Optimal Inflation Target in an Economy with Menu Costs and an Occasionally Binding Zero Lower Bound*

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JOB MARKET PAPER

Click here for updates

January 7, 2015

Abstract

This paper studies the optimal inflation target in a menu cost model with an occasionally binding zero lower bound on interest rates. I find that the optimal inflation target is 5%, much larger than the rates currently targeted by the Fed and the ECB, and also larger than in other time- and state-dependent pricing models. In my model resource misallocation does not increase greatly with inflation, unlike in previous sticky price models. The critical additions for this result are firms' idiosyncratic shocks. Higher inflation does indeed increase the gap between old and new prices, but it also increases firms' responsiveness to idiosyncratic shocks. These two effects are balanced using idiosyncratic shocks consistent with micro-price statistics. By increasing the inflation target, policymakers can reduce the probability of hitting the zero lower bound, avoiding costly recessionary episodes.

JEL: E3, E5, E6.

Keywords: menu costs, (S,s) policies, monetary policy, inflation target.

*I'm deeply indebted to Virgiliu Midrigan for his constant guidance and support at each stage of this project. I also thank Ricardo Lagos, Mark Gertler, John Leahy, Jaroslav Borovicka, Tom Sargent, Tim Cogley, Gaston Navarro, Isaac Baley, Axelle Ferrere, Anmol Bhandari, Joseba Martinez, Diego Anzoategui, Simon Mongey, David Backus and Venky Venkateswaran.

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1 Introduction

During the last recession, central banks around the world quickly reduced their short-term interest rates to zero in order to stimulate economic activity. Had they been able to, they would have decreased rates even further, but since nominal interest rates cannot be negative, they could not.[1] The zero lower bound (ZLB) constraint on nominal interest rates motivated a number of economists to argue in favor of increasing the inflation target. According to these commentators, a higher inflation target would increase average nominal interest rates, thus giving central bankers more room to react to adverse shocks.

The goal of my paper is to carefully quantify the benefits and costs of a higher inflation target. While the benefits of a higher inflation target are well-understood, much less is known about the costs of permanently higher rates of inflation. In existing sticky price models, the major cost of inflation is that it induces dispersion in relative prices across otherwise identical producers. Such dispersion implies an efficiency loss due to dispersion in the producer's marginal products. I build a menu cost model that is capable of reproducing the salient features of the micro-data on prices; this is critical to determine the optimal inflation target. In particular, I extend the Gertler and Leahy (2008) and Midrigan (2011) menu cost models with idiosyncratic shocks to a standard New Keynesian setting amenable to policy analysis. I incorporate a Taylor rule for monetary policy, occasionally subject to a zero lower bound, as well as a rich source of aggregate dynamics arising from several aggregate shocks.

My main result is that the optimal inflation target in this environment is about 5%, much greater than in other leading time- and state-dependent pricing models studied in the literature – Calvo (1983), Taylor (1980), and Dotsey, King, and Wolman (1999). Thus, my results sharply contrast with those of Coibion, Gorodnichenko, and Wieland (2012), who find a robust result of an optimal inflation target between 1-2 % by quantifying the optimal inflation target with an occasionally binding zero lower bound in several time- and state-dependent pricing models.[2]

The reason the optimal inflation target is so much higher in my model is that the misallocation costs do not greatly increase with a higher rate of inflation. Intuitively, a higher inflation target increases the gap between recently-adjusted prices and those of the producers that have not adjusted in a while. This effect, which is present in my model and all other leading sticky price models, implies that price dispersion greatly increases with a higher inflation target. However, my model with menu costs and firm-level shocks contains an offsetting force. Because firms optimally choose the timing of their price changes, a higher inflation target forces some firms that would otherwise not react to an idiosyncratic shock to do so. A higher inflation target thus forces firms to react more forcefully to large idiosyncratic shocks, thereby reducing the dispersion in the firms' marginal

[1] See Ball (2013), Blanchard, DellAriccia, and Mauro (2010), and Williams (2009) for a revival of this old proposal by Summers (1991)

[2] See also Billi (2011), Walsh (2009), Williams (2009) and Schmitt-Grohé and Uribe (2010)

productivity of labor. I find that this second effect is very strong quantitatively in versions of the model consistent with the micro-data, offseting the first effect almost entirely. Therefore, a higher inflation target in my model does not generate as much inefficiency as it does in economies without idiosyncratic shocks.

To understand the difference between my results and the previous results, I characterize output gap in the steady state for all nominal rigidity models. I characterize TFP losses due to price dispersion and markups as a function of observable micro-price statistics.[3] I show that price dispersion depends on five micro-price statistics: mean and variance of size of price changes as well as mean, variance, and skewness of time between price changes. Additionally, I can use these statistics to analyze the important dimensions of different pricing models to determine price dispersion. Moreover, I characterize aggregate markups as a function of time between price changes and inflation target.

Consider next the benefits of having a higher inflation target. A higher inflation target decreases the business cycle volatility of the output gap for two reasons. First, it decreases the probability of hitting the zero lower bound. Second, in menu cost models the inflation target determines the extent to which the aggregate price level reacts to aggregate shocks in periods where the ZLB is binding. In models with price rigidities, an initial drop in the output gap decreases inflation; if the zero lower bound is binding, this initial drop in inflation increases real interest rates, even further depressing the output gap.

The important property in my menu cost model is that the deflationary spiral decreases with the inflation target. At low levels of inflation, periods of ZLB binding are associated with deflations, triggering downward price adjustments and a higher deflationary spiral. But higher inflation does not trigger downward price adjustment in periods of binding ZLB. At 0% inflation target the deflationary spiral is larger in the menu cost model than in the Calvo model, but at 4 % they are the same.

I depart from the standard New Keynesian framework by using menu costs with fat-tailed idiosyncratic shocks to model price rigidities, as in Gertler and Leahy (2008) and Midrigan (2011). I also depart from standard menu cost models in two ways.[4] In my model, business cycles of output and inflation are generated by a rich set of aggregate shocks given by productivity, government expenditure, monetary and risk premium shocks. Additionally, I model monetary policy with a Taylor rule subject to an occasionally binding zero lower bound. Consenquently, nominal interest rates react to inflation and ouput fluctuations, stabilizing business cycle fluctuations generated by the structural shocks.

Without any additional assumptions, this model implies empirical counterfactual large inflation

[3]See Alvarez, Gonzalez-Rozada, Neumeyer, and Beraja (2011) and Burstein and Hellwig (2008) for an analytical and numerical characterization of price dispersion in a menu cost model. For empirical studies at different inflation targets see Gagnon (2009) and Wulfsberg (2010)

[4]See Golosov and Lucas (2007), Midrigan and Kehoe (2011), Nakamura and Steinsson (2010) and Vavra (2014).

volatility.[5] To yield a lower volatility of inflation, I use GHH preferences together with complementarities. Additionally, these modeling choices reduce the deflationary spiral due to the ZLB, and therefore lower the cost of the ZLB constraint closer to the empirical evidence during the 2007-2009 recession. Finally, I assume Epstein-Zin preferences to generate a non-trivial cost of business cycles.

These features of the model deliver three main quantitative results. First, macroeconomic dynamics in the menu cost model are similar to Calvo when the inflation target is near zero and there is no ZLB constraint on nominal interest rates. This result confirms Gertler and Leahy (2008) and Midrigan (2011) in a model with richer macroeconomic dynamics. The interaction between the ZLB constraint and inflation target changes macroeconomic dynamics in the following two ways: (i) it raises the mean level of output gap at higher inflation targets; and (ii) it increases the deflationary spiral when the ZLB is binding at low inflation targets. To show the latter result, I use non-linear impulse-responses.

There are two challenges to numerically solve a menu cost model in a New Keynesian framework. First, even without the ZLB constraint, the firm problem has kinks, eliminating perturbation methods as a means to solve these economies. Thus, I rely on global projection methods. Due to the curse of dimensionality, I use the Smoliak sparse-grid method as in Judd, Maliar, Maliar, and Valero (2014) and Krueger and Kubler (2004). Second, since standard application of the Krusell-Smith algorithm fails in these economies, I develop a modified version of the algorithm.[6] Typically, the Krusell-Smith algorithm projects price and quantities on a small set of moments of the distribution of the idiosyncratic state. In this model, it consists of projecting inflation to some moments of the distribution of relative prices. But, an exogenous inflation function and a Taylor rule on nominal interest rates imply that these depend only on the state of the economy – thus they don't react to inflation and output endogenously, generating indeterminancy (see Galí (2009)). To avoid this problem, I modify the Krusell-Smith algorithm to incorporate the intensive margin of the Phillips curve in the inflation function, making inflation partially react to output – thus nominal interest rates react to inflation and output endogenously, generating determinacy.

This paper is closely related to recent work that has emphasized the implications of the zero lower bound on nominal interest rates for the optimal inflation target. A thorough and rigorous description of the cost and benefits of a higher inflation target is given by Schmitt-Grohé and Uribe (2010). In their paper, they compute the probability of hitting the ZLB using a model with a calibrated magnitude of shocks and reach the conclusion that the probability of hitting the zero lower bound is zero. In my paper, I follow Barro (2006) in calculating the probability of rare events using panel data for different countries controlling for different levels of inflation target. I find that at 2 % inflation target, the probability of hitting the ZLB is 10 %.

Section 2 describes the model. Section 3 presents a simplified version and shows the modified

[5] See Christiano, Eichenbaum, and Evans (2005)

[6] See Krusell and Smith (2006), Midrigan (2011) and Khan and Thomas (2008) for applications of the Krusell-Smith algorithm.

Krusell-Smith algorithm. Section 4 calibrates the model. Section 5 explains steady state welfare losses and section 6 quantifies the optimal inflation target in a medium scale DSGE model. Section 7 concludes.

2 Model

Time is discrete. There is a continuum measure one of intermediate firms indexed by $i \in [0,1]$, a final competitive firm, a representative household, and a central bank.

Intermediate firms are monopolistically competitive. The producer of intermediate good i produces output y_i using labor l_i and material n_i, and the productivity of the firm is given by an idiosyncratic component A_i and an aggregate component η_Z acording to

$$y_i = A_i \eta_Z N_i^{\alpha} l_i^{1-\alpha} \tag{1}$$

The idiosyncratic productivity of the firm A_i follows a compound Poisson process given by

$$\Delta \log(A_{t,i}) = \begin{cases} \eta_{t+1} & \text{with prob. p} \\ 0 & \text{with prob. 1-p} \end{cases} \quad ; \quad \eta_t \sim_{i.i.d} h(\eta) \tag{2}$$

with $\mathbb{E}[\eta] = 0$, i.e. there is no growth in the idiosyncratic productivity.

Firms face a physical cost of changing their price. Every time the firm changes her nominal price she has to paid a fixed cost given by θ units of labor.

Intermediate firms are competitive with technology given by

$$Y_t = \left(\int_0^1 \left(\frac{y_{t,i}}{A_{t,i}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

where final output uses a Dixit-Stiglitz aggregator with elasticity γ. The main reason to add the productivity shock in the Dixit-Stiglitz aggregator is to decrease the state space of the firm (see below for explanation).[7]

Households' preferences are given by

$$U_t = u(C_t, L_t) + \beta \mathbb{E}_t [U_t^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}} \tag{4}$$

where σ_{ez} is the risk sensibility parameter and measures the departure from expected utility. The consumer faces the following budget constraint given by

$$P_t C_t + B_t = W_t L_t + \int \Phi_{t,i} di + \eta_{Q,t} R_{t-1} B_{t-1} + T_t \tag{5}$$

[7]This formulation is also used in Midrigan (2011), Midrigan and Kehoe (2011) and Alvarez and Lippi (2011)

where W_t and P_t are the nominal prices of labor and consumption, $\Phi_{t,i}$ are nominal profits for the intermediate producer and T_t are lump sum transfers from the government. B_{t-1} is the stock of one period nominal bonds with a rate of return $R_{t-1}\eta_t^Q$, where the second element generates a wedge between the nominal interest rate controlled by the central bank and the return of the assets held by households. This shock can be micro-funded as a net-worth shock in models with the financial accelerator and capital accumulation.

The behavior of monetary policy is described by a Taylor rule given by

$$\begin{aligned} R_t^* &= \left(\frac{1+\bar{\pi}}{\beta}\right)^{1-\phi_r} \left(R_{t-1}^*\right)^{\phi_r} \left[\left(\frac{P_t}{P_{t-1}(1+\bar{\pi})}\right)^{\phi_\pi} X_t^{\phi_y}\right]^{1-\phi_r} \left(\frac{X_t}{X_{t-1}}\right)^{\phi_{dy}} \eta_{R,t} \\ R_t &= \max\{1, R_t^*\} \end{aligned} \tag{6}$$

R is the nominal interest rate, $\bar{\pi}$ is the target inflation, η^R is a money shock, and X_t is the output gap, i.e. the ratio between current output and the natural level of output defined in an economy with zero menu cost (an economy without price rigidities).

Aggregate output is equal to aggregate consumption plus government expenditure

$$Y_t - \int N_{t,i} = C_t + \eta_{G,t} \tag{7}$$

where I used $C_t, \eta_{G,t}$ to denote consumption and government expenditure. The government follows a balanced budget each period

$$\eta_{G,t} P_t = T_t \tag{8}$$

All aggregate shocks follow a AR(1), i.e. $\log(\eta_j) \sim AR(1)$ where $i \in \{R, Z, G, Q, \}$. Next I will describe each agent's problem and the equilibrium definition.

Household: The representative consumer problem is given by

$$\max_{\{C,L,B\}_t} U_0 \tag{9}$$

subject to (4) and (5). From the problem of the representative consumer we have the stochastic nominal discount factor

$$Q_t = \beta \left(\frac{U_{t+1}}{\mathbb{E}_t[U_{t+1}^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \frac{U_c(C_{t+1}, L_{t+1})}{U_c(C_t, L_t)} \frac{P_t}{P_{t+1}} Q_{t+1} \tag{10}$$

with $Q_0 = 1$.

Final Producer: The final producer problem is given by

$$\max_{\{Y_t,\{y_{t,i}\}_i\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \left(P_t Y_t - \int_0^1 p_{t,i} y_{t,i} di \right) \right] \tag{11}$$

subject to (3). Given constant return to scale and zero profits conditions, we have that the aggregate price level and the firm's demand are given by

$$P_t = \left(\int_0^1 (p_{t,i} A_{t,i})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \qquad y_t(A_{t,i}, p_{t,i}) = A_{t,i} \left(\frac{A_{t,t} p_{t,i}}{P_t} \right)^{-\gamma} Y_t \tag{12}$$

Firms: The firm's problem is given by

$$\max_{p_{i,t}} \mathbb{E} \left[\sum_{t=0}^{\infty} Q_t \Phi_{t,i} \right] \qquad s.t. \qquad \Phi_t^i = y_t \left(A_{t,i}, p_{t,i} \right) \left(p_{i,t} - \frac{W_t^{1-\alpha} P_t^{\alpha} \iota}{\eta_t^Z} \right) - I(p_{t-1,i} \neq p_{t,i}) W_t \theta \tag{13}$$

subject to (2) and A_{-1}, p_{-1} given. Note that I've already included the optimal technique in the marginal cost of the firm with $\iota = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$.

Equilibrium definition *An equilibrium is a set of stochastic processes for (i) consumption, labor supply, and bonds holding $\{C, L, B\}_t$ for the representative consumer; (ii) pricing policy functions for firms $\{p_i\}_t$ and inputs demand $\{N_{t,i}, l_{t,i}\}$ for the monopolistic firms; (iii) final output and inputs demand $\{Y_t, \{y_{t,i}\}_i\}_t$ for the final producer and (iv) nominal interest rate $\{R\}_t$:*

1. *Given prices, $\{C, L, B\}_t$ solve the consumer's problem in (9).*

2. *Given prices, $\{Y_t, \{y_{t,i}\}_i\}_t$ solve the consumer's problem in (11).*

3. *Given the prices and demand schedule, the firm's policy $p_{t,i}$ solves (13) and inputs demand are optimal.*

4. *Nominal interest rate satisfies the Taylor rule (6).*

5. *Markets clear at each date:*

$$\begin{aligned} \int_0^1 (l_{t,i} + I(p_{t,i} \neq p_{t-1,i})\theta)\, di &= L_t \\ Y_t - \int_0^1 N_{t,i} di &= C_t + \eta^G \end{aligned}$$

3 Equilibrium Description

This section describes equilibrium conditions, the main trade-offs for optimal target inflation and the solution method. For exposition, I simplify the model in several dimensions in this section:

preferences are given by expected utility $\sigma_{ez} = 0$ with period utility $u(C, L) = \log(C) - L$; the only input of production is labor $y_i = A_i l_i$; the risk premium shock is the only structural shock; and the Taylor rule is given by $R_t = \frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi_t}{1+\bar{\pi}}\right)^{\phi_\pi}$ without ZLB constraint.

Firm's Problem: The relevant state variable for firm i at time t is $\tilde{p}_{t,i} = \frac{p_{t,i}A_{t,i}}{P_t}$, the relative price multiplied by productivity. The relative price is the important idiosyncratic variable for the firm since the firm's demand and the static profits depend on it. Let $v(\tilde{p}_-, S)$ be the present discounted value of a firm with previous relative price $\tilde{p}_-$ and current aggregate state S. Then $v(\tilde{p}_-, S)$ satisfies

$$\begin{aligned}
v(\tilde{p}_-, S) &= \mathbb{E}_{\Delta a}\left[\max_{\text{change,no change}}\left\{V^c(S), V^{nc}(\frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)}, S)\right\}\right] \\
V^{nc}(\tilde{p}, S) &= \tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\} \\
V^c(S) &= -\theta + \max_{\tilde{p}}\left\{\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \\
\Delta a &= \begin{cases} \eta & \text{with prob. } p \\ 0 & \text{with prob. } 1-p \end{cases}
\end{aligned} \tag{14}$$

where I use marginal utility of consumption as the numeraire. There are two sources of fluctuation in the relative price: inflation and idiosyncratic shocks. After the change in the relative price due to these components, the firm has the option either to change the price or keep it the same. If it changes the price, it has to pay the menu cost θ.

The firm's problem depends on $\tilde{p}$; any combination of nominal price and productivity that generates the same value of $\tilde{p}$ yields the same profits. This comes from the assumption that productivity shocks also affect the demand of the intermediate input.

The policy of the firm is characterized by two objects: (1) a reset price and (2) a continuation region. Let $P^*(S)$ be the reset price. Then

$$P^*(S) = \max_{\tilde{p}}\left\{\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta\mathbb{E}_{S'}\left[v\left(\tilde{p}, S'\right)|S\right]\right\} \tag{15}$$

$P^*(S)$ does not depend on the idiosyncratic shock; it only depends on the aggregate state of the economy and therefore is the same across resetting firms. The continuation region is given by all relative prices such that the value of changing the price is less than the value of not changing the price. Let $\Psi(S)$ be the continuation region. Then

$$\Psi(S) = \{\tilde{p} : V^{nc}(\tilde{p}, S) \geq V^c(S)\} \tag{16}$$

with the firm's policy given by

$$\text{change the price and set a relative price equal to } P^*(S) \text{ if and only if } \frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \notin \Psi(S) \tag{17}$$

As is typical in models with heterogeneity, the firm needs to forecast equilibrium prices, real wages and inflation, and the state law of motion. If the firm knows these functions, then it has all the elements to take the optimal decision in (14).

Aggregate State: Given that the repricing decision depends on the previous relative price and that inflation is the aggregation of these decisions, the distribution of relative prices is the state in the economy. I denote with S the state of the economy with the law of motion $\Gamma(S'|S)$. Therefore the state of the economy is $S = (f(\tilde{p}_{-}), \eta^{Q})$ with law of motion $\Gamma(S'|S)$.

Aggregate conditions: The aggregate conditions are given by the household optimality conditions, feasibility and the monetary policy rule

$$\begin{aligned} C(S)^{-1} &= \beta R(S)\mathbb{E}_{S'}[\frac{C(S')^{-1}}{\Pi(S')}\eta^{Q}(S')|S] \\ C(S) &= w(S) \\ R(S) &= \frac{1+\bar{\pi}}{\beta}\left(\frac{\Pi(S)}{1+\bar{\pi}}\right)^{\phi_{\pi}} \\ C(S) &= \frac{L(S)-\Omega(S)\theta}{\Delta(S)} \end{aligned} \tag{18}$$

where $\Omega(S)$ is the fraction of repricing firms and $\Delta(S)$ is labor productivity due to price dispersion given by

$$\Delta(S) = \int \tilde{p}^{\gamma} f(\tilde{p}) \tag{19}$$

where $f(\tilde{p})$ is the distribution of $\tilde{p}$.

In aggregate conditions (18), we can see the two key distortions in models with nominal rigidities. The first one comes from markups. In the efficient allocation, firms' marginal costs should be equal to the price, and therefore the aggregate real wage should be one. Monopolistic competition implies a real wage less than one, since firms charge a positive markup (the markup is the inverse of the real wage). Sticky prices together with monopolistic competition implies fluctuation in the aggregate markup, and therefore inefficient fluctuation in the marginal cost and consumption. The distortion from the first best given by markups is the first component of output gap.

The second component comes from feasibility. Aggregate consumption is given by

$$C_t = \left[\int l_{t,i}^{\frac{1-\gamma}{\gamma}} di\right]^{\frac{\gamma}{\gamma-1}} \tag{20}$$

In the first best it is optimal to have the same labor input across firms. Sticky price models create a misallocation in the input of production across firms, decreasing aggregate productivity.[8]

The level of target inflation affects these distorsions. Increasing target inflation decreases the volatility of the output gap coming from markup fluctuations. However, it also increases mean

[8]There is a third distortion coming from the physical cost of repricing, that it is almost insignificant in my model

price dispersion. Given that the main trade-off is between the mean and the variance of the output gap, I use Epstein-Zin preferences to have a non-trivial cost of the later component.

Aggregate equilibrium conditions (18) are triangular: consumption, real wage, and inflation can be solved first, and then used to solve for the labor supply in the simulation. Therefore, with quasi-linear preferences it is not necessary to know $\Omega(S)$ or $\Delta(S)$ to solve for the equilibrium nominal interest rate, real wage, and consumption.

Given that the distribution of $\tilde{p}$ is the state, I use the Krusell-Smith algorithm to solve this problem. However, the standard method to implement Krusell-Smith does not work for this problem. To understand why, we need to note two properties in the aggregate conditions. First, for a given inflation function, in principle, it is possible to solve the equilibrium conditions. But, *given that the equilibrium equations are dynamic, it is important to satify Blanchard-Khan conditions whenever solving for the aggregate conditions.* Next, I show that the standard method does not satisfy the Blanchard-Khan conditions.

The standard Krusell-Smith algorithm consists of projecting price and quantities to a small set of moments of the distribution and the exogenous state. In this problem, Krusell-Smith consists of projecting inflation on some moments of the distribution. With this common approach, it possible to solve the *aggregate conditions separate from idiosyncratic conditions*, dividing the system into two sub-systems. Formally, the algorithm is given by

1. Guess a function for inflation $\Pi(S)$ and a law of motion for $\Gamma(S'_{KS}|S_{KS})$ for the Krusell-Smith state denoted by S^{KS}.

2. Solve aggregate conditions and the real wage function from (18).

3. With the real wage and inflation function, solve the firm's value function (14).

4. Simulate and project inflation and $\Gamma(S'_{KS}|S_{KS})$ on the state. Update and check convergence.

To my knowledge, all the Krusell-Smith formulations have this approach.[9] The next proposition shows how the standard method generates multiplicity of equilibrium at the step of solving aggregate conditions.

Proposition 1 *For any $\Pi(S_{KS}), \tilde{\Gamma}(S'_{KS}|S_{KS})$ and $\lambda > 0$, if $\{C(S_{KS}), R(S_{KS}), w(S_{KS})\}$ is a solution for (18), then $\{\lambda C(S), R(S), \lambda w(S)\}$ is a solution.*

The proof of the previous proposition is trivial, but the economic intuition is not. The reason why Krusell-Smith fails is similar to the Taylor principle in the New Keynesian model with Calvo pricing: nominal interest rates should respond strongly to inflation and consumption[10]. Krusell-Smith implementation generates an exogenous function for inflation and therefore an exogenous

[9] See Krusell and Smith (2006), Midrigan (2011) and Khan and Thomas (2008)

[10] Given that I don't have any real shocks, in this section consumption is equal to output gap.

nominal interest rate. Importantly, nominal interest is not reacting to consumption whenever solving aggregate conditions. It is a standard result that if nominal interest rates do not react to consumption, then there exist infinite solutions to the aggregate conditions (See Galí (2009)).

The main problem is that in the aggregate equilibrium conditions, there is no information on the relationship between inflation and consumption, i.e. the Phillips curve. Replacing the Phillips curve with an exogenous function for inflation changes the eigenvalues of the system, implying multiplicity of equilibria.

To understand the solution to this problem we need to understand the cross-equation restriction for inflation. Inflation depends on three objects: (1) the reset price; (2) the continuation region; and (3) the distribution of $\tilde{p}$. Let $\mathcal{C}(S)$ be the set of relative prices and productivity growth such that the firm does not adjust the price

$$\mathcal{C}(S) = \left\{ (\tilde{p}_-, \Delta a) : \frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \in \Psi(S) \right\}$$

The next proposition characterizes the cross-equation restriction for inflation.

Proposition 2 *Inflation dynamic is given by*

$$\begin{aligned} \Pi(S) &= \left(\frac{1 - \Omega(S)}{1 - \Omega(S) P^*(S)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(S) \\ \Omega(S) &= \int_{(\tilde{p}_-, \Delta a) \notin \mathcal{C}(S)} f_-(d\tilde{p}_-) g(d\Delta a) \\ \varphi(S) &= \left(\int_{(\tilde{p}_-, \Delta a) \in \mathcal{C}(S)} \frac{\left(\tilde{p}_- e^{\Delta a}\right)^{1-\gamma}}{1 - \Omega(S)} f_-(d\tilde{p}_-) g(d\Delta a) \right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{21}$$

I define $\varphi(S)$ as menu cost inflation. Inflation is a function of three elements: reset price, the firm's inaction set, and the distribution of $\tilde{p}$. Inflation depends on two forward-looking variables (reset price and the Ss bands), and a backward-looking variable (the distribution of $\tilde{p}$). For this problem, the issue in the standard application of Krusell-Smith is not decomposing between the optimal decisions of the firms (forward-looking variables) and the distribution of relative prices (backward-looking variable). Without this decomposition at the time of solving the aggregate conditions, *the eigenvalues of the aggregate dynamic system do not satisfy the Blanchard-Khan conditions for a unique equilibrium.*

In this model, the main components of inflation fluctuation iare given by menu cost inflation and reset price. The fraction of resetting firms is quantitatively insignificant in the inflation dynamics. First, the first order effect of the fraction of repricing firms on inflation is zero whenever the reset price is equal to one

$$\frac{\partial \Pi(S)}{\partial \Omega(S)}|_{P^*(S)=1} = 0 \tag{22}$$

Moreover, the fraction of repricing firms is almost a symmetric function with respect to the state, and therefore the first order effect of the the structural shocks on the fraction of repricing firms is also zero.[11]

Menu cost inflation captures the inflation due to changes in the distribution of non-adjusting prices. I could not find a method to endogenize $\varphi(S)$ in the aggregate conditions equation, since it depends on the interaction between inflation, Ss bands, and distribution, but the method I propose below endogenizes the intensive margin of inflation in the Phillips curve whenever solving the aggregate conditions.

The solution I propose is to apply Krusell-Smith to the frequency of price change and menu cost inflation, and solve the aggregate conditions *together* with the firm's problem. Even if this method generates numerical challenges, it provides the central bank with the cross-equation restriction of the intensive margin of the Phillips curve, breaking the multiplicity mensioned before. I don't have a theorem showing that this method works,[12] but though numerical computation, it seem a reliable method.

Before describing the solution, we need to find the state. In models with nominal rigidities, the important object for the repricing of the firm is the markup, the ratio between $\tilde{p}$ and real marginal cost. Therefore I use real marginal cost in the previous period as the state in Krusell-Smith. This variable is significant to predict menu cost inflation. Next I describe the algorithm used to solve the model.

1. Guess a function $\Omega(w_-, \eta^Q), \varphi(w_-, \eta^Q)$ as a function of the state.

2. Solve for the equilibrium conditions: the joint system of (14), (18), and (21). Get the law of motion for inflation and real wage $(w(S), \Pi(s))$, and the continuation set and reset price $(\Psi(S), P^*(S))$.

3. Simulate a measure of firms and compute $\{\Omega, \varphi, S\}_t$.

4. Project Ω, φ on the state. Check convergence. If not, update and go to step 2.

Note that no law of motion of the state is given in step 3. The law of motion for the endogenous state comes from step 2 solving the aggregate conditions. Secondly, even without ZLB I need to solve the model globally given the kinked property of idiosyncratic policies. In the appendix, I describe the computational details in the quantitative model and Krusell-Smith evaluation.

[11] See Numerical Appendix

[12] I use one method to check Blanchard-Khan conditions. I solve the linear version of the model with the reset price from the Calvo model and the frequency and menu cost inflation as exogenous function in Krusell-Smith. In this case, (18) satisfies Blanchard-Khan conditions.

4 Calibration

I calibrate the model to perform the quantitative evaluation of optimal target inflation. For the majority of the parameters, I use standard values in the New Keynesian model with Calvo pricing. Table 8.1 shows the calibrated parameters.

A period in the model is a month. I choose $\beta = 0.96^{1/12}$ because it implies a risk-free annual interest rate of 4 %. I use GHH preferences

$$u(C, L) = \frac{\left(C - \kappa \frac{L^{1+\chi}}{1+\chi}\right)^{1-\sigma}}{1-\sigma} \tag{23}$$

and set $\sigma = 1.5$ and $\chi = 0.5$. The second parameter implies a Frisch elasticity of 2. I set the Epstein-Zin parameter equal to -45. A similar number was used in Rudebusch and Swanson (2012) and van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2008) to match features of asset prices.

For the production function, I choose an elasticity between inputs γ equal to 5, an upper bound in micro estimations. [13] For the production technology, I set the elasticity with respect to materials equal to 0.7.

The parameters for the Taylor rule are given by $(\phi_r, \phi_\pi, \phi_y, \phi_{dy}) = (0.8, 1.7, 0.05, 0.08)$ as in Alejandro Justiano and Tambalotti (2010) and Del Negro, Schorfheide, Smets, and Wouters (2007). Government expenditure process and money shocks are calibrated as in Del Negro, Schorfheide, Smets, and Wouters (2007). For the labor productivity, I used labor productivity by the Bureau of Labor Statistics. The key process to match the probability of hitting the zero lower bound is the risk premium shock's stochastic process. I set the persistence equal to Smets and Wouters (2007) and I choose the innovation to match the probability of hitting the ZLB in the data.

To calibrate the probability of hitting the ZLB, I construct a international database for CPI inflation and call rates of the majority of countries around the world from the International Financial Statistics of the IMF. After doing this, I discard time periods before 1990Q1, since before that date central banks did not incorporate implicit or explicit inflation targeting. Then, I drop all countries with average inflation more than 4 % such as Mexico and Peru. In the sample, there are countries that hit the ZLB around 30 % and 55 % of the times, such as Switzerland and Japan, but also countries such as South Korea, New Zeland and Australia that hadn't hit the ZLB. I compute the average inflation across countries and the probability of hitting the zero lower bound. The mean inflation is 2 % and the probability of hitting the ZLB is 0.11. I calibrate σ_Q to match this probability. Appendix 9 shows the probability of hitting the ZLB for the countries in the sample.

I set the resources devoted to price-adjustment equal to 0.55 percent of revenue.[14] This implies

[13] See Nevo (2001), Barsky, Bergen, Dutta, and Levy (2003) and Chevalier, Kashyap, and Rossi (2000) for micro-estimation and Burstein and Hellwig (2008) for an estimation in menu cost models

[14] See Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) and Levy and Venable (1997)

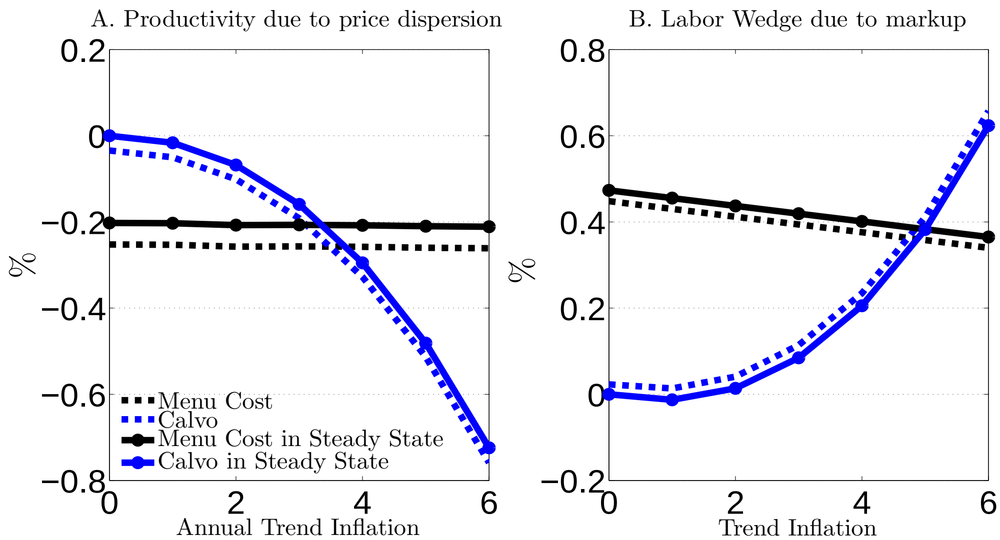

Figure 1: Panel A describes productivity changes due to price dispersion measure as $(\frac{1}{\mathbb{E}[\Delta_t]} - 1)100$. Panel B describes changes in the labor wedge due to markups measure as $(\frac{1}{\mathbb{E}[mc_t]} - 1)100$. Both panels describe these statistics in: (i) Calvo pricing model in steady state; (ii) Calvo pricing model with business cycle and no ZLB; (iii) Menu cost model in steady state; (iv) Menu cost model with business cycle and no ZLB.

$\theta\mathbb{E}[\frac{\Omega_t}{\eta^G+C_t}] = 0.0055$. I calibrate the idiosyncratic process with normal innovation given by $\eta_t \sim N(0, \sigma_a)$. For the idiosyncratic productivity process, I choose two moments of the micro-price behavior: frequency of price change and standard deviation of price change. These two moments pin down the idiosyncratic productivity shocks exactly as Baley and Blanco (2013) have shown.

Finally, to solve the New Keynesian model I assume $\sigma_a = 0$, since adding idiosyncratic shocks to the Calvo pricing model generates infinite price dispersion at low levels of target inflation.

5 Welfare Cost of inflation: Steady State

This section characterizes price dispersion and markups for different levels of target inflation in the steady state. There are two reasons to analyze the steady state. First, I provide a full analytical characterization of both distortions as a function of micro-price statistics. Second, I can focus mainly on the steady state because business cycles affect the mean of both distortions, but this relationship does not move with target inflation. Figure 1 describes price dispersion and markups in Calvo and menu cost models with and without business cycles. As we can see, adding business cycle fluctuations only changes the level of the distortions but not the slope with respect to target inflation. As I will explain in section 6, this result does not hold when the ZLB constraint affects business cycle dynamics.

To characterize the steady state distortions, I use a continuous time formulation of the model. In continuous time, the log relative price, $\hat{p}_t = \log(\tilde{p}_t)$, follows the stochastic differential equation given by

$$\hat{p}_t = \hat{p}^* - \bar{\pi}t + \sum_{i=1}^{N_t} \eta_i \tag{24}$$

where $\sum_{i=1}^{N_t} \eta_i$ is a compound Poisson process with arrival rate $\lambda = -\log(p)$.

Let τ be the time between price changes.[15] The time between price changes is the key firm decision that affects price dispersion. For example, in a Taylor pricing model the time between price changes is a fixed date ($\tau = \mathbb{E}[T]$); in Calvo it is given by an exponential distribution ($\tau \sim exp(\frac{1}{\mathbb{E}[\tau]})$) and in menu cost it is given by $\tau_{\underline{p},\overline{p}} = \min\{t : \hat{p}_t \notin [\underline{p}, \overline{p}]\}$.

Note that $\Delta_{ss} = \mathbb{E}_t[\tilde{p}^{-\gamma}]$ is not exactly price dispersion, but using a second order approximation we can show that it is proportional to price dispersion

$$\log(\Delta_{ss}) \approx \frac{\gamma \mathbb{V}[\hat{p}]}{2} \tag{25}$$

where γ is the demand elasticity. The next proposition characterizes the variance of relative prices where I use the notation Δp to denote the size of price changes.

Proposition 3 *Let τ be a stopping time with one of the following properties*

1. *$\exists c \in R$ s.t. $\tau < c$ almost surely.*
2. *Or $Pr[\tau < \infty] = 1$ and X_t^i for i=1,2 define as*

$$dX_t^i = \left[\left(\sum_{i=1}^{N_t} \eta_i \right)^i - \mathbb{E}[\eta^i]t \right] dt \tag{26}$$

with the property that $\mathbb{E}[|X_t^i|] < \infty$ and $\lim_{t\to\infty} \mathbb{E}[|X_t^i I_{\{\tau>t\}}|] = 0$ for $i = 1, 2$.

Then if there exists an ergodic distribution of relative prices we have that

$$\begin{aligned} \mathbb{V}_i^{\bar{\pi}}[\hat{p}_i] &= \frac{(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi})^2}{12} \mathcal{D}(\mathbb{V}^{\bar{\pi}}[\hat{\tau}], Ske^{\bar{\pi}}[\hat{\tau}]) + \frac{\mathbb{E}^x[\Delta p^2] - x^2 \mathbb{E}^x[\tau^2]}{2} \frac{\mathbb{E}^{\bar{\pi}}[\tau]}{\mathbb{E}^x[\tau]} (1 + \mathbb{V}^{\bar{\pi}}[\hat{\tau}]) \\ \mathcal{D}(\mathbb{V}^{\bar{\pi}}[\hat{\tau}], Ske^{\bar{\pi}}[\hat{\tau}]) &= 1 + \mathbb{V}^{\bar{\pi}}[\hat{\tau}]^{\frac{3}{2}} Ske^{\bar{\pi}}[\hat{\tau}] + 3\mathbb{V}^{\bar{\pi}}[\hat{\tau}](2 - \mathbb{V}^{\bar{\pi}}[\hat{\tau}]) \end{aligned} \tag{27}$$

where $\hat{\tau} = \frac{\tau}{\mathbb{E}^{\bar{\pi}}[\hat{\tau}]}$.

There are two requirements in this theorem. Conditions 1-2 are sufficient to use the optimal sampling theorem. The second condition assumes the existence of the ergodic distribution of relative prices. For example, in the Taylor pricing model the ergodic distribution of relative prices exists if and only if the initial distribution of relative prices is uniform.

[15] Formally $\tau : \Omega \to [-, +\infty) \cup +\infty$ is a measurable function on a probability space $(\Omega, \mathbb{F}, P)$ with the property that $\{\omega \in \Omega \leq t\} \in \mathcal{F}_t\}$

This formula is not the same as the one I described in the introduction. I have simplified equation (40) in the following way: if the ergodic distribution of relative prices exists, then we can apply the renewal theorem and show that $\mathbb{E}^{\bar{\pi}}[\Delta p]\frac{1}{\mathbb{E}^{\bar{\pi}}[\tau]} = \bar{\pi}$. Replacing $\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi} = \mathbb{E}^{\bar{\pi}}[\Delta p]$ in (40) and set $x = 0$, we get the formula in the introduction.

To understand the previous formula, let's focus on the case without idiosyncratic shocks where the second term of (40) is equal to zero. Under this assumption, the variance of the relative prices only depends increasingly on mean, variance, and skewness of the time between price changes. Higher time between price changes increases the dispersion in relative prices since on average they accumulate higher inflation. The variance in the relative time between price changes increases price dispersion since it increases the measure of firms with more accumulated inflation. To see this, let's assume the following pricing model

$$\tau_T = \begin{cases} \epsilon & \text{with prob.} 1/2 \\ 2 - \epsilon & \text{with prob. } 1/2 \end{cases} \tag{28}$$

In this example, the mean and skewness are the same as in the Taylor pricing model and the only difference is the variance. It is easy to show that the distribution of relative prices whenever $\epsilon \to 0$ is only composed by the time between price changes every two periods. Therefore this model is equivalent to a Taylor model with twice the mean between price changes. Using (40) we can see that in Taylor pricing model the variance of relative prices is equal to $\frac{(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi})^2}{12}$ and in the example described above with $\epsilon \to 0$, the variance of relative prices is equal to $\frac{(2\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi})^2}{12}$, i.e. a Taylor model with higher expected time between price changes. Finally, higher skewness of the relative time between price changes increases the tail of low relative prices.

To compare the sensitivity of different pricing models with respect to target inflation, it would be useful to see some examples of different pricing models without idiosyncratic shocks. In Taylor and menu cost models, price dispersion is given by $\mathbb{V}_i^{\bar{\pi}}[\hat{p}_i] = \frac{(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi})^2}{12}$, with the only difference that in the Taylor model the expected time is independent of target inflation. It is easy to see that these two models are the minimum price dispersion models, since they have zero variance and skewness. In Calvo the variance of relative prices is equal to $\mathbb{V}_i^{\bar{\pi}}[\hat{p}_i] = (\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi})^2$, much larger than the previous models.

Remember, for a model to have low sensitivity of price dispersion with respect to target inflation, the most important property is low variance and skewness. The previous proposition (40) highlights the low optimal target inflation obtained by Coibion, Gorodnichenko, and Wieland (2012). First, among all the models they analyze, the one with highest optimal target inflation is the Taylor pricing model. This is clear since there is no variance and skewness of the time between price changes in the Taylor pricing model. Notice that even though this model is time dependent, it has lower sensitivity with respect target inflation than the state-dependent models analyzed in Coibion, Gorodnichenko, and Wieland (2012).

Coibion, Gorodnichenko, and Wieland (2012) study two state-dependent models: Calvo with adjusted frequency of price adjustment and the Dotsey, King, and Wolman (1999) model. Calvo with data adjusted frequency has large sensitivity to target inflation since variance and skewness of relative time between price changes is constant. Moreover, the mean of the time between price changes has low elasticity with respect to target inflation at low levels.[16] The Dotsey, King, and Wolman (1999) model is similar to Calvo with a truncated tail. In this model, the time between price changes is given by [17]

$$f(t) = \frac{\bar{\pi}\varphi}{\underline{p}} t e^{-\frac{\pi\varphi}{\underline{p}\sqrt{\pi}} t^2} I(\bar{\pi} t < \underline{p}) \tag{29}$$

where $\underline{p}$ is the maximum Ss bands width and φ is the arrival rate of a lower menu cost. If the model is calibrated to match the expected time between price changes at 3 % target inflation, then $\underline{p}$ is large and therefore the distribution of time between price changes is similar to the Calvo pricing model, but with lower variance and skeness. For both models they found an optimal target inflation around 1 % – half as much as they found in Taylor.

As we can see in figure 1, even with zero inflation, menu cost models have positive price dispersion due to idiosyncratic shocks. This is the second term in the equation (40). To have an intuition of this formula, note that in time-dependent models the idiosyncratic component simplifies to

$$\frac{\mathbb{V}[\Delta p]}{2}(1 + \mathbb{V}[\hat{\tau}]) \tag{30}$$

where there is a penalization for the increase in the variance of relative time between price changes with a similar intuition as above. For example, if target inflation is zero, the variance of relative prices is $\frac{\mathbb{V}[\Delta p]}{2}$ in Taylor pricing model and $\mathbb{V}[\Delta p]$ in Calvo pricing model.

The key difference between my model and Coibion, Gorodnichenko, and Wieland (2012) comes from the assumption of idiosyncratic shocks and fixed menu costs. With these two features, variance and skewness are decreasing with respect to target inflation, as I will show next. This implies that the target inflation component increases more slowly than in Calvo with respect to target inflation. More importantly, the main component of price dispersion due to idiosyncratic shocks *decreases* with target inflation and almost offsets the increase in the target inflation component.

I solve numerically the micro-price statistics in the menu cost model with idiosyncratic shocks. The key property to understand the micro price statistics in this model is the relationship between the width of the Ss and target inflation. For low levels of target inflation, the width of the Ss bands are insensitive with respect to it. Given this result, higher target inflation triggers price changes for relative prices with sufficiently accumulated inflation. Therefore skewness, variance, and mean decrease with the property that skewness is the most affected and mean is the least affect.

[16] See Gagnon (2009), Alvarez, Gonzalez-Rozada, Neumeyer, and Beraja (2011) and Wulfsberg (2010)

[17] The continuous time formulation of Dotsey, King, and Wolman (1999) is given by $\tau_{g,\varphi} = \min\{t, \hat{p}_t \notin C_t\}$; $C_t = \begin{cases} [p^* - \underline{p}, p^*] & \text{if } D_t = D_{t-} \\ [p^* - x, p^*] & \text{if } D_t = D_{t-} + 1 \end{cases}$ where $x \sim U[0, \underline{p}]$ and D_t is a Poisson counting process with arrival rate φ.

The next proposition characterizes the width of Ss bands with respect to target inflation. To show the next proposition, I assume a quadratic static profit function $B\hat{p}_t^2$, $h(z) \in C^4$ and symmetric around zero, and a discount factor of the firm given by $\rho = -\log(\beta)$. Define $\tilde{\theta} = \frac{w_{ss}\theta}{B}$ as the normalized menu cost.

Proposition 4 *Define*

$$\begin{aligned} U &= \log\left(\frac{\max_{\tilde{p}}\{\Psi(S_{ss})\}}{P^*}\right) \\ L &= \log\left(\frac{\min_{\tilde{p}}\{\Psi(S_{ss})\}}{P^*}\right) \end{aligned}$$

Then $U = u(\frac{\bar{\pi}}{\rho+\lambda}, \tilde{\theta})$ *and* $U = l(\frac{\bar{\pi}}{\rho+\lambda}, \tilde{\theta})$ *with the following properties*

1. $u(\cdot), l(\cdot)$ *are increasing in both arguments.*
2. $\lim_{x\to 0^+} u_x(x,y) = \lim_{x\to 0^+} l_y(x,y) = 0$ *with*

$$\mathcal{E}_{u,x} \in (0, 1/3) \quad ; \quad \mathcal{E}_{l,x} \in (0, 1/3)$$

increasing in x.

The main intuition of this result is that for low levels of inflation target the main value for waiting, and not changing a suboptimal price, is given by the idiosyncratic shocks and not by inflation target. This implies that the width of the Ss bands is almost independent to inflation for low levels of inflation.

Given that Ss bands are almost constant for low levels of target inflation, higher target inflation generates a tighter bound of how old prices can get, decreasing the skewness, variance, and mean of time between price changes. The specific formula for the mean, variance, and skewness depend on the calibration of the Ss and the stochastic process for the idiosyncratic shocks. Figure 2 describes these moments for the final calibration in menu cost and Calvo pricing models, and it is easy to see that higher order moments are more sensitive with respect to the inflation target.

Now I study how markups react with respect target inflation. Let $\hat{\mathcal{M}}_{ss} = \log(\mathcal{M}\frac{\gamma-1}{\gamma})$. Next theorem characterizes the level of markups as function of observables.

Proposition 5 *For any model of firm behavior* (τ)*, the aggregate markup gap is given by*

$$\hat{\mathcal{M}}_{ss} = \bar{\pi}\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)s ds\right] + \left(\gamma - \frac{1}{\gamma-1}\right)\mathbb{E}[\int_0^\infty b(\rho,\tau,s)(\hat{p}_s + \hat{\mathcal{M}}_{ss})^2 ds] \tag{31}$$

Where

- $\mathbb{E}\left[\int_0^\infty a(0,\tau,s)ds\right] = 0$ *and* $\mathbb{E}[\int_0^\infty b(\rho,\tau,s)ds] = 1$.

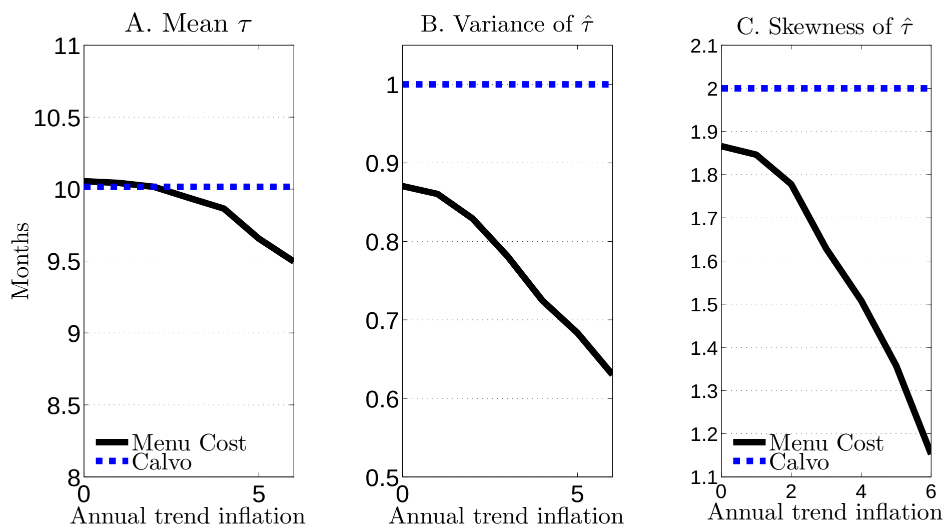

Figure 2: Expected time between price changes, variance of relative time between price changes and skewness of the relative time between price changes.

- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right] < 0$ *and* $\mathbb{E}\left[\int_0^\infty a(0,\tau,s)sds\right] = 0$.
- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]$ *is decreasing in* ρ.
- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau+k,s)sds\right] < \mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]$ *for all* $k > 0$.

Moreover $\rho \to 0$

$$\hat{\mathcal{M}}_{ss} \approx \left(\gamma - \frac{1}{\gamma - 1}\right)\mathbb{V}_i[\hat{p}_i] \tag{32}$$

The equilibrium level of markups depends on two effects that I denominate discounting and dispersion effect. The first term in (31) comes from the firm's inter-temporal marginal rate of substitution and reflects the discounting effect. After price adjustment, the firm's relative price falls over time; therefore at the time of the price adjustment the firm overadjusts its relative price to compensate for the expected fall. If the firm does not discount the future, then the price adjustment is the same as the expected fall and therefore the equilibrium level of markups does not change. If the firm intertemporal marginal rate of substitution is less than one, then the firm adjusts less than the expected fall in the relative price, decreasing the equilibrium level of markups.

The second term in (31) comes directly from the asymmetry of the static profit function and reflects the discounting effect. Given that the static profit function penalizes negative price gaps more[18] than positive price gaps, the firm always prefers a positive rather than a negative price gap with the same magnitude. If price dispersion if high, then the firm's relative price is more volatile and the firm increases the reset price as a precautionary motive to avoid negative price

[18]Price gap is the difference between current price and the static optimal price.

gaps. Therefore, an increase in the fluctuation of the relative price raises the reset price, increasing the equilibrium level of markups.

Figure (1) shows the level of markup in Calvo and menu cost model. Given that in menu cost models price dispersion is insensitive with respect to the inflation target, the discounting effect implies a decreasing aggregate markup with respect to the inflation target. In Calvo, for low levels of target inflation, price dispersion does not react since it depends up to a second order with respect target inflation. Therefore the discounting effect dominates and markups are decreasing. For higher target inflation, price dispersion start increasing and the discounted effect dominates.

Note that in models with nominal rigidities it is always optimal to have positive target inflation. The main intuition of this result is that price dispersion depends up to a second order with respect to the inflation target but the level of markups depends up to first order.

5.1 Business Cycle Without ZLB

What are the consequences of increasing the inflation target? Are inflation *and output* more volatile? Is inflation more or less forward-looking for a higher inflation target? These are important questions to answer before analyzing the optimal inflation target with a ZLB constraint. For example, in a recent paper, Ascari and Sbordone (2013) argue that a higher inflation target makes firms more forward-looking and therefore inflation dynamics become more unstable. This is not the case in the menu cost model as I show in this section. To keep it as simple as possible, I abstract from the ZLB constraint on nominal interest.

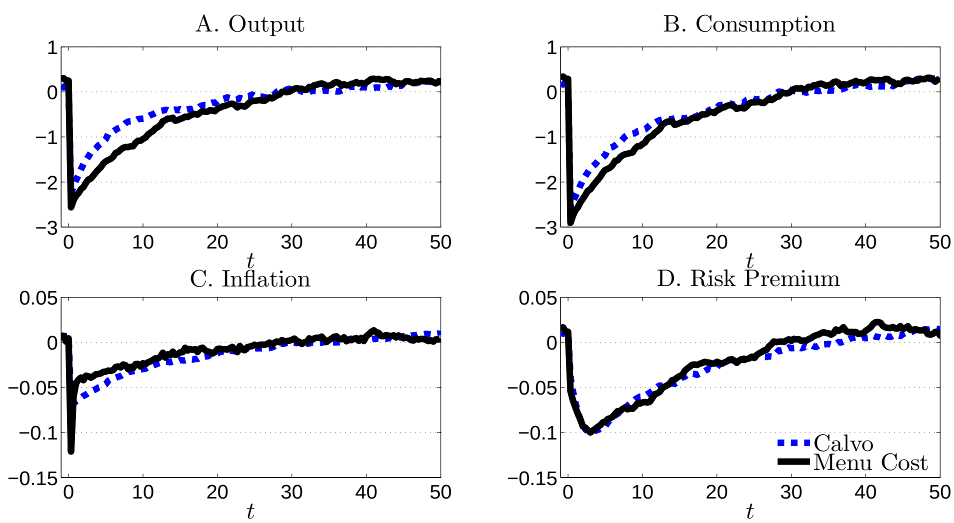

Figure 3: Risk premium shock impulse-response function from the estimated VAR in the simulation. Panel A. and B. plot consumption and output. Panel C. and D. plot inflation and nominal interest rate.

At zero inflation, aggregate dynamics in Calvo and menu cost models are similar. This is a quantitative statement that reinforces previous findings in Gertler and Leahy (2008) and Midrigan

(2011) in a model with richer aggregate dynamics. This is not obvious since in the case of Gertler and Leahy (2008), they assume that firms cannot change their price if they don't receive an idiosyncratic shock –an assumption that I don't have. In the case of Midrigan (2011), he use only small money shocks; in my model I have larger empirically relevant shocks for the US economy.[19] To show this, figure 3 presents the non-linear impulse-response to the risk premium shock.[20] I focus on this shock since it is the main driving shock that triggers the ZLB. The linear impulse-response for money, government expenditure, and productivity are in the appendix (See figures 8.3 to 8.3). Aggregate dynamics are not exactly the same since in menu cost models inflation reacts more with respect to real marginal cost, especially on impact. Once of the reasons I have this property is because the model overshoots the volatility of the frequency of price change. In the US economy the standard deviation of the frequency of price change is 3.2 % (See Klenow and Kryvtsov (2008)), but in the model at 2 % inflation target this statistic is 7.2 %, twice as much in the data.

A standard property of monetary policy is that it underreacts to the structural shocks. In practice, central banks adjust interest rate more cautiously than standard models predict (See Clarida, Gali, and Gertler (1997) and Rotemberg and Woodford (1997)). After a risk premium shock, the marginal utility of consumption increases, generating a drop in consumption and marginal cost. Inflation follows the marginal cost and nominal interest drops without offseting the structural shock. A higher inflation target changes this property since it increases the slope of the Phillips curve as I will show later (See impulse-response in figure 7).

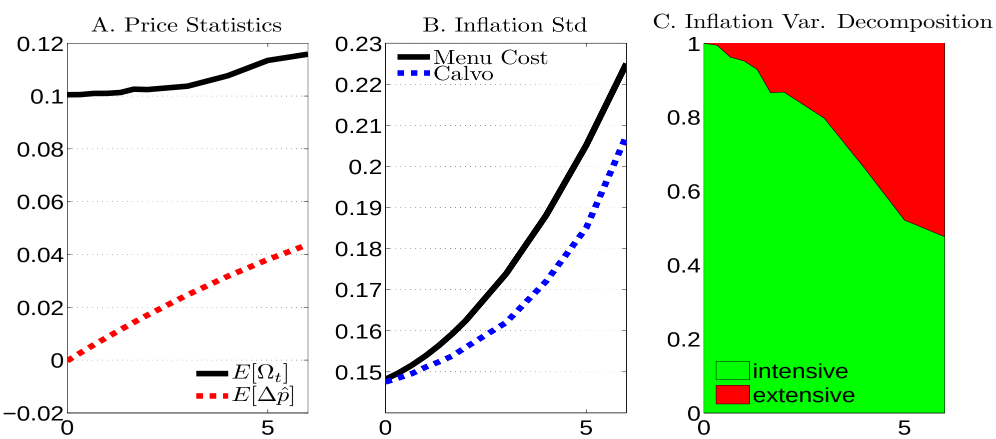

Figure 4: Panel A describes the mean frequency of price change and mean size of price change for the menu cost model. Panel B describes inflation variance for Calvo and menu cost models and Panel C decomposes the inflation variance into intensive and extensive margin in the menu cost model. The intensive margin is given by $\mathbb{E}[\Omega_t]^2\mathbb{V}[\Delta p]$ and the extensive margin is given by $\mathbb{E}[\Delta p]^2\mathbb{V}[\Omega_t] + 2\mathbb{E}[\Delta p]\mathbb{E}[\Omega_t]\mathbb{C}ov(\Omega_t, \Delta p_t)$.

At higher inflation targets, aggregate dynamics in the Calvo and menu cost models are not

[19] He uses only money shocks with a quarterly persistence 0.6 and innovations of 0.0018

[20] See quantitative section for an explanation of how to compute the impulse-response.

similar since the Phillips curve differs in both models. In figure 4, we can see that higher inflation target increases the frequency of price change slightly and also has the same effect on inflation volatility. It would be incorrect to conclude that inflation is slightly more volatile in the menu cost model than in Calvo since the equilibrium process for marginal cost also changes with inflation.

The first key difference from Calvo is that in the menu cost model a higher inflation target activates the extensive margin of inflation, i.e. the volatility of inflation that comes directly from movements in the frequency of price change. This is not the same in Calvo, since frequency of price change is constant. Following Klenow and Kryvtsov (2008), I measure the extensive margin for inflation as

$$\text{extensive margin} = \frac{\mathbb{E}[\Delta p]^2\mathbb{V}[\Omega_t] + 2\mathbb{E}[\Delta p]\mathbb{E}[\Omega_t]\mathbb{C}ov(\Omega_t, \Delta p_t)}{\mathbb{E}[\Delta p]^2\mathbb{V}[\Omega_t] + 2\mathbb{E}[\Delta p]\mathbb{E}[\Omega_t]\mathbb{C}ov(\Omega_t, \Delta p_t) + \mathbb{E}[\Omega_t]^2\mathbb{V}[\Delta p]} \tag{33}$$

In part, the extensive margin increases since the expected size of price change increases, but the key component is the variance of frequency of price change. [21]

The extensive margin becomes more important in the menu cost model since more prices are activated at business cycle frequency. Target inflation changes the distribution of relative prices, moving the mass of relative prices more closely to the lower Ss band. This property implies that more prices are activated at business cycle frequency and therefore the extensive margin of inflation becomes predominant. Figure 5 shows the ergodic mean distribution at 0 and 4 % inflation. As we can see in the figure, there is a higher inflation target near the lower Ss bands and therefore more price are activates at business cycle frequency. Given that these prices have a large size of price change, higher inflation activates what Golosov and Lucas (2007) call selection effect: a steeper Phillips curve since a small measure of firms makes large sized price change.

The fact that higher inflation activates prices near Ss bands is reflected in the business variance of the "menu cost" inflation. Remember that the menu cost inflation is given by the mean distribution of relative prices conditional on no adjustment. As we can see, a higher inflation target significantly increases the variance of the "menu cost" inflation. As we can see in table 8.2, this is the case with higher inflation targets.

The fact that a higher inflation target activates the tail of the time between price change –as explained in the previous section– affects the forward-looking behavior of the reset price. Importantly, firms are less forward in menu cost model for a higher inflation target. The next proposition shows how cross-equation restriction for the reset price.

[21] See table 8.2 in the appendix with micro-price statics.

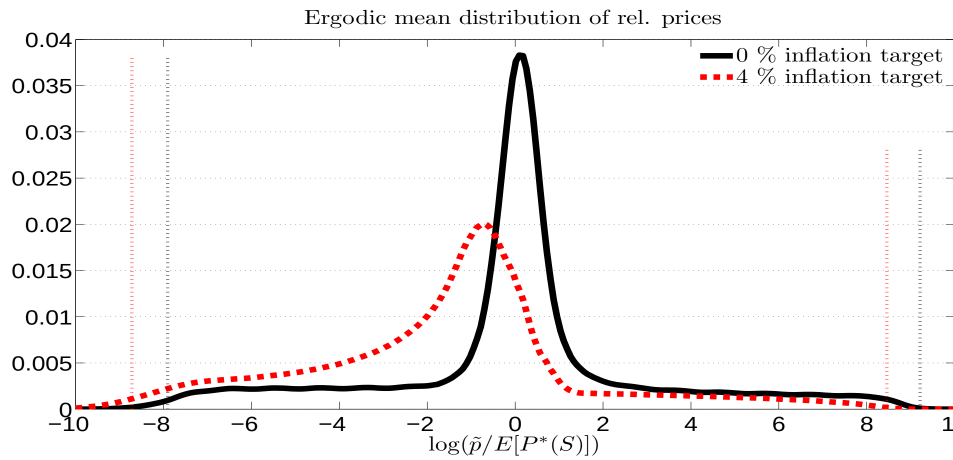

Figure 5: The black and red solid lines are the ergodic mean distribution of relative price relative to the reset price $\bar{f}(\tilde{p}) = \sum_{t=1}^{T} \frac{f_t\left(\log\left(\frac{\tilde{p}}{\mathbb{E}[P^*(S)]}\right)\right)}{T}$. The doted lines are the mean Ss bands with respect the reset price

Proposition 6 *In the menu cost model, the reset price is given by*

$$P^*(S_t) = \frac{\gamma}{\gamma-1}\mathbb{E}_{S_t}\left[\sum_{j=0}^{\infty}\tilde{\omega}\left(S^{t+j},\Delta a^{t+j}|S_t\right)F\left(S^{t+j},\Delta a^{t+j}|S_t\right)mc\left(S_{t+j}\right)\right]$$

$$F\left(S^{t+j},\Delta a^{t+j}|S_t\right) = \prod_{h=0}^{j-1}\frac{\Pi\left(S_{t+h+1}\right)}{e^{\Delta a_{t+h+1}}}$$

$$Z\left(S^{t+j},\Delta a^{t+j}|S_t\right) = \prod_{h=0}^{j-1}\frac{\beta u_c\left(S_{t+h+1}\right)}{u_c\left(S_{t+h}\right)}\left(\frac{U(S_{t+h+1})}{\mathbb{E}_{S_{t+h}}\left[U(S_{t+h+1})^{1-\sigma_{ez}}\right]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}}\left(\frac{\Pi\left(S_{t+h+1}\right)}{e^{\Delta a_{t+h+1}}}\right)^{\gamma-1}$$

$$\tilde{\omega}\left(S^{t+j},\Delta a^{t+j}|S_t\right) = \frac{Y(S_{t+j})Z\left(S^{t+j},\Delta a^{t+j}|S_t\right)I\left(\frac{P^*(S_t)}{F_{t,t+h}}\in\mathcal{C}(S_{t+h},\forall h\leq j)\right)}{\mathbb{E}\left[\sum_{j=0}^{\infty}Y(S_{t+j})Z\left(S^{t+j},\Delta a^{t+j}|S_t\right)I\left(\frac{P^*(S_t)}{F_{t,t+h}}\in\mathcal{C}(S_{t+h},\forall h\leq j)\right)\right]} \tag{34}$$

where

$$\mathbb{E}_{S_t}\left[\sum_{j=0}^{\infty}\tilde{\omega}\left(S^{t+j},\Delta a^{t+j}|S_t\right)\right] = 1 \tag{35}$$

I show this proposition for the menu cost model but it extends to all models of nominal rigidites; the only difference is the indicator function in 34. In models with nominal rigidities and forward-looking firms, reset prices follows a weighted average of future marginal cost adjusted by two elements: the level of markups and the expected fall in the relative price.

Reset price is a weighted average of the expected inflation and the expected marginal cost. The expected fall in the relative price between period S_t and S_{t+j} is given by $F\left(S^{t+j},\Delta a^{t+j}|S_t\right)$ (given that the idiosoyncratic shocks are i.i.d. with zero mean, the only drift comes from inflation). Ceteris paribus the weighting, a higher level of trend inflation increases the reset price since the firm internalizes the expected fall of the relative price.

Inflation target alters the weighting though two effects. The first effect is that higher trend inflation decreases the relative price over time, and therefore increases the firm's revenue over time. This effect generates more forward-looking behavior of the firm and is present in both Calvo and menu cost. This is the only effect in Calvo model, implying a higher forward-looking Phillips curve.

In menu cost models, there is a second effect. Higher trend inflation activates the frequency of price change for "older" prices, and therefore the firm will not internalize future marginal costs in their pricing decision today. This effect implies less forward-looking behavior. Quantitatively, this effect is larger since the revenue effect is third-order in profits (See Alvarez and Lippi (2011)) but the state where the firm changes the price is first order. Given that in Calvo the skewness of stopping times is constant for all levels of inflation, the opposite is present in the Calvo price model.

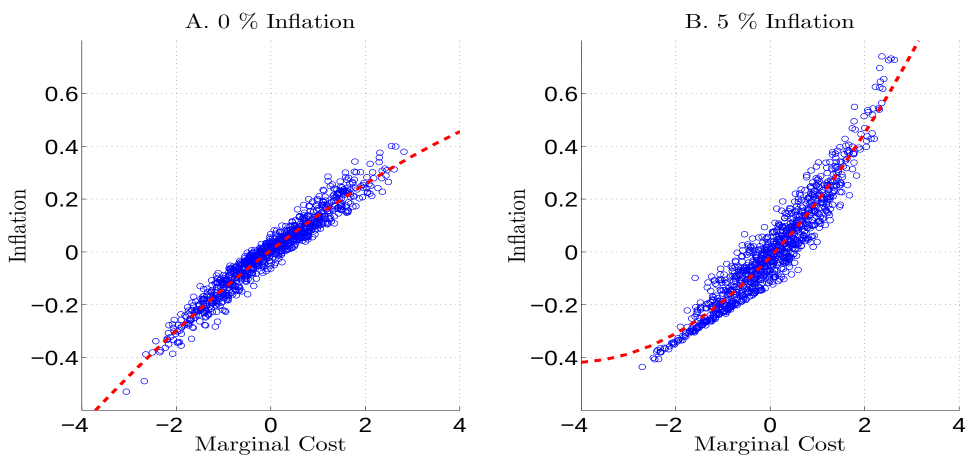

Figure 6: Panel A describes log-deviation of the business cycle fluctuation of marginal cost and inflation for zero inflation, together with a quadratic fit of inflation over real marginal cost. Panel B describes log-deviation of the business cycle fluctuation of marginal cost and inflation for zero inflation, together with a quadratic fit.

Given that we have characterized the reset price and the "menu cost" inflation, we can now undertand how the Phillips curve changes with inflation. It is almost linear with zero inflation target, while it is steeper and non-linear with higher inflation target. The relationship between inflation and real marginal cost is the key object, and is depicted in figure 6. At zero inflation level, it is easy to see that there is an almost linear relationship between inflation and marginal cost. To see this, note that if $\hat{x}_t$ is the log-deviation of a variable with respect to the mean level and assuming that inflation is approximately a martingale, Gertler and Leahy (2008) have shown that at zero inflation

$$\hat{\pi}_t = \lambda \hat{mc}_t + \beta \mathbb{E}[\hat{\pi}_{t+1}] \quad ; \quad \hat{\pi}_t = \frac{\lambda}{1-\beta} \hat{mc}_t \tag{36}$$

Therefore, the Phillips curve is linear for almost all values of marginal cost. Note that the nonlinearity at higher inflation targets comes at larger values of deviation of marginal cost with respect to the mean.

Higher inflation target increases the slope of Phillips curve and since the nominal interest rate depends on inflation, the nominal interest rate reacts more with respect to the structural shocks at higher inflation target. This implies that consumption is less volatile at higher inflation targets, even if inflation is more volatile. In figure 7 we can see the impulse-response for a positive/negative risk premium shock at 0 and 5% inflation target. As we can see, inflation reacts more to marginal cost (in this model almost colinear with consumption), and therefore the nominal interest rate reacts more to the same structural shock, implying less consumption volatility.

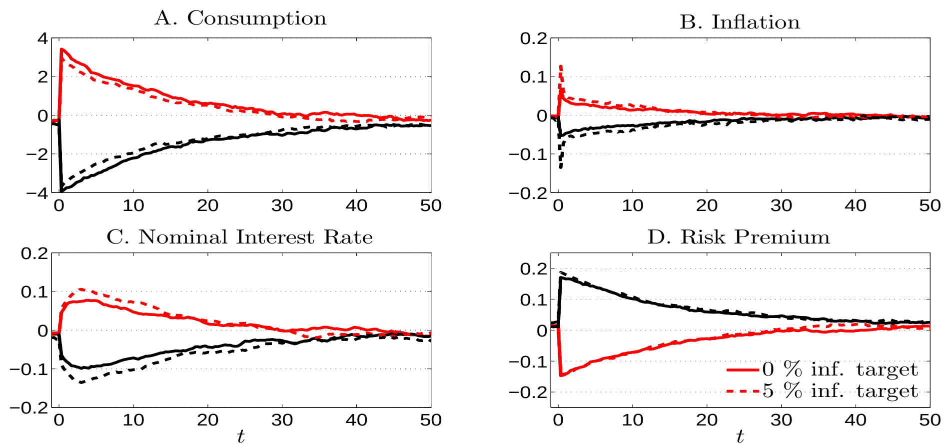

Figure 7: Panel A to D describe the non-linear impulse-response of consumption, inflation, nominal interest rate and risk premium.

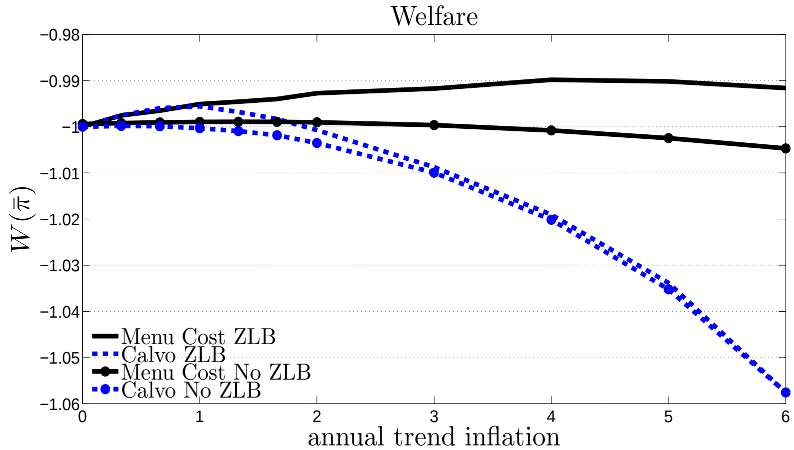

Figure 8: Normalize welfare for different levels of target inflation. The blue lines describe the welfare in the Calvo pricing model with and without ZLB. The black lines describe the welfare in menu cost pricing model with and without ZLB.

6 Quantitative Analysis

This section quantitatively studies my menu cost model to understand the optimal level of target inflation. I also solve the Calvo pricing model, the standard workhorse monetary model, to see the main difference with respect to it. Remember, this is the main model that allows Coibion, Gorodnichenko, and Wieland (2012) to claim that target inflation is not the right tool to reduce business cycle fluctuation of inflation and output gap due to the zero lower bound.

The optimal level of inflation with zero lower bound constraint in the Calvo model is 1% and 5% in the menu cost model. For this result, it is necessary to have the zero lower bound constraint on nominal interest rates since without it the optimal level of inflation is less than 1 % in both models. I compute the household welfare defined as

$$\mathcal{W}(\bar{\pi}) = \frac{\int U^{\bar{\pi}}(S)dF^{\bar{\pi}}(S)}{|\int U^{\bar{\pi}}(S)dF^{0}(S)|} = \frac{\left(\sum_{t=1}^{T} \frac{U^{\bar{\pi}}(S_t^{\bar{\pi}})}{T}\right)}{|\left(\sum_{t=1}^{T} \frac{U^{0}(S_t^{0})}{T}\right)|} \tag{37}$$

where $dF^{\bar{\pi}}$ is the ergodic distribution of the state and $U^{\bar{\pi}}(S)$ is the value function of the household. These variables depend on the Taylor rule parameter for target inflation $\bar{\pi}$.[22]

Figure 8 describes the welfare in Calvo and menu cost pricing models for different levels of target inflation. Two conditions hold at low levels of target inflation: without the ZLB constraint, both models have similar levels of welfare since price dispersion is insensitive. As I explain in section 5, this is given since price dispersion is a second order moment with respect to target inflation. Second, the menu cost model has a much higher welfare benefit than the Calvo model. As I will

[22]To compute this statistic and the ones described below, I use Monte-Carlo integration over $T = 2399980$ simulations.

explain below, in the menu cost model the cost of ZLB is much higher in this region.

To understand welfare, first I discuss the probability of hitting the zero lower bound at different levels of target inflation. Then, I explain how the zero lower bound constraint affects the mean level of markups and price dispersion. Finally, I explain output and inflation dynamics under the ZLB constraint. The main takeaway is that without the ZLB, the Calvo and the menu cost pricing model have similar macroeconomics dynamics at low target inflation. The ZLB constraint generates differences in output and inflation dynamics that I explain below.

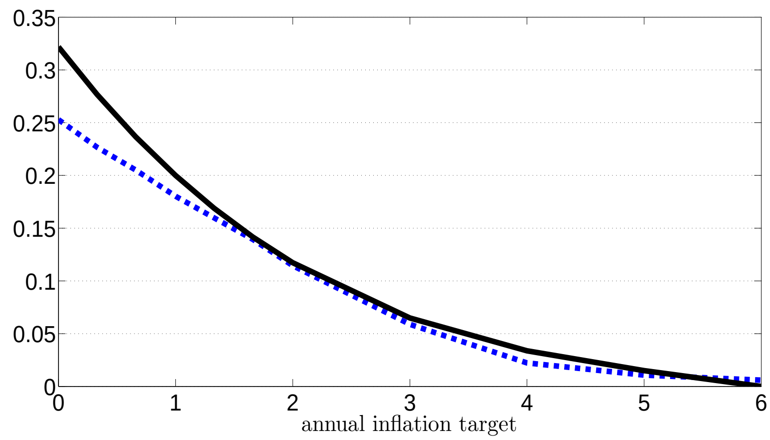

Figure 9: Probability of hitting the zero lower bound computed as $\sum_{t=1}^{T} \frac{I(R_t<1)}{T}$.

The probability of hitting the zero lower bound is decreasing with the level of target inflation. Moreover, at all levels of target inflation, the probability of hitting the zero lower bound is always higher in the menu cost than in the Calvo model. Figure 9 shows the probability of hitting the ZLB for different levels of target inflation. There are two reasons why nominal interest rates hit the ZLB more often in the menu cost model. First, given that slope of the Phillips curve is higher than in Calvo,[23] nominal interest rates are more volatile and therefore the probability of hitting the ZLB is higher. Additionally, at higher levels of target inflation, the extensive marginal component of inflation is higher, increasing the slope of the Phillips curve even more, as we can see in figure 4 in the appendix.[24] Second, at low levels of target inflation, the deflationary spiral in menu cost models is larger than in Calvo, increasing the probability of hitting the ZLB even further since ZLB periods become more absorbing.

What are the shocks that trigger the ZLB? The main shocks that trigger the ZLB are the risk premium shock and, at less magnitude, the TFP shock. Moreover, smaller shocks trigger the ZLB in the menu cost model. Figures 14 and 15 in the appendix describe the distribution of the structural shocks conditional on hitting ZLB.

The level of the target inflation affects the mean of both distortions, since it determines the

[23]See Gertler and Leahy (2008) for an analytical characterization of the Phillips curve in menu cost model with

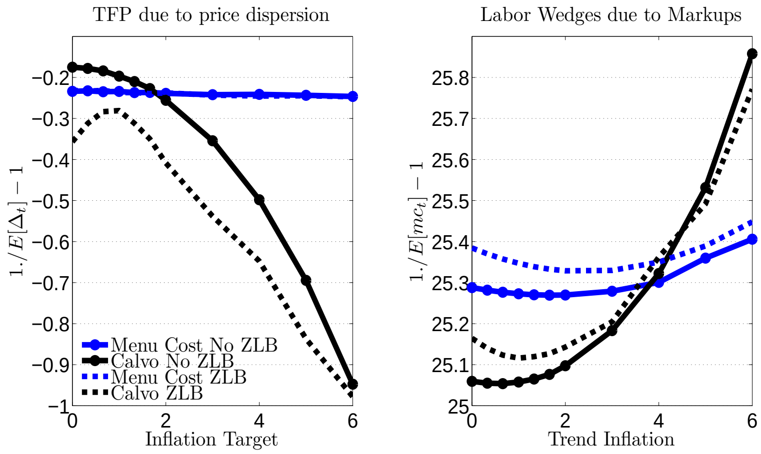

Figure 10: Panel A describes productivity changes due to price dispersion measured as $(\frac{1}{\mathbb{E}[\Delta_t]}-1)100$. Panel B describes changes in the labor wedge due to markups measured as $(\frac{1}{\mathbb{E}[mc_t]}-1)100$. Both panels describe these statistics in: (i) Calvo pricing model with ZLB; (ii) Calvo pricing model without ZLB; (iii) Menu cost model with ZLB; (iv) Menu cost model without ZLB.

probability of hitting the ZLB constraint. The ZLB constraint affects price dispersion more in the Calvo pricing model and affects markups more in the menu cost pricing model. We can see the mean in the ergodic distribution of price dispersion and markups as a function of target inflation in figure 10.

In the Calvo pricing model, the mean of price dispersion is highly sensitive to business cycle volatility of inflation; this is the main determinant of the optimal inflation target. Since the ZLB constraint increases inflation volatility, there is an additional gap in price dispersion due to the ZLB at low levels of inflation. In the case of the menu cost model, price dispersion is insensitive to changes in inflation volatility and therefore the ZLB constraint does not affect the level of this variable. The deflationary spiral due to the ZLB constraint is much larger in the menu cost model, affecting the mean markups much more in this model.

Given that I'm working in a menu cost model with fat-tailed shocks, aggregate dynamics without the ZLB constraint are similar in both models at low levels of inflation target. This property doesn't hold in the model with the ZLB constraint.

The deflationary spiral is larger in the menu cost model than in Calvo for low inflation targets, but becomes similar at higher inflation targets. My interest is in understanding the average effect of the ZLB over aggregate dynamics. To analyze this I will compute the impulse-response of the model. Positive inflation target and ZLB constraint generate non-linearities in the aggregate dynamics. In non-linear models, there are several different ways to construct the associated impulse-response. Ideally, I would like to choose a sequence of shocks that can lead to the ZLB with sufficient

fat-tailed shocks.

[24]See Klenow and Kryvtsov (2008) for the decomposition of inflation in intensive and extensive margin.

probability in the ergodic distribution, but I could not find a non-arbitrary way of constructing this sequence.[25]

To analyze the effect of the ZLB on aggregate dynamics, I proceed as follows: I simulate both economies for $T = 2399980$ and construct 5000 random draws of the state in both models. Each draw is going to be one economy. In the case of the menu cost model, the state includes the distribution of relative prices. Then I simulate over 100 months; at t=101, I feed all the 5,000 economies with the same shocks; and from then onward keep on independently simulating each economy. The impulse response is the average over the 5,000 economies for each period. Figure 11 describes the impulse response at 0 and 4 % target inflation. As discused above, the main driving shock that triggers the ZLB constraint is given by the risk premium shock. For this reason, at period 100, I hit all the economies with a large positive risk premium shock given by $10\sigma_Q$ to analyze the dynamics during a binding ZLB.

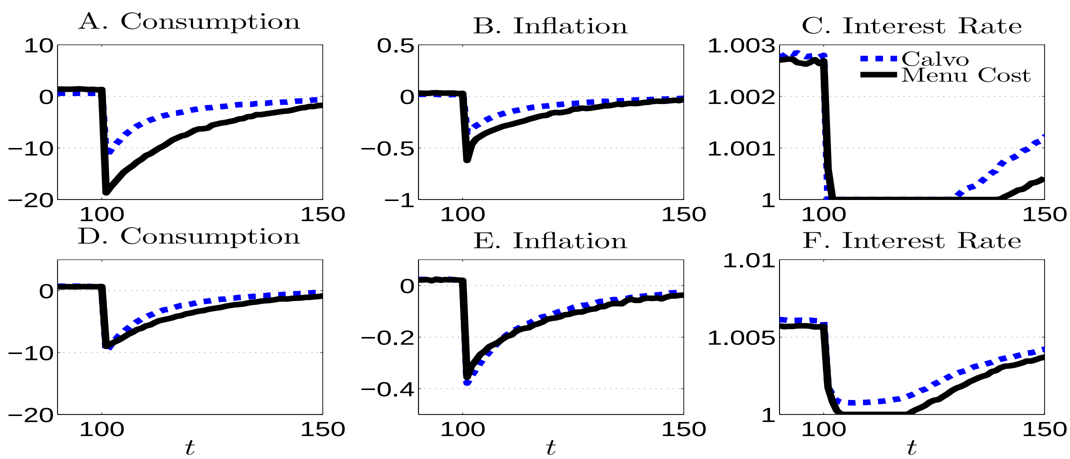

Figure 11: Panels A to C describe consumption, inflation, and nominal interest rates in the Calvo and in the menu cost models for 0 % target inflation. Panels D to F describe consumption, inflation, and nominal interest rates in the Calvo and in the menu cost models for 4 % target inflation.

An initial drop in consumption decreases inflation and if the zero lower bound is binding, this initial drop in inflation increases real interest rates, even further depressing the consumption. In figure 11, panels A to C describe consumption, inflation and nominal interest rates for a risk premium shock at zero target inflation. On impact, there is a large deflation, especially in the menu cost model. This large deflation does not affect aggregate dynamics since real interest rates depend on expected inflation and nominal interest rates is constraint and cannot react to inflation.

At higher inflation target Calvo and menu cost models have similar aggregate dynamics when the ZLB is binding. To understand this we have to see *which prices are activated* during periods

[25] Another method to compute the impulse-response is starting in an initial state where the ZLB is binding and analyzing the response to aggregate shocks conditional from this state. Given that the distribution of relative price is part of my state, I choose not to use this methodology.

where the monetary authority is binding. Remember that inflation dynamics is given by

$$\Pi(S) = \left(\frac{1-\Omega(S)}{1-\Omega(S)P^*(S)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(S) \tag{38}$$

Note that at zero inflation there is a *persistent* increase in the fraction of repricing firms. The key feature in menu cost models is that the increase in the fraction of repricing firms is not random across firms. This new mass of repricing firms hits the upper Ss band with a large downward price adjustment, which is not present at higher inflation target level as figure 12 shows.

Menu cost models during periods where the ZLB is binding are similar to a domino effect. An initial drop in consumption decreases the real marginal cost and triggers downward price adjustments with a large price change. If ZLB is not binding, this drop in inflation decreases nominal interest. Given that ZLB is binding, real interest rates increase, generating even further drops in the marginal cost and triggering new price adjustments. The key assumption to break this deflationary spiral is the elasticity of the real marginal cost with respect to real interest rates. Complementarities and GHH preferences decrease the elasticity of the real marginal cost significantly with respect to real interest rates.

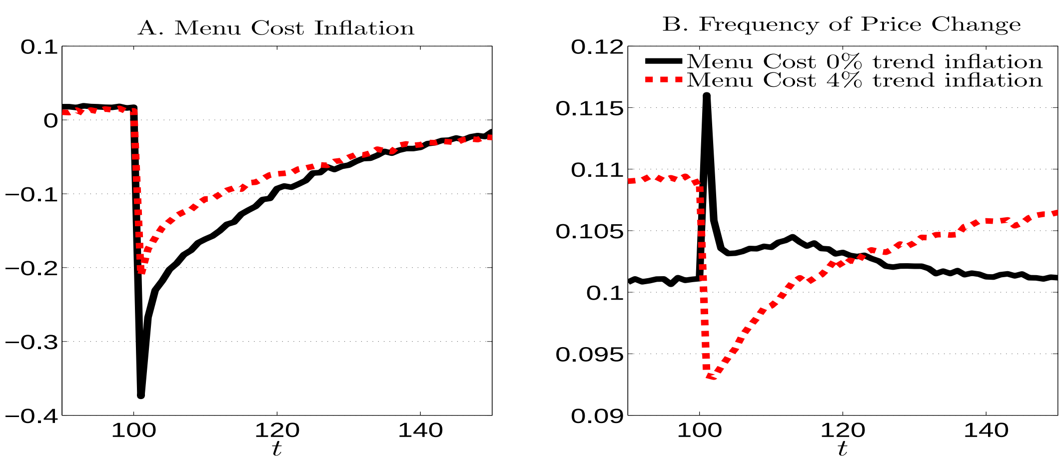

Figure 12: Panels A and B describe menu cost inflation and frequency of price change in menu cost models

There are two mechanisms to explain why dynamics in menu cost and Calvo are the same at 4 % inflation target. First, at higher target inflation, the measure in the upper Ss is smaller since on average there is positive inflation. In figure 13 we can see the ergodic mean distribution of relative prices at 0% and 4% inflation target under different conditional sets. When ZLB is not binding for higher levels of target inflation, the mass near the upper Ss bands is near zero, and therefore the mass with downward price adjustment is lower. Second, at 4% inflation target, a drop in inflation offsets the initial target inflation. The fact that during periods of binding ZLB there is no drift in the distribution implies that there are not price adjustments because prices hit the Ss bands.

Therefore the only prices changed during periods of binding ZLB are the ones that are hit by idiosyncratic productivity shocks. This is the main reason, as shown in figure 13, the distribution of relative prices at 0 % target inflation and no binding ZLB is equal to the distribution of relative prices at 4 % target inflation and binding ZLB.

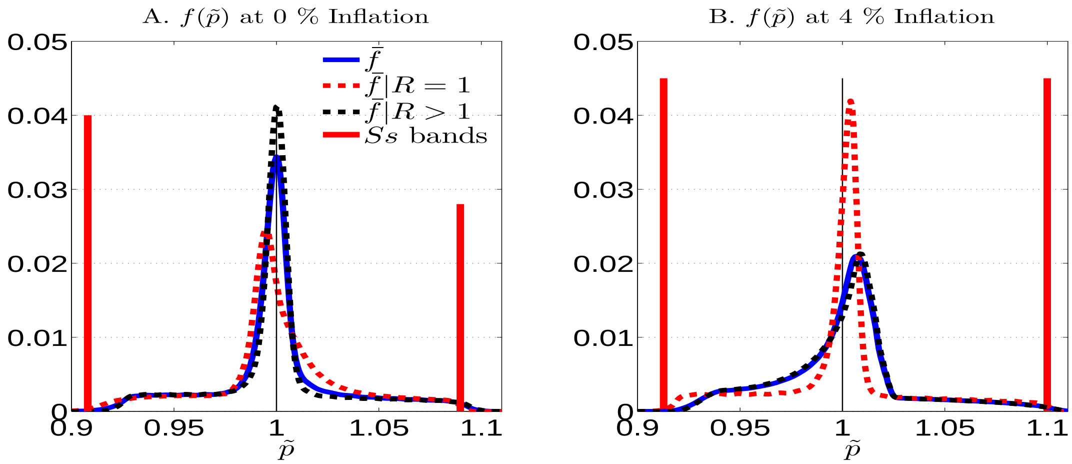

Figure 13: Panel A describes the ergodic mean distribution, the mean ergodic distribution conditional of no binding ZLB and the mean and the mean ergodic distribution whenever the ZLB is binding at zero inflation. The vertical read lines are the mean Ss bands conditional of a binding ZLB. Panel B is similar for 4 % target inflation. All the ergodic mean are given by $\frac{\sum_t^T f_t(\tilde{p}) I(X_t)}{\sum_t^T I(X_t)}$ where $X_t = \{R_t \geq 1, R_t > 1, R_t = 1\}$

In the Calvo pricing model, consumption falls during periods of binding ZLB because the marginal cost decreases and because price dispersion increases. This is not the same in the menu cost model; the only relevant wedge is given by markups and there is no additional contribution due to price dispersion. Moreover, inflation decreases at the same magnitude as the marginal cost in the Calvo model, since the slope of the Phillips curve is almost constant. This property doesn't hold in the menu cost model, since inflation targets affect the slope of the Phillips curve differently when the ZLB is binding, as I explain above. Table 1 describes business cycle statistics in Calvo and menu cost models. From row 3 to 8, I compute the mean of the logarithm of the variable conditional on hitting the ZLB and I subtract the unconditional mean.

7 Conclusion

As Golosov and Lucas (2007) observe, US firms change their prices once a year, with an average size of 10 % and half of these changes are downward. This cannot be rationalized in a model where firms react to aggregate inflation only, but it can be rationalized in a model with menu cost and idiosyncratic shocks. This paper used this framework, founded on micro data, to find an optimal inflation target of 5 %, twice as high as several leading sticky price models (See Coibion, Gorodnichenko, and Wieland (2012)). In order to do this, I extend a menu cost model with

Mean	Calvo $\bar{\pi}=0$	Calvo $\bar{\pi}=4$	Calvo % change	Menu Cost $\bar{\pi}=0$	Menu Cost $\bar{\pi}=4$	Menu Cost % change
Prob. of hitting ZLB	0.25	0.029	-88 %	0.31	0.037	-65 %
Monthly duration	19.55	4.3	-77 %	21.64	11	-43 %
Consumption drop	-5.21	-9.38	78 %	-7.52	-9.5	28 %
Price dispersion drop	0.28	0.71	153%	0.03	0.04	33%
Marginal cost drop	-0.84	-1.42	40 %	-1.42	-1.96	38%
Inflation drop	$-.57$	-0.82	43 %	-0.72	-0.81	12 %
Reset price drop	-1.34	-2.32	73 %	$-.91$	-1.12	38 %
Menu cost inflation drop	$(-)$	$(-)$	$(-)$	-0.15	-0.17	13 %

Table 1: Row 3 to 8 is computed as $\mathbb{E}[\log(x)|R=1]-\mathbb{E}[\log(x)]$ where x is consumption, price dispersion, marginal cost, inflation, reset price and menu cost inflation.

idiosyncratic shocks to a standard new Keynesian framework with a Taylor rule subject to a ZLB constraint and rich aggregate dynamics given by aggregate, government expenditure, money and risk premium shocks. The main reason for this result is that price dispersion –the main cost in sticky price model– has low sensitivity with respect to inflation target. Moreover, the likelihood of hitting the ZLB constraint and deflationary spirals during period when the ZLB is binding are decreasing with the inflation target.

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8 Table and Graphs

8.1 Calibration

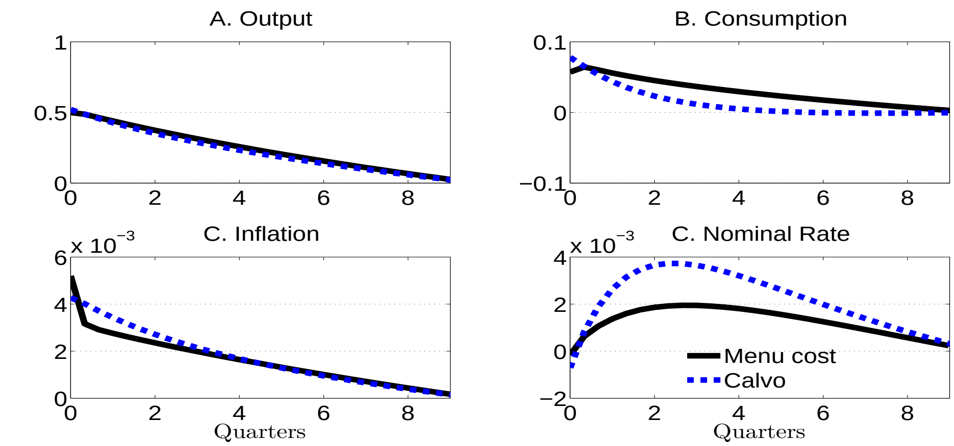
Calibrated Parameters

Parameter	Value	Target (data,model)
β	$0.96^{1/12}$	Interest Rate 4%
σ_{ez}	-45	Learning stage
(σ_{np}, χ)	$(1.5, 0.5)$	Fisch Elasticity 2.5
γ	5	Micro-estimate (4-6,5)
α	0.7	Kehoe-Midrigan (2014)
σ_a	0.15	Std log-price change (8%)
p	0.17	Mean frequency of price change (10m)
θ	0.8	Physical Cost of Menu Cost (0.5% of GDP)
$(\phi_r, \phi_\pi, \phi_y, \phi_{dy})$	$(0.8, 1.7, 0.05, 0.08)$	adjusted-Justiniano et al. (2010)
(ρ_Z, σ_Z)	$(0.91, 0.002)$	Output Per Person (BLS)
(ρ_G, σ_G)	$(0.97, 0.0035)$	Justiniano et al. (2010)
(ρ_R, σ_R)	$(0, 0.0006)$	Output Per Person (BLS)
(ρ_Q, σ_Q)	$(0.98, 0.0008)$	ZLB frequency and duration

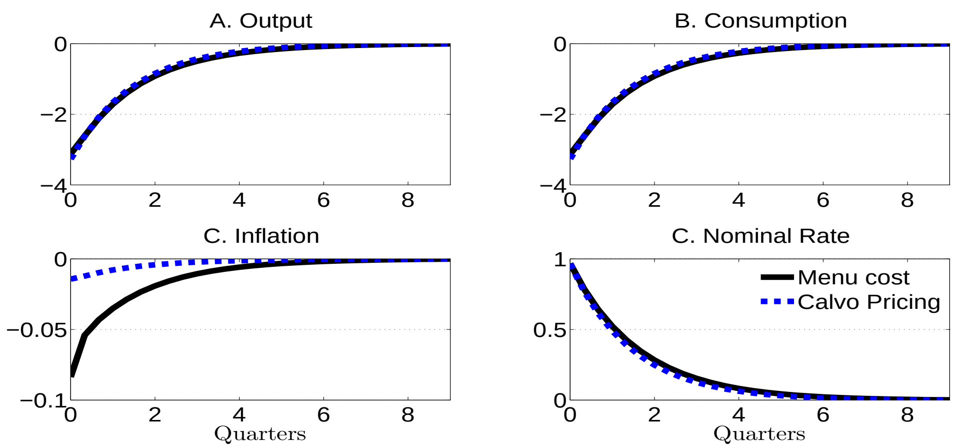
8.2 Welfare Cost of Inflation: Micro Price Statistics

Parameter	target 0	target 3	target 6
mean of abs. price change	17.89	17.77	17.2
std. dev. of of abs. price change	7.8	7.9	8
Kurtosis of abs. price change	1.18	1.19	1.26
min. of abs. price change	7	7.4	6.46
exp. time bet. price changes	9.91	9.72	8.79
std. of time bet. price changes	9.16	8.81	7.36
Ske. of time bet. price change	1.91	1.87	1.71
Std reset price	0.6	0.6	0.43
Std menu Cost inflation	0.08	0.09	0.12
Std frequency	3.36	6.54	11.86

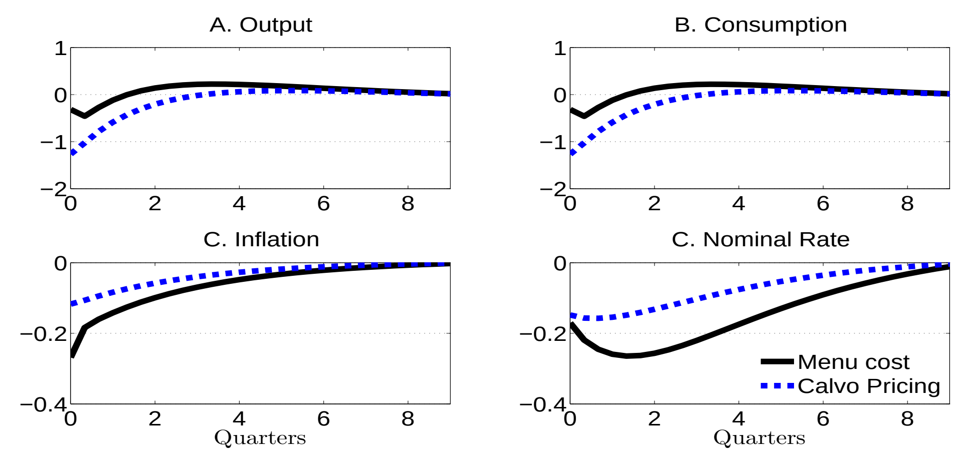
8.3 Welfare Cost of Inflation: Business Cycle without ZLB



government shock.



Money shock.



TFP shock.

8.4 Quantitative Analyzis appendix

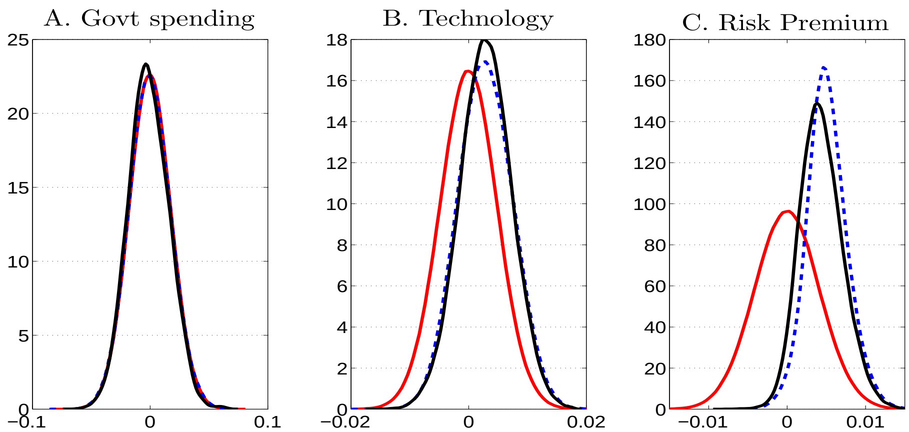


Figure 14: Panel A, B and C describe unconditional distribution of the strutural shocks and distribution of the structural shocks conditional of hitting the ZLB for 4 % level of trend inflation.

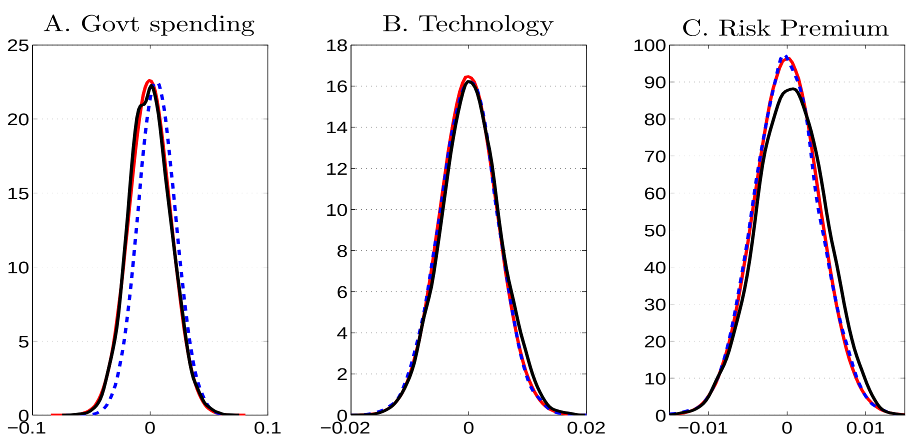


Figure 15: Panel A, B and C describe unconditional distribution of the strutural shocks and distribution of the structural shocks conditional of hitting the ZLB for 4 % level of trend inflation.

9 Data Appendix

To construct the probability of hitting the ZLB I construct a data base for all the countries of nominal interest rate using the call rates and CPI in the world. I discard countries where I didn't have data from 1990Q1-2013Q1. This data base consist of the following countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Israel, Japan, Korea, Mexico, New Zealand, Peru, Philippines, South Africa, Switzerland, United Kindom and United States. I use probability of hitting ZLB whenever the call rate is less than 0.07. Table 9 describes the data for each country where the last column and row compute the average across countries with inflation less than 4 %.

country	Time period	ZLB prob.	mean inflation	in/out
Australia	90Q1-13q1	2.573	0	in
Belgium	90Q1-13q1	2.133	.068	in
Canada	90Q1-13q1	1.987	.045	in
Denmark	90Q1-13q1	2.082	.038	in
Finland	90Q1-13q1	1.803	.068	in
France	90Q1-13q1	1.708	.068	in
Germany	90Q1-13q1	1.925	.0684	in
Israel	90Q1-13q1	5.273	.0228	out
Japan	90Q1-13q1	.2270	.540	in
Korea	90Q1-13q1	3.878	0	in
Mexico	90Q1-13q1	9.995	0	in
New Zealand	90Q1-13q1	2.169	0	out
Peru	90Q1-13q1	23.662	0	out
Philippines	90Q1-13q1	5.906	0	out
South Africa	90Q1-13q1	7.014	0	out
Switzerland	90Q1-13q1	1.222	.2663	in
United Kindom	90Q1-13q1	2.478	.1293	in
United States	90Q1-13q1	2.563	.038	in
Mean	90Q1-13q1	5.89	.144	2.058 (.1106)

10 Technical Appendix

10.1 Steady State

Proposition 7 *Let τ be a stopping time with one of the following properties*

1. *$\exists c \in R$ s.t. $\tau < c$ almost surely.*
2. *Or $Pr[\tau < \infty] = 1$ and X_t^i for i=1,2 define as*

$$dX_t^i = \left[\left(\sum_{i=1}^{N_t} \eta_i \right)^i - \mathbb{E}[\eta^i] t \right] dt \tag{39}$$

with the property that $\mathbb{E}[|X_t^i|] < \infty$ and $\lim_{t\to\infty} \mathbb{E}[|X_t^i I_{\{\tau>t\}}|] = 0$ for $i = 1, 2$.

Then if there exists an ergodic distribution of relative prices we have that

$$\begin{aligned} \mathbb{V}_i^{\bar{\pi}}[\hat{p}_i] &= \frac{\left(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi}\right)^2}{12} \mathcal{D}(\mathbb{V}^{\bar{\pi}}[\hat{\tau}], Ske^{\bar{\pi}}[\hat{\tau}]) + \frac{\mathbb{E}^x[\Delta p^2] - x^2 \mathbb{E}^x[\tau^2]}{2} \frac{\mathbb{E}^{\bar{\pi}}[\tau]}{\mathbb{E}^x[\tau]} (1 + \mathbb{V}^{\bar{\pi}}[\hat{\tau}]) \\ \mathcal{D}(\mathbb{V}^{\bar{\pi}}[\hat{\tau}], Ske^{\bar{\pi}}[\hat{\tau}]) &= 1 + \mathbb{V}^{\bar{\pi}}[\hat{\tau}]^{\frac{3}{2}} Ske^{\bar{\pi}}[\hat{\tau}] + 3\mathbb{V}^{\bar{\pi}}[\hat{\tau}](2 - \mathbb{V}^{\bar{\pi}}[\hat{\tau}]) \end{aligned} \tag{40}$$

where $\hat{\tau} = \frac{\tau}{\mathbb{E}^{\bar{\pi}}[\hat{\tau}]}$.

Proof. The stochastic process for the relative price is given by

$$\hat{p}_t = \hat{p}^* - \pi t + \sum_{i=1}^{N_t} \eta_i \tag{41}$$

Remember that by definition of relative price $\mathbb{E}[\hat{p}_i] = 0$. To compute the reset relative price note that

$$\begin{aligned} 0 &= \int \hat{p} f(d\hat{p}) \\ &= \frac{\int_0^\tau \hat{p}_t dt}{\mathbb{E}^{\bar{\pi}}[\tau]} \\ &= \frac{\mathbb{E}[\int_0^\tau \left(\hat{p}^* - \pi s + \sum_{i=1}^{N_s} \eta_i\right) ds]}{\mathbb{E}^{\bar{\pi}}[\tau]} \\ &= \hat{p}^* - \bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} + \frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau \sum_{i=1}^{N_s} \eta_i\right]}{\mathbb{E}^{\bar{\pi}}[\tau]} \end{aligned}$$

Where in the second line I use that the undiscounted first moment also describes the cross-sectional average (see Stokey (2008)). Now we'll show the the last term is zero. Define

$$X_t = \int_0^t \sum_{i=1}^{N_s} \eta_i ds \qquad X_0 = 0 \tag{42}$$

First it is easy to see that X_t is a martingale.

$$\mathbb{E}[X_{t+j}|F_t] = X_t + \mathbb{E}[\int_t^{t+j} \sum_{i=1}^{N_s} \eta_i ds] = X_t \tag{43}$$

Applying the optimal sampling theorem

$$\mathbb{E}[X_\tau|F_0] = X_0 = 0 \tag{44}$$

Therefore the reset price is given by

$$\hat{p}^* = \bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} \tag{45}$$

The variance of the relative prices is given by

$$\begin{aligned} \mathbb{V}^{\bar{\pi}}[\hat{p}_i] &= \int \hat{p}^2 f^{\bar{\pi}}(d\hat{p}) - \left(\int \hat{p} f^{\bar{\pi}}(d\hat{p})\right)^2 \\ &= \frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau \left(\hat{p}^* - \pi s + \sum_{i=1}^{N_s} \eta_i\right)^2 ds\right]}{\mathbb{E}^{\bar{\pi}}[\tau]} \\ &= (\hat{p}^*)^2 + \bar{\pi}^2 \frac{\mathbb{E}^{\bar{\pi}}[\tau^3]}{3\mathbb{E}^{\bar{\pi}}[\tau]} + \frac{\mathbb{E}^{\bar{\pi}}\left[(\sum_{i=1}^{N_s} \eta_i)^2 ds\right]}{\mathbb{E}^{\bar{\pi}}[\tau]} - (\hat{p}^*)\bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{\mathbb{E}^{\bar{\pi}}[\tau]} + (\hat{p}^*)\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau \sum_{i=1}^{N_s} \eta_i ds\right]}{\mathbb{E}^{\bar{\pi}}[\tau]} - \bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau s \sum_{i=1}^{N_s} \eta_i ds\right]}{\mathbb{E}^{\bar{\pi}}[\tau]} \end{aligned}$$

With simular argument as before the last two terms are zero. Given that

$$\mathbb{E}[\int_0^t (\sum_{i=1}^{N_s} \eta_i)^2 ds] = \int_0^t \mathbb{E}[\sum_{i=1}^{N_s} \eta_i^2] ds = \int_0^t \mathbb{V}[\sum_{i=1}^{N_s} \eta_i^2] ds = \lambda \mathbb{E}[\eta^2]\frac{t^2}{2} \tag{46}$$

Where in the last inequality I used that $\mathbb{E}[\eta^2] = 0$. Therefore $\int_0^t (\sum_{i=1}^{N_s} \eta_i)^2 ds - \lambda \mathbb{E}[\eta^2]\frac{t^2}{2}$ is a martingale. Using the optimal sampling theorem

$$\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau (\sum_{i=1}^{N_s} \eta_i)^2 ds\right]}{\mathbb{E}[\tau]} - \lambda \mathbb{E}[\eta^2]\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2} = 0 \tag{47}$$

The variance of relative prices are given by

$$\mathbb{V}[\hat{p}_i]^{\bar{\pi}} = \bar{\pi}^2\left(\frac{\mathbb{E}^{\bar{\pi}}[\tau^3]}{3\mathbb{E}^{\bar{\pi}}[\tau]} - \frac{\mathbb{E}^{\bar{\pi}}[\tau^2]^2}{4\mathbb{E}^{\bar{\pi}}[\tau]^2}\right) + \lambda \mathbb{E}[\eta^2]\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} \tag{48}$$

To get an expression for the idiosyncratic shocks as function of observables note that using Ito lemma at zero inflation

$$d\hat{p}_t^2 = 2\hat{p}_t dp_t + \left((\eta_t + \hat{p}_{t-})^2 - \hat{p}_{t-}^2\right) dq_t = -2\hat{p}_t \bar{\pi} dt + \left[\eta_t + (\eta_t p_{t-}) + \eta_t^2\right] dq_t \tag{49}$$

Using the previous equation

$$\mathbb{E}[(\hat{p}_t - p^*)^2] = -2\bar{\pi}\int_0^t \mathbb{E}[\hat{p}_s]ds + \lambda\mathbb{E}[\eta^2]t = \bar{\pi}^2 t^2 ds + \lambda\mathbb{E}[\eta^2]t \tag{50}$$

where we used that $\mathbb{E}[\eta] = \mathbb{E}[\eta\hat{p}_-] = 0$. Therefore $(\hat{p}_t - \hat{p}^*)^2 + (\bar{\pi}t)^2 - \lambda\mathbb{E}[\eta^2]t$ is a martingale and using the optimal sampling theorem

$$\mathbb{E}^{\bar{\pi}}[\Delta p^2] = \mathbb{E}^{\bar{\pi}}[(\hat{p}_\tau - \hat{p}_*)^2] = \bar{\pi}^2\mathbb{E}^{\bar{\pi}}[\tau^2] + \lambda\mathbb{E}[\eta^2]\mathbb{E}^{\bar{\pi}}[\tau] \tag{51}$$

Therefore

$$\lambda\mathbb{E}[\eta^2] = \frac{\mathbb{E}^{\bar{\pi}}[\Delta p^2] - \bar{\pi}^2\mathbb{E}^{\bar{\pi}}[\tau^2]}{\mathbb{E}^{\bar{\pi}}[\tau]} \tag{52}$$

and we have that

$$\mathbb{V}[\hat{p}_i] = \bar{\pi}^2\left(\frac{\mathbb{E}^{\bar{\pi}}[\tau^3]}{3\mathbb{E}^{\bar{\pi}}[\tau]} - \frac{\mathbb{E}^{\bar{\pi}}[\tau^2]^2}{4\mathbb{E}^{\bar{\pi}}[\tau]^2}\right) + \frac{\mathbb{E}^x[\Delta p^2] - x^2\mathbb{E}^x[\tau^2]}{\mathbb{E}^x[\tau]}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} \tag{53}$$

We can write second term as

$$= \frac{\mathbb{E}^0[\Delta p^2]}{\mathbb{E}^0[\tau]}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} \tag{54}$$

$$= \mathbb{E}^0[\Delta p^2]\frac{\mathbb{E}^{\bar{\pi}}[\tau]}{\mathbb{E}^0[\tau]}\frac{\mathbb{E}[\tau^2]}{2\mathbb{E}[\tau]^2} \tag{55}$$

$$= \frac{\mathbb{E}^0[\Delta p^2]}{2}\frac{\mathbb{E}^{\bar{\pi}}[\tau]}{\mathbb{E}^0[\tau]}(1 + \mathbb{V}[\hat{\tau}]) \tag{56}$$

where $\hat{\tau} = \frac{\tau}{\mathbb{E}^{\bar{\pi}}[\tau]}$. For the first term

$$= \bar{\pi}^2\left(\frac{\mathbb{E}^{\bar{\pi}}[\tau^3]}{3\mathbb{E}^{\bar{\pi}}[\tau]} - \frac{\mathbb{E}^{\bar{\pi}}[\tau^2]^2}{4\mathbb{E}^{\bar{\pi}}[\tau]^2}\right) \tag{57}$$

$$= \frac{\left(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi}\right)^2}{12}\left(4\mathbb{E}^{\bar{\pi}}[\hat{\tau}^3] - 3\mathbb{E}^{\bar{\pi}}[\hat{\tau}^2]^2\right) \tag{58}$$

$$= \frac{\left(\mathbb{E}^{\bar{\pi}}[\tau]\bar{\pi}\right)^2}{12}\left(1 + \mathbb{V}[\hat{\tau}]^{\frac{3}{2}}Ske(\hat{\tau}) + 3\mathbb{V}[\hat{\tau}](2 - \mathbb{V}[\hat{\tau}])\right) \tag{59}$$

Where we use that $\mathbb{V}[\hat{\tau}]^{\frac{3}{2}}Ske[\hat{\tau}] = \mathbb{E}^{\bar{\pi}}[\hat{\tau}^3] - 3\mathbb{V}[\hat{\tau}] - 1$ ■

Proposition 8 *Assume a quadratic approximation of the profit function $B\hat{p}_t^2$ with $h(\eta)$ symmetric around zero. Define*

$$U = \frac{\log(\max_{\tilde{p}}\{\Psi(S_{ss})\})}{P^*}$$
$$L = \frac{\log(\min_{\tilde{p}}\{\Psi(S_{ss})\})}{P^*}$$

Then $U = u(\frac{\bar{\pi}}{\rho+\lambda}, \tilde{\theta})$ and $U = l(\frac{\bar{\pi}}{\rho+\lambda}, \tilde{\theta})$ with the following properties

1. *$u(\cdot), l(\cdot)$ are increasing in both arguments.*
2. *$\lim_{x\to 0^+} u_x(x,y) = \lim_{x\to 0^+} l_y(x,y) = 0$ with*

$$\mathcal{E}_{u,x} \in (0, 1/3) \quad ; \quad \mathcal{E}_{l,x} \in (0, 1/3)$$

increasing in x.

Proof. This proof is not complicate but it is long. Therefore I'm going to skip some steps in the proof. It would be easier to redefine the firm's problem in the markup space, since the reset markup is a decision variable (reset price is an equilibrium condition, see above). Note that the Ss band in markups space is a re-scalation of the Ss in relative price space. The firm's problem is given by

$$V(\mu) = \max_{\tau} \mathbb{E}\left[\int_0^{\tau} -e^{-\rho t} B\hat{\mu}_t^2 dt + e^{-\rho\tau}(\max_x V(x) - \theta)\right] \tag{60}$$

subject to

$$d\hat{\mu}_t = -\bar{\pi}dt + \eta_t^a dq \text{ with } \hat{\mu}_0 = \mu \tag{61}$$

where q_t is a Poisson problem with arrival rate λ. The first step consist in showing that if $\tilde{\theta}, h''(0)$ small then the policy for low level of inflation can be computed as

$$L = \tilde{\pi}\bar{L}\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right) \quad \mu^* = \tilde{\pi}\bar{\mu}^*\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right) \quad U = \tilde{\pi}\bar{U}\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right)$$

where $\bar{L}(x), \bar{U}(x), \bar{\mu}^*(x)$ satisfy the triangular system

$$\begin{aligned}
x &= \bar{L}(x)^2 + 2\frac{\bar{L}(x)}{e^{\bar{L}(x)}-1}\left(-\left(e^{\bar{L}(x)}-1\right) + L\bar{(}x)\right) \\
x &= \bar{U}(x)^2 - 2\frac{\bar{L}(x)}{e^{\bar{L}(x)}-1}\left((e^{-\bar{U}(x)}-1) + \bar{U}(x)\right) \\
\bar{\mu}^* &= 1 - \frac{\bar{L}(x)}{\exp(\bar{L}(x)) - 1}
\end{aligned}$$

Note that the JBE is given by

$$(\rho+\lambda)V(\mu) = -\mu^2 + \lambda\int_{-\infty}^{\infty}\max\{V(\mu^*) - \frac{\theta}{B}, V(\mu+x)\}h(x)dx - \pi V_\mu'(\mu) \tag{62}$$

With the optimality and border conditions

$$V_\mu'(\mu^*(\pi)) = 0 \tag{63}$$

$$V(\mu^* - L) = V(\mu^* + U) = V(\mu^*) - \frac{\theta}{B} \tag{64}$$

and the smooth pasting condition

$$V'(\mu^* - L) = 0 \tag{65}$$

Using the assumption of symmetry ($h'(0) = h'''(0) = 0$) and optimality of μ^*

$$\begin{aligned}
&= \int_{-\infty}^{\infty}\max\{V(\mu^*) - \theta, V(\mu+x)\}h(x)dx \\
&= V(\mu^*) - \frac{\theta}{B} + \int_{[-L,U]}(V(\mu^*+x) - (V(\mu^*) - \theta))\,h(\mu^* - \mu + x)dx \\
&= V(\mu^*) - \frac{\theta}{B} + \left[\int_{[-L,U]}\left(\frac{\theta}{B} + \frac{V_{\mu^2}(\mu^*)}{2}x^2 + O(x^3)\right)(h(0) + \frac{h''(0)}{2}(\mu^* - \mu + x)^2) + O((\mu^* - \mu + x)^4)\right]dx \\
&= B_1 + \frac{h''(0)}{2}\left((\mu^* - \mu)(\theta\frac{U^2 - L^2}{2} + \frac{V''(\mu^*)}{2}\frac{U^4 - L^4}{4}) + (\mu^* - \mu)^2(\theta(U+L) + \frac{V''(\mu^*)}{2}\frac{U^3 - L^3}{3})\right) + \ldots \\
&\cdots + \int_{[-L,U]} O((\mu^* - \mu + x)^4 x^2)dx \\
&= B_0 + \frac{h''(0)}{2}\left((\mu^* - \mu)(\theta\frac{U^2 - L^2}{2} + \frac{V''(\mu^*)}{2}\frac{U^4 - L^4}{4}) + (\mu^* - \mu)^2(\theta(U+L) + \frac{V''(\mu^*)}{2}\frac{U^3 - L^3}{3})\right) + o((\mu^* - \mu)^4)
\end{aligned}$$

Note that if $h''(0)$ goes to zero then this term coverge to a constant. But even if $h''(0)$ does not converge to a constant, all the term in the expantion are small $U^2 - L^2, \frac{\theta}{B}$ with respect to μ^2 (the norminlize static profit). Therefore $V(\mu) = B_0 + G(\mu)$. Using a Taylor approximation over $G(\mu)$

$$G(\mu) = \sum_{n=0}^{\infty} \frac{G^n(\mu^*)}{n!}(\mu - \mu^*)^n \tag{66}$$

For $n \geq 3$ and $\mu \in [\mu^* - L, \mu^* + U]$ we have that

$$(r + \lambda) G^n(\mu^*) = -\pi G^{n+1}(\mu^*) \tag{67}$$

Skipping intermediate step, it is easy to see that the value function is given by

$$G(\mu) = G(\mu^*) + \frac{G^2(\mu^*)}{2}(\mu - \mu^*)^2 + G^3(\mu^*)\left[-\tilde{\pi}^3 \exp\left(-\frac{\mu - \mu^*}{\tilde{\pi}}\right) + \tilde{\pi}^3 - \tilde{\pi}^2(\mu - \mu^*) + \tilde{\pi}\frac{(\mu - \mu^*)^2}{2}\right] \tag{68}$$

$$= G(\mu^*) + \frac{G^2(\mu^*) + \tilde{\pi}G^3(\mu^*)}{2}(\mu - \mu^*)^2 + V^3(\mu^*)\left[-\tilde{\pi}^3 \exp\left(-\frac{\mu - \mu^*}{\tilde{\pi}}\right) + \tilde{\pi}^3 - \tilde{\pi}^2(\mu - \mu^*)\right] \tag{69}$$

Imposing smooth-pasting condition over G

$$\frac{L}{\tilde{\pi}} = \frac{G^3(\mu^*)\tilde{\pi}}{G^2(\mu^*) + G^3(\mu^*)\tilde{\pi}}\left(\exp\left(-\frac{L}{\tilde{\pi}}\right) - 1\right) \tag{70}$$

From the other two border conditions

$$-\tilde{\theta} = (r + \lambda)\frac{G^2(\mu^*) + G^3(\mu^*)\tilde{\pi}}{2}L^2 + (r + \lambda)G^3(\mu^*)\tilde{\pi}[-\tilde{\pi}^2(\exp\left(\frac{L}{\tilde{\pi}}\right) - 1) + \tilde{\pi}L] \tag{71}$$

$$-\tilde{\theta} = (r + \lambda)\frac{G^2(\mu^*) + G^3(\mu^*)\tilde{\pi}}{2}U^2 + (r + \lambda)G^3(\mu^*)\tilde{\pi}[-\tilde{\pi}^2(\exp\left(-\frac{U}{\tilde{\pi}}\right) - 1) - \tilde{\pi}U] \tag{72}$$

Using the previous conditions and working with the algebra

$$\begin{aligned}
\tilde{\theta} &= L^2 + 2\frac{L/\tilde{\pi}}{\exp(L/\tilde{\pi}) - 1}\left(-\tilde{\pi}^2\left(e^{L/\tilde{\pi}} - 1\right) + \tilde{\pi}L\right) \\
\tilde{\theta} &= U^2 + 2\frac{L/\tilde{\pi}}{\exp(L/\tilde{\pi}) - 1}\left(-\tilde{\pi}^2(e^{-U/\tilde{\pi}} - 1) - \tilde{\pi}U\right) \\
\mu^* &= \tilde{\pi}\left(1 - \frac{L/\tilde{\pi}}{\exp(L/\tilde{\pi}) - 1}\right)
\end{aligned}$$

or equivalently

$$\begin{aligned}
\frac{\tilde{\theta}}{\tilde{\pi}^2} &= \bar{L}^2 + 2\frac{\bar{L}}{e^{\bar{L}} - 1}\left(-\left(e^{\bar{L}} - 1\right) + \bar{L}\right) \\
\frac{\tilde{\theta}}{\tilde{\pi}^2} &= \bar{U}^2 - 2\frac{\bar{L}}{e^{\bar{L}} - 1}\left((e^{-\bar{U}} - 1) + \bar{U}\right) \\
\bar{\mu}^* &= 1 - \frac{\bar{L}}{\exp(\bar{L}) - 1}
\end{aligned}$$

where

$$L = \tilde{\pi}\bar{L}\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right) \quad \mu^* = \tilde{\pi}\bar{\mu}^*\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right) \quad U = \tilde{\pi}\bar{U}\left(\frac{\tilde{\theta}}{\tilde{\pi}^2}\right)$$

Now we need to show that $\bar{L}(x)$ and $\bar{U}(x)$ satisfy:

1. $\lim_{x\to\infty}\left(\frac{\bar{L}(x)}{\sqrt{x}},\frac{\bar{U}(x)}{\sqrt{x}}\right)=1.$
2. $\bar{L}(x)$ and $\bar{U}(x)$ are increasing in x.
3. $\Xi_{\bar{U},x},\Xi_{\bar{L},x}$ increases from 1/3 to 1/2.

Taking limit

$$\lim_{\bar{L}\downarrow 0}\bar{L}^2+2\overbrace{\frac{\bar{L}}{e^{\bar{L}}-1}}^{=1}\overbrace{\left(-\left(e^{\bar{L}}-1\right)+\bar{L}\right)}^{=-\bar{L}+\bar{L}=0}=0 \tag{73}$$

Therefore $L(x)$ finish $\sqrt{x}$, with $\lim_{x\downarrow 0}\bar{L}(x)=0$. The last two properties can be checked numerically or using Taylor approximation of arbitrary number. Similar arguments with $U(x)$. ■

Proposition 9 *For any model of firm behaviour (τ), the aggregate markup gap is given by*

$$\hat{\mathcal{M}}_{ss}=\bar{\pi}\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]+\left(\gamma-\frac{1}{\gamma-1}\right)\mathbb{E}[\int_0^\infty b(\rho,\tau,s)(\hat{p}_s-w_{ss})^2ds] \tag{74}$$

Where

- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]<0$ *and* $\mathbb{E}\left[\int_0^\infty a(0,\tau,s)sds\right]=0$
- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]$ *is decreasing in* ρ
- $\mathbb{E}\left[\int_0^\infty a(\rho,\tau+1,s)sds\right]<\mathbb{E}\left[\int_0^\infty a(\rho,\tau,s)sds\right]$

More over $\rho\to 0$

$$\hat{\mathcal{M}}_{ss}\approx\left(\gamma-\frac{1}{\gamma-1}\right)\mathbb{V}_i[\hat{p}_i] \tag{75}$$

Proof. For the proof it is easier to work with markups $\hat{\mu}=\hat{p}-\log(mc)$. With similar argument as the previous proof it easy to show that

$$\mathbb{E}_i^{\bar{\pi}}[\hat{\mu}_i]=\mu^*-\bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]} \tag{76}$$

Given that in steady state the level of output and marginal utility is constant the optimal program of the firm is given by

$$= \left\{\int_0^\tau e^{-\rho t}K\mu_t^{-\gamma}(\mu_t-1)\right\} \tag{77}$$

$$= K_1+K\left\{\int_0^\tau e^{-\rho t}\left(\Phi_2(\mu^*)^2\hat{\mu}_t^2+\Phi_3(\mu^*)^3\hat{\mu}_t^3\right)\right\} \tag{78}$$

Using the envelope condition we have that

$$\mu^*=\bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}sds\right]}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}ds\right]}-\frac{3\Phi_3(\mu^*)^3}{2\Phi_2(\mu^*)^2}\frac{\mathbb{E}[\int_0^\tau e^{-\rho s}\hat{\mu}_s^2ds]}{\mathbb{E}[\int_0^\tau e^{-\rho s}ds]} \tag{79}$$

$$=\bar{\pi}\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}sds\right]}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}ds\right]}+\left(\gamma-\frac{1}{\gamma-1}\right)\frac{\mathbb{E}[\int_0^\tau e^{-\rho s}\hat{\mu}_s^2ds]}{\mathbb{E}[\int_0^\tau e^{-\rho s}ds]} \tag{80}$$

Conving the two previous results we have that

$$\mathbb{E}_i[\hat{\mu}_i]=\bar{\pi}\left(\frac{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}sds\right]}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}ds\right]}-\frac{\mathbb{E}^{\bar{\pi}}[\tau^2]}{2\mathbb{E}^{\bar{\pi}}[\tau]}\right)+\left(\gamma-\frac{1}{\gamma-1}\right)\frac{\mathbb{E}^{\bar{\pi}}[\int_0^\tau e^{-\rho s}\hat{\mu}_s^2ds]}{\mathbb{E}^{\bar{\pi}}[\int_0^\tau e^{-\rho s}ds]} \tag{81}$$

we can redefine

$$a(\rho,\tau,s) = \left[\frac{e^{-\rho s}}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}ds\right]} - \frac{1}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau 1 ds\right]}\right] I(s<\tau) \tag{82}$$

$$b(\rho,\tau,s) = \frac{e^{-\rho s}}{\mathbb{E}^{\bar{\pi}}\left[\int_0^\tau e^{-\rho s}ds\right]} I(s<\tau) \tag{83}$$

$$\tag{84}$$

Note that $a(\rho,\tau,s)$ is a decreasing function in s with the property that $\mathbb{E}^{\bar{\pi}}\left[\int_0^\infty a(\rho,\tau,s)ds\right]$, therefore $\mathbb{E}^{\bar{\pi}}\left[\int_0^\infty a(\rho,\tau,s)sds\right] < 0$. The second properties with respect the discount factor are strightforward. For the last propery note that if we define $s(\tau) = \min_x a(\rho,\rho,x)$, then $a(a,\tau,x) > a(\rho,\tau+1,x)$ for all $x \leq s(\tau)$. From this property is straightforward the last property.

Taking the limit of $\rho \to 0$

$$\mathbb{E}_i[\hat{\mu}_i] = \left(\gamma - \frac{1}{\gamma-1}\right)\left(\mathbb{V}_i[\hat{p}_i] + \mathbb{E}_i[\hat{\mu}_i]^2\right) \approx \left(\gamma - \frac{1}{\gamma-1}\right)\mathbb{V}_i[\hat{p}_i] \tag{85}$$

■

11 Numerical Appendix

The cost function is given by

$$\min W_t l_t + P_t N_t \qquad s.t. \qquad A_t \eta_{Z,t} N_{t,i}^{\alpha} l_{t,i}^{1-\alpha} = y_{t,i} \tag{86}$$

Define w_t as the real wage. The demand functions are given, total cost and marginal cost are given by

$$N_{t,i} = \frac{y_{t,i}}{\eta_{Z,t}A_{t,i}} w_t^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \tag{87}$$

$$l_{t,i} = \frac{y_{t,i}}{A_{t,i}\eta_{Z,t}} w_t^{-\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \tag{88}$$

$$TC_t = \frac{y_{t,i}}{A_{t,i}\eta_{Z,t}} w_t^{1-\alpha}\left(\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right) \tag{89}$$

$$MC_t = \frac{1}{A_{t,i}\eta_{Z,t}} w_t^{1-\alpha}\left(\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right) \tag{90}$$

Let define $\iota = \left(\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \right)$. From the equation of labor demand

$$\begin{aligned}
l_{t,i} - \Omega_t \theta &= \int_0^1 \left(\frac{y_{t,i}}{\eta_{Z,t} N_{t,i}^{\alpha}} \right)^{\frac{1}{1-\alpha}} di & (91)\\
&= \left(\frac{(1-\alpha)}{\alpha w_t} \right)^{\alpha} \frac{1}{\eta_{Z,t}} \int_0^1 \left(\frac{y_{t,i}}{A_{t,i}} \right) di & (92)\\
&= \frac{Y_t}{\eta_{Z,t}} \left(\frac{(1-\alpha)}{\alpha w_t} \right)^{\alpha} \int_0^1 \tilde{p}_{t,i}^{-\gamma} di & (93)\\
&= \frac{Y_t}{\eta_t^Z} Y_t \left(\frac{(1-\alpha)}{\alpha w_t} \right)^{\alpha} \Delta_t & (94)\\
\eta_t^G + C_t &= Y_t - \int N_{t,i} dt & (95)\\
&= Y_t \left(1 - \left(\frac{w_t \alpha}{1-\alpha} \right)^{1-\alpha} \frac{\Delta_t}{\eta_{Z,t}} \right) & (96)
\end{aligned}$$

11.1 Computation of the Equilibrium in the Menu Cost Model

The equilibrium conditions are given by

$$
\begin{aligned}
mu(S) &= \beta \mathbb{E}_{S'}[R(S)\Xi(S')mu(S')\frac{\eta^Q(S')}{\Pi(S')}|S] \\
\kappa L(S)^\chi &= w(S) \\
u(S) &= K\frac{(C(S)-\kappa\frac{L(S)^{1+\chi}}{1+\chi})^{1-\sigma_n}}{1-\sigma_n} \\
mu(S) &= K(C(S)-\kappa\frac{L(S)^{1+\chi}}{1+\chi})^{-\sigma_n} \\
U(S) &= u(S)-\beta\mathbb{E}[\left(-U(S')\right)^{1-\sigma_{ez}}|S]^{\frac{1}{1-\sigma_{ez}}} \\
\Xi(S) &= \left(\frac{-U(S)}{\mathbb{E}_t[(-U(S))^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \\
\frac{R(S)\beta}{1+\bar{\pi}} &= \max\left\{\frac{\beta}{1+\bar{\pi}}, \tilde{R}_{-1}(S)\left(\left(\frac{\Pi(S)}{1+\bar{\pi}}\right)^{\phi_\pi}\left(\frac{mc(S)}{mc_-(S)}\right)^{\tilde{\phi_y}}\right)^{1-\phi_r}\right\} \\
\tilde{R}(S) &= \left(\frac{R(S)\beta}{1+\bar{\pi}}\right)^{\phi_r} e^{\sigma^R\epsilon^R} \\
\eta^Z(S)\left(L(S)-\Omega(S)\theta\right) &= Y(S)\left(\frac{(1-\alpha)}{\alpha w(S)}\right)^\alpha \Delta(S) \\
\eta^G(S)+C(S) &= Y(S)\left(1-\left(\frac{w(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta(S)}{\eta^Z(S)}\right) \\
\Pi(S) &= \left(\frac{1-\Omega(S)}{(1-\Omega P^*(S)^{1-\gamma})}\right)^{\frac{1}{1-\gamma}}\varphi(S) \\
mc(S) &= \frac{\iota w(S)^{1-\alpha}}{\eta^Z(S)} \\
\Delta(S) &= \Omega(S)P^*(S)^{-\gamma}+(1-\Omega(S))\Pi(S)^\gamma \upsilon(S) \\
P^*(S) &= \arg\max_x\{V(x,S)\} \\
v(\tilde{p},S) &= \mathbb{E}_{\tilde{p}',S'}\left[\Xi(S')\max_{c,nc}\{\max_x\{V(x,S')\}-w(S')mu(S')\theta, V(\tilde{p}',S')\}|\tilde{p},S\right] \\
V(\tilde{p},S) &= \Phi(\tilde{p},S)+\beta v(\tilde{p},S) \\
\Phi(\tilde{p},S) &= mu(S)Y(S)\tilde{p}^{-\gamma}\left(\tilde{p}-mc(S)\right) \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}}{\Pi(S')} & \text{with pr. } e^{-\lambda} \\ \frac{\tilde{p}eta}{\Pi(S')} & \text{with pr. } 1-e^{-\lambda} \end{cases} \\
\log(\eta^Z(S')) &= (1-\rho_z)\log(\eta^Z)+\rho_z\log(\eta^Z(S))+\sigma_z\epsilon^Z \\
\log(\eta^G(S')) &= (1-\rho_g)\log(\eta^G)+\rho_g\log(\eta^G(S))+\sigma_g\epsilon^G \\
\log(\eta^Q(S')) &= (1-\rho_q)\log(\eta^Q)+\rho_q\log(\eta^Q(S))+\sigma_q\epsilon^Q
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{C}(S) &= \left\{(p, \Delta a) : \frac{pe^{\Delta a}}{\Pi(S)} \in \Psi(S)\right\} \\
\Delta(S) &= \int \tilde{p}^{-\gamma} f(d\tilde{p}) \\
\Omega(S) &= \int_{(\tilde{p},\Delta a)\notin \mathcal{C}(S)} f(d\tilde{p}_{-}) g(d\Delta a) \\
\Pi(S) &= \left(\frac{1-\Omega(S)}{(1-\Omega(S)P^{*}(S)^{1-\gamma})}\right)^{\frac{1}{1-\gamma}} \varphi(S) \\
\varphi(S) &= \left(\frac{\int_{\Gamma(S)} (\tilde{p}_{-})^{1-\gamma}}{1-\Omega(S)} f(d\tilde{p}_{-})\right)^{\frac{1}{1-\gamma}} \\
\Delta(S) &= \Omega(S)P^{*}(S)^{-\gamma} + (1-\Omega(S))\Pi(S)^{\gamma} \upsilon(S) \\
\upsilon(S) &= \left[\frac{\int_{\mathcal{C}} \tilde{p}_{-}^{-\frac{\gamma}{1-\alpha}} f(d\tilde{p}_{-})}{1-\Omega(S)}\right]
\end{aligned}
$$

11.2 Solution Algorith: General outline

The solution algorith consist in 4 steps

Step 1: Solve the steady state and obtain initial guess for the steady state

$$
(C_{ss}, L_{ss}, P^{*}_{ss}, \Pi_{ss}, R_{ss}, \Delta_{ss}, \Omega_{ss}, \varphi_{ss}, mc_{ss}) \tag{97}
$$

Step 2: Use perturbations methods to approximate the equilibrium dynamics without the ZLB. Approximate reset price with Calvo pricing using as probability of changing the price the probability of receving an idiosyncratic shock.

Step 3: Solve equilibrium conditions with global methods ignoring the zero lower bound.

Step 4: Solve equilibrium conditions with global methods with the zero lower bound.

11.3 Step 1: Solve Steady State economy

1. Guess $(\Delta^i_{ss}; \Omega^i_{ss}, mc^i_{ss})$ and solve

$$
\begin{aligned}
w_{ss} &= \left(\frac{mc_{ss}\eta^Z_{ss}}{\iota}\right)^{\frac{1}{1-\alpha}} \\
L_{ss} &= \left(\frac{w_{ss}}{\kappa}\right)^{\frac{1}{\chi}} \\
Y_{ss} &= \eta^Z_{ss}\frac{L_{ss}-\Omega_{ss}\theta}{\Delta_{ss}}\left(\frac{\alpha w_{ss}}{1-\alpha}\right)^{\alpha} \\
C_{ss} &= Y_{ss}\left(1-\left(\frac{w_{ss}\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_{ss}}{\eta^Z_{ss}}\right)-\eta^G_{ss} \\
u_{ss} &= K\frac{(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{1-\sigma_n}}{1-\sigma_n} \\
mu_{ss} &= K(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{-\sigma_n} \\
U_{ss} &= \frac{u_{ss}}{1-\beta} \\
\Pi_{ss} &= 1+\bar{\pi} \\
R_{ss} &= \frac{1+\bar{\pi}}{\beta} \\
\tilde{R}_{ss} &= 1
\end{aligned}
$$

2. Given $(Y^i_{ss}, mu^i_{ss}, w^i_{ss}, L^i_{ss})$, solve

$$
\begin{aligned}
v(\tilde{p})^i &= \mathbb{E}_{\tilde{p}'}\left[\max_{c,nc}\left\{\max_x\{V^i(x)^i\}-w_{ss}\theta, V^i(\tilde{p}')^i\right\}|\tilde{p}\right] \\
V^i(\tilde{p})^i &= \Phi(\tilde{p})+\beta v(\tilde{p}) \\
\Phi(\tilde{p}) &= mu_{ss}Y_{ss}\tilde{p}^{-\gamma}\left(\tilde{p}-mc_{ss}\right) \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}}{1+\bar{\pi}} & \text{with pr. } e^{-\lambda} \\ \frac{\tilde{p}e^{\sigma_a\epsilon^a}}{1+\bar{\pi}} & \text{with pr. } 1-e^{-\lambda} \end{cases}
\end{aligned}
$$

and get $(P^*_{ss})^i, U^i_{ss}, L^i_{ss}$.

(a) Technical 1: For the firm problem of spline of 3 order to approximate the value function.

(b) Technical 2: Use contraction together with colotion to solve the Bellman equation of the firm.

(c) Technical 3: Use Golden search to solve the firm problem.

3. Fix a grid between $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$ with n_s points. With $(P^*_{ss})^i, U^i_{ss}, L^i_{ss}$, construct the three transition $F_{\Delta a}, F_{\Pi}, F_{p'}$ where

(a) $F_{\Delta a}$ is given by the transition probability $\tilde{p}_1 = \tilde{p}\eta_t$.

(b) F_{Π} is given by the transition probability $\tilde{p}_1 = \frac{\tilde{p}}{\Pi_{ss}}$.

(c) $F_{p'}$ is given by the transition probability $\tilde{p}_1 = P^*_{ss}I(\tilde{p}\in C) + I(\tilde{p}\notin C)P^*_{ss}$.

using linear splines over the grid $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$.and compute the eigenvector of the unit eigenvalue of $F_{\Delta a}F_{\Pi}F_{p'}$ that gives the ergodic distribution n_{ss}. After having the ergodic distribution compute the reset inflation, price dispersion and frequency of price change.

- $n_{aux} = (n_{ss} F_{\Delta a} F_{\Pi}) I(p \in C)$.
- $\Omega_{ss} = 1 - \sum_i n_{aux}(i)$.
- $\varphi_{ss} = (1+\bar{\pi}) \left(\sum_i \tilde{p}(i) \frac{n_{aux}(i)}{\sum_i n_{aux}(i)} \right)^{\frac{1}{1-\gamma}}$.
- $\Pi_{ss} = \left(\frac{1-\Omega_{ss}}{1-\Omega P_{ss}^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_{ss}$.
- $\Delta_{ss} = \sum_i \tilde{p}(i)^{-\gamma} n_{ss}(i)$

4. Update aggregate state from the simulation.

   (a) If $abs(\frac{mean(\Pi_t)}{1+\bar{\pi}}) < A$, then

$$mc_{ss}^{i+1} = mc_{ss}^{i} \left(\frac{1+\bar{\pi}}{\Pi_{ss}} \right)^{adj} \qquad \Omega_{ss}^{i+1} = \Omega_{ss}^{i} + adj(\Omega_{ss}^{i+1} - \Omega_{ss}^{i}) \qquad \Delta_{ss}^{i+1} = \Delta_{ss}^{i} + adj(\Delta_{ss}^{i+1} - \Delta_{ss}^{i}) \tag{98}$$

   (b) If $abs(\frac{mean(\Pi_t)}{1+\bar{\pi}}) > A$, then

$$mc_{ss}^{i+1} = mc_{ss}^{i} \left(\frac{1+\bar{\pi}}{mean(\Pi_t)} \right)^{adj} \tag{99}$$

5. Go to step 1 and check convergence of the policy

$$\max(abs(\frac{C_{ss}^{i+1} - C_{ss}^{i}}{C_{ss}^{i}}, \frac{L_{ss}^{i+1} - L_{ss}^{i}}{L_{ss}^{i}}, \frac{MC_{ss}^{i+1} - MC_{ss}^{i}}{MC_{ss}^{i}}, \Omega_{ss}^{i+1} - \Omega_{ss}^{i}, \frac{\Delta_{ss}^{i+1} - \Delta_{ss}^{i}}{\Delta_{ss}^{i}}, \frac{Y_{ss}^{i+1} - Y_{ss}^{i}}{Y_{ss}^{i}})) < tol_{conv} \tag{100}$$

   Continue the procedure if it doesn't converge.

1. After converge compute micro-price statistics with extended $F_{\Delta a}^{*}$, then we can compute the the distribution of price change as $f(p) = n_{ss} * F_{\Delta a} F_{\Pi} * (p_i \notin C)$.

11.4 Step 2: Approximation of the solution

1. Solve the steady state: Given

$$(\Delta^i_{ss}, \Omega^i_{ss}, \varphi^i_{ss}, mc^i_{ss}, (P^*_{ss})^i) \tag{101}$$

   solve

$$\begin{aligned}
w_{ss} &= \left(\frac{mc_{ss}\eta^Z_{ss}}{\iota}\right)^{\frac{1}{1-\alpha}} \\
L_{ss} &= \left(\frac{w_{ss}}{\kappa}\right)^{\frac{1}{\chi}} \\
Y_{ss} &= \eta^Z_{ss}\frac{L_{ss}-\Omega_{ss}\theta}{\Delta_{ss}}\left(\frac{\alpha w_{ss}}{1-\alpha}\right)^{\alpha} \\
C_{ss} &= Y_{ss}\left(1-\left(\frac{w_{ss}\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_{ss}}{\eta^Z_{ss}}\right)-\eta^G_{ss} \\
u_{ss} &= K\frac{(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{1-\sigma_n}}{1-\sigma_n} \\
mu_{ss} &= K(C_{ss}-\kappa\frac{L_{ss}^{1+\chi}}{1+\chi})^{-\sigma_n} \\
U_{ss} &= \frac{u_{ss}}{1-\beta} \\
\Pi_{ss} &= 1+\bar{\pi} \\
R_{ss} &= \frac{1+\bar{\pi}}{\beta} \\
\tilde{R}_{ss} &= 1
\end{aligned}$$

2. Given

$$\Omega(S) = L^i_{\Omega}(\log(S)) \qquad \Delta(S) = L^i_{\Delta}(\log(S)) \qquad \varphi(S) = L^i_{\varphi}(\log(S)) \tag{102}$$

   Solve the following system around

$$(\Delta_{ss}, \Omega_{ss}, \varphi_{ss}, w_{ss}, P^*_{ss}, C_{ss}, L_{ss}, Y_{ss}, mc_{ss}) \tag{103}$$

using 1 order perturvation methods. The system is given by

$$
\begin{aligned}
mu_t &= \beta \mathbb{E}_t[R_t \Xi_{t+1} mu_{t+1} \frac{\eta^Q_{t+1}}{\Pi_{t+1}} | S] \\
\kappa L_t^{\chi} &= w_t \\
u_t &= K \frac{(C_t - \kappa \frac{L_t^{1+\chi}}{1+\chi})^{1-\sigma_n}}{1-\sigma_n} \\
mu_t &= K (C_t - \kappa \frac{L_t^{1+\chi}}{1+\chi})^{-\sigma_n} \\
U_t &= u_t - \beta(-U_{ss}) \mathbb{E}_t[\left(-\frac{U_{t+1}}{U_{ss}}\right)^{1-\sigma_{ez}}]^{\frac{1}{1-\sigma_{ez}}} \\
\Xi_{aux,t} &= \mathbb{E}_t[\left(\frac{-U_{t+1}}{(-U_{ss})}\right)^{1-\sigma_{ez}} \\
\Xi_{t+1} &= \left(\frac{-\frac{U_{t+1}}{(-U_{ss})}}{\Xi_{aux,t}^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \\
\frac{R_t \beta}{1+\bar{\pi}} &= \tilde{R}_t \left(\left(\frac{\Pi_t}{1+\bar{\pi}}\right)^{\phi_\pi} \left(\frac{mc_t}{mc_{t-1}}\right)^{\tilde{\phi_y}} \right)^{1-\phi_r} \\
\tilde{R}_t &= \left(\frac{R_{t-1}\beta}{1+\bar{\pi}}\right)^{\phi_r} e^{\sigma^R \epsilon_t^R} \\
\eta_t^Z \left(L_t - \Omega_t \theta\right) &= Y_t \left(\frac{(1-\alpha)}{\alpha w_t}\right)^{\alpha} \Delta_t \\
\eta_t^G + C_t &= Y_t \left(1 - \left(\frac{w_t \alpha}{1-\alpha}\right)^{1-\alpha} \frac{\Delta_t}{\eta_t^Z}\right) \\
\Pi_t &= \left(\frac{1-\Omega_t}{(1-\Omega(P_t^*)^{1-\gamma})}\right)^{\frac{1}{1-\gamma}} \varphi_t \\
mc_t &= \frac{\iota w_t^{1-\alpha}}{\eta_t^Z} \\
\Delta_t &= \Omega_t (P_t^*)^{-\gamma} + (1-\Omega_t) \Pi_t^{\gamma} \upsilon_t \\
S_t &= \frac{\gamma}{\gamma-1} mu_t Y_t MC_t + \beta(1-p) \mathbb{E}_t[\Pi_{t+1}^{\gamma} \Xi_{t+1} S_{t+1}] \\
F_t &= mu_t Y_t + \beta(1-p) \mathbb{E}_t[\Pi_{t+1}^{\gamma-1} \Xi_{t+1} F_{t+1}] \\
P_t^* &= \frac{S_t}{F_t} \\
\log(\eta^Z(S')) &= (1-\rho_z)\log(\eta^Z) + \rho_z \log(\eta^Z(S)) + \sigma_z \epsilon^Z \\
\log(\eta^G(S')) &= (1-\rho_g)\log(\eta^G) + \rho_g \log(\eta^G(S)) + \sigma_g \epsilon^G \\
\log(\eta^Q(S')) &= (1-\rho_q)\log(\eta^Q) + \rho_q \log(\eta^Q(S)) + \sigma_q \epsilon^Q
\end{aligned}
$$

With the solution generate the coefficient for the policies using projection methods (smoliak) for the following functions

$$(C^i(S), L^i(S), Y^i(S), G^i(S), \tilde{R}^i(S), mc^i(S), \Delta^i(S), \Omega^i(S), \varphi^i(S), \Pi(S), \eta^Z(S), \eta^Q(S)) \tag{104}$$

(a) Technical 1: Note that we are not approximating the solution from the steady state, but we are approximating the solution from the mean in the simulation with respect to $(\Delta_{ss}, \Omega_{ss}, \varphi_{ss}, w_{ss}, P^*_{ss})$.

(b) Technical 2: I will never use $P^*(S)$ since this function comes from the optimality conditions of the firm.

(c) Technical 3: If blachard conditions are not satisfied put a stochastic discount factor to the firm of βD.

4. Given $(C^i(S), \Pi^i(S), mc^i(S), \tilde{R}^i(S), w^i(S), \Delta^i(S))$ solve

$$
\begin{aligned}
v(\tilde{p}, S) &= \mathbb{E}_{\tilde{p}', S'}\left[\max_{c,nc}\{\max_x\{V(x, S')\} - mu(S')w(S')\theta, V(\tilde{p}', S')\}|S\right] \\
V(\tilde{p}, S) &= \Phi(\tilde{p}, S) + \beta v(\tilde{p}, S) \\
\Phi(\tilde{p}, S) &= Y(S)mu(S)\tilde{p}^{-\gamma}(\tilde{p} - mc(S)) \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}}{\Pi(S')} & \text{with pr. } e^{-\lambda} \\ \frac{\tilde{p}e^{\sigma_a \epsilon^a}}{\Pi(S')} & \text{with pr. } 1 - e^{-\lambda} \end{cases}
\end{aligned}
$$

and get $P^i(S), V^i(S)$.

(a) Technical 1: For the firm problem I used Anisotropic construction of smoliak polinomial with coefficients $\mu = (3_{\tilde{p}}, 2_{mc_-}, 2_{\Delta_-}, 3_{\tilde{R}_-}, 3_{\eta^Z}, 3_{\eta^G}, 3_{\eta^Q})$.

(b) Technical 2: I generate the base before starting contraction and colocation, note that there is no decision variable for competing the integral!!!!!.

5. Given

$$\Pi^i(S), mc^i(S), \tilde{R}^i(S), V^i(\tilde{p}, S), v^i(\tilde{p}, S), P^{*,i}(S) \tag{105}$$

start from the previous simulation $sidio_0$ and $Saggre_0$.

(a) Compute $\Pi_t(Saggre_t)$ from the aggregate equilibrium conditions.

(b) With $\hat{\Pi}_t(Saggre_t)$ compute the following matrices

i. F_{Π_t} is given by the transition probability $\tilde{p}_1 = \frac{\tilde{p}}{\hat{\Pi}_t(Saggre_t)}$ using linear splines over the simulated grid.

ii. $F_{p'_t}$ is given by the transition probability $\tilde{p}_1 = P^*_t(Saggre_t)I(\tilde{p} \in C(Saggre_t)) + I(\tilde{p} \notin C(Saggre_t))\tilde{p} * \frac{\hat{\pi}_t}{\Pi^s_t}$.

(c) Compute micro-price statistics

i. $n_{aux,t} = (n_{t-1}F_{\Delta a}F_{\Pi_t})I(p \in C(Saggre_t))$.

ii. $\Omega_t = 1 - \sum_i n_{aux,t}(i)$.

iii. $\varphi_t = \hat{\Pi}_t\left(\sum_i \tilde{p}(i)\frac{n_{aux,t}(i)}{\sum_i n_{aux,t}(i)}\right)^{\frac{1}{1-\gamma}}$.

iv. $\Pi_t = \left(\frac{1-\Omega_t}{1-\Omega_t P_t^{1-\gamma}}\right)^{\frac{1}{1-\gamma}}\varphi_t$.

v. $\Delta^s_t = \sum_i \tilde{p}(i)^{-\gamma} n_t(i)$

vi. $n_t = (n_{t-1}F_{\Delta a}F_{\Pi_t})F_{p'_t}$

(d) Update the exogenous and endogenous states for the firm for period $t+1$

$$
\begin{aligned}
\eta^Z_{t+1} &= (\eta^Z_{ss})^{1-\rho_Z}(\eta^Z_t)^{\rho_Z}\exp(\sigma_Z\epsilon_{t+1}) && (106) \\
\eta^G_{t+1} &= (\eta^G_{ss})^{1-\rho_G}(\eta^G_t)^{\rho_G}\exp(\sigma_G\epsilon_{t+1}) && (107) \\
\eta^Q_{t+1} &= (\eta^Q_{ss})^{1-\rho_Q}(\eta^Q_t)^{\rho_Q}\exp(\sigma_Q\epsilon_{t+1}) && (108) \\
R_{t+1} &= \tilde{R}(S_t)e^{\sigma^R\epsilon_{t+1}} && (109) \\
mc_t &= mc(S_t) && (110) \\
\Delta_t &= \Delta^s_t && (111)
\end{aligned}
$$

4. Given the simulation $(\Omega_t, \varphi_t, \Delta_t)$ estimate

$$\Omega_t = \exp(P^(1)_\Omega(\log(S_t))) \qquad \Delta_t = \exp(P^1_\Delta(\log(S_t))) \qquad \varphi_t = \exp(P^1_\varphi(\log(S_t))) \tag{112}$$

using linear regression methods.

5. Update aggregate state from the simulation.

   (a) If $abs(\frac{mean(\Pi_t)}{1+\bar{\pi}}) < A$, then

$$\begin{aligned}
mc_{ss}^{i+1} &= mc_{ss}^{i}\left(\frac{1+\bar{\pi}}{mean(\Pi_t)}\right)^{adj} \qquad \Omega_{ss}^{i+1} = \Omega_{ss}^{i} + adj(\Omega_{ss}^{i+1} - \Omega_{ss}^{i}) \qquad \Delta_{ss}^{i+1} = \Delta_{ss}^{i} + adj(\Delta_{ss}^{i+1} - \Delta_{ss}^{i}) \\
\varphi_{ss}^{i+1} &= \varphi_{ss}^{i} + adj(\varphi_{ss}^{i+1} - \varphi_{ss}^{i}) \qquad (P_{ss}^{*})^{i+1} = (P_{ss}^{*})^{i} + adj((P_{ss}^{*})^{i+1} - (P_{ss}^{*})^{i}) \\
P_{\Omega}^{2,i+1}(\log(S_t)) &= P_{\Omega}^{2,i+1}(\log(S_t)) + adj(P_{\Omega}^{2}(\log(S_t)) - P_{\Omega}^{2,i}(\log(S_t))) \\
P_{\Delta}^{2,i+1}(\log(S_t)) &= P_{\Delta}^{2,i+1}(\log(S_t)) + adj(P_{\Delta}^{2}(\log(S_t)) - P_{\Delta}^{2,i}(\log(S_t))) \\
P_{\varphi}^{2,i+1}(\log(S_t)) &= P_{\varphi}^{2,i+1}(\log(S_t)) + adj(P_{\varphi}^{2}(\log(S_t)) - P_{\varphi}^{2,i}(\log(S_t))) \\
(\lambda_1, \lambda_2)^{i+1} &= (\lambda_1, \lambda_2)^{i} + adj((\lambda_1, \lambda_2) - (\lambda_1, \lambda_2)^{i})
\end{aligned}$$

   (b) If $abs(\frac{mean(\Pi_t)}{1+\bar{\pi}}) > A$, then

$$mc_{ss}^{i+1} = mc_{ss}^{i}\left(\frac{1+\bar{\pi}}{mean(\Pi_t)}\right)^{adj} \tag{113}$$

6. Go to step 1 and check convergence of the policy.

$$\max_{S\in S^{erg}}(abs(\frac{C^{i+1}(S)-C^{i}(S)}{C^{i}(S)}, \frac{L^{i+1}(S)-L^{i}(S)}{L^{i}(S)}, \frac{MC^{i+1}(S)-MC^{i}(S)}{MC^{i}(S)}, \Omega^{i+1}(S) - \Omega^{i}(S), \frac{\Delta^{i+1}(S)-\Delta^{i}(S)}{\Delta^{i}(S)})) < tol_{conv} \tag{114}$$

   where $S = \{S_1, S_2, S_3, ...\}$ is obtaint from the simulation to evaluate the model in the ergodic set. Continue the procedure if it doesn't converge.

11.5 Solve the model without ZLB

1. Guess

$$\Omega(S) = P^{2,i}_{\Omega}(\log(S)) \qquad \Delta(S) = P^{2,i}_{\Delta}(\log(S)) \qquad \varphi(S) = P^{2,i}_{\varphi}(\log(S)) \qquad \upsilon(S) = P^{2,i}_{\upsilon}(\log(S)) \tag{115}$$

I found that it is better doing KS directly on Δ than on υ. So I will describe the algorith with $\Delta(S)$. Let

$$\begin{gathered}(C^{j,i}(S), L^{j,i}(S), Y^{j,i}(S), R^{j,i}(S), \tilde{R}^{j,i}(S), u^{j,i}(S), mu^{j,i}(S), U^{j,i}(S), \ldots \\ \Delta^{i}(S), \Omega^{i}(S), \varphi^{i}(S), \ldots \\ mc^{j,i}(S), w^{j,i}(S), \Pi^{j,i}(S), P^{*,j,i}(S)), \upsilon^{j,i}(\tilde{p}, S), V^{j,i}(\tilde{p}, S)\end{gathered} \tag{116}$$

for j=0 an initial condition for the global solution. I use fixed point interation to solve global, explained below.

(a) Let K_{glo} be a positive integer, for $k = j, ..., j + K_{glo} - 1$ update the policy as

$$mu^{k+1,i}(S) = \beta R^{k,i}(S)\mathbb{E}\left[\frac{mu^{k,i}(S')\eta^{Q}(S')\Xi^{k,i}(S')}{\Pi^{j,i}(S)}|S\right] \tag{117}$$

$$\Xi_{k,i}(S') = \left(\frac{U^{k,i}(S')}{\mathbb{E}\left[(-U^{k,i}(S'))^{1-\sigma_{ez}}|S\right]^{\frac{1}{1-\sigma_{ez}}}}\right)^{-\sigma_{ez}} \tag{118}$$

$$\tag{119}$$

with $mu^{k+1,i}(S)$ solve

$$w^{k+1,i} = \kappa L^{k+1,i}(S)^{\chi} \tag{120}$$

$$\eta^{G}(S) + C^{k+1,i}(S) = Y^{k+1,i}(S)\left(\frac{(1-\alpha)}{\alpha w^{k+1,i}}\right)^{\alpha}\Delta^{i}(S) \tag{121}$$

$$\eta^{Z}(S)(L^{k+1,i}(S) - \theta\Omega^{i}(S)) = Y^{k+1,i}(S)\left(1 - \left(\frac{w^{k+1,i}(S)\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta^{i}(S)}{\eta^{Z}(S)}\right) \tag{122}$$

$$mu^{k+1,i} = K\left(C^{k+1,i}(S) - \frac{\kappa}{1+\chi}L^{k+1,i}(S)^{1+\chi}\right)^{-\sigma_{np}} \tag{123}$$

Then solve

$$mc^{k+1,i} = \frac{\iota(w^{k+1,i})^{1-\alpha}}{\eta^{Z}(S)} \tag{124}$$

$$u^{k+1,i} = K\frac{\left(C^{k+1,i}(S) - \frac{\kappa}{1+\chi}L^{k+1,i}(S)^{1+\chi}\right)^{1-\sigma_{np}}}{1-\sigma_{np}} \tag{125}$$

$$U^{k+1,i} = u^{k+1,i} - \beta\mathbb{E}\left[(-U^{k,i}(S'))^{1-\sigma_{ez}}|S\right]^{\frac{1}{1-\sigma_{ez}}} \tag{126}$$

$$R^{k+1,i}(S) = \frac{1+\bar{\pi}}{\beta}R_{-}(S)^{\phi_r}\left[\left(\frac{\Pi^{j,i}(S)}{1+\bar{\pi}}\right)^{\phi_\pi}\left(\frac{MC^{k+1,i}(S)}{MC_{-}(S)}\left(\frac{\Delta_{-}(S)}{\Delta^{i}(S)}\right)^{\frac{\chi}{1-\alpha}}\right)^{\frac{\phi_y}{\sigma+\frac{\chi+\alpha}{1-\alpha}}}\right]^{1-\phi_r} \tag{127}$$

$$\tag{128}$$

For $k = j + K_{glo}$ I solve equilibrium inflation given by

$$\begin{aligned}
v^{j+K_{glo},i}(\tilde{p},S) &= \mathbb{E}_{\tilde{p}',S'}\left[\max_{c,nc}\{\max_{x}\left\{V(x,S')^{j+K_{glo},i}\right\} - w^{j+K_{glo},i}(S')\theta, V^{j+K_{glo},i}(\tilde{p}',S')\right\} |S,\tilde{p}\right] \\
V^{j+K_{glo},i}(\tilde{p},S) &= \{\Phi(\tilde{p},S) + \beta v^{j+K_{glo},i}(\tilde{p},S)\} \\
\Phi(\tilde{p},S) &= mu^{j+K_{glo},i}(S)Y^{j+K_{glo},i}(S)\tilde{p}^{-\gamma}\left(\tilde{p} - mc^{j+K_{glo},i}(S)\right)\} \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}}{\Pi(S')} & \text{with pr. } e^{-\lambda} \\ \frac{\tilde{p}e^{\sigma_a \epsilon^a}}{\Pi(S')} & \text{with pr. } 1-e^{-\lambda} \end{cases} \\
P^*(S)^{j+K_{glo},i} &= \arg\max_x \left\{V(x,S)^{j+K_{glo},i}\right\} \\
\Pi^{j+K_{glo},i}(S) &= \left(\frac{1-\Omega^i(S)}{1-\Omega^i(P^*(S)^{j+K_{glo},i})^{1-\gamma}}\right)^{\frac{1}{1-\gamma}} \varphi^i
\end{aligned}$$

2. Idem. as equilbirium approximation.
3. Given the simulation $(\Omega_t, \varphi_t, \upsilon_t)$ estimate

$$\Omega_t = \exp(P^2_\Omega(\log(S_t))) \qquad \upsilon_t = \exp(P^2_\upsilon(\log(S_t))) \qquad \varphi_t = \exp(P^2_\varphi(\log(S_t))) \qquad \Delta_t = \exp(\Delta^2_\varphi(\log(S_t))) \tag{129}$$

using linear regression methods. Generate the coefficient of the policies as

$$\Omega^* = \max(\min(\exp(P^2_\Omega(\log(S_t))), Q^{99}(\Omega_t)), Q^1(\Omega_t)) \tag{130}$$

$$\Delta^* = \max(\min(\exp(P^2_\Delta(\log(S_t))), Q^{99}(\Delta)), Q^1(\Delta)) \tag{131}$$

$$\varphi^* = \max(\min(\exp(P^2_\varphi(\log(S_t))), Q^{99}(\varphi)), Q^1(\varphi)) \tag{132}$$

5. Update aggregate state from the simulation.

$$C^{i+1}_{ss} = C^i_{ss} + adj(C^{i+1}_{ss} - C^i_{ss})$$

$$\begin{aligned}
P^{2,i+1}_\Omega(\log(S_t)) &= P^{2,i+1}_\Omega(\log(S_t)) + adj(P^2_\Omega(\log(S_t)) - P^{2,i}_\Omega(\log(S_t))) \\
P^{2,i+1}_\upsilon(\log(S_t)) &= P^{2,i+1}_\upsilon(\log(S_t)) + adj(P^2_\upsilon(\log(S_t)) - P^{2,i}_\Delta(\log(S_t))) \\
P^{2,i+1}_\varphi(\log(S_t)) &= P^{2,i+1}_\varphi(\log(S_t)) + adj(P^2_\varphi(\log(S_t)) - P^{2,i}_\varphi(\log(S_t)))
\end{aligned}$$

6. Go to step 1 and check convergence of the policy.

$$\max_{S\in S^{erg}}(abs(\frac{C^{i+1}(S)-C^i(S)}{C^i(S)}, \frac{L^{i+1}(S)-L^i(S)}{L^i(S)}, \frac{MC^{i+1}(S)-MC^i(S)}{MC^i(S)}, \Omega^{i+1}(S)-\Omega^i(S), \frac{\Delta^{i+1}(S)-\Delta^i(S)}{\Delta^i(S)})) < tol_{conv} \tag{133}$$

where $S = \{S_1, S_2, S_3, ...\}$ is obtaint from the simulation to evaluate the model in the ergodic set. Continue the procedure if it doesn't converge.

11.6 Equilibrium Solution with ZLB

Change the equation of the zero lower bound.

$$R^{k+1,i}(S) = \max\left\{1, \frac{1+\bar{\pi}}{\beta}\tilde{R}_{-}(S)^{\phi_r}\left[\left(\frac{\Pi^{j,i}(S)}{1+\bar{\pi}}\right)^{\phi_\pi}\left(\frac{MC^{k+1,i}(S)}{MC_{-}(S)}\left(\frac{\Delta_{-}(S)}{\Delta^{i}(S)}\right)^{\frac{\chi}{1-\alpha}}\right)^{\frac{\phi_y}{\sigma+\frac{\chi+\alpha}{1-\alpha}}}\right]^{1-\phi_r}\right\} \tag{134}$$

11.7 Evaluation of Krusell-Smith

To evaluate Krusell-Smith, first I simulate the model and construct a time series of

$$(mc_{-}(S_t^s), \Delta_{t-1}^s, R_{-}(S_t^s), \eta_Z(S_t)^s, \eta_G(S_t)^s, \eta_Q(S_t)^s, \Pi_t^s, \Omega_t^s)_t \tag{135}$$

Then I construct the implied by the model in the simulation in the static equations

$$\hat{R}_t = \tilde{R}_{-}(S_t^s)^{\phi_\pi}\left(\frac{\Pi_t^s}{1+\bar{\pi}}\right)^{\phi_\pi(1-\phi_r)}\left(\frac{mc(S_t^s)}{mc_{-}(S_t^s)}\right)^{\phi_y(1-\phi_r)} \tag{136}$$

$$\tag{137}$$

Given $mu(S_t)^s$ in the simulation

$$\hat{w} = \kappa\hat{L}_t^{\chi} \tag{138}$$

$$\eta_G(S_t)^s + \hat{C}_t = \hat{Y}_t\left(\frac{(1-\alpha)}{\alpha\hat{w}_t}\right)^{\alpha}\Delta_t^s \tag{139}$$

$$\eta_Z(S_t)^s(\hat{L}_t - \theta\Omega_t^s) = \hat{Y}_t\left(1-\left(\frac{\hat{w}_t\alpha}{1-\alpha}\right)^{1-\alpha}\frac{\Delta_t^s}{\eta_Z(S_t)^s}\right) \tag{140}$$

$$mu(S_t)^s = K\left(\hat{C}_t - \frac{\kappa}{1+\chi}\hat{L}_t^{1+\chi}\right)^{-\sigma_{np}} \tag{141}$$

Finally for the dynamic equation, euler equation and belman equation of the firm, estimate a second order linear regression $\hat{\Pi}(S_t) = L(S_t^s)$ and with this function solve

$$\hat{mu}(S_t) = \beta\hat{R}_t\mathbb{E}_{S'}\left[\frac{mu(S')}{\hat{\Pi}(S')}\eta_Q(S')|S\right] \tag{142}$$

$$\hat{P}^s(S_t) = \arg\max_x\{V(x,S)\} \tag{143}$$

Figure **??** and **??** plots the Krusell-Smith evaluation for consumption, labor supply, reset price and nominal interest rate.

11.8 Computation of the Equilibrium in the New Keynesian Model

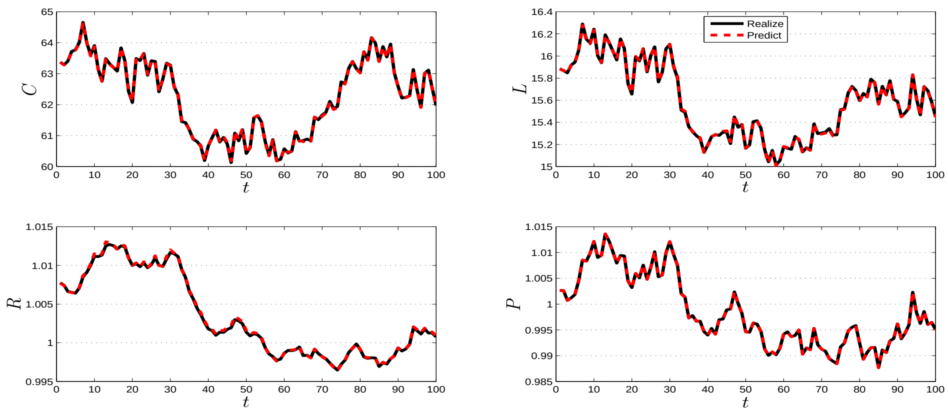


Figure 16: Predicted and simulated consumption, labor supply, nominal interest rate and reset price at 0 % trend inflation

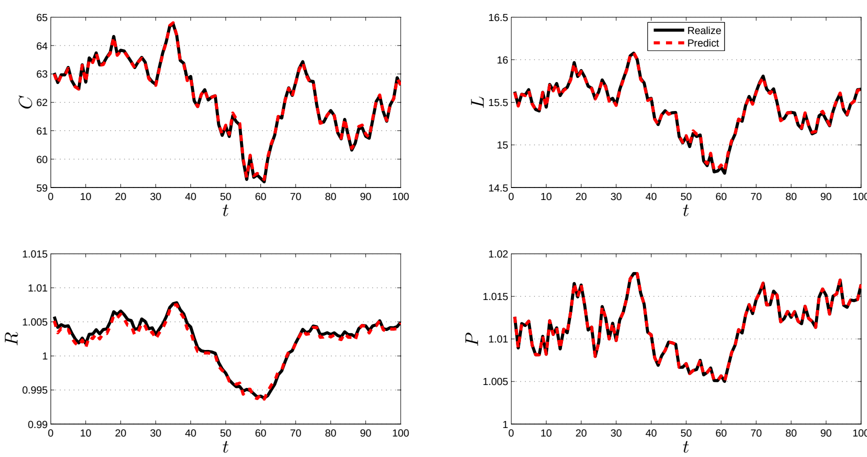


Figure 17: Predicted and simulated consumption, labor supply, nominal interest rate and reset price at 6 % trend infla