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Abstract

We analyze a sticky price model where firms choose a price plan, namely a set of two prices. Changing the plan entails a "menu cost", but either price in the plan can be charged at any point in time. We analytically solve for the optimal policy and for the output response to a monetary shock. The setup rationalizes the coexistence of many price changes, most of which are temporary, with a modest flexibility of the aggregate price level. We present evidence consistent with the model implications using CPI data for Argentina across a wide range of inflation rates.

JEL Classification Numbers: E3, E5

Key Words: sticky prices, menu cost models, temporary price changes, reference prices, price plans, price flexibility.

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1 Introduction

Transitory price changes, i.e. temporary deviations above or below a reference price level, appear in many datasets. Kehoe and Midrigan (2015) conclude that over 70% of price changes in the BLS data are temporary. Such transitory price changes do not fit neatly in simple sticky price models, though they might be important since they make a large difference in the measured frequency of price changes. This difference is apparent in the way different authors have approached the treatment of sales in the data: some authors, such as Bils and Klenow (2004), count sales as price changes since they view temporary price changes as a source of price flexibility. Others, such as Golosov and Lucas (2007) or Nakamura and Steinsson (2008), exclude sales from the counting of price changes believing they are not a useful instrument to respond to aggregate shocks.[1]

This paper analyzes a stylized version of the model in Eichenbaum, Jaimovich and Rebelo (2011), a model that produces transitory, as well as persistent, price changes. The objective is to provide a theoretical benchmark to asses whether the temporary price changes matter for the propagation of monetary shocks. The model extends Golosov and Lucas (2007) by assuming that upon paying the menu cost the firm can choose 2 prices, instead of 1, say $\mathscr{P} \equiv \{p_H, p_L\}$. We call the set $\mathscr{P}$ a price plan, which is a singleton in the standard model, and the firm is free to change prices as many times as it wishes within the plan. Instead, changes of the plan require paying a fixed cost. We analytically solve for the optimal policy of a representative firm, for several cross sectional statistics (e.g. frequency and size of temporary and reference price changes) and for the cumulative impulse response function of output to a once and for all monetary shock.

To highlight the importance of the model with temporary price changes, and whether it is consequential to abstract from them in the modeling of the monetary transmission mechanism, we compare the cumulated effect of a once and for all monetary shock on real output in two models: a model with temporary prices and a standard menu cost model. Since the menu cost model cannot accommodate both the large number of price changes *and* a small number of reference prices, the comparison hinges on what is being kept constant across models. We establish analytical results for two interesting cases. If we keep the same number of total price changes across models, the real effect of the monetary shock is larger in the model with temporary prices than in the standard menu cost model. Temporary price changes turn out to be an imperfect tool to respond to permanent policy shocks compared

[1] Klenow and Kryvtsov (2008) put it very clearly (page 871): "Focusing on regular prices begs the question of whether one should exclude sale prices for macro purposes. This is not obvious. First, sales may have macro content. Items may sell at bigger discounts when excess inventory builds up or when inflation has been low."

to regular price changes. The model with temporary price changes thus rationalizes the coexistence of a high frequency of price changes with a larger effect of monetary shocks. Instead, if we keep the same frequency of reference price changes across models, the effect of a monetary shock is smaller in the model with temporary prices than in the standard menu cost model. We find this last result useful since the the frequency of reference price changes is easily estimated in actual data. Thus, using a simple model that abstracts from temporary price changes and calibrating it to *all price changes* or to just *reference price changes*, leads to either underestimation or overestimation of the real effect of a monetary shock. Such differential effects, together with the inability of the simpler menu cost model to account for both the large number of price changes and small number of reference price changes, imply that one should not abstract from temporary price changes.

A few theoretical contributions analyze the relevance of temporary price changes using micro founded sticky price models. Guimaraes and Sheedy (2011) develop a model of sales where the firm's profit maximizing behavior implies a randomization between a regular price and a low price. They assume that the timing for the adjustment of the regular price follows an exogenous rule a la Calvo. The real effects of monetary shocks in their model are essentially identical to the real effects produced by a Calvo model. This is because their model features a strong strategic substitutability in the firm's price setting decisions that, in the quantitative parametrization chosen by the authors, completely mutes the individual firm's incentive to use less sales in order to respond to e.g. a positive monetary shocks. Such effect is not present in our framework since the firm's pricing decision is not subject to any form of strategic interaction with other firms. Our paper is also related to Kehoe and Midrigan (2015) who set up a model where the firm faces a regular menu cost for permanent price changes and a smaller menu cost for temporary price changes (which are reversed after 1 month). Their model implies that firms will not use the temporary price changes to respond to monetary policy shocks, from which they conclude that temporary price changes are not a relevant measure of the firm's price flexibility. Our result is different because the implementation of a temporary price change is not automatically reversed.[2] One novelty compared to those papers is that in our model firms use the temporary price changes to respond to shocks so that temporary prices cannot be ignored to understand the transmission of monetary policy shocks. Analyzing empirically whether firms use temporary price changes to respond to the aggregate macroeconomic conditions, as in the recent papers by Kryvtsov and Vincent (2014) and Anderson, Malin, Nakamura, Simester and Steinsson (2015), is a useful way to select between these alternative models.

We give an overview of the organization of the paper and a more detailed list of the main

[2]See Section 6 and Appendix G for a detailed analysis of the relation with the Kehoe and Midrigan paper.

results. The paper has three main parts. The first part analyzes the firm's problem and its aggregation in steady state, providing several comparative statics. The second part uses the result of the first to characterize the real effect of a monetary shock. The third part presents evidence consistent with the presence of price plans.

The first part of paper starts with Section 2 which sets up the price setting problem and derives the firm's optimal policy. The decision rules determine when to change plans and how to change prices within plans, which can be expressed in terms of four thresholds. In Proposition 1 and Proposition 2 we solve analytically for the decision rules for the case zero inflation and provide an expansion when the inflation is low. In Proposition 3 we solve for the case of very high inflation, which converges to the Sheshinsky-Weiss case. For the remaining cases this section provides a set of four equations whose solution characterize the optimal decision rules. Section 3 characterizes the model behavior in the steady state. In Proposition 4 we characterize the invariant distribution of the relevant state, and a key statistic: the frequency of plan changes. Proposition 5 and Proposition 6 analyze the frequency of price and plan changes at zero inflation. We consider the continuos time limit as well as its discrete time counterpart, which is important to clarify and quantify the sense in which the model model with plans can give lots of price changes that are temporary in nature. The results of these propositions are illustrated in Section 3.2. Proposition 7 and Proposition 8 analyze the time spent at reference prices and how the frequency of reference price changes is related to the frequency of price plans in the menu cost model and in the plan model, in all cases for inflation close to zero. In Proposition 9 we characterize the size of price changes around zero inflation in terms of the optimal thresholds.

The second part of the paper consists of Section 4, which analytically derives the cumulative output response of the economy to an unexpected small monetary shock. The results in this section allow several comparisons with the size of the real effects produced by the canonical menu cost model. We first develop a method to analytically evaluate the area under the cumulative impulse response, obtaining a semi-closed form solution. In Proposition 10 we specialize to the case around zero inflation, where we obtain a simple expression for the area under the IRF for both the menu cost model and the plan model. Importantly, this proposition shows that the real effect of monetary shocks depend on the frequency of plan changes, as opposed to the frequency of price changes. Furthermore, in Proposition 11 we show that a large part of this effect occurs on impact. We complement these result for zero inflation with an analysis for moderate inflation. Section 4.2 provides several comparison of the real effect between the menu cost and the plan model, which highlight the importance of temporary price changes. In particular, Proposition 12 compares the two economies keeping the same number of price plans, and Table 2 explores other relevant cases.

The third part of the paper consists of Section 5, which compares predictions from the model with evidence gathered using the micro data underlying the Argentine CPI across a very wide range of inflation rates. In particular we compare the predictions of the menu cost and plan model for the frequencies of regular and reference price changes, as well as the degree to which prices come back to old values, across a wide range of inflation. We also evaluate the degree of asymmetry of the model compared with the data for moderate inflation rates, and or robustness compare it with data from the Billions Price Project. Section 6 concludes by discussing the scope and robustness of our results.

2 Economic environment and the firm's problem

Consider a firm whose (log) profit-maximizing price at time t, $p^*(t)$, follows the process

$$\mathrm{d}p^*(t) = \pi dt + \sigma \; \mathrm{d}W(t) \tag{1}$$

where $W(t)$ is a standard brownian motion with variance σ^2 per unit of time and the drift is given by the inflation rate π. A firm that charges the (log) price $p(t)$ at time t has a loss, relative to what it will get charging the desired price, equal to $B\,(p(t) - p^*(t))^2$ where B is a constant that relates to the curvature of the profit function.[3]

The firm maximizes the present value of profits, discounted at rate $r \geq 0$. At any moment of time the firm has a price plan available. A price plan is given by two numbers $\mathscr{P} \equiv \{p^L, p^H\}$ so that the firm can charge either (log) price in this set at t, i.e. $p(t) \in \mathscr{P}$ at t. At any time the firm can pay a cost ψ and change its price plan to any $\mathscr{P} \in \mathbb{R}^2$. We let $\mathscr{P}_i$ be the i^{th} price plan and let τ_i be the stopping time at which this i^{th} price plan was chosen, so this plan will be in effect between τ_i and τ_{i+1}. The stopping times and the price plans can depend on all the information available until the time they are chosen. The problem for the firm is to choose the stopping times $\tau_1 < \tau_2 \cdots$, the corresponding price plans $\mathscr{P}_1, \mathscr{P}_2, ...$, as well as the two prices $p(t) \in \mathscr{P}_i$.

The firm maximizes the value of profits discounted at the rate r. The state of the problem is given by the triplet: $\{p^*, p^L, p^H\}$, where p^* is the current desired price level, and where $\mathscr{P} = \{p^L, p^H\}$ is the price plan currently available containing a low and a high price (p^L and p^H, respectively). Thus the firm's problem is

$$V(p^*, p^L, p^H) = \min_{\{\tau_i, \mathscr{P}_i\}_{i=1}^{\infty}} \mathbb{E}\left[\sum_{i=1}^{\infty} \int_{\tau_{i-1}}^{\tau_i} e^{-r\,t} \min_{p(t)\in\mathscr{P}_{i-1}} B\; (p(t) - p^*(t))^2\; dt + \sum_{i=1}^{\infty} e^{-r\tau_i}\psi \;\middle|\; p^* = p^*(0), \mathscr{P}_0 \right]$$

[3]See Appendix B in Alvarez and Lippi (2014) for a detailed derivation of these expressions as a second order approximation to the general equilibrium problem in which firms face a CES demand for their goods.

where $\tau_0 = 0$ and $\{\tau_i, \mathscr{P}_i\}_{i=1}^{\infty}$ are the (stopping) times and the corresponding price plans. The key novel element compared to the standard menu-cost problem is the *min* operator which appears inside the square bracket: at each point in time the firm can freely choose to charge any of the prices specified by the Plan, for instance the plan P_0 lets the firm freely choose either p^L or p^H at any point in time.

Normalization of the value function. Note that $V : \mathbb{R}^3 \to \mathbb{R}$ is symmetric in the following sense: $V(p^*, p^L, p^H) = V(p^* + \Delta, p^L + \Delta, p^H + \Delta)$ for all Δ, p^*, p^L, p^H,a property that follows directly from the fact that the period return function is a function of the difference between the price charged $p(t)$ and the desired price $p^*(t)$. Notice that the law of motion of the ideal price is irrelevant for this symmetry property, which amounts to a normalization of the desired (log) price level $p^*(t)$ with respect to the desired price level at the beginning of each plan $p_i^* \equiv p^*(\tau_i)$. Using this symmetry property we can rewrite the normalized value function for the i-th plan, with a current desired price p^* and desired price $p^*(\tau_i)$ when the plan started, as

$$v(g\,;\,\ell,\,\hbar) \;\;\equiv\;\; V(p^*\,,\,p_i^* + \ell\,,\,p_i^* + \hbar\;) \qquad \text{with} \quad g(t) \equiv p^*(t) - p_i^*$$

where $\ell < \hbar$, $p^L \equiv p_i^* + \ell$, $p^H \equiv p_i^* + \hbar$, and $\mathrm{d}g(t) = \pi dt + \sigma\ \mathrm{d}W(t)$. In words, g measures the current desired price relative to the desired price at the time of the last change in plans. With this new definition the state g is reset to zero every time a new price plan is chosen. We refer to the state g as the *desired price*, where it is understood that the desired price is normalized by the level p_i^* observed at the beginning of each plan.

2.1 The firm's optimal policy

Since at any time price plans can be changed by paying a cost ψ, the value function must satisfy the following equation for all g

$$r\,v(g;\ell,\hbar) = \min\left(\min_{g^* \in \{\ell,\hbar\}} B\,(g-g^*)^2 + \pi\,v'(g;\,\ell,\hbar) + \frac{1}{2}\sigma^2\,v''(g;\,\ell,\hbar)\;,\; \min_{\{\ell'\hbar'\}} v\left(0,\ell',\hbar'\right) + \psi \right)$$

where the outer min operator describes the firm optimal choice between sticking to the current plan vs. changing the plan.

We look for an optimal policy that is described by four numbers: $\underline{g} < \ell < \hbar < \bar{g}$, where $\underline{g}$ and $\bar{g}$ denote the boundaries of the inaction region. Simple algebra shows that $\hat{g} \equiv \frac{\ell+\hbar}{2}$ is the threshold below which it is optimal for the firm to charge the low price within the plan.[4]

[4]Notice $\min_{g^* \in \{\ell,\hbar\}} (g-g^*)^2 = (g-\ell)^2$ if $g \le \hat{g}$ where $\hat{g} \equiv \frac{\ell+\hbar}{2}$ and $(g-\hbar)^2$ otherwise. See Appendix A for the closed form solution of the value function.

The firm's value function then is

$$r\,v(g;\ell,\hbar) = \min \begin{cases} B(g-\ell)^2 + \pi\, v'(g;\,\ell,\hbar) + \frac{\sigma^2}{2}\, v''(g;\,\ell,\hbar) & \text{optimal for } g \in \left[\underline{g}, \hat{g}\right] \\ B(g-\hbar)^2 + \pi\, v'(g;\,\ell,\hbar) + \frac{\sigma^2}{2}\, v''(g;\,\ell,\hbar) & \text{optimal for } g \in \left[\hat{g}, \bar{g}\right] \\ r\left[\min_{\{\ell'\hbar'\}} v\left(0,\ell',\hbar'\right) + \psi\right] & \text{optimal for } g \notin \left[\underline{g}, \bar{g}\right] \end{cases} \tag{2}$$

The oscillations of g about the threshold $\hat{g}$ will generate a price change within the plan. When g crosses either of the barriers, $\underline{g}$ and $\bar{g}$, the price plan is changed and a price change occurs.

To determine the optimal policy parameters, $\underline{g}, \ell, \hbar, \bar{g}$ we use the following optimality conditions, where we use that $g = 0$ at the beginning of a plan, i.e. at the time when the optimal prices are chosen, and a prime denotes derivatives with respect to g:

$$v(\underline{g};\ell,\hbar) = \psi + v(0;\ell,\hbar) \quad , \quad v(\bar{g};\ell,\hbar) = \psi + v(0;\ell,\hbar) \qquad \text{value matching ,} \tag{3}$$

$$v'(\underline{g};\ell,\hbar) = 0 \quad , \quad v'(\bar{g};\ell,\hbar) = 0 \quad \text{smooth pasting ,} \tag{4}$$

$$\frac{\partial\, v(0;\ell,\hbar)}{\partial\, \ell} = 0 \quad , \quad \frac{\partial\, v(0;\ell,\hbar)}{\partial\, \hbar} = 0 \qquad \text{optimal prices .} \tag{5}$$

Fixing the value of $\ell, \hbar$ the set of equations (3) and equation (4) are the familiar value-matching and smooth-pasting conditions for a fixed cost problem (see Dixit (1991)). Lastly, the prices $\ell, \hbar$ within the plan should be optimally decided, which requires equation (5) to hold. These prices are the main novelty of compared to a traditional menu cost problem in which only 1 price is allowed. In this modified problem the firm can spread losses plan using 2 prices, instead of one. Analysis of the first order condition gives that the optimal prices satisfy

$$\ell = \frac{\mathbb{E}\left[\int_0^\tau e^{-rt}\iota(t)\, g(t)\, dt \;\middle|\; g(0) = 0\right]}{\mathbb{E}\left[\int_0^\tau e^{-rt}\iota(t)\, dt \mid g(0) = 0\right]} \quad , \quad \hbar = \frac{\mathbb{E}\left[\int_0^\tau e^{-rt}(1-\iota(t))\, g(t)\, dt \;\middle|\; g(0) = 0\right]}{\mathbb{E}\left[\int_0^\tau e^{-rt}(1-\iota(t))\, dt \mid g(0) = 0\right]} \tag{6}$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g} < g(t) < \hat{g}$ and zero otherwise and all expectations are conditional on the value of $g(0) = 0$, i.e. the value at the start of the plan (see Appendix B for analytic equations to solve for the optimal prices).

Equation (6) shows that the optimal prices are a weighted average of the desired prices $g(t)$ over the periods in which they will apply (as measured by the indicator function ι). Simple closed form solutions can be computed for special cases, for instance in the case of a small inflation (technically $\pi/\sigma^2 \to 0$) discussed in the next subsection, or in the deterministic problem which obtains when inflation diverges ($\pi/\sigma \to \infty$), discussed in Section 2.3. Yet another closed form solution for equation (6) is discussed in Appendix F where we study a

"Calvo" version of our problem in which plans change at an exogenous rate λ.

2.2 The small inflation case

This section discusses the optimal decision rules for the case of zero inflation. We first show that around zero inflation several features of the optimal decision rules are not sensitive to inflation. This results justifies the use of the optimal rules derived for the zero inflation case in a range of low inflations, a case that is suitable for most developed economies. Next we derive closed form solutions for the optimal decision rules. We have the following result (see Appendix D for the proof):

PROPOSITION 1. Let $\ell(\pi)$, $\hbar(\pi)$, $\bar{g}(\pi)$, $\underline{g}(\pi)$ denote the optimal thresholds that solve equation (4) and equation (5) when the inflation rate is π. We have:
(i) At $\pi = 0$ the width of the inaction region , $\bar{g} - \underline{g}$, and the width between the high and the low price, $\hbar - \ell$, have a zero sensitivity with respect to inflation, i.e.

$$\hbar'(0) - \ell'(0) = \bar{g}'(0) - \underline{g}'(0) = 0. \tag{7}$$

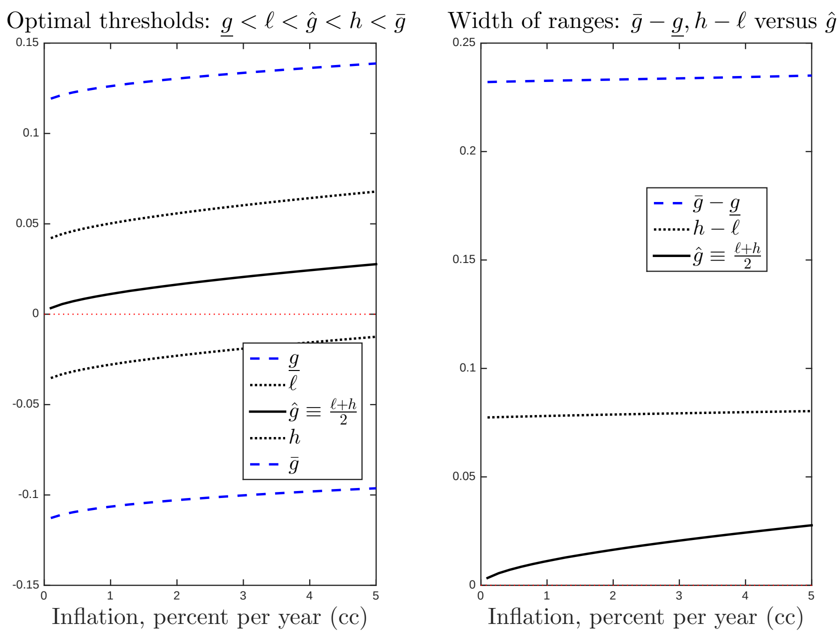
(ii) The optimal prices display "front loading" i.e. a positive elasticity with respect to inflation at $\pi = 0$, i.e.

$$\hat{g}'(0) \equiv \frac{\hbar'(0) + \ell'(0)}{2} = \ell'(0) = \hbar'(0) > 0 \quad . \tag{8}$$

(iii) The number of price changes per period, $N(\pi)$, has a zero sensitivity with respect to inflation, i.e. $N'(0) = 0$.

The proposition implies that, as long as $\pi > 0$, every optimal plan must start with the low price ℓ since $g = 0$ by definition at the beginning of a plan and $\hat{g} > 0$. It also shows that a small inflation, namely a small inflation rate above $\pi = 0$, will have a modest effect on the width of the inaction region as well as on the average frequency of price changes, N. These findings, common to several models where idiosyncratic shocks are present, justify using the limiting case of a zero inflation as a benchmark for a range of low inflation rates. Figure 1 illustrate the result of the proposition using numerical results for a calibrated version of the model. It is apparent that close to zero inflation the optimal prices are symmetrically distributed around zero, i.e. that $\ell(\pi) \approx -\hbar(\pi)$, and so are the optimal inaction thresholds $\underline{g}(\pi) \approx -\bar{g}(\pi)$. Moreover all policy variables have a positive sensitivity with respect to inflation (front loading, point (ii) of the proposition). The right panel shows that the width of the inaction intervals, $\bar{g}(\pi) - \underline{g}(\pi)$, and the width between the high and the low price $\hbar(\pi) - \ell(\pi)$, i.e. the size of a price change within the plan, are insensitive to inflation (point

Figure 1: Optimal decisions at low inflation



(i) of the proposition). The third part of the proposition states that the frequency of price changes $N(\pi)$, as measure by the number of price changes per year, is also insensitive to inflation at $\pi = 0$. This feature is in stark contrast with inflation behavior at high inflation, discussed in Section 2.3, where $N(\pi)$ has an elasticity of $2/3$ with respect to inflation. Both the low elasticity at low inflation and the $2/3$ elasticity at high inflation find strong support in the data, as will be shown in Section 5.

We now present an analytic characterization of the optimal thresholds at zero inflation. It is straightforward to notice that at zero inflation implies the following symmetry features of the optimal policy $\underline{g}(\pi) = -\bar{g}(\pi)$, and $\ell(\pi) = -\hbar(\pi)$. We first establish the following intermediate result:

Lemma 1. Assume $\pi = 0$ and let $\hbar = -\ell$ be the optimal decision rule for the high price within a plan given a barrier $\bar{g}$ for change of plans. The optimal price $\hbar$ is given by a function ρ of the variable $\phi \equiv r\,\bar{g}^2/\sigma^2$, satisfying:

$$\hbar = \bar{g}\;\rho(\phi) \quad \text{where} \quad \rho(\phi) = \frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)} \quad \text{where} \quad \rho(0) = \frac{1}{3} \quad \text{and} \quad \rho'(\phi) < 0\,.$$

This lemma shows that the ratio $\hbar/\bar{g}$ is equal to $1/3$ for small $\phi \equiv r\bar{g}^2/\sigma^2$, and an even smaller fraction for larger values.[5]

We now turn to the characterization of the optimal decision concerning the width of the inaction range, $\bar{g}$ given $\hbar$. This involves solving the value function explicitly, which is shows to be differentiable at $g = 0$ in spite of the fact that the objective function is not. The following proposition, together with Proposition 1, shows that indeed the optimal policy is given by the (symmetric) thresholds $\hbar, \bar{g}$ and provides a complete characterization.

Proposition 2. *Assume $\pi = 0$. The optimal policy rule is given by the symmetric thresholds $\bar{g} = -\underline{g}$ and $\hbar = -\ell$. The value of $\bar{g}$ is the unique solution to the equation:*

$$\eta^2 \, r \, \frac{\psi}{B} = \kappa\,(\eta \bar{g}) \quad \text{with} \quad \eta \equiv \sqrt{2\,r/\sigma^2}\,, \text{ and } \kappa\,(x) \equiv \left[1 - 2\,\rho\,(x^2/2)\right]\left[x^2 - 2\,x\,\frac{(e^x + e^{-x} - 2)}{(e^x - e^{-x})}\right]$$

where the function $\rho(\cdot)$ is given in Lemma 1. The function κ is strictly increasing, with $\kappa(0) = 0$, $\lim_{x\to\infty} \kappa(x) = \infty$, for small values we have: $\kappa(x) = x^4/36 + o(x^4)$, and for large values $\kappa(x)/x^2 \to 1$ as $x \to \infty$. As shown in Lemma 1 the value of $\hbar$ is given by $\hbar = \bar{g}\,\rho\,(\eta^2\,\bar{g}^2/2)$.

Proposition 2 provides simple approximate solutions for $\bar{g}$ and $\hbar$, which are accurate for small values of $(r^2/\sigma^4)\,\psi/B$, thus for small values of the fixed cost ψ and/or a small value of r. In this case we can disregard the terms of order higher than x^4 and write:

$$\hbar = \frac{1}{3}\,\bar{g} \quad \text{and} \quad \bar{g} = \left(18\,\frac{\psi}{B}\,\sigma^2\right)^{1/4} \tag{9}$$

Note also that this approximation for $\bar{g}$ does not invoke r.[6] It is interesting to compare the expression for $\bar{g}$ in equation (9) with the one that obtains in the standard menu cost model, which we refer to as the Golosov-Lucas model, or GL model for short. In the GL model $\hbar = 0$ since each price plan has only one price. The expression for $\bar{g}$ in such a model is identical except that instead of the factor 18 it has a factor 6, or in other words, it will lead to the same value of $\bar{g}$ if it had a fixed cost three times higher. This is intuitive: if the firm has the same fixed cost and has access to the price plan then it chooses to have a wider band, by a factor of $3^{1/4}$ or approximately 32% wider than in the case without access to price plans. Note that otherwise the formula for the inaction threshold is the same quartic root expression than in Barro (1972) or Dixit (1991). Indeed the quartic root is not obvious at all in this context, since the period objective function is not quadratic -as in these two papers

[5] Indeed as ϕ becomes arbitrary large, as implied by very large values $\bar{g}$ (hence of the fixed cost ψ), then the $\hbar/\bar{g}$ converges to zero and $\hbar$ converges to $\sigma/\sqrt{2r}$.

[6] Indeed differentiating the equation for the optimal $\bar{g}$ with respect to r one can show that $\partial\bar{g}/\partial r = 0$ when evaluated at $r = 0$.

-it is given by $(\hbar - |g|)^2$, which includes the absolute value.[7]

2.3 The high inflation case

This section briefly discusses the limit of the model for "high inflation" which occurs if π/σ diverges. This case provides a good description for economies in which inflation π is high relative to the volatility of the marginal costs σ. We have the following result (see Appendix D for the proof)

PROPOSITION 3. Consider a steady state model with $r \to 0$ in the limiting case where $\pi/\sigma \to \infty$. We have the following optimal decision rules:
(i) All plans have identical duration τ given by

$$\tau = 2\left(\frac{3\psi\,\pi^{-2}}{B}\right)^{\frac{1}{3}} \quad . \tag{10}$$

(ii) The optimal low and high prices within the plan are

$$\ell = \frac{\tau}{4}\pi \quad , \quad \hbar = \frac{3\tau}{4}\pi \tag{11}$$

(iii) All plans begin with $g(0) = 0$ and are terminated when $g(\tau) = \tau\pi$.

The proposition shows that with high inflation the model becomes deterministic as in Sheshinski and Weiss (1977) model where $\sigma = 0$. As in that model the frequency of price changes $N(\pi) = 1/\tau$ has an elasticity of $2/3$ with respect to inflation. The model thus converges to the Sheshinsky Weiss model with the only difference, irrelevant for the interpretation of the facts, that the menu cost is paid every two price changes. Contrast this result with the one obtained for zero inflation where we highlighted the lack of sensitivity of the frequency of price changes to inflation (see Proposition 1). In Section 5 we will show how these predictions are validated in the data as inflation moves from low to very high values.

3 Model behavior in the steady state

This section computes some statistics for the frequency and size of price changes observed in a steady state, i.e. under the invariant distribution of desired prices. For the limiting case of zero inflation, which provides a good benchmark for low inflation regimes, we present an analytic characterization of size and frequency of the price changes triggered by the change

[7] For instance, Dixit (1991) finds that when the period objective function is purely $|g|$ the approximation for the optimal rule has a cubic root.

of plans as well as of the price changes that occur *within* the plan. We also discuss the notion of *reference* price change, a benchmark statistic in the empirical literature.

Let π denote the steady state inflation rate and $f(g)$ denote the invariant density function of the desired prices $g \in [\underline{g}, \bar{g}]$. Moreover, let $N_p(\pi)$ denote the frequency of *plan* changes per period, i.e. the frequency with which g hits either plan-resetting barriers. We have:

Proposition 4. Given the inflation rate π and policy parameters $\ell(\pi)$, $\hbar(\pi)$, $\bar{g}(\pi)$, $\underline{g}(\pi)$. Define $\xi \equiv -\frac{2\pi}{\sigma^2}$ we have that: (i) the density for the desired prices is given by

$$f(g) \begin{cases} A\left(e^{-\xi \underline{g}} - e^{-\xi g}\right) & \text{for } g \in \left[\underline{g}, 0\right] \\ A\,\frac{e^{-\xi \underline{g}}-1}{1-e^{-\xi \bar{g}}}\left(e^{-\xi g} - e^{-\xi \bar{g}}\right) & \text{for } g \in \left[0, \bar{g}\right] \end{cases} \quad \text{where} \quad A = \left(-\underline{g} e^{-\xi \underline{g}} - \bar{g}\frac{e^{-\xi \underline{g}} - 1}{1 - e^{-\xi \bar{g}}} e^{-\xi \bar{g}}\right)^{-1} \tag{12}$$

(ii) The frequency of Plan changes per period, $N_p(\pi)$, is

$$N_p(\pi) = \frac{\pi\left(e^{-\xi \bar{g}} - e^{-\xi \underline{g}}\right)}{\bar{g}\left(e^{-\xi \underline{g}} - 1\right) - \underline{g}\left(e^{-\xi \bar{g}} - 1\right)}\,. \tag{13}$$

The density function for the desired prices in equation (12) shows that for finite values of π/σ^2 the density has zero mass at the boundaries of the support, $\{\underline{g}, \bar{g}\}$. This feature, common to models with idiosyncratic shocks ($\sigma^2 > 0$) is key to understand the small impact response of small shocks (see Alvarez et al. (2016a)). As $\pi/\sigma^2 \to 0$ the distribution has a triangular tent-shape. As π/σ^2 diverges, the shape of the distribution converges towards a rectangle, i.e. more ad more mass piles up near the inaction boundaries, so the impact response of the model converges to the one of the Caplin-Spulber model. Equation (13) gives a closed form solution for the number of plan changes per period as a function of inflation and of the optimal policy rule. Simple analysis reveals that this function has a zero sensitivity with respect to inflation at $\pi = 0$, a feature that echoes the behavior of the total number of price changes established in part (iii) of Proposition 1.

3.1 Analytic results for an economy with low inflation

This subsection presents several analytic results which are a useful guidance for economies with low inflation. Technically we consider an economy with $\pi > 0$ but small (i.e. $\pi \downarrow 0$) arguing that this benchmark is accurate in a range of small inflation rates. We first present results on the frequency of various types of price changes that appear in the model, i.e. the frequency of temporary price changes (price changes within a plan) and other notion of low frequency price changes, such a reference price changes. In Section 3.1.2 we briefly discuss the model's implications for the size of price changes. For completeness, in Appendix E we show that, unlike the menu cost model, the plan model has a hazard rate of price changes that is decreasing in its duration, and we give a closed form expression for it. Indeed for short duration we get that the hazard rate $h(t)$ is approximately $h(t) \approx 1/(2t)$.[8]

3.1.1 On the prevalence of temporary prices

This section provides some analytic results on the "prevalence" of reference prices, namely an analysis of how much time the actual prices will spend at the modal price (defined over a period of length T). This statistic is of interest because it has been analyzed empirically both by Eichenbaum et al. (2011) and by Kehoe and Midrigan (2015).

We start by setting up a discrete time / discrete state representation of the model, for three reasons. First, in the continuous time version the expected number of price changes within a price plan diverges to $+\infty$ (see equation (15) below for a proof). Second, the source of the empirical study in Eichenbaum et al. (2011) comes from a grocery chain where price changes are decided (and recorded) weekly. Third, a finite number of total price changes per period allows us to to compare the effect of introducing price plans into an otherwise stylized version of the Golosov-Lucas model by keeping the *total* number of price changes fixed.

Discrete time version of the model. The discrete state / discrete time representation has time periods of length Δ and the normalized desired price following

$$g(t+\Delta) - g(t) = \begin{cases} +\sqrt{\Delta}\,\sigma & \text{with probability } 1/2 \\ -\sqrt{\Delta}\,\sigma & \text{with probability } 1/2 \end{cases} \tag{14}$$

We assume that g reaches $\pm\bar{g}$ after an integer number of periods (or steps), we define this value as $\bar{n} = \bar{g}/\left[\sqrt{\Delta}\sigma\right]$, an integer greater than or equal to 2 (a requirement which allows us to have price changes within a plan). Let $g(t)$ follow equation (14) for $-\bar{g} < g < \bar{g}$ and let $\tau(\bar{g})$ be the stopping time denoting the first time at which $|g(t)|$ reaches $\bar{g}$.

We define N to be the total number of price changes per unit of time, N_p the number of price plans per unit of time, and N_w the number of price changes per unit of time without a price plan change, so that $N = N_w + N_p$. The next proposition characterizes the expected number of plans per unit of time, N_p:

Proposition 5. Let $\Delta > 0$ be the length of the discrete-time period. Assume that $\bar{g}/(\sigma\sqrt{\Delta})$ is an integer larger than 2. Given the threshold $\bar{g}$, the number of plan changes per

[8]This is because when the ideal price is close to $\hat{g}$, and it crosses this barrier it triggers a price change within the plan. This price has about half a chance to be reverted in a very short time.

unit of time is $N_p = \frac{\sigma^2}{\bar{g}^2}$.

It is immediate to realize that N_p is independent of Δ and hence its value coincides with the number of adjustments of the continuous time model, i.e. the limit for $\Delta \to 0$. We also notice that N_w depends only on N_p and Δ, and no other parameters. This is because the computation for N_w requires knowing the number of steps $\bar{n}$ that are necessary to get from $g = 0$ to $|g| = \bar{g}$ (at which time the plan is terminated), which is $\bar{n} = \sqrt{N_p/\Delta}$ as shown in the proposition. Thus the value of N_w, which in principle depends on $\bar{g}, \sigma, \Delta$, can be fully characterized in terms of 2 parameters only: N_p and Δ.

We now establish two inequalities bounding N_w as a function of the length of the time period Δ, and of the number of price plans per unit of time N_p. The inequality follows directly from Doob's uncrossing inequality applied to our set-up.

Proposition 6. Let $\Delta > 0$ be the length of the time period, and $\bar{g}$ be the width of the inaction band. The expected number of price changes within a plan N_w (per unit of time) has the following bounds

$$\frac{1}{\sqrt{\frac{\Delta}{N_p}} + \frac{\Delta}{2}\left[\frac{1+\sqrt{\Delta N_p}}{1-\sqrt{\Delta N_p}}\right]} \leq N_w \leq 2\sqrt{\frac{N_p}{\Delta}} - \frac{N_p}{2} \tag{15}$$

Note that both the lower and upper bounds for N_w are increasing in N_p and decreasing in Δ. As $\Delta \to 0$, then $N_w \to \infty$, and indeed N_w behaves as $\Delta^{-1/2}$ for small values of Δ.[9]

Next we use the lower bound in equation (15) to derive an approximation for N as a function of Δ and N_p which is accurate for small Δ. To this end we first define the function $N = \mathcal{N}(\Delta, N_p)$ which gives the total number of price changes as function of Δ and N_p. This implicitly defines the function $N_p = \mathcal{N}_p(\Delta, N)$. We have the following approximation:

$$N \approx \sqrt{\mathcal{N}_p/\Delta} \quad \text{for small } \Delta \quad \text{ or formally } \quad \lim_{\Delta\to 0} \frac{\mathcal{N}_p(\Delta, N)}{\Delta N^2} = 1 \tag{16}$$

The previous results provide novel insights on the measurement of flexibility for an actual economy featuring both temporary and permanent price changes. It is common practice to measure the flexibility of an economy by the measured frequency of the price changes, at

[9] Note also that both bounds for N_w are increasing in N_p, at least for small N_p. This is because as N_p becomes large, the price gap is reset to values closer to zero more often, which is the time when price changes without a price plan tend to happen. Finally note that fixing $\Delta > 0$, and letting $N_p \to 0$ then $N_w \to 0$, and hence $N = N_p + N_w \to 0$. Summarizing, letting $N \equiv N_w + N_p = \mathcal{N}(\Delta, N_p)$ we have $\mathcal{N}(0, N_p) = \infty$ for $N_p > 0$ and $\mathcal{N}(\Delta, 0) = 0$ for $\Delta > 0$, with the upper bound and lower bound of $\mathcal{N}(\Delta, N_p)$ being increasing in N_p and decreasing in Δ.

least since Blinder (1994). Indeed this statistic is central in Klenow and Malin (2010) who also carefully distinguish between permanent and short-lived price changes. An important result in our paper, presented below in Proposition 10, is key to interpret the importance of temporary versus permanent price changes. The result will establish that it is the number of permanent changes N_p (i.e. Plan changes) that concur to determine the output effect of a monetary shock, not the overall number of price changes.

Fraction of time spent at the reference price. Reference prices are defined as the modal price during an interval of time, say during $[0, T]$ a concept introduced by Eichenbaum et al. (2011). In this section we analyze the "prevalence" of reference prices: the idea is to highlight that while there are many price changes during a time interval, prices spend a large fraction of time at the modal value during this interval, i.e. prices are often at the reference price. The comparison between the large frequency of price changes and the prevalence of reference prices captures the idea that prices return to the previous values.

To analyze this effect, we compute a statistic that depends on 2 parameters: a time interval of length T and a fraction $\alpha \in [1/2, 1]$. The statistic $F(T, \alpha)$ is the fraction of sample periods of length T in which the firm price spends at least αT time at the modal price. The parameter T is the time-window chosen by the statistician who measures references prices in the data, for instance T is a quarter in Eichenbaum et al. (2011). The parameter α defines what fraction of time prices spend at the modal value in a sample path of length T. We will show that it is possible to have $F(T, \alpha) \approx 1$ even for α close to one, and at the same time that we have an arbitrarily large number of price changes, N. We have:

PROPOSITION 7. Fix $\sigma^2 > 0$ and let $\Delta \geq 0$ be the length of the time period, with $\Delta = 0$ denoting the case of a Brownian motion. Consider an interval length $T > 0$, a fraction $1/2 \leq \alpha < 1$ and a number $0 < \epsilon \leq 1$. Then there exists a threshold value $G > 0$ such that for all $\bar{g} \geq \sigma G$ then $F(T, \alpha) \geq 1 - \epsilon$. The threshold G depends on ϵ, α and T but it is independent of σ.

In words, the proposition states that for any fraction $\alpha \in (1/2, 1)$, it is possible to choose a value of $\bar{g}$ large enough so that the price will be at the reference price at least a fraction $1 - \epsilon$ of the times. Notice by equation (16) that a given $\bar{g}$ is consistent with a very large number of price changes (as Δ is small). Thus, our model can simultaneously have prices spending a large fraction of time at the reference price, as well as a very large number of price changes, a feature that is apparent in the micro data.

Frequency of Reference Price changes: model with Plans vs. menu cost. Next we compare the duration of *Reference Prices* in the model with N_p plans per unit of time, with the duration of *Reference Prices* in a model without plans with N^{GL} price changes per unit of time, imposing that $N^{GL} = N_p$. For short we refer to first model as the plans model and to the second model as the GL model –for Golosov and Lucas.[10] We fix the same initial condition $p^*(0)$, and the same sample path for the Brownian shocks, so that $\mathbf{p}^* \equiv [p^*(t)]$ for $t \geq 0$ is the same for both models. We specialize the comparison for the case where inflation is positive, but arbitrarily small.[11] We have:

PROPOSITION 8. Let N^{GL} be the number of price changes in a menu cost model and N_p the number of plan changes in a plans model. Assume $N^{GL} = N_p$ and fix an arbitrary interval $[T_1, T_2]$ with $0 \leq T_1 < T_2$ and a path $\mathbf{p}^*$. Let the duration of the modal price in the interval $[T_1, T_2]$ be $D(T_1, T_2; \mathbf{p}^*)$ for the plans model and $D_\pi^{GL}(T_1, T_2; \mathbf{p}^*)$ for the GL models. We have: $\lim_{\pi \to 0} D_\pi(T_1, T_2; \mathbf{p}^*) \leq \lim_{\pi \to 0} D_\pi^{GL}(T_1, T_2; \mathbf{p}^*) \leq 2\, D(T_1, T_2; \mathbf{p}^*)$

Note that since the inequalities in Proposition 8 hold for any path $\mathbf{p}*$ they also hold for the average and median durations. Computing the frequency of price changes as the reciprocal of its duration, the proposition implies that the number of reference price changes in GL is smaller than in a model with plans, namely

$$N_r^{GL} < N_r < 2N_r^{GL} \tag{17}$$

where N_r and N_r^{GL} denote, respectively, the number of *reference* price changes in the plans model and in the GL model. The intuition behind the proof of this result is that if the number of price changes in the GL model equals the number of plan changes in PL model, then the PL model will have more reference price changes than the GL model due to the price changes that occur within each plan, in fact up to twice as many.

The result in Proposition 8 that when the GL and PL models have the same number of plan changes, the GL model will have fewer reference price changes than the PL model has important implications to calibrate the model. In particular, this means that if one wants to calibrate the GL and PL models to have the same number of reference price changes, then the GL model has to have more price changes than the number of plan changes of the PL model. This property has important implications for the effect of monetary policy when we compare the PL and GL model calibrated to the same number of reference price changes, as we will see later.

[10] In particular we require that $\sigma^2/\bar{g}^2$ be the same for both models, and ensure this by choosing a larger value for the fixed cost in the plans model. Using the expression for the case where $r \downarrow 0$ it can be seen that this requires a fixed cost in the plans model that is a third of the cost in the GL model.

[11] More precisely, we use the limit as $\pi \to 0$ to break some ties that occur only when $\pi = 0$.

Table 1: Summary statistics on price setting behavior (weekly model)

# Plan changes per year (N_p)	5.8	3.3	1.4	0.4
# Reference Price changes per year*	4.1	3.3	2.4	1.2
# Price changes per year ($N = N_p + N_w$)	20	15	10	5
Freq. of Price changes (per week)	0.35	0.27	0.18	0.10
Freq. of Reference price change (per quarter)	1.0	0.8	0.6	0.30
Fraction of time spent at reference price	0.50	0.63	0.79	0.95
Fraction of time spent below the reference price	0.25	0.17	0.10	0.02

* The reference price is defined as the modal price in a quarter. All results refer to a discrete time model where the time period is one week, i.e. $\Delta = 1/52$ and inflation is zero.

3.1.2 On the size of price changes

Consider the distribution of price changes, Δp, in a model with positive but arbitrarily small inflation. Notice that at $\pi = 0$ the firm is indifferent between ℓ and $\hbar$ when starting a plan, but this indeterminacy is resolved as we take limit $\pi \downarrow 0$ which ensures the firm's optimal price at the start of the plan is ℓ. Recall from Proposition 1 that ℓ and $\hbar$, and the width of the interval between prices is insensitive to π at low inflation. We have the following

Proposition 9. *Let $\mathbb{E}[|\Delta p|]$ measure the size of price changes, as measured by the mean absolute value of price changes Δp. The size of price changes within a plan, $\frac{2}{3}\bar{g}$ is equal to the size of price changes between plans. Thus the mean absolute size of price changes is $\mathbb{E}[|\Delta p|] = \frac{2}{3}\bar{g}$.*

It is interesting to compare these predictions to the data. In most micro datasets it is found that the size of price changes excluding sales is smaller than the size of all price changes, as summarized by Klenow and Malin (2010). In scanner datasets the size of **Reference** Price Changes is smaller than the Size **All** price changes, suggesting that temporary price changes are larger that reference price changes.[12] In the more encompassing BLS data however the size of **Reference** price changes is essentially identical to the size of all price changes, as in Table 4 of Kehoe and Midrigan (2015) where it is equal to 0.11 in both samples.

3.2 Simulated cross-sectional moments.

Table 1 illustrates the main implications of our model concerning the patterns of price setting behavior. The statistics in the table depend only on 2 parameters: the length of the decision

[12] See Table 2 in Eichenbaum et al. (2011) where the standard deviation of reference price changes is 0.14 vs. 0.20 log pts. for all price changes.

period, Δ, which we assume to be weekly as in several datasets (i.e. $\Delta = 1/52$), and the frequency of plan changes, N_p, for which we consider the four values reported in the first line of the table.[13] The results are computed for an economy with low inflation (namely $\pi = 0$, virtually identical results obtain for inflation equal to 4 per cent). In Section 5 we extend the model simulations, and data comparison, to a large range of inflation rates.

The table also reports statistics for the "reference price", defined as the modal price in a quarter. It appears that the frequency of plan changes and of reference price changes do not coincide but that they are positively correlated. The entries in the second column are the ones that replicate more closely the frequency of total weekly price changes reported by Eichenbaum et al., with a total number of about 15 price changes per year.[14] In our model (which differs from theirs because, among others, we focus on unit root shocks) this implies about 3 reference price changes per year, a value that is almost two times the value they estimate. In spite of such differences, the model successfully reproduces the main feature of the data i.e. the presence of many price changes, most of which are due to temporary price changes.

We use the calibration to compute a measure of the prevalence of reference prices, i.e. what fraction of the time the prices spend at the reference price, as well as below it. The second to last row of the table shows that the stickier the plan (i.e. the smaller N_p), the larger the fraction of time that prices spend at their modal value, consistently with the theoretical result in Proposition 7. The remaining time is equally split in visits to other prices that are both "above" as well as "below" the reference price. The calibration in the second column shows that the prices spend about 63% of the time at their modal value, a statistic that is almost identical to the one by Eichenbaum et al. and not far from the BLS statistics of Kehoe and Midrigan (2015) on the prevalence of reference prices which indicate that prices spend about 75% of the time at the modal price (see their Table 4).[15]

4 Cumulative output response to a monetary shock

This section analyzes the propagation of a monetary shock in a menu cost models with plans. In particular, we consider an economy in steady state, i.e. with an invariant cross

[13] It is straightforward to use equation (9) and Proposition 5 to map these values to different primitives, e.g. different values of the fixed cost ψ.

[14] They estimate that weekly price changes have a duration of 0.18 quarters or, correcting for measurement error, a duration of 0.27 quarters. Those magnitudes imply between 15 to 22 price changes per year.

[15] We compare our model to the statistics in Eichenbaum et al. (2011) because their data is weekly. Instead the BLS data used by Kehoe and Midrigan (2015) is monthly, which requires further time aggregation. Also the value of the period length T to compute the fraction of time at the modal price is different, it is four times longer in statistics computed by Kehoe and Midrigan relative to those computed by Eichenbaum et al.

sectional distribution of desired prices, and analyze the effect of an unexpected once and for all monetary shock of size $\delta > 0$ on output. We consider the impulse response of output to such a shock, and focus on the area below such impulse response function as a summary measure of the propagation mechanism. As in Alvarez et al. (2016b) such a measure, combining the persistence of the response to the shock with the intensity (size) of the response, is convenient because it is easier to characterize than the full profile of the impulse response. The section provides a general framework, analytic results for the case of zero inflation, an accurate benchmark for low inflation rates, and numerical results for any inflation rate.

Let $P(t,\delta)$ denote the aggregate price level t periods after a monetary shock of size δ:

$$P(t,\delta) = \Theta(\delta) + \int_0^t \theta(s,\delta) ds \tag{18}$$

The notation in equation (18) uses that the aggregate price level, just before the shock, is normalized to zero. So that $\Theta(\delta)$ is the instantaneous jump in the price level at the time of the monetary shock and $\theta(\delta,t)$ is the contribution to the price level at time t.

We consider models where the effect of output is proportional to the difference between the monetary shock and the price level, i.e. denoting by $Y(t,\delta)$ the impulse response of aggregate output t units of time after the shock of size δ as: $Y(t,\delta) = (1/\epsilon)(\delta - P(t,\delta))$, where ϵ is an elasticity that maps the increase in real balances (or real wages) into increases in output. We denote the cumulated output response following a small monetary shock of size δ as follows:

$$\mathscr{M}(\delta) = \int_0^\infty Y(t,\delta)\, dt \equiv \int_0^\infty \frac{1}{\epsilon}\Big(\delta - P(t,\delta)\Big)\, dt \tag{19}$$

Our approach to characterize equation (19) is to compute the corresponding cumulated output measure for each firm, as indexed by its desired price g, and then aggregate over firms using the steady state distribution $f(g)$ from Proposition 4.

The firm's expected contribution to cumulative output. Consider the optimal policy parameters $\{\underline{g}, \bar{g}, \ell, \hbar\}$, let $\hat{g} \equiv (\ell + \hbar)/2$ and define the *price gap* as the difference between the price charged and the desired price. Denoting price gaps by $\hat{p}(t)$ we have

$$\hat{p}(t) = \hbar + (\ell - \hbar)\, \iota(t) - g(t)\,, \quad \text{for } \tau_i \leq t < \tau_{i+1} \tag{20}$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g} < g(t) < \hat{g}$ and zero otherwise, already introduced above.[16] The price gap measures the firm's deviation from the static profit max-

[16]This follows from the definition and simple algebra: $\hat{p}(t) \equiv p(t) - p^*(t) = [p^*(\tau_i) + \hbar + (\ell - \hbar)\,\iota(t)] - [p^*(\tau_i) + g(t)]$ which gives the equation in the text.

imizing price (in log points), so that a firm with a negative gap is charging a low price (i.e. it has a low markup) and thus contributes to above average output.

Define the expected cumulated output for a firm with desired normalized price g:

$$\hat{m}(g) = -\mathbb{E}\left[\int_0^\tau \hat{p}(t)\,dt \mid g(0) = g\right] \tag{21}$$

where τ is the stopping time indicating the next change of price plans. Appendix C provides an analytic solution for the expected value $\hat{m}(g)$ by solving the associated stochastic differential equation for the benchmark case with no discounting ($r \to 0$).

Given $\hat{m}(g)$ and the steady state density of desired prices in equation (12), $f(g)$, the cumulated aggregate output following a small monetary shock $\delta > 0$ is $\mathcal{M}(\delta) = \int_{\underline{g}+\delta}^{\bar{g}} f(g - \delta)\hat{m}(g)\,dg$ which, for a small $\delta > 0$, we approximate by

$$\mathcal{M}(\delta) = \delta\mathcal{M}'(0) + o(\delta) = -\delta \int_{\underline{g}}^{\bar{g}} f'(g)\hat{m}(g)\,dg + o(\delta) \tag{22}$$

Equation (22) lends itself to straightforward numeric analysis since we have analytic expressions for each of its components, as shown below. Next we derive a closed-form analytic expression for the case of zero inflation. This result provides a useful benchmark for low inflation economies. We then return to the general case with inflation and present some numerical results on the effect of π on $\mathcal{M}(\delta)$.

4.1 Analytic results at $\pi = 0$ and sensitivity to inflation

This section specializes the model to $\pi = 0$ to provide a simple analytic characterization of the cumulated output effect. We assume that the economy is in a steady state when an unexpected shock occurs. The shock permanently increases money, nominal wages, and aggregate nominal demand by δ log points. We use the function $\hat{m}$ derived above, as well as the invariant distribution of normalized desired prices $f(g)$, to compute the cumulative impulse of aggregate output for a once and for all shock to the money supply of size δ for the special case in which inflation is zero. We summarize the solution of the integration in equation (22) for the $\pi = 0$ case in the proposition, where we use $N_p = \sigma^2/\bar{g}^2$ to denote the expected number of plan changes per period (see Proposition 5):

Proposition 10. The cumulative output effect after a small monetary shock δ is $\mathcal{M}(\delta) = \delta\, \mathcal{M}'(0) + o(\delta)$ where

$$\mathcal{M}'(0) = \frac{\bar{g}^2}{18\,\sigma^2} = \frac{1}{18 N_p} \quad . \tag{23}$$

This proposition shows that the cumulated output effect is a decreasing function of the number of plan changes per period, N_p. It is interesting that only the number of *plan* changes, not the *total* number of price changes, appears in the formula. Below we will use this proposition to discuss the relevance of temporary price changes in macroeconomics. Next we provide a result that highlights one key difference between a canonical menu cost model and a model with plans, namely the impact response of aggregate prices to a monetary shock.

Impact effect. Consider the impact of a shock on the price level in the model with price plans and zero inflation, so that $\ell = -\hbar$ and $\hat{g} = 0$. On impact there are two types of price changes, those that come with a change of price plan, and those within the existing price plans. The mass of price changes triggered by a change of price plans is second order, as in canonical menu cost models. In spite of this, we show next that an aggregate shock triggers a non-neglible response of the aggregate price level on impact, a result triggered by price changes *within* the plan.

Let $\tilde{\Theta}(\delta)$ denote the impact effect on prices due to price changes within the existing price plan, which is given by the mass of firms whose negative desired price $g < 0$ becomes positive following the shock times the size of their price change, $2\hbar$. The next proposition summarizes the main result

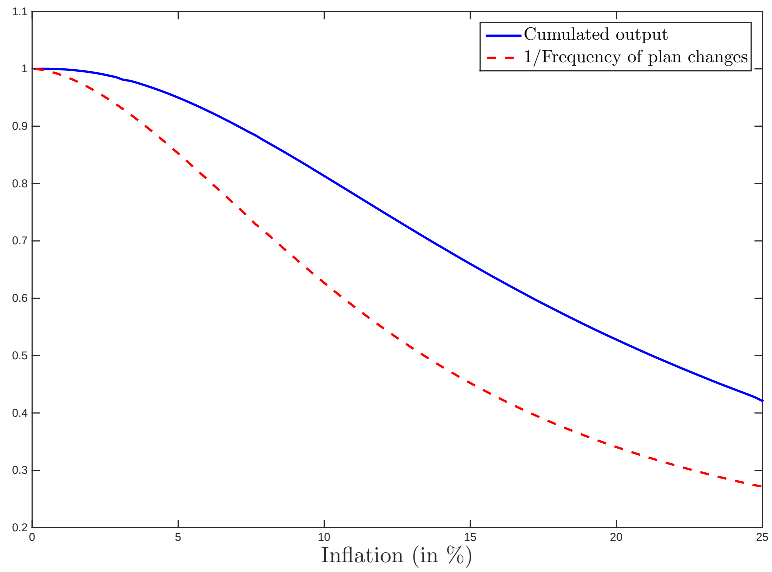
PROPOSITION 11. Let $\pi = 0$, δ be an aggregate nominal shock and $\tilde{\Theta}(\delta)$ denote the impact effect on prices due to price changes within the existing price plans. We have:

$$\lim_{\delta \to 0} \lim_{r \to 0} \frac{\tilde{\Theta}(\delta)}{\delta} = \lim_{\delta \to 0} \lim_{\frac{\psi}{B} \to 0} \frac{\tilde{\Theta}(\delta)}{\delta} = \frac{2}{3} \quad .$$

Thus, for either a small discount rate r or a small fixed cost ψ/B, the response of the aggregate price level on impact is 2/3 of the monetary expansion. This result is in sharp contrast with the zero impact effect that is a pervasive feature of time-dependent and state dependent models.[17] It is intuitive that a zero density at the boundaries of the inaction region implies that a small shift of the support of the distribution, say of size δ, triggers a very small mass of adjustments in canonical menu cost models, since this mass is roughly given by the product between the (near zero) density at the boundary and δ. While this basic mechanism

[17] This result can be seen analytically from equations (19) and (20) in Caballero and Engel (2007). See proposition 1 in Alvarez, Lippi and Passadore (2016a) for analytic results on a large class of models in which the impact effect is second order.

Figure 2: Cumulated output effect, $\mathscr{M}(\delta)$ at different inflation rates



continues to hold in the model with plans, concerning the mass of firms that adjust their plan, the key difference is that in the model with plans there is a non-negligible mass of firms that change their price *within* the plan.

How inflation affects the cumulated output effect. Next we briefly return to the general case with on zero inflation and discuss how inflation affects the cumulated output effect. Figure 2 reports the cumulated output effect at various inflation rates computed by a numerical evaluation of equation (22). Each point of the curves in the figure is obtained by solving the model at a given inflation rate, keeping all other fundamental parameters constant (such as ψ, B, σ, r).

The blue solid line plots the cumulated output effect, $\mathscr{M}$, as a function of the inflation rate π in a range from zero to 25%, relative to the value computed of $\mathscr{M}$ computed at zero inflation. It appears that the curve is flat around zero, a result that confirms that, as was shown analytically for several of the model's endogenous variables, that the zero inflation case provides a good approximation for a range of small inflation rates.[18] As inflation gets into high values the cumulative effect drops, mostly as a result of more frequent Plan changes, as suggested by the N_p term which appears in the denominator of equation (23). When inflation equals 20% the cumulated output effect is about 50% smaller than the effect at zero inflation.

[18] This result can be easily established analytically assuming the function $\mathscr{M}(\delta;\pi)$ is differentiable. Noting that $\mathscr{M}(\delta;\pi) = -\mathscr{M}(-\delta;-\pi)$ and differentiating both sides with respect to δ (denoted by a prime) and then to π, and evaluating the derivative at $\delta = \pi = 0$, gives $\frac{\partial \mathscr{M}'(0;0)}{\partial \pi} = 0$.

The red dashed line in the figure plots $1/N_p(\pi)$ as a function of inflation (normalized to 1 at $\pi = 0$). This curve can be used to gauge what part of the reduction is due to the higher frequency of Plan changes. The gap between the solid line and the dashed line indicates that other forces are at work, such as changes in the shape of the distribution of desired price changes, to mitigate the "output reducing" effect of higher inflation.

4.2 Do temporary price changes matter for macro?

This section uses the theoretical results gathered so far to discuss an important substantive macroeconomic question: do temporary price changes matter for the transmission of monetary shocks? To be even more prosaic, suppose we have a simple menu cost model and some data, containing both temporary and permanent price changes. Should the model be calibrated to match the total number of price changes or only the number of permanent price changes? Our model with plans of course embeds both temporary and permanent price changes and will be used to answer these questions. We will use the model with plans as a laboratory of the "true" monetary response and derive the implications for what approximate simple calibration of the menu cost model, e.g. including or excluding temporary price changes, best matches the true response of the economy.

We begin the analysis by computing the cumulated output effect in a standard Golosov-Lucas model (GL for short) with a threshold for price adjustment equal to $\bar{g}$. The cumulated output effect is $\mathcal{M}_{GL}(\delta) = \int_{-\bar{g}}^{\bar{g}} m\,(g+\delta)\; f\,(g)\; dg$, which is given by $\mathcal{M}_{GL}(\delta) = \delta\, \mathcal{M}'_{GL}(0) + o(\delta)$ where

$$\mathcal{M}'_{GL}(0) = \int_{-\bar{g}}^{\bar{g}} m'\,(g)\; f\,(g)\; dg = \frac{\bar{g}^2}{6\,\sigma^2} = \frac{1}{6\,N^{GL}} \tag{24}$$

where we used that the expected number of price changes per period in GL is $N^{GL} = \sigma^2/\bar{g}^2$. The next proposition compares the real cumulative effects in the model with plans to those in the GL model:

Proposition 12. Assume $\pi = 0$ and let N_p be the mean number of plan changes per-period and N^{GL} be the mean number of price changes in the canonical menu cost model without plans. The ratio of the cumulative output responses in the two models is:

$$\lim_{\delta \downarrow 0} \lim_{\frac{\psi}{B} \downarrow 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \lim_{\delta \downarrow 0} \lim_{r \downarrow 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \frac{3\,N_p}{N^{GL}} \quad .$$

The proposition is extremely useful because, for small fixed cost ψ/B or small discount factor r, it shows that the only parameters that matter for the comparison between the GL

Table 2: Three calibrations for a bi-weekly model and low inflation ($\Delta = 1/26$ and $\pi = 2\%$)

	$N^{GL} = N = 3.7$	$N^{GL} = N_p$	$N_r^{GL} = N_r$
Plans model	$N_p = 0.7, N_r = 1.4$	$N_p = 0.7, N_r = 1.4$	$N_p = 0.7, N_r = 1.4$
Menu cost	$N^{GL} = 3.7, N_r^{GL} = 2.3$	$N^{GL} = 0.7, N_r^{GL} = 0.7$	$N^{GL} = 1.5, N_r^{GL} = 1.4$
Ratio: $\frac{\mathcal{M}_{GL}}{\mathcal{M}\text{ Plans}}$	$\frac{3N_p}{N^{GL}} = \frac{3\times0.7}{3.7} \approx 0.5$	$\frac{3N_p}{N^{GL}} = 3$	$\frac{3N_p}{N^{GL}} = \frac{3\times0.7}{1.5} \approx 1.4$

model and the model with plans are the number of plan changes (N_p) and the number of price changes in the GL model (N^{GL}).[19]

In Table 2 we present 3 alternative comparisons. As a benchmark, and for consistency with the seminal analysis of Eichenbaum et al. (2011), the numerical results use a weekly Plans model ($\Delta = 1/52$) with a total number of about 1.4 reference price changes per year ($N_r = 1.4$, see the second row of the table). Note that given these data the plans model is fully determined. Instead, the critical choice concerns the frequency of price changes to be used for the GL model.

The first column of the table assumes that the *total* number of price changes in the GL model is the same as in the model with plans, $N^{GL} = N$, equal to 3.7 price adjustments per year. The second row of the table shows that this corresponds to about 0.7 plan changes per year, so that a straightforward application of Proposition 12 implies that the output effect in the model with plans is about two times bigger than in the Golosov-Lucas model (first column, third row). The economics behind this result is that the model with plans is stickier: many of its price changes are temporary, and revert to baseline. In comparison, a GL model with the same number of total price changes has no temporary price changes, which implies more flexibility of the aggregate price level and a smaller output effect.

An analytic approximation highlights how the total number of price changes N and the length-of-time period Δ affect the result of the first column (where $N = 3.7$ and $\Delta = 1/52$). Imposing that $N = N^{GL}$, applying Proposition 12 and rewriting N_p in terms of N and Δ,

[19]We refer to these magnitudes as "parameters" since one can always choose the underlying fixed costs to obtain those objects as the optimal firm choices.

by the approximation in equation (16), we can write

$$\lim_{\delta \to 0} \frac{\mathcal{M}_{GL}(\delta)}{\mathcal{M}(\delta)} = \frac{3\, N_p}{N^{GL}} = \frac{3\mathcal{N}_p\,(\Delta, N)}{N} \approx 3N\,\Delta \tag{25}$$

Indeed as $\Delta \to 0$ the ratio of the real effects goes to zero, i.e. the model with plans can be made arbitrarily stickier than the menu cost model.

Two alternative comparisons are presented in the second and third columns of the table. Both comparisons assume that the *total* number of price changes in the menu cost model is matched to a measure of low-frequency price changes: the number of plan changes (in the second column) or the number of reference price changes (third column). The motivation for these comparisons is to understand the consequences of the practice, followed by several economists, to calibrate the menu-cost model after discarding from the data some temporary price fluctuations, as in e.g. Golosov and Lucas (2007) and Nakamura and Steinsson (2008).[20] The second column thus assumes that the number of price changes in the menu cost model, N^{GL}, is matched to the number of plan changes, N_p. Naturally the model with plans will feature more price changes per period, due to the presence of the temporary changes. This comparison yields a clear analytic result: setting $N^{GL} = N_p$ in Proposition 12 shows that the high frequency price changes provide a significant amount of flexibility to respond to the shock in the short run compared to the menu cost model. The cumulative output effect in the plans model under this assumption is 1/3 of the effect of the menu cost model. The mechanism behind this result relies on the larger response of prices on impact that occurs in the model with plans (see Proposition 11).

While the comparison in the second column is interesting to highlight the workings of the different models, it is arguable that this is a proper interpretation of what previous authors (discarding temporary prices) have done since the frequency of *plan* changes is not directly observable from the data. For this reason the third column presents a calibration where the menu-cost model is calibrated to match the number of *reference* price changes observed in the data (formally: $N_r^{GL} = N_r$). The third row of the table shows that in this case the menu-cost model continues to over-estimate the effects produced by a model with plans, but by a smaller margin than happened in the second column. The reason is that the calibration in the second column produces too many reference price changes for the menu-cost model, as established by equation (17). In order to bring this statistic in line with the data it is necessary to increase the menu cost, thus making the model stickier.

Altogether, this analysis suggests two main points. First, taking into account the nature of

[20] The heuristic behind this practice is that temporary price changes are not seen as a regular form of price adjustment.

price changes, temporary vs. regular, is essential for a proper quantification of the monetary transmission mechanism. Failing to do so leads to a substantial under-estimation of the effectiveness of monetary shocks, as illustrated in the first column of Table 2. This result justifies Kehoe and Midrigan (2015) view that "prices are sticky after all", since a model with plans can indeed produce very many price changes (arbitrarily many as Δ gets smaller) and yet deliver a large output effect. Second, our model shows that approximating the effects of temporary price changes by calibrating a standard menu cost to selected low-frequency moments for price changes can be misleading. The model with plans shows that the output effect of monetary shocks in a model with N_r reference price changes is smaller than the output effect in a menu cost model with the same number of reference price changes.

5 Model vs the data at different inflation rates

In this section we compute several statistic using 9 years of micro-data underlying the CPI in Argentina during a period with very large variations in the inflation rates. This data gives qualitative support for the implications of the plan model at both high and low inflation rates. In particular, we compare the statistics in the Argentine data computed for every 4 month non-overlapping period with the same statistic computed for the invariant distribution of the plan and for the invariant distribution of the menu cost model. Since prices are gathered every two weeks, we use a discrete time version of the models with $\Delta = 1/26$. The first type of statistics measures the frequency of regular price changes and the frequency of reference price changes, where reference prices are defined as the modal price in 4-month period. The second type of statistics measures the extent to which at the time of price changes prices "come back" to old values recently used. At small inflation rates, the plan model displays reference prices that are sticky –roughly with duration equal to half of the duration of the price plans– and prices came back to previous values, as long as the plan is still in place. Instead, at least for low inflation rates, the menu cost model does not display neither of these features. Instead as inflation rates increases, the plan model and the menu cost model both converge to a menu cost model with no idiosyncratic shocks, and thus prices are not coming back to old values. We found that the level as well as the pattern of these statistics for the plan model are closer to the ones in the Argentine data than those of the menu cost model across a large range of inflation rates.

5.1 Statistics discriminating Price Plans vs Menu Costs

In each non-overlapping four month period we compute the annually continuously compounded inflation across all store x product combinations. We use continuously compounded inflation to be consistent with the model, where the log ideal price has a constant drift π. The choice of four month is a compromise so that we compare the average inflation in a period in the data with the steady state in the model. Additionally since we compute reference prices –see their definition below– we need a reference interval of time to define them.[21] For these statistics we use a discrete time version of both models where the period is two weeks, i.e. $\Delta = 1/26$ of a year. In each of the two models we need to set a value for σ and ψ/B so that, with a 2% percent inflation ($\pi = 0.02$) the average number of price changes and the standard deviation of price changes are equal to ones in the Argentine data for inflation rate of that magnitude, which is achieved in the last four years of our sample.

It is instructive to consider the variation in the inflation rates in the Argentine data we use. On the one hand, inflation is extremely high in the first three years of the sample, and there is even a period of a hyperinflation. On the other hand, about a year after the stabilization plan of April 1991, there is almost price stability. For instance, the four month period with highest inflation rate has an annualized continuously compounded inflation rate of 792 % during the second fourth months of 1989. Note that, if instead of continuously compounded, we use quarterly compounded inflation, the annualized inflation rate for the same period is higher than 275,000 % per year! Indeed the entire year of 1889 has a yearly continuously compounded inflation of 405 percent, which gives a compounded annual inflation higher than 5,300 percent.[22]The average continuously compounded inflation during the three years between 1994 to 1996 is 0.93 percent. Recall that we measure inflation in continuously compounded annualized term throughout to be consistent with the model.

The first two statistics for which we compare the Argentine data with data generated by the two models are two measures of the frequency of price changes. The first one is the probability of regular price changes, estimated as the average fraction of price changes in each two week period across all good $\times$ outlet combinations, excluding substitutions.[23] The second statistic is the probability of a reference price change, estimated as the fraction of good $\times$ outlets in which there is a different reference price different from the one in the previous

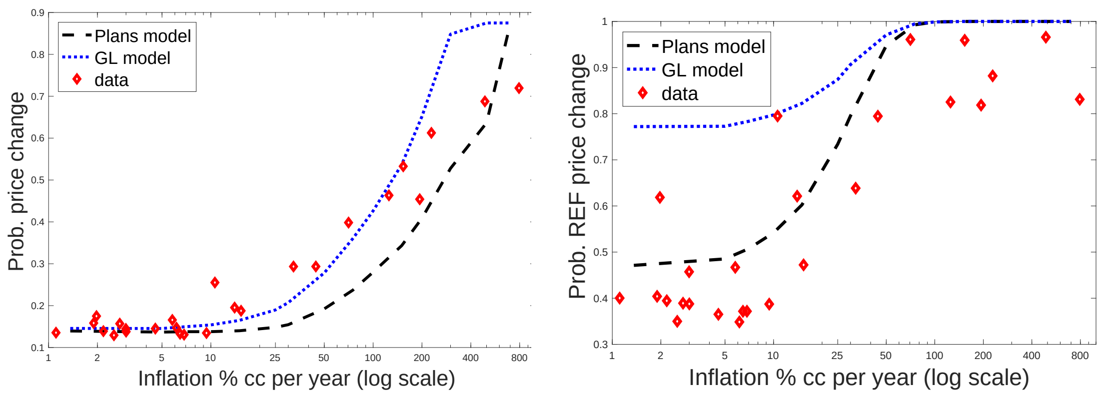
[21] Given the periodicity of our price data (two weeks) and the one used in US studies (weekly for the scanner data in Eichenbaum et al. (2011) and monthly CPI data in Kehoe and Midrigan (2015)), a reference interval of four months is a reasonable compromise. Eichenbaum et al. (2011) use a reference period of three months and Kehoe and Midrigan (2015) use one of five months.

[22] These figures comes from the following calculations: $275,077 \approx \left[(\exp(792/300))^3 - 1\right] \times 100$ and $5,641 \approx [\exp(405/100) - 1] \times 100$.

[23] We use regular to distinguish them from reference prices. But we include all price changes, even those which have a sale flag.

Figure 3: Probability of price changes vs inflation

Pr. price change* Pr. Ref. price change+

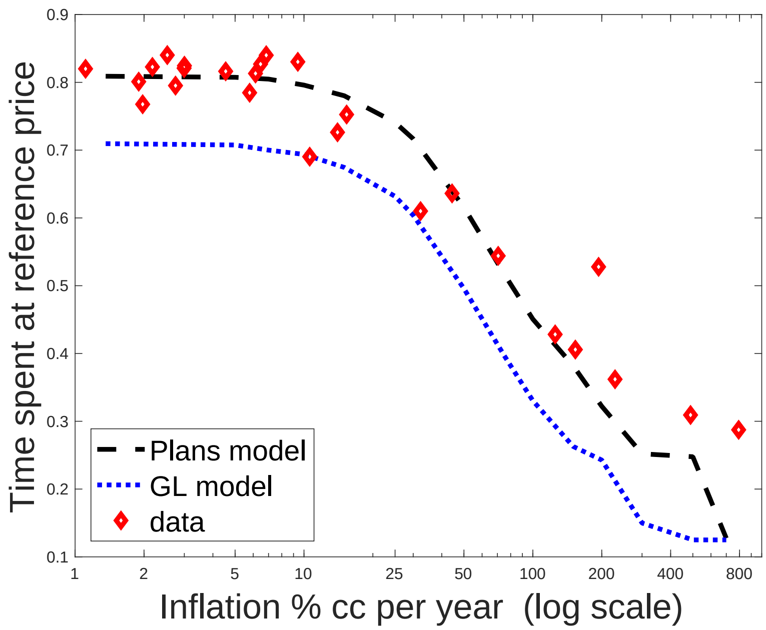


Note: * probability of a price change per two weeks period, average during four month period in the data. + probability of a price change per four month period. Reference prices computed as modal prices in a four month period.

4-months. The reference price in a given 4-month is the modal price for that good × outlet combination. Each of the panels of Figure 3 displays a scatter plot for the probability of price changes and the average inflation for each of the 36 non-overlapping 4-month periods with a red diamond. The figure also plots the corresponding statistic for the plan's model and for the menu cost model. In both models, as the steady state inflation increases, the benefit of increasing prices is higher, and thus prices are changed more often. Technically, the ideal price hits the top barriers more often, even if the barriers are wider as inflation increases. Both models show a patterns of the probability of regular price changes for different inflation rates roughly similar to the one in the Argentine data. Instead, for the probability of reference price changes, the plan model displays a pattern much closer to the data than the one for the menu cost model. In the menu cost model the probability of reference price changes varies with inflation much less than in the data.

Figure 4 displays the time spent at reference prices in the Argentine data and in each of the two models. The time spent at the reference price is defined for each good × outlet and 4-month non-overlapping time period. For each one we compute the fraction of two week periods for which the price equals the reference price. The statistic displayed is the average across good × outlets for each 4-month period. In the models it is the average under the invariant distribution. Both models display a pattern similar to the one in the Argentine data, as inflation increases prices tend to go up, and hence the time spent at the reference price decrease with the steady state inflation. Yet, the values for the plan model are closer to the ones in the data than those for the menu cost model–recall that the models are calibrated

Figure 4: Prevalence of reference prices vs inflation



Note: time spent at reference price measure as a fraction of the four month period. Reference prices computed as modal prices in a four month period for each product x store combination.

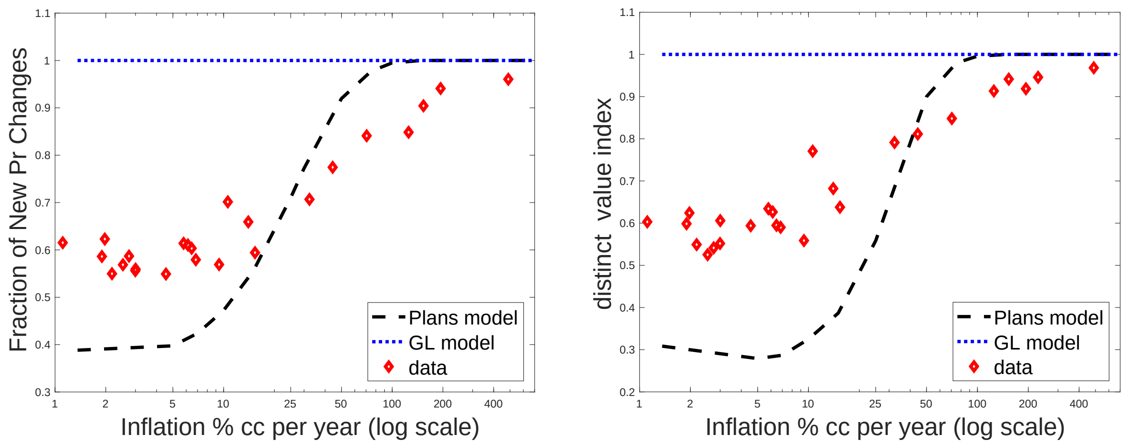
so that at 2% inflation they have the same total number of price changes that in the data.

Figure 5 displays two statistics that measure whether the values of prices right after a price change have been used as prices in the recent past. The first statistic is the fraction of price changes in a two week period for which the values of the prices after the change have not been used for the good $\times$ outlet in the past year. For each good $\times$ outlet and each 4-month period we compute the average of this fraction across the 8 two-week periods. The statistic displayed is the average across all good $\times$ outlets in each non-overlapping 4-month period. A related statistic which is not displayed in the figure but reported in the Table 4 and Table 5 under "novelty index", is the fraction of prices that in a given period are new, i.e. that have not been used by that good $\times$ outlet for the last year. The second statistic is the distinct value index, the ratio between the normalized number of distinct prices used in the last fourth month relative to the normalized number of price spells.[24] We compute the statistic only for 4-months periods where there are 3 or more price spells, since with fewer spells prices can't return to an old value. The statistic displayed for each 4-month non-overlapping period is the average across all good $\times$ outlet combinations. We normalize

[24] A price spell is the intervals with the same continuous price. In 4-month with prices gathered very two weeks, the number of price spells is between 1 and 8.

Figure 5: Indicators of comeback prices vs inflation

Fraction of Price Changes w/New Prices* Distinct value index^{+}:



Note: * Fraction of price changes where the new price has not been observed in the last year. $^{+}$Distinct value index: (# distinct prices during the for month period-2) divided by (# of price spells during the 4 month period - 2). Numerator and denominator computed for the eight two-weeks period in each 4 month period.

the number of distinct prices by subtracting two from the number of distinct price index and by subtracting two for the number of price spells, so that the distinct value index varies between 0 and 1, taking the value of 1 if after a change in prices the values are different, and 0 if there are at most two values for prices are used in all price spells.

Comparing the two statistics with the data it is clear that the model with plans has the same qualitative patterns than the Argentine data i.e., for low inflation price-changes feature prices that have been used in the past, but as inflation increases there are more distinct or new prices. In the plan model the logic is clear: as inflation increases the ideal price has a larger positive drift, and thus it hits the upper barrier much more often than the lower barrier. While the qualitative behaviour of both statistic is the same for the plan model as for the data, the model displays a wider range of variation than the data. Nevertheless, the pure menu cost model is clearly at odds with the data, since essentially all price changes lead to new values for the prices at all inflation rates.

5.2 Asymmetry and Sales

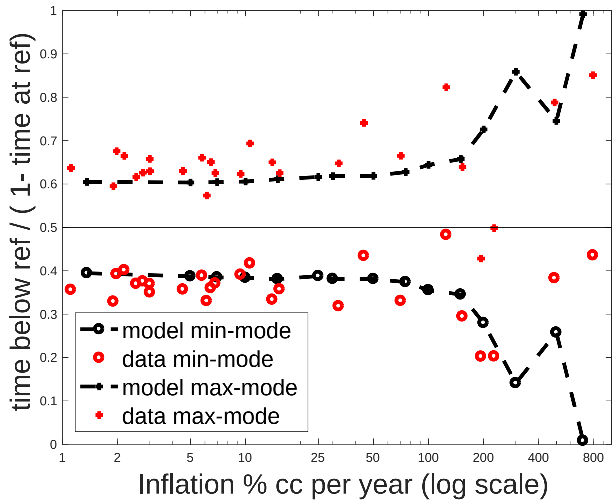
In this section we discuss the extent to which prices spend an equal amount of time above or below reference prices, whether the degree of asymmetry is related to "sales", and more broadly, whether temporary price changes and sales are exactly the same phenomenon. In the Argentine CPI data, at least for low inflation rates, the frequency of prices that are higher than reference prices is equal to the frequency of prices that are lower than reference prices, and hence it is consistent with the symmetry of the model at low inflation. The same pattern emerges using internet scraped data for Argentina in a later period, i.e. 2008-2010. Additionally, using internet scraped data for another 4 countries we find that the frequency of sales is correlated with the degree of asymmetry: countries with higher fraction prices with sales flags (such as the US) also have prices spending more time below reference prices. This correlation also holds for each country across different goods categories. From this discussion we conclude that temporary price changes and sales are not exactly the same phenomenon.

We measure the degree of symmetry by the fraction of time that the price of a given good × outlet spends below the reference price, relative to the time that the price spends at price different from the reference price in each non-overlapping four month period. At zero inflation rate the plan model is symmetric, so prices are equally likely to be above than below the reference price. It turns out that the Argentine CPI data is also quite symmetric. For this statistic it is important to specify what is the value of the reference price if there are multiple modes for the prices in a 4 month period. In Figure 6 we plot the fraction of time below reference prices, divided by the time not-at-the reference price using two versions of the value of reference prices: one that uses the maximum mode as a reference price and the other that uses the minimum mode as the reference price. We compute the time spent below the reference price in the two alternative ways for simulated data obtained from the plan model as well as for the CPI. It can be seen that outside high inflation rates the average of the two values for this statistic is close to 0.5, both for the Argentine CPI data and for the model. Table 8 and Table 9 in Appendix I displays the data for both definitions of reference prices.

We complement the empirical analysis by using data scraped from internet outlets, taken from the Billon Price Project (BPP) developed by Cavallo and Rigobon (2016). These data contains daily prices from selected internet outlets for Argentina, Brazil, Chile, Columbia and the US for the years 2008, 2009 and 2010.[25] In Figure 7 we display the time spent by prices below reference prices during the years 2008-2010 for each country × good's category in the vertical axis, and the fraction of times that the prices in country × goods category display a sale flag. Reference prices are computed as the average of the maximum mode and the minimum mode. There are two clear facts that emerge from Figure 7. First, while the average of the fraction of time spent below reference prices is higher than 50%, which is the value that correspond to the symmetric case, the differences are small. For instance, using an unweighted average across all countries and categories the fraction of time spent below

[25] We sample the BPP data every two weeks to make it comparable with the CPI data from Argentina which we use above. The source of this data, the data itself, and its detailed description can be found at http://www.thebillionpricesproject.com/our-research/

Figure 6: Fraction time spent below reference prices



Note: Fraction time spent below reference prices, out of the time spent outside reference prices.

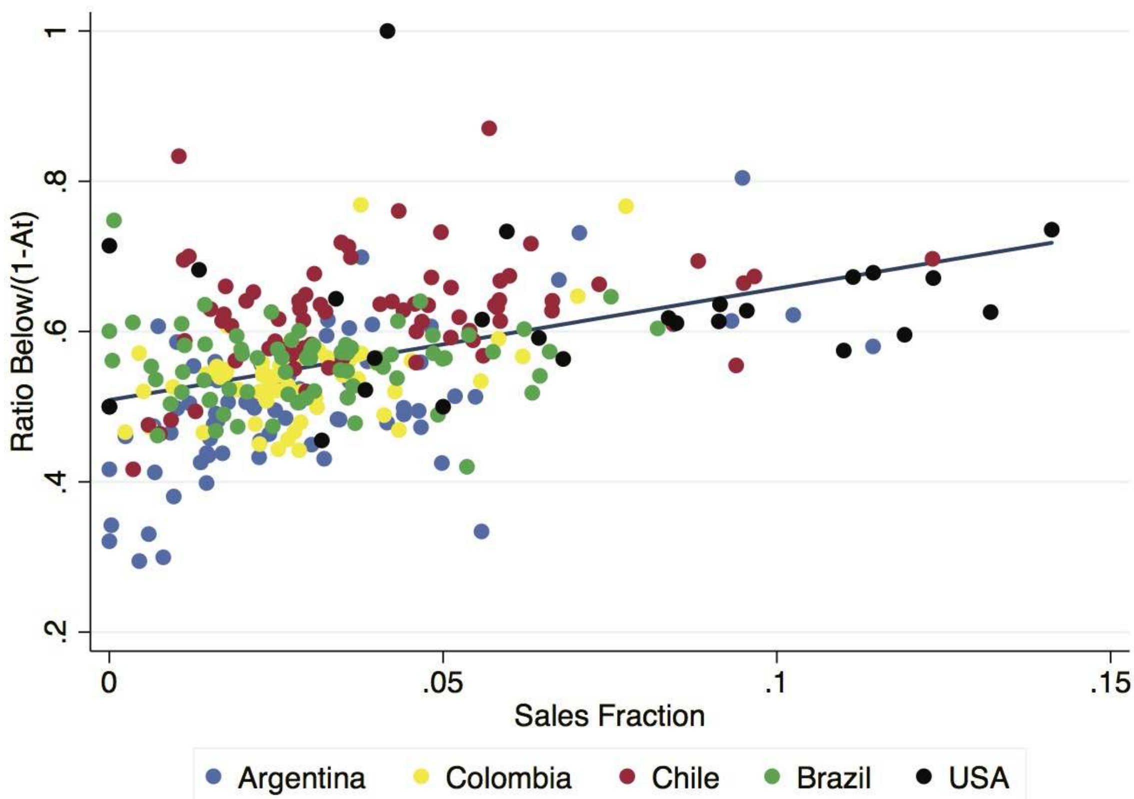
reference prices is 60%. Detailed statistics for these data can also by found in Table 14-Table 15 in Appendix I. Second, it is clear that the fraction of time where goods have a sale flag is positively correlated with the degree of asymmetry, in particular with the fraction of time spent below reference prices.

6 Conclusions, robustness, and extensions

This paper showed that the introduction of a 2-price plan in a standard menu cost environment generates a persistent reference price level and many short lived deviations from it, as seen in many datasets. We also showed that modeling temporary price changes substantially alters the real effect of monetary shock. In the continuous time model, the real effect of monetary shocks is inversely proportional to the number of plan changes, and independent of the number of price changes. Thus one can have a very modest amount of aggregate price response to a monetary shock and simultaneously have an arbitrarily large number of price changes. Our preferred exercise is that the net effect, over and above the standard menu cost model is to add some extra flexibility to the aggregate price level, and hence to have a smaller real effect relative to this benchmark. Yet this conclusion, as illustrated in our characterizations depend on the exact nature of the comparison, i.e. what is being kept constant when comparing a model with plans to one without them.

In this concluding section we discuss two substantive extensions that are useful to clarify which results are robust to changes in the modeling environment and which ones are not.

Figure 7: Normalized Fraction of Prices Below Reference versus Sales



Note: Each dot is the average of the statistic for a good category × country during the years 2008, 2009 and 2010. The underlying data are prices from few internet stores in each country, as produced by the Billion Price Project. The statistic are computed by first sampling these data every two weeks. In the vertical axis we display the fraction of time prices are below reference prices divided by the time prices spent outside reference prices. Reference prices are computed as the average of the maximum mode and the minimum mode.

The extensions, whose analytical details and formal propositions are given in dedicated appendices, are of intrinsic interest and also allow us to clarify the origin of the differences between our contribution and some important related papers in the literature.

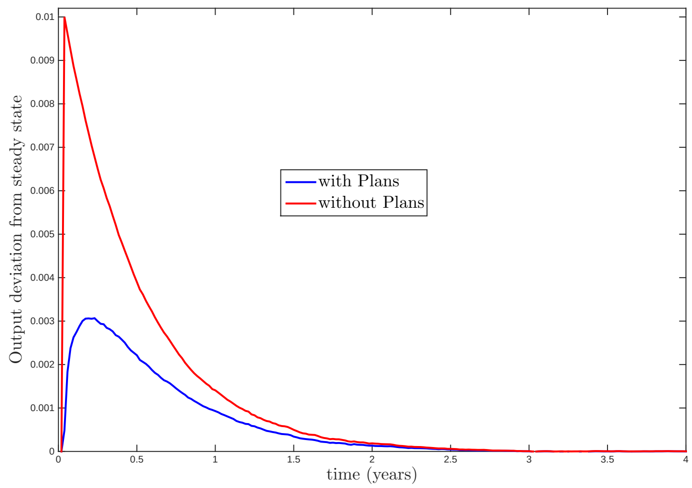
Price plans a la Calvo. Appendix F analyzes the consequences of introducing price plans $P \equiv \{p_H, p_L\}$ in a setting where the adjustment times for the plan are exogenous and follow the standard Calvo model in which the probability of adjusting the plan has a constant hazard rate. The model retains tractability and we can derive the optimal prices within the plan and analytically characterize the hazard rate of price changes, which is also decreasing as for the model in the main text.[26] There are three main differences of the "Calvo" version of the model compared to the version in the main body of the paper. The first difference is that the ratio of the area under the output impulse response of output of an economy with price plans and one without, but with the same number of price plans, is 1/2 instead of 1/3 as in the model in the body of the paper. The second difference is that the Calvo version of the model produces a hump shaped response of output to monetary shocks, as we briefly explain below. The third difference is that the Calvo model shows that some implications of the model in the main body of the paper for the size and the duration of price changes, measured at and just prior to a plan change, are not a robust feature of this class of models.

The hump shaped response of output to a monetary shock in the Calvo version of the model is due to its higher price flexibility on impact. In turn, this added flexibility comes because the mass of firms that are ready to switch from a "low" to a "high" price is higher with a Calvo plan, and thus the model produces *full* price flexibility on impact, i.e. that the price level increases by the full amount of the monetary shock in the moment that it occurs. This response is partially reversed in later periods, so that the impulse response function of output is hump shaped, a remarkable feature of this very simple model displayed in Figure 8.

The behavior of the size and duration of price changes prior (and at) the time of a plan change differ in the two versions of the price plan model. In the benchmark version of the model in the main body of the paper, since a price plan ends when the normalized desired price is large, the duration of a price spell immediately before a plan change cannot be very small. In words, just before a change of the plan, there cannot be many temporary price changes. Related to this feature, the size of a price change that coincides with a plan change should be large. Both implications come from the fact that in the model in the body of the paper price plans are changed when the (absolute value of the) normalized desired price reaches a threshold $\bar{g}$. In principle, one may try to devise a test of these implications, by

[26] In the Calvo version of the model the hazard rate function is simply $h(t) = 1/(2t) + \lambda$, where λ is the Poisson arrival rate of a price plan change.

Figure 8: Output impulse response in model with exponential adjustments



Note: Response to a 1 percent $\delta = 0.01$ monetary shock. Parameters are $\sigma = 0.20, \lambda = 2, r = 0$. The number of plan adjustment per period equal the number of price adjustments in Calvo: $N_p = N = \lambda$.

identifying the times at which price plans have occurred. Yet these implications are not central to models with price plans and temporary price changes, instead they come from the particular mechanism assumed to change price plans, namely by paying a fixed cost. If Instead price plans are changed at exponentially distributed time, i.e. a la Calvo, there are no implications of the typical length of price changes before a price change, and price changes can be small even if they coincide with a plan change.

Costly price changes within the plan. Appendix G discusses a modified model which assumes that price changes within the plan are not free, although they are cheaper than changes of the plan. This extension is useful to understand the differences with the model of Kehoe and Midrigan (2015) who allow for *costly* temporary price changes that *automatically* revert to the baseline price after one period. The main finding of this extension is a "continuity property" of our main result, namely that the model with price plans continues to feature a lot of additional price flexibility even if the price changes (within the plan) are not free.[27] Of course for the price plan to deliver a significant amount of additional flexibility the cost of price changes within the plan must be small, an assumption that is consistent with the large number of temporary price changes in the data. This extension highlights the important role of the assumption of the automatic-price reversion in the model of Kehoe and

[27]This result, formally given in Proposition 18 extends the result of Proposition 11. The economics is that even in the presence of a small adjustment cost within the plan there is a non-negligible mass of firms in the neighborhood of the price-adjustment threshold (within a plan).

Midrigan: under this assumption it becomes very costly for the firm to use the temporary price change to track the permanent monetary shock, since this implies paying the temporary cost several times (to offset the automatic reversal).

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WEB APPENDIX

Temporary Price Changes, Inflation Regimes and the Propagation of Monetary Shocks

F. Alvarez and F. Lippi

A Solving the value function in closed form

Let the optimal prices within the plan be denoted by $\ell, \hbar$, and the optimal plan-resetting thresholds $\underline{g}, \bar{g}$. The value function $v(g;\ell,\hbar)$ is given by two piecewise component functions v_0 , for $g \in [\underline{g}, \hat{g}]$ and v_1 for $g \in (\hat{g}, \bar{g}]$. These functions must satisfy the following conditions, where primes denote the partial derivatives with respect to the first argument of the function:

$$
\begin{aligned}
\frac{\partial\ v_0(0;\ell,\hbar)}{\partial\ \ell} = 0 \quad , \quad \frac{\partial\ v_0(0;\ell,\hbar)}{\partial\ \hbar} = 0\,, \\
v_0(\underline{g};\ell,\hbar) = \psi + v_0(0;\ell,\hbar) \quad , \quad v_1(\bar{g};\ell,\hbar) = \psi + v_0(0;\ell,\hbar)\,, \\
v_0'(\underline{g};\ell,\hbar) = 0 \quad , \quad v_1'(\bar{g};\ell,\hbar) = 0, \\
\lim_{g\uparrow\hat{g}} v_0(g;\ell,\hbar) = \lim_{g\downarrow\hat{g}} v_1(g;\ell,\hbar) \quad , \quad \lim_{g\uparrow\hat{g}} v_0'(g;\ell,\hbar) = \lim_{g\downarrow\hat{g}} v_1'(g;\ell,\hbar)\,.
\end{aligned}
$$

which is a system of 8 equations in 8 the unknowns $\ell, \hbar, \underline{g}, \bar{g}$ and the 4 unknowns from the 2 second order differential equations for v_0 and v_1.

The solution of the value function v_0 in inaction is given by the sum of a particular solution and the solution to the homogenous function. The particular solution $v^p(g)$ is:

$$v_0^p(g;\ell,\hbar) = \frac{B}{r}\left[\ell^2 + \frac{2\pi}{r}\left(\frac{\pi}{r} - \ell\right) + \frac{\sigma^2}{r} + 2g\left(\frac{\pi}{r} - \ell\right) + g^2\right]$$

the homogenous solutions $v_0^f(g)$, for $g > 0$ is :

$$v_0^f(g;\ell,\hbar) = \xi_{0,1}\, e^{\eta_1 g} + \xi_{0,2}\, e^{\eta_2 g} \;\; \text{where } \eta_i = \frac{-\pi \pm \sqrt{\pi^2 + 2\, r\sigma^2}}{\sigma^2} \;\; , \quad i = 1,2 \qquad (26)$$

Equivalent expressions obtain for v_1. Thus the solution for v is:

$$v(g;\ell,\hbar)=\begin{cases}\frac{B}{r}\left[\ell^2+\frac{2\pi}{r}\left(\frac{\pi}{r}-\ell\right)+\frac{\sigma^2}{r}+2g\left(\frac{\pi}{r}-\ell\right)+g^2\right] & \\ \quad +\ \xi_{0,1}\,e^{\eta_1\,g}+\xi_{0,2}\,e^{\eta_2\,g} & \text{for all } g\in[\underline{g},\frac{\ell+\hbar}{2}] \\ \frac{B}{r}\left[\hbar^2+\frac{2\pi}{r}\left(\frac{\pi}{r}-\hbar\right)+\frac{\sigma^2}{r}+2g\left(\frac{\pi}{r}-\hbar\right)+g^2\right] & \\ \quad +\ \xi_{1,1}\,e^{\eta_1\,g}+\xi_{1,2}\,e^{\eta_2\,g} & \text{for all } g\in(\frac{\ell+\hbar}{2},\bar{g}]\end{cases} \tag{27}$$

B Solving the optimal prices within the plan

We solve for $\ell,\hbar$, given a value of the inaction thresholds $\underline{g},\bar{g}$. Consider a firm that has just reset its price plan, that *takes as given* the value of the plan-resetting thresholds. Let τ be the stopping time associated with a change in the plan (which occurs as either $\underline{g}$ or $\bar{g}$ is hit). The relevant objective function for this problem, until the next time the plan is changed, is:

$$\min_{\ell,\hbar}\mathbb{E}\left[\int_0^\tau e^{-rt}\,\min\left((\ell-g(t))^2,\,(\hbar-g(t))^2\right)\,dt\;\middle|\;g(0)=0\right]$$

Note that this is a quadratic minimization problem, with a convex objective function. The first order conditions for this problem for ℓ and $\hbar$ give equation (6).

Consider the expected discounted values that appear in the numerator and denominator of the expression that determines ℓ equation (6), as function of an arbitrary initial g:

$$n_\ell(g)\equiv\mathbb{E}\left[\int_0^\tau e^{-rt}\iota(t)\,g(t)\,dt\;\middle|\;g(0)=g\right]\quad,\quad d_\ell(g)\equiv\mathbb{E}\left[\int_0^\tau e^{-rt}\iota(t)\,dt\;\middle|\;g(0)=g\right]$$

We are interested in evaluating them at $g=0$ to get: $\ell=n_\ell(0)/d_\ell(0)$. We have:

Proposition 13. Consider the steady state problem with $r\to0$. Given two arbitrary thresholds $\underline{g}<\bar{g}$ defining the stopping times for a change of plan, the optimal low and high prices within the plan, ℓ and $\hbar$ respectively, solve the system of equations

$$\ell=\frac{n_\ell(0)}{d_\ell(0)}\quad\text{and}\quad\hbar=\frac{n_\hbar(0)}{d_\hbar(0)}$$

where $\hat{g}\equiv(\ell+\hbar)/2$, $\xi\equiv-\frac{2\pi}{\sigma^2}$ and

$$n_\ell(0)=\frac{\underline{g}}{\pi\xi}+\frac{\underline{g}^2}{2\pi}+C\left(1-e^{\xi\underline{g}}\right)\quad,\ \text{with } C=\frac{1}{\pi}\left(\frac{\frac{\underline{g}-\hat{g}}{\xi}+\frac{\underline{g}^2-\hat{g}^2}{2}+\frac{\hat{g}\xi+1}{\xi^2}\left(1-e^{\xi(\bar{g}-\hat{g})}\right)}{e^{\xi\underline{g}}-e^{\xi\bar{g}}}\right) \tag{28}$$

$$d_\ell(0)=\frac{\underline{g}}{\pi}+A\left(1-e^{\xi\underline{g}}\right)\quad,\ \text{with}\quad A=\frac{1}{\pi}\left(\frac{\underline{g}-\hat{g}+\frac{1-e^{\xi(\bar{g}-\hat{g})}}{\xi}}{e^{\xi\underline{g}}-e^{\xi\bar{g}}}\right) \tag{29}$$

$$n_\hbar(0)=F\left(1-e^{\xi\underline{g}}\right)\quad,\ \text{with}\quad F=\frac{1}{\pi}\left(\frac{\frac{\bar{g}-\hat{g}}{\xi}+\frac{\bar{g}^2-\hat{g}^2}{2}+\frac{\hat{g}\xi+1}{\xi^2}\left(1-e^{\xi(\bar{g}-\hat{g})}\right)}{e^{\xi\bar{g}}-e^{\xi\underline{g}}}\right) \tag{30}$$

$$d_{\hbar}(0) = E\left(1 - e^{\xi \underline{g}}\right) \quad , \text{ with } \quad E = \frac{1}{\pi}\left(\frac{\bar{g} - \hat{g} + \frac{1 - e^{\xi(\bar{g} - \hat{g})}}{\xi}}{e^{\xi \bar{g}} - e^{\xi \underline{g}}}\right) \tag{31}$$

ODE for $n(g)$. Likewise, for $r \to 0$ the function $n_\ell(g)$ solves the ode

$$0 = \begin{cases} g + n'\pi + \frac{\sigma^2}{2} n'' & \text{for } \underline{g} < g < \hat{g} \\ 0 + n'\pi + \frac{\sigma^2}{2} n'' & \text{for } \hat{g} < g < \bar{g} \end{cases} \tag{32}$$

We solve for $n(g)$ using the boundary conditions $n(\underline{g}) = n(\bar{g}) = 0$. This gives:

$$n_\ell(g) = \begin{cases} \frac{1}{\pi\xi}(\underline{g} - g) + \frac{1}{2\pi}(\underline{g}^2 - g^2) + C\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ D\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases} \tag{33}$$

To pin down C and D use value matching and smooth pasting at $g = \hat{g}$, to get equation (28)

ODE for $d(g)$. The function $d(g)$ solves the ode (where obvious, we omit the ℓ subscript in what follows). Focus on the steady state case, i.e. $r \to 0$, we get (since τ is finite so that $d(g) < \infty$)

$$0 = \begin{cases} 1 + d'\pi + \frac{\sigma^2}{2} d'' & \text{for } \underline{g} < g < \hat{g} \\ 0 + d'\pi + \frac{\sigma^2}{2} d'' & \text{for } \hat{g} < g < \bar{g} \end{cases} \tag{34}$$

We solve for $d(g)$ using the boundary conditions $d(\underline{g}) = d(\bar{g}) = 0$. This gives:

$$d_\ell(g) = \begin{cases} \frac{1}{\pi}(\underline{g} - g) + A\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ B\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases} \tag{35}$$

To pin down A and B use value matching and smooth pasting at $g = \hat{g}$, to get equation (29).

C Solution for the cumulated output effect

Using the price gap definition given in the text we have

$$\hat{m}(g) = -\mathbb{E}\left[\int_0^\tau \left(\hbar + (\ell - \hbar)\,\iota(t) - g(t)\right) dt \mid g(0) = g\right]$$

so that $\hat{m}(g)$ solves the following ode $0 = g - \ell + \hat{m}'\,\pi + \frac{\sigma^2}{2}\hat{m}''$ for $g \in (\underline{g}, \hat{g})$ and $0 = g - \hbar + \hat{m}'\,\pi + \frac{\sigma^2}{2}\hat{m}''$ for $g \in (\hat{g} < \bar{g})$ where the function $\hat{m}(g)$ is continuous and differentiable at $\hat{g}$, with boundary conditions $\hat{m}(\underline{g}) = \hat{m}(\bar{g}) = 0$. This gives

$$\hat{m}(g) = \begin{cases} \frac{g - \underline{g}}{\pi}\left(\ell - \frac{1}{\xi}\right) + \frac{\underline{g}^2 - g^2}{2\pi} + A\left(e^{\xi g} - e^{\xi \underline{g}}\right) & \text{for } \underline{g} < g < \hat{g} \\ \frac{g - \bar{g}}{\pi}\left(\hbar - \frac{1}{\xi}\right) + \frac{\bar{g}^2 - g^2}{2\pi} + B\left(e^{\xi g} - e^{\xi \bar{g}}\right) & \text{for } \hat{g} < g < \bar{g} \end{cases} \quad \text{where} \quad \xi \equiv -\frac{2\pi}{\sigma^2} \tag{36}$$

Use continuity and differentiability at $\hat{g}$ to solve for A and B. Continuity gives

$$(\hat{g}-\underline{g})\left(\ell-\frac{1}{\xi}\right)+\frac{\underline{g}^2-\bar{g}^2}{2}+\pi A\left(e^{\xi\hat{g}}-e^{\xi\underline{g}}\right)=(\hat{g}-\bar{g})\left(\hbar-\frac{1}{\xi}\right)+\pi B\left(e^{\xi\hat{g}}-e^{\xi\bar{g}}\right) \quad .$$

Differentiability gives:

$$B=A+\frac{\ell-\hbar}{\pi\xi e^{\xi\hat{g}}}$$

which gives a simple linear system of 2 equations in A, B.

D Proofs

Proof. (of Proposition 1) Part (i). The symmetry of the quadratic period losses implies $-\ell(-\pi)=\hbar(\pi)$ and $-\underline{g}(-\pi)=\bar{g}(\pi)$. Assuming differentiability we get $\ell'(-\pi)=\hbar'(\pi)$ and $\underline{g}'(-\pi)=\bar{g}'(\pi)$ which gives equation (7) at $\pi=0$.

Part (ii). With a slight abuse of notation let $v(g;\ell,\hbar,\pi)$ denote the value function with inflation explicitly added as a fourth argument. Totally differentiate the value matching condition in equation (3) with respect to inflation and evaluate at $\pi=0$:

$$\begin{aligned}\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial g}\frac{\partial \underline{g}}{\partial\pi}+\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\ell}\frac{\partial\ell}{\partial\pi}+\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\hbar}\frac{\partial\hbar}{\partial\pi}+\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\pi}\\ =\frac{\partial v(0;\ell,\hbar,\pi)}{\partial\ell}\frac{\partial\ell}{\partial\pi}+\frac{\partial v(0;\ell,\hbar,\pi)}{\partial\hbar}\frac{\partial\hbar}{\partial\pi}+\frac{\partial v(0;\ell,\hbar,\pi)}{\partial\pi}\end{aligned}$$

Notice that the three terms in the second line are equal to zero due to the optimality of ℓ and $\hbar$ and the symmetry of v with respect to zero inflation. The first term of the first equation is also zero due to the optimal choice of $\underline{g}$. We thus have

$$\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\ell}\frac{\partial\ell}{\partial\pi}+\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\hbar}\frac{\partial\hbar}{\partial\pi}+\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\pi}=0$$

The key to the proof is to note that $\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\hbar}=0$ since at $\underline{g}$ the price $\hbar$ is not affecting the return , so it has a zero impact on the value function for $g\in(\underline{g},0)$ and it also has a zero effect at $g=0$ due to optimality. Then we get

$$\frac{\partial\ell}{\partial\pi}=-\frac{\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\pi}}{\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\ell}}>0$$

since the derivative $\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\pi}$ is negative (lower inflation at $\underline{g}$ will lower the losses (lowers the expected losses for $g\in(\underline{g},0)$ and has a zero effect at $g=0$ due to symmetry). By the same logic $\frac{\partial v(\underline{g};\ell,\hbar,\pi)}{\partial\ell}>0$ since a higher price at $\underline{g}$ worsens the losses (the firm is about to set a lower, not a higher price at that point).

Part (iii). Symmetry of the period losses implies that $N(\pi)=N(-\pi)$. Assuming differentiability then gives $N'(\pi)=-N'(-\pi)$ which at $\pi=0$ gives the result in the proposition.

Proof. (of Lemma 1). Let the functions $a(g)$ and $d(g)$ denote respectively the numerator and denominator of equation (6) for the optimal price $\hbar$ in the case of $\pi = 0$. These functions solve the following o.d.e.'s and boundary conditions:

$$r\, a(g) = |g(t)| + \frac{\sigma^2}{2} a''(g) \text{ for all } g \in [-\bar{g}, \bar{g}]\,,\, g \neq 0 \quad , \quad a(-\bar{g}) = a(\bar{g}) = 0 \text{ and } a'(0) = 0$$
$$r\, d(g) = 1 + \frac{\sigma^2}{2} d'(g) \text{ for all } g \in [-\bar{g}, \bar{g}] \quad , \quad d(-\bar{g}) = d(\bar{g}) = 0\,.$$

First we develop the expressions for a. The function a must be symmetric around $g = 0$ so that $a(g) = a(-g)$ for all $g \in [0, \bar{g}]$, thus:

$$a(g) = \begin{cases} +g/r + A_1\, e^{\sqrt{2r/\sigma^2}\, g} + A_2\, e^{-\sqrt{2r/\sigma^2}\, g} & \text{if } g \in [0, \bar{g}] \\ -g/r + A_2\, e^{\sqrt{2r/\sigma^2}\, g} + A_1\, e^{-\sqrt{2r/\sigma^2}\, g} & \text{if } g \in [-\bar{g}, 0] \end{cases}$$

The boundary conditions $a'(0) = 0$ and $a(\bar{g}) = 0$ give:

$$1 = r\,(A_2 - A_1)\,\sqrt{2r/\sigma^2} \quad , \quad 0 = \bar{g} + r\, A_1\, e^{\sqrt{2r/\sigma^2}\, \bar{g}} + r\, A_2\, e^{-\sqrt{2r/\sigma^2}\, \bar{g}}$$

Hence

$$-\bar{g} - \frac{e^{-\sqrt{2r/\sigma^2}\, \bar{g}}}{\sqrt{2r/\sigma^2}} = r\, A_1\, \left(e^{\sqrt{2r/\sigma^2}\, \bar{g}} + e^{-\sqrt{2r/\sigma^2}\, \bar{g}} \right)$$
$$r\,(A_1 + A_2) = \frac{1}{\sqrt{2r/\sigma^2}} + 2rA_1 = \frac{1}{\sqrt{2r/\sigma^2}} - 2\frac{\bar{g} + \frac{e^{-\sqrt{2r/\sigma^2}\, \bar{g}}}{\sqrt{2r/\sigma^2}}}{e^{\sqrt{2r/\sigma^2}\, \bar{g}} + e^{-\sqrt{2r/\sigma^2}\, \bar{g}}}$$

since we are interested in:

$$r\, a(0) = r\,\,(A_1 + A_2) = \bar{g} \left[\frac{1}{\sqrt{2r/\sigma^2}\, \bar{g}} - 2\frac{1 + \frac{e^{-\sqrt{2r/\sigma^2}\, \bar{g}}}{\sqrt{2r/\sigma^2}\bar{g}}}{e^{\sqrt{2r/\sigma^2}\, \bar{g}} + e^{-\sqrt{2r/\sigma^2}\, \bar{g}}} \right]$$

For d we have, as shown in Alvarez et al. (2016b) that:

$$r\, d(0) = \frac{e^{\sqrt{2r/\sigma^2}\bar{g}} + e^{-\sqrt{2r/\sigma^2}\bar{g}} - 2}{e^{\sqrt{2r/\sigma^2}\bar{g}} + e^{-\sqrt{2r/\sigma^2}\bar{g}}}$$

We can write:

$$r\, a(0) = \bar{g}\, \frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}\right)} \quad , \quad r\, d(0) = \frac{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2}{e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}}}$$

Thus

$$\hbar = \frac{r\,a(0)}{r\,d(0)} = \bar{g}\;\rho(\phi) = \bar{g}\,\frac{e^{\sqrt{2\phi}} - e^{-\sqrt{2\phi}} - 2\sqrt{2\phi}}{\sqrt{2\phi}\,\left(e^{\sqrt{2\phi}} + e^{-\sqrt{2\phi}} - 2\right)}$$

The properties of ρ follow directly from this expression. The limit as $\phi \to 0$ follows by letting $x = \sqrt{2\phi}$ into the expression for $\hbar/\bar{g}$ and expanding the exponentials, canceling to obtain:

$$\frac{\hbar}{\bar{g}} = \frac{e^x - e^{-x} - 2x}{x\,[e^x + e^{-x} - 2]} = \frac{2(x + x^3/3! + x^5/5! + \cdots) - 2x}{x\,[2(1 + x^2/2 + x^4/4! + \cdots) - 2]} = \frac{(2/3!) + x^2(2/5!) + x^4(2/7!) + \cdots}{(2/2!) + x^2(2/4!) + x^4(2/6!) + \cdots}$$

Taking the limit $x \to 0$ we obtain $\hbar/\bar{g} = (2/3!)/(2/2!) = 1/3$. That ρ is decreasing it follows by inspection of the previous expression since each of the coefficients of x is smaller in the numerator. That the limit as $\phi \to 0$ of $\rho(\phi)\sqrt{2\phi} \to 1$ follows immediately since $\phi > 0$. This also implies that $\rho \to 0$ as $\phi \to \infty$. $\square$

Proof. (of Proposition 2). We first establish an intermediate result in the following lemma.

Lemma 2. Let $\hbar \geq 0$ be an arbitrary width for the reset prices. We derive an equation solving for the optimal inaction threshold $\bar{g}$, where we write the width of the price threshold as $\hbar = \gamma\,\bar{g}$ for a constant $0 \leq \gamma$. The optimal inaction threshold $\bar{g}$ must solve:

$$\eta^2 r \frac{\psi}{B} = \varphi\,(x\,;\,\gamma) \quad \text{with } x \equiv \eta\,\bar{g}\,, \text{ and } \varphi\,(x\,;\,\gamma) \equiv (1 - 2\,\gamma)\left(x^2 - 2\,x\,\frac{[e^x + e^{-x} - 2]}{[e^x - e^{-x}]}\right) \tag{37}$$

The function $\varphi(x;\gamma)$ is: *i)* strictly increasing in $x \geq 0$ for each $0 \leq \gamma < 1/2$, *ii)* strictly decreasing in γ for each $x > 0$, and *iii)* for $0 \leq \gamma < 1/2$, then $\lim_{x\to\infty} \varphi(x;\gamma)/x^2 = 1$, and $\lim_{x\to 0} \varphi(x;\gamma)/\left(x^4/12\right) = 1$.

Using Lemma 2 we obtain the function κ by simply replacing ρ into φ and using that $x^2/2 \equiv (\eta\bar{g})^2/2 = r\,\bar{g}^2/\sigma^2 \equiv \phi$, where ϕ is defined in Proposition 1. Since $\varphi(x,\gamma)$ is increasing in x and decreasing in γ, and since ρ is decreasing in γ, then κ is strictly increasing in x. Since $\rho'(0)$ is finite, then we can just substitute the limit value of $\rho(0) = 1/3$ and obtain $\kappa(x) = (1 - 2/3)x^4/12 + o(x^4) = x^4/36 + o(x^4)$. For large x we established that $\rho(x) \to 0$ and hence $\lim_{x\to\infty} \kappa(x)/x^2 = \lim_{x\to\infty} \varphi(x)/x^2 = 1$. $\square$

Proof. (of Lemma 2). The solution of the value function in inaction is given by the sum of a particular solution and the solution to the homogenous function. The particular solution $v^p(g)$ is:

$$v^p(g) = \frac{B}{r}\left[g^2 + \hbar^2 - 2|g|\hbar + \frac{\sigma^2}{r}\right]$$

and the homogenous solutions $v^f(g)$, for $g > 0$ is:

$$v^f(g) = A_1\,e^{-\eta\,g} + A_2\,e^{\eta\,g} \quad \text{where } \eta = \sqrt{2\,r/\sigma^2} \tag{38}$$

Thus the solution of v is:

$$v(g;\hbar) = \begin{cases} (B/r)g^2 - (B\,2\,\hbar/r)\,g + (B/r)\left[\hbar^2 + \frac{\sigma^2}{r}\right] & \\ +A_1\,e^{\sqrt{2\,r/\sigma^2}\,g} + A_2\,e^{-\sqrt{2\,r/\sigma^2}\,g} & \text{for all } g \in (0,\bar{g}] \\ (B/r)g^2 + (B\,2\,\hbar/r)\,g + (B/r)\left[\hbar^2 + \frac{\sigma^2}{r}\right] & \\ +A_2\,e^{\sqrt{2\,r/\sigma^2}\,g} + A_1\,e^{-\sqrt{2\,r/\sigma^2}\,g} & \text{for all } g \in [-\bar{g},0) \end{cases} \tag{39}$$

Value matching and smooth pasting are:

$$A_1 + A_2 + \psi = (B/r)\bar{g}^2 - (B\,2\,\hbar/r)\,\bar{g} + A_1\,e^{\sqrt{2\,r/\sigma^2}\,\bar{g}} + A_2\,e^{-\sqrt{2\,r/\sigma^2}\,\bar{g}} \tag{40}$$

$$0 = 2(B/r)\bar{g} - (B\,2\,\hbar/r) + \sqrt{2\,r/\sigma^2}\left[A_1\,e^{\sqrt{2\,r/\sigma^2}\,\bar{g}} - A_2\,e^{-\sqrt{2\,r/\sigma^2}\,\bar{g}}\right] \tag{41}$$

If v is differentiable at $g = 0$, then equation (5) implies:

$$\sqrt{2\,(r/\sigma^2)}\,[A_1\, - A_2\,] = B\,2\,(\hbar/r) \tag{42}$$

For $r > 0$ we can rewrite this system as:

$$a_1 + a_2 + r\frac{\psi}{B} = (\bar{g} -\, 2\,\hbar)\,\bar{g} + a_1\,e^{\eta\,\bar{g}} + a_2\,e^{-\eta\,\bar{g}} \tag{43}$$

$$0 = 2\,(\bar{g} -\, \hbar) + \eta\left[a_1\,e^{\eta\,\bar{g}} - a_2\,e^{-\eta\,\bar{g}}\right] \tag{44}$$

$$a_1\, - a_2 = 2\;\hbar/\eta \tag{45}$$

Solving for a_1 and replacing it we get:

$$r\frac{\psi}{B} = (\bar{g} -\, 2\,\hbar)\,\bar{g} + a_2\left(\,e^{\eta\,\bar{g}} + e^{-\eta\,\bar{g}} - 2\right) + 2\,\hbar\left(\frac{e^{\eta\,\bar{g}} - 1}{\eta}\right) \quad , \quad a_2 = -2\;\frac{\bar{g} + \,\hbar\,(e^{\eta\,\bar{g}} - 1)}{\eta\,\left[e^{\eta\,\bar{g}} - e^{-\eta\,\bar{g}}\right]}$$

Solving for $\bar{g}$ we get

$$r\frac{\psi}{B} = (\bar{g} - 2\hbar)\,\bar{g} + 2\hbar\left(\frac{e^{\eta\bar{g}} - 1}{\eta}\right) - \frac{[e^{\eta\bar{g}} + e^{-\eta\bar{g}} - 2]}{(e^{\eta\bar{g}} - e^{-\eta\bar{g}})}\,2\left[\frac{\bar{g}}{\eta} + \,\hbar\,\left(\frac{e^{\eta\bar{g}} - 1}{\eta}\right)\right] \tag{46}$$

Thus we can define

$$\eta^2 r\frac{\psi}{B} = \varphi\,(x;\gamma) \quad \text{with } x \equiv \eta\,\bar{g}\,,\, \hbar \equiv \gamma\,\bar{g}$$

and $\varphi(\cdot)$ defined as

$$\varphi\,(x;\gamma) \equiv x\,(x - 2\,\gamma\,x) +\, 2\,\gamma\,x\,(e^x - 1) - 2\frac{[e^x + e^{-x} - 2]}{[e^x - e^{-x}]}\,(x + \gamma\,x\,(e^x - 1)) \tag{47}$$

Rewrite the first term as: $x\,(x - 2\,\gamma\,x) = x^2\,(1 - 2\,\gamma)$ and collecting the remaining terms we have (after simple algebra):

$$2\,\gamma\,x\,(e^x-1)\left[1-\frac{e^x+e^{-x}-2}{e^x-e^{-x}}\right]-2\left[\frac{e^x+e^{-x}-2}{e^x-e^{-x}}\right]\,x=(2\,\gamma\;-1)\;\left[\frac{e^x+e^{-x}-2}{e^x-e^{-x}}\right]\;2x$$

Thus we can write:

$$\varphi\,(x;\gamma)\equiv(1-2\,\gamma)\left(x^2-2\frac{[e^x+e^{-x}-2]}{[e^x-e^{-x}]}x\right)$$

To see that φ is increasing in x note that:

$$\frac{\partial\varphi\,(x;\gamma)}{\partial x}\equiv(1-2\,\gamma)\left(2\frac{[e^x+e^{-x}-2]}{[e^x-e^{-x}]}\right)\left(x\frac{[e^x+e^{-x}]}{[e^x-e^{-x}]}-1\right)\geq 0$$

since

$$2\,\frac{[e^x+e^{-x}-2]}{[e^x-e^{-x}]}=2\,\frac{(e^x-1)\,(1-e^{-x})}{[e^x-e^{-x}]}\geq 0\;\;\text{and}\quad x\,\frac{[e^x+e^{-x}]}{[e^x-e^{-x}]}=\frac{x+x^3/2!+x^5/4!+x^7/6!+\cdots}{x+x^3/3!+x^5/5!+x^7/7!+\cdots}\geq 1$$

To see that $\lim_{x\to\infty}\varphi\,(x;\gamma)=\infty$ note that:

$$\lim_{x\to\infty}\frac{\varphi\,(x;\gamma)}{x}=\lim_{x\to\infty}x-2\lim_{x\to\infty}\frac{[e^x+e^{-x}-2]}{[e^x-e^{-x}]}=\infty-2=\infty\;\text{ and thus: }\;\lim_{x\to\infty}\frac{\varphi\,(x;\gamma)}{x^2}=1\;.$$

Finally to obtain the expansion for small x we write:

$$\begin{aligned}\frac{\varphi\,(x;\gamma)}{(1-2\gamma)}&=x^2-2\frac{[e^x+e^{-x}-2]}{[e^x-e^{-x}]}x=x^2\left(1-2\,\frac{1}{x}\,\left[\frac{x^2/2+x^4/4!+\cdots}{x+x^3/3!+x^5/5!+\cdots}\right]\right)\\&=x^2\left(\frac{x^4/3!+x^6/5!+\cdots-x^4(2/4!)-x^6(2/6!)+\cdots}{x^2+x^4/3!+x^6/5!+\cdots}\right)=x^4\frac{1}{12}+o(x^4)\end{aligned}$$

□

Proof. (of Proposition 3) Fix a sequence of stopping times in the firm's sequence problem. The firm's expected losses within the plan are

$$\min_x\mathbb{E}\left(B\int_0^\tau e^{-rt}(x-g(t))^2\;\;dt\,\middle|\,g(0)=0\right)\quad\text{with}\quad x=\{\ell,\hbar\}$$

where τ is the length of the plan and $g(t)\equiv p^*(t)-p^*(0)$. Recall that for $\pi>0$ all plans begin with $g=0$ and price ℓ, and switch to the high price $\hbar$ when $g\in(\hat{g},\bar{g})$, where $\hat{g}\equiv(\ell+\hbar)/2$. Let us consider the steady state case where $r\to 0$ rewrite

$$B\min_{\ell,\hbar}\int_0^\tau\mathbb{E}\left(\iota(t)(\ell^2+g^2(t)-2g(t)\ell)+(1-\iota(t))(\hbar^2+g^2(t)-2g(t)\hbar)\,\middle|\,g(0)=0\right)\;dt$$

where $\iota(t)$ is an indicator function equal to 1 if $\underline{g}<g(t)<\hat{g}$ and zero otherwise. Computing the expectations using the law of motion for $g(t) = g(0) + \pi t + \sigma \int_0^t dW(s)$ we get

$$B \min_{\ell,\hbar} \int_0^{\tau} \left(\iota(t)(\ell^2 + \pi^2 t^2 - 2\pi\, t\ell + \sigma^2 t) + (1-\iota(t))(\hbar^2 + \pi^2 t^2 - 2\pi\, t\hbar + \sigma^2 t) \right) \, dt$$

We conjecture (and later verify) that the optimal rule is linear in inflation, so that $\ell = \pi\tilde{\ell}$, $\hbar = \pi\tilde{\hbar}$ which implies $\hat{g}/\pi = (\tilde{\ell} + \tilde{\hbar})/2 \equiv \hbar$. We can rewrite the problem as

$$\pi^2 B \min_{\tilde{\ell},\tilde{\hbar}} \int_0^{\tau} \left(\iota(t)(\tilde{\ell}^2 + t^2 - 2t\tilde{\ell} + \frac{\sigma^2}{\pi^2} t) + (1-\iota(t))(\tilde{\hbar}^2 + t^2 - 2\, t\tilde{\hbar} + \frac{\sigma^2}{\pi^2} t) \right) \, dt$$

For $\pi \to \infty$ the σ/π terms becomes infinitesimal, thus $\tilde{\ell}, \tilde{\hbar}$ solve the following deterministic problem (we omit the $\pi^2 B$ scaling term)

$$\min_{\tilde{\ell},\tilde{\hbar}} \int_0^{\tau_1} \left(\tilde{\ell}^2 + t^2 - 2\tilde{\ell}t \right) \, dt + \int_{\tau_1}^{\tau} \left(\tilde{\hbar}^2 + t^2 - 2\, \tilde{\hbar}\, t \right) \, dt \tag{48}$$

where τ_1 is the time elapsed until $g(t)$ hits $\hat{g}$ starting from $g(0) = 0$, or $\tau_1 = \frac{\ell + \hbar}{2\pi} = \frac{\tilde{\ell} + \tilde{\hbar}}{2}$. Using $\tau_1 = \hbar$ in the first order conditions for the optimal ℓ and $\hbar$ in equation (6) immediately gives equation (11) which verifies the conjecture that ℓ and $\hbar$ are linear in π.[28]

Let's us now turn to the optimal choice of τ, the duration of a plan. In the deterministic problem obtained when $\pi^2/\sigma^2 \to \infty$ the firm optimal choice of τ solves (again we assume $r \to 0$)

$$\min_{\tau} \frac{1}{\tau} \left(\psi/B + \int_0^{\tau/2} \left(\frac{\tau\pi}{4} - \pi t \right)^2 \, dt + \int_{\tau/2}^{\tau} \left(\frac{\tau\pi 3}{4} - \pi t \right)^2 \, dt \right)$$

symmetry of the period losses and optimal prices immediately gives

$$\min_{\tau} \frac{1}{\tau} \left(\psi/B + 2 \int_0^{\tau/2} \left(\frac{\tau\pi}{4} - \pi t \right)^2 \, dt \right)$$

The first order condition gives the optimal duration of each price within the plan $\frac{\tau}{2} = \left(\frac{3\psi\, \pi^{-2}}{B} \right)^{\frac{1}{3}}$ which shows the elasticity of the frequency of price changes, $1/\tau$ w.r. to inflation is 2/3, as in the Sheshinski and Weiss (1977) model.

Proof. (of Proposition 4). First we derive the invariant distribution of desired prices, whose density we denote by $f(g)$. Recall $dg(t) = \pi dt + \sigma dW$. The Kolmogorov forward equation gives $0 = \pi f' + (\sigma^2/2) f''$. Given the repricing rule with boundaries $(\underline{g}, \bar{g})$ and return point $g^* \in (\underline{g}, \bar{g})$. Note $g^* = 0$ in our model as the desired normalized price is zero at the beginning of a new plan. For $g \in (\underline{g}, g^*)$, the density $f(g)$ solving ODE and boundary $f(\underline{g}) = 0$ is $f(g) = A \left(e^{-\xi \underline{g}} - e^{-\xi g} \right)$ with $\xi \equiv -\frac{2\pi}{\sigma^2}$. For $g \in (g^*, \bar{g})$, the density $f(g)$ solving ODE, boundary $f(\bar{g}) = 0$, and continuity at g^* is $f(g) = C \left(e^{-\xi g} - e^{-\xi \bar{g}} \right)$ with $C \equiv A \frac{e^{-\xi \underline{g}} - e^{-\xi g^*}}{e^{-\xi g^*} - e^{-\xi \bar{g}}}$.

[28] The expressions for the optimal ℓ and $\hbar$ can equivalently be obtained taking first order conditions of equation (48) (note that the derivatives of the extremes of integration will cancel out at $\tau_1 = \hbar$).

Use that the density integrates to 1 to find A: $1 = \int_{\underline{g}}^{g^*} f(g)\, dg + \int_{g^*}^{\bar{g}} f(g)\, dg$, after simple algebra we get $A = \left((g^* - \underline{g}) e^{-\xi \underline{g}} - (\bar{g} - g^*) \frac{e^{-\xi \underline{g}} - e^{-\xi g^*}}{e^{-\xi g^*} - e^{-\xi \bar{g}}} e^{-\xi \bar{g}} \right)^{-1}$. Next we compute the number of plan changes per unit of time. Let the function $\mathcal{T}(\tilde{p})$ the expected time until g first reaches $\bar{g}$ or $\underline{g}$. The average number of plan adjustments, denoted by $N_p = 1/\mathcal{T}(0)$. The function $\mathcal{T}(g)$ satisfies the o.d.e.: $0 = 1 - \pi\, \mathcal{T}'(g) + \frac{\sigma^2}{2} \mathcal{T}''(g)$ with boundary conditions $\mathcal{T}(\underline{g}) = \mathcal{T}(\bar{g}) = 0$, where $\xi \equiv -\frac{2\pi}{\sigma^2}$. This gives $\mathcal{T}(g) = A_0 + \frac{g}{\pi} + A_1 e^{-\xi g}$ where $A_1 = -\frac{\bar{g} - \underline{g}}{\pi \left(e^{-\xi \bar{g}} - e^{-\xi \underline{g}} \right)}$, $A_0 = -A_1 e^{-\xi \bar{g}} - \frac{\bar{g}}{\pi}$ which gives equation (13).

Proof. (of Proposition 5) Let $T(g)$ be the expected time until the next change of price plan, i.e. until $|g_n|$ reaches $\bar{g}$. We can index the state by $i = 0, \pm 1, ..., \pm \bar{n}$. We have the discrete time version of the Kolmogorov backward equation:

$$T_i = \Delta + \frac{1}{2} \left[T_{i+1} + T_{i-1} \right] \text{ for all } i = 0, \pm 1, \pm 2, ..., \pm (\bar{n} - 1) \ .$$

and at the boundaries we have $T_{\bar{n}} = T_{-\bar{n}} = 0$. We use a guess a verify strategy, guessing a solution of the form:

$$T_i = a_0 + a_2\, i^2 \text{ for all } i = 0, \pm 1, \pm 2, ..., \pm \bar{n} \ .$$

for some constants a_0, a_2. Inserting this into the KBE we obtain

$$a_0 + a_2\, i^2 = \Delta + \frac{1}{2} \left[a_0 + a_2 (i+1)^2 + a_0 + a_2 (i-1)^2 \right] \text{ for all } i = 0, \pm 1, \pm 2, ..., \pm (\bar{n} - 1) \ .$$

so that $a_2 = -\Delta$. Using this value, into the equation for the boundary condition, we get:

$$a_0 - \Delta (\bar{n})^2 = 0 \,, \implies a_0 = \Delta (\bar{n})^2 \ .$$

and since $\bar{n} \sqrt{\Delta}\, \sigma = \bar{g}$ and $T_0 = a_0$ we have:

$$T_0 = a_0 = \Delta (\bar{n})^2 = \Delta \left(\frac{\bar{g}}{\sqrt{\Delta} \sigma} \right)^2 = \left(\frac{\bar{g}}{\sigma} \right)^2 = \frac{1}{N_p}$$

□

Proof. (of Proposition 6) We now derive formally the expression that give the inequalities described in equation (15). The proof focus on the case $\bar{n} \geq 2$ (see the discussion following equation (14)), which is equivalent to $N_p \Delta \leq 1/4$. We first obtain an upper bound on the number of price changes within a price plans. We first state the 2 parts of the inequality in two lemmas, and then prove each of them.

Lemma 3. *Let $\Delta > 0$ be the length of the time period, and $\bar{g}$ be the width of the inaction band. Let n_w be the expected number of price changes during a price plan. We have:*

$$n_w \leq \frac{2}{\sqrt{\Delta}} \frac{1}{\sqrt{N_p}} - \frac{1}{2} \qquad (49)$$

Hence the total number of price changes per unit of time within price plans, denoted by N_w, and equal to $n_w N_p$, satisfies:

$$N_w \leq 2\sqrt{\frac{N_p}{\Delta}} - \frac{N_p}{2}$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/(\sigma\sqrt{\Delta}) = 1/\sqrt{N_p \Delta}$ is an integer larger than 2.

It is straightforward to obtain an upper bound on expected number of all price changes $N = N_p + N_w$. We obtain:

$$N = N_w + N_p \leq 2\sqrt{\frac{N_p}{\Delta}} + \frac{N_p}{2}$$

Proof. (of Lemma 3) We first start with a lemma that relates the expected number of up-crossings within a plan to the expected number of plans.

LEMMA 4. In the discrete-time discrete-state model we have: $n_w = 2\, E\left[U(\tau)\right] - \frac{1}{2}$.

Proof. (of Lemma 4). We relate the price changes within a price plan to the number of up-crossing, $U(\tau)$, and number of down-crossings, $D(\tau)$, between $g = 0$ and $g = \sqrt{\Delta}\,\sigma$. We assume that the optimal policy within a plan that has just started at $g(0) = 0$ has a price $\hbar > 0$ if $g \geq \sqrt{\Delta}\,\sigma$ and price $-\hbar < 0$ if $g \leq 0$. We focus on upcrossing where g goes from $g(t) = 0$ to $g(t + \Delta) = \sqrt{\Delta}\,\sigma$, so there is a price increase. For a down crossing, $g(t)$ goes from $g(t) = \sqrt{\Delta}\,\sigma$ to $g(t + \Delta) = 0$, so there is price decrease. We will denote by $U(\tau)$ the number of up-crossings, and $D(\tau)$ the number of down-crossings at the time where the price plan ends. Notice that in any path from $g(0) = 0$ to $g(\tau) = +\bar{g}$ there are $U(\tau) = D(\tau) + 1$ up-crossings, while in any path where $g(\tau) = -\bar{g}$ there are $U(\tau) = D(\tau)$ up-crossings. Since the number of price changes is the sum of up-crossings plus down-crossings, and since the price plan is as likely to end with $g(\tau) = \bar{g}$ as well as with $g(\tau) = -\bar{g}$, thus

$$\Pr\left\{U(\tau) - D(\tau) = 1\right\} = \Pr\left\{U(\tau) - D(\tau) = 0\right\} = \frac{1}{2}\ .$$

and hence: $n_w = 2\, E\left[U(\tau)\right] - \frac{1}{2}$. This finishes the proof of the lemma.

We now return to the proof of Lemma 3 and use Doob's inequality for the expected number of up-crossings obtaining:

$$(b - a)E\left[U(\tau)\right] \leq \sup_{t=0,\Delta,2\Delta,\ldots} \left(a + E\left[\,|g(t)|\,\right]\right)$$

so that using the values $a = 0$, $b = \sqrt{\Delta}\,\sigma$ and that $E\left[\,|g(t)|\,\right] \leq \bar{g}$ we have

$$E\left[U(\tau)\right] \leq \frac{\bar{g}}{\sqrt{\Delta}\,\sigma}$$

Hence:

$$n_w = 2\, E\left[U(\tau)\right] - \frac{1}{2} \leq 2\,\frac{\bar{g}}{\sqrt{\Delta}\,\sigma} - \frac{1}{2} = \frac{2}{\sqrt{\Delta}}\,\frac{1}{\sqrt{N_p}} - \frac{1}{2}\ . \ \square$$

Next we obtain a lower bound on the number of price changes within a plan.

Lemma 5. The expected number of price changes per unit of time within a plan N_w has the following lower bound:

$$N_w \geq \frac{1}{\sqrt{\frac{\Delta}{N_p}} + \frac{\Delta}{2}\left[\frac{1+\sqrt{\Delta N_p}}{1-\sqrt{\Delta N_p}}\right]},$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/(\sigma\sqrt{\Delta}) = 1/\sqrt{N_p\Delta}$ is an integer larger than 2.

Proof. (of Lemma 5) The proof proceeds in several steps. First we define a stopping time that counts consecutive price changes, the first an increase of size $2\hbar$ and the second a decrease of $2\hbar$, starting from a normalized desired price $g = 0$ and ending in the same value $g = 0$. Call this event a cycle. Because of the Markovian nature of g and because it starts and ends at the same value then consecutive cycles are independent so that the expected number of cycles is, by the fundamental law of renewal theory, the inverse of the expected duration of such a cycle. We know that by construction each cycle has 2 price changes of the same absolute value, $2\hbar$. Second we decompose this into two events, whose expected values we compute separately. Third we use the fundamental theorem of renewal theory to compute the expected number of price changes per unit of time which do not involve a change in price plan. We use the following normalization for price changes within a plan:

$$p(t) = \begin{cases} p^*(t) + \hbar & \text{if } g(t) > 0 \\ p^*(t) - \hbar & \text{if } g(t) \leq 0 \end{cases} \tag{50}$$

Note that the normalization consists on charging $p(t) = p^*(t) - \hbar$ when $g(t) = 0$. The normalization affects the definition below, but not the final result.

1. Define the stopping times τ^u and τ^d as:

$$\tau^u = \min\left\{t : p(t) - p(t-\Delta) = +2\,\hbar\,,\ g(t) = \sqrt{\Delta}\sigma\,,\ g(0) = 0\,,\ t = \Delta, 2\Delta, ...\right\} \tag{51}$$

$$\tau^d = \min\left\{t : p(t) - p(t-\Delta) = -2\,\hbar\,, g(t) = 0\,,\ g(0) = \sqrt{\Delta}\sigma\,,\ t = \Delta, 2\Delta, ...\right\} \tag{52}$$

   In words τ^u is the time elapsed until the first price increase starting from a state where $g = 0$, i.e. at the beginning of a price plan. Instead τ^d is the time elapsed until the first price decrease starting from the state where $g = \sigma\sqrt{\Delta}$, i.e. after a price increase has just occurred. Note that at τ^d the state is the same as in the beginning of a price plan. The expected value of $\tau^u + \tau^d$ gives the expected value of a cycle of at least one price increase followed by a price decrease, within a price plan. In this cycle the initial state is equal to the final one, namely $g = 0$. Notice that in each cycle there are at least two price changes, one (or more) increases and one (or more) decreases. There could be more than two price changes because in each τ^u there could be price decreases and during each τ^d there can be price increases caused by changes of the plan.

2. We compute the expected value of τ^u and τ^d separately.

(a) We discuss how to compute $E[\tau^u]$. For this quantity we use the operator T^u, for which $T^u(0) = E[\tau^u]$. The operator T^u is the expected first time for which g goes from 0 to $\sqrt{\Delta}\,\sigma$, which coincides with a price increase, conditional on $g(0) = 0$. Note that there may be none or several plan changes before this event occurs. The function T_u solves:

$$T^u(i) = \Delta + \frac{1}{2}\left[T^u(i-1) + T^u(i+1)\right] \quad \text{for } i = -1, -2, ..., -\bar{n} + 1$$

which is a version of the backward Kolmogorov equation, and the boundary conditions: $T^u(-\bar{n}) = T^u(0)$, because when the price plan ends it is restarted at $g = 0$, or index $i = 0$, and $T^u(0) = \Delta + (1/2)T^u(-1)$, because at $g = \sqrt{\Delta}\sigma$, which is index $i = 1$ there is a price increase, and we stop counting time. We show that $T^u(i) = a + b\,i + c\,i^2$. First, the Kolmogorov Backward equation implies that $c = -\Delta$. We use this into the two boundary conditions. The boundary condition $T^u(-\bar{n}) = T^u(0)$ gives $a = a + b\bar{n} - \Delta(\bar{n})^2 = 0$ or $b = -\Delta(\bar{n})$. The boundary condition at $i = 0$ gives $a = \Delta + (1/2)[a - b - \Delta]$, or $a + b = \Delta$. These equations imply that $T^u(0) = a = \Delta - b = \Delta(1 + \bar{n})$.

(b) Now we discuss how to compute $E[\tau^d]$. For this quantity we use the operator T^d, for which $T^d(1) = E[\tau^d]$. The operator T^d is the expected time for which g goes from $\sqrt{\Delta}\,\sigma$ to 0, which coincides with a price decrease, conditional on $g(0) = \sqrt{\Delta}\,\sigma$. Note that there may be none or several price plan changes before this event occurs, as well as none, one, or more price increases. The function T^d solves:

$$T^d(i) = \Delta + \frac{1}{2}\left[T^d(i-1) + T^d(i+1)\right] \quad \text{for } i = 1, 2, ..., \bar{n} - 1$$

which is a version of the backward Kolmogorov equation, and the boundary conditions. At the top we have $T^d(\bar{n}) = T^u(0) + T^d(1)$, since at this point there is a price plan change which returns the process to $g = 0$ and thus there must be an increase in prices within a plan before we can have a decrease. The other boundary condition is $T^d(1) = \Delta + (1/2)T^d(2)$ which uses the fact that a price decrease within a price plan must occur when $g = \sqrt{\Delta}\sigma$ which correspond to the $i = 1$ index. In this event we stop counting time. We try a solution of the type $T^d(i) = \alpha + \beta\,i + \gamma\,i^2$. Using the Kolmogorov Backward equation we obtain that $\gamma = -\Delta$. Using the boundary condition at the top, as well as the solution for $T^u(0)$, we obtain:

$$\alpha + \beta\bar{n} - \Delta\bar{n}^2 = \Delta(1 + \bar{n}) + \alpha + \beta - \Delta$$

This implies that $\beta = \Delta(\bar{n}^2 + 1)/(\bar{n} - 1)$. The other boundary gives:

$$\alpha + \beta - \Delta = \Delta + (1/2)\left[\alpha + \beta 2 - \Delta 4\right]$$

or $\alpha = (1/2)\alpha$ which implies $\alpha = 0$. Hence we have

$$T^d(1) = \beta - \Delta = \Delta(\bar{n}^2 + 1 - \bar{n} + 1)/(\bar{n} - 1) = \Delta\bar{n} + \Delta\frac{2}{\bar{n} - 1}$$

3. Now we use the previous result to obtain the desired expression for N_w. First note that

$$T^u(0) + T^d(1) = E[\tau^u] + E[\tau^d] = 2\Delta\bar{n} + \Delta\frac{2 + \bar{n} - 1}{\bar{n} - 1} = 2\Delta\bar{n} + \Delta\frac{1 + \bar{n}}{\bar{n} - 1}$$

Because the cycles start and end at $g = 0$ and consecutive cycles are independent, we can use the Fundamental theorem of renewal theory. Hence the expected number of cycles per unit of time is $1/(E[\tau^u] + E[\tau^d])$. Also recall that in each cycle there are at least two price changes, hence the expected number of price changes N_w per unit of time is at least two times the (reciprocal of) expected duration of the cycle, i.e.:

$$N_w \geq \frac{2}{2\Delta\bar{n} + \Delta\frac{1+\bar{n}}{\bar{n}-1}} = \frac{1}{\Delta\bar{n} + \Delta\frac{1+\bar{n}}{2(\bar{n}-1)}} \quad .$$

Using $\sqrt{\Delta}\,\sigma\,\bar{n} = \bar{g}$ and $\bar{n} = \sqrt{1/(\Delta N_p)}$ we can write

$$N_w \geq \left(\sqrt{\frac{\Delta}{N_p}} + \frac{\Delta}{2}\left[\frac{1 + \sqrt{1/(\Delta N_p)}}{\sqrt{1/(\Delta N_p)} - 1}\right]\right)^{-1} \quad .$$

□

Proof. (of Proposition 7) We fix an interval $[0, T]$ and index each price-path in the interval by ω, so the prices for this path are denoted by $p(\omega, t)$ for each $t \in [0, T]$. We let μ_T be the measure of these sample paths. We will fix a path ω and define three concepts. First we define the set of prices observed in a interval of length T for a given price path ω: $\mathbb{P}(\omega) \equiv \{y : y = p(\omega, t) \text{ for some } t \in [0, T]\}$. Second we define the modal price in an interval of length T for a given price path ω, or the reference price:

$$p^{ref}(\omega) \equiv \text{ mode of } \mathbb{P}(\omega)$$

Third, we define the duration of the reference price as the time spent at the modal price in $[0, T]$ for a given sample path ω:

$$d^{ref}(\omega) \equiv \int_0^T 1_{\{p(\omega,t)=p^{ref}(\omega)\}}\, dt$$

Finally, the statistic $F(T, \alpha)$ measures the mass of price paths of length T for which the duration of the reference price is higher than αT:

$$F(T, \alpha) = \mu_T\left(\omega : d^{ref}(\omega) \geq \alpha\, T\right) \quad .$$

The proof proceeds by first defining a subset of the path at which the price spent at least αT of the time at the reference price. This will give a lower bound for F. The advantage is that this lower bound is easier to compute. Then we will show the proposition for the lower bound.

We first consider the most delicate case, i.e the continuous time case with $\Delta = 0$. We note that, without loss of generality given the symmetry in the model, we will consider that at the beginning $[0, T]$ the normalized desired price $g(0)$ is positive. Using the invariant distribution for the normalized desired prices, and conditioning that $g = g(0) > 0$ we have that it has density $f(g) = 2/\bar{g} - 2g/\bar{g}^2$. Fixing $g = g(0) > 0$ we can consider the path of price that will follow during $[0, T]$. If $0 < g(\omega, t) < \bar{g}$ for all $0 < t < \alpha T$ then the price will remain at $p(\omega, t) = p^*_-(\omega) + \hbar$ where $p^*_-(\omega)$ is the ideal price at the start of the current price plan corresponding to this price path. Thus if $\alpha > 1/2$ the reference price in this path is $p^*_-(\omega) + \hbar$. If otherwise for $g(\omega, t) = 0$ or $g(\omega, t) = \bar{g}$ at some $0 < t < \alpha T$, then the reference price may be a different number. If g reaches the upper bound there will be a new price plan. If g reaches zero then there will be a price change within the plan. Notice that in either of these two events it is possible that $p^*_-(\omega)$ will still be the reference price (depending of what happens subsequently), we will ignore this possibility so that we obtain a lower bound on F. We denote our lower bound as $\tilde{F}(T, \alpha)$ which is given by:

$$F(T, \alpha) \geq \tilde{F}(T, \alpha) \equiv \int_0^{\bar{g}} \Pr\left\{0 < B(t) < \frac{\bar{g}}{\sigma} \text{ for all } t \in [0, \alpha T] \,\middle|\, B(0) = \frac{g}{\sigma}\right\} f(g)\, dg$$

where B is a standard Brownian motion (BM). We compute the lower bound for the probability of a BM not hitting a barrier as follows. First we denote this probability as:

$$Q\left(\alpha T, \frac{\bar{g}}{\sigma} \,\middle|\, \frac{g}{\sigma}\right) \equiv \Pr\left\{0 < B(t) < \frac{\bar{g}}{\sigma} \text{ for all } t \in [0, \alpha T] \,\middle|\, B(0) = \frac{g}{\sigma}\right\}$$

so we can write:

$$\tilde{F}(T, \alpha) = \int_0^{\bar{g}} Q\left(\alpha T, \frac{\bar{g}}{\sigma} \,\middle|\, \frac{g}{\sigma}\right) f(g)\, dg$$

We break the interval $[0, \bar{g}]$ in three parts. Let $n \geq 2$ be an integer and let $a = \frac{1}{n}\frac{\bar{g}}{\sigma}$ so that:

$$Q\left(\alpha T, \frac{\bar{g}}{\sigma} \,\middle|\, \frac{g}{\sigma}\right) \geq \begin{cases} 0 & \text{if } \frac{g}{\sigma} \in [0\,,\, a) \\ Q\left(\alpha T, 2\,a \,\middle|\, a\right) & \text{if } \frac{g}{\sigma} \in [a\,,\, a(n-1)] \\ 0 & \text{if } \frac{g}{\sigma} \in (a(n-1)\,,\, a\,n] \end{cases}$$

where the inequality for the middle range follows immediately by the assumption that $\bar{g}/\sigma = na$ for $n \geq 2$. The density for the first hitting time of either of two barriers for a BM, from which we can obtain Q, is as follows:

$$Q\left(\alpha T, n\,a \,\middle|\, a\right) = \frac{2\pi}{a^2 n^2} \sum_{j=0}^{\infty} (2j+1)\,(-1)^j \cos\left[\pi(2j+1)\frac{(n-2)}{2n}\right] \int_{\alpha T}^{\infty} \exp\left(-\frac{(2j+1)^2\,\pi^2\,t}{2n^2a^2}\right) dt$$

$$= \sum_{j=0}^{\infty} (-1)^j \cos\left[\pi(2j+1)\frac{(n-2)}{2n}\right] \frac{4}{(2j+1)\,\pi} \exp\left(-\frac{(2j+1)^2\,\pi^2\,\alpha T}{2n^2a^2}\right)$$

and for $n=2$ we get:

$$Q\left(\alpha T, 2\,a \,\middle|\, a\right) = \frac{2\pi}{a^2 4} \sum_{j=0}^{\infty} (2j+1)\,(-1)^j \int_{\alpha T}^{\infty} \exp\left(-\frac{(2j+1)^2\,\pi^2\,t}{8a^2}\right) dt$$

$$= \sum_{j=0}^{\infty} (-1)^j \frac{4}{(2j+1)\,\pi} \exp\left(-\left(\frac{(2j+1)\,\pi}{2}\right)^2 \frac{\alpha T}{2a^2}\right)$$

Clearly $Q\left(\alpha T, 2\,a \,\middle|\, a\right)$ is increasing in a, since for larger a the BM starts further away from the two barriers. Using the Gregory-Leibniz's formula for π, for any $\alpha T > 0$ we have:

$$\lim_{a\to\infty} Q\left(\alpha T, 2\,a \,\middle|\, a\right) = \sum_{j=0}^{\infty} (-1)^j \frac{4}{(2j+1)\,\pi} = 1$$

Thus, for any $\delta > 0$ we can find an A_δ such that $Q\left(\alpha T, 2\,a \,\middle|\, a\right) \geq 1-\delta$ for $a > A_\delta$.

We also have:

$$\int_0^a f(g)dg + \int_{(n-1)a}^{na} f(g)dg = \frac{2}{n}$$

Thus

$$\tilde{F}(T,\alpha) \geq \left[1-\frac{2}{n}\right] Q\left(\alpha T, 2\,a \,\middle|\, a\right) \geq \left[1-\frac{2}{n}\right](1-\delta) \geq 1-\epsilon$$

Hence setting n and a large enough we show the desired result. In particular set n and δ to satisfy

$$\frac{n}{n-2} < \frac{1}{1-\epsilon} \text{ and } 0 < \delta < 1 - \frac{n}{n-2}(1-\epsilon)$$

and let G to be higher than $G \geq nA_\delta$.

Now we briefly comment on the differences with the discrete time model with period length $\Delta > 0$. The summary is that steps of the proof are identical. There are only two minor differences. One is that the invariant distribution of g takes a discrete number of values, but it is also triangular, so the calculations that involve f are virtually identical. The second difference is that probability $Q\left(\alpha T, 2\,a \,\middle|\, a\right)$ does not take the expression we used above.

Nevertheless, it is easy to see that for any $\Delta > 0$ one can in fact chose a large enough so that $Q\left(\alpha T, 2\,a \,\middle|\, a\right) = 0$. For instance, if $a/\left(\sigma\sqrt{\Delta}\right) > T\alpha/\Delta$ or equivalently if $a \geq T\,\alpha\,\sigma/\sqrt{\Delta}$ then $Q\left(\alpha T, 2\,a \,\middle|\, a\right) = 0$ since it will take at least $a/\left(\sigma\sqrt{\Delta}\right)$ consecutive ups (or downs) to hit either of the barriers. Thus in the discrete time case with length Δ we can take $n > 2/\epsilon$ (using the expression derived from the continuous case) and $G/\sigma \geq n\,T\,\alpha/\sqrt{\Delta} \geq 2\,T\,\alpha/\left(\sqrt{\Delta}\,\epsilon\right)$. $\square$

Proof. (of Proposition 8). We let $\{\tau_i\}_{i=0}^{\infty}$ denote the times at which the plans in the PL model change which are also the times at which the prices change in the GL model for the path $\mathbf{p}^*$. For each $i \geq 0$ define the interval $[t_i\,,\,t_{i+1}] \equiv [\tau_i\,,\,\tau_{i+1}] \cap [T_1\,,\,T_2] \neq \emptyset$. In the GL model there is only one price in the interval $[t_i\,,\,t_{i+1}]$. In the PL model there are, at most two difference prices in the interval $[t_i\,,\,t_{i+1}]$. Thus the duration of mode in the interval $[t_i\,,\,t_{i+1}]$ is at most the same for the PL model than the GL model, and at least half for the PL than the GL model, where the minimum duration is achieved if each of the two prices in the PL model have exactly the same duration. Thus, defining the $D^{PL}[a,b;\mathbf{p}^*]$ and $D^{GL}[a,b;\mathbf{p}^*]$ as the duration of the mode on the interval $[a,b]$ for path $\mathbf{p}^*$. We thus have: $D^{PL}[t_i\,,\,t_{i+1};\mathbf{p}^*] \leq D^{GL}[t_i\,,\,t_{i+1};\mathbf{p}^*] \leq 2D^{PL}[t_i\,,\,t_{i+1};\mathbf{p}^*]$ for all i.

Since $\pi > 0$, but small, then the prices that corresponds to different intervals $[t_i\,,\,t_{i+1}]$ with non-empty intersection with $[T_1\,,\,T_2]$ are different, both in the PL model and in the GL model. Thus the duration of the mode in $[T_1, T_2]$ can be computed as the highest duration across all intervals $[t_i, t_{i+1}]$. Thus for reference prices we define: $D^{PL}[T_1, T_2;\mathbf{p}^*] \equiv \max_i D^{PL}[t_i\,,\,t_{i+1};\mathbf{p}^*]$ and $D^{GL}[T_1, T_2;\mathbf{p}^*] \equiv \max_i D^{GL}[t_i\,,\,t_{i+1};\mathbf{p}^*]$. Taking the maximum in the previous inequality we have: $\max_i D^{PL}[t_i\,,\,t_{i+1};\mathbf{p}^*] \leq \max_i D^{GL}[t_i\,,\,t_{i+1};\mathbf{p}^*] \leq 2\max_i D^{PL}[t_i\,,\,t_{i+1};\mathbf{p}^*]$, which gives the desired result.$\square$

Proof. (of Proposition 9). Within a plan the size of price increases is $\Delta p = \hbar - \ell = \frac{\bar{g}}{3} - \left(-\frac{\bar{g}}{3}\right) = \frac{2}{3}\bar{g}$, while the size of price decreases is $\Delta p = -\frac{2}{3}\bar{g}$. Thus the mean absolute value of price changes within the plan is $\mathbb{E}\left[|\Delta p|\right] = \frac{2}{3}\bar{g}$. Next, the size of price increases between plans is $\Delta p = \frac{1}{3}\bar{g}$ (plan ends hitting upper barrier) while the size of price decreases is: $\Delta p = -\bar{g}$ (plan ends hitting lower barrier). Thus the mean absolute value of price changes between plans os $\mathbb{E}\left[|\Delta p|\right] = \frac{2}{3}\bar{g}$.

Proof. (of Proposition 10). Note that the invariant density function when $\pi = 0$ is triangular,[29] namely $f(g) = (\bar{g} - |g|)/\bar{g}^2$ for $g \in (-\bar{g}, \bar{g})$. Recall that at $\pi = 0$ then $\underline{g} = -\bar{g}$, $\ell = -\hbar$, so that $\hat{g} = 0$, and $\hbar = \bar{g}/3$. The ODE solved by $\hat{m}$ becomes $0 = g - \ell + \frac{\sigma^2}{2}\hat{m}''$ for $g \in (\underline{g}, 0)$ and $g - \hbar + \frac{\sigma^2}{2}\hat{m}''$ for $g \in (0, \bar{g})$ where the function $\hat{m}(g)$ is continuous and differentiable at $\hat{g}$, with boundary conditions $\hat{m}(-\bar{g}) = \hat{m}(\bar{g}) = 0$. This gives

$$\hat{m}(g) = \frac{-g^2}{\sigma^2}\left(\frac{\bar{g}+g}{3}\right) \quad \text{for} \quad g \in (-\bar{g}, 0) \quad , \quad \hat{m}(g) = \frac{g^2}{\sigma^2}\left(\frac{\bar{g}-g}{3}\right) \quad \text{for} \quad g \in (0, \bar{g}) \tag{53}$$

Next we use equation (23) and integrate $\mathscr{M}'(0) = \int_{\underline{g}}^{\bar{g}} f'(g)\hat{m}(g)\,dg$. Simple analysis gives $\mathscr{M}'(0) = \bar{g}^2/(18\sigma^2)$ or, using that $N_p = \sigma^2/\bar{g}^2$ we write $\mathscr{M}'(0) = 1/(18N_p)$. $\square$

[29]This follows since the invariant density solves the Kolmogorov forward equation: $f''(g) = 0$ which immediately implies the linearity, with the boundary conditions $f(\bar{g}) = 0$.

Proof. (of Proposition 12). The proof is immediate using the result of Lemma **??** and equation (24). □

Proof. (of Proposition 11). Using the expression for $f(g)$ given above we obtain

$$\tilde{\Theta}(\delta) = \hbar \left[1 - \frac{\delta^2}{2\,\bar{g}^2} - \frac{(\bar{g}-\delta)^2}{\bar{g}^2}\right] \tag{54}$$

Note that one can understand this simple expression by computing the fraction of firms with normalized desired price g that the shock shifts from a negative to a positive desired price. For a small δ this fraction is $f(0)\,\delta$. The effect on price of this is $2\,\hbar\,f(0)\,\delta$. Thus, we have:

$$\tilde{\Theta}(\delta) = \tilde{\Theta}(0) + \tilde{\Theta}'(0)\,\delta + o(\delta) = 2\,\hbar\,f(0)\,\delta + o(\delta) = 2\,\frac{\hbar}{\bar{g}}\,\delta + o(\delta)\,. \tag{55}$$

We notice that the first equality in equation (55) will hold for other cases, i.e. even if f is different (see for instance the extension that assumes costly price changes within the plan, developed in Appendix G). The proof of the proposition follows immediately from equation (54) after replacing the optimal value of $\hbar$ for $r \to 0$ or $\hbar = \bar{g}/3$. □

E Hazard rate of price changes

In this section we study the hazard rates of price changes and show that they are decreasing. We do this for two models. The first version is a model with price plans that change when the absolute value of the normalized desired price $|g|$ reaches a critical value, the threshold that we denote by $\bar{g}$. We refer to this version as the menu cost version, and we denote the hazard rate for a price with duration $t > 0$ as $h_{MC}(t)$. In Appendix F we also consider a version of the model where price plans are changed at (exogenous) exponentially distributed times, in which case we denote the hazard rate for price changes by $h_{exp}(t)$. In both cases we provide an analytical solution to the hazard rate of price changes. These analytical expressions depend on only one parameter, namely N_p: the expected number of plan changes per unit of time. In the benchmark price plan model, the expected number of price plan changes per unit of time has a simple expression $N_p = \bar{g}^2/\sigma^2$, an expression whose derivation and interpretation we return to in Section 3. In the version where price plans are changed at exponentially distributed times, N_p is simply the expected number of price plan changes per unit of time. Both hazard rates are downward slopping, very much so for low durations, behaving approximately as $1/2t$ for low t, and they asymptote to different constants. The asymptote for h_{MC} is a multiple of the number of price plan changes per year, namely $\pi^2/2\,N_p \approx 5\,N_p$. While the asymptote for in the exponential case is simply N_p. Appendix E.1 provides more information and the exact definition of the hazard rates, and on their analytical characterization. We summarize that analysis in the following proposition:

Proposition 14. The hazard rate h_{MC} for the baseline model with price plans is:

$$h_{MC}(t) = \sum_{m=1,3,5,\ldots}^{\infty} m^2 \, N_p \, \frac{\pi^2}{2} \, \theta(t, m; N_p) \quad \text{where} \tag{56}$$

$$\theta(t, m; N_p) \equiv \frac{e^{-t\, m^2 \, N_p \, \frac{\pi^2}{2}}}{\sum_{m'=1,3,5,\ldots}^{\infty} e^{-t\,(m')^2 \, N_p \, \frac{\pi^2}{2}}}$$

where for each $t > 0$, the $\theta(t, \cdot; N_p)$ are non-negative and add up to one over $m = 1, 3, 5, \ldots$. The hazard h_{MC} has the following properties:

$$h'_{MC}(t) < 0 \text{ for } t > 0\,, \ \lim_{t \to 0} h_{MC}(t) = \infty\,, \ \lim_{t \to 0} h_{MC}(t)\, t = \frac{1}{2}\,, \ \text{and} \ \lim_{t \to \infty} h_{MC}(t) = \frac{\pi^2}{2} N_p\,.$$

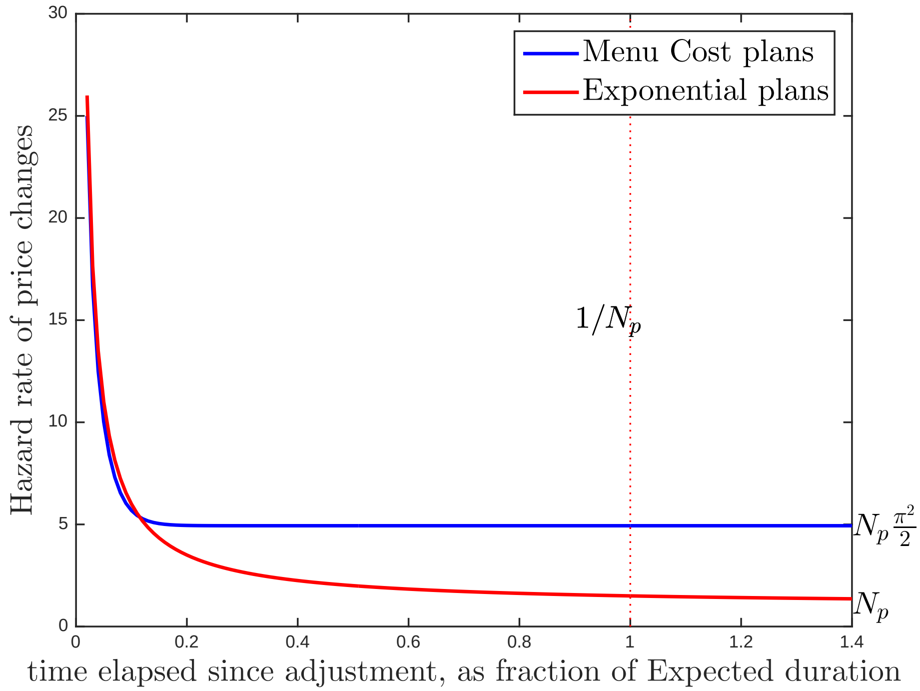
For the case with exponentially distributed price plans times we have:

$$h_{exp}(t) = N_p + \frac{1}{2\,t} \text{ for all } t > 0\,. \tag{57}$$

Figure 9 plots the two hazard rates. As explained in the proposition the hazard rate depends on one parameter, the expected number of plan changes, and hence $1/N_p$ is the expected time between price changes. In the figure we normalize N_p to one, so that duration, i.e. time, on the horizontal axis can be interpreted relative to the average duration of a plan. As it can be seen they are very similar for short durations, say for durations below 10% of the expected duration of a price plan, and very similar to the function $1/2t$. They differ in the level of asymptotic hazard rate, which is reached much sooner for the model with "state dependent" plans and is reached later for the model with exponential plans.

Next we provide an intuitive explanation of why the hazard rate of price plans are decreasing, while in the model without plans they are not. For instance, in the standard Calvo model of price setting without plans, hazard rates are constant by assumption. Likewise, hazard rates are increasing in the canonical menu cost model, such as in Golosov and Lucas (2007), since right after a price change the firm charges the profit maximizing price, so that the probability to observe a new price change right after an adjustment is near zero. Instead, in the case of price plans with with two prices, the firm is indifferent between charging $p_i^* \pm \hbar$ right after a price change. Given that the upper threshold is preferred when $g > 0$ and the lower threshold is preferred when $g < 0$, the fact that $g = 0$ right after a price change makes it very likely that its sign with reverse many times, which triggers lots of price changes. We can also understand why $h(t) \approx 1/(2t)$ for small duration t. The reason is that a Brownian motion has, for a small enough time interval, approximately the same probability of an increase as a decrease, so if $g(t) > 0$, but $g(t)$ is small, then with probability roughly 1/2 it returns to zero, and thus the hazard rate is $1/(2t)$.

Figure 9: The hazard rate of price changes in two models



E.1 Proofs for Hazard Rates

We compute the instantaneous hazard rate of price changes in two versions of our model. The first version has price plans that change when the (absolute value of the) normalized desired price $|g|$ reaches a critical value, a threshold that we denote by $\bar{g}$. We refer to this version as the menu cost version. We also consider another version where price plans are changed at exponentially distributed plans. In both cases we provide an analytical solution to the hazard rate of price changes. These analytical expressions depends on only one parameter, namely N_p: the expected number of price plans changes per unit of time. Both hazard rates are downward slopping, very much so for low durations (behaving approximately at $1/2t$), and they asymptote to different constants.

Hazard rate when price plans changes subject to menu cost. To describe the hazard rate in this case we discuss the mathematical objects we use to define them and compute them. These results comes from the analysis in Alvarez et al. (2015b), which themselves borrow some results from Kolkiewicz (2002). In our model a price change occurs when either a new price plan is in place or when within the same price plan prices are changed. In either case, at the instant right before price change takes place, the value of the desired normalized price satisfies $g = 0$. Thus, we compute the hazard rate for the following objects. We take

$g(0) = \epsilon$, with $0 < \epsilon < \bar{g}$ and consider the following three stopping times:

$$\tilde{\tau}(\epsilon) = \inf_t \{\sigma W(t) \leq 0 \,|\, \sigma W(0) = \epsilon\} \tag{58}$$

$$\bar{\tau}(\epsilon) = \inf_t \{\sigma W(t) \geq \bar{g} \,|\, \sigma W(0) = \epsilon\} \tag{59}$$

$$\tau(\epsilon) = \min \{\bar{\tau}(\epsilon)\,,\, \tilde{\tau}(\epsilon)\} \tag{60}$$

where W is a standard Brownian notion, so that we can use the desired normalized price until a price plan as $g(t) = \sigma W(t)$. The stopping time $\tilde{\tau}$ gives the first time that the desired normalized price g reaches back to 0, and hence the price changes, in logs, by $2\,\hbar$. Instead $\bar{\tau}$ gives the firms that the desired normalized price g reaches the upper barrier $\bar{g}$, and hence there is a new price plan, which new price. Thus, a price changes occurs, the first time that either event takes place, which is denoted by the stopping time τ. Note that in all cases we started with a normalized desired price equal to ϵ. Since right after price change $g = 0$, we will compute the limit of these stopping times as $\epsilon \to 0$. We require $g(0) = \epsilon$ to be small but strictly positive, because if we set $g(0)$ exactly equal to zero, then the distribution of τ is degenerate, i.e. $\tilde{\tau} = 0$ with probability one.[30] A convenient expressions for the distribution of $\bar{\tau}(\epsilon)$ and $\tilde{\tau}(\epsilon)$ can be found in Kolkiewicz (2002) expressions (15) and (16). In Alvarez et al. (2015b), we derive the hazard rates, and compute the limit as $\epsilon \to 0$. Letting $h(t)$ the hazard rate of price changes, and adapting the expression in Alvarez et al. (2015b) we obtain equation (56) in Proposition 14.

Hazard rate when price plans have exponentially distributed durations Again a price change occurs when either a new price plan is in place or within the same price plan. In this version we simply assume that price plans are changed at durations that are exponentially distributed, and independent of the normalized desired price g. This exponential distribution is assumed to have expected duration denoted by $1/N_p$, so N_p is the expected number of price plans per unit of time. Price changes within a plan are given by the stopping time as $\hat{\tau}$, define in equation (59). The price changes that occur within a price plan are described by the same (limit of) the stopping time $\tilde{\tau}$ define above. Thus the stopping time for price changes is given by:

$$\tau(\epsilon) = \min \{\hat{\tau}\,,\, \tilde{\tau}(\epsilon)\} \tag{61}$$

where W is $g(t)$ are defined as above. Since $\hat{\tau}$ and $\tilde{\tau}(\epsilon)$ are independent, then the hazard rate is simple the sum of the two hazard rates. The hazard rate corresponding to $\hat{\tau}$ is simply N_p. The hazard rate corresponding to $\tilde{\tau}$ can be computed as the hazard rate corresponding to the first time that a BM (with zero and volatility σ) and that starts at $\epsilon > 0$ and reaches 0. This stopping time is distributed according to the stable Levy law with density and CDF

[30] Give the symmetry of the problem we could have defined $\epsilon < 0$ and concentrate on the fist time that it comes back to zero, or it reaches $-\bar{g}$. Clearly we obtain the same stopping times.

equal to[31]:

$$f(t;\epsilon) = \frac{\epsilon}{\sigma\sqrt{2\,\pi\,t^3}}\, e^{-\frac{\epsilon^2}{2\,t\,\sigma^2}}$$

$$F(t;\epsilon) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{\epsilon^2}{2\,t\,\sigma^2}}} e^{-z^2}\, dz\,.$$

Defining the hazard rate in terms of f and F, and taking ϵ to zero we obtain:

$$\tilde{h}(t) \equiv \lim_{\epsilon\to 0} \frac{f(t;\epsilon)}{1-F(t;\epsilon)} = \lim_{\epsilon\to 0} \frac{\frac{\epsilon}{\sigma\sqrt{2\,\pi\,t^3}}\, e^{-\frac{\epsilon^2}{2\,t\,\sigma^2}}}{\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\frac{\epsilon^2}{2\,t\,\sigma^2}}} e^{-z^2}\, dz} = \frac{\frac{1}{\sigma\sqrt{2\,\pi\,t^3}}}{\frac{2}{\sqrt{\pi}}\sqrt{\frac{1}{2\,t\,\sigma^2}}} = \frac{1}{2\,t}\,,$$

where we use L'Hopital rule to evaluate the limit. Thus we have equation (57) in Proposition 14.

We briefly comment on the nature the limit hazard rates displayed in equation (56) and equation (57). We note that in continuous time both cases h_{MC} and h_{Exp} are not hazard rates that corresponds to a proper survivor function. The survivor function that correspond to $\epsilon = 0$ has $S(0) = 1$ and $S(t) = 0$ for all $t > 0$. The hazard rates in equation (56) and equation (57) are the limits of the approximation as $\epsilon \to 0$, so they should be regarded as approximations that are accurate for very small ϵ, or alternatively, as the hazard rates conditional on surviving a very small duration.[32]

F Plans with exponentially distributed duration

In this section we consider an alternative model to the menu cost model. Specifically, we assume that the duration of the price plans is exogenous and has a constant hazard rate λ, so that the duration of a plan is exponentially distributed. This version model corresponds to the well known Calvo (1983) pricing, if the price plan is a singleton. Thus this section can also be viewed as introducing price plans, or menu of prices, into the Calvo price setting. The reason for exploring this case is the pervasive use of the Calvo pricing in the sticky price literature. First we discuss the optimal value for $\hbar$. Then we characterize output's cumulative IRF to a monetary shock.

Optimal threshold $\hbar$. The determination of the optimal threshold $\hbar$ follows exactly the same logic as in the case where the firm must pay a fixed cost, and thus price plans has duration given by the first time a top or bottom thresholds $\bar{g}$ or $-\bar{g}$ is hit. Instead in this case the stopping time is given by an exponentially distributed random variable, independent of g. Using the same first order condition as in Section 2.2.

[31]See Alvarez et al. (2015a) for a derivation for the case of a BM with drift.

[32]The derivation in Alvarez et al. (2015a) takes the second limit, i.e. the hazard rates conditional on a strictly positive duration.

Proposition 15. The optimal threshold for the exponentially distributed price plan is:

$$\hbar = \frac{\sigma}{\sqrt{2\,(r+\lambda)}} \tag{62}$$

The result in equation (62) is intuitive: the threshold is increasing in σ since for higher values of it the deviations will be larger to each side. It is decreasing in $r+\lambda$ because this decreases the duration of the price plan, hence it is more likely that gaps will be smaller. Note also that it is the same as the limit obtained in Proposition 2 as $\bar{g} \to \infty$.

The firm's contribution to the IRF. The logic of the firm's contribution to the cumulative output response after a shock is the same as in the benchmark case discussed in the main text, so that the price gap is $\hat{p}(t) \equiv p(t) - p^*(t) = \hbar \ \text{sgn}\,(g(t)) - g(t)$ for $\tau_i \le t < \tau_{i+1}$ as in Section 4. The difference concerns the stopping time that determines the change of plan, so the definition of $\hat{m}$ is the same as in equation (21). In this set-up the we have that $s(g)$ is:

$$s(g) = \mathbb{E}\left[\int_0^\tau 1_{g(t)\ge 0}\, dt \mid g(0) = g\right] = \int_0^\infty e^{-\lambda\, t}\, \mathbb{E}\left[1_{g(t)\ge 0} \middle| \, g(0) = g\right] dt \tag{63}$$

where we use that $e^{-\lambda\, t}$ is the probability that the price plan survived at time t, and $\mathbb{E}\left[1_{g(t)\ge 0} \middle| \, g(0) = g\right]$ is the fraction of paths that at t have positive $g(t)$, conditional on $g(0) = g$. The following lemma gives an expression for s:

Lemma 6. The derivative of $D(g) = s(g) - s(-g)$, where s is equation (63) is given by:

$$D'(g) = \frac{2}{\sigma}\, \frac{e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}}{\sqrt{2\,\lambda}} \tag{64}$$

The invariant distribution of the normalized desired prices is described by the density $f(g)$ which is a Laplace distribution, i.e.:

$$f\,(g) = \frac{\sqrt{2\,\lambda}\,/\sigma}{2}\, e^{-\sqrt{2\,\lambda}/\sigma\,|g|} \quad \text{for all } g\,.^{33} \tag{65}$$

Notice that the definition of the cumulative real output effect in equation (**??**) is, again except for the specification of τ, the same. Likewise, equation (**??**) also holds. Simple computations than leads to

Lemma 7. With exponentially distributed revisions of plan the cumulative output effect after a small monetary shock δ is $\mathcal{M}(\delta) = \delta\ \mathcal{M}'(0) + o(\delta)$ where

$$\mathcal{M}'(0) = \frac{1}{2\,\lambda} = \frac{1}{2N_p} \tag{66}$$

[33] This is easily seen by noticing that the invariant density solves the Kolmogorov forward equation: $\lambda\, f\,(g) = \frac{\sigma^2}{2}\, f''\,(g)$ and also that $\int_0^\infty f(g)dg = 1/2$.

For comparison with the well known Calvo pricing with $N_C = \lambda$ price adjustments per period we define $\mathscr{M}_C(\delta) = \int_{-\bar{g}}^{\bar{g}} m\,(g+\delta)\; f\,(g)\; dg$ as the cumulative impulse response where $f(g)$ is the same exponential density defined above.[34] Simple analysis along the lines followed above reveals that the cumulative real effect of a small monetary shock in the Calvo model are given by:

$$\mathscr{M}_C(\delta) \;=\; \delta\,\frac{1}{\lambda} + o(\delta) \;\approx\; \delta\,\frac{1}{N_C}$$

PROPOSITION 16. Assume plans are adjusted at the exogenous constant rate λ. Let $N_p = \lambda$ be the mean number of plan changes per period. Let N_C denote the mean number of price changes per period in a Calvo model without plans. The ratio of the cumulative output responses in the two models is:

$$\lim_{\delta\downarrow 0, r\downarrow 0} \frac{\mathscr{M}(\delta)}{\mathscr{M}_C(\delta)} = \frac{N_C}{2\;N_p} \tag{67}$$

The proposition shows that, as was observed for the menu cost model, the introduction of the plans introduces a flexibility that reduces the real effects of monetary shocks *assuming the number of plan changes is the same across models*, i.e. $N_p = N_C$.

Table 3: Synopsis of theoretical effect of price-plans across models: $\mathscr{M}'(0)$

"Menu cost model"		"Calvo model"	
Without Price Plans	With Price Plans	Without Price Plans	With Price Plans
$\frac{1}{6\;N}$	$\frac{1}{18\;N_p}$	$\frac{1}{N}$	$\frac{1}{2\;N_p}$

Note: N denotes the total number of price changes, N_p denotes the total number of plan changes.

Table 3 provides a summary of the effects of introducing price plans in the various models where the notation there uses N_p the number of plans and N for the total number of price changes in the model without plans. The cumulative output response in a model with exponentially distributed plan's adjustments is 1/2 of the effect in the corresponding Calvo model, as it appears comparing the expressions in the third and fourth panels of the table with $N = N_p$. This result is to be compared with the one in Proposition 12 where, for small r the ratio was 1/3.[35]

[34]As noted above, the price gap in the Calvo model is $\hat{p} = -g$. Since the density f is symmetric around zero this is also the density of price gaps.

[35] The table also shows that for models without price plans, the area under the output's IRF in the menu cost model is 1/6 of the area in a Calvo model, a result first proved by Alvarez et al. (2016b). For models with price plans, the table shows that ratio of the cumulated real effects is even smaller: the real effects of the menu cost model with plans is 1/9 of the real effect of a Calvo model with plans.

Figure 8 reports the impulse response (numerically computed) to a 1 percent monetary shock ($\delta = 0.01$). The exercise assumes that the number of plan adjustment per period equals the number of price adjustments in the Calvo model (without plans): $N_p = N = \lambda$. The figure confirms that area under the IRF in Calvo is twice the area in the model with plans. Interestingly the figure shows that the introduction of plans gives rise to a hump-shaped profile of the output response: output does not respond on impact because the model with plans has a large impact effect of the monetary shock. This is very different from the model without plans and it is due to the fact that in the model with plans there is a mass point of firms responding on impact to the shock, as was also shown for the menu cost model in Section 4, whereas in the model with plans the max of firms responding on impact is typically negligible.

Impact effect It is immediate to see that, as was the case for the menu cost model, the introduction of the plans leads to a non-negligible mass of adjustments on impact when the shock occurs. This happens because the monetary shock δ shifts the distribution of the normalized desired prices $f(g)$ given in equation (65) and a mass of agents $\int_0^\delta f(g)\,dg$ switches from negative to positive values of g, therefore switching from the low to the high price within the price plan, i.e. each firm increases its price by $2\hbar$. The next proposition summarizes this result

Proposition 17. The impact effect of a monetary shock δ on the aggregate price level is:

$$\lim_{r\to 0} \tilde{\Theta}(\delta) = \lim_{r\to 0} \hbar \int_0^\delta f(g)\,dg = \delta \lim_{r\to 0} \sqrt{\frac{\lambda}{\lambda + r}} = \delta \quad .$$

The proof follows immediately by using the density in equation (65) and the expression for $\hbar$ in equation (62). This result shows that the impact effect that results from the firm adjustments on impact yields an immediate jump of the price level of the same size of the monetary shock, so that output does not change at all on impact, as seen in the impulse response of Figure 8.

F.1 Proofs for the model with "Calvo" plans

Proof. (of Proposition 15)

$$\begin{aligned}
\hbar &= \frac{\mathbb{E}\left[\int_0^\tau e^{-rt}\,|g(t)|\,dt \;\middle|\; g(0)=0\right]}{\mathbb{E}\left[\int_0^\tau e^{-rt}\,dt \middle|\; g(0)=0\right]} \\
&= \frac{\int_0^\infty \lambda\, e^{-(r+\lambda)t}\,\mathbb{E}\left[\,|g(t)|\,dt \;\middle|\; g(0)=0\right]\,dt}{\int_0^\infty \lambda\, e^{-(r+\lambda)t}\,dt} = \frac{\int_0^\infty e^{-(r+\lambda)t}\,\sigma\sqrt{t\,2/\pi}\,dt}{\int_0^\infty e^{-(r+\lambda)t}\,dt} \\
&= \int_0^\infty (r+\lambda)\,e^{-(r+\lambda)t}\,\sigma\sqrt{2\,t/\pi}\,dt = \int_0^\infty (r+\lambda)\,e^{-(r+\lambda)t}\,\sigma\sqrt{2\,t\,/\pi}\,dt \\
&= \frac{\sigma}{\sqrt{2\,(r+\lambda)}}
\end{aligned}$$

where we use that $g(t)$ is, conditional on $g(0)=0$, normally distributed with mean 0 and variance $\sigma^2\, t$, and hence $\mathbb{E}\left[\,|g(t)|\, dt \;\middle|\; g(0)=0\right] = \sigma\sqrt{2\, t/\pi}$. The last line follows by performing the integration. $\square$

Proof. (of Lemma 6) Since $g(t)$ in normally distributed with mean g and variance $\sigma^2\, t$, so denoting by Φ the CDF of a standard normal, we can write

$$s(g) = \int_0^\infty e^{-\lambda\, t} \left[1 - \Phi\left(\frac{-g}{\sigma\sqrt{t}}\right)\right] dt\,, \text{ and } s(-g) = \int_0^\infty e^{-\lambda\, t} \left[1 - \Phi\left(\frac{g}{\sigma\sqrt{t}}\right)\right] dt\,.$$

Thus we have:

$$s'(g) = \int_0^\infty \frac{e^{-\lambda\, t}}{\sqrt{\sigma^2\, t}}\, \phi\left(\frac{-g}{\sigma\sqrt{t}}\right) dt\,, \text{ and } s'(-g) = -\int_0^\infty \frac{e^{-\lambda\, t}}{\sqrt{\sigma^2\, t}}\, \phi\left(\frac{g}{\sigma\sqrt{t}}\right) dt$$

$$D'(g) = s'(g) + s'(-g) = 2\int_0^\infty \frac{e^{-\lambda\, t}\, e^{-\frac{1}{2}\left(\frac{g}{\sigma\sqrt{t}}\right)^2}}{\sqrt{2\,\pi\,\sigma^2\, t}}\, dt$$

Note we can write:

$$\begin{aligned} e^{-\lambda\, t - \frac{1}{2}\left(\frac{g}{\sigma\sqrt{t}}\right)^2} &= e^{-\frac{g^2+2\sigma^2\lambda t^2}{2\,\sigma^2\, t}} = e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}\, e^{-\frac{g^2+2\sigma^2\lambda t^2 - 2g\,\sigma\,\sqrt{2\,\lambda}\, t}{2\,\sigma^2\, t}} = e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}\, e^{-\frac{\left(g-\sigma\sqrt{2\,\lambda}\, t\right)^2}{2\,\sigma^2\, t}} \\ &= e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}\, e^{-\frac{\left(1-\sqrt{2\,\lambda}\, t(\sigma/g)\right)^2}{2\,(\sigma/g)^2\, t}} \end{aligned}$$

Hence we can write:

$$\begin{aligned} D'(g) &= 2\, e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}} \int_0^\infty \frac{e^{-\frac{\left(1-\sqrt{2\,\lambda}\, t(\sigma/g)\right)^2}{2\,(\sigma/g)^2\, t}}}{\sqrt{2\,\pi\,\sigma^2\, t}}\, dt \\ &= 2\, \frac{e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}}{g} \int_0^\infty \frac{e^{-\frac{\left(1-\sqrt{2\,\lambda}\, t(\sigma/g)\right)^2}{2\,(\sigma/g)^2\, t}}}{\sqrt{2\,\pi\,(\sigma/g)^2\, t}}\, dt = 2\, \frac{e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}}{g} \int_0^\infty \frac{e^{-\frac{\left(1-\sqrt{2\,\lambda}\, t(\sigma/g)\right)^2}{2\,(\sigma/g)^2\, t}}}{\sqrt{2\,\pi\,(\sigma/g)^2\, t^3}}\, t\, dt \end{aligned}$$

Using that the last term is the expected value of an inverse gaussian so that

$$\int_0^\infty \frac{e^{-\frac{\left(1-\sqrt{2\,\lambda}\, t(\sigma/g)\right)^2}{2\,(\sigma/g)^2\, t}}}{\sqrt{2\,\pi\,(\sigma/g)^2\, t^3}}\, t\, dt = \frac{g/\sigma}{\sqrt{2\,\lambda}}$$

we have:

$$D'(g) = 2\, \frac{e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}}{g}\, \frac{(g/\sigma)}{\sqrt{2\,\lambda}} = \frac{2}{\sigma}\, \frac{e^{-\frac{g\sqrt{2\,\lambda}}{\sigma}}}{\sqrt{2\,\lambda}}$$

$\square$

Proof. (of Proposition 16) Differentiating the definition of $\mathscr{M}$ we have:

$$\begin{aligned}
\mathscr{M}'(0) &= \frac{1}{\lambda} - 2\,\hbar \int_0^\infty D'(g)\; f(g)\; dg = \frac{1}{\lambda} - 2\,\hbar \int_0^\infty \frac{2}{\sigma} \frac{e^{-\frac{g\sqrt{2\lambda}}{\sigma}}}{\sqrt{2\lambda}} \frac{\sqrt{2\lambda}/\sigma}{2}\, e^{-\sqrt{2\lambda}/\sigma\, g}\, dg \\
&= \frac{1}{\lambda} - 2\,\frac{\hbar}{\sigma^2} \int_0^\infty e^{-\frac{g\sqrt{2\lambda}}{\sigma}}\, e^{-\sqrt{2\lambda}/\sigma\, g}\, dg = \frac{1}{\lambda} - 2\,\frac{\hbar}{\sigma^2} \int_0^\infty e^{-\frac{g\,2\sqrt{2\lambda}}{\sigma}}\, dg \\
&= \frac{1}{\lambda} - 2\,\frac{\hbar}{\sigma^2} \frac{1}{\frac{2\sqrt{2\lambda}}{\sigma}} = \frac{1}{\lambda} - 2\,\frac{\hbar}{\sigma^2} \frac{\sigma}{2\sqrt{2\lambda}} \\
&= \frac{1}{\lambda} - \frac{\hbar}{\sigma\sqrt{2\lambda}}
\end{aligned}$$

Replacing the optimal value of $\hbar$:

$$\mathscr{M}'(0) = \frac{1}{\lambda} - \frac{\sigma}{\sigma\sqrt{2\,(r+\lambda)}\,\sqrt{2\lambda}}$$

Considering the case when $r \downarrow 0$:

$$\mathscr{M}'(0) = \frac{1}{\lambda} - \frac{1}{2\lambda}$$

Using that $\mathscr{M}_C(\delta) = \delta\left(1 - e^{-\lambda t}\right)$,

$$\lim_{\delta\downarrow 0, r\downarrow 0} \frac{\mathscr{M}_C(\delta)}{\mathscr{M}(\delta)} = \frac{1/\lambda}{\frac{1}{\lambda} - \frac{1}{2\lambda}} = 2. \qquad \square$$

G Costly adjustments within plan

This appendix generalizes the model of the paper by assuming that prices changes within the plan, i.e. changes back and forth between the low and the high price within the plan, are also costly. In particular we assume the firm must pay a menu cost ν to change the price within the plan, and a larger menu cost ψ to change the plan.

Let $\nu > 0$ be the cost for a price change within the plan $-\hbar \rightleftarrows +\hbar$. Our baseline model assumes that $\nu = 0$. This modified problem gives rise to 2 value functions $v_h(\cdot), v_l(\cdot)$; symmetric : $\quad v_h(g) = v_l(-g)$, since the value of a given normalized price g depends on the price currently charged, i.e. $\pm\hbar$.

In such a setting the optimal policy is given by 3 thresholds: $-\underline{g} \leq 0 \leq \hbar < \bar{g}$, such that the profit maximizing firm sets the price $\hbar$ as long as $g \in (-\underline{g}, \bar{g})$ and $-\hbar$ for $g \in (-\bar{g}, \underline{g})$. We have that $\hbar, \bar{g}$ and value functions $v_h(\cdot), v_l(\cdot)$ solve for all g:

$$r\,v_h(g) \leq B\;(g-\hbar)^2 + \frac{\sigma^2}{2}\,v_h''(g)$$

$$r\,v_l(g) \leq B\;(g+\hbar)^2 + \frac{\sigma^2}{2}\,v_l''(g)$$

with equality if inaction is optimal, and

$$v_h(g) \le \nu + v_l(g) \text{ and } v_l(g) \le \nu + v_h(g; \hbar)$$
$$v_h(g) \le \psi + v_h(0) \text{ and } v_l(g) \le \psi + v_l(0; \hbar)$$

if either changing from high to low price (or vice-versa) or if changing the plan is optimal. Thus at least one of this inequality must hold with equality at each g.

Solving the problem requires solving a system of 5 equations in 5 unknowns: $\underline{g}, \hbar, \bar{g}$ and the 2 parameters of the second order ODE for the bellman equation. The five equations are given by:

$$\text{2 value matching at } \underline{g} \text{ and } \bar{g}: \quad v_h(-\underline{g}) = \nu + v_l(-\underline{g}) \quad , \quad v_h(\bar{g}) = \psi + v_h(0)$$

$$\text{smooth pasting at } \underline{g}: \quad v_h'(-\underline{g}) = v_l'(-\underline{g}) = -v_h'(\underline{g}) \quad \text{by symmetry}$$

$$\text{smooth pasting at } \bar{g}: \quad v_h'(\bar{g}) = 0 \quad \text{and the optimal return } v_h'(\tilde{g}) = 0 \; .$$

This system can be solved numerically to deliver the three optimal thresholds $-\underline{g} \le 0 < \hbar < \bar{g}$. The classic menu cost problem with one price is obtained when $\nu = \psi$ so that $\hbar = 0, \bar{g} > 0$ and $\underline{g} = \bar{g}$. The price plan model discussed in the paper has $\psi > 0$ and $\nu = 0$ so that $\hbar > 0, \bar{g} > 0$ and $\underline{g} = 0$.

Next, for given thresholds, we compute the density of the price gaps $f(g)$ as well as the density of high prices $\tilde{p}$, which we denote by $f_f(g)$, and the density of low prices $-\tilde{p}$, which we denote by $f_l(g)$. The density $f(g)$ is the usu The Kolmogorov forward equation $0 = f''(g)\sigma^2/2$, which implies a linear density function, and the boundary conditions $f(\bar{g}) = f(-\bar{g}) = 0$ (due to the fact that these are exit points and no mass can be accumulated here) imply that the density $f(g)$ is

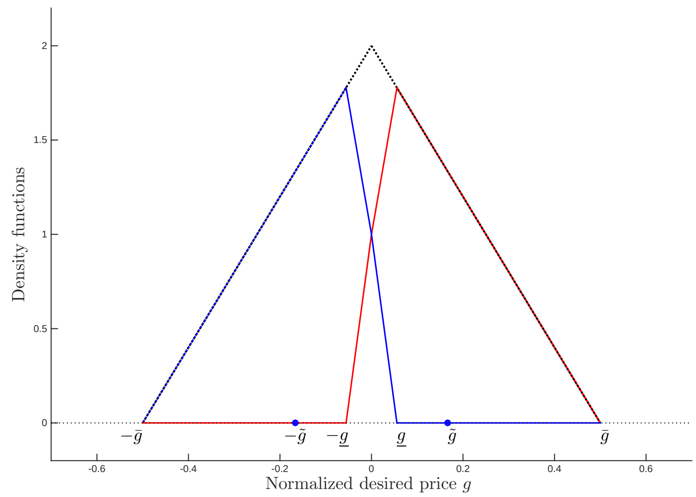
$$f(g) = \begin{cases} \frac{\bar{g}+g}{\bar{g}^2} & \text{for } g \in [-\bar{g}, 0] \\ \frac{\bar{g}-g}{\bar{g}^2} & \text{for } g \in [0, \bar{g}] \end{cases} \tag{68}$$

Notice that the densities $f_h(g), f_l(g)$ follow the same Kolmogorov equation, hence they are linear, but they have different boundaries. In particular we have that the density $f_h(g)$ is continuous in $(-\bar{g}, \bar{g})$, the density is zero between $[-\bar{g}, -\underline{g})$, it is upward sloping between $-(\underline{g}, 0)$, it is upward sloping between $(0, \underline{g})$, and it coincides with $f(g)$ between $(\underline{g}, \bar{g})$. Using linearity and the boundary conditions $f_h(-\underline{g}) = 0$, $f_h(0) = f(0)/2$ and $f_h(\underline{g}) = f(\underline{g})$ yields the following density for high prices

$$f_h(g) = \begin{cases} 0 & \text{for } g \in [-\bar{g}, -\underline{g}) \\ \frac{1}{2\bar{g}} + \frac{1}{2\bar{g}\underline{g}}\, g & \text{for } g \in [-\underline{g}, 0) \\ \frac{1}{2\bar{g}} + \left(\frac{1}{2\bar{g}\underline{g}} - \frac{1}{\bar{g}^2}\right) g & \text{for } g \in [0, \underline{g}) \\ \frac{1}{\bar{g}} - \frac{1}{\bar{g}^2} g & \text{for } g \in [\underline{g}, \bar{g}] \end{cases} \tag{69}$$

The density for low prices $f_l(g)$ is the symmetric counterpart of $f_h(g)$, in particular we have that $f_l(-g) = f_h(g)$. Figure 10 plots the two densities as an illustration of for the case

Figure 10: Density function for high and low prices when $\nu > 0$



in which $\nu > 0$ so the price gaps in the interval $(-\underline{g}, \underline{g})$ are associated with both high and low prices.

Next we use the solution in equation (69) to discuss the impact effect of a monetary shock. The next proposition shows that the impact effect is still approximately the same than the impact effect in a model where $\nu = 0$, i.e. that the shock δ has a first order effect on the aggregate price level, provided the fixed cost ν is small enough. More formally, the proposition states that the impact effect is continuous in ν, so that for small values of ν, which are necessary to get many temporary price changes as in the data, the impact effect is close to the impact of a model where $\nu = 0$:

Proposition 18. Continuity of the impact effect on $\underline{g}$. Fix a $0 < \delta < \bar{g}$ and $\epsilon > 0$. Then there exist a $\underline{G}(\epsilon, \delta)$ such that for all $0 < \underline{g} < \underline{G}(\epsilon, \delta)$ the impact effect $|\tilde{\Theta}(\delta; \underline{g}) - \tilde{\Theta}(\delta; 0)| < \epsilon$.

Note that the optimal threshold $\underline{g} \to 0$ as the fixed cost $\nu \to 0$, so that the impact effect can be made arbitrarily close to the impact discussed in Proposition 11 in the main text.

H Description of the Argentine CPI data for 1989-1997

Our dataset contains 8,618,345 price observations underlying the Argentinean CPI from December 1988 through September 1997. Each quote represents an item in a specific outlet for a specific time period. Goods and outlets are chosen to be representative of consumer expenditure in the 1986 consumer expenditure survey.[36] Goods are divided into two groups: homogeneous and differentiated goods. Homogeneous goods represent 49.5 percent of our sample and cover goods sold in super-market chains. Price quotes for differentiated goods

[36] For a more detailed description see Alvarez et. al (2017).

are collected every month and cover mainly services.[37] We focus on homogeneous goods, and exclude price quotes for baskets of goods, rents, and fuel prices. We focus on these goods for two reasons: first because their prices are sampled every two weeks – versus heterogeneous goods are sampled every month–, and second because homogenous goods are closer to the goods for which there are scanner price data, which increases the comparability of our study with ours. [38]

Next we discuss the sample that we use to compute different statistics. The main restrictions come from requiring that in each period of 4 months we can compute the reference prices, which are defined as the modal price for a store x good combination for a period of time. We discuss the definition of reference prices in detail below.

The data set has some missing observations and flags for stock-outs. We treat stock-outs and price quotes with no recorded information as missing observations. As a preliminary step we conduct two types of imputations. First, we impute missing observations when the price quotes before and after the missing value are the same, i.e. we "iron-out" the prices. Second, if in a given month a good × outlet has exactly one missing observation, we impute its price as the non-missing price of the same good × outlet for that month. The data set also contains a flag for price substitutions. The statistical agency substitutes the price quote of an item for a similar item when the good is either discontinued by the producer or not sold any longer by an outlet. We define the relevant sample of four-month periods for a given good × outlet as those that have at most one substitution, at most one month where we impute its price, and they have no other missing prices for any other reasons –such as the outlet dropping from the sample, etc. Our final sample contains 4,759,584 price quotes from 198 different items and a total of 2877 unique stores. Around 5 percent of items have a sale flag, 1 percent have a substitution flag, and 0.1 percent are imputed prices. Overall, we have 594,948 four-month period × item *times* outlet combinations, i.e. it has 594,948 reference prices. We have 36 non-overlapping four month periods, so in average we have about 132,000 price quotes and about 26,000 reference prices in each of the non-overlapping four month period.

I Additional moments: Argentine CPI and BPP data

This appendix provides additional detailed quantitative information on several price setting moments using two data sources: the Argentine CPI data as well as from the Billion Prices Project (BPP henceforth) by Cavallo and Rigobon (2016).

[37]This is similar to the BLS, except that the frequency to which prices are gathered is twice as high in Argentina during this period, mostly because Argentina has a history of sustained inflation since the 1950's. Incidentally, during this period the agency in charge of measuring inflation, INDEC, was very prestigious and well regarded by other agencies. The intervention of the INDEC agency and the manipulation of the CPI started in the mid 2000's.

[38]Baskets correspond to around 9.91% of total expenditure and are excluded because their prices are gathered for any good in a basket, i.e., if one good is not available, it is substituted by any another in the basket. Examples are medicines and cigarettes. Rents are sampled monthly for a fixed set of representative properties. Reported prices represent the average of the sampled properties and include what is paid on that month, as opposed to what is paid for a new contract. Rents represent 2.33 percent of household expenditure. Fuel prices account for 4 percent of total expenditure and we exclude them because they were gathered in a

Table 4: Pricing Statistics - Argentina CPI (largest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Date	Inflation	Freq. - Regular	Freq. - Reference	Freq.- Reference Adj.	Distinct	Fraction to New	Novelty
1989-1	228.6	0.613	0.832	0.882	94.6		
1989-2	792.9	0.720	0.831	0.831	98.7		
1989-3	193.7	0.454	0.788	0.819	91.9	0.941	0.660
1990-1	488.6	0.688	0.879	0.966	96.8	0.960	0.668
1990-2	153.3	0.533	0.859	0.959	94.1	0.904	0.482
1990-3	70.6	0.398	0.774	0.961	84.8	0.841	0.340
1991-1	125.2	0.463	0.763	0.825	91.3	0.848	0.393
1991-2	44.4	0.293	0.710	0.795	81.1	0.774	0.229
1991-3	10.6	0.255	0.628	0.795	77.1	0.701	0.167
1992-1	32.3	0.293	0.600	0.639	79.1	0.707	0.189
1992-2	14.0	0.195	0.556	0.621	68.2	0.659	0.112
1992-3	2.0	0.175	0.511	0.619	62.4	0.623	0.090
1993-1	15.4	0.188	0.442	0.472	63.8	0.594	0.094
1993-2	5.8	0.166	0.423	0.467	63.4	0.614	0.087
1993-3	3.0	0.144	0.383	0.457	55.2	0.556	0.066
1994-1	-2.7	0.157	0.361	0.389	54.1	0.587	0.075
1994-2	9.4	0.135	0.342	0.387	55.9	0.569	0.061
1994-3	3.0	0.138	0.325	0.388	60.6	0.559	0.064
1995-1	1.9	0.158	0.381	0.404	59.9	0.586	0.081
1995-2	-1.1	0.135	0.360	0.400	60.3	0.615	0.070
1995-3	2.2	0.139	0.340	0.395	54.9	0.550	0.063
1996-1	-4.5	0.145	0.341	0.365	59.4	0.549	0.065
1996-2	6.8	0.131	0.331	0.372	59.0	0.579	0.062
1996-3	-6.5	0.133	0.307	0.372	59.5	0.603	0.060
1997-1	-2.5	0.129	0.328	0.350	52.5	0.569	0.056
1997-2	6.1	0.147	0.294	0.349	62.6	0.610	0.074

separate database that we do not have access to.

Table 5: Pricing Statistics - Argentina CPI (smallest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Date	Inflation	Freq. - Regular	Freq. - Reference	Freq.- Reference Adj.	Distinct	Fraction to New	Novelty
1989-1	228.6	0.613	0.864	0.916	94.6		
1989-2	792.9	0.720	0.880	0.880	98.7		
1989-3	193.7	0.454	0.840	0.871	91.9	0.941	0.660
1990-1	488.6	0.688	0.882	0.969	96.8	0.960	0.668
1990-2	153.3	0.533	0.863	0.964	94.1	0.904	0.482
1990-3	70.6	0.398	0.777	0.965	84.8	0.841	0.340
1991-1	125.2	0.463	0.755	0.816	91.3	0.848	0.393
1991-2	44.4	0.293	0.707	0.792	81.1	0.774	0.229
1991-3	10.6	0.255	0.624	0.790	77.1	0.701	0.167
1992-1	32.3	0.293	0.594	0.632	79.1	0.707	0.189
1992-2	14.0	0.195	0.555	0.621	68.2	0.659	0.112
1992-3	2.0	0.175	0.510	0.618	62.4	0.623	0.090
1993-1	15.4	0.188	0.443	0.473	63.8	0.594	0.094
1993-2	5.8	0.166	0.426	0.471	63.4	0.614	0.087
1993-3	3.0	0.144	0.386	0.461	55.2	0.556	0.066
1994-1	-2.7	0.157	0.358	0.386	54.1	0.587	0.075
1994-2	9.4	0.135	0.339	0.383	55.9	0.569	0.061
1994-3	3.0	0.138	0.323	0.384	60.6	0.559	0.064
1995-1	1.9	0.158	0.376	0.398	59.9	0.586	0.081
1995-2	-1.1	0.135	0.356	0.396	60.3	0.615	0.070
1995-3	2.2	0.139	0.337	0.392	54.9	0.550	0.063
1996-1	-4.5	0.145	0.344	0.368	59.4	0.549	0.065
1996-2	6.8	0.131	0.333	0.374	59.0	0.579	0.062
1996-3	-6.5	0.133	0.309	0.375	59.5	0.603	0.060
1997-1	-2.5	0.129	0.319	0.339	52.5	0.569	0.056
1997-2	6.1	0.147	0.286	0.338	62.6	0.610	0.074

Table 6: Frequency of Price Adjustment by Types - Argentina CPI (largest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

		Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
1989-1	228.6	0.613	0.477	0.052	0.832	0.749	0.082	0.882	0.795	0.087
1989-2	792.9	0.720	0.598	0.072	0.831	0.701	0.130	0.831	0.701	0.130
1989-3	193.7	0.454	0.345	0.091	0.788	0.653	0.135	0.819	0.678	0.141
1990-1	488.6	0.688	0.501	0.165	0.879	0.841	0.038	0.966	0.924	0.042
1990-2	153.3	0.533	0.452	0.065	0.859	0.817	0.042	0.959	0.910	0.049
1990-3	70.6	0.398	0.295	0.091	0.774	0.738	0.036	0.961	0.914	0.047
1991-1	125.2	0.463	0.352	0.097	0.763	0.668	0.095	0.825	0.719	0.106
1991-2	44.4	0.293	0.214	0.070	0.710	0.609	0.101	0.795	0.673	0.121
1991-3	10.6	0.255	0.156	0.091	0.628	0.540	0.087	0.795	0.676	0.119
1992-1	32.3	0.293	0.203	0.081	0.600	0.470	0.130	0.639	0.496	0.142
1992-2	14.0	0.195	0.119	0.070	0.556	0.434	0.122	0.621	0.481	0.140
1992-3	2.0	0.175	0.098	0.072	0.511	0.399	0.112	0.619	0.480	0.139
1993-1	15.4	0.188	0.113	0.069	0.442	0.293	0.149	0.472	0.313	0.159
1993-2	5.8	0.166	0.094	0.067	0.423	0.283	0.140	0.467	0.313	0.154
1993-3	3.0	0.144	0.075	0.065	0.383	0.256	0.127	0.457	0.307	0.151
1994-1	-2.7	0.157	0.082	0.068	0.361	0.214	0.147	0.389	0.229	0.160
1994-2	9.4	0.135	0.077	0.053	0.342	0.204	0.138	0.387	0.231	0.156
1994-3	3.0	0.138	0.072	0.061	0.325	0.194	0.131	0.388	0.232	0.156
1995-1	1.9	0.158	0.084	0.069	0.381	0.226	0.155	0.404	0.238	0.166
1995-2	-1.1	0.135	0.067	0.064	0.360	0.215	0.146	0.400	0.238	0.163
1995-3	2.2	0.139	0.074	0.061	0.340	0.204	0.136	0.395	0.237	0.157
1996-1	-4.5	0.145	0.068	0.071	0.341	0.178	0.164	0.365	0.190	0.175
1996-2	6.8	0.131	0.071	0.055	0.331	0.173	0.157	0.372	0.196	0.176
1996-3	-6.5	0.133	0.061	0.068	0.307	0.160	0.146	0.372	0.197	0.175
1997-1	-2.5	0.129	0.061	0.064	0.328	0.166	0.163	0.350	0.178	0.172
1997-2	6.1	0.147	0.085	0.058	0.294	0.151	0.144	0.349	0.179	0.169

Table 7: Frequency of Price Adjustment by Types - Argentina CPI (smallest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

		Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
1989-1	228.6	0.613	0.477	0.052	0.864	0.819	0.045	0.916	0.868	0.048
1989-2	792.9	0.720	0.598	0.072	0.880	0.814	0.067	0.880	0.814	0.067
1989-3	193.7	0.454	0.345	0.091	0.840	0.770	0.069	0.871	0.799	0.073
1990-1	488.6	0.688	0.501	0.165	0.882	0.862	0.020	0.969	0.947	0.023
1990-2	153.3	0.533	0.452	0.065	0.863	0.840	0.022	0.964	0.937	0.027
1990-3	70.6	0.398	0.295	0.091	0.777	0.759	0.019	0.965	0.939	0.025
1991-1	125.2	0.463	0.352	0.097	0.755	0.677	0.077	0.816	0.729	0.087
1991-2	44.4	0.293	0.214	0.070	0.707	0.624	0.084	0.792	0.690	0.103
1991-3	10.6	0.255	0.156	0.091	0.624	0.551	0.073	0.790	0.688	0.102
1992-1	32.3	0.293	0.203	0.081	0.594	0.468	0.126	0.632	0.495	0.137
1992-2	14.0	0.195	0.119	0.070	0.555	0.438	0.117	0.621	0.486	0.135
1992-3	2.0	0.175	0.098	0.072	0.510	0.403	0.107	0.618	0.484	0.134
1993-1	15.4	0.188	0.113	0.069	0.443	0.295	0.148	0.473	0.314	0.159
1993-2	5.8	0.166	0.094	0.067	0.426	0.287	0.139	0.471	0.317	0.154
1993-3	3.0	0.144	0.075	0.065	0.386	0.259	0.127	0.461	0.309	0.152
1994-1	-2.7	0.157	0.082	0.068	0.358	0.211	0.147	0.386	0.226	0.159
1994-2	9.4	0.135	0.077	0.053	0.339	0.201	0.138	0.383	0.227	0.156
1994-3	3.0	0.138	0.072	0.061	0.323	0.192	0.131	0.384	0.228	0.156
1995-1	1.9	0.158	0.084	0.069	0.376	0.220	0.156	0.398	0.231	0.166
1995-2	-1.1	0.135	0.067	0.064	0.356	0.211	0.145	0.396	0.234	0.162
1995-3	2.2	0.139	0.074	0.061	0.337	0.202	0.135	0.392	0.235	0.157
1996-1	-4.5	0.145	0.068	0.071	0.344	0.176	0.168	0.368	0.188	0.180
1996-2	6.8	0.131	0.071	0.055	0.333	0.172	0.161	0.374	0.194	0.180
1996-3	-6.5	0.133	0.061	0.068	0.309	0.159	0.150	0.375	0.194	0.181
1997-1	-2.5	0.129	0.061	0.064	0.319	0.161	0.158	0.339	0.172	0.167
1997-2	6.1	0.147	0.085	0.058	0.286	0.146	0.140	0.338	0.173	0.165

Table 8: Time at the Reference Price - Argentina CPI (largest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. For each non-overlapping 4 month interval and product × store combinations there are 8 two-weeks periods. In each of these two week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. Reference prices are computed using the largest mode. The table also reports the fraction of sales which is the average number of product × store combinations with a sales flag in each two weeks period. The reported statistic is the average in each non-overlapping 4 month period.

Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
1989-1	228.6	0.362	0.318	0.993	0.498	0.023
1989-2	792.9	0.287	0.606	5.701	0.851	0.054
1989-3	193.7	0.528	0.202	0.749	0.428	0.064
1990-1	488.6	0.309	0.544	3.715	0.788	0.126
1990-2	153.3	0.406	0.380	1.771	0.639	0.048
1990-3	70.6	0.544	0.303	1.986	0.665	0.075
1991-1	125.2	0.428	0.471	4.650	0.823	0.071
1991-2	44.4	0.636	0.270	2.860	0.741	0.056
1991-3	10.6	0.690	0.215	2.262	0.693	0.076
1992-1	32.3	0.610	0.253	1.837	0.648	0.056
1992-2	14.0	0.726	0.178	1.856	0.650	0.055
1992-3	2.0	0.768	0.157	2.083	0.676	0.058
1993-1	15.4	0.753	0.155	1.667	0.625	0.049
1993-2	5.8	0.785	0.142	1.947	0.661	0.056
1993-3	3.0	0.821	0.118	1.924	0.658	0.052
1994-1	-2.7	0.795	0.128	1.675	0.626	0.048
1994-2	9.4	0.830	0.106	1.655	0.623	0.043
1994-3	3.0	0.824	0.110	1.699	0.629	0.048
1995-1	1.9	0.801	0.118	1.468	0.595	0.051
1995-2	-1.1	0.820	0.115	1.754	0.637	0.053
1995-3	2.2	0.823	0.118	1.984	0.665	0.049
1996-1	-4.5	0.816	0.116	1.702	0.630	0.055
1996-2	6.8	0.840	0.100	1.669	0.625	0.046
1996-3	-6.5	0.827	0.113	1.860	0.650	0.053
1997-1	-2.5	0.840	0.098	1.605	0.616	0.049
1997-2	6.1	0.813	0.107	1.345	0.573	0.047

Table 9: Time at the Reference Price - Argentina CPI (smallest mode)

The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. For each non-overlapping 4 month interval and product × store combinations there are 8 two-weeks periods. In each of these two week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. Reference prices are computed using the smallest mode. The table also reports the fraction of sales which is the average number of product × store combinations with a sales flag in each two weeks period. The reported statistic is the average in each non-overlapping 4 month period.

Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
1989-1	228.6	0.362	0.129	0.254	0.202	0.023
1989-2	792.9	0.287	0.310	0.771	0.435	0.054
1989-3	193.7	0.528	0.095	0.253	0.202	0.064
1990-1	488.6	0.309	0.264	0.620	0.383	0.126
1990-2	153.3	0.406	0.175	0.417	0.295	0.048
1990-3	70.6	0.544	0.151	0.493	0.330	0.075
1991-1	125.2	0.428	0.276	0.933	0.483	0.071
1991-2	44.4	0.636	0.158	0.767	0.434	0.056
1991-3	10.6	0.690	0.129	0.715	0.417	0.076
1992-1	32.3	0.610	0.124	0.467	0.318	0.056
1992-2	14.0	0.726	0.091	0.500	0.333	0.055
1992-3	2.0	0.768	0.091	0.645	0.392	0.058
1993-1	15.4	0.753	0.088	0.556	0.357	0.049
1993-2	5.8	0.785	0.084	0.635	0.388	0.056
1993-3	3.0	0.821	0.066	0.586	0.370	0.052
1994-1	-2.7	0.795	0.077	0.601	0.375	0.048
1994-2	9.4	0.830	0.066	0.642	0.391	0.043
1994-3	3.0	0.824	0.061	0.538	0.350	0.048
1995-1	1.9	0.801	0.065	0.490	0.329	0.051
1995-2	-1.1	0.820	0.064	0.553	0.356	0.053
1995-3	2.2	0.823	0.071	0.670	0.401	0.049
1996-1	-4.5	0.816	0.066	0.555	0.357	0.055
1996-2	6.8	0.840	0.059	0.589	0.371	0.046
1996-3	-6.5	0.827	0.062	0.562	0.360	0.053
1997-1	-2.5	0.840	0.059	0.587	0.370	0.049
1997-2	6.1	0.813	0.062	0.493	0.330	0.047

Table 10: Pricing Statistics - BPP (largest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is the fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Country	Date	Inflation	Freq. - Regular	Freq. - Reference	Freq. - Reference Adj.	Distinct	Fraction to New	Novelty
	2008	5.700	0.171	0.358	0.602	73.900	0.678	0.125
Argentina	2009	4.168	0.190	0.562	0.632	73.660	0.719	0.150
	2010	8.905	0.168	0.596	0.635	74.062	0.730	0.137
	2008	1.452	0.253	0.369	0.624	79.505	0.687	0.179
Brazil	2009	1.419	0.267	0.725	0.813	88.121	0.781	0.251
	2010	0.234	0.320	0.886	0.978	90.183	0.748	0.250
	2008	3.039	0.169	0.285	0.485	59.718	0.616	0.116
Chile	2009	0.102	0.156	0.311	0.341	49.207	0.510	0.085
	2010	0.919	0.147	0.284	0.297	41.876	0.420	0.066
	2008	2.397	0.236	0.394	0.650	77.577	0.814	0.226
Colombia	2009	1.762	0.258	0.690	0.714	80.778	0.778	0.205
	2010	1.486	0.250	0.771	0.791	72.438	0.732	0.188
	2008	2.910	0.299	0.000		47.124		
USA	2009	-1.651	0.207	0.350	0.386	41.167	0.336	0.064
	2010	4.328	0.275	0.343	0.370	66.713	0.374	0.145

Table 11: Pricing Statistics – BPP (smallest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is the fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Country	Date	Inflation	Freq. - Regular	Freq. - Reference	Freq. - Reference Adj.	Distinct	Fraction to New	Novelty
	2008	5.700	0.171	0.377	0.634	73.900	0.678	0.125
Argentina	2009	4.168	0.190	0.571	0.642	73.660	0.719	0.150
	2010	8.905	0.168	0.502	0.534	74.062	0.730	0.137
	2008	1.452	0.253	0.373	0.631	79.505	0.687	0.179
Brazil	2009	1.419	0.267	0.714	0.803	88.121	0.781	0.251
	2010	0.234	0.320	0.887	0.979	90.183	0.748	0.250
	2008	3.039	0.169	0.282	0.485	59.718	0.616	0.116
Chile	2009	0.102	0.156	0.339	0.371	49.207	0.510	0.085
	2010	0.919	0.147	0.294	0.307	41.876	0.420	0.066
	2008	2.397	0.236	0.392	0.648	77.577	0.814	0.226
Colombia	2009	1.762	0.258	0.695	0.721	80.778	0.778	0.205
	2010	1.486	0.250	0.791	0.811	72.438	0.732	0.188
	2008	2.910	0.299	0.000		47.124		
USA	2009	-1.651	0.207	0.382	0.416	41.167	0.336	0.064
	2010	4.328	0.275	0.304	0.332	66.713	0.374	0.145

Table 12: Frequency of Price Adjustment by Types – BPP (largest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is the fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

			Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Country	Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
	2008	5.700	0.171	0.121	0.041	0.358	0.331	0.027	0.602	0.555	0.048
Argentina	2009	4.168	0.190	0.129	0.058	0.562	0.465	0.096	0.632	0.526	0.106
	2010	8.905	0.168	0.130	0.037	0.596	0.563	0.032	0.635	0.599	0.035
	2008	1.452	0.253	0.130	0.111	0.369	0.206	0.163	0.624	0.349	0.275
Brazil	2009	1.419	0.267	0.167	0.096	0.725	0.485	0.239	0.813	0.540	0.273
	2010	0.234	0.320	0.133	0.183	0.886	0.427	0.459	0.978	0.465	0.513
	2008	3.039	0.169	0.102	0.059	0.285	0.240	0.045	0.485	0.408	0.077
Chile	2009	0.102	0.156	0.079	0.075	0.311	0.196	0.115	0.341	0.214	0.127
	2010	0.919	0.147	0.079	0.068	0.284	0.136	0.149	0.297	0.142	0.155
	2008	2.397	0.236	0.129	0.094	0.394	0.248	0.145	0.650	0.410	0.240
Colombia	2009	1.762	0.258	0.150	0.106	0.690	0.457	0.233	0.714	0.473	0.241
	2010	1.486	0.250	0.142	0.107	0.771	0.546	0.225	0.791	0.558	0.233
	2008	2.910	0.299	0.143	0.119	0.000	0.000	0.000			
USA	2009	-1.651	0.207	0.098	0.107	0.350	0.163	0.187	0.386	0.183	0.203
	2010	4.328	0.275	0.177	0.096	0.343	0.123	0.220	0.370	0.131	0.239

Table 13: Frequency of Price Adjustment by Types – BPP (smallest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We convert the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two week period without a substitution flag. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is the fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

			Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Country	Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
	2008	5.700	0.171	0.121	0.041	0.377	0.352	0.025	0.634	0.589	0.045
Argentina	2009	4.168	0.190	0.129	0.058	0.571	0.471	0.100	0.642	0.532	0.110
	2010	8.905	0.168	0.130	0.037	0.502	0.450	0.052	0.534	0.478	0.057
	2008	1.452	0.253	0.130	0.111	0.373	0.201	0.171	0.631	0.340	0.291
Brazil	2009	1.419	0.267	0.167	0.096	0.714	0.483	0.231	0.803	0.539	0.264
	2010	0.234	0.320	0.133	0.183	0.887	0.386	0.501	0.979	0.421	0.559
	2008	3.039	0.169	0.102	0.059	0.282	0.232	0.050	0.485	0.399	0.086
Chile	2009	0.102	0.156	0.079	0.075	0.339	0.215	0.124	0.371	0.233	0.137
	2010	0.919	0.147	0.079	0.068	0.294	0.154	0.140	0.307	0.161	0.146
	2008	2.397	0.236	0.129	0.094	0.392	0.247	0.145	0.648	0.408	0.239
Colombia	2009	1.762	0.258	0.150	0.106	0.695	0.457	0.238	0.721	0.474	0.247
	2010	1.486	0.250	0.142	0.107	0.791	0.556	0.235	0.811	0.569	0.242
	2008	2.910	0.299	0.143	0.119	0.000	0.000	0.000			
USA	2009	-1.651	0.207	0.098	0.107	0.382	0.174	0.209	0.416	0.191	0.225
	2010	4.328	0.275	0.177	0.096	0.304	0.124	0.180	0.332	0.135	0.197

Table 14: Time at the Reference Price – BPP (largest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. For each non-overlapping 4 month interval and product × store combinations there are 8 two-weeks periods. In each of these two week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. Reference prices are computed using the largest mode. The table also reports the fraction of sales which is the average number of product × store combinations with a sales flag in each two weeks period. The reported statistic is the average in each non-overlapping 4 month period.

Country	Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
	2008	5.7	0.769	0.144	1.655	0.623	0.029
Argentina	2009	4.2	0.759	0.131	1.197	0.545	0.031
	2010	8.9	0.722	0.201	2.599	0.722	0.023
	2008	1.5	0.713	0.189	1.940	0.660	0.031
Brazil	2009	1.4	0.671	0.215	1.894	0.654	0.026
	2010	0.2	0.590	0.201	0.961	0.490	0.044
	2008	3.0	0.798	0.137	2.113	0.679	0.039
Chile	2009	0.1	0.835	0.120	2.636	0.725	0.044
	2010	0.9	0.867	0.092	2.224	0.690	0.038
	2008	2.4	0.698	0.192	1.741	0.635	0.029
Colombia	2009	1.8	0.670	0.207	1.686	0.628	0.029
	2010	1.5	0.683	0.204	1.807	0.644	0.032
	2008	2.9	0.717	0.228	4.175	0.807	0.083
USA	2009	-1.7	0.799	0.151	3.025	0.752	0.069
	2010	4.3	0.730	0.142	1.107	0.526	0.061

Table 15: Time at the Reference Price – BPP (smallest mode)

The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. For each non-overlapping 4 month interval and product × store combinations there are 8 two-weeks periods. In each of these two week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non-overlapping four month interval, we compare the reference price of that 4-month interval with the one in the previous 4 month interval. Reference prices are computed using the smallest mode. The table also reports the fraction of sales which is the average number of product × store combinations with a sales flag in each two weeks period. The reported statistic is the average in each non-overlapping 4 month period.

Country	Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
	2008	5.7	0.769	0.094	0.690	0.408	0.029
Argentina	2009	4.2	0.759	0.095	0.648	0.393	0.031
	2010	8.9	0.722	0.100	0.557	0.358	0.023
	2008	1.5	0.713	0.129	0.820	0.451	0.031
Brazil	2009	1.4	0.671	0.187	1.313	0.568	0.026
	2010	0.2	0.590	0.137	0.500	0.333	0.044
	2008	3.0	0.798	0.099	0.964	0.491	0.039
Chile	2009	0.1	0.835	0.094	1.318	0.569	0.044
	2010	0.9	0.867	0.076	1.323	0.569	0.038
	2008	2.4	0.698	0.134	0.793	0.442	0.029
Colombia	2009	1.8	0.670	0.137	0.706	0.414	0.029
	2010	1.5	0.683	0.137	0.757	0.431	0.032
	2008	2.9	0.717	0.183	1.833	0.647	0.083
USA	2009	-1.7	0.799	0.110	1.212	0.548	0.069
	2010	4.3	0.730	0.103	0.616	0.381	0.061