Brothers in Arms: Monetary-Fiscal Interactions Without Ricardian Equivalence*

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Abstract

We study equilibrium outcomes in a model with monetary-fiscal interaction when Ricardian Equivalence fails and argue that the latter feature has fundamental consequences. First, there is no longer any clear distinction between active and passive fiscal policies - there is always some monetary-fiscal interaction. Second, the equilibrium displays local determinacy for a much larger range of monetary and fiscal rules than in representative agent models. Third, the Taylor principle no longer applies in the sense that it is neither necessary nor sufficient for local determinacy. Fourth, when Ricardian equivalence fails and the equilibrium is locally determinate, fiscal deficits are generally inflationary, and their timing matters. These results hold both in a simple three-equation New Keynesian model and in a medium-scale setting when agents have finite planning horizons.

JEL Codes: E32, E52, E62, E63

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1 Introduction

This paper examines monetary-fiscal policy interaction when Ricardian Equivalence fails, that is, when households perceive government debt to be net wealth. This – arguably realistic – feature is present in a very large set of models in modern macroeconomics, not least in the workhorse class of incomplete markets, heterogeneous agent models. We show that the absence of Ricardian Equivalence has profound implications for the existence and uniqueness of equilibrium as well as for the properties of equilibrium, such as the impact of fiscal transfer shocks, the importance of the timing of such shocks, or the effectiveness of policy to contain fiscally-driven inflation.

We study a perpetual youth model in which finitely-lived households operate in an otherwise standard sticky price New Keynesian economy – a framework used recently in important contributions of Angeletos, Lian and Wolf (2024), Angeletos, Lian and Wolf (2025) and earlier in Richter (2015) and Galí (2021). This approach offers an extremely transparent and tractable framework, allowing for analytical characterization of many results and building economic intuition when it comes to the mechanisms. Importantly, however, our findings are not specific to this model and apply more generally.

It is well-established that monetary and fiscal policy coordination is a requirement for macroeconomic stabilization. The classic contribution of Sargent and Wallace (1981) demonstrated the need for such coordination in a monetarist framework with nominal government debt finance where the monetary and fiscal policy authorities are connected through the consolidated public sector budget constraint. They showed that whether inflation is controlled by the monetary authority or not depends on which policy authority is "dominant," and that standard monetarist doctrines require monetary dominance. In another classic contribution, Leeper (1991) translated the concepts of Sargent and Wallace (1981) into "passive" and "active" policy authorities by considering policy regimes described by simple feedback rules. His work showed how combinations of active and passive authorities may allow for the existence of unique stable equilibria, multiple stable equilibria, or inexistence of stable rational expectations equilibria.

Much work that followed these classic contributions has analyzed the design of stabilization policies and monetary-fiscal interaction in representative agent New Keynesian frameworks, referred to as RANK models.[1] A key aspect of such RANK models is that government debt has no *direct* impact on household consumption choices due to Ricardian Equivalence. In NK models, this also implies that there is no direct impact of public debt on inflation dynamics. Inflation instead has direct consequences for the fiscal policymaker because of its impact on the real interest rates that the government must pay on its outstanding debt and because surprise changes in inflation affect the real value of outstanding debt.

Because of this one-way interdependence, the implications for monetary-fiscal coordination are

[1] Sargent and Wallace (1981) actually alluded to the importance of heterogeneous agents, see their Appendix A.

very stark in the RANK economy. In particular, local determinacy requires the two arms of government to settle on one of them being active and the other passive. When the monetary authority sets short-term nominal interest rates with a rule that satisfies the Taylor principle, the requirements for fiscal policy are very weak and simply entail the fiscal authority being willing to go marginally further than raising sufficient revenue to roll over the debt when it deviates from its target. In this case, debt-financed transfers have no real effects in the RANK model and their timing is irrelevant. Should the fiscal policymaker become active, which essentially implies refusing to adjust deficits to changes in debt, equilibrium uniqueness requires a potentially fundamental change in the monetary framework by implementing an interest rate rule that violates the Taylor principle. In this RANK-FTPL regime, an increase in the deficit financed through inflation induces a boom in the economy. Thus, the RANK model features a very discontinuous and abrupt form of policy coordination.

In a RANK setting, equilibrium multiplicity is the major threat to local determinacy. Such multiplicity arises when the fiscal authority stabilizes debt while the monetary authority does not adhere to the Taylor principle which implies that there are convergent paths of inflation and output dictated by sunspot expectations. Indeed, it is the Taylor principle that rules out such paths. Non-existence of bounded equilibrium instead is a relatively minor concern in the sense that it arises only in a relatively small part of the policy space: when the Taylor principle is violated while the fiscal authority is forever unwilling to adjust the primary surplus sufficiently to stabilize debt.

We show that each of these insights are fundamentally different when households have finite planning horizons and Ricardian Equivalence fails (a setting which we refer to as a HANK model).[2] The key difference between the HANK model and the RANK setting is that government debt impacts directly on consumption dynamics. Because of this aspect, the HANK model involves a two-way interaction between monetary and fiscal policy: Changes in government debt impact demand and inflation, and, as in the RANK setting, monetary policy has fiscal implications through inflation affecting real interest rates and the real value of nominal debt. Therefore, in this setting, there is always some *direct* monetary-fiscal interaction.

We first examine the equilibrium determinacy properties of the simple HANK model. We develop detailed analytical insights into how the failure of Ricardian Equivalence affects the roots of the characteristic polynomial which determine the local determinacy properties of the model. We discuss in detail the underlying economic mechanisms that underlie these determinacy properties. We also characterize in closed form the equilibrium dynamics of the model, focusing primarily on fiscal transfer shocks and monetary policy shocks. This allows us to develop a number of results.

To fix ideas, we summarize the monetary stance by an interest rate rule with the strength of the monetary policy response to changes in inflation indicated by the parameter $\phi_\pi \in \mathbb{R}_+$ and a fiscal rule

[2] We follow Angeletos, Lian and Wolf (2024) in referring to this model as HANK because the finite planning horizons can be thought of as a short cut to the modeling of binding liquidity constraints and allows one to model sizeable marginal propensities to consume (MPCs), see Farhi and Werning (2019).

which adjusts the primary surplus to changes in government with the parameter $\alpha_b \in [0,1]$. In the RANK model, the subsets of the policy parameter space that are consistent with local determinacy are disconnected and relate to $(\phi_\pi, \alpha_b) \in \mathcal{S}_{TP}^{RANK} \cup \mathcal{S}_{FTPL}^{RANK}$ where $\mathcal{S}_{TP}^{RANK} = ([1;\infty],[r/(1+r);1])$ and $\mathcal{S}_{FTPL}^{RANK} = ([0,1),[0,r/(1+r)])$ (r is the steady-state real interest rate). The first of these regions is the standard case in which the Taylor principle is satisfied and fiscal policy is passive, while the second region relates to the FTPL regime. Our first result is that there are no such distinct policy regions in the HANK setting (said alternatively, the regions define a connected set). Therefore, the usual distinction between active and passive policies does not apply to the HANK model. This result derives fundamentally from the two-way interaction between government debt and inflation dynamics in the HANK model (unless government indebtedness vanishes to zero).

Next, we show that local indeterminacy is *less* of a concern when Ricardian Equivalence fails, while non-existence of a bounded equilibrium is a *more serious* issue. Local indeterminacy of the equilibrium arises in RANK for policies belonging to the set $\mathcal{S}_{IDE}^{RANK} = ([0;1),(r/(1+r),1])$, i.e. when both policies are passive. We show that this set of policy parameters for which the equilibrium is locally indeterminate in the HANK model is a proper subset of $\mathcal{S}_{IDE}^{RANK}$.

In the RANK setting, local indeterminacy can arise when, in response to a pure expectational shock, the economy first reacts but ultimately converges back to the steady state. Conversely, such sunspot shocks can be ruled out when they take the economy on a divergent path away from the steady state. When fiscal policy is passive, taking debt back to the steady state after any shock, elimination of sunspots requires monetary policy to be such that demand and inflation diverge away from the steady state were a sunspot to be realized. This requires monetary policy to be active (that is, to satisfy the Taylor principle) which induces real interest rates to rise.[3] This leaves a large region in the parameter space with local indeterminacy, where expectations of higher future output result in bounded self-fulfilling equilibria.

In the HANK model, there are three new endogenous forces which tend to eliminate such convergent paths in response to expectational shocks. All three operate through the real wealth of households, determined by the effective stance of fiscal policy. First, higher expected inflation lowers real interest rates, and implies that households expect to accumulate wealth more slowly. Second, when realized, higher inflation devalues the real value of debt. Third, over time, the two previous forces accumulate and, if the government does not fully rebate the proceeds from the net-of-interest inflation tax, impacts on households' demand. We pin down these forces analytically and show how they combine to deliver the parametric condition for equilibrium determinacy.

This implies that in the HANK model, the Taylor principle is not necessary for the existence of

[3] Expectations of higher future output raise demand one-for-one today through the intertemporal Keynesian cross. A higher real interest rate then induces households to save, thus making current output rise less than future output which implies a rising (exploding) path of demand and inflation. Eliminating unbounded equilibria thus rules out the sunspot and delivers uniqueness of the "fundamental" solution.

a stable unique rational expectations equilibrium. We also show that it is not sufficient. The reason for this is that the subset of policy parameter space for which there is non-existence of a stable equilibrium in the HANK model is bigger than in the RANK model. In particular, as ϕ_π increases above unity, the requirements for the fiscal response to government debt become stricter. This result is very intuitive. Suppose that the central bank has a very aggressive stance on inflation while the fiscal authority does not react much to debt. In this case, a fiscal deficit stimulates private sector demand and therefore inflation. The rise in inflation triggers the central bank to increase the nominal interest rate, which induces even higher public debt, leading the economy on a divergent path.

In combination, these results imply that there is a much more intricate and less abrupt need for policy coordination in the HANK model. When the Taylor principle is satisfied, adopting a more aggressive monetary policy stance may require fiscal reforms too, a mechanism that is not present in the RANK economy. In addition, when $\alpha_b > r/(1+r)$, fiscal reforms may require monetary policy reforms but do not require extreme changes in central bank charters.

Next we show that in the HANK model, conditional on the equilibrium being locally determinate, fiscal deficits are in general inflationary, something that only happens in the FTPL region in the RANK model. Studying the response of the economy to fiscal transfer shocks, we uncover a close analytical parallel between the HANK model and the RANK-FTPL model. In the simple RANK-FTPL model, we show that in response to a fiscal transfer, there is a date 0 jump in public debt, output, and inflation and, from date 1 onward, a common persistence in each of these variables which is determined by the date 0 jump in public debt. The analytical representation of the solution in HANK turns out to be exactly identical, up to the persistence parameter. In the specific case where the central bank engages in a real interest rate peg, the two persistence parameters converge to the same value (of 1), and, as shown by Angeletos, Lian and Wolf (2024), the HANK and RANK models involve identical equilibrium responses of inflation and output to deficits.

This analytical symmetry does not necessarily mean that the positive properties of the equilibria are the same across these models. Specifically, we show that there is extreme sensitivity of what equilibria in RANK look like to even tiny variations in policy parameters. Instead, in the HANK economy, the properties vary much more smoothly with policy. This, of course, has important policy implications.

The analytical characterization of equilibrium allows us to prove a number of results. We show that the HANK model has the general property that, when the equilibrium is locally determinate, a more aggressive monetary policy stance leads to more persistent effects associated with a guaranteed worse inflation-output tradeoff. Moreover, our results demonstrate that in RANK-FTPL, conditional on $\alpha_b < r/(1+r)$, the fiscal stance impacts only on the date 0 jumps in inflation and output and ***not*** on the persistence of the responses of output and inflation. In HANK instead, changes in the fiscal policy rule have intuitive effects on the dynamics of debt (as well as demand and inflation).

In combination, these results paint a very different picture of monetary-fiscal coordination in the

HANK setting than in RANK. Furthermore, we show that the timing of fiscal deficits is crucial in HANK but irrelevant in RANK. In RANK-TP, fiscal deficits are neutral regardless of their timing. In RANK-FTPL, the inflationary and output effects of deficits announced and implemented today are identical to those of policies announced far out in the future (up to their present value being identical). In HANK instead, a surprise increase in the deficit stimulates current output and inflation, while announcements of future deficits may be inflationary but recessionary today. The timing matters in the HANK model because households perceive that those receiving transfers may not be the same households that have to pay for them be it through taxes or through inflation. A current transfer stimulates the economy exactly because the currently alive households perceive that future generations will partially finance it. An announcement of a future deficit instead may be recessionary because it is partially financed through a rise in current inflation, which reduces the real wealth of currently alive cohorts.

In the final part of the paper, we extend the setup to include many features present in so-called medium-scale models. This includes the introduction of sticky wages, productive capital, investment adjustment costs, long-term government debt, and interest rate smoothing. We show that our qualitative results are robust. In particular, in the medium scale HANK model, the Taylor principle is neither necessary nor sufficient and (surprise) fiscal deficits are inflationary (and stimulate output) whenever the equilibrium is locally determinate.

Related literature In a classic contribution, Sargent and Wallace (1981) argue that, because monetary and fiscal policies are connected through the government budget constraint, the equilibrium properties of rational expectations models depend on the coordination of monetary and fiscal policies. In particular, they show that whether deficits are neutral or not, and how monetary injections affect equilibrium inflation (and output), depend on which policy authority is "dominant." Assuming that policies can be described by rules, Leeper (1991) demonstrate that the dominance concept can be translated into "active" vs. "passive" policy authorities determined by the systematic policy response parameters, and that the combinations of such policies determine the existence, uniqueness and dynamic properties of rational expectations equilibria. Leeper (1991) derives results in a log-linearized model but Sims (1994) shows that they hold also more generally in a non-linear setting. These papers have had a fundamental impact on monetary economics and influenced the design of stabilization policies.

The literature on monetary-fiscal interactions that followed these two classic papers focused on Representative Agent settings and gave rise to very detailed discussions about the plausibility of the FTPL, the implications for monetary-fiscal policy coordination, and the selection criteria of equilibria. Classic contributions to this literature include Woodford (1995) who showed that the FTPL can generate price level determinacy even if the central bank pegs the nominal interest rate; Woodford (2001) who argued that fiscal deficits often may have inflationary effects due to unfunded fiscal programmes; Cochrane (2011) who argued that the use of the Taylor principle for the selection of equilibria in the New Keynesian model is problematic because it rules out equilibria that are not necessarily inconsistent with agent optimization; Bassetto (2002) who questioned aspects of the game-theoretic underpinning of the FTPL; and many others. A thorough and insightful review of this literature can be found in Canzoneri, Cumby and Diba (2011), and Cochrane (2023) covers in detail the theoretical background and implications of the FTPL in a lucid manner.

Our conceptual framework deviates from the earlier literature by replacing the representative agent setting with an overlapping generations model. In particular, we adopt a Blanchard-Yaari perpetual youth model, see Blanchard (1985) and Yaari (1965), embedded in an otherwise standard NK model with monetary-fiscal interaction. In this framework, Ricardian Equivalence fails because tax liabilities related to current spending obligations fall partially on future (unborn) cohorts. The analytical framework that we study is very similar to that in the very insightful contribution of Richter (2015), who also examines a NK model of monetary-fiscal interaction with finite planning horizons, but the focus of our paper is very different from his. In particular, Richter (2015) asks how trends in US public debt affect fiscal limits when Ricardian Equivalence fails, while we study how non-Ricardian Equivalence impacts the equilibrium properties of a NK model with monetary-fiscal interaction and the implications thereof for the determinants of inflation and the design of fiscal and monetary policies.[4] In a paper contemporaneous to ours, Dupraz and Rogantini Picco (2025) study a flexible price Blanchard-Yaari economy and characterize a fiscal limit beyond which monetary policy fails to stabilize prices.[5]

An important source of inspiration for our analysis is Angeletos, Lian and Wolf (2024) and Angeletos, Lian and Wolf (2025). In the latter of these papers, they demonstrate that there are circumstances under which fiscal deficits can be entirely self-financing. Closer to our paper, Angeletos, Lian and Wolf (2024) also study monetary-fiscal interaction in a NK model with finite planning horizons where monetary and fiscal policies are described by interest rate rules and deficit rules. They show how the NK model with finite planning horizons has similar predictions to the FTPL in terms of the relationship between deficits and inflation, but side-steps many of the controversial features of the FTPL because of the direct impact of deficits on aggregate demand in the setting without Ricardian Equivalence. Angeletos, Lian and Wolf (2024) also demonstrate that, in general, the model with non-Ricardian consumers implies more front-loading of the inflationary response to deficits than the RANK-FTPL theory. Our paper complements theirs by providing a more general characterization of the equilibrium properties of the non-Ricardian Equivalence model in terms of the combinations of monetary and fiscal policies that deliver local determinacy and closed-form

[4] Interestingly, in their seminal paper Sargent and Wallace (1981) actually also outline an overlapping generations model for explaining why there might be a limit to the amount of interest rate bearing debt that a fiscal authority can create.

[5] See also Leith and Wren-Lewis (2000) who study monetary-fiscal interactions in the context of the EMU.

solutions for the output and inflation responses to monetary and fiscal shocks. These allow us to sharply characterize how non-Ricardian Equivalence impacts the economy. We also discuss how pre-announced deficits in the model without Ricardian Equivalence may be inflationary but, at the same time, recessionary, an aspect that RANK-FTPL cannot generate.

Aguiar, Amador and Arellano (2024) also study monetary and fiscal policies in an OLG cum NK framework. Their analysis builds on the assumption that government debt is real so that the inflation channels between monetary and fiscal policy that we emphasize are absent. On the other hand, like in our analysis, there is a direct link between government debt and household demand deriving from the non-Ricardian Equivalence feature of the model. They analyze the optimal level of debt in that setting and how monetary policy should be designed in the light thereof.

Canzoneri and Diba (2005) study a flexible price model with monetary-fiscal interaction and derive conditions for local determinacy of equilibria that share a number of aspects of our results. These authors do not introduce non-Ricardian Equivalence features through finite planning horizons, but alternatively assume that government bonds provide liquidity services. Because of these liquidity services, the pricing of bonds induces properties of the Euler equation similar to the finite planning horizon model analyzed here, see also the discussion in Canzoneri, Cumby and Diba (2011). Our analysis goes further in pursuing the underlying economic mechanisms and in showing how the link from government deficits to aggregate demand shapes outcomes in the face of monetary-fiscal interaction.

More generally, a substantial amount of work has investigated how finite planning horizons have implications for fiscal and monetary policies. Indeed, the original contribution of Blanchard (1985) shows how government debt and deficits affect aggregate demand when households have finite planning horizons. A number of papers instead study fiscal policy in settings without Ricardian Equivalence features that derive from household heterogeneity as in the Campbell and Mankiw (1989) savers-spenders framework. Canova and Ravn (2000) use such a framework to look at the impact of social insurance through transfers, Galí, López-Salido, and Vallés (2007) look at the impact of government purchases, while Bilbiie (2009) use such a framework to consider the Taylor principle and optimal monetary policy constraining fiscal policy to be passive. Our analysis goes further in terms of analyzing how monetary-fiscal coordination matters for the equilibrium properties of the NK model. Finite planning horizons and overlapping generations models are also an often-studied framework in monetary economics, a recent example being Galí (2021) who studies how monetary policy can be designed to prevent the emergence of asset price bubbles.[6] Rachel and Summers (2019)

[6] Galí (2021) studies a Blanchard-Yaari model with retirement shocks as in Gertler (1999) in a NK setting. He shows that rational bubbles in this setting can arise only when there are nominal rigidities and that they influence real activity. In Galí (2014) he studies a two-period OLG model where bubbles have redistributive effects only. An important difference between Galí (2021) and our analysis is that we assume that firms are infinitely lived and that equity is not traded, while Galí (2021) assumes that households are firm owners and that their financial wealth consists of equity. In that set-up, the aggregate Euler equation reproduces the standard infinite horizon one.

use the Blanchard-Yaari-Gertler setting to study the evolution of the neutral real interest rates across advanced economies, and Beaudry, Cavallino and Willems (2024) use a similar model to argue that monetary policy itself can be a driver of the neutral rate.

A more recent literature based on models with idiosyncratic risk and incomplete markets in NK settings, so-called HANK models, has also considered the interaction between monetary and fiscal policies and examined how non-Ricardian Equivalence impacts on equilibrium properties of NK models. Kaplan, Moll and Violante (2018) study the impact of monetary policy shocks in a rich HANK setting and find that it is sensitive to the modeling of fiscal policy. Wolf (2025) derives conditions under which uniform fiscal transfer policy on its own can deliver the same output and inflation outcomes as interest rate policy in a HANK model. Bayer, Born and Luetticke (2023) instead study the impact of changes in public debt in an estimated HANK model with monetary-fiscal interaction. Our focus is different from these papers, but our results may be helpful for understanding some of their insights because the model that we study, although much simpler than the models in these papers, captures some of the same underlying economic forces that work through the introduction of sizeable MPCs.[7] Parts of this literature have also shown how deviations from Ricardian Equivalence invalidate the Taylor principle. Ravn and Sterk (2021), for example, show that depending on the cyclicality of idiosyncratic risk, monetary policy may have to be more or less aggressive than prescribed by the Taylor principle in order to bring about local determinacy of the equilibrium. Cui (2016) studies monetary-fiscal interactions in a model with idiosyncratic investment risk and endogenous liquidity. In his setting, higher issuance of perfectly liquid government debt raises investment. His work complements our paper, which focuses on the household side of the economy.

Bianchi, Faccini and Melosi (2023) introduce a distinction between funded and unfunded debt in a NK model. In their setting, the fiscal authority is committed to stabilizing deviations of debt from its unfunded level, but this is not the case for unfunded debt, and the monetary authority is assumed to be willing to accept some fiscal inflation (and therefore not satisfy the Taylor principle in response to such shocks). They show that in this setting, unfunded debt shocks can generate persistent inflation. Bigio, Caramp and Silva (2024) instead study a setting with uncertainty about monetary-fiscal reform, where such a reform is associated with a temporary inflation-financing of debt. It is shown that while there is uncertainty about such a reform, the Taylor principle is modified in a fashion similar to our results. Our analysis alternatively assumes that there is a route from fiscal shocks to inflation through demand due to the failure of Ricardian Equivalence. Therefore, in our setting, the inflationary-deficit nexus is a persistent feature of the equilibrium.

Much empirical work has investigated the potential fiscal origins of inflation and the evolution of policy regimes. Woodford (2001) points towards an episode from the early 1940s to the early

[7] Farhi and Werning (2019) argue that the perpetual youth model can be seen as a short-cut to the presence of liquidity constraints present in more sophisticated HANK models. For this reason, along with Angeletos, Lian and Wolf (2024), we will also refer to the NK model with finite planning horizons as a "HANK" model.

1950s when the U.S. Federal Reserve implemented an interest rate peg (in the form of bond price support policies) which would have led to FTPL type dynamics. Davig and Leeper (2006) estimate a regime switching model for US monetary and fiscal policies and find several switches over time. In particular, they find that monetary policy was consistently passive from 1948 until the end of the 1970s and thereafter becomes active with the exception of a more accommodative stance during the early 1990s and the early 2000s recessions. Leeper and Leith (2016) provide a thoughtful review of the empirical literature and the extent to which it allows one to reach strong conclusions about monetary vs. fiscal dominance. More recently, Bianchi, Faccini and Melosi (2023) argue that the fiscal transfers in the U.S. that followed after the Covid-19 pandemic were unfunded and a significant contributor to the post-pandemic rise in inflation. Relatedly, Barro and Bianchi (2023) find that, for a cross-section of 37 OECD countries, up to 80 percent of the fiscal expansion in the 2020-23 period was financed through inflation.

2 A Simple Model

We start by focusing on a simple New Keynesian model with monetary-fiscal interaction through the government budget constraint in which households are non-Ricardian. This model has significant analytical appeal, allows us to derive a number of closed form results, and makes it feasible to tease out the underlying economic forces in a lucid manner. We later extend the model with many features included in medium-scale models in order to explore more quantitative implications.

2.1 The Environment

2.1.1 Households

We examine an overlapping generations economy in the fashion of the Blanchard-Yaari perpetual youth model, see Yaari (1965) and Blanchard (1985), also studied recently in the fiscal-monetary interaction literature by Richter (2015), Angeletos, Lian and Wolf (2024). Angeletos, Lian and Wolf (2025), Aguiar, Amador and Arellano (2024), or Dupraz and Rogantini Picco (2025). At date t, a continuum of newborn agents of mass $X_{t,t}$ enter the economy. Agents face constant mortality risk $(1-q) \in (0,1]$ between any two consecutive periods.[8] We normalize the mass of the total population to 1 and assume that the cohort sizes are constant. The mass of newborn agents therefore equals $(1-q)$ and the measure of cohort $t \leq s$ households that are alive at date s follows as $(1-q)q^{s-t}$. Households are non-altruistic, have rational expectations, and derive utility from consumption of goods and leisure. The OLG structure introduces non-Ricardian Equivalence features and, as argued by Farhi and Werning (2019), can be seen as a short-cut to a richer incomplete markets setting.

[8] Moll, Rachel and Restrepo (2022) provide a range of reinterpretations, or microfoundations, of this shock, including wealth dissipation, capital income risk, population growth, or discount factor shocks.

Household preferences are given as:

$$\mathbf{U}_{s,t} = \mathbb{E}_t \sum_{h=0}^{\infty} (\beta q)^h \left(\log c_{s,t+h} + \psi \log \left(1 - n_{s,t+h}\right)\right) \tag{1}$$

where $s \leq t$ denotes the date at which the agent was born, $\mathbb{E}_t x_{t+h}$ denotes the mathematical expectation of x_{t+h} given all information available at date t, $\beta \in (0,1)$ is households' subjective discount factor, $c_{s,t}$ is the date t consumption of a cohort s household, $n_{s,t}$ denotes hours worked, and $\psi \in (0,1)$ is a constant preference weight. Note that, due to the presence of survival risk, households discount future utility with $\beta q < 1$.

Households have access to life insurance contracts. At the end of every period, the household transfers its financial wealth to a life insurance company. The next period, the life insurance pays surviving households a life insurance premium equal to $1/q^i$ times their deposits in return for the life insurance company taking ownership of the assets deposited by households that did not survive. The life insurance industry is competitive and there is free entry; the life insurance premium is therefore actuarially fair, i.e. $1/q^i = 1/q \geq 1$.

Furthermore, we allow for a social fund, see Sterk and Tenreyro (2018). The social fund runs a balanced budget and makes transfers to newborn agents financed by taxing "old" households. The transfer to each newborn agent is set as a constant proportion of the steady-state wealth of old households at a level so that, in steady state, all agents are equally wealthy. Due to this latter aspect, the presence of the social fund implies that the OLG structure does not impact the steady-state real interest rate which is determined by the intertemporal discount factor as in standard infinite horizon models.[9]

Households earn labor income from working, asset income from their savings, make contributions to the social fund, and may also pay lump-sum taxes (or receive lump-sum transfers). They can purchase one-period nominal government bonds, which we specify as pure discount bonds. Bonds are purchased at date t at the price $1/(1+i_t)$ and pay out one unit of currency at date $t+1$ (i.e. i_t is the net nominal interest rate). The flow budget constraint facing an agent of cohort s at date t is:

$$P_t c_{s,t} + \frac{B_{s,t}}{(1+i_t)} - P_t \tau_t = W_t n_{s,t} + \frac{B_{s,t-1}}{q} + P_t z_s + P_t d_t \tag{2}$$

where P_t denotes the price level, $B_{s,t}$ are purchases of government debt, τ_t are real lump-sum taxes, W_t is the nominal wage, $P_t d_t$ are nominal profit transfers from firms,[10] and z_s is the real social fund

[9]Because of this feature, the steady-state real interest rate in our model is independent of the level of government debt. Other lines of work have instead emphasized the potential importance of the long-run interest-rate debt nexus due to debt impacting on aggregate demand, see e.g. Hagedorn (2021) for an incomplete markets setting, or Mian,Straub and Sufi (2024) for a TANK model with downward sticky nominal wages where public debt provides a convenience yield.

[10]We assume that profit income from firms are equally distributed across households and that equity cannot be

transfer. Notice that the life insurance premium implies that the effective return on savings for agents who survive between two consecutive periods includes a capital gain, $(1-q)/q$.

We denote $v_{s,t} = B_{s,t}/P_t$ as the end of period t real wealth of the household so that the flow budget constraint can be expressed as:

$$c_{s,t} + \frac{v_{s,t}}{(1+i_t)} - \tau_t = w_t n_{s,t} + \frac{v_{s,t-1}}{q\pi_t} + z_s + d_{s,t}. \tag{3}$$

$\pi_t = P_t/P_{t-1}$ is the gross inflation rate between period $t-1$ and period t, and w_t is the real wage.

2.1.2 Firms

The supply side of the economy follows the usual New-Keynesian setup. Final output is produced by competitive firms using inputs of intermediate goods produced by monopolistically competitive firms that set prices subject to nominal price rigidity.

Final Goods Producers: There is a single homogeneous final good produced by competitive firms using a continuum of intermediate goods, $y_{j,t}$. The final good technology is given as:

$$y_t = \left(\int_0^1 y_{j,t}^{1-1/\phi} dj \right)^{1/(1-1/\phi)} \tag{4}$$

where $y_{j,t}$ denotes the input of intermediate good j and $\phi > 1$ is the elasticity of substitution between intermediate goods.

Intermediate Goods Producers: Intermediate goods producers are monopolistically competitive. They produce the intermediate goods with a linear technology:

$$y_{j,t} = n_{j,t} \tag{5}$$

where $n_{j,t}$ denotes their input of labor. Intermediate goods producers rent labor on a spot market at the nominal wage W_t which they take for given. They set the price of their product subject to a nominal rigidity which we specify by a standard Calvo staggered price contracting framework where $1-\theta \in [0,1]$ corresponds to the probability that an individual producer can adjust the price of its product in a given period.

2.1.3 Monetary and Fiscal Policy

There are two policy authorities, a monetary authority and a fiscal authority. The monetary authority sets the nominal interest rate while the fiscal authority is in charge of the primary surplus, issues

traded.

nominal bonds, and runs the social fund. We specify their actions with simple policy rules.

The Monetary Authority: We assume that monetary policy is dictated by a standard Taylor-type interest rate rule:

$$1 + i_t = \bar{R} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}} \right)^{\phi_x} \exp\left(e_t^i\right) \tag{6}$$

where $\bar{R}$ is a constant that is equal to the steady-state natural interest rate, $\bar{\pi}$ is an inflation target, ϕ_π captures the monetary policy response to deviations of inflation from its target, $\bar{y}$ is an output target, ϕ_x captures the central bank's response of the short-term nominal interest rate to deviations of output from its target, and e_t^i is an i.i.d. monetary policy shock with expectation 0. We impose that $\phi_\pi, \phi_x \geq 0$.

The Fiscal Authority: The fiscal authority chooses deficits, is in charge of nominal bond issuance, and runs the social fund. The social fund is assumed to be fully funded. The social fund payments are given as:

$$z_{s,t} = \begin{cases} v^{ss} & \text{for } s = t \\ -\frac{1-q}{q} v^{ss} & \text{otherwise} \end{cases}$$

where v^{ss} denotes steady-state per-capita real wealth. Because this fund is run separately and is balanced, it does not impact directly the government budget constraint.

The government budget constraint can be expressed as:

$$\frac{B_t}{(1 + i_t)} = B_{t-1} - P_t s_t \tag{7}$$

where $s_t = -\tau_t$ denotes the real budget surplus. Using the definition of real debt introduced in the household section, the government budget constraint in real terms can be expressed as:

$$\frac{v_t}{(1 + i_t)} = \frac{v_{t-1}}{\pi_t} - s_t \tag{8}$$

We assume that the government operates the following surplus rule:

$$s_t = \bar{s} \left(\frac{v_{t-1}}{\bar{v}} \right)^{\alpha_b} \left(\frac{y_t}{\bar{y}} \right)^{\alpha_x} \exp\left((1 - \alpha_b)\, e_t^s \right) \tag{9}$$

where $\bar{s}$ is a constant, α_b captures feedback from deviations of real government debt from its target, $\bar{v}$, to the surplus, α_x captures feedback from deviations of output its target to the deficit, and e_t^s is a shock to the surplus with mean 0. Consistency between the surplus rule and the law of motion of government debt implies that $\bar{s} = r/(1 + r)\bar{v}$ where r is the steady-state net real interest rate. We impose that $\alpha_b \in [0, 1)$.

2.2 Optimization

2.2.1 Households

Households of cohort s maximize (1) subject to a sequence of budget constraints given by (3) and to a no-Ponzi game restriction. The first-order necessary conditions can be expressed as:

$$\begin{aligned} w_t n_{s,t} &= w_t - \psi c_{s,t}, & (10) \\ \frac{1}{c_{s,t}} &= \beta \mathbb{E}_t \frac{1}{c_{s,t+1}} \frac{1+i_t}{\pi_{t+1}}, & (11) \end{aligned}$$

in addition to their budget constraint. These two equations determine households' labor supply and savings decisions. Importantly, due to the availability of life insurance, although households have finite planning horizons, the capital gain on savings that accrues to households that survive between periods t and $t+1$ neutralizes the impact of mortality risk on their impatience, and (11) therefore involves only the standard intertemporal discount factor and not the survival probability.

Iterating equation (3) forwards, using the intertemporal savings condition, and imposing a no-Ponzi game restriction, we can express time-t consumption of households born at $s < t$ as:

$$c_{s,t} = (1 - q\beta) \left(\frac{v_{s,t-1}}{q\pi_t} + \mathbb{E}_t \sum_{j=0}^{\infty} R_{t,t+j} \tilde{y}_{s,t+j} \right) \tag{12}$$

where $R_{t,t+j} = R_{t,t+j-1} \frac{q}{R_{t+j-1}}$, $R_{t,t} = 1$, $R_{t+j} = \frac{1+i_{t+j}}{\pi_{t+j+1}}$, and $\tilde{y}_{s,t} = w_t n_{s,t} + \tau_t + z_{s,t} + d_t$.

The perpetual youth assumption means that we can aggregate across households in a simple manner. Due to the availability of actuarially fair life insurance and the presence of the social fund, the *steady-state* of the economy is unaffected by the finite planning horizon so that the steady state interest rate equals $\frac{1}{\beta}$ irrespectively of q. Nonetheless, as we will see, debt dynamics affect private sector choices in response to shocks to the economy.

2.2.2 Final Goods Producers

Final goods producers are competitive, minimize costs, and take prices of inputs for given. They purchase intermediate goods at prices $P_{j,t}$ and sell the final good at marginal cost. Given the CES technology assumption, the intermediate goods demand is given as:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\phi} y_t \tag{13}$$

where $P_t = \left(\int_0^1 P_{j,t}^{1-\phi} dj \right)^{1/(1-\phi)}$ which is the price of the final good.

2.2.3 Intermediate Goods Producers

Intermediate goods producers set the price of their product, $P_{j,t}$, subject to the goods demand given above in (13) taking into account that they may be unable to change prices next period. Given the technology specified earlier, real marginal costs are determined by the real wage. The price chosen by a firm given the chance to reoptimize is therefore given as:

$$p_t^* = \frac{\frac{\phi}{\phi-1}\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{1}{c_{t+s}} \left(\frac{P_{t+s}}{P_t}\right)^{1+\phi} y_{t+s} w_{t+s}}{\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta_p)^s \frac{1}{c_{t+s}} \left(\frac{P_{t+s}}{P_t}\right)^{\phi} y_{t+s}} \tag{14}$$

where $\theta_p \in (0,1]$ is the probability that the firm cannot reoptimize the price of its product, and c_t denotes aggregate consumption.

2.2.4 Equilibrium

In equilibrium, households maximize utility subject to their budget constraints, firms maximize profits, fiscal and monetary authorities follow their respective policy rules, and all markets clear.

2.3 Aggregation

We define aggregate per-capita variables as:

$$x_t = \frac{\sum_{h=0}^{\infty} N_{t-h,t} x_{t-h,t}}{N}$$

where $N_{t-h,t} = (1-q)q^h N$ is the size of the cohort $t-h$ that is alive at the date t and N is the total (constant) population size, which we normalize to 1.

Consumption of the newborn agent. $c_{t,t}$ is the consumption of the newborn agent. Since at birth they receive $z_0 = v_{ss}$, we have:

$$c_{t,t} = (1-q\beta)\left[v^{ss} + \mathbb{E}_t \sum_{h=0}^{\infty} \mathcal{R}_{t,t+h}\left(y_{t+h} + \tau_{t+h} + d_{t+h} + \tilde{z}_{s,t+h}\right)\right]. \tag{15}$$

where

$$\tilde{z}_{s,t+h} = \begin{cases} 0 & if\ h = 0 \\ -\frac{1-q}{q} v^{ss} & if\ h > 0 \end{cases}$$

is the payment, from next period on, to the social fund.

Aggregate consumption. Combining the consumption function for the newborns together with the surviving cohorts, we find:

$$c_t = q \cdot c_t^{alive\ yesterday} + (1-q)c_t^{born\ at\ t} = (1-q\beta)\left(\frac{v_{t-1}}{\pi_t} + \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{R}_{t,t+j}\left(y_{t+j} + \tau_{t+j} + \tilde{z}_{s,t+j}\right)\right). \quad (16)$$

Aggregate Euler Equation. Combining the individual Euler equations and aggregating consumption across generations delivers the following "aggregate Euler equation"[11]:

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1} + \chi\left(\frac{v_t}{q\Pi_{t+1}} - \frac{v^{ss}}{q}\right)} \frac{1+i_t}{\Pi_{t+1}} \quad (17)$$

where we have defined the parameter χ as:

$$\chi = \frac{(1-q)(1-q\beta)}{q}$$

In steady state, the Euler equation is identical to the benchmark neoclassical growth model. This is a direct consequence of the assumptions we have made on the availability of life insurance and the presence of the social fund, and ensures that the steady state is unaffected by the degree of finite planning horizons.

The Euler equation is also identical to a standard representative agent model when $q = 1$. However, when $q < 1$, Ricardian Equivalence fails and government debt deviations from its steady-state affect aggregate consumption. This is because variations in public debt reallocates wealth between currently alive generations and future generations. In particular, for a given real interest rate, higher public debt raises demand today. Intuitively, when real public debt rises, the wealth of currently alive households rises relative to future households because currently alive generations perceive that they may not be alive when the rise in public indebtedness needs to be financed. Similarly, expected inflation impacts the intertemporal consumption allocation not only through real interest rates but also through the real value of government debt.

2.3.1 Log-linearization of the equilibrium conditions

We work with a log-linearized version of the model. We impose that the central bank's inflation target corresponds to price stability, $\bar{\pi} = 1$, and log-linearize around this non-inflationary steady state. Letting $\widehat{x}_t$ denote the percentage deviation of a variable x_t from this deterministic steady

[11] See Farmer, Nourry, and Venditti (2011) for details on its derivation.

state, the log-linearized equilibrium conditions are given as:

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - i_t + \chi\widehat{v}_t + (1-\chi\gamma)\mathbb{E}_t\widehat{\pi}_{t+1} \quad (18)$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t \quad (19)$$

$$\widehat{v}_t = \frac{1}{\beta}(\widehat{v}_{t-1} - \widehat{s}_t) + \gamma\left(i_t - \frac{1}{\beta}\widehat{\pi}_t\right) \quad (20)$$

$$\widehat{s}_t = \alpha_b(\widehat{v}_{t-1} - e^s_t) + \alpha_x\widehat{y}_t + e^s_t \quad (21)$$

$$i_t = \phi_\pi\widehat{\pi}_t + \phi_x\widehat{y}_t + e^i_t \quad (22)$$

where $\gamma = v^{ss}/y^{ss}$ is the steady-state public debt-to-GDP ratio, and $\kappa = ((1-\theta_p)(1-\beta\theta_p)/\theta_p)/(1-n^{ss})$ (where n^{ss} denotes aggregate steady-state hours worked per capita) is the slope of the "Phillips curve".

The log-linearized Euler equation for savings, equation (18), relates consumption growth to the real value of government debt, $\widehat{v}_t$, and adds an additional impact of expected inflation, $\chi\gamma\mathbb{E}_t\pi_{t+1}$, because of the non-Ricardian feature of the model discussed above. In the log-linearized model, the strength of the former of these effects depends on χ while the latter also depends on the steady-state public debt to GDP ratio. The latter aspect derives from the fact that the additional impact of expected inflation is scaled by the "tax base" which is γ. In the corner case $q = 1$, the model reproduces the standard New Keynesian model extended with a fiscal block. Equation (19) is the familiar forward-looking New Keynesian Phillips curve. Equation (20) is the log-linearized government budget constraint which relates the evolution of real government debt to budget surpluses and to the real costs of servicing the interest on public debt. Equation (21) is the fiscal rule for the surplus where α_b determines the strength of the feedback from variations in government debt to the surplus, and α_x the automatic feedback from output to the surplus. Finally, equation (22) is the monetary policy rule.

These equations jointly determine the dynamics of the economy in response to shocks to government transfers (deficits) and to monetary policy. It is straightforward to introduce many more shocks and frictions. We will do so in the last section of the paper, but will for now focus on the properties of this simpler model because it has considerable analytical appeal. In particular, it will allow us to characterize the determinants of the equilibrium properties and dynamic adjustment of the economy and how these depend on the failure of Ricardian Equivalence.

3 Determinacy Properties

In forward-looking models such as this one, *actual* current outcomes are intrinsically linked to *expectations* future outcomes. This feature presents a challenge to the uniqueness of equilibrium: non-fundamental shocks to expectations of future outcomes might be self-fulfilling and thus influence actual outcomes. If this is the case, the rational expectations equilibrium displays local indeterminacy, meaning that non-fundamental sunspot shocks cannot be ruled out. Moreover, not all combinations of monetary and fiscal rules are guaranteed to deliver a stable equilibrium in the sense that the economy will be prevented from diverging arbitrarily far away from the steady-state.

What delivers determinacy in the RANK environment is the pattern of systematic fiscal and monetary policy responses, which must be carefully designed to *rule out* such sunspot shocks by inducing diverging paths of the economy were non-fundamental expectational shocks to materialize. We will now examine how the non-Ricardian features impact on such conditions for policy rules. We will show that there are potentially fundamental implications of introducing this feature for how policy rules allow for the existence of a unique and (saddle-path) stable equilibrium.

3.1 The mathematics of determinacy

Mathematically, as we will first show, the determinacy properties depend on the roots of the characteristic polynomial of the log-linearized model. Next, we go beyond the mathematics and provide economic intuition for how the uniqueness of the equilibrium is achieved.

Substituting the surplus rule and the monetary policy rule into the Euler equation and into the law of motion for government debt to obtain a three-equation representation:

$$\widehat{v}_t = \frac{1}{\beta}(1-\alpha_b)\widehat{v}_{t-1} + (\gamma\phi_x - \alpha_x)\widehat{y}_t + \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t + \gamma e_t^i - \frac{1}{\beta}(1-\alpha_b)e_t^s \tag{23}$$

$$(1+\phi_x)\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \phi_\pi\widehat{\pi}_t + \chi\widehat{v}_t + (1-\chi\gamma)\mathbb{E}_t\widehat{\pi}_{t+1} - e_t^i \tag{24}$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t \tag{25}$$

Defining $z_t = [\widehat{v}_{t-1}, \widehat{\pi}_t, \widehat{y}_t]'$ and $e_t = [e_t^i, e_t^s]'$, the equilibrium conditions can be represented as:

$$\mathcal{A}\mathbb{E}_t z_{t+1} = \mathcal{B}z_t + \mathcal{C}e_t \tag{26}$$

which, when $\mathcal{A}$ is invertible, implies that the dynamics can be formulated as:

$$\mathbb{E}_t z_{t+1} = \mathcal{D}z_t + \mathcal{F}e_t \tag{27}$$

where $\mathcal{D} = \mathcal{A}^{-1}\mathcal{B}$ and $\mathcal{F} = \mathcal{A}^{-1}\mathcal{C}$.

The local determinacy properties of the model are determined by the roots of $\mathcal{D}$, which reflect the interdependencies between debt, output, and inflation. Since debt is backward-looking, while output and inflation are both forward-looking, local determinacy requires two roots of the characteristic polynomial of $\mathcal{D}$ to be outside the unit circle and one root to be inside the unit circle. The characteristic polynomial follows from:

$$\det(\mathcal{D} - \lambda\mathbb{I}_3) = 0$$

where $\mathbb{I}_3$ is a 3×3 identity matrix and λ is the vector of the eigenvalues.

3.2 Determinacy in RANK

To start, consider the familiar infinite-horizon RANK case, $\chi = 0$. Imposing this parameter restriction, the three-equation model becomes:

$$\widehat{v}_t = \frac{1}{\beta}(1-\alpha_b)\,\widehat{v}_{t-1} - \alpha_x \widehat{y}_t + \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t - \frac{1}{\beta}(1-\alpha_b)\,e_t^s \tag{28}$$

$$(1+\phi_x)\,\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \phi_\pi\widehat{\pi}_t + \mathbb{E}_t\widehat{\pi}_{t+1} - e_t^i \tag{29}$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t \tag{30}$$

We notice that the output and inflation block formed by equations (29)-(30) is independent of the fiscal block (28). However, the law of motion of government debt (28) depends on inflation and on output. Inflation impacts debt dynamics because government debt is nominal, so that a surprise increase in inflation depreciates government debt, and because inflation may impact real interest rates through the response of the monetary authority. Output impacts debt dynamics indirectly through its impact on inflation, and directly through government tax revenues as measured by α_x.

This one-way interdependence between inflation and debt dynamics matters for the determinacy properties of the model. In particular, the characteristic polynomial in RANK can be factorized into the product of two polynomials, $P_1(\lambda_1)$ and $P_2(\lambda_2, \lambda_2)$:

$$P(\lambda) = \underbrace{\left(-\frac{1}{\beta}(\alpha_b - 1) - \lambda\right)}_{P_1(\lambda_1)}\underbrace{\left(-\frac{1}{\beta}\left(-\beta\lambda^2 + (1+\beta+\kappa+\beta\phi_x)\,\lambda - (\phi_x + \kappa\phi_\pi + 1)\right)\right)}_{P_2(\lambda_2,\lambda_3)} \tag{31}$$

$P_1(\lambda_1)$ is a first-order polynomial that depends on the coefficients of the fiscal rule but is independent of the monetary policy rule, while $P_2(\lambda_2, \lambda_2)$ is a second-order polynomial that depends on the monetary policy rule, but not on the fiscal policy rule. Therefore, monetary-fiscal interaction leads to a particular stark combination of policy rules that deliver local determinacy. We summarize this in the following proposition which is well-known:

Proposition 1 *In the RANK model with monetary-fiscal interaction:*

1. ***(RANK-TP)*** *When $\alpha_b > 1 - \beta$ and $\frac{\kappa}{\beta}(\phi_\pi - 1) + \phi_x(\frac{1}{\beta} - 1) > 0$, there is local determinacy.*

2. ***(RANK-FTPL)*** *When $\alpha_b < 1 - \beta$ and $\frac{\kappa}{\beta}(\phi_\pi - 1) + \phi_x(\frac{1}{\beta} - 1) < 0$, there is local determinacy.*

3. *When $\alpha_b > 1 - \beta$ and $\frac{\kappa}{\beta}(\phi_\pi - 1) + \phi_x(\frac{1}{\beta} - 1) < 0$ there is local indeterminacy and there may be expectational equilibria.*

4. *When $\alpha_b < 1 - \beta$ and $\frac{\kappa}{\beta}(\phi_\pi - 1) + \phi_x(\frac{1}{\beta} - 1) > 0$, there is no stable rational expectations equilibrium.*

Proof: See the appendix.

We illustrate the determinacy properties of our models in Figure 1. The left panel corresponds to the RANK model. As indicated by Proposition 1, there are four different regimes that differ in the policy parameter constellations. These consist of two different regimes where there is local determinacy of the equilibrium, the RANK-TP regime and the RANK-FTPL regime, a regime with local indeterminacy, and a regime with non-existence of stable rational expectations equilibria.

In the RANK-TP regime, located in the north-east quadrant, the root of P_1 is inside the unit circle, $\alpha_b \geq 1 - \beta$, and fiscal policy is passive because the automatic adjustment of deficits to government debt is sufficient to bring about debt convergence at unchanged yields. In this case, fiscal policy by itself does not rule out sunspot equilibria and local determinacy is induced by the monetary policy rule. In particular, since both roots of P_2 must be outside the unit circle, monetary policy is active and satisfies the Taylor principle. Local determinacy is then induced by monetary policy committing to set the economy on an explosive path were extraneous output expectations to be realized. Such divergent paths, in turn, are ruled out thereby inducing local determinacy.[12]

The second region where there is local determinacy is the RANK-FTPL region, located in the south-west part of the left panel of Figure 1. In this "fiscal theory of the price level" region, fiscal policy is active and the root of P_1 is outside the unit circle, $\alpha_b < 1 - \beta$, while only one root of P_2 is outside the unit circle. The former of these implies that, at unchanged yields and in the absence of monetary actions, the fiscal authority's automatic adjustment of the deficit to changes in government debt is insufficient to bring about government debt convergence. From the latter, it follows that monetary policy is passive and the Taylor principle is violated so that the monetary authority is willing to accommodate inflation without moving short-term nominal interest rates much. This policy constellation brings about equilibrium uniqueness by allowing for well-defined equilibria in response to fundamental shocks while, at the same time, leading the economy on divergent paths in the case of non-fundamental shocks.

Consider first a non-fundamental expectational shock, for example, expectations of lower inflation, in the RANK-FTPL policy regime. With a weak response of nominal interest rates to inflation, such

[12] Consider for simplicity the case where $\phi_x = 0$. The Euler equation implies that $\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - (\phi_\pi\widehat{\pi}_t - \mathbb{E}_t\widehat{\pi}_{t+1})$, where the term in parentheses is the real interest rate. Suppose that the economy is in steady state and consider a sunspot shock to expectations of output at $t+1$. Through the Phillips curve, higher output expectations induce higher inflation and higher expected inflation. With $\phi_\pi > 1$ the central bank raises the nominal interest rate sufficiently that the real interest rate goes up. This implies that $\widehat{y}_t < \mathbb{E}_t\widehat{y}_{t+1}$. The pattern is repeated in future periods, meaning that output grows over time. Formally, it is easy to show (results available upon request) that output and inflation diverge if and only if $\phi_\pi > 1$. Thus, the monetary policy response induces a higher real interest rate which sets the economy onto an explosive path were non-fundamental expectations of future output and/or inflation to materialize. Ruling out such explosive paths then eliminates sunspot equilibria.

expectations induce a divergent government debt path because fiscal adjustments are insufficient to bring about debt convergence. Hence, extraneous shocks can be eliminated. What about a fundamental shock such as an increase in the government deficit? In the RANK model, consumers are Ricardian and deficits accordingly have no *direct* impact on output or inflation. To avoid government debt divergence, however, the deficit shock must be accompanied by expectations of a change in the price level that exactly allows for financing of the deficit shock. In the sticky price model, this corresponds to expectations of changes in inflation (accompanied by expected output changes) that precisely deliver a bounded path for government debt. Thus, inflation rises in such a way that the change in the deficit gets financed over time through inflation (which affects the real value of government debt and yields). This ensures that a fundamental shock to the deficit is consistent with a *bounded* equilibrium. The implication is that, even though consumers are Ricardian, government deficit shocks influence inflation and output dynamics in the RANK-FTPL region.

This tendency of debt to diverge in the absence of monetary responses also drives the equilibrium dynamics following other shocks. For example, a monetary policy shock affects the interest rate cost of government debt, which, if left unchecked, would diverge. Again, a particular expectational shock is needed to deliver convergence of debt back to steady state. For example, an expansionary monetary policy shock (a nominal rate cut) delivers a windfall gain for the government by lowering its financing costs, and so it must compensate debt holders by a decline in inflation that leads debt back towards the steady state. This fall in inflation occurs through a negative expectational shock to future output, which lowers inflation today (even if contemporaneously output booms, as a result of lower interest rates).

The two remaining regions have either many or no bounded equilibria. When both monetary and fiscal policy are passive, i.e. when the Taylor principle is violated and $\alpha_b > 1 - \beta$, there is local equilibrium indeterminacy. In this regime, located in the north-west part of Figure 1, one cannot rule out other types of stable equilibria. In particular, there may be self-fulfilling expectations of inflation that neither monetary nor fiscal policy are sufficiently aggressive to eliminate. Monetary policy allows real interest rates to decline in response to extraneous expectations of higher output and inflation, raising today's output by more than the future output, and thus resulting in a convergent path of inflation and output. Government debt convergence is guaranteed by the elastic response of fiscal surpluses to any changes in government debt. In effect, sunspot shocks result in bounded non-fundamental equilibria, implying indeterminacy.

Finally, when both policies are active, the equilibrium is unstable and any shocks to the economy will lead the economy on a divergent path from the steady-state. Consider a positive deficit shock in this regime. Given that $\alpha_b < 1 - \beta$, this shock requires the central bank to engineer higher inflation to induce government solvency. However, were inflation to rise, the central bank would respond by raising the nominal rate aggressively, which increases the real cost of financing the deficit. This takes the economy on an unstable path with ever-increasing debt and inflation that diverges from the steady-state.

3.3 Determinacy in HANK

We now allow for non-Ricardian Equivalence through the introduction of a finite planning horizon. Here we initially concentrate on the case in which $\alpha_x = \phi_x = 0$. The Euler equation and the law of motion of government debt become:

$$\widehat{y}_t = \mathbb{E}_t \widehat{y}_{t+1} - \phi_\pi \widehat{\pi}_t + \chi \widehat{v}_t + (1 - \chi\gamma)\, \mathbb{E}_t \widehat{\pi}_{t+1} - e_t^i \tag{32}$$

$$\widehat{v}_t = \frac{1}{\beta}(1-\alpha_b)\, \widehat{v}_{t-1} + \gamma \left(\phi_\pi - \frac{1}{\beta}\right) \widehat{\pi}_t - \frac{1}{\beta}(1-\alpha_b)\, e_t^s \tag{33}$$

As highlighted earlier, the Euler equation differs from the RANK case because households consider government debt to be wealth. A fundamental difference between the HANK model and the RANK model examined above is therefore that fiscal policy impacts output dynamics and therefore inflation when Ricardian Equivalence fails. In other words, there is a two-way interdependence between inflation and deficits in this model. As we shall see, this has fundamental consequences.

Proposition 2 *Under certain regularity conditions given in the appendix, the conditions for the existence and uniqueness of bounded, non-oscillatory equilibria in a HANK model with $q < 1$ are:*

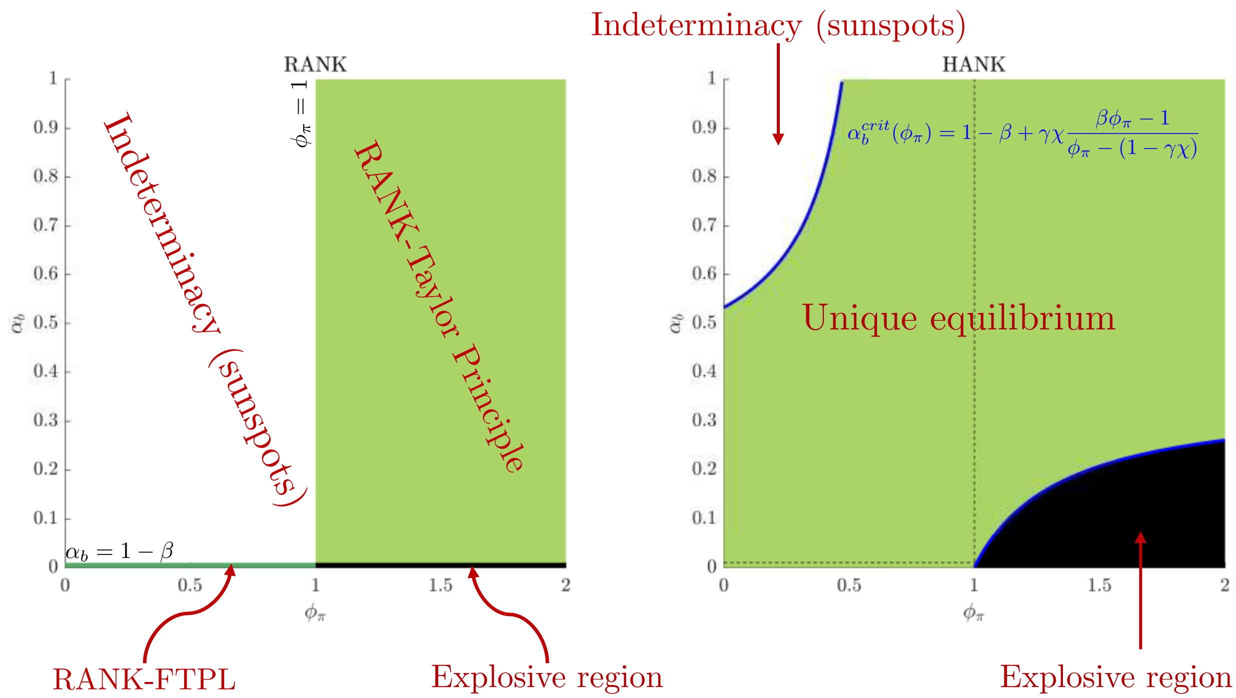
1. *When $\phi_\pi > 1 - \gamma\chi$, a necessary and sufficient condition for local determinacy is that $\alpha_b > \alpha_b^{crit}(\phi_\pi)$.*
2. *When $\phi_\pi < 1 - \gamma\chi$, a necessary and sufficient condition for local determinacy is that $\alpha_b < \alpha_b^{crit}(\phi_\pi)$.*
3. *When $\phi_\pi < 1 - \gamma\chi$, there is local indeterminacy for $\alpha_b > \alpha_b^{crit}(\phi_\pi)$.*
4. *When $\phi_\pi > 1 - \gamma\chi$, there is no stable equilibrium for $\alpha_b < \alpha_b^{crit}(\phi_\pi)$*

where the critical value of α_b is given by:

$$\alpha_b^{crit}(\phi_\pi) = 1 - \beta + \gamma\chi \frac{\beta\phi_\pi - 1}{\phi_\pi + \gamma\chi - 1}. \tag{34}$$

Proof: See the appendix.

The right panel of Figure 1 visualizes Proposition 2. The figure shows that the determinacy properties of the HANK model differ significantly from RANK in a number of important ways, which we will now discuss. In what follows, we first state the results and then provide the economic intuition.



Notes: The left panel shows the determinacy properties of the RANK model, the right panel the HANK model. There is local determinacy in the green shaded areas, local indeterminacy in the non-shaded areas, and non-existence of a stable rational expectations equilibria in the black areas. The figures are drawn assuming $\beta = 0.99$, and the left panel assumes $q = 0.75$. Annual steady state debt to GDP ratio is 100%.

Figure 1: Determinacy Properties

3.3.1 Determinacy in HANK: the results

We first state five results that are implied by Proposition 2.

Result 1: The subset of the parameter space for $(\phi_\pi, \alpha_b) \in \mathbb{R}_+ \times (0, 1)$ that delivers local determinacy is connected for $\chi\gamma > 0$. Thus, there are no sharply divided areas as in the RANK model.[13]

Thus, while RANK model features distinct RANK-TP and a RANK-FTPL regions, a similar logic does not hold true in the HANK setting. We will discuss the economics of this result below.

Result 2: When monetary policy does not satisfy the Taylor Principle, i.e. when $\phi_\pi < 1$, a unique equilibrium in HANK exists for a wider set of fiscal policy parameters than in RANK. In particular, there are fiscal policy parameters that induce a unique equilibrium even when $\alpha_b > 1 - \beta$.

We showed above that in the RANK setting, when the Taylor Principle fails, local determinacy requires fiscal policy to be active, $\alpha_b < 1 - \beta$. In HANK, there is local determinacy for a much wider

[13] Technically speaking, $\alpha_b^{crit}(\phi_\pi)$ defined in Proposition 3, asymptotes to infinity as $\phi_\pi \to (1-\gamma\chi)_-$ and to minus infinity as $\phi_\pi \to (1-\gamma\chi)_+$.

set of fiscal rules when the Taylor Principle is violated. Again, we will discuss the economics of this result below.[14]

Result 3: The threshold α_b^{crit} required for determinacy depends on ϕ_π and vice versa. Because of this interdependency, even if the Taylor Principle is satisfied, monetary policy reforms cannot in general be undertaken without consideration of the fiscal policy framework and vice versa.

The RANK model has the same qualitative implications for policy coordination, but in a very sharp fashion in the sense that, were monetary policy to become passive rather than active, fiscal policy has to be made active. In the HANK setting, the requirement for policy coordination is less drastic because policy interaction is much more gradual.

Result 4: When monetary policy satisfies the Taylor Principle by a sufficiently large margin, i.e. when $\phi_\pi > 1/\beta$, non-existence is more pervasive in HANK than in RANK. To ensure equilibrium existence, fiscal policy must act more forcefully (i.e. more passively) to stabilize debt.

In continuation of the point above, this result implies that, when a monetary policy reform has implications for how strongly interest rates should respond to inflation, one needs to consider the implications for the fiscal response to government debt. Failure to do so can potentially put the economy in the region where there are no stable equilibria, with a ratcheting up of debt as monetary actions lead to ever higher interest payments (which in turn further stimulate demand).

Result 5: When monetary policy *just* satisfies the Taylor Principle, i.e. when $1 < \phi_\pi < 1/\beta$, a unique bounded equilibrium can exist even if fiscal policy does not stabilize debt, i.e. even if $\alpha_b < 1 - \beta$ (as long as $\alpha_b > \alpha_b^{crit}$).

In this (small) region of the parameter space, debt converges back to steady state purely because of endogenous macroeconomic forces (debt erosion due to inflation).

3.3.2 Determinacy in HANK: the economics

To understand how policy shapes the determinacy properties of the HANK model, it is useful to consider a heuristic derivation of the analytical formula for α_b^{crit} in Proposition 2. Start by considering

[14] Such modifications of the Taylor principle have been found by others too; Dupraz and Rogantini Picco (2025) study a model similar to ours but with flexible prices and derive a condition similar to that in Proposition 2; Canzoneri, Cumby and Diba (2011) find similar equilibrium determinacy results in a setting where government bonds deliver transactions services, while Cui (2016) derive them in a setting with more general liquidity frictions; Bigio, Caramp and Silva (2024) show that the Taylor principle is invalid when there is uncertainty about monetary-fiscal reforms. Davig and Leeper (2007) show that bounded unique equilibria can exist even if there are temporary deviations from the Taylor principle as long as a long-run Taylor principle is satisfied.

the Euler equation in the long run, following some shock at time t. Iterating (32) forward to an arbitrary period j we obtain:

$$
\begin{aligned}
\widehat{y}_{t+j} &= \underbrace{\mathbb{E}_{t+j}\widehat{y}_{t+j+1}}_{\text{Keynesian cross}} - \overbrace{(i_{t+j} - \mathbb{E}_{t+j}\widehat{\pi}_{t+j+1})}^{\text{intertemporal subst.}} + \underbrace{\chi\gamma\left(i_{t+j} - \mathbb{E}_{t+j}\widehat{\pi}_{t+j+1}\right)}_{\text{real rate effect on wealth}} - \overbrace{\chi\gamma\frac{1}{\beta}\widehat{\pi}_{t+j}}^{\text{inflation effect on wealth}} \\
&+ \underbrace{\chi\gamma\left(\sum_{s=1}^{j}\left(\frac{1-\alpha_b}{\beta}\right)^s\left[\left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_{t+j-s}\right]\right)}_{\text{inflation tax rebate}}. \qquad (35)
\end{aligned}
$$

We discuss the labels of these effects momentarily. For now, substituting the monetary policy rule into the Euler equation, we can back out the set of parameter values for which demand and inflation stabilize in the long run (as $j \to \infty$) away from the steady state, at some values $\widehat{y}$ and $\widehat{\pi}$. Such a path would be internally consistent since, by the Phillips curve, if output is constant, so is inflation.[15] We obtain:

$$
\widehat{y} = \widehat{y} - (\phi_\pi - 1)\widehat{\pi} + \chi\gamma(\phi_\pi - 1)\widehat{\pi} - \chi\gamma\frac{1}{\beta}\widehat{\pi} + \chi\gamma\frac{\left(\phi_\pi - \frac{1}{\beta}\right)\frac{1}{\beta}(1-\alpha_b)}{1 - \frac{1}{\beta}(1-\alpha_b)}\widehat{\pi}.
$$

Canceling out the $\widehat{y}$ and $\widehat{\pi}$ terms and rearranging, we obtain the expression for α_b^{crit} in Proposition 2. This derivation explicitly illustrates that the set of parameters characterized analytically in the proposition forms a threshold beyond which demand and inflation diverge (or converge) back to (or away from) the steady state.

Just as in RANK, uniqueness in HANK follows if an expectational shock were to set the economy on an explosive path. Consider again a positive shock to expectations of future output. Such a shock raises demand today one-for-one through the Keynesian cross embedded in the NK model. An explosive path requires demand today to rise by less than the sunspot: the sum of the remaining four effects needs to be negative to rule out expectational equilibria.

The first of these additional effects is the intertemporal substitution effect, also present in the RANK model: If the central bank tightens policy sufficiently to raise the real interest rate, households postpone consumption and output today increases by less than the expectation of output tomorrow. Conversely, if ϕ_π is low, the real rate might fall, providing a further boost to demand today as households bring their spending forward in time.

In HANK, there are three additional effects. First, the real rate effect on wealth arises as changes in the real return on government bonds affect the wealth of bond holders. If, faced with a positive expectational shock, the central bank tightens monetary policy sufficiently aggressively to raise the real rate, households' asset returns increase, which stimulates demand today over and above the Keynesian cross, thus putting demand on a convergent path. All else equal, this works against the

[15] Recall that, solving the NKPC forward, $\widehat{\pi}_t = \kappa\sum_{j=0}^{\infty}\beta^j\widehat{y}_{t+j} = \kappa\frac{y}{1-\beta}$ if output is constant.

intertemporal substitution effect. Nonetheless, this effect is always dominated by the intertemporal substitution effect as long as $\gamma\chi < 1$, which we assume is the case.

Second, there is the inflation effect on wealth: higher inflation induces a capital loss on the real value of households' assets. Since the NK Phillips curve implies that an increase in expected output is inflationary, this effect always works towards making the equilibrium locally determinate.

The final effect recognizes that an expectational shock changes the effective fiscal stance, due to changing interest payments and the inflation tax. Specifically, if monetary policy does not tighten policy strongly in response to inflation ($\phi_\pi < 1/\beta$), the inflation tax rise is the dominant one of the two forces, and so the fiscal position of the government improves as a result of the positive extraneous expectational shock. This represents an effective fiscal tightening, which the government can offset to some degree if it rebates some of the proceeds back to households. The fiscal rule states that the share α_b of the proceeds is rebated, and so $1 - \alpha_b$ is the share of the fiscal tightening that is effectively borne by households. Unless there is full rebating, the inflation tax rebate term is negative, dampening demand today relative to the Keynesian cross alone, and assists in achieving determinacy.

These three novel effects present in HANK help us better understand the patterns of determinacy in Figure 1. Relative to RANK, the endogenous effect of inflation on wealth makes households poorer today, making the determinacy achievable even for monetary policy rules that do not satisfy the Taylor Principle. In particular, even with full rebating of the endogenous fiscal surplus, $\alpha_b = 1$, all that is required for determinacy is $\phi_\pi > 1 - \frac{1}{\beta}\frac{\chi\gamma}{1-\chi\gamma}$, a condition weaker than in RANK. This explains the horizontal shift to the left of the left boundary of the determinacy region. In addition, an incomplete rebating of the endogenous fiscal surplus further reduces the requirement for the central bank to respond with rate hikes. This accounts for the shape of the upper left boundary of the determinacy region.

This discussion also clarifies why the region of non-existence of a bounded equilibrium is larger than in RANK for $\phi_\pi > 1/\beta$. Recall that for existence, we need the equilibrium system to converge back to the steady state following any of the fundamental shocks. Consider an expansionary deficit shock. The rise in the deficit increases government debt which stimulates aggregate demand. Higher demand, in turn, increases inflation, triggering a monetary tightening. As monetary policy tightens, the fiscal problems worsen as the rise in the interest costs exceeds the inflation tax, and with the automatic fiscal tightening being too weak, the economy ends up on a divergent path that sees ever-increasing inflation and government debt. The direct feedback from fiscal policy to demand when Ricardian Equivalence does not hold means that, to ensure the existence of a bounded equilibrium, fiscal policy must be more stabilizing in HANK than in RANK.

Two corollaries stand out. First, monetary policy and fiscal policy work together, and reinforce each other, in bringing about determinacy. The traditional taxonomy of active vs. passive policies is not really useful in this context. At a fundamental level, this comes about because fiscal policies impact demand directly due to the failure of Ricardian Equivalence, and, as in the RANK model, monetary policies impact the fiscal space directly. Thus, there are always some fiscal aspects of monetary policy and vice versa. Therefore, the policies also act jointly to reduce the region of the parameter space where there is indeterminacy, but also enlarge the region where no bounded equilibrium exists.

Second, the fiscal response to changes in government debt may need to be adjusted in lieu of monetary policy reform, and vice versa. In particular, the more aggressive the monetary policy response to inflation, the stricter the fiscal requirements to ensure that the economy does not diverge. Suppose that government debt rises due to a budget deficit. In the HANK setting, higher public debt stimulates private sector spending and, therefore, induces inflation. The more aggressive the central bank's response to inflation, the more important it is that the fiscal authority implements a budget tightening since the hike in nominal rates increases the interest rate costs of government debt.

In summary, the equilibrium determinacy properties of the economy where Ricardian Equivalence fails are quite different from a standard RANK model. Such differences are larger the higher the public debt and/or the higher the MPC.

3.4 Extension to $\alpha_x > 0$

Proposition 3 *With $\alpha_x > 0$, the relevant threshold for α_b is:*

$$\alpha_b^{crit}(\phi_\pi, \alpha_x) = 1 - \beta + \chi\gamma\frac{\beta\phi_\pi - 1}{\phi_\pi + \chi\gamma - 1} - \chi\alpha_x\frac{1-\beta}{\kappa(\phi_\pi + \chi\gamma - 1)}.$$

Proof: See the appendix.

In the presence of automatic stabilizers, the critical threshold for α_b depends not only on the Taylor Rule coefficient α_b, but also on the strength of these stabilizing effects. In particular, when $\alpha_x > 0$, the indeterminacy region and the explosive region both shrink. This is intuitive: in the indeterminacy region, a sunspot of higher future demand induces fiscal tightening through the automatic stabilizers, lowers demand today, and helps to set demand on a divergent path. In the explosive region, automatic stabilizers help achieve government debt convergence over a wider range of the policy parameter space.

4 Equilibrium effects of fiscal deficit shocks

We now consider the effects of government deficits in our economy. Specifically, we systematically analyze the responses of macroeconomic variables to a deficit shock across the policy space. In this section, we continue to assume $\alpha_x = \phi_x = 0$. To set the scene, we briefly review the response of the RANK economy to such a shock.

4.1 Deficit shock in RANK

Consider a one-off unexpected and temporary deficit shock of 1% of GDP:

$$\widehat{s}_t = \alpha_b \widehat{v}_{t-1} + (1-\alpha_b) e_t^s$$

with $e_0^s = -1\%$.

4.1.1 RANK-Taylor Principle

In RANK, the deficit shocks are neutral for the macroeconomy:

Proposition 4 *Consider a fiscal deficit shock e_0^s in the RANK-TP model, that is, assume $\phi_\pi > 1$ and $\alpha_b > 1-\beta$. The unique bounded equilibrium takes the form:*

$$\widehat{y}_t = 0 \ \forall \ t$$

$$\widehat{\pi}_t = 0 \ \forall \ t$$

$$\widehat{v}_t = \left(\frac{1}{\beta}(1-\alpha_b)\right)^t \widehat{v}_0, \quad \widehat{v}_0 = \frac{1}{\beta}(1-\alpha_b) e_0^s.$$

Proof: See the appendix.

In the RANK model, Ricardian Equivalence holds, and so there is no direct effect of the deficit shock on demand. In a standard setting in which the monetary policy is active, deficit shocks are a wash from a macroeconomic point of view. It is straightforward to see this in the system of equations that describe the model. With $q = 1$, the system is:

$$\mathbb{E}_t \begin{bmatrix} \widehat{\pi}_{t+1} \\ \widehat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \phi_\pi - \frac{1}{\beta} & \frac{\kappa}{\beta} + 1 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_t \\ \widehat{y}_t \end{bmatrix} \tag{36}$$

$$\widehat{v}_t = \frac{1}{\beta}(1-\alpha_b)\widehat{v}_{t-1} + \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_t - \frac{1}{\beta}(1-\alpha_b) e_t^s \tag{37}$$

It is immediate that the macro-block of the model, described by the first two equations, is completely independent of the fiscal block. As a result, as long as fiscal policy stabilizes debt, i.e. as long as $\frac{1}{\beta}(1-\alpha_b) < 1$, fiscal shocks induce movements in deficits and debt, but have no bearing on the aggregate economy.

On the other hand, the fiscal block *is* affected by the macro block – the second term in equation (37) shows that the evolution of the real value of government debt is affected by the path of nominal interest rates (which in turn vary because of inflation via the Taylor Rule) and inflation itself. The fiscal theory of the price level builds on this dependence. We now analyze this case.

4.1.2 RANK-FTPL

When $\alpha_b < 1-\beta$, the government adjusts the primary surplus less in response to changes in government debt than what is required for simply rolling over debt.[16] In this case, a deficit shock sets government debt on an unsustainable, explosive path unless it is countered by a jump in inflation of an appropriate size to deliver debt convergence. The following proposition contains the full characterization of the response of the economy to a fiscal shock in the RANK-FTPL model.

Proposition 5 *Consider a fiscal transfer (surplus) shock of e_0^s% of GDP in the RANK-FTPL model ($\phi_\pi \leq 1$ and $\alpha_b < 1-\beta$). The unique bounded equilibrium is characterized by:*

$$\begin{aligned} \widehat{\pi}_0 &= -\frac{1}{\gamma}\frac{1-\alpha_b-\beta\lambda}{1-\beta\phi_\pi}\cdot e_0^s & \qquad \widehat{\pi}_t &= \lambda^t\widehat{\pi}_0 \\ \widehat{y}_0 &= -\widehat{\pi}_0\frac{1-\beta\lambda}{\kappa}\cdot e_0^s & \qquad \widehat{y}_t &= \lambda^t\widehat{y}_0 \\ \widehat{v}_0 &= -\lambda\cdot e_0^s & \qquad \widehat{v}_t &= \lambda^t\widehat{v}_0 \end{aligned}$$

where:

$$\lambda = \frac{1+\frac{1+\kappa}{\beta}-\sqrt{\left(1+\frac{1+\kappa}{\beta}\right)^2-4\frac{1+\kappa\phi_\pi}{\beta}}}{2} \leq 1$$

Proof: See the appendix.

We also have the following corollary:

Corollary 1 *When $\phi_\pi = 1$,*

$$\lambda = 1, \quad \widehat{y}_0 = -\frac{1}{\gamma\kappa}(1-\beta)e_0^s, \quad \widehat{\pi}_0 = -\frac{1}{\gamma}e_0^s.$$

The economic logic of how the economy responds to deficit shocks in the RANK-FTPL regime comes from the government budget constraint. In particular, when the government raises the deficit that is unbacked (in the sense that α_b is close to or equal to zero), output and inflation increase by precisely the right amount to ensure that the government intertemporal budget constraint is satisfied. To see this, solve equation (37) for $\widehat{v}_{t-1}$, forward one period, and plug back into (37) to obtain:

[16] Note that $1-\beta = r/(1+r)$ where r is the steady-state real interest rate. In analogy to the Permanent Income Hypothesis, when $\alpha_b = r/(1+r)$, corresponds to the government rolling over any shocks to debt, $\alpha_b > r/(1+r)$ to stabilizing debt dynamics through the budget surplus, and $\alpha_b < r/(1+r)$ to non-stabilization of debt dynamics through the surplus.

$$\widehat{v}_{t-1} = -\gamma \left(\phi_\pi - \frac{1}{\beta} \right) \sum_{j=0}^{\infty} \left(\frac{\beta}{1-\alpha_b} \right)^{j+1} \widehat{\pi}_{t+j} + e_t^s \tag{38}$$

Note also the following: $\widehat{v}_{t-1} = 0$ because the economy starts in steady state; NKPC implies that inflation is proportional to the discounted stream of output; and output converges back to steady state at rate λ: $\widehat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j \widehat{y}_{t+j} = \kappa \sum_{j=0}^{\infty} (\beta\lambda)^j \widehat{y}_0 = \frac{\kappa}{1-\beta\lambda} \widehat{y}_0$. Together, these observations imply that equation (38) uniquely pins down $\widehat{y}_0$ and hence $\widehat{\pi}_0$.

Note that output and inflation move in response to fiscal shocks despite the fact that Ricardian Equivalence holds. Ricardian behavior is perhaps most starkly captured by the independence of the macro block of the model in (36) from the fiscal variables. Instead, fiscal policy has real effects as it coordinates expectations at precisely the required value for the debt to converge back to zero and not explode. It is easy to see that debt would explode had inflation and output not reacted (the coefficient on $\widehat{v}_{t-1}$ in equation (37) is greater than 1). In fact, given the implied dynamic behavior of the model, there exists a unique jump in inflation that delivers debt convergence back to steady state. Due to the NK Phillips curve, inflation can only increase if output goes up. Thus, the fiscal deficit triggers an inflationary boom in the economy. And, since the Taylor Principle is not satisfied in the FTPL region, our analysis of the determinacy properties of the model implies that such a jump on the demand side is consistent with a bounded equilibrium – demand and inflation converge back to steady state at rate $\lambda \leq 1$.

The analytical characterization of the equilibrium allows us to study the comparative dynamics of the RANK-FTPL equilibrium:

Proposition 6 *In the RANK-FTPL model, the size of the responses of output and inflation is inversely proportional to the steady-state debt-to-GDP ratio γ, while the persistence of the responses does not depend on γ:*

$$\frac{\partial \lambda}{\partial \gamma} = 0 \qquad \frac{\partial \widehat{y}_t}{\partial \gamma} < 0 \qquad \frac{\partial \widehat{\pi}_t}{\partial \gamma} < 0.$$

A more hawkish monetary policy response increases the persistence of the effects of the transfer shock and worsens the contemporaneous inflation-output trade-off:

$$\frac{\partial \lambda}{\partial \phi_\pi} > 0 \qquad \frac{\partial \frac{\widehat{\pi}_t}{\widehat{y}_t}}{\partial \phi_\pi} > 0.$$

The dynamics of debt are independent from the surplus rule parameter α_b, and inflation and output responses are smaller, the larger α_b is:

$$\frac{\partial \lambda}{\partial \alpha_b} = 0 \qquad \frac{\partial \widehat{\pi}_t}{\partial \alpha_b} < 0 \qquad \frac{\partial \widehat{y}_t}{\partial \alpha_b} < 0.$$

Proof: See the appendix.

The comparative dynamics are very intuitive. First, the sizes of the responses of output and inflation are inversely proportional to the steady-state debt-to-GDP ratio. Since inflation is a tax on nominal bonds, the smaller the tax base (the debt-to-GDP ratio), the larger the required impact on inflation needed to finance a given deficit. The debt-to-GDP ratio does not affect the persistence of the responses of output and inflation; it only scales them up or down.

Monetary policy determines how the magnitudes of the rise in inflation and output that result from a fiscal transfer shock – but the direction of this dependence is perhaps surprising: a more hawkish monetary policy response leads to *higher* inflation, a *smaller* short-run output boom, and a larger long-run (NPV) output boom. Why? Observe first that a more hawkish monetary policy response – a higher ϕ_π – *increases the persistence* of the demand boom. To see why, recall the Euler Equation of this model:

$$\widehat{y}_t = \mathbb{E}_t \widehat{y}_{t+1} - \widehat{r}_t,$$

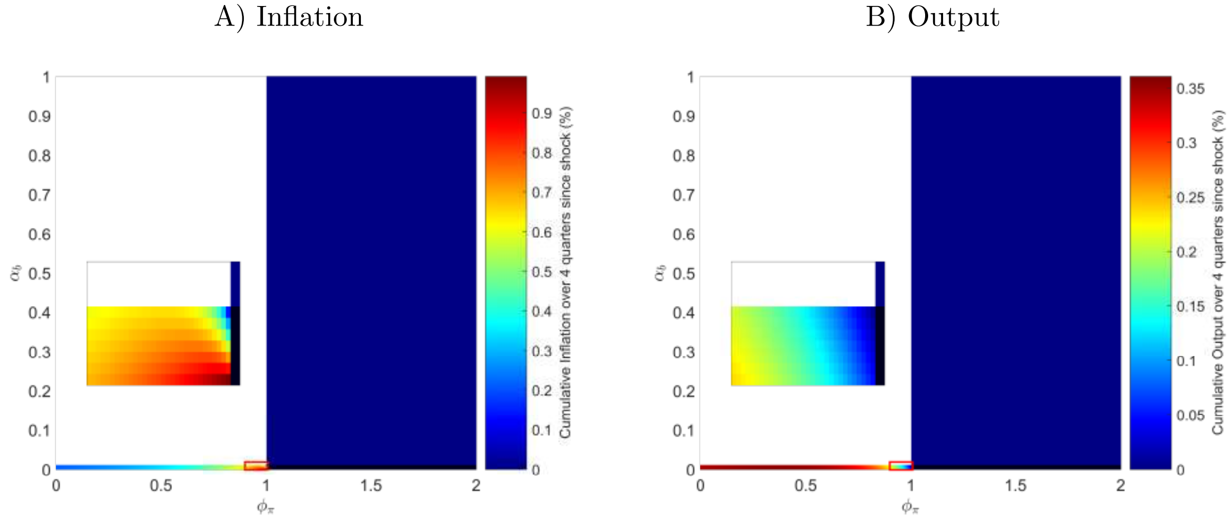
and note that the fiscal deficit shock triggers a jump in output and inflation. In the FTPL region of the parameter space, the real rate declines following a deficit shock, thus bringing some of the demand from the future into the present via intertemporal substitution. The stronger is the intertemporal substitution force, the larger the demand boom today relative to the future – i.e., the less persistence there is in the output response. If ϕ_π is close to 1, the real rate is approximately constant and there is little intertemporal substitution. Hence, in this case, the intertemporal Keynesian cross drives the persistent – but smaller at every t – output response.

Inflation, however, does not respond just to the contemporaneous output boom, but is instead proportional to the discounted sum of future output gaps, and the total amount of inflation needed to finance the debt depends on monetary policy. To see this, note that there are two ways in which an unbacked fiscal deficit can be paid for, through a surprise change in inflation and through a sequence of changes in expected future inflation rates. Equation (38) implies:[17]

$$-e_0^s = \gamma \left(\widehat{\pi}_0 - \sum_{j=0}^{\infty} \beta^{j+1} \widehat{r}_{t+j} \right). \tag{39}$$

The left-hand side is the unfunded deficit spending shock. The right-hand side shows the two ways of paying for it: On impact there is a surprise jump in inflation π_0 which devalues nominal debt, and after that, there is a period of low real rates (i.e., higher expected inflation). The monetary policy stance determines the mix of the two. If $\phi_\pi = 1$ so that real rates do not fall at all, all of the deficit shock must be paid for by the surprise increase in inflation, so that for one percent of GDP deficit,

[17]For simplicity, this expression assumes $\alpha_b = 0$. Without this assumption, we have $\widehat{\pi}_0 = \frac{1-\alpha_b-\beta\lambda}{\beta\lambda-1} \frac{1}{\gamma} \left(R + e_0^s\right)$ where $R := -\gamma \sum_{j=0}^{\infty} \left(\frac{\beta}{1-\alpha_b}\right)^{j+1} \left(\widehat{r}_{t+j}\right) \geq 0$.



Notes: The figures are drawn assuming $\beta = 0.99$, $\kappa = 0.24$, and $\gamma = 4$ implying an annual debt to GDP ratio of 100%.

Figure 2: Inflation and output response across the policy space in RANK

inflation rises by $\frac{1}{\gamma}$. If real rates decline, the initial rise in inflation is commensurately smaller. Thus, an important analytical property of the RANK-FTPL model is that a more hawkish monetary policy response worsens the contemporaneous inflation-output trade-off following a fiscal transfer shock.

The final set of comparative statics of the Proposition concerns how, within the FTPL region, the degree of debt stabilization (α_b) matters for dynamics. These results are also surprising: the equilibrium dynamics of debt do not depend on α_b at all; but both inflation and output are lower the more stabilization there is from the fiscal side. In fact, the dependence of the macro variables on α_b is very strong: there is extreme sensitivity of inflation and output to very small changes in the fiscal policy rule parameter:

Proposition 7 *In the RANK-FTPL model, when ϕ_π is equal to or is close to 1, inflation and output responses to the fiscal shock exhibit extreme sensitivity to changes in the surplus rule parameter α_b. Specifically, when $\phi_\pi = 1$,*

$$\widehat{\pi}_0 = \begin{cases} -\frac{1}{\gamma} \cdot e_0^s & \text{if } \alpha_b = 0 \\ 0 & \text{as } \alpha_b \to 1 - \beta \end{cases}.$$

Proof: See the appendix.

This result says that within the FTPL region, the equilibrium responses of the macro variables to the fiscal transfer shock change dramatically for small changes in α_b. Note that the model predicts maximum inflation response or no inflation response for changes in α_b in the region of 0.01, for a standard calibration of the β parameter.

4.1.3 Numerical illustration

To go beyond the analytics, and to systematically study the properties of equilibria, we now examine the responses of the economy to a fiscal transfer shock *across the entire policy space*. Figure 2 shows a numerical example illustrating the properties of the equilibrium. The two panels show the cumulative 4-quarter response of inflation and output following a 1% deficit shock for a baseline calibration of the model.

In the RANK-TP model – the top right quadrant of the policy space – Ricardian Equivalence holds, the Taylor Principle is satisfied, and fiscal policy stabilizes debt following the shock. Thus, both inflation and output are unaffected by deficits. In the FTPL region in the bottom left of the panels, the increase in the deficit is inflation financed and thus stimulates both inflation and output. In this region, inflation tends to respond more strongly, the more aggressive the monetary authority is; however, there is also the extreme sensitivity of inflation to α_b when ϕ_π is close to 1. The output response over the first 4 quarters exhibits the opposite pattern: it becomes smaller as monetary policy becomes more responsive to inflation.

Thus, equilibria in the RANK model exhibit a sharp dependence on the policy rule parameters, both across and within the policy regimes. Inflation responds strongly in the FTPL regime but not at all in the TP regime; and within the FTPL model, it responds strongly when $\alpha_b = 0$ but much less so when this parameter is only marginally higher. Taken literally, this sharp dependence carries stark implications for policy. But this sharp dependence also makes apparent the lack of robustness and close to knife-edge behavior of the RANK model. As we will next show, these sensitivities are much less stark in the HANK model.

4.1.4 HANK

Finite planning horizons break Ricardian Equivalence, meaning that deficit policies have a direct impact on household demand. We now systematically investigate how these features translate into a reaction of the economy to a fiscal shock. As for RANK, we start with an analytical characterization of the equilibrium:

Proposition 8 *Equilibria in HANK have analytically identical representation to those in RANK-FTPL, up to the persistence parameter λ. That is, analytically, the equilibria differ only through differences in λ.*
Consider a fiscal transfer (surplus) shock of e^s_0 percent of GDP in the HANK model ($q < 1$). The unique bounded equilibrium is characterized by:

$$\widehat{\pi}_0 = -\frac{1}{\gamma}\frac{1-\alpha_b-\beta\lambda}{1-\beta\phi_\pi}\cdot e_0^s \qquad \widehat{\pi}_t = \lambda^t\widehat{\pi}_0$$
$$\widehat{y}_0 = -\widehat{\pi}_0\frac{1-\beta\lambda}{\kappa}\cdot e_0^s \qquad \widehat{y}_t = \lambda^t\widehat{y}_0$$
$$\widehat{v}_0 = -\lambda\cdot e_0^s \qquad \widehat{v}_t = \lambda^t\widehat{v}_0$$

where λ solves:

$$\lambda_{HANK} = \frac{1}{\beta}(1-\alpha_b) + \frac{\chi\gamma\frac{\kappa}{\beta^2}(\phi_\pi - \frac{1}{\beta})(1-\alpha_b)}{\lambda_{HANK}^2 - \lambda_{HANK}\left(1+\frac{1}{\beta}+\frac{\kappa}{\beta}(1-\chi\gamma)\right)+\frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma)+\frac{1}{\kappa}+\frac{\chi\gamma}{\beta}\right)}.$$

Proof: See the appendix.

The key analytical result in Proposition 8 is a rather surprising one: there is a parallel between the HANK and the RANK-FTPL models. Equilibria in the two models have identical representations up to the persistence λ.

Thus, despite the complexity of the two-way general equilibrium interactions between macroeconomic variables and government debt in HANK, the dynamics of the equilibrium are mathematically straightforward: output and inflation are proportional to real wealth, and so they inherit the dynamic properties from the evolution of real debt in the economy. Put differently, the more persistent the response of debt to the shock, the more persistent the output and inflation responses are.

In the HANK case, the persistence parameter λ cannot be solved for in closed form, except in special cases. One such case is when $\alpha_b = 0$ and $\phi_\pi = 1$:

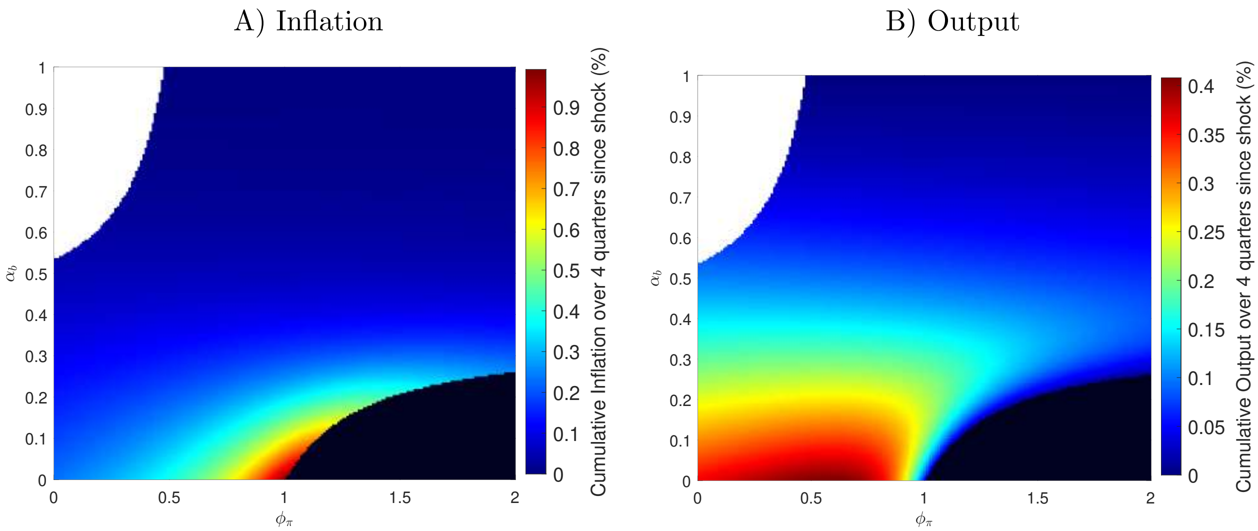
Corollary 2 *When $\alpha_b = 0$ and $\phi_\pi = 1$,*

$$\lambda_{RANK-FTPL} = \lambda_{HANK} = 1,$$

and thus the two models yield identical dynamic responses of all variables.

A closed-form solution emerges when the Taylor Principle is exactly satisfied, $\phi_\pi = 1$. This is the case described in Corollary 2. Here, the analytical representations highlighted in the proposition imply that the two models yield identical predictions for the entire dynamic paths of all variables. This is a generalization of the HANK-meets-FTPL result of Angeletos, Lian and Wolf (2025).

Given the analytically parallel representations, which properties of the RANK-FTPL model carry through to HANK, and which are qualitatively different? To answer this question, we turn to the comparative dynamics of the HANK model. The following proposition contains the main property that we focus on:



Notes: The figures are drawn assuming $\beta = 0.99$, $\kappa = 0.24$, $q = 0.75$ (implying MPC of 0.26) and $\gamma = 4$ (implying an debt-to-annual GDP ratio of 1).

Figure 3: Inflation and output response across the policy space in HANK

Proposition 9 *A more hawkish monetary policy response leads to more inflation relative to output at any time t:*

$$\frac{d\frac{\widehat{\pi}_t}{\widehat{y}_t}}{d\phi_\pi} > 0.$$

Assume $\phi_\pi > 1 - \frac{1}{\beta}\frac{\chi\gamma}{1-\chi\gamma}$ *(sufficient, but not necessary). Debt stays higher for longer if monetary policy reacts aggressively:*

$$\frac{d\lambda_{HANK}}{d\phi_\pi} > 0.$$

Proof: See the appendix.

Proposition 9 shows that the persistence of the government debt dynamics is increasing in the policy rule parameter ϕ_π. The condition in the proposition is sufficient for this result, but is not necessary, and the numerical explorations suggest that this result holds robustly across the parameter space. This result implies that a hawkish response makes government debt persistently higher, and via non-Ricardian behavior, translates into a more persistent demand boom. Since $\widehat{\pi}_t = \kappa\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \widehat{y}_{t+j}$, a more persistent demand boom adds to inflation. Since higher inflation reduces the size of the demand boom (as it devalues government debt), it follows that a more hawkish monetary response leads to a smaller but more persistent demand boom, and to higher inflation.

4.1.5 Numerical illustration: HANK

Figure 3 uses the heatmaps to illustrate the properties of the HANK equilibria across the policy space. Three results stand out.

First, whenever there is local determinacy, fiscal transfer shocks are inflationary, and such local determinacy exists in a large set of the parameter space in HANK. Second, the equilibrium responses of macroeconomic variables to the fiscal shock are less sharply dependent on the policy parameters than in RANK and vary smoothly and gradually with how policy is set in HANK. Third, for parameter configurations consistent with the RANK-FTPL region, the equilibria in HANK and in RANK are qualitatively similar. In line with Corollary 2, when $\alpha_b = 0$ and $\phi_\pi = 1$, the responses of output and inflation are identical across the two models – an extension of the equivalence result of Angeletos, Lian and Wolf (2025) to the case where central bank responses to inflation rather than to output. But even away from that benchmark, the economy exhibits similar qualitative patterns: inflation response is greater the more aggressive monetary policy is; and the opposite is true for output. Thus, while inflation and output co-move following a fiscal deficit shock, there is a negative correlation between the size of these responses across different policy rule parameters. Following fiscal transfer shocks, aggressive monetary policy worsens the contemporaneous inflation-output trade-off.

A standard (perhaps non-intuitive) property of RANK-FTPL is that deficits are more inflationary the lower is the level of (nominal) government debt. The reason for this is that the tax base of the inflation tax is the stock of nominal government debt. Hence, if inflation finances deficits, it needs to rise more when the tax base is lower. Barro and Bianchi (2023) find empirical evidence in favor of this prediction from examining the recent post-Covid-19 inflationary experience for a cross-section of countries. Inspecting Proposition 8, it is not immediately clear that this property is also present in the HANK model because of the very non-linear way that γ enters the equilibrium dynamics. Figure 14 in the Appendix illustrates by means of the heatmap the impact of deficits on inflation accumulated over the first four quarters for $\gamma = 2$ (a debt-to-GDP ratio 50 percent annually) and for an annual debt-to-GDP ratio equal to 1. We find that, consitently with the empirical evidence, lower government indebtedness implies more inflationary effects of deficits in the HANK model as in RANK-FTPL.[18]

4.2 Can the central bank kill the fiscally-driven inflation?

In the previous section, we showed that deficits are inflationary everywhere if Ricardian Equivalence does not hold and the central bank follows the standard Taylor Rule. But is there a way for the central bank to prevent fiscally driven booms and inflation? We explore one candidate strategy, which is for the central bank to respond by adjusting the interest rate in line with the implicit natural rate of interest, which moves directly with the level of debt – see Aguiar, Amador and Arellano (2024) and Dupraz and Rogantini Picco (2025). We now review this insight in the context of our model, highlighting that such a policy places a tight requirement on the fiscal policymaker who must be

[18]To make the two panels of Figure 14 comparable, we have used the same heatmap encoding in the two panels. Panel B of Figure 14 is identical to panel A of Figure 3 apart from the change in the coding of the heatmap.

sufficiently determined to stabilize debt (i.e. must have a high α_b) in the face of rising interest costs.

Suppose that the interest rate rule is of the form

$$i_t = \phi_\pi \widehat{\pi}_t + \phi_v \widehat{v}_t, \tag{40}$$

where ϕ_v indicates the central bank's response of nominal interest rates to deviations of government debt from its target. We assume implicitly that the central bank and the fiscal authority have a common public debt target.

The log-linearized Euler equation (18) then becomes

$$\widehat{y}_t = \mathbb{E}_t \widehat{y}_{t+1} - \phi_\pi \widehat{\pi}_t - \phi_v \widehat{v}_t + \mathbb{E}_t \widehat{\pi}_{t+1} + \chi \left(\widehat{v}_t - \gamma \mathbb{E}_t \widehat{\pi}_{t+1} \right).$$

Clearly, if the central bank sets $\phi_v = \chi$, the two debt terms cancel out – the non-Ricardian effect of debt on demand is neutralized by the central bank adjusting interest rates proportionately to any increase in debt (with the proportionality factor of χ, which is exactly the elasticity of demand with respect to debt). In other words, an interest rate rule of the type (40) can restore the independence of the macro block of the model from the fiscal block with an appropriate choice of ϕ_v. In that case, the Euler equation in this model differs from the standard RANK model only due to the effect of the expected inflation effect on the real value of debt. Just as we discussed above, this effect translates into a looser requirement for the central bank to uniquely determine inflation (recall the *inflation effect on wealth* in equation (35)). It is easy to show that to eliminate sunspots, the central bank's response to inflation must satisfy $\phi_\pi > 1 - \gamma\chi$, less stringent than the Taylor principle requirement.

Nonetheless, the requirements for stability of debt will naturally be more demanding if the central bank raises interest rates in response to debt directly. Imposing that $\phi_v = \chi$, the debt accumulation equation (20) becomes

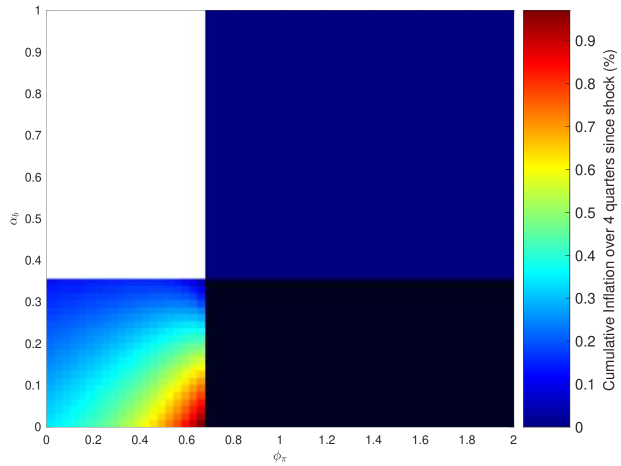
$$\widehat{v}_t = \frac{1}{1 - \gamma\chi} \left[\frac{1 - \alpha_b}{\beta} \widehat{v}_{t-1} + \gamma \left(\phi_\pi - \frac{1}{\beta} \right) \widehat{\pi}_t \right].$$

For debt to be stable, the coefficient on previous period debt in the preceding equation must be smaller than 1, and so the fiscal policy rule must now satisfy:

$$\alpha_b > 1 - (1 - \gamma\chi)\beta = \lim_{\phi_\pi \to \infty} \alpha_b^{crit}(\phi_\pi).$$

That is, the coefficient α_b must be as high as the limit of the bound of the explosive region as ϕ_π goes to infinity. Therefore, unless government indebtedness is low or consumers are approximately Ricardian, when the monetary authority responds directly to variations in government debt, the requirements for automatic surplus responses to the level of debt become much stricter.

We illustrate the workings of the HANK economy with such a rule in Figure 4. The central bank can stabilize inflation in the north-east region of the policy space even as consumers are non-



Notes: The figure shows the determinacy regions and cumulative 4-quarter inflation response to a fiscal transfer shock when the central bank follows the policy rule augmented with a debt term, as in equation (40), with $\phi_v = \chi$. The figure is drawn assuming $\beta = 0.99$, $\kappa = 0.24$, $q = 0.75$ (implying MPC of 0.26 and $\chi = 0.086$) and $\gamma = 4$ (implying an debt-to-annual GDP ratio of 1).

Figure 4: Determinacy regions and inflation responses when the central bank responds to debt and $\phi_v = \chi$

Ricardian. Essentially, this policy restores the distinction between active and passive policies and, like the RANK model, implies the existence of two distinct regions with local determinacy: a north-east region where deficits are neutral, and a south-west "FTPL-like" region.

Nonetheless, adding government debt to the monetary policy rule and setting $\phi_v = \chi$ comes along with two side effects. First, the region with explosive solutions in the HANK model with this augmented policy rule is much larger than the corresponding regions in both the RANK model and in the HANK model with a standard Taylor rule. This is intuitive: after an increase in the government deficit, the central bank hikes interest rates both because of its inflationary effects and because of the increase in government debt. To stabilize this, the fiscal authority needs to aggressively stabilize the deficit; otherwise, government debt and interest rates will be set on a non-convergent path. Secondly, the HANK model with this augmented rule also features a much larger region with FTPL-type dynamics and a unique equilibrium, where fiscal deficits are inflationary. Intuitively, it is the inflation effect on the real value of debt again. While there are self-fulfilling equilibria in the RANK model when $\phi_\pi < 1 - \gamma\chi$ and $1 - \beta < \alpha_b < 1 - (1 - \gamma\chi)\beta$, there is still an added impact of inflation on consumption dynamics in the HANK setting even when $\phi_v = \chi$ which induces divergent paths of output and inflation in response to non-fundamental shocks in this region.

It is thus clear that the policy coordination requirements for the fiscal authority become demanding when the central bank responds directly to variations in government debt through the interest rate rule. This observation may help explain why such automatic monetary policy responses to government debt are not commonly used in practice.

5 Monetary policy shocks

In the textbook New Keynesian model, a one-off cut to the nominal interest rate lowers the real interest rate, and, through intertemporal substitution, leads to a demand boom. This in turn translates into higher inflation. The textbook model lacks internal persistence, so that the macro variables – output, inflation, and the interest rate – are back to steady state next period (unless the shock itself is persistent, of course).

The intertemporal substitution mechanism is also present in the HANK economy, but it is no longer the only channel through which monetary policy affects demand. Instead, the effects on demand and inflation are also mediated through the fiscal consequences of a nominal interest rate cut. As the next proposition shows, the presence of the non-Ricardian demand block induces *dampening* of the effects of monetary policy on demand and inflation on impact, and adverse demand consequences beyond the period of the shock itself. This is because, in the simple model with one-period debt and no capital, the monetary policy shock constitutes an effective tightening of the fiscal stance (and a fall in the value of government debt). On impact, this force works against the intertemporal substitution channel. It also persists beyond the timing of the monetary policy shock itself.

The qualifier in the previous paragraph is important: the results developed here for the simple model highlight the workings of the specific channel from monetary policy through the value of households' holdings of government debt to demand. With some more realistic features – notably long-term debt and capital – there will be positive wealth effects working in the opposite direction. We show that this is indeed the case in the medium-scale model in Section 7: there, an expansionary monetary policy shock raises demand and inflation. Nonetheless, we find it interesting to explore the analytics of the interest-rate-cut-driven fiscal tightening mechanism in detail in the simple model here, given its conceptual importance in the study of fiscal-monetary interactions.

In the following proposition, we also include the characterization of the RANK-FTPL equilibria. This is easily done since the analytical parallel familiar from the earlier analysis continues to hold true here: analytically, RANK-FTPL and HANK models are identical up to the persistence parameter λ (even as the economic mechanisms involved are starkly different).

Proposition 10 *Consider a temporary time-0 interest rate shock of e_0^i percentage points. The unique bounded equilibrium takes the following form:*
In RANK-TP:

$$\widehat{v}_0 = \frac{\gamma(\beta+\kappa)}{\beta(1+\kappa\phi_\pi)} \cdot e_0^i, \qquad \widehat{v}_t = 0 \quad \textit{for } t > 0$$
$$\widehat{y}_0 = -\frac{1}{1+\kappa\phi_\pi} \cdot e_0^i, \qquad \widehat{y}_t = 0 \quad \textit{for } t > 0$$
$$\widehat{\pi}_0 = -\frac{\kappa}{1+\kappa\phi_\pi} \cdot e_0^i, \qquad \widehat{\pi}_t = 0 \quad \textit{for } t > 0$$

In RANK-FTPL and HANK:

$$\widehat{v}_0 = \frac{\gamma(\beta+\kappa)}{\Psi + \beta(1+\kappa\phi_\pi)} \cdot e_0^i, \qquad \widehat{v}_t = \lambda \widehat{v}_{t-1} \quad \textit{for } t > 0$$

$$\widehat{y}_0 = -\left[\frac{1}{1+\kappa\phi_\pi} + \frac{\beta+\kappa}{\kappa(1+\kappa\phi_\pi)(1-\beta\phi_\pi)}\left(1 - \frac{2-\alpha_b-\beta\lambda}{1+\frac{\Psi}{\beta(1+\kappa\phi_\pi)}}\right)\right] \cdot e_0^i, \qquad \widehat{y}_t = \chi_y \widehat{v}_{t-1} \quad \textit{for } t > 0$$

$$\widehat{\pi}_0 = -\left[\frac{\kappa}{1+\kappa\phi_\pi} + \kappa\left(\chi_y \frac{\beta}{1-\beta\lambda}\widehat{v}_0 + \widehat{y}_0 - \frac{1}{1+\kappa\phi_\pi}\right)\right] \cdot e_0^i, \qquad \widehat{\pi}_t = \chi_\pi \widehat{v}_{t-1} \quad \textit{for } t > 0$$

where

$$\Psi := (1-\alpha_b-\beta\lambda)(1-\beta\lambda+\beta+\kappa(1-\gamma\chi)) + (1-\phi_\pi\beta)\kappa\chi\gamma,$$

χ_π *and* χ_y *are as defined in Propositions 5 and 8, and* λ *equals* $\lambda^{RANK-FTPL}$ *or* λ^{HANK}*, as defined in Propositions 5 and 8, in the respective models.*

Proof: See the appendix.

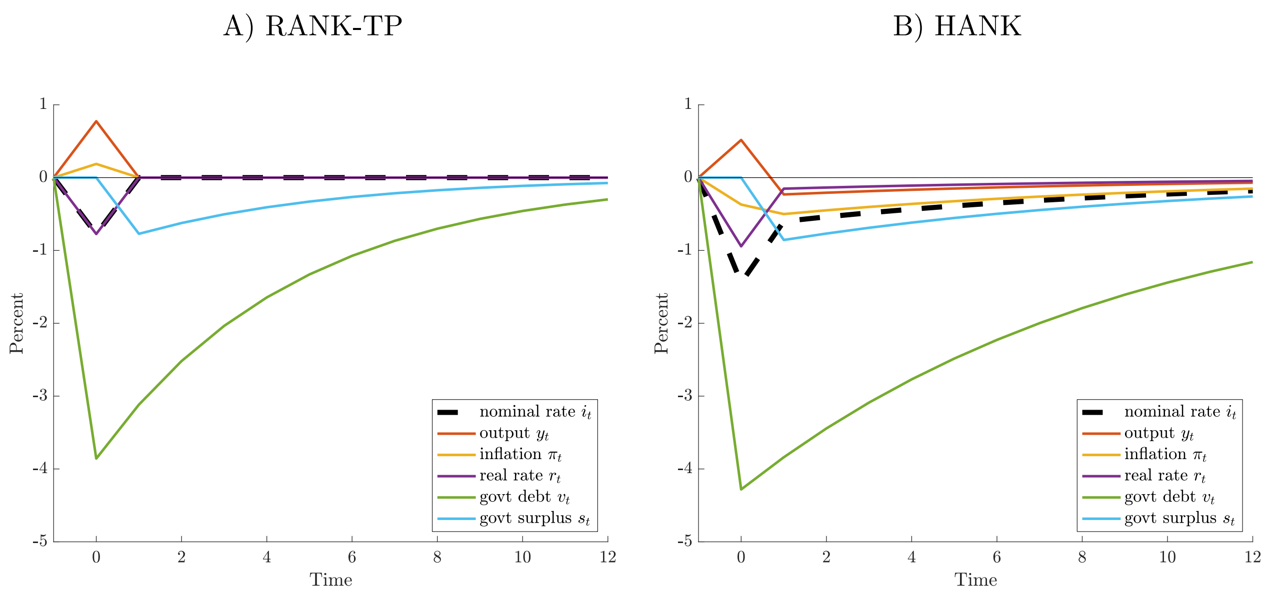
In any model we consider – RANK or HANK – a surprise interest rate cut lowers government debt, effectively tightening the fiscal stance. The crucial difference between the textbook RANK-TP and HANK is that this tightening has direct consequences for demand and inflation in the latter (and, through the equilibrium selection discussed in the previous sections, also drives demand and inflation in RANK-FTPL). Thus, fiscal implications of monetary policy cast a persistent shadow on the effects of an interest rate cut in HANK. As a result, short-run output increase is smaller than in RANK, and is followed by a persistent, negative output gap. This is because the fiscal adjustment back to the steady state takes time. Thus, the HANK model predicts persistent effects even of purely transitory monetary policy shocks.

The persistent negative demand reaction driven by the persistent fiscal tightening means that inflation – the NPV of output – can decline in response to this shock even on impact. When the shock hits, demand likely increases as the intertemporal substitution force dominates the fiscal consequences in the initial period when the interest rate is lowered. Thereafter, only the influence of the fiscal tightening remains.

Figure 5 illustrates the results in the Proposition by showcasing the impulse responses following an expansionary monetary policy shock in the RANK-Taylor Principle and in the HANK models. In line with the analytical results and the preceding discussion, on impact, demand response is smaller in HANK, and there is a persistent period of weak demand, driven by lower government debt. The latter effect is the dominant one in driving the inflation response: inflation is negative throughout.[19]

Of course, whether inflation actually falls on impact depends on the specific parameters of the

[19] Note that, as long as the coefficient on the contemporaneous inflation (ϕ_π) in the Taylor Rule is positive, the central bank lowers the nominal rate by more than the shock (while the opposite is true in RANK, where the central bank self-corrects within the period, given that inflation responds positively).



Notes: The figures show the response of the economies to a $e_0^i = -1\%$ unexpected, one-off cut in the nominal interest rate. The figures are drawn assuming $\beta = 0.99$, $\kappa = 0.24$, and $\gamma = 4$ (implying an debt-to-annual GDP ratio of 1), $\phi_\pi = 1.2$, $\alpha_b = 0.2$. In the HANK model we set $q = 0.75$.

Figure 5: Monetary policy shock in RANK-TP and in HANK

model, as well as model details, including many realistic features absent from the simple model we study here. Nonetheless, the analysis in this section illustrates how the non-Ricardian behavior can matter greatly for monetary policy and its fiscal implications, even if the central bank is focused solely on its inflation mandate.

6 Anticipation effects

So far, we have shown that the HANK model exhibits much more gradual dependence on policy parameters, compared to the RANK benchmark. We now show that quite the opposite is true for anticipation effects: in RANK, the effects of pre-announced fiscal transfer programs are the same as when transfers come as a surprise: the benchmark RANK model is remarkably insensitive to the timing of transfers. Instead, anticipation effects are crucially important in HANK.

6.1 RANK

Whether a deficit shock is anticipated or not does not matter in RANK, and the reason for that is straightforward: in RANK-Taylor Principle, current or future deficit shocks are irrelevant for the macro variables. And in RANK-FTPL, inflation adjusts to bring about the balance between the real value of government debt and the *net present value* of surpluses. The latter is not very sensitive to the timing of deficits, at least for standard values of the discount rate and anticipation horizon of up to several years.

To illustrate this, Figure 6 plots the response of the economy to an unanticipated and a 4-quarter-ahead anticipated fiscal shock in the FTPL regime (assuming $\phi_\pi = 1$ and $\alpha_b = 0$). The responses are virtually identical, except for the path of government debt, which in this model is irrelevant for the macro variables.

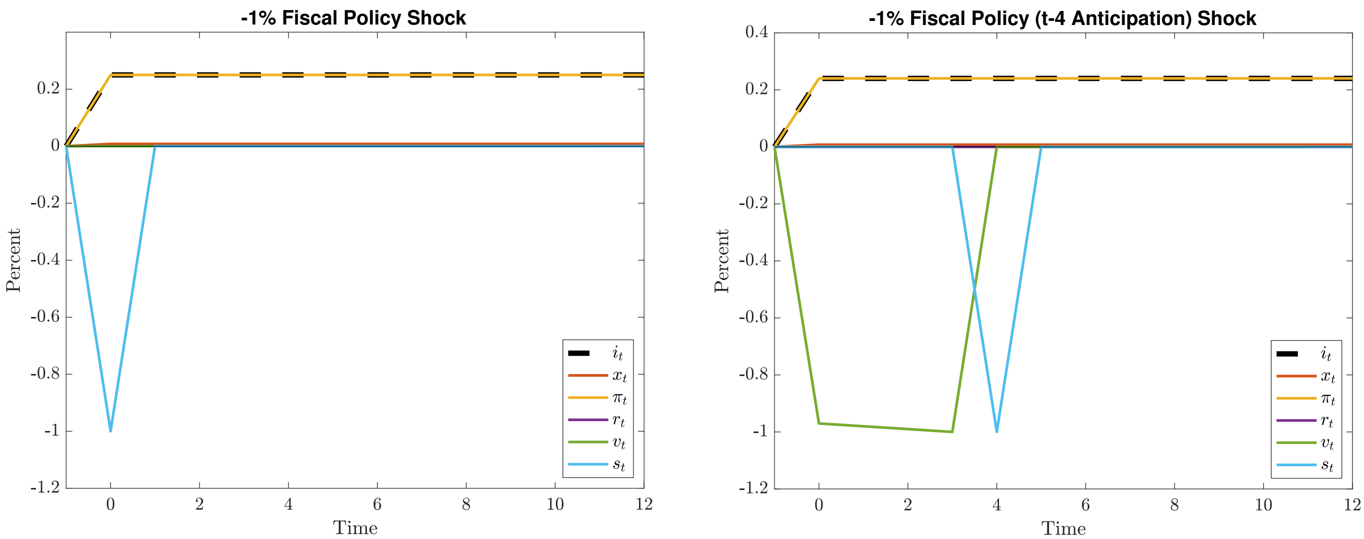


Figure 6: Unanticipated and anticipated fiscal transfer shock in the RANK-FTPL model

6.2 HANK

Anticipation effects matter greatly in HANK. An anticipated fiscal transfer generates an expectation of a boom in demand. This tends to raise inflation expectations and, through the forward-looking pricing decision of firms, pushes up prices and inflation even before the stimulus actually hits the pockets of the finitely-lived individuals. Consequently, during the anticipation episode, higher inflation reduces the real value of households' assets. This acts as a dampener on demand. As a result, an anticipated fiscal deficit can lead to a near-term recession followed by a subsequent boom – in other words, a pattern of heightened macroeconomic volatility.

This intuition is illustrated in Figure 7. The left panel shows the inflation and output boom following a time-0 fiscal transfer shock. The right panel of the figure shows that the economy reacts very differently to an anticipated shock, with inflation that reduces the real value of household assets and generates a decline in output in the near term.

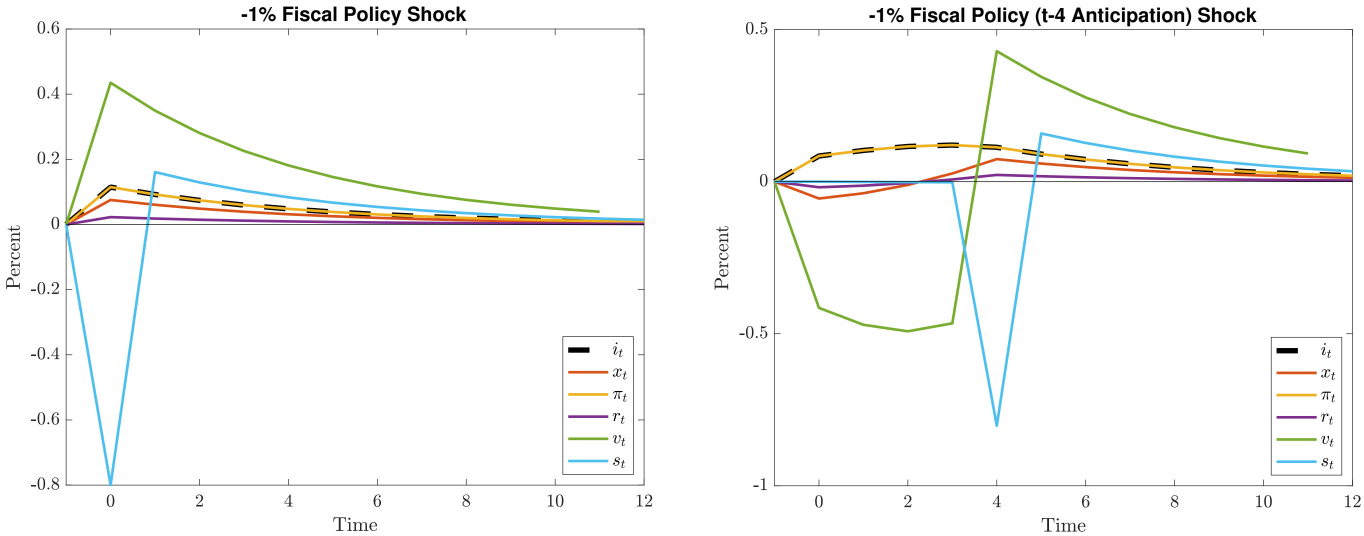


Figure 7: Unanticipated and anticipated fiscal transfer shock in HANK

7 A Medium Scale New Keynesian Model

We now relax a number of the simplifying assumptions made above and examine whether key results regarding the determinacy properties and the impact of government debt are robust to the extension to a medium-scale New Keynesian model.

7.1 The Extended Model

We extend the simple model with the following features. First, we allow households to save both in government bonds and in capital. Capital is an input to production that firms rent from the households. Secondly, we introduce investment adjustment costs, to deal with the well-known property that the economy otherwise becomes excessively volatile when capital is introduced. Third, we now also allow for sticky nominal wages. We do this by introducing labor unions with market power. Fourth, we generalize the fiscal framework by introducing government spending and distortionary tax finance. Fifth, we generalize the monetary policy side by allowing for interest rate smoothing. Finally, following Cochrane (2001) and Cochrane (2022), we allow for long-term government debt.

7.1.1 Households

We assume that household preferences are given as:

$$\mathbf{U}_{s,t} = \mathbb{E}_t \sum_{h=0}^{\infty} (\beta q)^h \left(\log c_{s,t} - \frac{\psi}{1+\kappa} n_{s,t}^{1+\kappa} \right)$$

which generalizes the earlier log-log preference specification allowing us to target a realistic value of the Frisch labor supply elasticity, $1/\kappa \geq 0$

Households can save in government bonds and in capital. They purchase new capital goods at the nominal price P_t^i from competitive capital producers. Capital depreciates at the proportional rate δ per period. As above, we allow for life insurance and for a social fund. Having purchased new capital at the beginning of the period, households deposit their assets with the life-insurance company. The life insurance company then collects the return on government bonds, rents out the capital stock to firms at the real rental rate r_t^k, and refurbishes the depreciated capital. If the household survives until next period, the life insurance company pays out a premium to the household that is proportional to its deposits including returns accrued during the period. If the household does not survive, the ownership of its assets passes over to the life insurance company. Free entry to the insurance industry implies that the premium is actuarially fair and equal to $1/q$ times deposits cum returns. The government then taxes all factor income at the proportional rate τ^y and makes transfers. Finally, the social fund taxes currently alive households and makes transfers to newly born households.

The sequence of budget constraints faced by households is given as:

$$\begin{aligned} c_{s,t} + p_t^i k_{s,t} &+ P_t^1 B_{s,t}^1/P_t + P_t^n B_{s,t}^n/P_t = (1-\tau^y) w_t n_{s,t} \\ &+ \frac{(p_t^i + (1-\tau^y) r_t^k - \delta) k_{s,t-1} + B_{s,t-1}/P_t + (1+\xi P_t^n) B_{s,t-1}^n/P_t}{q} + \tau_t + d_t^f + d_t^u + z_s \end{aligned}$$

where $p_t^i = P_t^i/P_t$ is the relative price of new capital goods, $k_{s,t}$ denotes the capital stock acquired by households in period t, r_t^k is the capital rental rate, d_t^f denotes profit transfers from firms. $d_{s,t}^u$ are transfers from labor unions discussed below. $B_{s,t}^1$ is a one-period pure discount bond which is purchased at price P_t^1 at date t and pays out one unit of currency at date $t+1$. Therefore, the gross short term nominal interest rate equals $1+i_t = 1/P_t^1$. $B_{s,t}^n$ is a long-term bond with average duration $n = 1/(1-\beta\xi)$ which is purchased at price P_t^n and pays out ξ^h units of currency in period $t+h$.

Iterating the budget constraint forwards, exploiting arbitrage between capital and bond investments, and imposing a no-Ponzi game restriction, implies that:

$$\sum_{j=0}^{\infty} R_{t,t+j} c_{s,t} = \sum_{j=0}^{\infty} R_{t,t+j} \Big((1-\tau^y) w_{t+j} n_{s,t+j} + d_{s,t+j} + d_{t+j}^f + d_{t+j}^u + \tau_{t+j} + z_s \Big) + \frac{\tilde{v}_{s,t-1}}{q}$$

where:

$$\tilde{v}_{s,t-1} = (p_{i,t} + (1-\tau^y) r_t^k - \delta) k_{s,t-1} + \frac{b_{s,t-1}^1 + (1+\xi P_t^n) b_{s,t-1}^n}{P_t}$$

is the beginning of the period t return inclusive value of savings from the period $t-1$. The first-order conditions for household choices of labor supply, one-period bonds, n-period bonds, and capital are given as:

$$\psi n_{s,t}^{\kappa} = \frac{(1-\tau^y)w_t}{c_{s,t}} \tag{41}$$

$$\frac{1}{c_{s,t}} = (1+i_t)\beta\mathbb{E}_t\frac{1}{c_{s,t+1}}\frac{1}{\pi_{t+1}} \tag{42}$$

$$\frac{1}{c_{s,t}} = \beta\mathbb{E}_t\frac{1}{c_{s,t+1}}\frac{1+\xi P_{t+1}^n}{P_t^n}\frac{1}{\pi_{t+1}} \tag{43}$$

$$\frac{1}{c_{s,t}} = \beta\mathbb{E}_t\frac{1}{c_{s,t+1}}\frac{p_{t+1}^i+(1-\tau_{t+1}^y)r_{t+1}^k-\delta}{p_t^i} \tag{44}$$

Arbitrage between short-term and long-term government bonds implies:

$$(1+i_t)\mathbb{E}_t\frac{1}{\pi_{t+1}} = \mathbb{E}_t\frac{1+\xi P_{t+1}^n}{P_t^n}\frac{1}{\pi_{t+1}}$$

so that changes in short-term nominal interest rates are reflected in capital gains and losses on long-term bonds.[20]

7.1.2 Labor unions and labor packers

Labor packers: Labor packers purchase a continuum of labor varieties from the labor unions, $n_{h,t}$, at nominal prices $W_{h,t}$. Labor packers are competitive and produce labor services, n_t, which they rent out to intermediate goods producers at the nominal wage, W_t^f. The technology for producing labor services is given as:

$$n_t = \left(\int_h n_{h,t}^{1-1/\phi_n}dj\right)^{1/(1-\phi_n)}$$

where $\phi_n > 1$. Given this, the demand for each variety of labor and the wage paid by firms are given as:

$$n_{h,t} = \left(\frac{W_{h,t}}{W_t^f}\right)^{-\phi_n} n_t$$

$$W_t^f = \left(\int_h W_{h,t}^{1-\phi_n}dj\right)^{1/1-\phi_n}$$

Labor Unions: There is a large set of wage setting labor unions indexed by h that purchase labor from households at the nominal wage W_t. Labor unions differentiate labor and set the nominal wage, $W_{h,t}$ to maximize their rents subject to the labor demand from the labor packers and then return

[20] It follows from the arbitrage condition that in a steady-state $P^n = \frac{1}{1+i-\xi} = \frac{\beta}{1-\xi\beta}$.

any rents to households. In a flexible wage setting, this would lead labor unions to set wages as:

$$W_{h,t}^{f} = \frac{\phi_n}{\phi_n - 1} W_t$$

where W_t is the wage paid to households which equals the marginal rate of substitution between consumption and work corrected for the income tax wedge. Wage setting is subject to a Calvo-style nominal wage rigidity. This introduces a Phillips curve-type relationship for nominal wage inflation which, to a first-order approximation, implies that:

$$\widehat{\pi}_{w,t} = \beta \mathbb{E}_t \widehat{\pi}_{w,t+1} + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w} w_t$$

where $\pi_{w,t} = W_t^f / W_{t-1}^f$ indicates nominal wage inflation, and θ_w is the degree of nominal wage stickiness.

7.1.3 Firms

On the supply side, there are competitive final goods producers, monopolistically competitive intermediate goods producers, and competitive capital goods producers.

Final goods producers: Firms in the final goods sector are competitive and produce output using inputs of intermediate goods according to the CES technology:

$$y_t = \left(\int_j y_{j,t}^{1-1/\phi_g} dj \right)^{1/(1-1/\phi_g)}$$

with the associated price index given as:

$$P_t = \left(\int_j P_{j,t}^{1-\phi_g} dj \right)^{1/(1-\phi_g)}$$

where $P_{j,t}$ denotes the nominal price of intermediate good j. Final goods are used for consumption, government purchases, and for investment. The resource constraint is:

$$y_t = c_t + g_t + ci_t$$

where ci_t denotes the total amount of resources spent on producing new capital.

Intermediate goods producers: Intermediate goods producers are monopolistically competitive and set prices subject to nominal price rigidity. Their technology is given as:

$$y_{j,t} = \left(n_{j,t}^{f} \right)^{s_n} k_{j,t-1}^{1-s_n} \tag{45}$$

where $n^f_{j,t}$ is intermediate goods producer j's input of labor, and $k_{j,t-1}$ is their use of capital. $s_n \in (0,1)$ is the output elasticity to labor and we impose constant returns.[21] Firms rent labor from labor unions at the nominal wage W^f_t and capital from households at the real rental rate r^k_t.

Capital Producers: Capital goods are supplied by competitive capital goods producers. Depreciated capital is refurbished costlessly, while new capital goods are produced subject to quadratic adjustment costs parametrized by $\omega_I > 0$. Let p^i_t denote the relative price of new capital goods (in units of the final good), and net investment by $i_{n,t}$. Net revenue of the capital producer is then:

$$v^I_t = \left(p^i_t - 1\right) i_{n,t} - \frac{\omega_I}{2}\left(\log\left(\frac{i_{n,t}+\psi}{i_{n,t-1}+\psi}\right)\right)^2 (i_{n,t}+\psi) \tag{46}$$

where $\psi \geq 0$ is a constant. p^i_t is determined as:

$$\frac{p^i_t - 1}{\omega_I} = \log\left(\frac{i_{n,t}+\psi}{i_{n,t-1}+\psi}\right) + \frac{1}{2}\left(\log\left(\frac{i_{n,t}+\psi}{i_{n,t-1}+\psi}\right)\right)^2 - \beta\mathbb{E}_t\left(\log\left(\frac{i_{n,t+1}+\psi}{i_{n,t}+\psi}\right)\right)\frac{i_{n,t+1}+\psi}{i_{n,t}+\psi} \tag{47}$$

Denote gross additions to the capital stock by x_t, and ci_t as total resources spent on capital production. It follows that:

$$k_t - k_{t-1} = i_{n,t}, \tag{48}$$

$$x_t = i_{n,t} + \delta k_{t-1}, \tag{49}$$

$$ci_t = x_t + \frac{\omega_I}{2}\left(\log\left(\frac{i_{n,t}+\psi}{i_{n,t-1}+\psi}\right)\right)^2 (i_{n,t}+\psi). \tag{50}$$

where $\delta \in (0,1)$ is the capital depreciation rate, and ψ is a constant.

7.1.4 Government

As above, we assume that the government sector is split into a monetary authority in charge of setting nominal interest rates and a fiscal authority that sets the fiscal instruments.

Monetary policy: We extend the monetary policy setting by allowing for interest rate smoothing:

$$1 + i_t = R\left(1 + i_{t-1}\right)^{\phi_i}\left(\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{\bar{y}}\right)^{\phi_x}\right)^{1-\phi_i}\exp\left(\varepsilon^i_t\right) \tag{51}$$

where $\phi_i \in [0,1)$ parametrizes the extent of interest rate smoothing.

Fiscal policy: The government issues government debt, makes transfers, purchases final goods, and

[21] It is straightforward to allow for productivity shocks. Our focus is on the impact of deficits and monetary policy shocks and therefore we eliminate other aggregate shocks from the medium-scale model.

collects tax revenue. The budget constraint is given as:

$$P_t^1 B_t^1 + P_t^n B_t^n = B_{t-1}^1 + (1 + \xi P_t^n) B_{t-1}^n - P_t s_t \tag{52}$$

where s_t denotes the real government primary surplus given as:

$$s_t = \tau^y (w_t n_t + r_t^k k_{t-1}) - g_t + \tilde{s}_t$$

where $n_t = \int_s n_{s,t} ds$ and $-\tilde{s}_t$ are transfers. We assume that transfers obey a fiscal rule as in the simple model:

$$\tilde{s}_t = \bar{s} \left(\frac{\tilde{b}_t}{b^T} \right)^{\alpha_b} \exp((1 - \alpha_b) e_t^s) \tag{53}$$

where

$$\tilde{b}_t = \frac{B_{t-1}^1 + (1 + \xi P_t^n) B_{t-1}^n}{P_t}$$

is the real value of outstanding government debt obligations and b^T is a target for government debt. We assume that the distortionary tax rate, τ_y, and government spending are constant.

7.2 Calibration

We calibrate the model to the quarterly frequency. We solve the model by a first-order (log) perturbation around a deterministic steady-state with price (and wage) stability. We neutralize price and wage markups by subsidies. We first discuss the baseline calibration of the model, but will vary several of these parameters to evaluate the importance of different mechanisms in the model.

We set $\beta = 0.99$ so that it implies an annual real interest rate of approximately 4 percent in the steady state. The preference weight on the disutility of work, ψ, is calibrated to be consistent with hours worked equal to 30 percent in the steady state. The Frisch labor supply elasticity is determined by $1/\kappa$ and we set this parameter equal to 0.75 which is in line with the consensus estimate of Chetty et al (2011). In the baseline, we calibrate q to target a MPC just around 25 percent per quarter in line with the calibration in Kaplan, Moll and Violante (2018) but significantly smaller than the estimates in the tax rebate literature such as Parker, Souleles, Johnson and McClelland (2013). In particular, we assume that $q = 0.75$ which implies a quarterly MPC of 25.7 percent.

On the firm side, we set the degree of price stickiness as measured by θ_p to target an average contract length of 4 quarters, which is a standard value in the literature. This implies that $\theta_p = 0.75$. Our baseline calibration adopts the same value for the degree of nominal wage stickiness, $\theta_w = 0.75$, but we will also examine how the results depend on the degree of wage stickiness relative to price stickiness.[22] We then set the output elasticity to labor equal to 0.65, which is a standard value in

[22]Since the steady-state price and wage markups are neutralized by subsidies, the elasticities of substitution,

the literature.

On the asset side, we set the level of government debt to GDP equal to 5 at the quarterly rate, which corresponds to the current US level. In the baseline case, we assume that all government debt is short-term, but will also look at the impact of assuming that the average maturity is 72 months, the current (weighted) average maturity of US government debt. We set the capital-output ratio equal to 4, which is low relative to standard calibrations. However, in this model, capital is a liquid asset (despite the presence of adjustment costs) while much of the HANK literature assumes that capital is, at least partially, illiquid. This matters for the results because the Euler equation central to the properties of the model relates to the liquid assets. Kaplan, Moll and Violante (2018), for example, assume that all of the physical capital stock is illiquid, in which case the return on liquid assets can move in the short run in a manner almost independently of the return on capital. Assuming, on the other extreme, that there are no adjustment costs and that capital is liquid, implies that the real return on assets is determined entirely by the marginal product of capital. To strike a balance in between these extremes, we calibrate to a capital-output ratio of 4.[23] We set the adjustment cost parameter equal to 10, but given the lower capital-output ratio, this parameter matters less for the results.

We assume that the level of government spending corresponds to 20 percent of GDP in the steady state. Given this and other parameters, we calibrate the average tax rate, τ_y, so that it is consistent with the level of government debt. In the baseline model, this implies an average income tax rate of 25 percent.[24] On the monetary policy side, we set $\phi_y = 0$ so as to focus on the importance of ϕ_π but also examine the importance of this parameter. We allow for interest rate smoothing and set the degree of interest rate smoothing equal to 0.7 following Rudebusch (1995).

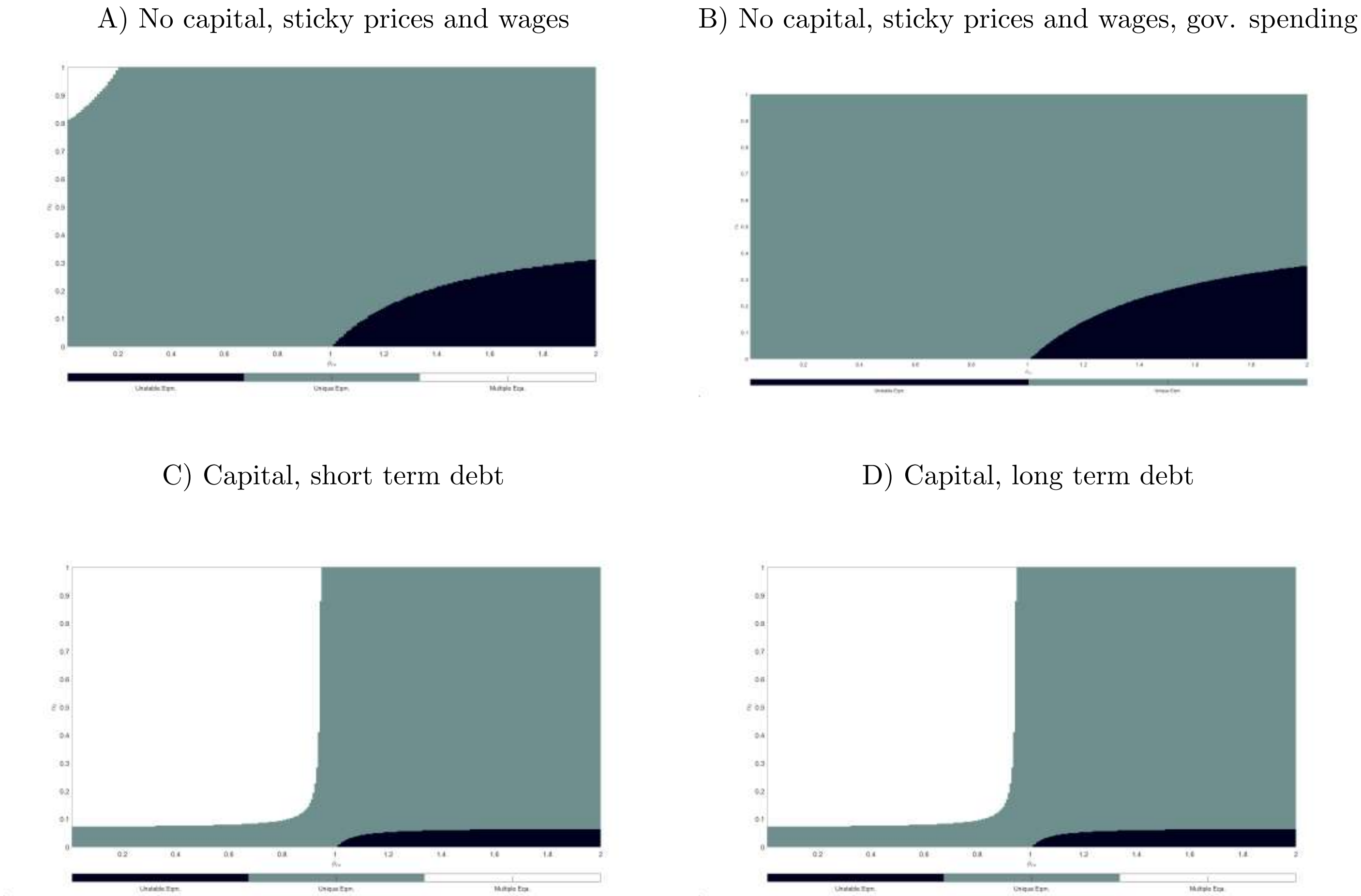
7.3 Determinacy Properties

We first examine the determinacy properties of the model. In Figure 8 we illustrate the determinacy properties of the model when introducing key extensions in steps. In Panel A we show the model without capital, sticky prices and sticky wages, but no government spending. In Panel B we add government spending, assuming it accounts for 20 percent of output in the deterministic steady state. In Panel C we introduce capital accumulation and assume that all government debt is one-period, while in Panel D we assume long-term government debt. In all cases, the areas shaded in green indicate policy parameter combinations that induce locally determinate equilibria; those in white correspond to local indeterminacy, and those in black correspond to policy parameter combinations

$1/\phi_g$ and $1/\phi_n$, do not enter the log-linearized model.

[23] Galí (2021) also shows that if firms are established by households, the capital stock does not enter the aggregate Euler equation.

[24] The steady-state tax rate is determined as the the government spending to GDP ratio plus the interest rate cost of rolling over the steady-state government debt to GDP ratio.



Notes: The panels illustrate the determinacy properties of the model. Green shaded areas correspond to policy parameter regions with a unique equilibrium. White areas correspond to policy parameter regions with local indeterminacy. In the black shaded area, there is no stable equilibrium.

Figure 8: Local Determinacy Properties

inconsistent with the existence of stable equilibria.

Allowing for both sticky prices and wages does not change the determinacy properties of the model relative to the simpler model analyzed earlier. When we add constant government spending, the small subset of the policy parameter space for which there is local indeterminacy is essentially eliminated. Thus, as in the simple model, we find that the introduction of non-Ricardian equivalence features fundamentally changes the determinacy properties of the model.

The introduction of capital is of more fundamental importance, see Panel C, while allowing for long-term government debt matters little, see Panel D. When we introduce capital, the determinacy properties are more similar to those of the standard RANK model, yet, qualitatively key insights from the simple model still hold true. There are two main reasons for why introducing capital matters for the determinacy properties. First, in this economy, government debt accounts for only a fraction of real household financial assets. Secondly, arbitrage between bonds and capital implies that the real interest rate is intrinsically linked to the (marginal products of the) capital stock and less sensitive to monetary policy actions.[25] Because of the first of these features, the demand channel of deficits is less powerful in the medium-scale model with capital. Nonetheless, Ricardian Equivalence still fails and there is a direct channel from fiscal deficits to aggregate demand so that monetary- and fiscal policies always interact. The second aspect implies a less strong impact of monetary policy actions on the fiscal policymaker thus requiring a stronger fiscal response to deficits in order to assure government solvency in the absence of the aggressive monetary responses to inflation.

Despite this sensitivity to the introduction of capital, key conclusions from the simple model survive. First, the Taylor principle is neither necessary nor sufficient for local determinacy. Secondly, there are no separate zones of local determinacy characterized by active and passive policies. Third, when the Taylor principle fails, there is local determinacy for a much larger set of fiscal policies than in the RANK model.[26]. The introduction of longer term government debt, in contrast, does not matter for the determinacy properties of the model. As we will discuss below, this does not mean, however, that the assumptions regarding the maturity structure of government debt are unimportant.

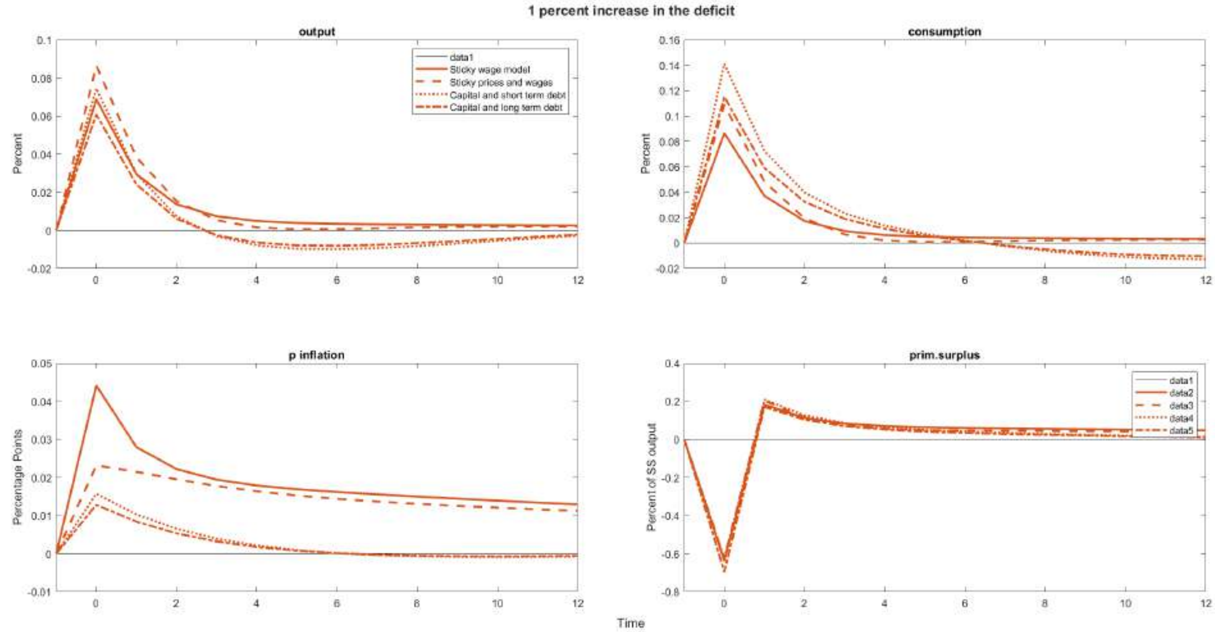
7.4 The Impact of Deficits

Next, we investigate the extent to which deficits impact the economy. In Figures 9 and 10 we illustrate the impulse responses of output, consumption, inflation, and the primary budget surplus after an increase in the deficit corresponding to one percent of steady-state output. For these exercises, we assume that government spending accounts for 20 percent of steady-state output and that there is interest rate smoothing. We first look at how the results depend on the source of nominal rigidities, assuming that labor is the only input to production (and that government bonds are the only financial assets). In particular, we compute impulse responses to the deficit shock, assuming either that only wages are sticky or that both prices and wages are sticky. Next, we introduce capital in the economy with both price and nominal wage stickiness. In these first three cases, we assume that government debt is short-term. Finally, we introduce longer-term government debt, setting the average maturity of government bonds equal to 72 months.

Moreover, we examine two regions of the policy parameter space. In the first region, we assume that $\phi_\pi = 1.5$ and $\alpha_b = 0.3$. This parameter constellation is therefore in the subset of the parameter space usually considered as referring to the case where monetary policy is active and fiscal policy is passive. In the RANK setting, this implies that deficits are neutral in this region of the parameter

[25]The presence of investment adjustment costs are important for providing monetary policy makers with the ability to impact on short term real interest rates in models with capital, see Rupert and Sustek (2019).

[26]Recall that in the RANK model, when $\phi_\pi < 1$, local determinacy requires $\alpha_b < 1 - \beta \sim 0.01$. In the medium-scale model, there is local determinacy of the equilibrium whenever $\alpha_b > 0.08$ and for a much larger set of fiscal policies as ϕ_π approaches unity.



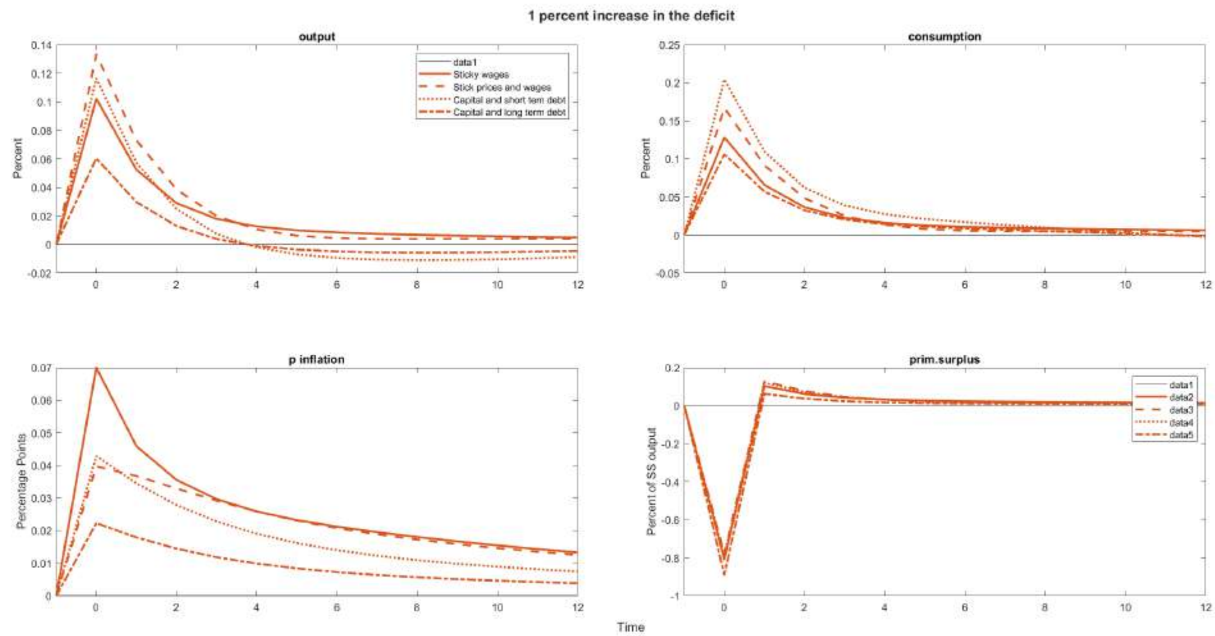
Notes: The panels illustrate the impulse responses when (a) there is no capital and only wages are sticky (full lines), (b) there is no capital and both wages and prices are sticky (dashed lines, (c) there is capital, both wages and prices are sticky and the government issues one period debt (dotted lines), (d) there is capital, both wages and prices are sticky and the government issues long-term debt (dash-dotted lines).

Figure 9: Impact of Deficit Shock when $(\phi_\pi, \alpha_b) = (1.5, 0.3)$

space, but due to the failure of Ricardian Equivalence, this is not the case in the medium-scale HANK model. In the second case, we assume that $\phi_\pi = 0.9$ and $\alpha_b = 0.1$. In this region, the RANK model implies local indeterminacy while the equilibrium is locally determinate in the medium-scale HANK model.

We find remarkable robustness of the impact of deficits on output and consumption across variations of the model and across the parts of the policy parameter space that we inspect here. A one-time increase in the deficit stimulates output and consumption regardless of whether we assume that nominal rigidities derive from prices or wages, whether there is capital or not in the model, and whether the government issues short-term or longer-term debt. Quantitatively, the consumption responses tend to be larger than the output responses and higher in the region with weaker policy responses to inflation and debt.

Moreover, regardless of which variation of the model that we look at, and whether we focus on the region with $(\phi_\pi, \alpha_b) = (1.5, 0.3)$ or with $(\phi_\pi, \alpha_b) = (0.9, 0.1)$, deficits are inflationary. For well-known reasons, the jump in inflation is smaller when the government issues long-term debt than in the one-period debt scenario, but qualitatively, the dynamics of inflation are very similar across policy and parameter regions. Thus, the results that we have discussed earlier about the inflation impact of deficits continue to hold in the medium-scale model.



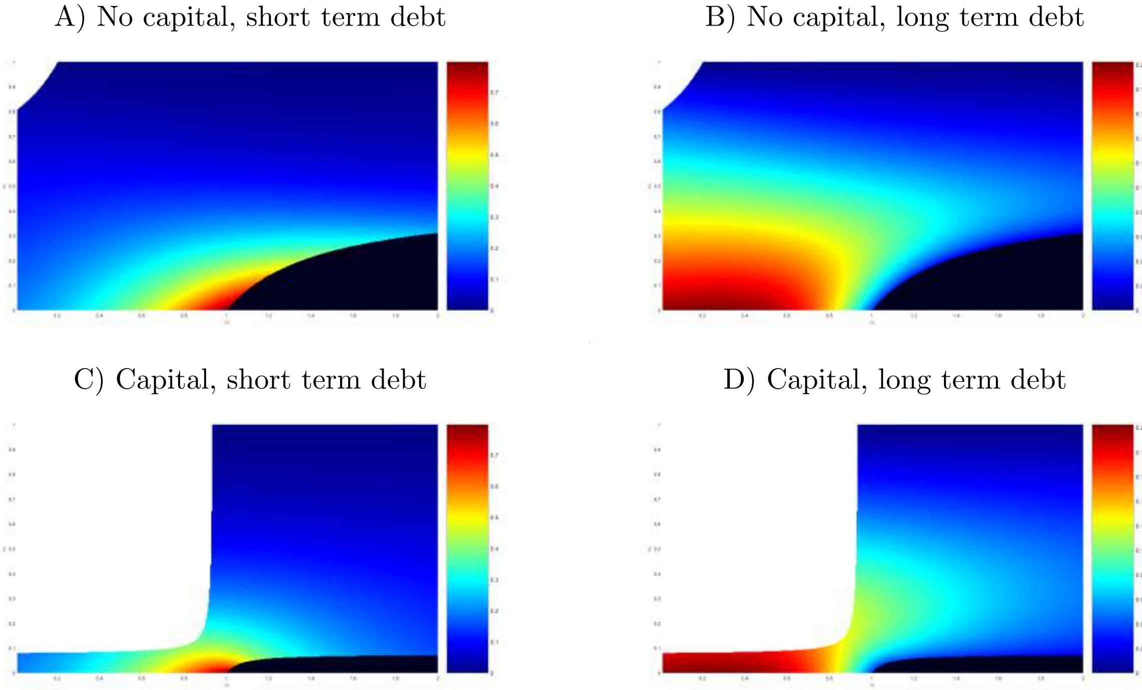
Notes: The panels illustrate the impulse responses when (a) there is no capital and only wages are sticky (full lines), (b) there is no capital and both wages and prices are sticky (dashed lines, (c) there is capital, both wages and prices are sticky and the government issues one period debt (dotted lines), (d) there is capital, both wages and prices are sticky and the government issues long-term debt (dash-dotted lines).

Figure 10: Impact of Deficit Shock when $(\phi_\pi, \alpha_b) = (0.9, 0.1)$

Next, we revisit the result from Section 4 that in the HANK setting, a more hawkish monetary response leads to higher inflation following deficits, a result that may be viewed as controversial. In Figure 11 we illustrate the cumulated inflation response over the first year after a one percent increase in the deficit across the policy parameter space for four different variations of the model. For all cases, we assume that prices are sticky while wages are flexible, that government spending accounts for a tiny fraction of output, and that there is no interest rate smoothing. These auxiliary assumptions are irrelevant for the points that we will make. The first variation of the model effectively reproduces the assumptions embedded in the simple model studied above: there is no capital and government debt is short term (has a maturity structure of one quarter).[27] In the second variation, we introduce longer term debt into this model, setting the average duration of government bonds equal to 72 months. Third, we introduce capital but assume short term government debt finance. Fourth, we combine the two features by allowing for both capital and longer-term government debt.

We find that whether or not a more aggressive monetary stance stabilizes the inflationary impact of deficits is sensitive to assumptions on the financing of deficits with short-term or longer-term debt, but not to the introduction of capital. When the government issues only short-term debt, a more

[27]This model only differs from the simple model analyzed earlier in that the Frisch labor supply elasticity is realistically calibrated.



Notes: The figures show the cumulated inflation response four quarters after a one percent increase in the deficit. All panels assume that $q = 0.75$, $\beta = 1/(1.01)$, $\kappa = 4/3$, $\gamma = 5$, $\phi_x = 0$, $\phi_R = 0$, $\phi_p = 0.75$, $g/y = 0.01$, and $\theta_w = 0.01$. In panels A and B we assume that $k/y = 0.01$ and $s_n = 0.99$. In panels C and D we assume $k/y = 4$ and $s_n = 0.65$. In panels A and C we assume that all government debt is one-period. In panels B and D we assume that the average duration of government debt is 24 quarters.

Figure 11: Inflation Responses and Policy Rules: Medium-Scale HANK

aggressive monetary stance produces a more inflationary response to deficit shocks. Intuitively, a more aggressive monetary policy stance on inflation makes the demand stimulus smaller but more persistent, thereby inducing a worse inflation-output trade-off. In HANK, this mechanism works through the impact of government debt on aggregate demand, as discussed earlier.

In contrast, when we set the average duration of government debt equal to 72 months, the more aggressively the central bank responds to inflation, the more it stabilizes inflation responses to budget deficits. The crucial difference between the model with short-term debt and the longer-term debt economy is the response of bond prices. Cochrane (2018) discuss in detail the underlying logic in the RANK-FTPL model. Here, when the government issues long-term bonds, the movement in bond prices are important for the transmission of monetary policy, and the fiscal consequences of adjustments of nominal interest rates are different from the standard NK model with one-period bonds. In the latter case, when the central bank raises short-term interest rates, it puts a drag on the fiscal authority, and this is the underlying reason for why a more aggressive monetary stance has the tendency of producing worse inflationary consequences of deficits in this model. In the face of longer-term government bond financing, long-term bond prices fall in response to hikes in the short-term nominal interest rates. These bond price movements effectively ease the fiscal burden of deficits implying that a more aggressive monetary stance now leads to less inflation in the face of fiscal deficits. In the HANK setting, rather than these effects working through equilibrium selection, they work through the induced demand effects. In particular, as the monetary policymaker responds more aggressively and the fiscal policymaker relies on longer-term bond finance, the decline in long-term bond prices induces a capital loss on households, which moderates the demand effects of the deficit, leading to inflation stabilization.

Thus, the results in Section 4 regarding the inflation-output trade-off in the face of shocks to the deficit are sensitive to the modeling of the maturity structure of government debt.

7.5 Monetary Policy Shocks

Next, we revisit the impact of monetary policy shocks in the medium-scale model. As discussed in Section 5, the simple HANK model shares with the RANK-FTPL model the aspect that a reduction in nominal interest rates reduces inflation. In RANK-FTPL, this outcome comes from the fact that lower short-term nominal interest rates reduce real interest rates and the present value of the government budget deficit. In other words, the decline in the nominal interest rate corresponds to a tightening of fiscal policy. For that reason, inflation falls, and along with it, output (due to the standard connection between output and inflation through the NK Phillips curve). In the simple HANK model, the fiscal tightening spills over to demand through the non-Ricardian feature of the model, and there is therefore a consistent downward pressure on inflation in the HANK model.

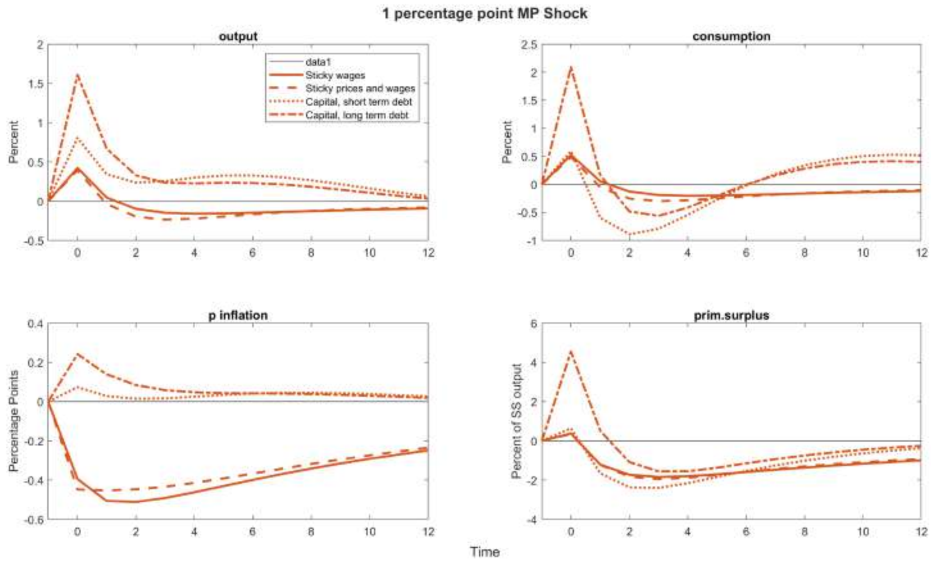
Such disinflationary effects of interest rate cuts are a controversial property of the RANK-FTPL model and therefore also of the simple HANK model. In particular, much of the monetary policy literature builds on the idea that central banks fight inflation by raising interest rates, and this is customarily considered to be the empirically plausible case.[28]

Figure 12 shows the impulse responses of the model to a monetary policy shock corresponding to a one percentage point drop in the short-term rate. We show impulse responses for the same parameter constellations as when we examined the fiscal transfer shock above.

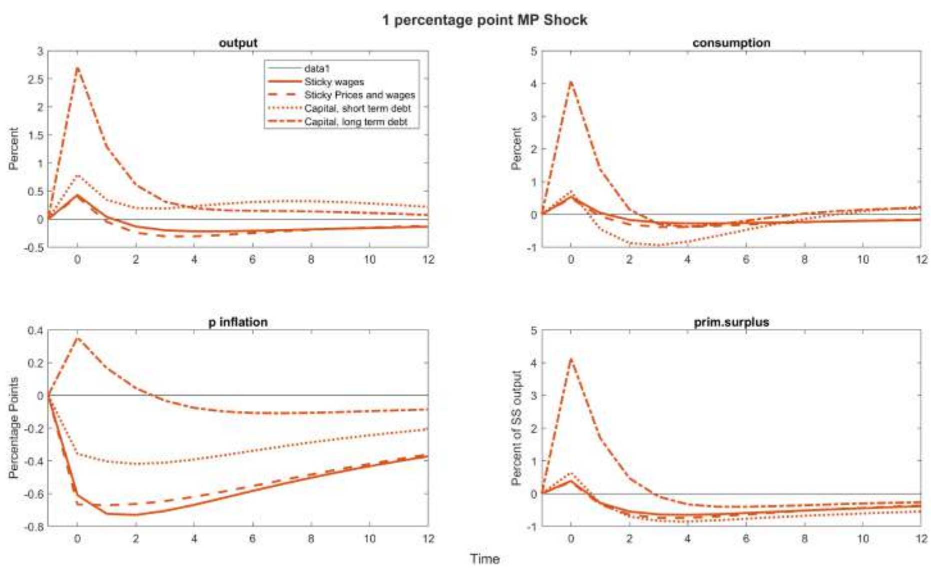
We find that the medium-scale HANK model extended with long-term government debt can produce "conventional" effects of monetary policy shocks. The reason is that households experience capital gains on their assets when the central bank cuts the short-term interest rates. When monetary policy is rather aggressive and the primary surplus responds elastically to variations in debt, $(\phi_\pi, \alpha_b) = (1.5, 0.3)$, capital gains on the capital stock are sufficient to produce a demand stimulus

[28]Nonetheless, it is well-known that many empirical estimates of the impact of monetary policy shocks are associated with a Price Puzzle, that is, a short term reduction in inflation in the face of interest rate cuts. It remains disputed whether this result derives from mis-specification of empirical models due to e.g. misalignment of the information set or inappropriate measures of monetary policy shocks.

A) When $(\phi_\pi, \alpha_b) = (1.5, 0.3)$



B) When $(\phi_\pi, \alpha_b) = (0.9, 0.1)$



Notes: The panels illustrate the impulse responses when (a) there is no capital and only wages are sticky (full lines), (b) there is no capital and both wages and prices are sticky (dashed lines, (c) there is capital, both wages and prices are sticky and the government issues one period debt (dotted lines), (d) there is capital, both wages and prices are sticky and the government issues long-term debt (dash-dotted lines).

Figure 12: Impact of Monetary Policy Shock in the Medium-Scale Model

that overturns the deflationary forces present in the simple HANK model. Adding long-term government debt adds to this inflationary channel. Hence, in this part of the policy regime, we obtain orthodox inflation responses to monetary policy shocks when including long-term assets. In the other policy regime, $(\phi_\pi, \alpha_b) = (0.9, 0.1)$, the deflationary pressures are stronger for the same reasons as in RANK-FTPL. Nonetheless, when government debt is long-term, we find a short run rise in inflation in response to a reduction in the policy rate, while over the medium- to long-term, inflation reverses. This result is reminiscent of the "stepping-on-a-rake" argument of Sims (2011), and derives from the capital gains on long-term bonds induced by the monetary policy shock.

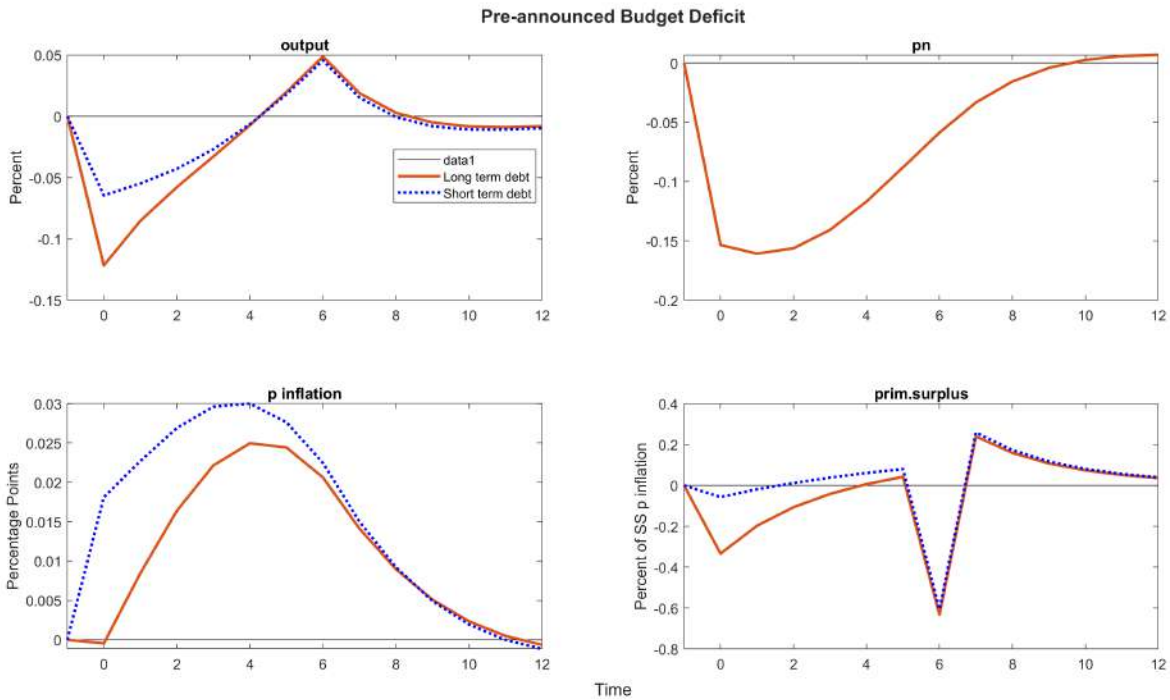
7.6 Long-Term Debt and Anticipation Effects

As we have seen above, the introduction of long-term government debt is important for its quantitative properties in terms of the impact of deficits (and monetary policy shocks). We will now show that this is also an important aspect of anticipated deficits. Figure 13 shows the impact of a one percent deficit shock pre-announced four quarters in advance when assuming that debt is either short-term or has an expected duration of 72 months. We show the impact on output, inflation, the budget surplus, and, for the model with long-term debt, the impact on the price of newly issued long-term debt.

As in the simple model, in the medium-scale model a pre-announced deficit is recessionary and inflationary prior to its implementation. When we allow for long-term debt, the size of the pre-implementation recession is much larger and the announcement of a future deficit induces a sudden and large drop in the long-term debt price. The bond price response derives from the persistent rise in current and expected future short-term interest rates that the policy induces. Households therefore experience a sudden drop in the value of their bond holdings which, through the demand effects, leads to a much sharper drop in activity than when government debt is of a short duration.

This sharp drop in bond prices resembles the dynamics of UK government bond prices in the aftermath of the announcement of the Liz Truss government's "mini-budget" in the UK. Together with an earlier £60 billion energy support package, the mini-budget was associated with a £161 billion tax cut to be implemented over a five-year period, the largest UK tax cut for more than 50 years. It was explicitly meant to be debt financed and not funded. The announcement was ill-received by financial markets, gave rise to a sharp drop in the value of the sterling and in the prices of longer-term UK government debt, prompting the Bank of England to intervene in bond markets. The turmoil that followed eventually led Liz Truss to replace the Chancellor of the Exchequer, Kwasi Kwarteng, with Jeremy Hunt who was seen as more fiscally responsible, and most of the tax cuts were abolished by October 5th. Two weeks later, Liz Truss resigned from office and her premiership is the shortest in British parliamentary history.

The turmoil in UK financial markets following the mini-budget announcement is usually associated



Notes: The figure illustrates the impact of a deficit shock announced four quarters in advance in the medium scale model. The dotted lines refer to the model with short term debt, the full lines to the model with long-term debt with an expected duration of 42 quarters.

Figure 13: Impact of a Pre-Announced Deficit in the Medium-Scale Model

with the use of LDI (liability driven investment) strategies of UK pension funds. These strategies essentially relate to leveraged investments in risky ("growth") assets financed by cash deposits. Our theory illustrates that underlying these market dynamics can lie a simple and intuitive fundamental story, namely that rising inflation expectations and falling longer-term nominal bond prices have direct and negative implications for growth , potentially putting government plans in peril.

Our results regarding the inflation-cum-recessionary effects of anticipated deficits echo those of Aguiar, Amador and Arellano (2024) who also study the impact of such shocks in a perpetual youth NK model with capital. The main difference with our analysis in that respect is that they assume that government debt is real and that households hold government bonds only while all capital is held by firm owners. In their setting, the announcement of a future deficit reduces bond demand and gives rise to an increase in the real interest rate which is accommodated by the monetary authority, leading to an increase in inflation. If the anticipation horizon is sufficiently long, it also produces a decline in activity. In our setting, inflation has the further effect of devaluing the real value of debt, and anticipation effects are further amplified through long-term bond prices as discussed above.

8 Conclusions and Summary

In this paper, we have shown that violations of Ricardian Equivalence due to finite planning horizons has fundamental implications for the equilibrium properties of an otherwise standard New Keynesian model with monetary-fiscal interaction. We have done so by adopting a simple HANK model where agent heterogeneity is modeled, assuming that households have finite planning horizons. This feature leads to a violation of Ricardian Equivalence because households perceive that the agents on who will finance public deficits are not the same as those who enjoy the benefits of transfers. Because of the violation of Ricardian Equivalence, deficits impact directly on demand and there is a general two-way direct interdependence of monetary and fiscal policies rather than the one-way direct impact of monetary policy on fiscal solvency present in models with Ricardian Equivalence.

A key aspect of our analysis comes from the fact that monetary and fiscal policies always interact in the HANK model. This has profound implications. One such implication is that the standard Taylor principle is neither necessary nor sufficient for equilibrium determinacy and that the usual distinction between active and passive policy authorities is no longer useful. In the HANK setting, there is local determinacy for a wide range of monetary and fiscal policy configurations; local indeterminacy is a smaller concern than in RANK models, while the non-existence of a stable equilibrium is a more prominent feature. Translating these results into the consequences for the coordination of monetary and fiscal policies, the abrupt changes in monetary and fiscal policies that are called for in RANK models when one policy changes from an active to a passive (or passive to active) stance are replaced by smooth changes in HANK. However, even when monetary policy satisfies the Taylor principle and the equilibrium is unique, a monetary policy reform may need to be accompanied by a fiscal policy reform and vice versa.

In this setting, monetary and fiscal policies work together because monetary policy actions have fiscal implications and vice versa. A key implication that follows from this is that fiscal deficits are in general inflationary. In RANK settings this is only the case for unfunded policies, i.e. when fiscal policy is active as in the FTPL region. Our results show that parts of the FTPL logic hold much more generally once we do away with Ricardian Equivalence. The reason is that an increase in transfers stimulates demand directly in settings without Ricardian Equivalence and therefore tends to be inflationary. The impact on output, however, depends crucially on the timing, and deficits may be, at the same time, inflationary and recessionary if pre-announced, a feature that often applies to fiscal policies because of democratic lags.

We have focused on the implications of violations of Ricardian Equivalence for the equilibrium properties of monetary economies, for the coordination of monetary and fiscal policies, and for the dynamic impact of deficits and monetary policy shocks. We think it would be interesting to look at optimal monetary and fiscal policies in this framework, and also to consider other issues such as fiscal multipliers. It would also be interesting to look at richer HANK settings, and at richer descriptions of policies that are less tied to simple feedback rules. We leave these issues for future research.

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9 Appendices

9.1 Appendix 1. Derivation of the Aggregate Euler Equation

The first-order condition for household purchases of government bonds can be expressed as:

$$1 = \mathbb{E}_t Q_{t,t+1}(1+i_t)/\Pi_{t+1}$$

where $Q_{t,t+1} = \beta c_{s,t}/c_{s,t+1}$ is the stochastic discount factor which is equalized across households because they face the same intertemporal prices. Next, allowing for complete markets as in **?** and aggregating across households implies that:

$$Q_{t,t+1} = \beta \frac{qc_t}{qc_{t+1} + (1-q)(1-q\beta)\left(\frac{v_t}{q\Pi_{t+1}} - \frac{v^{ss}}{q}\right)}$$

Combining this expression with the household Euler equation implies the aggregate Euler equation:

$$1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1} + \frac{(1-q)(1-q\beta)}{q}\left(\frac{v_t}{q\Pi_{t+1}} - \frac{v^{ss}}{q}\right)} \frac{1+i_t}{\Pi_{t+1}}$$

9.2 Appendix 2. Proofs of Propositions 1-2

Proposition 1: Recall that the characteristic polynomial can be written as

$$P(\lambda) = \underbrace{\left(-\frac{1}{\beta}(\alpha_b - 1) - \lambda\right)}_{P_1(\lambda_1)} \underbrace{\left(-\frac{1}{\beta}\left(-\beta\lambda^2 + (1+\beta+\kappa+\beta\phi_x)\lambda - (\phi_x + \kappa\phi_\pi + 1)\right)\right)}_{P_2(\lambda_2,\lambda_3)}$$

Local determinacy requires two roots outside the unit circle. When $\alpha_b > 1-\beta$, the root of P_1 is inside the unit circle, and therefore both roots of P_2 must be outside the unit circle. Alternatively, when $\alpha_b < 1-\beta$, one of the roots P_2 must be outside the unit circle and one inside. Let $\mathcal{D}_m$ denote the matrix:

$$\mathcal{D}_m = \begin{pmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \phi_\pi - \frac{1}{\beta} & \phi_x + \frac{\kappa}{\beta} + 1 \end{pmatrix} \tag{54}$$

P_2 then has two roots inside the unit circle when either of the parameter restrictions in Case 1 or Case 2 below are satisfied. These conditions are:

Case 1:

$$\begin{aligned}\det \mathcal{D}_m &> 1\\ \det \mathcal{D}_m - tr\mathcal{D}_m &> -1\\ \det \mathcal{D}_m + tr\mathcal{D}_m &> -1\end{aligned}$$

Case 2:

$$\begin{aligned}\det \mathcal{D}_m - tr\mathcal{D}_m &< -1\\ \det \mathcal{D}_m + tr\mathcal{D}_m &< -1\end{aligned}$$

The determinant and the trace are given as:

$$\begin{aligned}\det \mathcal{D}_m &= \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\phi_x}{\beta}\\ tr\mathcal{D}_m &= 1 + \frac{1}{\beta} + \frac{\kappa}{\beta} + \phi_x\end{aligned}$$

Checking **Case 1**, we get:

$$\det \mathcal{D}_m = \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\phi_x}{\beta} > 1 \tag{55}$$

which is satisfied since $1/\beta > 1$ and the other terms are non-negative. Next:

$$\begin{aligned}\det \mathcal{D}_m - trC_m = \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\phi_x}{\beta} - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta} + \phi_x\right) &> 0 \Rightarrow\\ \frac{\kappa}{\beta}(\phi_\pi - 1) + \phi_x\left(\frac{1}{\beta} - 1\right) &> 0\end{aligned} \tag{56}$$

which is satisfied under the restriction in Proposition 2. Finally:

$$\det \mathcal{D}_m + tr\mathcal{D}_m = \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\phi_x}{\beta} + \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta} + \phi_x\right) > -1 \tag{57}$$

which is satisfied since all terms are non-negative.

Since one of the roots of $\mathcal{D}_m$ is guaranteed to be outside the unit circle, we get the result in Proposition 2. Proposition 2 follows from the characteristic polynomial being identical to that of the RANK model.

Proposition 3: Here the characteristic polynomial follows from:

$$P(\lambda) = \det \begin{pmatrix} \frac{1}{\beta}(1-\alpha_b) - \lambda & \frac{v^{ss}}{y^{ss}}\left(\phi_\pi - \frac{1}{\beta}\right) & 0 \\ 0 & \frac{1}{\beta} - \lambda & -\frac{\kappa}{\beta} \\ -\frac{1}{\beta}\chi(1-\alpha_b) & \phi_\pi + \frac{1}{\beta}(\gamma\chi - 1) - \gamma\chi\left(\phi_\pi - \frac{1}{\beta}\right) & -\frac{\kappa}{\beta}(\gamma\chi - 1) + 1 - \lambda \end{pmatrix}$$

We can write this polynomial in the general form:

$$\lambda^3 + \Gamma_2\lambda^2 + \Gamma_1\lambda + \Gamma_0 = 0$$

Using this notation, we get the coefficients:

$$\begin{aligned} \Gamma_2 &= -\frac{1}{\beta}(1-\alpha_b) - \frac{1}{\beta}(1 + \beta + \kappa(1-\gamma\chi)) \\ \Gamma_1 &= \frac{1}{\beta^2}(1-\alpha_b)(1+\beta+\kappa(1-\gamma\chi)) + \frac{1}{\beta}\left(1 + \kappa\left(\phi_\pi - \gamma\chi\left(\phi_\pi - \frac{1}{\beta}\right)\right)\right) \\ \Gamma_0 &= -\frac{1}{\beta^2}(1-\alpha_b)\left(1 + \kappa\left(\phi_\pi - \gamma\chi\left(\phi_\pi - \frac{1}{\beta}\right)\right)\right) - \frac{1}{\beta^2}(1-\alpha_b)\kappa\chi\gamma\left(\phi_\pi - \frac{1}{\beta}\right) \\ &= -\frac{1}{\beta^2}(1-\alpha_b)(1+\kappa\phi_\pi) \end{aligned}$$

Following Woodford (2005), either of the following sets of conditions are sufficient to guarantee that $P(\lambda)$ has two roots outside the unit circle:

Case 3

$$1 + \Gamma_2 + \Gamma_1 + \Gamma_0 < 0 \tag{58}$$

$$-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 > 0 \tag{59}$$

Case 4

$$1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0 \tag{60}$$

$$-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 < 0 \tag{61}$$

$$\Gamma_0^2 - \Gamma_0\Gamma_2 + \Gamma_1 - 1 > 0 \tag{62}$$

Case 5

$$
\begin{aligned}
1+\Gamma_2+\Gamma_1+\Gamma_0 &> 0 && (63)\\
-1+\Gamma_2-\Gamma_1+\Gamma_0 &< 0 && (64)\\
\Gamma_0^2-\Gamma_0\Gamma_2+\Gamma_1-1 &< 0 && (65)\\
|\Gamma_2| &> 3 && (66)
\end{aligned}
$$

We note first that condition (59) is violated since:

$$
\begin{aligned}
-1+\Gamma_2-\Gamma_1+\Gamma_0 &= -1-\frac{1}{\beta^2}(1+\kappa\phi_\pi+\beta)(2+\beta+\kappa(1-\gamma\chi)-\alpha_b)\\
&- \frac{1}{\beta}(1+\kappa\phi_\pi(1-\gamma\chi)+\frac{1}{\beta}\kappa\gamma\chi)<0 \qquad (67)
\end{aligned}
$$

where the sign follows from $\gamma\chi < 1$, so that all terms are non-positive. Hence, condition (58) is violated and the parameter conditions associated with Case 3 are therefore not satisfied. Therefore, we focus on Cases 4 and 5.

We start with Case 4. Equation (67) implies that condition (61) (and therefore also (64)) is satisfied. Condition (60) requires that:

$$
\begin{aligned}
1+\Gamma_2+\Gamma_1+\Gamma_0 &> 0 \Rightarrow\\
\frac{\kappa}{\beta^2}\left((1-\beta-\alpha_b)(1-\phi_\pi-\gamma\chi)+\gamma\chi(1-\beta\phi_\pi)\right) &> 0 \qquad (68)
\end{aligned}
$$

This condition is satisfied when either

(a) $\phi_\pi > 1-\gamma\chi$ and $\alpha_b > \alpha_b^{crit}$, or

(b) $\phi_\pi < 1-\gamma\chi$ and $\alpha_b < \alpha_b^{crit}$

where:

$$
\alpha_b^{crit} = 1-\beta+\gamma\chi\frac{\beta\phi_\pi-1}{\phi_\pi+\gamma\chi-1} \qquad (69)
$$

Finally, (62) can be expressed as:

$$
\Gamma_0(\Gamma_0-\Gamma_2)+\Gamma_1-1>0
$$

Inserting and simplifying, this condition can be expressed as:

$$
\begin{aligned}
\tfrac{1}{\beta^4} &\quad (1-\alpha_b)(1+\kappa\phi_\pi-\beta)\left((1+\kappa\phi_\pi)(1-\alpha_b)-\beta(1+\beta+\kappa(1-\gamma\chi)\right)\\
+ &\quad \frac{1}{\beta}(1+\kappa\phi_\pi(1-\gamma\chi)+\kappa\gamma\chi\frac{1}{\beta})-1>0 \qquad (70)
\end{aligned}
$$

This condition involves a conic section in the (α_b, ϕ_π)-space the properties of which depends on the parameters. It is easy to see that the condition is always satisfied for large values of α_b, since:

$$\begin{aligned}\lim_{\alpha_b \to 1} \ \tfrac{1}{\beta^4} \ & (1-\alpha_b)(1+\kappa\phi_\pi - \beta)\left((1+\kappa\phi_\pi)(1-\alpha_b) - \beta(1+\beta+\kappa(1-\gamma\chi)\right) \\ + \ & \frac{1}{\beta}(1+\kappa\phi_\pi(1-\gamma\chi)+\kappa\gamma\chi\frac{1}{\beta}) - 1 \\ = \ & \frac{1}{\beta}(1+\kappa\phi_\pi(1-\gamma\chi)+\kappa\gamma\chi\frac{1}{\beta}) - 1 > 0 \end{aligned} \tag{71}$$

where the inequality follows from $1/\beta > 1$, and noting that the remaining terms inside the parenthesis are positive under the assumption that $\gamma\chi < 1$. For lower values of α_b condition (70) may be violated.

If conditions (60)-(61) are satisfied but (62) is violated, then there is still local determinacy if condition (66) satisfied (see Case 5). This latter condition can be expressed as:

$$\begin{aligned} |\Gamma_2| = 1 + \frac{1}{\beta} + \frac{1}{\beta}\left((1-\alpha_b) + \kappa\,(1-\gamma\chi)\right) \ &> \ 3 \Rightarrow \\ (1-\alpha_b) + \kappa\,(1-\gamma\chi) \ &> \ 2\beta - 1 \end{aligned} \tag{72}$$

Since a realistic calibration of β is close to one, this condition is unlikely to be satisfied unless the Phillips curve is very steep and α_b close to zero.

9.3 Proof of Proposition 3

Consider the system

$$\widehat{v}_t = \frac{1}{\beta}\Big(1-\alpha_b\Big)\widehat{v}_{t-1} - \frac{1}{\beta}\alpha_x \widehat{y}_t + \frac{v^{ss}}{y^{ss}}\Big(\phi_\pi - \frac{1}{\beta}\Big)\widehat{\pi}_t, \tag{73}$$

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \phi_\pi\widehat{\pi}_t + \mathbb{E}_t\widehat{\pi}_{t+1} + \chi\Big(\widehat{v}_t - \frac{v^{ss}}{y^{ss}}\mathbb{E}_t\widehat{\pi}_{t+1}\Big), \tag{74}$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa\,\widehat{y}_t. \tag{75}$$

We conjecture solutions of the form

$$\widehat{v}_t = V\,\lambda^t, \quad \widehat{y}_t = Y\,\lambda^t, \quad \widehat{\pi}_t = \Pi\,\lambda^t,$$

with constants V, Y, Π, and common factor λ. Rational expectations imply:

$$\mathbb{E}_t\widehat{v}_{t+1} = V\,\lambda^{t+1}, \quad \mathbb{E}_t\widehat{y}_{t+1} = Y\,\lambda^{t+1}, \quad \mathbb{E}_t\widehat{\pi}_{t+1} = \Pi\,\lambda^{t+1}.$$

Substituting these into (73) – (75) and canceling the common factors yields:

$$V\Big(1-\frac{1-\alpha_b}{\beta\lambda_V}\Big)=-\frac{1}{\beta}\alpha_x Y+\gamma\Big(\phi_\pi-\frac{1}{\beta}\Big)\Pi \tag{76}$$

$$Y=\lambda Y+(\lambda-\phi_\pi)\Pi+\chi\Big(V-\gamma\lambda\Pi\Big) \tag{77}$$

$$Y=\frac{1-\beta\lambda}{\kappa}\Pi \tag{78}$$

We now set $\lambda=1$. We obtain

$$V=\gamma\frac{\beta\phi_\pi-1}{\beta-1+\alpha_b}\Pi-\frac{\alpha_x}{\beta-1+\alpha_b}Y$$
$$Y=Y+(1-\phi_\pi)\Pi+\chi\left(\gamma\frac{\beta\phi_\pi-1}{\beta-1+\alpha_b}\Pi-\frac{\alpha_x}{\beta-1+\alpha_b}Y-\gamma\Pi\right)$$

Cancelling out the terms and re-arranging, we obtain:

$$\alpha_b=1-\beta+\chi\gamma\frac{\beta\phi_\pi-1}{\phi_\pi+\chi\gamma-1}-\chi\alpha_x\frac{1-\beta}{\kappa(\phi_\pi+\chi\gamma-1)}.$$

9.4 Proof of Proposition 4

Since the macro block is independent of fiscal policy, macro variables do not react to the shock. The starting value of debt and its evolution follow from substituting $\pi_t=0$ in the log-linearized law of motion for government debt.

9.5 Proof of Proposition 5

We begin with the log-linearized equilibrium conditions around the zero-deviation steady state, subject to a one-period deficit shock e_0^s with Taylor-rule $i_t=\phi_\pi\pi_t$ and $\alpha_x=0$. Recall the three equations:

$$\widehat{v}_t = \frac{1}{\beta}(1-\alpha_b)\widehat{v}_{t-1}+\gamma\left(\phi_\pi-\frac{1}{\beta}\right)\widehat{\pi}_t-\frac{1}{\beta}(1-\alpha_b)e_t^s \tag{79}$$

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1}-\phi_\pi\widehat{\pi}_t+\mathbb{E}_t\widehat{\pi}_{t+1} \tag{80}$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1}+\kappa\widehat{y}_t \tag{81}$$

We look for a solution of the form

$$\widehat{y}_t=\widehat{y}_0\,\lambda^t,\quad \widehat{\pi}_t=\widehat{\pi}_0\,\lambda^t,\quad \widehat{v}_t=\widehat{v}_0\,\lambda^t, \tag{82}$$

where $\lambda\in(0,1)$. Under this conjectured solution, expectations satisfy $E_t[\widehat{y}_{t+1}]=\lambda\,\widehat{y}_t$ and $E_t[\widehat{\pi}_{t+1}]=\lambda\,\widehat{\pi}_t$.

The macro block

Substituting the proposed solution (82) into the Euler equation (80) and the Phillips curve (81), we obtain:

$$\widehat{y}_t = \lambda\,\widehat{y}_t - \phi_\pi\,\widehat{\pi}_t + \lambda\,\widehat{\pi}_t,$$

$$\widehat{\pi}_t = \beta\,\lambda\,\widehat{\pi}_t + \kappa\,\widehat{y}_t. \tag{83}$$

Eliminating $\widehat{y}_t$ yields the following characteristic equation:

$$(1-\lambda)(1-\beta\lambda) = \kappa\,(\lambda - \phi_\pi).$$

which can be rearranged into a quadratic form in λ

$$\beta\,\lambda^2 - (1+\beta+\kappa)\,\lambda + (1+\kappa\,\phi_\pi) = 0 \tag{84}$$

This expression is consistent with $P_2(\lambda_2, \lambda_3)$ introduced in Section 9.2, reflecting the structural independence between the New Keynesian block and the Fiscal Block in *RANK*. From Section 9.2, we know that one root of $P_2(\lambda_2, \lambda_3)$ necessarily lies outside the unit circle. Moreover, in *RANK-FTPL*, it is assumed that $\phi_\pi \leq 1$ and $\alpha_b < 1-\beta$. It follows then that the stable root takes the form:

$$\lambda = \frac{1 + \frac{1+\kappa}{\beta} - \sqrt{\left(1 + \frac{1+\kappa}{\beta}\right)^2 - 4\frac{1+\kappa\phi_\pi}{\beta}}}{2}$$

Evaluating equation (83) at $t = 0$:

$$\widehat{\pi}_0 = \frac{\kappa}{1-\beta\lambda}\,\widehat{y}_0, \tag{85}$$

Government flow budget constraint

With $\widehat{v}_{-1} = 0$, equation (28) at time $t = 0$ becomes:

$$\widehat{v}_0 = \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\widehat{\pi}_0 - \frac{1}{\beta}\,(1-\alpha_b)\,e^s_0 \tag{86}$$

At $t = 1$, in the absence of additional shocks, the same equation becomes:

$$\lambda\widehat{v}_0 = \frac{1}{\beta}\,(1-\alpha_b)\,\widehat{v}_0 + \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\lambda\widehat{\pi}_0 \tag{87}$$

Equating (86) and (87) and solving for v_0 gives

$$\widehat{v}_0 = -\lambda e_0^s \tag{88}$$

Finally, substituting equations (85) and (88) into (86) yields the expression for the initial output response:

$$\widehat{y}_0 = \frac{1}{\gamma}\frac{(1-\alpha_b-\beta\lambda)(1-\beta\lambda)}{\kappa(\beta\phi_\pi-1)}e_0^s \tag{89}$$

9.6 Proof of Propositions 6 and 7

All the comparative statics follow immediately from the closed-form solution in Proposition 5.

9.7 Proof of Proposition 8

We write the equilibrium system as:

$$\mathbb{E}_t\begin{bmatrix}\widehat{v}_t\\ \widehat{\pi}_{t+1}\\ \widehat{y}_{t+1}\end{bmatrix} = \mathcal{D}\begin{bmatrix}\widehat{v}_{t-1}\\ \widehat{\pi}_t\\ \widehat{y}_t\end{bmatrix} + \mathcal{F}\begin{bmatrix}e_t^i\\ e_t^s\end{bmatrix}$$

where

$$\begin{aligned}\mathcal{D} &= \begin{bmatrix}\frac{1}{\beta}(1-\alpha_b) & \gamma(\phi_\pi-\frac{1}{\beta}) & 0\\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta}\\ -\frac{1}{\beta}\chi(1-\alpha_b) & \phi_\pi-\frac{1-\gamma\chi}{\beta}-\chi\gamma\left(\phi_\pi-\frac{1}{\beta}\right) & \frac{\kappa}{\beta}(1-\gamma\chi)+1\end{bmatrix}\\ \mathcal{F} &= \begin{bmatrix}\gamma & \frac{1-\alpha_b}{\beta}\\ 0 & 0\\ 1-\gamma\chi & \chi\frac{1-\alpha_b}{\beta}\end{bmatrix}.\end{aligned}$$

Diagonalizing the matrix $\mathcal{D}$, we obtain:

$$\mathbb{E}_t z_{t+1} = E\Lambda E^{-1}z_t + \mathcal{F}e_t \tag{90}$$

where Λ is a diagonal matrix with eigenvalues of $\mathcal{D}$ on the diagonal, and E is a matrix of corresponding eigenvectors. Next, define $w_t := E^{-1}z_t$ to obtain:

$$\begin{aligned}\mathbb{E}_t w_{t+1} &= \Lambda w_t + E^{-1}\mathcal{F}e_t\\ \mathbb{E}_t w_t &= \Lambda w_{t-1} + E^{-1}\mathcal{F}e_{t-1}\\ \mathbb{E}_t w_{t+1} &= \Lambda^2 w_{t-1} + \Lambda E^{-1}\mathcal{F}e_{t-1}\end{aligned}$$

Thus, focusing on the evolution of the transformed system after some shock were realized at time $t = 0$, we have

$$\mathbb{E}_t w_{t+1} = \Lambda^{t+1} w_0 + \Lambda^t E^{-1} \mathcal{F} e_0,$$

where $w_0 = E^{-1} z_0$.

In the determinate region of the parameter space, the model has one eigenvalue that is smaller than 1 in absolute value. Suppose we order the stable eigenvalue first, and denote it with λ_{HANK}. For $\lim_{t\to\infty} w_t = 0$ and so for $\lim_{t\to\infty} z_t = 0$ the system must move along the stable eigenvalue. Then for a stable solution to exist, w_0 must take the form

$$w_0 = \begin{bmatrix} w_0^1 \\ 0 \\ 0 \end{bmatrix},$$

which implies

$$z_0 = E w_0 = \begin{bmatrix} 1 & 1 & 1 \\ \chi_\pi & \cdot & \cdot \\ \chi_y & \cdot & \cdot \end{bmatrix} \begin{bmatrix} w_0^1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_0^1 \\ \chi_\pi w_0^1 \\ \chi_y w_0^1 \end{bmatrix}$$

where χ_π and χ_y are the second and third element of the eigenvector associated with λ_{HANK}. We can then write the system as:

$$\mathbb{E}_t z_{t+1} = \lambda_{HANK}^{t+1} \begin{bmatrix} 1 \\ \chi_\pi \\ \chi_y \end{bmatrix} w_0^1 + \begin{bmatrix} \gamma & \frac{1-\alpha_b}{\beta} \\ 0 & 0 \\ 1-\gamma\chi & \chi\frac{1-\alpha_b}{\beta} \end{bmatrix} \begin{bmatrix} e_t^i \\ e_t^s \end{bmatrix}.$$

This implies that the equilibrium takes the following form:

$$\begin{aligned} v_t &= \lambda_{HANK} v_{t-1} \\ \pi_t &= \chi_\pi v_{t-1} \\ y_t &= \chi_y v_{t-1}. \end{aligned}$$

Note that this also implies:

$$\begin{aligned} \pi_0 &= \chi_\pi \lambda_{HANK} v_0 \\ y_0 &= \chi_y \lambda_{HANK} v_0. \end{aligned}$$

In order to determine v_0, evaluate the government debt evolution equation at time zero, noting that the dynamics above and the NKPC imply $\widehat{\pi}_0 = \kappa \sum_{t=0}^{\infty} \beta^t \widehat{y}_t$:

$$\widehat{v}_0 = \gamma \left(\phi_\pi - \frac{1}{\beta} \right) \widehat{\pi}_0 - \frac{1}{\beta} (1 - \alpha_b) e_0^s = \gamma \frac{\chi_y}{\lambda_{HANK}} \left(\phi_\pi - \frac{1}{\beta} \right) \kappa \sum_{t=0}^{\infty} (\beta \lambda_{HANK})^t \widehat{v}_0 - \frac{1}{\beta} (1 - \alpha_b) e_0^s$$

Therefore

$$\widehat{v}_0 = \frac{-\frac{1}{\beta} (1 - \alpha_b)}{1 - \kappa\gamma \frac{\chi_y}{\lambda_{HANK}} \left(\phi_\pi - \frac{1}{\beta} \right) \frac{1}{1 - \beta\lambda_{HANK}}} e_0^s.$$

We can chcracterize the coefficients χ_π and χ_y by solving the characteristic equation:

$$\begin{bmatrix} \frac{1}{\beta}(1-\alpha_b) - \lambda_{HANK} & \gamma(\phi_\pi - \frac{1}{\beta}) & 0 \\ 0 & \frac{1}{\beta} - \lambda_{HANK} & -\frac{\kappa}{\beta} \\ -\frac{1}{\beta}\chi(1-\alpha_b) & \phi_\pi + \frac{1}{\beta}(\gamma\chi - 1) - \gamma\chi(\phi_\pi - \frac{1}{\beta}) & \frac{\kappa}{\beta}(1 - \gamma\chi) + 1 - \lambda_{HANK} \end{bmatrix} \begin{bmatrix} 1 \\ \chi_\pi \\ \chi_y \end{bmatrix} = 0.$$

This system implies the following:

$$\chi_\pi = \begin{cases} \frac{\lambda_{HANK} - \frac{1}{\beta}(1-\alpha_b)}{\gamma(\phi_\pi - \frac{1}{\beta})}, & \phi_\pi \neq \frac{1}{\beta} \\ \frac{\chi(1-\alpha_b)}{\gamma\chi(1-\alpha_b) + \alpha_b + \frac{\alpha_b}{\kappa}(\beta - 1 + \alpha_b)} & \phi_\pi = \frac{1}{\beta} \end{cases}$$

$$\chi_y = \frac{\beta}{\kappa} \left(\frac{1}{\beta} - \lambda_{HANK} \right) \chi_\pi$$

Note also that λ_{HANK} solves:

$$\left(\frac{1}{\beta}(1-\alpha_b) - \lambda_{HANK} \right) \det \Delta + \gamma(\phi_\pi - \frac{1}{\beta}) \frac{\kappa}{\beta^2} \chi(1 - \alpha_b) = 0$$

where Δ is the 2x2 matrix in the bottom right of $\mathcal{D}$, as defined in the proposition. Thus

$$\lambda_{HANK} = \frac{1}{\beta}(1-\alpha_b) + \frac{\gamma(\phi_\pi - \frac{1}{\beta})\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\det \Delta}.$$

This implies $\lambda_{HANK} = \frac{1}{\beta}(1-\alpha_b) + \frac{\gamma(\phi_\pi - \frac{1}{\beta})\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\lambda_{HANK}^2 - \lambda_{HANK}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma)\right) + \frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right)}$ and in particular $\chi_\pi = \frac{\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\det \Delta}$ and $\chi_y = \frac{\frac{1}{\beta}\chi(1-\alpha_b)\left(\frac{1}{\beta} - \lambda_{HANK}\right)}{\det \Delta}$. Finally, note that plugging in expressions for χ_π and χ_y into the expression for $\widehat{v}_0$ we obtain:

$$\widehat{v}_0 = \frac{-\frac{1}{\beta}(1-\alpha_b)}{1 - \kappa\gamma\frac{\beta}{\kappa}\left(\frac{1}{\beta\lambda_{HANK}} - 1\right)\frac{\lambda_{HANK} - \frac{1}{\beta}(1-\alpha_b)}{\gamma(\phi_\pi - \frac{1}{\beta})}\left(\phi_\pi - \frac{1}{\beta}\right)\frac{1}{1-\beta\lambda_{HANK}}} = -\lambda_{HANK} e_0^s,$$

where the second equality follows from simplifying the fraction. Thus:

$$\widehat{\pi}_0 = \kappa \frac{\chi_y}{\lambda_{HANK}} \frac{\widehat{v}_0}{1-\beta\lambda_{HANK}} = -\kappa \frac{\chi_y}{1-\beta\lambda_{HANK}} e_0^s \qquad \widehat{y}_0 = \frac{\chi_y}{\lambda_{HANK}} \widehat{v}_0 = -\chi_y e_0^s.$$

9.8 Proof of Proposition 9

Define the function:

$$F(\lambda_{HANK}, \phi_\pi) = \lambda_{HANK} - \frac{1}{\beta}(1-\alpha_b) - \frac{\gamma(\phi_\pi - \frac{1}{\beta})\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\lambda_{HANK}^2 - \lambda_{HANK}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma)\right) + \frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right)}. \tag{91}$$

We have:

$$\begin{aligned} F_{\phi_\pi} &= -\frac{\gamma\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\det\Delta} + \frac{\gamma(\phi_\pi - 1/\beta)\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{(\det\Delta)^2} \cdot \frac{\kappa}{\beta}(1-\chi\gamma), \\ F_{\lambda_{HANK}} &= 1 + \frac{\gamma(\phi_\pi - 1/\beta)\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{D(\lambda_{HANK}, \phi_\pi)^2} \cdot \left(2\lambda_{HANK} - 1 - \frac{1}{\beta} - \frac{\kappa}{\beta}(1-\chi\gamma)\right) \end{aligned}$$

Therefore, by the implicit function theorem:

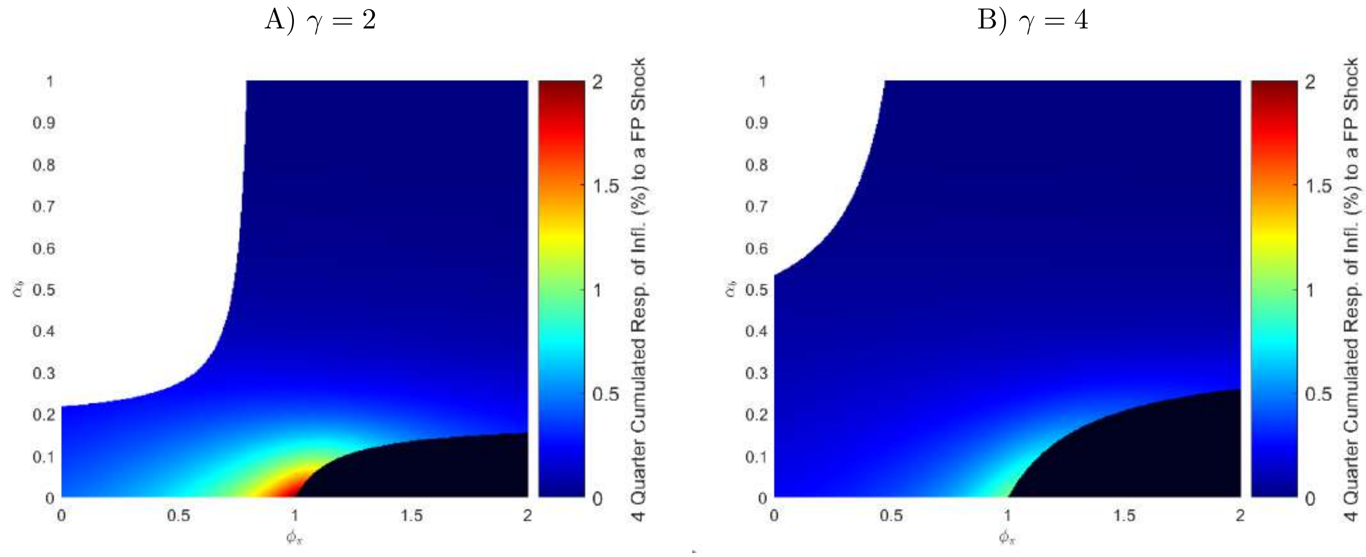
$$\frac{d\lambda_{HANK}}{d\phi_\pi} = \frac{\frac{\gamma\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{\det\Delta} - \frac{\gamma(\phi_\pi-1/\beta)\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{(\det\Delta)^2} \cdot \frac{\kappa}{\beta}(1-\chi\gamma)}{1 + \frac{\gamma(\phi_\pi-1/\beta)\frac{\kappa}{\beta^2}\chi(1-\alpha_b)}{(\det\Delta)^2} \cdot \left(2\lambda_{HANK} - 1 - \frac{1}{\beta} - \frac{\kappa}{\beta}(1-\chi\gamma)\right)}. \tag{92}$$

The denominator is positive for any $\phi_\pi < 1/\beta$ (since $\mid \lambda_{HANK} \mid < 1$ and $\chi\gamma < 1$). The numerator is positive if $\det\Delta > 0$. Recall that $\det\Delta = \lambda_{HANK}^2 - \lambda_{HANK}\left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma)\right) + \frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right)$. A sufficient condition for this to be positive is that the smaller root of $\det\Delta = 0$ is greater than 1 (since $\lambda_{HANK} < 1$). Therefore:

$$\lambda_{HANK}^0 = \frac{1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma) - \sqrt{\left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma)\right)^2 - 4\frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right)}}{2} > 1$$

which implies:

$$\begin{aligned} 1 - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma)\right) + \frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right) &> 0 \\ \frac{\kappa}{\beta}\left(\phi_\pi(1-\chi\gamma) + \frac{1}{\kappa} + \frac{\chi\gamma}{\beta}\right) &> \frac{1}{\beta} + \frac{\kappa}{\beta}(1-\chi\gamma) \\ \phi_\pi &> 1 - \frac{1}{\beta}\frac{\chi\gamma}{1-\chi\gamma} \end{aligned}$$



Notes: The panels illustrate the four quarter cumulated inflation response to a one percent deficit shock. In panel A we assume that the debt-to-GDP ratio is 50 percent on an annual basis, in panel B we assume a 100 percent annual debt-to-GDP ratio. We assume that $\beta = 0.99$ and $\kappa = 0.24$.

Figure 14: Government Indebtedness and the Inflationary Impact of Deficits

We also need a $b < -2a$ type of condition, so that the vertex of the parabola lies to the right of 1. This condition in our context is $\left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}(1 - \chi\gamma)\right) > 2$ which is satisfied.

9.9 The impact of deficits on inflation

Figure 14 illustrates by means of the heatmap the impact of deficits on inflation for low government indebtedness (panel A) and higher government indebtedness (panel B). In particular, panel A assumes that $\gamma = 2$ so that the level of government debt in the steady-state corresponds to 2 quarters of output, i.e. 50 percent of annual GDP. In panel B we assume that the annual debt-to-GDP ratio is 1. In both panels, we cumulate the inflation response over the first four quarters.

9.10 Proof of Proposition 10

We start from the Euler equation, and the government debt flow budget constraint. We also utilize the structure of the model that we derived in the proof of Proposition 8. We obtain:

$$\begin{aligned}
\widehat{y}_0 =& \chi_y \widehat{v}_0 - \left(\phi_\pi \widehat{\pi}_0 - \chi_\pi \widehat{v}_0 + e_t^i\right) + \chi\left(\widehat{v}_0 - \gamma\chi_\pi \widehat{v}_0\right) \\
\widehat{v}_0 =& \gamma\left(\phi_\pi - \frac{1}{\beta}\right)\kappa\left(\frac{\chi_y \beta \widehat{v}_0}{1 - \beta\lambda} + \widehat{y}_0\right) + \gamma e_t^i \\
\widehat{\pi}_0 =& \kappa\left(\frac{\chi_y}{\lambda}\frac{\widehat{v}_0}{1 - \beta\lambda} + \widehat{y}_0\right)
\end{aligned}$$

Solving the system we obtain:

$$
\begin{aligned}
\widehat{v}_0 =& \frac{\gamma\left(\frac{1}{\beta}+\frac{1}{\kappa}\right)}{\gamma\chi_y\left(\frac{1}{\beta}-\phi_\pi\right)\left(1+\frac{\beta+\kappa(1-\gamma\chi)}{1-\beta\lambda}\right)+\phi_\pi(1-\chi\gamma)+\frac{\chi\gamma}{\beta}+\frac{1}{\kappa}} e_0^i \\
\widehat{y}_0 =& \frac{\widehat{v}_0-\gamma e_0^i}{\gamma\left(\phi_\pi-\frac{1}{\beta}\right)\kappa} - \chi_y \frac{\beta}{1-\beta\lambda}\widehat{v}_0 \\
\widehat{\pi}_0 =& \kappa\left(\frac{\chi_y}{\lambda}\frac{\widehat{v}_0}{1-\beta\lambda}+\widehat{y}_0\right)
\end{aligned}
$$

Re-arranging and defining terms, yields the expressions in the propositions.