Wealth Sorting and Cyclical Employment Risk*

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Abstract

I present empirical evidence that U.S. workers with low liquid wealth face significantly higher cyclical employment risk. This finding cannot be fully explained by skills, socio-demographics, or past income. I propose a framework that generates a negative correlation between wealth and employment risk in equilibrium, through job sorting based on wealth. Job search provides an insurance mechanism for liquidity constrained and risk-averse unemployed workers. Asset-poor unemployed sort into relatively lower wage and lower security jobs, because these jobs offer a relatively higher job finding probability. I build a quantitative model to study the implications of wealth sorting for wages and job transitions over the business cycle, as well as its consequence for long-term inequalities. The interaction between employment risk and wealth accumulation generates a "poverty trap" which is amplified in bad times. The cost of entering unemployment during a recession amounts to 3% of lifetime consumption for the poorest, more than twice that of the wealthiest.

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1 Introduction

Wealth is closely linked to labor market outcomes for unemployed individuals, with financial distress often leading to worse employment prospects.[1] Understanding how wealth affects different labor market outcomes is important for deepening our understanding of economic inequalities and, in turn, for the design of labor market policies. Nevertheless, whether and how wealth might influence employment risk through the labor market remains largely unexplored – employment risk is generally assumed to be exogenous to an agent's wealth. The interaction between wealth and earnings risk is of particular interest from the perspective of the recent heterogeneous agent macroeconomic literature, given its potential implications for consumption risk.

In this paper, I empirically and theoretically examine the relationship between workers' wealth and their cyclical employment risk. I show that U.S. workers at the bottom of the liquid wealth distribution experience greater sensitivity of employment to changes in aggregate output. To explain this finding, I present a theory of wealth sorting that generates a negative correlation between liquid wealth and employment risk. Wealth sorting emerges from the presence of risk aversion and liquidity constraints, which incentivize asset-poor workers to look for jobs offering a high job finding probability – those that are more risky. I then build a quantitative model to study the implications of wealth sorting on wages and job transitions over the business cycle, as well as its consequence for long-term inequalities. The cost of starting an unemployment spell during a recession (relative to a normal time) amounts to 3% of lifetime consumption for the poorest and 1% for the richest workers.

I start by providing evidence that asset-poorer workers in the U.S. have employment probabilities that are more sensitive to aggregate fluctuations. Using the Panel Study of Income Dynamics (PSID), I compute the correlation between individual employment and changes in aggregate output, distinguishing workers based on their prior period liquid wealth holdings. I show that the negative correlation remains significant even after controlling for workers' fixed effects, skills, past income, and demographics. This paper focuses on this *residual* inequality, attributable to heterogeneous liquid wealth holdings. I find that asset-poorer workers not only face higher employment risk over the business cycle but also have a higher job loss probability in the cross-section. What matters is whether the worker is *liquid* wealth-poor in a certain period. I show that classifying agents according to their illiquid wealth or "permanent" wealth does not induce significant differences in employment elasticity. These findings support a theoretical link between heterogeneous employment risk and binding liquidity constraints. My empirical motivation closely aligns with Patterson (2023), who shows that workers with a high marginal propensity to consume (MPC) exhibit relatively greater employment elasticity to aggregate fluctuations. These empirical findings challenge models of job acceptance under risk aversion, which predict that poorer workers are relatively more likely to reject low-security jobs. A key contribution of this paper is to provide a theoretical explanation for the observed the negative correlation between liquid wealth and employment risk.

[1] See e.g. Bloemen and Stancanelli, 2001. In the U.S., more than a quarter of working-age households do not have sufficient savings to cover their expenditures after a month of unemployment (Catherine et al., 2020).

I then develop the simplest theoretical framework consistent with the empirical evidence above. Job search is directed, unlike in job acceptance models. Directed search introduces a trade-off between job-finding probability and job attributes, which is central to the sorting mechanism. I introduce two-sided heterogeneity on the labor market: workers differ in wealth, and jobs differ in their separation risk (security). As is common in directed search theory, the labor market is segmented in separated submarkets, each representing a subset of unemployed workers and firms with vacant jobs searching for each other. Time lasts for two periods. The aggregate state in the second period is uncertain, and heterogeneously affects jobs' separation risk. Firms post wages, and there is free entry for vacancy posting. Workers are risk-averse and can save at the (constant) risk-free interest rate, up to a borrowing constraint. Employed workers take a consumption-savings decision, while unemployed workers additionally need to search for a job.

In this economy, workers sort to different types of jobs based on their level of wealth. I provide sufficient conditions for having positive wealth sorting in equilibrium, i.e., for asset-poor workers to sort into low security jobs and wealthy workers to sort into high security jobs. Similarly to Eeckhout and Sepahsalari (2024), positive sorting comes from a *preference* complementarity between asset-poor risk-averse workers and jobs that offer a high job finding probability. Differently from their paper, in my model with heterogeneous job security, agents additionally need to consider the job separation probability (security). If high matching probability jobs also feature lower security, the sorting motive is weakened as asset-poor value job security relatively more. I show that positive sorting may still happen in equilibrium, as long as the trade-off between matching probability and job attributes is "good enough". In particular, it must be that workers are compensated enough in terms of job finding probability when applying to low wage, low security jobs. This is governed by the matching function elasticity and holds true for a reasonable parametrization. Whether low security jobs are those associated with high matching probability is governed by the free entry condition and relates to the shape of the separation and vacancy cost functions. If the cost of an unfilled vacancy is relatively larger for higher type jobs, firms opening high type jobs post relatively higher wages to attract applicants. In the positive wealth sorting equilibrium, wages and job security are thus endogenously positively correlated, and workers with lower assets not only have less secure jobs but also lower wages.[2]

More generally, wealth sorting is positive if workers' need to self insure outweighs the utility costs of matching in low wage and low security jobs ("bad" jobs). Any force that either increases the need for insurance or decreases the costs of searching for bad jobs strengthens the motive for positive wealth sorting. Infinite horizon and aggregate uncertainty ambiguously affect both the need for insurance and the long-term utility costs of searching for bad jobs through various channels. Understanding how wealth sorting behaves in such a framework is therefore a quantitative question.

[2] Note that the mechanism I propose can be thought to work within skill groups, or in addition to how skills may affect cyclical employment risk. While these dimensions of worker heterogeneity are surely important determinants of labor market choices, they have been the focus of some other papers already (e.g., Kramer, 2022; Griffy, 2021; Acabbi et al., 2022). Whether there are some interaction effects between wealth and skill accumulation in the current context is beyond the scope of this paper.

Next, I extend the simple model to an infinite horizon framework with aggregate uncertainty. I build a dynamic quantitative incomplete market heterogeneous agent model with wealth sorting to study the dynamics of wages, assets, and labor market flows over the business cycle. Relative to existing research, my model newly connects wealth heterogeneity with labor market sorting in an incomplete market setup. An interaction between the distributions of cyclical employment risk and wealth arises. Wealth influences job search decisions through the need for insurance, and job search decisions influence the workers' wealth through savings during the employment spell. My model therefore highlights an economic force that is present in models with incomplete markets and heterogeneous wealth and that operates on labor markets if one allows for sorting.
I show that the quantitative model is block recursive, i.e., that the individual value functions and market tightness in every submarket are independent of the distribution of workers over asset and employment states. This result allows me to solve for the full stochastic equilibrium in a relatively tractable way. I calibrate the model to the U.S. at a monthly frequency and assume firms with higher separation risk to be less productive. My calibration matches moments of the wealth distribution, as well as cross-sectional and cyclical job flows moments. In the steady state, relatively poorer workers look for relatively lower wage jobs and lower productivity jobs (with a larger separation risk). They use their labor market search as a way to smooth consumption. This mechanism is valid both across workers (low versus high assets) and also across aggregate productivity states. In bad times, asset-poor unemployed optimally decide to apply to even worse jobs to mitigate the decrease in job finding probability. As a consequence, sorting is relatively stronger in bad aggregate states, as measured by the larger correlation between individual wealth and separation risk, wages and job productivity. Moreover, the dispersion of wages and job productivity is relatively larger in bad aggregate states.

Finally, I use the quantitative model to study the effect of recessions and targeted transfers. I simulate a 1% negative aggregate productivity shock. First, I show that following a 1% negative aggregate productivity shock, the search of unemployed workers tilts towards worse jobs – average wages and productivity decrease by around 6%. This number hides substantial heterogeneity concentrated in the bottom 1% of the wealth distribution. In fact, these workers search for wages and job productivity around 1 percentage point lower than the richest 1%. This change in job search allows to greatly mitigate the drop in their consumption. Second, I study the effect of four types of targeted transfers implemented when the aggregate productivity shock hits. The first policy is a standard unemployment benefit and targets all unemployed. I compare this to policies targeting the asset-poor, as well as policies targeting only the employed or unemployed asset-poor. By targeting the transfer to workers with on average lower assets, the aggregate consumption drop is mitigated. This is especially true for the transfers targeted at the unemployed. However, this distorts their job search –allowing them to look for better jobs, which depresses the following period's aggregate consumption through a relatively larger share of workers who remain unemployed (matching probabilities are lower for good jobs). This backlash effect on consumption does not happen when targeting the poorest employed workers. Lastly, I show that in the presence of aggregate risk and liquidity constraints, wealth sorting creates a “poverty trap”. Asset-poor workers systematically face higher employment risk and struggle to escape their precarious state, as they are unable to save sufficiently during short and low-wage employment spells. Additionally, I show that starting an unemployment spell during a recession, as opposed to normal times, has long-lasting effects for all workers, though to a greater extent for the poorest. A job search during a recession leads to persistently lower wages and slower asset accumulation for years, especially so for the poorest workers. I quantify the lifetime cost of starting an unemployment spell during a recession (relative to a normal time) to represent 3% of lifetime consumption for the poorest, more than twice that of the wealthiest workers.

Related literature This paper relates to four strands of the literature. First, I contribute to the literature on sorting over the business cycle by studying *wealth* sorting. Lise and Robin (2017) study skill-sorting over the business cycles in a model with random search and alternating offers. Payne et al. (2024) instead develop a deep-learning algorithm to extend the class of random search models that can be solved with aggregate uncertainty. Acabbi et al. (2022) explore how sorting of heterogeneously skilled workers varies over the business cycle in a directed search framework. These papers do not model heterogeneity in wealth, which is my focus. I add to the understanding of how aggregate shocks affect sorting by studying the role of wealth heterogeneity for labor market transitions and wage dynamics over the business cycles. Moreover, in my model, sorting newly relates wealth to cyclical employment risk.[3] Unlike these papers, job sorting in my model is driven purely by the need for insurance; I assume no production complementarity between wealthy workers and good jobs. My mechanism is not in conflict with alternative sorting theories but rather works within dimensions of heterogeneity such as skills. My paper builds on the directed search frameworks developed in Menzio and Shi (2010, 2011), and extends the proof for block recursivity to an environment with aggregate risk, worker wealth heterogeneity, and firm productivity heterogeneity.[4]

Second, my paper speaks to the literature on incomplete market models with wealth heterogeneity and labor market frictions. Relative to other frameworks (e.g., Krusell et al. 2010; Shao and Silos 2017 or Ravn and Sterk (2021)), my model implies a selection effect of negative aggregate shocks on individual probability to get unemployed through sorting. In particular, asset-poor workers are relatively more likely to become unemployed. While risk is usually modeled to be exogenous to wealth, my mechanism maps wealth into cyclical risk and cyclical risk to wealth.

The coexistence of low wage and low security jobs resembles Jarosch (2023), though emerging from a different mechanism. While he assumes a joint distribution consistent with his estimated data, in my model the fact that low security jobs offer low wages is endogenous and depends on primitives.

[3]Kramer (2022) jointly studies wealth heterogeneity and income risk, but through a different mechanism – he does not have sorting.

[4]Existing related papers have proved the existence of a block recursivity in similar environments: with skill-sorting and aggregate shocks (Acabbi et al., 2022); with a wealth distribution (Chaumont and Shi, 2022); with a wealth distribution, worker heterogeneity, and aggregate shocks (Birinci and See, 2023; Herkenhoff et al., 2024).

Third, my work relates to theoretical and empirical literature analyzing the role of wealth for labor market outcomes. The theoretical literature, going back to Danforth (1979), studies the effect of wealth on job search behavior and various outcomes. Acemoglu and Shimer (1999) show that risk-averse agents use the labor market as an insurance mechanism against consumption risk due to unemployment. Rendon (2006) and Chaumont and Shi (2022) study the relationship between job search, unemployment spells, and asset accumulation. Eeckhout and Sepahsalari (2024) introduce the notion of wealth sorting and show that workers' wealth determines their productivity through labor market sorting. The mechanism at the heart of my paper builds on this line of research in the sense that wealthier individuals are more selective and are able to wait for better job offers. The novelty I propose is to explore a job search outcome that has not been explored yet (both empirically and theoretically), namely employment cyclical risk.[5] Importantly, I also introduce aggregate risk and study the effect of wealth over business cycles.

Many empirical papers explore related outcomes. Herkenhoff et al. (2024) estimate the role of credit constraints on unemployment duration, wages, and matched firm productivity. He and le Maire (2020) analyzes the effect of liquidity constraints on wages, human and physical capital accumulation in Denmark. Relatedly, Bednarzik et al. (2021) estimates the role of indebtedness on unemployment duration for U.S. households. Lammers (2014) analyzes the role of wealth on reservation wages and search effort using Dutch data. Chetty (2008); Card et al. (2007) and Fontaine et al. (2020) establish a negative relationship between liquidity and job-finding rates. My model is consistent with these results and additionally considers the effect of liquid wealth on workers' employment risk.

Fourth, understanding heterogeneous earnings cyclical risk across different dimensions of heterogeneity has been the focus of recent empirical studies.[6] I add to this literature by documenting a negative relationship between liquid wealth and employment cyclicality. In terms of empirics, my work is related to Patterson (2023) who studies the correlation between MPCs and earnings' elasticity to aggregate fluctuations. Differently from her, I focus on residual inequality – differences that are not explained by workers' characteristics beyond their liquid wealth.

The manuscript is organized as follows. Section 2 presents empirical findings on the correlation between cyclical employment and liquid wealth. Section 3 introduces a tractable theoretical framework where workers with different levels of wealth bear heterogeneous employment risk. I provide conditions for the model to be consistent with the negative empirical correlation between wealth and cyclical risk. Section 4 outlines a parsimonious quantitative model and presents the equilib-

[5] Other strands of the literature explored different search outcomes; see e.g., Lentz and Tranaes (2005); Lise (2013) who studies how wealth affects search effort; or Griffy (2021) and Kaas et al. (2023) who study how wealth inequality affect workers' climbing of the job ladder.

[6] Guvenen et al. (2014) focuses on idiosyncratic labor income risk across the earning distribution; Hoynes et al. (2012) analyzes how recessions impacts labor income differently by demographic groups; Dany-Knedlik et al. (2021) document that lower and higher income individuals experience significantly larger fluctuations across the business cycle than middle-income individuals; and Cairó and Cajner (2018) find that less educated individuals in the U.S. have less stable jobs, which is due to a more frequent job loss. Carrillo-Tudela et al. (2022) study cyclical earnings through the lens of a job ladder model.

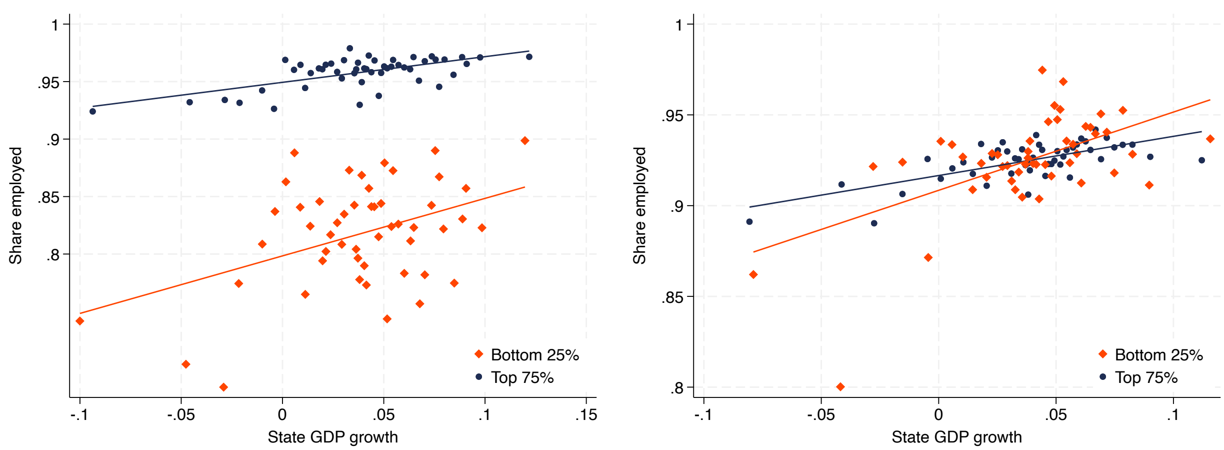
rium. Section 5 presents quantitative results on the short- and long-term effect of recessions, and implements different policy counterfactuals. Finally, Section 6 concludes.

2 Liquid wealth and employment in the data

The relationship between wealth and employment risk is a relevant moment in models with incomplete insurance, however it has received little attention in the literature. An exception is Patterson (2023), who shows that the earnings of high-MPC individuals are relatively more exposed to aggregate fluctuations. Figure 1 shows a similar correlation for individuals with low liquid wealth.[7] In particular, it plots the correlation between employment status and U.S. State real GDP growth, differentiating by liquid wealth holdings. The orange diamonds and lines represent workers who are in the bottom quartile of liquid wealth distribution – roughly corresponding to those with zero liquid wealth, and the blue dots and lines represent the rest of the workers.

Figure 1: Employment and aggregate output fluctuations by liquid wealth

(a) No controls **(b)** With controls



Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Dots represent mean of x and y values within 50 equally populated bins. The $x-$ axis represents log changes in U.S. states real GDP; the $y-$axis is the mean employment status. The left panel plots the unconditional correlation, while the right panel controls for individual fixed effects, socio-demographics, and a quadratic in age. Liquid wealth is measured as the amount in the savings accounts and stocks holdings.

The correlation between GDP and employment is positive for all workers, but stronger so for workers with low assets. This holds both unconditionally (left panel) and after controlling for observable characteristics (right panel). These figures suggest that on average, asset-poor workers' employment moves more with aggregate fluctuations, everything else being equal. This correlation

[7]MPCs and liquid wealth are closely related concepts in incomplete market models. Typically, individuals closer to their borrowing constraints have a higher MPC. However, this relationship can vary depending on specific modeling assumptions, such as the presence of assets with different levels of liquidity. Additionally, in empirical data, MPC and wealth are distinct concepts: MPC must generally be estimated (unless derived from an experiment), whereas wealth is directly measured in many large-scale surveys.

is consistent with Patterson (2023), and no existing theory can rationalize it.[8]

Asset-poor workers not only have more cyclical employment risk, but they also show a higher probability to change employment status in the cross-section (see Appendix Figure A1). In what follows, I examine the correlation between liquid wealth and cyclical employment in greater detail. My findings show that workers with low liquid wealth are statistically significantly more exposed to employment fluctuations during the business cycle. I then explain why existing models fail to rationalize this relationship.

2.1 Data and empirical specification

The PSID is the world's longest ongoing longitudinal household panel dataset. The survey is conducted in the U.S. and on a biannual basis since 1999. From that year onward, collected information of interest on workers include socio-demographics, employment, and detailed financial situation. See Appendix A1 for more information about the data and cleaning.

While wealth is an endogenous variable correlated with education, productivity, and skills, among others, the focus of this paper is on *residual* inequality, i.e., inequality that is directly attributable to wealth. I estimate the correlation between employment elasticity to aggregate fluctuations and liquid wealth, accounting for both observed and unobserved differences between asset-poor and wealthy agents. The results should be interpreted as operating *within* specific groups, such as those defined by skill or education. My baseline specification is as follows.

$$e_{it} = \gamma_i + X_{it}'\beta + +\chi a_{it-1} \times \Delta \log Y_t + \nu_{it}, \tag{1}$$

where e_{it} is a dummy capturing the employment status of the worker (1 if employed, 0 if unemployed), γ_i is the worker fixed effect, a_{it-1} is a dummy equal to 1 if the individual is asset-poor; X_{it} are controls including time-by-state fixed effects and the asset-poor dummy; and Y_t is the U.S. States' real GDP.

The coefficient of interest is χ: a positive value means that the employment of asset-poor individuals is more exposed to changes in aggregate output relative to wealthy individuals. If there is no role for a residual effect of wealth on the correlation with cyclical employment, the coefficient should become insignificant as we control for dimensions of heterogeneity underlying the correlation.

2.2 Asset-poor workers and cyclical employment

Table 1 presents baseline results. Columns (1) through (4) progressively saturate with fixed effects and controls. In Column (1), the estimates are displayed without the inclusion of controls or fixed effects. The first coefficient indicates a positive correlation between aggregate output and individual employment probability. Specifically, a 1% increase in aggregate GDP is associated with a 0.23% higher probability of employment. The second coefficient shows that individuals in

[8] It is consistent to the extent that liquid wealth and MPC are negatively correlated. Computing the MPC as she does, I find that workers at the bottom of the liquid wealth distribution in my sample indeed have a higher MPC.

Table 1: Liquid assets and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)
$\Delta \log Y_t$	0.23^{***}			
	(0.035)			
a_{it-1}	-0.15^{***}	-0.011	-0.011	-0.011
	(0.0093)	(0.0076)	(0.0077)	(0.0073)
$a_{it-1} \times \Delta \log Y_t$	0.32^{***}	0.27^{**}	0.26^{**}	0.27^{***}
	(0.12)	(0.11)	(0.11)	(0.097)
FEs	No	Yes	Yes	Yes
Worker controls	No	No	Yes	Yes
Past income	No	No	No	Yes
R^2	0.050	0.45	0.45	0.45
N	35961	35961	35961	35961

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. Liquid wealth is measured as the checks and savings account. a_{it-1} is equal to one if the household's liquid wealth lies in the first quartile of that year's liquid wealth distribution. Fixed effects are at the individual and year level. Worker controls include a quadratic in age, and all the respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years) and it is interacted with GDP.

the lowest asset quartile are, on average, 0.15% less likely to be employed. More relevant to this paper, the estimates reveal that asset-poor individuals exhibit a greater exposure of labor income to aggregate fluctuations, as evidenced by the significantly positive coefficient on the interaction term. Introducing fixed effects and socio-demographic controls decreases the magnitude of coefficients, as it isolates the direct effect of being asset-poor on the sensitivity of employment to changes in aggregate output. The coefficient on the asset-poor dummy decreases in magnitude and becomes insignificant, suggesting that the conditional correlation effectively controls for the direct effect of wealth on the employment probability. To mitigate concerns that the skills are not controlled for by the fixed effects and education, Column (4) further controls for workers' past income interacted with real log GDP.[9] The interaction coefficient of interest is of 0.27%, and can be interpreted as the effect of *residual* wealth inequality on the elasticity of individual employment to aggregate output fluctuations.[10]

This residual inequality is present not only within skill groups, but also within occupation and industries. In Appendix Table A2 I further control for workers' occupation and industry interacted with GDP fluctuations. The baseline coefficient is slightly lower (also due to a smaller sample), but remains significantly different from zero. Consistently with the evidence that unequal employment risk is present within industries, occupations and skill groups, in my model I assume an ex-ante

[9] If the worker was unemployed in the previous period, the "past income" variable corresponds to the previous period's income. If the worker was unemployed in that previous period, she is dropped out of the sample. Past income may capture idiosyncratic productivity, helping to disentangle the distinct effects of income and wealth.

[10] The covariance implied by these coefficients is 0.049, which is in line with Patterson (2023)'s results. It can be computed as the interaction coefficient times the variance of a_{it}.

distribution of job separation risk. In that sense, the mechanism I propose can be thought to work within skill, occupation and industry groups.

Job loss and job finding I then explore whether the heterogeneous cyclicality across the liquid wealth distribution is driven by job loss or job finding. I change the baseline regression's dependent variable to represent either a transition from employment to unemployment (job loss dummy) or a transition from unemployment to employment (job finding dummy). Results suggest that job loss is the more relevant margin, both in terms of magnitude and significance – see Appendix Table A6. Nevertheless, in the cross-section both the job loss and job finding margins matter. Liquid wealth poor workers, on average, have more chance of finding a job and loosing their jobs. Moreover, the magnitudes are similar – see Appendix Table A5. The model that I present in the next section is consistent with these findings.

Sensitivity Low levels of liquid wealth are closely related to tight liquidity constraints in a standard model. Tight liquidity constraints are also the underlying reason that agents with low liquid wealth have, everything else being equal, higher MPCs. Therefore, the fact that the evidence above is consistent with Patterson (2023) suggests that it is the liquidity constraints that matter for the residual differential employment exposure to aggregate fluctuations. While I do not have a good measure of liquidity constraints in the PSID, below I show evidence that supports this hypothesis. The wealth distribution exhibits a high concentration of observations at zero, with approximately 25% of individuals in the sample possessing no assets. In Appendix Figure A3, I compare different segments of the wealth distribution to higher wealth percentiles. The first data point illustrates the comparison of the bottom 20th percentile to the remaining distribution, followed by the comparison of the bottom 30th percentile, and so on. Coefficients are weakly decreasing up to the 80th percentile (with a magnitude of around one half of the baseline estimate). The fact that coefficients decrease with liquid wealth aligns with the idea that liquidity constraints underlie the baseline correlation, as the latter weakens when constraints become less binding. To distinguish the role of liquidity constraints from other mechanisms potentially linked to wealth holdings, I then run the baseline regression classifying agents according to their illiquid asset holdings. Appendix Table A3 shows results where I classify agents according to their illiquid wealth (instead of the baseline result that uses *liquid* wealth). Coefficients are smaller and not significant, indicating that illiquid wealth does not imply a differential employment elasticity to output fluctuations. In contrast, and supporting a role of liquidity constraints, I show that being hand-to-mouth is relevant; see Appendix Table A4. Another question that may arise is whether the correlation is driven by some permanent component of wealth or because of different histories of shocks. First, although liquid wealth is auto-correlated at the individual level (around 0.9), the auto-correlation of the probability to be in the lowest quartile is smaller (around 0.5), suggesting that the baseline regression does not merely capture a permanent component of being asset-poor. To verify whether there is an important role of the permanent component of wealth that would not be captured by the individual fixed effects, Appendix Table A4 shows baseline results changing the "poor" dummy to represent workers that are in the lowest quartile of the permanent liquid wealth distribution. The coefficients are smaller and the standard errors larger, implying no significantly higher employment response to aggregate fluctuations for the Fixed effects are at the individual and year level.ly poor" workers.

2.3 Employment risk and wealth in existing models

From the perspective of risk-averse agents, these findings are at odds with standard random search models. Intuitively, since asset-poor individuals are in a very concave part of their utility function, they value low volatility relatively more. In a world where agents can accept job offers (à la McCall, 1970), this basic economic force induces them to trade low wages for income security. Instead, wealthy individuals do not mind risk as much and are willing to accept riskier but better paid jobs. To understand this, consider a McCall structure built on a simple consumption-saving problem under borrowing constraints. Assume that unemployed workers may receive job offers that differ in their cyclical risk and have to decide to accept or reject them. The equilibrium is such that asset-poor agents are less likely to accept risky jobs, as their reservation wage for these jobs is relatively higher. On average, this thus predicts that more risky jobs are populated with more wealthy agents.[11] The same intuition carries over to more complex models of wage compensation differentials. For example, in Pinheiro and Visschers (2015), the wage compensating differential for additional job loss risk is larger for those workers who have most to lose from job loss, in their case workers who are at the top of the job ladder. Applying the same logic to a model with liquidity constraints predicts that workers with little savings are less likely to accept highly risky jobs – the opposite of what I find in the data.

This paper provides a theoretical foundation for the higher employment elasticity of asset-poor individuals to aggregate output fluctuations. The key distinction from the models discussed above is that I allow workers to direct their labor market search. I focus on a labor supply mechanism because, from an employer's perspective, there is no reason to expect different hiring or firing behavior based on workers' liquid wealth, and because the latter is not even necessarily observable.

3 Wealth-sorting and elasticity of employment

In this section, I show how asset-poor workers can decide to sort into jobs with relatively larger separation risk. My model focuses on residual inequalities – those unexplained by workers' characteristics aside from wealth. When agents are risk-averse, liquidity constrained, and can direct their job search, they may use the labor market to insure themselves by applying to jobs that offer a high matching probability.[12] But is it possible that poor workers apply to bad jobs even if this means bearing more cyclical employment risk? This feature directly conflicts with workers' need for insurance. I present the conditions under which sorting still happens because the insurance motive dominates the "bad" features of a job. In this model, as will become clear in the next section,

[11] The model and results are available upon request.

[12] See discussion of Acemoglu and Shimer 1999; Chaumont and Shi 2022 in the literature review.

exposure to cyclical risk is a by-product of workers' optimal labor market search. I start by laying down a two-period model that clarifies the trade-offs involved. For a more realistic and complete effect of wealth on sorting into jobs, Section 4 presents a quantitative model.

3.1 A two-period model

In this section, I present the simplest model conveying the core mechanism of the paper. In essence, I show that constrained risk-averse agents decide to apply to jobs with a higher probability of destruction if this allows them to increase the probability of matching.

Environment Time lasts two periods, i.e., $t = 1, 2$. Aggregate productivity z in the second period is uncertain. In particular, it may take two values, z^H and z^L, each materializing with respective probabilities p and $(1-p)$.

The economy is populated with a continuum of heterogeneous workers and firms. Workers are risk-averse and differ in their asset holdings $a_t \in \mathcal{A} = [\underline{a}, \bar{a}] \subset \mathbb{R}_+$. Their period utility function $u(c)$ is increasing and concave ($u'(c) > 0$, $u''(c) < 0\ \forall c \in \mathbb{R}_+$) and satisfies the Inada conditions. They face a liquidity constraint such that they cannot save less than $\underline{a}$. They discount the future at rate β, and the risk-free interest rate $R = 1 + r$ is determined by the world market and taken as given. Their labor market status is either employed or unemployed. When employed, workers earn a wage w and when unemployed they benefit from home production b.

Firms have one job, are risk neutral, and face the same discount rate as workers. As is common in the search literature, I refer to a job as a firm. Jobs are indexed by their (observable) type $x \in \mathcal{X} = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$, and differ in their cyclical separation probability $\lambda(x, z)$. Let $|\lambda_x(x, z^H) - \lambda_x(x, z^L)| < 0$, such that low-type jobs are those carrying relatively more cyclical separation risk. The production technology is independent of job type; all jobs produce output $\bar{y}$ using labor as the only factor of production.[13] A final good competitive firm pools individual output and transforms it to the consumption good.

Labor markets Job search is directed, and the labor market is segmented in submarkets that do not compete directly with one another. Like is common in directed search theory, every submarket represents a subset of unemployed and firms with vacant jobs that are searching for each other. Submarkets are denoted by the firm type and the posted wage, (x, w) – different firms posting the same wage operate in different submarkets. There is free entry for vacancy posting; firms opening a vacancy pay a cost that depends on their type, $\varphi(x)$.[14] Each entrant firm posts a wage to be paid in the production stage upon matching. Unemployed workers choose to apply to (only) one submarket, and employed workers do not search on the job. The firm-to-worker ratio in each submarket is denoted by $\theta(x, w)$. For ease of notation, I hereafter drop the arguments and write θ

[13] I will relax this assumption in the quantitative model.

[14] This assumption is needed for the existence of an interior solution for sorting. Generally, one can think about it as assuming that there exist deep-pocketed entrepreneurs who can invest in the quality of their jobs, and higher type jobs require a higher investment (in the same spirit as Acabbi et al., 2022).

to denote the tightness in a specific submarket. Matching is governed by a matching function that satisfies common assumptions (twice continuously differentiable, concave) and features constant returns to scale. The latter allows to write matching probability in every submarket as a function of tightness, $m(\theta)$, with $m'(\theta) > 0$ and $m''(\theta) < 0$. The probability of filling a vacancy is given by $q(\theta) = m(\theta)/\theta$, with $q'(\theta) < 0$. I assume a CES matching function, in the with the literature (see e.g., Menzio and Shi 2011).

Timing In $t = 1$, all agents start unemployed. Firms open vacancies and post wages. Workers apply to one job offer and take their consumption-savings decision. At the end of the period, the matching function determines the matching outcome in each open submarket. In the beginning of period 2, the aggregate productivity state materializes, and a lottery destroys some matches created in the end of period 1 according to the separation function $\lambda(x, z)$. If separated, workers remain unemployed and the job vacant (with zero value). Matched workers get their contract wage, and production takes place.

Value functions It is natural to write the value functions backwards. In the second (and last) period, the values of employment E and unemployment U are, (given that $a_3 = 0$), $E_2(a_2, w) = u(Ra_2 + w)$, and $U_2(a_2) = u(Ra_2 + b)$. For simplicity of exposure and without loss of generality, in what follows I set $b = 0$. The value of a filled job, given that wages and productivity are fixed, is simply $J(x, w) = \bar{y} - w$.

In period 1, the firm decides to open a vacancy in a certain market by posting a wage, according to the following value function.

$$V(x) = \max_{w} -\varphi(x) + \beta\Lambda(x)q(\theta)J(x, w),$$

where $\Lambda(x) = \mathbb{E}_z(1 - \lambda(x, z))$ is the expected retention rate. Given that in this model the aggregate state only affects separation probabilities and not the continuation value functions (J, E, U), what is relevant to agents is the *expected* retention, not its cyclicality.[15]

Unemployed workers' problem is to save a_2 subject to the liquidity constraint $\underline{a}$, and to choose a submarket (x, w) to which to apply. Their value function writes

$$U_1(a_1) = \max_{a_2 \geq \underline{a}, (x,w)} u(Ra_1 - a_2) + \beta[m(\theta)\Lambda(x)E_2(a_2, w) + (1 - m(\theta)\Lambda(x))U_2(a_2)]$$

Relevant to the unemployed' labor market search is only the *expected* retention. A search in a high-type job x affects the continuation values through the retention rate $\Lambda(x)$ (exogenously), as well as through the matching probability $m(\theta)$ (endogenously). As will become clear below, if these forces move in opposite directions, the sign of sorting depends on which of the two prevails.

[15] One may also assume that the aggregate productivity affects continuation value functions. In this case, agents would care about expected continuation value instead of expected retention rate only. This mechanism is present in the quantitative model.

Free entry The measure of entrant firms is determined by competitive entry. This implies that in equilibrium, the value of open vacancies is zero, i.e., the cost of opening a vacancy is equal to the expected value of the job. Formally,

$$\varphi(x) \geq \beta\Lambda(x)q(\theta)J(x,w) \qquad \forall x \in \mathcal{X} \tag{2}$$

and $\theta(x,w) \geq 0$ with complementary slackness:

$$\theta(x,w) = \begin{cases} q^{-1}\left(\frac{\varphi(x)}{\beta\Lambda(x)J(x,w)}\right) & \text{if } \beta\Lambda(x)J(x,w) > \varphi(x) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

For a given job type x, the free-entry condition pins down the tightness as a function of wages in any submarket that is visited by a positive number of workers. It provides firms' indifference curves in the menu of market tightness and wages; posting a high wage has to be compensated with high job filling probability.

3.2 Equilibrium and positive wealth sorting

To solve for the equilibrium allocation, and as is usual in the directed search literature, I consider the unemployed optimal search and the firms' optimal posting strategy jointly. Effectively, unemployed workers take the firms' indifference condition as given. We can then analyze the problem as a maximization with non-linear Pareto frontier $U(a,x)$, which denotes the value to the worker with assets a when matched with job type x. Conditional on a search with job type x, the maximization problem of an unemployed writes

$$\begin{aligned} U(a_1,x) = \max_{a_2,\theta} \quad & u(c_1) + \beta\left[m(\theta)\Lambda(x)u\left(Ra_2 + w\right) + (1 - m(\theta)\Lambda(x))u(Ra_2)\right] \\ \text{s.t.} \quad & c_1 + a_2 = Ra_1 \\ & w = \bar{y} - \frac{\varphi(x)}{\beta q(\theta)\Lambda(x)} \end{aligned}$$

The first order conditions (FOC) with respect to a_2 and θ are, respectively:

$$-u'(c_1^*) + \beta R\left[m(\theta^*)\Lambda(x)u'\left(c_{e,2}^*\right) + (1 - m(\theta^*)\Lambda(x))u'(c_{u,2}^*)\right] = 0 \tag{4}$$

$$\beta\Lambda(x)\left[m'(\theta^*)\left(u\left(c_{e,2}^*\right) - u(c_{u,2}^*)\right) + u'\left(c_{e,2}^*\right)\frac{\varphi(x)\theta^* q'(\theta^*)}{\beta q(\theta^*)\Lambda(x)}\right] = 0 \tag{5}$$

where $c_1^* = Ra_1 - a_2^*$, $c_{e,2}^* = Ra_2^* + \bar{y} - \frac{\varphi(x)}{\beta q(\theta^*)\Lambda(x)}$ and $c_{u,2}^* = Ra_2^*$.

The first condition is an Euler equation where $t = 2$'s consumption depends on the probability of match and match outcome; the second condition pins down market tightness for a given type x.

The worker takes the firm payoff as given – like a menu of value functions for all possible θ, and chooses the firm type that maximizes her expected utility. From the first order condition, it must hold that $\frac{\partial U(a_1,x)}{\partial x} = 0$. The optimality condition for the choice of x can be written as

$$\frac{\Lambda'(x)}{\Lambda(x)} = \frac{-u'(c_{e,2^*})}{u(c^*_{e,2}) - u(c^*_{u,2})} \left(- \frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta^*)(\Lambda(x))^2} \right) \tag{6}$$

In equilibrium, Eq. (25) - (27) determine whether wealth sorting is positive or negative. I define wealth sorting to be positive whenever ex-ante wealthier agents apply to relatively higher type jobs – negative sorting would induce them to apply to relatively lower type jobs. Proposition 1 gives the sufficient conditions for the equilibrium of the two-period model to feature positive wealth sorting.

Proposition 1. *Wealth sorting is positive if*

$$\frac{u'(c_{e,2}) - u'(c_{u,2})}{u(c_{e,2}) - u(c_{u,2})} < \frac{u''(c_{e,2})}{u'(c_{e,2})} \tag{7}$$

$$\frac{\varphi'(x)}{\varphi(x)} > \frac{\Lambda'(x)}{\Lambda(x)}(1+\gamma) \tag{8}$$

Moreover, the equilibrium is unique.

Proof. See Appendix B2. ■

Inequality (7) is satisfied for all utility functions featuring Decreasing Absolute Risk Aversion (DARA) (Eeckhout and Sepahsalari, 2024). In directed search models, workers choose which job offers to apply to by weighing the value of a job against the probability to get it (or the waiting time). When workers are ex-ante heterogeneous in their wealth levels, face liquidity constraints, and have a DARA utility, they value the matching probability in a heterogeneous way. In particular, poor agents use the labor market as an insurance mechanism by applying to short queues with a high probability of matching as they desire to smooth consumption. Risk aversion is thus a key force in this model, as it makes workers relatively more sensitive to consumption smoothing. To gain further insights into the mechanism, I solve the model numerically and parametrize it such that equations (7) and (8) hold. Figure D5 presents the equilibrium under different calibrations of risk aversion. While the equilibrium is qualitatively similar, when agents are more risk-averse, they direct their search towards relatively worse jobs, ceteris paribus. This effect is strongest the lower the unemployed workers' wealth. Higher risk aversion implies a higher probability of match, and the difference is largest at the bottom of the wealth distribution.

While the first condition determines which types of workers look for high matching probability jobs, Eq. (8) instead governs which types of jobs are the ones associated with a higher effective probability of being employed. First, this condition guarantees that high-type firms are those posting higher wages in equilibrium, and therefore having the longest queues. To understand why it is the case, it is useful to think about the firm indifference curves in the menu of wages and tightness. From the free entry condition (2), it is easy to show that the rate of substitution between wages and tightness decreases with firm type whenever the following holds

$$\frac{\varphi'(x)}{\varphi(x)} > \frac{\Lambda'(x)}{\Lambda(x)} \tag{9}$$

Intuitively, φ determines firms ex-post profits and Λ the expected duration (and thus value) of a filled vacancy. When the marginal benefit from opening a higher type job outweighs the marginal decrease in job duration, then the search costs in terms of foregone production increase with firm type. Therefore, to accept a lower job filling probability, high type jobs need to be compensated by relatively lower wages compared to low type jobs.

Second, introducing job separations that are decreasing in firm's type directly decreases the insurance of searching for low type jobs. While Eq. (9) ensures that $q(\theta)$ is increasing with type x, in a positive wealth sorting equilibrium it must be that the *effective* probability to find a job $m(\theta)\Lambda(x)$ is decreasing in x. Eq. (8) ensures that this is the case by correcting Eq. (9) for the different elasticity of the job finding and job filling probabilities. With CES matching function, this takes the form $(1+\gamma)$. Intuitively, the endogenous variable $q(\theta)$ must move enough so to counteract the exogenous force $\Lambda(x)$.

Relationship to exiting papers Besides a different notion of equilibrium, the key difference with Eeckhout and Sepahsalari (2024) is that I introduce a force that directly decreases the insurance motive of searching for low type jobs (the decreasing separation rate by job type). Through the free entry condition, I can moderate the conditions under which this force outbalances the endogenous force which is going through the market tightness. In the baseline model, if the separation function is such that the *expected* retention is equal across job types, i.e., $\Lambda'(x) = 0$, then, just like in Eeckhout and Sepahsalari (2024), Eq. (7) is a sufficient condition for positive wealth sorting. If cyclicality in the separation function impacts continuation values, as previously discussed, the condition is no longer sufficient, but positive wealth sorting may still obtain.

The coexistence of low wage and low security jobs is consistent with what Jarosch (2023) estimates in Germany. The dual labor market is also an example of the coexistence of low-wage, low-security jobs alongside high-wage, high-security jobs. More generally, the fact that low-paid jobs are relatively more likely to get destroyed is consistent with the theory of cleansing effects of recessions, if low wages reflect a lower firm productivity. Having heterogeneous firm productivity is irrelevant in the two-period model, but I will integrate it in the quantitative model.

Equilibrium allocation Positive wealth sorting implies that asset-poor agents self-select into high matching probability jobs as they smooth their consumption, consistently with empirical evidence.[16] Moreover, asset-poor workers' optimal search imply that upon matching, they get relatively lower wages and lower job security. Because of risk aversion and the liquidity constraint, workers' need to self insure outweighs the "costs" of matching in low wage and low security jobs. Any force that either increases the need for insurance (e.g., higher risk aversion, tighter liquidity constraints) or decreases the "cost" of searching for bad jobs (e.g., more homogeneous separation rates) strengthens the motive for positive wealth sorting.[17]

More generally, with an infinite horizon and aggregate risk, both the need for insurance and the "cost" of searching for bad jobs may increase. The former because in a recession it is overall more difficult to find a job, and the latter because bad jobs imply a higher chance of becoming unemployed, especially when it is more costly to do so (in bad times). In this two-period model, I assume that the aggregate state affects only the separation function and not the second period's values of consuming for employed and unemployed workers ($u\left(Ra_2 + w\right)$ and $u\left(Ra_2\right)$ are independent of z). Effectively, agents thus factor in the *expected* separation risk, and not directly its cyclicality. However, sorting in an infinite horizon model does depend on the cyclicality, through the continuation values of employment and unemployment. To illustrate how cyclicality matters for sorting, one straightforward extension is to allow the continuation value of employment to depend on the aggregate state so as to reduce the insurance that working provides. In particular, it is easy to assume that the value of working is lower in bad states and higher in good states, relative to the benchmark model. In this case, Eq. (7) is no longer sufficient to ensure positive wealth sorting, even when (8) holds. Instead, the condition need for positive sorting is more stringent, as it must hold in expectations (see Appendix G1 for more details). This illustrative model is useful to understand that both the cross-sectional and cyclical risk heterogeneity matter for sorting, and both dimensions make *positive* sorting more difficult to obtain.

The quantitative model fully accounts for these additional channels and allows studying the effects of positive wealth sorting on the dynamics of wages and employment over the business cycle.

4 Quantitative model

In this section, I embed the mechanisms outlined in the preceding section into a quantitative model featuring aggregate uncertainty.

Environment Time is discrete, and the economy is infinitely lived. As in the two-period model, the economy is populated by a continuum of risk-averse agents with asset holdings $a_t \in \mathcal{A} =$

[16] E.g., Card et al. (2007); Chetty (2008); Lammers (2014); Herkenhoff et al. (2024); He and le Maire (2020); Fontaine et al. (2020); Bednarzik et al. (2021). See the literature review for more information.

[17] Kaplan and Violante (2022) highlight the empirical and quantitative relevance of the so-called "wealthy hand-to-mouth" – workers with low liquidity but a significant amount of illiquid wealth (e.g., in housing). In this project, I abstract from illiquid assets for tractability reasons. However, my mechanism in theory would also be relevant for wealthy hand-to-mouth, as long as it is less costly for them to adapt their labor market search than transfer their illiquid asset.

$[\underline{a}, \bar{a}] \subset \mathbb{R}_+$, and a continuum of risk neutral one-employee firms, with (observable) productivity $x \in \mathcal{X} = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$. Agents have a period utility function $u(\cdot)$, which is increasing ($u'(\cdot) > 0$) and concave ($u''(\cdot) < 0$). They discount the future at rate β, they can save at the risk-free interest rate $R = 1 + r$ and can borrow up to a constraint $\underline{a}$. The interest rate is determined by the world market and taken as given. Firms have the same discount factor as workers.

When they work, employed agents earn a fixed wage $w \in \mathcal{W}$ which is match-specific. When unemployed, they earn unemployment benefits b_t which are financed by proportional wage taxes τ collected by the government.

Aggregate productivity $z \in \mathcal{Z}$ evolves according to a Markov transition probability $\pi(z_t, z_{t+1})$. Firms produce output $f(x_t, z_t)$ using labor as the only factor of production, and a final good competitive firm pools individual output and transforms it to a consumption good. The aggregate state of the economy, including the distribution of workers and the aggregate productivity state, is denoted with Γ.

Labor markets Job search is directed, and there is a separate submarket for each productivity and wage so that they do not compete directly with one another. There is free entry in every submarket and firms have to pay a vacancy cost $\varphi(x_t, z_t)$. I allow the entry cost to vary with type and aggregate productivity for the calibration. Each entrant firm posts wages w promised to the workers upon match formation and for the entire duration of the match. Firms have limited commitment in the sense that they can fire workers if the value of a filled job becomes negative.[18] Unemployed workers choose to apply to (only) one submarket $(x_{t+1}, w_{t+1}; \Gamma_t)$, and workers do not search on the job.[19] The firm-to-worker ratio in each submarket is denoted $\theta_t(x_{t+1}, w_{t+1}; \Gamma_t)$.[20] Matching is governed by a matching function $m(\theta_t)$ that is twice continuously differentiable, strictly increasing, strictly concave, and such that $m(0) = 0,\ m'(0) < \infty$. The probability of filling a vacancy is given by $q(\theta_t) = m(\theta_t)/\theta_t$, with $q'(\theta_t) < 0$. Matches are exogenously separated with a probability that is decreasing in the firm type and function of the aggregate state of the economy, i.e., $\lambda(x_t, z_t), \lambda_x < 0, \lambda_z < 0$.

Timing Within a period, there are the following four phases. (*i*) the aggregate shock materializes and some existing matches get endogenously destroyed, others get exogenously destroyed; (*ii*) firms with type x decide whether to open a vacancy, in which case they post wages; (*iii*) unemployed workers apply to a job offer and take their consumption-savings decision, employed workers choose optimal savings; (*iv*) agents potentially match on the labor market (the matching function determines the outcome) and production takes place. Matches become productive next period – if they do not get destroyed before production.

[18]This assumption is not material to the results. However, it allows to bound the expected surplus function from below, to allow firms to open vacancies even if in expectations they generate negative value. In the calibrated model, job destruction coming from this assumption is immaterial.

[19]These assumptions are crucial for the tractability of the model, in particular for it to be block recursive. See proposition 2 and Appendix C for further details.

[20]For notational simplicity and in an abuse of notation, I hereon write θ_t to denote the tightness in a specific submarket.

4.1 Value functions

Unemployed workers choose in which market to apply (summarized by the contract pair (x_t, w_t)) and solve a standard consumption saving problem

$$
\begin{aligned}
U(a_t;\Gamma_t) = & \max_{a_{t+1},(x_t,w_t)} u(c_{u,t}) + \beta \mathbb{E}_t \left[m(\theta_t) E(a_{t+1}, x_t, w_t; \Gamma_{t+1}) + (1-m(\theta_t)) U(a_{t+1};\Gamma_{t+1}) \right] \\
& \text{s.t.} \quad c_{u,t} + a_{t+1} = Ra_t + b_t, \qquad a_{t+1} \geq \underline{a} \qquad (10)
\end{aligned}
$$

Employed workers with wage w_t simply choose savings

$$
\begin{aligned}
E(a_t, x_t, w_t;\Gamma_t) = & \max_{a_{t+1}} \quad u(c_{e,t}) + \beta \mathbb{E}_t \left[\lambda(x_t, z_t) U(a_{t+1};\Gamma_{t+1}) + (1-\lambda(x_t,z_t)) E(a_{t+1}, x_t, w_t;\Gamma_{t+1}) \right] \\
& \text{s.t.} \quad c_{e,t} + a_{t+1} = Ra_t + (1-\tau) w_t, \qquad a_{t+1} \geq \underline{a} \qquad (11)
\end{aligned}
$$

, where τ are (exogenous) taxes.
The value of posting a vacancy in a specific submarket with productivity x is

$$
V(x_t, \Gamma_t) = \max_{w_t} \quad -\varphi(x_t, z_t) + \beta \mathbb{E}_t \left[q(\theta_t) J(x_t, w_t; \Gamma_{t+1}) + (1-q(\theta_t)) V(x_{t+1};\Gamma_{t+1}) \right] \qquad (12)
$$

The value of a filled job in submarket (x_t, w_t) is

$$
\begin{aligned}
J(x_t, w_t;\Gamma_t) = & \max\{0; \\
& f(x_t, z_t) - w_t + \beta \mathbb{E}_t \left[\lambda(x_t,z_t) V(x_{t+1};\Gamma_{t+1}) + (1-\lambda(x_t,z_t)) J(x_t, w_t;\Gamma_{t+1}) \right]\} \qquad (13)
\end{aligned}
$$

Note that both firm productivity and wages are fixed over the employment duration. The max operator comes from limited commitment by firms. At the beginning of the period, the aggregate productivity materializes, and existing matches may be subject to destruction. If firms exit the market, they send their workers to unemployment and get payoff 0. This ensures that firms do not refrain from opening vacancies even if the expected value is negative.

Free entry Free entry drives the value of posting a vacancy to zero. The expected value of the job is then equal to the fixed cost of opening a vacancy, i.e.,

$$
\beta \mathbb{E}_t \left[q(\theta_t) J(x_t, w_t; \Gamma_{t+1}) \right] = \varphi(x_t)
$$

Which pins down the market tightness for markets that are visited by unemployed workers. The concept follows the notion of subgame perfect equilibrium. For unvisited markets, the market tightness is equal to zero. Given the functional assumptions stated above, it holds

$$
\theta_t(x_t, w_t;\Gamma_t) = \begin{cases} q^{-1}\left(\frac{\varphi(x_t)}{\beta \mathbb{E}_t [J(x_t, w_t;\Gamma_{t+1})]} \right) & \text{if } \beta \mathbb{E}_t \left[J(x_t, w_t;\Gamma_{t+1}) \right] > \varphi(x_t) \\ 0 & \text{otherwise} \end{cases} \qquad (14)
$$

Government In an extension of the model, I introduce a government to fund the unemployment benefits. The government pools income taxes and redistributes them to the unemployed. I do not assume a balanced budget in every state, but that in the stochastic steady state it holds that

$$\mathbb{E}_z \int^{\Psi^e} \tau w = \mathbb{E}_z \int^{\Psi^u} b \tag{15}$$

, where Ψ^e denotes the stationary distribution of employed workers and Ψ^u that of unemployed workers. This condition implies a balanced budget in expectations. On the one hand, this avoids having a cyclical unemployment benefit or tax. On the other hand, it is also relevant for block recursivity, as will become clear in the next section.

4.2 Equilibrium characterization

Definition 1. *Let $\Gamma = \Omega \times \Psi \times \mathcal{Z}$. A recursive equilibrium for the baseline economy consists of a set of value functions for workers $U : \mathcal{A} \times \Gamma \to \mathbb{R}$ and $E : \mathcal{A} \times \mathcal{W} \times \Gamma \to \mathbb{R}$; a value function for entrant firms $J : \mathcal{X} \times \mathcal{W} \times \Gamma \to \mathbb{R}$; a set of policy functions c_u, c_e, x, w; a market tightness function $\theta : \mathcal{X} \times \mathcal{W} \times \Gamma \to \mathbb{R}_+$; a destruction productivity function $\ell : \mathcal{X} \times \mathcal{W} \times \Gamma \to \{0, 1\}$; such that:*

i) U and E satisfy, respectively, Eq. (10) *and* (11)*, taking taxes τ as given. c_u and $c_{e,x}$ are the associated policy functions.*

ii) V and J satisfy, respectively, Eq. (12) *and* (13)*, and w is the associated policy function.*

iii) Market tightness satisfies (14)*, for all submarkets (x, w, Γ).*

iv) Γ satisfies the policy and contract policy functions, as well as the exogenous low of motion for aggregate productivity, which evolves according to a Markov chain with transition probability $\Xi(z_t, z_{t+1})$.

Under some restrictive assumptions, the model admits a *block* recursive equilibrium.

Definition 2. *A Block Recursive Equilibrium (BRE) is a recursive equilibrium in which the functions $\{\theta, E, U, J, \ell\}$ only depend on the aggregate state of the economy Γ through the aggregate state of productivity z.*

In general, BRE obtains as long as workers' policy and value functions are independent of the distribution of workers across employment states (Menzio and Shi, 2010). Under some additional assumptions, this is true even when the asset distribution is a state variable of the steady state and in the presence of sorting.

Proposition 2. *The baseline quantitative model is block recursive.*

Proof. See Appendix C. ∎

The combination of separated submarkets and the free entry condition is central to the existence of a BRE. To get some intuition of why the equilibrium is BR, assume that J depends on Γ solely through the aggregate component of productivity z. Importantly, I assume that firms do not observe workers' wealth when posting job contracts. Then, free entry implies that θ is also independent of Ψ (see Eq. 14). In fact, free entry determines the number of vacancies in each submarkets, independently of the distribution of workers across the employment distribution. The support of market tightness, i.e., the open markets in equilibrium, is function of the distribution of workers across employment and asset states, but is immaterial for workers' value functions. The unemployed workers' jobs search problem is thus also independent of Γ, as it merely depends on exogenous parameters and the job finding probability (function of market tightness θ). It follows that the values of search and unemployment also only depend on z. Directed search is a crucial assumption here, as if it were undirected, the worker would need to evaluate the whole distribution of open vacancies. As for the consumption-savings decision of both employed and unemployed workers, it is independent of Γ given that we assume the interest rate r to be exogenous to their savings decision. Therefore, the workers value functions and policy functions are independent of Γ. We can then prove that the mapping is a contraction, therefore that the solution is such that the equilibrium is block recursive.

Block recursivity implies that one can solve for the equilibrium of the model without keeping track of the distribution of agents over asset or employment states. It therefore offers a relatively tractable way to solve a fully stochastic dynamic equilibrium. I solve the model numerically with a nested algorithm and using policy and value function iteration; see Appendix E for more details.
The extended model with a government taxing the working population to finance unemployment benefits is still BR. The assumption of having a tax rate balancing the stationary distribution and not the period distribution is key. In particular, the tax rate is fixed and can be treated as a parameter by agents. Numerically, I solve the model for a guess of τ, verify the government balance constraint, and adapt the initial guess of τ accordingly, until the constraint is met.

4.3 Calibration and steady state

I calibrate the model at a monthly frequency for the United States. Functional forms are listed in Appendix Table D7, externally calibrated parameters in Table 2 and internally calibrated parameters in Table 3. The utility and matching functions are taken from standards in the literature. I assume a CRRA utility function with an intertemporal elasticity of substitution σ equal to 2. I follow Menzio and Shi (2011) and choose a standard CES matching function. I set the interest rate to target a 2 percent annualized real rate in steady state. I follow the standard and set the borrowing constraint to $\underline{a} = 0$. Important for the mechanism is that the borrowing constraint exists, not necessarily that it is binding. In fact, because of unemployment benefits, unemployed workers never consume zero. As explained below in more detail, what matters ultimately is the liquidity when unemployed, which is the sum of assets and benefits. I calibrate the discount factor

β to target an average annual MPC consistent with the literature.[21] Given my focus on *residual* inequality, the model is in nature parsimonious. My calibration exercise focuses on calibrating the dispersion and cyclicality of matching and separation rates. I thus assume a separation function that depends both on the firm type and on aggregate productivity, following Birinci and See (2023). $\bar{\lambda}$ is the average separation rate, which I choose to target the average EU rates; η^{λ_z} captures the cyclicality of the job destruction rate; and η^{λ_y} captures the dispersion of the job destruction rate across firms. Similarly, I choose the vacancy cost function to have the same functional form and calibrate parameters $(\bar{\varphi}, \eta^{\varphi_y}, \eta^{\varphi_z})$ to match, respectively, the average matching probability, the cross-sectional dispersion in matching by assets, and the cyclicality of matching.

Unemployment benefits are important in this model, as they affect the utility cost for unemployed workers to be at the borrowing constraint. A generous unemployment benefit makes it generally less costly to be in that state, thereby decreasing the urgency for these workers to find a job. I set b to target a replacement rate of 40% for the lowest paid jobs.[22] I target γ to match an average elasticity of the matching function to tightness of 0.5, in line with the literature (see e.g., Chaumont and Shi, 2022). The average vacancy cost is tightly connected to the average matching rate, which I target to 25%. The average separation rate is targeted to replicate the monthly E-U flow of 1.7%. I calibrate the dispersion of vacancy cost by firm type to match the standard deviation of residual wages observed in the data (in progress). In particular, "total" residual wage dispersion is on average 0.46-0.56 (Morin, 2019), while "matching" dispersion is on average between 0.25 and 0.3 (Cannon and Mustre-del Rio, 2017). I match the standard deviation of matching and separations using the cyclicality of vacancy cost and separation rate, respectively.

Table 2: External calibration

Parameter	Definition	Value	Parameter	Definition	Value
Households			*Productivity*		
r	interest rate	0.0016	ρ_z	TFP, persistence	0.918
σ	risk aversion	2.0	σ_z	TFP, standard deviation	0.004
$\underline{a}$	borrowing limit	0.0			

[21] The corresponding annual MPC is 0.46, computed as $mpc_a = 1 - (1 - mpc_m)^{12}$.

[22] Unemployment benefits in this model are meant to capture only the pecuniary value of benefits.

Table 3: Internal calibration

Param	Definition	Value	Target moment	Model	Data
β	discount factor	0.990	average MPC	0.039	0.03-0.056 (Literature)
b	unemployment benefits	3.1	replacement rate	0.321	40% (Shimer, 2005)
γ	matching elasticity	0.38	elasticity of m to θ	0.50	[0.27-0.5] (Literature)
	Separation rate				
$\bar{\lambda}$	average separation rate	0.03	average λ	0.016	1.7% (Eeckhout and Sepahsalari, 2024)
η^{λ_x}	- dispersion by firm	-0.24	dispersion λ	0.0124	0.012 (CPS)
η^{λ_z}	- cyclicality	30.00	std. λ	0.0024	0.002 (CPS, std. EU rate)
	Vacancy cost				
$\bar{\varphi}$	average vacancy cost	1.00	average $m(\theta)$	0.172	25% (Eeckhout and Sepahsalari, 2024)
η^{φ_x}	- dispersion by firm	1.00	dispersion $m(\theta)$	0.117	0.063 (CPS)
η^{φ_z}	- cyclicality	65.00	std. $m(\theta)$	0.0100	0.069 (CPS, std. UE rate)

The model replicates well the median income to asset ratio, as I compute in the PSID – using the variables described in Appendix A.

Table 4: Non targeted moments

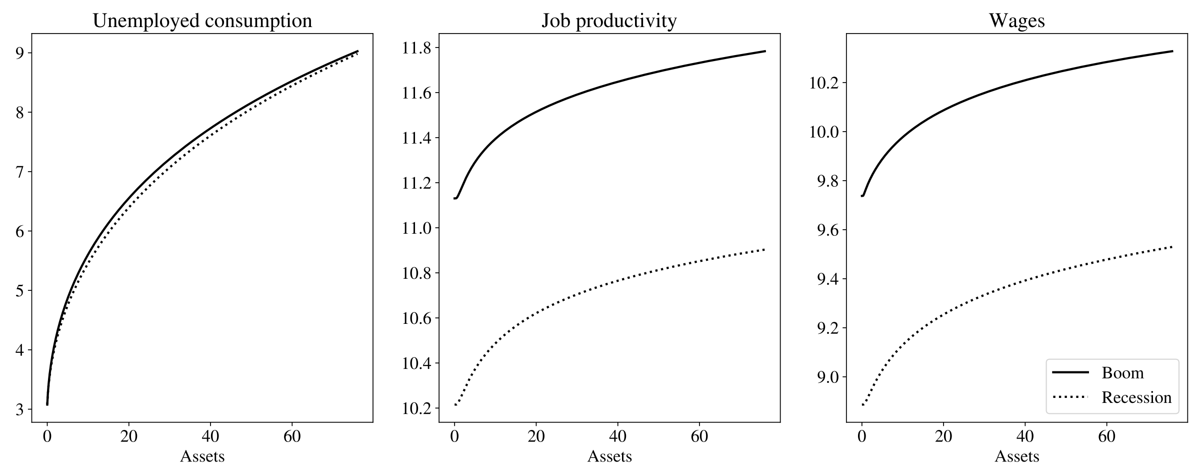
Moment	Model	Data
unemployment rate	0.085	6% (FRED)
std. unemployment rate	0.014	0.0165
avg (w/a)	0.256	53% (PSID)
sd(w)	0.333	0.3 (residual wage dispersion, Morin 2019)

4.4 Consumption and job search policy functions

Figure 2 illustrates the consumption and job search behavior of unemployed workers as a function of their asset holdings. The consumption policy function follows a standard pattern (left panel). The middle and right panels, respectively, plot the choice of wages and firm type as a function of assets. As shown by the monotonically increasing lines, the equilibrium exhibits positive wealth sorting, i.e., wealthier individuals apply to higher-productivity jobs that also offer higher-wage ("good" jobs), while poorer workers target lower-productivity jobs that offer lower-wage ("bad" jobs). The driving mechanism is the shorter queue length in bad jobs, which implies a higher probability of match in those submarkets. Since asset poor individuals are at risk of becoming hand-to-mouth and thereby off their Euler equation, they prefer to search in submarkets that offer a relatively higher probability of match. Instead, wealthy unemployed workers are far from the borrowing constraint and prefer to target a relatively high wage, thereby accepting a longer waiting time. The solid and dotted lines plot the policy functions under different states of aggregate productivity. Low aggregate productivity states mute the job creation motive. Together with the increased probability of separations (by assumption), this implies a slacker labor market and therefore an overall lower matching probability. If unemployed were to keep their job search unchanged, the lower tightness would induce them to decrease their consumption today. Agents thus balance consumption smoothing against job search and future income prospects. When aggregate productivity is low, unemployed workers shift their search towards "worse" jobs (lower wages and lower productivity jobs) in an effort to smooth their consumption. In the presence of incomplete markets, the labor market thus provides some insurance to unemployed workers – in the spirit of Acemoglu and Shimer (1999), at the same time implying a cyclicality of wages and employment risk. Wages decrease relatively more than productivity in bad times.

Appendix Figure D7 shows the consumption policy function for employed workers in different submarkets. Unsurprisingly, those employed in bad jobs consume less as they earn lower wages and face a higher separation probability.

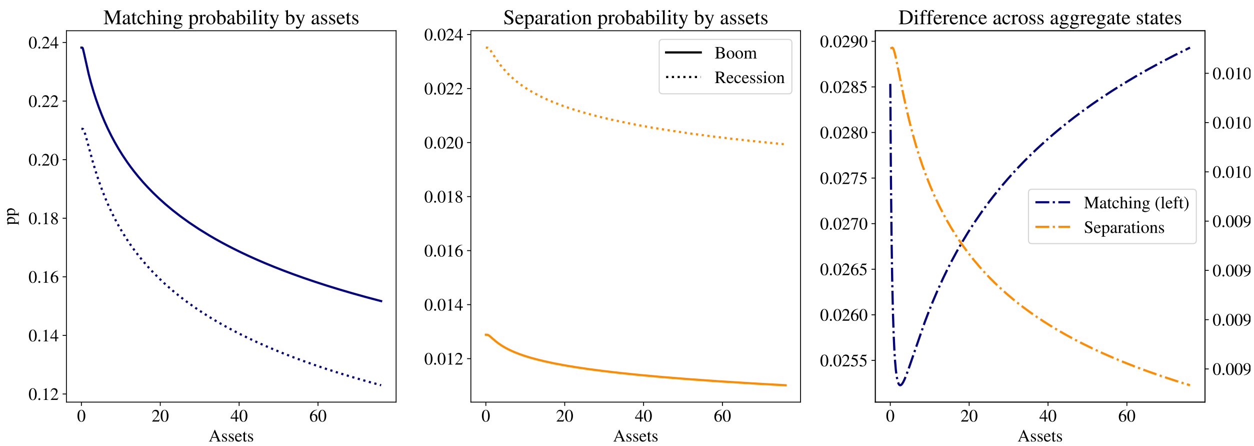
Figure 2: Unemployed workers policy functions



Note: The solid (dotted) lines represent unemployed workers' smoothed policy functions when the aggregate state is good (bad).

The left panel of Figure 3 plots the matching probabilities by assets, which reflects the choice of submarkets. By directing their search towards bad jobs, poor workers have around 20% probability of exiting their precarious unemployment state (versus around 15% for the wealthiest). Wealthier workers are willing to wait longer (direct their search towards markets featuring low probability of matches) to get good jobs. Although unemployed shift their search towards relatively lower productivity jobs in bad times, the matching probability of all unemployed remains lower than in good times – consistently with empirical evidence on the pro-cyclicality of market tightness. Given the assumption of decreasing job separation by firm productivity, positive wealth sorting implies a higher probability of separation for poorer employed workers, especially in times of low aggregate productivity (middle panel). The right panel plots the difference of the matching and separation probabilities across good and bad times. When moving from a boom to a recession, matching and separation probabilities of poor agents increase relatively more than for the wealthy. This can be seen as a theoretical counterfactual of the empirical evidence presented in Section 2.

Figure 3: Matching and separation probabilities



Note: All lines are smoothed. The right panel plots the difference between the solid and dotted lines of the left and middle panels.

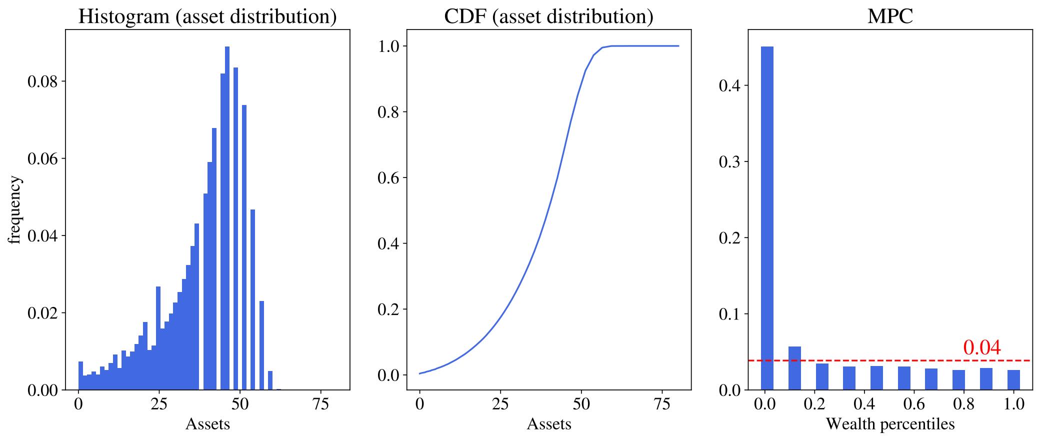
Appendix Figure D8 plots the value of filled vacancies ordered by the markets populated by workers with increasing assets. Jobs employing poor workers are also the ones that have the lowest value. This arises endogenously monitored by the calibration of the entry cost function of the firms.[23]

4.5 Stationary distribution

The (stochastic) steady state of the model supports a stationary distribution of agents of states. Figure 4 plots the distribution of agents over assets, as well as the cumulative distribution function (CDF). While the majority of agents have substantial savings, a non-negligible share of the population holds zero assets. On the right panel, I plot the marginal propensity to consume (MPC) by wealth percentiles. The fact that the lowest percentiles exhibit an MPC close to 1 indicates tight liquidity constraints, which in turn are associated with a strong incentive to look for high matching probability jobs.

[23] This result suggests the possibility of endogenizing the separation function λ within the framework. Suppose firms face the same separation risk, and endogenous job destruction is introduced through a productivity threshold for labor market entry. Firms at the productivity margin are vulnerable to aggregate shocks, with those at the threshold being destroyed following a negative shock. Since ex-post profits increase with productivity, this mechanism implies a higher layoff risk for lower-productivity workers. Only that in that case it would come from a firm endogenous exit and not the separation function.

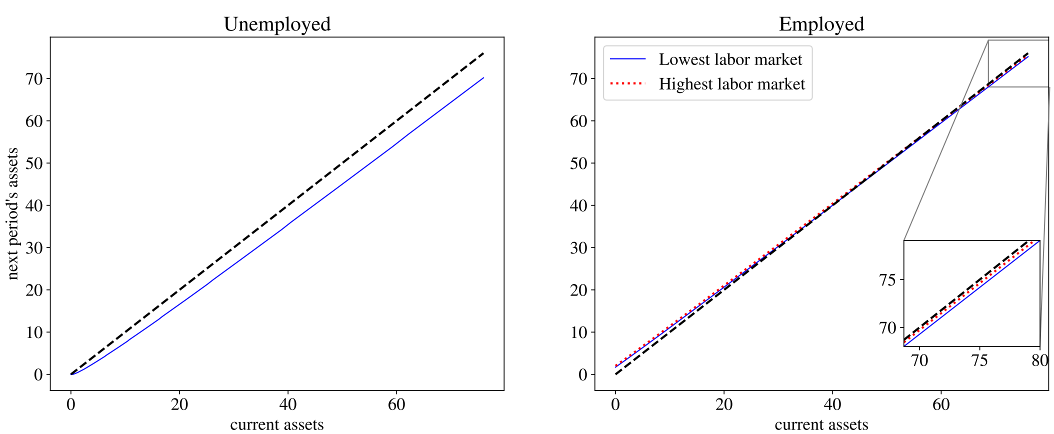
Figure 4: Steady state distribution of agents over assets



Note: The left and middle panels show the stationary asset distribution. On the right panel, the vertical bars show the marginal propensity to consume (MPC) of workers in different wealth percentiles, and the red dashed line represents the aggregate monthly MPC.

In equilibrium, workers with higher levels of assets are more likely to earn higher wages and have relatively more secure jobs. This dynamic could, in theory, set off a path of explosive asset accumulation, undermining the existence of a stationary equilibrium. However, stabilizing forces are sufficient to sustain a stationary equilibrium under the baseline calibration. Specifically, the asset policy function is not diverging even in the highest asset and labor market states, as illustrated in Figure 5.

Figure 5: Workers' asset accumulation



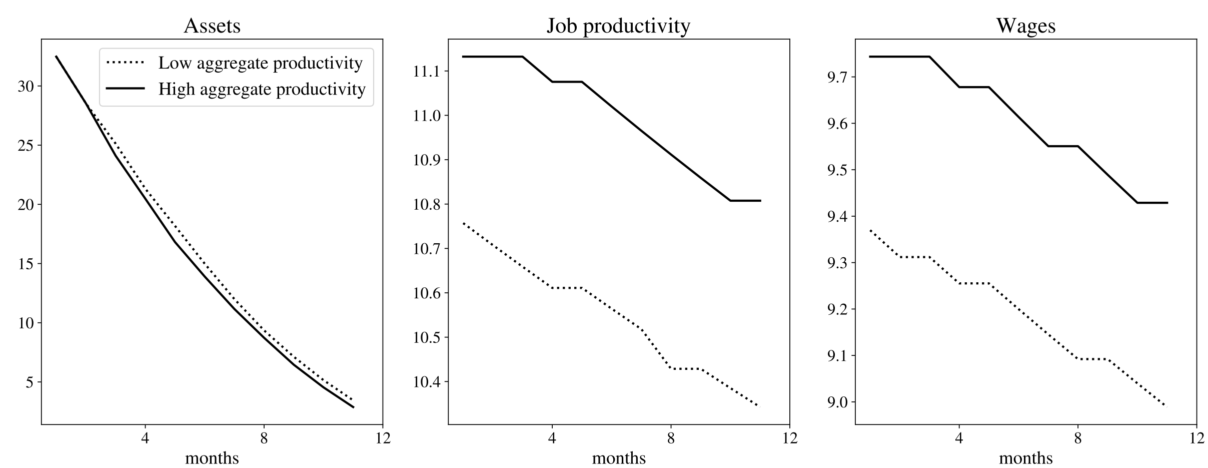
Note: The 45° line is represented in dashed black. On the right panel, the two lines represent the asset accumulation decisions of workers in the lowest and highest labor market, i.e., those with lowest (highest) wages and productivity.

First, all workers face a positive probability of job separation, with separation rates that do not exhibit large cross-sectional variation. Second, workers deplete their assets during periods of unemployment, especially if they enter unemployment with higher initial assets. Figure 6 plots the behavior of an unemployed worker holding the mean level of steady state assets, as she continues not to be selected by the matching function (she is unlucky and remains unemployed period after period). The left panel shows the worker's assets over time, while the middle and right panels illustrate her labor market search behavior. The dotted (solid) line represents the worker's choices if the aggregate state is low (high) throughout.

Workers with higher assets direct their search towards good jobs, which implies longer average unemployment duration due the relatively low matching probability associated to these jobs. Consequently, these workers experience longer unemployment spells in which they deplete their asset holdings. Moreover, they do so at a faster pace the higher the initial assets , as visible from the decreasing slope in absolute terms in the left panel. Therefore, although workers with higher assets face a relatively lower probability of job separation, when they become unemployed they deplete their assets faster and for a longer expected period. Furthermore, within individual workers, negative duration dependence in asset levels drives a shift in search behavior over time, towards lower wages and lower security jobs. In fact, as unemployment lengthens, asset holdings decline, increasing the relative preference for higher-matching probability jobs. This within-worker duration dependence contrasts with the positive duration dependence observed *across* workers, where higher-wage jobs correlate with longer unemployment spells.[24] Therefore, initially wealthier workers may ultimately accept the worst jobs if they experience extended unemployment. This outcome occurs after 12 months of unemployment for an average-wealth worker under baseline calibration.

The aggregate state influences the rate of asset depletion and the job search behavior. In low aggregate productivity states, workers exhibit a stronger precautionary savings motive, which slightly moderates the rate of asset depletion.

Figure 6: Average outcomes for cohort starting with the same level of assets

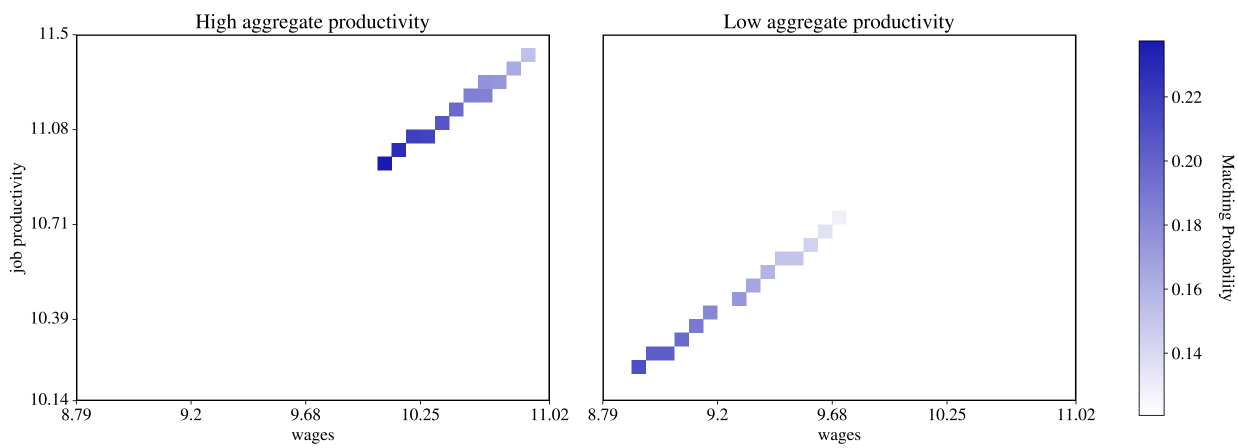


[24]This is a common feature of directed search models.

4.6 Wealth sorting over the business cycle

Figure 7 plots the distribution of labor markets that are open in equilibrium, in low and high aggregate productivity states. The color bars show the probability of match, $m(\theta)$. Because of both the muted job creation motive and the need for insurance, recessions are times when labor markets feature worse jobs (wages and job productivity are lower).

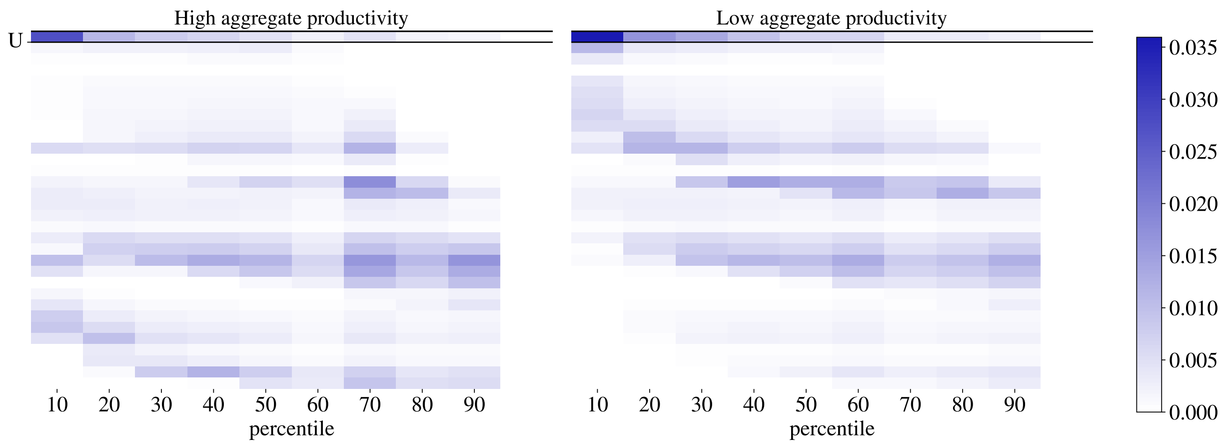
Figure 7: Distribution of open labor markets



Note: The left panel shows the distribution of open labor markets (indexed by job productivity x and wages w), under in the "recession" state, while the right panel shows that of the "boom" state. The intensity of coloring represents the matching probability in each open equilibrium labor market – darker color being associated with higher matching probability.

Moreover, the distribution of agents over labor markets also changes with aggregate productivity, as illustrated in Figure 8. The first row shows the unemployed, and the rest show the open labor markets ordered in ascending order (the rows at the top are bad jobs, and quality improves as we read rows more to the bottom). Two features are worth mentioning. First, as expected, recessions are times with relatively more unemployed. Second, the distribution is more scattered during booms than in recessions. Looking in the top left corner, we see an increased density of agents that are poor and working in low-quality jobs. At the same time, workers at the top of the asset distribution are less likely to work in these low paid, low security jobs. The density becomes more concentrated on the diagonal, which also means that sorting is stronger in recessions.

Figure 8: Steady state distribution of agents over assets



Note: The $x-$axis represents deciles of the asset distributions. The $y-$axis represents workers' labor market status; the first row is the unemployment status, while the other rows represent open labor markets (in ascending order, i.e., the bottom rows are higher productivity and wages jobs relative to the first rows). The intensity of coloring represents the density of agents in each asset bin and labor market state.

Table 5: Sorting in booms and recessions

Moment	Aggregate productivity		Difference
	recessions	*booms*	
$\text{corr}(a_i, w_i)$	0.9373	0.9112	0.0261
$\text{corr}(a_i, x_i)$	0.9445	0.9298	0.0147
$\text{corr}(a_i, \Lambda_i)$	0.9382	0.9221	0.0161
$\text{sd}(w_i)$	0.1657	0.1413	0.0245
$\text{sd}(x_i)$	0.1826	0.1617	0.0210

Table 5 quantifies the dynamics of wealth sorting during booms and recessions. The strength of sorting can be evaluated by the relative correlation in good and bad aggregate states. As we can read from the positive differences, sorting is stronger in bad aggregate states. This is true by all measures of sorting (assets with wages, productivity, and retention rates). The correlation difference is relatively larger for wages, which is the margin that workers use more to insure their consumption. Moreover, the dispersion of wages and firm productivity is slightly higher in bad aggregate states.

5 Effects of recessions and targeted transfers

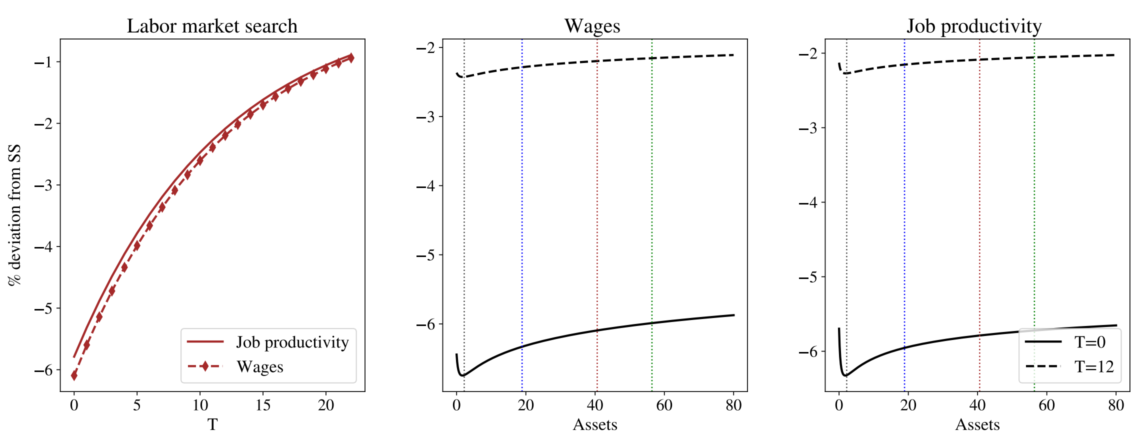
In this section, I first present the impulse response functions (IRFs) to a negative aggregate productivity shocks. I show that unemployed workers primarily adjust their job search behavior rather than their consumption, particularly among the asset-poor. A policy that provides an unexpected one-off transfer is most effective in boosting consumption if it targets the poorest unemployed workers. I then show that wealth sorting has implications for long-run inequalities, as it generates a "poverty trap".

5.1 Short-term responses to a recessionary shock

I simulate the effect of a recession on consumption and job search, both at the individual level and on aggregate. I assume that aggregate productivity follows an AR(1) process such that $z_t = \rho_z z_{t-1} + (1-\rho_z)z^{ss} + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0,1)$; and implement a 1% negative aggregate productivity shock.

Figure 9 plots the job search response. On the left panel, we see that following a negative aggregate productivity shock, the median job search decreases both in terms of wages and job productivity. Wages and productivity decrease by around 6% on impact. This aggregate response hides important heterogeneity, as shown in the middle and right panels (for wages and productivity, respectively). The vertical lines in the middle and right panels flag percentiles of the distribution (1^{st}, 10^{th}, 50^{th} and 99^{th} percentiles, from left to right). The bottom 1% of the distribution decreases their search by relatively more, and the difference between the bottom and top 1% is of around 1% of wages, and 0.5% of productivity. After a year, the unemployed search shifts up (dashed lines) and also becomes more homogeneous across the wealth distribution. When aggregate productivity is at its lowest, its effect on the search of unemployed workers is thus the largest both in terms of magnitude and heterogeneity across workers.

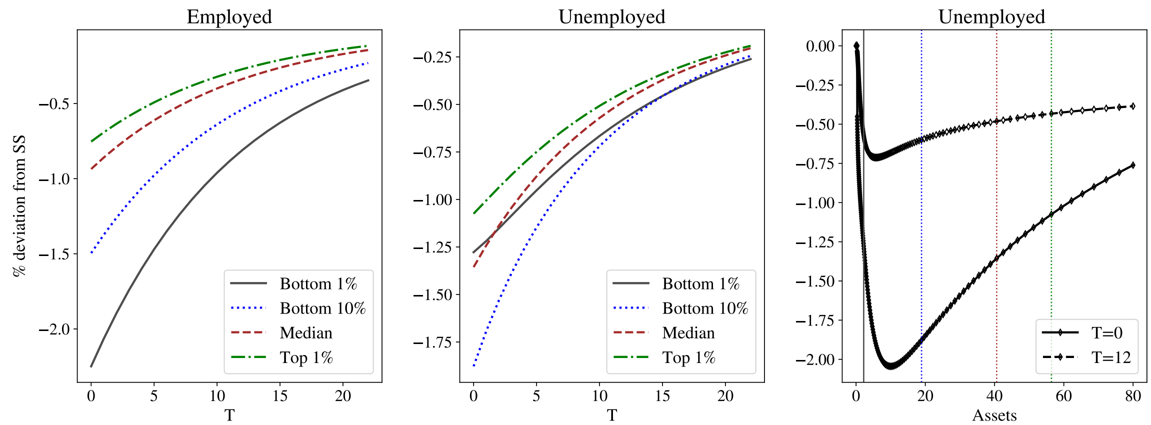
Figure 9: Unemployed workers' job search



Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. On the left panel, the two lines correspond to the labor market search of the median unemployed worker. The vertical lines correspond, from left to right, to the bottom 1%, 10%, the median, and the top 1% of the wealth distribution.

The reason why the poorest unemployed adapt their search more can be seen in Figure 10 (right panel)– it allows mitigating the drop in their consumption. The left panel of Figure 10 instead plots the consumption response for employed workers. Consistently with more standard models, the poorest agents decrease their consumption by relatively more. In the middle panel, we see that besides for the poorest who are hand-to-mouth, the unemployed workers' consumption behavior mirrors that of the employed.

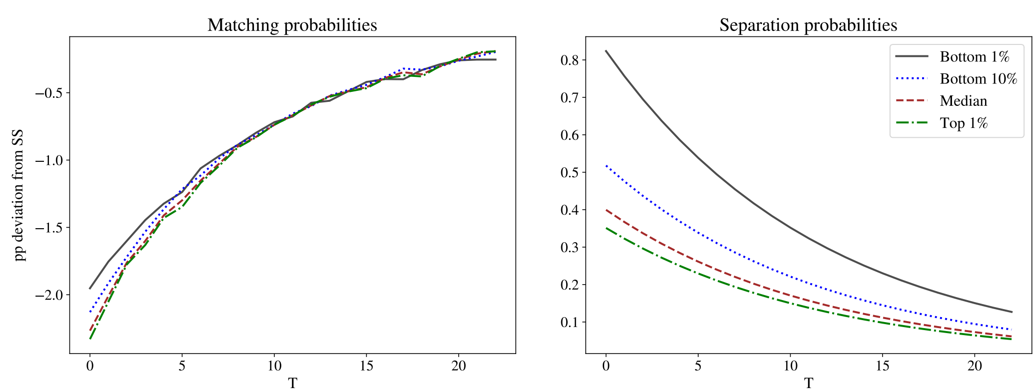
Figure 10: Workers' consumption response



Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. On the left panel, lines correspond to workers working in the same submarket but holding different wealth. The vertical lines correspond, from left to right, to the bottom 1%, 10%, the median, and the top 1% of the wealth distribution.

Positive wealth sorting implies a difference matching and separation probability across the wealth distribution. As shows in Figure 11, on average, matching decreases and separations increase. Effects are heterogeneous across the wealth distribution. The change in job search of bottom 1% unemployed allows them to mitigate the drop in their matching probability, and they experience the lowest decrease.

Figure 11: Matching and separation probability by asset



Note: All panels plot the IRFs to a 1% negative aggregate productivity shock. The different lines represent percentiles of the wealth distribution

However, the heterogeneous effect are stronger in the separation probabilities. On impact, the separation for the poorest 1% is twice as larger relative to the median worker. This difference stays present throughout the shock – which is not the case for the matching probability. Therefore, recessions heterogeneously affect workers mostly through an increase separation probability, and through the consumption response. Positive wealth sorting may thereby serve as a microfoundation of the empirically observed higher employment elasticity of asset-poor to aggregate output fluctuations.

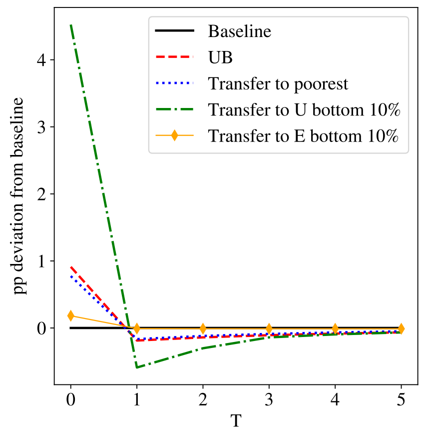
5.2 Targeted transfers and aggregate consumption response

I study the effect of four types of targeted transfers implemented when the aggregate productivity shock hits for aggregate consumption response. Figure 12 plots the differential consumption response under policy counterfactuals. In the baseline, the social planner distributes an untargeted transfer to all workers in the first period of the negative aggregate productivity shock.[25] Agents do not expect it ("MIT" policy shock) and they correctly do not expect the transfer to be reiterated. I set the total value of the transfer to be equal to one-fifth of average wages (this would correspond to a transfer of roughly $1000).

The first policy is a standard unemployment benefit and targets all unemployed workers (dashed line). The second is a policy targeting the asset-poor (dotted line), and for comparability with the first policy, I set the threshold such that the same amount of workers get the transfer. Then, I also implement policies targeting only the unemployed or employed asset-poor (dash-dotted line and diamond line, respectively). There, I divide the total transfer to the 10% poorest. By targeting the transfer to workers with on average lower assets, the aggregate consumption drop is mitigated in the first period by more than 4 percentage points. This is especially true for the transfers targeted at the asset-poor unemployed workers, as these are the group with the largest share of hand-to-mouth. The targeted transfer also allows them to look for better jobs, which depresses aggregate consumption in the following periods through a relatively larger share of workers who remain unemployed (matching probabilities are lower for good jobs). This backlash effect on consumption (0.5 percentage points) is strongest when targeting the unemployed poor, and does not happen when targeting the poorest employed workers.

[25] Here I do not impose the government to balance its budget for simplicity. For this reason, in this section I always compare counterfactuals holding the total amount of transfer fixed.

Figure 12: Consumption effect of targeted transfers



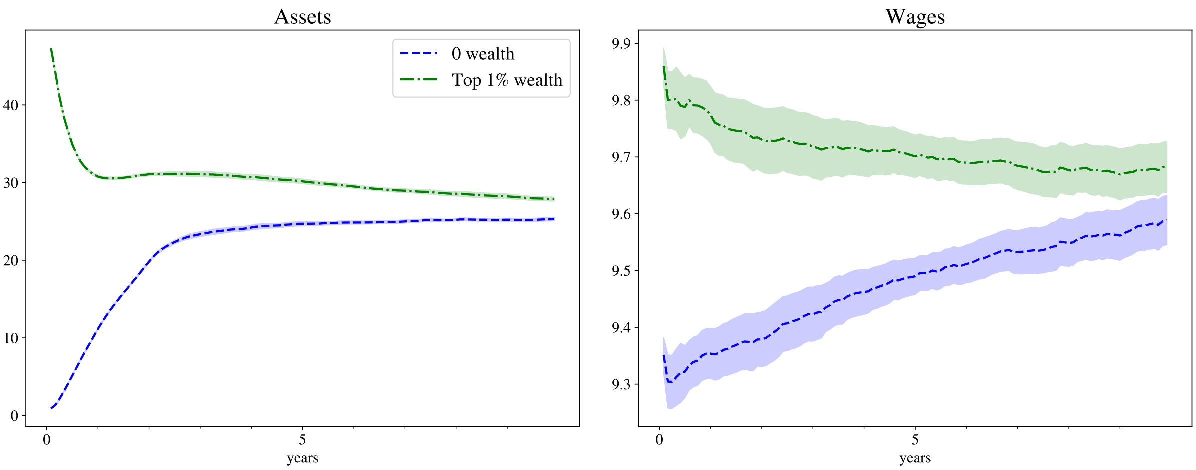
Note: All lines correspond to the same aggregate transfer, which amounts to one tenth of average equilibrium wages. The solid line represents the aggregate consumption response when the transfer is distributed to all agents (baseline). The other lines represent differences from this baseline. The dashed line distributes the transfer equally among unemployed; the dotted line to all workers at the bottom of the wealth distribution (the mass is chosen to be equal to the mass of unemployed); the dash-dotted line to the unemployed workers at the bottom 10% of the wealth distribution; and the diamond line to the employed worker at the bottom 10% of the wealth distribution.

5.3 Cyclical shocks have long term consequences

Whether the increase in inequality following an negative aggregate productivity shock is associated with long term consequences is an important policy question. In this model, cyclical fluctuations have long-lasting effects because of workers' assets accumulation and job search.

In Figure 13, I simulate two cohorts of 10'000 individuals, each starting either with the bottom or the top 1% wealth. I track them over time as aggregate productivity evolves according to different draws of the stochastic process. The left panel plots mean asset holdings, and the right panel plots mean wages; while the shaded areas represent 90% confidence interval (across aggregate productivity states). A striking result is that the initial asset position determine different asset and wages for at least the next 10 years. Comparing the poorest cohort with the richest, the right panel reveals that after 10 years, the cohort that started with the highest wealth continues to benefit from wages that are % higher. Wealth sorting leads asset-poor workers to accept lower wages, limiting their ability to accumulate savings during employment and thereby increasing their probability to apply to low-wage jobs if they become unemployed. In that sense, the presence of liquidity constraints in this setting generates a "poverty trap". Heterogeneous separation risk aggravates this phenomenon, because it disproportionately affects wealth-poor workers. Bad aggregate states intensify the poverty trap for wealth-poor unemployed workers, who are then driven to seek riskier, lower-wage jobs. It is worth mentioning that the model does not embed *permanent* inequalities; all heterogeneity in asset and wages are merely due to workers' initial condition in wealth and its interaction with labor market outcomes. Given the randomness of shock processes, average effects for the different cohorts converge to the same value in the long-run.

Figure 13: Average outcomes for cohort starting with different levels of assets



Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$. I then randomly simulate 20 different paths for aggregate productivity and follow each worker in the different cohorts. Lines represent the average worker, while shaded areas are the 90% confidence interval.

Next, I quantify the scarring effects of recessions along the wealth distribution by comparing two simulated aggregate states: a "recession" scenario, where the aggregate state is negative for the first two quarters (following the NBER definition) and then goes back to the unconditional mean, and a "tranquil" scenario, where the aggregate state is constant at the mean throughout. In Figure 14, I show the percentage difference in assets (left panel) and wages (right panel) for cohorts who began their unemployment spells in the "recession" scenario relative to a "tranquil" scenario. Moreover, I show how these differences vary between workers at the bottom and top of the wealth distribution (blue and green lines, respectively).

Figure 14: Long-term effects of recessions for cohort starting with different levels of assets

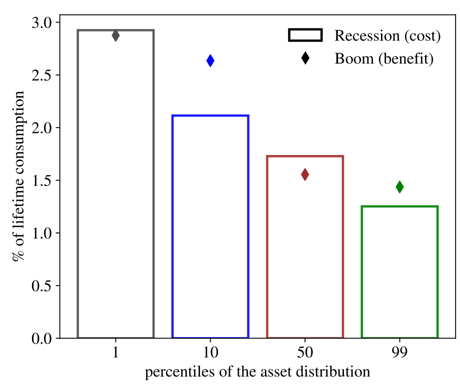
Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$. Lines represent the difference between starting the unemployment spell in a recession (2 quarters negative growth) versus a tranquil period, separately for each wealth cohort. After the first 2 quarters, the aggregate state is the same across simulations.

Starting the unemployment spell in a recession has long-lasting effect for all workers. Wages are around 4.5% lower on impact, and are still 1% lower after 5 years.[26] Asset accumulation is positive in the very beginning, reflecting precautionary savings motives, but then becomes and stays negative for all workers. Those suffering from larger negative effects are the wealth-poor, whose wages are between 0.5 and 1 percentage points lower compared to the wealthiest. Moreover, those with initially zero assets accumulate much less in the "recession" scenario, by as much as 8% in the first year. The wealthiest also start decumulating assets, though to a smaller extent. As a consequence, wealth inequality between these two groups increases in the first years.

Finally, I calculate the welfare cost of beginning a job spell in a recession versus a boom across the wealth distribution. The solid bars in Figure 15 represent the lifetime cost of starting a job search during a recession, computed in a similar fashion as Lucas (1987) – see Appendix F for more details. The poorest workers would sacrifice 3% of lifetime consumption to avoid the recession scenario. This is more than two times larger than the wealthiest workers, whose utility losses of starting in a recession are equivalent to 1.25% of lifetime consumption.

The diamonds show the benefits of starting in a boom respective to the tranquil scenario. The heterogeneity is slightly lower, mostly driven by the top 1%. In fact, these workers benefit more from a positive aggregate state as they suffer in a bad aggregate state – they would give up 1.5% of their lifetime consumption for the boom scenario. Instead, the benefits and symmetric to the cost of recession for the bottom 1%, while the 10^{th} percentile gains relatively more in booms than what they lose from recessions.

Figure 15: Scarring effect of recessions and welfare gains from booms



Note: Each cohort is of 10'000 workers who share the same level of assets in $t = 0$ (respectively 5^{th} and 95^{th} percentiles). I compute the welfare cost of starting in a boom or recession, compared to tranquil times. I follow each worker in the different cohorts, and show outcomes for the median worker in the cohort.

[26] The strong persistence is also due to the assumption of fixed wages throughout the employment duration. Wages therefore adjust to aggregate conditions only when the employed worker gets separated and finds a new job.

6 Conclusion

This paper explores the relationship between wealth, job search behavior, and labor income risk over the business cycle. First, I present suggestive evidence that asset-poor workers have a relatively larger employment elasticity to aggregate output fluctuations. I then rationalize this finding with a theory based on risk aversion, liquidity constraints, and labor market search. I show that in the presence of incomplete markets, risk-aversion drives agents to sort positively on the labor market, even though bad jobs are more likely to be destroyed. This implies that wealthier individuals can afford to be more selective in their job search, thereby obtaining better job matches with higher wages and higher security. During economic downturns, wealthier workers are able to cushion a potential job loss through savings, and their resulting job search implies more stable and higher income. Conversely, poor workers seek insurance and use the labor market to smooth their consumption. Wealth sorting is relatively stronger in bad aggregate states, and so are inequalities (in terms of wages and security). Positive wealth sorting generates a higher employment elasticity to aggregate shocks for low-wealth workers, in line with the empirical evidence. Moreover, it generates a "poverty trap", in which asset-poor workers struggle to accumulate assets due to short and low-wage employment contracts. Following a negative aggregate productivity shock, I show that a transfer targeted at the unemployed asset-poor is the most effective way to mitigate the drop in aggregate consumption but induces a small backlash in the next period due to more workers staying unemployed.

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Appendix

A Empirical analysis

A1 Panel Study of Income Dynamics

The PSID is the world's longest ongoing longitudinal household panel dataset. This survey was conducted in the U.S. yearly from 1968 to 1997, and biennially afterwards. In total, more than $82,000$ individuals (coming from around $9,500$ families) were interviewed over 50 years. The panel consists of different subsamples. The first is the so-called SRC sample, which is representative of the U.S. population. The SEO samples add an over-sample of low-income families. Together, the SRC and SEO are referred to as the Core PSID sample, which is the sample I work with. My main variables of interest are individual demographics, employment, income, and wealth.

Sample selection I restrict the PSID to the heads in the Core sample, that are aged between 25 and 62. Data on wealth is available only from 1999 onward, which restricts my sample year to 1999 to 2017. I discard individuals with inconsistent data, e.g., those having positive hourly labor income lower than half of the federal minimum wage.

Demographics Household heads are individuals whose sequence number is equal to 1 – spouses instead are denoted with a sequence number equal to 2. Until 2017, heads are always male unless the household does not feature a spouse. The education level is not reported consistently throughout the years. I compute a consistent variable for each respondent and year and then attribute the education to that individual that is the highest computed over all observed years.

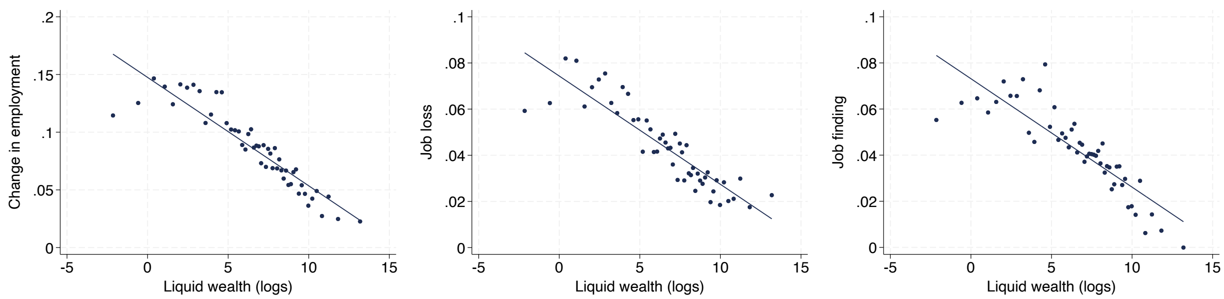
Employment and income The employment status is reported consistently after 1999, I can directly use the variable. I use the household head individual labor income, as reported in the data. It is defined as "Labor part of farm income and business income, wages, bonuses, overtime, commissions, professional practice, labor part of income from roomers and boarders or business income."

Wealth Wealth is reported at the household level since 1999. I use the total of savings and checks accounts as a measure of liquid wealth. Results are similar when using a constructed measure of liquid wealth that sums savings and checks accounts to stock holdings.

A2 Sensitivity analysis

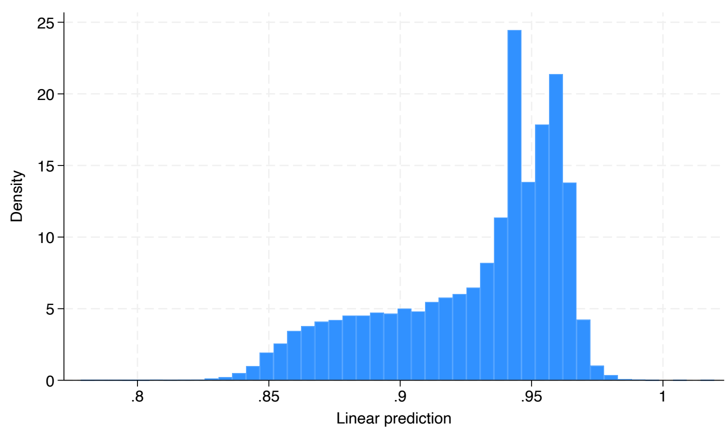
Figure A1: Employment and liquid wealth

(a) Change in employment **(b)** Job loss **(c)** Job finding



Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Dots represent mean of x and y values within 50 equally populated bins. The $x-$ axis represents log liquid wealth holdings; the $y-$axis is the mean change in employment status. All panels control for a quadratic in age interacted with socio-demographs and past income. Liquid wealth is measured as the amount in the savings accounts and stocks holdings.

Figure A2: Histogram of predicted individual employment status



Note: The $x-$axis represents a linear prediction on the individual employment status from Eq. (1). The $y-$axis shows the density.

I also perform a sensitivity analysis using GDP measures at the aggregate level – Y_t represents the U.S real GDP.

Columns (1) to (4) of Table A1 present the estimates as the model is gradually saturated. The results are similar to the baseline estimates, though the magnitudes are approximately doubled.

Figure A3 reproduces the decile figures for the state-level analysis. Again, we can see that the pattern is essentially unchanged.

Table A1: Liquid assets and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)
$\Delta \log Y_t$	0.61***			
	(0.084)			
a_{it-1}	-0.17***	-0.027**	-0.026**	-0.025**
	(0.014)	(0.011)	(0.011)	(0.011)
$a_{it-1} \times \Delta \log Y_t$	0.89***	0.77***	0.74***	0.68***
	(0.23)	(0.23)	(0.23)	(0.23)
FEs	No	Yes	Yes	Yes
Worker controls	No	No	Yes	Yes
Past income	No	No	No	Yes
R^2	0.056	0.47	0.47	0.47
N	32144	32144	32144	32144

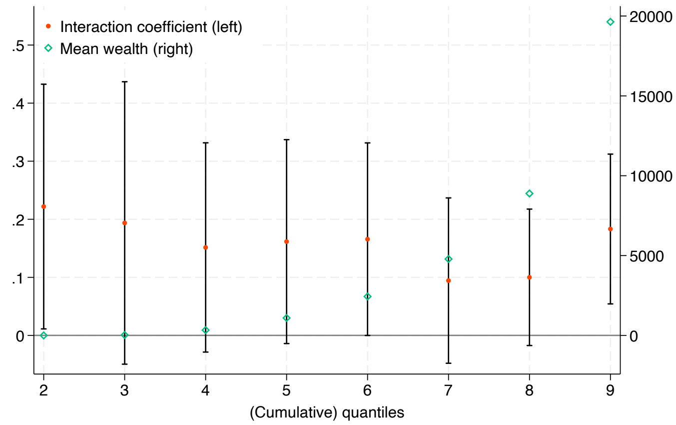
Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$, clustered at the state by year level. The outcome variable is the employment status of the household head. Liquid wealth is measured as the checks and savings account and stocks holdings. $\bar{a}$ is equal to one if the household's liquid wealth lies in the first quartile of that year's liquid wealth distribution. Fixed effects are at the individual and year level. Worker controls include a quadratic in age, and all the respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years). GDP is at the state level, and the last column disentangles expansions (positive growth) to recessions (negative growth).

Table A2: Liquid assets and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)
$a_{it-1} \times \Delta \log Y_{s,t}$	0.25**	0.25**	0.24*	0.24*
	(0.12)	(0.12)	(0.12)	(0.12)
Industry	No	Yes	No	Yes
Occupation	No	No	Yes	Yes
Industry $\times$ Occupation	No	No	No	Yes
R^2	0.37	0.37	0.37	0.37
N	30241	30241	30241	30241

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. Liquid wealth is measured as the checks and savings account. a_{it-1} is equal to one if the household's liquid wealth lies in the first quartile of that year's liquid wealth distribution. All columns feature fixed effects at the worker and year by state level, and the following controls: a quadratic in age, and all the respective interactions with education, gender, race, and past income. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years). The first column is the baseline regression.

Figure A3: Employment elasticity to state output fluctuations by liquid wealth deciles



Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$, clustered at the state level. The $x-$axis represents different thresholds for the dummy variable indicating "asset-poor" status. The left $y-$axis displays the coefficient of interest which is the interaction term between log-changes in States' GDP and liquid wealth-poor dummy, estimated in the baseline model of Eq. (1). The red dots plot the point estimates, and the bars indicate 95% confidence intervals. The diamonds on the right $y-$axis plot the average levels of liquid wealth for each income decile.

Figure A4: Permanent asset position and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)	(5)
$\Delta \log Y_t$	0.25^{***}				
	(0.042)				
$\bar{a}_i$	-0.18^{***}				
	(0.010)				
$\bar{a}_i \times \Delta \log Y_t$	0.12	0.19	0.20	0.21	0.095
	(0.13)	(0.18)	(0.18)	(0.18)	(0.39)
$\bar{a}_i \times \Delta \log Y_t \times \Delta \log Y_t^{>0}$					0.22
					(0.42)
FEs	No	Yes	Yes	Yes	Yes
Worker controls	No	No	Yes	Yes	Yes
Past income	No	No	No	Yes	Yes
R^2	0.064	0.46	0.46	0.46	0.46
N	32591	32150	32150	32150	32150

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. Liquid wealth is measured as the checks and savings account and stocks holdings. $\bar{a}$ is equal to one if the household's liquid wealth lies in the first quartile of the average liquid wealth distribution. Fixed effects are at the individual and year level. Worker controls include a quadratic in age, and all the respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years). GDP is at the state level, and the last column disentangles expansions (positive growth) to recessions (negative growth).

Table A3: Iliquid assets and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)	(5)
$\Delta \log Y_t$	0.26***				
	(0.039)				
a_{it-1}	-0.14***	-0.013*	-0.012*	-0.012*	-0.022
	(0.0096)	(0.0066)	(0.0066)	(0.0065)	(0.019)
$a_{it-1} \times \Delta \log Y_t$	0.25*	0.14	0.15	0.16	0.22
	(0.13)	(0.13)	(0.13)	(0.12)	(0.27)
$a_{it-1} \times \Delta \log Y_t \times \Delta \log Y_t^{>0}$					-0.34
					(0.36)
FEs	No	Yes	Yes	Yes	Yes
Worker controls	No	No	Yes	Yes	Yes
Past income	No	No	No	Yes	Yes
R^2	0.047	0.45	0.45	0.45	0.45
N	35961	35961	35961	35961	35961

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1, ^{**} p < 0.05, ^{***} p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. Iliquid wealth is measured as the sum of real estate, pensions savings, other assets, vehicles and farm business. a is equal to one if the household's illiquid wealth lies in the first quartile of that year's illiquid wealth distribution. Fixed effects are at the individual and year level. Worker controls include a quadratic in age, and all the respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years). GDP is at the state level, and the last column disentangles expansions (positive growth) to recessions (negative growth).

Table A4: H2M and individual employment elasticity to aggregate fluctuations

	(1)	(2)	(3)	(4)
$\Delta \log Y_t$	0.29***			
	(0.055)			
$H2M_{t-1}$	-0.058***	-0.0015	-0.0013	0.00047
	(0.0065)	(0.0073)	(0.0075)	(0.0073)
$H2M_{t-1} \times \Delta \log Y_t$	0.14	0.16**	0.17**	0.16**
	(0.087)	(0.078)	(0.077)	(0.077)
FEs	No	Yes	Yes	Yes
Worker controls	No	No	Yes	Yes
Past income	No	No	No	Yes
R^2	0.011	0.47	0.47	0.47
N	32144	32144	32144	32144

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1, ^{**} p < 0.05, ^{***} p < 0.01$, clustered at the state level. The outcome variable is the employment status of the household head. $H2M$ is a dummy equal to 1 if the household's net worth is lower than a sixth of their labor income, following Aguiar et al. (2024). Fixed effects include workers, and year by state. Worker controls include a quadratic in age, and all th respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years) and it is interacted with GDP.

Table A5: Liquid assets and individual change in employment status

	(1) Change	(2) Job loss	(3) Job finding
a_{it}	0.074***	0.036***	0.038***
	(0.0045)	(0.0034)	(0.0033)
R^2	0.067	0.024	0.057
N	36774	36774	36774

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Robust standard errors in parentheses; $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$. The outcome variable is the change in employment status of the household head. In column (1), the variable captures both job loss and job finding, while columns (2) and (3) only capture the former and the latter, respectively. Liquid wealth is measured as the checks and savings account. a_{it-1} is equal to one if the household's liquid wealth lies in the first quartile of that year's liquid wealth distribution. Regressions include year by state fixed effects and the following worker controls: a quadratic in age, and all the respective interactions with education, gender, race, and past income. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years) and it is additionally interacted with GDP.

Table A6: Liquid assets and individual change in employment status

	(1) Change	(2) Job loss	(3) Job finding
Cross-section			
a_{it-1}	0.074***	0.036***	0.038***
	(0.0045)	(0.0034)	(0.0033)
R^2	0.067	0.024	0.057
N	36774	36774	36774
Cyclical exposure			
$a_{it-1} \times \Delta \log Y_t$	0.28**	-0.22**	0.061
	(0.13)	(0.099)	(0.078)
R^2	0.016	0.025	0.057
N	36634	36634	36634

Note: Observations come from the Core PSID dataset. The sample covers years from 1999 to 2017. Standard errors in parentheses; $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$, clustered at the state level. The outcome variable is the change in employment status of the household head. Liquid wealth is measured as the checks and savings account. a_{it-1} is equal to one if the household's liquid wealth lies in the first quartile of that year's liquid wealth distribution. Fixed effects are year by state. Worker controls include a quadratic in age, and all the respective interactions with education, gender and race. Past income is the lagged labor income (if missing for the prior two years, it is the past income from the prior 4 years) and it is interacted with GDP.

B Proofs of two-period model

B1 Matching function

As Menzio and Shi (2011); Eeckhout and Sepahsalari (2024), I assume a CES matching function.

$$
\begin{aligned}
m(\theta) &= \theta(1+\theta^\gamma)^{\frac{-1}{\gamma}} \\
m'(\theta) &= (1+\theta^\gamma)^{\frac{-1}{\gamma}} - \theta^\gamma(1+\theta^\gamma)^{\frac{-1}{\gamma}-1} \\
m''(\theta) &= -(1+\theta^\gamma)^{\frac{-1}{\gamma}-1}\left[1+\gamma\theta^{\gamma-1}\right] + \left(\frac{-1}{\gamma}-1\right)\theta^{2\gamma-1}(1+\theta^\gamma)^{\frac{-1}{\gamma}-2}
\end{aligned}
$$

$$
\begin{aligned}
q(\theta) &= (1+\theta^\gamma)^{\frac{-1}{\gamma}} \\
q'(\theta) &= -\theta^{\gamma-1}(1+\theta^\gamma)^{\frac{-1}{\gamma}-1} \\
q''(\theta) &= -\theta^{\gamma-1}(1+\theta^\gamma)^{\frac{-1}{\gamma}-1}\theta^{-1}\left[\gamma-1-(\gamma+1)\theta^\gamma(1+\theta^\gamma)^{-1}\right]
\end{aligned}
$$

We can thus show that $(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta < 0$.

$$
\begin{aligned}
(q'(\theta)+\theta q''(\theta))q(\theta) - (q'(\theta))^2\theta &= \left(q'(\theta) + q'(\theta)\left[\gamma-1-(\gamma+1)\theta^\gamma(1+\theta^\gamma)^{-1}\right]\right)q(\theta) - (q'(\theta))^2\theta \\
&= q'(\theta)\left(1+\left[\gamma-1-(\gamma+1)\theta^\gamma(1+\theta^\gamma)^{-1}\right]\right)(1+\theta^\gamma)^{\frac{-1}{\gamma}} - (q'(\theta))^2\theta \\
&= q'(\theta)\left(\gamma(1+\theta^\gamma)^{\frac{-1}{\gamma}} - (\gamma+1)\theta^\gamma(1+\theta^\gamma)^{\frac{-1}{\gamma}-1}\right) - (q'(\theta))^2\theta \\
&= q'(\theta)\left(\gamma\left((1+\theta^\gamma)^{\frac{-1}{\gamma}} - \theta^\gamma(1+\theta^\gamma)^{\frac{-1}{\gamma}-1}\right) - \theta^\gamma(1+\theta^\gamma)^{\frac{-1}{\gamma}-1}\right) - (q'(\theta))^2\theta \\
&= q'(\theta)\left(\gamma m'(\theta) + \theta q'(\theta)\right) - (q'(\theta))^2\theta \\
&= \gamma q'(\theta)m'(\theta) < 0
\end{aligned}
$$

B2 Proposition 1 : positive wealth sorting

Proof. Let $\phi(a_2^*, \theta^*)$ be the maximand of $U(a_1, x)$

$$
\phi(a_2^*,\theta^*) = u(Ra_1 - a_2^*) + \beta\left[m(\theta^*)\Lambda(x)u\left(Ra_2^* + \bar{y} - \frac{\varphi(x)}{\beta q(\theta^*)\Lambda(x)}\right) + (1-m(\theta^*)\Lambda(x))u(Ra_2^*)\right]
$$

By the implicit function theorem, we have

$$
\begin{aligned}
\frac{\partial a_{t+1}}{\partial x_t} &= -\frac{\begin{vmatrix} \phi_{a_{t+1}x_t} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t x_t} & \phi_{\theta_t\theta_t} \end{vmatrix}}{|H|} = -\frac{\phi_{a_{t+1}x_t}\phi_{\theta_t\theta_t} - \phi_{\theta_t x_t}\phi_{a_{t+1}\theta_t}}{|H|} \\
\frac{\partial \theta_t}{\partial x_t} &= -\frac{\begin{vmatrix} \phi_{\theta_t x_t} & \phi_{\theta_t a_{t+1}} \\ \phi_{a_{t+1}x_t} & \phi_{a_{t+1}a_{t+1}} \end{vmatrix}}{|H|} = -\frac{\phi_{\theta_t x_t}\phi_{a_{t+1}a_{t+1}} - \phi_{\theta_t a_{t+1}}\phi_{a_{t+1}x_t}}{|H|}
\end{aligned}
$$

Where $|H| > 0$ is the determinant of the Hessian:

$$
H = \begin{bmatrix} \phi_{a_{t+1}a_{t+1}} & \phi_{a_{t+1}\theta_t} \\ \phi_{\theta_t a_{t+1}} & \phi_{\theta_t\theta_t} \end{bmatrix} \tag{16}
$$

We first derive the partial derivatives of the surplus function $U(a_2, x)$. Taking the envelope theorem into account, we have that

$$U_{a_1} = Ru'(Ra_1 - a_2)$$
$$U_{a_1 x} = -Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial x}$$

The condition for PAM is

$$\begin{aligned} U_{a_1 x} > 0 \quad &\Leftrightarrow \quad -Ru''(Ra_1 - a_2)\frac{\partial a_2}{\partial x} > 0 \\ \frac{\partial a_2}{\partial x} > 0 \quad &\Leftrightarrow \quad -\frac{\phi_{a_{t+1}x_t}\phi_{\theta_t\theta_t} - \phi_{\theta_t x_t}\phi_{a_{t+1}\theta_t}}{|H|} > 0 \end{aligned}$$

$$\phi_{a_2 x}\ \phi_{\theta\theta} < \phi_{\theta x}\ \phi_{a_2\theta} \tag{17}$$

Where we used the implicit function theorem in the second line. Eq. (17) is the condition for PAM, and we now compute the relevant partial derivatives

$$\begin{aligned} \phi_{a_2,x} &= \beta R m(\theta^*)\left[\Lambda'(x)(u'(c^*_{e,2}) - u'(c^*_{u,2})) + \Lambda(x)u''(c^*_{e,2})\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\right] \\ &= \beta R m(\theta^*)\left[\Lambda'(x)(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \Lambda'(x)u''(c^*_{e,2})\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c^*_{e,2})}\right] \\ &= \beta R m(\theta^*)\Lambda'(x)\left[(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}(u(c^*_{e,2}) - u(c^*_{u,2}))\right] \end{aligned} \tag{18}$$

$$\begin{aligned} \phi_{\theta,a_2} &= \beta R\Lambda(x)\left(m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)}\right) \\ &= \beta R\Lambda(x)m'(\theta)\left(\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] - u''(c^*_{e,2})\frac{(u(c^*_{e,2}) - u(c^*_{u,2}))}{u'(c^*_{e,2})}\right) \end{aligned} \tag{19}$$

With DARA utility function, it holds that $\frac{u'(c^*_{e,2})-u'(Ra_2)}{u(c^*_{e,2})-u(Ra_2)} < \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}$ (Eeckhout and Sepahsalari, 2024). Therefore, Eq. (18) and Eq. (19) are negative, i.e., $\phi_{a_2,x} < 0$ and $\phi_{\theta,a_2} < 0$.

$$\begin{aligned} \phi_{\theta,\theta} = \beta\Lambda(x)\Bigg[&m''(\theta^*)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] + m'(\theta^*)u'(c^*_{e,2})\left(\frac{\varphi(x)q'(\theta)}{\beta\Lambda(x)(q(\theta))^2}\right) \\ &+ u''(c^*_{e,2})\left(\frac{\varphi(x)q'(\theta)}{\beta\Lambda(x)(q(\theta))^2}\right)\frac{\varphi'(x)\theta q'(\theta)}{\beta q(\theta)\Lambda(x)} + u'(c^*_{e,2})\frac{\varphi(x)}{\beta\Lambda(x)}\left(\frac{(q'(\theta) + \theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{(q(\theta))^2}\right)\Bigg] \end{aligned}$$

$$= \beta\Lambda(x)\Bigg[m''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] \tag{20}$$

$$+ \frac{\varphi(x)q'(\theta)}{\beta\Lambda(x)(q(\theta))^2}\left(m'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta) + \theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right)\Bigg] \tag{21}$$

Under CES matching function, Eq. (21) is negative as all terms are negative. We assume that it is the case.

$$
\begin{aligned}
\phi_{\theta,x} = \beta \Bigg(& \Lambda'(x)\left[m'(\theta)(u(c^*_{e,2}) - u(c^*_{u,2})) + u'(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)}\right] \\
& + \Lambda(x)m'(\theta)u'(c^*_{e,2})\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right) \\
& + \Lambda(x)u''(c^*_{e,2})\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} + \Lambda(x)u'(c_{e,2})\frac{\theta q'(\theta)}{\beta q(\theta)}\left(\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{(\Lambda(x))^2}\right)\Bigg) \\
= \beta & \left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\left(\Lambda(x)m'(\theta)u'(c^*_{e,2}) + \Lambda(x)u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} - \Lambda(x)u'(c_{e,2})\theta q'(\theta)\right) \\
= \beta & \left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\left(\Lambda(x)u'(c^*_{e,2})[m'(\theta) - \theta q'(\theta)] + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)}\right)
\end{aligned} \tag{22}
$$

All terms in the second bracket are positive. A necessary condition for PAM is $\phi_{\theta,x} < 0$, and therefore

$$
\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2} > 0 \qquad \Leftrightarrow \qquad \frac{\varphi'(x)}{\varphi(x)} > \frac{\Lambda'(x)}{\Lambda(x)} \tag{23}
$$

To understand whether the condition is also sufficient, I now put terms together and write down the full condition for PAM

$$
\begin{aligned}
& \beta R m(\theta^*)\Lambda'(x)\left[(u'(c^*_{e,2}) - u'(c^*_{u,2})) - \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}(u(c^*_{e,2}) - u(c^*_{u,2}))\right] \\
& \beta\Lambda(x)\Bigg[m''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] \\
& + \frac{\varphi(x)q'(\theta)}{\beta\Lambda(x)(q(\theta))^2}\left(m'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta) + \theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right)\Bigg] \\
& < \beta\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)}\left(u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{\Lambda'(x)}{\Lambda(x)}\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right) \\
& \quad \beta R\Lambda(x)m'(\theta)\left(\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] - u''(c^*_{e,2})\frac{(u(c^*_{e,2}) - u(c^*_{u,2}))}{u'(c^*_{e,2})}\right)
\end{aligned}
$$

With DARA utility functions, we know that the last parenthesis is negative. Simplifying on both sides yields

$$
\begin{aligned}
& \Lambda'(x)m''(\theta)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] \\
& + \frac{\Lambda'(x)}{\Lambda(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(m'(\theta)u'(c^*_{e,2}) + u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} + u'(c^*_{e,2})\frac{(q'(\theta) + \theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{q'(\theta)}\right) \\
& > \frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{\Lambda'(x)}{\Lambda(x)}\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right)m'(\theta) \\
& \quad = \frac{\Lambda'(x)}{\Lambda(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\left(\frac{\Lambda(x)}{\Lambda'(x)}u'(c^*_{e,2})\frac{\varphi'(x)}{\varphi(x)} - u''(c^*_{e,2})\frac{u(c^*_{e,2}) - u(c^*_{u,2})}{u'(c_{e,2^*})}\right)m'(\theta)
\end{aligned}
$$

Where in the last line we simply extended the RHS so that we can then take the difference

$$\Lambda'(x)m''(\theta)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]+\frac{\Lambda'(x)}{\Lambda(x)}\frac{\varphi(x)q'(\theta)}{\beta(q(\theta))^2}\Bigg(m'(\theta)u'(c^*_{e,2})\left(1-\frac{\Lambda(x)}{\Lambda'(x)}\frac{\varphi'(x)}{\varphi(x)}\right)$$
$$+u'(c^*_{e,2})\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)}+u''(c^*_{e,2})\left(\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)}+m'(\theta)\frac{u(c^*_{e,2})-u(c^*_{u,2})}{u'(c_{e,2^*})}\right)\Bigg)>0$$

The last term in parentheses is equal to zero. Then, we can write (still using Eq. (26))

$$\Lambda'(x)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]\left[m''(\theta)-\frac{m'(\theta)}{m(\theta)}\left(m'(\theta)\left(1-\frac{\Lambda(x)}{\Lambda'(x)}\frac{\varphi'(x)}{\varphi(x)}\right)+\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)}\right]>0$$

$$\Lambda'(x)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]\left[m''(\theta)-\frac{(m'(\theta))^2}{m(\theta)}\left(1-\frac{\Lambda(x)}{\Lambda'(x)}\frac{\varphi'(x)}{\varphi(x)}+\frac{(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)m'(\theta)}\right)\right]>0$$

The sufficient condition is thus

$$\frac{\Lambda(x)}{\Lambda'(x)}\frac{\varphi'(x)}{\varphi(x)}>\frac{q'(\theta)m'(\theta)+(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)m'(\theta)}$$
$$\frac{\varphi'(x)}{\varphi(x)}>\frac{\Lambda'(x)}{\Lambda(x)}\left[\frac{q'(\theta)m'(\theta)+(q'(\theta)+\theta q''(\theta))q(\theta)-(q'(\theta))^2\theta}{q'(\theta)m'(\theta)}\right]$$

With CES matching function B1, this becomes

$$\frac{\varphi'(x)}{\varphi(x)}>\frac{\Lambda'(x)}{\Lambda(x)}\left(1+\gamma\right)$$

■

Uniqueness follows from the negative-semidefinite Hessian matrix (16).

Whether matching probability is cyclical can be verified by looking at the "strength" of sorting in different aggregate states. Assume that the matching function M features a state-dependent matching efficiency, $M(U,V)=A(z)\left[U^\gamma+V^\gamma\right]^{\frac{1}{\gamma}}$, such that in bad times (z^L) average matching efficiency is lower. Since A enters as a multiplicative term, it gets carried through in all derivatives of the job filling and job finding functions. I adapt the equation of sorting to include the average matching efficiency.

$$\Lambda'(x)A(z^L)m''(\theta)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]+\frac{\Lambda'(x)}{\Lambda(x)}\frac{\varphi(x)q'(\theta)m'(\theta)}{\beta(q(\theta))^2}u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)A(z^L)m'(\theta)}\lesseqgtr$$
$$\Lambda'(x)A(z^H)m''(\theta)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]+\frac{\Lambda'(x)}{\Lambda(x)}\frac{\varphi(x)q'(\theta)m'(\theta)}{\beta(q(\theta))^2}u''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)A(z^H)m'(\theta)}$$

$$[A(z^L)-A(z^H)]\Lambda'(x)m''(\theta)\left[u\left(c^*_{e,2}\right)-u(c^*_{u,2})\right]+\frac{\Lambda'(x)}{(\Lambda(x))^2}\frac{(\varphi(x))^2\theta(q'(\theta))^2}{\beta^2(q(\theta))^3}u''(c^*_{e,2})\left[\frac{1}{A(z^L)}-\frac{1}{A(z^H)}\right]\lesseqgtr 0$$

Which is indefinite because the LHS is positive while the RHS is negative.

C Block recursive equilibrium

The proof follows Karahan and Rhee (2019) which is an extension of Menzio and Shi (2010, 2011), and proceeds in two steps. First, we show that there exists a value function for firms as well as market tightness functions that depend on the aggregate state of the economy only through exogenous shocks, and not through any endogenous distributions. This implies that we can reduce the state space for firms to only the exogenous shocks. In the baseline model without aggregate uncertainty, this means effectively that the value functions for firms and the market tightness do not depend on any aggregate state.
Second, we collapse the employed and unemployed problems and show that the overall household problem is a contraction. If the rental rate (which is exogenous) and the tightness functions are independent of the endogenous asset distribution, we show that the functional equation maps the set of functions that do not depend on the endogenous distribution into itself. Hence, the firm value function, market tightness functions and household problem constitute a block-recursive equilibrium.

C1 Existence

i) Let $\mathcal{J}(\mathcal{X} \times \mathcal{W} \times \mathcal{Z})$ be the set of continuous and bounded firm's value functions: $J(\mathcal{X} \times \mathcal{W} \times \mathcal{Z}) \to \mathbb{R}$, and denote $\mathbb{T}_j$ as an operator associated with the firm's value function (Eq. (13)).The operator $\mathbb{T}_j$ maps $\mathcal{J}$ into itself; applying Blackwell's conditions it is straightforward to show that $\mathbb{T}_j : \mathcal{J} \to \mathcal{J}$ is a contraction. Let $J^* \in \mathcal{J}$ be the fixed point of $\mathbb{T}_j$. J^* is a continuous, bounded function that does not depend on any endogenous aggregate state.

Then, we can rewrite the market tightness functions for open markets, from Eq. (14), as

$$\theta^*(x, w; z) = q^{-1}\left(\frac{\varphi(x)}{\mathbb{E}_t[J^*(x, w; z)]}\right)$$

ii) Denote the set of functions $\mathcal{R} : \{0,1\} \times \mathcal{A} \times \mathcal{W} \times \mathcal{Z} \to \mathbb{R}$. Let e be an indicator of the employment status ($e = 1$ if the worker is employed), and let $R : \{0,1\} \times \mathcal{A} \times \mathcal{W} \times \mathcal{Z}$ denote the combined households' value functions:

$$R(e = 0, a, w; \Gamma) = U(a; \Gamma)$$
$$R(e = 1, a, w; \Gamma) = E(a, w; \Gamma)$$

If $R \in \mathcal{R}$, then the employed and unemployed value functions are also only a function of z and not Ψ. Let $\mathbb{T}_r$ be an operator such that

$$\begin{aligned}(\mathbb{T}_r R)(e, a, x, w; \Gamma) = (1-e)\Big\{&\max_{a',x,w} u(c_u) + \beta\mathbb{E}\big[m(\theta(x, w; z))R(e' = 1, a', x, w; \Gamma') \\ &+ (1 - m(\theta(x, w; z)))R(e' = 0, a'; \Gamma')\big]\Big\} \\ &+ e\left\{\max_{a'} \quad u(c_e) + \beta\mathbb{E}\left[\lambda(x, z)R(e' = 0, a'; \Gamma') + (1 - \lambda(x, z))R(e' = 1, a', w'; \Gamma')\right]\right\} \\ \text{s.t.} \quad & c_e + a' = Ra + w, \quad c_u + a' = Ra + b, \qquad a' \geq \underline{a}\end{aligned}$$

Where we used the fact that market tightness does not depend on Γ (point i)). Assuming that $u(\cdot)$ is bounded and continuous, $\mathcal{R}$ is a set of bounded and continuous functions. We can show that $\mathbb{T}_r : \mathcal{R} \to \mathcal{R}$, so that Blackwell's sufficiency conditions imply that $\mathbb{T}_r$ is a contraction. Being the space complete and non-empty, there exists a unique fixed point $R^* \in \mathcal{R}$ such that $\mathbb{T}_r(R^* \in \mathcal{R}) = R^*$. R^* is the solution to the household problem, and does not depend on any aggregate state.

C2 Uniqueness

The uniqueness follows from the strict concavity of value functions. Given the assumptions on $u(\cdot)$, $R(e' = 1, a', x, w; \Gamma')$ is strictly concave in w, while $R(e' = 0, a'; \Gamma')$ is constant in w. With

C3 Discussion

Under some different modeling assumptions, block recursivity breaks. Here are some examples.

1. If firms observe workers' wealth and post optimal contracts;
2. with on-the-job search;
3. if the interest rate is endogenous.

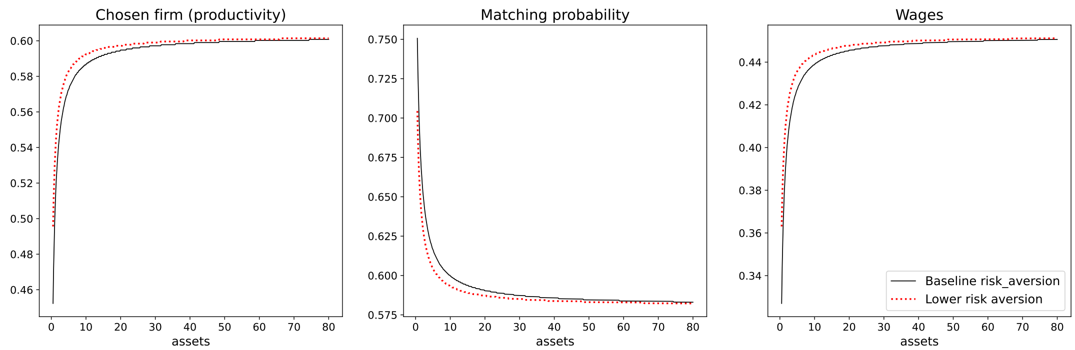
First, if firms would observe workers' wealth, intuitively they would post contracts to attract specific worker types; hence, the distribution of workers over asset and employment states would not be independent of their value function. However, it is in reality unlikely that the employer observes the assets of applicants. Second, on-the-job search would create differential incentives to search across the wealth distribution, and again firms would post contracts to attract targeted workers. In this case as well, firms' value functions would not only depend on the aggregate state through aggregate productivity. This assumption is a restriction on the equilibrium, as it is an established fact that workers search on the job and that job-to-job transitions account for a lot of labor market flows (see e.g., Faberman et al., 2022). Third, if the interest rate is endogenous it directly depends on the distribution of workers across assets. Interest rates influence the trade-off of labor market search – differently across the asset distribution – and therefore value functions of workers directly depend on the distribution of agents over states.

D Additional figures and tables

Table D7: Functional forms

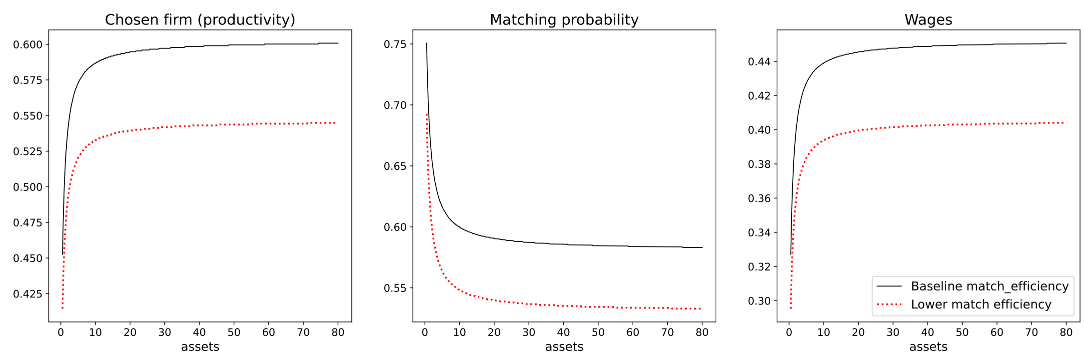
Matching function (CES)	$m(\theta) = \theta(1 + \theta^\gamma)^{\frac{-1}{\gamma}}$
Utility function	$\frac{c^{1-\sigma}}{1-\sigma}$
Separation function	$\bar{\lambda} \exp(\eta^{\lambda_z}(z-1)) \exp(\eta^{\lambda_x}(x-\bar{x}))$
Vacancy cost function	$\bar{\varphi} \exp(\eta^{\varphi_z}(z-1)) \exp(\eta^{\varphi_x}(x-\bar{x}))$
Production function	$x - w$

Figure D5: Equilibrium allocations with different risk aversion parameters



Notes: Baseline risk aversion: $\nu = 2$, counterfactual risk aversion: $\nu = 1.3$.

Figure D6: Equilibrium allocations with different matching friction parameters



Notes: Baseline matching efficiency: $\gamma = 0.41$, counterfactual matching efficiency: $\gamma = 0.36$.

Figure D7: Consumption policy functions

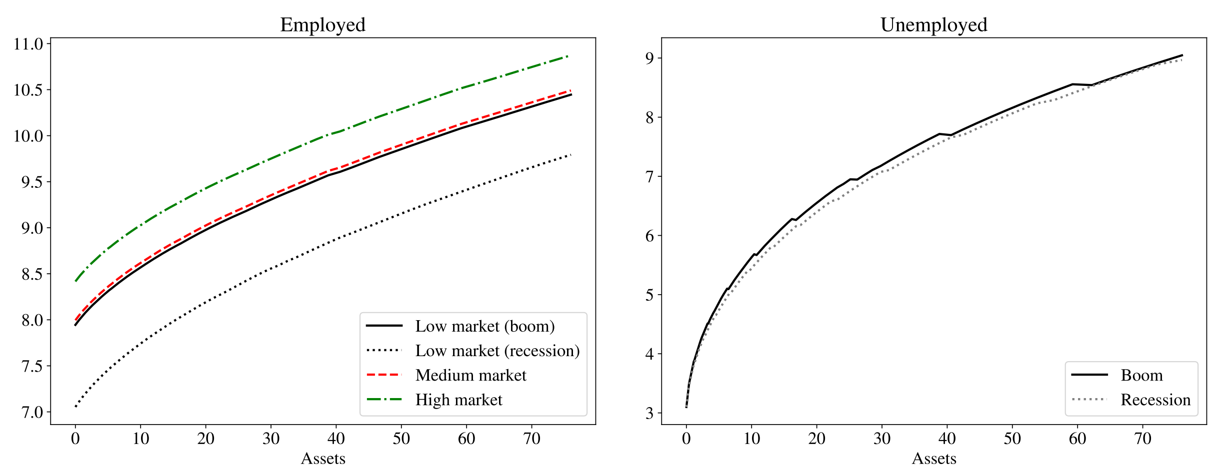
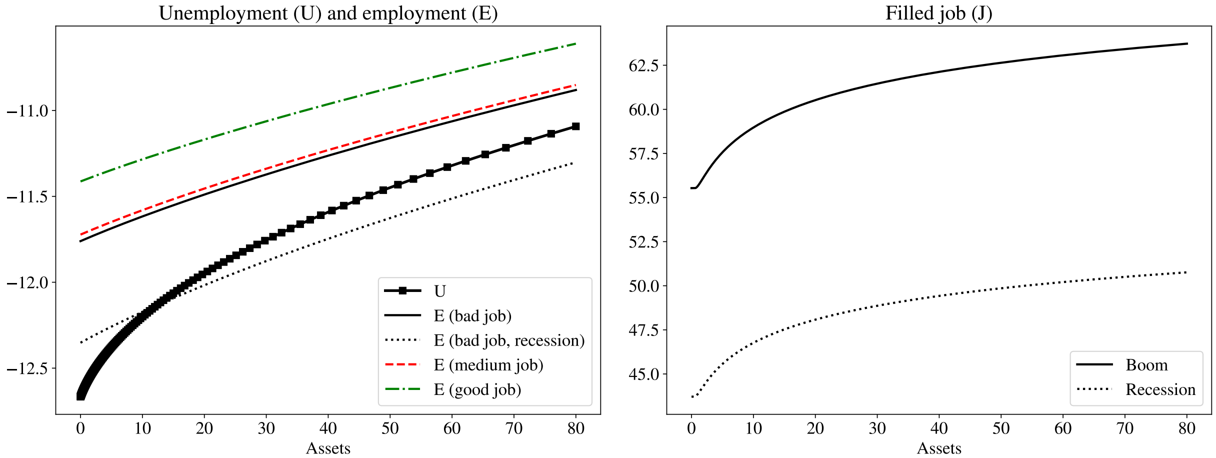


Figure D8: Workers' and firms' value functions



E Solution method

With Block Recursivity (BR), the solution method is greatly simplified, as one can solve the labor market problem orthogonal to the distribution of workers across employment states. In fact, the latter is not considered by agents in their optimal decision making.
In the baseline calibration, I use 90 grid points for assets, wages, and productivity, and 5 aggregate states of productivity.
The algorithm works as follows.

1. Given $V(x) = 0$, find the fix point of $J(x, w, z)$ using VFI on Eq. (13).
2. Compute the market tightness according to Eq. (14). Take a guess on $U^0(a, z)$, $E^0(a, x, w, z)$, $a_0^u(a, z)$, $a_0^e(a, x, w, z)$.
3. Take a guess for the tax rate τ. Solve for the workers' problem.
   (a) Apply policy function iteration (PFI) procedure on the employed workers' Euler equation.
      i. Fix a submarket (x, w). Write the employed workers' EE using the asset policy function initial guesses $a_0^u(a, z), a_0^e(a, z)$

$$u'\left(Ra + w - h^e(a, z)\right) = \beta R\, \mathbb{E}_{z'} \Bigg\{ \lambda(x, z) u'\left(Ra_0^e(a, z) + b - a_0^u\left(a_0^e(a, z), z'\right)\right) \\ + (1 - \lambda(x, z)) u'\left(Ra_0^e(a, z) + w - a_0^e\left((a_0^e(a, z), z'\right)\right) \Bigg\}$$

      Where we interpolate a_0^u and a_0^e on the value of assets given by $a_0^e(a, z)$.
      The new update for the asset policy function is constructed as

$$a_1^e(a, z) = \begin{cases} h^e(a, z) & \text{if } h^e(a, z) \geq \underline{a} \\ 0 & \text{otherwise} \end{cases}$$

      ii. Update on the policy function a^e in a next iteration k, until $a_k^e - a_{k-1}^e <$ tol. Denote the fix-point policy function for a given guess on the unemployed policy function as $a^e(a, z)$.

iii. Loop over all submarkets and retrieve the consumption policy function as $c_e(a,x,w,z) = Ra + w - a^e(a,x,w,z)$.

(b) Apply PFI procedure on the unemployed workers' Euler equation.

i. Fix a submarket (x,w), construct the RHS of the EE using the asset policy function guesses $a_0^u(a,z), a^e(a,x,w,z)$

$$u'\left(Ra + b - h^u(a,z)\right) = \beta R\, \mathbb{E}_{z'} \Big\{ m(\theta(x,w,z))u'\left(Ra_0^u(a,z) + w - a^e\left(a_0^u(a,z), x, w, z'\right)\right) \\ + (1 - m(\theta(x,w,z)))u'\left(Ra_0^u(a,z) + b - a_0^u\left(a_0^u(a,z), z'\right)\right) \Big\}$$

Where we interpolate a^e and a_0^u on the value of assets given by $a_0^u(a,z)$.
The new update for the asset policy function is constructed as

$$a_1^u(a,z) = \begin{cases} h^u(a,z) & \text{if } h^u(a,z) \geq \underline{a} \\ 0 & \text{otherwise} \end{cases}$$

ii. Update on the policy function a^u in a next iteration k, until $a_k^u - a_{k-1}^u <$ tol. Denote the fix-point policy function for a given guess on the unemployed policy function as $a^u(a,z)$.

iii. Loop over all submarkets and retrieve the consumption policy function as $c_u(a,x,w,z) = Ra + b - a^u(a,x,w,z)$.

(c) Apply value function iteration (VFI) to find the fixed point of the workers' VF.

i. Solve for the employed VF given the policy functions a^u and a^e.

A. Compute the RHS using policy function from above and the guesses on the workers' VFs.

$$E^1(a,x,w;z) = u(c_e(a,x,w,z)) + \beta \mathbb{E}_{z'} \Big[\lambda(x,z) U^0\Big(a^e(a,x,w,z); z'\Big) \\ + (1 - \lambda(x,z)) E^0\Big(a^e(a,x,w,z), x, w; z'\Big) \Big]$$

We need to interpolate U_0 and E_0 on the next value of assets given by the policy function a^e.

B. Update the VF until $E^k - E^{k-1} <$ tol.

ii. Apply VFI to find the fixed point of the unemployed worker' VF.

A. Compute the RHS using the policy function evaluated for all possible labor market choices, and the new guess for the employed VF.

$$U^1(a;z) = \max_{\tilde{x},\tilde{w}} u(c_u(a,\tilde{x},\tilde{w},z)) + \beta \mathbb{E}_{z'} \Big[m(\theta(\tilde{x},\tilde{w};z)) E^1\Big(a^u(a,\tilde{x},\tilde{w},z), \tilde{x}, \tilde{w}; z'\Big) \\ (1 - m(\theta(\tilde{x},\tilde{w};z))) U^0\Big(a^u(a,\tilde{x},\tilde{w},z); z'\Big) \Big]$$

The relationship between tightness and wages is pinned down by combining Eq. (13)

and (12).

B. Update the VF until $U^k - U^{k-1} < \text{tol}$. For every (a, z), store the submarket policy function $\vartheta(x^*, w^*)$.

(d) Verify that $c_u^1(a, z) = c_u^1(a, x^*, w^*, z)$. If not, update the guess $c_u^0(a, z) = c_u^1(a, x^*, w^*, z)$, as well as value functions and other policy functions, and go back to point 3.

4. Verify that the government budget constraint holds. If not, say taxes are too low (high) to finance unemployment benefits, increase (decrease) the tax rate and go back to 3.

F Welfare costs

Similarly to Lucas (1987), I compute the welfare gain from an alternative model, as solving the following

$$\sum_{t=0}^{T} \beta^t \frac{\left(c_t(1-\delta)\right)^{1-\sigma}}{1-\sigma} = \sum_{t=0}^{T} \beta^t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} \tag{24}$$

, where $\tilde{c}_t$ is the consumption in the alternative model. In these computations, I solve the model under a given path of aggregate productivity. Then, the welfare cost is

$$\begin{aligned}
\sum_{t=0}^{T} \beta^t \left(c_t(1-\delta)\right)^{1-\sigma} &= \sum_{t=0}^{T} \beta^t \tilde{c}_t^{1-\sigma} \\
(1-\delta)^{1-\sigma} \sum_{t=0}^{T} \beta^t (c_t)^{1-\sigma} &= \sum_{t=0}^{T} \beta^t \tilde{c}_t^{1-\sigma} \\
(1-\delta)^{1-\sigma} &= \frac{\sum_{t=0}^{T} \beta^t \tilde{c}_t^{1-\sigma}}{\sum_{t=0}^{T} \beta^t (c_t)^{1-\sigma}} \\
\delta &= 1 - \left(\frac{\sum_{t=0}^{T} \beta^t \tilde{c}_t^{1-\sigma}}{\sum_{t=0}^{T} \beta^t (c_t)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

G Alternative two-period models

G1 Model where aggregate productivity affects continuation value functions

Let $\phi(a_2^*, \theta^*)$ be the maximand of $U(a_1, x)$

$$\phi(a_2^*, \theta^*) = u(Ra_1 - a_2^*) + \beta\left[m(\theta^*)\mathbb{E}_z\left[(1-\lambda(x,z))u\left(Ra_2^* + z + \bar{y} - \frac{\varphi(x)}{\beta q(\theta^*)\Lambda(x)}\right)\right] + (1 - m(\theta^*)\Lambda(x))u(Ra_2^*)\right]$$

Let me simplify notation and define $\tilde{u}(c) = \mathbb{E}_z\left[(1-\lambda(x,z))u(c,z)\right]$, and $\bar{u}(c) = \mathbb{E}_z\left[(1-\lambda_x(x,z))u(c,z)\right]$.

The FOCs w.r.t to a_2 and θ become:

$$-u'(c_1^*) + \beta R\left[m(\theta^*)\tilde{u}'\left(c_{e,2}^*\right) + (1 - m(\theta^*)\Lambda(x))u'(c_{u,2}^*)\right] = 0 \tag{25}$$

$$\beta\left[m'(\theta^*)\left(\tilde{u}\left(c_{e,2}^*\right) - \tilde{u}(c_{u,2}^*)\right) + \tilde{u}'\left(c_{e,2}^*\right)\frac{\varphi(x)\theta^* q'(\theta^*)}{\beta q(\theta^*)\Lambda(x)}\right] = 0 \tag{26}$$

$$\beta m(\theta^*)\left[\left(\bar{u}(c_{e,2}^*) - \bar{u}(c_{u,2}^*)\right) + \tilde{u}'(c_{e,2^*})\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta^*)(\Lambda(x))^2}\right)\right] = 0$$

$$1 = \frac{-\tilde{u}'(c_{e,2}^*)}{\bar{u}(c_{e,2}^*) - \bar{u}(c_{u,2}^*)}\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta^*)(\Lambda(x))^2}\right) \tag{27}$$

$\phi_{a_2 x}\ \phi_{\theta\theta} < \phi_{\theta x}\ \phi_{a_2\theta}$

The partial derivatives become

$$\begin{aligned}
\phi_{\theta,x} = \beta\Bigg(&\left[m'(\theta)(\bar{u}(c_{e,2}^*) - \bar{u}(c_{u,2}^*)) + \tilde{u}'(c_{e,2}^*)\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\right] \\
&+ \tilde{u}''(c_{e,2}^*)\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} + \tilde{u}'(c_{e,2})\frac{\theta q'(\theta)}{\beta q(\theta)}\left(\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{(\Lambda(x))^2}\right)\Bigg) \\
= \beta&\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\left(\tilde{u}'(c_{e,2}^*)[m'(\theta) + 1] + \tilde{u}''(c_{e,2}^*)\frac{\varphi(x)\theta q'(\theta)}{\beta\Lambda(x)q(\theta)} - \tilde{u}'(c_{e,2})\theta q'(\theta)\right) \\
= \beta&\left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2}\right)\left(\tilde{u}'(c_{e,2}^*)[m'(\theta) + 1 - \theta q'(\theta)] + \tilde{u}''(c_{e,2}^*)\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)}\right)
\end{aligned} \tag{28}$$

The second condition for positive wealth sorting is thus unchanged (first bracket). Moreover, as before under CES matching function, $\phi_{\theta,\theta} < 0$. We assume that it is the case. Therefore, positive sorting requires that

$\phi_{a_2,x}$, $\phi_{\theta,a_2} < 0$. It holds

$$\begin{aligned}\phi_{a_2,x} &= \beta R m(\theta^*) \left[(\bar{u}'(c^*_{e,2}) - \bar{u}'(c^*_{u,2})) + \tilde{u}''(c^*_{e,2}) \left(-\frac{\varphi'(x)\Lambda(x) - \varphi(x)\Lambda'(x)}{\beta q(\theta)(\Lambda(x))^2} \right) \right] \\ &= \beta R m(\theta^*) \left[(\bar{u}'(c^*_{e,2}) - \bar{u}'(c^*_{u,2})) - \tilde{u}''(c^*_{e,2}) \frac{\bar{u}(c^*_{e,2}) - \bar{u}(c^*_{u,2})}{\tilde{u}'(c^*_{e,2})} \right]\end{aligned} \tag{29}$$

$$\begin{aligned}\phi_{\theta,a_2} &= \beta R \left(m'(\theta) \left[\tilde{u}'(c^*_{e,2}) - \tilde{u}'(c^*_{u,2}) \right] + \tilde{u}''(c^*_{e,2}) \frac{\varphi(x)\theta q'(\theta)}{\beta \Lambda(x) q(\theta)} \right) \\ &= \beta R \Lambda(x) m'(\theta) \left(\left[\tilde{u}'(c^*_{e,2}) - \tilde{u}'(c^*_{u,2}) \right] - \tilde{u}''(c^*_{e,2}) \frac{\tilde{u}(c^*_{e,2}) - \tilde{u}(c^*_{u,2})}{\tilde{u}'(c^*_{e,2})} \right)\end{aligned} \tag{30}$$

Now, even when Eq. (8) holds, condition (7) is not sufficient. In fact, it is necessary that it holds in expectations, both in terms of the derivative of the separation function and for the value function.

G2 Model with heterogeneity in firm productivity

A different, and under some assumption similar, way to model sorting into cyclical jobs is to assume that there is a firm productivity distribution and that separations are a function of productivity (should it be endogenously or exogenously). Below, I derive the model under homogeneous separation and show under what conditions the model would endogenously destroy low productivity jobs in bad times. In the case of separations that exogenously depend on productivity, the model is equivalent to the model exposed in the main text, as the only difference is the production function $f(x)$ and the latter does not matter for any proof.

Assume now that there is a continuum of risk neutral one-employee firms, with (observable) type $x \in \mathcal{X} = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$. A firm of type x produces output $f(x)$ using labor as the only factor of production.
The values of employment and unemployment in $t = 2$ are unchanged, while the value of a filled job, given that wages and productivity are fixed, is now

$$J(x, w) = f(x) - w$$

We can then analyze the problem as a maximization with non-linear Pareto frontier $U(a, x)$, which denotes the value to the worker with assets a when matched with firm x

$$\begin{aligned}U(a_1, x) = \max_{a_2,\theta}\ & u(Ra_1 - a_2 + b) + \\ & \beta \left[m(\theta)(1-\lambda) u \left(Ra_2 + f(x) - \frac{\varphi(x)}{\beta q(\theta)(1-\lambda)} \right) + (1 - m(\theta)(1-\lambda)) u(Ra_2 + b) \right]\end{aligned}$$

The FOCs are

$$u'(Ra_1 - a_2 + b) = \\ \beta R\left[m(\theta)(1-\lambda)u'\left(Ra_2 + x - \frac{\varphi(x)}{\beta q(\theta)(1-\lambda)}\right) + (1 - m(\theta)(1-\lambda))u'(Ra_2 + b)\right] \tag{31}$$

$$0 = m'(\theta_t)(1-\lambda)\left[u\left(Ra_2 + x - \frac{\varphi(x)}{\beta q(\theta)(1-\lambda)}\right) - u(Ra_2 + b)\right] \\ + u'\left(Ra_2 + x - \frac{\varphi(x)}{\beta q(\theta)(1-\lambda)}\right)\frac{\varphi(x)\theta q'(\theta)}{\beta q(\theta)(1-\lambda)} \tag{32}$$

The first condition is an Euler equation where $t = 2$'s consumption depends on the probability of match and match outcome; the second condition pins down market tightness given productivity x.
The optimality condition for the choice of x is $U_x = 0$, i.e.,

$$\beta R m(\theta)(1-\lambda)u'(c_{e,2^*})\left[f'(x) - \frac{\varphi'(x)}{\beta(1-\lambda)q(\theta)}\right] = 0 \\ \Leftrightarrow \quad \varphi'(x) = f'(x)\beta(1-\lambda)q(\theta) \tag{33}$$

PAM in this model only depends on the preference forms, there are thus less restrictions on the model to generate it.

Proposition 3. *A necessary and sufficient condition for positive assortative matching is*

$$\frac{u'(c^*_{e,2}) - u'(Ra_2)}{u(c^*_{e,2}) - u(Ra_2)} < \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})}$$

Proof. The proof follows Eeckhout and Sepahsalari (2024).
The condition for PAM is unchanged

$$\phi_{a_{t+1}x_t}\phi_{\theta_t\theta_t} < \phi_{\theta_t x_t}\phi_{a_{t+1}\theta_t} \tag{34}$$

We now compute the partial derivatives. In optimum (using Eq. (33)), it holds that

$$\phi_{a_2,x} = 0 \tag{35}$$

$$\phi_{\theta,x} = u'(c_{e,2})\frac{\varphi'(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)} < 0 \tag{36}$$

$$\phi_{\theta,\theta} = m''(\theta)(1-\lambda)\left[u\left(c^*_{e,2}\right) - u(c^*_{u,2})\right] + m'(\theta)(1-\lambda)u'(c^*_{e,2})\left[\frac{\varphi(x)q'(\theta)}{\beta(1-\lambda)(q(\theta))^2}\right] \\ + u''(c^*_{e,2})\left[\frac{\varphi(x)q'(\theta)}{\beta(1-\lambda)(q(\theta))^2}\right]\frac{\varphi'(x)\theta q'(\theta)}{\beta q(\theta)(1-\lambda)} + u'(c^*_{e,2})\frac{\varphi(x)}{\beta(1-\lambda)}\left(\frac{(q'(\theta) + \theta q''(\theta))q(\theta) - (q'(\theta))^2\theta}{(q(\theta))^2}\right) \tag{37}$$

$$\phi_{\theta,a_2} = Rm'(\theta)(1-\lambda)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + Ru''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)} \tag{38}$$

The LHS of Eq. (34) is equal to 0, and $\phi_{\theta_t x_t} < 0$. The condition for PAM is thus $\phi_{a_2\theta} < 0$.

From the FOC, it holds

$$\phi_{a_2\theta} = Rm'(\theta)(1-\lambda)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + Ru''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)}$$

The sign so far is ambiguous, as the first term is negative and the second positive. However, from the market tightness FOC we know that

$$\frac{\varphi(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)} = -\frac{m'(\theta)(1-\lambda)\left(u(c^*_{e,2}) - u(c^*_{u,2})\right)}{u'(c^*_{e,2})}$$

Therefore, the condition for PAM boils down to

$$\begin{aligned} m'(\theta)(1-\lambda)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] &< u''(c^*_{e,2})\frac{m'(\theta)(1-\lambda)\left(u(c^*_{e,2}) - u(Ra_2)\right)}{u'(c^*_{e,2})} \\ \frac{u'(c^*_{e,2}) - u'(Ra_2)}{u(c^*_{e,2}) - u(Ra_2)} &< \frac{u''(c^*_{e,2})}{u'(c^*_{e,2})} \end{aligned}$$

Which is satisfied for DARA utility functions (Eeckhout and Sepahsalari, 2024).

■

Whether matching probability is cyclical can be verified by looking at the "strength" of sorting in different aggregate states. Assume that the matching function M features a state-dependent matching efficiency, $M(U,V) = A(z)\left[U^\gamma + V^\gamma\right]^{\frac{1}{\gamma}}$, such that in bad times (z^L) average matching efficiency is lower. Since A enters as a multiplative term, it gets carried through in all derivatives of the job filling and job finding functions. I adapt the equation of sorting to include the average matching efficiency.

$$U_{a_1x}(z) = u'(c_{e,2})\frac{\varphi'(x)\theta A(z) q'(\theta)}{\beta(1-\lambda)A(z)q(\theta)}\left(R(1-\lambda)A(z)m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + Ru''(c^*_{e,2})\frac{\varphi(x)\theta A(z)q'(\theta)}{\beta(1-\lambda)A(z)q(\theta)}\right)$$

It is straightfoward to see that

$$\begin{aligned} & u'(c_{e,2})\frac{\varphi'(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)}\left(R(1-\lambda)A(z^L)m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + Ru''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)}\right) \\ & \quad > u'(c_{e,2})\frac{\varphi'(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)}\left(R(1-\lambda)A(z^H)m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] + Ru''(c^*_{e,2})\frac{\varphi(x)\theta q'(\theta)}{\beta(1-\lambda)q(\theta)}\right) \\ \Leftrightarrow \quad & A(z^L)m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] < A(z^H)m'(\theta)\left[u'(c^*_{e,2}) - u'(c^*_{u,2})\right] \\ \Leftrightarrow \quad & U_{a_1x}(z^L) > U_{a_1x}(z^H) \end{aligned}$$

G3 Model with homogeneous vacancy cost

In this section, I expose why a heterogeneous vacancy cost is a necessary condition for PAM. Below, I derive the model with a homogeneous cost. The conclusion is that PAM cannot obtain under this specification.

With a homogeneous vacancy cost, the joint unemployed and firm problem becomes

$$U(a_1,x) = \max_{a_2,\theta} u(Ra_1 - a_2) + \beta\left[m(\theta)\Lambda(x)u\left(Ra_2 + \bar{y} - \frac{\varphi}{\beta q(\theta)\Lambda(x)}\right) + (1 - m(\theta)\Lambda(x))u(Ra_2)\right]$$

where $\Lambda(x) = 1 - \lambda(x, z)$ and $\bar{y}$ is the common production output.

The FOCs with respect to a_2 and θ are, respectively:

$$-u'(c^*_{u,1}) + \beta R \left[m(\theta^*)\Lambda(x)u'\left(c^*_{e,2}\right) + (1 - m(\theta^*)\Lambda(x))u'(c^*_{u,2}) \right] = 0 \tag{39}$$

$$\beta\Lambda(x) \left[m'(\theta^*) \left(u\left(c^*_{e,2}\right) - u(c^*_{u,2}) \right) + u'\left(c^*_{e,2}\right) \frac{\varphi\theta^* q'(\theta^*)}{\beta q(\theta^*)\Lambda(x)} \right] = 0 \tag{40}$$

where $c^*_{u,1} = Ra_1 - a^*_2$, $c^*_{e,2} = Ra^*_2 + \bar{y} - \frac{\varphi}{\beta q(\theta^*)\Lambda(x)}$ and $c^*_{u,2} = Ra^*_2$.

The first condition is an Euler equation where $t = 2$'s consumption depends on the probability of match and match outcome; the second condition pins down market tightness given productivity x.

Proposition 4. *There is no equilibrium featuring positive assortative matching.*

Proof. The first order condition for the choice of x is

$$\begin{aligned}
\frac{\partial U}{\partial x} &= \beta m(\theta^*) \left[\Lambda'(x) \left(u(c^*_{e,2}) - u(c^*_{u,2}) \right) + \Lambda(x)u'(c_{e,2^*}) \left(\frac{\varphi\Lambda'(x)}{\beta q(\theta^*)(\Lambda(x))^2} \right) \right] \\
&= \beta m(\theta^*)\Lambda'(x) \left[\left(u(c^*_{e,2}) - u(c^*_{u,2}) \right) + u'(c_{e,2^*}) \left(\frac{\varphi}{\beta q(\theta^*)\Lambda(x)} \right) \right]
\end{aligned} \tag{41}$$

The squared bracket of Eq. (41) is always positive, i.e., all workers always prefer the same type of jobs. There is no interior solution and hence no sorting. Depending on the sign of $\Lambda'(x)$, all workers will apply to the maximum (if positive) or minimum (if negative) type. ∎