providing a definition of capital overaccumulation. The *golden rule* capital stock of this model, or the capital stock that maximizes welfare on a steady-state, is given by[10]

$$\left.\frac{\partial Y}{\partial K}\right|_{K=K_{GR}} = 1$$

or, equivalently, $K_{GR} = (\alpha A)^{1/(1-\alpha)}$. The economy will be characterized by overaccumulation of capital when it converges to a steady-state $K^* > K_{GR}$. Alternatively, when $\partial Y/\partial K|_{K=K^*} < 1$, as it formalized in the following definition.

Definition. (Overaccumulation of Capital) The bubbleless steady-state K^* features overaccumulation of capital if

$$\left.\frac{\partial Y}{\partial K}\right|_{K=K^*} < 1$$

Proposition (3) states the conditions for capital overaccumulation.

Proposition 3. *(Conditions for Overaccumulation of Capital) The bubbleless steady-state features overaccumulation of capital if*

$$\gamma > \max\left\{\frac{\alpha}{1-\alpha}, \frac{1}{\beta}\right\}$$

Proof. We have $\partial Y/\partial K = \alpha A K^{\alpha-1}$. From (16), we have $\partial Y/\partial K|_{K=K^*} = (\beta\gamma)^{-1}$ when $\alpha/(1-\alpha) < 1/\beta$, and $\partial Y/\partial K|_{K=K^*} = \alpha/\left[\gamma(1-\alpha)\right]^{-1}$ otherwise.

Proposition 4 states the conditions under which the equilibrium is Pareto inefficient. As the proposition shows, the decentralized and bubbleless equilibrium is inefficient whenever there (i) is overaccumulation of capital or (ii) when there is first period consumption in a steady-state. ■

[10]Note that this model features zero population growth and full depreciation of capital.

Proposition 4. *(Pareto Inefficiency) The bubbleless equilibrium is Pareto inefficient if and only if*

$$\gamma > \max\left\{\frac{\alpha}{1-\alpha}, \frac{1}{\beta}\right\}$$

or

$$\frac{\alpha}{1-\alpha} < \frac{1}{\beta}$$

Proof. If $\gamma > \max\{\alpha/(1-\alpha), 1/\beta\}$, we have $\partial Y/\partial K|_{K=K^*} < 1$ and the bubbleless steady-state is above the *golden rule* ($K^* > K_{GR}$). Starting from K^*, it is possible to implement a transfer scheme that makes the economy converge to K_{GR}, such that all generations are better off.

If $\alpha/(1-\alpha) < 1/\beta$, we have $R^* = 1/\beta$. Thus, there is first-period consumption in the steady-state ($C^{y*} > 0$). Suppose that the young can give away their consumption level C^{y*} to old. This does not change the capital stock and, given $\beta > 1$, results in a Pareto improvement.

If $\gamma < \alpha/(1-\alpha)$ and $1/\beta < \alpha/(1-\alpha)$, we have $\partial Y/\partial K|_{K=K^*} > 1$ and $R^* > 1/\beta$. Thus, the bubbleless steady-state is below the *golden rule* ($K^* < K_{GR}$), and does not feature first-period consumption ($C^{y*} = 0$). Any transfer from the old to the young (e.g. a policy that reduces profits) will hurt the welfare of the old. Any transfer from the young to the old will reduce the capital stock and hurt the welfare of some generation. Let the economy start at some K_0 such that $\partial Y/\partial K|_{K=K_0} > 1$ and $C_0^y = 0$ (which must be reached in finite time). Suppose that the young give $\lambda > 0$ to the old. Let $\tilde{\Delta} X_t := \tilde{X}_t - X_t$ be the difference of X_t between the new and the old allocation. Thus, $\tilde{\Delta} K_1 = -\tilde{\Delta} C_0^o = \lambda$. Suppose further that $\tilde{\Delta} C_{t+1}^o \geq 0\, \forall t$ (no future generation is worse off). Thus, we also have $\partial Y/\partial K|_{K=\hat{K}_1} > 1$, which implies $\tilde{\Delta} Y_1 < \tilde{\Delta} K_1$. Combining the last inequality with $\tilde{\Delta} K_2 \leq \tilde{\Delta} Y_1$, we have $\tilde{\Delta} K_2 < \tilde{\Delta} K_1$. This implies that the capital stock will eventually reach a value of zero, which makes this plan unfeasible. This establishes that, in this case, the equilibrium is Pareto efficient. ■

A comparison between Propositions 2, 3 and 4 shows that they are equivalent if and only if $\gamma = 1$ (i.e. the economy features perfect competition). However, if $\gamma < 1$, rational bubbles can emerge even when there is no overaccumulation of capital and the equilibrium is Pareto efficient. Similarly, when $\gamma < 1$, it is possible to have an equilibrium that does not feature overaccumulation of capital and which is Pareto inefficient. To make these points clear, Figure 3 illustrates these propositions in the (α, γ) space. In region III, we have $\alpha > 1/2$. Bubbles cannot emerge in this region, there is no overaccumulation of capital and the equilibrium is Pareto

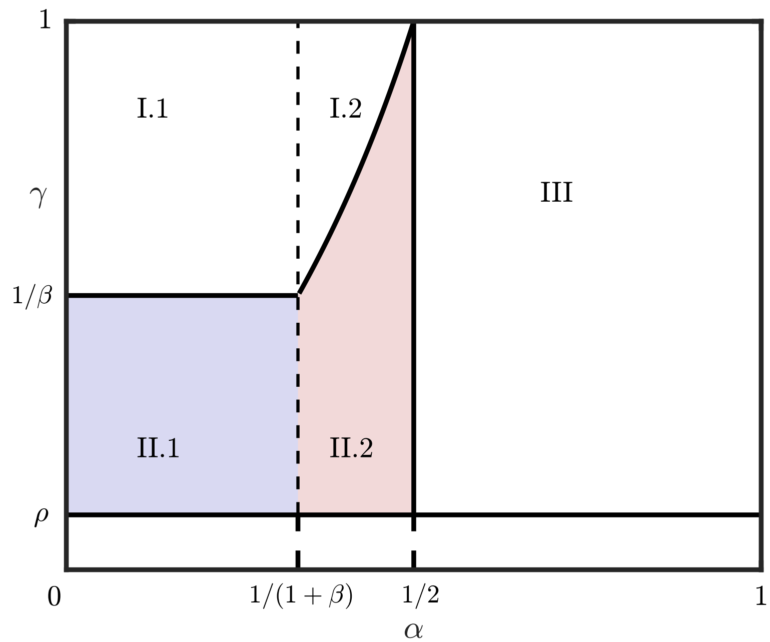


Figure 3: Bubbles and Overaccumulation of Capital

Region	Interest Rate	Overaccumulation	Pareto Efficiency
I.1	$R^* = \beta^{-1} < 1$	Yes	No
I.2	$R^* \in (\beta^{-1}, 1)$	Yes	No
II.1	$R^* = \beta^{-1} < 1$	No	No
II.2	$R^* \in (\beta^{-1}, 1)$	No	Yes
III	$R^* > 1$	No	Yes

Table 1: Steady-State Characterization

efficient. Bubbles can appear in regions I and II, where $\alpha < 1/2$. In region I, the conditions of Propositions 3 and 4 are also satisfied — there is overaccumulation and the equilibrium is Pareto inefficient.[11] In region II, we have a more interesting case: bubbles are possible, even if there is no overaccumulation of capital. There are two interesting cases to distinguish in region II. In region II.2, we have $\alpha > 1/(1+\beta)$, and young agents save all their income; the equilibrium is Pareto efficient. In region II.1, we have $\alpha < 1/(1+\beta)$. This implies $R^* = 1/\beta$ and so young agents consume part of their income. Even if there is no overaccumulation of capital,

[11] In region I.1, we have $R^* = 1/\beta$, implying that young agents consume part of their income. In region I.2, we have $R^* \in (1/\beta, 1)$, implying that young agents save all of their income.

the equilibrium is Pareto inefficient — every young generation could give their consumption to the contemporaneous old, which would generate a Pareto improvement. This region is also interesting because it is characterized by underinvestment. If the young were to give up unit of consumption and use it for investment, this would generate a change in utility of

$$-1 + \beta \underbrace{\partial Y / \partial K|_{K=K^*}}_{=(\beta\gamma)^{-1}} > 0$$

Therefore, in this region, a social planner can raise welfare by increasing investment.[12]

Discussion Before exploring some of the general equilibrium properties of the model with bubbles, two aspects should be highlighted. First, in region II.2, bubbles are possible even when the bubbleless equilibrium is Pareto efficient. An immediate consequence of this fact is that, in this economy, bubbles may not always be Pareto improving. This is illustrated in more detail below, in the discussion of the general equilibrium consequences of government debt. Second, in region II.1, there is underinvestment. Thus, when raising investment demand, bubbles can potentially lead to an efficient increase in capital accumulation. This will be shown with the discussion of bubbly stocks in general equilibrium.

3.2 General Equilibrium with Bubbles

I next discuss the aggregate consequences of asset bubbles. I will assume that $\alpha < 1/2$, so that rational bubbles are possible.

3.2.1 Government Debt

I start by discussing the general equilibrium effects of the government debt scheme introduced in (8). In the OLG model of Tirole (1985) with competitive markets, such a debt scheme would (i) not be expansionary, but (ii) would be Pareto-improving. In the current model, as in Tirole

[12] In region I.1, there is overaccumulation of capital; thus, it is possible to increase welfare by transferring resources from the young to the old. However, this region also features first-period consumption and $\partial Y / \partial K|_{K=K^*} = (\beta\gamma)^{-1}$. Therefore, a Pareto improvement is also possible if the young reduce consumption and increase investment. Intuitively, the young decide their investment based on R^*, but this only reflects part of $\partial Y / \partial K|_{K=K^*}$.

(1985), this debt scheme will never increase capital accumulation. However, it may or may not be Pareto-improving. These results are established in Propositions 5 and 6.

Proposition 5. *Government debt is never expansionary.*

Proof. The aggregate resource constraint implies that $K_{t+1} + D_t + C_t^y = W_t$. Note that W_t is predetermined at t (from (14)). When $C_t^y > 0$, the interest rate is $R_{t+1} = \beta^{-1}$ and young individuals are indifferent between saving or consuming; an infinitesimal increase in D_t thus crowds out C_t^y. When $C_t^y = 0$, a marginal increase in D_t crowds out K_{t+1}. Similarly, a higher D_{t+1} has no impact on R_{t+1} (from (14)) and hence on the savings decisions of young agents. ∎

Proposition 5 says that the introduction of bubbly government debt will never increase capital accumulation. When the economy is characterized by $C_t^y > 0$, a sufficiently small increase in D_t crowds out young-age consumption and has no impact on K_{t+1}. When $C_t^y > 0$, an increase in D_t will necessarily crowd-out K_{t+1}.

Contrarily to Tirole (1985), however, government debt may not always be Pareto improving. To illustrate this point, suppose that the economy starts at its bubbleless steady-state defined by (16) and that the government makes a one time debt issuance such that

$$d_0 > 0$$

$$d_t = 0 \,\forall\, t > 0$$

Suppose that d_0 is the maximum level of debt that the economy can sustain. In this case, the economy will converge to a steady-state with interest rate $R^{**} = 1$, capital stock

$$K^{**} = (\gamma\,\alpha\,A)^{1/(1-\alpha)} \tag{18}$$

and debt level

$$D^{**} = (\gamma\,\alpha\,A)^{1/(1-\alpha)}\,\frac{1-2\alpha}{\alpha} \tag{19}$$

As the next proposition highlights, such a government debt scheme will lead to an increase in steady-state welfare if the economy is characterized by a sufficiently high level of competition.

Proposition 6. *(Aggregate Consequences of Government Debt) Suppose that the economy starts at the bubbleless steady-state given by* (16) *and that, at $t = 0$, the government issues the largest amount of debt* *that the economy can sustain. This bubble generates an increase in steady-state welfare if and only if*

$$\gamma > \begin{cases} \dfrac{1-\beta^{-\alpha/(1-\alpha)}}{1-\alpha\beta^{-\alpha/(1-\alpha)}-(1-\alpha)\,\beta^{-1}} & \textit{if} \quad \dfrac{\alpha}{1-\alpha} < \dfrac{1}{\beta} \\ \dfrac{(1-\alpha)^{\alpha/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)}}{(1-\alpha)^{1/(1-\alpha)}-\alpha^{1/(1-\alpha)}} & \textit{if} \quad \dfrac{\alpha}{1-\alpha} \geq \dfrac{1}{\beta} \end{cases}$$

Proof. When $\alpha/(1-\alpha) < 1/\beta$, steady-state consumption in a bubbleless steady-state is $u^* = w^* + \beta\,\pi^* = A\,(\beta\,\gamma\,\alpha\,A)^{\alpha/(1-\alpha)}\,[\gamma\,(1-\alpha)+\beta\,(1-\gamma)]$ where (14) and (16) have been used. When $\alpha/(1-\alpha) \geq 1/\beta$, steady-state consumption in a bubbleless steady-state is $u^* = R^*w^* + \pi^* = A\,[\gamma\,(1-\alpha)\,A]^{\alpha/(1-\alpha)}\,[1-\gamma\,(1-\alpha)]$ where (14) and (16) have been used. Instead, in a steady-state with the government debt level (19), total welfare is $u^{**} = R^{**}W^{**} + \pi^{**} = A\,(\gamma\alpha A)^{\alpha/(1-\alpha)}\,(1-\gamma\alpha)$, where (14) and (18) have been used. Thus, $u^{**} > u^*$ if and only if the condition in Proposition 6 satisfied. ■

Proposition 6 states that, if the economy converges to a steady-state with the maximum debt level, this will lead to a reduction in welfare when γ is low. In this case, there is a large wedge between interest rates and the marginal product of capital — hence, the economy can be characterized by $R^* < 1$ even if the marginal product of capital is high. Bubbles can emerge and crowd-out efficient investment, thereby resulting in a reduction in welfare for future generations.

Even when raising steady-state welfare, the previous debt policy may reduce the welfare of generations born during the transition. Indeed, Proposition 9 in Appendix A states that, when $\alpha/(1-\alpha) \geq 1/\beta$ and $\gamma < 1/(2-\alpha)$, such a debt policy will hurt the welfare of at least one generation.[13] However, when the conditions of Proposition 6 are satisfied, the government can guarantee that such a policy increases the welfare of all generations if the transition takes place within a period. To achieve this, at period t_0, the government can issue the steady-state level of debt $d_0 = D^{**}$ and levy a lump sum tax equal to $\tau_0 = (W^* - D^{**}) - K^{**}$ to the young. The economy will immediately converge to the new steady-state and generations born from t_0 onwards will enjoy the new steady-state utility level.

Figure 4 illustrates Proposition 6. The previous policy (i.e. maximum debt level and a transition that takes place in one period) will be Pareto improving in the green region. It will

[13] Note that, as $\alpha \to 0$, we can have $\gamma < 1/(2-\alpha)$ while the conditions of Proposition 6 are also satisfied.

however lead to a decline in welfare in the yellow region.[14]

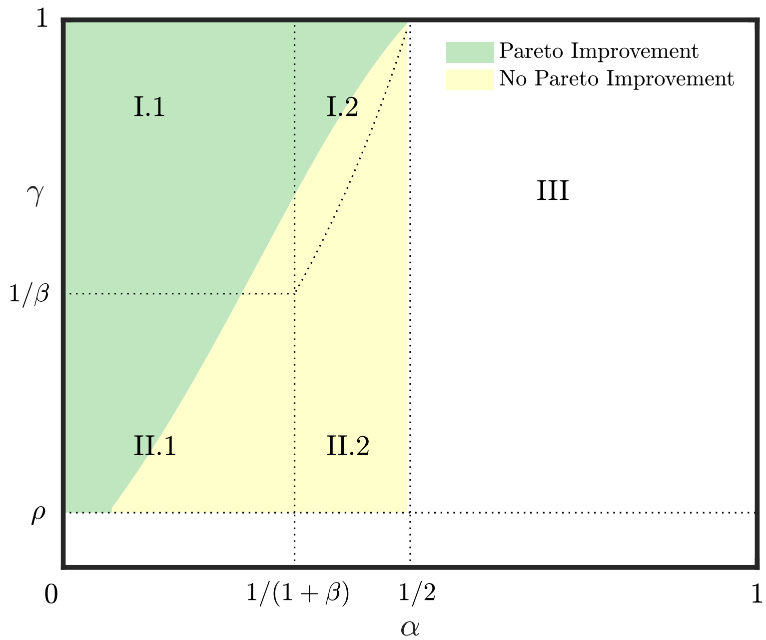


Figure 4: Welfare consequences of the maximum government debt level

The previous proposition shows that bubbles can have negative welfare consequences in this economy. Proposition 7 characterizes optimal government debt issuance in a steady-state.

Proposition 7. *(Optimal Government Debt Issuance) The optimal level of government debt issuance in a steady-state is equal to*

$$d^* = \begin{cases} (1-\gamma)\left(\alpha A\right)^{1/(1-\alpha)}\left(\dfrac{1-\alpha}{\alpha}\gamma - 1\right) & \textit{if} \quad \gamma > \max\left\{\dfrac{\alpha}{1-\alpha}, \dfrac{1}{\beta}\right\} \\ \dfrac{\beta-1}{\beta}\left(\beta\gamma\alpha A\right)^{1/(1-\alpha)}\left(\dfrac{1-\alpha}{\beta\alpha} - 1\right) & \textit{if} \quad \gamma < \dfrac{1}{\beta} \textit{ and } \dfrac{\alpha}{1-\alpha} < \dfrac{1}{\beta} \\ 0 & \textit{otherwise} \end{cases} \tag{20}$$

When $\gamma = 1$ and the conditions of Proposition 3 are satisfied, the optimal level of steady-state new debt issuance is $d^* = 0$, while the optimal level of steady-state debt is given by (19).

[14]Note that this can also happen in region I, which is characterized by excessive investment in a *bubbleless* steady-state. In this region, sufficiently small bubbles can eliminate overaccumulation of capital and result in a Pareto improvement. However, if bubbles are large enough, they can eliminate efficient investment and hurt the welfare of some generations.

Proof. If $\gamma > \max\left\{\frac{\alpha}{1-\alpha}, \frac{1}{\beta}\right\}$, there is overaccumulation of capital. The optimal level of government debt is such that $\partial Y/\partial K|_{K=K^{**}} = 1$. This is achieved with a level of capital $K^{**} = K_{GR} = (\alpha A)^{1/(1-\alpha)}$, implying an interest rate $R^{**} = \gamma$ and a wage $W^{**} = \gamma(1-\alpha)(\alpha A)^{\alpha/(1-\alpha)}$. Given this interest rate there is no first-period consumption. If $\gamma < 1/\beta$ and $\alpha/(1-\alpha) < 1/\beta$, there is no overaccumulation of capital, but there is young-age consumption and $R^{**} = 1/\beta < 1$. The optimal level of government debt must absorb all young-age consumption, i.e. $C^{y**} = 0$. Both the resulting interest rate, capital stock and wage are unchanged, i.e. $R^{**} = 1/\beta$, $K^{**} = (\beta\gamma\alpha A)^{1/(1-\alpha)}$, and $W^{**} = \gamma(1-\alpha)(\beta\gamma\alpha A)^{\alpha/(1-\alpha)}$. If $\gamma < 1/\beta < \alpha/(1-\alpha)$, the equilibrium is Pareto efficient and the optimal level of government debt is zero. Combining these expressions with capital market clearing $W^{**} = K^{**} + d^{**}/(1-R^{**})$, (20) obtains. ∎

Proposition 7 characterizes the optimal level of government debt issuance in a steady-state. Note that each level of debt issuance d^{**} will be associated with a steady-state level of debt equal to $D^{**} = d^{**}/(1-R^{**})$, where R^{**} is characterized in the proof of the proposition. If the government can ensure that the transition to the new steady-state takes place within a period, this policy will necessarily lead to a Pareto improvement (i.e. all generations will be better off).

When $\gamma > \max\left\{\frac{\alpha}{1-\alpha}, \frac{1}{\beta}\right\}$, there is overaccumulation of capital, and the government would like to ensure that the economy converges to $K^{**} = K_{GR}$ (i.e. the *golden rule* capital stock). The transition can take place within a period if the government initially issues the steady-state level of debt $d_0 = D^{**}$ and levies a lump sum tax equal to $\tau_0 = (W^* - D^{**}) - K^{**}$ to the young (which can be distributed to the old). Then, every period, it must issue a new debt level d^{**} according to (20) to sustain this steady-state. Since the *golden rule* capital stock maximizes steady-state welfare, this policy generates a Pareto improvement.

When $\gamma < 1/\beta$ and $\alpha/(1-\alpha) < 1/\beta$, government debt will simply be absorbing first-period consumption. The steady-state level of capital will be unchanged. The government can issue the steady-state level of debt $d_0 = D^{**}$ (without the need to levy any tax), and the transition to the new steady-state equilibrium will take place within a period. Every period, the government must issue a new debt level d^{**} according to (20) to sustain this steady-state.

3.2.2 Bubbly Stocks

A distinctive feature of bubbly stocks, which makes them different from government debt, is that they can result in higher aggregate investment. As I show below, this can happen when (i) there is new bubbly stock issuance and (ii) the economy is characterized by positive young-age consumption. Note that, when all industries are identical and characterized by the same amount of new stock issuance $b_{it} = b_t$, they will feature the same price $p_{it} = 1$ and output $y_{it} = y_t$. Thus, aggregate output is still given by (13). From (10), we obtain an expression for the aggregate factor share (which corresponds to the inverse of markups)

$$\omega_t := \frac{R_t K_t + W_t}{Y_t} = \left(1 + \frac{b_t}{Y_t}\right)\gamma \tag{21}$$

Equilibrium factor prices are equal to

$$\begin{aligned} W_t = &\ \ \omega_t\ (1-\alpha)\ A\, K_t^{\alpha} &&= \gamma\,(1-\alpha)\,(A\,K_t^{\alpha} + b_t) \\ R_t = &\ \ \omega_t\, \alpha\, A\, K_t^{\alpha-1} &&= \gamma\,\alpha\ (A\,K_t^{\alpha} + b_t)\,K_t^{-1} \end{aligned} \tag{22}$$

As (22) shows, a larger amount of bubbly stock issuance b_t results in higher factor prices (for fixed K_t). One observation should be made. Even if leading to a higher wage W_t (and hence higher income for the young), a larger b_t will not lead to higher K_{t+1}. To see this, combine (12) with (9) and (22) to write

$$K_{t+1} + R_t B_{t-1} + C_t^y = \gamma\,(1-\alpha)\,A\,K_t^{\alpha} - b_t\,[1 - \gamma\,(1-\alpha)]$$

Intuitively, even if a higher b_t leads to a higher wage W_t, these bubbles have to be purchased by the young, using the same wage income W_t. Even if K_{t+1} does not increase with b_t, it can however increase with b_{t+1}. A higher b_{t+1} results in a higher interest rate R_{t+1} (through (22)), and may induce young agents to reduce consumption C_t^y and increase investment K_{t+1}. Thus, bubbly stocks can be expansionary only when young-age consumption takes place.

Having made this observation, suppose that the industries always issue the same amount of bubbly stocks $b_t = b$ and suppose that the economy is in an equilibrium with $C_t^y > 0$. Combining $R_{t+1} = 1/\beta$ with (21) and (22), we can find an equation that implicitly defines K_{t+1} as a function of $b_{t+1} = b$

$$K_{t+1}\left[(\gamma\beta\alpha)^{-1} - A K_{t+1}^{\alpha-1}\right] = b \tag{23}$$

It immediately follows from the previous equation that, when $C_t^y > 0$, K_{t+1} is increasing in b.

These results are formally established in the next proposition.

Proposition 8. *(Expansionary Bubbly Stocks) Suppose that all industries feature a constant amount of new bubbly stock issuance $b_t = b \geq 0$ $\forall t$. These bubbles result in higher aggregate investment at all periods if and only if $\alpha < 1/(1+\beta)$ and $b \leq \overline{b} := \eta^{-1}\left[(A\beta\alpha\gamma)^{-1} - (A\eta)^{-1}\right]^{-1/(1-\alpha)}$ where $\eta := \dfrac{\beta}{\beta-1}\dfrac{\beta\alpha}{1-\alpha(1+\beta)}$. For any $b \leq \overline{b}$, the aggregate capital stock satisfies (23). In particular, the capital stock associated with the maximum expansionary bubble $b = \overline{b}$ is equal to $\overline{K} = \eta\,\overline{b}$.*

Proof. The aggregate resource constraint requires that $K_{t+1} + B_t + C_t^y = W_t$. Thus, bubbly stocks can lead to an expansion only if $C_t^y > 0$ in a bubbleless steady-state, which happens if $\alpha < 1/(1+\beta)$. The largest expansionary bubble is such that (i) $C^{y**} = 0$, (ii) $R^{**} = 1/\beta$ and (iii) $B^{**} = b/(1-R^*)$. Combining these conditions with the aggregate resource constraint and equations (22) and (23), $\overline{b}$ and $\overline{K}$ obtain. ■

Bubbles can be expansionary if first-period consumption takes place in a bubbleless steady-state (regions I.1 and II.1). In that case, as firms issue bubbly stocks, their demand for capital increases. Bubbles reduce young-age consumption and crowd-in capital. Note that $\overline{b}$ is the maximum amount of new bubbly stocks that firms can issue on a stationary symmetric equilibrium with $R^{**} = 1/\beta$. Therefore, the capital stock is increasing in b provided that $b < \overline{b}$. An analysis of the expression of $\overline{b}$ shows that the largest expansionary bubble is increasing in the level of aggregate TFP (A) and in the degree of competition (γ).[15] As shown in section 2.4, sufficiently large bubbles will induce market leaders to charge a price below their unit cost of production. That result was, however, obtained under partial equilibrium. I conclude by showing that the economy can also sustain a stationary symmetric equilibrium where all industries are characterized by negative earnings. To see this, let us focus on an equilibrium in which the economy is characterized by the largest expansionary bubble $\overline{b}$, so that capital is equal to $\overline{K} = \eta\,\overline{b}$. Combining the expressions for $\overline{b}$ and $\overline{K}$ (from Proposition 8) with (21), we obtain an aggregate factor share $\omega > 1$ if and only if

$$\gamma > \left[1 + (\beta - 1)\frac{1-\alpha(1+\beta)}{\beta}\right]^{-1}. \tag{24}$$

[15] This happens because the aggregate capital stock is increasing in both A and γ (as it follows from (23)), and so the economy can accommodate larger bubbles.

Therefore, when the degree of competition γ is large (so that the aggregate factor share is already high in a bubbleless steady-state), the economy can experience a bubble-driven expansion which makes firms grow too much and exhibit negative production profits. In such a case, it is in the interest of regulatory authorities to intervene, for example imposing a minimum price (as discussed in section 2.4).

3.3 Discussion and Extensions

This model provides two main insights. The first is that, when firms charge a price-cost markup, interest rates will be below the marginal product of capital. In this case, rational bubbles can be possible even if there is no overaccumulation of capital. The second is that, when the issuance of rational bubbles depends on production decisions, bubbles can stimulate production and result in lower price-cost markups.

In the appendix, I provide two main extensions. In the first extension, I modify preferences and consider a general CRRA utility function. The assumption of linear utility is convenient for analytical purposes, but it can be seen as restrictive. I show that under CRRA the main results of this paper hold. In particular, rational bubbles can emerge when there is no overaccumulation of capital. Furthermore, corporate bubbles can be expansionary provided that the intertemporal elasticity of substitution is sufficiently high (a necessary condition is that it is greater than one).

In the second extension, I consider a different market structure. I assume that firms need to pay a fixed cost of production and compete à la Cournot (via quantities). There are two main differences in this alternative environment. First, contrarily to the previous setting with limit pricing, there can be variation in the number of firms. Second, bubbly stocks can boost entry even when their division firms is not linked to size/market shares. If firms can issue an exogenous amount of bubbly stocks upon entering, entry will become more attractive. As a result, more firms will decide to enter, pay the fixed cost and produce; total output expands, while price-cost markups shrink.

4 Conclusion

Financial history shows that stock market boom/bust episodes are often an industry phenomenon that can be accompanied by changes in the market structure. Motivated by this observation, this paper developed a framework to investigate the interactions between asset bubbles and product market competition. The model shows that bubbles can reduce barriers to entry and force firms to expand, to the ultimate benefit of consumers. An interesting aspect of the theory is that asset bubbles may force productive firms to expand only when potentially unproductive competitors can also get overvalued. This observation helps us think about different questions. For instance, how will a large company react to a bubble on its stock prices? Will Apple lower the price of its *iPhones* if investors suddenly become excited about the company alone and its market value doubles? This paper suggests that it will probably not. Instead, Apple is more likely to expand and cut its profit margins in the presence of a generalized boom in which potential competitors (perhaps smaller and less innovative) can also get overvalued. In such a case, as barriers to entry decrease, Apple may be forced to expand to preserve its market share.

The model developed in this paper also gives a novel perspective on famous stock market overvaluation episodes. For instance, it provides a simple rationale for the low and negative profit margins reported by high-tech firms at the peak of the dotcom bubble. Rather than the realization of a negative technology shock (as argued by Pastor and Veronesi (2006)), this paper suggests that negative profits may have been a rational reaction to an environment characterized by high stock prices.

I conclude by pointing to some avenues for future research. The first concerns the role of policy. This paper provides a stylized model that connects financial and product markets. Its theoretical simplicity allowed me to uncover new mechanisms, but makes it unsuitable for a quantitative policy analysis. The second concerns bubbles and innovation. In the model explored in this paper, bubbles can be pro-competitive and correct a market failure. However, if market leaders could innovate (in order to increase their productivity advantage), such a pro-competitive effect might reduce firms' innovation incentives and growth.

Appendix

A Additional Results

Proposition 9. *(Welfare Consequences of Government Debt) Suppose that the economy starts at the bubbleless steady-state and the government issues a debt level at d_0. This can never be Pareto improving if*

$$\frac{\alpha}{1-\alpha} \geq \frac{1}{\beta}$$

and

$$\gamma < \frac{1}{2-\alpha}.$$

Proof. When $\alpha > 1/(1+\beta)$, the young do not consume in a steady-state and welfare can be written as $u_t = \beta \left(R_{t+1} W_t + \pi_{t+1}\right)$. Government debt can be Pareto-improving only if $\partial u_t / \partial k_{t+1} < 0$. This happens if and only if $K_{t+1}/W_t > (1-\gamma) / [(1-\alpha)\gamma]$. Thus, if the right-hand side of the previous inequality is greater than one, we have $\partial u_t / \partial k_{t+1} > 0$. This happens if and only if $\gamma < 1/(2-\alpha)$. ∎

B CRRA Utility

In this section, I consider an extended version of the model, where individuals have general CRRA preferences. The model is as in section 2, except for the utility function. Suppose that individuals born at t have utility

$$u\left(c_t^y, c_{i,t+1}^o\right) = \frac{\left(c_{it}^y\right)^{1-\theta}}{1-\theta} + \beta \frac{\left(c_{i,t+1}^o\right)^{1-\theta}}{1-\theta}$$

where $\theta \geq 0$ is the inverse of the intertemporal elasticity of substitution (IES). Note that the model with linear utility obtains as the particular case of $\theta = 0$. As before, young individuals face the budget constraint in (2). Denoting by $s_{it} := W_t - c_{it}^y$ their savings level, they solve

$$\max_{s_{it} \in [0, W_t]} \frac{\left(W_t - s_{it}\right)^{1-\theta}}{1-\theta} + \beta \frac{\left(R_{t+1} s_{it} + \pi_{i,t+1}\right)^{1-\theta}}{1-\theta}$$

which yields an optimal savings rate

$$\frac{s_{it}}{W_t} = \frac{1}{1+\beta^{-1/\theta} R_{t+1}^{(\theta-1)/\theta}} - \frac{(\beta R_{t+1})^{-1/\theta}}{1+\beta^{-1/\theta} R_{t+1}^{(\theta-1)/\theta}} \frac{\pi_{i,t+1}}{W_t}$$

Bubbleless Dynamics and Steady-State

Absent the existence of bubbles, the economy is characterized by a law of motion

$$K_{t+1} = \frac{\gamma (1-\alpha) AK_t^{\alpha} - (\beta R_{t+1})^{-1/\theta} \pi_{t+1}}{1+\beta^{-1/\theta} R_{t+1}^{(\theta-1)/\theta}}$$

and will converge to a steady-state that is implicitly defined by

$$\frac{A\beta^{1/\theta}\gamma (1-\alpha) - (\alpha A\gamma)^{-1/\theta} (1-\gamma) AK^{(1-\alpha)/\theta}}{\beta^{1/\theta}K^{1-\alpha} + (\alpha A\gamma)^{(\theta-1)/\theta} K^{(1-\alpha)/\theta}} = 1 \tag{25}$$

The steady-state can be shown to be unique, and to be increasing in the degree of competition γ, as stated in the next proposition.

Proposition 10. *(Steady-State Capital Stock) There is a unique steady-state K^* defined by (25), which is increasing in the level of competition γ.*

Proof. Rewrite (25) as

$$\underbrace{\frac{\beta^{1/\theta}\gamma (1-\alpha) - (\alpha A\gamma)^{-1/\theta} (1-\gamma) K^{(1-\alpha)/\theta}}{\beta^{1/\theta}K^{1-\alpha} + (\alpha A\gamma)^{(\theta-1)/\theta} K^{(1-\alpha)/\theta}} A - 1}_{\equiv F(K,\gamma)} = 0$$

The steady-states of the model are given by the values K^* such that $F(K^*,\gamma) = 0$. We have $\partial F(K,\gamma)/\partial K < 0$, with $F(0,\gamma) = \infty$ and $F(\infty,\gamma) = -1$ for $\gamma \in (0,1]$. This establishes that there is a unique K^* such that $F(K^*,\gamma) = 0$. To prove that K^* is increasing in γ, we can use the implicit function theorem. We have that

$$\frac{\partial K^*}{\partial \gamma} = -\left(\frac{\partial F(K,\gamma)}{\partial \gamma}\right) / \left(\frac{\partial F(K,\gamma)}{\partial K}\right)$$

Given that $\partial F(K,\gamma)/\partial K < 0$, it suffices to show that $\partial F(K,\gamma)/\partial \gamma > 0$. We have that

$$\begin{aligned} & \frac{\partial F(K,\gamma)}{\partial \gamma} > 0 \\ \Leftrightarrow \quad & \beta^{1/\theta}(1-\alpha)A + \frac{AK^{(1-\alpha)/\theta}}{\theta}(\alpha A\gamma)^{-1/\theta}\left[\gamma^{-1} + (\theta-1)(1-\alpha)\right] > 0 \end{aligned}$$

which is always satisfied given that $\theta \geq 0$, $\gamma < 1$ and $\alpha < 1$. ■

The steady-state interest rate R^* can be obtained by combining the expression for R_t in (14) with (25). It is implicitly defined by

$$\alpha (R^*)^{-1} + \left[\gamma^{-1} - (1-\alpha)\right](\beta R^*)^{-1/\theta} - (1-\alpha) = 0 \tag{26}$$

Contrarily to the baseline model with linear utility, the steady-state interest rate now depends on the degree of competition γ. In particular, it follows directly from the analysis of (26) that R^* decreases in γ.[16] Asset bubbles are possible when $R^* \leq 1$. The condition for investment efficiency can again be written as

$$\left.\frac{\partial Y}{\partial K}\right|_{K=K^*} = \alpha A (K^*)^{\alpha-1} > 1$$

which can be restated in terms of the steady-state interest rate as

$$R^* > \gamma$$

The next two propositions characterize the conditions for the existence of bubbles and for capital overaccumulation.

Proposition 11. *(Existence of a Bubble Equilibrium) Rational bubbles are possible if and only if*

$$\alpha < \frac{\beta^{1/\theta} - \gamma^{-1} + 1}{2\beta^{1/\theta} + 1}$$

Proof. Using (26), we have $R^* < 1$ if and only if the previous inequality is satisfied. ■

[16] As discussed before, the degree of competition γ has a dual role on interest rates. On the one hand, it increases the capital share and R^*. On the other hand, it results in larger K^* and lower R^* because of decreasing returns. In the present setting, the second effect always dominates.

Proposition 12. *(Condition for Capital Overaccumulation) The economy features overaccumulation of capital in a bubbleless steady-state if and only if*

$$\alpha < \frac{(\beta\gamma)^{1/\theta} - \gamma^{-1} + 1}{(\beta\gamma)^{1/\theta}\,(1+\gamma^{-1}) + 1}$$

Proof. Using (26), we have $R^* < \gamma$ if and only if the previous inequality is satisfied.

Figure 5 illustrates Propositions 11 and 12 in the (α, γ) space, for the particular case of $\theta = 1$ (logarithmic utility) and $\beta = 1$. Bubbles can appear only in regions I and II. In region I, $R < \gamma$ and there is overinvestment. However, in region II, $R > \gamma$ and there is no overaccumulation. ■

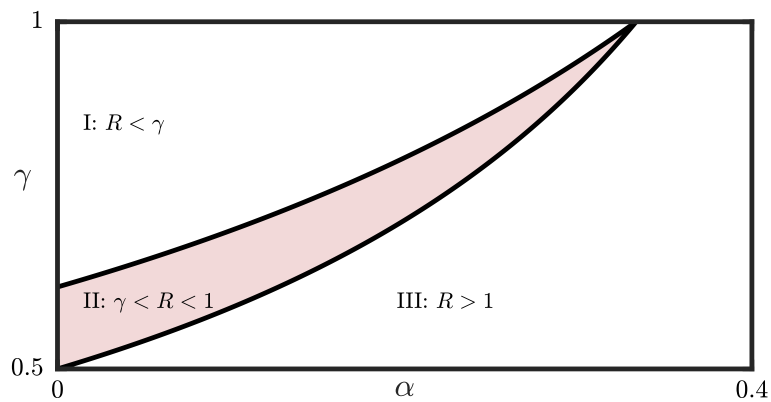


Figure 5: Bubbles and Investment Efficiency with $\theta = 1$ and $\beta = 1$

Proposition 13 says that when firms have market power ($\gamma < 1$), the decentralized and bubbleless equilibrium is always Pareto inefficient. Under perfect competition, the equilibrium is Pareto inefficient if and only if $R^* < 1$, as in Tirole (1985).

Proposition 13. *(Pareto Inefficiency) When $\gamma < 1$, the decentralized and bubbleless equilibrium is always Pareto inefficient. When $\gamma = 1$, it is Pareto inefficient if and only if $\alpha < \left(2 + \beta^{-1/\theta}\right)^{-1}$.*

Proof. From the steady-state Euler equation we have

$$-\left(\partial u / \partial c^y\right) + \beta R \left(\partial u / \partial c^o\right) = 0$$

Now suppose that all agents were to reduce young-age consumption and increase investment.

The change in utility would be equal to

$$
\begin{aligned}
& -\left(\partial u / \partial c^{y}\right)+\beta \underbrace{(\partial Y / \partial K)}_{R / \gamma}\left(\partial u / \partial c^{o}\right) \\
= & \beta R\left(\partial u / \partial c^{o}\right)\left(-1+\frac{1}{\gamma}\right)>0
\end{aligned}
$$

The last condition is always satisfied provided that $\gamma<1$. When $\gamma=1$, the aggregate economy is as in Tirole (1985). It is Pareto inefficient if and only if there is overaccumulation (which is equivalent to $R^{*}<1$). From Proposition 12, this happens if and only if $\alpha<\left(2+\beta^{-1 / \theta}\right)^{-1}$. ■

General Equilibrium with Bubbly Stocks

I next discuss the general equilibrium consequences of bubbly stocks.[17] When all industries issue a stock of new bubbles $b_{it}=b$, equilibrium in the capital market requires that

$$
K_{t+1}+\underbrace{R_{t} B_{t-1}+b}_{B_{t}}=\frac{1}{1+\beta^{-1 / \theta} R_{t+1}^{(\theta-1) / \theta}} \underbrace{\left(1+\frac{b}{A K_{t}^{\alpha}}\right) \gamma(1-\alpha) A K_{t}^{\alpha}}_{W_{t}}
$$

where B_{t} is the aggregate stock of bubbles at time t (issued at t and before). As in the baseline economy studied in section 2, the aggregate factor share is $\left(1+\frac{b}{A K_{t}^{\alpha}}\right) \gamma$, which is increasing in the size of b relative to output $Y_{t}=A K_{t}^{\alpha}$. The aggregate bubble dynamics are given by

$$
B_{t+1}=R_{t+1} B_{t}+b
$$

and the equilibrium interest rate is equal to

$$
R_{t+1}=\left(1+\frac{b}{A K_{t+1}^{\alpha}}\right) \gamma \alpha A K_{t+1}^{\alpha-1}
$$

[17] Government debt can be shown to be always contractionary. There is no additional insight from this process under general CRRA utility. For this reason, I just focus on bubbly stocks.

Using the previous three equations, we can define the level of steady-state capital K to which the economy will converge

$$\frac{\gamma\left(1-\alpha\right)\left(AK^{\alpha}+b\right)K^{-1}}{1+\beta^{-1/\theta}\left[\gamma\alpha\left(AK^{\alpha}+b\right)K^{-1}\right]^{(\theta-1)/\theta}}-\frac{b}{K-\gamma\alpha\left(AK^{\alpha}+b\right)}-1=0 \tag{27}$$

Under CRRA preferences, it becomes harder to characterize the conditions under which bubbly stocks can be expansionary. Figure shows the set of values for θ under which this can happen (for fixed α, γ and β). Bubbles can be expansionary when θ is low, that is, when the IES is high. Intuitively a high IES is required so that individuals are willing to substitute consumption intertemporally. When a bubble appears, young individuals must be willing to reduce first period consumption and increase savings (allocated to both capital accumulation and bubbly stocks), to enjoy higher consumption in the second period of life.

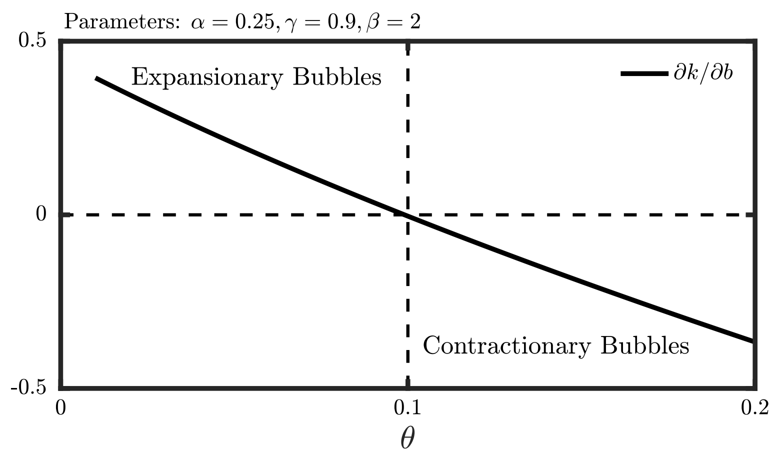


Figure 6: Condition for expansionary industry bubbles

Proposition 14 says that bubbly stocks cannot be expansionary when the intertemporal elasticity of substitution is equal or lower than one.

Proposition 14. *(Expansionary Industry Bubbles) Industry bubbles cannot be expansionary when*

$$\frac{1}{\theta}\leq 1$$

Proof. A steady-state where firms issue a new value of bubbles b is implicitly defined by

$$\underbrace{K+\frac{b}{1-R}-\frac{W-\left(\beta R\right)^{-1/\theta}\pi}{1+\beta^{-1/\theta}R^{(\theta-1)/\theta}}}_{F(K,b)}=0 \tag{28}$$

with $R = \gamma\alpha\left(AK^{\alpha}+b\right)K^{-1}$, $W=\gamma\left(1-\alpha\right)\left(AK^{\alpha}+b\right)$ and $\pi=\left(1-\gamma\right)\left(AK^{\alpha}+b\right)$. We want to show that

$$\left.\frac{\partial K}{\partial b}\right|_{b=0} = -\frac{\left.\frac{\partial F\left(K,b\right)}{\partial b}\right|_{b=0}}{\left.\frac{\partial F\left(K,b\right)}{\partial K}\right|_{b=0}} < 0$$

when $\theta \geq 1$.[18] First, note that, when $b=0$, $F\left(K,b\right)$ can be rewritten as

$$F\left(K,0\right) = K^{\alpha}\left(K^{1-\alpha}+\frac{\left(1-\gamma\right)\left(\beta\alpha\gamma A\right)^{-1/\theta}AK^{(1-\alpha)/\theta}-\gamma\left(1-\alpha\right)A}{1+\beta^{-1/\theta}\left(\alpha\gamma A\right)^{(\theta-1)/\theta}K^{-(1-\alpha)(\theta-1)/\theta}}\right)$$

which is increasing in K when $\theta \geq 1$. Thus, when $\theta \geq 1$ and $b=0$

$$\left.\frac{\partial F\left(K,b\right)}{\partial K}\right|_{b=0} > 0$$

Second, note that

$$\left.\frac{\partial F\left(K,b\right)}{\partial b}\right|_{b=0} = \frac{1}{1-R}-\frac{\frac{1}{\theta}\gamma\left(1-\alpha\right)+\frac{\theta-1}{\theta}\frac{\gamma\alpha}{R}}{1+\beta^{-1/\theta}R^{(\theta-1)/\theta}}$$

When $\theta \geq 1$, the above expression is weakly declining in R. From (26), we have $R > \alpha/\left(1-\alpha\right)$. Thus, when $\theta \geq 1$, we have

$$\left.\frac{\partial F\left(K,b\right)}{\partial b}\right|_{b=0} > \underbrace{\frac{1}{1-R}}_{>1}-\underbrace{\frac{\gamma\left(1-\alpha\right)}{1+\beta^{-1/\theta}R^{(\theta-1)/\theta}}}_{<1} > 0$$

This completes the proof. ■

[18] I focus on $b=0$ to consider the impact of an infinitesimally small bubble on the capital stock.

C Cournot Competition and Fixed Costs

In this section, I consider an alternative model of the market structure. Preferences and demographics are as in the baseline model, and so equations (1) to (3) hold. The only differences concern technology and the market structure.

Technology and Market Structure

Firms are identical and have the same production function $f_{ij}(k, l) = k^{\alpha} \, l^{1-\alpha}$ (i.e. there are no productivity differences). However, now production entails a fixed cost $f > 0$, which is in units of the final good. To give a role to fixed costs, and allow for a variable number of firms, I shall however depart from limit pricing. For this reason, I assume Cournot competition. Let $n_{it} \subseteq \mathbb{N}$ be the number of active firms in industry i. All firms $j \in \{1, \ldots n_{it}\}$ that decided to enter and pay the fixed cost $f > 0$ solve

$$\max_{y_{jit}} \quad (p_{it} - \theta_t)\, y_{jit} \qquad \text{s.t.} \quad p_{it} = \left(\frac{Y_t}{y_{it}}\right)^{1-\rho}$$
$$y_{it} = \sum_{k=1}^{n_{it}} y_{kit}$$

The solution to this problem yields a markup

$$\mu_{it} := \frac{p_{it}}{\theta_t} = \frac{n_{it}}{n_{it} - (1-\rho)}$$

which is decreasing in the number of active firms. In every period, each active firm can issue an amount of bubbly stocks $b_{jit} = b \geq 0$. This is assumed to be exogenous and independent of output.[19] The equilibrium number of firms n_{it} is determined by two conditions: (i) all active firms must break even, and (ii) no additional firm can profitably enter. Formally,

$$[\pi(n_{it}, \theta_t, Y_t) - (f - b)]\,[\pi(n_{it}+1, \theta_t, Y_t) - (f - b)] \leq 0 \tag{29}$$

[19] As before, one could assume that there is a fixed amount of bubbly stocks at the industry level, which is distributed according to market shares. This is considered in Queirós (2021).

where $\pi(n_{it}, \theta_t, Y_t) := (p_{it} - \theta_t) y_{jit}$ are production profits (revenues minus variable costs). The profit function $\pi(n_{it}, \theta_t, Y_t)$ can be shown to be decreasing in the number of active firms n_{it}, and to approach zero as $n_{it} \to \infty$, as stated in the following lemma.

Lemma 1. *The profit function* $\pi(n_{it}, \theta_t, Y_t) := (p_{it} - \theta_t) y_{jit}$ *decreases in the number of firms* n_{it}*. Furthermore,* $\lim_{n_{it} \to \infty} \pi(n_{it}, \theta_t, Y_t) = 0$.

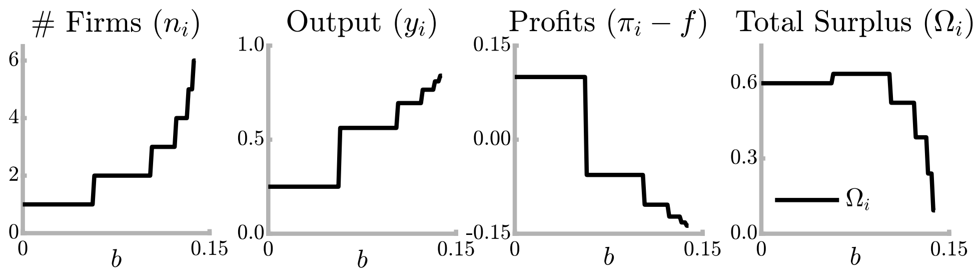
Proof. We have $\pi(n_{it}, \theta_t, Y_t) = (1-\rho)[n_{it} - (1-\rho)]^{\rho/(1-\rho)} n_{it}^{(\rho-2)/(1-\rho)} \theta_t^{-\rho/(1-\rho)} Y_t$. Furthermore, $\partial \pi(\cdot) / \partial n_{it} < 0 \Leftrightarrow 2(n_{it} - 1) + \rho > 0$, which is always satisfied for $n_{it} \geq 1$. This proves the first statement. To prove the second, note that $\lim_{n_{it} \to \infty} \mu_{it} = 1$. ■

It immediately follows from Lemma 1 that, if there is a value n_{it} satisfying (29), such a value is unique and increasing in b. Figure 7 shows some equilibrium variables as a function of b. Given the parameters chosen, absent the formation of bubbles ($b = 0$), the industry consists of a monopoly ($n_i = 1$). For sufficiently large values of b, more firms will enter, even if they make negative operating profits (third panel). Even when firms make an operating loss, their entry necessarily results in higher consumer welfare, as total output y_{it} increases. To assess the efficiency gains associated with the entry of additional firms, we must evaluate the change in the total industry surplus[20]

$$\Omega_{it} := \underbrace{\int_0^{y_{it}} \left[\left(\frac{Y_t}{x} \right)^{1-\rho} - p_{it} \right] dx}_{\text{consumer surplus}} + \underbrace{(p_{it} - \theta_t) y_{it} - n_{it} f}_{\text{producer surplus}}$$

The last panel of Figure 7 shows Ω_i as a function of b. As b increases, output y_i increases and the price p_i decreases. This results in higher consumer welfare, but in a lower producer surplus. When b is small, few firms produce, and the increase in consumer welfare exceeds the decrease in producer surplus. As b becomes large, the increase in consumer welfare is outweighed by the reduction in the producer surplus. In the example of Figure 7, the total surplus is maximized when two firms produce.

[20] This is a measure of economic efficiency that ignores the rents stemming from the issuance of bubbly stocks.



Parameters: $\rho = 0.5$, $f = 0.15$, $Y = 1$ and $\theta = 1$

Figure 7: Industry Equilibrium with Exogenous Firm Bubbles

C.1 General Equilibrium

Definition. An aggregate equilibrium consists of a non-negative sequence for aggregate bubbles (government debt and stocks), capital, labor and consumption $\left\{D_t, B_t, K_t, L_t, C_t^y, C_t^o\right\}_{t=0}^{\infty}$, factor prices $\{W_t, R_t,\}_{t=0}^{\infty}$, a set of active firms $\mathcal{I}_{it}$ and firm policies $\left\{p_{it}, y_{it}^j, k_{it}^j, l_{it}^j, B_{it}^j\right\}_{t=0}^{\infty}$, $\forall\, i \in [0,1]$ and $j \in \mathcal{I}_{it}$, for all $t \geq 0$ such that (i) individuals optimize, (ii) all active firms maximize profits, (iii) the number of firms is given by (29), (iv) government debt and bubbly stocks evolve according to (8) and (9), and (v) labor and capital markets clear, i.e.

$$L_t = 1 \tag{30}$$

and

$$K_{t+1} = W_t - \left(D_t + B_t + C_t^y\right) \tag{31}$$

Next, I characterize the within-period equilibrium (in which I take K_t as given). Then, I characterize the equilibrium dynamics.

Within-Period Equilibrium

The next proposition characterizes the equilibrium number of firms as a function of the aggregate capital stock (K_t). It states that, when $\rho \geq 1/2$, there is a unique equilibrium. Moreover, the aggregate number of firms (weakly) increases in K_t.

Proposition 15. *(Equilibrium Number of Firms) Let $\underline{K}(n)$ and $\overline{K}(n)$ be defined as*

$$\underline{K}(n) := \left(\frac{f}{A}\frac{n^2}{1-\rho}\right)^{1/\alpha}$$
$$\overline{K}(n) := \left[\frac{f}{A}\frac{(n+1)^2}{1-\rho}\left(\frac{n+1}{n}\frac{n-(1-\rho)}{n+\rho}\right)^{\rho/(1-\rho)}\right]^{1/\alpha}$$

When $\rho \geq 1/2$ and no bubbles are traded, there is a unique within-period equilibrium. In particular, if

1. *$K_t \in \left[\underline{K}(n), \overline{K}(n)\right]$, then all industries have n firms.*
2. *$K_t \in \left[\overline{K}(n), \underline{K}(n+1)\right]$, then a fraction $\eta_t \in (0,1)$ of the industries features $n+1$ firms, and the remaining fraction $1-\eta_t$ features n firms. η_t is increasing in the aggregate capital stock K_t.*

Proof. In a symmetric equilibrium with aggregate capital K and n firms in every industry, each firm makes production profits equal to

$$\pi(n,K) = \frac{1-\rho}{n^2}AK^\alpha$$

If one industry were to have $n+1$ firms, profits in this industry would be

$$\tilde{\pi}(n,K) = \frac{1-\rho}{(n+1)^2}\left(\frac{n}{n+1}\frac{n+\rho}{n-(1-\rho)}\right)^{\rho/(1-\rho)}AK^\alpha$$

The thresholds $\underline{K}(n)$ and $\overline{K}(n)$ are such that $\pi(n, \underline{K}(n)) = f$ and $\tilde{\pi}\left(n, \overline{K}(n)\right) = f$. It follows that $\underline{K}(n) < \overline{K}(n)$, given $\rho < 1$.

For $K \in \left(\overline{K}(n), \underline{K}(n+1)\right)$, a fraction $\eta \in (0,1)$ of all industries contains $n+1$ firms, while the remaining fraction $1-\eta$ contains n firms. The equilibrium value of η is such that, in industries with $n+1$ firms, these firms exactly break even. This no profit condition can be written as

$$\frac{1-\rho}{(n+1)^2}\nu(n)^\rho \underbrace{\frac{\left[1+\eta\left(\nu(n)^\rho-1\right)\right]^{(1-\rho)/\rho}}{1+\eta(\nu(n)-1)}}_{g(\eta)} AK^\alpha = f$$

with $\nu(n) := \left(\frac{n}{n+1}\frac{n+\rho}{n-(1-\rho)}\right)^{1/(1-\rho)}$. If $g(\eta)$ is monotone in η, the previous equation defines a unique η as a function of K. Furthermore, if $g(\eta)$ decreases in η, η is increasing in K. We have $(dg/d\eta) < 0$ if and only if $\nu(n)^\rho(1-\rho) - 1 < (2\rho-1)\eta\left(\nu(n)^\rho - 1\right)(\nu(n)-1)$, which is always satisfied if $\rho \geq 1/2$. ∎

$\underline{K}(n)$ and $\overline{K}(n)$ correspond to the minimum and maximum values of K_t that are consistent with n in all industries. When $K_t < \underline{K}(n)$ or $K_t > \overline{K}(n)$, not all industries can have n firms.

When $\rho < 1/2$, there can be multiple equilibria. In particular, for the same capital stock K_t, it is possible to have a symmetric equilibrium with n firms in every industry, and an asymmetric equilibrium in which some industries have n firms, while others have only $n-1$ firms. In the first equilibrium, aggregate productivity is high and equal to A. In the second, aggregate productivity is lower than A (because industries are asymmetric and there is hence misallocation). Thus, it is possible to sustain one equilibrium with high output and a large number of firms. And another equilibrium with low output and fewer firms. Note that, when ρ is low, there are strong complementarities across varieties and the CES aggregator features a strong aggregate demand externality.

Aggregate Output When a fraction $\eta_t \in (0,1)$ of the industries features $n+1$ firms, and the remaining fraction $1-\eta_t$ features n firms, aggregate output is equal to

$$Y_t = \underbrace{\frac{\left[1+\eta_t\left(\nu(n)^{\rho}-1\right)\right]^{1/\rho}}{1+\eta_t\left(\nu(n)-1\right)}}_{=\varphi_t} A\, K_t^{\alpha}$$

where $\nu(n) := \left(\dfrac{n}{n+1}\dfrac{n+\rho}{n-(1-\rho)}\right)^{1/(1-\rho)}$.

For what follows, it will be useful to define net output, as the difference between aggregate output Y_t and the total value of resources spent in fixed costs.

Definition. Let $N_t := \int_0^1 n_{it}\, di$ denote the aggregate mass of firms. Net output is the difference between total output and the total mass of resources spent in fixed costs, i.e.

$$Y_t^{\text{net}} = Y_t - N_t \cdot f$$

Factor Shares and Factor Prices When a fraction $\eta_t \in (0,1)$ of the industries features $n+1$ firms, and the remaining fraction $1-\eta_t$ features n firms, the aggregate factor share is given by[21]

$$\sigma_t := \frac{R_t K_t + W_t L_t}{Y_t} = \frac{n-(1-\rho)}{n} \frac{1+\eta_t\left(\nu\left(n\right)-1\right)}{1+\eta_t\left(\nu\left(n\right)^{\rho}-1\right)} \tag{32}$$

The interest rate is equal to

$$R_t \;=\; \alpha\,\frac{n-(1-\rho)}{n}\;\left[1+\eta_t\left(\nu\left(n\right)^{\rho}-1\right)\right]^{(1-\rho)/\rho}\;A\,K_t^{\alpha-1} \tag{33}$$

Recall that η_t (i.e. the fraction of industries with $n+1$ firms) increases in K_t. Thus, from the previous equation, one can see that K_t can have an ambiguous effect on R_t. On the one hand, a higher K_t leads to lower R_t because of decreasing returns. On the other hand, it leads to a higher η_t and hence a higher capital share; this leads to higher R_t. Lemma 2 states sufficient conditions for the interest rate to be monotonically decreasing in the capital stock.

Lemma 2. *(Equilibrium Interest Rate) If* $\rho \geq 1/2$ *and* $\alpha \leq 1-\dfrac{1-\rho^2}{\rho}\dfrac{(1+\rho)^{\rho/(1-\rho)}-(2\,\rho)^{\rho/(1-\rho)}}{(1+\rho)^{1/(1-\rho)}-(2\,\rho)^{1/(1-\rho)}}$, *then the interest rate in (33) is monotonically decreasing in the capital stock* K_t.

Proof. If $K_t \in \left[\underline{K}\left(n\right), \overline{K}\left(n\right)\right)$, then $\eta_t = 0$ and R_t decreases in K_t. If $K_t \in \left[\overline{K}\left(n\right), \underline{K}\left(n+1\right)\right)$, the no profit condition in industries with $n+1$ firms can be written as

$$\frac{1-\rho}{n+\rho}\frac{\nu\left(n\right)A}{\alpha}\left[\frac{K_t}{1+\eta_t\left(\nu\left(n\right)-1\right)}\right]R_t \;=\; f$$

It suffices to show that the term in square brackets is increasing in K_t. This term can be written as

$$\left[1+\eta_t\left(\nu\left(n\right)-1\right)\right]^{(1-\alpha)/\alpha}\;\left[1+\eta_t\left(\nu\left(n\right)^{\rho}-1\right)\right]^{-(1-\rho)/(\alpha\rho)}$$

Under $\rho \geq 1/2$, η_t is increasing in K_t. The previous expression is thus increasing in K_t if

$$1-\alpha > \frac{1-\rho}{\rho}\,\underbrace{\frac{1+\eta_t\left(\nu\left(n\right)-1\right)}{1+\eta_t\left(\nu\left(n\right)^{\rho}-1\right)}}_{g(\eta_t)}\,\frac{\nu\left(n\right)^{\rho}-1}{\nu\left(n\right)-1}$$

$g\left(\eta_t\right)$ can be shown to be increasing in η_t. Thus, the previous condition is implied by

[21] Aggregate gross profits are equal to $\Pi_t := \left(1-\sigma_t\right)Y_t$, while net profits are equal to $\Pi_t^{net} := \left(1-\sigma_t\right)Y_t - N_t \cdot f$, where $N_t := \int_0^1 n_{it}\,di$ is the aggregate mass of firms.

$$1-\alpha > \frac{1-\rho}{\rho}\frac{\nu\left(n\right)-\nu\left(n\right)^{1-\rho}}{\nu\left(n\right)-1}$$

The left hand-side can be shown to be increasing in $\nu\left(n\right)$. Since $\nu\left(n\right)$ is decreasing in n, a sufficient condition is

$$\alpha \leq 1-\frac{1-\rho}{\rho}\frac{\nu\left(1\right)-\nu\left(1\right)^{1-\rho}}{\nu\left(1\right)-1}$$

■

Let $R\left(K\right)$ describe the equilibrium interest rate as a unique function of K. Thus, under the conditions of Lemma 2, we have $R'\left(K\right) < 0$.

Capital Dynamics and Steady-State

The dynamics of the capital stock is described in Proposition 16. It states that under the conditions of Lemma 2, the capital stock follows a unique path and converges to a steady-state that can be shown to be weakly decreasing in fixed costs.

Proposition 16. *(Equilibrium Dynamics) Assume that the conditions of Lemma 2 are satisfied. Then the economy follows a unique path, where the dynamics of the capital stock satisfy*

$$K_{t+1} \;=\; \min\left\{W_t\,,\,R^{-1}\left(\frac{1}{\beta}\right)\right\} \tag{34}$$

Moreover, it converges to a unique steady-state K^{*} implicitly defined by

$$R\left(K^{*}\right)=\max\left\{\frac{\alpha}{1-\alpha},\,\frac{1}{\beta}\right\} \tag{35}$$

The steady-state K^{*} is weakly decreasing in fixed costs f.

Proof. From Lemma 2, the interest rate is monotonically decreasing in K_t. Therefore, there is a unique $\widetilde{K}$ such that $R\left(\widetilde{K}\right)=1/\beta$ and $R\left(K\right)\geq 1/\beta$ if and only if $K<\widetilde{K}$. All savings are converted into capital ($K_{t+1}=W_t$) provided that $R\left(W_t\right)\geq 1/\beta$. Otherwise, only a fraction of all savings are converted into capital ($K_{t+1}<W_t$) and $R\left(K_{t+1}\right)=1/\beta$, so that young individuals are indifferent between consuming when young and old. Then (16) obtains. Since $W_t = \left(1-\alpha\right)\alpha^{-1}R\left(K_t\right)K_t$, on a steady-state where all savings are converted into capital, we have $R\left(K^{*}\right)=\alpha/\left(1-\alpha\right)$. Then (35) obtains.

To prove that K^* is weakly decreasing in f note the following. If, on a steady-state, we have $K^* \in \left(\underline{K}(n), \overline{K}(n)\right)$ for some $n \subseteq \mathbb{N}$, then

$$K^* = \left(\frac{n-(1-\rho)}{n} A \min\{\alpha\beta, 1-\alpha\}\right)^{1/(1-\alpha)}$$

and a marginal change in f will not affect K^*. If the steady-state is such that $K^* \in \left[\overline{K}(n), \underline{K}(n+1)\right]$, for some $n \subseteq \mathbb{N}$, then K^* satisfies

$$\underbrace{\frac{1-\rho}{(n+1)^2}\left[1+\left(\frac{K^*}{f}\frac{R^* A \nu(n)}{\alpha}-1\right)\frac{\nu(n)^{\rho}-1}{\nu(n)-1}\right]^{(1-\rho)/\rho}(K^*)^{\alpha-1}}_{F(K^*,f)} = \frac{R^* \nu(n)^{1-\rho}}{\alpha}$$

R^* is constant and independent of K^* and f. We have $\partial F(K^*, f)/\partial f < 0$. Thus, from the implicit function theorem, we have that K^* decreases in f if and only if $\partial F(K^*, f)/\partial K^* < 0$, which is equivalent to

$$1-\alpha > \frac{1-\rho}{\rho}\frac{1+\eta^*(\nu(n)-1)}{1+\eta^*\left(\nu(n)^{\rho}-1\right)}\frac{\nu(n)^{\rho}-1}{\nu(n)-1}$$

which is satisfied given the conditions of Lemma 2 (see proof of Lemma 2). ■

It is possible to give an analytical characterization of the steady-state capital stock K^* when it is characterized by full symmetry across industries. Proposition 17 says that, when fixed costs are within a certain interval, the steady-state will be characterized by n firms in all industries.

Proposition 17. *(Symmetric Steady-State) Let $\underline{f}^*(n)$ and $\overline{f}^*(n)$ be defined as*

$$\begin{aligned}
\underline{f}^*(n) &:= \frac{1-\rho}{(n+1)^2} A \left[\frac{n-(1-\rho)}{n} A \min\{\alpha\beta, 1-\alpha\}\right]^{\alpha/(1-\alpha)} \left(\frac{n}{n+1}\frac{n+\rho}{n-(1-\rho)}\right)^{\rho/(1-\rho)} \\
\overline{f}^*(n) &:= \frac{1-\rho}{n^2} A \left[\frac{n-(1-\rho)}{n} A \min\{\alpha\beta, 1-\alpha\}\right]^{\alpha/(1-\alpha)}
\end{aligned}$$

The economy converges to a symmetric steady-state with n firms in all industries if

$$f \in \left[\underline{f}^*(n), \overline{f}^*(n)\right] \tag{36}$$

In this case, the steady-state is equal to

$$K^* = \left(\frac{n - (1-\rho)}{n} A \min\{\alpha\beta, 1-\alpha\} \right)^{1/(1-\alpha)} \tag{37}$$

Proof. When n firms are active in all industries, the aggregate factor share is $\sigma = \dfrac{n-(1-\rho)}{n}$. Combining the definitions of $\underline{K}(n)$ and $\overline{K}(n)$ from Proposition 15, with (34) and (35) from Proposition 16, (36) and (37) obtain. ∎

Equation (37) is the counterpart of (16). In an equilibrium with n firms in every industry, the aggregate factor share is $[n-(1-\rho)]/n$, while in the model of section 2 it was equal to γ.

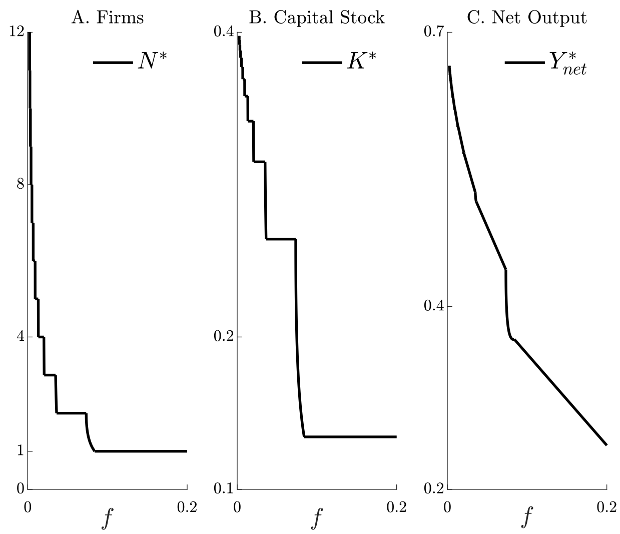


Figure 8: Condition for overaccumulation of capital

Figure 8 shows the steady-state values of the number of firms (panel A), the capital stock (panel B) and net output (panel C) as a function of fixed costs f. Higher fixed costs result in a lower (steady-state) number of firms, capital stock and net output.

Overaccumulation of Capital With fixed costs, the definition of capital overaccumulation must be adjusted. It will reflect the impact of a marginal increase in investment on net output.

Definition. (Overaccumulation of Capital) The bubbleless steady-state K^* features overaccumulation of capital if

$$\left.\frac{\partial Y^{\text{net}}}{\partial K}\right|_{K=K^*} < 1$$

Contrarily to the model of section 2, it is not possible to give a full analytical characterization of the conditions for capital overaccumulation. One can only characterize these conditions in the particular case of a steady-state with identical industries. Proposition 18 states that, if the economy converges to a steady-state with $n \in \mathbb{N}$ firms in all industries, it will be characterized by excessive capital when fixed costs are sufficiently low.

Proposition 18. *(Conditions for Overaccumulation of Capital) Suppose that the economy converges to a steady-state where all industries are symmetric and have $n \in \mathbb{N}$ firms. Then, the bubbleless steady-state features overaccumulation of capital if*

$$\max\left\{\frac{1}{\beta}, \frac{\alpha}{1-\alpha}\right\} < 1 \tag{38}$$

and

$$f < A\left[1 - \max\left\{\frac{1}{\beta}, \frac{\alpha}{1-\alpha}\right\}\right]^2 \frac{(A\alpha)^{\alpha/(1-\alpha)}}{1-\rho} \tag{39}$$

Proof. In a symmetric equilibrium with n firms per industry, we have $\partial Y/\partial K = \alpha A K^{\alpha-1}$. From (37), we have $\partial Y/\partial K|_{K=K^*} = \left[\frac{n-(1-\rho)}{n}\min\left\{\beta, \frac{1-\alpha}{\alpha}\right\}\right]^{-1}$. Using $\max\left\{\frac{1}{\beta}, \frac{\alpha}{1-\alpha}\right\} < 1$, the condition for capital overaccumulation becomes equivalent to

$$n > (1-\rho)\left[1 - \max\left\{\frac{1}{\beta}, \frac{\alpha}{1-\alpha}\right\}\right]^{-1} \equiv \underline{n}$$

Combining this condition with $f < \overline{f}^*(\underline{n})$, the condition obtains. ■

Let us interpret Proposition 18. First note that, when (38) is not satisfied, there is never overaccumulation of capital (since $R^* > 1$). Second, when (38) is satisfied, the steady-state features overaccumulation when fixed costs are low. Suppose, for example, that fixed costs take initially a value $f \in \left(\underline{f}^*(1), \overline{f}^*(1)\right)$, but then decrease to some $f' \in \left(\underline{f}^*(2), \overline{f}^*(2)\right)$. The initial steady-state is characterized by a monopoly in all industries, but the new steady-state is characterized by a full set of duopolies. From Proposition 17, the new steady-state level of capital is greater than the previous one, i.e.

$$\underbrace{\left(\rho A \min\{\alpha\beta, 1-\alpha\}\right)^{1/(1-\alpha)}}_{K^*} < \underbrace{\left(\frac{1+\rho}{2} A \min\{\alpha\beta, 1-\alpha\}\right)^{1/(1-\alpha)}}_{K'^*}$$

implying

$$\underbrace{\frac{1}{\rho}\max\left\{\frac{1}{\beta},\frac{\alpha}{1-\alpha}\right\}}_{\partial Y/\partial K|_{K=K^*}} > \underbrace{\frac{2}{1+\rho}\max\left\{\frac{1}{\beta},\frac{\alpha}{1-\alpha}\right\}}_{\partial Y/\partial K|_{K=K'^*}}$$

Proposition 18 gives the conditions for overaccumulation when all industries are symmetric. When industries are not identical, it becomes harder to give an analytical characterization of $\partial Y^{\text{net}}/\partial K|_{K=K^*}$. One can however obtain a numerical characterization. Figure 9 shows $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ as a function of f (for fixed values of ρ, α and β). Some aspects are worth mentioning. First, $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ is not defined at $\left\{\underline{f}^*(n), \overline{f}^*(n)\right\}_{n\in\mathbb{N}}$ and is flat at $f \in \left(\underline{f}^*(n), \overline{f}^*(n)\right)$.[22] Second, the value of $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ in a symmetric steady-state increases in f (as already discussed). Third, within $f \in \left(\overline{f}^*(n), \underline{f}^*(n+1)\right)$, $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ happens to be decreasing in f. To understand this, note that an increase in f has a dual impact on $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ in these intervals. On the one hand, as f increases, K^* declines; because of decreasing returns, this translates into a higher marginal product of capital. On the other hand, as f increases, more resources are absorbed in fixed costs whenever entry increases. Note that a marginal increase in K will boost entry when $f \in \left(\overline{f}^*(n), \underline{f}^*(n+1)\right)$; as a consequence, $\partial Y^{\text{net}}/\partial K|_{K=K^*}$ may actually decrease in these intervals.

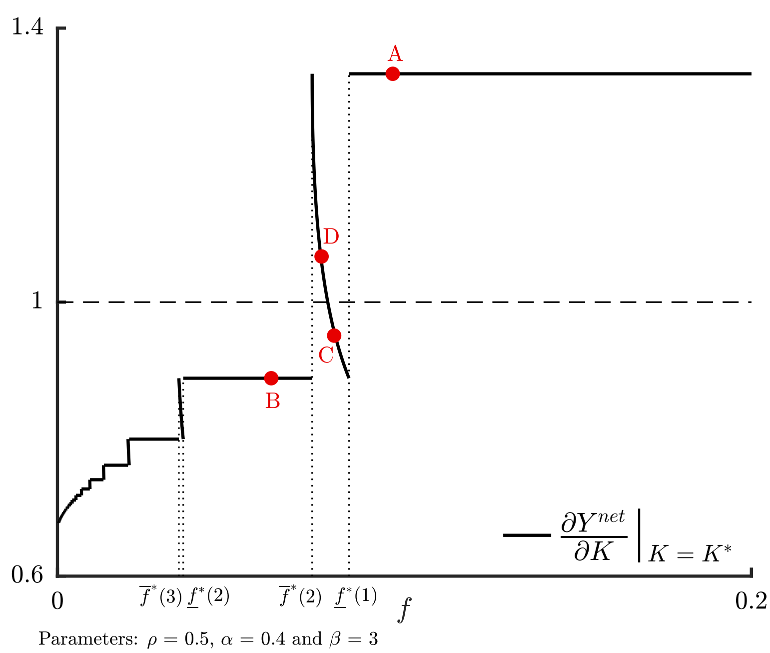


Figure 9: Condition for overaccumulation of capital

[22] For any $f \in \left(\underline{f}^*(n), \overline{f}^*(n)\right)$, the steady-state K^* is constant across f (and given by (37)). In this region, a marginal increase in K has always the same impact on Y^{net}, since it does not affect entry.

Consider again Figure 9. In point A, where all industries consist of a monopoly, we have $\partial Y^{\text{net}}/\partial K|_{K=K^*} > 1$. In point B, where all industries consist of a duopoly, $\partial Y^{\text{net}}/\partial K|_{K=K^*} < 1$. An increase of fixed costs from B to A will hence make the economy transition from a steady-state with overaccumulation of capital to a lower steady-state without overaccumulation.

Let us now consider points C and D. In both cases we have $f \in \left(\overline{f}^*(2), \underline{f}^*(1)\right)$; thus, both economies converge to steady-states where some industries consist of a monopoly, while some others consist of a duopoly.[23] In point D we have $\partial Y^{\text{net}}/\partial K|_{K=K^*} > 1$, while in point C $\partial Y^{\text{net}}/\partial K|_{K=K^*} < 1$. Therefore, an increase in fixed costs from D to C will make the economy transition from a steady-state without capital overaccumulation, to a steady-state with capital overaccumulation. Note that, starting from any of the steady-states represented by C and D, a marginal increase in the capital stock will boost entry (contrarily to what happens in A and B). Since point C is characterized by higher f, more resources will be spent in fixed costs as entry increases. Therefore, a marginal increase in the capital stock will have a lower impact on net output in point C.

Possibility of Rational Bubbles

Proposition 19 characterizes the conditions for rational bubbles to be possible.

Proposition 19. *(Possibility of Rational Bubbles) Rational bubbles are possible if and only if* $\alpha < \dfrac{1}{2}$.

Proof. Using (35) and given that $\beta > 1$, it follows that $R^* < 1$ if and only if $\alpha < 1/2$. ■

As in the model of section 2, this condition only depends on the capital elasticity α.

Underinvestment Asset bubbles will be contractionary when, in a bubbleless equilibrium, all savings are converted into capital, i.e. $K_{t+1} = W_t$. However, as in the model of section 2, they can lead to higher investment if not all savings are converted into capital, so that $R_{t+1} = 1/\beta$. From (35), the bubbleless steady-state is characterized by $R^* = 1/\beta$ if and only if

$$\alpha < \frac{1}{1+\beta}$$

Therefore, when the previous condition is verified and $\partial Y^{\text{net}}/\partial K|_{K=K^*} > 1$, the steady-state K^* features underinvestment.

[23] The fraction of monopolies is higher in point C, since fixed costs are also higher.

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