Either or Both Competition: A "Two-sided" Theory of Advertising with Overlapping Viewerships*

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May 2014

Abstract

We propose a new model of competition between media platforms. Most of the existing literature assumes that consumers select only the outlet they like most. Therefore, platforms' choices, such as advertising intensity, prices and content are shaped solely by "business stealing" considerations. We argue that when consumers satisfy their content needs on multiple platforms, additional forces come into play that reflect platforms' incentives to control the composition of the customer base. We label these forces the "business sharing" and "duplication" effects. We derive empirically testable conditions on preferences and technology that determine the impact of competition on advertising intensities and prices, and draw implications for entry, mergers and content choice. Our model can explain empirical observations from media markets that are difficult to reconcile with existing models.

Keywords: Platform Competition, Two-Sided Markets, Multi-Homing, Viewer Composition, Viewer Preference Correlation

JEL-Classification: D43, L13, L82, M37

[*]This paper partially builds on results from the manuscript "Exclusive vs. Overlapping Viewers in Media Markets" by Ambrus and Reisinger. We would like to thank Simon Anderson, Bruno Jullien, Elena Argentesi, Rossella Argenziano, Mark Armstrong, Susan Athey, Alessandro Bonatti, Drew Fudenberg, Doh-Shin Jeon, Martin Peitz, Jesse Shapiro, Gabor Virag, Helen Weeds. We would also like to thank participants at the Eleventh Annual Columbia/Duke/MIT/Northwestern IO Theory Conference, the MaCCi and ICT conference in Mannheim, the NUS Multi-sided Platforms Workshop, the CESifo conference in Munich for their useful comments. We also thank Vivek Bhattacharya and especially Peter Landry for careful proofreading.

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1 Introduction

A central question in the ongoing debate about the changing media landscape is how competitive forces shape advertising levels and revenues and, as a result, the incentives to entry and the amount of diversity in the marketplace.[1] In media markets, platforms fight for consumer attention and in particular for the accompanying stream of advertising revenues. Online advertising networks, such as the Google and Yahoo ad-networks, and traditional broadcasting stations, such as CNN and Fox News, are among the most prominent examples.

The traditional approach in media economics posits that consumers have idiosyncratic tastes about media platforms, and stick to those they like best.[2] So, if anything, consumers choose *either* one platform *or* some other. Competition is for *exclusive* consumers as all platforms are restricted a priori to be (imperfect) substitutes. A common argument regarding the impact of competition in this framework goes as follows. The amount of advertising supplied by platforms can be considered the shadow price that consumers pay to satisfy their content needs. Competition typically lowers prices as platforms try to woo consumers from their rivals. So, as the argument goes, increased competition ought to reduce advertising levels.

While compelling, this line of reasoning fails to account for the fact that many consumers satisfy their content needs on *multiple* platforms. This is increasingly so as content moves from paper and TV towards the Internet. In fact, many contend that a distinguishing feature of "online consumption" is the users' increased tendency to spread their attention across a wide array of outlets. Table 1 shows the reach of the six largest online advertising networks, that is, the fraction of the U.S. Internet users who, over the course of December 2012, visited a website belonging to a given network. This table shows that while Google can potentially deliver an advertising message to 93.9% of all Internet users, the smallest of the six networks (run by Yahoo!), can deliver a whopping 83.3%. The table highlights a key feature of these markets whose implications for market outcomes are left unexplored: different platforms provide advertisers alternate means of reaching the same users.

Rank	Property	Unique Visitors (000)	% Reach
	Total Internet : Total Audience	*221,486*	*100.0*
1	Google Ad Network**	208,074	93.9
2	Specific Media**	198,119	89.5
3	Federated Media Publisher Network**	193,453	87.3
4	AOL Advertising**	186,595	84.2
5	AT&T AdWorks**	185,757	83.9
6	Genome from Yahoo!**	184,556	83.3

Table 1. Top 6 Online Ad Newtworks by reach.[3]

In addition, the traditional approach fails to rationalize extensive evidence of advertising levels rising

[1]The issue is whether the wider array of choices and media formats can potentially upend the traditional cross-subsidization business model in which advertising pays the bills. For example, see the Federal Trade Commission's report, 'The Information Needs of Communities' (2011).

[2]For example, see Anderson and Coate (2005) and several follow-up papers. We provide a detailed literature review in the next section.

with competition[4] (the so called "Fox News Puzzle," documented in the appendix, is one well known illustration) and remaining fairly high in competitive environments.[5]

Motivated by these concerns we develop a theory of market provision of informative advertising that allows consumers to consume content on *multiple* platforms. So with two platforms, consumers can choose *either* one platform or *both* (or none). Specifically, we work under the (extreme) assumption that consumer demand for one platform does not affect the demand for another platform. This is what we call *either or both* competition in contrast with the *either/or* framework discussed above.

We claim that this model of competition is an appealing alternative to existing ones for several reasons. First, it is a good approximation of reality in some non-trivial contexts where substitutability is limited. For example choosing Facebook.com for online social networking services is arguably orthogonal to choosing Yelp.com as one's supplier of restaurant reviews.[6] Moreover, and somewhat surprisingly, Gentzkow, Shapiro and Sinkinson (2014) document limited substitutability even in traditional media markets such as that of US newspapers. They show that on average 86% of an entrant's circulation comes from households reading multiple newspapers or households who previously did not read at all. Furthermore, independence of demand across platforms allows generality in other dimensions, including the advertising technology, and the consumer preference correlation across different platforms. This in turn allows us to address questions that are not tractable in the traditional framework, such as how consumer preference correlation affects equilibrium advertising levels, and how it influences the effect of platform entry and content choice.[7]

The baseline model features two platforms, with continuums of consumers and advertisers. Consumers "dislike" ads in the sense that they would rather get ad-free content. Advertisers instead want to reach more consumers, as greater consumer exposure increases demand for their products. In line with the literature, platforms simultaneously choose the total quantity of ads. These ads are subsequently allocated to advertisers according to a simple contracting environment in which each platform offers a contract specifying a price for a given advertising intensity. We do not impose a specific functional form on either the distribution of viewer preferences or on the advertising technology.

A key equilibrium property of our model is that viewers who are exclusive to a platform are more valuable than overlapping (also called "multi-homing") viewers. As these latter are catered by more than one platform, no individual platform can extract from the advertisers more than the incremental value of reaching these same viewers via an additional platform. This implies that platforms do not only care about the overall viewer demand *level*, as in existing models, but also about its *composition*, i.e., the

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[4]Several observers contend that the wave of channel entry during the 1990s in the US cable TV industry coincided with an *increase* in advertising levels on many channels (this is what is usually referred to as the Fox News puzzle). In the Appendix, using a dataset provided by Kagan-SNL, we show that an increase in the number of US cable TV channels market is on average indeed associated with an increase in advertising levels on incumbent channels.

[5]Jeziorski (2012) documents high advertising levels even in markets that are widely considered fairly competitive, such as the US local radio broadcasting markets.

[6]According to the source supra cited, Facebook and Yelp are among the top 10 most-visited U.S. websites and in fact belong to different advertising networks.

[7]Existing models either assume a Hotelling framework, imposing perfect negative correlation in viewer preferences for two platforms, or a representative consumer framework. In contrast, our framework allows for viewer preferences to be correlated in any way between platforms.

fraction of exclusive versus overlapping viewers.[8]

Indeed it is common for ad-networks to assess the extent of overlap and common for advertisers to take into account the extent of duplication in large cross-outlet campaigns. To stress the importance of these considerations the appendix includes a screen-shot from the sales pitch for Google's Display Network (figure 4). It employs proprietary data to assess the effect for an advertising campaign on auto insurance. A "key takeaway" according to the sales pitch is that the GDN "exclusively reaches 30% of the auto-insurance seekers" that do not visit Yahoo, 36% that do not visit Youtube and so on.

We propose a characterization of the incentives to provide advertising opportunities in duopoly and draw the implications for the advertising intensity, prices, entry, mergers and content choice. In particular, we show that two forces come into play when some consumers are shared. In duopoly, multihomers receive advertising messages from two different sources. This fact together with diminishing returns from advertising implies that the marginal ad is less valuable than in monopoly. This *duplication effect* induces platforms to supply *fewer* ads. Second, as discussed above, in duopoly common consumers are of lower value. So the opportunity cost, in terms of lost consumers, of increasing the advertising level is lower relative to that of a monopolist whose customers are all exclusive by definition. As a result, duopolists are more aggressive in the sense that they are less wary of increases in the advertising level. This *business-sharing* effect induces higher equilibrium advertising levels. We provide an intuitive and full characterization of how these effects interact and shape equilibrium outcomes, in terms of the elasticities of viewer demand and of the properties of the communication technology, that is, in terms of empirical objects.

To understand under what conditions either effect prevails, we trace out the impact of competition to two sources: a preference-driven and a technology-driven source.

On the preference-side the key question turns out to be: are overlapping consumers more responsive to changes in the advertising level relative to exclusive ones? If yes, then the business-sharing effect dominates. So a follow-up question is: when is it that increasing the amount of advertising disproportionately repels exclusive viewers? To illustrate our results, consider the preference correlation of consumers for media platforms. For example, suppose Fox News were to enter, MSNBC being the incumbent. As these stations do not share the same ideological affiliation, one can reasonably conjecture that a consumer who likes the former dislikes the latter, and vice versa. That is viewer preferences are negatively correlated. This implies that a large portion of viewers of each platform will be exclusive. When reducing the advertising level, a platform attracts more viewers, both single-homers and multi-homers. Compared to its viewer base, these marginal viewers are comprised to a larger portion of multi-homers. Since multi-homers are less valuable—the business-sharing effect—incentives to lower advertising are small, and equilibrium advertising levels are large. Conversely, if the viewer preference correlation is positive, for example think about the Fox Sports Channel and ESPN, advertising levels fall with entry. We provide a first empirical

[8]That multi-homing viewers are worth less to advertisers is consistent with the well-documented fact that the per-viewer fee of an advertisement on programs with more viewers is larger. In the U.S., e.g., Fisher, McGowan and Evans (1980) find this regularity. ITV, which is the largest TV network in the UK, enjoys a price premium on its commercials, which, despite entry of several competitors, increased steadily in the 1990s. This trend is commonly referred to as the "ITV premium puzzle". Our model can account for this puzzle since reaching the same number of eyeball pairs through broadcasting a commercial to a large audience implies reaching more viewers than reaching the same number of eyeball pairs through a series of of commercials to smaller audiences, because the latter audiences might have some viewers in common. See Ozga (1960) for an early observation of this fact.

pass using data from the U.S. cable TV industry that provides suggestive evidence for these results.

On the technological-side instead the key question is: are exclusive consumers relatively harder to inform than common ones? If yes, then, other things held constant, the duplication effect prevails. To gain intuition, consider entry of a new internet platform. Some previously exclusive users of an incumbent platform are now common users. Suppose that the fact that a user is active on an additional platform makes tracking and targeting easier, implying that common users are easier to inform. By lowering its advertising level, the incumbent platform attracts single-homers and multi-homers but advertising to multi-homers is more effective. By contrast, without competition, the platform could only attract single-homers by definition. Therefore, the platform has a stronger incentive to reduce its level of ads.

We use this model to gauge the strength of the outlets' incentives to differentiate their content, i.e. to enter and supply diverse content. These incentives are governed by two considerations. First, differentiation allows to increase the share of the (valuable) exclusive viewers. Second, more differentiation affects the strategic environment by making the rival more aggressive. Since the two forces point in opposite direction, we find that diversification does not necessarily enhance revenues per viewer. If the strategic effect prevails, then entrants would find it more profitable to provide similar content. This is the opposite result to a common theme in the literature on differentiation, that the direct effect induces firms to move closer to each other to secure more demand while the strategic effect induces them to differentiate in order to soften competition.

Finally we consider a natural policy experiment. We show that if advertising to exclusive and overlapping viewers is equally effective, a merger between two platforms would not change the equilibrium advertising levels. The intuition for this result is similar to why monopoly versus competition might not influence outcomes in common agency models (Bernheim and Whinston, 1985 and 1986). The result is important both for economic theory and for policy discussions as it shows that mergers in media markets can be neutral with respect to social welfare. As a corollary to this neutrality result, we should not expect policies that require spin-offs to mitigate inefficient overprovision of advertising, as the caps and bans on advertising proposed by policy makers seem to suggest. Also competition between channels does not necessarily lead to a fall in advertising levels and might even increase the inefficient overprovision.

The rest of the paper is organized as follows: Section 2 discusses the related literature. Section 3 introduces the model and Section 4 presents some preliminary analysis. Section 5 analyzes platform competition and presents the main trade-offs of our model. Section 6 considers the effects of viewer preference correlation and Section 7 explores the advertising technology. Sections 8 considers platform mergers. Section 9 analyzes welfare implication and Section 10 presents an extension to heterogeneous advertisers. Section 11 concludes.

2 Literature Review

The traditional framework in media economics makes the assumption that viewers do not switch between channels, but rather select the program they like most, e.g. Spence and Owen (1977) or Wildman and Owen (1985). These early works usually do not allow for endogenous advertising levels or two-sided externalities between viewers and advertisers.

The first paper accounting explicitly for these externalities is Anderson and Coate (2005).[9] In their model, viewers are distributed on a Hotelling line with platforms located at the endpoints. Similar to early works, viewers watch only one channel while advertisers can buy commercials on both channels.[10] In this framework, Anderson and Coate (2005) show, among several other results, that the number of entering stations can either be too high or too low compared to the socially optimal number, or that the advertising level can be above or below the efficient level.

The framework of Anderson and Coate (2005) has been used to tackle a wide array of questions. Gabszewicz, Laussel and Sonnac (2004) allow viewers to mix their time between channels, Peitz and Valletti (2008) analyze optimal locations of stations, and Reisinger (2012) considers single-homing of advertisers. Dukes and Gal-Or (2003) explicitly consider product market competition between advertisers and allow for price negotiations between platforms and advertisers, while Choi (2006) and Crampes, Haritchabalet and Jullien (2009) consider the effects of free entry of platforms. Finally, Anderson and Peitz (2012) allow advertising congestion and show that it can also lead to increased advertising rates after entry of new platforms. These papers do not allow viewers to watch more than one station, i.e., they assume either/or competition, and usually consider a spatial framework for viewer demand.[11] As discussed, we add to this literature by providing a model in which business stealing considerations are replaced by business sharing ones.

On the empirical side, Sweeting (2013) provides a rich dynamic structural framework that allows to back out the drivers of product variety in the commercial radio industry. In line with previous works and to simplify the analysis consumers are assumed to listen to at most one station. He finds that a fee of 10% of the revenues levied from the copyright holders would result in a 9.4% drop of music stations in the long-run. In light of our theory, an open question is how business sharing considerations, which in our model play a key role at the content choice stage, affect the results.

There are a few recent studies which also consider multi-homing viewers.[12] Anderson, Foros and Kind (2013) consider a model similar in spirit to ours, i.e., both papers share the insight that overlapping viewers are less valuable in equilibrium. In contrast to our paper, they consider a different contracting

[9]For different applications of two-sided market models, see e.g., Ellison and Fudenberg (2003), Ellison, Fudenberg and Möbius (2004), Rochet and Tirole (2003, 2006), and Weyl (2010).

[10]In Section 5 of their paper Anderson and Coate (2005) extend the model by allowing a fraction of viewers to switch between channels.

[11]A different framework to model competition in media markets is to use a representative viewer who watches more than one program. This approach is developed by Kind, Nilssen and Sørgard (2007) and is used by Godes, Ofek and Savary (2009) and Kind, Nilssen and Sørgard (2009). These papers analyze the efficiency of the market equilibrium with respect to the advertising level and allow for viewer payments. Due to the representative viewer framework, they are not concerned with overlapping viewers or viewer preference correlation.

[12]For a demand structure, which also allows overlapping of consumers, albeit in a very different context, see Armstrong (2013).

environment (per-unit pricing) and use a different equilibrium concept (rational instead of adaptive expectations). The latter implies that platforms cannot attract consumers via lower ad-levels.[13] In addition, the questions addressed in Anderson, Foros and Kind (2013) are different and complementary to those investigated in our paper. They mainly analyze public broadcasting and genre selection, and show e.g., that the well-known problem of content duplication is ameliorated with multi-homing viewers.[14] By contrast, our analysis provides implications of viewer composition on market outcomes and characterizes the marginal incentive of a media platform to supply advertising opportunities. In this respect, our analysis is in line with the canonical two-sided market framework, in which a change in the quantity on one side changes the platform's attractiveness on the other side.

Athey, Calvano and Gans (2013) and Bergemann and Bonatti (2011, in Sections 5 and 6) also consider multi-homing viewers but are mainly concerned with different tracking/targeting technologies and do not allow for advertisements generating (negative) externalities on viewers, which is at the core of our model. Specifically, in Athey, Calvano and Gans (2013) the effectiveness of advertising can differ between users who switch between platforms and those who stick to one platform because of imperfect tracking of users, whereas Bergemann and Bonatti (2011) explicitly analyze the interplay between perfect advertising message targeting in online media markets and imperfect targeting in traditional media.

Gentzkow, Shapiro and Sinkinson (2014) develop a structural empirical model of the newspaper industry that embeds the key prediction found here that advertising-market competition depends on the extent of overlap in readership. They find that competition increases diversity significantly, offsetting the incentive to cater to the tastes of majority consumers (George and Waldfogel, 2003).[15]

Finally, in a different context (insurance markets), Weyl and Veiga (2014) also examines how the composition of the marginal consumers affects incentives. They study the problem of choosing the characteristics of a product, such as the co-insurance rate, in order to attract the "most valuable" consumers.

3 The Model

The basic model features a unit mass of heterogeneous viewers, a unit mass of homogeneous advertisers and two platforms indexed by $i \in \{1, 2\}$.[16]

Viewer Demand

Viewers are parametrized by their reservation utilities $(q_1, q_2) \in \mathbb{R}^2$ for platforms 1 and 2, where (q_1, q_2) is distributed according to a bivariate probability distribution with smooth joint density denoted $h(q_1, q_2)$. A viewer of (q_1, q_2)-type joins platform i if and only if $q_i - \gamma n_i \geq 0$, where n_i is the advertising

[13] Anderson, Foros and Kind (2014) provide a justification for doing so, based on the inability of consumers to observe the contractual terms offered to the advertisers.

[14] For related ideas, see also Anderson, Foros, Kind and Peitz (2012).

[15] Rüdiger (2013) also applies the idea that multi-homers are worth less to study the implications of "cross checking" on media-bias. He finds that diminishing returns to scale from advertising increase incentives of platforms to move towards extreme positions.

[16] We cast our model in terms of the television context. The model also applies to internet or radio, where the term viewers would be replaced by users or listeners.

level on platform i and $\gamma > 0$ is a nuisance parameter. Given the advertising level on each platform, we can back out the demand system:

$$\begin{aligned}
\text{Multi-homers:} \quad & D_{12} := \text{Prob}\{q_1 - \gamma n_1 \geq 0;\ q_2 - \gamma n_2 \geq 0\}, \\
\text{Single-homers}_1\text{:} \quad & D_1 := \text{Prob}\{q_1 - \gamma n_1 \geq 0;\ q_2 - \gamma n_2 < 0\}, \\
\text{Single-homers}_2\text{:} \quad & D_2 := \text{Prob}\{q_1 - \gamma n_1 < 0;\ q_2 - \gamma n_2 \geq 0\}, \\
\text{Zero-homers:} \quad & D_0 := 1 - D_1 - D_2 - D_{12}.
\end{aligned}$$

The demand system is discussed at length at the end of this section. To ensure uniqueness of the equilibrium and interior solutions, we need the demand functions to be well-behaved. Ultimately, this boils down to assumptions on the joint density function $h(q_1, q_2)$. However, it is not necessary to spell out assumptions on this function, since we will later only work with the resulting demand functions. Hence, we make the following assumptions directly on the demand functions:

$$\frac{\partial^2 D_i}{\partial n_i^2} \leq 0, \quad \frac{\partial^2 D_{12}}{\partial {n_i}^2} \leq 0 \quad \text{and} \quad \left|\frac{\partial^2 D_i}{\partial {n_i}^2}\right| \geq \left|\frac{\partial^2 D_i}{\partial n_i \partial n_j}\right|, \forall i = 1, 2 \text{ and } j = 3 - i.$$

These regularity assumptions are stricter than necessary. If instead each of the three inequalities were violated but only slightly so, we still have interior solutions.[17]

Timing and Platforms' Choices

Platforms compete for viewers and for advertisers. Platforms receive payments only from advertisers but not from viewers. To make the model more transparent we develop a four-stage game. When discussing the modeling assumptions, we relate this model's equilibrium outcome to a canonical two-stage model of platform competition à la Armstrong (2006).

At stage 1, platforms simultaneously set the *total* advertising levels n_1 and n_2. At stage 2, viewers observe n_1 and n_2 and choose which platform(s) to join, if any. At stage 3, platforms simultaneously offer menus of contracts to advertisers. A contract offered by platform i is a pair $(t_i, m_i) \in \mathbb{R}^2_+$, which specifies an advertising intensity $m_i \geq 0$ in exchange for a monetary transfer $t_i \geq 0$. Finally, at stage 4, advertisers simultaneously decide which contract(s), if any, to accept. Below we will show that in our basic model with homogeneous advertisers, each platform only offers one contract in equilibrium, and it is accepted by all advertisers.[18] This implies that, in equilibrium $m_i = n_i$ for the unique advertising intensity m_i offered by platform i.

To make sure that the announced advertising levels are consistent with realized levels after stage 4, we assume that if total advertising levels accepted by advertisers at platform i exceed n_i then platform i obtains a large negative payoff.[19] Therefore, our game is similar to Kreps and Scheinkman (1983),

[17]For a more detailed discussion of why these assumptions ensure concavity of the objective functions and uniqueness of the equilibrium, see Vives (2000).

[18]In Section 11 we consider a model with heterogeneous advertisers in which platforms do offer multiple contracts in equilibrium.

[19]Our results would remain unchanged if we instead assume that, if there is an ecxess demand for a platform's advertising intensities, given the contracts offered, then actual advertising intensities are rationed proportionally for participating advertisers. We stick to the current formulation as it simplifies some of the arguments in the proofs.

i.e., in the first stage platforms choose an advertising level that puts an upper bound on the advertising intensities they can sell subsequently.

The extensive form captures actual practice in US and Canadian broadcasting markets. On a seasonal basis, broadcasters and advertisers meet at an "upfront" event to sell commercials on the networks' upcoming programs. At this point the networks' supply of commercial breaks is already determined. Also, the Nielsen rating system, which measures viewership for different programs and platforms (and advertisers) supplies viewership estimates. Contracts that specify, among other things, the number of ads (so called "avails") in exchange for a fixed payment are then signed between broadcasters and advertisers.

The solution concept we use throughout the paper is subgame perfect Nash equilibrium (SPNE).

Advertising Technology

Advertising in our model is informative. We normalize the return of informing a viewer about a product to 1.[20] In line with the literature, e.g., Anderson and Coate (2005) or Crampes, Haritchabalet and Jullien (2009), we assume that viewers are fully expropriated of the value of being informed.

The mass of informed viewers (also known as "reach") is determined by the number of advertising messages (m_1, m_2) a particular advertiser purchases on each platform. Without loss of generality, we decompose the total reach in the sum of the reach within the three different viewers' subsets. We denote the probability with which a single-homing viewer on platform i becomes informed of an advertiser's product by $\phi_i(m_i)$. We assume that ϕ_i is smooth, strictly increasing and strictly concave, with $\phi_i(0) = 0$. That is, there are positive but diminishing returns to advertising. By definition, ϕ_{12} equals the probability that a multi-homing viewer becomes informed on *some* platform. In what follows, we decompose ϕ_{12} as one minus the probability that the viewer is not informed on either outlet $\phi_{12}(m_1, m_2) := 1 - (1 - \hat{\phi}_1(m_1))(1 - \hat{\phi}_2(m_2))$, where $\hat{\phi}(m_i)$ is the probability that an overlapping viewer becomes informed *on platform* i, where $\hat{\phi}(m_i)$ is also smooth, strictly increasing and strictly concave. Note that ϕ_i, $\hat{\phi}_i$, ϕ_j and $\hat{\phi}_j$ may all be different. We discuss the interpretation below.

Payoffs

A platform's payoff is equal to the total amount of transfers it receives (for simplicity, we assume that the marginal cost of ads is zero). An advertiser's payoff, in case he is active on both platforms, is $u(n_1, n_2, m_1, m_2) - t_1 - t_2$, where

$$u(n_1, n_2, m_1, m_2) := D_1(n_1, n_2)\phi_1(m_1) + D_2(n_1, n_2)\phi_2(m_2) + D_{12}(n_1, n_2)\phi_{12}(m_1, m_2)$$

and t_1 and t_2 are the transfers to platforms 1 and 2, respectively. If he only joins platform i, the payoff is $u(n_i, n_j, m_i, 0) - t_i = \phi_i(m_i)\left(D_i(n_i, n_j) + D_{12}(n_i, n_j)\right) - t_i$, since the advertiser reaches viewers only via platform i. Advertisers' reservation utilities are normalized to zero.

Discussion of Modeling Choices

Conditional on the realization of the his utility parameters (q_1, q_2), a viewer's choice of whether to join platform i is assumed to depend neither on n_j nor q_j. This 'demand independence' assumption

[20] In Section 11, we allow advertisers be different with respect to this return.

should not be confused with nor does it imply *statistical* (or unconditional) independence between q_i and q_j. For instance the model allows preferences for i (say Facebook) and j (say Yelp) to be correlated to account for some underlying common covariate factor (say 'internet saviness'). In fact the model nests those specifications which add structure to preferences by positing a positive or negative relationship between valuations of different platforms. A extreme example is the Hotelling-type spatial model with the two platforms at the opposite ends of a unit interval and viewers distributed along the interval. Thus, the Hotelling specification is captured by the above setup with the restriction $q_1 = 1 - q_2$.

An important property of the demand schedules, following directly from the way we defined them, is that if n_i changes but n_j is unchanged, the choice of whether to join platform j remains unaffected. This property contrasts either/or formulations in which viewers choose one platform *over* the other. In our framework, if n_i increases, then platform i loses some single-homers and some multi-homers; these single-homers become zero-homers while the multi-homers become single-homers on platform j. The latter effect implies that $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$. We refer to this formulation as pure either/both competition.

The ϕ functions capture, in a parsimonious way, several relevant aspects of consumer behavior, platform asymmetry, and advertising technology. For example, if one platform is more effective at reaching viewers for all nonzero levels, or if viewers spend more time on one platform than on the other, this could be captured by the restriction $\phi_i(m) > \phi_j(m)$ for all $m > 0$. The assumption that $\hat{\phi}_i(m)$ is not necessarily equal to $\phi_i(m)$ allows us to capture heterogeneity in behavior across viewer types. While we are agnostic here as to the source of this heterogeneity, one obvious way to motivate this broader formulation is accounting for multi-homers allocating a different amount of attention. If the marginal returns from an additional unit of time spent on either platform are decreasing then we would naturally expect multi-homers to spread this limited time across outlets so that $\hat{\phi}_i(m) < \phi_i(m)$ for all m. Although the former scenario is more natural, in certain settings multi-homing might create synergies inducing viewers to pay more time and attention to the outlets, resulting in $\hat{\phi}_i(m) > \phi_i(m)$ for all m.

The game presented is equivalent to a three-stage game whereby advertisers and viewers simultaneously make their choices. In turn we argue below that this game is equivalent (with one important caveat) to a two-stage duopoly model in which platforms simultaneously make offers and, upon observing the offers, all agents simultaneously make their choices. The role of stage 1 is that of insulating viewers from the advertisers' choices. Indeed, viewerships are fixed before platforms sell their advertising slots. The assumption that the aggregate advertising level is fixed at the contracting stage greatly simplifies the analysis. In Appendix 12.2 we relax it by considering a version of the model in which platforms *do not* announce total advertising levels, but instead offer contracts of the form (t_i, m_i) to advertisers, and afterwards viewers and advertisers simultaneously decide which platform to join. We show that under some additional conditions on preferences, there exists an outcome-equivalent SPNE to that of our game. This two-stage game is much harder to analyze since a deviation by one platform leads to simultaneous changes in viewers' and advertisers' decisions that are influenced by each other. For this reason, and due to the outcome-equivalence under certain conditions, we stick to the easier formulation.

4 Preliminaries: Contracting Stage

To identify the competitive forces, we proceed by contrasting the market outcome of the game just described, in which two platforms compete, with the monopoly case, that is, only one platform is present in the market. We first solve the contracting stage which is critical to tackle the strategic interaction.

A key observation is that after any pair of first stage announcements (n_1, n_2), in any continuation equilibrium, platforms spread their advertising level equally across all advertisers. This result follows due to diminishing returns from advertising. As there is a unit mass of advertisers, the number of advertising intensities offered to each advertiser by platform i is equal to n_i. In turn, the equilibrium transfer is the incremental value that advertising intensity n_i on platform i generates for an advertiser who already advertises with intensity n_j on the other platform.[21]

Claim 1: *In any SPNE of a game with competing platforms, given any pair of first-stage choices (n_1, n_2), each platform i only offers one contract (t_i, m_i). Moreover, this contract is accepted by all advertisers, and has the feature that $m_1 = n_1$, $m_2 = n_2$, $t_1 = u(n_1, n_2) - u(0, n_2)$ and $t_2 = u(n_1, n_2) - u(n_1, 0)$.*

The next claim establishes a parallel result for the single platform (that is, monopoly) case, whose proof we omit because it follows along the same lines as the proof of Claim 1 above. In particular the monopolist offers a single contract that is accepted by all advertisers.

Claim 2: *In any SPNE of a game with a monopolistic platform, given first-stage choice n_i, the monopolist offers a single contract (t, m_i). Moreover, this contract is accepted by all advertisers, and has the feature that $m_i = n_i$, and $t = u(n_i, 0)$.*

Claims 1 and 2 imply that, since in equilibrium viewers correctly anticipate the unique continuation play following stage 1, in any SPNE viewer demand of platform i is $D_i(n_1, n_2) + D_{12}(n_1, n_2)$, $i = 1, 2$. In what follows, we denote $D_i(n_1, n_2) + D_{12}(n_1, n_2)$ by $d_i(n_i)$, that is, $d_i(n_i) := \text{Prob}\{q_i - \gamma n_i \geq 0\}$. Furthermore, platforms' equilibrium profits under duopoly are lower than the equilibrium profit obtained by the monopolist. Under duopoly, platforms can only charge the incremental value of an advertiser who is also active on the other platform, whereas a monopolist can extract the whole surplus. Specifically, a monopolist platform i obtains a profit of $d_i(n_i)\phi_i(n_i)$, since it has only exclusive viewers, while platform i in duopoly only obtains $D_i(n_i)\phi_i(n_i) + D_{12}(n_1, n_2)\left(\phi_{12}(n_1, n_2) - \hat{\phi}_j(n_j)\right)$, because it *shares* some viewers with its rival.

5 Platform Competition

We proceed by contrasting the choice of a monopolist:[22]

$$n_i^m := \arg\max_{n_i} \quad d_i(n_i)\phi_i(n_i), \tag{1}$$

with the duopoly outcome, that is with the fixed point of the best reply correspondences:

$$n_i^d := \arg\max_{n_i} \quad D_i(n_i)\phi_i(n_i) + D_{12}(n_1, n_2)\left(\phi_{12}(n_1, n_2) - \hat{\phi}_j(n_j)\right) \qquad i = 1, 2; \quad j = 3 - i. \tag{2}$$

Our goal is to determine the effects that drive competition in this model. For this purpose it is useful to rewrite the duopolist's profit *as if* all viewers were exclusive plus a correction term that accounts for the

[21] In what follows, to simplify notation we denote $u(n_i, n_j, n_i, n_j)$ by $u(n_i, n_j)$ and $u(n_i, n_j, n_i, 0)$ by $u(n_i, 0)$.

[22] Here we adopt the convention that i denotes the monopolist platform.

fact that platform i can only extract the incremental value from its shared viewers:

$$n_i^d := \arg\max_{n_i} \quad d_i(n_i)\phi_i(n_i) + D_{12}(n_1, n_2)\left(\phi_{12}(n_1, n_2) - \hat{\phi}_j(n_j) - \phi_i(n_i)\right). \tag{3}$$

First consider problem (1). Its solution is characterized by the first order condition:

$$\frac{d\phi_i}{dn_i}d_i + \frac{dd_i}{dn_i}\phi_i = 0. \tag{4}$$

When increasing n_i, platform i trades off profits on inframarginal viewers due to increased reach with profits on marginal viewers who switch off. If we introduce the advertising elasticities of the total demand d_i and of the advertising function ϕ_i with respect to n_i,

$$\eta_{d_i} := -\frac{dd_i}{dn_i}\frac{n_i}{d_i} \qquad \text{and} \qquad \eta_{\phi_i} := \frac{d\phi_i}{dn_i}\frac{n_i}{\phi_i},$$

then the optimal quantity is characterized by the following simple and intuitive condition:

$$\eta_{\phi_i} = \eta_{d_i}.$$

Consider now problem (3). In duopoly, condition (4) should be augmented to account for the fact that some of the previously exclusive consumers are now shared:

$$\frac{d\phi_i}{dn_i}d_i + \frac{dd_i}{dn_i}\phi_i + D_{12}\frac{\partial(\phi_{12} - \phi_i - \hat{\phi}_j)}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_i - \hat{\phi}_j) = 0. \tag{5}$$

To build intuition, consider the simplest case in which the two platforms are symmetric, i.e., $d_i(n) = d_j(n)$, $\phi_i(n) = \phi_j(n)$, $\hat{\phi}_i(n) = \hat{\phi}_j(n)$, and $\phi_{12}(n_1, n_2) = \phi_{12}(n_2, n_1)$ for all n, n_1, and n_2, and suppose that competing platforms behave as a monopolist: $n_j = n_i = n_i^m$. Can these advertising levels constitute an equilibrium? First, recall that overlapping viewers receive advertising messages from two platforms. At these levels, the advertising levels they are exposed to *doubles* in duopoly. Other things held constant, decreasing marginal returns give an incentive to scale back advertising on platform i, whose marginal contribution to the advertisers' surplus drops as a result of such duplication. This *duplication effect* is captured by the third term of (5) which is negative. The second, arguably more subtle, effect is captured by the fourth term in (5). It is typically[23] positive as $\partial D_{12}/\partial n_i < 0$ and $\phi_i + \hat{\phi}_j > \phi_{12}$. Recall that $d_i(n_i) = D_i(n_i) + D_{12}(n_1, n_2)$. So the total variation in demand due to a small increase in n_i decomposes as $\partial D_i/\partial n_i + \partial D_{12}/\partial n_i$. This term captures the fact that a monopolist is wary of the total variation of d_i regardless of how this variation is spelled out. A duopolist distinguishes instead between the two sources of variation. In duopoly the opportunity cost of losing shared business is lower than that of losing exclusive business. Other things held constant, this *business-sharing effect* gives the platform an incentive to increase its advertising levels.

Before moving on we stress that this force points in the *opposite* direction as the one brought about

[23] $\phi_{12} - \hat{\phi}_j - \phi_i$ is equivalent to $\hat{\phi}_i\hat{\phi}_j - (\hat{\phi}_i - \phi_i)$. So in principle, with strong complementarities ($\hat{\phi}_i$ much larger than ϕ_i) the last term of (5) would be negative. We think of this as a mere theretical possibility. Accrodingly we deliberately neglect this possibility while interpreting the results, impliclty assuming that the correction term in (3) is negative.

by competition in traditional two-sided single-homing setups. A key insight there is that competitive pressure induces competing platforms to put *more* emphasis on lost business than monopolists do. (See, for example, the discussion in Armstrong (2006), section 4). Lost business on one side if captured by the rival would lower revenues on the other side of the market, as consumer find the rival more attractive because of the indirect network effects. As a consequence, advertising levels, which act as a price for viewers, fall if competitive pressure increases. By contrast, in our model the business sharing effect leads to higher advertising levels. Intuitively, if multi-homing viewers are relatively more responsive than single-homing viewers, that is, if they account for a relatively high portion of the variation, losing viewers is less detrimental for the duopolist. Therefore, platforms put *less* emphasis on lost business in duopoly relative to monopoly, leading to the opposite result. The following proposition takes stock. Let

$$\eta_{D_{12}} := -\frac{\partial D_{12}}{\partial n_i}\frac{n_i}{D_{12}} \qquad \text{and} \qquad \eta_{\phi_i+\hat{\phi}_j-\phi_{12}} := \frac{\partial(\phi_i+\hat{\phi}_j-\phi_{12})}{\partial n_i}\frac{n_i}{\phi_i+\hat{\phi}_j-\phi_{12}}.$$

Proposition 1: *An incumbent monopolist's advertising level increases (decreases) upon entry of a competitor if and only if*

$$\frac{\eta_{D_{12}}}{\eta_{d_i}} > (<) \frac{\eta_{\phi_i+\hat{\phi}_j-\phi_{12}}}{\eta_{\phi_i}}, \tag{6}$$

where all functions are evaluated at $n_i = n_i^m$ *and* $n_j = n_j^d$.

The left-hand side of (6) is the ratio of the demand elasticity of overlapping viewers to the demand elasticity of viewers in monopoly. To interpret the right-hand side, let us first rewrite the profit function of a platform in the duopoly case. Defining $\Delta_{\phi_i}(n_i) \equiv \phi_i(n_i) - \hat{\phi}_i(n_i)$ and plugging in $\phi_{12} = \hat{\phi}_1(n_1) + \hat{\phi}_2(n_2) - \hat{\phi}_1(n_1)\hat{\phi}_2(n_2)$, we can write this profit function as (arguments omitted):

$$\pi_i^d = d_i\phi_i - D_{12}(\hat{\phi}_i\hat{\phi}_j + \Delta_{\phi_i}). \tag{7}$$

Here, $\hat{\phi}_i\hat{\phi}_j$ is a measure of wasted (or duplicated) advertising. It is the probability that a given overlapping viewer is independently informed twice: once on *each* outlet. In equivalent terms, this is the fraction of multi-homers whose value cannot be extracted in duopoly as they can be delivered by both platforms. This term is adjusted by an amount Δ_{ϕ_i} to account for viewer heterogeneity. The right-hand side of (6) then is equal to:

$$\frac{\eta_{\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i}}}{\eta_{\phi_i}}, \tag{8}$$

with

$$\eta_{\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i}} := \frac{\partial(\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i})}{\partial n_i}\frac{n_i}{\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i}}.$$

Loosely speaking the numerator is a measure of duplication. It tells what fraction of advertising messages gets wasted due to duplication *across outlets* following a one percentage point increase in the amount of messages sent. Clearly, other things held constant the marginal value of an extra ad decreases with the numerator. We will come back to the determinants of this term in Section 7.

We note that a similar intuition holds if we start from any number of incumbent platforms, not just a monopoly platform. For example, if there are two incumbent platforms and a third one enters, there will be viewers who formerly were exclusives of a incumbent platform but now will be shared with the entrant. In addition, some of the formerly overlapping viewers will now watch all platforms. For both viewer types, a duplication and a business-sharing effect occurs. Therefore, whether advertising levels fall or rise depends on the strength of these effects and how large each viewer group is.[24]

An important merit of (6) is that it spells out the effect of applying competitive pressure in terms of empirical objects. However, its insightfulness is limited without a theory that suggests when the condition should have a particular sign. We address this issue in the next two sections. Note that at large this condition asks if there are any systematic differences between the two pools of consumers that could tilt the trade-off one way or the other. The two sides of the inequality stress two different sources of dissimilarities, both of which playing a role in duopoly only. The left-hand side focuses on relative *preferences*, expressed by demand elasticities. The right-hand side focuses on potential wedges in the advertising *technology*, expressed by elasticities of the advertising function. Being these two very different mechanisms, we tackle them separately. In Section 6 we add structure on the ϕ functions in a way that guarantees that the right-hand side of (6) equals one. This shuts down the technological source. Results there are purely driven by systematic differences in preferences across types. In Section 7 we carry over the mirror exercise. That is, we shut down the preferences source by using the insights gained in Section 6. As we shall see, it is possible to add structure to the joint distribution in a way that guarantees a left-hand side of (6) equal to one for all (n_1, n_2). Our findings there will hinge solely on technological factors. This break-up is implemented for illustrative purposes only. In principle, we could carry over the two exercises simultaneously.

6 Viewer Preference Correlation

To isolate how relative preferences shape the effect of competition, in this section we assume $\hat{\phi}_i(n_i) = \phi_i(n_i)$ for $i = 1, 2$. Loosely speaking, this amounts to considering the case where overlapping customers are neither harder nor easier to inform than exclusive ones. For example, this would be verified under the widely used Butters' (also called exponential) form: $\phi_i(n) = \phi_j(n) = 1 - e^{-n}$ and $\phi_{12}(n_1, n_2) = \phi_i(n_1 + n_2) = 1 - e^{-(n_1+n_2)}$.[25] Using $\hat{\phi}_i(n_i) = \phi_i(n_i)$ one can easily verify that the right hand side of (6) equals 1 for all (n_1, n_2). So the condition (6) simplifies to:

$$\frac{\eta_{D_{12}}}{\eta_{d_i}} > (<) 1.$$

We seek now to identify meaningful features of the joint distribution of preferences that could lead to systematic differences in the relative elasticities of demand. In light of our interest in diversity, a striking feature of (6) is that the effect of competition depends on the joint distribution of preferences through $\eta_{D_{12}}$ *only.* So any change in the joint distribution that results in a decrease of $\eta_{D_{12}}$ for equal marginal distributions is, loosely speaking, 'more likely' to yield downward competitive pressure. We proceed down this road by adding structure to preferences through a scalar which is meant to capture content 'likeness.' Specifically we assume that (q_1, q_2) is drawn from a bivariate Normal distribution with

[24] See 12.2 in the Appendix for a formal analysis.

[25] This functional form was firstly introduced in a seminal paper by Butters (1977) and widely used since then in applied work on advertising. It can be derived from natural primitive assumptions on the stochastic process that governs the allocation of messages to consumers.

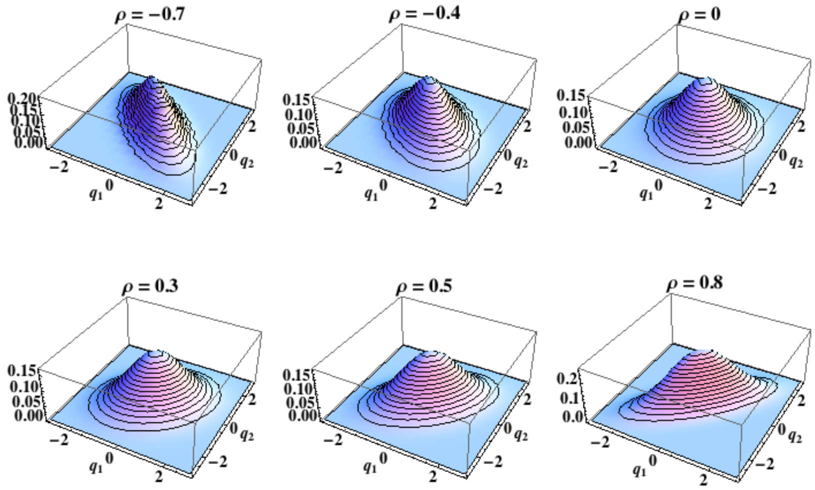


Figure 1: PDF of a standard bivariate normal with mean $(0, 0)$ and variance-covariance Σ_q

mean $(0, 0)$ and variance-covariance matrix:

$$\Sigma_q = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

The parameter ρ is the coefficient of linear correlation between q_1 and q_2 (Figure 1). A key observation is that a higher correlation coefficient, ceteris paribus, *is equivalent* to an increase in the extent of overlap D_{12}.[26] In other words an increase in correlation is equivalent to a change in the demand composition, whereby a higher fraction of platform i's total demand is comprised of overlapping viewers.[27]

At first thought, this suggests a *negative* relationship between ρ and the equilibrium advertising level. After all, the higher the number of overlapping viewers the stronger the duplication effect must be. However, this argument is not conclusive. The reason is that a larger ρ could enhance the business-sharing effect as well. A larger D_{12} indeed leads to a larger fraction of the variation coming from shared viewers. This, other things held constant, suggests a *positive* relationship. The resulting indeterminacy is reflected in the fact that what matters is how the *elasticity* of the demand D_{12} changes with correlation. The next lemma proves that a systematic relationship between η_{D12} and joint preferences as captured by ρ indeed exists.[28]

Lemma 1 $\eta_{D_{12}}$ decreases with ρ for all $n_1, n_2 > 0$ with $n_1 = n_2$.

It follows that η_{D12}/η_{d_i} decreases with ρ. So the lemma basically says that the set of marginal viewers is composed of relatively more exclusive viewers when correlation is higher. Figure 2 builds some geometric intuition by showing how changes in ρ affect the relevant variable for a discrete increase in n_1 given n_2. In Figure 2, when $n_1 = 0.5$, the measure of the square $A + B$ corresponds to overlapping

[26] Recall that the total demand of platform i depends only on the marginal distribution. To the best of our knowledge the result is not part of the basic collection of results on multivariate normals. A proof is therefore provided in the appendix.

[27] So, if $q_1 - \gamma n_1 > 0$ then an increase in ρ makes it more likely that $q_2 - \gamma n_2 > 0$ as well.

[28] The proof of this lemma is included in the proof of Proposition 2.

viewers, whereas $C + D$ corresponds to exclusive viewers. Instead, when $n_1 = 1$, the measure of the square B corresponds to overlapping viewers, whereas D corresponds to exclusive viewers. Consider first $\rho = 0.9$. At $n_1 = 0.5$, platform 1 has a balanced composition of overlapping and exclusive viewers. It is evident from the right-hand side of the figure that after the increase in n_1, the ratio of overlapping to exclusive viewers changes from $(A + B)/(C + D)$ to B/D, which is much larger than $(A + B)/(C + D)$. That is, after the increase in n_1 the demand of platform 1 consists only of a small portion of exclusive viewers. In other words, following an increase in the advertising level, a platform's demand composition is tilted towards overlapping viewers if the viewer preference correlation is positive. This implies that, with positive correlation, as n_1 increases, platform 1 loses the valuable exclusive viewers at a higher rate. So the marginal set of viewers is comprised of relatively more exclusives. Therefore, the business sharing effect is small and is dominated by the duplication effect. This results in a downward pressure of the advertising levels. By contrast, for $\rho = 0$, the composition of marginal viewers is much more balanced. The ratio B/D is similar to $(A+B)/(C+D)$. Indeed, as Proposition 2 will show, the two effects exactly off set each other.

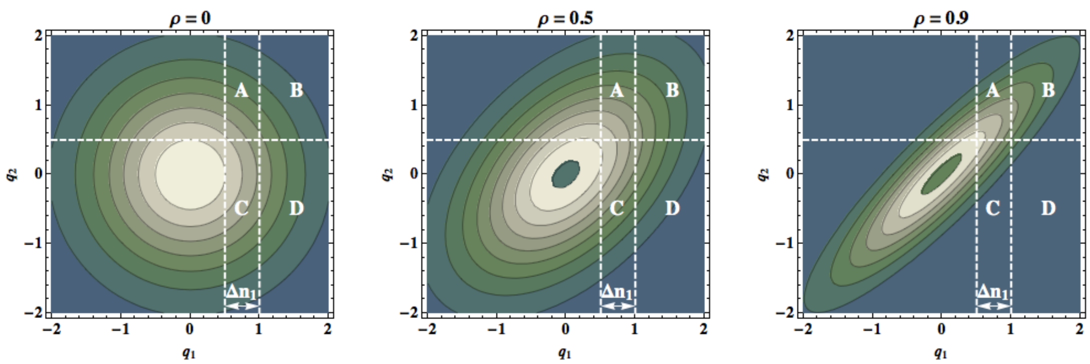


Figure 2: Contour map of h. Dashed lines correspond to $n_1 = n_2 = 0.5$ and $\Delta n_1 = 0.5$.

While we have not attempted to extend the proof to the larger class of exponential family of multivariate distributions, we conjecture that this property holds more extensively. To substantiate this claim we show in appendix that the above statement is equivalent to the bivariate normal satisfying a generalization to random *vectors* of the familiar Increasing Hazard Rate Condition. Specifically, we show in the appendix that η_{D12} decreasing is equivalent to $h(n_1, n_2)/\bar{H}(n_1, n_2)$ increasing in n_1 and n_2 along the $n_1 = n_2$ trajectory, where $\bar{H} \equiv D_{12}$ is the survival function and h is the density. Applying Lemma 2 allows to establish the following result:

Proposition 2 *The comparison between advertising levels in monopoly and in duopoly depends solely on the correlation coefficient ρ. In particular, an incumbent monopolist's advertising level increases upon entry of a competitor if and only if ρ is negative. That is,*

$$\text{sign}(n_i^d - n_i^m) = \text{sign}(\eta_{D_{12}} - \eta_{d_i}) = -\text{sign}(\rho).$$

where all functions are evaluated at $n_i = n_i^m$ and $n_j = n_j^d$.

A positive correlation coefficient leads to a fall in advertising levels with competition while a negative one leads to higher advertising levels. Before moving on we use this result in two different ways. First, we discuss the implied strategic considerations that platforms would have take into account when choosing which *kind* of content to produce. Second we discuss how this result, interpreted as a prediction provided we can indeed observe correlation, can be used as a first empirical test of the theory.

Implications for Content Choice

While content has been kept exogenous so far, a natural application of this model is content choice. In particular as it allows to add to the ongoing debate on "competition and diversity" in the media which, for obvious reasons, is often spelled out as "ideological" diversity. The exercise relies on two premises: 1) we conjecture that potential entrants can affect the degree of correlation at some 'content production' stage. This stage is akin to product positioning in standard models of product differentiation and 2) a decrease in ρ can be read as an increase in the supply of more diverse content. Our aim is not to provide a full-fledged model of differentiation, which would be largely outside the scope of the paper. Rather we seek here to identify broad mechanisms that we do not expect to be sensitive to a particular model specification: 1) Would an entrant that caters to the same viewers as the incumbent be more or less profitable than a platform that caters to those who find the incumbent unappealing? 2) In light of Proposition 2, do strategic considerations enhance or reduce the incentives to differentiate one's content from the rival's?

What we have in mind is a simple two-stage game. At stage 1 an entrant observes the content of the incumbent and chooses the extent of differentiation $-\rho$ so as to maximize its profits minus an investment cost that possibly depends itself on ρ.[29] At stage 2 competition takes place as described in Section 3. Given a well-behaved problem, in equilibrium the marginal benefit of differentiation:

$$\frac{d\pi_i^d}{d-\rho} = \left(\frac{\partial D_{12}}{\partial \rho} - \frac{\partial n_j}{\partial \rho}\left(\frac{\partial(\phi_{12}-\hat{\phi}_j)}{\partial n_j}\frac{1}{\phi_i+\hat{\phi}_j-\phi_{12}} - \frac{\partial D_{12}}{\partial n_j}\right)\right)(-\phi_{12}+\phi_i+\hat{\phi}_j),$$

is equated to the marginal cost. The key observation here is that this marginal benefit is not necessarily monotonically decreasing in ρ, so that $\rho > 0$ can indeed obtain in equilibrium as a result of firms trying to 'soften' competition much as in standard models of product differentiation. To build intuition suppose that the entrant is considering to supply similar content (i.e. $\rho > 0$), evaluate the marginal benefit and consider its sign which shapes the incentives to differentiate. The first term is positive. It captures the basic insight that decreasing correlation allows to capture relatively more exclusive viewers, which are obviously more valuable to advertisers. So other things held constant, the entrant has an incentive to invest in 'diverse' content (or diminish ρ). The second term in parenthesis accounts for the rival's reaction. As discussed a lower ρ induces an equilibrium in which the rival platform competes more aggressively for advertising dollars by increasing its supply of ads. This mechanism, which we conventionally refer to as the 'strategic' one, may point in the opposite direction, as more ads from one's rival reduce the extent of rent extraction from overlapping viewers.[30] In general the overall effect is ambiguous and which one prevails is ultimately an empirical question.

Interestingly, our 'direct' and 'strategic' effect can have opposite forces than in standard models of differentiation (see, for example, d'Aspremont et al. (1979) on horizontal differentiation or Shaked and

[29] For instance $c(1-\rho)$ with $c' > 0$ and $c'' > 0$ would capture the idea that duplication $\rho = 1$ is costless while differentiation is increasingly costly.

[30] For the case $\hat{\phi}_i = \phi_i$ it can be shown that the term reflecting the strategic effect is negative when $\rho > 0$.

Sutton (1982) on vertical differentiation). In these models, the strategic effect is that firms become more differentiated to 'soften' competition while the direct effect is that firms have a smaller secured demand (a smaller 'hinterland') which lowers profits. In our case, the opposite could hold, 'escaping' competition resulting from 'less' differentiation.

Note that these questions have not been addressed in existing works of either/or competition, as these models rely either on Hotelling spatial models or assume a representative viewer. In the first case the correlation between viewers' preferences is assumed to be perfectly negative, i.e., the viewer who likes platform i most likes platform j least, while in the second case viewers are homogeneous by assumption.

Empirical Analysis

As a reality check, Appendix 12.3 contains a first pass to test Proposition 2. We conduct there an empirical exercise using data on the U.S. Broadcasting TV industry. The exercise exploits variation in the extent of competitive pressure brought about by entry and exit of TV channels in the Basic Cable lineup in the 80s and 90s. The empirical strategy is to regress strategic choices, which we observe, (here the logarithm of the average number of advertising slots per hour supplied by the networks) on a measure of entry and a number of controls. There are mainly two difficulties with the analysis, which is why we decided to relegate it to the appendix. A first one is the issue of entry endogeneity on incumbent performance. In general it is hard to instrument for entry (see e.g., Berry and Reiss, 2007) and unfortunately we do not have exploitable variation for this purpose nor we are aware of the existence of natural experiments that could come to our rescue. A second one, specific to our theory, is the impossibility to *observe* consumers' preferences and thus their correlation across different outlets. This said, we believe it is reasonable to assume that those who watch ESPN are more likely to watch ESPN2 or FoxSports. So we make assumptions on how the content of the entrants in a particular segment relates to that of the incumbents. Also, while it is hard to establish a causal link due to endogenous entry, we can nonetheless show that a decrease in preference correlation is associated to an increase in the advertising levels as predicted (and vice versa). Furthermore this evidence is in line with the anecdotal evidence on the positive impact of the entry of FOX News on the advertising level supplied by MSNBC or CNN, which is often referred to as the "Fox News Puzzle". In summary, while this evidence cannot be considered definitive, we regard it as suggestive.

7 Advertising Technology

As outlined above, competition comes hand-in-hand with duplication. In duopoly multi-homers receive same ads from two different sources. We now explore if this fact together with diminishing marginal returns suffices to inject downward pressure on advertising levels. To this end assume throughout this section that $\eta_{D_{12}} = \eta_{d_i}$. For example, if $(q_1, q_2) \sim \mathcal{N}(0, \Sigma_q)$ as above than this assumption holds if and only if the valuations for the two platforms are independently distributed, (i.e., if and only if $\rho = 0$). Condition (6) tells us that in this particular case competition reduces advertising levels if and only if:

$$1 < \frac{\eta_{\hat{\phi}_i \hat{\phi}_j + \Delta_{\phi_i}}}{\eta_{\phi_i}}.$$

Interestingly, if the above condition is violated, in duopoly the advertising level on each platform is higher, despite preference independence and overlapping consumers redundantly being reached multiple times. (Precisely, they get at least one message from both sources with probability $\hat{\phi}_i \hat{\phi}_j$).

The next proposition shows that competition is shaped only by the platform's relative ability to inform different kinds of viewers, that is, in the difference to reach exclusive versus overlapping viewers. Recall that $\Delta_\phi := \phi_i - \hat{\phi}_i$. Employing the following decomposition

$$\frac{\eta_{\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i}}}{\eta_{\phi_i}} = \frac{\alpha\eta_{\hat{\phi}_i\hat{\phi}_j} + \beta\eta_{\phi_i} - \gamma\eta_{\hat{\phi}_i}}{\eta_{\phi_i}}, \tag{9}$$

with $\alpha+\beta+\gamma=1$ allows us to derive a result analogous to that of Proposition 2:

Proposition 3 *The comparison between advertising levels in monopoly and in duopoly depends solely on the relative elasticity of the reach of exclusive versus overlapping consumers. In particular, an incumbent monopolist's advertising level increases upon entry of a competitor if and only if:*

$$\text{sign}(n_i^d - n_i^m) = \text{sign}(\eta_{\hat{\phi}_i} - \eta_{\phi_i}). \tag{10}$$

where all functions are evaluated at $n_i = n_i^m$ and $n_j = n_j^d$.

If the technology takes the exponential form, that is, $\phi_i(n) = 1 - e^{-bn}$ and $\hat{\phi}_i(n) = 1 - e^{-\hat{b}n}$,[31] then (10) simplifies to

$$\text{sign}(n_i^d - n_i^m) = \text{sign}(\eta_{\hat{\phi}_i} - \eta_{\phi_i}) = \text{sign}(b - \hat{b}). \tag{11}$$

where $b - \hat{b}$ parametrizes the relative efficiency. If $b > \hat{b}$, the business-sharing effect dominates. In particular, the lower $\hat{b}$, the lower is the value of multi-homers, implying that a platform loses less when viewers switch off due to higher advertising levels.

The thrust of this result is that the effect of competition does *not* depend directly on the extent of duplication as represented by $\eta_{\hat{\phi}_i\hat{\phi}_j}$ or $\hat{\phi}_i\hat{\phi}_j$. In other words, for competition to have an impact on the incentives of platform i, it is necessary that there is consumer heterogeneity with respect to the reaching probabilities *within* platform i: $\hat{\phi}_i \neq \phi_i$. This heterogeneity could come from multiple sources. As discussed, if the same underlying factor that induces joint consumption (e.g. internet saviness) is also responsible of a differential intensity (for example overlapping consumers browsing relatively more pages on average on each outlet), then $\hat{b} > b$. Alternatively, think about a technology that allows the platforms to direct advertising messages to specific subsets of users that are sought by specific advertisers. How do these technologies shape comepetition? Shall we expect improvements in the communication techniques to enhance or reduce the market provision of advertising opportunities? Bergemann and Bonatti (2013), in a related setting employ the exponential form to model this feature. A higher targeting ability in their model translates here into a higher coefficients b and $\hat{b}$ corresponding to a higher ability of the platform to target advertising messages. If $\hat{b}$ increases by a larger extent than b, e.g., because targeting is more effective if a consumer is active on more platforms, then competition is more likely to decrease advertising levels.

[31] Calvano and Polo (2013) provide a micro-foundation for this generalized version of the Butter's technology.

8 Platform Mergers

An important question in media economics is the effect of platform mergers on market outcomes. To address this question, in this section we contrast the duopoly outcome with the outcome that a hypothetical monopolist who controls *both* platforms would implement. We obtain the following result:

Proposition 4: *The equilibrium advertising level in duopoly is larger than under joint ownership (i.e., $n_i^d > n_i^{jo}$), if and only if $\hat{\phi}_j > \phi_j$.*

To build intuition, suppose first that $\hat{\phi}_j = \phi_j$. When marginally increasing n_i, a monopolistic owner controlling both platforms loses some multi-homing viewers, who become single-homing viewers on platform j. With the first kind of viewers the monopolist loses ϕ_{12}, while with second he gains ϕ_j. In duopoly, when a platform increases n_i, it loses some multi-homing viewers whose values are $(\phi_{12} - \phi_j)$. But this implies that the trade-offs in both market structures are the same.[32] As a consequence, we obtain that the ownership structure is neutral for advertising levels.

If instead $\hat{\phi}_j < \phi_j$, overlapping consumers can be reached with a lower probability by advertisers than single-homing consumers. Therefore, competing platforms can then extract $(\phi_{12} - \hat{\phi}_j) > (\phi_{12} - \phi_j)$. This implies that when losing overlapping consumers, a platform in duopoly loses more than a joint owner does. The business-sharing effect for a joint owner is therefore larger than for competing platforms. Hence, advertising levels in joint ownership are larger. The result therefore runs counter the standard intuition that monopolists keep the advertising levels low to extract higher transfers from advertisers. In our model, the business-sharing effect points in the opposite direction. As we will show later, this implies that platform mergers can often be welfare enhancing.

To gain further intuition, let us explain the neutrality result that $n_i^d = n_i^{jo}$ for $\hat{\phi}_j = \phi_j$ in more detail. The following decomposition of Π_i^d, which is derived from (28) and (30), may aid intuition for the neutrality result:

$$\Pi_i^d = \Pi^{jo} - \phi_j(D_j + D_{12}).$$

The above profit is reminiscent of the payoff induced by Clarke-Groves mechanisms (Clarke (1971), Groves (1973)). Each agent's payoff is equal to the entire surplus minus a constant term—since the sum of D_j and D_{12} is unaffected by platform i's choices in pure either or both competition—which is equal to the payoff that the other agents would get in his absence. Clarke mechanisms implement socially efficient choices, here represented by the joint monopoly solution. If $\hat{\phi}_j \neq \phi_j$, this result no longer holds.

The result that advertising levels do not depend on the ownership structure for $\hat{\phi}_j = \phi_j$ is also reminiscent of common agency models (e.g., Bernheim and Whinston, 1985 and 1986), that predict the same outcomes in monopoly and in duopoly. However, common agency models feature a single agent who contracts with multiple principals instead of a continuum of agents, as in our framework. In particular, if there is only a single advertiser—or, equivalently, if all advertisers can coordinate their choices[33]—even in models featuring either/or competition, the equilibrium advertising level is the same with duopoly and

[32] Note that in both cases increasing n_i also implies losing some single-homing viewers on platform i. But the loss from this is exactly the same for the monopolist and the duopolist.

[33] For an analysis of consumer coordination in platform competition in a setting with positive network externalities, see Ambrus and Argenziano (2009).

with joint monopoly. To see this consider the case in which viewers join either platform i or j, implying that $D_{12} = 0$. If there is only a single advertiser, the transfer that platform i can charge to make the advertiser accept is the incremental value of the platform, i.e., $u(n_1^d, n_2^d) - u(0, n_j^d)$. In the either/or framework, $u(n_1^d, n_2^d) = D_1(n_1^d, n_2^d)\phi_1(n_1) + D_2(n_1^d, n_2^d)\phi_2(n_2)$, while $u(0, n_j^d) = D_j(0, n_j^d)\phi_j(n_j)$. Hence,

$$\Pi_i^d = D_1(n_1^d, n_2^d)\phi_1(n_1) + D_2(n_1^d, n_2^d)\phi_2(n_2) - D_j(0, n_j^d)\phi_j(n_j).$$

The first two terms are equivalent to a monopolist's profit, while the last term is independent of n_i^d. Therefore, the first-order conditions for monopoly and duopoly coincide and neutrality obtains. It follows that competition is disabled in common agency models independent of the agent being able to single- or to multi-home.

9 Is There too much Advertising?

A common theme in media markets is that the market provides an inefficiently high quantity of advertising. To address this concern we proceed by characterizing the socially optimal allocation. As mentioned, $q_i - \gamma n_i$ is the utility of a single-homing viewer of platform i and $q_1 - \gamma n_1 + q_2 - \gamma n_2$ is the utility of a multi-homing viewer. Social welfare is given by

$$W = \int_{\gamma n_1}^{\infty} \int_0^{\gamma n_2} (q_1 - \gamma n_1)\, h(q_1, q_2) dq_2 dq_1 + \int_0^{\gamma n_1} \int_{\gamma n_2}^{\infty} (q_2 - \gamma n_2)\, h(q_1, q_2) dq_2 dq_1$$

$$+ \int_{\gamma n_1}^{\infty} \int_{\gamma n_2}^{\infty} (q_1 - \gamma n_1 + q_2 - \gamma n_2)\, h(q_1, q_2) dq_2 dq_1 + D_1\phi_1 + D_2\phi_2 + D_{12}\phi_{12}.$$

Comparing the equilibrium advertising level with the socially efficient advertising level we obtain the following result for the case when multi-homing decreases the amount of attention a viewer pays to a particular outlet (presumably the more empirically relevant case):

Proposition 5: *Equilibrium advertising levels are inefficiently high as long as* $\hat{\phi}_j \leq \phi_j$.

The condition in Proposition 5 is sufficient, but not necessary: the result also holds when ϕ_j is larger than but close enough to ϕ_j. To grasp this result, it is useful to consider the incentives of the joint monopoly platform. Note that under our assumptions the monopolist fully internalizes advertisers' welfare. On the contrary, it does not internalize viewers' welfare. More precisely, it only cares about viewers' utilities inasmuch as they contribute to advertising revenue, while the nuisance costs from advertising are not taken into account. This leads to inefficiently high advertising levels. From our last section, we know that competing platforms implement even larger advertising levels as long as $\hat{\phi}_j < \phi_j$. Therefore, the equilibrium allocation in duopoly is worse from a social welfare perspective than with joint ownership.

Proposition 5 should be interpreted with caution. The overprovision result hinges on the assumption that advertisers are homogeneous. If advertisers are heterogeneous with respect to their product qualities, an extensive margin comes into play in addition to the intensive margin considered so far. This extensive margin arises because, as in previous literature, a platform owner trades off the marginal profit from an additional advertiser with the profits from inframarginal advertisers. This effect coupled with our result can either lead to socially excessive or socially insufficient advertising levels.

Propositions 4 and 5 together imply that with either/both type competition, advertising levels tend to be inefficiently high. This is consistent with the existence of regulatory "caps" or ceilings on the number of commercials per hour in many countries, suggesting concerns of overprovision of advertising.

We note that our conclusions differ from those obtained in models with either/or competition. For instance, in Anderson and Coate (2005) competition for viewers always reduces advertising levels relative to monopoly, which can lead to inefficiently low advertising levels, even with homogeneous advertisers. In addition, the redistributive impact of joint ownership is very different in both models. In our model, a joint owner can fully expropriate advertisers, whereas competing platforms cannot, implying that advertisers are hurt by a merger. In contrast, in Anderson and Coate (2005) a merger leads to an increase in the advertising level and a lower advertising price. Hence, advertisers are better off after a merger.

10 Heterogeneous Advertisers

In this section, we discuss how the trade-off characterized in Proposition 1 extends to advertisers with heterogeneous product values ω, as in Anderson and Coate (2005). The key insight is that, despite the information rent that comes with the inability to perfectly discriminate among advertisers, in equilibrium platform i's optimal transfer demanded from an advertiser of type ω is proportional to the incremental value of this type. So by the same token, the trade-off at the core of the analysis carries trough with all the due caveats. Needless to say, our results also hold when platforms can perfectly discriminate between advertisers. In that case, the results for each type are the same as the ones in case of homogeneous advertisers.

The analysis is more involved when ω is private information held by individual advertisers. We need to characterize an entire schedule (i.e. the optimal screening contracts) offered by platforms $(m_i(\omega), t_i(\omega))$, instead of only a single transfer-quantity pair (m_i, t_i).

Consider the following extension of our baseline model. The value of informing a viewer, ω, is distributed according to a smooth c.d.f. F with support $[\underline{\omega}, \overline{\omega}]$, $0 < \underline{\omega} \leq \overline{\omega}$, that satisfies the monotone hazard rate property. The value ω is private information to each advertiser. The timing of the game is the same as before. In the first stage, each platform i announces its total advertising level n_i. Afterwards, consumers decide which platform to join. Given these decisions, each platform offers a menu of contracts consisting of a transfer schedule $t_i := [0, \overline{m}] \to \mathbb{R}$ defined over a compact set of advertising levels. $t_i(m)$ is the transfer an advertiser has to pay to get an allocation m from platform i. In the final stage, as before, advertisers decide which platform to join. In what follows, we define $n = (n_1, n_2)$.

Let us start with the monopoly case. With an abuse of notation we still use $\omega u(m_i, n_i)$ to denote the surplus of advertiser type ω from an advertising quantity m_i. The overall utility of an advertiser depends on the transfer schedule in addition to the surplus. If $m_i(\omega)$ denotes the optimal quantity chosen by type ω, then platforms i's problem in case of monopoly is

$$\Pi = \max_{t_i(\cdot)} \int_{\underline{\omega}}^{\overline{\omega}} t_i(m_i(\omega)) dF(\omega). \tag{12}$$

By choosing the optimal menu of contracts, the monopolist determines which advertiser types to exclude, that is, $m_i(\omega) = 0$ for these types, and which advertiser types will buy a positive intensity. We denote the marginal advertiser by ω_0^m. Problem (12) can be expressed as a standard screening problem:

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\overline{\omega}} t_i(m_i(\omega))dF(\omega)$$

$$\begin{aligned} \text{subject to} \quad & m_i(\omega) = \arg\max_{m_i} v_i^m(m_i, \omega, n_i) - t_i(m_i), \\ & v_i^m(m_i(\omega), \omega, n_i) - t_i(m_i(\omega)) \geq 0 \;\; \text{for all } \omega \geq \omega_0^m, \\ & \textstyle\int_{\omega_0^m}^{\overline{\omega}} m_i(\omega)dF(\omega) \leq n_i, \end{aligned}$$

where $v_i^m(m_i, \omega, n_i) \; := \omega d_i(n_i)\phi_i(m_i)$ denotes the net value of advertising intensity m_i to type ω in the monopoly case. The first constraint is the incentive-compatibility constraint and the second one is the participation constraint. The third one is the capacity constraint specifying that the aggregate advertising level cannot exceed the one specified by the platform in the first stage. Provided that the function $v_i^m(m_i, \omega, n_i)$ satisfies the standard regularity conditions in the screening literature, we can apply the canonical methodology developed by Mussa and Rosen (1978) or Maskin and Riley (1984). Our assumptions on the viewer demand $d_i(n_i)$ and on the advertising technology $\phi_i(m_i)$ ensure that v_i^m is continuous and increasing in ω. It also has strict increasing differences in (m, ω).

Evidently, the capacity constraint will be binding at the optimal solution since it can never be optimal for the monopolist to announce a strictly larger advertising level than the one it uses. Applying this methodology, we can transform the maximization problem to get

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) d_i(n_i)\phi_i(m_i(\omega))dF(\omega)$$

subject to $n_i = \int_{\omega_0^m}^{\overline{\omega}} m_i(\omega)dF(\omega)$.

We show in the Appendix that the optimal advertising level n_i can be characterized by the following equation:

$$\int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) \left(\tilde{d}_i \frac{\partial \phi_i}{\partial m_i} + \frac{\partial d_i}{\partial n_i}\phi_i\right) dF(\omega) = 0, \tag{13}$$

with $\tilde{d}_i := (1 - F(\omega_0^m))d_i$. We can compare this characterization with the one for homogeneous advertisers given by (4). Due to the information rent that is required for incentive compatibility, the platform can no longer extract the full rent from advertisers but only a fraction of it. This is expressed by the first bracket in the integral. Inspecting the second bracket, the expression is analogous to the one with homogeneous advertisers. Note that in the latter case $m_i = n_i$ implying that the derivative was taken with respect to n_i in both terms. The above expression instead accounts for the fact that the optimal allocation $m_i(\omega)$ is heterogeneous across types. A second difference comes from the first term in the second bracket where we have $\tilde{d}_i$ instead of d_i. This is because when changing m_i only those advertisers which participate are affected which is only a mass of $1 - F(\omega_0^m)$, while with homogeneous advertisers all of them are active in equilibrium.

So, with heterogeneous advertisers the equation characterizing n_i trades off the cost and benefits of increasing n_i over the whole mass of participating advertisers, implying that the average costs and benefits are important. However, the basic trade-off in case of homogeneous advertisers and heterogeneous advertisers is the same. In particular, the first term in the second bracket represents the average marginal profit from increased reach on infra-marginal consumers, whereas the second term represents the average marginal loss from marginal consumers, who switch off.

Let us now turn to the optimal advertising levels in duopoly.

The goal is to characterize the best-reply tariff $t_i(m_i)$ given platform j's choice $t_j(m_j)$. As in the monopoly case, it is possible to rewrite this problem as a standard screening problem. To this end denote by $\omega u(m_1, m_2, n)$ the surplus of type ω from advertising intensities (m_1, m_2). If $m_i(\omega)$ denotes the optimal quantity chosen by type ω, then platforms i's optimization problem is:

$$\Pi = \max_{\omega_0^i, m_i(\omega)} \int_{\omega_0^i}^{\overline{\omega}} t_i(m_i(\omega)) dF(\omega)$$

$$\begin{aligned} \text{subject to} \quad & m_i(\omega) = \arg\max_{m_i} v_i^d(m_i, \omega, n) - t_i(m_i), \\ & v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) \geq 0 \ \text{ for all } \omega \geq \omega_0^i, \\ & \textstyle\int_{\omega_0^i}^{\overline{\omega}} m_i(\omega) dF(\omega) \leq n_i, \end{aligned}$$

where $v_i^d(m_i, \omega, n) := \max_y \omega u(m_i, y, n) - t_j(y) - \max_{y'}(\omega u(0, y', n) - t_j(y'))$, with $u(m_i, y, n) := D_i(n_1, n_2)\phi_i(m_i) + D_j(n_1, n_2)\phi_j(y) + D_{12}(n_1, n_2)\phi_{12}(m_i, y)$.

Note that the sole difference with respect to the monopoly case is that each advertiser's outside option accounts for the option of accepting the rival's offer. Hence, $v_i^d(m_i, \omega, n)$ is larger than $v_i^m(m_i, \omega, n_i)$. Again, our assumptions on the viewer demands $D_i(n_1, n_2)$ and $D_{12}(n_1, n_2)$ and on the advertising technology $\phi_i(m_i)$ and $\phi_{12}(m_1, m_2)$ ensure that v_i^d is continuous and increasing in ω. It also has strict increasing differences in (m, ω).

In the appendix we show by following the methodology of Martimort and Stole (2009) that it is possible to characterize the best-reply allocation as the solution to

$$\int_{\omega_0^i}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) \left(\tilde{d}_i \frac{\partial \phi_i}{\partial m_i} + \frac{\partial d_i}{\partial n_i}\phi_i + \tilde{D}_{12} \frac{\partial(\phi_{12} - \phi_i - \hat{\phi}_j)}{\partial m_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_i - \hat{\phi}_j)\right) dF(\omega) + \kappa = 0, \tag{14}$$

with $\tilde{d}_i := (1 - F(\omega_o^i))d_i$ and $\tilde{D}_{12} := (1 - F(\omega_o^i))D_{12}$ and κ defined in the Appendix. Ignoring κ for the moment, it is evident that this optimal duopoly solution (14) is the analogous of condition (5) accounting for the business sharing and duplication effect with heterogeneous advertisers.

Let us finally turn to κ. When changing the allocation of type ω, platform i has to take into account that such a different allocation also affects the advertisers' demand from the rival platform, m_j, given the posted schedule $t_j(\cdot)$. Intuitively, the higher the number of advertising messages on platform i, the lower the utility from one additional ad on platform j. This channel brings in new competitive forces that are absent with homogeneous advertisers. These forces are specific to the contracting environment considered and *in addition* to the ones discussed so far. To stress this we note in the appendix that if the rival platform were to offer a single quantity-transfer pair (or in other words, were to implement an incentive compatible allocation flat across all active types) then $\kappa = 0$ and the best-reply would still be characterized by (14).

11 Conclusion

This paper presented a platform competition model with either/both competition on the viewer side, allowing for fairly general viewer demand and advertising technologies. We emphasize the role that viewer composition plays for market outcomes, and identify novel competitive effects, such as the business sharing and the duplication effect.

The generality of the framework allows the model to serve as a useful building block to tackle a variety of questions. For example, we took the quality of platforms exogenous in our analysis, yet competition in media markets (and in many other industries) often works through quality. Our model can be used to investigate whether markets in which users can be active on multiple platforms lead to higher or lower quality than those in which users are primarily active on a single platform. Another interesting question pertains to pricing tools. We considered the case in which platforms offer contracts consisting of an advertising level and a transfer, but in some industries firms primarily charge linear prices. How then do our results depend on the contracting environment? Also, do linear prices lead to a more or less competitive outcome? We leave these questions for future research.

Our model is also not restricted to the media markets context. In particular, a characterizing feature of our model is that consumers are multi-stop shoppers, i.e., can patronize multiple firms, but that a firms revenue is lower for a consumer who buys from several other firms. The model is a first pass at understanding competition in settings where firms care not only about the overall demand but also about its composition. Such settings arise naturally when serving different types of customers yield different (indirect) revenues from other sources (as in our model), as well as when there are consumption externalities among customers.

12 Appendix

12.1 Proof of Propositions

Proof of Claim 1:

First suppose that there is a non-singleton menu of contracts $(t_i^k, m_i^k)_{k=1}^K$ offered by platform i such that each of these contracts are accepted by some advertisers. Then advertisers have to be indifferent between these contracts. Let $F(k)$ denote the cumulative density of advertisers accepting some contract $(t_i^{k'}, m_i^{k'})$ for some $k' \leq k$. Then, by strict concavity of ϕ_i and ϕ_{12}, if platform i instead offered a single contract $(F(K)E(t_i^k), F(K)E(m_i^k))$, where the expectations are taken with respect to F, each advertiser would strictly prefer to accept the contract, resulting in the same total advertising level and profit for the platform. But then platform i could increase profits by offering a single contract $(F(K)E(t_i^k) + \varepsilon, F(K)E(m_i^k))$, for a small enough $\varepsilon > 0$, since such a contract would still guarantee acceptance from all advertisers. The same logic can be used to establish that it cannot be in equilibrium that a single contract (t_i, m_i) is offered but only a fraction of advertisers $F(1) < 1$ accept it, since offering $(F(1) \times t_i + \varepsilon, F(1) \times m_i)$ for small enough $\varepsilon > 0$ would guarantee acceptance by all advertisers and generate a higher profit for platform i.

The above arguments establish that the total realized advertising level on platform i is m_i, the intensity specified in the single contract offered by i. It cannot be that $m_i > n_i$, since then by assumption the platform's payoff would be negative. Moreover, since ϕ_i and ϕ_{12} are strictly increasing, it cannot be that $m_i < n_i$, since then the platform could switch to offering a contract $(t_i + \varepsilon, n_i)$, which for small enough $\varepsilon > 0$ would guarantee acceptance by all advertisers and generate a higher profit for platform i. Thus $m_i = n_i$.

Finally, note that $t_1 < u(n_1, n_2) - u(0, n_2)$ implies that platform 1 could charge a higher transfer and still guarantee the acceptance of all advertisers, while $t_1 > u(n_1, n_2) - u(0, n_2)$ would contradict that all advertisers accept both platforms' contracts. Hence, $t_1 = u(n_1, n_2) - u(0, n_2)$. A symmetric argument establishes that $t_2 = u(n_1, n_2) - u(n_1, 0)$.

The proof of Claim 2 proceeds exactly along the same lines and is therefore omitted. ■

Proof of Proposition 1:

We know that the advertising level in case of duopoly is given by (3), while the advertising level of a single platform monopolist is given by (4). To check if advertising levels rise with entry, let us evaluate (3) at n_i^m and n_j^d. Since the first terms in both equations are the same, we get $n_i^d > n_i^m$ if and only if

$$\frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_i - \hat{\phi}_j) + D_{12}\left(\frac{\partial \phi_{12}}{\partial n_i} - \frac{\partial \phi_i}{\partial n_i}\right) > 0.$$

This is because, due to the fact that the objective functions are single-peaked, it follows that if the marginal profit evaluated at the pre-entry advertising level is positive, given that platform j sets n_j^d, then the incumbent's equilibrium advertising level in duopoly must be larger. Rearranging this inequality gives (acknowledging the fact that $\phi_{12} - \phi_i - \hat{\phi}_j < 0$)

$$-\frac{\partial D_{12}}{\partial n_i}\frac{n_i}{D_{12}} > \left(\frac{\partial \phi_{12}}{\partial n_i} - \frac{\partial \phi_i}{\partial n_i}\right)\frac{n_i}{\phi_{12} - \phi_i - \hat{\phi}_j}.$$

Since $\partial\phi_j/\partial n_i = 0$, we can write this inequality as

$$-\frac{\partial D_{12}}{\partial n_i}\frac{n_i}{D_{12}} > \frac{\partial(\phi_i+\hat{\phi}_j-\phi_{12})}{\partial n_i}\frac{n_i}{\phi_i+\hat{\phi}_j-\phi_{12}}. \tag{15}$$

Using our definitions

$$\eta_{D_{12}} := -\frac{\partial D_{12}}{\partial n_i}\frac{n_i}{D_{12}}$$

and

$$\eta_{\phi_i+\hat{\phi}_j-\phi_{12}} := \frac{\partial(\phi_i+\hat{\phi}_j-\phi_{12})}{\partial n_i}\frac{n_i}{\phi_i+\hat{\phi}_j-\phi_{12}},$$

we can rewrite (15) as $\eta_{D_{12}} > \eta_{\phi_i+\hat{\phi}_j-\phi_{12}}$. Dividing this expression by $\eta_{d_i} > 0$, we obtain $\eta_{D_{12}}/\eta_{d_i} > \eta_{\phi_i+\hat{\phi}_j-\phi_{12}}/\eta_{d_i}$. Finally, note that from (4) we have $\eta_{d_i} = \eta_{\phi_i}$, which yields

$$\frac{\eta_{D_{12}}}{\eta_{d_i}} > \frac{\eta_{\phi_i+\hat{\phi}_j-\phi_{12}}}{\eta_{\phi_i}}.$$

■

Proof of Proposition 2:

The proof consists of two lemmas. To simplify the exposition, we normalize γ to 1 in what follows.

Lemma *$\partial D_{12}/\partial\rho$ is strictly positive.*

Proof

For the bivariate normal distribution with mean (0,0) and variance $\Sigma = ((1,\rho),(\rho,1))$, we can write

$$D_{12} = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{n_2}^{\infty}\int_{n_1}^{\infty} e^{-\frac{q_1^2-2\rho q_1 q_2+q_2^2}{2(1-\rho^2)}}dq_1dq_2. \tag{16}$$

We can now perform integration with respect to q_1 and then differentiate with respect to ρ. Performing first the q_1 integration leads to the following expression,

$$D_{12} = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{n_2}^{\infty}\frac{\sqrt{\pi(1-\rho^2)}e^{-\frac{q_2^2}{2}}}{\sqrt{2}}\left[\mathrm{erf}\left(\frac{\rho q_2 - n_1}{\sqrt{2(1-\rho^2)}}\right) - \lim_{q_1\to\infty}\mathrm{erf}\left(\frac{\rho q_2 - q_1}{\sqrt{2(1-\rho^2)}}\right)\right]dq_2,$$

where erf$(\cdot)$ is an error function. Since erf$(-\infty) = -1$, we can write the above expression as

$$D_{12} = \frac{e^{-\frac{q_2^2}{2}}}{2\sqrt{2\pi}}\int_{n_2}^{\infty}\left[\mathrm{erf}\left(\frac{\rho q_2 - n_1}{\sqrt{2(1-\rho^2)}}\right)+1\right]dq_2.$$

Taking the derivative with respect to ρ yields

$$\frac{\partial D_{12}}{\partial\rho} = \frac{1}{2\pi(1-\rho^2)^{3/2}}\int_{n_2}^{\infty} e^{\frac{n_1^2-2\rho n_1 q_2+q_2^2}{2(1-\rho^2)}}(q_2-\rho n_1)dq_2. \tag{17}$$

We can integrate the right-hand side of (17) directly to obtain

$$\frac{\partial D_{12}}{\partial \rho} = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\frac{n_1^2 - 2\rho\gamma^2 n_1 n_2 + (n_2)^2}{2(1-\rho^2)}} > 0.$$

It is evident that the right-hand side of the last expression is the partial density function of D_{12}. We can also revert the order of integration and differentiation to obtain the same result. As a consequence, we have that $\partial D_{12}/\partial \rho > 0$ for all (n_1, n_2). ■

Lemma *$\partial \eta_{D_{12}}/\partial \rho$ is strictly negative.*

Proof Taking the derivative of $\eta_{D_{12}}$ with respect to ρ, we obtain

$$\frac{\partial \eta_{D_{12}}}{\partial \rho} = -\frac{\partial\left(\frac{\partial D_{12}}{\partial n_i}\right)}{\partial \rho}\frac{n_i}{D_{12}} + \frac{\partial D_{12}}{\partial \rho}\frac{\partial D_{12}}{\partial n_i}\frac{n_1}{D_{12}^2} = -\frac{n_i}{D_{12}}\left(\frac{\partial^2 D_{12}}{\partial n_i \partial \rho} - \frac{\partial D_{12}}{\partial \rho}\frac{\partial D_{12}}{\partial n_i}\frac{1}{D_{12}}\right), \tag{18}$$

Rearranging (18) and using $\partial D_{12}/\partial \rho > 0$ yields $\partial \eta_{D_{12}}/\partial \rho < 0$ if and only if

$$\frac{\frac{\partial D_{12}}{\partial n_i}}{D_{12}} < \frac{\frac{\partial^2 D_{12}}{\partial n_i \partial \rho}}{\frac{\partial D_{12}}{\partial \rho}}. \tag{19}$$

We can write D_{12} as $D_{12} = \int_{n_2}^{\infty}\int_{n_1}^{\infty} f(q_1, q_2) dq_2 dq_1$, where $f(q_1, q_2)$ denotes the density function of the distribution. As we have shown in the proof of the last Lemma, for the bivariate normal distribution we know that $\partial D_{12}/\partial \rho$ is just the partial density function, implying that

$$\frac{\partial D_{12}}{\partial \rho} = f(n_1, n_2).$$

We can therefore rewrite (19) as

$$-\frac{\int_{n_j}^{\infty} f(n_i, q_j) dq_j}{\int_{n_2}^{\infty}\int_{n_1}^{\infty} f(q_1, q_2) dq_2 dq_1} < \frac{\frac{\partial f(n_1, n_2)}{\partial n_i}}{f(n_1, n_2)}. \tag{20}$$

We note here that due to the fact that $\partial D_{12}/\partial \rho = f(n_1, n_2)$, condition (19) is equal to

$$\frac{\frac{\partial^2 D_{12}}{\partial n_1 \partial n_j}}{D_{12}}$$

decreasing in n_i, which is the familiar Monotone Hazard Rate Condition.

Using the fact that $f(n_1, n_2) = e^{-\frac{n_1 - \rho n_1 n_2 + n_2}{2(1-\rho^2)}}$, we can rewrite the right hand-side of (20) as

$$-\frac{n_i - \rho n_j}{1-\rho^2}.$$

Now we turn to the left-hand side of (20). The numerator is $\int_{n_j}^{\infty} f(n_i, q_j) dq_j$. We can rearrange this to

$$\int_{n_j}^{\infty} f(n_i, q_j) dq_j = \int_{n_j}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{n_i^2 - 2\rho n_i q_j + q_j^2}{2(1-\rho^2)}} dq_j$$

$$= \left| \int_{n_j}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_j \right|_{\infty}^{n_i}$$

$$= \int_{n_j}^{\infty} \int_{\infty}^{n_i} \frac{1}{2\pi\sqrt{1-\rho^2}} \frac{\partial e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}}}{\partial q_i} dq_i dq_j$$

$$= \int_{n_j}^{\infty} \int_{\infty}^{n_i} \frac{1}{2\pi\sqrt{1-\rho^2}} \frac{\rho q_j - q_i}{1-\rho^2} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j$$

$$= \int_{n_j}^{\infty} \int_{n_i}^{\infty} \frac{q_i - \rho q_j}{2\pi\sqrt{1-\rho^2}(1-\rho^2)} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j.$$

The denominator is given by

$$\int_{n_j}^{\infty} \int_{n_i}^{\infty} f(q_1, q_2) dq_2 dq_1 = \int_{n_j}^{\infty} \int_{n_i}^{\infty} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_2 dq_1.$$

We can therefore write the left-hand side of (20) as

$$-\frac{\int_{n_j}^{\infty} \int_{n_i}^{\infty} (q_i - \rho q_j) e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_i dq_j}{\int_{n_j}^{\infty} \int_{n_i}^{\infty} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_j dq_i} \frac{1}{1-\rho^2}. \tag{21}$$

For $n_j = n_i$ we can rewrite this as

$$\frac{\int_{n_j}^{\infty} \int_{n_i}^{\infty} q_j e^{-\frac{q_i^2 - 2\rho q_i n_j + n_j^2}{2(1-\rho^2)}} dq_i dq_j}{\int_{n_2}^{\infty} \int_{n_1}^{\infty} e^{-\frac{q_i^2 - 2\rho q_i q_j + q_j^2}{2(1-\rho^2)}} dq_2 dq_1} \frac{1-\rho}{1-\rho^2} = -E(q_j | q_j \geq n_j) \frac{1-\rho}{1-\rho^2} \tag{22}$$

For $n_i = n_j$, the right hand-side of (20) is given by

$$-n_j \frac{1-\rho}{1-\rho^2}.$$

Since

$$-E(q_j | q_j \geq n_j) \frac{1-\rho}{1-\rho^2} < -n_j \frac{1-\rho}{1-\rho^2}.$$

the inequality in (20) is always fulfilled, implying that $\partial \eta_{D_{12}} / \partial \rho < 0$. ■

To complete the poof note that η_{d_i} is unaffected by ρ. Hence, the left-hand side of (6) is strictly decreasing in ρ. Now let us look at the case $\rho = 0$. The left-hand side of (6) is given by $\eta_{D_{12}} / \eta_{d_i}$. The denominator is given by

$$\eta_{d_i} = \frac{e^{-\frac{n_i^2}{2}}}{\int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i} n_i, \tag{23}$$

while the numerator is given by

$$\eta_{D_{12}} = \frac{\int_{n_j}^{\infty} e^{-\frac{n_i^2+q_j^2}{2}} dq_j}{\int_{n_j}^{\infty}\int_{n_i}^{\infty} e^{-\frac{q_i^2+q_j^2}{2}} dq_i dq_j} n_i.$$

For $n_j = n_i$, the last equation can be written as

$$\eta_{D_{12}} = \frac{e^{-\frac{n_i^2}{2}} \int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i}{\left(\int_{n_i}^{\infty} e^{-\frac{q_i^2}{2}} dq_i\right)^2} n_i. \tag{24}$$

Dividing (24) by (23), it is easy to see that this equals 1, which implies that $\eta_{D_{12}}/\eta_{d_i} = 1$ at $\rho = 0$.

Finally, it is readily checked that for $n_i = n_j$, we have $\eta_{\phi_i+\phi_j-\phi_{12}}/\eta_{\phi_i} = 1$, implying that the right-hand side of (6) is equal to 1, independent of ρ. This result coupled with the fact that the left-hand side equals 1 at $\rho = 0$ and that it is strictly decreasing in ρ yields the result. ■

Proof of Proposition 3:

We know that $n_i^d > n_i^m$ if and only if

$$1 > \frac{\eta_{\hat{\phi}_i\hat{\phi}_j+\Delta_{\phi_i}}}{\eta_{\phi_i}} = \frac{\eta_{\hat{\phi}_i\hat{\phi}_j+\phi_i-\hat{\phi}_i}}{\eta_{\phi_i}}.$$

or

$$\eta_{\phi_i} > \eta_{\hat{\phi}_i\hat{\phi}_j+\phi_i-\hat{\phi}_i}.$$

Writing out the respective expressions for the elasticities gives

$$\frac{\partial\phi_i}{\partial n_i}\frac{n_i}{\phi_i} > \frac{\partial\left(\phi_i - \hat{\phi}_i + \hat{\phi}_i\hat{\phi}_j\right)}{\partial n_i}\frac{n_i}{\phi_i - \hat{\phi}_i + \hat{\phi}_i\hat{\phi}_j}. \tag{25}$$

We have that $\partial(\hat{\phi}_i\hat{\phi}_j)/\partial n_i = \hat{\phi}_j(\partial\hat{\phi}_i)/\partial n_i$. Inserting this into (25) and rearranging yields

$$\frac{\partial\phi_i}{\partial n_i}\left(\frac{n_i}{\phi_i} - \frac{n_i}{\phi_i - \hat{\phi}_i + \hat{\phi}_i\hat{\phi}_j}\right) > -(1-\hat{\phi}_j)\frac{\partial\hat{\phi}_i}{\partial n_i}\frac{n_i}{\phi_i - \hat{\phi}_i + \hat{\phi}_i\hat{\phi}_j}. \tag{26}$$

Simplifying and dividing (26) by $\hat{\phi}_i(\hat{\phi}_j - 1) < 0$ yields

$$\frac{\partial\phi_i}{\partial n_i}\frac{n_i}{\phi_i} < \frac{\partial\hat{\phi}_i}{\partial n_i}\frac{n_i}{\hat{\phi}_i}$$

or

$$\eta_{\phi_i} < \eta_{\hat{\phi}_i}.$$

■

Proof of Proposition 4:

Consider first the case of competing platforms. From (2), we know that platform i's profit maximization is

$$\max_{n_i} \ \Pi_i^d = \left[D_i(n_i,n_j)\phi_i(n_i) + D_{12}(n_i,n_j)(\phi_{12}(n_i,n_j) - \hat{\phi}_j(n_j))\right]. \tag{27}$$

The equilibrium advertising levels are therefore characterized by the following system of first-order conditions (arguments omitted for ease of exposition):

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \hat{\phi}_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i,j = 1,2;\ j = 3-i. \tag{28}$$

Consider now the case of joint ownership. The joint monopolist's problem is

$$\max_{n_i,n_j} \ \Pi^{jo} = D_1\phi_1 + D_2\phi_2 + D_{12}\phi_{12}, \quad i,j = 1,2;\ j = 3-i. \tag{29}$$

Taking the first-order condition of (29) with respect to n_i we obtain

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_j}{\partial n_i}\phi_j + \frac{\partial D_{12}}{\partial n_i}\phi_{12} + D_{12}\frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i,j = 1,2;\ j = 3-i. \tag{30}$$

After using $\partial D_j/\partial n_i = -\partial D_{12}/\partial n_i$, we can rewrite (30) to

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i} = 0, \quad i,j = 1,2;\ j = 3-i. \tag{31}$$

Comparing (31) with (28), it is evident that at $n_i = n_i^d$, (31) is positive if and only if $\hat{\phi}_j < \phi_j$. This implies that sign $\left\{n_i^d - n_i^{jo}\right\} = $ sign $\left\{\hat{\phi}_j - \phi_j\right\}$. ■

Proof of Proposition 5:

We first look at the last three terms in W, i.e., $D_1\phi_1 + D_2\phi_2 + D_{12}\phi_{12}$. Taking the derivative of these terms gives (with arguments omitted):

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\frac{\partial \phi_i}{\partial n_i} + \frac{\partial D_j}{\partial n_i}\phi_j + \frac{\partial D_{12}}{\partial n_i}\phi_{12} + D_{12}\frac{\partial \phi_{12}}{\partial n_i}. \tag{32}$$

We can now substitute $\partial D_{12}/\partial n_i = -\partial D_j/\partial n_i$ into (32) to obtain

$$\frac{\partial D_i}{\partial n_i}\phi_i + D_i\phi_i' + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_j) + D_{12}\frac{\partial \phi_{12}}{\partial n_i}.$$

From (28) we know that at $n_i = n^d$ the last expression equals zero.

However, the first terms in W are the utilities of the viewers which are strictly decreasing in n_i. As a consequence, the first-order condition of W with respect to n_i evaluated at $n_i = n_i^d$ is strictly negative, which implies that there is too much advertising. ■

12.2 Further Material

Two-stage game

Consider the following assumptions:

$A1$ Platforms are symmetric.

$A2$ For any $\alpha \in [0,1]$, the following inequality holds

$$t_i^\star(1-\alpha) > \alpha\Big\{d_i(\alpha n_i^d)\phi(n_i^d) - d_i((1-\alpha)n_i^\star)\phi(n_i^\star)\Big\}, \tag{33}$$

where $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, $n_i^d = \arg\max_{n_i} d_i(\alpha n_i)\phi(n_i)$, $n_i^\star$ is implicitly defined by (28) and $t_i^\star$ is implicitly defined by (27) with $n_i = n_i^\star$ and $n_j = n_j^\star$.

We provide a discussion of these assumptions after the proof of the following proposition. There we explain that $A1$ can be weakened while $A2$ is a relatively natural assumption in our framework.

Proposition *Suppose that $A1$ and $A2$ hold. Then, there is an equilibrium in the two-stage game game with posted contracts, that is outcome-equivalent to the equilibrium of the game defined in Section 3.*

Proof:

Suppose that in the two-stage game with posted contracts each platform offers a contract with $n_i = n_i^\star$, where $n_i^\star$ is implicitly determined by (28), and a transfer

$$t_i^\star = D_i(n_i^\star, n_j^\star)\phi_i(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \hat{\phi}_j(n_j^\star)\right).$$

By the same argument as we used for the original model, these contracts will be accepted by all advertisers. As this is anticipated by viewers, viewerships are $D_i(n_i^\star, n_j^\star)$ and $D_{12}(n_i^\star, n_j^\star)$. Since advertising levels are the same as in the equilibrium of the original model, viewerships are also the same. Therefore, this candidate equilibrium is outcome-equivalent to the equilibrium of the original model.

Let us now consider if there exists a profitable deviation from this candidate equilibrium. We first show that there can be no profitable deviation contract of platform i that still induces full advertiser participation on platform j but a smaller participation on platform i. Let x_i denote the fraction of advertisers who accept the offer of platform i.

Consider a candidate contract (n_i, t_i). Suppose that platform i's equilibrium profit from this contract is $t_i x_i$. Now consider the following alternative contract: $(x_i n_i, x_i t_i)$. Note that total advertising on platform i is still equal to $x_i n_i$. So platform i is at least as attractive as with the candidate equilibrium contract. Note moreover that because ϕ_i and ϕ_{12} are strictly concave in n_i, the incremental value of accepting offer $(x_i n_i, x_i t_i)$ must exceed $x_i t_i$ for all levels of advertiser participation. So all advertisers would accept $(x_i n_i, x_i t_i)$ regardless. It follows that platform i can marginally increase $x_i t_i$ while still getting full participation and therefore profits would strictly increase. It follows that no offer inducing a level of participation $x_i < 1$ can be part of a best reply.

Now suppose platform i deviates from the candidate equilibrium in such a way that it induces a fraction α of the advertisers to single-home on its platform while the remaining fraction $1-\alpha$ single-homes on platform j. Defining $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, the largest possible transfer that platform i can ask is then bounded above by

$$t_i^d = d_i(\alpha n_i^d, (1-\alpha)n_j^\star)\phi_i(n_i^d) - u_{shj},$$

where n_i^d denotes the optimal deviation advertising level and u_{shj} denotes the payoff of an advertiser who chooses to reject the contract of platform i and instead single-homes on platform j. To determine u_{shj}

we determine the advertiser's payoff when accepting only platform j's contract, which is the platform's equilibrium contract after platform i has deviated to induce a fraction α of advertisers to single-home on platform i. We obtain

$$u_{shj} = d_j((1-\alpha)n_j^\star, \alpha n_i^d)\phi_j(n_j^\star) - t_j^\star =$$
$$d_j((1-\alpha)n_j^\star, \alpha n_i^d)\phi_j(n_j^\star) - D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star) - D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \hat{\phi}_i(n_i^\star)\right).$$

Platform i's profit is then αt_i^d. Hence, deviating is not profitable if

$$\alpha\Big\{d_i(\alpha n_i^d)\phi_i(n_i^d) - d_j((1-\alpha)n_j^\star)\phi_j(n_j^\star) + D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \hat{\phi}_i(n_i^\star)\right)\Big\}$$
$$< D_i(n_i^\star, n_j^\star)\phi_i(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \hat{\phi}_j(n_j^\star)\right).$$

Now suppose that the two platforms are symmetric. Then the above condition reduces to

$$\alpha\Big\{d_i(\alpha n^d)\phi(n^d) - d_i((1-\alpha)n^\star)\phi(n^\star)\Big\} - (1-\alpha)\left(D_i(n^\star, n^\star)\phi(n^\star) + D_{12}(n^\star, n^\star)\left(\phi_{12}(n^\star, n^\star) - \hat{\phi}(n^\star)\right)\right) < 0,$$

where $n_i^\star = n_j^\star = n^\star$, $n_i^d = n^d$, and $\phi_i(\cdot) = \phi_j(\cdot) = \phi(\cdot)$. This can be rewritten as

$$t_i^\star(1-\alpha) > \alpha\Big\{d_i(\alpha n_i^d)\phi(n_i^d) - d_i((1-\alpha)n_i^\star)\phi(n_i^\star)\Big\}.$$

which is fulfilled by $A2$. As a consequence, a deviation is not profitable. ■

We now shortly explain why the assumptions $A1$ and $A2$ are not very restrictive in our framework. First, consider $A1$. Since the game is continuous, $A1$ can be relaxed to some extent without affecting the result, implying that the proposition still holds if platforms are not too asymmetric. Now consider $A2$. It is evident from (33), that the assumption is clearly fulfilled for α low enough. In this case the right-hand side is close to 0, while the left-hand side is strictly positive. Now consider the opposite case, i.e., $\alpha \to 1$. In that case the left-hand side goes to zero, while the right-hand side goes to $d_i(n_i^d)\phi(n_i^d) - d_i(0)\phi(n_i^\star)$. Evidently, $d_i(0) > d_i(n_i^d)$. Hence, the right-hand side is negative if $\phi(n_i^d)$ is not much larger than $\phi(n_i^\star)$. In general, $n_i^\star$ can be larger or smaller than n_i^d, implying that the difference can be either way. However, even in case $n_i^d > n_i^\star$, if the slope of the advertising functions ϕ_i and ϕ_{12} is relatively small, we obtain that the difference between $n_i^\star$ and n_i^d is small and so the right-hand side is negative. Finally, consider intermediate values of α. Again, if the difference between $n_i^\star$ and n_i^d is relatively small, the term in the bracket on the right-hand side of (33) is close to zero. Since the left-hand side is strictly positive, $A2$ is then fulfilled as well.

Entry in case of two incumbent platforms[34]

Consider the case of two incumbents and entry of a third platform. After the entry the profit of

[34] We note that the neutrality result extends to any finite number of incumbent platforms, not just two. As the arguments needed to establish this more general result are long and tedious, we did not include them here, but it is included in a previous version of the paper (available by request from the authors).

platform i is

$$\Pi_(n_1, n_2, n_3) = D_i(n_1, n_2, n_3)\phi_i(n_i) + D_{ij}(n_1, n_2, n_3)\left(\phi_{ij}(n_i, n_j) - \phi_j(n_j)\right)$$

$$+D_{ik}(n_1, n_2, n_3)\left(\phi_{ik}(n_i, n_k) - \phi_k(n_k)\right) + D_{123}(n_1, n_2, n_3)\left(\phi_{ij}(n_i, n_j, n_k) - \phi_{jk}(n_j, n_k)\right)$$

As in the case of entry of a second platform, we can rewrite this profit function such that it is the profit without entry plus a negative correction term leads to (dropping arguments)

$$\Pi = (D_i + D_{ik})\phi_i + (D_{ij} + D_{ijk})(\phi_{ij} - \phi_j)$$

$$-D_{ik}(\phi_i + \phi_k - \phi_{ik}) - D_{ijk}\left(\phi_{ij} - \phi_j - (\phi_{ijk} - \phi_{jk})\right).$$

First two terms are the profit of in duopoly. Note that without entry D_{ik} did not exist since there was no platform k and so platform i could get ϕ_i for these viewers. The last two terms are the negative correction terms.

Taking the derivative with respect to n_i yields

$$\frac{\partial \Pi}{\partial n_i} = \frac{\partial \Pi^d}{\partial n_i} + D_{ik}(\phi_i + \phi_k - \phi_{ik})\left[\eta_{D_{ik}} - \eta_{\phi_i + \phi_k - \phi_{ik}}\right] + D_{ijk}\left(\phi_{ij} - \phi_j - (\phi_{ijk} - \phi_{jk})\right)\left[\eta_{D_{ijk}} - \eta_{\phi_{ij} - \phi_j - (\phi_{ijk} - \phi_{jk})}\right] = 0.$$

So we obtain that for $\eta_{D_{ik}} > \eta_{\phi_i + \phi_k - \phi_{ik}}$ and $\eta_{D_{ijk}} > \eta_{\phi_{ij} - \phi_j - (\phi_{ijk} - \phi_{jk})}$, the business-sharing effect dominates the duplication effect. The formula now consists of two terms since entry of third platform leads to change in two viewers groups, namely, the exclusive ones and the overlapping ones before entry. Each term is multiplied by the absolute profits of the respective viewer group.

Equilibrium with Heterogeneous Advertisers

We first determine the solution to the more complicated duopoly problem. (Solving the monopoly problem proceeds along very similar lines and we will describe it very briefly towards the end.) The problem of a duopolist i who chooses a transfer schedule to maximize profits is $\int_{\underline{\omega}}^{\overline{\omega}} t_i(m_i(\omega))dF(\omega)$, given its rival's choice $t_j(m_j)$. From the main text, this problem can be rewritten as in (12). Denote by $m_j^\star(m, \omega)$ the quantity that type ω optimally buys from platform j when buying quantity m from platform i. Then, the net contracting surplus for type ω is

$$\begin{aligned} v_i^d(m, \omega, n) &= \max_y \left[\omega u(m, y, n) - t_j(y)\right] - \left(\max_{y'} \left[\omega u(0, y', n) - t_j(y')\right]\right) \\ &= \omega u(m, m_j^\star(m, \omega), n) - t_j(m_j^\star(m, \omega)) - \omega u(0, m_j^\star(0, \omega), n) - t_j(m_j^\star(0, \omega)) \end{aligned} \tag{34}$$

Incentive compatibility requires $m_i(\omega) = \arg\max_m v_i^d(m, \omega, n) - t_i(m)$, which implies

$$v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) = \max_{y, y', m} \left\{\omega u(m, y, n) - t_j(y) - (\omega u(0, y', n) - t_j(y')) - t_i(m)\right\}$$

By the envelope theorem the derivative of the above with respect to ω is

$$u(m, m_j^\star(n_i(\omega), \omega), n) - u(0, m_j^\star(0, \omega), n)$$

Since the above pins down the growth rate of the advertiser's payoff, we have that $\max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0}^{\overline{\omega}} t_i(\omega)$ subject to the first two constraints of (12) equals

$$\max_{m_i(\cdot),\omega_0^i} \int_{\omega_0}^{\overline{\omega}} \Big\{ \omega u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - \omega u(0, m_j^\star(0,\omega), n) - t_j(m_j^\star(m_i(\omega),\omega)) + t_j(m_j^\star(0,\omega))$$
$$- \int_{\omega_0^i}^{\omega} \left[\omega u(m, m_j^\star(m_i(z), z), n) - \omega u(0, m_j^\star(0,z), n)\right] dz \Big\} dF(\omega)$$
$$= \max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\overline{\omega}} \Big\{ v_i^d(m_i,\omega,n) - \underbrace{\int_{\omega_0^i}^{\omega} \left[\omega u(m, m_j^\star(m_i(z), z), n) - \omega u(0, m_j^\star(0,z), n)\right] dz}_{\text{information rent}} \Big\} dF(\omega),$$

Integrating the double integral by parts gives

$$\max_{m_i(\cdot),\omega_0^i} \int_{\omega_0^i}^{\overline{\omega}} \omega u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - \omega u(0, m_j^\star(0,\omega), n) - t_j(m_j^\star(m_i(\omega),\omega)) + t_j(m_j^\star(0,\omega)) +$$
$$- \frac{1-F(\omega)}{f(\omega)} (u(m, m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)) \, dF(\omega)$$

The duopolist's best reply allocation $m_i^d(\omega)$ then solves

$$\max_{m_i(\cdot),\omega_0^i} \int_{\omega_0^i}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right) (u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n))$$
$$- \big(t_j(m_j^\star(m_i(\omega),\omega)) - t_j(m_j^\star(0,\omega))\big) \, dF(\omega), \tag{35}$$
$$\text{subject to} \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega') dF(\omega') \le n_i.$$

From now on we will denote the integrand function by $\Lambda^d(m_i(\omega),\omega,n)$. Recall that solving a canonical screening problem usually involves maximizing the integral over all types served of the "full utility" of type ω minus its informational rent, expressed as a function of the allocation. The "full utility" here is the incremental value $u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)$, minus the difference in transfers.

The maximization problem in the first stage with respect to n_i as

$$\max_{n_i} \left(\max_{m_i(\cdot),\omega_0} \int_{\omega_0^i}^{\overline{\omega}} \Lambda^d(m_i(\omega),\omega,n) dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega) dF(\omega) \right). \tag{36}$$

Let us first determine $u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)$. Abbreviating $m_j^\star(m_i(\omega),\omega)$ by $m_j^\star$ and $m_j^\star(0,\omega)$ by $(m_j')^\star$ we can write

$$u(m_i(\omega), m_j^\star, n) - u(0, (m_j')^\star), n)$$

$$= D_i(n_1,n_2)\phi_i(m_i(\omega)) + D_j(n_1,n_2)\phi_j(m_j^\star) + D_{12}(n_1,n_2)\phi_{12}(m_i(\omega), m_j^\star)$$
$$- D_j(n_1,n_2)\phi_j((m_j')^\star) - D_{12}(n_1,n_2)\phi_j((m_j')^\star)$$
$$= d_i(n_i)\phi_i(m_i(\omega)) + D_{12}(n_1,n_2)\left(\phi_{12}(m_i(\omega), m_j^\star) - \phi_i(m_i(\omega)) - \hat{\phi}_j((m_j')^\star)\right) + D_j(n_1,n_2)\left(\phi_j(m_j^\star) - \hat{\phi}_j((m_j')^\star)\right),$$

where $\phi_{12}(m_i(\omega), m_j^\star) = \hat{\phi}_i(m_i(\omega)) + \hat{\phi}_j(m_j^\star) - \hat{\phi}_i(m_i(\omega))\hat{\phi}_j((m_j')^\star)$.

Adapting results from Martimort and Stole (2009) we have that at the optimal solution $m_i(\omega) = 0$ for all $\omega < \omega_0$ and that $m_i(\omega) = \arg\max_m \Lambda^d(m_i(\omega), \omega, n)$. By our assumptions on the demand and advertising function, the optimal solution involves a schedule $m_i(\omega)$ that is non-decreasing.

We can write the maximization problem with respect to the optimal allocation of advertising intensities, given n_i, as

$$\max_{m_i(\cdot),\lambda} \quad \int_{\omega_0^i}^{\overline{\omega}} \Lambda^d(m_i(\omega), \omega, n) dF(\omega) + \lambda \left(n_i - \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega) dF(\omega) \right). \tag{37}$$

Pointwise maximization with respect to $m_i(\cdot)$ yields

$$\left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) \left[d_i(n_i) \frac{\partial \phi_i}{\partial m_i} + D_{12}(n_1, n_2) \left(\frac{\partial \left(\phi_{12}((m_i, m_j^\star)) - \phi_i(m_i) - \hat{\phi}_j((m_j')^\star) \right)}{\partial m_i} \right) \right.$$

$$\left. +(D_j(n_1, n_2) - D_{12}(n_1, n_2)) \frac{\partial \phi_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial m_i} \right] - \frac{\partial t_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial m_i} = \lambda. \tag{38}$$

Denoting the left-hand side of (38) by ρ, and integrating both sides from ω_0^i to $\overline{\omega}$, we obtain

$$\frac{\int_{\omega_0^i}^{\overline{\omega}} \rho dF(\omega)}{1 - F(\omega_0^i)} = \lambda. \tag{39}$$

The maximization problem of the first stage with respect to n_i is

$$\max_{m_i(\cdot),\lambda} \quad \int_{\omega_0^i}^{\overline{\omega}} \Lambda_i^d(\omega, m_i(\omega)^\star, n_i) dF(\omega) + \lambda \left(n_i - \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega)^\star dF(\omega) \right).$$

Differentiating with respect to n_i and using the Envelope Theorem yields

$$\int_{\omega_0^i}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) \left[\frac{\partial d_i}{\partial n_i} \phi_i + \frac{\partial D_{12}}{\partial n_i} \left(\phi_{12}((m_i, m_j^\star)) - \phi_i(m_i) - \hat{\phi}_j((m_j')^\star) \right) \right.$$

$$\left. +D_{12} \left[\frac{\partial \phi_{12}}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^\star} \frac{\partial (m_j')}{\partial n_i} \right] \right] + D_j \left[\frac{\partial \phi_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^\star} \frac{\partial (m_j')}{\partial n_i} \right] \Bigg] dF(\omega) \tag{40}$$

$$- \frac{\partial t_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} + \frac{\partial t_j}{\partial (m_j')^\star} \frac{\partial (m_j')^\star}{\partial n_i} = -\lambda.$$

Combining (38) and (40) to get rid of λ yields expression (14) of the main text, where κ is defined as

$$\kappa \equiv \int_{\omega_0}^{\overline{\omega}} \left(\omega - \frac{1 - F(\omega)}{f(\omega)}\right) \left\{ \frac{1}{1 - F(\omega)} (D_j - D_{12}) \frac{\partial \phi_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial m_i} + D_{12} \left[\frac{\partial \phi_{12}}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \hat{\phi}_j}{\partial (m_j')^\star} \frac{\partial (m_j')^\star}{\partial n_i} \right] \right.$$

$$\left. +D_j \left[\frac{\partial \phi_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \phi_j}{\partial (m_j')^\star} \frac{\partial (m_j')^\star}{\partial n_i} \right] + \frac{\partial D_j}{\partial n_i} \left(\phi_j(m_j^\star) - \phi_j((m_j')^\star) \right) - \frac{\partial t_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} \right\} dF(\omega)$$

$$- \frac{\partial t_j}{\partial m_j^\star} \frac{\partial m_j^\star}{\partial n_i} + \frac{\partial t_j}{\partial (m_j')^\star} \frac{\partial (m_j')^\star}{\partial n_i}.$$

It is evident that if platform j offers a single transfer-quantity pair, then $m_j^\star$ equals $(m_j')^\star$ and both are invariant to changes in $m_i(\cdot)$ and n_i. This implies that $\kappa = 0$.

Proceeding in the same way for the monopoly platform, we obtain that its profit function is given by

$$\max_{n_i} \left(\max_{m_i(\cdot),\omega_0^m} \int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)} \right) d_i(n_i)\phi_i(m_i(\omega))dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^m}^{\overline{\omega}} m_i(\omega)dF(\omega) \right). \tag{41}$$

The solution is then characterized by (13).

12.3 Empirical Analysis

Here we empirically investigate the link between entry and correlation in advertising level. As our data is limited, we regard this exercise as providing suggestive evidence, as opposed to a careful empirical analysis of the investigated issues.

The dataset are provided by Kagan-SNL a highly regarded proprietary source for information on broadcasting markets. It consists of an unbalanced panel data set of 68 basic cable channels from 1989 to 2002. The channels cover almost all cable industry advertising revenues (75% of all revenue is generated by the twenty biggest networks in our data set). We know the date for each new network launch within our sample period (a total of 43 launches), and for each network active in each year we have information on the average number of 30-second advertising slots per hour of programming (in jargon 'avails'). We also have a good coverage for other network variables, such as subscribers, programming expenses and ratings.

We first use our panel data set to study the relationship between the avails broadcasted by each channel and the number of incumbents. As our model characterizes the effects of varying competition, we consider each channel within its *own* competitive environment. That is, we define a relevant market segment for each of the 68 channels. The hypothesis is that channels with content tailored to the same segment 'compete' for viewers and advertisers. For this purpose, we divide channels in three segments: (i) sports channels (henceforth Sports), (ii) channels broadcasting mainly movies and TV series (henceforth Movies&Series), and (iii) all remaining channels, which is used as a reference group. To test whether viewer preference correlation affects the relationship between entry and advertising levels, we estimate separate parameters for the Sports and the Movies&Series segments. Our working assumption is that the viewers' preferences within these segments are positively correlated. Our model predicts that avails would fall after entry in the Sports and Movies&Series segments relative to the reference group.[35,36]

We use two different empirical approaches, that lead us to similar conclusions. First, we use a panel analysis, that pools all channel-year observations from 1989-2002, so it relies on within- and across-channel

[35]Our data does not include viewer prices. This should not be a problem because their impact was not particularly important during our sample period (see for example, Strömberg (2004)). In addition, viewer prices were highly regulated in the 1990s, see Hazlett (1997).

[36]We note that we intended to create a separate News segment as well, as this segment provides a natural counterpart to the others in that viewer preferences can be reasonably assumed to be negatively correlated. Unfortunately, the number of channels here is too small to obtain statistically meaningful results. The point estimates one can obtain are consistent with Propositions 5 and 6 - contact the authors for details.

variation.

We estimate the following linear regression model:

$$\begin{aligned} log(Avails_{it}) =& \beta * Platforms_{it} + \beta_M * Platforms_{it} * MoviesSeries\_dummy \\ & + \beta_S * Platforms_{it} * Sports\_dummy + \gamma * x_{it} + \alpha_i + \delta_t + \epsilon_{it}, \end{aligned}$$

where $Avails_{it}$ is the average number in year t of 30-second advertising slots per hour of programming by channel i, $Platforms_{it}$ is the number of channels in channel i's segment at the end of year t, $Sports\_dummy$ and $MoviesSeries\_dummy$ are dummy variables equal to 1 when channel i belongs to the Sport and to the Movies&Series segments respectively (and zero otherwise), x_{it} is a vector of channel-time controls, α_i is a channel fixed effect and δ_t is a time fixed effect. Given that the dependent variable is transformed in logs, while the main explanatory variable is measured in units of channels, β has the following interpretation: when a new channel enters the control segment, the incumbents increase their 30-second advertising slots by $100\beta\%$. The coefficients β_M and β_S measure the *additional* effect that the number of channels has on the avails in the Movies&Series and Sports segments respectively.

Table 1: Number of Platforms and Avails - Average Effect

	(1)	(2)	(3)	(4)	(5)	(6)
Platforms	0.00268**	0.00104	0.00974***	0.0105***	0.0103***	0.00857**
	(0.00123)	(0.00140)	(0.00339)	(0.00364)	(0.00362)	(0.00407)
Real GDP		0.00185***	-0.000294			
		(0.000602)	(0.00134)			
Rev Mkt Share					0.178	
					(0.255)	
Rating						-0.0828
						(0.0769)
Constant	3.017***	2.825***	2.913***	2.434***	2.403***	2.969***
	(0.0209)	(0.0642)	(0.112)	(0.143)	(0.160)	(0.106)
Observations	416	416	416	416	415	279
R^2	0.016	0.027	0.275	0.303	0.307	0.276
Channel FE	NO	NO	YES	YES	YES	YES
Time FE	NO	NO	NO	YES	YES	YES
No. of Platforms	56	56	56	56	56	33

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1 reports the estimation results when we restrict the coefficient on the number of channels to be homogeneous across segments. We find evidence that entry is associated with an increase in the advertising levels on incumbent channels. The coefficient is positive and significant across almost all specifications. Starting from the single variable model in column (1), we progressively add controls and fixed effects: column (2) controls for the real GDP to capture the business cycle's effect on the advertising market, starting from column (3) we report estimates for a fixed-effect model where the units of observations are the single channels. From column (4) we introduce time dummies, while in columns (5) and (6) we add channel-time controls: the channel's share of revenues in its segment and its rating. Since we only have US data, the real GDP control is dropped whenever time controls are included. All regressions are estimated with robust standard errors. The average effect estimated is on the order of 1%.

Table 2: Number of Incumbents and Avails - Effect by Segment

	(1)	(2)	(3)	(4)	(5)	(6)
Platforms	0.00615***	0.00139	0.00648**	0.00672**	0.00676***	0.00160*
	(0.00164)	(0.00496)	(0.00104)	(0.000878)	(0.000517)	(0.000459)
Platforms×Movies&Series	-0.00613	-0.0137*	-0.00902**	-0.00811**	-0.00788**	-0.0137***
	(0.00379)	(0.00705)	(0.00164)	(0.00161)	(0.00177)	(0.000325)
Platforms×Sports	-0.00261	-0.00692	-0.00687**	-0.00619***	-0.00547	-0.0154***
	(0.00600)	(0.00700)	(0.000913)	(0.000377)	(0.00200)	(0.000117)
Real GDP		0.00283	0.00188*			
		(0.00247)	(0.000620)			
MoviesSeries_dummy	0.191***	0.257***				
	(0.0595)	(0.0766)				
Sports_dummy	0.106	0.0493				
	(0.0690)	(0.0866)				
Rev Mkt Share					0.156	
					(0.398)	
Rating						-0.0904
						(0.0542)
Constant	2.908***	2.685***	2.760***	3.242***	3.226***	3.273***
	(0.0351)	(0.190)	(0.0469)	(0.0372)	(0.0621)	(0.0258)
Observations	416	416	416	416	415	279
R^2	0.048	0.050	0.284	0.307	0.311	0.299
Channel FE	NO	NO	YES	YES	YES	YES
Time FE	NO	NO	NO	YES	YES	YES
No. of Platforms	56	56	56	56	56	33

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 2 reports the estimation results when we allow for heterogeneous effects in the number of channels across segments. The coefficients of interest are β_M and β_S. Given our theory and our assumption that preferences are correlated within segments, we expect these coefficients to be negative. That is, we expect the effect of entry within the Sports and Movies&Series segment to be diminished compared to the average industry effect (and possibly negative over all). Indeed, the coefficients have the expected sign in all regressions: the effect of the number of channels on advertising levels is positive for channels in the reference group (β is again positive and significant), while it is significantly lower for channels in the other two segments. This additional negative effect is particularly strong for Movies&Series where $|\beta_M| > |\beta|$ in almost all specifications. Standard errors are clustered at the segment level.

To summarize, we obtain evidence of a positive relationship between entry and advertising levels. We also find a systematic reduced impact of entry on advertising levels within the same market segments. Based on our theory, we speculate that this difference comes from viewers' tastes for content which induce a good deal of overlap among viewers of the channels belonging to each of these segments. We leave a more careful empirical investigation of these issues to future research.

The regressions above have the advantage of pooling data on different channels without taking a stance on the time it takes for entry to impact the incumbent choices. However, this strategy does not allow to account for within channel omitted variables that vary over time. These variables may also operate at the segment level. To account for this, as an alternative way to address the same issues empirically, we also estimate a model for entry episodes, where our sample is now reduced to the periods when a given segment experiences the entry of a new channel. We estimate the following model:

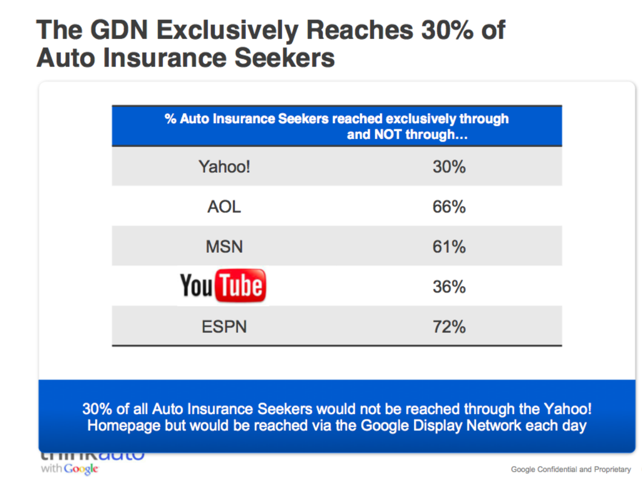
$$\begin{aligned}\Delta log(Avails_{it}) =& \beta + \beta_M * MoviesSeries\_dummy + \beta_S * Sports\_dummy \\ & + \gamma * x_{it} + \delta_t + \epsilon_{it}\end{aligned}$$

This model can be obtained by first differencing the previous model around the years when entry occurs. In fact $\Delta log(Avails_{it}) = log(Avails_{it+1}) - log(Avails_{it-1})$ and the effect of entry (changed number of incumbents) is captured by the constant terms. Channel fixed effects are now excluded (as they cancel out in taking first differences), but we keep time fixed effects and also add some channel controls. The constant β measures the effect of entry on the reference group (infotainment), while β_M and β_S measure the additional effect for the Movies&Series and Sports segments, respectively. The estimates reported in Table 3 confirm our previous results: entry episodes are associated with an increase in the quantity of avails in the reference group, while the effect is lower in the Sports and Movies&Series segments. Since there are half as many observations in this setup, the point estimates are less precisely estimated than in Tables 1 and 2. Furthermore, because here we are looking at the effect one year after entry (t+1), the magnitude of the parameters are notably bigger. The point estimate of the % variation in avails due to an additional channel is on the order of 5% in column (3). Notably, the interaction term that captures the differential impact of entry in sports is 11% less than the industry average. The difference is statistically and economically significant.

Table 3: Entry Episodes - Average Effect and Effect by Segment

	(1)	(2)	(3)	(4)
MoviesSeries dummy		-0.0327***	-0.0494**	-0.0314
		(0.000919)	(0.00841)	(0.0124)
Sports dummy		-0.0563***	-0.110***	-0.0172
		(0.000457)	(0.00460)	(0.0222)
Δ GDP[t-1,t+1]	0.00380	0.00328*		
	(0.00133)	(0.00103)		
Rating			-0.0392***	
			(0.00128)	
Rev Mkt Share				-0.171
				(0.0987)
Constant	0.00393	0.0178	0.0615**	0.263***
	(0.0213)	(0.00901)	(0.00875)	(0.000141)
Observations	219	219	158	219
R^2	0.009	0.028	0.121	0.091
Time FE	NO	NO	YES	YES

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1



(a)

Figure 3: Exclusive GDN consumers in the Auto-Insurance market (2011)
Source: Excerpt from the Google's Reasearch study, "Google Display Network vs. Portal Takeovers for Auto Insurance seekers' available at http://www.google.com/think/research-studies/google-display-network-vs-portal-takeovers-for-auto-insurance-seekers.html' (Last accessed 5/17/2013)

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