# Risk-taking with Financing Constraints*

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**Abstract**

We analyze how financing constraints affect risk-taking behaviors of non-financial firms. We show that improved credit conditions can either promote or deter socially productive risk-taking. Following an interest rate cut, firms are less likely to take risks in a low interest rate environment but more inclined to do so in a high interest rate setting. As the interest rate rises, the dispersion of firm returns and total factor productivity may initially increase but then decrease after reaching a certain point. Since higher interest rates often lead to less investment, the relationship between economic output and interest rates (the IS curve) forms a "crawl-dive" hump, where output rises slowly at first due to more risk-taking but then drops sharply. An optimal interest rate may exist, determined by the extent of return associations and credit limits.

Key Words: financing constraints; risk-taking; valuation effect; search-for-yield effect; optimal interest rate

JEL code: E22; E44; E61; G32

# 1 Introduction

Socially productive risk-taking is crucial to driving opportunities and progress within society. It fuels the exploration of new ideas and ventures, which are essential for aggregate productivity. However, the propensity for non-financial firms to take risks varies according

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to numerous factors, such as the nature of a project's risk, the individual's risk tolerance, leverage limits, and the prevailing interest rates.

How do debt and equity financing shape firms' willingness to take socially desirable risks? In the wake of the global financial crisis, persistently low real interest rates have reshaped the financial landscape, raising important questions about the role of financing constraints in corporate risk-taking. Do lower interest rates spur firms to take more risks, or do they reinforce caution? Does a further decline in rates encourage innovation and investment, or distort incentives? And how might financial regulations affect the power of interest rate policy, especially in environments where firms' successes and failures are closely correlated? These debates gained renewed attention recently, when the current US government administration repeatedly pushed for lower interest rates, arguing that they would boost US competitiveness and spur innovation among American firms.

This paper answers these questions by studying the joint impact of interest rates and leverage restrictions on firms' risk-taking incentives. Financial conditions produce non-linear/non-monotonic effects on risk-taking, indicating the existence of an optimal interest rate under different degrees of financing constraints and return associations among firms. In short, although a high interest rate hurts capital accumulation and output, an overly low rate deters risk-taking, reducing overall productivity and negatively impacting the economy as well.

Using multiple proxies for risk-taking, we begin with an empirical analysis of how changes in interest rates influence risk-taking in the US. We study different periods marked by varying interest rates and leverage conditions. Our findings indicate a strong, non-linear, and non-monotonic relationship between the interest rates and the risk-taking measures. For example, although interest rate levels and policy cuts were similar in 2001 and 2007, risk-taking measures fell in 2001 but rose in 2007 when firms held higher leverage. This underscores the critical role that financial conditions can play in shaping risk-taking behaviors.

To understand the impact of financial conditions on risk-taking, we explore a model scenario featuring risk-averse entrepreneurs with two investment options: a risk-free project yielding a higher return than the risk-free rate, and a riskier project that can be either successful or unsuccessful and promises the highest expected return. For either type of project, firms can borrow up to a limit. Undertaking a risky project incurs idiosyncratic running costs, such as those associated with testing and prototyping in research and development, which can vary significantly across industries and project types. The availability of a risk-free project provides an alternative option, affecting entrepreneurs' choice to pursue riskier ventures based on borrowing capacity and running costs. As a result, some firms choose to take risks, while others do not.

We obtain two key effects. First, easing financial conditions (through higher leverage limits or lower interest rates) increases firm profitability and firm value; This "valuation effect" reduces the incentive to pursue risky high-return projects, as the need for higher expected growth through risk-taking decreases when the firm value increases. However, easing liquidity also triggers a traditional "search-for-yield" effect, making riskier projects with a higher expected return more attractive, sometimes understood as the substitution effect. In particular, the model establishes a critical threshold of the idiosyncratic costs to execute the risky project, below which the firms choose the risky option. We analyze how leverage constraints and interest rates influence this threshold via the valuation and search-for-yield effects.

Suppose that firms are financially constrained if they implement the risky project, though we also consider cases in which they are not. Reducing interest rates primarily boosts the value of firms, leading to fewer firms taking risks. This happens when firms' equity is only marginally leveraged, or low debt servicing costs in general, since the reduction of interest rate increases the (expected) return of the safe project more in percentage terms, making the safe project more attractive. Therefore, lowering the policy rate may not incentivize firms to pursue socially beneficial but individually risky projects under strict leverage constraints. In contrast, when firms can leverage extensively or when debt servicing costs are high, the (leveraged) valuation effect is outweighed by the substitution effect, driven by an improved leveraged return relative to leveraged risk. In this case, lowering the policy rate encourages greater risk-taking. As a result, the "search-for-yield" behavior (e.g., Rajan (2006)) is more pronounced in a low interest rate environment only if firms are highly leveraged.

Our theory thus suggests that further rate reductions in an environment with low interest rates can discourage firms from pursuing socially beneficial but risky projects. This effect could cause aggregate economic indicators to show hump-shaped patterns. For example, the "IS" curve, which illustrates the relationship between output and the real interest rate, *can* slope upward when rates fall to very low levels. Consequently, an overly accommodating monetary (or fiscal) policy could be contractionary.

In summary, although a lower interest rate generally stimulates investment and output constrained by financial frictions, it can discourage productive risk-taking, diminish productivity, and ultimately decrease output if it drops too much. In this sense, financial frictions bring about the usual endogenous capital misallocation and selection in the existing exogenous productivity distribution, such as in Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2014);[1] But more importantly, our model shows that frictions can influence the productivity distribution itself through risk-taking, creating the possibility of an optimal in-

[1]Gopinath et al. (2017) found substantial capital misallocation in southern Europe, showing that the decline in real interest rates, often linked to euro convergence, expanded the dispersion of capital returns and reduced total factor productivity (TFP) by channeling capital to less productive firms in underdeveloped financial markets. See also recent work on the counterproductive general equilibrium effect of a low interest rate by Benigno and Fornaro (2018), Reis (2021), Kiyotaki et al. (2021), and Asriyan et al. (2024).

terest rate level because many aggregate variables are hump-shaped when we vary the target interest rate level.

Calibrating to macro- and micro-data, we indeed find a hump-shaped total factor productivity (TFP) curve and a hump-shaped IS curve by varying the interest rate. When interest rates are high, reducing them increases both investment and risk-taking by lowering borrowing costs. However, when interest rates are already low, further cuts still encourage investment by reducing borrowing costs, but hurt risk-taking. Consequently, the humped IS curve has a flat slope on the left branch (where interest rates are low and sensitivity is low) and a steep slope on the right branch (where interest rates are high and sensitivity is high). We label this hump as a *"crawl-dive"* hump.

In addition, the calibrated model replicates the decline in risk-taking measures observed in 2001 and the increase in 2007, despite similar interest rate levels and cuts. This pattern is closely tied to the higher firm leverage in 2007 than in 2001. Thus, stricter leverage requirements introduced after the global financial crisis, together with persistently low real interest rates, could lead to sustained reductions in risk-taking and lower equilibrium marginal products of capital, further reinforcing the low interest rate environment.

Finally, our exercises identify an optimal target interest rate based on the extent of the interdependence of firms' risky projects and specific leverage constraints. Although this paper leaves aside long-term influences like demographic changes or technological shifts on the long-run interest rate, it focuses on the real rate component that can be targeted by monetary-fiscal policies at least in the medium run. Besides savers, the interdependence of firm production and credit limits shape this optimal target rate.

In modeling interdependence as an extension, we borrow from the Statistics literature by using a generalized binomial distribution that considers the interdependence of the Bernoulli trials. Specifically, risk-taking behaviors of firms can influence the success probabilities of other firms. However, each entrepreneur takes their probability of success as given, so the aggregate economy contains an externality. The positive association captures the spillover of knowledge (for example, Jones and Williams (2000)) when the project is successful and the spillover of risk (for example, the conditional value at risk along the line of Adrian and Brunnermeier (2016)). In contrast, negative association captures crowding out and learning from others' mistakes.

When project outcomes are positively associated, interest rates should be lowered to direct resources toward firms engaged in productive projects, which generally feature beneficial knowledge spillover. In contrast, raising interest rates can be advantageous when project outcomes are negatively associated. However, a lower rate, which boosts investment, may also be optimal if the negative association is particularly severe, since the lower rate can deter risk-taking too because of the valuation effect. Credit limits also influence the optimal interest rate. This suggests that financial regulation policies (e.g., targeting asset collaterability) significantly impact monetary or fiscal policy effectiveness. Coordination should balance efficiency and risk in the economy. For realistic credit limits in our model, e.g. within 10% above or below our calibrated benchmark, and a broad range of association scenarios, the optimal real lending rate falls between 2.2% and 3.1%.

*Related literature.* Our theory of risk choices in the context of financial conditions is closely aligned with the literature on entrepreneurship with financial frictions, as seen in works such as Cagetti and De Nardi (2006), Buera and Shin (2013), and Buera et al. (2015). In these studies, agents face the decision to become an entrepreneur or a worker (or a serial entrepreneur in Brandt et al. (2022)), and financial frictions influence this decision. Notably, agents' optimization problems often exhibit non-convexity due to discrete choices, which can intertwine risk-taking with career decisions. For example, in Vereshchagina and Hopenhayn (2009), where agents make occupational and risky project choices under no-borrowing constraints, those in the intermediate wealth range who opt to be entrepreneurs may exhibit risk-seeking behavior, as this "lottery" mechanism can help smooth value functions and may bring them to the top wealth. Compared to this literature, our emphasis lies in illustrating how varying financial conditions impact firms' risk-taking behavior, the related macroeconomic effects, and the optimal interest rate.

Our framework connects naturally to financial frictions[2] and risks in general. Based on Bernanke et al. (1999), Christiano et al. (2014) show that risk shocks in a quantitative business cycle model with financing constraints generate countercyclical spreads and account for a large fraction of macroeconomic fluctuations. Miao and Wang (2010) include long-term risky defaultable debt in a macroeconomic model with financial shocks to the recovery rate. Cui and Kaas (2020) develop a framework to analyze belief risks in credit contracts. We add to this literature by examining the effect of financing constraints/interest rates on *endogenous* risk-taking behaviors.

Our work also adds to the literature on active risk management: the seminal paper by Froot et al. (1993) in the static case and Bolton et al. (2011) in the dynamic case show that financing constraints generate the rationale for active risk management. In studying risk-taking with banks, Dell Ariccia et al. (2014) find the trade-off between risk shifting and search-for-yield effects in providing credit. When the borrowing cost falls, risk shifting can occur because the marginal return on the monitoring is higher, inducing banks to monitor more and reduce the risk of a loan. However, in subsequent empirical studies, Dell Ariccia et al. (2017) and Jiménez et al. (2014) (and the references therein) find that the loan yield data mostly suggest the search-for-yield effect. Unlike banks, non-financial firms may have

[2] The literature on financial frictions is vast, and the seminal contribution at least includes Kiyotaki and Moore (1997), Bernanke et al. (1999), Mendoza (2010), and Brunnermeier and Sannikov (2014).

more curvature in their preferences because they are more risk-averse or face more dividend smoothing requirements (Jermann and Quadrini, 2012); they may also face tighter financing constraints as part of the demand side for credit. This feature makes the valuation effect more salient, and consequently, in the data, we observe a non-monotonic relationship between interest rate and risk-taking of non-financial firms. The model shows that the non-monotonic effect via a project selection problem differs from the usual bank's risk-monitoring problem.

We use a highly tractable framework that yields closed-form individual risk-taking policy functions. At the aggregate level, even project associations are accounted for without complicating individual firm decisions, thanks to the flexible generalized binomial distribution with a dependence structure of each Bernoulli trial. Although this technical feature borrowed from the Statistics literature may be of independent interest, its primary value is simplifying the optimal target interest rate analysis in response to return associations among risky projects.

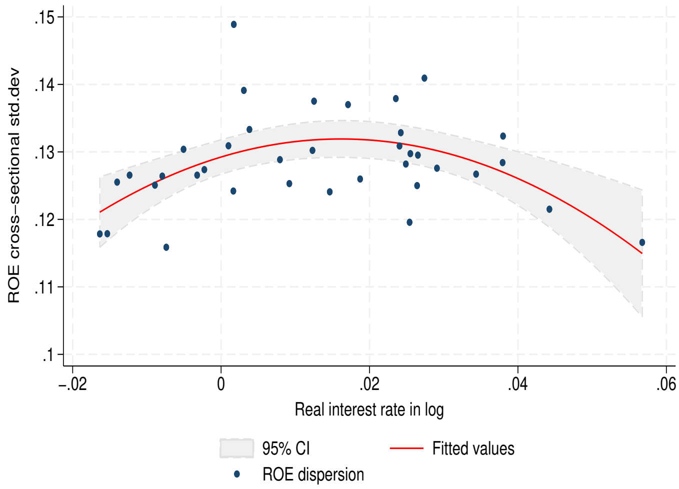
The remainder of the paper is organized as follows. We first present stylized facts about the non-monotonic risk-taking measures to variations in interest rates. Section 3 presents a simple two-period model, illustrating the key effects of interest rate and leverage on risk-taking. Section 4 extends the two-period model to an infinite-horizon economy. Section 5 shows quantitatively how the target interest rate and the borrowing constraint shape macroeconomic outcomes and the optimal level of interest rate. Section 6 considers the association/correlation between project returns, which significantly influences the policy discussion. Section 7 concludes.

# 2 Motivating Evidence

We start with some evidence about the non-monotonic relationship between firms' risk-taking behavior and interest rates. As a proxy for firm risk-taking behavior, for a given year, we first look at the cross-sectional dispersion of return on equity (ROE). ROE is calculated by net income over total equity in book value (see Appendix A for data description). When more firms undertake riskier ventures, the ex-post measured ROE should exhibit a higher cross-sectional dispersion since some firms will be successful and some other firms will fail.

Figure 1 plots the cross-sectional ROE standard deviation (ROE dispersion) against the logarithmic level of the gross Cleveland real interest rate, a widely used proxy for real rates provided by the Federal Reserve Bank of Cleveland. The figure also includes a quadratic fit and a 95% confidence interval. The relationship appears non-linear: at higher levels of the real interest rate, ROE dispersion declines with the rate, while at lower levels, the relationship reverses and becomes positive. This suggests a potential non-monotonic relationship between real interest rates and firm risk-taking behaviors. This pattern holds at the industry

Figure 1: Risk-taking and Real Interest Rate




Note: This figure plots the dispersion (cross-sectional standard deviation) of return on equity (ROE) against the real interest rate series provided by the Federal Reserve Bank of Cleveland.

level, and we include the plot in Appendix A.

What might contribute to the interest rate's non-monotone effect? Firm leverage can be an important factor. To see this, we examine 2001 and 2007, marking the onset of respective recessions. During both periods, the Cleveland rate was around $1.7 - 1.8\%$, yet the median firm leverage ratio in 2007 surpassed that of 2001 by approximately $19\%$. Despite encountering similar interest rate reductions (about 150 basis points) in both years, they triggered differing responses in the ROE dispersion. The measure declines $7\%$ after 2001, whereas it rises by $3.24\%$ after 2007.

Further, we construct a firm-level metric, following John et al. (2008), which captures the firm-level variation in return on equity (ROE) over an eight-year window following a given year. This measure is used to test whether firm-level patterns are consistent with the evidence discussed above. We estimate the following baseline specification:

$$y_{ij,t} = \beta_0 + \beta_1(R_t)\Delta R_t + X_{ij,t-1} + f_i + h_j + \Gamma_t, \tag{1}$$

where $y_{ij,t}$ is the firm-level risk-taking measure for firm $i$ in sector $j$ in a given year $t$, $R_t$ is the level of interest rate, $\Delta R_t$ is a measure of interest-rate shock, $\Gamma_t$ is an aggregate control which includes the current and lagged terms of inflation, unemployment rate, real GDP growth rate, and the lagged interest rate.[3] The vector of firm-level controls $X_{ij,t-1}$ includes the firm leverage ratio (long-term debt over fixed assets such as property, plant, and

[3]The results remain robust when using only one-period lagged aggregate controls.

Table 1: Variation in ROE with respect to Changes in Interest Rate

| VARIABLES | (1)<br>ROE std in log | (2)<br>ROE std in log | (3)<br>ROE std in log | (4)<br>ROE std in log | (5)<br>ROE std in log | (6)<br>ROE std in log |
|---|---|---|---|---|---|---|
| mpshock | 1.392*** | -0.099 | 1.786*** | -1.155*** | 1.490*** | -0.987*** |
| | (0.441) | (0.268) | (0.417) | (0.226) | (0.294) | (0.182) |
| Observations | 50,163 | 52,324 | 49,629 | 53,085 | 48,415 | 53,244 |
| R-squared | 0.614 | 0.709 | 0.612 | 0.697 | 0.579 | 0.726 |
| Firm FE | YES | YES | YES | YES | YES | YES |
| Interest use | ffr | ffr | ffr2 | ffr2 | Cleveland rate | Cleveland rate |
| Interest level | low | high | low | high | low | high |

Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1

Note: Results from estimating equation (1). Alternative measures of the real interest rate are employed to classify the rate regime: the real federal funds rates, using the federal funds rate deflated by the past inflation (ffr, columns 1 and 2) and deflated by the expected inflation (ffr2, columns 3 and 4); and the Cleveland real rate (columns 5 and 6).

equipment), size (total assets), market-to-book ratio, ROE, return on assets (ROA), return on investment (ROI), Tobin's Q, and cash holdings (cash and short-term investments over total assets). These firm-level data are drawn from US COMPUSTAT covering 1984 to 2019. Firm fixed effects ($f_i$) and sector fixed effects ($h_j$) are included. We cluster standard errors two ways to account for correlations within firms and within years.

Our primary coefficient of interest, $\beta_1$, captures how interest rate shocks influence firm risk-taking behavior, as measured by $y_{ij,t}$. Importantly, $\beta_1(R_t)$ may vary with the level of interest rates. Motivated by the non-linearity observed in Figure 1, we split the sample into two sub-samples based on the level of interest rates: one for high rate periods and one for low rate periods. We use the median real interest rate as the threshold, classifying years with rates above (below) the median as high- (low-) rate periods. We consider several measures of the real interest rate, including the federal funds rate deflated by past inflation ("ffr") and expected inflation ("ffr2"), the Cleveland real rate. To capture exogenous interest rate fluctuations, we use the monetary policy shock series from Jarociński and Karadi (2020).[4]

Table 1 presents the findings derived from estimating specification (1). The coefficient $\beta_1$ is significantly positive when the interest rate is low and, conversely, negative when the interest rate is high. A one-unit monetary policy shock—corresponding to an unexpected interest rate cut of approximately 3 to 6 basis points, as estimated by Jarociński and Karadi (2020)—reduces firm-level ROE variation in low-interest-rate environments, with a semi-elasticity of approximately 1.4 to 1.8. In contrast, under high-interest-rate conditions, the same shock leads to an increase in ROE variation, with semi-elasticities ranging from –0.1 to –1.1 across different real interest rate measures.

Notice that the ROE measures above are "book" measures. In a previous version of the paper, we also looked at "market" measures.[5] We examined the standard deviation or in-

[4]Alternative shock measures, such as those from Swanson (2021), yield similar results.

[5]We also tried other measures such as research and development (R&D), which show similar patterns.

terquartile range (IQR) of individual daily stock returns over a year as an alternative measure of firm-level risk-taking. Using this proxy, we find similar patterns. In the aggregate, we also observe a humped relationship between the cross-sectional median of this risk measure and the real interest rate; At the firm level, the coefficient $\beta_1$ is positive in periods of low interest rates and turns negative in periods of high interest rates. Furthermore, even though interest rates dropped similarly, the 5-year moving average of the (cross-sectional) median IQR fell by 7.5% after 2001 but rose by 2% after 2007, again highlighting how firm leverage can influence risk-taking.

The above findings prompt us to theoretically investigate the influence of interest rates and leverage, or broader financial conditions, on firm risk-taking.

# 3 A Simple Leveraged Risk-taking Model

In this section, we examine the effect of financing constraints on risk-taking behaviors in a simple two-period model. The model shows that the relationship between risk-taking incentives and financial conditions is non-monotonic.

## 3.1 An Entrepreneur's Problem

There are two periods and two types of projects. An entrepreneur discounts between two periods with a discount factor $\beta$ and picks one of the two types of projects. The risky type has two possible returns: with probability $p$, the return is $\Pi^h$; with probability $1-p$, the return is $\Pi^l$. The expected return of the risky project is denoted by

$$\Pi \equiv p\Pi^h + (1-p)\Pi^l.$$

The entrepreneur can also choose the safe type with return $\Pi^f$. The entrepreneur can borrow at a gross interest rate $R$ with a financing constraint specified later.

For exposition reasons, we assume that

$$\Pi^h > \Pi > \Pi^f > R > \Pi^l. \quad \text{A1}$$

The expected return of the risky project $\Pi$ is higher than the return $\Pi^f$ of a safe project. Unlike Vereshchagina and Hopenhayn (2009), who study the choices of projects with the same expected return,[6] we impose $\Pi > \Pi^f$ so that socially it may be optimal to take risks while it may not be the case at individual levels. The return of the risky project is higher than

However, we prefer the return measures since firms can take "ordinary" risks without R&D.

[6]They study why entrepreneurs take risks even when expected returns are the same as the safe return.

$\Pi^f$ when the project is successful (that is, with the return $\Pi^h$), while the return is lower than $\Pi^f$ if the project is unsuccessful (that is, with the return $\Pi^l$). Also, $\Pi^f > R$ so that the safe project has a higher return than the risk-free rate. This might be natural, as running a firm generally should have a higher return than the risk-free rate. Finally, our analysis abstracts from the saving in risk-free assets, since they are dominated by running the safe project. In the macroeconomic model, we will consider a case in which an entrepreneur could be indifferent between a safe project and risk-free deposits.

That is, the entrepreneur *decides between two leveraged projects*, with the safe one hitting the borrowing constraint specified later, while the risky one may or may not have a binding borrowing constraint. In addition to A1, we assume that

$$\Pi^f \hat{\Pi} < \Pi^h \Pi^l, \text{ where } \hat{\Pi} \equiv p\Pi^l + (1-p)\Pi^h \tag{A2}$$

so that the risk-free project return is not too high, which captures reasonable features in practice and turns out to be useful in simplifying the analysis.

Let $\omega$ denote the wealth level of the entrepreneur at the beginning of each period, and let $s$ be the amount invested in a project which can be used as collateral. We specify the utility function of the entrepreneur as

$$u(c) = \log c,$$

where consumption/dividends $c = \omega - s$. It may initially seem that the entrepreneur owns the firm, but the entrepreneur can also be its manager, so $c$ could also be interpreted as a dividend payout. Besides closed-form solutions, there are several advantages of assuming $u(c) = \log c$. The curvature in $u(.)$ captures the manager's preference for dividend smoothing. Lintner (1956) first showed that managers consider dividend smoothing over time, a fact further confirmed by subsequent studies. In addition, increasing $c$ can be interpreted as equity share repurchases while reducing $c$ can be thought of as sales of new shares.

Putting curvature in $u(.)$ is thus a simple way of modeling the speed with which firms can vary the funding source when financial conditions change.[7] The model can thus quickly show how financial structures influence risk-taking. Notice that what is crucial is the concavity that generates the valuation effect below, but log utility simplifies the algebra and delivers reasonable quantitative results in the macro model later.[8]

Importantly, there is a running effort cost to implement the risky project. This cost requires $e$ units of time efforts of the entrepreneur, generating $v(e)$ disutility where $v'(e) > 0$. The level of $e$ is project-specific, so it is a state variable, and $e$ follows a certain distribution. For instance, developing a novel hardware prototype might demand 500 hours of engineer

[7] Jermann and Quadrini (2012) also assume dividend adjustment costs, supported by evidence cited therein.

[8] In the infinite horizon model, it implies $(1-\beta)$ fraction of net worth is used for paying dividends and rewarding managers. The calibrated $\beta$ and log utility deliver reasonable quantitative results (see Section 5).

and founder time, as well as external testing effort and expense that are wholly idiosyncratic to the specifications and complexity of that particular design. Likewise, launching a bespoke cloud-based software service could incur 600 developer-hours before any customer feedback arrives, highlighting how each project's $e$ (and thus $v(e)$) varies uniquely with its scope and requirements. We will solve for the threshold in the distribution, below which the entrepreneur decides to take risks.

Notice that the cost resembles the entrepreneurial choice in the Lucas (1978) span-of-control model. In a classic span-of-control setup, the heterogeneity lies in the entrepreneur's ability to manage workers; here, the capability is reflected by the disutility. The smaller $e$, the more efficient the entrepreneur is in running the risky project.[9] It is more straightforward to replace the state $e$ by $\eta \equiv v(e)$ since $v(e)$ will enter linearly into the utility function. Note that the effort $e$ is drawn exogenously from an i.i.d. distribution, we say $\eta \in [\underline{\eta}, +\infty)$ is drawn from an i.i.d. distribution (note: the lower bound does not have to be positive or even finite).

Let $b$ denote the level of borrowing. The value of implementing the risky project, $V^r(\omega, \eta)$, can be written as

$$\begin{aligned} V^r(\omega,\eta) = \max_{s,b} \Big\{ & \log(\omega - s) - \underbrace{\eta}_{v(e)} + \beta p \log(\Pi^h(s+b) - Rb) \\ & + \beta(1-p)\log(\Pi^l(s+b) - Rb) \Big\} \\ \text{s.t. } & 0 \le b \le \bar{\theta} s, \end{aligned}$$

where $\bar{\theta}$ is a parameter that governs the tightness of the borrowing constraint. The entrepreneur can put in internal savings $s$, together with the borrowing $b$; next period, the entrepreneur earns either $\Pi^h(s+b)$ if the project turns out to be productive, or $\Pi^l(s+b)$ if the project turns out to be unproductive. The interest payment is naturally $Rb$. The amount of borrowing, $b$, is limited due to financial frictions. The entrepreneur can pledge some of the capital $s$ as collateral, i.e., borrow up to $\bar{\theta}$ fraction of its capital at the time of borrowing. The value function of implementing the safe project, $V^f(\omega)$, can be written as

$$\begin{aligned} V^f(\omega) = \max_{s,b} & \Big\{ \log(\omega - s) + \beta \log(\Pi^f(s+b) - Rb) \Big\} \\ \text{s.t. } & 0 \le b \le \bar{\theta} s. \end{aligned}$$

Compared to the value function $V^r(\omega, e)$, the cost of implementing the risk-free project is normalized to 0, and the return in the next period is certain. The value of an entrepreneur

[9] Cui and Kaas (2020) prove that $\eta$ is equivalent to goods costs proportional to the wealth of entrepreneurs (given the utility and technology in this environment).

with effort cost unit $\eta$ and wealth $\omega$ is thus

$$V(\omega,\eta)=\max\{V^r(\omega,\eta),V^f(\omega)\}.$$

So, the realization of $\eta$ controls whether the firm chooses a risky or a risk-free project in period $t$. We abstract from the default issue to focus on the project choices, and firms are required and able to repay all their debt regardless of the realization of the project return. Appendix D introduces the potential of default. Optimization shows that $s$ is linear in the wealth level $\omega$, independent of the choice of projects. That is, $s=\varphi\omega$, with $\varphi=\frac{\beta}{1+\beta}$. When the entrepreneur chooses the safe project, their firms will borrow to the limit $\theta=\bar{\theta}$ since $\Pi^f>R$. When the entrepreneur chooses the risky project, depending on the interest rate $R$, $\theta$ can be one of the three: $0$, $\bar{\theta}$, and $\theta^*$ as the level of leverage when the firm is unconstrained:

$$\theta^*\equiv-\frac{\Pi^h\Pi^l-R\hat{\Pi}}{(\Pi^l-R)(\Pi^h-R)}=-\left[p\frac{\Pi^l}{\Pi^l-R}+(1-p)\frac{\Pi^h}{\Pi^h-R}\right]. \tag{2}$$

Assumptions A1 and A2 imply that $\theta^*>0$. The following proposition determines the threshold cost of taking on the risky project. In simple words, the threshold is the difference in logarithmic leveraged returns.

**Proposition 1.** *Suppose A1 and A2 hold. The leverage of a firm that implements the risky project satisfies*

$$\theta=\min\{\theta^*,\bar{\theta}\},$$

*where $\theta^*$ is defined in (2) and $\theta^*$ decreases in interest $R$. An entrepreneur with $\eta\leq\max\{\underline{\eta},\tilde{\eta}\}$ chooses the risky project and the threshold level $\tilde{\eta}$ satisfies*

$$\tilde{\eta}=\beta p\log\left(\frac{\Pi^h+(\Pi^h-R)\theta}{\Pi^f+(\Pi^f-R)\bar{\theta}}\right)+\beta(1-p)\log\left(\frac{\Pi^l+(\Pi^l-R)\theta}{\Pi^f+(\Pi^f-R)\bar{\theta}}\right). \tag{3}$$

*Proof.* See Appendix B. □

Next we investigate how the interest rate $R$ and leverage limit $\bar{\theta}$ affect the risk-taking behavior characterized by $\tilde{\eta}$.

## 3.2 Non-monotonic Risk- taking

First, we show that risk-taking may not be monotonous to the interest rate (borrowing cost). A falling borrowing cost does not necessarily induce more risk-taking.

To simplify the discussion, let us first focus on the case where the financing constraint is binding if a risky project is implemented, and we will relax this assumption later.[10] Define the log–leverage returns of the "safe" and "risky" projects by

$$
\begin{aligned}
r^f(R) &\equiv \log\big(\Pi^f + \bar{\theta}\,(\Pi^f - R)\big),\\
r^r(R) &\equiv p\,\log\big(\Pi^h + \bar{\theta}\,(\Pi^h - R)\big) + (1-p)\,\log\big(\Pi^l + \bar{\theta}\,(\Pi^l - R)\big),
\end{aligned}
$$

which are the key components of the value of taking the safe project and risky project, respectively. Under Assumption A1, both $r^f$ and $r^r$ are strictly decreasing and concave in the borrowing rate $R$, since

$$
\begin{aligned}
\frac{dr^f}{dR} &= -\frac{\bar{\theta}}{\Pi^f + \bar{\theta}\,(\Pi^f - R)} < 0, & \frac{d^2 r^f}{dR^2} < 0,\\
\frac{dr^r}{dR} &= -\sum_{j\in\{h,l\}} p^j\,\frac{\bar{\theta}}{\Pi^j + \bar{\theta}\,(\Pi^j - R)} < 0, & \frac{d^2 r^r}{dR^2} < 0.
\end{aligned}
$$

**Low-rate case.** When $R \approx 0$, the gap in log–leverage returns coincides with the gap in unconditional log-returns:

$$
r^r(0) - r^f(0) = p \log \Pi^h + (1-p)\log \Pi^l - \log \Pi^f.
$$

By Assumption A2 (ensuring that $\Pi^f$ is not excessively large), the initial sensitivity of the safe project exceeds that of the risky one:

$$
\left.\frac{dr^f}{dR}\right|_{R=0} = -\frac{\bar{\theta}}{(1+\bar{\theta})\,\Pi^f} \;<\; -\frac{\bar{\theta}}{1+\bar{\theta}}\sum_j p^j \frac{1}{\Pi^j} = \left.\frac{dr^r}{dR}\right|_{R=0}.
$$

Hence, as $R$ increases from zero, $r^f$ *declines more rapidly than* $r^r$, making the risky project relatively more attractive. This occurs because the risky project's expected return is higher, so the curvature in utility implies that log-return is less sensitive to the rise in $R$. In other words, when $R$ is low, the equity holders take the majority of the return, and, as $R$ increases, the excess return of the risky project becomes relatively more attractive compared to the safe project, since the percentage of decline in the return (after the interest rate rise) is lower.

**High-rate case** For sufficiently large $R$ (with $\Pi^l < R < \Pi^f$), it may instead hold that

$$
\frac{dr^f}{dR} > \frac{dr^r}{dR},
$$

[10] Proposition 1 implies that as the interest rate increases, $\theta^*$ decreases so that the financing constraints for entrepreneurs who choose the risky project could change from binding to non-binding.

which is algebraically equivalent to

$$\left[\Pi^f + \bar{\theta}\,(\Pi^f - R)\right]\left[\Pi + \bar{\theta}\,(\Pi - R)\right] > \left[\Pi^h + \bar{\theta}\,(\Pi^h - R)\right]\left[\Pi^l + \bar{\theta}\,(\Pi^l - R)\right].$$

The right-hand product can be close to zero (or even become negative) when $R$ is high, while the left-hand product remains positive for $R < \Pi^f$. This implies that at high borrowing costs, the poor outcome of the risky project appears exceptionally severe to the entrepreneur (as marginal utility spikes). When $R$ is high, most of the return is captured by the debt holder, causing the entrepreneur's value to fall sharply and their marginal utility to become highly sensitive to $R$, especially in the poor state. As a result, an increase in $R$ significantly reduces the attractiveness of the risky project, leading entrepreneurs to prefer the safe project instead.

The above two scenarios illustrate that the risk-taking incentives can be non-monotonic in $R$. Of course, within the range of interest rate implied by A1 and A2, we may not see the non-monotonic relationship. Thus, it is important to identify the conditions under which non-monotonicity emerges. Formally, we examine the derivative

$$\frac{\partial \tilde{\eta}}{\partial R} = \beta\left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - p\frac{\bar{\theta}}{\Pi^h + (\Pi^h - R)\bar{\theta}} - (1-p)\frac{\bar{\theta}}{\Pi^l + (\Pi^l - R)\bar{\theta}}\right], \quad (4)$$

and, as will become clear, the sign of $\partial\tilde{\eta}/\partial R$ depends on the upper bound of the leverage $\bar{\theta}$. A higher interest rate will reduce the leveraged return of both projects; however, whether the safe project or the risky project is more attractive depends on $\bar{\theta}$. If $\bar{\theta}$ is low (high) enough, the threshold $\tilde{\eta}$ increases (decreases) in the interest rate $R$, and the risky (safe) project is thus preferred. In intermediate cases, $\tilde{\eta}$ has a hump-shaped relationship with $R$.

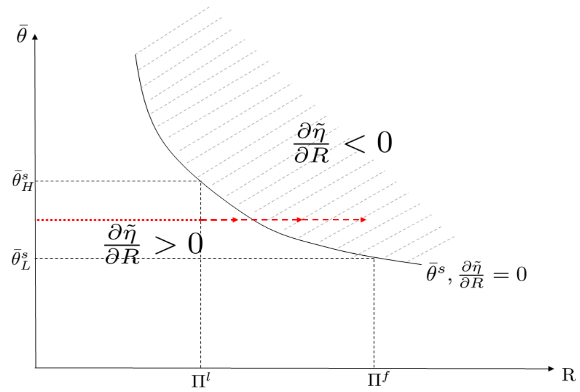
**Proposition 2.** *Suppose A1 and A2 hold. Assume the financing constraint is binding if the risky project is implemented (that is, $\bar{\theta} < \theta^*$). There exist cut-off levels $\bar{\theta}^s_L < \bar{\theta}^s_H$ such that the threshold level $\tilde{\eta}$ increases in $R$ when $0 < \bar{\theta} \le \bar{\theta}^s_L$, decreases in R when $\bar{\theta}^s_H \le \bar{\theta} < \theta^*$, and is hump-shaped in $R$ when $\bar{\theta}^s_L < \bar{\theta} < \bar{\theta}^s_H$.*

*Proof.* See Appendix B. □

The proposition can be illustrated by Figure 2. For a given leverage $\bar{\theta}$, the downward-sloping curve is denoted by $R^s(\bar{\theta})$, corresponding to the interest rate such that $\partial\tilde{\eta}/\partial R = 0$ and separates the two regions with different signs of the derivative $\partial\tilde{\eta}/\partial R$.[11] The shaded region above the curve has the feature of $\partial\tilde{\eta}/\partial R < 0$, and the blank area under the curve has the feature of $\partial\tilde{\eta}/\partial R > 0$. When $\bar{\theta} < \bar{\theta}^s_L$, as $R$ increases from the low return realization $\Pi^l$ to the safe return $\Pi^f$, $\partial\tilde{\eta}/\partial R$ is always positive. When $\bar{\theta} > \bar{\theta}^s_H$, as $R$ increases from the low

[11] Alternatively, the downward-sloping curve also represents $\bar{\theta}^s(R)$, which corresponds to the leverage upper bound such that $\partial\tilde{\eta}/\partial\bar{\theta} = 0$ for a given interest rate $R$.

Figure 2: Hyperplane of $\frac{\partial \tilde{\eta}}{\partial R} = 0$ in $(\bar{\theta}, R)$ Space




return realization $\Pi^l$ to the safe return $\Pi^f$, $\partial\tilde{\eta}/\partial R$ is always negative. In these two cases, the change in interest rate has a monotonic effect on the threshold $\tilde{\eta}$.

However, if $\bar{\theta}$ is in the middle range, that is, $\bar{\theta} \in (\bar{\theta}^s_L, \bar{\theta}^s_H)$, the effect of the interest rate on risk-taking is non-monotone, and has a *hump-shaped* relationship with $R$. To see this key result, for the points $(R, \bar{\theta})$ on the red dashed line below the separating curve, $\partial\tilde{\eta}/\partial R$ is positive since $\Pi^l < R < R^s(\bar{\theta})$; for the points $(R, \bar{\theta})$ on the red dashed line above the separating curve, $\partial\tilde{\eta}/\partial R$ turns negative since $R^s(\bar{\theta}) < R < \Pi^f$. In this middle range case, an increase in the interest rate $R$ first encourages more risk-taking behavior and then discourages risk-taking once the interest rate $R$ exceeds $R^s(\bar{\theta})$. The illustration at the beginning of this subsection is the basis of the proposition. When $R$ is low, an increase in $R$ reduces the equity value of the safe project by a higher percentage, making the risky option comparatively more attractive. But when $R$ is already high, pushing it up further makes the downside of failure much more severe, which in turn weakens the incentive to take on risk.

Notice that the non-monotonic effect is not unique to the interest rate; we also find the non-monotonic effect of leverage as in Appendix B.2. As illustrated below, the key behind the non-monotonicity is how debt servicing costs influence risk-taking incentives.

## 3.3 Understanding Further the Non-monotonicity

The intuitive reasoning in Section 3.2 provides a heuristic illustration using very low and very high values of $R$ to show how the relative attractiveness of the risky project might depend on the interest rate. The possibility of a humped relationship in Proposition 2 is crucial for an optimal level of interest rate in the interior, as discussed in the macroeconomic model below. To gain further insight, we approximate the threshold $\tilde{\eta}$ to the second order around the point $\tilde{\eta}_s$ (see below) and decompose it into two components. These two components represent what we label the valuation effect and the search-for-yield effect.

First, if we replace the risky project with a (hypothetical) safe project with a return $\Pi$ (the same as the expected return of the risky project), the entrepreneur borrows to the credit limit with this hypothetical safe project (that is, $\theta = \bar{\theta}$) due to the return $\Pi > \Pi^f > R$. Then, the threshold below which this hypothetical project is chosen can be defined as

$$\begin{aligned}\tilde{\eta}_s &\equiv \beta \log(\Pi(1+\theta) - R\theta) - \beta \log(\Pi^f(1+\theta) - R\theta) \\ &= \beta \log \left( \frac{\Pi - \frac{\bar{\theta}R}{1+\bar{\theta}}}{\Pi^f - \frac{\bar{\theta}R}{1+\bar{\theta}}} \right),\end{aligned}$$

$\tilde{\eta}_s$ is associated with the ratio between the leveraged returns on risky and risk-free projects; this term represents the "valuation effect" of the leverage or the interest rate since

$$\frac{\partial \tilde{\eta}_s}{\partial R} = \frac{\beta\bar{\theta}(1+\bar{\theta})(\Pi - \Pi^f)}{[\Pi^f(1+\bar{\theta}) - R\bar{\theta}]^2} > 0,$$

given $\bar{\theta} > 0$. When $R$ increases, financing costs rise, and the entrepreneur's firm will be less valued regardless of the chosen project. However, the ratio of the expected net return on the risky project, whose expected return is higher, to that of the safe project increases with the interest rate $R$. In other words, the value of the safe project declines more rapidly with $R$ than the value of the risky project, making the risky project relatively more attractive. This is again due to the decreasing marginal utility of wealth; as the interest rate rises, the drop in expected return in percentage terms is greater under the safe option. Consequently, the higher expected return for the risky project becomes marginally more attractive.

Therefore, the valuation effect makes the risky project with a higher expected return more preferred, raising $\tilde{\eta}_s$ and explaining $\frac{\partial \tilde{\eta}_s}{\partial R} > 0$. This term, which only looks at the logarithmic expected return, precisely reflects the intuition for the low-rate case described in Section 3.2.

Second, we expand $\tilde{\eta}$ to the second order around $\tilde{\eta}_s$ by using the cutoff (3):

$$\begin{aligned}\tilde{\eta} \approx \;\; & \tilde{\eta}_s \;\; +\beta p \left[ \frac{1+\bar{\theta}}{\Pi(1+\bar{\theta}) - R\bar{\theta}}(\Pi^h - \Pi) - \frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^2}(\Pi^h - \Pi)^2 \right] \\ & \qquad +\beta(1-p) \left[ \frac{1+\bar{\theta}}{\Pi(1+\bar{\theta}) - R\bar{\theta}}(\Pi^l - \Pi) - \frac{1}{2}\frac{(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^2}(\Pi^l - \Pi)^2 \right] \\ = \;\; & \tilde{\eta}_s \;\; \underbrace{-\frac{1}{2}\frac{\beta\sigma^2}{\left[\Pi - R\frac{\bar{\theta}}{1+\bar{\theta}}\right]^2}}_{\tilde{\eta}_\sigma},\end{aligned} \tag{5}$$

where $\sigma^2 \equiv p(\Pi^h - \Pi)^2 + (1-p)(\Pi^l - \Pi)^2$ is the variance of the risky project itself. After we adjust for discounting, $\tilde{\eta}_\sigma$ is a *negative* half of the *inverse* of the squared (leveraged) Sharpe ratio; this term reflects the disutility related to the volatility of the leveraged return.[12] It represents the "search-for-yield" effect or the substitution effect since

$$\frac{\partial \tilde{\eta}_\sigma}{\partial R} = \frac{-\beta\bar{\theta}(1+\bar{\theta})^2}{[\Pi(1+\bar{\theta}) - R\bar{\theta}]^3}\sigma^2 < 0,$$

given $\bar{\theta} > 0$. A higher Sharpe ratio, possibly driven by a higher leverage or a lower interest rate, increases the term $\tilde{\eta}_\sigma$, reducing the disutility of risk-taking and thus increasing the probability of taking risks (searching for a higher yield).

When $R$ increases, the poor outcome of the risky project significantly reduces the value of the leveraged risky project and the leveraged Sharpe ratio. For a given project volatility $\sigma$, the effect of a higher borrowing cost is amplified due to the leverage, which appears in the denominator of $\tilde{\eta}_\sigma$. This search-for-yield effect makes the risky project less attractive as $R$ increases, which explains why $\frac{\partial \tilde{\eta}_\sigma}{\partial R} < 0$, and leverage amplifies this effect. Therefore, the search-for-yield effect will be particularly strong when $R$ or $\bar{\theta}$ is already high, so that this term captures the intuition for the high-rate case discussed in Section 3.2.

Depending on the parameter values, a rise in the interest rate $R$ can have a humped effect on $\tilde{\eta}$ because the valuation and search-for-yield effects operate in opposite directions (Proposition 2). As $R$ increases, the *absolute* values of both $\tilde{\eta}_s$ and $\tilde{\eta}_\sigma$ increase, driven by the valuation effect and the search-for-yield effect, respectively. However, the search-for-yield effect dominates at high levels of $R$ as illustrated above, while the valuation effect is likely stronger when $R$ is low. The non-monotonic relationship can thus be a hump-shaped, *instead of a U-shaped*, relationship between risk-taking and the interest rate.

In addition to the possibility of a hump-shaped relationship, the valuation effect tends to dominate for all $R \in (\Pi^l, \Pi^h)$ when the equity is large relative to the debt, that is, when $\bar{\theta}$ is low. Intuitively, when the entrepreneur finances a larger share of the firm internally, the safer project can always become less attractive when $R$ increases at any level in $(\Pi^l, \Pi^h)$. So, in this case, the risk-taking increases with $R$. This possibility comes from the fact that the drop in return in percentage terms is greater under the safe option when the firm's leverage is low. However, when the entrepreneur relies heavily on external finance, that is, when $\bar{\theta}$ is large, the search-for-yield effect tends to dominate, as the volatility of the return is substantially magnified under high leverage; thus, the entrepreneur becomes particularly wary of the amplified downside risk (when $\Pi^l$ occurs) as $R$ increases from a high level. So, in this case, risk-taking decreases with the interest rate $R$.

More generally, a higher debt servicing cost per unit of equity, captured by $\bar{\theta}R/(1+\bar{\theta})$,

[12] As $\tilde{\eta}_\sigma$ is negative, $\tilde{\eta}_s$ is above $\tilde{\eta}$. This is intuitive because entrepreneurs will be more cautious (reflected in the lower threshold $\tilde{\eta}$ relative to $\tilde{\eta}_s$) for a riskier project with the same expected return, due to risk aversion or risk management needs.

discourages risk-taking through the search-for-yield effect, while encouraging it through the valuation effect. Importantly, we adopt log utility for its analytical tractability. However, both the valuation and the search-for-yield effects persist under alternative CRRA utility specifications, although the degree of curvature may influence their relative strength.

**Remark:** Finally, we further generalize that the non-monotonicity can persist even when the liquidity constraint is not binding. Notice that we previously focused on $\bar{\theta} < \theta^*_{min}$, where

$$\theta^*_{min} \equiv \frac{\Pi^h \Pi^l - \Pi^f \hat{\Pi}}{(\Pi^h - \Pi^f)(\Pi^f - \Pi^l)}$$

is the minimal leverage when the entrepreneur is not financing constrained when implementing the risky project. This level is obtained by assuming $R = \Pi^f$ in (2). For $\bar{\theta} \geq \theta^*_{min}$, we have:

**Proposition 3.** *Suppose A1 and A2 hold. Let* $\bar{R}(\bar{\theta})$ *be such that* $-\left[p\frac{\Pi^l}{\Pi^l - \bar{R}} + (1-p)\frac{\Pi^h}{\Pi^h - \bar{R}}\right] = \bar{\theta}$ *when entrepreneurs just become unconstrained. If* $\bar{\theta} \geq \theta^*_{min}$*, there exists an interest rate level* $R^u(\bar{\theta})$ *such that the risk-taking threshold* $\tilde{\eta}$ *for unconstrained entrepreneurs decreases in* $R$ *when* $\bar{R}(\bar{\theta}) \leq R < R^u(\bar{\theta})$ *and increases in* $R$ *when* $R^u(\bar{\theta}) \leq R < \Pi^f$*.*

*Proof.* See Appendix B. □

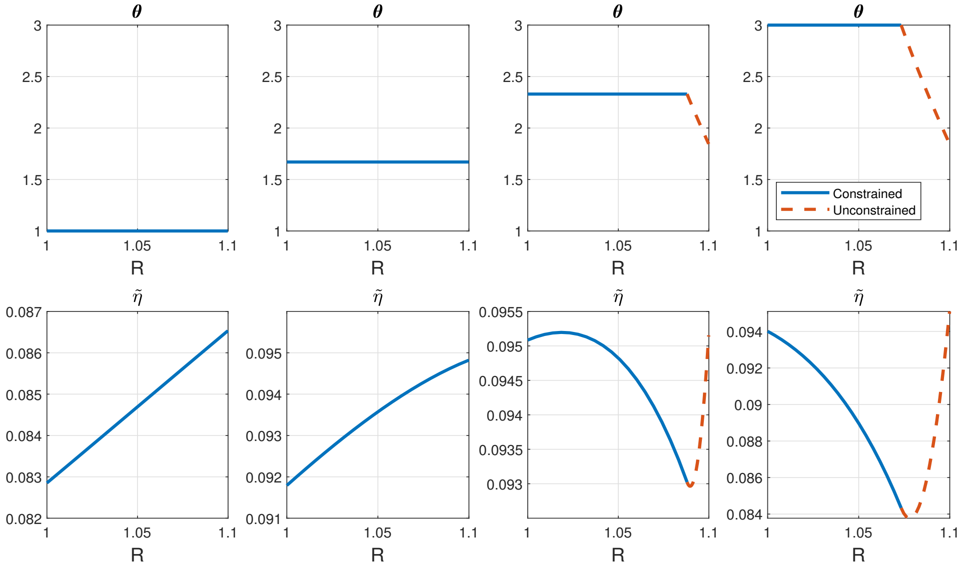
## 3.4 Examples

The entrepreneur may or may not be financing constrained when implementing the risky project because of the project's risk profile and the entrepreneur's risk preference. Figure 3 is a numerical illustration in which the parameterization is as follows: $\Pi^f = 1.1$, $\Pi^h = 1.3$, $\Pi^l = 0.95$, $p = 0.65$, and $\beta = 0.96$. Note that this specification satisfies A2. We do not have to specify $F(\cdot)$ for individual decision problems. However, it will be used later in the general equilibrium analysis. The figure delivers the key message of the model: a cut in the interest rate reduces the incentive to take risks (a fall in the threshold $\tilde{\eta}$) when the leverage limit $\bar{\theta}$ is low, while it encourages risk-taking when $\bar{\theta}$ is high.

Under a low-leverage limit, a reduction in the interest rate $R$ operates mainly through the "valuation effect" (the first two columns of Figure 3). Debt servicing costs are already small, so when $R$ falls (e.g., from 5 percent to 4 percent), the proportional increase in the safe project's return outpaces the expected return of the risky project: the entrepreneur is more likely to move to the safe option because the marginal gain from safety exceeds any incremental benefit from risk.

In contrast, under a high-leverage limit (the last two columns of Figure 3), the same cut in $R$ triggers a pronounced search-for-yield effect, which outweighs the pure valuation gains. Leverage magnifies improvements in the Sharpe ratio of the risky project (leveraged):

Figure 3: Effect of $R$ on $\tilde{\eta}$ for Alternative $\bar{\theta}$



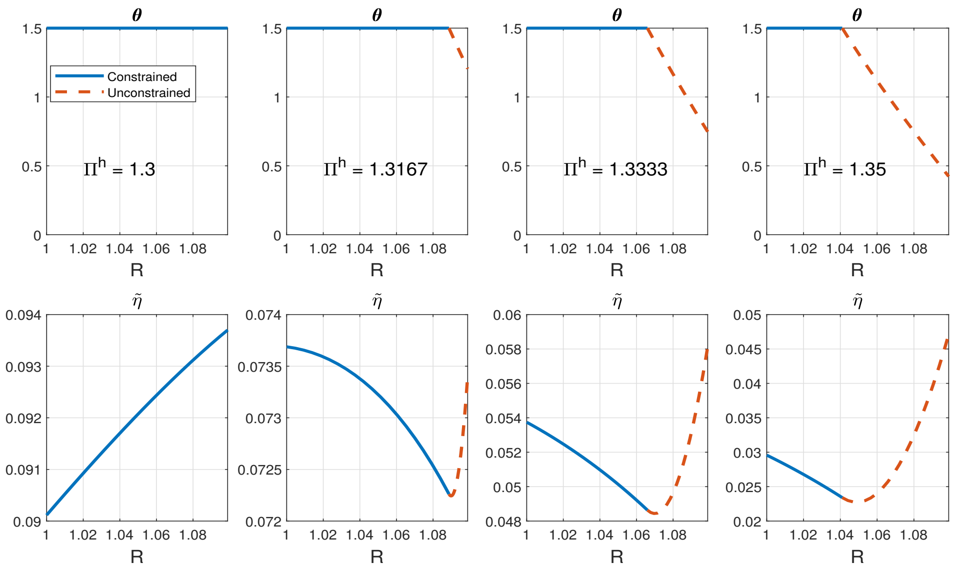

Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different leverage upper bound (from the left to the right: $\bar{\theta} = 1.00, 1.67, 2.33,$ and $3.00$).

Each drop in basis points in $R$ reduces the penalty of failure on the leveraged equity claim by more than it increases the leveraged return of the safe project. In other words, a lower interest rate under high leverage makes downside risk less costly, so the entrepreneur chases extra yield in the leveraged risky project (even if the leveraged safe project also becomes slightly more valuable).

Note that the financing constraint becomes slack for high leverage limits and interest rates, as shown in the third and fourth columns of Figure 3. Consistent with Proposition 3, when the borrowing constraint ceases to be binding, risk-taking first decreases and then increases with $R$. Furthermore, we show that the non-monotonic relationship is also robust if we endogenize the credit limit $\bar{\theta}$ (see Appendix D).

Finally, Figure 4 further shows that the effect of the borrowing cost $R$ on the risk-taking behavior also depends on the profile of the risky project return $\Pi^l$ and $\Pi^h$. In this experiment, we fix $\bar{\theta} = 1.5$. The first column uses the previously specified values of $\Pi^h$ and $\Pi^l$. Now, we increase $\Pi^h$ and decrease $\Pi^l$ simultaneously to keep the mean return of the risky project $\Pi$ unchanged. If the project becomes more risky, as the interest rate $R$ increases, the Sharpe ratio decreases more, and the risky project becomes less attractive. In other words, the search-for-yield effect dominates when the risky project becomes riskier. This result is intuitive because, when risk goes up, the leveraged volatility and the Sharpe ratio are more sensitive to interest rate changes.

Figure 4: Effect of $R$ on $\tilde{\eta}$ under Different Risk Profiles




Note: The top panel of each column shows the leverage associated with the risky project; the bottom panel of each column shows the threshold cost for risk-taking. Each column corresponds to a different risk profile (from the left to the right: $\Pi^h = 1.3, 1.32, 1.33,$ and $1.35$), while the expected return $\Pi = p\Pi^h + (1-p)\Pi^l$ is kept the same across all panels.

We also note that as the project becomes riskier, the interest-rate range in which the leverage constraint ceases to binding expands. As risks increase, the financially unconstrained interest rate region with a dominating valuation effect also expands.

# 4 A General Equilibrium Model

Now, we extend the previous two-period model into an infinite-horizon general equilibrium model. The next section will use the equilibrium model for policy discussion. Our analysis includes questions such as whether policy should encourage risk-taking and how the optimal policy is shaped by factors such as credit limits often regarded as financial policy.

## 4.1 The Environment

Time is discrete and infinite. We use recursive notation, that is, let the variable $x$ denote $x_t$ and let $x_+$ denote $x_{t+1}$. The economy is populated by a unit measure of entrepreneurs who may run firms. The economy also has a household sector and a government that runs a flat-rate value-added tax policy[13] and credit policy.

[13]VAT tax policy is equivalent to taxing labor income and capital income at the same rate. We also experimented with a lump-sum tax policy; the quantitative results are similar.

**Entrepreneurs.** Entrepreneurs can choose among the risk-free project, the risky project, and risk-free savings. The firm technology is represented by

$$y = (zk)^{\alpha} (\ell)^{1-\alpha},$$

where $\alpha \in (0,1)$ is the capital share, $k$ is capital input, $\ell$ is labor input, capital productivity $z \in \{z^h, z^l\}$ is risky if an entrepreneur chooses the risky project, and $z = z^f$ is a constant number if the entrepreneur chooses the safe project.

Consider a firm with capital $k$. In the labor market, the firm hires workers at the competitive wage rate $w$, which leads to a labor demand proportional to the firm's effective capital input. The firm thus maximizes its capital return by solving the problem

$$\max_{\ell}\{(1-\tau^y)(zk)^{\alpha}(\ell)^{1-\alpha} - w\ell\},$$

where $\tau^y$ is the VAT tax rate set by the government. The optimal labor choice is as follows:

$$\ell = zk\left(\frac{(1-\alpha)}{w/(1-\tau^y)}\right)^{\frac{1}{\alpha}}. \tag{6}$$

Therefore, the firm's capital return (before debt repayment) is $rzk$, where $r$ can be shown as

$$r \equiv (1-\tau^y)\alpha\left[\frac{(1-\alpha)}{w/(1-\tau^y)}\right]^{\frac{1-\alpha}{\alpha}}. \tag{7}$$

Denote the capital depreciation rate as $\delta$. To make the decision problem introduced below similar to the problems discussed in the simple model, we specify $\{\Pi^f, \Pi^h, \Pi^l\}$ as

$$\Pi^f = rz^f + 1 - \delta, \quad \Pi^h = rz^h + 1 - \delta, \text{ and } \Pi^l = rz^l + 1 - \delta. \tag{8}$$

We retain the one-period project and debt structure from the previous two-period model for tractability. Although incorporating long-term projects and debt would be an interesting extension, it would introduce additional complexities, such as discounting effects from long-term interest rates and shifting asset allocations driven by search-for-yield incentives. Our current framework already captures these channels effectively. Moreover, idiosyncratic implementation costs reflect differences in project duration and credit risk, while the i.i.d. structure (including success probability $p$) captures the unpredictable nature of risky opportunities, justifying the absence of full self-financing and the simplification of excluding risk persistence.

Let $V(\omega)$ denote the expected value of an entrepreneur with wealth $\omega$ before observing the cost of implementing the risky project. As before, $V^r(\omega, \eta)$ and $V^f(\omega)$ represent the values associated with choosing risky and risk-free projects, respectively. The value of saving in deposits with return $R^d$ is given by $V^d(\omega)$. We have the following relationship:

$$V(\omega) = \max\{V^d(\omega), \mathbb{E}\left[\max\{V^r(\omega,\eta), V^f(\omega)\}\right]\},$$

where the expectation is taken over the distribution of $\eta$. Entrepreneurs decide first whether to save or become active entrepreneurs. If they decide to become active entrepreneurs, they can choose between safe and risky projects. The value function of an entrepreneur who takes the risky project can be rewritten as

$$\begin{aligned} V^r(\omega,\eta) = \max_{s,b} \Big\{&(1-\beta)\log(\omega - s) - \eta + \beta p V(\Pi^h(s+b) - Rb) \\ &+ \beta(1-p)V(\Pi^l(s+b) - Rb)\Big\} \\ \text{s.t. } & 0 \le b \le \bar{\theta}s, \end{aligned}$$

where $(1-\beta)$ serves as a normalization. As before, the entrepreneur can invest with internal savings $s$, together with the borrowing $b$; next period, the entrepreneur earns $\Pi^h(s+b)$ (if the project turns out to be productive) or $\Pi^l(s+b)$ (if the project turns out to be less productive). The interest payment is naturally $Rb$.

For flexibility in model calibration, we assume a fixed cost (utility) $\eta^f$ associated with running the risk-free project. The value function of an entrepreneur who takes a safe project or saves in deposits can be rewritten as

$$\begin{aligned} V^f(\omega) &= \max_{s,b} \Big\{(1-\beta)\log(\omega - s) - \eta^f + \beta V(\Pi^f(s+b) - Rb)\Big\} \\ &\text{s.t. } 0 \le b \le \bar{\theta}s; \\ V^d(\omega) &= \max_{s} \Big\{(1-\beta)\log(\omega - s) + \beta V(R^d s)\Big\}, \end{aligned}$$

respectively. Naturally, when entrepreneurs choose the risk-free project, they will borrow up to the limit $\theta = \bar{\theta}$ if $\Pi^f > R$. We allow a constant $\tau \ge 0$ to be the difference between the deposit rate $R^d$ and the lending rate $R$:

$$R = R^d\left(1 + \tau\right). \tag{9}$$

The parameter $\tau$ represents the intermediation costs or the interest rate markup, making the calibration below also more flexible.

It turns out that the value functions satisfy the following forms $V^r(\omega,\eta) = \log(\omega) - \eta + v^r$, $V^f(\omega) = \log(\omega) - \eta^f + v^f$, and $V^d(\omega) = \log(\omega) + v^d$ for some endogenous $v^r$, $v^f$, and $v^d$. As in the two-period model, the saving function is linear in wealth $s = \beta\omega$, and we have a similar characterization of the threshold cost:

**Proposition 4.** *Define* $l^d \equiv \log R^d$, $l^r \equiv p \log\left(\Pi^h + \theta(\Pi^h - R)\right) + (1-p)\log\left(\Pi^l + \theta(\Pi^l - R)\right)$ *and* $l^f \equiv \log\left(\Pi^f + \bar{\theta}\left(\Pi^f - R\right)\right) - \beta^{-1}\eta^f$. *Those entrepreneurs with* $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ *choose the risky project, where* $\tilde{\eta}$ *satisfies,*

$$\tilde{\eta} = \beta(l^r - l^f), \tag{10}$$

*and the leverage* $\theta = \min\{\theta^*, \bar{\theta}\}$. *Additionally, letting* $\phi$ *be the fraction of entrepreneurs who choose to save in deposits, we have the following condition:*

$$\begin{cases} \phi = 1 & \text{if } l^d > \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right] \\ \phi \in (0,1) & \text{if } l^d = \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right] \\ \phi = 0 & \text{if } l^d < \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right] \end{cases}. \tag{11}$$

*Proof.* See Appendix B. □

The first branch of (11) with $\phi = 1$ does not arise in equilibrium, as it implies zero production in the economy. In an interior equilibrium with $\phi \in (0,1)$, those entrepreneurs must be indifferent between saving through risk-free deposits and becoming active entrepreneurs who choose the two types of projects. That is, $l^d = \mathbb{E}\left[\max\{l^r - \frac{\eta}{\beta}, l^f\}\right]$, or more specifically,

$$l^d = (1-F)l^f + Fl^r - \beta^{-1}\int_{\underline{\eta}}^{\tilde{\eta}} \eta dF(\eta),$$

which governs how the capital return rate $r$ varies with $R$. Entrepreneurs who save in risk-free deposits can be considered as unconstrained firms in practice. In the model, we can allow them to operate a different technology, which, in equilibrium, features the same rate of return as safe deposits.[14]

**Households.** Households are hand-to-mouth consumers and supply labor to firms.[15] To focus on the firm side, we assume a Greenwood-Hercowitz-Huffman CRRA utility function and a representative household maximizes

$$(1-\beta)\sum_{t=0}^{\infty} \beta^t U\left(C_t - \frac{\kappa L_t^{1+\gamma}}{1+\gamma}\right),$$

[14] With this modelling choice, $\phi = 1$ is possible in equilibrium. However, with $\phi = 1$, there is no risk-taking, and the model becomes a standard neo-classical model.

[15] The assumption is less strong than it seems. In incomplete market models, the interest rate is lower than the time-preference rate. Households would like to borrow, but (unlike entrepreneurs) households do not have collaterals, so they will not borrow in equilibrium.

where $U(.)$ is an increasing and concave function, $C_t$ is the consumption level of households, $L_t$ is the labor supply, $\kappa > 0$ is the disutility parameter and $\gamma$ is the inverse of the Frisch labor elasticity. The household chooses consumption and labor supply according to (where again we ignore the time subscript):

$$C = wL; \tag{12}$$

$$w = \kappa L^{\gamma}. \tag{13}$$

**The (consolidated) government agency.** A consolidated government conducts joint monetary and fiscal policies with the following budget constraint:

$$G + R^d B = T + B_+. \tag{14}$$

The expenditure side includes government spending $G$ and debt repayment $R^d B$. For simplicity, we assume that the government can borrow without the intermediation cost so that the interest rate is also $R^d$ because of no-arbitrage. The revenue side includes tax $T = \tau^y Y$ (where $Y$ is the aggregate output shown below) and newly issued debt $B_+$.

$B$ represents a broad liquidity policy, as we consider the government agency as a consolidated identity that includes a monetary authority, a fiscal authority, and financial intermediaries, which lend to the firm sector. Therefore, $B$ is the net debt position of the joint agency; when $B < 0$, the consolidated identity lends to the firm sector. Alternatively, if we interpret the consolidated agency with only monetary and fiscal authorities, $B$ can be considered an outcome of the quantitative easing (QE) policy. That is, we simplify the institutional details of credit/liquidity policy to focus on the equilibrium effect of the interest rate on entrepreneurs' risk-taking and its implications for the optimal interest rate.

The government agency sets $(G, R)$ for each period. Then $R^d = R/(1+\tau)$ and $B_+$ will be determined in the credit market, while the tax rate $\tau^y$ will be governed by the government budget constraint.

## 4.2 Equilibrium

Denote the total wealth of private agents by $\Omega$. Wealth accumulation implies:

$$\Omega_+ = \beta\Omega\left\{\phi R^d + (1-\phi)\left\{ [1 - F(\tilde{\eta})]\left[\Pi^f(1+\bar{\theta}) - R\bar{\theta}\right] + F(\tilde{\eta})\left[\Pi(1+\theta) - R\theta\right] \right\}\right\}, \tag{15}$$

where again $\phi$ is the probability of saving entrepreneurs (note: they save with the rate $R^d$). The wealth of the next period $\Omega_+$ will come from three sources. All entrepreneurs put a

$\beta$ fraction of their wealth aside, which explains $\beta\Omega$. The group of entrepreneurs have a $\phi$ fraction of savers (who have a return $R^d$) and $(1-\phi)(1-F(\tilde{\eta}))$ fraction implementing the safe project with a return $\Pi^f(1+\bar{\theta})-R\bar{\theta}$.[16] The remaining entrepreneurs take risks, and the project return after leverage is $\Pi(1+\theta)-R\theta$.

Capital productivity is endogenous in the aggregate because of credit-induced investment in different technologies. That is, capital reallocation linked to credit conditions determines the endogenous productivity of capital. Define $Z$ as the endogenous capital efficiency unit that takes into account the technology choices of entrepreneurs:

$$Z = z^f(1-F)(1+\bar{\theta}) + \bar{z}F(1+\theta), \text{ where } \bar{z} = pz^h + (1-p)z^l. \tag{16}$$

Then, the aggregate output can be written as

$$Y = \int (z_i k_i)^\alpha \ell_i^{1-\alpha} di = ((1-\phi)Z\beta\Omega)^\alpha L^{1-\alpha}. \tag{17}$$

Notice that each firm produces $z_i k_i/\alpha$ in which $z_i k_i$ is retained. Furthermore, $(1-\phi)Z\beta\Omega$ can be regarded as the effective capital stock used in production,

To define equilibrium, we look at market clearing conditions for labor and for credits. For the labor market, hours of work $L$ should be the labor supply of households, so that the "rental rate" $\frac{r}{1-\tau^y}$ can be written in a conventional way, which is the marginal product of the effective capital stock (using $L$ in the product).

$$\frac{r}{1-\tau^y} = \alpha\left((1-\phi)Z\beta\Omega\right)^{\alpha-1} L^{1-\alpha}. \tag{18}$$

For the credit market, the consolidated agency conducts the credit/liquidity policy, determining the market-clearing interest rate. As mentioned, we can think of the consolidated policy maker conducting an interest rate policy by choosing $R$. An interest rate $R$ corresponds to a particular debt/liquidity policy $B$ according to the market clearing condition. The total liquidity savers provide for period $t$ is $\phi\beta\Omega$. The total credit demanded by the entrepreneurs who borrow is $\left[(1-F(\tilde{\eta}))\bar{\theta}+F(\tilde{\eta})\theta\right](1-\phi)\beta\Omega$: a fraction $[1-F(\tilde{\eta})](1-\phi)$ of whom use safe technology and borrow to the credit limit implied by $\bar{\theta}$ and a fraction $F(\tilde{\eta})(1-\phi)$ of whom use the risky technology and leverage to $\theta \leq \bar{\theta}$. Therefore, clearing the credit market implies

$$\left[(1-F(\tilde{\eta}))\bar{\theta}+F(\tilde{\eta})\theta\right](1-\phi)\beta\Omega = \phi\beta\Omega - B_+. \tag{19}$$

**Definition.** Given states including the entrepreneurs' wealth $\Omega$, a government spending $G$, and an interest-rate policy target $R$, a recursive competitive equilibrium is a collection of

[16] Here, we focus on the case of $\Pi_f > R$. The numerical exercises below allow $\Pi_f \leq R$, in which case $\theta = 0$.

variables $\{L, C, \tilde{\eta}, \theta, \phi, \Pi^h, \Pi^l, \Pi^f, r, w, \tau^y, B_+, R^d, \Omega_+\}$ such that

- Households supply labor according to (13) ; their budget constraint (12) holds;
- $\tilde{\eta}$ and $\theta$ solve the entrepreneurs' problem with risk-taking choice and leverage choice as shown in Proposition 4;
- project returns (8) are satisfied, with $r$ given by (7);
- The wealth dynamics (15) holds;
- The consolidated government budget constraint is satisfied, i.e., (14) holds with tax revenue $T = \tau^y Y$ where output is calculated in (17);
- The labor market clears, i.e., (18) holds; The credit market clears, i.e., (19) holds with (11) satisfied, and the deposit rate $R^d$ is determined by (9).

In addition to showing the short-run effects of an interest rate cut, the following quantitative exercises are mostly steady-state analyses because they are tractable and already contain many useful results. Note that the steady-state social welfare of entrepreneurs $V(\Omega)$ and households $W$ can be seen as follows. Given the value functional form and the nature of the steady state economy, the social welfare measure, which is related to the present value of utility of consumption from all groups, thus becomes (see Appendix C.3 for details):

$$V(\Omega) + W = \log(\Omega) + \tilde{V} + U\left(C^h - \frac{\kappa L^{1+\gamma}}{1+\gamma}\right) + \text{constants},$$

where $\Omega$, $\tilde{V}$, $C^h$, and $L$ are all endogenous. $\tilde{V}$ includes the base return (on the risk-free project or the safe deposits) and the relative value gain from choosing the risky project.

# 5 Macroeconomic Effects of Target Interest-rate Policy

Using the macroeconomic environment introduced above, we now analyze the aggregate effects of varying the (real) interest rate target. Note that the consolidated government agency implements the interest rate policy (with government debt management in the background), and it can thus be considered a joint monetary-fiscal policy outcome in practice. It becomes too complex to study the optimal interest rate fully analytically. We choose to calibrate the model and assess the policy effects.

To clarify, we do not consider external factors (such as demographic trends) that can affect the real interest rate, potentially through impacts on the subjective discount factor, $\beta$. Here, we treat the long-term interest rate target as given. Instead, our focus is on an interest rate component, which can be influenced by monetary and fiscal policy. For example, this can be managed through medium-term targets for inflation and nominal interest rates. Our analysis includes questions such as whether policy should encourage risk-taking and how the optimal policy is shaped by factors like the credit limit.

## 5.1 Parameterization

Suppose that one period in the model represents one year. Table 2 reports the calibrated parameters. Some parameters are exogenous. The capital share is set to a conventional value $\alpha = 0.33$. The inverse of the Frisch elasticity of the labor supply $\gamma$ is set to $1/1.5$, also a conventional number for macroeconomic models. We assume a CRRA utility for $U$, and the relative risk aversion is set to 2, so households are slightly more risk-averse than entrepreneurs. Note this number does not affect macroeconomic variables; it only slightly influences the aggregate welfare comparison. The discussion below illustrates the key steps in calibrating other parameters.

Table 2: Calibration

| | Value | Explanation/Target | | Value | Explanation/Target |
|---|---|---|---|---|---|
| $\beta$ | 0.9437 | Discount factor | $z^f$ | 1 | Normalization |
| $\gamma$ | 1/1.5 | Inverse Frisch elasticity | $z^h$ | 1.5989 | ROE top 80th per. |
| $\kappa$ | 1.6179 | Hours 0.33 | $z^l$ | 0.0785 | ROE bottom 20th per. |
| $\alpha$ | 0.33 | Capital share | $p$ | 0.6803 | ROE std. dev. |
| $\delta$ | 0.0938 | Investment-to-output 17% | $\mu$ | -3.1785 | Debt-to-output ratio: 65.2% |
| $\tau$ | 0.02 | Interest-rate spread | $\sigma$ | 0.0031 | Elasticity of ROE: 3.71 |
| $\bar{\theta}$ | 0.5600 | Median leverage ratio | $G$ | 0.0828 | Gov-spending-to-output 18% |
| $\eta^f$ | 0.0305 | Median ROE | $R$ | 1.0370 | Prime rate |

$\kappa$, which governs the disutility of labor, is calibrated to hit the labor hours such that $L = 0.33$ after normalizing the total hours to unity. The model targets the ratio of government spending to GDP $G/Y$ as $18\%$, which pins down $G$. The capital depreciation rate $\delta$ is calibrated so that the investment-to-output ratio is $17\%$.

We choose the real prime rate to be the status quo interest rate $R$, since there is no default in the model and the prime rate is a benchmark interest rate used for high-quality borrowers. We use the period $1984$-$2019$ (see Appendix A for data description). The average annualized real (gross) prime rate during this period is $1.037$, while the average annualized gross federal funds rate during this period is $1.017$. Therefore, the interest rate markup $\tau$ is set to $2\%$. In addition, we set the discount factor as $\beta = 0.9437$ so that the gap between $1/\beta$ and $R$ is

2.2%, which targets the average gap between BAA corporate bonds and 10-year Treasury bonds in the sample period. This gap reflects reasonable liquidity and risk premia that are not modeled in the economy. In addition, the model implies that $(1-\beta) \approx 5.6\%$ of net worth is used for the consumption of entrepreneurs, which should be interpreted as paying dividends or rewarding managers in practice. Notice that the average S&P 500 dividend yield in the sample period is 2.4%; this implies the rest (3.2%) could be considered as being used to incentivize managers, which seems conservative.[17]

Next, we use the top $80th$ percentile value ($\Pi^{80th}$) and the bottom $20th$ percentile value ($\Pi^{20th}$) of cross-sectional ROE to represent the net leveraged return for high realization and low realization, i.e.,

$$\Pi^{80th} \equiv (1+\bar{\theta})\Pi^{h} - R\bar{\theta}; \quad \Pi^{20th} \equiv (1+\bar{\theta})\Pi^{l} - R\bar{\theta}.$$

As a normalization, we map the ROE's median return to the net leveraged risk-free return $\Pi^{m} \equiv (1+\bar{\theta})\Pi^{f} - R\bar{\theta}$. The top $80th$ percentile, the bottom $20th$ percentile, and the median net returns for this period are $18\%$, $-15\%$, and $5\%$, respectively, so that $\Pi^{80th} = 1.18$, $\Pi^{20th} = 0.85$, and $\Pi^{m} = 1.05$. The benchmark exercise normalizes $z^{f} = 1$ and assumes that some entrepreneurs save. Then, we obtain $\eta^{f}$ from the indifference condition, and we obtain $r$ as a function of $\bar{\theta}$:

$$r = \frac{\Pi^{m} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta).$$

Furthermore, $z^{h}$ and $z^{l}$ are also functions of $\bar{\theta}$ by substituting the above expression for $r$ into the expression for $\Pi^{80th}$ and $\Pi^{20th}$:

$$z^{h} = \frac{1}{r}\left[\frac{\Pi^{80th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right]; \quad z^{l} = \frac{1}{r}\left[\frac{\Pi^{20th} + R\bar{\theta}}{1+\bar{\theta}} - (1-\delta)\right].$$

Since the leverage parameter $\bar{\theta}$ is needed for $r$, $z^{h}$, and $z^{l}$, it is set according to the firm leverage ratio used in Section 2. The median of the leverage ratio is $0.56$. Then $r$, $z^{h}$, and $z^{l}$ are identified immediately as shown above.

We assume that the cost distribution follows a log-normal distribution with the mean as $\mu$ and variance $\sigma^{2}$ after log transformation. We calibrated other distributions with positive support and found it is not crucial for the qualitative conclusion in the following. Therefore, the remaining key parameters are: $\{\mu, \sigma, p\}$. They are chosen to match the three key moments below.

[17] The estimates of CEO pay to company equity ratios are around 3% to 12% according to the literature about CEO compensation and equity performance.

(1). The model targets the debt-to-output ratio $0.652$ as in the data[18], calculated based on the total debt of non-financial businesses in the data.

(2). Assuming risk-taking firms are financially constrained in the status quo, we have the standard deviation of the overall return in the model

$$\sigma_I = \sqrt{F(\tilde{\eta})}(1+\bar{\theta})\sqrt{p(1-p)}\left(\Pi^h - \Pi^l\right) = \sqrt{F(\tilde{\eta})}\sqrt{p(1-p)}\left(\Pi^{80th} - \Pi^{20th}\right),$$

corresponding to the average cross-sectional standard deviation ($0.13$) of ROE in the sample.

(3). Its sensitivity to the interest rate becomes

$$\frac{\partial \sigma_I}{\partial R}\frac{R}{\sigma_I} = \left[\frac{1}{2}\frac{f(\tilde{\eta})}{F(\tilde{\eta})}\frac{\partial \tilde{\eta}}{\partial R} + \frac{1}{\Pi^h - \Pi^l}\left(\frac{\partial \Pi^h}{\partial R} - \frac{\partial \Pi^l}{\partial R}\right)\right]R, \tag{20}$$

corresponding to $3.71$ in the data. Note that when the financing constraint is binding, the derivative $\partial\tilde{\eta}/\partial R$ can be computed as

$$\frac{\partial \tilde{\eta}}{\partial R} = \begin{cases} \beta p \frac{1}{\Pi^{80th}}\left(\frac{\partial \Pi^h}{\partial R} + \left(\frac{\partial \Pi^h}{\partial R} - 1\right)\bar{\theta}\right) \\ +\beta(1-p)\frac{1}{\Pi^{20th}}\left(\frac{\partial \Pi^l}{\partial R} + \left(\frac{\partial \Pi^l}{\partial R} - 1\right)\bar{\theta}\right) \\ -\beta\frac{1}{\Pi^m}\left(\frac{\partial \Pi^f}{\partial R} + \left(\frac{\partial \Pi^f}{\partial R} - 1\right)\bar{\theta}\right) \end{cases}.$$

Since this feature is unique to our model, we elaborate the procedure. First, as in the calibration above, entrepreneurs are indifferent between saving in safe assets and implementing projects when $\phi \in (0,1)$. That is, we have the second branch of (11), and with some algebra we obtain

$$\frac{\partial \Pi^f}{\partial R} = \frac{\partial r}{\partial R} = \frac{\frac{1}{R^d}\frac{1}{1+\tau} + \bar{\theta}\left[(1-F)\frac{1}{\Pi^m} + F\left(p\frac{1}{\Pi^{80th}} + (1-p)\frac{1}{\Pi^{20th}}\right)\right]}{(1+\bar{\theta})\left[(1-F)\frac{1}{\Pi^m} + F\left(p\frac{z^h}{\Pi^{80th}} + (1-p)\frac{z^l}{\Pi^{20th}}\right)\right]}.$$

$\partial\Pi^h/\partial R$ and $\partial\Pi^l/\partial R$ follow immediately. Second, we examine when risk-taking entrepreneurs may not be financing-constrained, but the result points to the constrained scenario. It turns out that $p$ is about $0.68$, which means that the high return is more likely to be observed, so being financing constrained is more likely when entrepreneurs take risks.

The calibrated economy features $\phi = 0.153$, implying that $84.7\%$ of firms are financing-constrained. The fraction of constrained firms is not crucial for at least two reasons. First, the consolidated "government agency" in equilibrium saves, which can include savings from unconstrained firms and households in reality. Second, we experimented with adding another group of entrepreneurs with a different probability of success $p$, so their firms are uncon-

[18] In the model, debt-to-output ratio is $\frac{D}{Y} = \frac{\bar{\theta}}{\frac{r}{\alpha}\left[z^f(1-F(\tilde{\eta}))(1+\bar{\theta}) + \bar{z}F(\tilde{\eta})(1+\bar{\theta})\right]}$. In the data, we use the total debt of non-financial businesses over their total output. Note that $\tilde{\eta}$ is solved by (10).

strained if they choose the risky project. By varying the relative population of this group, we can hit a certain target fraction of unconstrained firms in the economy. Still, we have the non-monotonic result, as illustrated in the examples of the two-period model. The optimal interest rate levels shown below differ, but the qualitative conclusions below are robust.

## 5.2 Medium Run Effects of Interest Rate Policy

Timing is critical to evaluate government policies. In the long run, persistent structural forces, such as demographic shifts that influence savings and interest rates, dominate, so policy has little permanent impact. In the short run, price rigidities delay adjustment, so policy can exert strong but temporary effects. Our focus is on medium-run equilibrium, where policy interventions meaningfully affect markets with prices already adjusted. Specifically, consider a policymaker altering the supply of government debt $B$ in the background to influence the target interest rate (Figure 5). Using the steady-state outcomes of the model, we examine the effects of this medium-term target rate and, later, explore whether an optimal level exists.

This exercise confirms a humped relationship between the risk-taking threshold $\tilde{\eta}$ and the interest rate. The hump-shaped relationship also applies to the dispersion of ROE as $R$ increases, as in the data. It is worth noting that the total factor productivity (TFP), the residual after considering firm capital and labor inputs in the output definition (17),

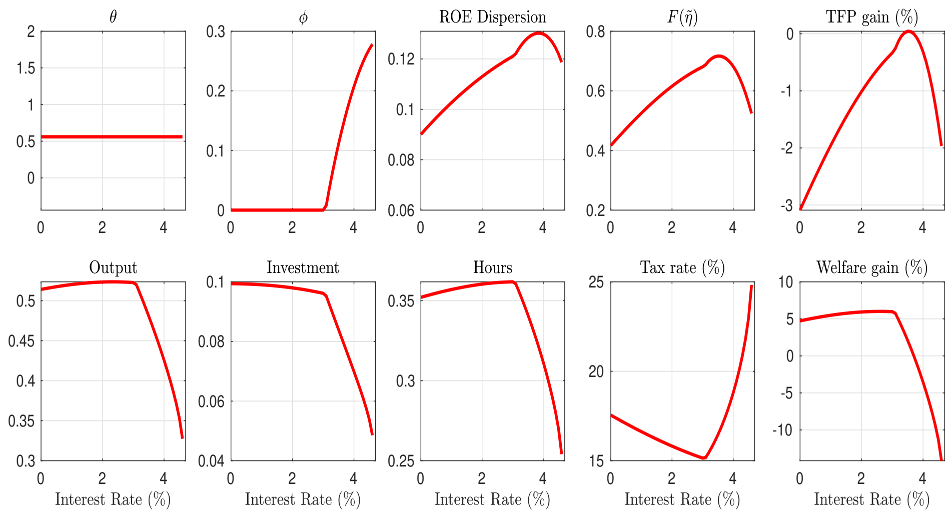
$$TFP \equiv Z^{\alpha} = \left[z^{f}(1-F(\tilde{\eta}))(1+\bar{\theta}) + \bar{z}F(\tilde{\eta})(1+\theta)\right]^{\alpha},$$

also inherits the humped shape of risk-taking, thanks to the presence of $F(\tilde{\eta})$. At the calibrated interest rate level (3.70%), TFP is close to its peak, but it reaches its maximum at the interest rate level of $3.55\%$. Not all variables exhibit a hump. When the interest rate increases, saving in secure deposits becomes more appealing. The proportion ($\phi$) of savers who provide liquidity increases weakly with the interest rate.

A lower interest rate relaxes borrowing conditions and encourages investment. However, it may also lead to inefficiency. The fall in productivity at the aggregate level is caused by less risk-taking and falling labor demand, which eventually reduces output. In this sense, expansionary interest rate policy can eventually become contractionary. In other words, the conventional "IS" curve slopes downward when the interest rate is not too low; but once the interest rate is below 2%, the output starts to fall quickly with the falling interest rate, and the IS curve slopes upward. For example, compared to the highest output (roughly when the rate is around 2.5%), a -2% rate implies almost a 5% drop in output.

To be more precise, when the interest rate falls, investment typically increases because it becomes easier to invest when financing costs drop. The issue is that productivity-enhancing

Figure 5: The Effects of Interest Rate Policy




Note: This figure plots equilibrium variables as functions of the (net) interest rate $R-1$ at the calibrated parameters. The variables shown are the optimal leverage $\theta$, the share of saving firms $\phi$, the standard deviation of project returns, the share of risk-taking firms among active firms $F(\tilde{\eta})$, TFP, output, investment, hours of work, tax rate, and the consumption-equivalent (in the calibrated economy) welfare gain.

risk-taking may also fall when financing costs drop, leading to falling hours and output. The second effect can only dominate when *productivity distribution is endogenous*, due to endogenous risk-taking. Of course, the model still features the usual capital misallocation channel, but the endogenous productivity distribution is essential.

Therefore, investment typically declines as interest rates rise, while total factor productivity (TFP) follows a humped pattern, peaking at moderate interest rates. At low interest rates, output grows slowly as higher rates encourage more productive risk-taking, boosting TFP. However, when rates exceed approximately 3%, output declines rapidly. This occurs because the search-for-yield effect dominates, discouraging productive risk-taking, while higher rates simultaneously suppress investment. As a result, the IS curve exhibits a "crawl-dive" hump, with a gradual rise followed by a sharp drop.

When assessing the policy effect on the social welfare of a target economy, we calculate the corresponding consumption equivalence measure of the calibrated economy.[19] Figure 5 shows that welfare closely tracks output. When the interest rate rises, the government borrows more (or saves less), since firms borrow less and/or save more. As explained above, initially, risk-taking behavior increases with $R$. Productivity rises and the economy expands, so the government can reduce the tax rate given a larger tax base. Eventually, the government

[19] Denote this measure as $\psi$. Given that entrepreneurs' consumption is a fraction $(1-\beta)$ of their wealth, we calculate the gain/loss of wealth of entrepreneurs ($\Omega_{base}$) and the consumption of households ($C^h_{base}$) such that the baseline economy with $(1+\psi)\Omega_{base}$ and $(1+\psi)C^h_{base}$ (and everything else stays at their calibrated level) has the same welfare measure as in the target economy.

has to raise more taxes from private agents to satisfy the government budget constraint. Risk-taking also decreases significantly when the interest rate becomes high enough. Therefore, the social welfare eventually falls, and it also shows a crawl-dive hump shape, suggesting that the optimal interest rate is around $2.7\%$.

In sum, our theory implies that excessively low interest rates can be detrimental to production and overall welfare. Although the traditional "Keynesian" mechanism remains present (especially for investment), a sharp decline in interest rates can quickly dampen risk-taking incentives, ultimately harming productivity, output, and social welfare.

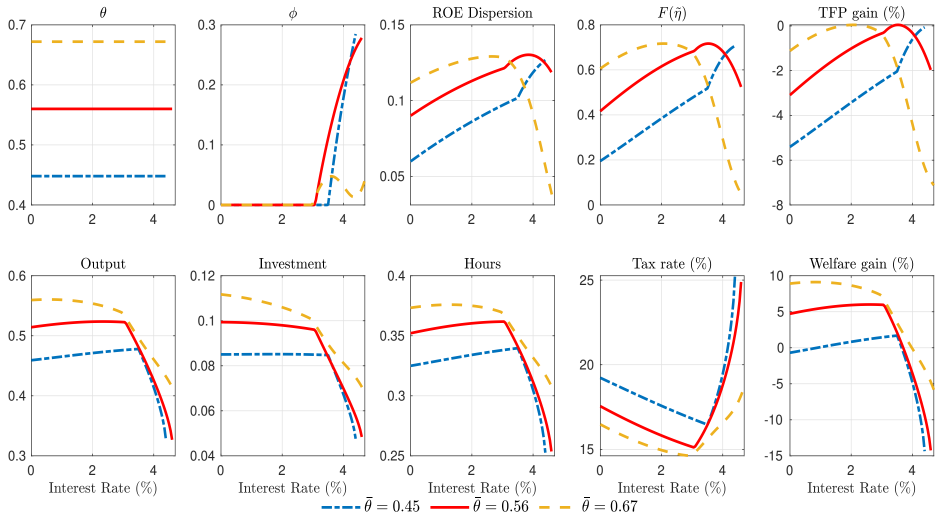
**The role of leverage limit.** Notice that financial regulation (e.g., macro-prudential policy and systematic risk oversight) can significantly affect the level of credit limit. The borrowing limit influences the effectiveness of interest rate policy. Therefore, we assess the effects of interest rate policy with different $\bar{\theta}$.

When $\bar{\theta}$ declines by $20\%$ from the calibrated value (0.56) to $\bar{\theta} = 0.45$ (the blue dashed dotted lines in Figure 6), the proportion of risk-taking entrepreneurs, $F(\tilde{\eta})$, monotonically increases with the interest rate. This finding reaffirms the earlier observation that the valuation effect dominates when leverage is low. As a result, the equilibrium proportion of savers, $\phi$, decreases for most levels of $R$ as credit demand decreases due to the fall in the external financing limit $\bar{\theta}$. Only when the interest rate is above 4.2%, $\phi$ increases above the case when $\bar{\theta} = 0.56$. Moreover, with the falling leverage, TFP decreases for almost all levels of interest rate, resulting in a downward shift in output. Thus, the IS curve's crawl-dive hump pattern is still preserved.

For comparison, the yellow dashed lines correspond to an increase of $20\%$ in $\bar{\theta}$ to $0.67$. With this adjustment, $F(\tilde{\eta})$ now shows a downward slope over a wider interest rate range, reflecting the dominance of search-for-yield when leverage is high, consistent with Proposition 2. Furthermore, as $\bar{\theta}$ increases, the risk-taking incentive (captured by $F(\tilde{\eta})$) decreases when the interest rate is high and increases when the interest rate is low compared to the baseline. This pattern is illustrated by the left shift of the $F(\tilde{\eta})$ curve. Intuitively, debt service costs are low at a low rate $R$, so increased leverage significantly impacts the leveraged Sharpe ratio, greatly increasing the opportunity cost of choosing the safe project, and thus promoting risk-taking. This observation is in line with the corollary of Proposition 2 (see Corollary B.2 in the Appendix).

At lower rates, increasing leverage directly encourages entrepreneurs to take on more risk, channeling additional resources into productive ventures, and thus generally pushing TFP upward. However, this leverage-driven boost is only part of the story: the overall trajectory of TFP is governed primarily by the function $F$, which reacts strongly to even small changes in the interest rate. In other words, even if we loosen credit, allowing leverage

Figure 6: Interest Rate Effects under Alternative Financing Constraints




Note: This figure shows the equilibrium outcomes under three possible leverage limits of $\bar{\theta}$. Variables are the same as in Figure 5.

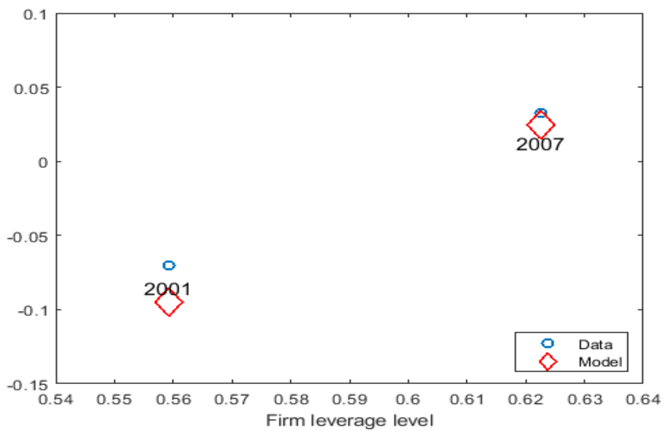
to rise, TFP can still fall at a given interest rate because $F$ itself declines. But the impact of higher leverage remains valuable. It continues to boost production, investment, hours worked, and public finances. By improving these aggregates, increased leverage also reduces the need for entrepreneurs to accumulate savings, which in turn reduces $\phi$.

**Remark:** As in the 2-period model, we have attempted to endogenize $\bar{\theta}$ by introducing, for example, a limited commitment framework where entrepreneurs can abscond with their firm's value. The fundamental hump-shaped relationship between risk-taking and the interest rate remains intact. This is expected since even if a decrease in the interest rate causes $\bar{\theta}$ to increase, Figure 6 already contains the message that the additional leverage effect shifts the risk-taking curve $F(\tilde{\eta})$ (and the TFP curve) to the left, thus preserving the overall shape of the hump. As a result, the "dive" part of the IS curve is widened if the leverage is higher.

**Cross validation.** The model's quantitative implications regarding varying leverage exhibit alignment with the data. Figure 7 shows this comparison for 2001 and 2007 with comparative static analysis. Given that we only fit the model with relevant interest rates and leverages while keeping other parameters fixed, a perfect fit should not be expected. The association of project returns, which will be discussed later, can improve the fit. However, the results in the following demonstrate that the impact of an interest rate cut significantly relies on the borrowing constraint.

We focus on the model's steady-state outcomes by using medium-term input and comparing them to medium-term output. Specifically, we fit the model using 5-year moving

Figure 7: Changes of Standard Deviations of Cross-sectional ROE




Note: The model results are calculated with leverage and interest rates from the data, keeping other parameters fixed. The left points are for 2001 (with a lower leverage ratio), and the right points are for 2007 (with a higher leverage ratio). The data counterparts are 5-year moving averages.

averages of leverage and compare the model's cross-sectional standard deviation (dispersion) of ROE to the 5-year moving average observed in the data. Additionally, as noted in Section 2, both the levels and changes in interest rates were similar in 2001 and 2007. Thus, these two years are the main natural targets.

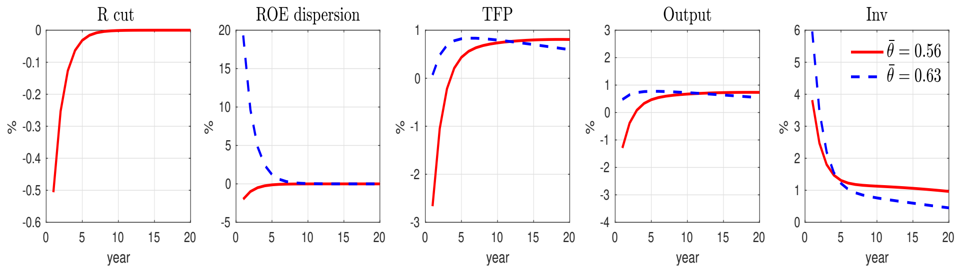
The Cleveland rate was approximately 1.7% in 2001, and between 2001 and 2002, it dropped by 142 basis points. The average leverage during the 2001-2006 period was approximately 0.559, and the cross-sectional dispersion of ROE decreased by 7% compared to 2000. Using the steady-state model with the same leverage, interest rate, and observed rate cut, the model predicts a 9.55% fall in ROE dispersion. Although the model slightly overshoots, it is directionally consistent with the data.

In 2007, the Cleveland rate was approximately 1.8%. Between 2007 and 2008, it fell by 150 basis points, making the interest rate levels and cuts in 2001 and 2007 comparable. However, the 5-year moving average of leverage in 2007 was around 0.623, higher than in 2001. Using these values in the steady-state model predicts a 2.43% increase in the ROE dispersion. In the data, the moving average of the ROE dispersion increased by 3.24% compared to 2006, showing that the prediction of the model is closely aligned with the observed data.

## 5.3 Short-run Dynamics of Interest-rate Cuts

Finally, we briefly illustrate the short-run reaction of the economy to a 50 basis point cut in the interest rate in Figure 8. Although our main focus is on the medium-run implications of

Figure 8: Impulse Responses to An Interest Rate Cut.




Note: This figure shows impulse response functions to the same interest-rate cut under two possible leverage limits.

interest rate policy, the same qualitative forces emerge immediately.

When leverage is initially low, cutting rates reduces financing costs without dramatically encouraging additional borrowing; as a result, the cross-sectional dispersion of ROE, aggregate output, and TFP all contract. In other words, firms face cheaper credit but lack the balance-sheet capacity to ramp up debt driven risk-taking. In contrast, in a high-leverage setting, the drop in rates liberates already geared firms to borrow even more, fueling risk-taking and amplifying short-run fluctuations in the cross-sectional dispersion of ROE, output, and TFP. However, in both environments, firms seize the lower cost of capital to expand investment (the last panel), as cheaper debt makes new projects more profitable.

# 6 Extension with Project Outcomes' Association

We now extend the model to introduce associations of project successes/failures without changing the key ingredients of the model. We investigate how the optimal interest rate is jointly shaped by the credit limit and the degree of project-return association, illustrating the importance of financial policy.

## 6.1 Modeling Project Outcomes' Association

The economy has infinitely many similar risky projects. Once a project is selected, each risk-taker is randomly assigned to one of these projects. The number of risky projects that achieve high returns at the aggregate level may display associations. These associations represent externalities when individual firms treat market prices (such as the wage and lending rates) and the success probability $p$ as given. However, all market participants jointly determine these factors.

A positive association suggests that when the risky project of one entrepreneur succeeds, it increases the likelihood that the project of another entrepreneur succeeds. This implies that successful projects create knowledge spillovers, where one entrepreneur's success enhances the chances of favorable outcomes for others. Similarly, if one entrepreneur's project fails, others are more likely to face failures, aligning with the concepts of adverse risk spillover and conditional value at risk (CoVaR) as discussed in Adrian and Brunnermeier (2016).

Conversely, a negative association indicates the opposite relationship: the success of one project may crowd out opportunities for another, while the failure of one project provides lessons that help reduce the likelihood of failure in others.

Notice that we follow the Statistics literature to label the interdependence as *an association instead of a correlation*, since the relationship between the outcomes of two entrepreneurs may not be linear. To characterize the association structure in the Bernoulli trials, we follow the Conway-Maxwell Binomial / Poisson (CMB / CMP) distribution approach as in Shmueli et al. (2005) and Kadane (2016). This specification (see below) modifies the baseline model with minimal departure because the individual problem stays the same as a Bernoulli trial. The specification also allows for straightforward aggregation. To the best of our knowledge, this specification is new in a macroeconomic setup.

We characterize the structure of the overall dependence through an exogenous parameter $\nu \in (0, +\infty)$. For CMB distribution:

$$Prob\{m = k\} = \frac{\binom{n}{k}^{\nu} p^k (1-p)^{n-k}}{\sum_{j=0}^{n} \binom{n}{j}^{\nu} p^j (1-p)^{n-j}} \equiv \frac{\binom{n}{k}^{\nu} p^k (1-p)^{n-k}}{D(\nu, p, n)},$$

where $m$ is the total number of successes in $n$ trials, $\binom{n}{k} \equiv \frac{n!}{k!(n-k)!}$, and we have used

$$D(\nu, p, n) \equiv \sum_{j=0}^{n} \binom{n}{j}^{\nu} p^j (1-p)^{n-j}.$$

The expression stands for the probability of $k$ realization of high returns among $n$ risky projects where the probability of realizing high return $\Pi^h$ for each project is $p$. If $\nu = 1$, it becomes the standard binomial distribution. When $0 < \nu < 1$, risky projects' returns are positively associated, and when $\nu > 1$, risky projects' returns are negatively associated.[20] To see this, denote $X_i$ as the random variable of result at $i^{th}$ position. Consider $P(X_2 = z^l | X_1 = z^l)$, the conditional probability of the second project achieving a low

[20]See Kadane (2016) for more detailed discussions.

return conditional on the low return of the first project is[21]

$$\begin{aligned}\frac{P(X_2=z^l,X_1=z^l)}{P(X_1=z^l)} &= \frac{P(X_2=z^l,X_1=z^l)}{P(X_2=z^l,X_1=z^l)+P(X_2=z^h,X_1=z^l)}\\ &= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2+\frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^{\nu}p(1-p)}\\ &= \frac{(1-p)}{(1-p)+2^{\nu-1}p}. \end{aligned} \tag{21}$$

If $\nu < 1$, we have $2^{\nu-1} < 1$, then $P(X_2 = z^l|X_1 = z^l) > 1-p$, implying a positive association. If $\nu > 1$, we have $2^{\nu-1} > 1$, then $P(X_2 = z^l|X_1 = z^l) < 1-p$, implying a negative association. Similarly, $P(X_2 = z^h|X_1 = z^h) > p$ if $\nu < 1$ and $P(X_2 = z^h|X_1 = z^h) < p$ if $\nu > 1$.

The association of risky outcomes influences the macroeconomy. Denote the overall proportion of success to be $p^s(\nu)$ when the number of projects $n$ goes to infinity. When $n \to \infty$ and $p$ is small/large enough, we know that the CMB distribution converges to the CMP distribution (see Shmueli et al. (2005) and Daly and Gaunt (2016)):

$$Prob\{m=k\} = \frac{\frac{(\lambda)^k}{(k!)^{\nu}}}{\sum_{j=0}^{j=n}\frac{(\lambda)^j}{(j!)^{\nu}}},$$

where $\lambda = n^{\nu}p$ when $p$ is small and $\lambda = n^{\nu}(1-p)$ when $p$ is large. Additionally, the overall success probability $p^s(\nu)$ satisfies the following (see Appendix C.1 for more details):

$$p^s(\nu) = \begin{cases} p^{\frac{1}{\nu}} & \text{if } p < p^* \\ (1-(1-p)^{\frac{1}{\nu}}) & \text{if } p \geq p^{**}, \end{cases} \tag{22}$$

for some $p^* > 0$ and close to zero and for some $p^{**} < 1$ and close to one, respectively. When $p$ is small, high returns are relatively more scarce. The number of high-return realizations converges to the CMP distribution. Taking into account the association, the social planner realizes that the overall probability of $\Pi^h$ is $p^{1/\nu}$. When $p$ is large, $1-p$ is small and counting the low realizations $\Pi^l$ will converge to the CMP distribution with $\lambda = n^{\nu}(1-p)$. The social planner realizes that the overall probability of $\Pi^l$ is $(1-p)^{1/\nu}$. Finding $p^*$ and $p^{**}$ is a numerical question which could be unnecessary; we obtain $p^s(\nu)$ by simulating CMB samples with large $n$.

The central implication of CMB/CMP is that when individuals choose a project, they perceive the *ex-ante probability* of achieving a high return, $\Pi^h$, as $p$, accurately reflecting their own project's success likelihood. Each entrepreneur considers their firm to be small

[21] The result uses the property of the exchangeability: $P(X_2 = z^h, X_1 = z^l) = P(X_2 = z^l, X_1 = z^h)$.

and, therefore, does not believe its return is correlated with those of others. This assumption preserves the solution to the decision problem of each entrepreneur. However, while project returns are correlated/associated at the aggregate level, individuals understand this correlation/association and treat $p^s(\nu)$ (the average probability of success) as given when making their choices, assuming only others' projects are correlated/associated. In rational expectation equilibrium, after we aggregate all individual choices, $p^s(\nu)$ is the consistent average probability of success across all projects. This modeling approach effectively captures a *pecuniary externality*: individuals overlook the impact of their own choices on the overall correlation/association of project returns. This externality influences aggregate credit demand and, consequently, the interest rate that clears the credit market.

We can write the aggregate expected return of risky projects $\Pi(\nu)$ as

$$\Pi(\nu) = p^s(\nu)\Pi^h + (1 - p^s(\nu))\Pi^l.$$

The aggregate expected return affects the social welfare as we aggregate entrepreneurs' choices. Therefore, (22) implies that $\Pi(\nu)$ becomes

$$\Pi(\nu) = \begin{cases} p^{\frac{1}{\nu}}\Pi^h + (1 - p^{\frac{1}{\nu}})\Pi^l & \text{if } p < p^* \\ (1 - (1-p)^{\frac{1}{\nu}})\Pi^h + (1-p)^{\frac{1}{\nu}}\Pi^l & \text{if } p \geq p^{**} \end{cases}. \tag{23}$$

To illustrate, first note that $\Pi = \Pi(1)$ represents no association effect. Now suppose there is a positive association, i.e., $\nu < 1$. When $p > p^{**}$, $p^s(\nu)$ takes the second branch of (22). That is, $p^s(\nu) = 1 - (1-p)^{\frac{1}{\nu}} > p$ and thus $\Pi(\nu) > \Pi$. When $p < p^*$, $\Pi(\nu) < \Pi$ holds similarly. Intuitively, when $p$ is large, high returns are relatively more abundant, and the knowledge spillover dominates the adverse risk spillover. Therefore, the overall probability of success viewed by the social planner becomes higher. That is why the overall aggregate expected return is above $\Pi(1)$. Note that one can verify that when $p$ is large, $\nu \to 0$ means $p^s(\nu) \to 1$ because of the strong positive association, and $\nu \to +\infty$ means $p^s(\nu) \to 0$ because of the strong negative association. Our calibration above suggests a relatively large $p$, leading to a social return $\Pi(\nu)$ exceeding the private return $\Pi(1)$. This finding aligns with a large literature on R&D, demonstrating that the social return rate to R&D is typically higher than the private rate of return (see Griffith (2000)).

Now, the endogenous capital efficiency unit $Z$ that takes into account the technology choices of entrepreneurs becomes:

$$Z = z^f(1-F)(1+\bar{\theta}) + \bar{z}F(1+\theta), \text{ where } \bar{z} = p^s(\nu)z^h + (1 - p^s(\nu))z^l. \tag{24}$$

One can immediately see that the measure of association, $\nu$, matters. It affects the market demand for credit/liquidity and, therefore, the tax revenue. For example, when $v < 1$ and $p$ is small, according to (23) we know that $\Pi(\nu) < \Pi(1) = \Pi$ and the tax revenue is lower. We will also show how the outcome association affects other variables and social welfare.

## 6.2 The Role of the Association of Project Outcomes

We now examine scenarios involving positive association (e.g., $\nu = 0.85$) and negative association (e.g., $\nu = 1.15$). We interpret changes in $\nu$ as exogenous shocks to the economy.

Notice that when projects exhibit positive association and $p$ is relatively high, as discussed in Section 4, we know that $\Pi(\nu) > \Pi$. The reason is that the knowledge spillover of the higher return dominates the risk spillover effect of the low return when the high return is relatively more abundant. The opposite is true if project outcomes are negatively associated. The "social" probability of success, $p^s(\nu)$ is thus a decreasing function of $\nu$, using the benchmark $p = 0.6803$.

To better understand the outcome association, we obtain the correlation of two consecutive draws and compute the correlation of the two draws as $z^l$ (see Appendix C.2 for more details). Under benchmark parameters, $\nu = 0.85$ corresponds to a correlation of $0.045$, and $\nu = 1.15$ corresponds to a correlation of $-0.045$.[22] For $\nu = 0.85$, the pairwise correlation ($0.045$) may appear small initially, but the cumulative effect (or the association of project returns) across millions of projects cannot be seen from that. The "social" probability of success thus better reflects this cumulative effect and $p^s(0.85) = 0.71$, about $4.4\%$ more than the independent probability $p$. Another way to see this is to examine the conditional probability. When $\nu = 0.85$, (21) implies that the probability of the failing second project conditioning on the failing first project is about $0.34$, about $7.2\%$ more than the independent probability of failing $1 - p$. Therefore, pairwise correlation does not serve the purpose of measuring the overall association effect. Thus, we use $\nu$, and it can be shown that the social probability $p^s(\nu)$ is close to unity when $\nu$ approaches zero.

When project outcomes become positively associated (the blue dashed-dotted lines in Figure 9), the social return assigns a higher probability $p^s(\nu) > p$ to $\Pi^h$. However, people still believe in the probability $p$ of their own project, and they understand that the aggregate capital efficiency unit is higher due to (16), putting downward pressure on the marginal product $r$ according to (18). This channel reduces the expected return gap between the risky project and the risk-free project. As a result, the risk-taking measure $\tilde{\eta}$ decreases. With a lower $r$, the gross return of the safe project $\Pi^f$ also diminishes, making it less appealing. Thus, we observe a weakly higher $\phi$ across all interest rate levels. However, TFP increases due to the positive association and the relatively large $p$, leading to higher investment, pro-

[22] The correlation further increases as $\nu$ decreases. For example, $\nu = 0.4$ produces a correlation of $0.18$, $\nu = 0.2$ produces a correlation of $0.24$, and $\nu = 0$ results in a correlation of $0.29$.

duction, and hours for low interest rate values.

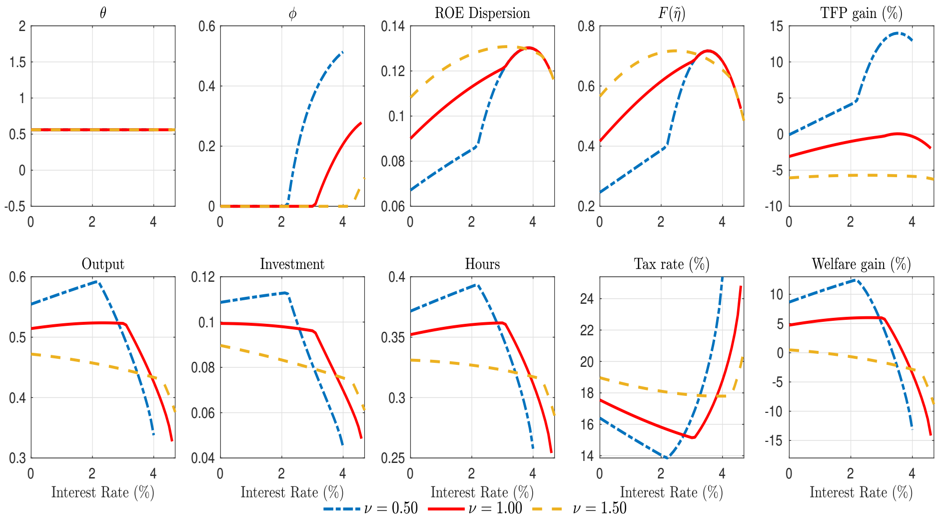
Positive associations increase social welfare compared to baseline (solid lines in Figure 9). However, as more firms save in deposits, greater government borrowing is required, leading to higher taxes and reduced social welfare at high interest rates. This results in a lower optimal interest rate than in the baseline scenario. The optimal rate must balance the welfare of entrepreneurs with projects and those saving in deposits. Note that the optimal welfare with a positive association exceeds the optimal welfare under baseline parameters due to the higher social return.

As $\nu$ increases, the projects are negatively associated with each other, and we see the opposite reaction compared to the above. In particular, more risk-taking is observed for most levels of interest rate. This excessive risk-taking comes from lower capital efficiency units $Z$, putting upward pressure on the marginal product $r$. Since individuals believe in the same probability of success $p$, the risky project becomes more attractive as $\nu$ increases. Also, notice that since the interest rate cannot exceed $\beta^{-1}$, $\phi$ eventually remains at zero across all interest rate levels, reducing savings and investment, which puts another upward pressure on $r$. As a result, the risk-taking measure $\tilde{\eta}$ increases. This effect is similar to reducing $R$ while keeping $r$ constant: for any given $R$, the values of $\Pi^h$, $\Pi^l$, and $\Pi^f$ all increase. Consequently, the search-for-yield effect dominates for a high interest rate $R$, as Proposition 2 illustrates. This dominance widens the downward-sloping part of $F(.)$ (the dashed yellow line), representing the fraction of risk-takers.

With the projects negatively associated, the above discussion suggests excessive risk-taking in the private sector due to externality. Therefore, given the hump-shaped relationship between risk-taking and the interest rate $R$, monetary policy can reduce excessive risk-taking by adjusting the interest rate either upward or downward. Because of continuity, for a small negative association, we should expect the optimal rate to rise (as the opposite of the case of a positive association) to deter excess risk-taking. However, when the negative association is more severe, the benefit of correcting externality by raising the interest rate is outweighed by the fall in investment and output because of the high financing cost. A low rate thus turns out to be optimal. Therefore, for $\nu = 1.5$, both the output-maximizing and welfare-maximizing interest rates are significantly lower than the rate that maximizes the risk-taking fraction of entrepreneurs. We find that $R = 1.026$ maximizes $F(\tilde{\eta})$ and $R = 1.032$ maximizes the standard deviation. However, even with an interest rate as low as -2%, output still does not reach its peak.

To further illustrate, note that in the cases of no association and positive association (represented by the solid and dashed-dotted lines), the optimal rate places the economy at the intersection of the flatter section and the steeper section of the $F(\tilde{\eta})$ curve. In addition, the flatter section features $\phi = 0$, and the steeper section features $\phi > 0$. In this way, risk-

Figure 9: Macroeconomy under Different $\nu$.




Note: This figure plots the equilibrium outcomes for different levels of project correlation. $\nu = 1$ is the benchmark case (no correlation) as shown in Figure 5. $\nu < 1$ means project outcomes are positively correlated; $\nu > 1$ means project outcomes are negatively correlated.

taking is highly sensitive to interest rate changes. Of course, when project outcomes are only slightly negatively associated, the policymaker should raise the interest rate. However, if the negative association is severe (e.g. $\nu = 1.5$), the flatter section of $F(.)$ spans all possible interest rates below $1/\beta$. Avoiding risk-taking altogether and focusing on the level of investment, thus, may be better for society, as the yellow dashed line shows. We should examine the implications of a continuum of $\nu$ below.

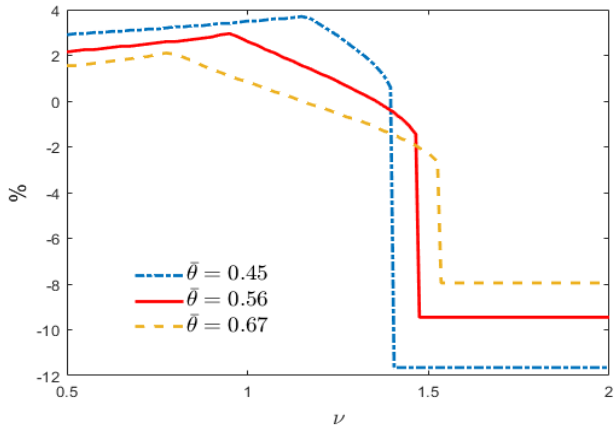
## 6.3 Further on Medium Run Optimal Interest Rate

Next, we study how the credit limit and the association of the project jointly influence the optimal rate (Figure 10).

First, with risk taking (e.g., for $\nu \leq 1.4$ and more generally for those $\nu$ levels below the levels when there is a discrete jump), the optimal interest rate falls for any degree of association when leverage is higher. As leverage increases, we have already learned that the region in which the search-for-yield effect dominates expands. This means that, for example, the $F(.)$ curve and the welfare curve in Figure 5 shift to the left, and thus the optimal interest rate will have to fall.

Second, in a risk-taking economy, the optimal interest rate is non-monotonic with respect to the degree of association in project returns, for any given credit limit $\bar{\theta}$. This non-monotonicity mirrors the hump-shaped welfare profile shown in Figure 9. The key trade-off lies between promoting risk-taking (to enhance productivity) and stimulating investment (to grow the economy following a rate cut).

Figure 10: Optimal Rate under Different Associations and Credit Limits




When risky project returns are highly positively associated, and this association strengthens further (i.e., $\nu$ *decreases* from an already low level), the aggregate social return $\Pi(\nu)$ exceeds the average private return $\Pi$ even more. In such cases, a lower interest rate is optimal to channel more funds to entrepreneurs, encouraging risk-taking. Conversely, when returns become less positively correlated—or even negatively correlated—the optimal interest rate tends to rise. However, if negative association becomes too pronounced, the planner may prefer to reduce risk-taking also via cutting $R$. In that case, as $\nu$ increases beyond a certain threshold, the planner may lower the interest rate to capitalize on the valuation effect: reducing risk-taking while still promoting investment. These opposing forces result in a peak in the optimal interest rate at an intermediate level of $\nu$.

The peak of this optimal rate depends on the credit limit (Figure 10). For instance, when $\bar{\theta} = 0.45$, the rate peaks at about 3.7%, around $\nu = 1.15$; for $\bar{\theta} = 0.67$, it peaks at approximately 2.12%, near $\nu = 0.79$. More broadly, greater leverage widens the range of association values for which the planner favors the benefits of the investment scale. In other words, the downward-sloping portion of the optimal interest rate curve becomes more pronounced as $\bar{\theta}$ increases. In this portion of the curve, a rise of $\nu$ leads to a fall of rate. When we restrict the credit limit to within 10% of its calibrated value, $\bar{\theta} = 0.56$ (which aligns with the sample data), the optimal interest rate peaks between 2.2% and 3.1%. In particular, these peaks occur close to $\nu = 1$, indicating that social returns are approximately 5% above or below the private return.

Third, an economy without risk-taking can be optimal under certain conditions. When $\nu$ is sufficiently high, that is, when the project returns are strongly negatively associated, the optimal interest rate converges to a stable low level. As the rate falls, entrepreneurs eventually stop pursuing risky projects, and investment becomes dominated by safe projects. Further rate cuts then only stimulate safe investment.

To maintain such a low rate, the government must save, financed through distortionary taxation. In this case, a local optimum is achieved without risk-taking, and the optimal rate becomes essentially independent of $\nu$. Since another local optimum exists with risk-taking, the overall optimal rate may exhibit a discrete jump, reflecting a switch between two equilibria based on the degree of return association.

As leverage increases, the range of rates that trigger the search-for-yield effect, where lower rates encourage risk-taking, also expands. Consequently, the jump to a regime without risk is induced at a higher level $\nu$. Meanwhile, in this no-risk-taking case, the optimal rate in the should rise to reflect higher credit demand.

# 7 Conclusion

In this paper, we show that the effect of interest rates on risk-taking non-financial firms could be non-monotonic overall, which depends on leverage conditions influenced by financial regulations. Specifically, when firm leverage is low (high), the firm valuation effect is strong (weak), and a rate cut discourages (encourages) risk-taking. When the leverage condition is moderate, an interest rate cut encourages risk-taking only when the interest rate level is high, and discourages risk-taking otherwise. Our analysis may shed light on why earlier studies found mixed results regarding the relationship between financial conditions and firm volatility. Therefore, with a further cut in the interest rate in a low interest rate environment, firms may not pursue risky but socially desirable projects.

Apart from leverage conditions, we also show that whether an interest rate policy incentivizes firm risk-taking depends crucially on the association of project returns among risk-takers. The analysis highlights the need for a careful mix of interest rate policy and financial regulations, such as macro-prudential policies. Future research can study whether monetary/financial policy could have endogenous growth effect through this risk-taking channel, although technological progress still largely determines long-term growth. Additionally, optimal capital taxation that alters projects' risk profiles seems crucial for risk-taking, productivity, and welfare.

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# Online Appendix

## A Data Source and Further Empirical Results

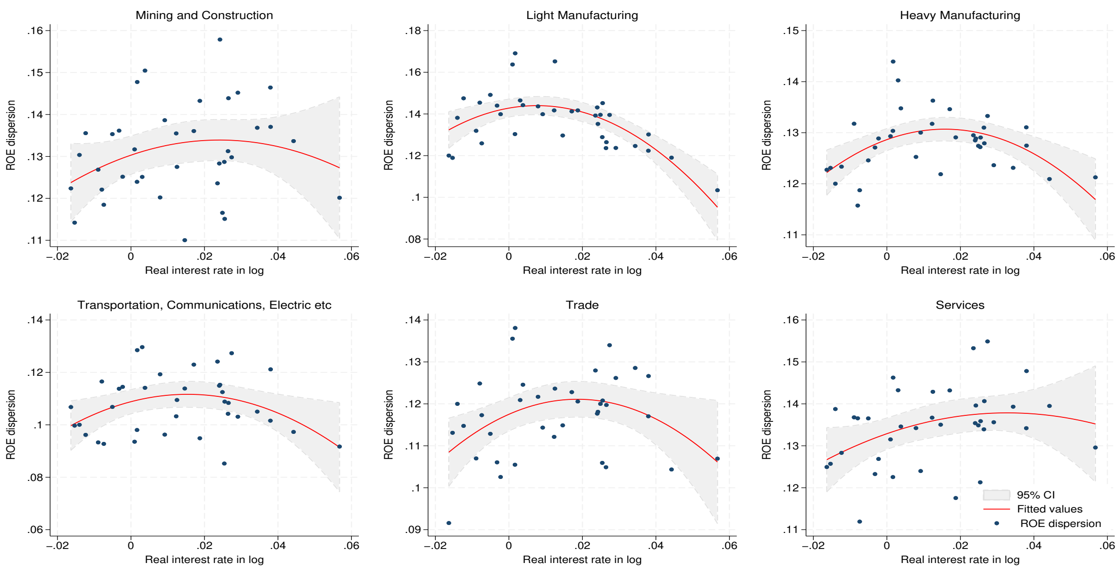
For interest rate measures, we use the federal funds rate, inflation, the Federal Reserve Bank of Cleveland's real interest rate, expected inflation, and the prime rate from FRED, maintained by the Federal Reserve Bank of St. Louis.

Financial data (mentioned in the main text) between 1984 and 2019 is sourced from COMPUSTAT. Firms in the financial sector are excluded from the sample. Return on equity (ROE) is measured as net income over total equity (total assets minus total liabilities). In Figure 11, we plot the ROE dispersion against the interest rate at industry level (SIC one-digit). We exclude the Agriculture, Forestry and Mining, Finance, and Public Administration sectors.

The ROE variation of a firm is measured by its standard deviation of a window from year $t$ to $t+7$.[23] We exclude extreme observations by dropping the firm observation if any of its illiquid ratio, size, Tobin's Q, cash holdings, ROA, ROE, etc, are below the 5th or above the 95th percentile.

For "market"-based risk-taking measures, we use daily stock returns between 1984 and 2019 from CRSP to calculate the inter-quartile range (IQR) and the standard deviation of the daily returns. CRSP database maintains the most comprehensive collection of security price, return, and volume data for the NYSE, AMEX, and NASDAQ stock markets.

Figure 11: Risk-taking and Real Interest Rate at Industry Level




Note: This figure plots the cross-sectional (SIC industry level) standard deviation of return on equity (ROE) against the real interest rate provided by the Federal Reserve Bank of Cleveland.

[23] We also examined the variation using the difference between the maximum value and the minimum value in this time window. The results are similar.

# B Derivations and Proofs

## B.1 Proof of Proposition 1

If the entrepreneur chooses a safe project, we have the value function:

$$\begin{aligned} V^f(\omega) &= \max_{s, 0 \le b \le \bar{\theta} s} \{\log(\omega - s) + \beta \log(\Pi^f(s+b) - Rb)\} \\ &= \max_{s, 0 \le \theta \le \bar{\theta}} \{\log(\omega - s) + \beta \log s + \beta \log(\Pi^f(1+\theta) - R\theta)\}. \end{aligned}$$

The optimal saving is $s = \varphi\omega$ with $\varphi = \frac{\beta}{1+\beta}$. Since $\Pi^f > R$, the credit constraint is binding: $\theta = \bar{\theta}$. If the entrepreneur chooses a risky project, the value function becomes:

$$\begin{aligned} V^r(\omega, \eta) &= \max_{s, 0 \le b \le \bar{\theta} s} \left\{ \log(\omega - s) - \eta + \beta p \log(\Pi^h(s+b) - Rb)\} + \beta(1-p) \log(\Pi^h(s+b) - Rb) \right\} \\ &= \max_{s, 0 \le \theta \le \bar{\theta}} \left\{ \log(\omega - s) + \beta \log s - \eta + \beta p \log \left( \Pi^h(1+\theta) - R\theta \right) \right. \\ &\quad \left. + \beta(1-p) \log \left( \Pi^l(1+\theta) - R\theta \right) \right\}. \end{aligned}$$

The optimal saving is $s = \varphi\omega$ again. Let $\theta^*$ denote the level of unconstrained optimal leverage that a firm would choose in the absence of financing constraints. Then, $\theta^*$ solves the first-order condition (ignoring $\theta > 0$, but we will come back to that):

$$p \frac{\Pi^h - R}{\Pi^h(1+\theta) - R\theta} + (1-p) \frac{\Pi^l - R}{\Pi^l(1+\theta) - R\theta} = 0.$$

Therefore, the cutoff interest rate level $\Pi^h \Pi^l / \hat{\Pi}$ (above which $\theta^* < 0$) is obtained by setting $\theta = 0$ above and it is straightforward to verify that $\Pi^h \Pi^l / \hat{\Pi} \in \left( \Pi^l, \Pi^h \right)$. If $R > \Pi^h \Pi^l / \hat{\Pi}$, then the interest rate is too high to justify borrowing. However, under Assumption A1 and A2, $\theta^* > 0$ because $(\Pi^l - R)(\Pi^h - R) < 0$ and $R < \Pi^f \le \Pi^h \Pi^l / \hat{\Pi}$. Then we can express $\theta^* > 0$ as

$$\theta^* \equiv -\frac{\Pi^h \Pi^l - R(p\Pi^l + (1-p)\Pi^h)}{(\Pi^l - R)(\Pi^h - R)} = -\left[ p \frac{\Pi^l}{\Pi^l - R} + (1-p) \frac{\Pi^h}{\Pi^h - R} \right]. \tag{25}$$

The optimal leverage $\theta = \min\{\theta^*, \bar{\theta}\}$. We now show that $\theta^*$ decreases in $R$. Notice that

$$\begin{aligned} \frac{\partial \theta^*}{\partial R} &= \frac{\hat{\Pi}(\Pi^l - R)(\Pi^h - R) + (-\Pi^h \Pi^l + R\hat{\Pi})(+\Pi^h + \Pi^l - 2R)}{(\Pi^l - R)^2 (\Pi^h - R)^2} \\ &= \frac{\Pi^l \Pi^h (R - \Pi) + (\Pi^h \Pi^l - \hat{\Pi} R) R}{(\Pi^l - R)^2 (\Pi^h - R)^2}. \end{aligned}$$

The numerator above is quadratic and concave in $R$. With some algebra, the maximum value of the numerator is $\frac{\Pi^h \Pi^l (\Pi^h \Pi^l - \hat{\Pi} \Pi)}{\hat{\Pi}} < 0$. To verify this, we can use the convexity feature, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h \Pi^l < \hat{\Pi}\Pi$. Thus, the numerator is negative, which implies that $\frac{\partial \theta^*}{\partial R} < 0$. The optimal leverage $\theta = \min\{\theta^*, \bar{\theta}\}$.

Finally, we determine the threshold for taking risky project $\tilde{\eta}$. By taking the difference between $V^r(\omega, \eta)$ and $V^f(\omega)$ above, we obtain (3).

## B.2 Proof of Proposition 2

Assuming $\theta^* > \bar{\theta}$, we have $\theta = \bar{\theta}$ when an entrepreneur chooses the risky project. Define $x \equiv R\frac{\bar{\theta}}{1+\bar{\theta}}$ as the debt servicing cost per unit of capital used in production, then $\tilde{\eta}$ in (3) can be rewritten as

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h - x}{\Pi^f - x}\right) + \beta(1-p)\log\left(\frac{\Pi^l - x}{\Pi^f - x}\right).$$

We then obtain

$$\frac{\partial \tilde{\eta}}{\partial x} = \beta \frac{\Pi^h \Pi^l - \Pi^f \hat{\Pi} + (\Pi^f - \Pi)x}{(\Pi^h - x)(\Pi^l - x)},$$

where we use $\hat{\Pi} \equiv (1-p)\Pi^h + p\Pi^l$ and $\Pi + \hat{\Pi} = \Pi^h + \Pi^l$. Under Assumptions A1 and A2, one can verify

$$\Pi^h > \Pi^l > R\frac{\theta^*}{1+\theta^*} > R\frac{\bar{\theta}}{1+\bar{\theta}}.$$

Then, the denominator of $\partial\tilde{\eta}/\partial x$ is positive. Additionally, since $(\Pi^f - \Pi)$ is negative, we can thus reach $\partial\tilde{\eta}/\partial x = 0$ if $x = x^s$, where

$$x^s \equiv \frac{\Pi^h \Pi^l - \Pi^f \hat{\Pi}}{\Pi - \Pi^f}.$$

And $\partial\tilde{\eta}/\partial x > 0$ if $x < x^s$, and $\partial\tilde{\eta}/\partial x < 0$ if $x > x^s$.

Notice that

$$\frac{\partial \tilde{\eta}}{\partial R} = \frac{\partial \tilde{\eta}}{\partial x}\frac{\partial x}{\partial R}.$$

Since $\partial x/\partial R > 0$, the sign of $\partial\tilde{\eta}/\partial R$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let $R^s$ be the interest rate such that $x = x^s$, i.e.,

$$R^s = R^s(\bar{\theta}) \equiv x^s(1/\bar{\theta} + 1).$$

Note that $R^s$ depends on the leverage upper bound $\bar{\theta}$ and the superscript $s$ indicates that the debt servicing cost is $x^s$ (a parameter determined by the project returns as shown above). Therefore, we define two boundary values

$$\bar{\theta}^s_H \equiv \frac{\Pi^h \Pi^l - \Pi^f \hat{\Pi}}{(\Pi - \Pi^f)\Pi^l - (\Pi^h \Pi^l - \Pi^f \hat{\Pi})}, \quad \bar{\theta}^s_L \equiv \frac{\Pi^h \Pi^l - \Pi^f \hat{\Pi}}{(\Pi - \Pi^f)\Pi^f - (\Pi^h \Pi^l - \Pi^f \hat{\Pi})}$$

determined by setting $R^s(\bar{\theta}^s_H) = \Pi^l$ and $R^s(\bar{\theta}^s_L) = \Pi^f$. Based on the two boundary values, we have

1). If $\bar{\theta} > \bar{\theta}^s_H$, then $R^s(\bar{\theta}) < \Pi^l$. For the range of interest rate under assumptions, $R > \Pi^l > R^s(\bar{\theta})$, the debt servicing cost $x$ is above $x^s$. In this case, $\partial\tilde{\eta}/\partial x < 0$ and thus $\partial\tilde{\eta}/\partial R < 0$;

2). If $\bar{\theta} < \bar{\theta}^s_L$, then $R^s(\bar{\theta}) > \Pi^f$. For the range of interest rate under assumptions, $R < \Pi^f < R^s(\bar{\theta})$, the debt servicing cost $x$ is below $x^s$. In this case, $\partial\tilde{\eta}/\partial x > 0$ and thus $\partial\tilde{\eta}/\partial R > 0$;

3). Otherwise, $\tilde{\eta}$ is hump-shaped in $R$ and $\Pi^l < R^s(\bar{\theta}) < \Pi^f$ when $\bar{\theta}^s_L < \bar{\theta} < \bar{\theta}^s_H$.

The above completes the proof. However, we can extend the reasoning by varying the leverage limit while holding the interest rate $R$ fixed. To see this, notice that

$$\frac{\partial \tilde{\eta}}{\partial \bar{\theta}} = \frac{\partial \tilde{\eta}}{\partial x}\frac{\partial x}{\partial \bar{\theta}}.$$

Since $\partial x/\partial\bar{\theta} > 0$, the sign of $\partial\tilde{\eta}/\partial\bar{\theta}$ depends on the sign of $\partial\tilde{\eta}/\partial x$. Let $\bar{\theta}^s$ be the leverage upper bound such that $x = x^s$, i.e.,$\bar{\theta}^s \equiv \bar{\theta}^s(R) = \frac{x^s}{R - x^s}$. One can show that $0 < \bar{\theta}^s < \theta^*$. Therefore, $\tilde{\eta}$ is hump-shaped in $\bar{\theta}$, and $\bar{\theta}^s$ decreases in $R$. Finally, we have the following corollary:

**Corollary.** *Suppose A1 and A2 hold and suppose the financing constraint is binding when the risky project is implemented (i.e., $\bar{\theta} < \theta^*$). We have that $\bar{\theta}^s(R) \in (0, \theta^*)$, the leverage upper bound* such that $\partial\tilde{\eta}/\partial R = 0$, *decreases in $R$. When $0 < \bar{\theta} < \bar{\theta}^s(R)$, the cutoff $\tilde{\eta}$ is increasing in $\bar{\theta}$; when $\bar{\theta}^s(R) < \bar{\theta} < \theta^*$, the cutoff $\tilde{\eta}$ is decreasing in $\bar{\theta}$.*

Unlike Proposition 2 with three scenarios based on the level of $\bar{\theta}$, there are no multiple cases here. This is because $R^s$ can be lower than the lower bound $\Pi^l$, higher than the upper bound $\Pi^f$, or in between, depending on the level of $\bar{\theta}$. However, $\bar{\theta}^s$ is always between the lower and upper bounds, i.e., $0 < \bar{\theta}^s < \theta^*$.

## B.3 Proof of Proposition 3

When the borrowing constraint is slack under the entrepreneur's risky choice, we have

$$\tilde{\eta} = \beta p \log\left(\frac{\Pi^h + (\Pi^h - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right) + \beta(1-p)\log\left(\frac{\Pi^l + (\Pi^l - R)\theta^*}{\Pi^f + (\Pi^f - R)\bar{\theta}}\right),$$

where $\theta^* = -\left[p\frac{\Pi^l}{\Pi^l - R} + (1-p)\frac{\Pi^h}{\Pi^h - R}\right]$. Then, we can express $\tilde{\eta}$ in this case as

$$\begin{aligned}\tilde{\eta} =& \beta[p\log p + (1-p)\log(1-p) + \log R + \log(\Pi^h - \Pi^l) \\ &- p\log(R - \Pi^l) - (1-p)\log(\Pi^h - R) - \log(\Pi^f + (\Pi^f - R)\bar{\theta})].\end{aligned}$$

Taking the derivative with respect to $R$, we obtain

$$\frac{\partial\tilde{\eta}}{\partial R} = \beta\left[\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}} - \frac{\theta^*}{R}\right]. \tag{26}$$

To prove the non-monotonic cutoff level for an unconstrained entrepreneur in Proposition 3 we proceed in two steps, each with a Lemma: 1) we show that $\tilde{\eta}$ is convex in the interest rate $R$ when the financing constraint is slack; 2) if $\bar{\theta}$ is large enough we will see $\tilde{\eta}$ decreases and then increases in $R$ when the credit constraint is slack.

**Lemma 5.** *For an financially unconstrained entrepreneur, the cutoff level $\tilde{\eta}$ is convex in $R$.*

*Proof.* With the expression for $\theta^*$ equation (25), we can rewrite $\frac{\partial\theta^*}{\partial R}$ as

$$\frac{\partial\theta^*}{\partial R} = \frac{\theta^*}{R\hat{\Pi} - \Pi^h\Pi^l}\left[\frac{\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2)}{R}\right].$$

From Proposition 1, we know $\frac{\partial\theta^*}{\partial R} < 0$ and $\theta^* > 0$. With these inequalities and Assumption A2 ($\Pi^h\Pi^l > \Pi^f\hat{\Pi}$), we have $\Pi^h\Pi^l - R\hat{\Pi} > \Pi^h\Pi^l - \Pi^f\hat{\Pi} > 0$, which implies $\Pi^h\Pi^l + \theta^*(\Pi^h\Pi^l - R^2) > 0$. Then, it follows that

$$-\frac{\partial\frac{\theta^*}{R}}{\partial R} = \frac{\theta^*\left[\Pi^l\Pi^h + \theta^*(\Pi^h\Pi^l - R^2)\right] + \theta^*(\Pi^h\Pi^l - R\hat{\Pi})}{R^2(\Pi^h\Pi^l - R\hat{\Pi})} > 0 \text{ and } \frac{\partial^2\tilde{\eta}}{\partial R^2} = \frac{\partial\frac{\bar{\theta}}{\Pi^f + (\Pi^f - R)\bar{\theta}}}{\partial R} - \frac{\partial\frac{\theta^*}{R}}{\partial R} > 0.$$

Thus, the cutoff level $\tilde{\eta}$ is convex in the interest rate $R$ for an unconstrained entrepreneur. □

**Lemma 6.** *At the interest rate $\bar{R}(\bar{\theta})$, where the borrowing constraint shifts from binding to non-binding, $\tilde{\eta}$ decreases in the interest rate $R$, i.e., $\frac{\partial\tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$.*

*Proof.* We use the non-monotonic result from the binding case in Proposition 2. Since when the interest rate is higher than $R^s(\bar{\theta})$, the rate at which $\tilde{\eta}$ reaches its maximum under a binding borrowing constraint, we have $\frac{\partial \tilde{\eta}}{\partial R} < 0$. Therefore, if $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$, it follows that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$. We need to show $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$. It turns out more convenient to compare the debt servicing cost, $R\frac{\bar{\theta}}{1+\bar{\theta}}$ in both cases. From the proof of Proposition 2 in Appendix B.2

$$R^s(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} = x^s = \frac{\Pi^h\Pi^l - \Pi^f\hat{\Pi}}{\Pi - \Pi^f} = g(\Pi^f),$$

where $g(s) \equiv \frac{\Pi^h\Pi^l - s\hat{\Pi}}{\Pi - s}$. Meanwhile

$$\bar{R}(\bar{\theta})\frac{\bar{\theta}}{1+\bar{\theta}} = \frac{-p\Pi^l(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)\Pi^h(\Pi^l - \bar{R}(\bar{\theta}))}{-p(\Pi^h - \bar{R}(\bar{\theta})) - (1-p)(\Pi^l - \bar{R}(\bar{\theta}))} = \frac{\Pi^h\Pi^l - \bar{R}(\bar{\theta})\hat{\Pi}}{\Pi - \bar{R}(\bar{\theta})} = g(\bar{R}(\bar{\theta})).$$

The rest of the proof shows $g(.)$ is decreasing and $\bar{R}(\bar{\theta}) < \Pi^f$, implying $\bar{R}(\bar{\theta}) > R^s(\bar{\theta})$.

By convexity, $\frac{p}{\Pi^h} + \frac{1-p}{\Pi^l} > \frac{1}{p\Pi^h + (1-p)\Pi^l} = \frac{1}{\Pi}$ which implies $\Pi^h\Pi^l < \hat{\Pi}\Pi$. Then, $g(s)$ decreases in $s$, since the first order condition of $g(s)$ with respect to $s$ is $\frac{-\hat{\Pi}(\Pi - s) + \Pi^h\Pi^l - s\hat{\Pi}}{(\Pi - s)^2} = \frac{\Pi^h\Pi^l - \hat{\Pi}\Pi}{(\Pi - s)^2} < 0$.

Notice that if the entrepreneur is unconstrained, then it must be that $\bar{R}(\bar{\theta}) < \Pi^f$. To see this claim, first observe that $\theta^*$ is a decreasing function of $R$. By the definition of $\bar{R}(\bar{\theta})$, we have $\theta^*(R) = \bar{\theta}$ when $R = \bar{R}(\bar{\theta})$. By the definition of $\theta^*_{min}$, we have $\theta^*(R) = \theta^*_{min}$ when $R = \Pi^f$. Second, if $\bar{R}(\bar{\theta}) \geq \Pi^f$ that is opposite of the claim, then $\bar{\theta} = \theta^*(\bar{R}(\bar{\theta})) \leq \theta^*(\Pi^f) = \theta^*_{min}$ as $\theta^*(R)$ decreases in $R$, which contradicts with our assumption that $\bar{\theta} > \theta^*_{min}$, and thus we prove the claim. $\square$

Finally, using Lemma 5 and Lemma 6, we can finish the proof of Proposition 3. To show $\tilde{\eta}$ changes from decreasing to increasing in $R$, we need to find further an interest rate level such that $\frac{\partial \tilde{\eta}}{\partial R} > 0$.

In the proof of (B.2), the derivative of $\tilde{\eta}$ with respect to the debt servicing cost $x$ is zero when $x = x^s$; we see that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} = 0$ when $\bar{\theta}$ takes the value of $\bar{\theta}^s_L$, since the debt servicing cost is indeed $x = x^s$ when $\bar{\theta} = \bar{\theta}^s_L$ and $R = \Pi^f$. It is straightforward to verify that $\frac{\partial^2 \tilde{\eta}}{\partial R \partial \bar{\theta}} > 0$ according to (26), which means that $\frac{\partial \tilde{\eta}}{\partial R}$ increases in $\bar{\theta}$. Thus $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$ when $\bar{\theta} > \theta^*_{min}$ since we already know that $\theta^*_{min} = \bar{\theta}^s_L$. Then by Lemma 5, Lemma 6, and the mean value theorem, when $\bar{\theta} > \theta^*_{min}$ there exists an interest rate $R^u(\bar{\theta})$ where $\bar{R}(\bar{\theta}) < R^u(\bar{\theta}) < \Pi^f$ such that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=R^u(\bar{\theta})} = 0$, since we already know that $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\bar{R}(\bar{\theta})} < 0$ (from Lemma 6), $\frac{\partial \tilde{\eta}}{\partial R}|_{R=\Pi^f} > 0$, and $\frac{\partial \frac{\partial \tilde{\eta}}{\partial R}}{\partial R} > 0$ (from Lemma 5).

Thus, the implied risk-taking threshold for unconstrained entrepreneurs first decreases with $R$ and then increases with $R$ later. That is, $\frac{\partial \tilde{\eta}}{\partial R} < 0$ if $\bar{R}(\bar{\theta}) \leq R \leq R^u(\bar{\theta})$ and $\frac{\partial \tilde{\eta}}{\partial R} > 0$ if $R^u(\bar{\theta}) < R(\bar{\theta}) \leq \Pi^f$.

## B.4 Proof of Proposition 4

Guess that $V^r(\omega,\eta) = \log(\omega)+v^r-\eta$, $V^f(\omega) = \log(\omega)+v^f-\eta^f$, $V^d(\omega) = \log(\omega)+v^d$. Plugging these guessed forms into the entrepreneur's Bellman equation, we have the following

$$\begin{aligned} v^r =& \mathcal{B} + \beta p \log\left(\Pi^h(1+\theta) - R\theta\right) + \beta(1-p)\log\left(\Pi^l(1+\theta) - R\theta\right) \\ &+ \beta \max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\}; \end{aligned} \tag{27}$$

$$v^f = \mathcal{B} + \beta\left[\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta})\right] + \beta\max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\}; \tag{28}$$

$$v^d = \mathcal{B} + \beta\log(R^d) + \beta\max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\}, \tag{29}$$

where $\mathcal{B} \equiv (1-\beta)\log(1-\beta) + \beta\log\beta$.

One can solve for the three unknowns $v^r$, $v^f$, and $v^d$ from the above three equations, and verify the guess. Their exact solutions are, however, not essential for our purposes. The choice (between saving in deposits or being an active entrepreneur) depends on the maximal value of the two, $\mathbb{E}\left[\max\{v^r-\eta, v^f-\eta^f\}\right]$. As for the share of firms saving in deposits, we have

$$\phi = \begin{cases} 0 & \text{if } v^d < \mathbb{E}\left[\max\{v^r-\eta, v^f-\eta^f\}\right] \\ (0,1) & \text{if } v^d = \mathbb{E}\left[\max\{v^r-\eta, v^f-\eta^f\}\right] \\ 1 & \text{if } v^d > \mathbb{E}\left[\max\{v^r-\eta, v^f-\eta^f\}\right] \end{cases}.$$

Using the expressions for $v^r$, $v^f$, and $v^d$ , we have $\phi$ as follows:

$$\phi = \begin{cases} 0 & \text{if } l^d < \mathbb{E}\left[\max\{l^r-\beta^{-1}\eta, l^f\}\right] \\ (0,1) & \text{if } l^d = \mathbb{E}\left[\max\{l^r-\beta^{-1}\eta, l^f\}\right] \\ 1 & \text{if } l^d > \mathbb{E}\left[\max\{l^r-\beta^{-1}\eta, l^f\}\right] \end{cases}.$$

Notice that when $\phi = 0$, the active projects strictly dominate; when $\phi = 1$, the risk-free deposits strictly dominate; when $0 < \phi < 1$, an entrepreneur is indifferent between saving via the risk-free deposits and becoming an active entrepreneur implementing projects.

The choice of $\theta$ follows the same reasoning as in the two-period model, for which the proof has already been provided. For the cutoff, there exists a level below which entrepreneurs choose the risky project. Similar to the proof before, we have $\tilde{\eta} = v^r - (v^f - \eta^f)$ as:

$$\tilde{\eta} = \beta p\log\left(\Pi^h + (\Pi^h - R)\theta\right) + \beta(1-p)\log\left(\Pi^l + (\Pi^l - R)\theta\right) - \beta(\log(\Pi^f(1+\bar{\theta}) - R\bar{\theta}) - \eta^f/\beta),$$

where we have used the value functions and $\theta = \min\{\theta^*, \bar{\theta}\}$ (with $\theta^*$ being the unconstrained solution as given in (25)). If the right-hand side is below $\underline{\eta}$, we set $\tilde{\eta} = \underline{\eta}$. Therefore, entrepreneurs with $\eta \leq \max\{\underline{\eta}, \tilde{\eta}\}$ choose the risky project.

# C Omitted Details in the Macroeconomic Model

## C.1 The average return in the society

Let $m$ denote the number of realizations of $\Pi^h$ in the economy, and let the proportion of success be $p^s(\nu) = \mathbb{E}\left[\frac{m}{n}\right]$ as $n$ goes to infinity, which is the share of realized return $\Pi^h$.

It has been shown in Shmueli et al. (2005) that for the Conway-Maxwell Poisson distribution,

$\mathbb{E}[m]$ converges to $\lambda^{1/\nu} - \frac{\nu-1}{2\nu}$ when $n$ goes to infinity and $p$ is small i.e. $p < p^*$. This suggests that $p^s(\nu)$ converges to $p^{1/\nu}$ when $n$ goes to infinity after substituting $\lambda = n^\nu p$ as illustrated in Daly and Gaunt (2016). Here, $p^{1/\nu}$ represents the probability of high realizations assessed by the social planner, which differs from the individual's assessment $p$ whenever there is an association in the projects (i.e., $\nu \neq 1$). As a result, the average return from the society's perspective becomes $p^{\frac{1}{\nu}}\Pi^h + (1-p^{\frac{1}{\nu}})\Pi^h$. The case where $p$ is large, i.e. $p > p^{**}$, follows a similar reasoning but one should replace $p$ by $1-p$ in the argument above.

## C.2 Pairwise correlation for association

Let $\mathbb{I}_{x_1=l}$ be the indicator function for $x_1 = l$ (the first trial fails) and $\mathbb{I}_{x_2=l}$ for $x_2 = l$ (the second trial fails), respectively. The covariance of the indicator functions is

$$\begin{aligned} Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l}) &= \mathbb{E}[\mathbb{I}_{x_1=l}\mathbb{I}_{x_2=l}] - \mathbb{E}[\mathbb{I}_{x_1=l}]\mathbb{E}[\mathbb{I}_{x_2=h}] \\ &= Pr(x_1 = l, x_2 = l) - Pr(x_1 = l)P(x_2 = l) \\ &= Pr(x_1 = h)Pr(x_1 = l)\left[Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)\right], \end{aligned}$$

With $\sigma(\mathbb{I}_{x_1=l}) = \sqrt{Pr(x_1 = h)Pr(x_1 = l)}$ and $\sigma(\mathbb{I}_{x_2=l}) = \sqrt{Pr(x_2 = h)Pr(x_2 = l)}$ being the standard deviation of these two indicator functions, then the correlation coefficient $\rho_\nu$ is

$$\begin{aligned} \rho_\nu &= \frac{Cov(\mathbb{I}_{x_1=l}, \mathbb{I}_{x_2=l})}{\sigma(\mathbb{I}_{x_1=l})\sigma(\mathbb{I}_{x_2=l})} \\ &= \frac{\sigma(\mathbb{I}_{x_1=l})}{\sigma(\mathbb{I}_{x_2=l})}\left(Pr(x_2 = l|x_1 = l) - Pr(x_2 = l|x_1 = h)\right) \\ &= \frac{Pr(x_2 = l, x_1 = l)}{Pr(x_1 = l)} - \frac{Pr(x_2 = l, x_1 = h)}{Pr(x_1 = h)} \\ &= \frac{\frac{1}{D(\nu,p,2)}(1-p)^2}{\frac{1}{D(\nu,p,2)}(1-p)^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} - \frac{\frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)}{\frac{1}{D(\nu,p,2)}p^2 + \frac{1}{2}\frac{1}{D(\nu,p,2)}\binom{2}{1}^\nu p(1-p)} \\ &= \frac{1-p}{1-p+2^{\nu-1}p} - \frac{2^{\nu-1}(1-p)}{p+2^{\nu-1}(1-p)}, \end{aligned}$$

where the second line uses the property of exchangeability (which implies $\sigma(\mathbb{I}_{x_1=l}) = \sigma(\mathbb{I}_{x_2=l})$) and the fifth line uses the definition of the CMB distribution and the property of exchangeability.

## C.3 Social Welfare

We first derive entrepreneurs' welfare, which consists of the contribution of three types: those who take risks, those who implement the risk-free project, and those who save in safe deposits:

$$V_e(\omega) = (1-\phi)\int^{\tilde{\eta}} V^r(\omega, \eta)dF(\eta) + (1-\phi)\int_{\tilde{\eta}} V^f(\omega, \eta^f)dF(\eta) + \phi\int_{\tilde{\eta}} V^d(\omega)dF(\eta). \quad (30)$$

We have two scenarios below, each of which determines the value $\max\{v^d, \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]\}$ in (27) - (29) differently. For exposition simplicity, $\tilde{\eta}$ is assumed to be above $\underline{\eta}$, which is always verified in our numerical exercises. In addition, the lower bound $\underline{\eta}$ is set to zero since it simplifies the derivation and does not affect welfare comparison.

Scenario 1: When $\phi = 0$, no one chooses the safe deposit option. That is, $v^d < \mathbb{E}\left[\max\{v^r - \eta', v^f - \eta^f\}\right]$.

Then, we can explicitly express $\mathbb{E}\max\{v^r-\eta', v^f-\eta^f, v^d\}$ as follows

$$
\begin{aligned}
\max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\} &= \mathbb{E}\max\{v^r-\eta', v^f-\eta^f\} \\
&= (1-F)\left(v^f-\eta^f\right)+Fv^r-\int^{\tilde{\eta}}\eta dF(\eta) \\
&= v^f-\eta^f+\int^{\tilde{\eta}}F(\eta)d\eta,
\end{aligned}
$$

since $\tilde{\eta}=v^r-v^f+\eta^f$. This means that we can explicitly solve the values in (27) - (29). It turns out that we only need to express

$$
\begin{aligned}
v^f &= \mathcal{B}+\beta\left[\log(\Pi^f(1+\bar{\theta})-R\bar{\theta})\right]+\beta\left[v^f-\eta^f+\int^{\tilde{\eta}}F(\eta)d\eta\right] \\
&= \frac{\mathcal{B}}{1-\beta}+\frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta})-R\bar{\theta})\right]+\frac{\beta}{1-\beta}[\int^{\tilde{\eta}}F(\eta)d\eta-\eta^f],
\end{aligned}
$$

where $\mathcal{B}$ is defined above in the proof of Proposition 4, the second equality results from rearranging $v^f$ on the right-hand side of the first equation and dividing both sides by 1-$\beta$. Therefore, we can rewrite (30) as

$$
\begin{aligned}
V_e(\omega) &= \int^{\tilde{\eta}}(V^r(\omega,\eta))dF(\eta)+\int_{\tilde{\eta}}V^f(\omega,\eta^f) \\
&= \log(\omega)+v^rF(\tilde{\eta})-\int^{\tilde{\eta}}\eta dF(\eta)+[1-F(\tilde{\eta})](v^f-\eta^f) \\
&= \log(\omega)+\tilde{\eta}F(\tilde{\eta})+(v^f-\eta^f)-F(\tilde{\eta})\tilde{\eta}+\int^{\tilde{\eta}}F(\eta)d\eta \\
&= \log(\omega)+\frac{\mathcal{B}}{1-\beta}+\frac{\beta}{1-\beta}\left[\log(\Pi^f(1+\bar{\theta})-R\bar{\theta})\right]+\frac{1}{1-\beta}\left[\int^{\tilde{\eta}}F(\eta)d\eta-\eta^f\right],
\end{aligned}
$$

where the third equality uses $v^r=v^f-\eta^f+\tilde{\eta}$ and the last equality substitutes the expression for $v^f$ derived above.

Scenario 2: When $0<\phi<1$, entrepreneurs are indifferent between investing in risk-free deposits and taking on projects. Then, we can explicitly express $\max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\}$ as

$$
\max\{v^d, \mathbb{E}\left[\max\{v^r-\eta', v^f-\eta^f\}\right]\}=v^d.
$$

Using the result above and following a similar procedure as in Scenario 1, we can express $v^d$ as

$$
v^d=\mathcal{B}+\beta\log R^d+\beta v^d=\frac{\mathcal{B}}{1-\beta}+\frac{\beta\log(R^d)}{1-\beta},
$$

where the second equality results from rearranging $v^d$. Therefore, we can rewrite (30) as

$$
V_e(\omega)=V^d(\omega)=\log(\omega)+v^d=\log(\omega)+\frac{\mathcal{B}}{1-\beta}+\frac{\beta\log(R^d)}{1-\beta},
$$

where we used the expression for $v^d$ in the last equality.

**Remark:** In a steady-state economy, project choice is independent of the wealth level. The distribution of the wealth level $\omega$ is always preserved. Denote $V$ as the value function of the entrepreneurs. Thus, $\omega = \Omega$ and $V = V_e(\Omega)$. We, therefore, conclude that entrepreneurs' welfare can be represented by $\log(\Omega) + \tilde{V} + constants$ as shown in the main text, since the results in Scenarios 1 and 2 above imply the constant term is $\mathcal{B}/(1-\beta)$ and

$$\tilde{V} = (1-\beta)^{-1}\left(\beta \max\{l^d, l^f + \beta^{-1}\int^{\tilde{\eta}} F(\eta)d\eta\}\right).$$

Note that both $\Omega$ and $\tilde{V}$ are endogenous to policy variations.

# D Endogenous leverage

This section introduces the concept of endogenous leverage, where the maximum leverage an entrepreneur can sustain depends on the risk of default. Here, we allow entrepreneurs engaged in risky projects to have the option to default upon realizing a return of $\Pi^l$. Entrepreneurs with safe projects may also choose to default under certain conditions.

Let $\bar{\theta}^r$ and $\bar{\theta}^f$ represent the maximum allowable leverage for entrepreneurs undertaking risky and safe projects, respectively. We demonstrate how these leverage bounds, $\bar{\theta}^r$ and $\bar{\theta}^f$, can be determined endogenously by incorporating the potential of default. In the event of default, we assume that entrepreneurs retain a fraction $\varphi^r < 1$ of total revenue from risky projects and $\varphi^f < 1$ from safe projects, pocketing these amounts without repaying any debt. However, after default, entrepreneurs face a probability $\xi$ of permanent exclusion from entrepreneurial activities, losing access to both safe and risky project opportunities.

The value of risk-taking entrepreneurs in the low state $\Pi^l$, conditional on not defaulting, is denoted as $V^{rl}_{expost}$, which is given by:

$$\begin{aligned} V^{rl}_{expost} &= \log(\Pi^l(s+b) - Rb) + \mathbb{E}\max\{v^r - \eta', v^f\} \\ &= \log(1+\bar{\theta}^r) + \log\left(\Pi^l - \frac{\bar{\theta}^r R}{1+\bar{\theta}^r}\right) + \log(s) + \mathbb{E}\max\{v^r - \eta', v^f\}. \end{aligned}$$

If they choose to default, the value becomes:

$$\begin{aligned} V^D(\Pi^l(s+b) - Rb)) &= \xi\left[\log(\varphi^r \Pi^l(s+b)) + \left(\log(1-\beta) + \frac{\beta}{1-\beta}(\log\beta + \log R)\right)\right] \\ &\quad + (1-\xi)\left[\log(\varphi^r \Pi^l(s+b)) + \mathbb{E}\max\{v^r - \eta', v^f\}\right] \\ &= \log(1+\bar{\theta}^r) + \log(\Pi^l) + \log(\varphi^r) + \log(s) \\ &\quad + \xi\left(\log(1-\beta) + \frac{\beta}{1-\beta}(\log\beta + \log R)\right) + (1-\xi)\mathbb{E}\max\{v^r - \eta', v^f\}, \end{aligned}$$

Therefore, the non-default condition becomes:

$$V^{rl}_{expost} \geq V^D\left(\Pi^l(s+b) - Rb)\right),$$

or

$$\xi\left[\frac{\beta}{1-\beta}\log(\Pi^l(1+\bar{\theta}^r)-R\bar{\theta}^r)+\frac{1}{1-\beta}\int_{\underline{\eta}}^{\tilde{\eta}}F(\eta)d\eta-\frac{\beta}{1-\beta}\log R\right]$$
$$\geq \underbrace{\log(\Pi^l)+\log(\varphi^r)-\log(\Pi^l-\frac{\bar{\theta}^r R}{1+\bar{\theta}^r})}_{\text{immediate gain}}.$$

In this inequality, the left-hand side represents the expected future benefits from not defaulting, weighted by the probability of losing entrepreneurial status $\xi$, while the right-hand side captures the immediate gain from defaulting under low returns $\Pi^l$. This condition ensures that entrepreneurs prefer not to default as long as the expected benefits of maintaining their status exceed the immediate gain from default.

Similarly, the non-default condition for entrepreneurs with safe projects becomes:

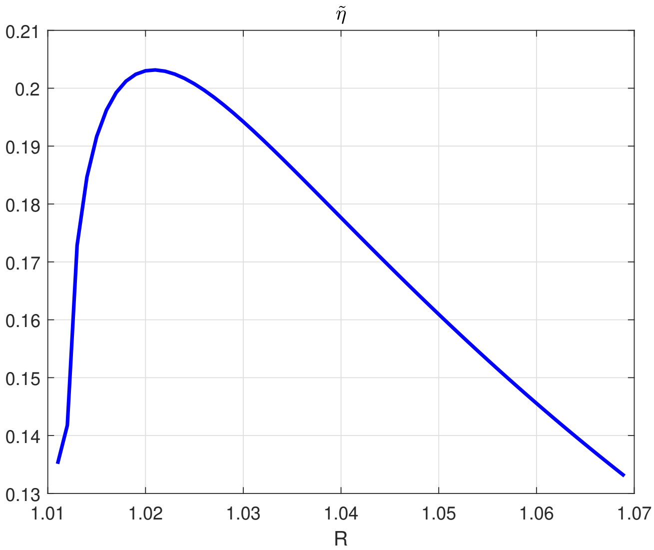
$$V^f_{expost} \quad \geq \quad V^D(\Pi^f(s+b)-Rb))$$

or

$$\xi\left[\frac{\beta}{1-\beta}\log(\Pi^f(1+\bar{\theta}^f)-R\bar{\theta}^f)+\frac{1}{1-\beta}\int_{\underline{\eta}}^{\tilde{\eta}}F(\eta)d\eta-\frac{\beta}{1-\beta}\log R\right]$$
$$\geq \underbrace{\log(\Pi^f)+\log(\varphi^f)-\log(\Pi^f-\frac{\bar{\theta}^f R}{1+\bar{\theta}^f})}_{\text{immediate gain}}.$$

The endogenous leverage upper bounds $\bar{\theta}^r$ and $\bar{\theta}^f$ can be determined by setting the non-default conditions derived above to hold as equalities. For numerical illustration, Figure 12 shows how $\tilde{\eta}$ varies with the interest rate $R$, and a similar non-monotonic relationship remains.

Figure 12: Effect of $R$ on $\tilde{\eta}$ for endogenous $\bar{\theta}$




Note: A numerical illustration with $\beta = 0.96, \Pi^h = 1.3, \Pi^l = 0.95, \Pi^f = 1.07, p = 0.6, \xi = 0.12, \varphi^r = 0.2,$ $\varphi^f = 0.5$, and $F(.)$ being log-normal distribution with $\mu = 0, \sigma = 1.2$.